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Design of Class 4 Hollow Structural Section Compression Members

by

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Abstract

This paper presents a study on the CSA S16:19, AISC 360-16, and EN 1993-1-1 approaches to determine the nominal capacity (C_n) of Class 4 hollow structural section (HSS) compression members. Class 3 limits and effective width (b_e) equations are compared, and the effect of member slenderness (KL/r), width-(or diameter-)to-thickness ratio, and height-to-width (or aspect) ratio on the relative C_n predictions are evaluated. For Class 4 rectangular hollow sections, CSA S16:19 is shown to under-predict C_n by up to 34% relative to AISC 360-16. A more economical, yet still safe, method to calculate b_e (and hence, C_n) is proposed. For Class 4 circular hollow sections (CHS), a new method to calculate C_n utilizing the “effective area method” is proposed. This new method removes the need for having an “effective yield stress method” in CSA S16 Clause 13.3.4.

Keywords

Hollow structural sections, compression members, columns, local buckling, global buckling, slender elements, effective widths, design.

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1. Introduction

There is presently a trend towards using large-size hollow structural section (HSS) columns in building construction. For many years, these sections have been imported from the Republic of Korea, South Africa, and Turkey (CISC 2016); however, larger sections, ranging in outer dimensions up to 864 mm, will soon be produced in the United States in the near future (Atlas Tube 2020). Over this new product range, a large number HSS will be “Class 4”. That is, by definition, they will contain plate elements, or be sections, that are susceptible to local buckling (CISC 2016).

In CSA S16:19, local buckling is typically avoided by adherence to maximum width-(or diameter)-to-thickness ratios (or “Class 3 limits”). However, for Class 4 sections, the interaction of local and global (i.e. flexural, torsional, or distortional) buckling must be considered. In that case, CSA S16:19 Clause 13.3.4 (previously Clause 13.3.5, in CSA S16-14) provides two methods to determine compressive resistance (C_r):

- (a) the “effective area method”; and
- (b) the “effective yield stress method”.

In Method (a), C_r is determined using an effective area calculated with reduced element widths that meet the corresponding Class 3 limit. In Method (b), C_r is determined using an effective yield stress determined from the width-to-thickness ratio meeting the same limit. The results (i.e. C_r) obtained by each method are not necessarily the same (CISC 2016) and, presently, designers can choose to use the one that produces the larger value of C_r .

Owing to the broad simplicity of these methods, CSA S16:19 Clause 13.3.4 can under-predict C_r for Class 4 members by a substantial margin (up to 34% relative to other specifications). In recognition of this, plus current trends in HSS production, this paper evaluates the CSA S16:19: (a) Class 3 limits; (b) effective property equations; and (c) local-global buckling interaction provisions for HSS compression members. Results from (c) (i.e. nominal compressive capacities, C_n , predicted by CSA S16:19 Clause 13.3.4) are compared to AISC 360-16 and EN 1993-1-1, and a more economical (yet, still safe) design method for Class 4 members is proposed.

2. Background

2.1. Local Buckling of Plates

Under ideal conditions, a doubly symmetric column, of length L , will buckle elastically at the Euler critical stress (F_e):

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \quad (1)$$

where K = column effective length factor (which depends on the manner of end support); E = elastic modulus; and r = radius of gyration. The column slenderness (KL/r) is taken about the axis that produces the largest value.

Similarly, under ideal conditions, a thin plate of width b_{el} (Fig. 1a) that is longitudinally compressed will buckle elastically at the critical plate-buckling stress (F_{el}):

$$F_{el} = k \frac{\pi^2 E}{(1-\nu^2)(b_{el}/r)^2} \quad (2a)$$

$$= k \frac{\pi^2 E}{12(1-\nu^2)(b_{el}/t)^2} \quad (2b)$$

where k = plate-buckling coefficient; ν = Poisson's ratio; and t = thickness of plate. In Fig. 1a, C = compression force.

(a) Deformed shape

(b) Longitudinal stress distribution

Fig. 1. Plate buckling (*two-column figure*)

The term involving the Poisson's ratio in Eqs. (2a,b) comes from the fact that a plate extends in two dimensions, in contrast to a column, which can be said to extend in only one. Additionally, for a plate of thickness t , the radius of gyration $r = t/\sqrt{12}$, which is substituted into Eq. (2a) to get F_{el} in the form of Eq. (2b). For a plate with both longitudinal edges supported by thin webs, such as that in Fig. 1a, k is usually (and conservatively) taken as 4.

Unlike columns, plates do not collapse (or fail) when the buckling stress F_{el} is reached. Instead, they develop slight, lateral deflections (Fig. 1a) and continue to support increasing load. The resistance in excess of F_{el} is known as “post-buckling strength”.

After the onset of buckling, various “struts” (or vertical strips of plate, as shown in Fig. 1a) deflect unequal amounts in proportion to their distance from the supports. Struts near the centre of the plate deflect more (because they are less stiff), contribute less to post-buckling strength, and may even shed some of their pre-buckling load to struts nearer to the supports. In contrast, those struts near the supports remain more-or-less straight and continue to resist increasing load. As a result, the hitherto uniform compression stress in the plate can become non-uniform. The plate fails (i.e. refuses to carry any more load) when the maximum/edge stress (f) (i.e. f_1 or f_2 , in Fig. 1b), reaches the yield stress (F_y).

It has long been recognized that it is difficult to take explicitly into account the non-uniform stress distribution in a plate. Thus, since the 1930s, the concept of an “effective design width” (b_e in Fig. 1b) has been widely used. Strictly speaking, the force resisted by a plate (C , in Fig. 1a) is equal to the integral of the (perhaps non-uniform) stress over the plate area ($b_{el} \times t$). However, with an “effective design width”, the non-uniform stress distribution is replaced by a constant stress equal to f (i.e. the maximum stress under non-uniform loading), and the total plate width, b_{el} , is replaced by b_e , such that $b_e \times t \times f$ gives the same result for C . This concept is illustrated in Fig. 1b (for two different values of the maximum/edge stress, f_1 and f_2 , and corresponding effective widths, $b_{e,1}$ and $b_{e,2}$) and has become widely used for the design of steel members, connections, and welds that experience non-uniform loading (Tousignant and Packer 2020).

