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**Determination of 3D PSFs from computed tomography
reconstructed x-ray images of spherical objects and the effects of
sphere radii**

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Abstract

Purpose: A method was developed to obtain three dimensional (3D) point spread functions (PSFs) of reconstructed x-ray volumetric images using spheres of known diameters. The algorithm consists of a sphere localization step using template matching applied to the entire volume.

20 Richardson Lucy (RL) deconvolution is used atypically to determine the PSF from the reconstructed x-ray image and a model of the sphere. The resulting PSF is arbitrary *i.e.*, there are no assumptions of separability or symmetry. Oversampling is not used, and sample spacing matches the image. The effect of sphere radius on PSF estimate reproducibility is investigated.

Methods: Phantoms were constructed by suspending 5 polytetrafluoroethylene (PTFE) spheres
25 having known radii equal to 4.77, 7.95, 9.52, 12.68 and 19.53 mm in an agar solution. The phantom included a 25 μm steel wire to calculate a line spread function (LSF). The phantom was imaged and reconstructed with a Medtronic surgical O-Arm 23 times and a Toshiba Aquilion One CT 20 times. A sharp reconstruction kernel exhibiting a non-monotonic PSF was used with the Toshiba CT. PSFs and LSFs were computed for all of the images and repeated estimates were used to
30 compute mean and standard deviation values for every point of the PSFs and LSFs. The PSFs from spheres were converted to LSFs and compared to the wire LSF.

Results: The standard deviations of the PSF estimates exhibit a decreasing trend as the sphere radius is increased. The PSF from the smallest 4.77 mm sphere is the least reproducible. The normalized root mean square difference between the mean LSF derived from the 4.77 mm radius
35 sphere and the mean wire LSF is 2.0% for the O-arm and 1.2% for the CT.

Conclusion: RL deconvolution provides a method to estimate generalized (no separability or other simplifying assumptions) 3D PSFs from spheres. X-ray noise in images acquired with typical clinical protocols cause noticeable variations in PSF estimates which can be mitigated by selecting larger spheres and combining PSF estimates from different images.

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I. INTRODUCTION

Resolution and noise are principal determinants of image quality. The resolution of an image can be characterized by its point spread function (PSF) or equivalently via its spatial frequency counterpart known as the modulation transfer function (MTF). Noise is commonly characterized via the frequency space noise power spectrum (NPS). This paper investigates a method to calculate PSF estimates from three dimensional reconstructed x-ray images of spherical test objects and examines how sphere radius affects these estimates. Determination of PSFs or resolution surrogates such as the resolvable line-pair distance are used to evaluate and optimize the performance of new detector designs. Detective quantum efficiency, a fundamental measure of detector performance is calculated from the MTF (PSF) and NPS. Measurements of resolution are also performed as part of image quality assurance programs to insure that equipment is functioning properly. Our interest is in using the PSF to perform non-blind restoration using deconvolution techniques. This is useful in the presence of small and/or thin bony structure which exhibit a drop in reconstructed voxel values causing biases in the determination of bone interfaces which affect segmentation used in surgical planning. Pakdelet *et al.*[1] have demonstrated how corrections reduce these biases.

Conceptually, the PSF can be obtained directly by imaging an infinitesimally small object. However, this is not practical as it results in a noisy estimate due to the small number of voxel values used to obtain the PSF. Thus, other methods have been developed whereby a measure of resolution is computed from a large feature encompassing more voxels to reduce noise. For example, in x-ray imaging, a wire can provide a two dimensional measure of resolution known as the line spread function (LSF) [2]. Other approaches have been developed using a slit[3] and a thin sheet of foil[4]. Ohkubo *et al.* [5] used a cylinder and assumed z separability to obtain a PSF in the transverse plane. Many of these approaches typically also involve tilting the object relative to some principal axes of the imaging system to yield oversampled response functions.

Several methods have been proposed to determine PSFs using spheres composed of materials having different attenuating properties than the surrounding medium[6–10]. Spheres are compact, require no alignment and can potentially provide PSF estimates in all directions. Chen *et al.*[6] used a 6 mm diameter polytetrafluoroethylene (PTFE) plastic sphere with the assumption of cartesian separability to obtain a 3D Gaussian PSF. PTFE has

x-ray attenuation properties similar to bone. Thornton *et al.*[7] used PTFE spheres with diameters of 0.5, 1.0 and 1.5 inches to obtain surface spread functions (SPF)s over a conic section. The SPF's are smoothed using b-splines, differentiated and combined to yield PSF's.

75 This approach requires the diameter of the sphere to be large compared to the width of the PSF. Steiding *et al.*[9] developed a quality assurance framework and phantom whereby a 12 mm diameter aluminum sphere is used to obtain a PSF using the aforementioned method described by Thornton *et al.*[7].

