Sinking Satellites and Tilting Disk Galaxies

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Abstract

I perform fully self-consistent disk+halo+satellite N-body simulations to investigate the dynamical interaction between a disk galaxy and an infalling satellite. In particular, I study the following three different dynamical responses of the disk to the infalling satellite: tilting, warping, and thickening, as well as the dynamical effects of the parent galaxy on the infalling satellite: orbital decay and tidal disruption. The model in this thesis is characterized with two cosmologically significant improvements. First, the satellite starts at a distance more than three times of the radius of the optical disk. This ensures a realistic interaction among the satellite, the disk, and the halo in the course of the satellite infall. Secondly, evolution of the structure and velocity ellipsoid of the disk due to internal heating is allowed. I study the commonly arising case of satellites having density profiles comparable to that of the parent galaxy in contrast to that of compact satellites considered in previous work.

I find that a disk is mainly tilted rather than heated by infalling satellites. Satellites of 10\%, 20\%, and 30\% of the disk mass tilt the disk by angles of \((2.9 \pm 0.3)\)\(^\circ\), \((6.3 \pm 0.1)\)\(^\circ\), and \((10.6 \pm 0.2)\)\(^\circ\), respectively. However, only 3.1\%, 6.9\%, and 11.1\% of the orbital angular momentum is transferred to the parent galaxy. The kinetic energy associated with the vertical motion in the initial coordinate frame of the disk is increased by \((6 \pm 3)\)%, \((26 \pm 3)\)%, and \((51 \pm 5)\)%, respectively, whereas the corresponding thermal energy associated with the vertical random motion in the tilted coordinate frame is only increased by \((4 \pm 3)\)%, \((6 \pm 2)\)%, and \((10 \pm 2)\)%, respectively. I find that satellites are mainly accreted onto the parent halo. Satellites having up to 20\% of the disk mass produce no observable thickening, whereas a satellite with 30\% of the disk mass produces little observable thickening inside the half-mass radius of the disk but significant thickening beyond this radius. Hence, a high cosmological accretion rate and thin disks can coexist if most infalling satellites have density profiles comparable to that of the parent galaxy.
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Dedication

To Giovanni for his encouragement, support, and love.
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Chapter 1

Introduction

Galactic mergers, which were once thought to be interesting but rare phenomena, are now considered to be one of the dominant processes governing the formation and evolution of galaxies. In any hierarchical cosmological model, merging is inevitable because small mass perturbations collapse before large ones and the evolution proceeds as a cascade of mergers from small to large scales. Therefore, most galaxies have had major mergers during their formation and a large fraction of galaxies have had minor mergers subsequently. In the cold dark matter (CDM) model, most galaxies accreted at least 10% of their mass over past 5 billion years (Bahcall & Tremaine 1988, Kauffmann, White & Guiderdoni 1993, Lacey & Cole 1993). By employing the two-point correlation function (Groth & Peebles 1977) for the galaxy distribution and Schechter’s luminosity function (Schechter 1976) for the galaxy mass distribution, with the assumption that the mass-to-light ratio for galaxies is a constant. Tremaine (1980) estimated the total mass accreted by a typical galaxy. Tremaine assumed that a spiral galaxy is surrounded by an extended isothermal halo, and the decay of the satellite orbit is caused by dynamical friction arising from the halo. If a typical galaxy is defined to be one having the luminosity \( L = 6.8 \times 10^9 h^{-2} L_{\odot} \) in Schechter’s luminosity function, where \( h = H_0/(100 \text{km/s/Mpc}) \), then

\[
m_{\text{acc}} = 4.1 \times 10^8 [(H_0 t) \ln (100 \text{km s}^{-1}/\sigma)]^{0.6} (M/L)^{1.6} M_{\odot} h^{-2.6}, \quad (1.1)
\]
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where $\sigma$ represents the one-dimensional velocity dispersion of the halo, $t$ is the age of the galaxy, and $\Lambda$ denotes the ratio of the size of the parent galaxy to that of a typical satellite. By assuming the reasonable values $\sigma = 140 \text{km/s}$, $M/L = 6M_\odot/L_\odot$, $H_0t = 1$, $h = 1$, and $\ln \Lambda = 3$, I obtain

$$m_{acc} = 6 \times 10^9 M_\odot. \quad (1.2)$$

This is about 10% of the disk mass of the Milky Way, and it is also comparable to the mass of the Large Magellanic Cloud (Kunkel, Demers & Irwin 1996).

Observational evidence for past and ongoing minor mergers between disks and satellite galaxies is numerous. The recent observational results from the Hubble Telescope show that some quasar activity is associated with minor mergers (Bahcall, Kirhakos, & Schneider 1995). A large fraction of disk galaxies are warped, which may relate to recent accretion (Binney). and grand design spiral structure is often associated with small close companions, such as M51 (Toomre 1981). The recently discovered Sagittarius dwarf is at a distance of only 16 kpc from the center of the Milky Way. It is a large, highly elongated dSph galaxy which appears to be in the process of tidal disruption (Ibata, Gilmore & Irwin 1995). Most galaxies have a number of satellite companions (Zaritsky et al. 1993) at close orbits, for example, the Large and Small Magellanic Clouds (LMC and SMC) are at distances of 51 Mpc and 63 Mpc respectively from the Sun. These satellites, if located inside the dark matter halo of their parent galaxy, will inevitably spiral into the center of the parent galaxy due to dynamical friction.

On the other hand, the stellar component of a galactic disk is built up over a period comparable to the Hubble time. The resulting stellar disk is so thin that the vertical scale height is only about 10% of the radial scale length. Although the details of vertical heating processes remain controversial (Wielen 1977, Lacey 1984, Carlberg 1987, Binney & Lacey 1988), the vertical extent of the disk is probably consistent with internal heating processes alone. The contribution from external sources of vertical
heating must therefore remain very small. Previous studies of the infall of a high density satellite onto a thin disk showed that the disk can be significantly thickened by a 10% disk-mass satellite. Quinn & Goodman (1986, hereafter QG) first performed simulations of an infalling satellite onto a spiral galaxy. Their N-body simulations were mainly focused on the orbital decay of rigid body satellites, but they noted strong disk heating. Further N-body simulations by Quinn, Hernquist, & Fullagar (1993, hereafter QHF) showed that a thin disk is thickened by a factor of two at the solar neighborhood by a 10% disk-mass infalling satellite, and they pointed out that the minor merger may be the mechanism for the formation of the thick disk of the Galaxy. Very recently, Walker, Mihos, & Hernquist (1996, hereafter WMH) performed high quality, fully self-consistent simulations and presented accurate results for the disk heating and sinking time of the satellite. They found that a 10% disk-mass infalling satellite thickens a disk at the solar circle about 60%, and the sink time of the satellite is $\sim 1$ Gyr. Tóth & Ostriker's (1992, hereafter TO) analytical calculation showed that the Milky Way cannot have accreted more than 4% of its mass inside the solar circle in the last 5 Gyr. They concluded that the high merging rate derived from the $\Omega = 1$ Universe is incorrect.

However, all these studies have treated the satellites as having extremely high central density, so the satellites are relatively immune to tidal disruption. Since a large fraction of the satellite mass reaches the center of the disk, tremendous heating in the disk is inevitable. For example, in TO and QHF's model, the density profiles of satellites are described by the Jaffe model, $\rho \propto \frac{1}{r^2(1+r)^2}$. Such a functional form leads to an extremely high density at the center of the satellite as compared to the density of the parent galaxy. Therefore, the core of the satellite could survive and reach the center of the disk. The infall of such a high density satellite may well represent the infall of a compact dwarf galaxy, such as the infall of M32 onto M31, but certainly does not represent the infall of satellites onto the Milky Way. Observations show that most of the dwarf galaxies in our Local Group have low central densities. The exception is M32, which is sometimes called an underluminous normal elliptical, or compact
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elliptical. Kormendy (1985) demonstrated that elliptical galaxies fall into a two-
family classification, as first proposed by Wirth & Gallagher (1984): high luminosity
normal ellipticals have a surface brightness that increases with decreasing luminosity
ending at M32: while the dwarf sequence is characterized by a surface brightness
that decreases with decreasing luminosity. Most of the companions of the Milky Way
Galaxy are low luminosity and non-nucleated dwarf elliptical galaxies (or, as they
are traditionally called, dwarf spheroidal galaxies) except the LMC, an irregular, and
the SMC, a dwarf irregular. The surface brightness of all these dwarf ellipticals is
well described by the King model (King 1966), with low central concentration or by
the exponential law which is often used to describe the surface brightness of dwarf
irregulars. Furthermore, the density profiles, derived from the rotation curves of
several dwarf spirals, are well fitted by an isothermal density function. \( \rho \propto 1/(r_c^2 + r^2) \)
with large core radius, \( r_c \) (Moore 1994). The nearly constant central densities of these
dwarf spirals are very similar to the King model profile, which is employed to describe
both the satellite and halo density profiles in this thesis.

Furthermore, observations show that accretion onto the outer halo and outer disk
are common but the fact that the inner disk remains thin suggests that infalling
satellites are tidally disrupted before they reach the inner disk. There is accumulating
observational evidence showing that the outer stellar halo may form by the accretion
of small, metal-poor fragments like gas-rich dwarf galaxies, as proposed by Searle &
Zinn (1978). They found that the chemical and orbital properties of the outer globular
clusters are decoupled. Preston, Beers & Shectman (1994) discovered a population
of blue metal-poor (BMP) stars in the solar neighborhood. Their ages are \( > 3 \) Gyr.
they have \([\text{Fe}/\text{H}] < -1\), and they are kinematically intermediate between the rapidly
rotating disk and the slowly rotating halo. Since Preston, Beers & Shectman noted
that nearby satellites, like the Carina dSph galaxy, have significant intermediate-age,
metal-poor components, they suggested that the galactic BMP stars may come from
similar dwarf galaxies. Rodgers et al. (1981) and Lance (1988) found young main
sequence A stars with \([\text{Ca}/\text{H}] > -0.5\) at heights up to 11 kpc from the galactic
CHAPTER 1. INTRODUCTION

plane. Since these A stars are a kinematically unusual population, they argued that the formation of these stars is associated with the accretion of somewhat metal poor gas, perhaps from a dwarf galaxy. The recently discovered Sagittarius dwarf is at a distance of only 16 kpc from the center of the Milky Way. It is a large, highly elongated dSph galaxy which appears to be in the process of tidal disruption (Ibata, Gilmore, & Irwin, 1995). Other accretion evidence is metal-poor, retrograde moving groups in the field (Eggen 1979, Majewski et al. 1994) and young retrograde globular clusters (Zinn 1993, van den Bergh 1993a,b. Da Costa & Armandroff 1995). Zaritsky (1995) found evidence of recent accretion in the outer disks of nearby late-type galaxies by estimating the duration of steep abundance gradients, elevated rates of star formation, or outer disk asymmetries. The fact that accretion onto the outer disk is common and the inner disk apparently remains undisturbed suggests that the infalling satellites are tidally disrupted before they reach the inner disk, and therefore they are low density satellites.

Due to tidal stripping, low density satellites are mainly accreted onto the halos or outer disks rather than the inner disks. Therefore, the low accretion rate onto the inner disk alone cannot be used to set the limit on the accretion rate of the galaxy. In order to set a limit on the rate of galactic accretion, it is necessary to include satellite accretion onto not only the inner disk, but also the outer disk and the outer halo. Therefore, in this thesis, infalling satellites are introduced inside the parent halo at cosmologically appropriate distances from the edge of the disk, whereas in previous simulations (WMH and QHF), satellites started at the edge of the disk. The physical complication of starting satellites further away from the edge of the disk is not the longer CPU time required in the simulations, but rather the accumulated disk internal heating the internal evolution of the disk. The disk model in this thesis is designed in such a way that the disk internal heating produces a velocity ellipsoid similar to that observed in the Galaxy. Furthermore, by moving satellites further away from the edge of the disk, I can study, in addition to the external heating of the disk, other dynamical responses of the disks to infalling satellites, namely disk
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tilting and warping. Binney & May (1986) demonstrated, by using straightforward test particle orbital integrations, that disks are extremely resilient in the presence of a slow rotating external torque: they tilt nearly as a unit, and in extreme cases, they can be turned "upside down" over time. However, the important issues, such as the rate of tilting, disk heating, and dynamical friction in the system, are not discussed in their study, and these issues will be investigated in this thesis. I will show in fully self-consistent simulations that thin disks are mainly tilted, rather than thickened by infalling low density satellites. Since disks tilt towards the orbital plane of a satellite on a direct orbit, the disk thickening in this work is much decreased, compared with that in previous work in which disk tilting is either not allowed (TO), or very small (QHF and WMH) due to small initial separation between the disk and the satellite.

Galactic warps are very common and remain a long standing puzzle in galactic dynamics. Attempts to resolve this puzzle are summarized by Toomre (1983) and Binney (1992), and the solutions are classified into two groups: warps can either be excited during the formation of an isolated galaxy or during the interaction of galaxies. However, studies show that warps are damped in a time period much shorter than the Hubble time (Nelson & Tremaine 1995a, Dubinski & Kuijken 1995). Thus, the fact that 50% of galaxies are warped could be an indication that disks are in fact being subjected to the addition of considerable misaligned angular momentum. In this thesis, a disk subjected to the addition of misaligned angular momentum from an infalling satellite is investigated.

This thesis is divided into 10 chapters. In Chapter 2, the initial conditions for the disk, halo, and satellite components, as well as the relation among these three components will be discussed. The tree code employed in the simulations of this thesis will be briefly described. The N-body simulation model in this work has several features which are significantly different from those considered in previous work (WMH, QG, QHF, and TO). First, the initial separation between satellites and the parent galaxy is cosmologically realistic. This ensures a true dynamical interaction between the satellite, the disk, and the halo. Secondly, internal evolution of the disk
CHAPTER 1. INTRODUCTION

is allowed. Thirdly, infalling satellites have density profiles comparable to that of the parent galaxy. Finally, a small softening length is employed in the simulation so that the disk is strongly self-gravitating. This minimizes the coupling between the vertical motion of the satellite and the vertical oscillations of stars within the disk. that is, the disk responds to the infalling satellite as a unit.

In Chapter 3, I will discuss the evolution of the structure and kinematics of an isolated galaxy model in order to carry out a comparison between the galaxy model in this thesis and spiral galaxies, especially the Milky Way. The resulting distributions of both the surface density and the velocity dispersions are exponential. The measured scale length is a constant, but the scale height of the asymptotic exponential distribution increases with time as observed in the Milky Way. For the solar neighborhood, the measured scale height and velocity dispersions for the model disk are similar to what is observed in the Milky Way. At a given location, the squared velocity dispersions increase proportionally with increasing time. The similarity of the structure and kinematics between the model disk and the Milky Way indicates that the model of the disk is dynamically reasonable. This model is used to further study the external heating of the disk caused by infalling satellites.

In Chapter 4, the interaction between an infalling satellite and a weak bar which forms in the disk will be investigated. I will show that the growth rate of the \( m = 2 \) bar in a disk with an infalling satellite is somewhat higher than that in the same disk but without the infalling satellite. However, the growth rates of the \( m = 2 \) bar mode in both cases are very low due to the high halo-to-disk mass ratio employed in the model of this thesis. I will show that the pattern speed of a weak bar decays very slowly compared with that of a strong bar studied in previous work. The outer Lindblad resonance of a weak bar remains in the inner disk, whereas the satellite is located far from the resonance. There is no angular momentum transfer from the bar to the outer disk and the satellite. The satellite orbital decay rate is not affected by the formation of a weak bar in the disk.

The results on three major dynamical effects of sinking satellites on disk galaxies,
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namely tilting, warping, and heating will be presented in Chapter 5, Chapter 6, and Chapter 7 respectively. First, I will demonstrate, in Chapter 5, that a disk is mainly tilted rather than heated by infalling satellites. Satellites of 10\%, 20\%, and 30\% of the disk mass tilt the disk by angles of (2.9 \pm 0.3)\degree, (6.3 \pm 0.1)\degree, and (10.6 \pm 0.2)\degree respectively. However, only 3.4\%, 6.9\%, and 11.1\% of the orbital angular momentum is transferred to the parent galaxy. The kinetic energy associated with vertical motion in the initial coordinate frame of the disk is increased by (6 \pm 3)\%, (26 \pm 3)\%, and (51 \pm 5)\% respectively, whereas the corresponding thermal energy associated with the vertical random motion in the tilted coordinate frame is increased by only (4 \pm 3)\%, (6 \pm 2)\%, and (10 \pm 2)\% respectively. Next, I will show, in Chapter 6, that infalling satellites can excite warps in the disk. However, the warps eventually fade away due to the fact that both the inner disk and outer disk are tilted towards the same plane. Finally, in Chapter 7, the vertical velocity dispersion and the vertical scale height of a disk in a local tilted coordinate frame are measured. Inside the half-mass radius of the disk, the vertical heating of the disk measured with both methods is equivalent. however, outside this radius, the vertical heating inferred from the vertical velocity dispersion is greater than that determined from the vertical scale height. I will show that the infall of a 20\% disk-mass satellite causes heating near the center and edge of the disk. On the other hand, the infall of a 30\% disk-mass satellite causes the entire disk to be heated, with the outer regions of the disk heated much more than the inner ones.

In Chapter 8, two dynamical effects of a parent galaxy on an infalling satellite will be discussed. One is satellite mass stripping which is caused by the tidal force from the parent galaxy, and the other is satellite orbital decay which results from the dynamical friction exerted on the satellite by the parent galaxy. First, I will show that the mass loss of the satellite is roughly proportional to the halo mass which the satellite traverses, if self-similar density profiles are employed for both the satellite and the halo. Then, I will show that the orbital decay rate, obtained from both theoretical estimates and simulation results, is constant because of the mass
stripping of the infalling satellite. In the case of a solid satellite, the orbital decay rate is inversely proportional to the distance between the center of the satellite and the center of the parent galaxy. The orbital decay rates for satellites on nearly circular orbits, but different orbital inclinations with respect to the parent disk plane, and for satellites on direct and retrograde orbits will be studied. In physical space, the final shape of the tidally stripped satellite particles resembles a warped torus. The planes of the outermost and innermost parts of the torus are aligned respectively with the planes of the initial and final orbits of the satellite. In phase space, the satellite particles are distinguished from halo particles by their high rotational speed and clumpy distribution.

In Chapter 9, I will calculate, by employing both analytical approximations and numerical integration methods, the orbital decay time of a satellite which experiences tidal disruption as its elliptical orbit decays. The orbital decay times estimated with both methods are similar. Therefore, based on the analytical result, I will calculate the galactic accretion for the case of low mass infalling satellites. In the calculation, I will follow Tremaine's (1980) method. However, I will introduce the following modifications: high peculiar velocity cutoff, high mass cutoff, and non-constant mass-to-light ratio for dwarf galaxies. I will show that for a median eccentricity, $\epsilon = 0.5$, a typical galaxy like our own Milky Way Galaxy has absorbed about 30% of its disk mass in the form of infalling satellites. According to results drawn from the simulations discussed in this thesis, it can be accommodated without unacceptably thickening the disk inside the half-mass radius.

Finally, in Chapter 10, the implications of the main results of this thesis on the density of satellites, the Holmberg effect, the rotation of the stellar halo of the Galaxy, and the theoretical merger rate for the $\Omega = 1$ CDM cosmology will be discussed. At the end, some concluding remarks and suggestions for future work will be presented.
Chapter 2

Numerical Methods

2.1 Introduction

The gravitational interaction between a disk galaxy and a satellite is investigated both analytically and numerically. Analytical treatments of the interactions mainly use the perturbation theory and the impulse approximation. However, the perturbation theory is generally restricted to weak interactions (Lyden-Bell & Kalnajs 1972, Goldreich & Tremaine 1979, Palmer & Papaloizou 1982, Palmer 1983), and the impulse approximation is useful only when the satellite moves sufficiently rapidly with respect to the parent disk (Aguilar & White 1985). In order to study the dynamical responses of a disk to an infalling satellite, namely disk tilting, warping, and thickening, as well as to investigate the dynamical effects of the parent disk on the infalling satellite, such as the orbital decay and tidal stripping of satellites, one must employ numerical simulations.

Simulating gravitational interactions between galaxies was initiated on an analog computer by Holmberg (1941) who found that a close encounter caused observable tidal distortion and resulted in capture due to orbital energy loss. Two decades later, Pfleiderer & Siedentopf (1961) and Pfleiderer (1963) revisited the subject on an electronic digital computer. Due to the fact that a number of observed galaxies show signs of interaction (Arp 1966), many simulations of galaxy interactions were performed after 1970, including those by Yabushita (1971), Wright (1972), Toomre
& Toomre (1972), and Eneev, Kozvol. & Sunyaev (1973). Amongst these works, the paper of Toomre & Toomre is most cited because they successfully modeled several pairs of interacting galaxies, such as Arp295, M51+NGC1915, NGC4038/39 and NGC4676. The good agreement of the bridges and tails between their models and observations showed that the gravitational interaction is responsible for the observed morphological distortions. Since the late 1980's, simulating galaxy interactions has become an active field due to both the accumulation of merging evidence and the theoretical support from hierarchical cosmological models, as well as the development of hierarchical tree code in conjunction with available fast computers. In this chapter, I will first explain, in Section 2.2, the detailed initial conditions of my simulation models, then I will briefly describe, in Section 2.3, the tree code employed in the simulations.

