Isometric Data Sequences and Data-Modulation Schemes in Fading Channels

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Abstract—In multiplicative fading channels, joint channel estimation and data detection (CE/DD) schemes cannot differentiate among certain sequences of amplitude- and/or phase-modulated (AM/PM) symbols drawn from rotationally invariant signal constellations. This paper identifies these so-called isometric sequences as the main source of performance degradation, and introduces a unifying framework that effectively solves the problem by using asymmetric signal constellations (ASC) and a normalized innovations-based detector. The encompassing nature of the solution is clearly demonstrated by showing that seemingly unrelated previous results, such as training-based solutions, can be viewed as special cases of the modulation-based solution discussed here. A comprehensive analysis, supported by simulation studies, of the relationships among modulation schemes, isometry, and detection performance is provided. Results indicate that the proposed ASC solution offers excellent performance without incurring significant complexity or reducing the transmission rate. Furthermore, it is shown to be robust in various fading rates, and for different signal constellations.

Index Terms—Asymmetric modulation, innovations, isometry, Kalman filtering, maximum-likelihood detection.

I. INTRODUCTION

IN A NARROWBAND mobile communication system, the transmitted amplitude- and/or phase-modulation (AM/PM) signal is distorted by multiplicative fading and corrupted by additive white Gaussian noise (AWGN) [1]–[3]. Joint channel estimation and data detection (CE/DD), often used in fading channels to detect data sequences, suffers from an irreducible detection error floor and a large estimation error, especially in deep fading [1], [4]. Deep-fading reversal phenomenon refers to the introduction of an unknown phase rotation to a contiguous block of symbols due to a detection error [1]. Differential-phase encoding and decoding (DED), used in [1] and [5] to combat the deep-fading reversal phenomenon, reduces, but does not eliminate, the error floor. As identified in this paper, severe detection and estimation errors result from the inability to uniquely detect the data sequence, because sequences of AM/PM symbols drawn from rotationally invariant signal constellations, which are phase-rotated versions of each other, are *isometric* and cannot be differentiated [5], [6]. Reversal

phenomenon is, in fact, a special case of isometry when the beginning and the end of the isometric sequences are unknown.

Previously suggested techniques, such as training symbols [4], which reduces transmission rate, and extra channel knowledge [7], which incurs significant complexity, can be used to improve performance. They attempt to resolve phase ambiguity and isometry by employing *a priori* knowledge of the transmitted sequence and the fading process, respectively. Since isometry is a modulation-induced phase ambiguity, it is natural to consider a modulation-based solution. Asymmetric signal constellation (ASC), previously used for phase estimation in AWGN channels [8], [9], is shown to be a novel, bandwidth-efficient, and robust modulation-based solution to isometry and phase ambiguity in CE/DD by ensuring that no sequence is the phase-rotated version of another.

The main contributions of this paper are: 1) identification of modulation as the cause for isometry, and isometry as the cause for the irreducible detection error floor and severe estimation error in CE/DD schemes; 2) introduction of a unifying framework for the use of ASC to combat isometry. Other solutions are shown to be special cases of the ASC solution; and 3) derivation of normalized innovations-based maximum *a posteriori* (MAP) and maximum-likelihood (ML) estimation-assisted data detectors for fading channels. The detector performance is shown to be related to modulation (thus isometry), past detected symbols, channel model parameters, and estimation performance.

This paper is organized as follows. Section II presents the CE/DD system model and discusses the properties of a signal constellation related to isometry. The cause and effect of isometry and the limitation of DED as an isometry solution are discussed in Section III. Section IV discusses the use of ASC as a solution to isometry, provides a procedure to design ASC, and discusses how the use of training symbols and extra channel knowledge to break isometry can be seen as special cases of the modulation-based solution. In Section V, simulation results for various isometry solutions in slow flat-fading channels verify the analysis and identify that the use of ASC is an effective and robust solution to isometry. Conclusions are drawn in Section VI.

II. SYSTEM MODEL FOR AMPLITUDE AND PHASE-SHIFT KEYING (PSK) MODULATION

In the sequel, *real* variables are in italics; complex variables are in regular font; **vectors** are in bold lowercase; **MATRICES** are in bold uppercase; x^* , $x^\top$, and x^H denote the complex conjugate, transpose, and Hermitian transpose of x , respectively;

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Fig. 1. CE/DD system in a multiplicative fading channel with additive noise for an AM/PM modulation scheme.

$\mathbf{I}_k$ is the identity matrix of dimension k ; $\mathbf{x} \sim N(\mu, \sigma^2)$ specifies that $\mathbf{x}$ is a complex circularly symmetric Gaussian random variable with mean μ and variance σ^2 ; $E\{\mathbf{x}\}$ is the expectation of the random variable $\mathbf{x}$; $\hat{\mathbf{x}}(k|k)$ and $\hat{\mathbf{x}}(k|k-1)$ denote the minimum mean-square estimation and prediction of $\mathbf{x}$ at time k , respectively; $\|D\|$ is the cardinality of the set D ; and $A \times B$ denotes the Cartesian product of the sets A and B . $D(k)$ specifies the signal constellation at time instance k (e.g., $D(a)$ is the constellation at $k = a$); D_{TYPE} specifies the constellation of the type “TYPE” (e.g., D_{QPSK} specifies a quaternary phase-shift keying (QPSK) constellation); if neither time nor type is specified, then D is used instead.

The system considered in this paper is shown in Fig. 1. For slow flat Rayleigh fading, matched filtering and symbol-rate sampling are performed to obtain sufficient statistics, with the overall discrete system model described as follows [10]–[12]:

$$y(k) = c(k)d(k) + v(k) \quad (1)$$

where $y(k)$ is the observation at time $t = kT$, T is the symbol duration, $c(k)$ is the fading coefficient, $d(k) \in D(k)$ is the independent transmitted symbol, and $v(k) \sim N(0, R)$ is the AWGN. For convenience, it is assumed that $E\{|c(k)|^2\} = 1$ and $E\{|d(k)|^2\} = 1$, with the signal-to-noise ratio (SNR) defined as R^{-1} . Channel estimation is required for coherent detection of the transmitted symbols in fading channels; hence, the CE/DD structure in Fig. 1 is employed. Due to the slow flat-fading assumption, the discrete system model in (1) is assumed free of intersymbol interference (ISI), and symbol detection is performed instead of sequence detection.