Lee *et al.*[10] described a method to determine the MTF in any arbitrary direction that
80 inspired our approach. Reconstructed x-ray image data encompassing a sphere of known diameter is integrated along 2 orthogonal directions defining a plane and resulting in a one dimensional plane integral function called $i_{pl}(\alpha)$. A model of the sphere is integrated along the same directions yielding $m_{pl}(\alpha)$. It can be shown that if a 3D image can be described by the convolution of a 3D model with a 3D filter/kernel function then $m_{pl}(\alpha)$ is similarly related
85 to $i_{pl}(\alpha)$ via a one dimensional convolution operation such that $i_{pl}(\alpha) = m_{pl}(\alpha) * PLSF(\alpha)$ where $*$ is a 1D convolution operator. $PLSF(\alpha)$ is called a plane spread function. The 1D Fourier transform of $PLSF(\alpha)$ is related to the MTF along a direction orthogonal to said plane by the central slice theorem. Richardson and Lucy deconvolution [11, 12] is typically used to estimate a model knowing the filtered image and filtering function but in Lee *et al.*
90 *al.* it is used atypically to estimate the $PLSF(\alpha)$ kernel/filter function instead knowing the plane integrals of the image $i(\alpha)$ and model $m(\alpha)$. The method is advantageous as it uses all of the information available in the image to obtain the MTF and is thus less sensitive to noise than a method that only uses a subset of data such as a conic section. This approach is also applicable if the PSF width is comparable or greater than the sphere diameter. The
95 method was validated with experimental and simulated data demonstrating its effectiveness.

Our primary interest is image restoration and not imaging system characterization. We require a system transfer function estimate with the same sampling structure and sampling density as the image so not along arbitrary radial directions as shown by Lee *et al.*[10]. Resampling these results would be inefficient because, as we will explain, the PSF's can
100 be obtained directly by instead applying the same atypical usage of RL deconvolution as described in Lee *et al.* directly to the 3D image data. Thus, we propose a method to obtain arbitrary/generic PSF from spheres using RL deconvolution and investigate the performance of this approach for spheres of differing diameters.

II. THEORY

A. Sphere localization

A robust strategy was developed to localize spheres in the presence of other image features without user interaction such as seeding of the sphere positions or limiting searches to volumes of interest. Our approach would allow for the spheres to be localized if used in conjunction with a clinical scan.

It is required that the spheres have known diameters and uniform compositions with x-ray attenuation properties that differ from the surrounding media in which they are suspended to create contrast. It is assumed that none of the voxel values for the various components in the phantom are known *a-priori*. Thus, the method is also applicable with voxel values having arbitrary scaling and offset *i.e.* values not expressed in Hounsfield units. These conditions are consistent with template matching using a three dimensional normalized cross-correlation (NCC)[13].

We define the target image $I(x, y, z)$ having $L_I \times M_I \times N_I$ voxels such that $x \in [0, L_I)$, $y \in [0, M_I)$, $z \in [0, N_I)$. The template image $T(x, y, z)$ has $L_T \times M_T \times N_T$ voxels such that $x \in [0, L_T)$, $y \in [0, M_T)$, $z \in [0, N_T)$. The template contains a noise free model of the sphere and has a region of support just large enough to hold said sphere and is usually smaller than the image. The NCC of the target image and template image is written

$$C(x, y, z) = \frac{\sum_{x', y', z'} I(x', y', z')(T(x' - x, y' - y, z' - z) - \bar{T})}{\sqrt{\sigma_I^2(x, y, z)\sigma_T^2}}, \quad (1)$$

where

$$\sigma_I^2(x, y, z) = \sum_{x'=x}^{x+L_T-1} \sum_{y'=y}^{y+M_T-1} \sum_{z'=z}^{z+N_T-1} [I(x', y', z') - \overline{I(x, y, z)}]^2, \quad (2)$$

$$\sigma_T^2 = \sum_{x', y', z'} [T(x', y', z') - \bar{T}]^2, \quad (3)$$

$$\overline{I(x, y, z)} = \sum_{x'=x}^{x+L_T-1} \sum_{y'=y}^{y+M_T-1} \sum_{z'=z}^{z+N_T-1} I(x', y', z') / (L_T M_T N_T), \quad (4)$$

and

$$\bar{T} = \sum_{x', y', z'} T(x', y', z') / (L_T M_T N_T). \quad (5)$$

The *NCC* is a measure of how well the template shifted by (x, y, z) resembles the target image whereby $C(x, y, z) \in [-1., 1.]$. Due to rotational symmetry of the template which contains a sphere, there is no need to repeat the NCC calculation for different rotations of the template. If the target image is $I(x, y, z) = aT(x - x_0, y - y_0, z - z_0) + b$, with
135 $a \neq 0$ then the NCC operation between $I(x, y, z)$ and $T(x, y, z)$ would yield a maximum for C at (x_0, y_0, z_0) such that $C(x_0, y_0, z_0) = 1.0$ indicating a perfect fit independent of a and b which is the behavior we want. The NCC like other template matching methods is computationally intensive but it can be speed-ed up using FFTs and sum tables also known as prefix sums[14, 15]. Localization is used to define a VOI encompassing the sphere. The
140 VOI for each sphere is padded to allow for sampling of the effects of the PSF tail but limited in extent to reduce noise[16].