## 2.2 Initial conditions

Assuming that a dark matter halo is formed at a redshift, \( z \), by a spherical top hat perturbation (Gunn & Gott 1972) from a spherical region of comoving radius, \( r_0 \), in current units. Narayan & White (1988) show that the mass, \( M \), the mean density, \( \rho \), and the velocity dispersion, \( \sigma \), of the dark matter halo are \( M = \frac{4\pi}{3} \rho_0 r_0^3 \), \( \bar{\rho} = 178\rho_0(1+z)^3 \), and \( \sigma \propto (1+z)^{1/2} H_0 r_0 \) respectively. In these expressions, \( \rho_0 \) and \( H_0 \) are respectively the critical density and the Hubble constant at present. Therefore, the size of the halo is \( r_{\text{size}} = (\frac{3}{4\pi} M/\bar{\rho})^{1/3} \propto (r_0/(1+z))^{1/3} \). Thus, at a given redshift, the relation among the size, \( r_{\text{size}} \), the mass, and the velocity dispersion of the dark matter halo is

\[
r_{\text{size}} \propto \sigma \propto M^{1/3}.
\]

In other words, all dark matter halos formed at the same redshift \( z \) have the same mean density and structure. However, since low mass halos are, on the average, formed earlier, their mean densities are higher than those of high mass halos. Eisenstein & Loeb (1996) derived that the relation between \( M \) and \( \sigma \) is \( M \propto \sigma^a \), with
\( \alpha = 3.1 - 3.2 \) instead of 3.0 in Equation 2.1. Recently, Navarro, Frenk, & White (1996) showed that the high mass halo is less centrally concentrated than the low mass one, but they indicated that their halos are too concentrated to be consistent with the halo parameters inferred from dwarf irregulars. In this thesis, I assume that the halos of the satellite and disk are formed at the same redshift, and I choose self-similar density profiles for both halos. Since the total mass of most low density dwarf galaxies is mainly contributed by dark matter halos, the density profile of the satellite halo is called the satellite density profile in the following discussion.

Observations show that the observed mass-to-light ratio and relation between luminosity and velocity dispersion from 17 dwarf elliptical galaxies (6 are galactic dwarf spheroidals) are \( M/L \propto L^{-0.40 \pm 0.06} \) and \( L \propto \sigma^{5.8 \pm 0.9} \) respectively (Peterson & Caldwell 1993). This is in accord with the corresponding theoretical predictions \( M/L \propto L^{-0.37} \) and \( L \propto \sigma^{5.3} \) (Dekel & Silk 1986). The resulting relation between mass and velocity dispersion is \( M \propto \sigma^{3.3-3.4} \). On the other hand, since the observed relation between luminosity and core radius is \( L \propto R^{5.0 \pm 0.5} \) (Peterson & Caldwell 1993), the resulting relation between mass and core radius is \( M \propto R^3 \). Based on these observed relations among core radius, velocity dispersion, and mass, the following scaling relation, \( R \propto \sigma \propto M^{1/3} \), is employed to scale models. In the model scaled under this relation, the central densities of both the satellite and the host are similar, therefore, this model cannot be applied to study the infall of a compact dwarf.

As a reasonable approximation to a relaxed halo, I employ a strongly concentrated King model with the dimensionless central potential \( W_0 = 8.0 \), which corresponds to the central concentration (King 1966), \( c = \log(r_t/r_K) = 1.833 \), in which, \( r_K \) is the King radius defined by

\[
r_K = \sqrt{\frac{9\sigma_0^2}{4\pi G\rho_0}}.
\]

(2.2)

and \( r_t \) is the tidal radius. In Equation 2.2, \( \rho_0 \) is the central density, and \( \sigma_0 \) is the central velocity dispersion.

I choose the initial orbital parameters of satellites, such as the distances from
the center of the parent galaxy, the orbital eccentricities, etc., to be as realistic as possible. For example, a satellite starts at the apocenter $r_+$ of an $\epsilon = 0.2$ elliptical orbit. The radius $r_+$ is chosen to be more than three times of the initial disk radius. a significant increase relative to the values employed in previous work (WMH and QHF), in which satellites started at the edge of the disk. In my models, satellites are initially set so far away from the edge of the disk that they do not penetrate the disk when they pass the disk plane.

Primarily to create an equilibrium disk, I choose one out of each eight halo particles to become disk particles. Since all halo particles ($i = 1, N$) are initially sorted from zero to the tidal radius, each chosen particle located at the spherical coordinates $r(i), \theta(i), \phi(i)$ is moved to the corresponding half-mass radius at the cylindrical coordinates $R(i) = r(i/2), \phi(i), and z(i) = 0$. The new disk is embedded inside the half-mass radius of the halo, and it has zero thickness, but the thickness increases with increasing time. The total mass ratio of the halo to the disk is 7, and the mass ratio inside the disk radius is 3.5. The newly formed disk has zero thickness, but the thickness increases with increasing time.

Initially, all disk particles have only rotational velocities. The rotational velocity of particle $i$, located at $R(i)$ from the center of the disk, is defined by $v_{rot}(i) = \sqrt{GM(R(i))/R(i)}$. In this equation, $M(R(i))$ is the total mass contained within the radius $R(i)$. The post-setup rotation curve of the disk, or the rotation velocity of a satellite versus the distance between the satellite and the disk, is shown in Figure 2.1. The resulting disk and halo system has the initial dimensionless spin $\lambda = 0.023$, which is lower than the average value of 0.05-0.07 found in papers by Efstathiou & Jones (1979), Barnes & Efstathiou (1987), and Warren et al. (1992), due to the non-rotating halo and the high mass ratio of halo to disk in my models.

Galactic disks are usually treated as quasi-equilibrium systems for periods much shorter than the Hubble time. However, since I am studying the interaction between a disk and an infalling satellite over a period comparable to the Hubble time, it is necessary to consider the internal dynamical evolution of the disk. Observational
CHAPTER 2. NUMERICAL METHODS

evidence of this internal evolution consists of the variation of the velocity dispersion and the scale height from the youngest to the oldest disk stars (Wielen 1977, Carlberg et al. 1985, Edvardsson et al. 1993). For example, the relation between the velocity dispersions of disk stars and their ages can be described as \( \sigma \propto t^\alpha \), where \( \alpha \) is approximately equal to 0.5 (Wielen 1977). There are many internal heating sources in the disk, but it is widely accepted that the disk stars are mainly heated by scattering with giant molecular clouds (Spitzer & Schwarzschild 1953, Lacey 1984, Villumsen 1985) and spiral arms (Barbanis & Woltjer 1967, Sellwood & Carlberg 1984, Carlberg & Sellwood 1985).

In N-body simulations, the two-body relaxation, due to the accumulation of many small deflections of the orbit of a particle arising from encounters with other particles, increases the velocity dispersion \( \sigma \) of a spherical system as \( \sigma \propto (T/T_r)^{0.5} \) (Huang, Dubinski, & Carlberg 1993). In the relation, \( \sigma \) is velocity dispersion, \( T \) is time and \( T_r \) is the two-body relaxation time which is defined as (Binney & Tremaine 1987)

\[
T_r = \frac{N}{8 \ln N} T_c. \tag{2.3}
\]

in which \( N \) is the number of particles and \( T_c \) is the crossing time. In a simulation with the softened potential \( \phi_{ij} = -Gm_i m_j/(r_{ij}^2 + \epsilon^2)^{1/2} \), where \( \epsilon \) is the softening length, the two-body relaxation time is redefined as (Huang, Dubinski, & Carlberg 1993)

\[
T_r = \frac{N}{8 \ln(R/\epsilon)} T_c. \tag{2.4}
\]

in which \( R \) is the size of the system.

However, I will show, in Chapter 3, that Equation 2.4 cannot be applied to a differentially rotating disk because the disk responds to non-axisymmetric initial disturbances in a remarkably spiral like and vigorous manner (Julian & Toomre 1966, hereafter JT). Therefore, each disk particle can be equivalent to a spiral like density wake in the disk, and the amplitude of the density wake is related to the Toomre parameter \( Q \). When \( Q \) is very low at the beginning of a simulation, the amplitude of
CHAPTER 2. NUMERICAL METHODS

Table 2.1: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( N )</th>
<th>( M_H/M_D )</th>
<th>( M_S/M_D )</th>
<th>( r_+ )</th>
<th>( \theta^e )</th>
<th>( \epsilon )</th>
<th>( v_+/c_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 0</td>
<td>80.000</td>
<td>7</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td>81.000</td>
<td>7</td>
<td>0.1</td>
<td>4.0</td>
<td>30</td>
<td>0.2</td>
<td>+</td>
</tr>
<tr>
<td>Model 2</td>
<td>82.000</td>
<td>7</td>
<td>0.2</td>
<td>4.0</td>
<td>30</td>
<td>0.2</td>
<td>+</td>
</tr>
<tr>
<td>Model 3</td>
<td>83.000</td>
<td>7</td>
<td>0.3</td>
<td>4.0</td>
<td>30</td>
<td>0.2</td>
<td>+</td>
</tr>
<tr>
<td>Model 4</td>
<td>81.000</td>
<td>7</td>
<td>0.1</td>
<td>2.5</td>
<td>30</td>
<td>0.0</td>
<td>+</td>
</tr>
<tr>
<td>Model 5</td>
<td>81.000</td>
<td>7</td>
<td>0.1</td>
<td>2.5</td>
<td>60</td>
<td>0.0</td>
<td>+</td>
</tr>
<tr>
<td>Model 6</td>
<td>81.000</td>
<td>7</td>
<td>0.1</td>
<td>2.5</td>
<td>90</td>
<td>0.0</td>
<td>+</td>
</tr>
<tr>
<td>Model 7</td>
<td>81.000</td>
<td>7</td>
<td>0.1</td>
<td>4.0</td>
<td>30</td>
<td>0.0</td>
<td>-</td>
</tr>
<tr>
<td>Model 8</td>
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<td>7</td>
<td>0.2</td>
<td>4.0</td>
<td>30</td>
<td>0.0</td>
<td>-</td>
</tr>
<tr>
<td>Model 9</td>
<td>83.000</td>
<td>7</td>
<td>0.3</td>
<td>4.0</td>
<td>30</td>
<td>0.0</td>
<td>-</td>
</tr>
<tr>
<td>Model 10</td>
<td>81.000</td>
<td>7</td>
<td>0.1</td>
<td>1.0</td>
<td>30</td>
<td>0.0</td>
<td>+</td>
</tr>
<tr>
<td>Model 15</td>
<td>160.000</td>
<td>7</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

the density wake is very high. The initial noise of the particle distribution through the swing amplification (Toomre 1981), immediately develops into the visible spiral structures, and the wakes of particles become strongly correlated. The strongly correlated wakes heat the disk very rapidly in the first few disk rotations. As \( Q \) increases and the spiral structure fades away, the amplitude of density wakes decreases, and the wakes of disk particles become uncorrelated or weakly correlated due to the transient weak spiral structures. The interaction between these uncorrelated or weakly correlated wakes perform as a continuous internal heating source in the simulated disk. Further discussion about the evolution of the velocity ellipsoid of the disk is discussed in Chapter 3.

Table 2.1 lists the values of the parameters associated with a series of simulations, for which I present results in this thesis. The number of particles, \( N \), is listed in column 2. In our standard models, the number of disk particles is chosen to be 10,000, and the number of halo particles, 70,000. The mass ratios of halo to disk, \( M_H/M_D \), and satellite to disk, \( M_S/M_D \), are listed in columns 3 and 4 respectively. The parameters of the satellite orbit, such as the apocenter \( r_+ \), the orbital inclination
Figure 2.1: Initial conditions imposed on the disk and the satellite. The initial position and the orbit of the satellite are shown in the top panel, whereas the rotation curve of the disk, or the rotation velocity of a satellite versus the distance between the satellite and the disk is shown in the bottom panel.
Table 2.2: Parameters in Standard Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Galaxy mass</td>
<td>$M = M_D + M_H$</td>
<td>$3.2 \times 10^{11} M_\odot$</td>
</tr>
<tr>
<td>Half-mass radius of halo</td>
<td>$R$</td>
<td>16 kpc</td>
</tr>
<tr>
<td>Velocity</td>
<td>$V = \sqrt{G M/R}$</td>
<td>262 km/s</td>
</tr>
<tr>
<td>Dynamical time</td>
<td>$t = R/V$</td>
<td>$6 \times 10^7$ yr</td>
</tr>
<tr>
<td>Timestep</td>
<td>$\Delta t$</td>
<td>2.4 Myr</td>
</tr>
<tr>
<td>Softening length</td>
<td>$\epsilon$</td>
<td>400 pc</td>
</tr>
</tbody>
</table>

The direction of satellite rotation $v_+ / v_-$, either direct (+) or retrograde (-), is listed in column 8.

In the previous fully self-consistent N-body simulation (WMH), halo particles were treated as heavier particles in order to provide better sampling of the disk and satellite components. However, this resulted in an extra heating source in the lighter particle components due to the scattering between light and heavy particles. In this thesis, all particles in the disk, the halo, and the satellite have the same mass so that there is no extra heating caused by the scattering between the light and heavy particles. In the simulation, I employ model units with the gravitation constant $G = 1$, the total mass of the parent galaxy, including the mass of the disk and the halo $M_D + M_H = 1$, and the half mass radius of the halo $R = 1$. For purposes of presentation, units are scaled to facilitate comparison with observed galaxies such that $M_D + M_H = 3.2 \times 10^{11} M_\odot$ and $R = 16$ kpc. The resulting velocity unit $\sqrt{G M/R}$ is 262 km/s which leads to $V_{R_{\odot}} = 8.5$ kpc $\approx 220$ km/s (see Table 2.2).

### 2.3 N-body Code

The algorithms for computing the potential of a system of $N$ particles can be divided into "direct" and "field" methods. The former explicitly treats interactions between individual particles while the latter does so only indirectly, through the contribution
of particles to the global gravitational field. The “direct” method is flexible but has a CPU cost per step, scaling as $N^2$. A variety of methods have been proposed to reduce the cost of computing the self-consistent potential, such as test-particle methods (Schwarzschild 1979, Quinn 1984), the restricted three-body method (Pfleiderer & Siedentopf 1961, Toomre & Toomre 1972), or semi-restricted N-body codes (Lin & Tremaine 1983, Quinn & Goodman 1986, Hernquist & Weinberg 1989). However, the dynamics of interacting galaxies will not be represented faithfully without including self-gravity (Barnes 1988). Early self-consistent methods, known as grid methods, are not useful for studying interaction of galaxies (Sellwood 1987, Hockney & Eastwood 1988) because of the low efficiency in increasing resolution. The recent development of the “hierarchical tree” method (Appel 1985, Jernigan 1985, Barnes & Hut 1986) is the best technique for studying the interaction of galaxies because it provides high resolution in high density regions and low resolution in low density regions.

For all simulations presented in this thesis, I employ a “tree” N-body simulation code which includes quadruple correction in the cell-particle force (Barnes & Hut 1986, Hernquist 1987, Dubinski 1988). The tree code is an algorithm which sorts particles in a N-body system into groups in a hierarchy of cubes. Each cube of particles is subdivided into eight subcubes with half the length of the parent. The cube hierarchy forms an octal tree data structure which is used to calculate the gravitational force on a particle. The net force exerted on a particle by the rest of the particles in the N-body system is calculated as the sum of the forces due to nearby particles and distant groups in hierachical cubes. The force from a distant group is determined from a quadruple order of expansion of the potential about the mass center of the group. A group is considered distant if $s/d < \theta$, in which $d$ is the distance from the particle to the mass center of the group, $s$ is the size of the cube, and $\theta$ is called tolerance parameter which is the maximum allowed angle subtended by the cube as seen from the particle. If a group is too close to the particle, it is subdivided into eight cubic subgroups and the subgroups can be further subdivided until the criterion $s/d < \theta$ is reached. A tree code can determine the force on a particle in a time of
order $N \log N$ compared with $N^2$ for a direct force calculation.

The evolution of the system is followed by a leapfrog integrator with a constant timestep $\Delta t = 0.04$ (or 2.4 Myr in Table 2.2) and a tolerance parameter $\theta = 0.7$. The softening length $\epsilon$ is usually chosen to be of order $R / N^{1/3}$, in which $R$ is the size of the system. I therefore select $\epsilon = 0.025$ (or 400 pc in Table 2.2) in the simulations of this thesis. A small softening length is essential in order that the disk remains strongly self-gravitating. I find that the resulting conservation of total energy is satisfied to better than 0.158% over 4,000 timesteps or 9.6 Gyr.

2.4 Summary

The N-body simulation model in this work has several features which are significantly different from those considered in previous work (WMH, QG, QHF, and TO). First, the initial separation between satellites and the parent galaxy is cosmologically realistic. This ensures a true dynamical interaction between the satellite, the disk, and the halo. Secondly, the model disk is unconventionally constructed from a cosmological perspective: the internal evolution of the disk is allowed. Thirdly, the density of infalling satellites is cosmologically scaled. The infalling satellites have density profiles comparable to that of the parent galaxy. Finally, a small softening length is employed in the simulation so that the disk is strongly self-gravitating. This minimizes the coupling between the vertical motion of the satellite and the vertical oscillations of stars within the disk, that is, the disk responds to the infalling satellite as a unit.
Chapter 3

The Evolution of an Isolated Galaxy

3.1 Introduction

Surface photometry of spiral systems (Freeman 1970, De Jong 1996) shows that almost all of them have exponential surface brightness profiles $\mu(R) \propto \exp(-R/h_R)$, where $h_R$ is called scale length. The vertical light distribution of edge-on spiral galaxies (van der Kruit & Searle 1981) can be expressed by the asymptotic exponential relation $L_z \propto \text{sech}^2(z/h_z)$, corresponding to isothermal sheets (Spitzer 1942). In the above relation, $h_z$ is called scale height, which increases with increasing age of the stellar group and is believed to be a constant as a function of disk radius. However, increasing evidence shows that the scale height of disk galaxies may not be constant (Rohlf & Wiemer 1982, Kent, Dame, & Fazio 1991 (hereafter KDF), Fux & Martinet 1994 (hereafter FM)) but increases with increasing radius. In most N-body simulations, the initial particle distribution of a disk model is given by the following relation, $\rho(R,z) \propto \exp(-R/h_R)\text{sech}^2(z/2h_z)$, with constant $h_R$ and $h_z$. However, in this thesis, the model disk is unconventionally constructed from a cosmological perspective as described in Chapter 2. It is therefore important to ensure that the prescription employed for constructing the disk galaxy yields a galaxy similar to a present day spiral galaxy in terms of its structure and kinematics.

The velocity dispersions of disk stars increase with increasing age (Spitzer & Schwarzschild 1953, Wielen 1977, Carlberg et al. 1985, Edvardsson et al. 1993),
\( \sigma \propto t^\alpha \) with \( \alpha = 0.3 - 0.5 \). In this thesis, the velocity dispersion of disk particles increases with increasing time as \( \sigma \propto t^{0.5} \) due primarily to the weakly correlated interaction between particle wakes. It is important to ensure that the amount of the disk heating in the disk model employed in this thesis is similar to the amount of internal heating in the Galaxy. In other words, the resulting velocity ellipsoid in the disk model should be similar to what is observed in the Galaxy.

In this chapter, I discuss the evolution of the structure and kinematics of an isolated galaxy (Model 0) in order to carry out a comparison with spiral galaxies, especially with the Milky Way. First, in Section 3.2, I compare the disk surface density and the vertical distribution of disk particles of Model 0 with those of spiral galaxies. Then, in Section 3.3, I compare the three dimensional velocity dispersions of the model disk with those observed in the Milky Way. The Toomre parameter \( Q \) which characterizes the stability of disks (Toomre 1964) is also discussed.

### 3.2 Structure

In the following discussion, the time clock is reset to zero after the disk evolves for three disk rotation periods because within this time period, the initial cold and zero thickness disk is rapidly heated by the newly formed spiral arms. The disk then enters a slow evolution phase in which two-body relaxation is the major source of internal heating. The disk rotation period defined at \( R = R_\odot = 8.5 \) kpc is \( 2.4 \times 10^8 \) yr or 4 model time units.

