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A mathematical framework to describe the effect of beam incidence angle on metrics derived from airborne LiDAR: the case of forest canopies approaching turbid medium behavior

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Abstract

Airborne laser scanning (LiDAR) is used in forest inventories to quantify stand structure with three dimensional point clouds. However, the 3D distribution of the point clouds depends not only on stand structure, but also on scan angle, because the probability for an oblique beam to be reflected by the canopy increases with the distance it must travel through the canopy. Thus, the canopy appears to increase in density as the incidence angle increases, all else being equal. The resulting variation between and within datasets can induce bias in LiDAR metrics derived from the vertical distribution of points. In this study, we modelled the effect of scan angle on the vertical structure of the point clouds to predict the bias of metrics derived from points sampled off-nadir. Comparison with paired observations from different flightlines (off- and at-nadir observations of the same point) demonstrated that the model accurately reproduced the bias of metrics calculated for a northern hardwood forest with relatively continuous canopy. Thus, the model could be used to correct the bias of LiDAR metrics, and provides a mathematical framework that could be used to inform the selection of maximum incidence angle in LiDAR surveys, considering the trade-off between decreasing acquisition costs and obtaining unbiased measurements.

Keywords: LiDAR, Scan angle, LiDAR parametrization, Forest, Resource monitoring, Canopy structure

1. Introduction

Airborne light detection and ranging (or LiDAR) technology is increasingly being used in the field of forestry as a complement to traditional field inventories. This technology provides forest managers with detailed, continuous information on forest structure that can cover large areas and be processed rapidly with little need for human

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interpretation. Data processing relies partly on automated algorithms (e.g. Pyysalo and Hyyppä, 2002; Morsdorf et al., 2004; Reitberger et al., 2009; Kwak et al., 2010; Yao et al., 2012; Vega et al., 2014) and partly on empirical statistical models (e.g. Holmgren, 2004; Ioki et al., 2009; Lim et al., 2014; Bouvier et al., 2015). Height and density metrics derived from the point cloud can be used to estimate the horizontal and vertical distribution of vegetation, which have various applications in forestry and ecology (Vauhkonen et al., 2014). For example, forest managers use LiDAR to predict product recovery under different harvest prescriptions (Maltamo et al., 2014).

LiDAR has brought a fundamental improvement in the quality of aerial inventories, which explains the rapid uptake of this technology by practitioners (Popescu et al., 2002; Gleason and Im, 2012). It can be argued, however, that some aspects of this technology are not fully understood. For example, the literature does not provide adequate understanding of the effect of LiDAR sensor parametrization and flight pattern on the three-dimensional structure of the point cloud (Goodwin et al., 2007). Thus, empirical statistical models of stand structure may only be applicable to a single forest, a single device and a single set of acquisition parameters.

While locally calibrated models meet user needs at a given point in time, each new LiDAR survey may require another calibration with new ground data, given that both the device and acquisition parameters change through time. Thus, the use of LiDAR technology for forest monitoring requires a better understanding of how changes in device settings and flight patterns affect the structure of the point cloud. Ultimately, users would benefit from being able to normalize any two sets of lidar-derived metrics as if they were acquired the same way.

While several studies have been dedicated to understanding how the density of emitted pulses affects various metrics and their prediction accuracy (Lovell et al., 2005; Anderson et al., 2006; Thomas et al., 2006; Gobakken and Næsset, 2008; Lim et al., 2008; Pirotti and Tarolli, 2010; Jakubowski et al., 2013), few have examined the effect of incidence angle. Widening the scanning angle allows a larger area to be surveyed more rapidly and at a lower cost (Goodwin et al., 2007; Evans et al., 2009). However, the financial advantage of a wide scanning angle could be offset by significant biases in the derived metrics.

Despite uncertainties regarding its magnitude, the effect of scanning angle is unarguable when one considers the extreme case of a 89° angle of incidence, in which case metrics are obviously biased compared to those obtained from a vertical beam simply because of shadowing effects or because of the increased distance between the top of the canopy and the ground. There is therefore a gradual increase in bias between 0 and 89°, such that

the canopy appears to increase in density as the **incidence** angle increases, all else being equal. Yet, the physical or geometrical phenomena leading to such effects, the magnitude of these effects at a given angle, and their consequences are still largely unknown. Thus, the ongoing debate about the choice of the **maximum incidence** angle would benefit from a better mechanistic understanding of angular bias, so that the trade-off between cost and prediction bias could be quantified.

[Holmgren \(2004\)](#) recommended limiting the scanning angle to 10° to prevent effects on forest metrics estimates, while [Disney et al. \(2010\)](#) proposed limiting it to less than 15° to avoid ground detection problems. However, these suggestions are hardly applicable in the case of large survey areas and in practice the **maximum incidence** angle is typically $\pm 15\text{-}20^\circ$, with a will to increase it further.

