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Abstract

A semi-analytical model for predicting heat transfer and pressure drop in annular flow regime for saturated flow boiling in a horizontal micro-tube at a uniform heat flux has been developed based on one-dimensional separated flow model. More than 600 two-phase heat transfer, 498 two-phase pressure drop, and 153 void fraction experimental data points for annular flow regime have been collected from the literature for validating the present model. The collected data were recorded for various working fluids, R134a, R1234ze, R236fa, R410a, R113, and CO2 for round macro and micro single horizontal tubes with a range of inner diameter of $0.244 \text{ mm} \leq D_h \leq 3.1 \text{ mm}$, a heated length to diameter ratio of $90 \leq (L_h/D_h) \leq 2000$, a range of saturation temperature of $-10 \leq T_{sat} \leq +50 \text{ °C}$, and liquid to vapor density ratios in the range $6.4 \leq (\rho_f/\rho_g) \leq 188$. The model was tested for laminar and turbulent flow boiling conditions corresponding to an equivalent Reynolds number, $1,900 \leq \text{Re}_\text{eq} \leq 48,000$, and a range of confinement number of $0.27 \leq C_{conf} \leq 3.4$. In the scope of annular flow regime, the present model predicted the collected data of the heat transfer, pressure drop, and void fraction with a mean absolute error (MAE) of 18.14 %, MAE of 23.02 %, MAE of 3.22 %, respectively.

Keywords: saturated flow boiling heat transfer, two-phase frictional pressure drop, annular flow regime, flow pattern maps, liquid film thickness, equivalent Reynolds number, and Bland-Altman Plot.
1. Introduction

Two-phase micro-channels are widely used and have many applications, extensively in heat sinks and compact heat exchangers. Typically, the main principle of micro-channel heat sinks is to dissipate a high heat flux at a lower mass flux. Additionally, predicting heat transfer coefficient and two phase pressure drop of flow boiling in micro-channels is one of the most important steps for designing micro-channel heat sinks. Furthermore, it has been reported in the literature that the channel size influences the applicability of the developed correlations or models. Therefore, the proposed correlations of two-phase frictional pressure drop prediction for large tubes are not applicable for small and mini-channel as has been proven in many previous studies (e.g. Tran et al. (2000)). There are several experimental investigations of two-phase heat transfer and pressure drop in micro-channels that have been reported recently for various working fluids and wide range of operating flow conditions. Some of them were performed for horizontal micro-channels at adiabatic flow conditions such as those presented by Cavallini et al. (2009), Zhang et al. (2004), Yun et al. (2005), Basu et al. (2011), Oh et al. (2011), Ali et al. (2012), Mahmoud and Karayiannis (2013), and Ducoulombier et al. (2011), and few experiments were performed at diabatic flow conditions, as those presented by Tibiriçá et al. (2011), Yan and Lin (1998), and Wu et al. (2011). Furthermore, there are some experiments that were performed for vertical tubes such as the ones reported by Maqbool et al. (2012). Some of the previous experimental investigations for predicting heat transfer and pressure drop of flow boiling in micro-channels have resulted in either predicting empirical parameters for developing and modifying some existing correlations or adding more experimental data points for flow boiling in mini-micro channels. However, most of the available correlations and approaches are either applicable only for a specific flow pattern or a
limited range of working fluids and operating conditions, or are too poor in scope to predict the experimental data correctly.

On the other hand, a few analytical models of flow boiling heat transfer and pressure drop in micro-channels have been presented in the literature (e.g. Moriyama and Inoue (1992), LaClair and Mudawar (2009), Na and Chung (2011), Das et al. (2006), Jacobi and Thome (2002), Thome et al. (2004)). The heat transfer model of an elongated bubble in slug flow regime in micro channels was proposed by Jacobi and Thome (2002), and is reported to be good for predicting local heat transfer coefficient for the elongated bubble/liquid slug pair regime in a circular micro channel. The three-zone flow boiling model that was developed by Thome et al. (2004) contains three empirical parameters which are difficult to estimate theoretically. These parameters are minimum liquid film thickness that is acceptable at dry-out ($\delta_{\text{min}}$), initial value of the liquid film thickness ($\delta_o$) and bubble frequency.

Heat transfer mechanisms for flow boiling in micro-channels play an important role in developing more realistic flow boiling models. Many researchers, (e.g. Kew and Cornwell (1997), Bao et al. (2000), Yen et al. (2003), and Kandlikar (2004)) have reported that dominant heat transfer mechanisms for boiling in small-and micro-channels are flow pattern dependent. Furthermore, the dominant heat transfer mechanism is affected by channel size as it has been studied by Park and Hrnjak (2007). They found that nucleate boiling heat transfer is dominant in the conventional tube with 6.1 mm inner diameter, whereas convective boiling heat transfer becomes dominant as the tube diameter decreases. Moreover, it has been noted that due to the difference of the properties of the working fluids, channel size, as well as the operating conditions, different flow patterns can be observed. According to some other investigators (e.g. Lee and Lee
(2001), Qu and Mudawar (2004), and Thome and Consolini (2010)), it has been established that at intermediate and high vapor quality, where the annular flow pattern is usually observed (e.g. Tran et al. (2000)), the evaporation of liquid film is the dominant heat transfer mechanism. Additionally, the forces acting at the liquid-vapor interface of the annular flow regime influence the heat transfer mechanism of flow boiling in micro-channels. Kandlikar (2004, 2010) studied and analyzed the significance of various forces that are important during rapid evaporation at high heat flux conditions and also influence the liquid-vapor interface in micro-channels. He reported that five main forces, namely, surface tension, viscous shear, inertia, evaporation momentum, and gravity forces play significant role for two phase flow boiling. Additionally he pointed out that the momentum change of evaporation and surface tension forces are dominant and significantly influence flow boiling characteristics, mainly for micro channel size, while the gravity force effects decreases by decreasing the channel size. Abadi et al. (2016) investigated experimentally the influence of gravity force on heat transfer, pressure drop and flow pattern of two phase flow boiling of R245fa in a mini inner tube diameter of 3.0 mm. They found that the gravity has an impact for the flow conditions where the confinement number is about 0.33.

Annular flow pattern is promised to be one of the main dominant flow patterns for flow boiling in micro-channels as it has been verified in many previous studies (e.g. Jiang et al. (2001), Celata et al. (2001), Thome (2004), Revellin and Thome (2007a), Cioncolini and Thome (2012)). Moreover, the ability to identify the flow pattern and its boundaries is an essential capability for developing a suitable heat transfer coefficient and pressure-drop prediction method. Consequently, several researchers have developed flow pattern maps for flow boiling in micro-channels. More prominent reported works are: Kandlikar (2002), Garimella et al. (2002), Wu and Cheng (2003), Revellin and Thome (2007a), Saisorn and Wongwises (2008), Karayiannis et al. (2010), and
Harirchian and Garimella (2010, 2012). Furthermore, having a comprehensive flow pattern map helps in developing a generalized flow boiling model for the basic flow patterns in micro-channels. For instance, Harirchian and Garimella (2010) established a flow pattern map for a wide range of channel size and experimental operating conditions for FC – 77 and recently, Harirchian and Garimella (2012) developed a flow regime-based model of heat transfer and pressure drop for flow boiling in micro-channels based on their comprehensive flow regime map.

However, several of these flow pattern maps were established for adiabatic flow database. Thome and El Hajal (2003) have developed a two-phase flow pattern map, proposed by Zürcher et al. (2002), which was able to predict fairly well, the two-phase flow regimes of seven different refrigerants. Recently, Revellin and Thome (2007a) developed a new diabatic flow pattern map for flow boiling of R134a in single micro-tubes applicable for the range of tested working conditions. In 2008, Cheng et al. (2008) developed a flow pattern map for flow boiling of CO2 in small and mini-tubes which was proven to be applicable for a wide range of operating conditions. Thus, the two flow pattern maps presented by Revellin and Thome (2007a), and Cheng et al. (2008) have been used in this work for predicting two-phase flow regimes of flow boiling of R134a and CO2 respectively in macro/micro horizontal channels. In the meantime, a systematic process for determining the flow pattern transition criterion is still not clear or available. However, there are some correlations developed for predicting the transition line, $x_{CB/A}$ for transition from coalescence flow pattern to annular flow pattern, (the onset of annular flow), in single horizontal channels. For instance, those proposed by Thome and El Hajal (2003), Revellin and Thome (2007a), and Ong and Thome (2009). Recently, Ong and Thome (2009) modified the correlation of the transition line $x_{CB/A}$, presented by Revellin and Thome (2007a), so they can be made applicable to three working fluids (R134a, R236fα, and R245fα) and wider range of channel sizes and operating
conditions. The correlation modified by Ong and Thome (2009) that has been reported is described as follows:

\[ \frac{x_{CB}}{A} = \left( \frac{P_r}{P_{sat}(R134a)} \right)^{0.45} Re_{lo}^{1.47} We_{lo}^{-1.23} \] (1)

Where \( P_r \) and \( P_{sat} \) are the reduced and saturated pressures, respectively. \( Re_{lo} \) is the Reynolds number of the liquid phase, and \( We_{lo} \) is the Weber number of the liquid phase. Harirchian and Garimella (2012) developed a flow regime map for flow boiling of FC-72 in multiple micro-channel heatsinks, and proposed a correlation for estimating the position of the transition from slug flow to annular flow along the micro-channels as follow:

\[ L_{ao} = 96.65 (Bo^{0.5} Re)^{-0.258} Bl^{-1} \frac{\rho_g}{\rho_f - \rho_g} D_{hH} \] (2)

Where \( L_{ao} \) is the length for the onset of annular flow (assuming saturation condition exists for the all liquid flow at the inlet), \( Bo \) is the Bond number, \( Bl \) is the Boiling number, and \( D_{hH} \) is the hydraulic diameter based on the heated perimeter.

Two-phase frictional pressure drop for flow boiling in micro-channels has been investigated by many researchers, and many predictive methods have been presented in literature some of them have been developed on the basis of experimental data of mini and micro-scale tubes and channels (e.g. Tran et al. (2000); Hwang and Kim (2006); Revellin and Thome (2007b); Cioncolini et al (2009); Kim and Mudawar (2013); Li and Hibiki (2017); and Sempértegui-Tapia and Rebatski (2017)). On the other hand, according to some existing flow pattern maps, there are few phenomenological models that have been developed for two-phase pressure drop prediction in annular flow regime. Recently, Cioncolini and Thome (2017), developed a new pressure drop
prediction method on the basis of 6291 experimental data points collected for 13 different fluids including vertical, horizontal, and annuli flows in a range of 3.0 to 25.0 mm inner diameter tubes. They considered the influence of gravity force and the interaction, at the liquid-vapor interface, between the surface tension and aerodynamic forces. Although, the model has been simple and validated to be much better than more than twelve existing correlations, but it is limited to macroscale tubes and channels.

