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Non-relativistic Quark Model under External Magnetic and Aharanov-Bohm (AB) Fields in the Presence of Temperature-Dependent Confined Cornell Potential

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Abstract

The dissociation of quarkonia in a thermal QCD medium in the background of an AB and strong magnetic fields is investigated. For this purpose, the Schrödinger equation with a charged quarkonium in the Cornell potential under the influence of AB flux and an external magnetic fields directed along the z-axis is employed. By using the Nikiforov-Uvarov (NU) method, the energy eigenvalue is obtained. The effect of temperature, AB flux, and an external magnetic field is studied. The study shows that the dissociation energy of 1S states of charmonium and bottomonium decreases with increasing temperature and AB flux, and external magnetic field. Also, the quarkonium melts faster in a hot medium in the presence of AB flux and external magnetic field. We found that the charmonium melts at $13.79 m_{\pi}^2 \text{ GeV}^2$ and the bottomonium melts at $99.48 m_{\pi}^2 \text{ GeV}^2$. A comparison is studied with other works. Thus, the present non-relativistic model gives satisfactory results for dissociation binding energy in a hot medium when AB flux and external magnetic fields are included.

Keywords: Quarkonia, non-relativistic model; Nikiforov-Uvarov method; external magnetic field; Aharanov-Bohm (AB) field.

1-Introduction

It is well established in the literature that quarkonium systems provide deep insight into the fundamental description of quantum chromodynamics (QCD) theory, the standard model and particle physics [1–3]. It is noted in Ref. [4] that quarkonia with heavy quark and antiquark and their interaction are well described by the Schrödinger equation (SE). The solution of this non-relativistic model with a given potential is one of the most pertinent problems in quarkonium systems. It has also been stated that potentials considered should take into account two important features of the strong interaction, namely, asymptotic freedom and quark confinement [5–9]. To this end, the Cornell potential is one the most used potential in studying quarkonium system. Several studies have considered the properties of baryon and meson in different media such as in Refs. [10-13]. Abu-shady [11] studied the N -dimensional radial SE analytically at finite temperature. The analytic exact iteration method (AEIM) was employed to obtain the energy eigenvalues and wave functions for any n and l states. The obtained energy spectrum was applied to study charmonium and bottomonium masses at finite temperature. The behavior of the charmonium and bottomonium masses was found to agree with other theoretical methods. Also, the quark sigma model was extended to finite temperature and density [12]. The Cornell potential modified to include the finite temperature parameter is given as [13];

$$V(T, r) = a(T, r)r - \frac{b(T, r)}{r}, \quad (1a)$$

where, $a(T, r) = \frac{a}{m_D(T)r} (1 - e^{-m_D(T)r})$, $b(T, r) = be^{-m_D(T)r}$, $m_D(T)$ is the Debye mass that vanishes at $T \rightarrow 0$. a and b are positive parameters. Rewriting Eq.(1a), we expand

$e^{-m_D(T)r}$ up to second order, which gives a good accuracy when $m_D r \ll 1$. We obtain a working form for the Cornell potential with finite temperature as in [13]

$$V(T, r) = \left(a - \frac{bm_D^2(T)}{2} \right) r - \frac{am_D(T)r^2}{2} + bm_D(T) - \frac{b}{r} \quad (1b)$$

The force between a heavy quark and its anti-quark, is weakened due to the presence of hot medium which leads to the dissociation of quarkonium bound states. This medium effect can be studied through temperature-dependent heavy quark potentials [14-16].

Recently, some authors have scrutinized the effects of the AB flux and magnetic fields on several quantum mechanical systems. Some of these studies include; Ikhdair and Sever [17] solved the Klein-Gordon (KG) equation for the two-dimensional scalar-vector harmonic oscillator plus Cornell potentials in the presence of external magnetic and AB flux fields using the wave function ansatz method. The exact energy eigenvalues and the wave functions were obtained in terms of potential parameters, magnetic field strength, AB flux field, and magnetic quantum number [17]. Hamzavi and Ikhdair [18] solved the Killingbeck potential which is also known as the extended Cornell. They studied the energy levels and wave function by solving the SE in two-dimensions (2D) with the Killingbeck potential under the influence of strong external uniform magnetic and AB flux fields. Ikhdair *et al.* [19] solved the 2D SE with various power interaction potentials in the presence of magnetic and AB flux fields. They calculated the energy levels of some diatomic molecules in the presence and absence of external magnetic and AB flux fields using different potential models. It was found that the effect of the AB field is much as it creates a wider shift for $m \neq 0$ and its influence for $m=0$ states was found to be greater than that of the magnetic field. Falaye *et al.* [20] presented a detailed study on the confinement influences of AB flux and electric and magnetic fields directed along the z axis and encircled by quantum

plasmas on the hydrogen atom. It was noted that the all-inclusive effects result in a strongly attractive system while the localizations of quantum levels change and the eigenvalues decrease.