B. Sphere Model

A template voxel volume is created to hold a sphere model at its center. The voxel count is made sufficient to support the sphere (no truncation). The model of the sphere is
145 calculated by giving voxels within the radius of the sphere a value of 1 and points outside a value of 0 given by

$$S(x, y, z, x_0, y_0, z_0, R) = \begin{cases} 1 & \text{if } (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 < R^2 \\ 0 & \text{otherwise} \end{cases}, \quad (6)$$

The sphere can be generated with voxel spacings Q times smaller in all directions than the voxel spacing used for $I(x, y, z)$. The high resolution sphere model is subsequently
150 downsampled to exhibit the same voxel spacing as $I(x, y, z)$ producing a linear partial volume effect. The sphere model is used to locate spheres and also to determine PSFs. When determining the PSF, we set $Q = 1$ so the model has hard edges because PSF determination only needs an approximate seed position for the sphere. Furthermore, sphere localization may be done on a downsampled image to speed up calculations. The PSF centers are
155 identified when data from different scans are combined in a subsequent step described in Sec. II E.

C. PSF determination

Richardson and Lucy deconvolution [11, 12] involves convolution operations which can be computed via fast Fourier transforms and parallelized. Many computing environments and libraries provide RL implementations with varying degrees of optimization. Given an object $o(x, y, z)$, *i.e.* the truth, and a filter function $p(x, y, z)$, the filtered image is given by $i(x, y, z) = o(x, y, z) * p(x, y, z)$ where $*$ is the 3D convolution operator. RL can be used to estimate $o(x, y, z)$ knowing $i(x, y, z)$ and $p(x, y, z)$ but as alluded to in Section I, it can also be used atypically to estimate $p(x, y, z)$ knowing $i(x, y, z)$ and $o(x, y, z)$. Fish *et al.*[17] employed both usages of the RL formulation to perform so called blind deconvolution. It is possible that others have used RL to estimate blurring kernels when it was known via different names. We use the Matlab (Version R2015a, The MathWorks Inc., Natick, MA) implementation of RL which includes modifications to accelerate restoration[18].

The function estimated by RL must either be all positive or all negative, otherwise the method does not converge. Depending on the range of values in the image and sphere models, a PSF with values of both signs *i.e.* exhibiting zero crossings is possible and cannot be estimated correctly. In our usage scenario, the negative values of the PSF become clipped. Lee *et al.*[10] add an offset $O\Sigma_{x,y,z}o(x, y, z)$ to the image so the PSF is increased by O and is all positive *i.e.* $\min_{x,y,z}(PSF(x, y, z)) > 0$ and subsequently correct the PSF by subtracting the offset. We found that PSF accuracy is degraded for larger radii if the offset is larger than required to obtain a positive PSF. This behavior is not seen with simulated data. To mitigate this additional problem, the offset is subsequently reduced until $\min_{x,y,z}(PSF(x, y, z)) \in [0, \epsilon]$ where ϵ is a small positive value (0.2). The offset applied to the image to achieve a mid-range target value of $\min_{x,y,z}(PSF(x, y, z)) = \epsilon/2$ is predictable by the properties of the convolution. The j th iteration corrected offset is $O_{j+1} = O_j - (\min_{x,y,z}(PSF_j(x, y, z)) - \epsilon/2) * \lambda$ where λ is a relaxation parameter (0.8). The parameter is needed prevent overshoot due to errors in offset prediction which could (again) cause PSF clipping.

Deconvolution with a sphere model is sensitive to sphere radius accuracy and image voxel spacing being in concordance with values specified in the DICOM tags. An incorrect sphere radius will cause systematic errors in the PSF. Our approach introduces a multiplicative geometrical scaling factor, M , for the sphere model radius to eliminate said errors. This

factor is determined by repeating the PSF calculation and sphere position are varied in a loop driven by simplex optimization whereby RMS differences between the image of the sphere and sphere model are minimized. Linear regression is performed to match image and sphere model voxel values at each loop step by rescaling the model such that $o'(x, y, z) = o(x, y, z)m + b$

D. Comparison to LSF via wire method

The phantom contains a wire allowing a determination the line spread function (LSF) in the transverse plane for each image. We use the method of Fujita *et al.*[3] to estimate the oversampled (sampling density higher than voxel spacing) LSF of an imaging system from a wire. The wire is selected to be small relative to the voxel spacing and the chosen material has an attenuation coefficient much greater than the surrounding media. The wire is tilted by small angles relative to one of the three principal image axes. As one travels along the wire length, the wire cross-section in the plane (nearly) orthogonal to the wire shifts by sub-pixel amounts, each plane providing an LSF estimate. These estimates can be combined to obtain an oversampled LSF approximating its pre-sampled counterpart.