2.2. Effective Width of RHS Elements

The expression for b_e that underlies local buckling criteria in North America and Europe is based on previous research by von Kármán et al. (1932) and Winter (1947). Some 150 tests were performed at Cornell University on steel sections with stiffened compression elements, covering $14.3 \leq b_{el}/t \leq 440$ (where b_{el}/t is the element “width-to-thickness ratio”), and from these tests the following lower-bound formula for b_e was derived (Winter 1970):

$$b_e = 1.9t \sqrt{\frac{E}{f}} \left(1 - \frac{0.475}{b_{el}/t} \sqrt{\frac{E}{f}} \right) \quad (3)$$

Eq. (3) is simply an “experimental modification” of the theoretical formula originally developed by von Kármán et al. (1932); i.e.:

$$b_e = \left(\frac{\pi}{3(1-\nu^2)} \sqrt{\frac{E}{f}} \right) t \quad (4)$$

From the 1940s to the 1960s, equations for b_e , such as those given in the *Light Gage Cold-Formed Steel Design Manual* (AISI 1961), were based, conservatively, on Eq. (3). In the 1968 Edition of the *Specification for the Design of Cold-Formed Steel Structural Members* (AISI 1968), which followed AISI (1961), a liberal adjustment was made to Eq. (3), which consisted solely of changing the constant in the last term from 0.475 to 0.415; i.e.:

$$b_e = 1.9t \sqrt{\frac{E}{f}} \left(1 - \frac{0.415}{b_{el}/t} \sqrt{\frac{E}{f}} \right) \leq b_{el} \quad (5)$$

Eq. (5) can be combined with Eq. (2b) (taking $k = 4$ and $\nu = 0.3$ in the latter), to give:

$$b_e = \sqrt{\frac{F_{el}}{f}} \left(1 - 0.22 \sqrt{\frac{F_{el}}{f}} \right) b_{el} \leq b_{el} \quad (6)$$

which is an arguably simpler form of Eq. (5) (and the basis for the EN 1993-1-5 effective width formula) (CEN 2006).

In AISI (1968), a slightly more liberal adjustment was made to Eq. (5) for RHS members; i.e.:

$$b_e = 1.9t \sqrt{\frac{E}{f}} \left(1 - \frac{0.386}{b_{el}/t} \sqrt{\frac{E}{f}} \right) \leq b_{el} \quad (7)$$

or:

$$b_e = \sqrt{\frac{F_{el}}{f}} \left(1 - 0.20 \sqrt{\frac{F_{el}}{f}} \right) b_{el} \leq b_{el} \quad (8)$$

This adjustment aimed to recognize that RHS are “strictly standardized” closed shapes, in contrast to the variety of mostly open shapes that were being produced by cold-formed steel manufacturers at that time.

Throughout most of the 1960s, the RHS effective width provisions of AISI’s counterpart steel code, AISC (1963), did not align with their own. But in 1969, AISC (1969) adopted a new approach that was based on Eq. (7)/(8) [see Eq. (C3-1) in AISC (1969)]. Today, after over 50 years of progress, Eq. (8) (with the same constant, 0.20) is still the basis of the b_e formula for RHS elements in AISC 360-16 Section E7.1 (AISC 2016).

2.2.1. Limiting Width-to-Thickness Ratios

Eqs. (3)-(8) illustrate that the effective width (b_e) of any element depends not only on b_{el}/t of that element, but also on the maximum stress f (and the load, C). Moreover, b_e decreases as f becomes larger, as shown in Fig. 1b. The effective width of a given plate element is hence highly variable. It is also apparent from Eqs. (3)-(8) that for any value of f , there is a width-to-thickness ratio b_{el}/t for which $b_e = b_{el}$.

As discussed in Section 1, local buckling of a compression member can be avoided in one of two ways: by selecting a Class 3 or better section [i.e. one that adheres to the appropriate “Class 3 limit” in CSA S16:19 Table 1, denoted $(b_{el}/t)_{\max}$ herein], or by designing the member in accordance with CSA S16:19 Clause 13.3.4. CSA S16:19 Table 1 is based on the notion above (that for any value of f , there is a width-to-thickness ratio b_{el}/t for which $b_e = b_{el}$), with f conservatively taken as F_y to cover all possible cases.

For example: setting $f = F_y$ and $b_e = b_{el}$ in Eq. (8), and taking $k = 4$ and $\nu = 0.30$ (as commonly done in North America and Europe) gives $(b_{el}/t)_{\max} = 615/\sqrt{F_y}$. This can be compared to the CSA S16:19 Table 1 limit of $670/\sqrt{F_y}$, as discussed in Section 3.

Eqs. (6) and (8), and their respective $(b_{el}/t)_{\max}$ values for CSA G40.20/21 Grade 350W steel (which is common for HSS in Canada), are also compared, in Fig. 2. As both equations serve as the basis for b_e equations outside of CSA S16:19 (in EN 1993-1-1 and AISC 360-16, respectively, as mentioned in Section 2.2), it is important to recognize that they produce similar results for the same steel grade.

Fig. 2. b_e/t vs. b_{el}/t for various steel grades assuming $E = 200,000$ MPa (*one-column figure*)

2.3. Local Buckling of CHS

It is well known that classical buckling theory (Timoshenko and Gere 1985) can overestimate the ultimate stress capacity (F_{ult}) of CHS (and other thin-walled cylinders) by up to 200% due to: (a) inevitable imperfections, and (b) loading eccentricity present in practical applications (CSA 2016, AISC 2016). Hence, CHS local buckling criteria have long been determined empirically (e.g. Plantemea 1946). As shown in Fig. 3, the ratio of F_{ult} to F_y for thin-walled tubes is typically plotted as a function of E , F_y , t and the mean tube diameter, d_m .