A. Signal Constellations for AM/PM

The AM/PM signal constellation $D(k)$, from which $d(k)$ in (1) is drawn, is a set of signal points in the two-dimensional complex plane. Since many practical constellations, such as 64 multiring differential phase-shift keying (64-MR-DPSK) and quadrature amplitude modulation (QAM) (Fig. 2), can be viewed as aggregations of a PSK signal, M -PSK, defined as $D_{M\text{-PSK}} = \{e^{j2\pi k/M}, k = 0, \dots, M-1\}$, is the main focus in this paper [13], [14].

The amplitude properties of a signal constellation include average symbol power, peak symbol power, and direct current (DC) component. The average symbol power, which changes the SNR, of all signal constellations in this research is unity. The peak symbol power dictates the linear range of the amplifier, and it is one for M -PSK, resulting in the lowest peak power

and the least expensive linear amplifier. A nonzero DC component acts as a pilot tone for synchronization but increases power consumption; M -PSK has zero DC component and is, hence, power efficient [8], [9].

DED can be used to change the phase property of a sequence of M -PSK symbols by putting the information in the phase differences instead of absolute phases. Differential phase encoding is defined as $d(k) = d(k-1)a(k)$, where $d(k)$ is the encoded symbol, $d(0) = 1$, and $a(k)$ is the uncoded symbol. Differential phase decoding is defined as $a(k) = d(k)d^*(k-1)$.

In addition, a signal constellation can be phase rotated. Let $\phi \in D$ be an independent random variable with probability mass function $P(\phi)$; then D is said to be *rotationally invariant* with respect to $0 < \vartheta < 2\pi$ if [8], [9]

$$\forall g(\cdot) \{ \exists (0 < \vartheta < 2\pi) \{ E\{g(\phi)\} = E\{e^{j\vartheta}g(\phi)\} \} \}. \quad (2)$$

Also, if $g(\phi) = \phi$, then $E\{\phi\} = E\{e^{j\vartheta}\phi\} = e^{j\vartheta}E\{\phi\}$. Hence, $E\{\phi\}$ must be zero, and rotationally invariant constellations have a zero-average DC component.

If D is uniformly distributed, i.e., $P(\phi) = (1/\|D\|)$, then condition (2) simplifies to

$$\begin{aligned} \exists (0 < \vartheta < 2\pi) \{ \forall \phi \{ (\phi \in D) \rightarrow (e^{j\vartheta}\phi \in D) \} \} \\ \text{or } \exists (0 < \vartheta < 2\pi) \{ e^{j\vartheta}D = D \}. \end{aligned} \quad (3)$$

Clearly, a uniformly distributed rotationally invariant constellation is a special type of geometrically uniform signal set, where the isometry transform is $u(D) = e^{j\vartheta}D$ [6]. The degree of rotational invariance r_D is defined as one plus the number of distinct ϑ 's that satisfy (2)–(3). It can be easily shown that $D_{M\text{-PSK}} = e^{j(2\pi k/M)}D_{M\text{-PSK}}, \forall k = 1, \dots, M-1$. Therefore, assuming that the symbols are equiprobable, M -PSK is rotationally invariant with degree M .

The definition of rotational invariance in (3) can be extended to an ordered set of uniformly distributed signal constellations. Let $d(k) \in D(k)$ with $P(d(k)) = (1/(\|D(k)\|))$, $\mathbf{d}_1^L = [d(1), \dots, d(L)]$ for $L \geq 1$. The ordered set of signal constellations $\Delta = D(1) \times \dots \times D(L)$ is rotationally invariant with respect to $0 < \vartheta < 2\pi$, if

$$\begin{aligned} \exists (0 < \vartheta < 2\pi) \{ \forall \mathbf{d}_1^L \{ (\mathbf{d}_1^L \in \Delta) \rightarrow (e^{j\vartheta}\mathbf{d}_1^L \in \Delta) \} \} \\ \text{or } \exists (0 < \vartheta < 2\pi) \{ e^{j\vartheta}\Delta = \Delta \} \end{aligned} \quad (4)$$

The degree of rotational invariance r_Δ is one plus the number of distinct ϑ 's that satisfy (4).

Given some phase rotation $0 < \vartheta < 2\pi$, it is observed from (1) that

$$\begin{aligned} \forall (1 \leq k \leq L) \{ y(k) &= c(k)d(k) + v(k) \\ &= [e^{-j\vartheta}c(k)][e^{j\vartheta}d(k)] + v(k) \}. \end{aligned}$$

Therefore, if Δ is rotationally invariant with respect to ϑ , then the observation in the multiplicative fading channel with AWGN is not uniquely associated with a fading random process and a sequence of transmitted symbols. In fact, the observation is associated with r_Δ sets of fading process and symbol sequence.

Fig. 2. AM/PM modulation signal constellations.

B. Channel Estimation

Though various filters can be used to estimate the fading coefficients in (1), the Kalman filter (KF) is used in this research due to its simplicity and demonstrated good performance in decision-directed channel estimation [2], [15], [16]. The KF assumes a state-space model, and thus, requires a rational approximation to the Rayleigh fading power spectral density (PSD) [1], [17]. Among the many state-space models discussed in [2], the autoregressive second-order (AR-2) model is used, due to its reasonable complexity and good performance [1], [2].

Using the AR-2 rational approximation to the Rayleigh PSD, the state-space model for (1) is

$$\mathbf{x}(k) = \mathbf{A}\mathbf{x}(k-1) + \mathbf{G}w(k) \quad (5)$$

$$y(k) = d(k)\mathbf{F}\mathbf{x}(k) + v(k) \quad (6)$$

where

$$\mathbf{x}(k) = \begin{bmatrix} c(k) \\ c(k-1) \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{F} = [1 \quad 0], \quad \mathbf{G}w(k) \sim N(0, \mathbf{Q})$$

$$\mathbf{Q} = \mathbf{G}\beta\mathbf{G}^H, \quad v(k) \sim N(0, R), \quad w(k) \sim N(0, \beta)$$

is the white Gaussian driving noise, and $a_1 = -2r_d \cos((2\pi f_D T)/(\sqrt{2}))$, $a_2 = r_d^2$, $\beta = ((1+a_2)^2 - a_1^2)/(1+a_2)$ and $r_d = 1 - 0.2\pi f_D T$ are chosen according to the normalized fading rate $f_D T$ of the channel to best approximate the fading characteristics [1], [2].