To compare our results to the wire method, the PSF is converted to an LSF. Let us define an infinite wire along the z direction as $w(x, y, z) = \delta(x)\delta(y)$ where $\delta()$ is a 1D Dirac delta function. The xy -LSF is computed by convolving the PSF with the z directed wire such that

$$LSF(x, y, z) = PSF(x, y, z) * \delta(x)\delta(y) \quad (7)$$

where $*$ is the 3D convolution operator. By the sifting properties of the delta function

$$LSF(x, y) = \int_{-\infty}^{\infty} PSF(x, y, z) dz, \quad (8)$$

and thus the LSF is computed from the PSF by integrating over z . A z separable PSF is written as

$$PSF(x, y, z) = PSF_{xy}(x, y)PSF_z(z). \quad (9)$$

Substituting Eq. 9 into Eq. 7 yields

$$LSF(x, y) = PSF_{xy}(x, y) \int_{-\infty}^{\infty} PSF_z(z) dz \quad (10)$$

215 and thus the LSF equals the PSF in the xy direction multiplied by a constant. If the PSF is z separable, agreement between the xy LSF derived from our method and an LSF determined via a reference method (*e.g.* using a wire) infers that the xy component of PSF is consistent with the reference. In general, the reverse operation of computing the PSF from the LSF is not possible unless the PSF fulfills some separability or symmetry conditions or
 220 LSF estimates are obtained from a multitude of directions.

E. PSF/LSF noise reduction by combining multiple estimates

Given a volumetric x-ray image of a sphere, the localization of the resultant PSF in a 3D sampled volume is subject to many factors by the properties of convolution integrals. These include: the location of sphere itself in the x-ray image, the extent of the volume of
 225 support containing the model sphere, the location of the sphere in said model. Uncontrolled localization is problematic in one dimensional plots extracted from 3D data combining different conditions like changing sphere radii. One or more of the one dimensional plots could miss the 3D peak of the function by varying amounts. Interpolation solves this problem by enabling data to be shifted so their respective central features occur exactly at prescribed
 230 discrete spatial indices. The prescribed indices may correspond for example to those closest to the center of the volume or any other arbitrary definition.

We identify the center of mass (COM) of the PSF for the first estimate and interpolate the PSF so its center corresponds to prescribed indices. Lanczos interpolation, a windowed sinc-interpolation method, is used because it provides an accurate reconstruction of signals
 235 that satisfy the Nyquist-Shannon sampling theorem. Subsequent estimates are shifted by the same amount with an additional correction for motion of the sphere. The motion correction is defined as the difference in COM of the sphere from the current image minus the estimate from the first image. This correction requires that variations in COM from repeated scans be mostly due to acquisition geometry variations with each scan and not x ray noise. The
 240 interpolated PSF are averaged together and the COM is computed. The residual distance from the prescribed center indices is re-computed and interpolation is done from the original data as before but with the addition of said residual distance. This latter step is repeated until the COM estimate is within a small voxel spacing fraction from the prescribed voxel indices but with the interpolation always done from the original data with increasingly

refined shift estimates. The PSFs are averaged together yielding a motion corrected PSF with a peak at the prescribed indices. This approach is also used for LSF estimates.

III. EXPERIMENTAL METHODS

A. Sphere and Wire Phantom

Fisher Scientific BD Difco Agar was added to deionized degassed water (1.5% by weight) to create a phantom medium. The mixture was heated to 85C, allowed to cool to 40C before being poured into a plastic container in three steps allowing each layer to set. The second layer includes PTFE spheres (Precision Plastic Balls, Franklin Park, IL. USA) having radii of 4.770 ± 0.008 , 7.946 ± 0.007 , 9.509 ± 0.008 , 12.678 ± 0.004 , and 19.053 ± 0.009 mm. The sphere diameters are the result of multiple caliper measurements obtained from each sphere under different orientations providing measurements of diameter reproducibilities. The phantom occupies a volume of $17\times12\times11$ cm³. PTFE was chosen because its linear attenuation coefficient is similar to cortical bone. The final agar layer includes a 0.001 inch (25.4 μ m) diameter steel wire tensioned via two plastic screws attached to a Lucite plate. Any bubbles forming on the spheres and wire were removed manually using a wire probe before the newest layer set. A projection image of the phantom is shown in Fig. 1.

B. X-ray imaging

The method was tested with a Medtronic O-arm (Louisville, Colorado, USA). The acquisition was performed in "HD" mode at 120 kVp, 25 mA, 3.91 s resulting in 740 projection images over 360°, reconstructed to a $512\times512\times192$ voxels volume with a voxel spacing of $0.415\times0.415\times0.833$ mm³. Our PSF implementation supports voxels having different spacings in different directions by including sample spacing information derived from DICOM metadata along with the PSF data. A total of 23 scans of the phantom were acquired allowing reproducibility estimates (standard deviations) to be computed for every quantity derived from the images. The DICOM header does not provide dose information. The phantom was also imaged 20 times with a Toshiba Aquilion One 320 Slice CT scanner with $512\times512\times801$ voxels. The spiral acquisition was a "Lung 0.5" series with a FC55 convolution kernel, 0.828 pitch factor, TCOT reconstruction algorithm, AIDR 3D STD filter, voxel

spacing of (0.421,0.421,0.30) mm, 120 kVP, 0.9 mm focal spot size, 500 ms exposure, 250 mA tube current and a CTDI of 14 mGy. In this manuscript, image directions 1 and 2 refer
275 the transverse imaging plane. The x-ray source and detector rotation axis corresponds to direction 3.