One of the phenomenological pressure-drop models is, the model proposed by Cheng et al. (2008) on the basis of boundaries of annular flow regime. The authors correlated two-phase pressure drop data of annular flow, $\Delta p_A$, and it is given by Eq. (3) as follow,

$$\Delta p_A = 4f_A \frac{L \rho_g u_g^2}{D_{eq}}$$

Where $D_{eq}$ is the equivalent diameter which is defined as $D_{eq} = \sqrt{4A/\pi}$, and $f_A$ is the friction factor coefficient of annular flow which is expressed by Eq. (4) as follows:

$$f_A = 3.128Re_g^{-0.454}We_f^{-0.0308}$$

Where $Re_g$ is Reynolds number of the vapor phase, and $We_f$ is Weber number of the liquid film.

In summary, the evidence from this review of previous works shows that most of the investigations conducted for predicting two-phase heat transfer and pressure drop for flow boiling in micro-channels are experimental investigations that presented either empirical correlations, or semi-analytical models which are applicable for a specific working fluid, or a certain flow pattern, or operating working condition. In contrast, only few analytical models for flow boiling in horizontal micro-channels have been developed. Furthermore, it has been known that developing an analytical model based on the physical phenomenon of the flow boiling feature is an effective
and essential first step for designing flow boiling micro-channels. Therefore, development of analytical models for flow boiling in micro-channels, which can be applicable for wider range of operating conditions and fluids, is needed. Consequently, the aim of this work is to develop a one dimensional semi-analytical model for predicting the saturated two-phase heat transfer coefficient, frictional pressure drop, and void fraction for the annular flow regime for saturated flow boiling in a single horizontal micro-tube subjected to a uniform heat flux. Moreover, it is important to mention here that the present study is a fundamental study for two phase flow boiling in a single horizontal micro-tube and not in multiple channels.

2. **Modeling Methodology**

A physical description of the two-phase annular flow model is shown in Figure 1. As can be seen, the annular flow consists of two continuous phases, the vapor core which flows with an average vapor velocity, $u_g$, and the liquid film phase, whose thickness is $\delta(z)$, which moves with an average liquid film velocity, $u_f$, in the axil direction, $z$, of the working fluid flow. The micro-tube is subjected to a uniform heat flux, $q_w$. Furthermore, the applied heat flux is transferred through the micro-tube wall to the liquid film. While, the evaporation of the liquid film is considered to be the dominant heat transfer mechanism in the annular flow regime.

2.1. **Basic Assumptions:**

Two-phase annular flow is still a complex flow and has many challenges in modeling its real features. Thus, for model simplification, the following assumptions have been taken into account:
4. The flow is assumed to be a one dimensional steady state and fully developed flow. Thus, the dependent variables adopted vary only in axial flow direction.

5. The dominant flow pattern for flow boiling in micro-tubes at intermediate and high vapor quality is an annular flow, and the dominant heat transfer mechanism in the annular flow regime is evaporation at the liquid/vapor interface.

6. The micro-tube is subjected to a circumferential and axial uniform heat flux, and the working fluid enters the micro-tube at saturation conditions.

7. The liquid and vapor phases, at the interference locations at any z, are considered to be at local thermodynamic equilibrium.

8. As the channel size decreases, the effects of the transverse or axial component of the gravity vector decreases. Therefore, in the present model the influence of gravity force is assumed to be negligible.

9. The pressure gradient in the radial direction is neglected, thus, the pressure of the liquid phase and vapor phase at the same radial cross section are equal.

10. No entrained liquid droplets exist in the vapor core and no nucleating bubbles exist in the liquid film trapped in the annular flow regime for flow boiling in micro-channels.

11. Any conduction heat transfer in the axial flow direction is neglected.

12. The liquid and vapor densities can be modeled as constants throughout the channel for operating conditions under consideration.

13. At the interface, the interfacial liquid enthalpy equals the liquid phase enthalpy, and the interfacial vapor enthalpy equals to vapor phase enthalpy. The specific enthalpy of each phase varies only in flow direction.
Taking the above assumptions into account, the basic equations of the model can be derived by applying the mass, momentum, and energy balances for each phase in the chosen control volume, as follows.

### 2.2. Equations of Mass Conservation

The mass balance has been applied for each phase, liquid film and vapor core phases, in the chosen control volume as shown in Figure 2. Consequently, the mass conservation equations of the liquid film phase and vapor phase for one dimensional and steady flow can be deduced and expressed by Eqs. (5) and (6), respectively.

\[
\frac{d}{dz} \left[ \rho_f \alpha_f u_f \right] = -\Gamma \frac{P_i}{A} \tag{5}
\]

\[
\frac{d}{dz} \left[ \rho_g \alpha_g u_g \right] = \Gamma \frac{P_i}{A} \tag{6}
\]

Where, \( u_f \) and \( u_g \), are the liquid film and vapor phases’ mean velocities, respectively, and \( \rho_f \) and \( \rho_g \), are the densities of the liquid phase and vapor phase respectively. \( \alpha_f \) and \( \alpha_g \) are the liquid and vapor phases’ void fractions, where, \( \alpha_g = 1 - \alpha_f \). The term \( \Gamma \) appears on the right side of the above Eqs. (5, 6) is defined as the rate of evaporation of the liquid film at the liquid-vapor interface per unit area of the interfacial surface, and it will be evaluated based on the gas kinetic theory. \( P_i \) is the interfacial perimeter at the liquid vapor interphase. By adding Eq. (5) to Eq. (6) and expanding the derivatives, the vapor mass equation and whole mixture mass equation can be as expressed by Eq. (7) and Eq. (8) respectively,
2.3. Equations of Momentum Conservation

The forces acting on the chosen control volume consisting of the two phases, liquid film and vapor core, are shown in Figure 3. The body and virtual mass forces were neglected here. For flow boiling in micro-channels, the effect of gravitational body force is small and can be neglected in comparing to viscous and inertial forces as it has been concluded in several previous studies, (e.g. Kandlikar (2004), Serizawa et al. (2002)). The virtual mass force was neglected for model simplifications. Newton’s Law can be applied to each phase in the control volume for deriving the liquid and vapor momentum equations respectively as follows,

\[
\frac{d}{dz}\left(\rho_f \alpha_f u_f^2\right) = -\alpha_f \frac{dp}{dz} - \Gamma u_f \frac{P_i}{A} - \tau_w \frac{P_i}{A} + \tau_{fi} \frac{P_i}{A} \tag{9}
\]

\[
\frac{d}{dz}\left(\rho_g \alpha_g u_g^2\right) = -\alpha_g \frac{dp}{dz} + \Gamma u_g \frac{P_i}{A} - \tau_g \frac{P_i}{A} \tag{10}
\]

Where, \(\tau_w\) is the wall shear stress, \(\tau_{fi}\) and \(\tau_{gi}\) are liquid and vapor phase interfacial shear stresses respectively. They are assumed here to be the same and denoted as \((\tau_{fi} = \tau_{gi} = \tau_i)\). The interfacial
phasic velocities of the liquid and vapor phase are assumed to be the same also ($u_{fi} = u_{gi} = u_i$), where $u_i$, is the interfacial velocity. $P_w$ is the tube perimeter, and $p$ is the cross sectional pressure at $z$. Adding Eq. (9) to Eq. (10), and expanding the derivatives, the momentum equations of the vapor phase, and the whole mixture are represented by Eq. (11), and Eq. (12), respectively as follows,

$$
[2\rho_g u_g \alpha_g] \frac{du_g}{dz} + [\rho_g u_g^2] \frac{d\alpha_g}{dz} + \alpha_g \frac{dp}{dz} = \Gamma u_i - \tau_i \frac{P_i}{A}
$$

(11)

$$
[2\rho_f u_f \alpha_f] \frac{du_f}{dz} + [2\rho_g u_g \alpha_g] \frac{du_g}{dz} + [\rho_g u_g^2 - \rho_f u_f^2] \frac{d\alpha_g}{dz} + \frac{dp}{dz} = \frac{P_w}{A}
$$

(12)

2.4. Equations of Energy Conservation

By applying the first law of thermodynamic for the liquid film and the vapor core phases, the basic energy conservation equations of the liquid and vapor phases are expressed by Eq. (13), and Eq. (14) as follows,

$$
d \left[ \rho_f \alpha_f u_f \left( h_f + \frac{u_f^2}{2} \right) \right] = q_w \frac{P_w}{A} - q_f \frac{P_i}{A} - \Gamma (h_{fi} + \frac{u_{fi}^2}{2}) \frac{P_i}{A} + \tau_f u_{fi} \frac{P_i}{A}
$$

(13)
\[
\frac{d}{dz} \left[ \rho_g u_g \alpha_g (h_g + \frac{u_g^2}{2}) \right] = q_{gi} \frac{P_i}{A} + \Gamma (h_{gi} + \frac{u_{gi}^2}{2}) \frac{P_i}{A} - \tau_{gi} u_{gi} \frac{P_i}{A}
\] (14)

Where \( q_{wi} \) is the uniform heat flux applied at the external surface of the micro-channel. For the evaporation conditions, the term \( q_{fi} \) is the heat flux transfers from the liquid phase to the interface, and \( q_{gi} \) is the heat flux transfers from the interface to the vapor core. The enthalpies, \( h_f \) and \( h_g \) are associated with the liquid film and the vapor core respectively. The enthalpies, \( h_{fi} \) and \( h_{gi} \) are the liquid and vapor interfacial enthalpies at the interphase, which are assumed to be the same at thermodynamic equilibrium conditions. By expanding the derivatives of the energy equations of liquid and vapor phases, the energy equation of the vapor phase, and the whole mixture can be rearranged and represented by Eqs. (15, and 16), respectively, as follows,

\[
\left[ \rho_g \alpha_g (h_g + \frac{u_g^2}{2}) \right] \frac{du_g}{dz} + \left[ \rho_g u_g (h_g + \frac{u_g^2}{2}) \right] \frac{d\alpha_g}{dz} + \left[ \rho_g u_g \alpha_g \right] \frac{dh_g}{dz} = q_{gi} \frac{P_i}{A} - \Gamma (h_{gi} + \frac{u_{gi}^2}{2}) \frac{P_i}{A} + \tau_{gi} u_{gi} \frac{P_i}{A}
\] (15)

\[
\left[ \rho_f \alpha_f (h_f + \frac{u_f^2}{2}) \right] \frac{du_f}{dz} + \left[ \rho_g \alpha_g (h_g + \frac{u_g^2}{2}) \right] \frac{du_g}{dz} + \left[ \rho_g u_g \left( \frac{u_g^2}{2} \right) - \rho_f u_f \left( \frac{u_f^2}{2} \right) \right] \frac{d\alpha_g}{dz} + \left[ \rho_g u_g \alpha_g \right] \frac{dh_g}{dz} + \left[ \rho_f u_f \alpha_f \right] \frac{dh_f}{dz} = q_w \frac{P_w}{A}
\] (16)
2.5. Estimation of Interfacial Parameters

Considering the assumptions listed above in section 2.1., six main interfacial parameters at the liquid-vapor interphase, $\Gamma$, $q_{gi}$, $q_{fi}$, $\tau_i$, $u_i$, and $f_i$, in the present work still need to be evaluated for closing the formulation of this modeling approach. The mass flux of liquid film evaporation at the interface, $\Gamma$, is calculated based on the gas kinetic theory which has been discussed in many previous studies in the literature (e.g. Wayner et al. (1976), Carey (1992), Schonberg et al. (1995), and Vij and Dunn (1996)). Since, most of the proposed models for calculating mass flux at the interface are dependent of the thermo-physical properties of the phases, as well as the interfacial and vapor phase temperatures, it has been assumed in the present study, that the interfacial temperature equals the vapor temperature which is evaluated here based on the modeled values of vapor enthalpy. On the other hand the liquid temperature is calculated based on the liquid enthalpy. Accordingly, the mass flux of evaporation at the interface can be expressed as follows,

$$\Gamma = p \left( \frac{M}{2\pi R} \right)^{1/2} \left[ \left( \frac{C_{pg}}{h_g} \right)^{1/2} - \left( \frac{C_{pf}}{h_f} \right)^{1/2} \right]$$

Where $p$ is the pressure, $M$ is the molecular weight of the working fluid, $R$ is the universal gas constant, $C_{pg}$ is the vapor specific heat capacity, $C_{pf}$ is the liquid specific heat capacity, $h_g$ is the average specific enthalpy of the vapor phase, and $h_f$ is the average specific enthalpy of the liquid film.