The initial surface density distribution of the disks is a projected King model, which is very close to the exponential distribution, \( \Sigma(R) = \Sigma(0) \exp(-R/h) \). Figure 3.1 shows the surface density of Model 0 at \( t=0 \), 3.6, and 7.2 Gyr. The surface density distribution of the disk changes little, except at the center region \( R < 1 \) kpc, where the surface density increases with time as shown in the figure. The resulting surface density distribution is consistent with the observed surface brightness distribution of disk galaxies: \( \Sigma(R) = \Sigma(0) \exp(-R/h_R) \). For \( t = 0 \), 3.6, and 7.2 Gyr, \( h_R \)
Figure 3.1: Surface density of Model 0 at $t = 0$ (dash-dotted line), $t = 3.6$ Gyr (dashed line), and $t = 7.2$ Gyr (solid line). The dotted straight line represents an exponential fit, with scale length, $h_R = 3.8 \pm 0.1$ kpc, of the surface density at $t = 7.2$ Gyr.

is $3.8 \pm 0.1$, $3.7 \pm 0.1$, and $3.8 \pm 0.1$ kpc respectively. The observed scale length for the Milky Way varies from 1.8 kpc to 6.0 kpc (see the review by KDF) depending on the data set (see discussion in Appendix A). However, the value of 3.5 kpc (de Vaucouleurs & Pence 1978) is often used. I therefore conclude that the surface density distribution of the disk model employed in this thesis is similar to that observed in
CHAPTER 3. THE EVOLUTION OF AN ISOLATED GALAXY

the Milky Way.

In order to measure the vertical distribution of disk particles, I divided the disk from \( R = 0 \) to \( R = 20 \) kpc into 10 rings with \( \Delta R = 2 \) kpc. The histograms of particle distribution in the vertical direction of 10 rings at \( t = 7.2 \) Gyr is shown in Figure 3.2. The solid lines represent asymptotic exponential fit. \( N(z) = N(0) \text{sech}^2(z/2h_z) \), where both \( N(z = 0) \) decreases and \( h_z \) increases with increasing radius. Figure 3.2 shows that the particle distribution in the vertical direction fits the asymptotic exponential distribution very well. Therefore, the parameter \( h_z \) is a good description of the thickness of disk.

Figure 3.3 shows the measured scale height and its standard deviation as a function of disk radius and time. The measured scale height is not a constant but increases slowly with increasing disk radius: \( h_z(R) = h_z(0) \exp(R/h) \), in which \( h \) is called the radial variance of vertical scale height and equals \( 32 \pm 3, 20 \pm 1 \), and \( 21 \pm 1 \) kpc for \( t = 0, 3.6, \) and \( 7.2 \) Gyr respectively. For the Galaxy, KDF proposed a non-constant scale height model in which the scale height is constant at a value \( h_{z, \text{min}} \) inside a characteristic radius \( R_{\text{min}} \) and increases linearly with radius for radii larger than \( R_{\text{min}} \). They found that the outwards increasing \( h_z \) significantly improved the fit of their observation data, and the measured \( h_R \) is 3.0 kpc instead of 2.7 kpc in constant \( h_z \) model. Inspired by KDF and using the similar model with \( h_R = 3.0 \) kpc but outwards increasing \( h_z \), FM pointed out that the model may fit the pioneer 10 data as well as van der Kruit's (1986) model with \( h_R = 5.5 \) kpc and constant \( h_z \). By employing the values of \( h_{z, \text{min}} \) and \( h_z(R_{\odot}) \) in KDF's model and assuming that \( h_z(R = 0) = h_{z, \text{min}}^{(KDF)} = 165 \) pc and \( h_z(R_{\odot}) = h_z(R = 0) \exp(R_{\odot}/h) = h_z(R_{\odot})^{(KDF)} = 247 \) pc, with \( R_{\odot} = 8.0 \) kpc, I obtain \( h = 20 \) kpc for the Galaxy. The estimated \( h \) is similar to that measured in the disk model. Based on the exponential scale height model \( h_z \propto \exp(R/20\text{kpc}) \) derived from the disk model of this thesis, I explain how the constant \( h_z \) assumption causes the discrepancy among varies photometric determinations of scale length in Appendix A.

The thickness of the disk, defined as \( h_z(R_{\odot}) \), is 144, 447, and 614 pc for \( t = 0 \).
Figure 3.2: Histograms of particle distribution in the vertical direction of 10 rings. The solid lines represent asymptotic exponential fit. The particle distribution in vertical direction fits the asymptotic exponential distribution well.
Figure 3.3: Scale height of the asymptotic exponential distribution versus radius and time. The measured scale height increases slowly with increasing disk radius: $h_z(R) = h_z(0) \exp(R/h)$, in which $h$ is equal to $32 \pm 3$, $20 \pm 1$, and $21 \pm 1$ kpc for $t = 0$, 3.6, and 7.2 Gyr respectively. It is consistent with observation that the scale height of different stellar groups increases from the youngest group to the oldest group (Blaauw 1965). For example, the scale heights measured from O stars, F stars, and dK and dM stars are 50, 190, and 350 pc (Mihalas & Binney 1981). Note that the exponential scale height is equivalent to half of the asymptotic exponential scale height. Therefore,
Figure 3.4: Distributions of velocity dispersions, $\sigma_R$, $\sigma_\phi$, and $\sigma_z$, and of the Toomre parameter, $Q$ at $t=0$ ($\cdot$), 3.6 ($+$), and 7.2 Gyr ($\ast$) respectively. The solid lines, which are the least squares fit of the velocity dispersions at $t = 7.2$ Gyr, show that, to a good approximation, the velocity dispersions decrease exponentially with increasing disk radius.

the structure of the disk model in this thesis is similar to the structure of the Milky Way.
CHAPTER 3. THE EVOLUTION OF AN ISOLATED GALAXY

3.3 Kinematics

In this section, I compare the kinematics of the disk model with those observed in the Milky Way. Figure 3.4 shows the velocity dispersions, \( \sigma_\phi, \sigma_r, \sigma_z \), and the Toomre parameter, \( Q \), as a function of disk radius and time for Model 0. The solid lines, which are the least squares fit of the velocity dispersions at \( t = 7.2 \) Gyr, show that to a good approximation the velocity dispersions decrease exponentially with increasing disk radius. Note that at 7.2 Gyr, the velocity dispersions in regions \( R < 5 \) kpc evolve away from the exponential trend, which is due to the extra heating caused by the scattering of disk particles with the weak bar formed at the center. The formation of a weak bar is discussed in the next chapter.

Direct measurements of radial velocity dispersion of the Galaxy (Lewis & Freeman 1989, hereafter LF) also show an exponential distribution. By assuming \( \sigma^2_R \propto \exp(-R/h_R^M) \), LF estimated that \( h_R^M \) of the Galaxy is 4.4 kpc which is 47% longer than the photometric measurement of \( h_R^L = 3.0 \) kpc (KDF). Note that \( h_R^M \) and \( h_R^L \) represent respectively the kinematic measurement and the photometric determination of scale length. Van der Kruit & Freeman (1986) also showed that in NGC 5247 the relation between \( h_R^M \) and \( h_R^L \) is: \( h_R^M = 1.2h_R^L \) (or \( h_R^M = 1.4h_R^L \) after deleting a discrepant point). The discrepancy between \( h_R^L \) and \( h_R^M \) may be due to the assumptions of a constant scale height and of a constant mass-to-light ratio. In the disk model of this thesis, \( h_R^M \), measured from vertical velocity dispersion of disk particles, \( \sigma_z^2 \propto \exp(-R/h_R^M) \), is equal to \( 1.4h_R^L \), where \( h_R^L \) is measured from the disk particle surface density, \( \Sigma_d \propto \exp(-R/h_R^L) \). This is because in the disk model, the measured scale height increases exponentially with increasing disk radius, \( h_z \propto \exp(R/h) \), and the mass-to-light ratio defined as the surface density ratio of both disk and halo particles to disk particles only, \( M/L = \Sigma_{d+h}/\Sigma_d \propto \exp(R/h) \). Therefore, I obtain the following relation among \( h_R^M, h_R^L \) and \( h, 1/h_R^M = 1/h_R^L - 2/h \). Independent measurements of these three parameters from vertical velocity distribution, surface density distribution, and scale height distribution in the disk model of this thesis satisfy the
above relation. The observed $h_R^H = 4.4$ (LF), $h_R^K = 3.0$ kpc (KDF), and $h = 20$ kpc estimated from KDF's data for the Galaxy also satisfy the above relation.

The Toomre parameter which characterizes the stability of disks (Toomre 1964) is defined as

$$Q = \frac{\kappa(R)\sigma_R}{3.36G\Sigma_d(R)},$$

(3.1)

where $\kappa(R)$ is epicycle frequency which is derived from rotation velocity $\Omega(R)$. $\kappa = \sqrt{R \frac{d\Omega^2}{dR} + 4\Omega^2}$. $\sigma_R$ and $\Sigma_d(r)$ represent respectively radial velocity dispersion and surface density of disks. The observed value of $Q$ at $R_{\odot}$ depends on the measurement of local surface density $\Sigma_d(R_{\odot})$. $Q = 1.5 \pm 0.5$ with $\Sigma_d(R_{\odot}) = 80 \pm 20 M_\odot pc^{-2}$ (Bahcall 1984), and $Q = 2.7 \pm 0.9$ with $\Sigma_d(R_{\odot}) = 45 \pm 9 M_\odot pc^{-2}$ (Kuijken & Gilmore 1989). For other spiral galaxies, it is believed that $Q$ is a constant as a function of disk radius (Sellwood & Carlberg 1984, Martinet 1988, Bottema 1993). By assuming $(M/L)_B = 2.0$. Bottema (1993) found that $Q = 2 - 2.5$ for 12 spiral galaxies. This result coincides with the general stability criterion for galaxies as derived in numerical simulations (Athanassoula & Sellwood 1986). $Q$ measured in the disk model of this thesis is approximately constant, however, the average value of $Q$, which is equal to 2.0, 2.5, and 2.9 at $t = 0, 3.6, \text{and } 7.2 \text{ Gyr.}$ is higher than the observed value for the Galaxy.

As I have discussed in Chapter 2, due to the fact that a differentially rotating disk responds to non-axisymmetric initial disturbances in a remarkably spiral like and vigorous manner (JT), each disk particle performs as a spiral like density wake in the disk. Therefore, the main disk heating could be due to the interaction between density wakes, rather than the two-body relaxation. By measuring the relaxation time, the disk heating rate, and the shape of the velocity ellipsoid, I will investigate whether the main disk heating is due to the interaction between the density wakes of disk particles, or due to the uncorrelated orbital deflection between particles. Furthermore, by comparing the equivalent mass of density wakes measured in my simulation with that estimated by JT, I will show whether the interaction between the particle wakes are correlated.
Figure 3.5: Evolution of the velocity dispersions, $\sigma_R$, $\sigma_\phi$, $\sigma_z$, and the ratios of $\sigma_\phi/\sigma_R$ and $\sigma_z/\sigma_\phi$ at $R_\odot = 8.5$ kpc. On the top panel, the dashed lines represent a least squares fit to the data with $\sigma \propto t^{1/2}$.
CHAPTER 3. THE EVOLUTION OF AN ISOLATED GALAXY

First, I measure the relaxation time from the evolution of the velocity dispersions at $R_\odot = 8.5$ kpc. The top panel of Figure 3.5 shows that the velocity dispersions, $\sigma_R$, $\sigma_\phi$, $\sigma_z$, and $\sigma$ at $R_\odot$ increase as the square root of time. This is consistent with the age and velocity dispersion relation derived by Wielen (1977) who assumed that the local fluctuations of the gravitational field are the cause of the evolution of the stellar velocity dispersion. I define the relaxation time to be the time for the kinetic energy associated with the random motion to double. At time $t'$, the square of the velocity dispersion is modified from its expression at time $t$ according to the relation

$$\sigma^2(t') = \sigma^2(t) + \frac{d\sigma^2}{dt}(t' - t).$$

If $\sigma^2(t') = 2\sigma^2(t)$, then $T_r = t' - t = \sigma^2(t)/\frac{d\sigma^2}{dt}$. Therefore, the relaxation time depends on time $t$. At $t = 0$, 3.6 and 7.2 Gyr, the relaxation time is 4.7 Gyr, 8.3 Gyr, and 11.9 Gyr, respectively. On the other hand, the two-body relaxation time can be estimated from Equation 2.4. Using $R = 1$ model radial unit, $N=80,000$ particles, $\epsilon = 0.025$ model radial unit, and $T_c = 1$ model time unit. I obtained from Equation 2.4 the two-body relaxation time $T_{2r} = 2704$ model time units, or 162.2 Gyr. The fact that the relaxation time measured from the velocity dispersion is much shorter than the two-body relaxation time estimated from Equation 2.4 implies that the two-body relaxation is not the main source of heating in the disk. Instead, it is the interaction between particle wakes. Since the amplitude of particle wakes decreases with increasing Toomre parameter $Q (JT)$, and that $Q$ itself increases with simulation time, the measured relaxation time increases with time. Table 3.1 lists the relaxation time measured at three different times and the corresponding values of $Q$.

Secondly, I measure the disk heating rate and the shape of the velocity ellipsoid at $R_\odot = 8.5$. If the disk is mainly heated by two-body relaxation, its heating rate should be isotropic: $\frac{d\sigma_R^2}{dt} : \frac{d\sigma_\phi^2}{dt} : \frac{d\sigma_z^2}{dt} = 1 : 1 : 1$. However, the measured heating rate is $\frac{d\sigma_R^2}{dt} : \frac{d\sigma_\phi^2}{dt} : \frac{d\sigma_z^2}{dt} = 1 : 0.44 : 0.24$. On the other hand, I find that the ratio of $\sigma_R/\sigma_\phi$ measured from Figure 3.5 is 0.66, which is approximately equal to the ratio of $\kappa/\sqrt{2}\Omega = 0.67$ measured from the disk rotation curve at $R_\odot = 8.5$ kpc. The fact that $\sigma_R/\sigma_\phi$ is determined by the rotation curve also shows that the disk heating is
CHAPTER 3. THE EVOLUTION OF AN ISOLATED GALAXY

Table 3.1: The relation among $Q$, $T_r$ and $m_{\text{wake}}/m_{\text{particle}}$

<table>
<thead>
<tr>
<th>$t$(Gyr)</th>
<th>$Q$</th>
<th>$T_r$(Gyr)</th>
<th>$m_{\text{wake}}/m_{\text{particle}} = T_{2r}/T_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.0</td>
<td>4.7</td>
<td>34.5</td>
</tr>
<tr>
<td>3.6</td>
<td>2.5</td>
<td>8.3</td>
<td>19.5</td>
</tr>
<tr>
<td>7.2</td>
<td>2.9</td>
<td>11.9</td>
<td>13.6</td>
</tr>
</tbody>
</table>

not due to the two-body relaxation, but due to the disk dynamics. I also find that $\sigma_z/\sigma_\phi$ increases with the thickness of the disk, with a value of 0.73 at 10 Gyr.

I conclude from the relaxation time, the rate of disk heating, and the shape of the velocity ellipsoid, that the main sources of heating are not due to orbital deflections between disk and halo particles or deflections between disk particles only, but due to the interactions of particle wakes. This could either be due to the interactions of uncorrelated wakes of disk particles or weakly correlated wakes in the form of weak transient spiral arms. In order to investigate whether the interaction between particle wakes are correlated, I compare the equivalent mass of particle wakes computed in my simulations with that calculated by JT. The equivalent mass of particle wakes is defined as $m_{\text{wake}} = T_{2r}/T_r m_{\text{particle}}$. The values of $m_{\text{wake}}/m_{\text{particle}}$ at different $Q$ are listed in Table 3.1. For $Q = 2.0$, $m_{\text{wake}} = 34.5 m_{\text{particle}}$. On the other hand, JT found that a disk with a flat rotation curve and $Q = 2.0$ responds to a perturbative density transform $D_0$ with a maximum density transform of $10.8 D_0$. Since the spatial density at a given location is proportional to the maximum density transform, the equivalent mass of density wakes is defined as $m_{\text{wake}} \approx D_{\text{max}}/D_0 m_{\text{particle}} = 10.8 m_{\text{particle}}$. The higher equivalent mass of particle wakes found in my simulations shows that particle wake interactions are correlated in the form of weak transient spiral structures.

Finally, the total amount of heating and the heating distribution in $R$, $\phi$, and $z$ directions in the simulations of this thesis is similar to those observed in the Galaxy. For example, at $t = 5$ Gyr, the measured velocity dispersions, $\sigma_R$, $\sigma_\phi$, and $\sigma_z$, are 40, 28, and 16 km/s, which are similar to the observed velocity dispersions. 39, 23,
CHAPTER 3. THE EVOLUTION OF AN ISOLATED GALAXY

and 20 km/s. of old disk stars in the solar neighborhood (Wielen 1977). I therefore conclude on this basis that the number of disk particles in the galaxy model of this thesis is adequate for simulating thin disk dynamics.

3.4 Summary

In summary, the distributions of both surface density and velocity dispersions of the disk model are exponential, which are consistent with observed distributions for spiral galaxies. The scale length measured from the disk surface density \( \Sigma \propto \exp(-R/h_R^L) \) is shorter than that measured from the vertical velocity dispersion \( \sigma_z^2 \propto \exp(-R/h_R^M) \). \( h_R^M = 1.4 h_R^L \). This discrepancy between photometric and kinematic measurements, which is also observed in the Galaxy and NGC5247, may be due to both the non-constant scale height and mass-to-light ratio. The vertical distribution of particles follows an asymptotic exponential distribution very well, however, the scale height increases exponentially with disk radius, and it increases proportionally to the square root of time. I find that at \( R_z \), it is equal to 144, 447, and 614 pc for \( t = 0, 3.6, \) and 7.2 Gyr, respectively, which is coincident with the result obtained from O. F. and dM+dK stars. At a given location, the squared velocity dispersions increase proportionally with increasing time. For the solar neighborhood, at \( t = 5 \) Gyr, the velocity dispersions, \( \sigma_R, \sigma_\phi, \) and \( \sigma_z \) are 40, 28, and 16 km/s respectively, whereas the observed results are 39, 23, and 20 km/s (Wielen 1977). The Toomre parameter is approximate constant, and the average value of \( Q \) is approximately equal to 2.0, 2.5, and 2.9 at \( t = 0, 3.6, \) and 7.2 Gyr, respectively. It is slightly higher than that observed in the Galaxy, \( Q=1.5 \) (Bahcall 1984) and 2.7 (Kuijken & Gilmore 1989), or in other spiral galaxies \( Q = 2 - 2.5 \) (Bottema 1993). The similarity of the structure and kinematics between the disk model employed in this thesis and the Milky Way indicates that the disk model is dynamically reasonable. In the following chapters, I will employ this model to further investigate the external heating of the disk caused by infalling satellites.
Chapter 4

Satellite-Bar Interaction

4.1 Introduction

It is widely believed that bars in galaxies are formed through a global dynamical instability, discovered in N-body simulations (Hohl & Hockey 1969, Miller, Prendergast & Quirk 1970). Toomre (1981) proposed that the instability is caused by a swing-amplified feedback loop, and there are two methods to suppress the instability. One is to reduce the gain of the swing-amplifier through a high velocity dispersion in the disk or by immersing the disk in a massive halo (Ostriker & Peebles 1973). The other is to suppress the feedback through the center by adding a bulge-like mass component with a small core radius (Sellwood 1989). The method of immersing a disk in a massive halo is widely used to prevent bar instability in disks because the presence of dark halos is inferred from the observed flat rotation curves of spiral galaxies (Robin et al. 1985, van Albada et al. 1985, Kent 1987).

In the disk model of this thesis, a high halo-to-disk mass ratio, $M_H/M_D = 3.5$ is employed in order to suppress the bar instability for a very long period. However, a weak inner bar slowly develops in Model 0 after the disk evolves for 25 disk rotations as shown in Figure 4.2. Increasing the number of particles can decrease the shot noise caused by random distribution of particles, which can consequently suppress the formation of a bar for a longer period. However, for simulations run as long as 50 disk rotations in this thesis, very large $N$ simulations are computationally expensive.
CHAPTER 4. SATELLITE-BAR INTERACTION

and not necessary for the main purpose of this thesis which is to study the dynamical responses of a disk to infalling satellites, including the interaction between infalling satellite and bar. The bar phenomenon is common among the disk galaxies, including our own galaxy. De Vaucouleurs (1963) found among 994 spirals that 31% are SA, 28% are SAB, and 37% are SB. It is not unrealistic that the disk model of this thesis has a weak bar formed at the center.

It is important to consider the possible interaction between an infalling satellite and the weak bar. This is because if a satellite is introduced into the system, the satellite may excite a stronger bar in the disk than that in the isolated disk (Athanassoula 1996). Consequently, the stronger bar would heat the inner disk more than the weak bar would. Therefore, the infalling satellite could heat the outer disk directly by depositing energy, and heat the inner disk indirectly by exciting a stronger bar. On the other hand, the satellite-bar interaction may affect the orbital decay rate of the satellite, which may consequently alter the disk heating rate caused by the satellite.

The purpose of this chapter is to investigate the interaction between a weak bar formed in the disk and an infalling satellite in the standard model of this thesis. More general satellite-bar interactions were discussed by Athanassoula (1996). The effect of an infalling satellite on the formation of a weak bar in the parent disk is studied in Section 4.2, and the effect of a weak bar on the orbital decay rate of an infalling satellite is discussed in Section 4.3. A brief summary is given in Section 4.4.