No consensus can be drawn from studies that have attempted to quantify the effects of **incidence** angle on LiDAR-derived forest metrics. [Holmgren et al. \(2003b\)](#) and [Lovell et al. \(2005\)](#) provided simulations of non-divergent beams hitting conical, ellipsoidal or half-ellipsoidal solid (i.e. impermeable) digitally reconstructed trees. With such simplifications of the reality [Lovell et al. \(2005\)](#) showed a dependency of the predominant height of the canopy on maximum **incidence** angle. They pointed out that increasing the maximum **incidence** angle and keeping all other parameters unchanged leads to an overall decrease in pulse density. The measure of the predominant height was therefore biased, as was mathematically demonstrated by [Roussel et al. \(2017\)](#). Within a given scan, [Holmgren et al. \(2003b\)](#) showed that the percentiles of height tended to decrease with increasing **incidence** angle, using a simulation which assumed that trees were solid, impermeable objects. [Lovell et al. \(2005\)](#) also highlighted the importance of the **incidence** angle by showing that maximum tree height retrieval is less accurate at the scanning edges due to a more uneven spacing of LiDAR points. [Goodwin et al. \(2007\)](#) improved these simulations by using permeable half-ellipsoidal digital trees. Their results showed that larger **incidence** angles “*produced a higher number of foliage hits and increased beam interception probability at the forest stand scale*”. They also demonstrated that higher **incidence** angles increase the crown area visible to a LiDAR pulse.

In their comparison of field measurements and LiDAR data, [Holmgren et al. \(2003a\)](#) did not find a statistically significant effect of the **incidence** angle on the estimation of a metric related to the height of dominant trees (i.e. mean of tree heights weighted by their basal area). Similarly, [Næsset \(1997\)](#) showed that the effect of the **incidence** angle was non-significant in a regression model used to predict forest basal area. These results do not necessarily contradict the previous results because all metrics may not be equally dependent on **incidence** angle.

It is also possible that different effects can compensate for one another, so that some predictive models appear to be insensitive to the **incidence** angle. Morsdorf et al. (2008) also concluded that the effect of the incidence angle is not as evident as it may first appear.

Montaghi (2013) presented an interesting study on the effect of **incidence** angle, taking advantage of perpendicular flightlines -typically used for strip adjustment and calibration- to compare a large amount of data sampled at nadir and off nadir. Although the overly large number of t-test comparisons (~1300) implies that some of the statistically significant effects reported may only be attributed to random variation (1 out of 20 test at $\alpha = 0.05$). Even with stronger statistical methods, this approach would lead to as many statistical models as metrics, i.e. one for each metric as they each have their own dependency to the **incidence** angle. Furthermore, these statistical models would be specific to a given dataset, which entails the same data-dependency issue described above.

An important issue with the current understanding of angular bias is that it has been acquired mainly through descriptive approaches. Indeed, mathematical models have rarely been used to explain how and why LiDAR metrics vary with **incidence** angle. Goodwin et al. (2007) and Disney et al. (2010) proposed that the probability a beam is reflected by the canopy increases as **incidence** angle increases, simply because the distance it travels through the canopy increases. Despite the plausibility of this explanation, a formal demonstration is still required.

In this study, we hypothesize that angular bias can be normalized by the distance a beam must travel through the canopy. We suggest a simple physical formalization of this hypothesis to quantify the effect of **incidence** angle on the vertical distribution of points. We then compare our theoretical model against real data to validate its relevance. Thus, rather than a very large number of empirical models derived for each metric, we propose a single overarching model that applies to all metrics.

2. Material and methods

2.1. Study area

The study area is located within the Haliburton Forest and Wildlife Reserve. The forest is a 32 000 ha privately owned property located in the Great Lakes - St. Lawrence Forest Region of central Ontario, Canada (45°13' N, 78°35' W). Elevation ranges from approximately 400 to 500 m above sea level. The forest is a mixture of hardwoods and

Figure 1: Map of study areas. Brown area represent the boudaries of the LiDAR dataset.

conifers typical of northern hardwood forests, and sugar maple (*Acer saccharum* Marsh) is the dominant species, comprising 60% of the basal area. Most of the forest has been managed under selection silviculture for the past 50 years, and was selectively harvested before then. Thus, most of the stands are uneven-aged, with average canopy heights ranging from 20 to 25 m.

2.2. LiDAR data

The LiDAR dataset was acquired in August 2009 covering the whole 320 km² area of the Haliburton forest. It was acquired with a pulse density of approximatively 2 pulses/m² on average. The complete set of parameters is given in table 1.

2.2.1. Data pre-processing

The normalization of the dataset (i.e. the subtraction of the digital terrain model) was done by the provider. The method was based on triangular irregular network construction from returns classified as “ground”. Each point was interpolated, which implies that the normalization was not based on a digital terrain model, thereby giving a virtually infinite resolution. Further details about the algorithm used to determine point classes could not be obtained. We did not have access to the raw data.

Lakes and wetlands were filtered from the dataset in an attempt to retain only forested areas. The process was based on geographic data from the latest official cartography of Ontario, which spatially matched very closely with observed lakes and wetlands from our LiDAR datasets.

Table 1: Flight parameters for the datasets. PRF: pulse repetition frequency

Parameter	Value
Sensor	Optech ALTM 3100
Altitude	1500 m
Swath overlap	30 %
Speed	120 kts
Scan Frequency	36 Hz
System PRF	70 kHz
Max. off-nadir angle	16 °
Cross track resolution	0.89 m
Along track resolution	0.86 m
Point density	$\approx 2 \text{ m}^{-2}$
Pulse density	$\approx 1.6 \text{ m}^{-2}$
Footprint size	0.14 m ²

2.3. Conceptual framework

We aim to quantify the changes in the height distribution of returns that result from increasing the incidence angle, and hence both the distance a beam travels through the canopy and the probability that the beam is reflected by the canopy. In particular, we aim to develop a model that reproduces the resulting increase in canopy returns and the corresponding decrease in ground returns. Successful description of this incidence angle effect should enable normalizing the point distribution, and consequently every existing metric derived from elevations within the point cloud (i.e. classical metrics derived from z coordinates) as if all data had been sampled at-nadir.