With regard to the energy transfers from the liquid at the interface, $q_{fi}$, one can assume that all the heat flux applied at external surface of the channel wall transfers by conduction through the channel wall into the liquid phase in the radial direction and no heat transfer in axial direction,
consequently it will equal the liquid heat flux at the interface, \( q^\prime\prime_w = q^\prime\prime_i \). Considering the mass
flux of liquid film evaporation, and the latent heat of evaporation, \( h_{fg} \), the energy balance at the
liquid-vapor interface can be represented as follows,

\[
q^\prime\prime_{gi} = q^\prime\prime_w - \Gamma h_{fg}
\]  

(18)

The interfacial shear stress is one of the most important parameters in the annular flow
regime, and has a significant effect on the liquid film thickness. In the present study, the interfacial
shear stress is calculated based on the friction factor modeling approach as follows,

\[
\tau_i = \frac{1}{2} f_i \rho_g (u_g - u_f) |u_g - u_f|
\]  

(19)

For modeling the interfacial friction factor, several previous studies have discussed the
correlations of interfacial friction factor (e.g. Chisholm (1967), Wallis (1969), Henstock and
Hanratty (1976), Brauner and Maron (1992), Andritsos and Hanratty (1987), Wongwises and
Kongkiatwanitch (2001), and Quiben and Thome (2007)). The correlation of interfacial friction
factor as proposed by Quiben and Thome (2007) for annular flow (boiling or adiabatic), is used in
the present work and represented as follows,

\[
(f_i)_{annular} = 0.67 \left( \frac{\delta}{D} \right)^{1.2} \left( \frac{\rho_f - \rho_g}{g \delta^2} \right)^{-0.4} \left( \frac{\mu_g}{\mu_f} \right)^{0.08} \left( We_f \right)^{-0.034}
\]  

(20)

Where \( D \) is the channel diameter, \( g \) is the gravity, \( \mu_g \) is the vapor dynamic viscosity, \( \mu_f \) is the
liquid dynamic viscosity, \( \rho_g \) and \( \rho_f \) are the vapor and liquid densities respectively. \( \delta \) is the liquid
film thickness and can be defined in terms of vapor void fraction as follows,
\[
\delta = \frac{D}{2} \left(1 - \alpha_g^{0.5}\right) \quad (21)
\]

Liquid Weber number, \(We_f\), is calculated as

\[
We_f = \left(\frac{\rho_f u_f^2 D}{\sigma}\right) \quad (22)
\]

The interfacial velocity at the interface plays a significant role for considering the influence of interfacial parameters in momentum and energy equations. In the present study, the interfacial velocity is calculated in terms of working fluid properties and liquid film thickness using the relation proposed by Vij and Dunn (1996), which is represented as follows,

\[
u_i = 2 \left[\frac{(\mu_f)}{(D - 2\delta)^2} u_f - \left(\frac{\mu_g}{(D - 2\delta)^2}\right) u_g\right] \left[\left(\frac{(\mu_f)}{(D - 2\delta)^2}\right) - \left(\frac{\mu_g}{(D - 2\delta)^2}\right)\right] \quad (23)
\]

### 2.6. Wall Shear Stress and Friction Factor:

The wall shear stress \(\tau_w\), in the right side hand of the momentum equation of the whole mixture, Eq. (12), is given in terms of liquid film velocity and the working fluid density as,

\[
\tau_w = \frac{1}{2} \rho_f u_f^2 \quad (24)
\]

For wide range of flow conditions which cover laminar transitional, and turbulent flow conditions, different correlations discussed in Li and Wu (2010) for predicting friction factor have been
employed including the correlation of Churchill (1977) which takes into account the effect of
surface roughness. The frictional correlation are expressed as follows,

\[ f_f = \begin{cases} 
  f_1 & \text{for } Re_f \leq 1600 \\
  f_2 & \text{for } Re_f \geq 3,000 \\
  f_3 & \text{for } 1600 > Re_f > 3,000 
\end{cases} \tag{25} \]

Where

\[ f_1 = \frac{16}{Re_f} \tag{26} \]

\[ f_2 = \left[ \left( \frac{8}{Re} \right)^{12} + \frac{1}{(a + b)^{3/2}} \right]^{1/12} \tag{27} \]

\[ a = \left[ 2.457 \ln \left( \frac{1}{(7/Re) + 0.27(\varepsilon/D)} \right) \right]^{16}; \quad b = \left[ \frac{37530}{Re_f} \right]^{16} \]

\[ f_3 = f_1(1600) + \frac{Re - 1600}{1400} \left[ f_2(3000) - f_1(1600) \right] \tag{28} \]

Where \( Re_f \) is the base liquid film Reynolds number which is calculated in the present model using
the predicted values of average liquid film thickness \( \bar{\delta} \), and average liquid film velocity \( \bar{u_f} \) for any
heat flux \( q_w^* \) and associated mass flux \( G \), and is given as,

\[ Re_f = \frac{\rho_f \bar{u_f} D_h}{\mu_f} \tag{29} \]
2.7. Main Structure of the Proposed Model:

In summary of this analytical formulation, six expanded equations represent the final form of the present model. These equations are as follows: two equations from the mass balance which are the mass conservation equation of the vapor phase Eq. (7), and the mass conservation equation of the whole mixture, Eq. (8), and two momentum equations, which are the momentum conservation equation of the vapor phase, Eq. (11), and the momentum conservation equation of the whole mixture, Eq. (12), and the energy conservation equation of the vapor phase, Eq. (15), and the energy conservation equation of the whole mixture, Eq. (16). Consequently, the final form of the present model equation is organized and formulated in a mass matrix form as follows:

\[ A Y = B \]  

(30)

Where \( A \) is a square mass matrix which contains the coefficients of the dependent variables obtained by expanding the differential terms of the main equations. The gradient of the main dependent variables is represented by the column vector \( Y \), and the column vector \( B \) represents the nonhomogeneous part of the present model. The main model equation can be rewritten, in details, as given in Eq. (31).
\[
\begin{bmatrix}
\rho_g \alpha_g & 0 & \rho_g u_g & 0 & 0 & 0 \\
\rho_g \alpha_g & \rho_f u_f & (\rho_g u_g - \rho_f u_f) & 0 & 0 & 0 \\
2\rho_g \alpha_g u_g & 0 & \rho_g u_g^2 & \alpha_g & 0 & 0 \\
2\rho_g \alpha_g u_g & 2\rho_f \alpha_f u_f & (\rho_g u_g^2 - \rho_f u_f^2) & 1 & 0 & 0 \\
\rho_g \alpha_g \left(h_g + \frac{3}{2} u_g^2\right) & 0 & \rho_g u_g \left(h_g + \frac{1}{2} u_g^2\right) & \rho_g u_g \left(h_g + \frac{1}{2} u_g^2\right) - \rho_f u_f \left(h_f + \frac{1}{2} u_f^2\right) & 0 & \rho_g \alpha_g u_g & \rho_f \alpha_f u_f \\
\rho_g \alpha_g \left(h_g + \frac{3}{2} u_g^2\right) & \rho_f \alpha_f \left(h_f + \frac{3}{2} u_f^2\right) & \rho_g u_g \left(h_g + \frac{1}{2} u_g^2\right) & 0 & \rho_g \alpha_g u_g & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Gamma \frac{P_i}{A} \\
0 \\
(\Gamma u_i - \tau_i) \frac{P_i}{A} \\
\tau_w \frac{P_w}{A} \\
q_g \frac{\Gamma}{A} \left(h_g + \frac{u_i^2}{2}\right) - \tau_i u_i \frac{P_i}{A} \\
q_w \frac{\Gamma}{A} \left(h_w + \frac{u_g^2}{2}\right) \\
\end{bmatrix}
\]

(31)
2.8. Equivalent Reynolds Number:

It is very convenient and helpful in two-phase flow to use the equivalent Reynolds number \( Re_{eq} \) instead of liquid-phase and vapor-phase Reynolds numbers. The equivalent Reynolds number was defined in several previous studies (e.g. Akers et al. (1959), Yan and Lin (1998), and Laohalertdecha and Wongwises (2011)). In the present study, the equivalent Reynolds number is used to describe the range of database collected from the literature and employed for model validation as well as to describe the applicability of the present model. \( Re_{eq} \) is defined in terms of average vapor quality \( x_m \) and properties of the working fluid as developed by Akers et al. (1959) and ,as given below, by Eq. (32),

\[
Re_{eq} = \frac{GD_h}{\mu_f} \left[ (1 - x_m) + x_m \left( \frac{\rho_f}{\rho_g} \right)^{0.5} \right]
\] (32)

3. Initial Values of the Model Variables

In order to solve the main model equation, Eq. (31), the initial values of the dependent variables, \( u_{go}, u_{fo}, \alpha_{go}, p_o, h_{go}, \) and \( h_{fo} \) at the entrance of the micro-tube should be estimated properly. The initial values of the pressure, \( p_o \), the vapor phase enthalpy, \( h_{go} \), and the liquid phase enthalpy, \( h_{fo} \), are evaluated based on the inlet saturation temperature. The initial values of the rest of the main parameters are evaluated as explained in the following sections.
3.1. Initial Value of Void Fraction