Given the model in (5)–(6), the channel state can be estimated using the KF recursion, $\mathcal{KF}(\cdot)$, as follows (Appendix I):

$$[\hat{\mathbf{x}}(k|k), \mathbf{P}(k|k)] = \mathcal{KF}(\hat{\mathbf{x}}(k-1|k-1), \mathbf{P}(k-1|k-1), d(k), y(k)). \quad (7)$$

The initial condition and its corresponding covariance are $\hat{\mathbf{x}}(0|0)$ and $\hat{\mathbf{P}}(0|0)$. In the absence of any prior information, they are assumed to match the statistical properties of the

fading process, i.e., $\hat{\mathbf{x}}(0|0) = [0, 0]^T$ and $\mathbf{P}(0|0) = \mathbf{I}_2$ for the Rayleigh channel [17].

Since the KF recursively estimates the channel states by propagating the mean and covariance according to (7), it preserves rotational invariance and introduces a counter phase rotation in the channel estimate in response to the phase rotation in the symbol, as follows.¹

Lemma 1: For the Kalman recursion in (7), given any phase rotation $0 < \vartheta < 2\pi$

$$[e^{-j\vartheta}\hat{\mathbf{x}}(k|k), \mathbf{P}(k|k)] = \mathcal{KF}(e^{-j\vartheta}\hat{\mathbf{x}}(k-1|k-1), \mathbf{P}(k-1|k-1), e^{j\vartheta}d(k), y(k)). \quad (8)$$

It should be noted that other channel estimators behave similarly.

C. Symbol Detection

Given the channel prediction $\hat{\mathbf{x}}(k|k-1)$, the observation $y(k)$ in (1) can be rewritten as

$$y(k) = d(k)\mathbf{F}(\hat{\mathbf{x}}(k|k-1) + \tilde{\mathbf{x}}(k|k-1)) + v(k) \quad (9)$$

where $\tilde{\mathbf{x}}(k|k-1) = \mathbf{x}(k) - \hat{\mathbf{x}}(k|k-1)$ is the prediction error. The ML symbol detector $\hat{d}(k)$ in the model (5)–(6) is given as follows (Appendix II):

$$\hat{d}(k) = \arg \min_{\phi \in \mathcal{D}} \left\{ \frac{|y(k) - \phi \mathbf{F} \hat{\mathbf{x}}(k|k-1)|^2}{\phi \mathbf{F} \mathbf{P}(k|k-1) (\phi \mathbf{F})^H + R} \right\}$$

$$= \arg \min_{\phi \in \mathcal{D}} \left\{ \frac{|\tilde{z}(k|k-1)_\phi|^2}{P_{\tilde{z}}(k|k-1)_\phi} \right\}. \quad (10)$$

The innovation $\tilde{z}(k|k-1)_\phi$ and the innovation covariance $P_{\tilde{z}}(k|k-1)_\phi$ are calculated via the KF recursion (17)–(23) in Appendix I [16]. The detector in (10), hereafter called *normalized innovations-based detector*, can also be derived using various predictors, estimators, or smoothers. Normalized

¹The proofs for all the lemmas, propositions, and corollaries are found in Appendix III.

innovations-based detectors preserve rotational invariance, and introduce a counter phase rotation in the detected symbol in response to the phase rotation in the channel estimate, as follows.

Lemma 2: Given the observation $y(k)$, the pair of detected symbol and channel estimate $(\hat{d}(k), \hat{\mathbf{x}}(k-1|k-1))$ and its phase-rotated counterpart $(e^{j\vartheta}\hat{d}(k), e^{-j\vartheta}\hat{\mathbf{x}}(k-1|k-1))$ both minimize (10).

Since the discrete system model in (1) is assumed to be ISI free, and the normalized innovations in (10) is an additive white process [17], the path metric for the detected sequence $\hat{\mathbf{d}}_1^L = [\hat{d}(1), \dots, \hat{d}(L)]$ is simply the sum of branch metrics of each detected symbol, $\tilde{p}(\hat{d}(k))$

$$\tilde{p}(\hat{\mathbf{d}}_1^L) = \sum_{k=1}^L \tilde{p}(\hat{d}(k)) = \sum_{k=1}^L \dots \frac{|y(k) - \hat{d}(k)\mathbf{F}\mathbf{A}\hat{\mathbf{x}}(k-1|k-1)|^2}{\hat{d}(k)\mathbf{F}[\mathbf{A}\mathbf{P}(k-1|k-1)\mathbf{A}^H + \mathbf{Q}]\mathbf{F}^H\hat{\mathbf{d}}^*(k) + R}. \quad (11)$$

For fast or frequency-selective fading channels, the state-space model in (5)–(6) can be modified to include the ISI induced by channel or pulse shaping, and the normalized innovations detector and the path metric can be derived similarly, based on the new model [4], [18].

It can be easily shown that the probability of detection error of (10) for a two-dimensional signal constellation (e.g., QPSK or QAM),² assuming perfect detection of $\mathbf{d}_1^{k-1}$, is

$$P_e \approx K \cdot \mathcal{Q}\left(\frac{|\mathbf{F}\hat{\mathbf{x}}(k|k-1)|d_{\min}}{2\sqrt{P_z(k|k-1)}}\right) \quad (12)$$

where K is the average number of nearest neighbors, $d_{\min}$ is the minimum distance between adjacent signal points of the constellation, and $\mathcal{Q}(x) = \int_x^\infty (e^{-t^2/2})/(\sqrt{2\pi})dt$, $x \geq 0$.

From (12), it is observed that the detection performance is governed, either explicitly or implicitly, by: 1) the modulation ($d_{\min}$); 2) the assumed fading-channel model (elements of $P_z(k|k-1)$); 3) the channel estimate ($\hat{\mathbf{x}}(k|k-1)$); and 4) the detection of past symbols, namely, the assumption that all past symbols are detected correctly.