Interest in the effects of strong magnetic fields in heavy ion collisions has become a hot topic recently following the prediction of a non-trivial quantum chromodynamics (QCD) effect dubbed by the chiral magnetic effect which stems from small P- and CP-odd interactions inducing an electromagnetic current when a quark-gluon plasma (QGP) is placed in an external magnetic field [21]. There has been much work related to this in recent years and in addition it has been shown to self-consistently take into account this effect through Berry curvature flux in the presence of magnetic field [22, 23]. The existence of such high magnetic fields has also prompted many research groups to study how the finite temperature deconfinement and chiral phase transitions are affected by the presence of a strong background magnetic field. These studies have included direct numerical investigations using lattice QCD [24-28] and theoretical investigations using a variety of methods including, for example, perturbative QCD studies, model studies, and string-theory inspired anti-de Sitter/conformal field theory (AdS/CFT) correspondence studies [29-31].

In the present study, the SE for a temperature-dependent Cornell potential is extended to include the presence of external magnetic and AB flux fields. To the best of our knowledge, no study has considered the SE in the presence of external magnetic and AB flux fields by using the Nikiforov-Uvarov formalism when quarkonium system is considered.

The outline of our paper is as follows. In Section 2, we give a brief review of the NU method. In Section 3, we solve the Schrödinger equation for a temperature-dependent Cornell potential in the presence of magnetic and AB flux fields.

Results and discussion are presented in the section 4. Finally, a brief concluding remarks is given in section 5.

2-Review of Nikiforov-Uvarov (NU) Method

The NU method was proposed by Nikiforov and Uvarov [32-33] to transform Schrödinger-like equations into a second order differential equation via a coordinate transformation $z = z(r)$, of the form

$$\psi''(z) + \frac{\tilde{\tau}(z)}{\sigma(z)}\psi'(z) + \frac{\tilde{\sigma}(z)}{\sigma^2(z)}\psi(z) = 0, \quad (2)$$

where $\tilde{\sigma}(z)$, $\sigma(z)$ are polynomials, at most second degree, and $\tilde{\tau}(z)$ is a first-degree polynomial. The exact solution of Eq. (2) can be obtained by using the transformation

$$\psi(z) = \Phi(z)y_n(z). \quad (3)$$

This transformation reduces Eq. (2) into a hypergeometric-type equation of the form

$$\sigma(z)y_n''(z) + \tau(z)y_n'(z) + \lambda y_n(z) = 0. \quad (4)$$

The function $\Phi(z)$ can be defined as the logarithm derivative

$$\frac{\Phi'(z)}{\Phi(z)} = \frac{\pi(z)}{\sigma(z)}, \quad (5)$$

with $\pi(z)$ being at most a first-degree polynomial. The second part of $\psi(z)$ being $y_n(z)$ in Eq. (3), is the hypergeometric function with its polynomial solution given by Rodrigues relation

$$y_n(z) = \frac{B_n}{\rho(z)} \frac{d^n}{dz^n} [\sigma^n(z) \rho(z)], \quad (6)$$

here, B_n is the normalization constant and $\rho(z)$ is the weight function which must satisfy the condition

$$\frac{d}{dz}[\sigma(z)\rho(z)] = \tau(z)\rho(z), \quad (7)$$

with

$$\tau(z) = \tilde{\tau}(z) + 2\pi(z). \quad (8)$$

It should be noted that the derivative of $\tau(z)$ with respect to z should be negative. The eigenfunctions and eigenvalues can be obtained using the definition of the following function $\pi(z)$ and parameter λ , respectively:

$$\pi(z) = \frac{\sigma'(z) - \tilde{\tau}(z)}{2} \pm \sqrt{\left(\frac{\sigma'(z) - \tilde{\tau}(z)}{2}\right)^2 - \tilde{\sigma}(z) + k\sigma(z)}, \quad (9)$$

and

$$\lambda = k + \pi'(z). \quad (10)$$

The value of k can be obtained by setting the discriminant of the square root in Eq. (9) equal to zero. As such, the new eigenvalue equation can be given as

$$\lambda + n\tau'(s) + \frac{n(n-1)}{2}\sigma''(s) = 0, \quad (n=0,1,2,\dots) \quad (11)$$