It should be noted that the initial vapor void fraction is mainly dependent on the initial liquid film thickness, which is an essential parameter for studying the annular flow regime. Furthermore, predicting its initial values properly is very significant. Thus, several investigations for predicting liquid film variation either in slug flow or annular flow have been done. Moriyama and Inoue (1996) investigated experimentally the variation of liquid film thickness of flow boiling of R113 considering the effects of bubble frequency. They recognized two different zones, the first one is when the bubble acceleration is low and the liquid film thickness is mainly controlled by Capillary number, and the second zone is when the bubble acceleration is high and the liquid film thickness is mainly controlled by the viscous boundary layer. In other words, they developed two empirical correlations to predict the liquid film thickness on the basis of the Bond number. Most of available correlations for liquid film thickness prediction either for slug or annular flow regime were developed based on the experimental data measured at adiabatic flow conditions (e.g., Irandoust and Andersson (1989), Han and Shikazono (2009a, 2009b), Kanno et al (2010)). On the other hand, some other correlations of liquid film models developed based on boiling conditions have been used, but these over predicted the experimental data due to the influence of interfacial instability as reported by Revellin et al. (2008). In general, it would be more realistic to employ a correlation for predicting the initial value of the liquid film thickness considering the above mentioned effects, (i.e., bubble acceleration, laminar, and turbulent flow). Additionally, the initial liquid film thickness at boiling conditions may be estimated fairly well using the correlations developed at adiabatic conditions, as it has been argued by Han et al. (2012). Consequently, in the
present model, the initial liquid film thickness is predicted using the correlations developed by Han and Shikazono (2009b) as given in Eq. (33):

\[
\left( \frac{\delta_o}{D} \right)_{\text{steady}} = \left\{ \begin{array}{ll}
0.67 \, Ca^{2/3} & \text{Re} < 2000 \\
1 + 3.13 \, Ca^{2/3} + 0.504 \, Ca^{0.672} Re^{0.589} - 0.352 \, We^{0.629} & \text{Re} \geq 2000
\end{array} \right.
\]

The correlation developed by Han and Shikazono (2009b), Eq. (33), was able to predict the experimental data measured by the authors for various working fluids and micro-tube sizes with an accuracy of ± 15%. Furthermore, by considering the bubble acceleration effects, Han and Shikazono (2010), expressed the initial liquid film thickness by Eq. (34).

\[
\left( \frac{\delta_o}{D} \right)_{\text{accel}} = \frac{0.968 \, Ca^{2/3} \, Bo_{\text{accel}}^{-0.414}}{1 + 4.838 \, Ca^{2/3} \, Bo_{\text{accel}}^{-0.414}}
\]

Where, \( Bo \) is the Bond number based on the bubble acceleration that is expressed as \( Bo = \left( \rho_g a D_h^2/\sigma \right) \). In the present model, the bubble acceleration is determined based on the mean vapor velocity, and heated length, \( a = \left( \frac{\bar{u}_g^2}{2 L_h} \right) \), instead of bubble velocity since the adopted flow pattern is the annular flow regime. Consequently, based on the two values obtained from Eqs. (33, and 34) of the ratio, \( \left( \frac{\delta_o}{D} \right) \), the initial liquid film thickness is given by the minimum value as reported by Han and Shikazono (2009b, 2010) as expressed by Eq. (35),

\[
\left( \frac{\delta_o}{D} \right) = \min \left( \left( \frac{\delta_o}{D} \right)_{\text{steady}}, \left( \frac{\delta_o}{D} \right)_{\text{accel}} \right)
\]

The initial value of the vapor void fraction is calculated as follows,
\[ \alpha_{go} = \left[ \frac{(D - 2\delta_o)^2}{D^2} \right] \tag{36} \]

### 3.2. Initial Values of Liquid Film and Vapor Velocities

The initial values of vapor and liquid film velocities at the onset of annular flow are calculated in terms of initial liquid film thickness using Eqs. (37, and 38):

\[ u_{go} = (G x_e / \rho_g) \left( \frac{D^2}{(D - 2\delta_o)^2} \right) \tag{37} \]

\[ u_{fo} = \left( G \left( 1 - x_e \right) / \rho_f \right) \left( \frac{D^2}{4\delta_o(D - \delta_o)} \right) \tag{38} \]

Where, \( x_e \) is the vapor quality estimated at the exit of the micro-tube. In the present model, the exit vapor quality, \( x_e \), is calculated based on the energy balance of a micro-tube subjected to a uniform heat flux, and it is given by Eq. (39) in terms of the heated length-to-diameter ratio for zero-inlet sub-cooling enthalpy (inlet saturated conditions), and the Boiling number.

\[ x_e = 4Bl \left( \frac{L_h}{D_h} \right) \tag{39} \]

The Boiling number is calculated as expressed by Eq. (40) as follows,

\[ Bl = \left( \frac{q \dot{w}}{G h_{fg}} \right) \tag{40} \]

The initial value of the pressure at the inlet of the micro-tube is evaluated based on inlet saturation conditions of the experimental data.
4. Solution Procedure

The main differential equations represented in the mass matrix form which are expressed by Eq. (31) are solved using a MATLAB code based on explicit Runge-Kutta method. The solution procedure is described as follows:

1. Firstly, the operating conditions such as, inlet saturation temperature, mass flux, and heat flux, channel geometry, and heated length are defined, as well as the properties of the working fluid which are estimated at the inlet saturation temperature or pressure.

2. The domain of the micro-channel is divided on the basis of the heated length into small segments of length Δz, and the local vapor quality is calculated using Eq. (39).

3. The non-dimensional numbers, Re, We, Bl, and Ca are calculated.

4. The initial liquid film thickness is calculated by Eq. (35), and consequently the vapor void fraction, αgo, the initial values of vapor core velocity ugo, liquid film velocity ufo are estimated by Eqs. (36, 37, and 38), respectively.

5. The based-liquid film Reynolds number, Re_f, the liquid film friction factor f_f, and the wall shear stress τ_w, are calculated using Eqs. (29, 25, and 24), respectively.

6. The liquid Weber number, We_f, is calculated as defined by Eq. (22), while the mass flux of evaporation at the interface, Γ, and the vapor heat flux at the interface, q̇_gi are calculated by Eqs. (17, and 18), respectively.

7. The interfacial velocity, u_i, the interfacial friction factor, f_i, and interfacial shear stress, τ_i, are calculated by Eqs. (23, 20, and 19), respectively.
8. Solving the set of equations of the present model represented by Eq. (31), considering the initial values as they have been estimated above.

9. Update the values of the main dependent parameters, $u_g$, $u_f$, $\alpha_g$, $p$, $h_g$, and $h_f$, then repeat steps 3-7 for the next segment along the heated length. Consequently, the local values of the main parameters are determined.

10. The local and average heat transfer coefficients can be determined, as explained in the following sections, by Eqs. (42, and 43), respectively.

5. Results and Discussion

Heat transfer coefficient, pressure drop, and void fraction have been predicted and discussed in the following sections. The experimental data collected from the literature which fall in the annular flow regime have been used for validating the present model. Furthermore, to ensure that the collected data from the literature can be classified as macro/micro scale data, the classification suggested by Ong and Thome (2011) based on the confinement number, $C_o$, has been addressed and added. According to Ong and Thome (2011), the flow can be considered as a micro-scale flow when the confinement number reaches one, ($C_o \approx 1$), where the surface tension effect is dominant. It is of interest to represent the confinement number here which is defined as,

$$C_o = \left(\frac{1}{D}\right)\sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}} \quad (41)$$
5.1. **Two Phase Heat Transfer Coefficient**

It has been reported by many investigators that convective liquid film evaporation is expected to be the main dominant heat transfer mechanism for annular flow regime in horizontal micro-channels (particularly when \( C_o \geq 0.5 \)). Furthermore, there is a significant relation between the liquid film thickness and heat transfer coefficient in this regime. Based on the values of vapor void fraction predicted by the present model, and considering the relation between the vapor void fraction and liquid film thickness as represented by Eq. (21), the variation of liquid film thickness \( \delta(z) \) in terms of heated length can be calculated. Assuming that the heat transferred by conduction through the channel wall equals that transferred by convection through the liquid film. Thus, the local two-phase heat transfer coefficient \( h_{tp}(z) \), under uniform heat flux conditions may be represented by Eq. (42).

\[
h_{tp}(z) = \frac{k_f}{\delta(z)} \tag{42}
\]

Where \( k_f \) is the thermal conductivity of the liquid phase, which is always much larger than that of the vapor phase, of the working fluid estimated at the inlet saturation conditions.

By obtaining the local two-phase heat transfer coefficient along the heated length of the micro-channel, the average two-phase heat transfer coefficient can be calculated by the following expression, Eq. (43), for the vapor quality, \( x(z) \), range over \( 0 \leq z \leq L_H \) that is being assumed to cover the annular flow regime:

\[
\overline{h_{tp}} = \frac{1}{N} \int_{0}^{L_H} h_{tp}(z)dz \tag{43}
\]
Two-phase heat transfer coefficient has been calculated by the present model and compared with various experimental heat transfer data for a wide range of operating conditions for flow boiling in single mini/micro-channels. The range of operating conditions of experimental heat transfer data points collected from the literature and used for heat transfer comparison are listed in Table (1).

Figure 4 shows the variation of two-phase average heat transfer coefficient $h_{tp}$ of saturated flow boiling of the refrigerant R-134a in terms of vapor quality as predicted by the present model for flow through a 1.12 mm inner diameter tube, and a ratio of heated length to inner diameter $(L_h/D) = 835$ at an inlet saturation temperature $T_{sat} = 10 ^\circ C$, corresponding to a confinement number $C_{conf} = 0.81$. It is apparent that, by increasing the mass flux and the vapor quality, the heat transfer coefficient increases. Compared to the heat transfer data measured by Saitoh et al. (2005), it can be seen that the model predicted the heat transfer coefficient data well at low mass flux, 150 [kg/m$^2$.s], over the range of vapor quality of $(0.38 \leq x_e \leq 0.70)$ with a MAE of 5.90 %, and for mass flux 300 [kg/m$^2$.s], in the range of vapor quality of $(0.38 \leq x_e \leq 0.90)$ with a MAE of 10.72 %. At low mass flux 150 [kg/m$^2$.s], the experimental data showed a decrease in heat transfer coefficient by increasing the vapor quality beyond the vapor quality line $x_e > 0.7$. A possible explanation for this, is that this could be the effect of the transition from annular to dry-out flow regime at this vapor quality. On the other hand, the present model over predicted the heat transfer data measured for 150 kg/m$^2$.s beyond the vapor quality line $(x_e > 0.7)$, where it showed an increase in heat transfer coefficient by increasing the vapor quality. The corresponding equivalent Reynolds numbers for this range of mass fluxes are $3820 \leq Re_{eq} \leq 7370$. Additionally, the corresponding $Re_{eq}$ for the mass flux 300 [kg/m$^2$.s] is $Re_{eq} = 7370$ where the
present model provided a good agreement with experimental data even beyond the vapor quality $x_e > 0.7$, the liquid film evaporation dominance may explain the good match with experimental data in this regime.

For higher inlet saturation temperature $T_{\text{sat}} = 30^\circ\text{C}$, higher mass fluxes, $G = 270 - 500$ kg/m$^2$.s corresponding to $(3,175 \leq \text{Re}_{eq} \leq 6,077)$, and smaller inner diameter tube, $D=0.781$ mm, and lower density ratio, $(\rho_l/\rho_g) = 31.64$, and $C_{\text{conf}} = 1.04$, the present model was compared with heat transfer data of Ali et al. (2012) as shown in Figure 5. From this figure, the experimental data shows that the average two-phase heat transfer coefficient for the mentioned range of mass fluxes increases by increasing the mass flux and the vapor quality. The model predicted well the experimental data for mass fluxes 270, and 400 kg/m$^2$.s, with MAE of 6.87 %, and 11.51 % respectively. While for the mass flux, 500 kg/m$^2$.s, it can be seen that there is a slight underprediction of the data.