The symbol-error rate (SER) in (12) is calculated assuming the perfect detection of all past symbols. Assuming that $\mathbf{d}_1^{k-2}$ is detected correctly, but $\hat{d}(k-1)$ is detected erroneously, an

²For one-dimensional signal constellations, such as BPSK, $P_e \approx K \mathcal{Q}((|\mathbf{F}\hat{\mathbf{x}}(k|k-1)|d_{\min})/\sqrt{2}\sqrt{P_z(k|k-1)})$.

additional disturbance n_d is introduced to (9), affecting the detection of $\hat{d}(k)$ [19]

$$y(k) = \hat{d}(k)\mathbf{F}(\hat{\mathbf{x}}(k|k-1) + \tilde{\mathbf{x}}(k|k-1)) + v(k) + n_d.$$

The error propagation effect n_d can be approximated by a Gaussian process with zero mean and variance $E\{|\hat{d}(k-1) - \hat{d}(k-1)|^2\}$. An optimistic but reasonable assumption when an error occurs is that $\hat{d}(k-1)$ is an adjacent symbol of $\hat{d}(k-1)$; thus, $E\{|n_d|^2\} = d_{\min}^2$. Therefore

$$P_z(k|k-1) = E\{|\hat{d}(k)\mathbf{F}\tilde{\mathbf{x}}(k|k-1) + v(k) + n_d|^2\} \\ = \hat{d}(k)\mathbf{F}\mathbf{P}(k|k-1)(\hat{d}(k)\mathbf{F})^H + R + d_{\min}^2.$$

So, a past detection error increases $P_z(k|k-1)$ and degrades the current detection performance through (12), and the amount of degradation is affected by the modulation.

III. ISOMETRY FOR AMPLITUDE AND PSK MODULATION

For CE/DD, the KF operates in decision-directed mode, so the actual symbols $\mathbf{d}(k)$ in (7) are replaced by the detected symbols $\hat{\mathbf{d}}(k)$. *Proposition 3* shows that the coupled channel estimation and symbol-detection system in (7) and (10) (Fig. 1) propagates rotational invariance recursively, and generates detected data sequences and their corresponding channel-estimate sequences with phase ambiguity.

Proposition 3: Let $\hat{\mathbf{d}}_1^L = [\hat{d}(1), \dots, \hat{d}(L)]$ and $\hat{\mathbf{x}}_1^L = [\hat{\mathbf{x}}(1|1), \dots, \hat{\mathbf{x}}(L|L)]$. For the CE/DD scheme in (7) and (10), given the observation sequence $\mathbf{y}_1^L = [y(1), \dots, y(L)]$, the set of detected symbol sequence, channel-estimate sequence, and initial condition $(\hat{\mathbf{d}}_1^L, \hat{\mathbf{x}}_1^L, \hat{\mathbf{x}}(0|0))$ and its phase-rotated counterpart $(e^{j\vartheta}\hat{\mathbf{d}}_1^L, e^{-j\vartheta}\hat{\mathbf{x}}_1^L, e^{-j\vartheta}\hat{\mathbf{x}}(0|0))$ both minimize (10).

Corollary 4: According to the CE/DD system in (7) and (10), given the observation sequence $\mathbf{y}_1^L$ and the initial condition matching the statistical characteristics of the fading process $\hat{\mathbf{x}}(0|0) = [0, 0]^T$, $\hat{\mathbf{d}}_1^L$ and $e^{j\vartheta}\hat{\mathbf{d}}_1^L$ are both equally likely detected symbol sequences, and cannot be differentiated.

From *Corollary 4*, it is shown in (13) at the bottom of the page that $\hat{\mathbf{d}}_1^L$ and $e^{j\vartheta}\hat{\mathbf{d}}_1^L$ are *isometric* as defined in (11) [5], [6]. Isometry causes an irreducible error floor and increases the estimation error. Let us assume that $\mathbf{d}_1^L$ is the sequence of transmitted symbols, and all L transmitted symbols are detected correctly, except for an unknown phase ambiguity $0 \leq \vartheta < 2\pi$, i.e., $\hat{\mathbf{d}}_1^L = e^{j\vartheta}\mathbf{d}_1^L$. It is obvious that $\forall (1 \leq k \leq L)\{\hat{d}(k) \neq e^{j\vartheta}d(k)\}$. When the correct isometric sequence is selected ($\vartheta = 0$), i.e., $\hat{\mathbf{d}}_1^L = \mathbf{d}_1^L$, then all L symbols are detected correctly. Selection of another isometric sequence results in L

$$\begin{aligned} \tilde{p}(\hat{\mathbf{d}}_1^L) &= \sum_{k=1}^L \frac{|y(k) - \hat{d}(k)\mathbf{F}\mathbf{A}\hat{\mathbf{x}}(k-1|k-1)|^2}{\hat{d}(k)\mathbf{F}[\mathbf{A}\mathbf{P}(k-1|k-1)\mathbf{A}^H + \mathbf{Q}]\mathbf{F}^H\hat{\mathbf{d}}^*(k) + R} \\ &= \sum_{k=1}^L \frac{|y(k) - e^{j\vartheta}\hat{d}(k)\mathbf{F}\mathbf{A}e^{-j\vartheta}\hat{\mathbf{x}}(k-1|k-1)|^2}{e^{j\vartheta}\hat{d}(k)\mathbf{F}[\mathbf{A}\mathbf{P}(k-1|k-1)\mathbf{A}^H + \mathbf{Q}]\mathbf{F}^He^{-j\vartheta}\hat{\mathbf{d}}^*(k) + R} \\ &= \tilde{p}(e^{j\vartheta}\hat{\mathbf{d}}_1^L) \end{aligned} \quad (13)$$

Fig. 3. Examples of asymmetric signal constellations used to break isometry.

erroneous detected symbols and 100% SER. For the rotationally invariant ordered set of signal constellations Δ , since all r_Δ isometric sequences are equally likely to be selected (*Corollary 4*), the probability of selecting an erroneous isometric sequence is $(1 - (1/r_\Delta))$, resulting in an irreducible symbol error floor of $F_e = (1 - (1/r_\Delta))$.

Employing DED, the decoded sequence of $e^{j\vartheta} \hat{\mathbf{d}}_1^L$ is $[e^{j\vartheta} a(1), a(2), \dots, a(L)]$, resulting in the first symbol being erroneous and a $(1/L)$ SER. Therefore, with DED, the irreducible symbol-error floor is reduced from $F_e = (1 - (1/r_\Delta))$ to $F_e = (1/L)(1 - (1/r_\Delta))$ [5]. However, as the parameter L is generally controlled by the system standard ($L \leq 162$ for IS-136), DED cannot eliminate the isometry-induced error floor.