3-The energy eigenvalues of the Temperature-dependent Cornell potential in the presence of the AB and External Magnetic Fields

In Eq. (12), the SE is written as in Ref. [19] considering the quarkonium system, in which we consider two charged particles with charge e interacting via radially symmetrical interaction potential that is given in Eq. (1b) under the influence of external uniform magnetic field $\vec{B} = B\hat{z}$ and AB flux. In the present consideration, we use natural units ($\hbar = c = 1$)

$$\left[\frac{1}{2\mu} \left(-i\vec{\nabla} + e\vec{A} \right)^2 - \left(a - \frac{bm_D^2(T)}{2} \right) r + \frac{am_D(T)r^2}{2} - bm_D(T) + \frac{b}{r} \right] \psi(r, \phi) = E\psi(r, \phi), \quad (12)$$

where E denotes the energy of particle, μ is the effective mass of the quark and antiquark, the vector potential $\vec{A}$ can be written as a sum of two terms $\vec{A} = \vec{A}_1 + \vec{A}_2$ having the azimuthal components [19] external magnetic field with $\vec{\nabla} \times \vec{A}_1 = \vec{B}, \vec{\nabla} \times \vec{A}_2 = 0$, where $\vec{B}$ is the applied external magnetic field.

$\vec{A}_1 = \frac{Br}{2} \phi$ and $\vec{A}_2 = \frac{\phi_{AB}}{2\pi r} \phi$, represents the additional magnetic flux ϕ_{AB} created by a solenoid with $\vec{\nabla} \cdot \vec{A}_2 = 0$. Let us take the wave function in cylindrical coordinates as $\psi(r, \phi) = \frac{1}{\sqrt{2\pi r}} e^{im\phi} H(r)$, where m denotes the magnetic quantum number $m = 0, \pm 1, \pm 2, \dots$. Inserting this wave function into Eq. (12), we find a second-order differential equation

$$\frac{d^2 H(r)}{dr^2} + 2\mu \left[E + \frac{am_D(T)r^2}{2} - \left(a - \frac{bm_D^2(T)}{2} \right) r - bm_D(T) + \frac{b}{r} \right] H(r) = 0 \quad (13)$$

$$- \frac{1}{2\mu} \left(\frac{(m+\xi)^2 - \frac{1}{4}}{r^2} \right) - \frac{\omega_c}{2} (m+\xi) - \left(\frac{\mu\omega_c^2}{8} \right) r^2$$

where $\xi = \frac{\phi_{AB}}{\phi_0}$ is an integer with the flux quantum $\phi_0 = \frac{1}{e}$ and $\omega_c = \frac{eB}{\mu}$ denotes the cyclotron frequency.

For mathematical simplicity, we define the following parameters

$$\beta_1 = \left(\frac{am_D(T)}{2} - \left(\frac{\mu\omega_c^2}{8} \right) \right), \beta_2 = - \left(a - \frac{bm_D^2(T)}{2} \right), \quad (14)$$

$$\beta_3 = bm_D(T) + \frac{\omega_c}{2} (m+\xi), \gamma = (m+\xi)^2 - \frac{1}{4}$$

which simplifies equation (13) as follows,

$$\frac{d^2 H(r)}{dr^2} + \left[2\mu E + 2\mu\beta_1 r^2 + 2\mu\beta_2 r - 2\mu\beta_3 + \frac{2\mu b}{r} - \frac{\gamma}{r^2} \right] H(r) = 0 \quad (15a)$$

Furthermore, we chose the following dimensionless parameters;

$$\begin{aligned} -\varepsilon &= 2\mu E, \delta_1 = 2\mu\beta_1, \\ \delta_2 &= 2\mu\beta_2, \delta_3 = 2\mu\beta_3, \eta = 2\mu b \end{aligned} \quad (15b)$$

and this allows us to write equation (15a) as

$$\frac{d^2 H(r)}{dr^2} + \left[-\varepsilon + \delta_1 r^2 + \delta_2 r - \delta_3 + \frac{\eta}{r} - \frac{\gamma}{r^2} \right] H(r) = 0 \quad (16)$$