The solid black lines in Fig. 3 correspond to an early tube buckling solution by Plantema (1946), which is based on several lines of best fit through test data points. (These data points are illustrated in Fig. C-E3.1.1.1-2 of the Commentary to CSA S136-16) (CSA 2016). In that solution, there are three lines that correspond to sections that fail by elastic local buckling (Line 1), inelastic local buckling (Line 2), and yielding (Line 3). Line 3 in Fig. 3 (the label for which is circled) corresponds to the range of Class 3 or better tubes (i.e. tubes that yield before locally buckling). Hence, the maximum d_m/t ratio (or “Class 3 limit”), denoted $(d_m/t)_{max}$ herein, is given by Point A. Plantema’s (1946) solution placed Point A at $(E/F_y)(t/d_m) = 8$ [and a second point, Point B, at $(E/F_y)(t/d_m) = 2.5$ (with $F_{ult}/F_y = 0.83$)].

Fig. 3. F_{ult}/F_y vs. $(E/F_y)(t/d_m)$ for longitudinally compressed cylindrical tubes (*one-column figure*)

In code provisions that followed (i.e. AISI 1968), slight, conservative modifications were made to Plantema's (1946) solution, by moving Point A to $(E/F_y)(t/d_m) = 8.93$ and Point B to $(E/F_y)(t/d_m) = 2.27$ (with $F_{ult}/F_y = 0.75$). After dividing by an Allowable Stress Design safety factor ($= 1.67$), the resulting dashed line (between Points A1 and B1 in Fig. 3) gave the equation for the allowable stress of Class 4 tubes.

In the same code (AISI 1868), there was a notable lack of provisions for tubes with " $d_m/t > 13,000/F_y$ " (or about $88,100/F_y$ in S.I. units). This "absolute maximum" d_m/t value, which corresponds to $(E/F_y)(t/d_m) = 2.27$ (or Point B1, in Fig. 3), was intended to prevent the use of "extremely thin" tubes, which require sharply reduced (and thus uneconomical) design stresses and are sensitive to imperfections (Winter 1970). In specifications that followed, d_m was replaced by the outside CHS diameter, d_o , as a matter of convenience.

3. Classification of Cross Sections

According to CSA S16:19, Class 4 HSS are those with b_{el}/t (for any element) $> 670/\sqrt{F_y}$ or an outside diameter-to-thickness ratio $(d_o/t) > 23,000/F_y$ (Table 1), which correspond to $(b_{el}/t)_{\max}$ and $(d_o/t)_{\max}$ [akin to $(d_m/t)_{\max}$], respectively. For RHS in CSA S16:19, b_{el} is the clear width of an RHS element, calculated by assuming an outside corner radius (r_o) of $2t$. Hence, $b_{el} = b_o - 4t$ or $h_o - 4t$, where b_o and h_o are the outside width and height of the RHS (Fig. 4a).

Table 1. Limiting width-(or diameter-)to-thickness ratios for elements in HSS under axial compression

Shape	Specification/Reference	$(b_{el}/t)_{\max}$ or $(d_0/t)_{\max}$
RHS	CSA S16 Table 1	$670 / \sqrt{F_y}$
	AISC 360 Table B4.1a	$626 / \sqrt{F_y}$
	EN 1994-1-1 Table 5.2	$644 / \sqrt{F_y}$
CHS	CSA S16 Table 1	$23,000 / F_y$
	AISC 360 Table B4.1a ¹	$22,000 / F_y$
	EN 1994-1-1 Table 5.2	$21,150 / F_y$

Note: The above values assume a modulus of elasticity (E) = 200,000 MPa for CSA S16:19 and AISC 350-16, and E = 210,000 MPa for EN1993-1-1, commensurate with the value for E used in each code.

¹ AISC 360-16 Table B4.1a. AISC 360-16 also gives an “absolute maximum” value of $d_0/t = 90,000/F_y$.

Fig. 4. Cross-sectional dimensions (*one-column figure*)

In Chapter B of AISC 360-16, Class 4 sections are referred to as “slender” sections, and $(b_{el}/t)_{\max}$ and $(d_0/t)_{\max}$ are given as $626/\sqrt{F_y}$ and $22,000/F_y$ (Table B4.1a) (AISC 2016). Unlike CSA S16:19, AISC 360-16 [via Section B4.1(b)] specifies that $b_{el} = b_0 - 3t$ or $h_0 - 3t$, which assumes that $r_0 = 1.5t$. Hence, AISC’s Class 3 limits $[(b_{el}/t)_{\max}$ and $(d_0/t)_{\max}]$ and their method for calculating b_{el} are more conservative than CSA’s.

Like CSA S16:19, EN 1993-1-1 adopts the “Class 4” designation for sections with slender elements, but it also assumes $r_0 = 1.5t$ for calculating b_{el} [as noted in EN 1993-1-5 Clause 4.4(2)] (CEN 2006). The EN 1993-1-1 Class 3 limits are shown in Table 1, where it can be seen that they are generally more similar to those of AISC than CSA.

Of all three codes, only AISC 360-16 gives an “absolute maximum” d_o/t value for CHS ($= 90,000/F_y$), which closely corresponds to the “absolute maximum” d_m/t value of about $88,100/F_y$ given in AISI (1968). As noted in Section E7.2 of the AISC 360-16 Commentary (AISC 2016), this limit is provided for the same reason. It is worth noting that while none of the CHS in the 11th edition of the CISC *Handbook of Steel Construction* (CISC 2016), the 15th edition of the AISC *Manual* (AISC 2018), or that are currently planned for production (Atlas Tube 2020) have d_o/t values close to this limit, it is relevant for fabricated/custom CHS and pipe columns.