Let us assume that the transmitted symbols are $\mathbf{d}_1^L$, and the actual channel states are $\mathbf{x}_1^L = [\mathbf{x}(1), \dots, \mathbf{x}(L)]$. From *Proposition 3*, the channel estimate corresponding to the correct detected sequence, i.e., when $\hat{\mathbf{d}}_1^L = \mathbf{d}_1^L$, is $\hat{\mathbf{x}}_1^L \approx \mathbf{x}_1^L$, while the channel estimate corresponding to $\hat{\mathbf{d}}_1^L = e^{j\vartheta} \mathbf{d}_1^L$ is $e^{-j\vartheta} \hat{\mathbf{x}}_1^L$. Since

$$\sum_{k=1}^L |\mathbf{F}\mathbf{x}(k) - \mathbf{F}\hat{\mathbf{x}}(k|k)|^2 \ll \sum_{k=1}^L |\mathbf{F}\mathbf{x}(k) - \mathbf{F}e^{-j\vartheta} \hat{\mathbf{x}}(k|k)|^2$$

isometry also significantly increases estimation error.

IV. SOLUTIONS FOR ISOMETRY IN AMPLITUDE AND PSK MODULATION

A. Asymmetric Signal Constellation (ASC)

Section III identifies isometry as the main cause for performance degradation of CE/DD systems employing modulation schemes with rotationally invariant signal constellations in

fading channels. The solutions to isometry are now discussed. Since isometry is induced by modulation, it is natural to consider a modulation-based solution.

Let the sequence of symbols $\mathbf{d}_1^L$ be drawn from Δ , where $P(\mathbf{d}(k)) = (1/||D(k)||), \forall 1 \leq k \leq L$. Recall that if Δ is rotationally invariant with respect to $0 < \vartheta < 2\pi$, then $\mathbf{d}_1^L$ and $e^{j\vartheta} \mathbf{d}_1^L$ are isometric. Therefore, to combat isometry, such that $\forall(\mathbf{d}_1^L \in \Delta) \{ \exists(0 < \vartheta < 2\pi) \{ e^{j\vartheta} \mathbf{d}_1^L \in \Delta \} \}$, the following condition must be satisfied:

$$\exists(a, b: 1 \leq a < b \leq L) \{ \forall(\mathbf{d}_{a,b} \in \Delta_{a,b}) \{ \exists(0 < \vartheta < 2\pi) \{ e^{j\vartheta} \mathbf{d}_{a,b} \in \Delta_{a,b} \} \} \} \quad (14)$$

where $\mathbf{d}_{a,b} = [\mathbf{d}(a), \mathbf{d}(b)]$ (a subsequence of $\mathbf{d}_1^L$), and $\Delta_{a,b} = D(a) \times D(b)$ (a subset of Δ).

Since rotational invariance is a sufficient, but not a necessary, condition for isometry, only signal constellation pairs that satisfy the condition in (14) are guaranteed to break isometry. For example, let $D(a) = \{+1, +j, -1, -j\}$ and $D(b) = \{+1, e^{j(\pi/4)}, -1, e^{-j(\pi/4)}\}$. Although $\Delta_{a,b}$ is not rotationally invariant, the sequences $\{+1, +1\}$ and $\{-1, -1\}$ are still isometric.

Any constellation can be modified to an isometry-breaking ASC by displacing the signal points on the complex plane until (14) is satisfied (Fig. 3).

Now, suppose $D(a)$ and $D(b)$ are N -PSK and M -PSK signal constellations, then the phase difference between adjacent signal points are $(2\pi/N)$ and $(2\pi/M)$, respectively. To ensure that $\mathbf{d}_{a,b}$ and $e^{j\vartheta} \mathbf{d}_{a,b}$ are not isometric, $D(a)$ and $D(b)$ must meet the following condition:

$$\begin{aligned} \forall(0 < k < M) \{ e^{j\frac{2k\pi}{M}} D(a) \neq D(a) \} \\ \forall(0 < k < N) \{ e^{j\frac{2k\pi}{N}} D(b) \neq D(b) \}. \end{aligned} \quad (15)$$

This is satisfied if N and M have no common prime factor; unfortunately, in practical applications, both N and M are powers of two. Therefore, to break isometry, a new $\Delta_{a,b}$ should be introduced by modifying appropriately one of the signal constellations. Let us assume that $D(a)$ remains N -PSK; to satisfy (14), $D(b)$ must be modified to the ASC $\check{D}(b)$ as follows:

$$\forall(\phi \in \check{D}(b)) \left\{ \check{A}(0 < k < N) \left\{ e^{j\frac{2k\pi}{N}} \phi \in \check{D}(b) \right\} \right\}. \quad (16)$$

In general, $\check{D}(b)$ can be constructed from $D(b)$ (M -PSK) by arbitrarily moving the signal points on the unity circle (Fig. 3(a)–(d)) until (16) is met. A procedure to construct an asymmetric M -PSK, although not unique, is outlined below as *Algorithm 1*.

- 1) Divide the unity circle into $M \times M$ equal sectors.
- 2) Mark the intersections, which are the potential new signal points, counterclockwise around the unity circle sequentially by $1, \dots, M$.
- 3) Select one point from each set with the same number to obtain the new M signal points.

Fig. 3(a) and (b) illustrate how the asymmetric BPSK and asymmetric QPSK employed in this paper are constructed using *Algorithm 1*.

Different ASCs have different peak powers, DC components, and $d_{\min}$'s, as shown in Fig. 3. It is easy to see from Fig. 3 that $\check{D}(b)$ has smaller $d_{\min}$, hence, higher P_e , than $D(b)$ does. The performance degradation is not significant, since only one of the L symbols needs to be asymmetric. The degradation caused by the reduced $d_{\min}$ is further mitigated by placing the asymmetric symbol at the end of the block, thereby eliminating any potential error propagation due to the recursive nature of the CE/DD shown in *Proposition 3*.