By employing the transformation $r = \frac{1}{z}$, Eq.(16) takes the form;

$$\frac{d^2 H(z)}{dz^2} + \frac{2}{z} \frac{dH(z)}{dz} + \frac{1}{z^4} \left[-\varepsilon + \frac{\delta_1}{z^2} + \frac{\delta_2}{z} - \delta_3 + \eta z - \gamma z^2 \right] H(z) = 0 \quad (17)$$

Following Ref.[13], the scheme is based on the expansion of $\frac{\delta_2}{z}$ and $\frac{\delta_1}{z^2}$ in a power series around a distance parameter r_0 that fitted with data. Setting $s = z - \alpha$, where $\alpha = \frac{1}{r_0}$, thus, we expand the $\frac{\delta_2}{z}$ and $\frac{\delta_1}{z^2}$ into a series of powers around $s = 0$.

$$\frac{\delta_2}{z} = \frac{\delta_2}{s + \alpha} = \frac{\delta_2}{\alpha} \left(1 + \frac{s}{\alpha} \right)^{-1} = \delta_2 \left(\frac{3}{\alpha} - \frac{3z}{\alpha^2} + \frac{z^2}{\alpha^3} \right) \quad (18)$$

$$\frac{\delta_1}{z^2} = \frac{\delta_1}{(z + \alpha)^2} = \frac{\delta_1}{\alpha^2} \left(1 + \frac{s}{\alpha} \right)^{-2} = \delta_1 \left(\frac{6}{\alpha^2} - \frac{8z}{\alpha^3} + \frac{3z^2}{\alpha^4} \right) \quad (19)$$

By substituting Eqs. (18) and (19) into Eq. (16). Eq. (17) takes the following form

$$\frac{d^2 H(z)}{dz^2} + \frac{2z}{z^2} \frac{dH(z)}{dz} + \frac{1}{z^4} \left[\begin{aligned} & - \left(\gamma - \frac{3\delta_1}{\alpha^4} - \frac{\delta_2}{\alpha^3} \right) z^2 + \left(\eta - \frac{8\delta_1}{\alpha^3} - \frac{3\delta_2}{\alpha^2} \right) z \\ & - \left(\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3 \right) \end{aligned} \right] H(z) = 0 \quad (20)$$

Comparing Eq. (20) and Eq. (2), with the basic knowledge of the NU method, we can obtain $\pi(s)$ that defined in Eq. (9) as;

$$\pi(s) = \pm \sqrt{\left(k + \gamma - \frac{3\delta_1}{\alpha^4} - \frac{\delta_2}{\alpha^3} \right) - B \left(\eta - \frac{8\delta_1}{\alpha^3} - \frac{3\delta_2}{\alpha^2} \right) + \varepsilon_{nm} - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3} \quad (21)$$

To find the constant k , the discriminant of the expression under the square root of Eq. (21) must be equal to zero, we obtain

$$k = \frac{\left(\eta - \frac{8\delta_1}{\alpha^3} - \frac{3\delta_2}{\alpha^2}\right)^2}{4\left(\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3\right)} - \left(\gamma - \frac{3\delta_1}{\alpha^4} - \frac{\delta_2}{\alpha^3}\right) \quad (22)$$

Substituting Eq. (22) into Eq. (21) yields

$$\pi(z) = \pm \frac{1}{2\sqrt{\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3}} \left(\left(\eta - \frac{8\delta_1}{\alpha^3} - \frac{3\delta_2}{\alpha^2} \right) z - 2 \left(\varepsilon_{nm} - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3 \right) \right) \quad (23)$$

By using Eq. (8), we can obtain $\tau(s)$ as follows

$$\tau(z) = 2z - \frac{\left(\eta - \frac{8\delta_1}{\alpha^3} - \frac{3\delta_2}{\alpha^2}\right)z}{\sqrt{\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3}} + \frac{2\left(\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3\right)}{\sqrt{\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3}} \quad (24)$$

Taking the derivative of Eq. (24) w. r. t. z , we have;

$$\frac{d\tau(z)}{dz} = 2 - \frac{\left(\eta - \frac{8\delta_1}{\alpha^3} - \frac{3\delta_2}{\alpha^2}\right)}{\sqrt{\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3}}, \quad (25)$$

we take the derivative of $\pi(z)$ with respect to z from Eq.(23), we obtain;

$$\frac{d\pi(z)}{dz} = \pm \frac{\left(\eta - \frac{8\delta_1}{\alpha^3} - \frac{3\delta_2}{\alpha^2}\right)}{2\sqrt{\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3}} \quad (26)$$