While the $(d_o/t)_{\max}$ values adopted by CSA S16:19, AISC 360-16 and EN 1993-1-1 for CHS are conservative relative to Point A1 in Fig. 3, the $(b_{el}/t)_{\max}$ values in Table 1 are notably unconservative relative to the value noted in Section 2.2.1 [i.e. $(b_{el}/t)_{\max} = 615/\sqrt{F_y}$]. The reason for this is the assumption used there that $k = 4$, which applies to plates that have simply supported longitudinal edges (e.g. Fig. 1a) (Peköz 1987). For RHS, this assumption is conservative, since adjacent, orthogonal elements (webs to flanges, and flanges to webs) provide some degree of fixity. A recent study by Seif and Schafer (2010) showed, using finite-strip analysis, that $k = 4.85$ can be considered more appropriate for a wide range of RHS flange elements under uniform compression.

The methodology described by Seif and Schaefer (2010), and presented in Section E7.1 of the AISC 360-16 Commentary (AISC 2016), can be applied to show that the $(b_{el}/t)_{\max}$ values of CSA S16:19, AISC 360-16 and EN 1993-1-1 are all generally safe (i.e. inherent $k < 4.85$) (Seif and Schafer 2010); however, it should be noted that CSA S16:19’s “Class 3 limit” is quite liberal compared to AISC 360-16’s when both the $(b_{el}/t)_{\max}$ values (in Table 2) and the b_{el} formula(e) (which incorporate different assumptions about r_o) are considered.

4. Local-Global Buckling Interaction Criteria

Several research projects in the 1970s (DeWolf et al. 1974; Kalyanaraman and Peköz 1978; Desmond et al. 1978) extended local buckling criteria to local-global buckling interaction. These projects found that the nominal buckling capacity ($C_{n,b}$) of Class 4 sections can be accurately computed by multiplying the effective area (A_e) by an *effective* nominal buckling stress ($F_{n,b}$) that (a) considers the relevant global buckling mode (e.g. flexural or Euler buckling, for HSS) and (b) is computed with *effective* properties (e.g. an effective radius of gyration, r_e). As explicitly stated in Section C4 of AISC (1969):

$$A_e = A - \sum (b_{el} - b_e) t \quad (9)$$

where A = gross cross-sectional area.

Eq. (9) can be applied to RHS by using the dimensions shown in Figs. 5a,b.

Fig. 5. Effective area of RHS members (shaded): (a) four slender elements; (b) two slender elements (*two-column figure*)

The drawback of the aforementioned approach is that it is iterative (i.e. $F_{n,b}$ depends on r_e which, in turn, depends on $F_{n,b}$). To address this, Peköz (1987) established the “unified approach” (from results of more than 508 experimental tests). According to this approach, $C_{n,b} = A_e \times F_{n,b}$, but where $F_{n,b}$ is computed by using *gross* (rather than *effective*) section properties, which eliminates the need for iteration.

The “unified approach” is similar to the approach(es) used today in CSA S16:19 and AISC 360-16 for local-global buckling interaction (Clause 13.3.4 and Section E7, respectively). It is simpler than the “iterative approach”, and it closely approximates it for the following reasons (Peköz 1987):

1. For short columns, $F_{n,b}$ is large relative to F_y , and $F_{n,b}$ is insensitive to changes in r (or r_e).
2. For long columns, $F_{n,b}$ is small relative to F_y , and local buckling itself is negligible. (Recall that b_e depends on f , which is physically limited to the small value of $F_{n,b}$ in this case).

The overall buckling behaviour of the column is thus well-represented by calculating the critical/buckling stress based on gross properties, and only afterwards multiplying it by A_e to find the ultimate load.

5. Design Specifications

5.1. CSA S16:19

In CSA S16:19, Clause 13.3.5 gives two methods to determine C_n ($= C_r/\phi = C_r/0.9$, where ϕ = resistance factor) for Class 4 members (i.e. to address local-global buckling interaction): (a) the “effective area method”; and (b) the “effective yield stress” method. It is well-known that different C_r values can be obtained by using each of these two methods.

5.1.1. Method (a): Effective Area Method

In Method (a) [CSA S16:19 Clause 13.3.5a)], C_n is calculated by using an effective area (A_e) [Eq. (9)] determined by using b_e values for each element that meet the CSA S16:19 Class 3 limit for RHS (Table 1); i.e.:

$$C_n = \frac{A_e F_y}{\left(1 + \lambda^{2n}\right)^{\frac{1}{n}}} \quad (10)$$

where:

$$\lambda = \sqrt{\frac{F_y}{F_e}} \quad (11)$$

where λ = non-dimensional slenderness parameter; n = column curve parameter (= 1.34 for HSS Class C and 2.24 for HSS Class H); and F_e = Euler critical stress, calculated according to Eq. (1), using gross section properties.

In Eq. (11) (for A_e):

$$b_e = \frac{670t}{\sqrt{F_y}} \quad (12)$$

The “effective area” method does not apply to CHS (as it is based on “effective widths”) and calculating A_e for CHS based on a reduced, effective diameter (which may appear logical) is grossly over-conservative. This is addressed by a new note in CSA S16:19 Clause 13.3.4b), stating that Method (b) (the effective yield stress method) is “intended for shapes with a circular cross section”. For clarity, however, it should be noted that Method (b) is applied by CISC to RHS as well (as discussed in Section 1).

5.1.2. Method (b): Effective Yield Stress Method

In Method (b) [CSA S16:19 Clause 13.3.5a)], C_n is calculated by using an effective yield stress (F_{ye}) determined in accordance with the appropriate CSA S16:19 Class 3 limit (Table 1); i.e.:

$$C_n = \frac{AF_{ye}}{\left(1 + \lambda_{ye}^{2n}\right)^{\frac{1}{n}}} \quad (13)$$

where:

$$\lambda_{ye} = \sqrt{\frac{F_{ye}}{F_e}} \quad (14)$$

where λ_{ye} = effective non-dimensional slenderness ratio.

For RHS:

$$F_{ye} = \left(\frac{670t}{b_{el}}\right)^2 \quad (15)$$

where b_{el} is the largest of all elements in the cross section (hence, conservative for “rectangular shapes”).