Our approach consisted of developing a set of two mathematical expressions to predict how a point distribution sampled at-nadir would be altered if it had been obtained from another off-nadir angle. To validate the model, the predicted bias of various LiDAR metrics derived from such an "off-nadir" point cloud was compared to the bias observed between paired observations from different flightlines. Among the infinite number of metrics that could be derived, we chose nine representative metrics that describe various aspects of a distribution. We believe that if our model is capable of predicting the behaviour of these diverse metrics, then it can be considered an adequate description of physical reality.

2.4. Metric computation

We rasterized the dataset at the level of a “plot raster” i.e. a 20×20 m pixel, which is a commonly used resolution both in the literature and in applications that map quantities of interest using an area based approach (Woods et al., 2011; White et al., 2013). The flightlines were treated individually to avoid introducing variation related to the existence of overlaps in which rasters were sampled twice. For each plot raster we computed one control metric (the mean absolute incidence angle of the returns) and, using all returns, nine metrics derived from the distribution of elevations in the point cloud:

- The mean height of the returns;
- The standard deviation of the heights;
- The coefficient of variation of the heights;
- The 30, 50 and 70th percentiles;
- The kurtosis and the skewness of the distribution;
- The entropy of the height distribution as labelled in the context of information theory. This index often called the “Shannon index” or “Shannon evenness index” in forestry or ecology applications. van Ewijk et al. (2011) name it “vertical complexity index” in a paper dedicated to LiDAR.

These metrics were not chosen for their relevance in forest inventory applications, but instead because they represent classical descriptors of central tendencies, deviation and heterogeneity applicable to any distribution.

The mean absolute incidence angle was based on the scan angle rank i.e. the actual data stored according to LAS format specifications ASPRS (2013), which does not represent the real nadir angle. Zero may in reality be off-nadir, depending on aircraft roll angle. However, we did not consider this effect as it would only create negligible noise in our analysis.

2.5. Software

Data pre-processing and processing was done in the R programming environment (R Core Team, 2015). A package named lidR specifically developed for LiDAR data processing was used (Roussel and Auty, 2017). The R source code to compute the model is given as a supplementary material.

3. Model development

3.1. Initial considerations

The model relies on probability theory and on three simplifying assumptions: the first two relate to the proportion of energy that is backscattered toward a LiDAR sensor, the third relates to the distribution and orientation of material within the canopy.

A quantity E_0 of energy emitted by a LiDAR instrument will either i) be absorbed by the canopy, ii) be backscattered in any direction other than towards the sensor, iii) be backscattered towards the sensor with insufficient energy to generate a point (when using discrete LiDAR) and, finally, vi) be backscattered towards the sensor with sufficient energy to generate a point. The first three quantities are considered to be “non-contributing”, or “lost” energy, while the last quantity will be referred to as “contributing energy”.

First, we assume that beams have an infinitesimal width and carry only contributing energy. Indeed, we worked with a point cloud that, by definition, resulted exclusively from such contributing energy. Following this assumption, when such a beam encounters an object of the canopy, it can only be reflected towards the sensor and generate a point. Multi-returns were considered to come from multiple beams and we neglected multiple scattering.

Second, we assume that the proportion of contributing energy is not affected by the beam incidence angle. Under this assumption, what changes between two different angles is not the amount of contributing energy but only the distribution of this energy throughout the canopy, and therefore the distribution of the triggered returns, or points.

Third, we assume that at a given elevation, the probability of reflecting contributing energy is proportional to the number of canopy elements, or material density. As we did not have prior information about the vertical distribution of canopy elements within a canopy layer, we assumed a random distribution at any given elevation.

Under these assumptions, the model describes how to recompute a point distribution sampled at one angle “as if it were sampled at another angle”. It relies on a set of two equations - the gap fraction profile function as presented in Bouvier et al. (2015) and its reciprocal function.

3.2. Notation

θ incidence angle

196 I event “beam interacts with a canopy element”
 197 R event “beam is reflected”
 198 A event “beam is absorbed”
 199 X a random variable
 200 $\bar{E}$ complementary event of the event E
 201 $\mathcal{P}(E)$ probability of the event E
 202 p_k probability for a given beam to generate a point in layer k
 203 i_k probability of I in layer k for a given beam
 204 r_k probability of R in layer k for a given beam
 205 a_k probability of A in layer k for a given beam
 206 Δz layer thickness

207 3.2.1. Gap fraction profile

208 The gap fraction describes the probability for a beam to reach the ground without encountering canopy
 209 elements. Bouvier et al. (2015) proposed an equation adapted to point clouds for computing the gap fraction
 210 profile as a function of height within the canopy (eq. 1). The authors used this equation to define a metric that
 211 could be used in a predictive model of biomass. We believe that it has further potential applications for modelling
 212 the behaviour of the LiDAR signal. The following lines provide a description of the Bouvier *et al.* equation and its
 213 interpretation. The gap fraction profile is defined in equation 1 (using the original notations):

$$P_k = \frac{N_{[0;z]}}{N_{[0;z+dz]}} \quad (1)$$

214 where $N_{[0;z]}$ refers to the number of returns below z , and $N_{[0;z+dz]}$ refers to the number of returns below $z + dz$
 215 with dz the thickness of a layer of forest. The equation expresses the number of laser returns that actually reached
 216 the layer $z + dz$ and those that passed through the layer $[z; z + dz]$. P_k represents the gap fraction of the k^{th} layer.