Referring to Eq. (10), we define the constant λ as

$$\lambda = \frac{\left(\eta - \frac{8\delta_1}{\alpha^3} - \frac{3\delta_2}{\alpha^2}\right)^2}{4\left(\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3\right)} - \left(\gamma - \frac{3\delta_1}{\alpha^4} - \frac{\delta_2}{\alpha^3}\right) - \frac{\left(\eta - \frac{8\delta_1}{\alpha^3} - \frac{3\delta_2}{\alpha^2}\right)}{2\sqrt{\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3}} \quad (27)$$

Substituting Eqs. (25) and $\sigma''(s)=2$ into Eq. (11), we obtain;

$$\lambda_n = -n^2 - n + \frac{\left(\eta - \frac{8\delta_1}{\alpha^3} - \frac{3\delta_2}{\alpha^2}\right)n}{\sqrt{\varepsilon - \frac{6\delta_1}{\alpha^2} + \frac{3\delta_2}{\alpha} + \delta_3}} \quad (28)$$

By comparing ($\lambda_n = \lambda$), Eqs. (27) and (28), the exact energy eigenvalue equation in 2D is obtained as

$$E_{nm} = \frac{3\beta_2}{\alpha} - \frac{6\beta_1}{\alpha^2} + \beta_3 - \frac{1}{2\mu} \left[\frac{2\mu b - \frac{8\beta_1}{\alpha^3}(2\mu) - \frac{3\beta_2}{\alpha^2}(2\mu)}{2n+1 + \sqrt{1+4\left((m+\xi)^2 - \frac{1}{4} - \frac{3\beta_1}{\alpha^4}(2\mu) - \frac{\beta_2}{\alpha^3}(2\mu)\right)}} \right]^2. \quad (29)$$

where $\beta_1, \beta_2, \beta_3$ are defined in Eq. (14) and μ , m are reduced mass of quarkonium and magnetic quantum number, respectively. Also, b is a parameter that was defined after Eq. (1a).

4 - Result and Discussions

In this section, we apply the results of energy eigenvalue that we derived in Eq. (29). Thus, we use Eq. (29) to define dissociation energy (E_d), where

$$E_d = V(r \rightarrow \infty) - E_{nm} = \frac{a}{M_d} - E_{nm}. \text{ We take charm mass } m_c=1.48 \text{ GeV and } m_b=4.823$$

GeV. We calculate dissociation of energy of charmonium and bottomonium. We discuss the melting energy in a hot medium when magnetic field and AB field are included. In Fig. (1), the energy dissociation of charmonium is plotted as a function in the magnetic field (eB) GeV². We note that the dissociation energy gives maximum value at $eB = m_\pi^2$ where, $m_\pi = 0.14$ GeV. The curve of charmonium decreases with increasing magnetic field and tends to zero at $eB = 13.79 m_\pi^2$ where the temperature of the hot medium is taken $T = 1.5T_c$ (T_c is critical temperature equals 0.17 GeV as in Ref. [34]). Therefore, the magnetic field melts

the charmonium at a critical magnetic field about $13.79 m_\pi^2$ when thermal medium has temperature equals 255 MeV. The effect of AB field is not considered in Fig. (1). The impact on the dissociation is considered with the magnetic field only. In Fig. (2), the 1S state of bottomonium is plotted as a function of magnetic field. The figure shows that the dissociation energy gives maximum value at lower values of magnetic field and then the dissociation energy decreases with increasing magnetic field and drops to zero at $eB = 99.49 m_\pi^2 \text{ GeV}^2$ when thermal medium has temperature of $T_d = 1.5T_c$. Therefore, the bottomonium needs an increased external magnetic field up to $eB = 99.49 m_\pi^2 \text{ GeV}^2$ for melting of its energy. In Ref. [35], the effect of strong and homogeneous magnetic field on properties of quarknonium is studied by employing medium-modified potential as a function in magnetic field and thermal medium. The authors found that 1S state of charmonium dissociated at $eB = 10 m_\pi^2 \text{ GeV}^2$ and 1S state of bottomonium dissociates at $eB = 100 m_\pi^2 \text{ GeV}^2$ when temperature of medium is taken as 300 MeV. By using another technique in same Ref. [35], the authors found that charmonium dissociated at $eB = 13 m_\pi^2$ and bottomonium dissociates at $eB = 130 m_\pi^2 \text{ GeV}^2$. Thus, the present finding agrees qualitatively with Ref. [35].

