Fig. 2. Direct cross coupling produces the unusual negative resistance characteristic shown between collectors. The voltage swing is limited to $(+/-) V_{BE}$ with no Zener diode. $V = 5$ V then 10 V; $R_L = 2.2$ K.

Fig. 3. With the inductor voltage limited to $\pm$ the base-emitter drop (V_{BE}), the inductor voltage (top) and current (bottom) waveforms shown are typical. $V = 10$ V, $R_L = 2.2$ K.

negative resistance region corresponding to (2). The negative resistance region is never traversed as the inductor switches states by moving in near zero time horizontally along a constant current path. A half-period corresponds to the inductor moving vertically along the V_{BE} trace between the current limits [3].

WAVEFORMS

In Fig. 3, waveforms are shown where the Zener diode is removed and $\pm V_{BE}$ is the constant inductor voltage. The voltage waveform does droop as the V_{BE} decreases with drive current and shows its nonconstant characteristic. Current flowing in the inductor in proper phase with the voltage, is shown as the lower trace. This exponential current waveform looks more like a ramp/triangular waveshape as the voltage (V_Z) stays constant.

These waveforms are typical for the normal mode of behavior and support the analysis developed. The symmetry of the waveform is a reflection of the circuit symmetry, in effect the components and transistor match. At higher frequencies some waveform deterioration can take place as the transistor frequency response limits the transition or switching time and the inductor

distributed capacitance limits the rate of voltage change between collectors. Because of the constant voltage base drive arrangement, this circuit displays excellent high-frequency characteristics.

APPLICATIONS

Certain potential applications of the inductively tuned astable can be suggested and these may offer viable alternatives to existing circuits [4], [5].

Since the collector to collector impedance characteristic is similar to that of a tunnel diode, many of the oscillator applications can be identical. For example, an antiresonant tank circuit, placed between collectors, will give rise to a sinusoidal circulating current where the amplitude is related to the voltage supply. The mode of oscillation and analysis follows the classical Van der Pol equations except that the negative resistance is piecewise linear instead of cubic. Going to the constant (V_Z) voltage mode extends the frequency range and with an inductor alone, operating essentially square wave, relaxation oscillations were obtained into the megahertz range. Varying the voltage will now result in frequency modulation in the manner of a VCO.

The inductor can be the primary of a transformer and the secondary isolated voltage available can be used as a trigger signal. It is also possible to inject a signal back into the inductor by means of this secondary winding and lock or synchronize the astable multivibrator to some external signal. Because the magnetic field of the inductor is relatively easy to couple into, the circuit can be used to detect changes in the reluctance of an air gap and convert these changes directly into a period or frequency change. The movement of a core inside an inductor can be converted to a frequency change directly for displacement or position indication. Similar applications based on the perturbation of the inductor's magnetic field are conceivable.

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On the Dynamic Range of Wave Digital Filters

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Abstract—Dynamic range performance is studied for a group of two-port wave digital filters, all of which satisfy the same attenuation specifications. Two dual prototype networks are used as well as four different sampling frequencies. The design is a 12th-order bandpass filter with high performance requirements.

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Fig. 1. 12th-order bandpass filter prototype—dual 1.

Fig. 2. Ideal response of 12th-order bandpass filter prototype.

I. INTRODUCTION

Wave digital filters (WDF's) have been of interest for the past decade [1]–[4]. They have been shown to combine low multiplier coefficient sensitivity with low roundoff noise. Fettweis was the first to publish a design procedure [2], but more recently an alternate approach has appeared [3], [4]. This latter type of WDF is a two-port cascade design which offers the advantage of a more straightforward implementation. This paper deals with dynamic range considerations for these WDF's under a variety of conditions; scaled versus unscaled networks, the use of different prototypes, and changing of sampling frequency. A more detailed discussion on the subject may be found in [5].^{1,2}

Eight WDF's were designed using four values of sampling frequency and two dual networks as LC prototypes. Fig. 1 shows one of the LC ladder prototypes [13] chosen for this work. The ideal frequency response is illustrated in Fig. 2 with the pass-band ripple being 0.18 dB. Design of the WDF consists of cascading the digital sections in a one for one replacement with their respective analog component(s). Each filter was then scaled for three kinds of input signals; an impulse, Gaussian white noise, and a sinusoid. A computer program WDFDAP was written to design and scale the WDF and utilizes Jackson's L_p norm equations [6]. Scaling of the filter is done via wave digital transformers [7]. Finally, another program WDFSP was written to perform a finite-arithmetic simulation of each WDF and thereby determine the best signal-to-noise ratio for each case.