217
 218 The gap fraction can be interpreted as the probability, for a single beam carrying contributing energy and
 219 reaching the layer k , of passing through this layer without interacting with canopy elements. In our model, we
 220 consider an alternative event I which expresses the probability i_k that a beam reaching layer k interacts with a
 221 canopy element in this layer.

$$i_k = 1 - P_k \quad (2)$$

As described further below, this probability is related to the height distribution of points, which itself can be interpreted as the probability, for a single beam carrying contributing energy, of generating a point in the k^{th} layer.

Let $p_k(\theta)$ be the probability that a beam generates point in the k^{th} layer at a **incidence** angle θ . According to equation 2, the probability of interacting with canopy elements in layer k is $i_k(\theta)$. If we assume that a beam interacting with a canopy element is always reflected towards the LiDAR sensor with sufficient energy to generate a return (contributing energy), events I and R are equal and $i_k = r_k$:

$$r_k(\theta) = 1 - \frac{\sum_{i=1}^{k-1} p_i(\theta)}{\sum_{i=1}^k p_i(\theta)} \quad (3)$$

Figure 2 illustrates how equation 3 can be used to calculate the probability of interaction from the height distribution of points in a hypothetical canopy with 5 layers that generate either 20 or 0 returns when sampled vertically. This example shows that to obtain the same number of points in each layer, the canopy must increase in density as the beams approach the ground. Accordingly, the probability of a beam interacting with a canopy element has to increase from top to bottom. The probability associated with the bottom layer is always 1 because of the presence of the ground (gap fraction is 0).

Figure 2: Vertical sampling scenario illustrating how the probability of interaction is calculated from the height distribution of points, and vice versa. Function f (eq 3) is used to calculate the probability of interaction from the height distribution, and the inverse function f^{-1} (eq 4) is used to recover the height distribution of points from the interaction probabilities.

Figure 2 also illustrates how the interaction probabilities can in turn be used to recover the height distribution

of points, which is the product of two probabilities - the probability that a beam reaching a layer interacts with it (i_k), and the probability of reaching the layer (i.e. the probability of passing through each previous layer). Thus, the height distribution can be recovered using the reciprocal function, f^{-1} :

$$p_k(\theta) = r_k(\theta) \prod_{i=k+1}^{n+1} (1 - r_i(\theta)) \quad (4)$$

with n being the number of layers. Note that for the equation to apply to the case where $k = n$, we added a virtual $n + 1$ layer with a probability of interaction of 0 (see [AppendixA](#) for explanations and a mathematical demonstration).

3.3. Effect of *incidence* angle on the height distribution of points

When a beam arrives at an angle of θ degrees, its travel distance through each layer is $1/\cos(\theta)$ times longer than that of a vertical beam. Thus, the probability of interacting with canopy elements increases with the *incidence* angle, and there is a corresponding change in the height distribution of points (i.e. the probability that a beam generates a point in any given layer).

As demonstrated further below, an approximate value of the probability of interaction can be calculated from $p_k(0)$ by including the factor $1/\cos(\theta)$ in function f^{-1} (eq. 4). For $k > 1$, $p_k(\theta)$ is the probability that a beam arriving at angle θ generates a point in layer k :

$$p_k(\theta) = \frac{r_k(0)}{\cos\theta} \prod_{i=k+1}^{n+1} \left(1 - \frac{r_i(0)}{\cos\theta}\right) \quad (5)$$

Thus, the probability of interaction increases in each layer by a factor of $1/\cos(\theta)$, except the ground layer, which by definition always has a probability of interaction of 1.

This *incidence* angle effect is illustrated in figure 3, which shows that increasing the *incidence* angle from 0 to 45° increases the probability of interaction by $\sqrt{2}$, thereby shifting the expected height distribution upwards (increasing the number of canopy while decreasing the number of ground points).

To demonstrate that dividing by $\cos(\theta)$ in equation 5 provides a reasonable approximation of the increase in interaction probability at larger incident angles, we must consider the spatial distribution of elements in the canopy. If we assume that the spatial distribution is random, the number of elements in the path of a beam is

Figure 3: Oblique sampling scenario illustrating how the expected height distribution of points sampled at 45° is calculated from the height distribution of points observed when sampled vertically (Fig. 1). The function f is used to calculate the interaction probabilities for each layer from the height distribution of points sampled at 0° . The inverse function f^{-1} is used to convert the interaction probabilities back into a height distribution specifying the probabilities that a beam generates a point in any given layer, given that it enters at an angle of 45° . The function g is the composition of f and f^{-1} ($g = f^{-1} \circ f$).

a random variable X that follows a Poisson distribution (Nilson, 1971). Thus, the probability, when traveling a distance d through layer k , of encountering n elements is:

$$\mathcal{P}_k(X = n) = \frac{d \lambda_k^n e^{-d \lambda_k}}{n!} \quad (6)$$

and the probability of interacting with at least one canopy element in layer k is i_k :

$$\begin{aligned} i_k &= \mathcal{P}(X > 0) \\ &= 1 - \mathcal{P}(X = 0) \\ &= 1 - e^{-d \lambda_k} \end{aligned} \quad (7)$$

The quantity λ_k is related to the density of leaves in the k^{th} layer, as well as their orientation and spatial distribution. The meaning of this quantity is not required to solve the problem at hand, though readers can refer to Nilson (1971), Campbell and Norman (1990) or the discussion section for more details.