C. Image quality

We record several measures of image fidelity/quality due to their effects on PSF accuracy and reproducibility. The spatial mean and standard deviation of voxels value are computed
280 for each scan. The RMS variations in these statistics are measures of signal reproducibility. There are many possible causes for variations in voxel values including fluctuations in supply voltage to the imaging unit, heating of components affecting their behavior as well as changes in detector gain. By the properties of convolutions, a change in signal scaling or offset will respectively affect the PSF value scaling and offset.

285 The reproducibility in sphere positions is estimated by computing standard deviations of the COM of the spheres for the repeated scans. Mechanical play can cause variations in source-detector orbits relative to the object to be imaged e.g. couch motion in a spiral scan. These variations cause shifts in the positions of PSF and LSF estimates. Uncontrolled variations in sphere positions would increase the standard deviation of the PSFs and LSFs
290 unless these variations are estimated and corrected.

D. PSF/LSF generation

The images are read using Matlab (MathWorks, Natick, MA, USA) DICOM utilities wrapped within DICOM classes we have developed. The three dimensional voxel spacing is obtained from the DICOM files so that non isotropic spacing is taken into account *i.e.*
295 sphere models and PSF estimates have the correct scaling in every direction. These data are passed to our PSF sphere class as well as another class implementing the wire LSF method. The 5 spheres are found and the mean voxel values and standard deviation within 2.5 mm radius volumes of interests (VOI)s centered on each sphere are obtained to investigate how changing sphere radius affects image noise. For each sphere, the sphere model and VOI
300 encompassing the sphere are used to obtain a PSF estimate. The VOIs containing the

sphere image volumes include 5.4 to 6.4 mm of padding on all sides to allow for sampling for the PSF "tail". The padding was constrained by the separation of the spheres placed along a line so their projections do not overlap and such that the entire phantom can be imaged with the O-arm which has no spiral acquisition option. The sphere model is generated with
 305 a fourfold increase in voxel density and subsequently downsampled to create partial volume effects.

PSFs from spheres of different diameters and in different locations exhibit some differences in scaling and offset due to local changes in the images. Given n PSF functions obtained under different conditions (*e.g.* different sphere radii), we transform the PSF values for the
 310 i th imaging condition where $i \in (2, n)$ such that $PSF'_i(x, y, z) = PSF_i(x, y, z)m_i + b_i$ where m_i and b_i are calculated via linear regression with the first PSF. Linear regression distributes difference between PSF evenly over their domains.

There are different preferred conventions for normalizing PSFs including a central peak value of one, normalization by the volume under the PSF, which can be done in voxel units
 315 or spatial units *e.g.* mm. We chose the latter, such that

$$PSF'(x, y, z) = PSF(x, y, z) / [\sum PSF(x, y, z) \Delta x \Delta y \Delta z] \quad (11)$$

where Δx , Δy , Δz are the voxel spacings in the x , y , z directions yielding 3D PSFs in mm^{-3} and similarly, 2D LSFs in mm^{-2} . The "raw" PSFs from our approach (sphere model $\rightarrow$ image) have values ranging from approximately zero in the periphery to hundreds at the
 320 centre, a consequence of the sphere model with values ranging between zero and one, the phantom materials, which are determinants of voxel value ranges in Hounsfield Unit (HU) for CT and in arbitrary units (AU) for the O-arm. PSFs and LSFs should be adjusted to have values that tend to zero in the periphery otherwise normalization may not yield the expected results. In our process, whereby several results are shown together, we calculate the
 325 normalization factor (volume under the curve) from the first PSF or LSF in the sequence and apply said factor to the first and remaining linearly regressed PSFs identically. Therefore, the relative closeness between the PSFs is unaffected by normalization and is still governed by linear regression.

E. PSF versus RL iterations

330 RL requires the prescription of the number of iterations. If too few iterations are used, the estimated PSF will be significantly different from the true PSF. RL can alternatively be susceptible to producing estimates with high frequency noise with some data sets when too many iterations are used[19]. PSFs are calculated using 30, 60, 800, 1600 RL iterations. The PSF computation time on a legacy Intel Core i7 CPU 930 2.80 GHz circa 2010 for 800
335 iterations with offset correction for the small sphere is 90s.

F. PSF accuracy surrogate

In the absence of an independent PSF estimate that can be treated as truth, an indicator of accuracy of our approach is computed by convolving our PSF estimate with the object model and subtracting the result from the x-ray volume image to yield a difference volume
340 as was done by Ohkubo *et al.* [5]. Residuals in a difference volume particularly at the sphere boundary indicate that the PSF estimate cannot account for the x-ray data.