Fig. 1: The Dissociation of 1S state of Charmonium is plotted as a function of magnetic field when AB field is absent

Fig. 2: The Dissociation of 1S state of Bottomonium is plotted as a function of magnetic field when AB field is ignored.

In Fig. (3, 4), we analyse the effect of temperature on dissociation of energy of charmonium and bottomonium at fixed value of magnetic field. In Fig. (3), we took three values of $eB = m_\pi^2$, $10 m_\pi^2$, $20 m_\pi^2$. The Figure shows that the dissociation energy of 1S state charmonium decreases with increasing temperature and drops to lower values with increasing magnetic field.

Fig. 3. The dissociation of 1S state of charmonium is plotted as a function of temperature ratio for different values of magnetic field in the absence of AB field

Fig. 4. The dissociation of 1S state of bottomonium is plotted as a function of temperature ratio for different values of magnetic field in the absence of AB field

This indicates that dissociation of energy of charmonium melts faster with increasing magnetic field in the hot medium. We note that $T_d = 1.54T_c$ at $eB = m_\pi^2$, $T_d = 1.52T_c$ at $eB = 10m_\pi^2$, $T_d = 1.45T_c$ at $eB = 20m_\pi^2$. A similar situation is accrued in Fig. 4. The 1S state of bottomonium is plotted as a function of temperature ratio when the different values of magnetic field is considered. The figure shows that dissociation of temperature decreases with increasing temperature and drops to zero at some values of temperature. When we increase the impact of magnetic field, we note that dissociation energy drops to lower values. This indicates that the bottomonium melts faster in magnetic-thermal medium. We note that $T_d = 2.15T_c$ at $eB = m_\pi^2$, $T_d = 2.11T_c$ at $eB = 20m_\pi^2$, $T_d = 1.92T_c$ at $eB = 50m_\pi^2$. The decreasing of dissociation energy with increasing magnetic field is noted in Refs. [36]. In Ref. [36], the effects of strong magnetic field on the properties of quarkonia immersed in a thermal medium of quarks and gluons is discussed. For that purpose, the quarkonium dissociation was studied by

investigating its properties by solving the SE numerically with potential derived from by thermalizing the Schwinger proper-time fermion propagator and then calculate the resumed retarded and symmetric propagators by the Schwinger-Dyson equation. We noted in Ref. [35] that the dissociation tends to lower values by increasing magnetic field. The J/ψ is dissociated at smaller temperatures, $1.8 T_c$ and $1.5 T_c$ for higher magnetic fields, $eB = 27$ and $68 m_\pi^2$, respectively. In present work, we obtained dissociation temperature at $T_d = 1.5 T_c$ and $T_d = 2 T_c$ for 1S state of charmonium and bottomonium at $eB = 13.79 m_\pi^2$ and $eB = 41.32 m_\pi^2$, respectively. From figures of (1-4), we note when we fixed the magnetic field and increase the temperature gradually from critical temperature up to higher temperatures, the dissociation energy melts faster than when we fixed the thermal medium at a specific temperature and increase the external magnetic field gradually.

Fig. 5. The dissociation energy of 1S state of charmonium is plotted as a function of temperature ratio for different values ξ at zero magnetic field.

Fig. 6. The dissociation energy of 1S state of bottomonium is plotted as a function of temperature ratio for different values ξ at zero magnetic field.

In the present work, we include the effect of the additional magnetic flux that comes from vector potential $\vec{A}_2$. We express this effect by using parameter $\xi = \frac{\phi_{AB}}{\phi_0}$ which is an integer with the flux quantum $\phi_0 = \frac{1}{e}$. In Fig. (5), the dissociation energy of 1S state bottomonium for different values of ξ . We note that the energy dissociation decreases with increasing temperature in the presence of magnetic flux. By increasing parameter ξ , we note that the dissociation energy drops to lower values. Thus, additional magnetic flux strongly melts the charmonium state. A similar situation is observed for bottomonium state in Fig. 6, the figure shows that the dissociation of energy decreases with increasing temperature and shifts to lower values by increasing the parameter ξ . This means that the parameter ξ increases melting energy of bottomonium. In Table (1), we note that the dissociation temperature of charmonium decreases by increasing parameter ξ in

the absence of external magnetic field. By taking the $eB= 10 m_{\pi}^2$, we note that the dissociation of charmonium decreases more in comparison with the case of absence of external magnetic field. A similar situation is found for bottomonium state. In addition, we note that the dissociation of bottomonium is greater than the charmonium state in the absence and the presence of external magnetic field. The effect of parameter ξ due to AB flux field is not considered in quarkonium system in the previous works.