4.2 Infalling satellite

A tidal interaction with an external perturber can cause the formation of a bar in the disk (Noguchi 1987) or can excite a stronger bar in the disk (Athanassoula 1996). I first compare the \( m = 2 \) bar modes of an isolated disk and of a disk with an infalling satellite. Figure 4.1 shows the evolution of the amplitude, \( |A(m = 2)| \), of the bar mode for the isolated disk (Model 0, dashed line) and for the disk with a 30% disk-mass satellite infall (Model 3, solid line). Here \( |A(m = 2)| \) is defined by
Figure 4.1: Evolution of the amplitude of the $m = 2$ bar mode for Model 3 (solid line) and Model 0 (dashed line). The bar mode of Model 3 is stronger and formed slightly earlier than that of Model 0.

$|A(m=2)| = 1/N \sqrt{\left(\Sigma_i^N \cos(2\theta_i)\right)^2 + \left(\Sigma_i^N \sin(2\theta_i)\right)^2}$, in which $N$ is the number of particles inside the turnover radius because the bars in both models end inside this radius. The bar mode for Model 3 grows slightly earlier than that for Model 0, and the resulting growth rate and the amplitude of the bar mode for Model 3 are slightly higher than those for Model 0. These results can be seen directly from Figure 4.2 which shows the evolution of the disk particles of Model 0 and Model 3. The bars in
CHAPTER 4. SATELLITE-BAR INTERACTION

Figure 4.2: Evolution of an isolated disk. Model 0 (top two rows) and of a disk with a 30% disk-mass infall, Model 3 (bottom two rows). Although weak bars formed in both models, the one in Model 3 is slightly stronger than the one in Model 0.

both models are weak but the one in Model 3 formed slightly earlier and is slightly stronger than the one in Model 0. This is because in Model 0, the m=2 bar mode is triggered by the density fluctuation arising from random distribution of particles. whereas in Model 3, both the density fluctuation and the infalling satellite are the origin of the m=2 bar mode in the disk. I therefore conclude that an infalling satellite can excite a slightly stronger bar in the parent disk than that in an isolated galaxy.
4.3 Slowdown of weak bars

It is believed that during the evolution of a barred galaxy, angular momentum redistribution among the bar, the disk, and the halo, leads to a gradual slowdown of the bar. Linear theory estimates (Weinberg 1985, Hernquist & Weinberg 1992) predict that a strong bar transfers a significant fraction of its angular momentum to the halo in only a few rotation periods. Similarly high slowdown rates were also found in the numerical simulations of Hernquist & Weinberg (1992) and Sellwood & Debattista (1996). However, Combes et al. (1990), Little & Carlberg (1991), and Athanassoula (1996) found in their simulations a lower slowdown rate: of order 30 to 40 initial bar periods are needed to slow down the bar by a factor of two. Obviously, the slowdown rate of a bar depends on the simulation models. For example, in Sellwood & Debattista’s model, the gravitation of the disk is dominant. A strong bar forms quickly, within three disk rotations, and it slows down at a high rate. However, in one of Athanassoula’s models, the gravitation of the halo is dominant. The bar forms slowly and slows down at a low rate.

In the disk model of this thesis, the bar forms extremely slowly due to the high mass ratio of the halo to disk employed in the model. Therefore, the bar slows down at an extremely low rate. I estimate the bar pattern speed, $\Omega_b$, by performing Fourier transform of $|A(t, m = 2)|$. I found that the pattern speed of bar decays linearly with time, and the bar pattern speed of Model 3 decays slightly faster than that of Model 0. This is because the bar in Model 3 is slightly stronger than that in Model 0. It is consistent with the results presented by Athanassoula (1996). However, the decay rates for both Model 0 and Model 3 are very low: $\Delta\Omega_b/\Omega_{ini}$ equals 0.11 and 0.16 respectively in 50 initial bar rotation periods.

Due to the slow decay of the pattern speed, the outer Lindblad resonance (OLR) of the weak bar of Model 3, which is located at the inner disk $R < 10$ kpc as shown in Figure 4.3, has little outwards shifting. On the other hand, the infalling satellite in Model 3 never reaches the inner disk because it is rapidly disrupted as soon as it
Figure 4.3: Rotation curve of disk and satellite and the angular speed of bar for Model 3 at t=6.0 Gyr (top line) and 10.8 Gyr (bottom line). There is little coupling between the rotation of the bar and the rotation of the satellite.

enters the disk. Therefore, there is no direct angular momentum transfer from the inner weak bar to infalling satellite. The satellite orbital decay rate is therefore not affected by the formation of a weak bar at the center of the disk.
CHAPTER 4. SATELLITE-BAR INTERACTION

4.4 Summary

In the standard model of this thesis, an infalling satellite causes a slightly stronger bar than that in an isolated disk. However, in both cases, the bars are weak and form very slowly, in approximate 25 disk rotations. This is due to the high halo-to-disk mass ratio employed in the disk model. The pattern speed of the stronger bar in Model 3 decays slightly faster than that of the weaker one in Model 0, however, the pattern speeds of weak bars in both models decay very slowly relative to those of strong bars found in previous work. Since the OLR of the weak bar remains at the inner disk, and the infalling satellite is located far from the OLR, there is no direct angular momentum transfer from the bar to the infalling satellite. Therefore, the satellite orbital decay rate is not affected by the formation of a weak bar in the disk. In other words, disk heating caused by the infalling satellite is not underestimated in the models of this thesis.
Chapter 5

Disk Tilting and Heating

5.1 Introduction

As the orbit of an infalling satellite decays, part of the orbital energy and orbital angular momentum is transferred into the disk and the halo, and the rest is carried by the satellite remnants. The orbit of the infalling satellite and its host galactic disk are usually not in the same plane. The resulting dynamical issues are what fraction of the orbital energy associated with the vertical motion of the satellite is transferred to the disk coherently: and what fraction is thermalized in the disk. In the former case, the energy transfer leads to the tilting of the disk. whereas in the latter case, it causes the thickening of the disk. In Section 5.2, I compute the disk tilting angle through the direction of the angular momentum of disk. In Section 5.3, I calculate the total kinetic energy associated with the vertical motion of the disk particles in both the initial coordinate frame and the tilted coordinate frame. so that I will be able to answer the question of whether the disk is more likely to be tilted or thickened by an infalling satellite.

5.2 Angular Momentum Transfer

First, I compare a perturbed disk (Model 3) in the initial coordinate frame and in the tilted coordinate frame with an isolated disk (Model 0) in the initial coordinate
Figure 5.1: Disk particles of Model 0 projected in $xy$, $xz$ and $yz$ planes (top row). Disk particles of Model 3 projected in both $xy$, $xz$ and $yz$ planes (middle row) and tilted $x'y'$, $x'z'$, $y'z'$ planes (bottom row) at $T=144$ time model units or 36 disk rotation periods.

In Figure 5.1, the disk particles of Model 0 are projected on the $xy$, $xz$, and $yz$ planes (top panels), and the disk particles of Model 3 are projected on the $xy$, $xz$, and $yz$ planes (middle panels), as well as on the tilted $x'y'$, $x'z'$, and $y'z'$ planes (bottom panels) at $T = 144$ model time units or $t = 8.64$ Gyr. Here $x$, $y$, and $z$ are the initial coordinates, and $x'$, $y'$, and $z'$ are tilted coordinates which are determined...
Figure 5.2: Evolution of the disk (D), satellite (S), and total (T) angular momentum directions. $x^\circ = \theta^\circ \cos \phi$ and $y^\circ = \theta^\circ \sin \phi$. After 40 disk rotations, the changes in the disk and satellite angular momentum directions are 10.6° and 9.6° respectively, and the direction drift of the total angular momentum is about 0.8°.

by the three principal axes of inertia of the tilted disk. Because the disk of Model 3 is tilted by the infalling satellite along both the $x$ and $y$ axes, the disk viewed in the initial coordinate frame as shown in the middle panels seems quite thickened by the infalling satellite, however, the real thickening in the tilted coordinate frame is not very large, as shown in the bottom panels.
CHAPTER 5. DISK TILTING AND HEATING

In order to show how the infalling satellite gradually tilts the disk to the position as shown in Figure 5.1, I plot in Figure 5.2 the evolution of the angular momentum directions of the disk, the satellite, and the whole system. For a given angular momentum \( \vec{L}(\theta, \phi) \), its direction is plotted as a point in polar coordinates \((\theta, \phi)\). In the figure, \( x^\circ = \theta^\circ \cos \phi \) and \( y^\circ = \theta^\circ \sin \phi \). The changes of the angular momentum directions of the disk and the satellite in the \( x \) coordinate show that the angle between the disk plane and the satellite orbital plane decreases due to the angular momentum transfer between them. Also it can be seen from the figure that the disk plane and the satellite orbital plane are precessing in opposite directions because the disk and the satellite exert an opposite torque on each other. I find that after 40 disk rotations, the directions of the disk and satellite angular momenta shifted 10.6° and 9.6° respectively. During the same period, the drift of the direction of the total angular momentum, due to the accumulated integration error, is very small. It is 0.8° in 40 disk rotations, or on the average, 0.7" per timestep.

In order to follow the angular momentum transfer among the satellite, the disk, and the halo, I measure the relative changes in both the magnitude and the direction of the angular momentum of each component. I first consider the results of a simple case which involves the angular momentum transfer between a disk and a halo in an isolated galaxy, namely Model 0. Since the initial rotation of the halo is nearly zero, the disk rotation angular momentum is almost the total angular momentum of the whole system. As the galaxy evolves with time, the disk gradually transfers its angular momentum to its surrounding halo at a steady, low rate of 0.12% per rotation. By the end of the simulation, 40 disk rotations, due to the formation of the weak bar in the center of the disk as discussed in Chapter 4, the disk has transferred 4.6% of its angular momentum to its surrounding halo. Conservation of total angular momentum is very good: there is no more than a 0.06% variation during the 40 disk rotations.

When a satellite is accreted onto the isolated galaxy, there is angular momentum transfer among the satellite, the disk, and the halo. Figure 5.3 shows the evolution of
the angular momenta of the satellite, the disk, and the halo for Model 1, Model 2, and Model 3. The left and right panels show respectively the evolution of the magnitudes and the direction of the various angular momenta. Compared with the isolated disk, the disks with satellites transfer more of their angular momenta to their halos due to the stronger bars excited by the satellites. The amount of the angular momentum loss depends on the strength of the bars. The disks also absorb angular momenta from their satellites, and the amount of angular momentum gain depends on the masses of the satellites. As a result, by the end of 40 disk rotations, the magnitudes of the disk angular momenta of Model 1, Model 2, and Model 3 have decreased by 6.8%, 5.3%, and 6.2%. Although the bar in Model 2 is stronger than that in Model 1, the net angular momentum loss of the disk in Model 2 is less than that in Model 1. This is because the disk in Model 2 absorbs some angular momentum from its satellite, whereas the disk in Model 1 absorbs no angular momentum from its satellite, which is tidally disrupted before it reaches the disk. The disk in Model 3 absorbs more angular momentum from its satellite than does the disk in Model 2. However, in this case, the net loss of angular momentum is greater due to the stronger bar, which transfers more of its angular momenta to the surrounding halo.

Due to the large initial distances between satellites and disks, the ratios of the disk angular momenta to the satellite angular momenta are much larger than their corresponding mass ratios. In Model 1, Model 2, and Model 3, the ratios of satellite angular momentum to disk angular momentum are 43%, 90%, and 130%, whereas the corresponding satellite to disk mass ratios are only 10%, 20%, and 30%. The large angular momenta of the satellites make disk tilting very easy. The disks of Model 1, Model 2, and Model 3 are tilted by angles of $(2.9 \pm 0.3)^\circ$, $(6.3 \pm 0.1)^\circ$, and $(10.6 \pm 0.2)^\circ$ respectively.

Now, I discuss the evolution of the angular momenta of the satellites. Since the dynamical friction exerted on an infalling satellite is proportional to the square of the satellite mass, a heavier satellite is subjected to a stronger dynamical friction. This stronger dynamical friction, in turn, results in greater angular momentum loss...
Figure 5.3: Angular momenta of the satellite, the disk, and the halo versus time. The evolution of the angular momenta of the three components of Model 1, Model 2, and Model 3 is shown from top to bottom. Most of the initial angular momenta of the satellites remain in the satellite remnants, and the halos absorb angular momentum from both the disks and the satellites.
from the satellite. For a rigid or point mass satellite, the orbital angular momentum of the satellite is completely transferred to the disk and halo system. However, for a self-consistent low density satellite, most of the orbital angular momentum is kept as rotational angular momentum of the satellite remnants due to tidal stripping. In the case of 10%, 20%, and 30% disk-mass satellites, I find that by the end of the simulations they have lost respectively 3.4%, 6.9%, and 11.1% of their angular momenta. The majority of the angular momenta of the satellites is carried by their remnants. The directions of the satellite angular momenta have changed respectively by angles of 6.3°, 7.9°, and 9.6° relative to their initial directions in Model 1, Model 2, and Model 3. As for the halo angular momentum, it reaches a fraction approximately equal to 10% of the total angular momentum of the system for all three models. But in the heavier satellite model, the halo angular momentum is slightly larger because the halo absorbs more angular momentum from the satellite. Clearly, the halo absorbs angular momentum very efficiently from both its satellite and its embedded disk.

5.3 Energy Transfer

The infalling satellite not only tilts the disk through the transfer of angular momentum between the satellite and the disk, but also heats the disk by depositing energy. In this thesis, I only investigate the disk heating in the vertical direction. Figure 5.4 shows the kinetic energy associated with the vertical motion of disk particles in the initial coordinate frame, $k_z$, and in the tilted coordinate frame, $k_{\gamma}$, for Model 1, Model 2, and Model 3. Here the tilted coordinate frame is constructed by employing the principal axes of the inertia tensor of the disk. I find that the 10%, 20%, and 30% disk-mass satellite infall increases the thermal energy associated with random motion of disk particles in the tilted coordinate frames by only $(4 \pm 3)\%$, $(6 \pm 2)\%$ and $(10 \pm 2)\%$ respectively, as compared to the isolated model. However, in contradistinction, the kinetic energy associated with the vertical motion of the disk particles in the initial coordinate frames is increased by $(6 \pm 3)\%$, $(26 \pm 3)\%$, and $(51 \pm 5)\%$
Figure 5.4: Kinetic energy associated with the vertical motion in the initial coordinate frame $k_z$ and in the tilted coordinate frame $k_{z'}$ as a function of time for, from top to bottom, Model 1, Model 2, and Model 3. After a satellite which has 10%, 20% or 30% disk mass falls into the disk, the kinetic energy associated with the vertical random motion of the disk $k_{z'}$ is increased by only $4 \pm 3\%$, $6 \pm 2\%$ or $10 \pm 2\%$ respectively, as compared to an isolated galaxy.
respectively, as compared to the isolated model. Consequently, the disks are mainly tilted rather than heated by the infalling satellites.

In order to study the distribution of the thermal energy associated with the random vertical motion of disk particles. I plot, in Figure 5.5, the evolution of the vertical velocity dispersion of the disks as a function of disk radius in the global tilted coordinate frame. The disks are not heated uniformly as a function of radius by the infalling satellites. I find that both the outer and the inner regions of the disks are heated more than the other regions. The satellites heat the outer disks directly by transferring energy to the outer disks and heat the inner disks indirectly by exciting slightly stronger bars. Note that a fraction of the heating of the inner disk shown in Figure 5.5 is artificial due to the mismatch between the inner local tilted coordinate frame and the one employed in the computation. As there is warping in the disk, the tilting angles of the inner and outer parts of the disk are different from each other, and they are also different from the tilting angle of the disk as measured in the tilted coordinate frame, defined by the principal axes of the total inertia tensor. In fact, the direction of the axes in the tilted coordinate frame is mainly fixed by the outer part of the disk (inertia increases quadratically with distance). To explicitly verify which fraction of the thickening of the inner part of the disk is caused by real thermal effects and which part is an artifact associated with the choice of reference frame. I study the vertical velocity dispersion in the local tilted coordinate frame in Chapter 7.

The results of Figure 5.5 can be summarized as follows: the evolution of the vertical velocity dispersion in the disk subjected to the infall of a 10% disk-mass satellite is almost the same as the one in the isolated galaxy, and there is no detectable thickening in the disk. A disk is only slightly thickened by the 20% disk-mass satellite. However, the infall of a 30% disk-mass satellite definitely causes detectable thickening of the disk, especially in its outer region. Finally, in order to compare this work with previous work (TO, QHF and WMH) that suggested that a dense 10% disk-mass satellite is sufficient to thicken a thin disk, I show in the last panel of Figure 5.5 the disk heating caused by a dense 10% disk-mass satellite (Model 10). The satellite is
CHAPTER 5. DISK TILTING AND HEATING

Figure 5.5: Vertical velocity dispersion as a function of radius and time for Model 1, Model 2, Model 3, and Model 10 (solid line) compared with that for Model 0 (dashed line). In each panel, from bottom to top, the distribution of the vertical velocity dispersion at $t = 0$, $3.6$, and $7.2$ Gyr is plotted. Only the 30% disk-mass satellite and the dense 10% disk-mass satellite can cause detectable thickening in the outer part of the disks.
CHAPTER 5. DISK TILTING AND HEATING

introduced into the system at the edge of the disk, \( r_+ = 1.0 \), and its half-mass radius. compared with that of the satellite in Model 1. is decreased by 50\%. I find that the disk heating caused by the 10\% disk-mass high density satellite is even larger than that caused by the 30\% disk-mass but low density satellite. Therefore. the density of an infalling satellite is a crucial parameter in determining the amount of external heating caused by the infalling satellite.

5.4 Summary

Due to the angular momentum transfer between the disks and their satellites. the disks in Model 1. Model 2 and Model 3. are tilted by angles of \((2.9 \pm 0.3)^\circ\), \((6.3 \pm 0.1)^\circ\) and \((10.6 \pm 0.2)^\circ\) respectively by the end of 40 disk rotation periods. In the case of the satellites. they have changed respectively by angles of \(6.3^\circ\), \(7.9^\circ\) and \(9.6^\circ\) relative to their initial direction in Model 1. Model 2 and Model 3 in the same period. As for the magnitude of these satellite angular momenta. they have lost respectively 3.4\%, 6.9\% and 11.1\% of their angular momenta. The remaining angular momenta of those satellites are left with their remnants.

The answer to the question that I posed at the beginning of this chapter is that a large fraction of the orbital energy associated with the vertical motion of the satellite is added to the disk coherently and a small fraction is thermalized in the disk. A 10\%. 20\% and 30\% disk-mass satellite infall increases the thermal energy associated with random motion of disk particles in the tilted coordinate frames by only \((4 \pm 3)\%\), \((6 \pm 2)\%\) and \((10 \pm 2)\%\) respectively. as compared to the isolated model. However. in contradistinction, the kinetic energy associated with the vertical motion of disk particles is increased respectively by \((6 \pm 3)\%\), \((26 \pm 3)\%\) and \((51 \pm 5)\%\) in the initial coordinate frames as compared to the isolated model. Therefore, there is more added energy related to the tilting of the disk than that related to the thickening of the disk.

The evolution of the vertical velocity dispersion in the disk with the infall of a 10\% disk-mass satellite is almost the same as the one in the isolated galaxy so there
is no detectable thickening in the disk. The disk is slightly thickened by the 20% disk-mass satellite and the 30% disk-mass satellite infall definitely causes detectable thickening in the disk, especially at the outer part of the disk.
Chapter 6

Warping

6.1 Introduction

Galactic warps are very common. For example, all three spiral galaxies in the local group are warped, and more than half of all disk galaxies appear to be warped (Sánchez-Saavedra, Battaner & Florido 1990, Bosma 1991). Therefore, warps are either long-lived or recently excited (excellent review papers are written by Toomre (1983), Binney (1990) and Nelson & Tremaine (1995b)). However, studies show that warps are damped in a time period much shorter than the Hubble time (Nelson & Tremaine 1995a, Dubinski & Kuijken 1995) due to the fact that short-wavelength bending waves are damped by wave-particle resonances (Hunter & Toomre 1969, Weinberg 1991, Nelson & Tremaine 1995b), while long-wavelength bending waves are strongly damped by dynamical friction from an oblate halo that is either non-rotating or rotating in the same direction as the disk (Nelson & Tremaine 1995a). Therefore, warps are unlikely to be long-lived, and they must be excited recently. Nelson & Tremaine (1995b) summarized the following four mechanisms which can excite warps: excitation by dynamical friction from the halo, if the halo and disk rotate in the opposite direction (Nelson & Tremaine 1995a); gravitational noise from halo (Nelson & Tremaine 1995b); Coriolis force from a twisting halo (Ostriker & Binney 1989); tidal fields from satellites (Burke 1975, Kerr 1975, Hunter & Toomre 1969, Bertin & Mark 1980, Lynden-Bell 1985, Weinberg 1995). In this chapter, the
warps excited by the tidal field of a satellite will be discussed.

The simulations in this thesis demonstrate that warps are excited by an infalling satellite. In Figure 6.1, I show the evolution of the disk and satellite particles of Model 2, projected on the $xz$ plane. Since the orbit of the satellite is inclined $30^\circ$ with respect to the disk, the angular momentum transfer between the disk and the satellite causes the outer part of the disk to tilt faster than the inner part. Therefore, a slight warp appears after $T=60$. As the system evolves in time, the inner part of the disk is tilted in the same plane without precessing because the gravitational influence of the infalling satellite is small. However, the outermost part of the disk has a greater tilting angle than the inner part, and it is subject to a slow precession under the torque imparted by the infalling satellite. The projected disk appears slightly thicker than it actually is in Figure 6.1, because the disk is not only tilted along the $x$ axis, but also along the $y$ axis.