G. LSF comparison

LSFs were computed by spatial integration of PSFs along direction 3 as written in Eq. 8 matching the direction of the wire. LSFs derived from sphere PSFs have different amplitudes
345 and level than the wire LSFs. The wire LSFs also exhibits higher sampling densities. Sample point harmonization via interpolation of the sphere LSF data facilitates alignment of central features from both methods which is useful for plotting. It also allows linear regression of the sphere LSF with a 2 parameter voxel value transformation model of the form $LSF'(x, y, z) = LSF(x, y, z)m + b$ so these data match the wire LSFs. A difference image is calculated
350 between the modified LSF from the 4.77 mm sphere and wire LSF as well RMS differences.

IV. RESULTS AND DISCUSSION

A. Image quality and reproducibility

The RMS variations in voxel value mean and standard deviation over the entire images between repeated scans were 1.5 and 1.6 voxel units respectively for the O-arm and 0.11 HU and 0.11 HU respectively for the CT scanner. These RMS variations are small relative to the voxel value range and x-ray noise and are thus not expected to affect PSF reproducibility. The O-arm voxel values are approximately 2.7 times greater than the Hounsfield units CT values for PTFE which has similar attenuation property as bone.

Table I shows the spatial means and standard deviations of voxel values for a spherical volumes of interest (VOI) having a 2.5 mm radius centered on each PTFE sphere. The number of voxels in the VOIs vary slightly depending on the fractional location of the sphere center in voxel units. For each sphere diameter, VOIs were extracted from each of the images allowing the inclusion of reproducibility estimates for the mean and standard deviation i.e. "±" shown in Table I. The standard deviations of the voxel values are a larger fraction of the mean values for the CT than the O-arm with the imaging protocols we have chosen. The VOI standard deviation, an estimate of image noise, exhibits little change as a function of sphere diameter. Although larger attenuation spheres absorb more x rays, the loss of x-ray photons has little effect on the noise of the reconstructed x-ray image. These results suggest that increasing the sphere diameter would be advantageous due to an increase in sphere boundary surface area with no associated changes in image noise.

Table II provides estimates of COM standard deviations along the 3 principal image axes as a measure of scan position reproducibility for different radii and scanners. These estimates are affected by deviations from the prescribed scanning geometry as well as x-ray noise. The deviations are larger for the CT data. In most instances, the deviations are small fraction of the voxel sizes.

B. PSF and RL iterations

Figure 2 shows PSF estimates for the O-arm along one principal axis of the x-ray image for a 12.68 mm radius sphere computed with 30, 60, 800 and 1600 RL iterations. The numerical values of the PSF have not been normalized and the large values

($\gg 1$) are due to differences in scaling between the sphere model and the x-ray image. Each point on the PSFs is the mean value from 23 estimates computed from the 23 images. The error bars are the standard deviations of the repeated estimates (not the standard error of the mean). The PSF values stabilize around 400-800 iterations and additional iterations have little noticeable effect.

Figure 3 shows differences

between the sphere model convolved with the PSF and the O-arm reconstruction for the 12.68 mm sphere. A slice bisecting the sphere was chosen to illustrate these differences. The aspect ratio of the images have been corrected to compensate for non isotropic voxel spacing resulting in non square voxels. The difference images are computed for 30, 60, 800 and 1600 RL iterations. The differences are reduced as the number of iterations is increased and little change is observed between 800 and 1600 iterations.

C. Image spatial scaling from sphere PSFs

The image scaling estimates obtained from the 12.68 mm radius sphere were 1.0013 ± 0.00020 for the O-arm and 1.0008 ± 0.00024 for the CT. These values are closer to unity than we expected indicating a well prescribed gantry geometry and/or a careful system calibration. We also confirmed that PSFs computed with and without spatial corrections were nearly identical.

D. PSF center of mass variation

Table III shows variations in COM of the PSFs along the three principal image directions for 3 of the 5 radii. The superscripts ^c indicates the correction as detailed in Sec. IIE whereby image data is shifted via interpolation according to the COMs of the spheres prior to calculating measures of resolution and combining them. For the O-arm data all reproducibility estimates (5x3) except for three are smaller with motion correction. O-arm results are mixed which we suspect is due to the greater x-ray noise reducing sphere COM reproducibility.

E. PSF and sphere diameter

Figure 4 shows the variability in the 23 PSF estimates obtained from repeated image O-arm acquisitions.

Figure 5 shows O-arm and CT PSF estimates along the 3 principal axes of the

410 x-ray image for the 4.77, 7.95, 9.52, 12.68 and 19.05 mm radius spheres. Linear regression was performed to match the PSF scaling and offset. The O-arm PSFs exhibit the expected monotonic decrease within a few pixels from their centers associated with a "soft" reconstruction kernel. The CT PSFs exhibits valleys on both sides of the peak associated with a sharp reconstruction kernel which required adding an offset to the convolved images and
415 compensating for this offset so that a zero crossing PSF is calculated correctly (Sec. II C). The CT PSFs are in general agreement but the valleys are attenuated at larger diameter. The valley should mostly be radii invariant as they are governed by the reconstruction kernel and we can see edge enhancement for all sphere diameters. We suspect that in this simple two material image, non-linear effect such as beam hardening, are causing systematic errors.