In order to investigate the influence of the working fluid on the heat transfer coefficient, the model was compared with the experimental heat transfer data available in the literature for flow boiling of CO2 in 0.529 mm inner diameter horizontal smooth micro-tube, measured by Ducoulombier et al. (2011) for low heated length to inner diameter ratio, $(L_h/D_h) = 302$, and low saturation temperature, $T_{\text{sat}} = 0^\circ\text{C}$, corresponding to $C_{\text{conf}} = 1.41$, and much lower density ratio, $(\rho_l/\rho_g) = 9.5$, as depicted in Figure 6. It is apparent that for the intermediate exit vapor quality, $0.25 \leq x_e \leq 0.75$, the model gave similar trend for the two-phase heat transfer coefficient for the tested mass fluxes, 200, 400, 600 kg/m$^2$.s, and corresponding equivalent Reynolds numbers, 2,170 $\leq \text{Re}_{eq} \leq 7,320$. While beyond a certain value of vapor quality (e.g. $x_e > 0.72$ for mass flux 200 kg/m$^2$.s), the model showed a steep increase in heat transfer coefficient. This value of
vapor quality increases by increasing the mass flux. Additionally, it has been noticed that the model slightly underpredicted the experimental heat transfer data for higher mass fluxes. The reason for this could be attributed to the significant effects of mass flux on the flow pattern and the boundary of annular flow regime as well as the onset of dry-out. Overall, for these flow conditions, the model predicted the tested heat transfer data of the annular regime with MAE = 10.98%.

For a larger tube diameter, \(D_h = 1.5\) mm, and same working fluid, CO\(_2\), at higher saturation temperature, \(T_{\text{sat}} = 10^\circ\text{C}\), corresponding to \(C_{\text{conf}} = 0.41\), and much lower density ratio, \((\rho_f/\rho_g) = 6.37\), the model was compared to the data of Choi et al. (2007), as depicted in Figure 7. According to the flow pattern map, updated by Cheng et al. (2008) for CO\(_2\), the boundaries of annular flow regime for mass fluxes 300, and 400 \(\text{kg/m}^2\cdot\text{s}\) are \((0.23 \leq x \leq 0.72)\), and \((0.23 \leq x \leq 0.68)\), respectively. From the Figure 7, a good agreement with the tested data at an intermediate and a high vapor quality range, \((0.15 \leq x_{\text{exit}} \leq 0.75)\), can be clearly seen for mass fluxes 300, and 400 \(\text{kg/m}^2\cdot\text{s}\). However, for mass flux 500 \(\text{kg/m}^2\cdot\text{s}\), and vapor quality, \(x \geq 0.45\), the model slightly underpredicted the experimental data. One reason of this underestimation is that the change of flow pattern of the tested data at this vapor quality. A second reason can be attributed to the flow instability where it is a quite hard to have a stable annular flow pattern. In general, for the tested range of equivalent Reynolds numbers, \(8,700 \leq \text{Re}_{\text{eq}} \leq 12,060\), the model predicted the heat transfer data of the CO\(_2\) with MAE = 10.67%.

For low reduced pressure conditions, the present model has been tested to predict the experimental heat transfer data measured by Mastrullo et al. (2018) for flow boiling of Anhydrous Ethanol in a 6 mm inner diameter single tube at low reduced pressures \(p_r = 0.009, \text{and } 0.02\) as shown in Figure 8. It is clear that the present model underestimates the experimental heat transfer
data of Mastrullo et al. (2018) at the both low reduced pressure for low and intermediate vapor quality ranges. However, at high vapor quality region, the model overestimates the experimental heat transfer data. This can be related to the heat transfer mechanism considered in this model. As it is known that in annular flow regime in microchannels, liquid film evaporation is considered as the dominant heat transfer mechanism. While for flow boiling in this tube, nucleate boiling mechanism influence the heat transfer.

Overall, the present model has been compared with 602 two-phase heat transfer data points which have been collected from different sources of annular flow regime for various working fluids and operating flow conditions that correspond to a wide range of equivalent Reynolds number of \(2,150 \leq Re_{eq} \leq 18,350\), as shown in Table (1). The model predicted the whole experimental heat transfer data of the annular flow regime with a MAE of 18.14 \% as depicted in Figure 9. Bland-Altman plots have been employed here to analyze and assess the ability of the present model in predicting the average heat transfer experimental data value points. The upper limit of agreement (ULOA) is +9.85, while the lower limit of agreement (LLOA) is -6.14. As it can be seen, both the data points predicted analytically and experimentally are uniformly scattered around the mean line of the difference, (Mean=1.85). However, as the average of the two results increases the difference increases and passes beyond the limits of the agreement. The results plotted over the upper limit of agreement (ULOA), are for the CO2 flows with \(Re_{eq} > 12,500\), where the model underestimated the experimental data points. Therefore, the range of model applicability is restricted to the limits of agreements (ULOA=+9.85, LLOA=-6.14).
5.2. Two Phase Pressure Drop

Two-phase pressure drop for flow boiling in micro-channels have been predicted by the present model for the working fluids and operating flow conditions which are listed in Table (2). Figure 10 compares the two-phase pressure drop results obtained by the present model with those measured by Revellin and Thome (2007b) for flow boiling of the R134a through a micro-tube has an inner diameter of D=0.509 mm and a heated length to inner diameter ratio of \( (L_h/D) = 138 \), at an inlet saturation temperature \( T_{sat} = 30^\circ C \), that matches a confinement number, \( C_{conf} = 1.59 \), for a wide range of mass fluxes, 350 to 1500 \( kg/m^2.s \), corresponding to a range of equivalent Reynolds number of \( 2,400 \leq Re_{eq} \leq 8,800 \). According to the diabatic flow pattern map developed by Revellin and Thome (2007a) for flow boiling in micro-channels, for the same operating conditions, the transition lines from coalescing bubble to annular flow, and from annular to dry-out flow regimes vary by varying the mass flux. For instance, the boundaries of annular flow for mass fluxes 350, 700, and 1000 \( kg/m^2.s \), are \( 0.36 \leq x \leq 0.87 \), \( 0.18 \leq x \leq 0.63 \), and \( 0.13 \leq x \leq 0.54 \), respectively. It is of interest to note that the boundaries of the annular flow regime become narrower as the mass flux increases. It can be seen from Figure 10 that the pressure drop increases as the mass flux and vapor quality increase. Additionally, for the annular flow regime, the present model predicted well the effect of mass flux and vapor quality on the pressure drop, and gave a good agreement with the experimental pressure drop data of Revellin and Thome (2007b), with a MAE of 23.24 %.

Furthermore, Figure 11 compares the results of the pressure drop obtained by the present model with those measured by Hwang and Kim (2006) for flow boiling of R-134a in a 0.486 mm inner diameter micro-tube at an inlet saturation temperature \( T_{sat} = 26.7^\circ C \), matches a confinement...
number, $C_{conf} = 1.93$, and a ratio of heated length to inner diameter ($L_h/D = 415$) for a range of mass fluxes 270 to 900 $kg/m^2.s$, corresponding to a range of equivalent Reynolds number of $(2,113 \leq Re_{eq} \leq 7,611)$. It has been seen that, for the mass flux 270 $kg/m^2.s$ which corresponds to an equivalent Reynolds numbers $Re_{eq} \approx 2,100$, the model showed a smooth and gradual increase in pressure drop with the exit vapor quality over the range $(0.28 \leq x \leq 0.84)$ and the predicted trend agreed very well the experimental pressure drop data. On the other hand, for the mass flux 510 $kg/m^2.s$, and $Re_{eq} \approx 4,000$, the model, noticeably, overpredicted the pressure drop data at low and intermediate vapor quality, and a slight decline in the trend can be observed at the vapor quality $x \approx 0.60$, where beyond this vapor quality the model gives a good agreement with the pressure drop data. With regard to the mass flux, 900 $kg/m^2.s$, and $Re_{eq} \approx 7,600$, a very good agreement with pressure drop data can be seen over the range of vapor qualities $(0.40 \leq x \leq 0.70)$, while beyond this range a slight decrease in pressure drop was predicted by the model. The decline positions, observed in the line graphs for the mass fluxes 510 and 900 $kg/m^2.s$ may represent the transition lines from slug flow to annular flow, and from annular flow to dry-out flow regime, respectively. Overall, for the equivalent Reynolds numbers, $2,000 \leq Re_{eq} \leq 7,600$ the model gave a good agreement with the measured pressure drop data of Hwang and Kim (2006) with MAE of 16.98 %.

Additionally, in order to investigate the influence of fluid properties on the two-phase pressure drop in micro-channels, the present model has been compared with the two-phase pressure drop data measured for CO2. For instance, Figure 12 shows a comparison between two-phase pressure drop data predicted by the present model and those measured by Ducoulombier et al. (2011) for flow boiling of the CO2 in a 0.529 mm inner diameter micro-tube at low inlet saturation temperature $T_{sat} = -5$ °C, and a confinement number, $C_{conf} = 1.52$, with a ratio of
heated length to inner diameter \((L_h/D) = 360\), and a low liquid to vapor density ratio of \((\rho_f/\rho_g = 11.47)\). Noticeably, the figure shows that a gradual increase in pressure drop by increasing the mass and vapor quality can be seen. Moreover, the model predicted well the pressure drop data for the range of mass fluxes, \((200 \text{ to } 800 \text{ kg/m}^2\cdot\text{s})\), and equivalent Reynolds number range of \((2,260 \leq \text{Re}_{eq} \leq 9,025)\). However, the model slightly, overpredicted the pressure drop data for the mass flux, \(1000 \text{ kg/m}^2\cdot\text{s}\) and \((\text{Re}_{eq} \approx 11,280)\) for the vapor quality range \((0.40 \leq x \leq 0.80)\). Again, a noticeable decline can be observed at different positions on the line graphs, which are varied by increasing the mass flux. A possible explanation for this is the influence of mass flux on the boundaries of flow pattern regimes. Moreover, a sharp decline in pressure drop can be seen for low mass flux, \(200 \text{ kg/m}^2\cdot\text{s}\) at very high vapor quality, \(x > 0.96\), and this could be attributed to the dry-out of the liquid film thickness at this vapor quality. The results of this comparison showed that the proposed model gave a fairly good prediction of the pressure drop data of Ducoulombier et al. (2011) which fall in the range of equivalent Reynold number \((2,260 \leq \text{Re}_{eq} \leq 11,280)\), with a MAE of 10.18 %.

Figure 13 presents the pressure drop of flow boiling of CO2 in a 1.42 mm inner diameter mini-tube predicted by the present model, and compared with pressure drop data of Wu et al. (2011) measured at a much lower inlet saturation temperature \((T_{sat} = -10 \degree \text{C})\), and a confinement number, \(C_{conf} = 0.6\), with a lower ratio of heated length to inner diameter \((L_h/D = 211)\), and a low liquid to vapor density ratio \((\rho_f/\rho_g = 13.83)\), for mass flux range of 300 to 600 \text{ kg/m}^2\cdot\text{s}\), and \((9,300 \leq \text{Re}_{eq} \leq 38,500)\). The figure showed that there has been a noticeable underestimation by the model of the pressure drop data. However, the model predicted the pressure drop data in the mentioned range of equivalent Reynolds number with a MAE of 18.97 %. The degree of underestimation increases as the mass flux increases, and this can be attributed to the effect of the
flow mode on the smoothness of the interface, and consequently on the flow instability of the flow pattern regime. Another possible explanation, for this underestimation, is that for $C_{conf} = 0.6$, the gravity force still plays a significant role and influences the model accuracy since it was neglected.