In this chapter, I first investigate the kinematics of the warps in this chapter by employing two models: one, which I call the ring model discussed in Section 6.2, in which the disk is subdivided into rings and the evolution of these rings is studied; the other, which I call the particle model discussed in Section 6.3, in which the motion of an individual particle is followed. A brief summary is given in Section 6.4.

6.2 The Ring Model

I first construct a ring model in order to investigate the kinematics of the warps. A disk is subdivided into rings and the evolution of these rings is studied. I regroup disk particles into 10 rings ($i=0-9$). The particles located between $R(i)$ and $R(i+1)$ form Ring $i$, where $R(i)$ is defined by $R(i) = 0.2 \times 1.25^i$. The disk particles are divided in this way so that there is a sufficient number of particles in each ring to make a reliable estimate of the moment of inertia tensor. For each Ring $i$, the principal axes of its inertia tensor are used as local coordinate axes $(x'_i, y'_i, z'_i)$. In Figure 6.2, I plot 10 rings according to their new coordinates $(x'_i, y'_i, z'_i)$ at $T = 100$ model time units.
Figure 6.1: Evolution of the disk and satellite particles of Model 2 as viewed in the $xz$ plane. The dominant halo particles are not plotted. Note the slight warp for $T > 60$ model time units, or $t > 3.6$ Gyr. The size of the boxes is 6 model radial units, or 96 kpc.
or \( t = 6 \) Gyr. The figure shows that the larger the radius of the ring, the more it is tilted. In other words, the warps can be seen very clearly in the figure. The `+` symbol on each ring indicates the direction of the \( y'_i \) axis. The outer rings, Rings 7, 8, and 9, show a clear warp, and the directions of their \( y'_i \) axes indicate the trend of precession.

I then study the evolution of the warps by following the orientations of individual rings. The orientation of ring \( i \) is specified by the two Eulerian angles \( \theta_i \) and \( \phi_i \).
CHAPTER 6. WARPING

Figure 6.3 shows the evolution of $\theta$ and $\phi$ for two typical rings. I select Ring 6 to show the typical behavior of inner rings and Ring 9 to represent the typical behavior of outer rings. I observe that Ring 6 is tilted with some nutation and without precession, which means that the inner part of the disk is gradually tilted in one direction without precession. On the other hand, Ring 9 is tilted while undergoing some slow precession because the outer ring is tilted away from the rest of disk and starts to precess under the torque imparted by its displacement. Since the inner disk is slowly tilted towards the outer disk, the warps can last for a very long time. As shown in Figure 6.1, the warp caused by the satellite in Model 2 lasts for more than 30 disk-rotation periods. I conclude that infalling satellites can excite warps in the disk. However, the warps eventually fade away due to the fact that both the inner disk and outer disk are tilted towards the same plane.

6.3 The Particle Model

An alternative way to study the kinematics of warps is to follow the directions of orbital angular momenta of individual disk particles which are located at different radii. I have plotted in Figure 6.4 the evolution of the directions of the angular momenta in polar coordinates $(\theta, \phi)$ for four typical particles as well as the traces of their positions. In the upper left panel, I consider the case in which a particle is located in the disk. The motion of the particle is mainly circular with little inwards shifting caused by the decreasing of rotation velocity and the increasing of random velocity due to two-body relaxation. However, the direction of the angular momentum shows clear evolution. The mean value of $\theta$ increases with time, which means that the orbit of the particle is gradually tilted by the infalling satellite. An increase of the dispersion of $\theta$, with movement from the origin, indicates that the particle is gradually heated due to two-body relaxation. There is no precession of the angular momentum for such a particle. In the disk, the angular momentum of most particles evolves in this way, which implies that most of the disk does not precess. This result
Figure 6.3: Evolution of the two Eulerian angles (φ, θ) plotted in the polar coordinates frame of Ring 6 and Ring 9. Ring 6 is tilted with some nutation, but does not undergo precession. Ring 9, however, precesses very slowly.
Figure 6.4: Evolution of the angular momentum directions ($\theta^o, \phi^o$) plotted in the polar coordinate frame for four typical disk particles. The traces of their positions ($r, \phi^o$) are also plotted in the polar coordinate frame, which are shown in the upper right corner of each panel. The symbol * indicates the initial positions and angular momentum directions of the particles. The simulation is run for 50 rotation periods.

is consistent with the conclusion of our ring model study, which suggests that the innermost rings do not precess.

In the upper right panel of Figure 6.4, I consider the case of a particle which is
very close to the edge of the disk. The particle orbits with increasingly greater radius around the center of the disk due to the increment of angular momentum caused by the infalling satellite. As the particle moves outwards, its mean orbital plane is gradually tilted away from the rest of the disk because the direction of the angular momentum absorbed from the satellite is not aligned with the direction of the initial angular angular momentum. Its angular momentum starts to precess under the torque imparted by the infalling satellite. This result is also consistent with the result that I have described for the ring model, namely that the outermost rings precess.

The particle shown in the lower right panel has a motion and an evolution of the angular momentum which is similar to the one depicted in the upper left panel, except that the inward motion and the standard deviation of \( \theta \) are larger in this case. This follows the fact that since \( \theta \) is defined as \( \cos^{-1}(l_z/l) \), \( \theta \) becomes very large for a particle with small \( l_z \), as is the case in this example of a particle located near the center of the disk.

Finally, in the lower left panel, I consider a particle which is initially located at the edge of the disk. I observe that its mean orbital plane is easily tilted by the infalling satellite and that the angular momentum precesses under the torque imparted by the infalling satellite.

### 6.4 Summary

I conclude, from the analysis of the results obtained with both the ring and particle models that the infalling satellite can excite warps in the disk. However the warps eventually fade away due to the fact that both the inner disk and outer disk are finally tilted towards the same plane. Therefore, the warps are unlikely to be long lived phenomena. The fact that most galaxies are warped could be an indicator that disks are, in fact, being subjected to a recent infall of satellites, though the infalling satellites may not be easily detected because of tidal disruption.
Chapter 7

Measurement of Disk Heating

7.1 Introduction

Since warping in the disk is excited by an infalling satellite, it is necessary to consider a local tilted coordinate system in order to study the disk thickening and its time dependence. Traditionally, there are two methods to measure the disk thickening. One is to measure the scale height of the disk, \( h_z \), and the other is to measure the vertical velocity dispersion, \( \sigma_z \). These two measurements are usually equivalent because if the presence of a dark matter halo is ignored, the relation between \( h_z \) and \( \sigma_z \) is

\[
\sigma_z^2 \propto \Sigma(R,z) h_z, \tag{7.1}
\]

where \( \Sigma(R,z) \) is the surface density of the disk. This relation is derived from the vertical component of the Jeans equation,

\[
-\frac{\partial \Phi}{\partial z} = \frac{1}{\nu} \frac{\partial}{\partial z} (\nu \sigma_z^2), \tag{7.2}
\]

in which \( \Phi \) is the potential field, and \( \nu \) is the space density of particles. On the left hand side of the equation, \(-\frac{\partial \Phi}{\partial z} = 4\pi G \int_{-z}^{z} \rho(R,z) = 4\pi G \Sigma(R,z)\), and on the right hand side of the equation, \( \frac{1}{\nu} \frac{\partial}{\partial z} (\nu \sigma_z^2) = \sigma_z^2/h_z \) under the assumption that \( \sigma_z \) does not vary with \( z \). By substituting the above two equations in Equation 7.2, I obtain
Figure 7.1: Densities of disk and halo versus radius. The dashed line is an exponential fit of the disk density.

Relation 7.1.

However, Relation 7.1 may not be valid across the disk in a typical model of this thesis because the contribution from halo particles may not be ignored, especially near the edge of the disk where the density of the halo is much larger than that of the disk as shown in Figure 7.1. In the figure, the densities of the disk and halo versus radius for Model 0 are plotted. By replacing $\nu$ with $\nu_d + \nu_h$ and still assuming that $\sigma_z$ does not vary with $z$, I obtain that the right hand side of Equation 7.2 is equal
to \( \sigma^2 = \frac{1}{v_d + \nu_h} \frac{\nu_d^2}{\frac{\nu_d}{\nu_h}} (\nu_d + \nu_h) \). If \( \nu_d \approx \nu_h \) and \( \frac{\nu_d}{\nu_h} \gg \frac{\nu_d}{\nu_h} \), Relation 7.1 is still valid, but if \( \nu_d \ll \nu_h \) near the edge of disk, Relation 7.1 is no longer valid.

I will measure both \( h_z \) and \( \sigma_z \) in this chapter. To measure either \( h_z \) or \( \sigma_z \) is a two-step procedure: first, to determine the local tilted coordinate system via the direction of angular momentum or the direction of the maximum principal axis of the inertia moment: second, to measure the thickness in the determined local tilted coordinate system. I will develop, in Section 7.2, an one-step method in which the vertical velocity dispersion is measured in the local tilted coordinate system, but without computing the exact orientation of the tilted coordinate system. The thickness of the disk measured from the traditional two-step method is also given in Section 7.3. Finally, I will briefly summarize the results in Section 7.4.

### 7.2 Vertical Velocity Dispersion

First, I demonstrate in Figure 7.2 the one-step measuring method by using a simple example. I assume that some particles are uniformly distributed in a ring, as shown in Figure 7.2a, and that their average velocities in spherical coordinates are \( \bar{v}_r = 0 \), \( \bar{v}_\phi = 0 \), and \( \bar{v}_\theta = 0 \). The velocity dispersions of the particles are \( \sigma_{v_r} \), \( \sigma_{v_\phi} \), and \( \sigma_{v_\theta} \). The direction of the angular momentum \( \bar{L}(\theta_i, \phi_i) \) of particle \( i \) is plotted in Figure 7.2c as a point \((\bar{\theta}_i, \phi_i)\) in polar coordinates. The plotted circles correspond to the standard deviation of the direction of the angular momentum, \( \sigma_\theta \), of the particles located in the ring. Here, \( \theta_i \) is defined by the following equation \( \tan(\theta_i) = l_\phi/l_\theta = v_\phi/v_\theta \), in which \( v_\theta \) and \( v_\phi \) denote the vertical and circular velocities respectively. Since \(|v_\theta|/v_\phi \leq 0.1 \) usually, I can write \( \theta_i \approx \tan(\theta_i) = v_\phi/v_\theta \). Therefore, \( \sigma^2_\theta = \sigma^2_{v_\phi}/v^2_\phi + \sigma^2_{v_\phi}/v^4_\phi \). Furthermore, since \( v^2_\phi/v^2_\phi \approx 0.01 \), and \( \sigma^2_{v_\phi}/\sigma^2_{v_\phi} \approx 4.0 \), the second term is very small in comparison to the first term and can therefore be ignored. Finally, I obtain the following relation, \( \sigma_\theta \approx \sigma_{v_\phi}/v_\phi \), which indicates that the standard deviation in \( \theta \) is proportional to the vertical velocity dispersion.

If the ring of particles is simply tilted by an infalling satellite by an angle \( \Theta \)
Figure 7.2: Particles located on the original ring (a) and the tilted ring (b). The directions, in polar coordinates \((\theta, \phi)\), of the angular momenta \(\vec{L}\) of these particles are plotted in the panels of c and d.

As shown in Figure 7.2b, the resulting \(\sigma_{\theta'}\) remains unchanged, though \(\bar{\theta}_i\) of particle \(i\) is changed into \(\bar{\theta}'_i = \bar{\theta} + \tilde{\theta}_i\) (Figure 7.2d). However, if the particles in the ring are also heated by the infalling satellite, the value of \(\sigma_{\theta'}\) increases proportionally with increasing \(\sigma_{v'_{\theta}}\), where \(v'_{\theta}\) is the vertical velocity dispersion in the local tilted coordinates. Therefore, I can obtain the increment of vertical velocity dispersion in the local tilted coordinates by measuring the increment of \(\sigma_{\theta'}\).
CHAPTER 7. MEASUREMENT OF DISK HEATING

Finally, I compare the evolution of $\sigma_\theta$ of Model 2 and Model 3 with that of Model 0, to calculate their relative increments of velocity dispersion caused by infalling satellites in Figure 7.3. I find that the infall of the 20% disk-mass satellite mainly causes heating near the center and edge of the disk. In the case of the infall of the 30% disk-mass satellite, the vertical velocity dispersion is increased in the entire disk, but the outer regions of the disk are heated much more than the inner regions. I find that, when averaged over radii greater than the half-mass radius, the velocity dispersion of the disk of Model 2 is increased by 5%, whereas that of Model 3 is increased significantly more to 28%. For radii smaller than the half-mass radius, the average increment in velocity dispersion of Model 3 is 6% whereas that of Model 2 is 4%.

7.3 Thickness

In this section, I measure the thickness of disk in local tilted coordinate frames following a two-step procedure. First, the disk is divided into rings, and their local tilted coordinate frames are determined by the principal axes of their inertia tensors. The thickness of each ring is then measured in the local tilted coordinate frame. The disk is uniformly divided into 10 rings from radius, $R = 0.0$ to $R = 20$ kpc. For each ring, the principal axes of its inertia tensor are employed as its local tilted coordinate axes, and the thickness of the disk is measured in the local tilted coordinate frame. The thickness of disk is defined by the scale height, $h_z$, of asymptotic exponential distribution, $\text{sech}^2(z/h_z)$. It is shown in Figure 3.2 that the particle distribution in vertical direction fits the asymptotic exponential distribution very well. Therefore, $h_z$ is a good description of the thickness of disk.

I plot, in Figure 7.4, the scale height, $h_z$, versus radius, $R$, for Model 2 and Model 3 (solid lines) compared with that for Model 0 (dash lines) at $T=0$, 60, and 120 time model units. I find that when averaged over radii greater than the half-mass radius, the thickness of the disk of Model 2 is increased by 7% compared with that of Model
Figure 7.3: Evolution of the dispersions of $\theta$ of Model 2 and Model 3 in comparison with that of Model 0. In each panel, the distributions of the dispersion of $\theta$ at $T = 0$, 60, and 120 model time units are plotted.
Figure 7.4: Disk scale height, $h_z$, versus radius, $R$, for Model 2 and Model 3 (solid line) compared with that of Model 0 (dashed line) at $T=0.60$, and 120 time model units.
CHAPTER 7. MEASUREMENT OF DISK HEATING

0. whereas that of Model 3 is increased significantly more to 21%. For radii smaller than the half-mass radius, the averaged increment in the thickness of disk of Model 3 relative to that of Model 0 is 12% whereas that of Model 2 is only 6%.

7.4 Summary

I have measured the disk heating caused by infalling satellites in Model 2 and Model 3 through both the increments of vertical velocity dispersion and scale height relative to those of Model 0. I find in Model 2 that the infall of the 20% disk-mass satellite mainly causes heating near the center and edge of the disk. In the case of the infall of the 30% disk-mass satellite in Model 3, the entire disk is heated, but the outer regions of the disk are heated much more than the inner regions. Inside the half-mass radius of the disk, Relation 7.1 is approximately valid because the thickness increment is approximately twice that of the increment of the vertical velocity dispersion. In that region, although the densities of halo and disk are similar, the derivative of density in vertical direction of the halo are much smaller than that of the disk, and therefore is negligible. However, outside the half-mass radius of disk, half of the thickness increment is much less than the increment of vertical velocity dispersion. This is because the density of the halo is larger than that of disk, and Relation 7.1 is not valid.
Chapter 8

Orbital Decay and Tidal Stripping of Satellites

8.1 Introduction

I have investigated the dynamical responses of a disk to infalling satellites in previous chapters. In this chapter, I will study the dynamical effects of the parent galaxy on infalling satellites, namely orbital decay and tidal stripping. In the models of this thesis, the orbital decay of infalling satellites is mainly caused by dynamical friction against the dark matter halo because the satellites are rapidly disrupted when they enter the parent disk. However, in previous work (QG, QHF, WMH), dense satellites are introduced at the edge of their parent disk, so the dynamical friction against the halo is not important, and the drag on the satellites is contributed by the disk. In those cases, the drag is due to the following three important mechanisms (QG): dynamical friction against the disk, second-order perturbative torque at Lindblad resonances (Lynden-Bell & Kalnajs 1972, Goldreich & Tremaine 1982), and nonperturbative horseshoe orbits. A more detailed discussion can be found in QG.

In order to understand the simulation results which will be presented in this chapter, I will briefly estimate, in Section 8.2, the orbital decay rate for a satellite with deceasing mass due to tidal stripping. In Section 8.3, I will first present results of orbital decay and mass stripping for satellites on nearly circular orbits but different
orbital inclinations with respect to the parent disk plane. I will then discuss the results for satellites having different masses on an elliptical orbit. Finally, I will compare the orbital decay rates for a satellite on both direct and retrograde orbits. The distribution of satellite remnants in physical and phase space is studied in Section 8.4. Finally, a brief summary is given in Section 8.5.

8.2 Theoretical Estimates

As the orbit of a satellite decays inside the halo, the mass of the satellite decreases due to tidal stripping. I will first show that the fraction of remaining mass of the satellite, \( M(r_t)/M_s \), is proportional to the fraction of halo mass within the location of the satellite, \( M(r)/M_h \), if the satellite and the parent galaxy have self-similar density profiles. Based on this result and Chandrasekhar's formula, I will show that the orbital decay rate is a constant as a function of the distance between the center of the satellite and the center of its parent galaxy, and it is proportional to the initial mass of the satellite.

Since the density profile of the King model cannot be expressed explicitly, I use the following similar density profile as a substitute, \( \rho = \frac{3}{4\pi G} \sigma_s^2/(r^2 + r_h^2) \) for \( r \leq r_{size} \) and \( \rho = 0 \) for \( r > r_{size} \). I assume that the tidal radius of the satellite, \( r_t \), is defined by the Jacobi limit (Binney & Tremaine 1987). I then have the relation between the mean density of the satellite inside the tidal radius and the mean density of the halo inside \( r \) where the satellite is located.

\[
\bar{\rho}_s(r_t) \approx 3\bar{\rho}_h(r).
\] (8.1)

By substituting \( \bar{\rho}_s(r_t) = \frac{3}{4\pi G} \sigma_s^2(1 - (r_K, s/r_t)\tan^{-1}(r_t/r_{K,s}))/r_t^2 \approx \frac{3}{4\pi G} \sigma_s^2/r_t^2 \) for \( r_t > r_{K,s} \) and \( \bar{\rho}_h(r) \approx \frac{3}{4\pi G} \sigma_h^2/r^2 \) for \( r >> r_{K,h} \) in the above equation, I obtain the relation between \( r_t \) and \( r \), \( r_t \approx (1/\sqrt{3})(\sigma_s/\sigma_h)r \). By using Equation 2.1 in the above relation, I have the following relation: \( r_t/r_s \approx (1/\sqrt{3})(r/r_h) \), where \( r_s \) and \( r_h \) are the sizes of the satellite and the halo respectively. Since the ratio of the remaining mass to the
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total mass of the satellite is \( M(r_t)/M_s \approx (r_t/r_s) \), and the ratio of the mass inside \( r \) to the total mass of the halo is \( M(r)/M_h \approx (r/r_h) \). I have the relation between these two mass ratios.

\[
M(r_t)/M_s \approx (1/\sqrt{3})M(r)/M_h.
\]

(8.2)
or \( 1 - M(r_t)/M_s \approx (1/\sqrt{3})(1 - M(r)/M_h) \). Thus, as the orbit decays, the mass loss of the satellite is roughly proportional to the halo mass which the satellite traverses. In reality, the mass loss of the satellite can be higher than our estimate because of the eccentric orbit, tidal distortion and evaporation of the satellite.

Chandrasekhar (1943) showed that a particle of mass \( M_s \) moving through a homogeneous background of individually much lighter particles with an isotropic velocity distribution suffers a drag force.

\[
F_d = -\frac{4\pi G^2 M_s^2 \rho(< v_s) \ln \Lambda}{v_s^2}.
\]

(8.3)

where \( v_s \) is the speed of the satellite with respect to the mean velocity of the field, and \( \rho(< v_s) \) is the total density of the field particles with speed less than \( v_s \). The parameter \( \Lambda = p_{\text{max}}/p_{\text{min}} \), where \( p_{\text{max}} \) is conventionally taken to be the half-mass radius of the field system, and \( p_{\text{min}} \) is the larger of the two scales \( p_{90} \approx GM_s/v_s^2 \) and the half-mass radius of the satellite (White 1976). Detailed studies (Lin & Tremaine 1983, White 1983, Tremaine & Weinberg 1984, Weinberg 1986, Bontekoe & van Albada 1987, Zaritsky & White 1988) show that Equation 8.3 gives a reliable estimate of the rate of orbital decay. On the other hand, Weinberg (1989), Hernquist & Weinberg (1989), and Prugniel & Combes (1992) showed that Chandrasekhar's formula may not adequately represent the orbital decay of an extended satellite in general due to self-gravitating of the halo and tidal deformation of the satellite. Since I will discuss the relation among the orbital decay rate, the distance between the satellite and parent galaxy, and the initial mass of the satellite, rather than calculate accurate decay time, Equation 8.3 is employed in following discussion.