Since $i_k = r_k$, and the thickness of the layers Δz tends towards 0, we can simplify this expression using the first order Taylor expansion of the exponential near 0:

$$\begin{aligned}
r_k(0) &= 1 - e^{-\Delta z \lambda_k} \\
&= 1 - 1 + \Delta z \lambda_k + O((\Delta z \lambda_k)^2) \\
&\approx \Delta z \lambda_k
\end{aligned} \tag{8}$$

For oblique angles, the distance Δz is increased by the inverse of cosine θ , and the probability of intersecting at least one element becomes:

$$r_k(\theta) = 1 - e^{-\frac{\Delta z}{\cos(\theta)} \lambda_k} \tag{9}$$

Finally, Taylor expansion in the neighbourhood of 0 yields the factor included in function f^{-1} (eq 5):

$$r_k(\theta) \approx \frac{\Delta z \lambda_k}{\cos(\theta)} \approx \frac{r_k(0)}{\cos(\theta)} \tag{10}$$

Without the first order Taylor expansion the expression would be difficult to manipulate, but it must be noted that the approximation remains valid only if $\frac{\Delta z \lambda_k}{\cos(\theta)}$ approaches 0. In other words, the approximation is correct only for thin layers and narrow incidence angles, as discussed in section 5.

With this approximation, we can then define a function, g (fig 3), using the composition of f and f^{-1} ($g = f^{-1} \circ f$), that enables us to calculate the expected height distribution of points that are sampled obliquely (fig. 3), taking into account the distance required to go through each layer. The source code of this function can be found in the supplementary materials.

3.4. Observed decrease in number of points per pulse: an unknown effect of incidence angle

We observed that in addition to increasing the probability of interaction, increasing the incidence angle also decreases the number of returns per beam (figure 4), which could result in an upward shift in the height distribution of points. Thus, we modified our model to provide an empirical way to account for this effect on the height distribution of points. The model does not provide a mechanistic explanation of this phenomenon, though we do provide two plausible explanations in the discussion section.

Given that a point is a spike of energy, the observed decrease in the number of points per pulse represents a decrease either in the amount of energy that is backscattered to the sensor, or in the amplitude of spikes, some

Figure 4: Number of points per pulse as a function of the **incidence** angle modelled empirically using the linear relationship in 600 000 plot rasters covering the all of Haliburton forest. **For better readability, the colour scale represents the density of observations. The blue line is the fitted linear regression. The confidence interval is narrower than the line at this scale.**

of which become too low to be registered as a point. Thus, the decrease in the number of returns per beam can be represented empirically by allowing foliage to absorb contributing energy, and allowing the probability of absorption to vary with **incidence** angle, similar to the probability of reflection. This implies relaxing the second assumption presented in section 3.1.

Let A be the event “beam is absorbed” and $a_k(\theta)$ its probability in layer k at angle θ . Thus far, we have assumed that a beam interacting with the canopy is always reflected ($I = R$), but now the energy is either reflected *or* absorbed: $I = R \cup A$. R and A are two disjoint events, thus:

$$\begin{aligned}\mathcal{P}(I) &= \mathcal{P}(R) + \mathcal{P}(A) \\ \Leftrightarrow i_k(\theta) &= r_k(\theta) + a_k(\theta)\end{aligned}\tag{11}$$

and equation 5 becomes:

$$p_k(\theta) = \left(\frac{r_k(0)}{\cos \theta} + a_k(\theta) \right) \prod_{i=k+1}^{n+1} \left(1 - \frac{r_i(0)}{\cos \theta} - a_i(\theta) \right)\tag{12}$$

Now, the height distribution of points depends on both the **incidence** angle and vertical variation in absorption. While it is unknown how a changes as a function of k , the probability of being absorbed was assumed to be

proportional to the probability of being reflected. Thus, we introduced a **proportionality** function, α , to describe the variability in absorbed vs reflected energy at different incidence angles:

$$a_k(\theta) = \alpha(\theta) r_k(\theta) \quad (13)$$

While the proportionality function is constant with respect to k , the probability of absorption can vary as a linear function of θ , consistent with the pattern shown in figure 4.

307

Substituting this term into equation 11 leads to:

$$i_k(\theta) = r_k(\theta)(1 + \alpha(\theta)) \quad (14)$$

Integrating this new expression into equation 12 leads to the final form of the expression:

$$p_k(\theta) = \left(\frac{r_k(0)}{\cos(\theta)} (1 + \alpha(\theta)) \right) \prod_{i=k+1}^{n+1} \left(1 - \frac{r_i(0)}{\cos(\theta)} (1 + \alpha(\theta)) \right) \quad (15)$$

Using the data shown in figure 4, we can obtain an empirical estimate of the function α from $\epsilon(\theta)$, the overall probability of being absorbed by the canopy at a given **incidence** angle, as:

$$\begin{aligned} \epsilon(\theta) &= \sum_{k=1}^n a_k(\theta) \\ \epsilon(\theta) &= \alpha(\theta) \sum_{k=1}^n r_k(\theta) \\ \alpha(\theta) &= \frac{\epsilon(\theta)}{\sum_{k=1}^n r_k(\theta)} \end{aligned} \quad (16)$$

In this equation the only unknown term is the function ϵ . Based on our data and assuming that the relationship is almost linear at least between 0 and 15°, we have:

$$\epsilon(\theta) = \frac{\theta}{15} \epsilon(15) \quad (17)$$

with $\epsilon(15) \approx 6\% = 0.06$. Note that ϵ is the only empirical parameter of our model.