For CHS:

$$F_{ye} = \frac{23000t}{D} \quad (16)$$

5.2. AISC 360-16

In Section E7 of AISC 360-16, $C_n (= C_r/\phi = C_r/0.9)$ is given for Class 4 (or “slender”) sections as:

$$C_n = F_{cr} A_e \quad (17)$$

where F_{cr} = critical stress, akin to C_n/A_e in Eq. (12). That value [i.e. C_n/A_e in Eq. (12) or F_{cr} in Eq. (19)] represents the so-called “column curve expression” for flexural buckling. AISC’s column curve expression is as follows:

$$F_{cr} = \left(0.658^{\lambda^2}\right) F_y \text{ when } \lambda \leq 1.5 \quad (18)$$

$$F_{cr} = 0.877 F_e \text{ when } \lambda > 1.5 \quad (19)$$

where λ is given by Eq. (11) and F_e is calculated from Eq. (1) using gross section properties.

Both the AISC and CSA column curve expressions are plotted in Fig. 6, for $F_y = 350$ MPa and $A_e = A$, and using $n = 1.34$ for CSA. These expressions were developed based on statistical analyses of column strength data

by the Structural Stability Research Council (SSRC) and represent mean fits to the data (Bjorhovde and Birkemoe 1979; Ziemian 2010). Hence, their similarity.

Fig. 6. Comparison of CSA S16:19, AISC 360-16, and EN 1993-1-1 column curve expressions with $F_y = 350$ MPa (*one-column figure*)

For RHS, A_e in Eq. (17) is calculated by using Eq. (9) with the following equations for b_e :

$$b_e = b_{el} \text{ when } \frac{b_{el}}{t} \leq \frac{626}{\sqrt{F_{cr}}} \quad (20)$$

$$b_e = b_{el} \left(1 - c_1 \sqrt{\frac{F_{el}}{F_{cr}}} \right) \sqrt{\frac{F_{el}}{F_{cr}}} \text{ when } \frac{b_{el}}{t} > \frac{626}{\sqrt{F_{cr}}} \quad (21)$$

where c_1 = imperfection adjustment factor (= 0.20), and F_{el} = critical plate buckling stress, calculated according to Eq. (2) with $k = 4.13$ and $E = 200,000$ MPa (AISC 2016).

By virtue of being based on F_{cr} (rather than F_y alone), the effective width of Class 4 (or “slender”) elements in AISC 360-16 depends on the member slenderness, KL/r , and hence f , in accordance with the notion Eq. (8). This is much unlike CSA S16:19. It should also be noted that:

1. Eq. (21) is analogous to Eq. (8) with f limited to F_{cr} . This results in large values of b_e for longer columns (with all else being equal).

2. According to Eq. (21), the b_e/t value at which A_e (and hence C_n) is reduced is $626/\sqrt{F_{cr}}$, not $626/\sqrt{F_y}$ or the so-called “Class 3 limit” (see Table 1). This implies that Class 4 (or “slender”) sections will not always have a reduced stress capacity compared to Class 3 (or “non-slender”) sections.

These observations agree with the underlying research on post-buckling strength of plates (Winter 1970) and local-global buckling interaction (Peköz 1987).

For CHS:

$$A_e = A \text{ when } \frac{d_0}{t} \leq \frac{22000}{F_y} \quad (22)$$

$$A_e = \left[\frac{7600}{F_y (d_0 / t)} + \frac{2}{3} \right] A \text{ when } \frac{22000}{F_y} < \frac{d_0}{t} < \frac{90000}{F_y} \quad (23)$$

Eqs. (22) and (23), and their respective limits, borrow from Fig. 3 (Plantemea 1946; AISI 1968). It is therefore obvious that they are not strictly equations for “effective areas”, but rather “effective stresses” (Ziemian 2010). Tests on fabricated and manufactured tubes have shown that the above equations and AISC 360-16 provisions for Class 4 (or “slender”) CHS are conservative (AISC 2016).

5.3. EN 1993-1-1:2004

For Class 4 sections, EN 1993-1-1 Clause 6.3.1.1 gives C_n ($= \gamma_{M1} C_r = 1.0 C_r$, where γ_{M1} = partial factor for resistance of members to instability) as:

$$C_n = \chi F_y A_e \quad (24)$$

where χ = reduction factor for the relevant buckling mode, akin to F_{cr}/F_y in Eq. (18) or (19). For flexural buckling:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \leq 1.0 \quad (25)$$

$$\Phi = 0.5 \left[1 + \alpha (\bar{\lambda} - 0.2) + \bar{\lambda}^2 \right] \quad (26)$$

where α = imperfection factor ($= 0.49$ for cold-formed HSS) (EN 1993-1-1 Table 6.2); Φ = stability factor; and $\bar{\lambda}$ = non-dimensional slenderness parameter for Class 4 sections; i.e.:

$$\bar{\lambda} = \sqrt{\frac{A_e F_y}{N_{cr}}} \quad (27)$$

where N_{cr} = elastic critical buckling force based on gross sectional properties ($= AF_e$, for HSS).

The non-dimensional slenderness parameter $\bar{\lambda}$ is similar to λ [Eq. (11)] except it is based on forces (rather than stresses), and has the yield load ($= A_e F_y$) based on effective (rather than gross) section properties. This results in a lower (more liberal) prediction of the slenderness parameter (or an apparent “raise” in the EN 1993-1-1 column curve) for Class 4 sections. Unlike the North American column curve expressions, the EN 1993-1-1 column curve expression (given by C_n/A_e) represents a lower bound of column strength data (Bjorhovde and Birkemoe 1979). This is largely apparent in Fig. 6.

For RHS, A_e in Eqs. (24) and (27) is calculated by using Eq. (9) with:

$$b_e = b \text{ when } \frac{b_{el}}{t} \leq \frac{644}{\sqrt{F_y}} \quad (28)$$

$$b_e = \sqrt{\frac{F_{el}}{F_y}} \left(1 - 0.22 \sqrt{\frac{F_{el}}{F_y}} \right) b_{el} \leq b_{el} \text{ when } \frac{b_{el}}{t} > \frac{644}{\sqrt{F_y}} \quad (29)$$

These equations are given in EN 1993-1-5 Sections 4.4 (CEN 2006) in a considerably more complex form.