By assuming that the halo is an isothermal sphere with one dimensional velocity
dispersion $\sigma$, and the satellite follows a slowly decaying circular orbit. Binney & Tremaine (1986) obtained (Equation 7.25)

$$\frac{dr}{dt} = -0.428 \frac{G M_s}{r v_c^2} \ln \Lambda, \quad (8.4)$$

where $v_c = \sqrt{2}\sigma$. Equation 8.4 indicates that the orbital decay rate $\frac{dr}{dt}$ is inversely proportional to the distance between the center of the satellite and the center of its parent galaxy, however, it is proportional to the mass of the satellite. In the models of this thesis, since self-similar density profiles are employed for the satellite and the halo, tidal stripping is unavoidable. The mass of the satellite decreases as its orbit decays, as shown in Equation 8.2. By replacing $M_h(r) \approx \frac{v_c^2 r}{G}$ in Equation 8.2. I obtain the following equation

$$M_s(r_t) = \frac{1}{\sqrt{3}} \frac{M_s v_c^2 r}{M_h G}, \quad (8.5)$$

Equation 8.4 therefore can be rewritten as

$$\frac{dr}{dt} = -0.246 \frac{M_s}{M_h} v_c \ln \Lambda. \quad (8.6)$$

Equation 8.6 shows that, due to the tidal stripping as approximated for the model of this thesis, the decay rate of the orbital radius is a constant as a function of the distance between the center of the satellite and the center of its parent galaxy, and it is proportional to the initial mass of the satellite.

### 8.3 Simulation Results

I first study the orbital decay rate of a satellite on a nearly circular orbit. However the orbital inclination with respect to the parent galactic plane is 30°, 60°, and 90° for Model 4, Model 5, and Model 6 respectively. The satellites have 10% of the disk mass and start at a distance of 2.5 model radial units from the center of the parent galaxy. Figure 8.1 shows orbital decay and mass stripping for Model 4 (top), Model 5 (middle), and Model 6 (bottom) respectively. In the figure, the mass inside
Figure 8.1: Orbital decay and mass stripping of satellites in Model 4 (top), Model 5 (middle), and Model 6 (bottom). The first column shows that the mass inside the initial half-mass radius of satellite decreases linearly with the decreasing distance between the center of the satellite and the center of the parent galaxy. The second column shows that the orbital decay rate is approximately constant. Hence, the mass loss rate is also approximately constant, as shown in the third column.
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the initial half-mass radius of the satellite is normalized according to the following relation. \( f = m(r_{\text{half}})/m_{\text{total}} \). For example, at \( t = 0 \), \( f = 0.5 \). Due to tidal stretching and stripping, the center of mass may not be located at the center of the satellite. I therefore redefine the density peak of the satellite as the center of the satellite. As the orbit decays, the satellite is gradually disrupted, and the location of the density peak of the satellite becomes uncertain. If the standard deviation of the position of the density peak becomes larger than the initial half-mass radius of the satellite, the satellite is considered completely stripped.

In the first column of Figure 8.1, I plot the distance between the center of the satellite and the center of the parent disk, \( r \), versus the mass inside the initial half-mass radius of the satellite, \( f \). It shows that \( f \) decreases linearly with decreasing \( r \). In the second column, \( r \) versus \( t \) is plotted. It shows that the orbital decay rate is approximately constant, which is in accord with the prediction of Equation 8.6. Finally, in the last column, \( f \) versus \( t \) is plotted. The mass loss rate, \( \frac{df}{dt} \), is also approximately constant. The satellites in these three models start at a distance of 2.5 model radial units and are completely disrupted at a distance of approximately 1.6 model radial units. The stripped satellite remnants are distributed in an annular ring, between 1.0 and 5.0 model radial units from the center of the parent galaxy, whereas the disk particles are distributed between 0 and 1.2 model radial units from the center. Since there is little interaction between these satellites and the disks of their parent galaxies, the orbital decay is independent of the orbital inclination.

Secondly, I investigate the orbital decay rate and mass stripping of a satellite on an initially elliptical orbit with \( e = 0.2 \), but the satellite has 10%, 20%, and 30% of the disk mass for Model 1, Model 2, and Model 3 respectively. From top to bottom, Figure 8.2 shows the mass inside the initial half-mass radius of the satellite versus the distance between the center of the satellite and the center of the parent galaxy for Model 1, Model 2, and Model 3 respectively. I find that during the same time period, the apocenter \( r_+ \) of the satellite orbit decreases by 1.0, 2.0, and 3.0 model radial units for Model 1, Model 2, and Model 3 respectively. In other words, the
Figure 8.2: Mass inside the half-mass radius of the satellite versus the distance between the satellite and the parent galaxy for Model 1, Model 2, and Model 3. The heavier the satellite, the faster the orbit decays, and the faster the orbital eccentricity decreases.
orbital decay rate measured at the apocenter is proportional to the initial mass of the satellite. On the other hand, the orbital decay rate measured at the pericenter $r_-$ is also proportional to the initial mass of the satellite. It is also consistent with the prediction of Equation 8.6.

The eccentricity of the satellite orbit is defined by $e = (r_+ - r_-)/(r_+ + r_-)$. I find that in Model 1, the final $r_+$ and $r_-$ of the satellite orbit are 3.0 and 2.0 model radial units respectively. Therefore, the final eccentricity the orbit is 0.2 which remains unchanged from its initial value. On the other hand, in Model 3, the evolution of orbital eccentricity can be divided into two stages. One is a slow decay stage in the halo where the orbital eccentricity remains almost unchanged. And the other is a fast decay stage in the disk where the orbital eccentricity rapidly decreases to zero. The fast decay stage in the disk can be explained as following. First, it is due to the additional dynamical friction against the disk. Secondly, the eccentricity decay rate, according to Equation 9.21, is $\frac{d\ln e}{d\ln r} \propto -\frac{M_f}{r}$. For large $r$, the decay rate is very small, the orbital eccentricity is nearly constant, but for small $r$, the eccentricity decays very fast.

Finally, another interesting point is to compare the orbital decay of direct (Model 8) and retrograde (Model 9) satellites. This is done in Figure 8.3. The similar orbital decay for both satellites show that the dynamical friction exerted on the satellite by the halo is the major factor that causes the orbital decay of the satellites. However, the fact that the retrograde orbit decays faster than the direct one suggests that the distant interaction between the satellite and the galactic disk is not completely negligible, although a 10% disk-mass satellite is completely disrupted in the halo before it enters the galactic disk. This is due to the fact that the Lindblad resonances of the disk exert a perturbative torque on the satellite (QG). Lynden-Bell & Kalnajs (1972) have shown that a uniformly rotating, perturbing potential exerts a negative torque on inner Lindblad resonances and a positive torque on outer Lindblad resonances. In other words, the torque on the satellite due to the inner Lindblad resonances is positive and that due to the outer Lindblad resonances is negative (QG). Figure 4.3
Figure 8.3: Orbital decay of the direct and retrograde orbital satellites as a function of time. The retrograde orbit decays faster than the direct one.

shows that the outer Lindblad resonances are not present in the model disks, but that there are two inner Lindblad resonances. Therefore, the inner Lindblad resonances exert positive torque on the satellite. This small positive torque increases the angular momentum of a direct orbit satellite resulting in a slower decay of the satellite orbit than would otherwise occur if only dynamical friction was involved. On the other hand, I find that for the satellite moving on retrograde orbit, the torque is opposite to the angular momentum. This, as expected, decreases the angular momentum and
the orbit decays faster. The fact that the retrograde orbit decays faster than the direct one when the satellite is located outside the disk is opposite to what is found when the satellite is located inside the disk (QG).

8.4 Satellite Remnants

I show in Figure 8.4 the evolution of the satellite particles projected on the $xy$ and $xz$ planes for Model 3. As the orbit of the satellite decays inside the halo, the satellite is partially tidally stripped by the halo. Once the satellite remnant enters the disk, it is rapidly disrupted by the tidal force due to the high density of the disk. The small core of the satellite survives for about 35 disk rotation periods or 8.4 Gyr. The final shape of the tidally stripped satellite particles resembles a warped torus. The planes of the outermost and innermost parts of the torus are aligned with the planes of the initial and final orbits of the satellite.

I find that the 10% disk-mass satellite of Model 1 is completely stripped before it enters the disk. Only 2% of the satellite particles are accreted onto the edge of the disk. The 20% disk-mass satellite of Model 2 is completely stripped at the edge of the disk. About 11% of the satellite particles are accreted onto the disk, and the particles are located outside of the half-mass radius of the disk. Finally, the 30% disk-mass satellite of Model 3 is rapidly stripped when it enters the dense disk. Approximately, 18% of the satellite particles are accreted onto the disk, and the particles, as in the previous case, are mostly located outside of the solar circle.

Figure 8.5 shows that the distribution of the satellite particles of Model 3 in phase space ($V_{rot}, r$) is very different from that of the halo particles, not only because of their high rotation speeds, but also because of their nonuniform distribution. If the stellar halo of a galaxy is produced by multiple satellite accretions, the stellar distribution in phase space should be clumpy. This is in accord with the recent observation of halo stars (Majewski, Hawley & Munn 1996) which shows a high degree of clumping in the U-V-W-[Fe/H] distributions.
Figure 8.4: Satellite particles of Model 3 projected on both the $xy$ (top two rows) and $xz$ (bottom two rows) planes. The size of the boxes is 6 model radial units or 96 kpc.
Figure 8.5: Distribution of the satellite particles of Model 3 in phase space \((V_{rot}, r)\). Only limited phase space is filled by the satellite particles.
CHAPTER 8. ORBITAL DECAY AND TIDAL STRIPPING OF SATELLITES

8.5 Summary

For the examples considered in this chapter, I have shown that all 10% disk mass satellites are completely tidally stripped before they penetrate the disks. Since the orbital decay is mainly caused by the dark matter halo instead of the parent disk, the orbital parameters, such as orbital inclination of the satellite relative to the parent galactic disk and the direction of the satellite rotation relative to the direction of disk rotation only play a small role in the satellite orbital decay. The orbital decay rate is constant for both circular and elliptical orbits, and it is proportional to the initial mass of the satellite. The orbital eccentricity is nearly constant in the halo but decreases rapidly in the disk. The mass inside the initial half-mass radius of the satellite decreases with increasing time and decreasing distance between the center of the satellite and the center of the parent disk. Most of the tidally disrupted satellite particles are accreted onto the halo and the outer disk rather than onto the inner disk. In physical space, the final shape of the tidally stripped satellite particles resembles a warped torus. The planes of the outermost and innermost parts of the torus are aligned with the planes of the initial and final orbits of the satellite. In phase space, the satellite particles are distinguished from halo particles by their high rotational speed and clumpy distribution.
Chapter 9

Orbital Decay Rate of Satellites and Galactic Accretion

9.1 Introduction

Most satellites orbit around their parent galaxy on elliptical orbits. For example the LMC and SMC orbit around our own Galaxy on elliptical orbits (Murai & Fujimoto 1980, Lin & Lynden-Bell 1982). It can be understood this way (Holmberg 1940, Tremaine 1980): when a small galaxy has a close encounter with a big galaxy, the small galaxy will lose kinetic energy due to tidal disturbance and may become a bound satellite galaxy, i.e., its parabolic orbit changes into an elliptical orbit with a rather large eccentricity. Then, because of the dynamical friction, every subsequent passage of the satellite galaxy through the pericenter tends to decrease the eccentricity, and eventually, the satellite galaxy will spiral into the parent galaxy. It is obvious that satellite galaxies spend nearly all their lifetime on elliptical orbits. However, in analytical calculations, their orbits are usually treated as circular for simplicity. Another simplification in traditional estimates of orbital decay rate is the assumption of a rigid satellite, which is subject to no tidal disruption. However, if both the satellite and its host have self-similar density profiles, the mass loss of the satellite is proportional to the host mass which the satellite traverses. It is therefore necessary to calculate the orbital decay rate of a decreasing mass satellite on an elliptical orbit.
CHAPTER 9. ORBITAL DECAY RATE OF SATELLITES AND GALACTIC ACCRETION

I will first investigate, in Section 9.2, the orbital decay rate of a decreasing mass satellite on an elliptical orbit by employing analytical approximations and direct integrations. Secondly, based on the resulting orbital decay rate, I will then, in Section 9.3, calculate the galactic accretion rate by following Tremaine's estimate, but include the following modifications: tidal disruption of satellites, elliptical orbits, non-constant mass-to-light ratio for dwarf galaxies, upper mass limit for satellites, and high peculiar velocity cutoff. Finally, a brief summary is given in Section 9.4.

9.2 Satellite Orbital Decay

The flatness of many observed rotation curves of spiral galaxies suggests that the density distribution of a galactic halo is given by the following expression

\[ \rho(r) = \frac{v_c^2}{4\pi G r^2}. \]  

(9.1)

which holds for a singular isothermal sphere with circular velocity \( v_c \) and velocity dispersion \( \sigma = v_c/\sqrt{2} \). As a satellite of mass \( M_s(r) \) orbits through the galaxy, it is subject to dynamical friction

\[ \vec{F}_{fric} = -\frac{4\pi G^2 \rho M_s(r)^2 \ln \Lambda}{v_M^2} \left[ \text{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2} \right] \vec{v}_M. \]  

(9.2)

in which \( X = v_M/v_c \) and \( \Lambda \) is the ratio of the size of the parent galaxy to that of a satellite, and \( \text{erf} \) is the error function, \( \text{erf}(X) = \frac{2}{\sqrt{\pi}} \int_0^X e^{-X^2} dX \). The angular momentum loss of the satellite can therefore be written as

\[ \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}_{fric}. \]  

(9.3)

By substituting Equation 9.1 and Equation 9.2 in Equation 9.3, I obtain the following equation.

\[ \frac{d\ln L}{dt} = -\frac{GM_s(r)^2 \ln \Lambda}{r^2 v_c X^3} \left[ \text{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2} \right]. \]  

(9.4)
For a circular orbit, $X = 1$. $L = L_z = v_c r$. Equation 9.4 can be rewritten as

$$\frac{d \ln r}{dt} = -0.428 \frac{GM_s(r) \ln \Lambda}{r^2 v_c}.$$  \hspace{1cm} (9.5)

For a rigid satellite of $M_s(r) = M_s$, the dynamical friction time is

$$t_{fric} = \frac{1.17 r^2 v_c}{GM_s \ln \Lambda}. \hspace{1cm} (9.6)$$

However, the decay of an elliptical orbit is much more complicated than that of the circular one because there is no explicit expression of $\frac{d \ln r}{dt}$ as a function of $r$ for the elliptical orbit. I therefore have to use a few approximations in the following calculation. The orbital time of a satellite is $T_{orb} \sim \frac{r}{v_c}$, and the ratio of $T_{orb} \sim \frac{r_0^2}{GM_s} \sim \frac{M_h}{M_s} \ll 1$. Since $T_{orb} \ll T_{fric}$, a continuous orbital decay (the solid line in Figure 9.1) can be treated approximately as a series discontinuous ellipses (the dash line in Figure 9.1). Each of the ellipses is characterized by their apocenter $r_+$. By approximating the approximate conservation of angular momentum per unit mass $L_+ / M_s(r_+) \approx L_- / M_s(r_-)$, i.e., $r_+ v_+ \approx r_- v_-$. and the approximate conservation of energy per unit mass $E_+ / M_s(r_+) \approx E_- / M_s(r_-)$, i.e., $\frac{1}{2} v_+^2 + v_c^2 \ln(r_+) \approx \frac{1}{2} v_-^2 + v_c^2 \ln(r_-)$, in which the expression for potential $\Phi(r) = v_c^2 \ln(r)$ is employed. I then obtain the following relation. $\frac{v_+^2}{v_c^2} = \frac{(1-\epsilon)^2}{2\epsilon} \ln \frac{1+\epsilon}{1-\epsilon}$, in which $\epsilon$ is the eccentricity determined by $\epsilon = \frac{r_+ - r_-}{r_+ + r_-}$. For a given $\epsilon$, the ratio of $\frac{v_+}{v_c}$ is listed in column 2 of Table 9.1. On the other hand, I assume that the eccentricity $\epsilon$ is a constant as the orbit decays. This assumption is discussed later in this section.

Since the satellite velocity, $v_M$, is not constant on elliptical orbits. I calculate the average angular momentum loss rate on each ellipse according to following equation,

$$\frac{d \ln(L)}{dt} \bigg|_{ave} = \frac{\int d \ln L \frac{dt}{T_{orb}}}{T_{orb}}. \hspace{1cm} (9.7)$$

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Figure 9.1: Satellite orbital decay. The solid line shows elliptical orbital decay of a satellite, and the dash line is the approximate orbital decay. The circular orbit decay referred to in this thesis is shown by the dash-dot line.

Substituting Equation 9.4 in Equation 9.7, I obtain

$$
\frac{d \ln L}{dt} \Big|_{ave} = -G \ln \Lambda \int_{r^+}^{r^-} M_s(r) \frac{[\text{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2}]}{r^2 v_c X^3} \frac{dt}{dr} dr / \int_{r^+}^{r^-} \frac{dt}{dr} dr. \quad (9.8)
$$

On a given ellipse, using the approximate conservation of energy per unit mass $E \approx E_+$, I obtain that the square of the satellite velocity at any distance $r$ from the
parent galaxy is given by

\[ v^2 = v_c^2 \left[ \frac{(1 - \epsilon)^2}{2\epsilon} \ln \frac{1 + \epsilon}{1 - \epsilon} + 2 \ln \frac{r_+}{r} \right]. \] (9.9)

and the square of the radial velocity of the satellite is given by the following expression

\[ \left( \frac{dr}{dt} \right)^2 = v_c^2 \left[ \frac{(1 - \epsilon)^2}{2\epsilon} \ln \frac{1 + \epsilon}{1 - \epsilon} \left( 1 - \frac{r_+^2}{r} \right) + 2 \ln \frac{r_+}{r} \right]. \] (9.10)

Assuming that infalling satellites have density profiles similar to that of the host, and according to Equation 8.2, I obtain

\[ M_s(r) \approx (1/\sqrt{3}) M_h(r), \frac{M_s}{M_h} = \frac{v_c^2}{\sqrt{3} Gr} \frac{M_s}{M_h}. \] (9.11)

Combining Equation 9.11, Equation 9.9, and Equation 9.10 into the integration Equation 9.8. I then have

\[ \frac{d \ln L}{dt} \bigg|_{\text{ave}} = -f(\epsilon) v_c M_s \ln \Lambda \left( \frac{r_+}{r} \right) \frac{M_s}{M_h}. \] (9.12)

where \( f(\epsilon) \) is listed in column 3 of Table 9.1. The dynamical friction time is

\[ t_{fric} = \frac{\sqrt{3} M_h r_+}{f(\epsilon) M_s v_c \ln \Lambda}. \] (9.13)

Equation 9.13 can also be rewritten as

\[ t_{fric} = \frac{3.87 \times 10^8 \left( \frac{r}{50 \text{kpc}} \right) \left( \frac{v_c}{220 \text{km/s}} \right)^{-1} \left( \frac{M_h}{M_s} \right)}{\ln \Lambda} \text{ yr.} \] (9.14)

Assuming \( \ln \Lambda = 3.0 \), \( v_c = 220 \text{ km/s} \), \( M_s/M_h = 0.1 \), and \( r = 50 \text{ kpc} \). I obtain in column 4 of Table 9.1, the dynamical friction time \( t_{app} \) for satellites on elliptical orbits with \( \epsilon = 0 \) to 0.9.

The orbital decay time ratio between an elliptical orbit and a circular orbit is

\[ t_{ellip}/t_{cir} = f(0)/f(\epsilon), \] with \( r_+ = r_{cir} \). For a typical elliptical orbit, \( v_+ \approx v_c/\sqrt{2} \), i.e., \( \epsilon \approx 0.3 \), the dynamical friction time is 78% of that for the circular one. I have
employed $r_+$ and $v_+$ to describe the satellite orbit because in my simulations, satellites are introduced at a distance $r_+$ from a parent galaxy with a tangential velocity $v_+$. In terms of mean radius, $a$, of an ellipse, Equation 9.13 can be rewritten as

$$t = \frac{a(1 + e)v_c}{f(\varepsilon)GM^2 \ln \Lambda}.$$  \hspace{1cm} (9.15)

The value of $f(\varepsilon)/(1 + \varepsilon)$ varies from 0.428 to 0.389 for $\varepsilon = 0$ to 0.9 (Table 9.1). It shows that most of the elliptical orbits can be simplified as circular ones with less than 10% error.