3.5. Model validation

The bias of LiDAR metrics can be expected to vary locally because they depend on the local forest structure. However, the number of returns available at the plot scale was insufficient to allow us to accurately apply our

model to each 400 m² raster. Thus, to validate our model, we first predicted the average bias of each metric over the whole study area, then compared it to the average bias observed between paired flightlines. For this reason, our model only captures the average bias observed at the scale of the entire forest, not the plot-scale bias that will remain and be manifest as noise.

To implement our model for the whole study area, we first extracted all points sampled at-nadir. Considering the las format specification (ASPRS, 2013) in which the incidence angle information is an integer, nadir (0°) corresponds to a incidence angle ranging from -0.5 to 0.5 °. This subset of the data provided an average height distribution of points sampled at-nadir, which was assumed to be representative of the entire forest because the sampling design and the forest structure were completely independent. This reference distribution was then used to recompute the expected height distributions for incidence angles between 0 and 15°, using equations 3 and 5 or 15. Finally, the nine metrics were calculated using each of the distributions, and the expected bias at each angle was calculated as the difference from the reference distribution.

For comparison, we calculated the observed bias by extracting 150 000 rasters centred on (x, y) coordinates that were sampled twice in the overlap of adjacent flightlines. The observed bias was then calculated as the difference between the two values obtained for each metric (fig. 5).

4. Results

4.1. Effect of scan angle on the height distribution of points

As shown in Figure 6, the expected height distribution of points sampled at 30° includes more returns in the upper canopy than observed at-nadir, which implies more interactions with canopy elements. This pattern is reversed in the lower canopy due to the conservation of energy. The proximity of the green and the blue lines shows that taking into account the reduction of points per beam has a relatively small effect on the expected height distribution of points. In contrast, the difference between the red line and the other two shows that the angular bias described by the probabilistic part of the model is comparatively large.

4.2. Comparison of observed and expected bias

Our model accurately reproduced the bias observed in the data (figure 7). Both the sign and magnitude of the bias were correctly predicted for each of the 9 metrics. Including the reduction in the number of points per beam

Figure 5: Illustration of the method used to quantify the bias of LiDAR metrics using 150 000 plot rasters sampled twice from different flightlines. The bias is the observed difference in the LiDAR metrics obtained from the two flightlines (x axis of inset), which varies as function of the difference in **incidence** angle (y axis of inset).

Figure 6: The average height distribution of points sampled at-nadir (0°) and the expected height distribution of points sampled at 30° , with and without considering the fewer number of points per beam (calculated using equations 3 and 15, respectively). An angle of 30° was chosen to visualize the magnitude of the predicted angular bias. The histograms appear continuous because they were computed with 1 cm bins.

only had a small effect on the predicted bias, but doing so brought the expected value closer to the observed value in every case. The observed bias varied considerably from one plot to the next, as shown by the whiskers on either side of the boxes. This is residual bias that is not captured by the model, because it was not implemented at the scale of a plot raster. As mentioned previously, it was implemented using the average height distribution of all the points in the dataset that were sampled at-nadir.

4.3. The effect of stand structure on bias

While we were unable to quantify the effect of stand structure empirically, we did calculate the expected bias for three hypothetical stand structures, as shown in Figure 8. The stand structure on the left is unrealistic, but serves to show that the biases can be non monotonic with respect to scan angle. In this example the magnitude of the bias in the standard deviation is low, but increases until 15° and then decreases until 30° . Thus, despite its apparent simplicity, the model is able to predict complex patterns of bias.

The next two structures are more realistic as they are derived from modified gamma distributions that produce patterns similar to those observed in our data. Both are symmetric and represent typical stand structures in which most returns occur in the lower or upper canopy (but never on the ground). Comparing the patterns of bias in these two stand structures shows that the sign of the bias can switch from positive to negative (or vice versa), depending on the structure of the stand. The relative magnitude of bias can also switch: in the middle panel, for example,

Figure 7: Bias observed in the plot rasters that were sampled twice from different flightlines, as shown in (fig. 5). The observed bias (boxplots) of the nine LiDAR metrics is compared to the the expected bias, both with (red points) and without (blue points) including the reduction in the number of points per beam (calculated using equations 3 and 15, respectively).

Figure 8: Expected bias in three hypothetical stand structures with a maximum height of 25 m. The expected height distribution of points is plotted in the insets, which show (from left to right) a purely theoretical structure in which the number of returns increases linearly towards the ground, and two more realistic structures in which most returns occur in the lower and upper layers of the canopy. The units of the main y axis are either in meters or dimensionless (depending on the metric units).

the 30th percentile is more biased than the 70th percentile, while the opposite is observed in the right-hand panel. The only constant pattern is the monotonic increase in bias for the height percentiles and the mean height, which was expected at the model development stage.

5. Discussion

5.1. Effect on forest resource inventory

We started from the hypothesis proposed by Goodwin et al. (2007) and Disney et al. (2010) that the increased travel distance through the canopy increases the probability a beam is reflected as incidence angle increase and we modelled that effect. Our model accurately reproduced the observed bias despite our simplifying assumption that canopy elements are randomly distributed in space. This likely reflects the fact that canopies are relatively continuous in northern hardwood forests. By enabling prediction of the bias attributable to incidence angle for various metrics, our model can be used to normalize LiDAR datasets acquired in forests that meet the assumptions made to build the model.

In our study the average overestimation of the mean height reached 40 cm at 15°. Bias of this magnitude could conceivably affect the accuracy of area-based models that use LiDAR metrics to predict other stand-level variables of interest, such as stand height, wood volume or aboveground biomass. However, we cannot make general

statements about the influence of bias on the accuracy of area-based models because it is highly dependent on which LiDAR statistics are used. The bias of some metrics may compensate for one another, which could explain the absence of significant effects in the study of Næsset (1997), while bias may be cumulative in other cases. Our study also showed that the effects of **incidence** angle depend on the forest structure, which implies that the practical importance of the phenomenon described in our study is very site specific.