Algorithm 1 constructs an asymmetric M -PSK by selecting a subset of M^2 -PSK signal points, so that the peak power remains unity and the linear range of the amplifier remains unchanged. Furthermore, symbol detectors for M^2 -PSK can also be used to detect the asymmetric symbol, reducing additional complexity in employing ASC. The nonzero DC component of ASC, which results in additional power consumption, can be seen as a pilot tone. Hence, the pilot-tone solution to isometry can be viewed as a special case of the ASC solution [8], [9]. Fortunately, since only one of the L symbols is asymmetric, the DC component averaged over L symbols is not significant. Also, if $\check{D}(b)$ is employed for the first block of L symbols, and $-\check{D}(b)$ is employed for the second block, then the overall average DC component becomes zero.

An example is now described to illustrate the use of ASC. Let 64-MR-DPSK symbols be transmitted in a channel with $f_D T = 0.00637$. Because the asymmetric 64-MR-DPSK has a very small $d_{\min}$, thus, a very high P_e , asymmetric QPSK is used instead. It is well known that the $P_z(k|k-1)$ decreases and converges asymptotically to the steady-state value as k increases [17]. Experiments show that two QPSK symbols should be placed at the beginning of the block, and the asymmetric QPSK should be placed at the end of the block to compensate for the larger transient $P_z(k|k-1)$ [12]. So, $D(1) = D(2) = D_{\text{QPSK}}$, $D(L) = D_{\text{Asymmetric QPSK}}$, and

$D(k) = D_{64\text{-MR-DPSK}}, \forall 2 < k < L$. The degree of rotational invariance of this ordered set of constellations is four; so, the receiver operates four KFs in parallel, each assuming one possible QPSK symbol as $\hat{d}(1)$. The subsequences of detected symbols $\hat{d}_1^{L-1}, j\hat{d}_1^{L-1}, -\hat{d}_1^{L-1}$, and $-j\hat{d}_1^{L-1}$ are isometric. The asymmetric symbol at $k = L$ breaks isometry, and the sequence of detected symbols with the least path metric is selected.

According to (2), the use of nonuniformly distributed signal constellations can also break isometry [8], [9], but it does not necessarily result in the selection of the correct sequence. For example, let us consider a sequence of two BPSK symbols with $P(d(1) = 1) = (1/2)$ and $P(d(2) = 1) = (1/2) + \epsilon$, where $\epsilon > 0$. The sequences $\{1, 1\}$ and $\{-1, -1\}$ are phase-rotated versions of each other, but $\{1, 1\}$ is always selected due to its higher *a priori* probability, even when $\{-1, -1\}$ is transmitted. Hence, although isometry is broken, the wrong isometric sequence may be selected. This scheme only works properly if $\epsilon = (1/2)$, i.e., $d(2) = 1$ is a known training symbol.

B. Pilot Symbol

From (12), the probability of correctly detecting the isometry-breaking symbol is strictly less than one, because of the nonzero $P_z(k|k-1)$. Although P_e may be reduced in many ways, it can never be made exactly zero unless the isometry-breaking symbol is known *a priori*. By *Proposition 3*, if $\hat{d}(1) = d(1)$ is known *a priori*, then the sequence of detected symbols $\hat{d}_1^L$ may be uniquely identified, thereby breaking isometry. Therefore, the pilot-symbol solution is, in fact, a special case, when $P_e = 0$, of the ASC solution to isometry.

Pilot symbols are inserted periodically in the information-bearing sequence, and the spacing between successive pilot symbols L depends on the normalized fading rate $f_D T$: $L \leq \lfloor (1/2 f_D T) \rfloor$ [4]. If Ψ pilot symbols are inserted periodically, then the effective symbol-transmission rate reduces from 100% to $(1 - 2\Psi f_D T)$. So, asymmetric symbols are more bandwidth efficient than pilot symbols, and they can replace pilot symbols in other phase-ambiguity-resolving applications, such as synchronization. For a higher data-rate constellation, more pilot symbols are needed to compensate for the smaller $d_{\min}$. The number of pilot symbols needed depends on $d_{\min}$. For the slow flat-fading Rayleigh channels in this paper, experiments show that one pilot symbol is needed for BPSK and QPSK, and two are needed for the other constellations in this paper.

C. Initial Condition

From *Proposition 3*, instead of $d(1)$, knowing $\mathbf{x}(0)$ can also break isometry. Therefore, for a KF-based CE/DD, the actual initial channel condition $\mathbf{x}(0)$ is the extra knowledge about the observation generation mechanism that can be used to break isometry [7]. When $\hat{\mathbf{x}}(0|0) = \mathbf{x}(0)$, by (5) and (17), $\hat{\mathbf{x}}(1|0) = \mathbf{A}\mathbf{x}(0) = \mathbf{x}(1) - \mathbf{G}\mathbf{w}(1) \approx \mathbf{x}(1)$, and $\mathbf{P}(1|0)_{\{\hat{\mathbf{x}}(0|0)=\mathbf{x}(0)\}} = E\{|\mathbf{F}(\mathbf{x}(1) - \mathbf{A}\mathbf{x}(0))|^2\} = \beta \ll E\{|\mathbf{F}(\mathbf{x}(1) - \mathbf{A}[0, 0]^T)|^2\} = \mathbf{P}(1|0)_{\{\hat{\mathbf{x}}(0|0)=[0,0]^T\}}$. So, given $\mathbf{P}(0|0)$ and the state-space model parameters, the use of $\mathbf{x}(0)$ not only improves the transient KF performance, but also enhances the probability of correctly detecting $d(1)$, thereby breaking isometry. Hence, the initial condition solution is a special case of the ASC solution.

TABLE I
SUMMARY OF SIMULATION PARAMETERS

Parameter	Value
center frequency	$f_c = 1.8 \times 10^9$ Hz (high tier IS-136)
symbol period	$T = 4.12 \times 10^{-6}$ s [20]
pulse shaping	square root raised cosine with 0.35 roll-off [20]
frame length	162 symbols [20]
Rayleigh fading model	Jakes model [1], [21]
normalized fading rates	$f_D T = 0.01, 0.00637, 0.00318 \leq 0.03$ (i.e. slow fading) [2], [3], [11]
vehicular speeds	145.8, 92.8, 46.4 km/h
# sub-blocks/frame	4, 3, 2 sub-blocks
Signal constellation	Grey coded DPSK, 64-MR-DPSK, QAM [13]
ASC	Asymmetric QPSK (Figure 3, example B)
# Monte Carlo runs	1000

Fig. 4. SERs, prior to differential-phase decoding, of various solutions to isometry for DQPSK modulation at $f_D T = 0.00637$.