Table (1): The dissociation temperatures of 1S state of charmonium and bottomonium at different values of ξ in the absence and the presence of magnetic field

	eB = 0			eB = 10 m_{π}^2		
ξ	0	1	2	0	1	2
$\frac{J}{\psi}$	1.547T _c	1.364T _c	1.282T _c	1.518T _c	0.737 T _c	T _d <T _c
Y	2.15T _c	1.588T _c	1.376T _c	2.141T _c	1.324 T _c	0.947 T _c

5-Summary and Conclusion

In the present study, the Schrödinger equation for a temperature-dependent Cornell potential is extended to include the presence of external magnetic and AB flux fields by using the Nikiforov-Uvarov formalism. We obtained the energy eigenvalue that depends on the temperature, external magnetic and AB flux fields. We applied the present results to study the melting of charmonium and bottomonium and the effect of these parameters on melting of charmonium and bottomonium. This study shows the importance of these parameters on quarkonium

in a hot medium which are not considered in the previous works. The effect of external magnetic is shown on the dissociation of energy. We found that the charmonium and bottomonium melts faster in the presence of external magnetic field in a hot medium. Beside, we found that the charmonium melts when $T_d = 1.54T_c$ at $eB = m_\pi^2$, $T_d = 1.52T_c$ at $eB = 10m_\pi^2$, $T_d = 1.45T_c$ at $eB = 20m_\pi^2$. Also, we found that the dissociation of temperature of 1S state of bottomonium, we note that $T_d = 2.15T_c$ at $eB = m_\pi^2$, $T_d = 2.11T_c$ at $eB = 20m_\pi^2$, $T_d = 1.92T_c$ at $eB = 50m_\pi^2$. The effect of the additional magnetic flux that comes from vector potential $\vec{A}_2$. We express this effect by using parameter $\xi = \frac{\phi_{AB}}{\phi_0}$ which is an integer with the flux quantum $\phi_0 = \frac{1}{e}$. We found that parameter ξ plays an important role in dissociation of charmonium and bottomonium. As discussed in section (4), we noted that the additional magnetic flux increases the melting of quarkonium in a hot medium. This effect is not considered in other recent works such as [35, 36]. Therefore, in this work, we conclude that the new treatment for non-relativistic model gives satisfactory results for quarkonium in a hot medium and gives a new effect of additional magnetic flux which is not considered in previous studies. We hope to extend this model to include anisotropic plasma which plays an important role for analysis of quark gluon plasma as a future work. In addition, thermodynamics properties of heavy mesons in the non-relativistic quark model in the presence of AB and magnetic fields could also be considered as an extension of our study [37-42].

6-Referneeces

[1] M. Abu-Shady, T. A. Abdel-Karim, and E. M. Khokha, Adv. High Energy Phys. **2018**, 7356843 (2018).

- [2] M. Abu-Shady and E. M. Khokha, Adv. High energy Phys. **2018**, 7032041(2018).
- [3] N. Brambilla and A. Vario. Acta. Phys. Polon. B, **38**, 3429 (2007).
- [4] M. Abu-shady and S. Y. Ezz-Alarab, Few body systems **60**, 66 (2019).
- [5] E. E. Ibekwe, U. S. Okorie, J. B. Emah, E. P. Inyang, & S. A. Ekong (2021). Eur. Phys. J. Plus **136**, 87 (2021)
- [6] E.P. Inyang, E.P. Inyang, J. E. Ntibi, E. E. Ibekwe & E. S. William (2021). Indian J Phys (2021). <https://doi.org/10.1007/s12648-020-01933-x>
- [7] D. Griffiths, Introduction to Elementary Particles, JohnWiley& Sons, New York, NY,USA, 1987.
- [8]M. Abu-shady and A. N. Ikot, Eur. Phys. J. plus **134**, 321 (2019).
- [9] J. Rathsmann, Quark and Lepton Interactions, Acta Universitatis Up salensis Uppsala, (1996).
- [10] M. Abu-Shady, Int. J. Theor. Phys. **49**, 2425 (2018).
- [11] M. Abu-Shady, Adv. Math. Phys.**2016**, 4935940, (2016).
- [12] M. Abu-shady and M. Soleiman, Phys. Particles and nuclei letters 10, 683 (2013).
- [13] M. Abu-Shady, J. Egyptian Math. Society **25**, 86 (2017).
- [14]Á. Mócsy and P. Petreczky Phys. Rev. D **77**,014501(2008).
- [15] A. Mocsy Eur. Phys. J. C **61**, 705 (2009).
- [16] L. Thakur, U. Kakade and B. K. Patra.Proc Indian NatnSci. Acad. **8**, 75(2015)
- [17] S. M. Ikhdair and R. Sever Adv. High Energ.Phys. 2013, **562959**(2013).
- [18] M. Hamzavi and S. M. Ikhdair, Mod. Phys. Lett. B **27**, 1350176 (2013).
- [19]S. M. Ikhdair, B. J. Falaye and M. Hamzavi.Annals of Phys **353**, 282 (2015).