¹The work in [14] is similar as presented here but does not include the simulation results of this work.

²[15] and [16] are also similar in approach to the work presented here. However, the empirical results described are not as extensive as those detailed here.

Fig. 3. WDF with scaling multipliers.

II. SCALING BY THE TRANSFORMER METHOD

There have been examples in the literature of scaling WDF's [8]–[10], but without any mention of the details involved. All the examples cited used pairs of inverse multipliers as wave digital transformers in order to perform the necessary signal scaling. This technique is readily applicable to any ladder-like structure (in this case WDF's), but suffers from a degree of nonoptimality.

As shown in Fig. 3, each digital two-port or section has a pair of inverse multipliers associated with it. This allows each section to be scaled independently of all the other two-ports (provided the scaling multipliers are assigned successively [14]). However there may be up to seven adders in each block [4], thus the signal scaling can only be optimum for one of these nodes. If there is a large variation in signal amplitudes within a two-port, and in particular if there are nodes where overflows are predicted (according to Jackson's equation), scaling by the transformer method reduces the dynamic range at those nodes where there is no danger of overflow.

Despite this problem, scaling by the transformer method does yield good results. As illustrated by Figs. 4–6, the signal-to-noise ratios are improved by from 7 to 35 dB.

However, when the gains (from each input to each node) for a given two-port have a large variance (as is the case for the WDF in [5]), the improvement in signal-to-noise ratio is not as great as it could conceivably be.

III. DUAL NETWORKS

Digital filters based on analog networks (in this case WDF's) can (and intuitively should) have varying levels of performance depending on the prototype selected. In [5] the two dual networks of a 12th-order bandpass filter were used for the WDF realizations. The signal-to-noise ratios for the various circuits are shown in Figs. 4–6. Except for the case of the impulsive input (Fig. 4), neither prototype shows consistently better results. However, the difference in signal-to-noise between the two networks can be as much as 15 dB and so it certainly seems worthwhile to consider different prototypes when designing a WDF.

IV. SAMPLING FREQUENCY

It has been shown that certain types of digital filters have increasingly worse signal-to-noise ratios as the normalized bandwidth is decreased (i.e. sampling frequency is increased) [11], [12]. This is true for the filters studied in [5] and is due to two factors. The first cause has been noted elsewhere [12] and is an increase in the output noise power due to the bandwidth reduction. The second problem is due to the structure of the building blocks which comprise the WDF. As the sampling frequency is increased, the maximum input level, which does not lead to overflow, is reduced. This is caused by a rapid increase in the gains³ to some of the nodes in some of the two-ports, particularly in the shunt LC sections. Scaling should allow the same

³Using Jackson's notation, these "gains" are the L_p norms of the frequency responses from the input of the filter to the various nodes [6].

Fig. 4. Signal-to-noise ratio—Impulsive input—16 bits.

Fig. 5. Signal-to-noise ratio—White noise input—16 bits.

Fig. 6. Signal-to-noise ratio—Sinusoidal input—16 bits.

Figs. 4–6 show the rapid decrease in signal-to-noise ratio as the sampling frequency is increased for the unscaled filters. In the scaled cases, the rate of decrease is lessened. Nevertheless the best results are obtained at the lowest value of sampling frequency.

V. SIMULATION OF WAVE DIGITAL FILTERS

Computer program WDFSP performs a finite-arithmetic simulation of the WDF's described in [5]. Integer arithmetic and special control logic are used to emulate a typical hardware WDF with variable signal and coefficient wordlengths. Each addition or subtraction is checked to ensure that no overflow has occurred. In this way the input signal level can be reduced just to the point where no overflows result and hence the maximum undistorted output signal power can be calculated. Then the simulation is repeated using a longer (25 bits) wordlength. The two output sequences are then subtracted to form an error sequence which is used to compute the output noise power and hence the signal-to-noise ratio. As seen in Figs. 4–6, the original wordlengths used are 16 bits long. Thus a 25-bit wordlength simulation can be considered to be relatively noise free. The sequence lengths used were 512, 1024, 2048, and 4096 points for sampling frequencies of 125, 250, 500, and 1000 kHz, respectively.