An important term in the model is the inverse of the cosine, which renders bias non-linear with respect to **incidence** angle, indicating that there is a threshold angle beyond which the effect becomes extremely strong. For metrics expressed in meters in our analysis, differences began to reach values larger than one meter at 30°. However, caution must be used in extrapolating to larger angles because of the Taylor expansion that was used to linearize the expression. As the cosine tends towards 0 when θ tends towards 90°, the term $\frac{\Delta z \lambda_k(\theta)}{\cos(\theta)}$ tends towards an undetermined limit. Therefore, the Taylor expansion cannot be used for any combination of variables that make $\frac{\Delta z \lambda_k}{\cos(\theta)}$ depart too far from 0, in which case the equation is no longer amenable to analysis.

In reality, bias would not reach the maximum value associated with the largest **incidence** angle due to the overlap between flightlines. Approximately 30% of our area was surveyed in two flightlines, which implies that plots were rarely sampled with a single, large **incidence** angle. In our dataset, a plot sampled at the maximum angle of 15° in one flightline was likely to have been scanned at a lower angle of about 10°, for example, in a separate flightline. This should limit the effects of the **incidence** angle in practice. Having points from two amalgamated flightlines would alter their vertical distribution in a way that is a linear combination of the two effects weighted by the respective local point densities (see supplementary materials).

For future practical applications, we recommend that users first determine whether or not the effect of **incidence** angle can be neglected. This can be achieved by analysing each flightline separately and comparing the metrics obtained from different angles.

5.2. Linearity of the observed bias

We quantified the expected bias at a given angle as the difference between two metrics, one of which was calculated using a 0° reference distribution - the average height distribution of points sampled at-nadir (Figure 6). In contrast, the two metrics we used to quantify the observed bias were both calculated using rasters that may have been sampled obliquely (fig. 5). This implies that a **incidence** angle difference of 4°, for example, can

originate from rasters sampled at 0 and 4°, but also 1 and 5°, 8 and 12°, 10 and 14° etc. The same applies to all incidence angle differences except 15°, which was necessarily the result of a raster sampled at-nadir and another one sampled at 15° because the maximum incidence angle was 15°.

Because our model always used 0° as a reference, our comparison of the expected and observed bias (figure 7) is only valid if the effects are linear. Figure 7 showed this was not strictly true, but within the range of our observations the angles are small enough, and the effects linear enough, that we consider the comparison to be valid. Using all the rasters sampled in two flightlines was necessary to obtain enough data to highlight the overall pattern hidden in the noise. If we had only used pairs that included one raster sampled at-nadir, the dataset of observed variation would have decreased from 150 000 to only 600 rasters, which was insufficient to show the signal.

5.3. Accounting for the reduction of number of returns per beam

Despite the limited magnitude of this effect on the expected bias, the data clearly showed that the number of points per pulse decreases with increasing incidence angle. To our knowledge, this phenomenon has not yet been reported in the scientific literature. One hypothesis is that as incidence angle increases, a pulse is more likely to be intercepted by vertical tree trunks because they become more exposed. Trunks are covered in bark that is more likely than foliage to absorb energy at 1064 nm. Thus, an oblique beam is less likely to be reflected when it encounters a tree trunk, and if absorbed there will be no subsequent returns. According to this hypothesis, the average number of points per beam can only decrease as a function of the incidence angle. Moreover the average intensity of first returns should be constant with respect to the incidence angle (if a range correction is applied).

Another hypothesis is that the probability of being absorbed by foliage increases with incidence angle. This can happen if the spatial distribution and orientation of leaves are not random. However, in contrast to the previous hypothesis, this could also lead to an increase in the number of pulses per beam in certain circumstances.

To appreciate why, consider the parameter λ_k that was first introduced in equation 6. λ_k is the product of the leaf area density μ , the G function and the clumping factor Ω (e.g. Nilson, 1971; Campbell and Norman, 1990). Thus, λ_k is not only a function of elevation (k) but also a function of incidence angle θ :

$$\lambda_k(\theta) = \mu_k G_k(\theta) \Omega_k(\theta) \quad (18)$$

However, we assumed that the spatial distribution of foliage is random, in which case $\Omega(\theta)$ equals unity and (given a spherical leaf angle distribution) $G(\theta) = 0.5$, for any incidence angle. λ was therefore only a function of the elevation (λ_k) in equation 6.

If these assumptions are relaxed equation 9 becomes:

$$\begin{aligned}
 i_k(\theta) &= \frac{\Delta z \lambda_k(\theta)}{\cos(\theta)} \\
 &= \frac{\Delta z \mu_k G(\theta) \Omega(\theta)}{\cos(\theta)} \\
 &= \frac{\Delta z \mu_k G(0) \Omega(0) G(\theta) \Omega(\theta)}{\cos(\theta) G(0) \Omega(0)} \\
 &= \frac{r_k(0) G(\theta) \Omega(\theta)}{\cos(\theta) G(0) \Omega(0)}
 \end{aligned} \tag{19}$$

Comparing the above to equation 14,

$$i_k(\theta) = \frac{r_k(0)}{\cos(\theta)} (1 + \alpha(\theta)) \tag{20}$$

we note that what is known as the extinction coefficient K –the G function multiplied by the clumping factor– is the function $1 + \alpha$ in our model. The G function depicts the azimuthal angle distribution of the foliage. G can be an increasing or decreasing function of the view angle and K as well. Thus, contrary to the first hypothesis, this second hypothesis allows the bias to be positive or negative i.e. either an increasing or decreasing number of returns per beam.