However, $\mathbf{x}(0)$ is not readily available, and extensive channel identification preprocessing is required to obtain knowledge of the initial condition.

In conclusion, the initial condition and the pilot-symbol solutions can be seen as duals: initial condition breaks isometry in the channel estimation domain, and the pilot symbol breaks isometry in the symbol detection domain. Both can be seen as special cases of the ASC solution.

V. RESULTS

In this section, experimental results are used to verify the design and analysis of the proposed isometry-breaking framework. The KF-based CE/DD scheme (Fig. 1) in an IS-136 environment is considered. The experiment parameters are listed in Table I.

In the absence of isometry-breaking solutions, Figs. 4–6 show that isometry causes an irreducible error floor [(SER) = 0.75, bit-error rate (BER) = 2×10^{-3}], and severe channel estimation mean-square error (MSE) = 2), even with DED. The isometry-breaking solutions improve both detection and estimation performances significantly (at 50 dB, SER is reduced by three orders of magnitude, BER is reduced by two orders of magnitude, and MSE is reduced by four orders of magnitude). Hence, ASC is an effective solution to isometry.

Fig. 7 verifies that a faster normalized fading rate results in a higher estimation MSE, and hence, higher BER (at 50 dB, BER

Fig. 5. BERs, after differential-phase decoding, of various solutions to isometry for DQPSK modulation at $f_D T = 0.00637$.

Fig. 6. Channel estimation MSE of various solutions to isometry for DQPSK modulation at $f_D T = 0.00637$.

Fig. 7. BERs of solutions to isometry for DQPSK modulation at various normalized fading rates.

increases by an order of magnitude as $f_D T$ increases by an order of magnitude) [2]. For fading rates where the discrete-system model in (1) remains valid, the ASC solution performs as well

Fig. 8. BERs of solutions to isometry for 64-MR-DPSK at $f_D T = 0.00637$ for various configurations. For the pilot-symbol solution, the configurations are: 1) one pilot symbol per subblock; 2) two pilot symbols per subblock; and 3) three pilot symbols per subblock. For the asymmetric signal constellation solution, the configurations are: 1) one leading QPSK symbol per subblock; and 2) two leading QPSK symbols per subblock. For $f_D T = 0.00637$, each frame of 162 symbols is segmented into three subblocks.

as the pilot-symbol solution [11]. Hence, the ASC solution is a robust solution to isometry for various fading rates.

Section IV-A states that QPSK symbols replaces 64-MR-DPSK symbols at the beginning of each subblock to compensate for poorer transient $P_z(k|k-1)$, and Section IV-B states that more than one pilot symbol may be needed for constellations with small $d_{\min}$'s. Fig. 8 shows that two leading QPSK symbols are needed for the ASC solutions, and two symbols are needed for the pilot-symbol solution for $f_D T = 0.00637$ (the BER reduces by almost two orders of magnitude when the number of pilot or leading QPSK symbols increases from one to two).

ASC is shown in Fig. 9 to be an effective solution to isometry for various rotationally invariant signal constellations (the BER performance is comparable with that of the pilot-symbol solution). The QPSK and asymmetric QPSK constellation pair is used for the ASC solution (Section IV-A). In addition, two trends are observed: 1) constellations with lower data rates, thus higher $d_{\min}$'s, perform better (Section II-C); and 2) DPSK performs better than QAM because DED can mitigate the reversal phenomenon and enhance detection performance [1], [5].

VI. CONCLUSION

The rotational invariance of commonly used AM/PM signal constellations causes phase ambiguity and isometry in CE/DD system in multiplicative Rayleigh fading channels. Isometry causes severe detection and estimation errors, which can be partially mitigated by DED. However, DED does not eliminate isometry. Three solutions to isometry have been analyzed in this paper; ASC, the pilot symbol, and the initial condition. The latter two are shown to be special cases of the ASC solution, so ASC can be used in applications, such as synchronization, where pilot symbols are used. The mechanism and performances of the three solutions have been discussed, and a procedure for designing asymmetric M -PSK has been

Fig. 9. BERs of solutions to isometry for various rotationally invariant signal constellations at $f_D T = 0.00637$.

outlined. The novel modulation-based ASC solution has been shown both analytically and experimentally to be an effective and robust solution to modulation-induced isometry without incurring significant complexity or reducing transmission symbol rate.

APPENDIX I KF RECURSION ALGORITHM

$$\hat{\mathbf{x}}(k|k-1) = \mathbf{A}\hat{\mathbf{x}}(k-1|k-1) \quad (17)$$

$$\mathbf{P}(k|k-1) = \mathbf{A}\mathbf{P}(k-1|k-1)\mathbf{A}^H + \mathbf{Q} \quad (18)$$

$$\hat{\mathbf{z}}(k|k-1) = \mathbf{y}(k) - \mathbf{d}(k)\mathbf{F}\hat{\mathbf{x}}(k|k-1) \quad (19)$$

$$\mathbf{P}_{\hat{\mathbf{z}}}(k|k-1) = \mathbf{d}(k)\mathbf{F}\mathbf{P}(k|k-1)(\mathbf{d}(k)\mathbf{F})^H + \mathbf{R} \quad (20)$$

$$\mathbf{K}(k) = \mathbf{P}(k|k-1)(\mathbf{d}(k)\mathbf{F})^H \mathbf{P}_{\hat{\mathbf{z}}}^{-1}(k|k-1) \quad (21)$$

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)\hat{\mathbf{z}}(k|k-1) \quad (22)$$

$$\mathbf{P}(k|k) = [\mathbf{I} - \mathbf{K}(k)\mathbf{d}(k)\mathbf{F}]\mathbf{P}(k|k-1) \quad (23)$$

APPENDIX II DERIVATION OF THE ML SYMBOL DETECTOR

Given the past observation record $\mathbf{y}_1^{k-1} = [y(1), \dots, y(k-1)]$ and the past transmitted symbols $\mathbf{d}_1^{k-1} = [d(1), \dots, d(k-1)]$