420 The standard deviations of PSF values were calculated over their central regions encompassing $9 \times 9 \times 9$ voxels. The said deviations for sphere radii of 4.77, 7.95, 9.52 and 19.05 mm were respectively 0.80, 0.12, 0.07, 0.01 for the O-arm and 0.24, 0.06, 0.02, 0.01 for the CT. This behavior is expected given that the image noise mostly independent of sphere size for spheres ranging from 4.77 to 19.05 mm and the increase in diameter provides a larger surface
425 area boundary from which to estimate the PSF. While this measure may seem arbitrary due to the definition of the domain over which it is calculated, this measure which is applied consistently for the different PSF demonstrates how PSF noise relates to sphere diameter.

Figure 6 shows differences between the

sphere model convolved with the PSF and the O-arm reconstruction as explained in Sec-
430 tion IV B for Figure 3. In this instance, the number of RL iterations is fixed at 800 and the sphere radii are varied. We have identified three possible mechanisms causing spherical artifacts in the difference images to become more noticeable with increasing diameter. Firstly, we know from contrast detail phantoms that features become more noticeable in noisy images with an increase in size. Secondly, larger spheres exacerbate non-linearities in
435 the imaging system due to the polychromatic nature of the x-ray beam. Thirdly, we know that PSFs in cone beam geometries vary spatially and the use of a larger spheres results in an

averaged PSF which may not be applicable at any given location. A different experimental protocol and phantom design is needed to explore this effect.

The spheres as shown in Fig.1 define an axis that is almost parallel to the rotation axis of the detector. As a result, they are subject to similar amounts of geometrical magnification resulting in similar amounts of focal spot and detector blurring. Therefore we expect modest change in the PSF in the transverse plane for different spheres in the case of the O-arm cone beam geometry due to incomplete ray sampling away from the central transverse plane of the volume. We expect little variation in the case of the CT spiral acquisition geometry which is akin to consecutive fan beam acquisitions. A different phantom design allowing the substitution of spheres of different radii at the same location would better resolve spatial variations of PSF estimates.

F. Line spread function comparison

Figure 7 shows the wire LSF as well as LSFs computed from PSFs for spheres of varying radii along the two principal orthogonal directions. The wire LSF was calculated for a segment of the wire closest to the 9.52 mm radius spheres. For the O-arm images, the sphere LSFs are in good agreement. The wire LSF peak is somewhat reduced compared to the sphere estimates. For the CT images, the sphere LSFs are in agreement but the valleys on either side of the peak are again reduced with increasing radii. The wire LSF peak is also somewhat reduced.

Figure 8 shows the wire LSF, the 4.77 mm radius sphere derived LSF and the difference between the two as images. The O-arm LSFs exhibits similar spiculated features. The normalized root mean square difference between the mean LSF derived from the sphere and the mean wire LSF is 2.0% for the O-arm and 1.2% for the CT.

V. SUMMARY AND CONCLUSIONS

Richardson Lucy deconvolution can be used to estimate three dimensional PSFs from spheres of known diameters. RL implementations are ubiquitous making this approach easy to implement. We verified whether repeated scans were reproducible in terms of positional

variations, spatial scaling, signal values and we developed corrections to prevent these variations from affecting PSF reproducibility and accuracy. Our design incorporating many images allowed the calculation of the mean and standard deviations of all points of the PSFs and LSFs. Sphere COM corrections improved PSF reproducibility for o-arm data but not for the CT data. COM estimates maybe more robust if confined to a shell encompassing the sphere boundary. The standard deviations of the PSF estimates exhibit a decreasing trend as the sphere radius was increased suggesting that the method requires a minimum sphere radius to yield the desired threshold of reproducibility. In the case of the O-arm scan, the PSFs are similar for all diameters. For the CT scan, the PSF shapes are similar but with a reduction in the valleys seen on both sides of the peak for larger radii. Our results suggest that there exists a trade-off between reproducibility and accuracy when the radius is increased as demonstrated by the structured noise of Fig. 6 indicating that the transfer function can not account for the image data. The experimental design was focused on obtaining measures of PSF reproducibility but sphere derived LSF measurements were compared to a wire LSF obtained in physical proximity. PSF are mostly z separable and thus LSF agreement in this instance implies some agreement of the PSF in the xy direction.

Noise has been identified as a challenge in all methods to determine PSFs and various regularization strategies have been proposed. Our approach uses an extended object relative to the voxel size and all voxels within a VOI to obtain the PSF reducing sensitivity to x-ray noise. Our approach also shows how PSF reproducibility can be improved by combining corrected estimates from different scans. At the same time, the absence of assumptions regarding the PSF such as separability or even-ess increase sensitivity to noise. Our approach does not provide oversampled or pre-sampled estimates of the PSF although this information can be extracted from sphere phantoms. Additional experiments are required to determine the level of PSF reproducibility required for our application involving the restoration of signal values and thicknesses of thin bones.