In the above calculation, for simplicity, I assume that the orbital eccentricity is constant as an orbit decays. I discuss here the decay rate of an orbital eccentricity by employing the first order perturbation theory. I rewrite the dynamical friction

$$\vec{F}_{fric} = -\frac{M_s \ln \Lambda v_c}{\sqrt{3}rM_h} g(X) \vec{r}_M.$$ \hspace{1cm} (9.16)

in which $g(X) = [\text{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2}] / X^3$. In plane polar coordinates $(r, \psi)$, the equation of motion for a satellite located at $(r, \psi)$ is
\[
\begin{align*}
\begin{cases}
r - r\dot{v}_r^2 = -\left(\frac{v_c}{r}\right)^2 - \frac{M_s \ln \Lambda \nu_c}{\sqrt{3}M_h r} g(X)\dot{r}, \\
r \ddot{v}_r + 2\dot{v}_r \dot{v}_v = -\frac{M_s \ln \Lambda \nu_c}{r} g(X) r \dot{v}_v.
\end{cases}
\end{align*}
\tag{9.17}
\]

By assuming \(v_r = \dot{r}\) and \(r \dot{v}_v = v_c + v_v\), where both \(v_r\) and \(v_v\) are small perturbations. Equation 9.17 can be rewritten into
\[
\begin{align*}
\begin{cases}
\dot{v}_r - 2\omega \nu_c = -k v_r, \\
\dot{v}_v + v_r \omega = -k (v_c + v_v).
\end{cases}
\end{align*}
\tag{9.18}
\]
in which \(\omega = v_c/r\), \(k = \frac{M_s \ln \Lambda \nu_c}{\sqrt{3}M_h r} g(X)\), and the second order terms \(v_r^2\) and \(v_r v_v\) are neglected in the equation.

Since \(\omega\) and \(k\) change very slowly with time, they are treated as constants when I solve Equation 9.18. The solutions for the equation are
\[
v_r = A \exp(-kt) \cos(2\sqrt{2}\omega t + \nu) - \frac{2v_c k \omega}{2\omega^2 + k^2},
\tag{9.19}
\]
and
\[
v_v = A' \exp(-kt) \cos(2\sqrt{2}\omega t + \nu') - \frac{v_c k^2}{2\omega^2 + k^2}.
\tag{9.20}
\]
In which \(A\) and \(\nu\) are constant, and they depend on the initial conditions \(\dot{r}_0, \dot{v}_0\), and \(A'\) and \(\nu'\) are also constant and can be determined by the knowledge of \(A\) and \(\nu\). For a circular orbit, \(A, A', \nu,\) and \(\nu'\) are equal to zero. The first terms in both Equation 9.19 and Equation 9.20 show how an elliptical orbit decays towards a circular one.

Integrating Equation 9.19, I obtain that
\[
r(t) = B \exp(-kt) \sin(2\sqrt{2}\omega t + \nu') + r.
\]
In which \(r\) is the integration of the second term in Equation 9.19, and it varies very slowly compared with \(r1(t)\). Orbital eccentricity is then defined as
\[
\frac{d \ln \epsilon}{dt} = -k = -\frac{M_s \ln \Lambda \nu_c}{\sqrt{3}M_h r} g(X) \times \frac{1}{t_{fric}(r, X)}.
\tag{9.21}
\]

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or
\[ \frac{d\epsilon}{dt} \propto -\frac{\epsilon}{t_{fric}(r, X)} \] (9.22)

For large \( r \), the decay rate is very small, and the orbital eccentricity is nearly constant, however, for small \( r \), the assumption of constant eccentricity for elliptical orbital decay is no longer valid.

For a given set of \( r_0 \), \( \nu_0 \), \( \dot{r}_0 \) and \( \dot{\nu}_0 \), the orbital decay time of a satellite can be calculated by integrating Equation 9.18 numerically. Assume \( \ln \Lambda = 3.0 \), \( M = 2.0 \times 10^{10} M_\odot \) for \( r_0 = 50 \) kpc. \( \nu_0 = 0.0 \), \( \dot{r}_0 = 0.0 \), and \( \dot{\nu}_0 = \nu_0 / r_0 \). the numerically integrated dynamical friction time \( t_{int} \) is listed in column 5 of Table 9.1. Comparing the dynamical friction time \( t_{app} \) in column 4 with \( t_{int} \) in column 5, I find that the dynamical friction times calculated from both the analytical approximation and direction integration are consistent, except those of high eccentricities \( \epsilon = 0.8 \) and 0.9. This is because for low \( \epsilon \), \( \frac{d\epsilon}{dt} \propto \epsilon \approx 0 \), and the constant eccentricity assumption in the analytical approximation is valid, however, for high \( \epsilon \), the constant eccentricity assumption in the analytical approximation is no longer valid. In general, the analytical approximation predicts a very good approximation of orbital decay time. Therefore, based on the analytical approximation, I will estimate the galactic accretion in the following section.

### 9.3 Galactic Accretion

Employing the two-point correlation function for galaxy distribution Groth & Peebles (1977) and Schechter's (1976) luminosity function for galaxy mass distribution, Tremaine (1980) estimated the total accretion of a typical galaxy,

\[ M_{acc}/M_* = \int_0^\infty n_0(M_s)(M_s/M_*)dM_s \int_0^{r_{fric}} 4\pi (r/r_0)^7 r^2 dr \] (9.23)

in which the number density of satellites of mass \( M_s \), at a distance \( r \) from a given galaxy is \( n(r, M_s)dM_s = n_0(M_s)dM_s(r/r_0)^{-\gamma} \). Based on Schechter's luminosity func-
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and constant mass-to-light ratio. The field density of satellites is \( n_0(M_s) dM_s = n_*(M_s/M_*)^{-\alpha} \exp(-M_s/M_*) d(M_s/M_*) \), in which \( \alpha = 1.25 \). \( n_* = 1.2 \times 10^{-2} h^3 \text{Mpc}^{-3} \).

and \( L_\ast = 1.0 \times 10^{10} L_\odot \). \( r(t_H) \) is derived from Equation 9.6 by employing \( t_{fric} = 1/H_0 \).

\( r(t_H) = \sqrt{\ln \Lambda GM/H_0 r_0^3 v_\perp} \). Equation 9.23 is then rewritten as

\[
\frac{M_{acc}/M_*}{9.5} = \left( \frac{G(M/L)L_* \ln \Lambda}{H_0 r_0^3 v_\perp} \right)^{0.6} r_0^3 n_*(1.6 + \alpha) \ln \Lambda GM/H_0 r_0^3 v_\perp.
\] (9.24)

which is Equation 7.31 of Binney & Tremaine (1986). For \( r_0 = 3h^{-1} \text{Mpc} \). \( v_\perp = 220 \).

\( H_0 = 100h \text{ km/s/Mpc} \). \( M/L = 12hM_\odot/ L_\odot \). Equation 9.24 gives \( M_{acc}/M_* = 0.16 \).

In this section, I calculate the galactic accretion rate by following Tremaine's (1980) estimate, but include the following modifications. The first modification is to include tidal disruption and elliptical orbits of satellites. By assuming that satellites have density profiles comparable to that of the host, I obtain \( r(t_H) \) from Equation 9.13.

\( r(t_H) = \frac{r_+ v_c M_\bullet \sqrt{3}}{f(r)GM_\bullet \ln \Lambda H_0 r_0} \).

Secondly, some nearby galaxies have rather high relative peculiar velocities, and they are not likely to merge with each other. The relative line-of-sight peculiar velocity distribution from CfA1 (Davis & Peebles 1983) and CfA2+SSRS2 redshift surveys (Markze et al. 1995) is

\[
f(V_r) \propto \exp\left(-2^{1/2}|V_r|/\sigma_{12}\right).
\] (9.25)

In which \( V_r \) is the relative line-of-sight peculiar velocity, and \( \sigma_{12} \) is the rms relative line-of-sight velocity. Davis & Peebles found that \( \sigma_{12} = 340 \pm 40 \text{ km/s} \), and Markze et al. measured \( \sigma_{12} = 540 \pm 180 \text{ km/s} \). But \( \sigma_{12} \) drops to \( 295 \pm 99 \text{ km/s} \) if all Abell clusters with richness class \( R \geq 1 \) are removed. For the case discussed in this thesis, a reasonable value for \( \sigma_{12} \) is \( 300 \text{ km/s} \). At a given distance from a typical galaxy, nearby galaxies on elliptical orbits are likely to merge. The upper limit for the relative peculiar velocity is \( \sqrt{V_r^2 + V_\phi^2 + V_\sigma^2} = \sqrt{2}v_\perp \), which corresponds to a parabolic orbit.

Therefore, the fraction of the galaxies which are going to merge is

\[
P = \frac{1}{\sqrt{2}v_\perp} \frac{1}{(\sqrt{2}\sigma_{12})^3} \exp -\frac{\sqrt{2}(|V_r| + |V_\phi| + |V_\sigma|)/\sigma_{12}}{dV_r/dV_\phi/dV_\sigma}.
\] (9.26)
in which $V_x^2 + V_y^2 + V_z^2 \leq 2v_c^2$, and I have assumed isotropic rms relative velocity $\sigma_r = \sigma_\theta = \sigma_\phi = \sigma_{12}$. The integration gives $P = 35\%$, where $v_c = 220$ km/s is used. Due to high relative peculiar velocity cutoff, only 35\% of galaxies are likely to merge.

Thirdly, non-constant mass-to-light ratio for dwarf ellipticals is employed. I assume $L/L_* = (M/M_*)^{5/3}$, which is based on the relation $M/L \propto L^{-0.4}$ for dwarf ellipticals (Dekel & Silk 1986, Peterson & Caldwell 1993). Then the mass distribution of satellites becomes

$$n_0(M_s) dM_s = \frac{5}{3} n_* (M_s/M_*)^{-\frac{5}{3} + \frac{2}{3}} \exp(-M_s/M_*)^{\frac{5}{3}} d(M_s/M_*) \quad (9.27)$$

Finally, I integrate the satellite mass from 0 to $0.3M_*$ rather than from 0 to $\infty$ in Tremaine’s estimate because I only consider the infall of low mass satellites. Therefore, a typical galaxy accretion is

$$M_{acc}/M_* = P \int_0^{0.3M_*} n_0(M_s)(M_s/M_*) dM_s \int_0^r 4\pi (r/r_0)^2 r^2 dr \quad (9.28)$$

Combining all the modifications into the integration Equation 9.28. I obtain

$$M_{acc}/M_* = 0.37 \left( \frac{f(e) \ln \Lambda v_c}{\sqrt{3}H_0 r_0} \right)^{1.2} n_0 r_0^3. \quad (9.29)$$

Assuming that $v_c = 220$ km/s, $H_0 = 100$ km/s/Mpc, $r_0 = 3.0$ Mpc, and $\ln \Lambda = 3$ are reasonable. I obtain the final result $M_{acc}/M_* = 0.02 h(e)$, where $h(e) = (f(e)/f(0))^{1.2}$, which is listed in the last column of Table 9.1. varies from 1 to 2. Furthermore, $M_{acc}/M_{disk} = (M_{acc}/M_*)/(M_{disk}/M_*)$. For $M_{disk}/M_* = 0.1$, $M_{acc}/M_{disk} = 0.2 h(e)$. For a median value of $e = 0.5$, the accretion rate is $M_{acc}/M_{disk} \approx 30\%$. According to the simulations in this thesis, the infall of a 30\% disk-mass satellite can produce observable thickening only beyond the half-mass radius of the parent disk.
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9.4 Summary

I calculate the orbital decay time of a satellite which experiences tidal disruption as its elliptical orbit decays, by employing both analytical approximation and numerical integration methods. For a median eccentricity, $e = 0.5$, the orbital decay time is approximately 50% shorter than that of the circular one if $r_+(t = 0) = r(t = 0)$. The orbital decay times estimated by both methods are similar. Therefore, based on the analytical result, I calculate the galactic accretion for the case that the infalling satellites have low masses. In estimating the galactic accretion, I follow Tremaine's (1980) method, but include the following modifications: high peculiar velocity cutoff, high mass cutoff, and non-constant mass-to-light ratio for dwarf galaxies. In this case, the value for the accretion rate varies between 13% and 25% of Tremaine's estimate. For a median eccentricity, $e = 0.5$, a typical galaxy, like our own Milky Way Galaxy, has absorbed about 3% of its own mass, or 30% of its disk mass, if the disk mass is assumed to be 10% of the total mass, in the form of low mass infalling satellites. According to the results drawn from the simulations in this thesis, it can be accommodated without unacceptable disk thickening within the half-mass radius of the disk.
Chapter 10

Discussion and Conclusion

10.1 Density of Satellites and Galaxy Accretion Rate

The infall of a high density satellite on its parent disk is of interest because the high density satellite could produce some observed features in the disk, such as the formation of the thick disk and the bulge. On the other hand, a low density satellite, which is tidally disrupted before it reaches the inner disk, produces little observable evidence in the inner disk, and it has been neglected in the past. However, Zaritsky (1996) found evidence of recent accretion in the outer disks of nearby late-type galaxies by estimating the duration of steep abundance gradients, elevated rates of star formation, and outer disk asymmetries. The fact that accretion onto the outer disks is common, and the inner disks remain undisturbed suggests that the infalling satellites are tidally disrupted before they reach the inner disk, therefore, they are low density satellites. This is not surprising since all but one of low mass galaxies in our local group are low density galaxies. Therefore, accretion of low density satellites appears to be more common than that of high density satellites.

By using the thickness and the Toomre parameter $Q$ of the Galaxy at the solar circle, TO estimated an upper limit for the disk accretion rate. In their model, density profiles of satellites are described by the Jaffe Model, $\rho \propto \frac{1}{r^2(1+r)^2}$. Such functional
forms lead to an extremely high density at the center of the satellite in comparison to the density of the parent galaxy. Therefore, the core of such a satellite could survive and reach the center of the disk. For example, in the two cases considered by TO, 84% and 43% of the satellite mass respectively, cannot be tidally stripped: it reaches the center, and tremendous heating in the disk is unavoidable. Therefore, a low disk accretion rate is required. TO further suggested a low density universe in which the galaxy accretion is suppressed at the current epoch. However, I found that a low density satellite is mainly accreted onto the outer halo and the outer disk. Therefore, I argue that first, the disk accretion rate is much smaller than the galaxy accretion rate, and secondly, that the accretion rate at the inner disk is much smaller than that at the outer disk. Therefore, the thinness and coldness of the inner disk can be used to set a limit on the special high density satellite infall rate, but certainly not on the general satellite infall rate. In the case of satellites having density profiles similar to that of the host galaxy, a high galaxy accretion rate and thin disks can coexist.

10.2 Disk Tilting, Holmberg Effect, and Rotation of Stellar Halo

In the course of a search for companions of nearby bright spiral galaxies, Holmberg (1969) found that essentially none of the physical companions were within 30° of the major axis of the parent disk. QG discussed a possible explanation for this effect and attempted to explain it by employing the relation between the orbital decay rate and the orbital inclination with respect to the plane of the parent disk. However, their simulation results predicted a very weak effect. Here, I find that disk tilting caused by an infalling satellite can produce a somewhat stronger effect.

I have shown that disks respond to infalling satellites primarily by tilting. I found that a disk tilts towards the orbital plane of a direct satellite (Model 3) and away from the orbital plane of a retrograde satellite (Model 9), as shown in Figure 10.1. Model 9 has the same model parameters as those of Model 3, except for the rotational
Figure 10.1: Evolution of the disk and satellite particles of Model 3 (top two rows) and Model 9 (bottom two rows) as viewed in the $xz$ plane. The size of the boxes is 6 model radial units, or 96 kpc. The disk tilts towards the orbital plane of the direct satellite and away from the orbital plane of the retrograde satellite. Note that due to disk precession, the disk appears much thicker than it actually is.
direction of its satellite. In other words, as the orbit of the direct satellite decays, the orbital inclination with respect to the plane of the disk decreases. On the other hand, as the retrograde orbit decays, the orbital inclination with respect to the plane of the disk increases. Since the orbital decay of distant satellites is caused by the dynamical friction arising from the halo, the decay rate does not depend on the orbital direction and inclination. Therefore, my simulation results imply that a rather uniform spatial distribution of distant satellites is expected unless the halo rotates significantly. This is in accord with the observational result of Zaritsky et al. (1993). However, the orbital decay of close satellites is mainly caused by the dynamical friction arising from the disk. Therefore, low inclination and direct orbits decay faster than high inclination and retrograde ones. As a result, there should be a net excess of surviving close satellites on high inclination and retrograde orbits. This is supported by the satellite sample of Zaritsky et al. (1993) but a larger sample is needed to further confirm our suggestion. In their sample, I found that there are five satellites located at distances less than 20 kpc from their parent galaxies. Three of them have unknown directions of orbit rotation, and two of them have retrograde orbits with 59° and 60° inclinations. In QG’s calculation, all satellite orbits are assumed to be direct, which implies that only the relation between orbital decay rate and orbital inclination is counted as the cause for the Holmberg effect. However, the more important relation between orbital decay rate and orbit rotation direction of the satellite is neglected. Usually, the decay time of a retrograde orbit is a few times longer than that of a direct one. Therefore, my simulation results on disk tilting, suggest a much stronger anisotropy of satellite distribution than that found by QG.

Remnants of tidally disrupted satellites may become part of the stellar halo. Searle & Zinn (1978) proposed that the stellar halo formed by the accretion of small, metal-poor fragments like gas-rich dwarf galaxies, because they found that the chemical and orbital properties of the outer globular clusters are decoupled. Direct evidence for present and past accretion (Freeman 1996a) is: the Sgr dwarf galaxy (Ibata et al. 1994): young main sequence A stars with \( [\text{Ca/H}] > -0.5 \) at heights up to 11 kpc.
CHAPTER 10. DISCUSSION AND CONCLUSION

from the galactic plane (Rodgers et al. 1981, Lance 1988): blue metal-poor main sequence stars with [Fe/H]< -1 at age > 3 Gyr (Preston, Beers, & Shectman 1994): metal-poor, retrograde moving groups in the field (Eggen 1979, Majewski et al. 1994): young retrograde globular clusters (Zinn 1993, van den Bergh 1993a,b. Da Costa & Armandroff 1995). The mean rotation of the metal-poor halo near the galactic plane is direct. and at a large height from the plane is retrograde (Carney 1996). This phenomenon can be explained by disk tilting due to infalling satellites. Our simulation results show that a disk tilts towards the orbital plane of a direct satellite, but away from the orbital plane of a retrograde satellite. This implies that the remnants of a direct satellite are located near the disk plane, whereas the remnants of a retrograde satellite are distributed away from the disk plane. If part of the stellar halo consists of remnants from many infalling satellites which were assumed uniformly distributed, my simulation results suggest that there should be a net excess of retrograde remnants with high orbital inclinations and a net excess of direct remnants with low orbital inclinations.

10.3 Merger Rate and $\Omega = 1$ CDM Cosmology

In any hierarchical cosmological model, merging is inevitable because small mass perturbations collapse before large ones and the evolution proceeds as a cascade of mergers from small to large scales. Most galaxies have had major mergers during their formation and a large fraction of galaxies have had minor mergers during their evolution. Under the cold dark matter (CDM) model, most galaxies accreted at least 10% of their mass over past 5 billion years (Bahcall & Tremaine 1988, Frenk et al. 1988, Carlberg & Couchman 1989, Kauffmann, White & Guiderdoni 1993, Lacey & Cole 1993). However, studies showed that a thin disk can be damaged by an infall of 10% disk-mass, high density satellite (TO, QHF, WMH). TO therefore suggested that the theoretical merger rate for $\Omega = 1$ CDM cosmology is too high.

By employing Carlberg's (1990a,b) formula for galaxy merger rate. TO obtained.
for $\Omega = 1$ CDM cosmology, the probability of a merger between a normal $L_*$ galaxy with one having a mass $M = M_{12} \times 10^{12} M_\odot$ between redshift $z_i$ and $z = 0$.

$$\chi(z_i, M_{12}) = \frac{0.15 M_{12}^{-5/6}}{(2/3)n - 1} [(1 + z_i)^{n-3/2} - 1].$$  \hspace{1cm} (10.1)

and the probability of at least one merger back to $z = z_i$ is

$$P(z_i, M_{12}) = 1 - \exp[-\chi(z_i, M_{12})].$$  \hspace{1cm} (10.2)

They found that the probability of at least one merger between the Milky Way and a satellite of $M_{12} = M_b/\Omega_b = 0.029$ for $M_b = 2 \times 10^9 M_\odot$ and $\Omega_b = 0.07$ after the Sun was born at $z_i = 0.33$ ($\Omega = 1$ and $H_0 = 50$) is 90%. However, they found in their calculation that no more than 4% of the Galaxy's mass can have accreted inside the solar circle in the last 5 Gyr. They therefore conclude that Carlberg's formula gives too high a merger rate, and $\Omega = 1$ CDM cosmology is incorrect.