1)] (i.e., all the past symbols are assumed to be detected perfectly), $y(k)$ in (9) is Gaussian distributed with the following conditional mean and variance [16]:

$$\begin{aligned} E\{y(k) | \mathbf{y}_1^{k-1}, d(k), \mathbf{d}_1^{k-1}\} &= d(k)\mathbf{F}\hat{\mathbf{x}}(k|k-1), \\ E\{|y(k) - d(k)\mathbf{F}\hat{\mathbf{x}}(k|k-1)|^2 | \mathbf{y}_1^{k-1}, d(k), \mathbf{d}_1^{k-1}\} &= d(k)\mathbf{F}\mathbf{P}(k|k-1)(d(k)\mathbf{F})^H + R. \end{aligned}$$

Given the current observation $y(k)$, the *a posteriori* probability of the independent random variable $d(k) \in D$ is $P(d(k) | y(k), \mathbf{y}_1^{k-1}, \mathbf{d}_1^{k-1}) = ([P(y(k) | \mathbf{y}_1^{k-1}, d(k), \mathbf{d}_1^{k-1}) P(d(k))] / P(\mathbf{y}_1^{k-1}, \mathbf{d}_1^{k-1})) / (P(y(k), \mathbf{y}_1^{k-1}, \mathbf{d}_1^{k-1}))$ [16]. Therefore, given $y(k)$, $\mathbf{y}_1^{k-1}$, and $\mathbf{d}_1^{k-1}$, the MAP symbol detector is

$$\begin{aligned} \hat{d}(k) &= \arg \max_{\phi \in D} \{P(y(k) | \mathbf{y}_1^{k-1}, d(k), \mathbf{d}_1^{k-1}) P(d(k))\} \\ &= \arg \max_{\phi \in D} \left\{ \frac{\exp\left(-\frac{|y(k) - \phi\mathbf{F}\hat{\mathbf{x}}(k|k-1)|^2}{2(\phi\mathbf{F}\mathbf{P}(k|k-1)(\phi\mathbf{F})^H + R)}\right)}{2\pi\sqrt{\phi\mathbf{F}\mathbf{P}(k|k-1)(\phi\mathbf{F})^H + R}} P(\phi) \right\} \end{aligned}$$

reducing to the ML symbol detector in (10) when $P(\phi) = (1/|D|)$.

APPENDIX III

PROOFS FOR LEMMAS, PROPOSITIONS, AND COROLLARIES

Proof [Lemma 1]: Equation (8) can be easily proved by substituting $e^{j\vartheta}d(k)$ and $e^{-j\vartheta}\hat{\mathbf{x}}(k-1|k-1)$ for $d(k)$ and $\hat{\mathbf{x}}(k-1|k-1)$, respectively, in (17)–(23). ■

Proof [Lemma 2]: From (10), it is observed that

$$\begin{aligned} \frac{|y(k) - \hat{d}(k)\mathbf{F}\hat{\mathbf{x}}(k|k-1)|^2}{\hat{d}(k)\mathbf{F}\mathbf{P}(k|k-1)(\hat{d}(k)\mathbf{F})^H + R} &= \frac{|y(k) - [e^{j\vartheta}\hat{d}(k)]\mathbf{F}[e^{-j\vartheta}\hat{\mathbf{x}}(k|k-1)]|^2}{e^{j\vartheta}\hat{d}(k)\mathbf{F}\mathbf{P}(k|k-1)(e^{j\vartheta}\hat{d}(k)\mathbf{F})^H + R}. \end{aligned}$$

Thus, if

$$\hat{d}(k) = \arg \min_{\phi \in D} \left\{ \frac{|y(k) - \phi\mathbf{F}\hat{\mathbf{x}}(k|k-1)|^2}{(\phi\mathbf{F}\mathbf{P}(k|k-1)(\phi\mathbf{F})^H + R)} \right\}$$

then

$$e^{j\vartheta}\hat{d}(k) = \arg \min_{\phi \in D} \left\{ \frac{|y(k) - \phi\mathbf{F}[e^{-j\vartheta}\hat{\mathbf{x}}(k|k-1)]|^2}{(\phi\mathbf{F}\mathbf{P}(k|k-1)(\phi\mathbf{F})^H + R)} \right\}. \quad \blacksquare$$

Proof [Proposition 3]: For $L = 1$, Proposition 3 is reduced to Lemma 2 for $k = 1$, and it has already been proven. Assume that Proposition 3 is true for $L = n$. For $L = n + 1$, it is given that for $y(n+1)$ and $\hat{\mathbf{x}}(n|n)$, $\hat{d}(n+1)$ is the detected symbol, and $\hat{\mathbf{x}}(n+1|n+1)$ is the channel-state estimate. By Lemma 2, given $y(n+1)$ and $e^{-j\vartheta}\hat{\mathbf{x}}(n|n)$, $e^{-j\vartheta}d(n+1)$ is the detected symbol. Then, by (8), $e^{-j\vartheta}\hat{\mathbf{x}}(n+1|n+1)$ is the channel-state estimate. So, by mathematical induction, given

$\mathbf{y}_1^L$, the CE/DD scheme obtains $\hat{d}_1^L$ and $\hat{\mathbf{x}}_1^L$ for $\hat{\mathbf{x}}(0|0)$, but $e^{j\vartheta}\hat{d}_1^L$ and $e^{-j\vartheta}\hat{\mathbf{x}}_1^L$ for $e^{-j\vartheta}\hat{\mathbf{x}}(0|0)$. ■

Proof [Corollary 4]: Given $\hat{\mathbf{x}}(0|0) = e^{-j\vartheta}\hat{\mathbf{x}}(0|0) = [0, 0]^T$, Proposition 3 states that the CE/DD scheme obtains either $\hat{d}_1^L$ and $\hat{\mathbf{x}}_1^L$, or $e^{j\vartheta}\hat{d}_1^L$ and $e^{-j\vartheta}\hat{\mathbf{x}}_1^L$. The detection criterion for $d(1)$ in (10) reduces to $\hat{d}(1) = \arg \min_{\phi \in D} \{(|y(1)|^2)/(P_z(1|0)_\phi)\}$, making $\hat{d}(1)$ and $e^{j\vartheta}\hat{d}(1)$ equally likely detected symbols, and hence, $\hat{d}_1^L$ and $e^{j\vartheta}\hat{d}_1^L$ equally likely sequences. ■

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