It should be noted that Eq. (29) is akin to Eq. (6) with $f = F_y$, which is conservative for intermediate and long columns (in the same way that CSA S16:19 Clause 13.3.4 is conservative for such columns, since it does not depend on member slenderness KL/r). In Eq. (29), F_{el} is calculated according to Eq. (2) with $k = 4.82$ and $E = 210,000$ MPa – the Eurocode values (see Table 2). As illustrated in Fig. 7, CSA S16:19 Method (a), AISC 360-16 and EN 1993-1-1 can give radically different values for b_e (and hence, C_n).

No straightforward rules are given in EN 1993-1-1 for the design of Class 4 CHS.

Fig. 7. Comparison of CSA S16:19, AISC 360-16, and EN 1993-1-1 b_e expressions with $F_y = 350$ MPa (*one-column figure*)

6. Comparison of CSA S16:19, AISC 360-16 and EN 1993-1-1

6.1. Range of Parameters

A range of parameters over which to compare the forgoing provisions was identified based on the HSS listed in the 11th Edition of the CISC *Handbook* (CISC 2016) and Atlas Tube's technical literature (Atlas Tube 2020). For RHS, a summary of the section designations, limiting b_{el}/t ratios, h_0/b_0 (or aspect) ratios and yield stress(es) of the Class 4 sections therein (according to CSA S16:19 Table 1) is provided in Tables 3 and 4 in the supplementary data. For CHS, the most slender section contained in the above references is HSS 610×9.5 made to ASTM A500 Gr. C. With $F_y = 317$ MPa (ASTM 2018) and using a design wall thickness of 0.9 times the nominal wall thickness, $d_0/t = 71.1 < 23,000/F_y = 72.6$, this section is Class 3 or better.

As shown in Tables 3 and 4 in the supplementary data, when available Class 4 RHS according to CSA S16:19 are considered, the following parameter ranges apply:

- $1.00 \leq h_0/b_0 \leq 3.40$;
- $39.9 \leq b_0/t$ or $h_0/t \leq 71.1$ (for slender walls of sections only); and
- $35.9 \leq b_{el}/t \leq 67.1$ (again, for slender walls of sections only).

These ranges, and the limit of $0 \leq KL/r \leq 200$ (CSA S16:19 Clause 10.4.2.1), are reflected in the comparisons in Section 6.2, which have been done for CSA G40.20/21 Grade 350W Class C HSS ($F_y = 350$ MPa and $n = 1.34$) (CSA 2018). The general conclusions from Section 6.2 also extend to ASTM A500 HSS (ASTM 2018), and other specifications/grades.

6.2. Results

The nominal compressive stress capacity (C_n/A) predicted by CSA S16:19 Methods (a) and (b), AISC 360-16 and EN 1993-1-1 is plotted in Figs. 8a-d for various (square) RHS with $h_0/b_0 = 1$. For clarity, these capacities are termed F_{CSA} , F_{AISC} and F_{EN} in the following discussion, where F_{CSA} refers to the larger of the predicted C_n/A values using CSA S16:19's Methods (a) and (b). Also plotted are the “unreduced” column curves for CSA S16:19 and AISC 360-16 (as dotted lines). These are provided only for reference.

For (square) RHS with $h_0/t = 39.9$ (Fig. 8a), which represent sections that are “almost Class 3”, the ratio of F_{CSA} to F_{AISC} ranges from 0.88 to 1.09. (In this case, a value less than 1.0 indicate CSA S16 is conservative relative to AISC 360-16). Similarly, the ratio of F_{CSA} to F_{EN} ranges from 1.03 to 1.10. For short columns (with low KL/r) CSA S16:19 is relatively liberal compared to AISC 360-16 because $b_e \approx b_{el}$ for both methods (and b_{el} is greater for CSA S16:19 than it is for AISC 360-16) (see Section 3).

As h_0/t increases (Figs. 8b-d), the effect of CSA S16:19's conservative b_e equation [Eq. (12)] becomes more pronounced. In Fig. 8d, for example, the ratio of F_{CSA} to F_{AISC} is as low as 0.66 (at $KL/r = 100$). On the other hand, the ratio of F_{CSA} to F_{EN} remains relatively constant, ranging from 0.89 to 1.17. This is unsurprising, since b_e in both CSA S16:19 and EN 1993-1-1 depends on the yield stress (F_y), rather than F_{cr} . It is also interesting to note that the apparent “raise” in the EN 1993-1-1 column curve (discussed in Section 5.3) has a relatively small effect on C_n .

For RHS with $h_0/b_0 = 1.67$ (Figs. 9a-d), similar trends can be observed. When $h_0/t = 39.9$ (Fig. 9a), corresponding to the lower end of the range identified in Section 6.1, F_{CSA}/F_{AISC} ranges from 0.88 to 1.09 and F_{CSA}/F_{EN} ranges from 1.02 to 1.10 (nearly the same as for the square RHS in Fig. 8a). When $h_0/t = 59.1$ (Fig. 9d), F_{CSA}/F_{AISC} ranges from 0.73 (at $KL/r = 110$) to 0.98 and F_{CSA}/F_{EN} ranges from 0.89 to 1.0. Aside from these extreme values, CSA S16:19 is very conservative relative to AISC 360-16 in the range of $35 \leq KL/r \leq 95$, which covers most steel building columns (Ziemian 2010).

For CHS, C_n/A predicted by CSA S16:19 Method (b) and AISC 360-16 are compared in Figs. 10a and 10b, for sections with $d_o/t = 65.7$ and 71.1 , respectively. (Note that for CHS with $F_y = 350$ MPa, “the Class 3 limit” is 65.7). To illustrate trends, Fig. 10c shows the same curves for CHS with $d_o/t = 90$ (which falls outside the range of available HSS).

By examining Figs. 10a-c, it is clear that CSA S16:19 Method (b) becomes increasing conservative relative to AISC 360-16 as d_o/t increases. (Note, in Section 5.2, that AISC’s method is conservative on its own) (AISC 2016). In Fig. 10c, the ratio of C_n/A according to CSA S16:19 Method (b) to F_{AISC} ranges from 0.80 (at $KL/r = 0$) to 1.16 (at $KL/r = 200$), representing an under-prediction of C_n by up to 20% .