The present model has been tested at low reduced pressure flow boiling conditions. For instance, Figure 14 Shows a comparison of pressure drop predicted by the present model with those measured by Grauso et al. (2013) for flow boiling of R134a at a low reduced pressure, $p_r = 0.119$, and an inlet saturation temperature of 7.0 °C through a small-tube with an inner diameter of $D=6.0$ mm and a heated length of 780 mm, at the range of mass fluxes of 200 to 500 $[kg/m^2.s]$. It can be seen that the present model underestimates the experimental pressure drop data of Grauso et al. (2013) at low mass flux while it gives much better prediction as the mass flux increases. This underestimated prediction can be attributed to the tube size effect where the gravity has a significant influence at low mass fluxes particularly at this tube diameter, and it was not considered in the present model, or because of the fluid properties which varies a lot and affect the friction factor at low reduced pressure. Moreover, due to the lack of experimental data at low reduced pressure for flow boiling in micro-scale tubes, the present model compared with the data available in literature at these conditions.

Additionally, in order to check the accuracy of the present model, it has been compared with some existed correlation. For instance, Figure 15 shows a comparison between two-phase pressure drop data measured by Sempértegui-Tapia and Rebatski (2017) for flow boiling of the R134a in a 01.1 mm inner diameter micro-tube at an inlet saturation temperature $T_{sat} = 31$ °C, and a mass flux, (800 $kg/m^2.s$), and those predicted by the present model and four other pressure drop correlations developed by Tran et al. (2000), Hwang and Kim (2006), Li and Hibiki (2017), and
Sempértegui-Tapia and Rebatski (2017). Noticeably, the present model predicted well the pressure drop data in the range of vapor quality, (0.1 to 0.75), much better than the other tested correlations.

In summary, the present model has been compared with 498 two-phase pressure drop data points of annular flow regime collected from the available literature for the working fluids and operating conditions shown in Table (2), corresponding to equivalent Reynolds numbers, $1,900 \leq \text{Re}_{\text{eq}} \leq 38,140$, heated length to inner diameter ratios, $90 \leq \left(\frac{L_h}{D_h}\right) \leq 580$, and saturation temperature range of $-10^\circ C \leq T_{\text{sat}} \leq 40^\circ C$, that covers a wide range of liquid to vapor density ratio of $11.45 \leq \left(\frac{\rho_l}{\rho_g}\right) \leq 35.0$. The results predicted by the present model, along with collected experimental data have been plotted using Bland-Altman plot as shown in Figure 16. The calculated mean difference is Mean=-3.59, while the upper and lower limits of agreements are (ULOA=+18.75) and (LLOA=-25.95) respectively. As it can be seen, most of the data points (93.8 %) have fallen within the limits of the agreement. In general, the present model gave a good prediction for the two-phase pressure drop data points with a MAE of 23.02 %.

5.3. Void Fraction

Vapor void fraction for flow boiling in single micro-tubes has been calculated by the present model and compared with the experimental data measured by Shedd (2012) for flow boiling of the refrigerant R410a in 0.508, 1.19, and 2.92 mm inner diameter single micro-tubes at inlet saturation temperature range of 43.5 to 50°C and a range of mass flux from 200 to 800 \[\text{[kg/m}^2\text{.s]}\]. The results of vapor void fraction are presented as here,
Figure 17 shows the variation of vapor void fraction versus vapor quality predicted by the present model and those measured by Shedd (2012) for flow boiling of R410a in a 0.508 mm inner diameter tube at inlet saturation temperature 44 °C, and a confinement number, $C_{conf} = 1.14$, with mass flux of 600 [kg/m².s], which corresponds to an equivalent Reynolds number $Re_{eq} \approx 6,050$. It can be seen that the model predicted well the void fraction data in the range of vapor quality of $(0.10 \leq x \leq 0.90)$. However, a slight overprediction of the void fraction data can be observed beyond the vapor quality $x \geq 0.70$, which can be attributed to the transition from annular flow to mist flow regime. For larger tube size, 1.19 mm, and the same mass flux, 600 [kg/m².s], and similar inlet saturation temperature, 43.5 °C, which matches a confinement number, $C_{conf} = 0.49$, and corresponds to a higher equivalent Reynolds number $Re_{eq} \approx 14,300$, the average vapor void fraction of the R410a, has been predicted and plotted together with the void fraction data of Shedd (2012) as depicted in Figure 18. A steady increase in the void fraction predicted by the model can be observed when the void fraction fall in the vapor quality range of $(0.20 \leq x \leq 0.90)$, and a good agreement with the void fraction data points, which fall in the range of vapor quality of $(0.35 \leq x \leq 0.70)$, can be seen. Figure 19 presents the variation of vapor void fraction in terms of vapor quality predicted by the present model and those measured by Shedd (2012) for flow boiling of R410a in a 2.92 mm inner diameter tube, with a higher inlet saturation temperature, 50 °C, and mass flux, 600 [kg/m².s], which matches an equivalent Reynolds number of $Re_{eq} \approx 35,400$, and confinement number, $C_{conf} = 0.27$. It has been seen that the model predicts the trend of void fraction in terms of vapor quality. Additionally, a good agreement with void fraction data can be seen for high vapor quality regime, $(0.65 \leq x \leq 0.90)$, which suggests the dominance of annular flow regime in this range of vapor quality. Furthermore, according to a flow pattern map developed by Shedd (2012) for the flow of R410a in the same tube size and exact working conditions, the
annular flow regime boundaries fall in the range of vapor quality of \((0.60 \leq x \leq 1.0)\). However, an obvious overprediction can be observed when the void fraction data fall in low and intermediate vapor qualities \((0.15 \leq x \leq 0.65)\). A possible explanation for this is that for the slug flow regime the void fraction is lower than that for the annular flow, and the present model was developed based on the annular flow. One more explanation is that the scale effect plays a significant role for \(C_{\text{conf}} = 0.27\), where the gravity factor cannot be neglected anymore. Therefore, this result suggests the limit of the present model at this equivalent Reynolds number and tube size.

Overall, the vapor void fraction of the flow boiling of the R410a has been calculated by the present model and compared with 153 void fraction data points measured by Shedd (2012) for the working conditions listed in Table (3), which correspond to a range of equivalent Reynolds number, \((2,160 \leq Re_{eq} \leq 48,000)\), and confinement number, \((0.27 \leq C_{\text{conf}} \leq 1.14)\). The results are shown in Figure 20, and a good agreement with void fraction data can be observed where the majority of the void fraction data points were predicted and fall within the limits of agreement, with a MAE of 3.22 %.

5.4. Vapor and Liquid Film Velocities

Vapor core, and liquid film velocities are modeled and calculated by the main model equation, Eq. (31). It is of interest to present a sample of calculated velocities here. Therefore, the vapor and liquid film velocities for flow boiling of CO2 in 1.42 mm inner diameter tube are presented. Figure 21 shows the variation of average vapor velocity versus the vapor quality predicted by the present model for flow boiling of CO2 in 1.42 mm inner diameter tube with a ratio of heated length to inner diameter \((L_h/D=210)\), at a low inlet saturation temperature, -10 °C,
for a range of mass fluxes, 300 to 500 $[\text{kg/m}^2\text{s}]$, which correspond to a range of equivalent Reynolds number, $(8,700 \leq \text{Re}_{eq} \leq 14,800)$. The figure shows that there has been a steady increase in vapor velocity by increasing the vapor quality. Furthermore, the model predicted the influence of mass flux on the vapor velocity, where it can be seen that vapor velocity increases by increasing the mass flux.

For the same working conditions, the average liquid film velocity has been calculated by the present model, and plotted as shown in Figure 22. A similar trend of the liquid film velocity can be noticed at the three mass fluxes. For the mass flux 300 $[\text{kg/m}^2\text{s}]$, ($\text{Re}_{eq} \approx 8,700$), the liquid film velocity tends to slightly increase at the low vapor quality, ($x \leq 0.3$), and then it is likely to level off in the vapor quality range $(0.30 \leq x \leq 0.5)$, after this, there has been a gradual decrease in liquid film velocity with vapor quality. According to the updated flow pattern map of Cheng et al. (2008) for CO2, the boundaries of annular flow regime correspond to mass flux 300 $[\text{kg/m}^2\text{s}]$ fall in the vapor quality range, $(0.23 \leq x \leq 0.72)$. It is of importance to mention here that these boundary values of vapor quality represent the center line of transition lines, since the transition from flow pattern to another does not occur instantly. With regard to the other two mass fluxes, 400, and 500 $[\text{kg/m}^2\text{s}]$, a slow decrease in liquid film velocity for $(0.4 \leq x \leq 0.68)$, and $(0.45 \leq x \leq 0.62)$, can be observed, and then a sharp decline in the liquid film velocity can be seen. The significance of this result in addition to showing the trend of liquid film velocity, is that it would be helpful to predict the transition line from annular flow to mist/dry-out flow regime. In other words, a slight fluctuation at $x=0.65$, and $x=0.67$ for the mass fluxes 400, and 500 respectively, has been observed, and these points are very close to those suggested by the flow pattern map updated by Cheng et al. (2008) for CO2.
5.5. Liquid Film Thickness

Liquid film thickness for various working fluids listed in Table (1), and Table (2) have been calculated by the present model, based on the relation between the modeled vapor void fraction and liquid film thickness which are represented by Eq. (21). A sample of obtained results for flow boiling of CO2 in 1.42 mm inner diameter is presented and discussed in this section. Figure 23 shows the variation of liquid film thickness versus vapor quality predicted by the present model for flow boiling of CO2 in 1.42 mm inner diameter, with a mass flux, 300 [kg/m².s], which corresponds to, Reₐq ≈ 8,700, at an inlet saturation temperature of -10 °C. It can be noticed from the figure that the liquid film thickness decreases by increasing the vapor quality, and the mass flux. Moreover, the relationship between the liquid film thickness and vapor quality is nonlinear in the annular flow regime (i.e. for this case 0.2 ≤ x ≤ 0.8), after this, a sharp decrease in liquid film thickness can be observed, and this of course is due to the onset of dry-out phenomenon.