VI. CONCLUSIONS

The study of a series of WDF's shows the value of considering different prototype networks and sampling frequencies as well as the improvements brought about by suitable scaling of the filters. The signal-to-noise ratios differ by as much as 15 dB for different prototypes, 40 dB for different sampling frequencies, and 35 dB for scaled versus unscaled networks.

The unscaled networks show the characteristic drop in signal-to-noise ratios as the bandwidth is decreased. Scaling tends to reduce the rate of this drop but cannot prevent it (at least not in most cases). The transformer method of scaling is easily implemented for the WDF's but is nonoptimal in that all nodes in any given two-port are scaled identically. However the method does result in improved performance and any further attempts would require a redesigning of the two-port networks which constitute the WDF.

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maximum input level for all bandwidths. However since these large gains only occur in some nodes, the global scaling of a particular two-port results in a poorer signal-to-noise ratio. This follows from the discussion in Section II.

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Effects of Excess Phase in Multiple-Feedback Active Filters

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Abstract—The paper presents theoretical and experimental results showing that lagging "excess phase" of second-order active filter sections causes potentially large errors in the form of a characteristic skewing of the passband shape of multiple-feedback (MF) bandpass functions. It is shown that simple capacitive phase-lead methods eliminate the deviations. The situation is discussed in detail for the specific case of "follow-the-leader-feedback" bandpass filters, but applies equally to other MF topologies. Experimental data are presented to demonstrate the effects of excess phase and to verify the correctness of the design process.

I. INTRODUCTION

Because of low sensitivities, the design of transfer functions of order $N > 4$ is implemented preferably by use of multiple feedback (MF) structures [1], or alternatively, by use of active simulations of passive LC ladder filters [2]–[4]. Both methods have been found to work well at low frequencies; in either approach, however, the magnitude of the realized function will, for high-frequency operation, generally exhibit noticeable deviation from the ideal in the form of a characteristically skewed appearance of the passband. For MF filters, this behavior will be shown to be a direct consequence of lagging "excess phase" of the second-order sections.

A similar problem in passive RLC filters is caused by parasitic dissipation in the filter reactances [5], [6]; results of these studies have been applied by analogy to correct the performance of certain active ladder structures [4], [7].

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Fig. 1. FLF 2nth order BP filter.

This paper examines the effects of excess phase and proposes correction methods specifically for "follow-the-leader-feedback" (FLF) filters [1], which have no passive analog so that phase effects are not predictable from previous work. Although the "modified leapfrog" (MLF) topology [1] appears to have a slight sensitivity advantage over all other MF structures [8], FLF filters are used in this study because they are easier to design, tune, and especially optimize [1]. Furthermore, FLF filters, in many cases, have better dynamic range than all other MF topologies [9], a property especially important for high-frequency applications where phase effects become critical and need to be eliminated.

II. SECOND-ORDER PHASE EFFECTS IN HIGH-FREQUENCY MF FILTERS

Second-order sections used in the design of MF filters introduce phase shifts that must be added to that predicted by the ideal transfer function. This "excess phase" has been found experimentally to cause distortions of filter characteristics even at audiofrequencies in any system employing feedback.

Excess phase is caused by parasitic capacitors [10] and by operational amplifier (OA) dynamics, usually neglected in filter design; both have long been known to give rise to deviations in the pole positions of second-order filters [10], [11, ch. 4.4] and corrective measures in terms of improved design, predistortion or compensation have been proposed. Excess phase is particularly easy to explain in the large number of filter sections whose gain is directly proportional to one or more OA gains [11], since in these sections amplifier phase is simply added to the total phase of the filter.

Concentrating on bandpass (BP) functions in this paper, excess phase ϕ_i can be modeled as

$$T_i(j\omega) = \frac{j\omega H_{Bi}/Q_i}{1 - \omega^2 + j\omega/Q_i} e^{-j\phi_i(\omega)}. \quad (1)$$

In (1), the frequency ω is normalized to the pole frequency ω_0 , and H_{Bi} and Q_i are the midband gain and the pole quality factor, respectively.

Further, phase errors that can cause problems in MF filters arise from the feedback paths which in all coupled filters are assumed to be purely resistive [1]. If these feedback resistors are associated with parasitic reactances, the feedback factors are complex and the feedback signals at the various summing nodes are slightly out of phase. To be specific, consider the FLF topology in Fig. 1, described by

$$H(s) = V_{ON}/V_{IN} = -a \prod_{i=1}^N T_i / \left(1 + \sum_{i=1}^N F_{i1} \prod_{j=1}^i T_j \right) \quad (2)$$