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(a) with $h_0/b_0 = 1$ and $h_0/t = 39.9$ (b) with $h_0/b_0 = 1$ and $h_0/t = 48.0$ (c) with $h_0/b_0 = 1$ and $h_0/t = 53.1$ (d) with $h_0/b_0 = 1$ and $h_0/t = 59.1$

Fig. 8. Comparison of CSA S16:19, AISC 360-16, and EN 1993-1-1 nominal strength predictions for Class 4 (square) RHS (*two-column figure*)

Fig. 9. Comparison of CSA S16:19, AISC 360-16, and EN 1993-1-1 nominal strength predictions for Class 4 RHS (*two-column figure*)

Fig. 10. Comparison of CSA S16:19 and AISC 360-16 nominal strength predictions for Class 4 CHS (*two-column figure*)

7. Proposed Modifications to CSA S16

To address the inherent conservatism of CSA S16:19 for design of Class 4 RHS compression members, it is proposed to modify the current b_e expression [Eq. (12)] to:

$$b_e = b_{el} \text{ when } \frac{b_{el}}{t} \leq \frac{670}{\sqrt{F_{cr}}} \quad (30a)$$

$$b_e = \frac{670t}{\sqrt{F_{cr}}} \text{ when } \frac{b_{el}}{t} > \frac{670}{\sqrt{F_{cr}}} \quad (30b)$$

where, for CSA S16:

$$F_{cr} = \frac{F_y}{\left(1 + \lambda^{2n}\right)^{\frac{1}{n}}} \quad (31)$$

$C_n (=C_r/\phi = C_r/0.9)$ can then be determined by using Method (a) [Eq. (10)] with A_e from Eq. (9). By being based on F_{cr} (rather than F_y , as it is presently), the effective width of Class 4 RHS elements would then correctly depend on KL/r , and hence f , and provide more liberal predictions of C_n for typical columns. The recommended b_e expressions for CSA S16 [Eqs. (30a,b) and (31)] and those of AISC 360-16 [Eqs. (20) and (21)] are compared in Fig. 11 in the supplementary data (across a range of different KL/r values). Fig. 11 in the supplementary data is akin to Fig. 7.

For CHS, it is recommended to determine C_n from Eq. (10) with:

$$A_e = A \text{ when } \frac{d_0}{t} \leq \frac{23000}{F_y} \quad (32a)$$

$$A_e = \left[\frac{7600}{F_y (d_0/t)} + \frac{2}{3} \right] A \text{ when } \frac{23000}{F_y} < \frac{d_0}{t} < \frac{90000}{F_y} \quad (32b)$$

As noted in Section 2.3, the “absolute maximum” d_0/t value ($= 90,000/F_y$) in Eq. (32b) is proposed to prevent the use of “extremely thin” tubes that require uneconomical design stresses and are sensitive to imperfections (Plantema 1946; AISC 2018).

Curves for C_n/A given by the above-proposed method(s) are shown (as solid black lines) in Figs. 8-10, where it can be seen that they result in a more liberal, yet safe, approach (compared to CSA S16:19). For RHS in Figs. 8a-d and 9a-d, the ratio of C_n/A for the proposed method to F_{AISC} ranges from 0.89 (in Fig. 8b, at $KL/r = 90$)

to 1.08. This reflects an improvement of up to 35%. For CHS in Fig. 10c, the same ratio ranges from 0.89 (at $KL/r = 90$) to 1.08 (at $KL/r = 200$), reflecting an improvement of up to 11%.

8. Conclusions

Previous research has been reviewed, and the CSA S16:19, AISC 360-16, and EN 1993-1-1 approaches to determine the nominal capacity (C_n) of Class 4 HSS compression members have been compared. Based upon this work, the following conclusions can be made (for CSA S16:19):

- For RHS and CHS, the CSA S16:19 $(b_e/t)_{\max}$ and $(d_o/t)_{\max}$ values (or “Class 3 limit”) are generally safe (Seif and Schafer 2010; Plantema 1946); however, an absolute upper limit of $(d_o/t)_{\max} = 90,000/F_y$ is recommended in CSA S16 to prevent the use of “extremely thin” tubes (see Section 2.3).
- For RHS, the CSA S16:19 Clause 13.3.5 “effective area method” [Method (a)] can be over-conservative by up to 34% relative to AISC 360-16; it is thus recommended to allow calculation of b_e according to Eqs. (30a,b) which are more accurate in considering b_e at the critical/buckling stress.
- For CHS, the CSA S16:19 Clause 13.3.5 “effective yield stress method” [Method (b)] has the potential to be over-conservative. It is thus recommended to adopt expressions for A_e [based on Eqs. (32a,b)] for use with Method (a) [thus removing the need for Method (b)].

The above recommendation for design of Class 4 RHS can be applied to non-RHS members provided that F_e and F_{cr} are calculated appropriately using Clause 13.1, 13.2 or 13.3 (CSA 2016).