6. Sensitivity Analysis of the Model

The influence of the initial liquid film thickness which is calculated by Eq. (33), and Eq. (34) on the two phase heat transfer coefficient has been discussed by performing the sensitivity analysis of the model. The analysis was performed for results predicted by the present model for flow boiling of CO2 in a 1.42 mm inner tube diameter for saturation conditions at -10 °C, with a mass flux, 300 [kg/m².s], and for a range of vapor quality of 0.3 ≤ x ≤ 0.8. Figure 24 shows the influence of initial liquid film thickness on the performance of the present model for predicting the two-phase heat transfer coefficient in annular flow regime.
It can be seen that for the vapor quality, x=0.7, decreasing or increasing the initial liquid film thickness by ± 50 %, the deviation in the heat transfer coefficient does not exceed ± 10 %. On the other hand, for the vapor quality, x=0.4, it can be seen that the heat transfer model is more sensitive to the initial liquid film thickness. One possible explanation for this is that, at the vapor quality, x=0.4, the flow pattern could be slug flow pattern or coalescence bubble pattern which is close to the transition line, where the liquid film thickness fluctuates and becomes unsteady. As the vapor quality increases, the model is less sensitive to the initial liquid film thickness. Therefore, in the range of annular flow regime, the sensitivity level of the model to the initial liquid film is reasonable, and acceptable.

7. Conclusions

A semi-analytical model to predict the two-phase heat transfer, pressure drop, and void fraction in annular flow regime for flow boiling in a horizontal micro-channel subjected to a uniform heat flux, has been developed on the basis of a separated flow model for the two phase flow. The influences of working fluid, channel size, and operating conditions on the two-phase heat transfer, pressure drop, and void fraction have been considered. It is observed that, in the annular flow regime, the two-phase heat transfer coefficient, pressure drop, and void fraction increase with mass flux and exit vapor quality. The model has been compared with 1,253 experimental data points, collected from the available literature, for model validation. The model was compared with 602 two-phase heat transfer data points for flow boiling conditions that cover a range of equivalent Reynolds number $2,000 \leq Re_{eq} \leq 18,350$, and a range of confinement number, $0.3 \leq C_{conf} \leq 1.8$. The heat transfer data were predicted by the model with a MAE of 18.14 %. Furthermore, the model was compared with 498 pressure drop data points, which fall in
the range of equivalent Reynolds number \(1,900 \leq Re_{eq} \leq 38,400\), and confinement number, \(0.32 \leq C_{conf} \leq 3.4\), and showed a good agreement with the data with a MAE of 23.02 \%. With regard to the average void fraction, the model was compared with 153 void fraction data points which corresponded to \(2,150 \leq Re_{eq} \leq 48,000\), and \(0.27 \leq C_{conf} \leq 1.14\), and predicted the data with a MAE of 3.22 \%. The applicability of the present model can be limited in terms of the above described ranges of equivalent Reynolds number and confinement number. Overall, the present model is applicable for \(1,900 \leq Re_{eq} \leq 18,000\), and \(0.5 \leq C_{conf} \leq 3.4\). Moreover, two main applications of the present model can be addressed here. Firstly, estimating the cooling load of the micro-channel heat sink based on the operating condition. Secondly, predicting the two-phase pressure drop in the same flow regime, and hence, estimating the pumping power required for circulating the working fluid at the applied conditions. It is of interest to point out, that the present model may be extended in the future work by considering the surface tension effect, and gravity vector component. This may be needed to improve the model accuracy and expand its applicability range over a predicted range of the annular flow regime boundaries.

8. **Numenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>Cross section area of the channel, ((m^2)).</td>
</tr>
<tr>
<td>(Bo)</td>
<td>Bond number.</td>
</tr>
<tr>
<td>(Bl)</td>
<td>Boiling number, (Bl = \left(\frac{q}{\Delta h} \right)).</td>
</tr>
<tr>
<td>(Ca)</td>
<td>Capillary number, (Ca = \left(\frac{\mu f U}{\sigma}\right)).</td>
</tr>
<tr>
<td>(C_o)</td>
<td>Confinement number, (C_o = \left(\frac{1}{D}\right)\sqrt{\frac{\sigma}{g}} \frac{\rho_f - \rho_g}{\rho_g}.)</td>
</tr>
<tr>
<td>(D)</td>
<td>Tube diameter, ((m)).</td>
</tr>
<tr>
<td>(D_h)</td>
<td>Hydraulic diameter, ((m)).</td>
</tr>
<tr>
<td>(f)</td>
<td>Friction factor.</td>
</tr>
</tbody>
</table>
\( G \) \( \text{mass flux, (kg/m}^2\cdot \text{s).} \)
\( h_f \) \( \text{saturated liquid enthalpy, (J/kg).} \)
\( h_g \) \( \text{saturated vapor enthalpy, (J/kg).} \)
\( k_f \) \( \text{Thermal conductivity of the working fluid, (W/m.K)} \)
\( L_h \) \( \text{heated length, (m).} \)
\( MAE \) \( \text{Mean Absolute Error,} \)
\( \dot{m} \) \( \text{mass flow rate, (kg/s).} \)
\( N \) \( \text{Number of Experimental data points} \)
\( Pr \) \( \text{Prandtl number.} \)
\( p_r \) \( \text{Reduced pressure.} \)
\( P_i \) \( \text{interfacial surface perimeter, (m)} \)
\( p \) \( \text{pressure, (N/m}^2\). \)
\( q''_w \) \( \text{heat flux, (W/m}^2\). \)
\( q''_c \) \( \text{critical heat flux, (W/m}^2\). \)
\( R \) \( \text{internal radius, (m)} \)
\( Re \) \( \text{Reynolds number.} \)
\( u_i \) \( \text{interfacial velocity, (m/s).} \)
\( u_g \) \( \text{vapor velocity, (m/s).} \)
\( u_f \) \( \text{liquid film velocity, (m/s).} \)
\( x \) \( \text{vapor quality.} \)
\( We \) \( \text{Weber number We = (G}^2\text{D}_h/\rho \sigma). \)
\( We_m \) \( \text{Average Weber number.} \)
\( z \) \( \text{Flow direction.} \)

**Greek symbols**

\( \alpha \) \( \text{void fraction, } \alpha = ((D - 2\delta)/D)^2 \)
\( \Gamma \) \( \text{mass flow rate per unit area, (kg/m}^2\cdot \text{s).} \)
\( \delta \) \( \text{liquid film thickness, (m).} \)
\( \mu \) \( \text{dynamic viscosity, (N.s/m}^2\). \)
\( \rho \) \( \text{density, (kg/m}^3\). \)
\( \sigma \) \( \text{surface tension, (N/m).} \)
\( \tau \) shear stress, \((N/m^2)\).

**Subscripts**
- \( c \) critical.
- \( eq \) equivalent
- \( Exp \) experimental.
- \( Pre \) predicted.
- \( f \) liquid or liquid film.
- \( g \) gas or vapor phase.
- \( i \) inlet.
- \( o \) initial.
- \( out \) outlet.
- \( sat \) saturated.
- \( sub \) subcooling
- \( TP \) two phase.
- \( w \) wall.
- \( h \) hydraulic or heated.

9. **References**


Grauso, S., Mastrullo, R., Mauro, A.W., Thome, J.R.; and Vanoli, G.P. 2013. Flow pattern map, heat transfer and pressure drops during evaporation of R-1234ze(E) and R134a in a

https://mc06.manuscriptcentral.com/tcsme-pubs


<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Geometry</th>
<th>Working fluid(s)</th>
<th>Hydraulic diameter $D_h$, [mm]</th>
<th>Heated length $L_h$, [mm]</th>
<th>Saturation Temperature $T_{sat}$, [°C]</th>
<th>Mass flux $G$, [kg/m².s]</th>
<th>Heat flux $q''$, [kW/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mahmoud &amp; Karayiannis</td>
<td>Circular</td>
<td>R134a</td>
<td>0.52, 1.10</td>
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<td>300</td>
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<td>(2013)</td>
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<tr>
<td>Tibirica (2011)</td>
<td>Circular</td>
<td>R134a, R1234ze</td>
<td>1.10</td>
<td>180</td>
<td>31</td>
<td>300-600</td>
<td>15-35</td>
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<td>0.781</td>
<td>191</td>
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<td>5.0-60.0</td>
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<tr>
<td>Ong &amp; Thome (2009)</td>
<td>Circular</td>
<td>R134a, R236fa</td>
<td>1.03</td>
<td>180</td>
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<td>21.5-43.0</td>
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<td>Saitoh et al. (2005)</td>
<td>Circular</td>
<td>R134a</td>
<td>0.51, 1.12, 3.10</td>
<td>550, 935, 3235</td>
<td>5.0, 10.0</td>
<td>150-300</td>
<td>12.0-27.0</td>
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<td>Yan &amp; Lin (1998)</td>
<td>Circular</td>
<td>R134a</td>
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<td>31.0</td>
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<tr>
<td>Ducoulombier et al. (2011)</td>
<td>Circular</td>
<td>CO2</td>
<td>0.529</td>
<td>160</td>
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<td>200-1200</td>
<td>10.0-30.0</td>
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<td>Wu et al. (2011)</td>
<td>Circular</td>
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<td>1.42</td>
<td>300</td>
<td>-10.0</td>
<td>300-500</td>
<td>7.5-29.8</td>
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<tr>
<td>Choi et al. 2007</td>
<td>Circular</td>
<td>CO2</td>
<td>1.50</td>
<td>2000</td>
<td>+10.0</td>
<td>300-500</td>
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<tr>
<td>Lee &amp; Lee (2001)</td>
<td>Rectangular</td>
<td>R113</td>
<td>0.784</td>
<td>300</td>
<td>50.0</td>
<td>104-208</td>
<td>5.0-15.0</td>
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Table (2): Test conditions of two-phase pressure drop data points used for model validation

<table>
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<tr>
<th>Author(s)</th>
<th>Geometry</th>
<th>Working fluid(s)</th>
<th>Hydraulic diameter $D_h$, [mm]</th>
<th>Heated length $L_h$, [mm]</th>
<th>Saturation Temperature $T_{sat}$, [°C]</th>
<th>Mass flux $G$, [kg/m².s]</th>
<th>Heat flux $q_w$, [kW/m²]</th>
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<tr>
<td>Tran et al. (2000)</td>
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<td>2.2-49.8</td>
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<td>Tibiriçá et al. (2011)</td>
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<td>R134a</td>
<td>2.32</td>
<td>464</td>
<td>31.3</td>
<td>200 – 600</td>
<td>10-55</td>
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<tr>
<td>Revellin &amp; Thome (2007b)</td>
<td>Circular</td>
<td>R134a</td>
<td>0.509, 0.790</td>
<td>70.7, 914</td>
<td>30</td>
<td>350 – 1000</td>
<td>3.1-415</td>
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<td>Hwang &amp; Kim (2006)</td>
<td>Circular</td>
<td>R134a</td>
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<tr>
<td>Cavallini et al. (2009)</td>
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<td>R134a</td>
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<td>40</td>
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<tr>
<td>Ducoulombier et al. (2011)</td>
<td>Circular</td>
<td>CO2</td>
<td>0.529</td>
<td>191</td>
<td>-5, -10</td>
<td>200-1400</td>
<td>-</td>
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<tr>
<td>Wu et al. (2011)</td>
<td>Circular</td>
<td>CO2</td>
<td>1.42</td>
<td>300</td>
<td>-10</td>
<td>300-600</td>
<td>7.5-29.8</td>
</tr>
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Table (3): Test conditions of vapor void fraction data of R410a measured by Shedd (2012) and used here for model validation