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CONFLICT OF INTEREST DISCLOSURE

The authors have no relevant conflicts of interest to disclose.

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FIG. 1. A projection of a 3D x-ray image of an agar phantom containing PTFE spheres of varying diameters and a steel wire.

t				
Scanner	Radius (mm)	VOI Mean	VOI Std Dev	# Voxels
O-arm	4.77	2691 ± 5.2	24.1 ± 1.1	460×23
O-arm	7.95	2561 ± 3.9	19.0 ± 1.3	466×23
O-arm	9.52	2442 ± 3.0	19.7 ± 1.1	462×23
O-arm	12.68	2430 ± 3.5	19.5 ± 0.8	465×23
O-arm	19.05	2610 ± 4.8	23.8 ± 1.1	464×23
CT	4.77	920 ± 1.0	19.3 ± 1.1	1217×20
CT	19.05	940 ± 1.0	21.2 ± 1.2	1223×20

TABLE I. The mean and standard deviation of voxel values within polytetrafluoroethylene spheres of different radii imaged using the Medtronic O-arm and Toshiba Aquilion One 320 CT. Errors in these estimates ($\pm$) are available because each estimate was obtained from 23 image for the O-arm and 20 images for the CT.

FIG. 2. Mean PSF estimates and standard deviations via error bars calculated along one principal axis of the O-arm images for a 12.68 mm radius sphere computed with 30, 60, 800 and 1600 RL iterations.

FIG. 3. Differences between the sphere model convolved with the PSF and the O-arm reconstruction for the 12.68 mm sphere with varying number of Richardson Lucy iterations. All images are windowed/leveled to same voxel value range as indicated by the color bar.

t				
Scanner	Radius (mm)	Sphere COM Std Dev (voxel/mm)		
O-arm	4.77	(0.0027/0.0022,	0.0024/0.0010,	0.0032/0.0013)
O-arm	9.52	(0.0034/0.0028,	0.0074/0.0031,	0.0031/0.0013)
O-arm	19.05	(0.0062/0.0052,	0.0113/0.0047,	0.0048/0.0020)
CT	4.77	(0.1189/0.0357,	0.0933/0.0393,	0.1242/0.0523)
CT	9.52	(0.0585/0.0175,	0.1381/0.0581,	0.1778/0.0748)
CT	19.05	(0.0856/0.0257,	0.0367/0.0155,	0.0492/0.0207)

TABLE II. Standard deviations in center of mass sphere positions for varying radii (3 of 5 are shown) expressed in voxel units and mm along the 3 principal image axes for 23 image from the O-arm and 20 images from the CT

FIG. 4. PSF estimates from the 23 repeated O-arm images calculated along the second principal image axis for a 12.68 mm radius sphere.

FIG. 5. Mean PSF estimates and standard deviations via error bars calculated along the 3 principal axes for spheres of different radii for images acquired with the O-arm (top row) and CT (bottom row)

FIG. 6. Differences between the sphere model convolved with the PSF and the x-ray reconstruction for 800 Richardson Lucy iterations and varying radii r . The first row shows results obtained with the O-arm and the second row show CT results. The VOI sizes increase to encompass the spheres as their radii increase resulting in smaller apparent voxel sizes.

FIG. 7. Mean line spread function estimates and standard deviations via error bars calculated from 5 spheres of different radii and a wire for images acquired with the O-arm (top row) and CT (bottom row)

FIG. 8. Line spread functions shown as images. The LSF estimated from a wire segment and the LSF estimated from a 4.77 mm radius sphere are shown as well as the difference between the 2 estimates.

t				
Scanner	Radius (mm)	PSF COM Std Dev (voxel/mm)		
O-arm ^c	4.77	(0.055/0.046,	0.088/0.037,	0.066/0.027)
O-arm ^c	9.52	(0.034/0.028,	0.072/0.030,	0.055/0.023)
O-arm ^c	19.05	(0.037/0.031,	0.072/0.030,	0.048/0.020)
O-arm	4.77	(0.057/0.047,	0.090/0.037,	0.065/0.027)
O-arm	9.52	(0.037/0.031,	0.079/0.033,	0.054/0.022)
O-arm	19.05	(0.044/0.037,	0.084/0.035,	0.051/0.021)
CT ^c	4.77	(0.093/0.028,	0.095/0.040,	0.068/0.029)
CT ^c	9.52	(0.013/0.004,	0.151/0.064,	0.175/0.073)
CT ^c	19.05	(0.035/0.010,	0.028/0.012,	0.038/0.016)
CT	4.77	(0.052/0.016,	0.039/0.017,	0.087/0.037)
CT	9.52	(0.070/0.021,	0.018/0.008,	0.085/0.036)
CT	19.05	(0.073/0.022,	0.045/0.019,	0.057/0.024)

TABLE III. Standard deviations of PSF center of mass obtained along the three principal image axes with sphere motion correction indicated via the superscript ^c for 23 image from the O-arm and 20 images from the CT. Results for 3 of 5 radii are shown.