In the simulations of this thesis, an infalling satellite having up to 20% of the disk mass produces little observable thickening in the disk. A 30% disk-mass satellite produces little observable thickening inside the half-mass radius of the disk but great thickening beyond the half-mass radius. The satellite mass for Model 1, Model 2, and Model 3 is $4 \times 10^9 M_\odot$, $8 \times 10^9 M_\odot$, and $12 \times 10^9 M_\odot$, respectively. According to Equation 10.2, the probability of at least one merger between the Milky Way and a satellite having a mass equal to that of Model 1, Model 2, and Model 3 is 71%, 47%, and 38% respectively. In other words, 38% of spiral galaxies have merged with a 30% disk-mass satellite in last 5 Gyr, and in these galaxies, disk thickening can be observed beyond their half-mass radii. Therefore, according to the simulation results presented in this thesis, the theoretical merger rate for $\Omega = 1$ CDM cosmology is not unreasonable.
CHAPTER 10. DISCUSSION AND CONCLUSION

10.4 Conclusions

I found that our thin disks primarily respond to infalling satellites by tilting. A disk tilts towards the orbital plane of a direct satellite and away from the orbital plane of a retrograde satellite. Disk heating in the vertical direction, or equivalently disk thickening, is less in the models of this thesis relative to that computed in other models. In these latter cases, the disk tilting is either not allowed (TO) or very small due to the limited initial separation between the satellite and the disk (QHF, WMH). Satellites with 10%, 20%, and 30% of the disk mass tilt the disk by angles of $(2.9 \pm 0.3)^\circ$, $(6.3 \pm 0.1)^\circ$, and $(10.6 \pm 0.2)^\circ$ respectively, although only 3.4%, 6.9%, and 11.1% of the orbital angular momentum of these satellites is transferred to the corresponding parent galaxy. In these cases, the kinetic energy associated with vertical motion in the initial coordinate frame of the disk is increased by $(6 \pm 3)\%$, $(26 \pm 3)\%$, and $(51 \pm 5)\%$ respectively, whereas the corresponding thermal energy associated with the random vertical motion in the tilted coordinate frame is increased by only $(4 \pm 3)\%$, $(6 \pm 2)\%$, and $(10 \pm 2)\%$ respectively.

I found that in the case of satellites having densities comparable to that of the parent galaxy, a 10% disk-mass satellite is completely disrupted by tidal force before it enters the galactic disk. Since 98% of the satellite mass is accreted onto the halo, the damage to the disk due to such an infalling satellite is not observable. A 20% disk-mass satellite is completely tidally disrupted at the edge of the disk, and 11% of the satellite mass is accreted onto the outer disk. Hence, only the fraction of the disk outside the half-mass radius is directly heated by the infalling satellite. Finally, a 30% disk-mass satellite is completely disrupted in the outer disk. Although only 18% of the satellite mass is accreted onto the disk, the disk beyond the half-mass radius is significantly heated by the satellite. Both 20% and 30% disk-mass satellites cause a slightly stronger bar in the disk than that in an isolated disk, therefore, the disk inside the half-mass radius is slightly heated indirectly by these infalling satellites.

I concluded that the accretion onto the halo is more common than that onto the
disk, and the accretion onto the outer disk is more common than that onto the inner
disk. Therefore, a high cosmological infall rate is compatible with the existence of
thin inner disks, given our result that the infall of low density satellites causes little
disturbance to the inner disk.

I have shown that an infalling satellite can excite warps in the disk. However the
warps eventually fade away due to the fact that both the inner disk and outer disk
are tilted towards the same plane. Therefore, the warps are unlikely to be long lived
phenomena. The fact that most galaxies are warped could be an indicator that disks
are in fact being subjected to a recent infall of satellites, though the infalling satellites
may not be easily detected because of tidal disruption. The results of disk warping
presented in this thesis only represent the warps in stellar disks.

I found that an infalling satellite causes a somewhat stronger bar than that formed
in an isolated disk. However, in both cases, the bars are weak and evolve very slowly,
taking approximately 25 disk rotations to form. This is due to the high halo-to-disk
mass ratio employed in the models of this thesis. The pattern speed of weak bars
decays very slowly relative to that of strong bars in previous work (Weinberg 1985,
Hernquist & Weinberg 1992, Sellwood & Debattista 1996). Since, in my model, the
OLR of the weak bar remains at the inner disk, and the infalling satellite is located
far from the OLR, there is no direct angular momentum transfer from the bar to
the infalling satellite. Therefore, the satellite orbital decay rate is not affected by
the formation of a weak bar in the disk. In other words, disk heating caused by the
infalling satellite is not underestimated in the simulations of this thesis. The results
obtained from my simulations do not represent the dynamical interaction between a
strong bar and an infalling satellite.

Since self-similar density profiles are employed for both the satellite and the halo
in my models, the satellite mass decreases proportionally to the decreasing distance
between the center of the satellite and the center of the parent galaxy. In this case,
I found that the orbital decay rate is constant for both circular and elliptical or-
bits, whereas the orbital decay rate for a solid satellite in previous work (Binney &
Tremaine 1986) increases with decreasing distance between the center of the satellite and the center of the parent galaxy. \( \frac{d\theta}{dt} \propto -\frac{1}{r} \). The orbital decay rate is proportional to the initial mass of the satellite. For elliptical orbits, the decay of the orbital eccentricity is very slow in the halo but decreases rapidly in the disk. Most of the tidally disrupted satellite particles are accreted onto the halo and the outer disk rather than onto the inner disk. In physical space, the final shape of the tidally stripped satellite particles resembles a warped torus. The planes of the outermost and innermost parts of the torus are aligned with the planes of the initial and final orbits of the satellite. In phase space, satellite particles are distinguished from halo particles by their high rotational speed and clumpy distribution. This is in accord with the recent observation of halo stars (Majewski, Hawley & Munn 1996) which shows a high degree of clumping in the \( U-V-W-[Fe/H] \) distributions.

I calculated the orbital decay time of a satellite which experiences tidal disruption as its elliptical orbit decays by employing both analytical approximations and numerical integration methods. For a median eccentricity, \( e = 0.5 \), the orbital decay time is approximately 50% shorter than that for the circular orbit if \( r_+(t = 0) = r(t = 0) \). Orbital decay times estimated with both methods are similar. Therefore, based on the analytical result, I calculated the galactic accretion for the case of low mass satellites. In estimating the galactic accretion, I followed Tremaine's (1980) method. However, I introduced the following modifications: high peculiar velocity cutoff, high mass cutoff, and non-constant mass-to-light ratio for dwarf galaxies. In this case, the value for the accretion rate which I obtained represents 13% to 25% of Tremaine's estimate. For a median eccentricity, \( e = 0.5 \), a typical galaxy like our own Milky Way Galaxy has absorbed about 30% of its disk mass in the form of low mass infalling satellites. According to the simulation results discussed in this thesis, such an amount of accretion can be accommodated without unacceptable disk thickening within the half-mass radius.
CHAPTER 10. DISCUSSION AND CONCLUSION

10.5 Future Work

The interaction between a disk galaxy and an infalling satellite can trigger star formation in both the disk and the satellite. However, star formation cannot be studied in the purely stellar models of this thesis. An important refinement to simulations of this kind would be to construct models which explicitly incorporate gas dynamics. Hernquist and Mihos (1995) studied mergers between gas rich disks and stellar dwarf galaxies with high densities and found that a large fraction of the gas is driven into the inner regions of the disk. The high density gas may cause starburst in the center of the disk. On the other hand, Zaritsky (1996) found elevated rates of star formation in the outer disks of nearby late-type galaxies, which may result from the infall of low density, gas rich satellites. In future models which explicitly include a gas component, it would be interesting to study the gas remnants of the infalling satellites. This may explain star formation in the outer disks of nearby late-type galaxies, as well as provide an explanation for the formation of the halo peculiar A stars with [Ca/H] > -0.5 at heights up to 11 kpc from the galactic plane (Rodgers et al. 1981, Lance 1988).

Usually, warps observed in gas disks are stronger than those observed in stellar disks. I have performed some simulations in which gas dissipation is modeled by applying a friction in the radial direction on disk particles. These preliminary simulation results show that both spiral structures and warps last longer in models which include this cooling. However, in order to truly understand warps in both gas disks and stellar disks excited by infalling satellites, it would be necessary to use models incorporating gas dynamics. It is also very interesting to study the disk tilting rate of both the gas disk and the stellar disk.

Optical surface photometry of barred spiral galaxies reveal that there are two types of bars: large bars tend to have a nearly constant surface brightness (flat bar), whereas smaller bars tend to have a decreasing surface brightness with a scale length similar to the disk (exponential bar) (Elmegreen 1996). By carrying out a statistical
CHAPTER 10. DISCUSSION AND CONCLUSION

study of the distribution of galaxies in different environments. Elmegreen, Bellin, & Elmegreen (1990) found a strong link between close interactions and flat bars. However, such a link cannot be investigated in the models of this thesis because only weak bars are formed due to the fact that a high halo-to-disk mass ratio is employed. I suggest that it may be interesting study the link between close interactions and flat bar in future models in which a low halo-to-disk mass ratio is employed.
Appendix A

The Scale Length of the Galaxy

A.1 Introduction

The space emissivity profile for the Galaxy, $\nu(R, z)$, can be written in the following form

$$\nu(R, z) = \mu(0) \exp(-R/h_R) \frac{\lambda^2(z/h_z)}{(2h_z)}. \quad (A.1)$$

where $R$ and $z$ are cylindrical coordinates, $\mu(0)$ is the central surface brightness, and $h_R$ and $h_z$ are called scale length and scale height respectively. There are photometric and kinematic methods to determine the scale length of the Galaxy. However, the measured scale length based on both methods shows a very large spread, from 1.8 kpc to 6 kpc (see review of the photometric determinations by KDF and the review of the kinematic estimates by FM). For photometric determinations, it was believed that the measured scale length depends on the wavelength of observations: $h_R = 2 - 3$ kpc for around 2.2 $\mu$m, 3.5-5.5 kpc for optical bands, and 4.5-6 kpc for IRAS OH/IR stars. However, UBV photometry towards the anti-center down to a magnitude $m_V = 25$ led Robin et al. (1992) to reach a value $h_R = 2.5$ kpc, compatible with the near IR values. Therefore, the measured value of $h_R$ depends not only on the wavelength, but also on the location of the observed sources. For observations carried out at a wavelength of approximate 2.2 $\mu$m, sources are located at low latitude $|b| < 10^\circ$. On the other hand, the OH/IR stars of IRAS observations are located at $|b| > 2^\circ$. In
optical bands, the location of sources varies: for example, $l = 179^\circ$, $b = 2.5^\circ$ (Robin et al. 1992) and $|b| > 20^\circ$ (van der Kruit 1986). FM raised the issue that the discrepancy between the various photometric determinations of $h_R$ of the Galaxy may arise from the assumption of a constant scale height. The constant scale height which is inferred from surface photometry on two edge-on spiral galaxies NGC1244 and NGC5907 (van der Kruit & Searle 1981) may not be valid for the Galaxy.

KDF proposed a non-constant scale height model for the Galaxy. In their model, the scale height is constant with a value $h_{\text{min}}$ inside a characteristic radius $R_{\text{min}}$, and increases linearly with radius for radii larger than $R_{\text{min}}$. Under such an assumption for the radial dependence of $h_z$, KDF found that the description of their observational data was significantly improved. They inferred a value $h_R = 3.0$ kpc in comparison to the smaller $h_R = 2.7$ kpc inferred on the basis of a constant $h_z$ assumption. Inspired by the model of KDF and using a similar model with $h_R = 3.0$ kpc but outwards increasing $h_z$, FM found that their non-constant scale height model could fit the pioneer 10 data as well as van der Kruit's (1986) model with $h_R = 5.5$ kpc and constant $h_z$. Therefore, the assumption of a constant $h_z$ for the Galaxy may not be well-founded. The purpose of this appendix is to show how the discrepancy between photometric determinations of $h_R$ for the Galaxy arises from the assumption of a constant scale height.

### A.2 The scale length of the Galaxy

By replacing $h_z$ in Equation A.1 by the form $h_z(0)\exp(R/h)$ discussed in Chapter 3 of this thesis. I obtain the following equation

$$
\nu(R, z) = \mu(0)e^{-R(1/h_R+1/h)}\text{sech}^2(z/(h_z(0)\exp(R/h)))/(2h_z(0)). \quad (A.2)
$$

I introduce the identity

$$
\text{sech}^2(z/(h_z(0)\exp(R/h))) = \text{sech}^2(z/h_z(0))\exp(-f(R/h, z/h_z(0))R/h). \quad (A.3)
$$
where \( f(R/h, z/h_z(0)) \) is a function of \( R/h \) and \( z/h_z(0) \) defined by the following expression.

\[
\begin{align*}
\ln 
\frac{
\left.
\text{sech}^2(z/((h_z(0) \exp(R/h)))
\right)
}{
\text{sech}^2(z/((h_z(0)))
}\end{align*}
\]  

Then Equation A.2 can be rewritten as

\[
\nu(R, z) = \mu(0)e^{-R/(h_R + (1 - f(R/h, z/h_z(0)^{1/h_R})/h))},
\]  

which in form is very similar to Equation A.1. By comparing these two equations, I obtain the following relation between two measured scale lengths \( h_{\text{const}}^R \) and \( h_R \).

\[
1/h_{\text{const}}^R = 1/h_R + (1 - f(R/h, z/h_z(0))/h).
\]  

In Equation A.6, if \( h >> h_R \), the effect of a non-constant scale height can be ignored. For example, in NGC4244, my estimate of \( h \) is \( 3R_{\text{max}} >> h_R \). Although the non-constant \( h_z \) model fits the observed \( z \)-profile of NGC4244 slightly better than constant \( h_z \) model, the latter is a very good approximation as shown in Figure A.1. However, in the Galaxy, according to the estimate based on KDF's data given in Chapter 3, \( h \approx R_{\text{max}} = 20 \) kpc. therefore, the effect of a non-constant scale height can not be ignored in this case. In my model, if the average value of \( f(R/h, z/h_z(0)) \) is equal to 1. then the effect of a non-constant scale height is not noticeable. However, if the average value of \( f(R/h, z/h_z(0)) \) departs from unity, then \( h_{\text{const}}^R \) is no longer equal to \( h_R \). I find that if \( \overline{f(R/h, z/h_z(0))} \) > 1. then \( h_{\text{const}}^R > h_R \), and for \( \overline{f(R/h, z/h_z(0))} < 1. \) then \( h_{\text{const}}^R < h_R \). Therefore, the function \( f(R/h, z/h_z(0)) \) gauges whether \( h_{\text{const}}^R \) is over- or underestimated. By choosing values \( h = 20 \) kpc, \( h_z(0) = 200 \) pc. and \( R_0 = 8 \) kpc. I plot in Figure A.2 \( f(R/h, z/h_z(0)) \) as a function of \( R/h \) and the galactic latitude \( b \). I also calculate the averaged value of \( f(z/h_z, R/h) \) for some simple cases in Table A.1: from \( R = 2 \) kpc to 8 kpc in \( l = 0^\circ \) direction; from 8 kpc to 18.3 kpc in \( l = 90^\circ \); and from 8 kpc to 20 kpc in \( l = 180^\circ \) direction. These data will be used in the following discussion concerning the discrepancy that exists between the different
Figure A.1: Observed z-profile of NGC4244 (taken from van der Kruit & Searle 1981) and constant $h_z$ fitting (dashed lines) and non-constant $h_z(R) = h_z(0)\exp(R/h)$ fitting (solid lines). The non-constant $h_z$ fits the data only slightly better that constant $h_z$ in NGC4244.

Determinations of $h_R$, based on 2.2 $\mu$m observations of $h_R$ are quite short: 2.0 kpc (Jones et al. 1981) and 3.0 kpc (Eaton et al. 1984, KDF 1991). In all cases, sources are located at low latitude. Therefore, the average value of $f(R/h, z/h_z(0))$ is smaller than 1. Hence, $h_R^{\text{const}}$ is smaller that $h_R$. In the first two cases, stars located near $b = 0$
Figure A.2: Function $f(z/h_z, R/h)$ of Equation A.4 versus $R/h$ for different values of $b$, where $b$ varies from $0^\circ$ to $10^\circ$.

are counted, and $\bar{f}(R/h, z/h_z(0)) \approx 0.0$. KDF averaged the surface brightness profiles in four cuts of constant latitude covering the range $|b| < 1^\circ$ ($f(R/h, z/h_z(0)) < 0.14$), $1^\circ < |b| < 2^\circ$ ($f(R/h, z/h_z(0)) < 0.55$), $2^\circ < |b| < 5^\circ$ ($f(R/h, z/h_z(0)) < 1.26$), and $5^\circ < |b| < 10^\circ$ ($f(R/h, z/h_z(0)) < 1.58$). Since the total averaged $\bar{f}(R/h, z/h_z(0))$ is less than 1, $h_R^{\text{const}} < h_R$, which is consistent with KDF's results that $h_R^{\text{const}} = 2.7$ kpc and $h_R = 3.0$ kpc. Therefore, under the assumption of a constant scale height, the $2.2\mu$m photometric determinations of $h_R$ are very low.
APPENDIX A. THE SCALE LENGTH OF THE GALAXY

Table A.1: $f(R/h, z/h_z(0))$

<table>
<thead>
<tr>
<th>$b^\circ$</th>
<th>$l = 0^\circ$</th>
<th>$l = 90^\circ$</th>
<th>$l = 180^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.18</td>
<td>0.40</td>
<td>0.11</td>
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<td>0.56</td>
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<td>0.33</td>
</tr>
<tr>
<td>3</td>
<td>0.90</td>
<td>1.51</td>
<td>0.55</td>
</tr>
<tr>
<td>4</td>
<td>1.14</td>
<td>1.78</td>
<td>0.72</td>
</tr>
<tr>
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<td>0.84</td>
</tr>
<tr>
<td>6</td>
<td>1.37</td>
<td>1.91</td>
<td>0.92</td>
</tr>
<tr>
<td>7</td>
<td>1.41</td>
<td>1.86</td>
<td>0.98</td>
</tr>
<tr>
<td>8</td>
<td>1.44</td>
<td>1.77</td>
<td>1.02</td>
</tr>
<tr>
<td>9</td>
<td>1.46</td>
<td>1.65</td>
<td>1.05</td>
</tr>
<tr>
<td>10</td>
<td>1.47</td>
<td>1.51</td>
<td>1.07</td>
</tr>
</tbody>
</table>

On the other hand, $h_R$ determined from IRAS stars counts are located at the high end of values: 4.5 kpc (Habing 1988) and 6.0 kpc (Rowan-Robinson & Chester 1987). This is due to the fact that the sources are located between $2^\circ < b < 10^\circ$, $10^\circ < l < 30^\circ$ (Rowan-Robinson & Chester 1987) and $15^\circ < l < 180^\circ$, but mainly $15^\circ < l < 90^\circ$ (Habing 1988). In both cases, $f(R/h, z/h_z(0)) > 1.0$, therefore, $h_R^{\text{const}} > h_R$.

UVB photometry towards the anti-center ($l = 179^\circ, b = 2.5^\circ$) led Robin et al. (1992) to the estimate $h_R^{\text{const}} = 2.5$ kpc. By using $f(R/h, z/h_z(0)) = 0.44$ ($l = 179^\circ, b = 2.5^\circ$) in Equation A.6. I obtain $h_R = 2.7$ kpc, which is very close to the value $h_R = 2.8$ kpc obtained by Robin et al. (1996) based on the thick disk model, but without the assumption of a constant $h_z$.

Van der Kruit (1986) analyzed the surface brightness of the Galactic background in the pioneer 10 background starlight experiment and found that the surface brightness ratio to be

$$SB(-90)/SB(180,0) = h_z/h_R = 1/17.0 \quad (A.7)$$

By choosing $h_z = 325$ pc for old dwarfs in the disk, van der Kruit obtained $h_R = 5.5$
kpc. If the scale height is not a constant. Equation A.7 can be written as

\[ SB(-.90)/SB(180.0) = h_z(0)(1/h_R + 1/h) = 1/17.0. \]  

(A.8)

in which \( h_z(0) \) can be calculated from the following equation. \( h_z(0) \exp(R_0/h) = 325 \) pc. By assuming \( h = 20 \) kpc and \( R_0 = 8.5 \) kpc. I determine \( h_z(0) = 212 \) pc. which yields. when substituted in Equation A.8. a value \( h_R = 4.4 \) kpc. It is consistent with FM’s conclusion that a model with shorter \( h_R \). but radially increasing \( h_z \). may fit the pioneer 10 data as well as van der Kruit’s (1986) model with \( h_R^{const} = 5.5 \) kpc and a constant \( h_z \).

### A.3 Summary

I have shown that the assumption of a constant scale height for the Galaxy may be the reason for the discrepancy which has emerged among the different photometric determinations of the scale length of the Galaxy. By using a non-constant scale height model. \( h_z(R) = h_z(0) \exp(R/h) \). which is derived on the basis of my numerical simulations. I have demonstrated that the scale length determined under the assumption of a constant scale height can either be underestimated or overestimated. depending on the location of the sources. In general. the scale height determined by \( \sim 2.2\mu m \) observations is underestimated. whereas that inferred from IRAS OH/IR star counts is overestimated. The optical measurements of the scale length can either be underestimated (Robin et al. 1992) or overestimated (van der Kruit 1986) depending on the location of the sources. Future observational data reduction. based on my non-constant scale height model. \( h_z(R) = h_z(0) \exp(R/h) \). perhaps result in a measured scale length which weakly depends on or may even be independent of the wavelength of observations.
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