<table>
<thead>
<tr>
<th>Hydraulic diameter $D_h$, [mm]</th>
<th>Saturation Temperature $T_{sat}$, [°C]</th>
<th>Mass flux $G$, [kg/m².s]</th>
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<tr>
<td>0.508</td>
<td>44.0</td>
<td>200, 400, 600, 800</td>
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<tr>
<td>1.19</td>
<td>44.0</td>
<td>200, 400, 600, 800</td>
</tr>
<tr>
<td>2.92</td>
<td>50.0</td>
<td>200, 400, 600, 800</td>
</tr>
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FIGURE 18: VAPOR VOID FRACTION VERSUS VAPOR QUALITY FOR FLOW BOILING OF R410A IN 1.19 mm INNER DIAMETER TUBE AT Tsat = 43.5 °C, AND 600 [KG/M2.s] MASS FLUX

FIGURE 19: VOID FRACTION VERSUS VAPOR QUALITY FOR FLOW BOILING OF R410A IN 2.92 mm INNER DIAMETER TUBE AT Tsat = 50.0 °C, AND 600 [KG/M2.s] MASS FLUX

FIGURE 20: COMPARISON OF VAPOR VOID FRACTION PREDICTED BY THE PRESENT MODEL WITH THOSE OF SHEDD (2012) FOR FLOW BOILING OF R410A IN 0.508, 1.19, AND 2.92 mm INNER DIAMETER TUBES AT Tsat = 44.0 TO 50.0 °C, AND RANGE OF MASS FLUXES, 200 TO 800 [KG/M2.s]

FIGURE 21: AVERAGE VAPOR VELOCITY VERSUS VAPOR QUALITY PREDICTED BY THE PRESENT MODEL FOR FLOW BOILING OF CO2 IN 1.42 mm INNER DIAMETER TUBE AT Tsat = −10 °C, AND OPERATING CONDITIONS SIMILAR TO THOSE OF WU ET AL. (2011) AS LISTED IN TABLE (1)

FIGURE 22: LIQUID FILM VELOCITY VERSUS VAPOR QUALITY PREDICTED BY THE PRESENT MODEL FOR FLOW BOILING OF CO2 IN 1.42 mm INNER DIAMETER TUBE AT Tsat = −10 °C, AND OPERATING CONDITIONS SIMILAR TO THOSE OF WU ET AL. (2011) AS LISTED IN TABLE (1)

FIGURE 23: LIQUID FILM THICKNESS VERSUS VAPOR QUALITY PREDICTED BY THE PRESENT MODEL FOR FLOW BOILING OF CO2 IN 1.42 mm INNER DIAMETER TUBE AND THE WORKING CONDITIONS OF WU ET AL. (2011) PRESENTED IN TABLE (1)

FIGURE 24: SENSITIVITY ANALYSIS OF THE TWO-PHASE HEAT TRANSFER COEFFICIENT PREDICTED BY THE PRESENT MODEL TO THE INITIAL LIQUID FILM THICKNESS.
Figure 1: Sketch of an annular flow model in horizontal circular channel.
Figure 2: Symmetric sketch shows basic terms for deriving mass conservation equations for liquid film and vapor core phases.

\[
\begin{align*}
\frac{dm_g}{dz} &= [\alpha \rho_g Adz] \\
\frac{dm_f}{dz} &= [(1 - \alpha) \rho_f Adz] \\
m_{ev} &= [\Gamma P_i dz] \\
m_{g, in} &= u_g \Psi_g A_d x \\
m_{f, in} &= u_f \Psi_f A_d x \\
m_{g, out} &= m_{g, out} \\
m_{f, out} &= m_{f, out}
\end{align*}
\]
Figure 3: Symmetric sketch shows the forces acting on the control volume.
Figure 4: Heat transfer coefficient for flow boiling of the refrigerant R134-a in 1.12 mm inner diameter micro-channel versus vapor quality for various mass fluxes, and comparison of the heat transfer data predicted by the present model with those of Saitoh et al. (2005) at $T_{sat} = 10$ °C
Figure 5: Heat transfer coefficient for flow boiling of the refrigerant R134-a in 0.781 mm inner diameter micro-channel versus vapor quality for various mass fluxes, and comparison of the heat transfer data predicted by the present model with those of Ali et al. (2012) at $T_{sat} = 30 \, ^{\circ}C$. 

\[ \text{Heat Transfer Coefficient} \left[ \text{[kW/m}^2\text{.K]} \right] \]

\begin{align*}
\text{Vapor quality} & \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \\
\text{Heat Transfer Coefficient} & \quad 10^0 \quad 10^1 \quad 10^2 \\
\end{align*}

- Ali et al. [2012], $G=270$ [kg/m$^2$.s]
- Ali et al. [2012], $G=400$ [kg/m$^2$.s]
- Ali et al. [2012], $G=500$ [kg/m$^2$.s]
- Present Model, $G=270$ [kg/m$^2$.s]
- Present Model, $G=400$ [kg/m$^2$.s]
- Present Model, $G=500$ [kg/m$^2$.s]
Figure 6: Heat transfer coefficient versus vapor quality for flow boiling of CO2 in 0.529 mm inner diameter micro-channel for various mass fluxes, and comparison of the present model with the data of Ducoulombier et al. (2011) at $T_{sat} = 0$ °C.
Figure 7: Heat transfer coefficient versus vapor quality for flow boiling of the refrigerant CO2 in 1.5 mm inner diameter micro-tube for various mass fluxes, and compared with the experimental data of Choi et al. (2007) at $T_{\text{sat}} = 10 \, ^\circ\text{C}$.
Figure 8: Heat transfer coefficient versus vapor quality for flow boiling of Anhydrous Ethanol in 6.0 mm inner diameter tube at mass flux 90 [kg/m².s], and low reduced pressures $p_r = 0.009$, and 0.02, compared with the experimental data of Mastrullo et al. (2018) at saturation temperatures $T_{sat} = 66 \, ^\circ C$, and $85 \, ^\circ C$. 
Figure 9: Using Bland-Altman Plot method for assessing the agreement between the average two-phase heat transfer coefficient predicted by the present model and those measured experimentally (602 data points) for various working fluids and flow conditions as shown in Table (1) for the annular flow regime.
Figure 10: Comparison of pressure drop predicted by the present model with that measured by Revellin and Thome (2007b) for flow boiling of R-134a at inlet saturation temperature of 30 °C through a micro-tube with an inner diameter of D=0.509 mm and heated length of 70.7 mm, for range of mass fluxes of 350 to 1500 [kg/m².s].
Figure 11: Comparison of pressure drop predicted by the present model with those measured by Hwang and Kim (2006) for flow boiling of R-134a at inlet saturation pressure of 700 kPa through a circular micro-tube with an inner diameter D=0.430 mm.
Figure 12: Comparison of pressure drop predicted by the present model with the experimental data of Ducoulombier et al. (2011) for flow boiling of CO2 at $T_{\text{sat}} = -5.0 \, ^\circ\text{C}$, through a circular micro-tube with $D=0.529 \, \text{mm}$, and heated length of 191 mm for mass fluxes range of $200 - 1000 \, [\text{kg/m}^2\cdot\text{s}]$. 

Ducoulombier et al. (2011), $G=200 \, [\text{kg/m}^2\cdot\text{s}]$
Ducoulombier et al. (2011), $G=400 \, [\text{kg/m}^2\cdot\text{s}]$
Ducoulombier et al. (2011), $G=600 \, [\text{kg/m}^2\cdot\text{s}]$
Ducoulombier et al. (2011), $G=800 \, [\text{kg/m}^2\cdot\text{s}]$
Ducoulombier et al. (2011), $G=1000 \, [\text{kg/m}^2\cdot\text{s}]$

Present Model, $G=200 \, [\text{kg/m}^2\cdot\text{s}]$
Present Model, $G=400 \, [\text{kg/m}^2\cdot\text{s}]$
Present Model, $G=600 \, [\text{kg/m}^2\cdot\text{s}]$
Present Model, $G=800 \, [\text{kg/m}^2\cdot\text{s}]$
Present Model, $G=1000 \, [\text{kg/m}^2\cdot\text{s}]$
Figure 13: Pressure drop predicted by the present model versus vapor quality and compared with experimental data of Wu et al. (2011) for flow boiling of CO2 at \( T_{\text{sat}} = -10 \, ^\circ C \), through a circular micro-tube with \( D=1.42 \) mm, and heated length of 300 mm, for mass fluxes range 300 – 600 [\( kg/m^2.s \)].
Figure 14 Shows a comparison of pressure drop predicted by the present model with those measured by Grauso et al. (2013) for flow boiling of R134a at a low reduced pressure, $p_r = 0.119$, and an inlet saturation temperature of $7.0 \degree C$ through a small-tube with an inner diameter of $D=6.0 \text{ mm}$ and a heated length of $780 \text{ mm}$, at the range of mass fluxes of $200$ to $500 \text{ [kg/m}^2\cdot\text{s]}$. 
Figure 15: Shows a comparison of pressure data measured by Sempértegui-Tapia and Rebatski (2017) for R134a with the present model and some available correlations including Sempértegui-Tapia and Rebatski (2017) correlation.
Figure 16: Using Bland-Altman Plot method for assessing the agreement between the average two-phase pressure drop predicted by the present model and those collected (498 data points) for working conditions shown in Table (2) for annular flow regime.
Figure 17: Void fraction versus vapor quality for flow boiling of R410a in 0.508 mm inner diameter tube at \( T_{\text{sat}} = 44.0 \, ^\circ\text{C} \), and 600 [kg/m\(^2\).s] mass flux.
Figure 18: vapor void fraction versus vapor quality for flow boiling of R410a in 1.19 mm inner diameter tube at $T_{\text{sat}} = 43.5 \, ^\circ\text{C}$, and 600 [kg/m$^2$.s] mass flux.
Figure 19: Void fraction versus vapor quality for flow boiling of R410a in 2.92 mm inner diameter tube at $T_{\text{sat}} = 50.0 \, ^\circ\text{C}$, and 600 [kg/m$^2$.s] mass flux.
Figure 20: Comparison of vapor void fraction predicted by the present model with those of Shedd (2010) for flow boiling of R410a in 0.508, 1.19, and 2.92 mm inner diameter tubes at $T_{sat} = 44.0$ to 50.0 °C, and range of mass fluxes, 200 to 800 [$kg/m^2.s$].
Figure 21: Average vapor velocity versus vapor quality predicted by the present model for flow boiling of CO2 in 1.42 mm inner diameter tube at $T_{\text{sat}} = -10^\circ \text{C}$, and operating conditions similar to those of Wu et al. (2011) as listed in Table (1).
Figure 22: Liquid film velocity versus vapor quality predicted by the present model for flow boiling of CO2 in 1.42 mm inner diameter tube at $T_{sat} = -10 \, ^{\circ}C$, and operating conditions similar to those of Wu et al. (2011) as listed in Table (1).
Figure 23: Liquid film thickness versus vapor quality predicted by the present model for flow boiling of CO2 in 1.42 mm inner diameter tube and the working conditions of Wu et al. (2011) presented in Table (1).
$\frac{\delta}{\delta_0}$

**Figure 24:** Sensitivity analysis of the two-phase heat transfer coefficient predicted by the present model to the initial liquid film thickness.