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References

- AISC. 1963. Specification for the design, fabrication and erection of structural steel for buildings. American Institute of Steel Construction. New York, USA.
- AISC. 1969. Specification for the design, fabrication and erection of structural steel for buildings. American Institute of Steel Construction. New York, USA.
- AISC. 2016. Specification for structural steel buildings. AISC 360-16, American Institute of Steel Construction, Chicago, USA.
- AISC. 2018. Steel construction manual, 15th ed., 2nd printing. American Institute of Steel Construction, Chicago, USA.
- AISI. 1961. Light gage cold-formed steel design manual. American Iron and Steel Institute, New York, USA.
- AISI. 1968. Specification for the design of cold-formed steel structural members. American Iron and Steel Institute, New York, USA.
- ASTM. 2018. Standard specification for cold-formed welded and seamless carbon steel structural tubing in rounds and shapes. ASTM A500/A500M-18, American Society for Testing and Materials International, West Conshohocken, USA.
- Atlas Tube. 2020. Coming soon: World's largest continuous ERW tube mill (brochure). Atlas Tube Inc., Chicago, USA.
- Bjorhovde, R. and Birkemoe, P.C. 1979. Limit states design of HSS columns. Canadian Journal of Civil Engineering, Canadian Society for Civil Engineering, **6**: 276 – 291.
- CEN. 2004. Design of steel structures – Part 1-1: General rules and rules for buildings. EN 1993-1-1, European Committee for Standardization, Brussels, Belgium.
- CEN. 2006. Design of steel structures – Part 1-5: Plated structural elements. EN 1993-1-5, European Committee for Standardization, Brussels, Belgium.
- CISC. 2016. Handbook of steel construction, 11th ed. Canadian Institute of Steel Construction, Toronto, Canada.
- CSA. 2014. Design of steel structures. CSA S16-14, Canadian Standards Association, Toronto, Canada.
- CSA. 2016. North American specification for the design of cold-formed steel structural members. CSA S136-19, Canadian Standards Association, Toronto, Canada.

- CSA. 2018. General requirements for rolled or welded structural quality steel / Structural quality steel. CSA G40.20-13/G40.21-13 (R2018), Canadian Standards Association, Toronto, Canada.
- CSA. 2019. Design of steel structures. CSA S16:19, Canadian Standards Association, Toronto, Canada.
- Desmond, T.P., Peköz, T., and Winter, G. 1978. Local and overall buckling of cold formed compression members. Department of Structural Engineering Report, Cornell University, Ithaca, USA.
- DeWolf, J.T., Peköz, T. and Winter, G. 1974. Local and overall buckling of cold formed steel members. Journal of the Structural Division, American Society of Civil Engineers, **100**(10): 2017–2036.
- Kalyanaraman, V. and Peköz, T. 1978. Analytical study of unstiffened elements. Journal of the Structural Division, **104**(ST9): 1507-1524.
- Peköz, T. 1986. Development of a unified approach to the design of cold-formed steel members. Report S6 86-4, American Iron and Steel Institute, Washington, USA.
- Plantema, F.J. 1946. Collapsing stresses of circular cylinders and round tubes. Report S.280, Nat. Luchtvaartlaboratorium, Amsterdam, Netherlands.
- Seif, M. and Schafer, B.W. 2010. Local buckling of structural steel shapes. Journal of Constructional Steel Research, **66**: 1232-1247.
- Timoshenko, S.P. and Gere, J.M. 1985. Theory of elastic stability, 2nd ed. McGraw-Hill, New York, USA.
- Tousignant, K., and Packer, J.A. 2020. Weld design for hollow structural section connections: Application to Canadian standards. Canadian Journal of Civil Engineering, Canadian Society for Civil Engineering, in press.
- von Kármán, T., Sechler, E.E., and Donnell, L.H. 1932. The strength of thin plates in compression. Transactions, American Society of Mechanical Engineers, **54**: 53–58.
- Winter, G. 1947. Strength of thin steel compression flanges. Transactions, American Society of Civil Engineers, **122**: 527 – 554.
- Winter, G. 1970. Commentary on the 1969 edition of the specification for the design of cold-formed steel structural members. American Iron and Steel Institute, New York, N.Y.
- Ziemian. 2010. Guide to stability design criteria for metal structures, 6th ed. John Wiley & Sons, Inc. Hoboken, USA.

List of Symbols

A	gross cross-sectional area of HSS
A_e	effective cross-sectional area of HSS
C	compressive force
C_n	nominal compressive capacity
C_r	compressive resistance
E	elastic modulus
F_{AISC}	nominal compressive stress capacity according to AISC 360-16
F_{CSA}	nominal compressive stress capacity according to CSA S16:19
F_{EN}	nominal compressive stress capacity according to EN 1993-1-1
F_{cr}	critical stress
$F_{cr,e}$	effective critical stress
F_e	Euler critical stress
F_{el}	critical plate buckling stress
$F_{n,b}$	nominal buckling stress
F_{ult}	ultimate stress capacity
F_y	yield stress
F_{ye}	effective yield stress
K	effective length factor
L	length of column
N_{cr}	elastic critical buckling force based on gross sectional properties ($= AF_e$)
b_0	outside width of RHS
b_e	reduced effective width
$b_{e,1}$	reduced effective width at load 1
$b_{e,2}$	reduced effective width at load 2
b_{el}	width of plate; width of compression element
$(b_{el}/t)_{\max}$	Class 3 limit for elements of RHS
c_1	imperfection adjustment factor
d_0	outside diameter of CHS
$(d_0/t)_{\max}$	Class 3 limit for CHS, using outside diameter
d_m	mean diameter of cylinder
$(d_m/t)_{\max}$	Class 3 limit for cylinders, using mean diameter
f	maximum (or edge) stress
f_1	maximum (or edge) stress at load 1
f_2	maximum (or edge) stress at load 2

h_0	outside height of RHS
k	plate buckling coefficient
n	column curve parameter (in CSA S16:19)
r	radius of gyration
r_0	outside corner radius of RHS
r_e	effective radius of gyration
t	thickness of plate; design wall thickness of HSS
Φ	stability factor (in EN 1993-1-1)
α	imperfection factor (in EN 1993-1-1)
γ_{M1}	partial factor for resistance of members to instability
λ	non-dimensional slenderness parameter
λ_{ye}	effective non-dimensional slenderness ratio
$\bar{\lambda}$	non-dimensional slenderness parameter for Class 4 sections (in EN 1993-1-1)
ν	Poisson's ratio (= 0.30 for steel)
ϕ	resistance factor
χ	reduction factor for the relevant buckling mode (in EN 1993-1-1)

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Fig. 2. b_e/t vs. b_{el}/t for various steel grades assuming $E = 200,000$ MPa (*one-column figure*)

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Fig. 6. Comparison of CSA S16:19, AISC 360-16, and EN 1993-1-1 column curve expressions with $F_y = 350$ MPa (*one-column figure*)

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