- [20] B. J. Falaye, G. H. Sun, R. S. Ortigoza and S. H. Dong. Phys Rev. E **93**, 053201 (2016).
- [21] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys.Rev. D**78**, 074033 (2008).
- [22] D. T. Son and N. Yamamoto, Phys. Rev. Lett. **109**, 181602 (2012).
- [23] J.-W. Chen, S. Pu, Q. Wang, and X.-N. Wang, Phys.Rev. D **110**, 262301 (2013)
- [24] M. D. Elia, S. Mukherjee, and F. S. Lippo, Phys. Rev. D **82**, **051501**(2010).
- [25] M. D. Elia and F. Negro, Phys.Rev. D**83**, **114028** (2011).
- [26] G.S., Bali, F. Bruckmann, G.Endrődi, et al. J. High Energ. Phys. **44**, 2012 (2012).
- [27] G. S. Bali, F. Bruckmann, G. Endrődi, Z. Fodor, S. D. Katz, and A. Schäfer Phys. Rev. D **86**, 071502 (2012).
- [28] Bali, G.S., Bruckmann, F., Endrődi, G. et al. J. High Energ. Phys. **130**, 2013 (2013).
- [29] N. Agasian and S. Fedorov, Phys.Lett. B **663**, 445 (2008).
- [30] E. S. Fraga and A. J. Mizher, Phys. Rev. D **78**, 025016 (2008).
- [31] A. J. Mizher, M. Chernodub, and E. S. Fraga, Phys. Rev. D **82**, 105016 (2010).
- [32] C. O. Edet, K. O. Okorie, H. Louis and N. A. Nzeata-Ibe. Indian J. Phys. Indian J Phys **94**, 243 (2020).
- [33] C. O. Edet, U. S., Okorie, A. T. Ngiangia and A. N. Ikot, Indian J. Phys. **94**, 425 (2020).
- [34] M. Abu-shady, H. M. Mansour, and A. I. Ahmadov, Adv. High Energy Phys., **2019**, 4785615 10 (2019).
- [35] M. Hasan, B. Chatterjee, B. K. Patra, Eur. Phys. J. C **77**, 767 (2017).

- [36] M. Hasan, B. K. Patra, B. Chatterjee, and P. Bagchi, hep-ph/1802.0687 (2019).
- [37] C. O. Edet, A. N. Ikot, M. C. Onyeaju, U. S. Okorie, G. J Rampho, M. L. Lekala, S. Kaya, *Physica E*, **131**, 114710 (2021)
- [38] E. P Inyang, J. E Ntibi, E. A Ibanga, F. Ayedun, E. P. Inyang, E. E Ibekwe, E. S. William, Ita O. Akpan, *Commun. Phys. Sci.*, **7**, 97 (2021)
- [39] C. O. Edet, A. N. Ikot, U. S. Okorie, G. J. Rampho, M. Ramantswana, R. Horchani, H. Abdullah, J. A. Vinasco, C. A. Duque, Abdel-Haleem Abdel-Aty, *Int. J. Thermophys.* **42**, 138 (2021).
- [40] G. J. Rampho, A. N. Ikot, C. O. Edet and U. S. Okorie, *Mol. Phys.* **119**, 5 (2021).
- [41] C. O. Edet & A. N. Ikot, *Mol. Phys.* (2021) DOI: [10.1080/00268976.2021.1957170](https://doi.org/10.1080/00268976.2021.1957170)
- [42] C. O. Edet and A. N. Ikot, *J. Low Temp. Phys.* **203**, 84 (2021)