Because both hypotheses lead to the same mathematical formulation i.e. a factor that multiplies the term $\frac{r_k}{\cos(\theta)}$, further analysis of the reduction in the number of points per beam in other datasets is required to distinguish between them. If decreases in number of returns per pulse are consistently observed at oblique incident angles, then it could be likely attributed to absorption by bark.

In our model, absorption was proportional to the density of canopy elements and the proportionality function (eq. 13) was constant with respect to height. This was sufficient to reproduce the behaviour of the data, but in reality it is likely that this coefficient varies between layers. A deeper inspection of the sequence of multiple returns would be necessary to refine our understanding of the reduction in the number of points per beam. In addition to evaluating how this reduction is distributed along the sequence of returns, it would be interesting to

examine how it varies with height. For example, Næsset (2009) found that that single echoes tend to occur in the densest parts of the tree crowns. Since our model assumes the forest is *perceived* to be denser off-nadir, the results are therefore compatible.

Over and above all the considerations presented in this section, it must be highlighted that the part of the model accounting for the reduction in the number of returns per beam remains only an empirical add-on. It should not be considered as an intrinsic part of the model that was developed using a hypothesis-driven approach. Instead, we have used this model to propose one way to address the question of point loss. Other explanations can be proposed to explain this phenomenon, such as a loss of energy backscattered due to the increasing path length, for example.

5.4. Model applicability

An important limit of our study is that the model was only validated using a single dataset from a northern hardwood forest, and thus for a specific instrument and specific survey settings. Despite this, we believe the model has more general applicability over any type of forest that meets our initial assumptions.

Under our hypothesis-driven approach, our model was derived from a few initial assumptions made about the forest canopy structure and the way energy is spread and backscattered, independently of any site- or device-specific principles. The model is self-contained and does not rely on empirical data. This is the key to justify the empirical validation using a single dataset, which is only deemed to provide a demonstration that the model can be applicable in reality, in one forest type that meets our initial assumptions. The fact the model fits well with our validation dataset provides a good indication that it can offer a plausible representation of the physical reality.

We therefore expect a similar applicability in other forest types that meet the same assumptions. However, with the infinity of forest structures that can be found globally, it would not be possible to provide an exhaustive analysis of the limits of applicability of the model. For this reason, our approach was to provide 1) one example and 2) a source code that enables future users to determine if the model applies or not to their specific context.

Listing all types of forest canopies that could be adequately represented as a set of horizontal turbid layers is

beyond the scope of this study. However, we believe the model may be applicable to any closed-canopy forests dominated by broadleaved trees. This includes temperate hardwood forests, but also to tropical humid or even dry tropical forests. Obviously, model applicability remains to be empirically demonstrated in other ecosystems.

5.5. *Alternative approaches for discontinuous canopies*

Our initial assumption that the forest canopy can be represented as a set of horizontal turbid layers would not be valid for forests with a clumped canopy structure such as conifer or savannah canopies. Coniferous forests, for example, exhibit hierarchical clumping structure at different levels (Wenge et al., 1997), and individual conifers are more analogous to large, solid geometrical objects (Li and Strahler, 1985). This suggests that the influence of incidence angle is determined more by geometrical effects than by probabilistic effects.

Our model is analogous to radiative transfer (RT) models because it describes the probability of interacting with components of turbid homogeneous horizontal layers. Another way to model canopies is the geometrical optic (GO) approach, which was first developed for discontinuous conifer canopies that can be represented as an assemblage of three-dimensional, solid objects. The conceptual foundations of both the RT and GO approach were formalized decades ago (e.g. Li and Strahler, 1985; Strahler and Jupp, 1990). To develop a GO model of incidence angle effects discontinuous canopies, the first step would be to transfer existing GO equations to LiDAR applications, similarly to what we attempted in this study with the RT approach. A more advanced approach could even rely on equations that use both GO and RT principles (GORT approach) in the spirit of studies proposed by Wenge et al. (1997) or Haverd et al. (2012).

6. Conclusion

We examined the changes in the height distribution of returns that result from increasing the incidence angle, and hence both the distance a beam travels through the canopy and the probability the beam is reflected by the canopy. We developed a conceptual framework for understanding and predicting the resulting bias of LiDAR metrics, and demonstrated that our model accurately reproduced the bias calculated for northern hardwoods with relatively continuous canopies. The model allows a point distribution sampled at-nadir to be recomputed “as if it were sampled at another incidence angle”.

The model also suggests that the non-random spatial distribution of foliage may be responsible for fewer returns per beam at large incidence angles. Alternatively, this may reflect the fact that oblique pulses are more

likely to be intercepted by vertical tree trunks, resulting in the end of a return sequence. Nevertheless, our model predicts the number of points per beam has a small effect on the height distribution of points, compared to increasing the length of the path a beam travels through the canopy.

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625 AppendixA. Function f^{-1} , basic form

626 We try to demonstrate that the function f^{-1} can be written:

$$\forall k \in \llbracket 1, n \rrbracket, p_k = r_k \prod_{i=k+1}^{n+1} (1 - r_i)$$

627 Let's consider n layers, with the ground layer being layer 1, and the highest layer the layer n . The probability
 628 to find a beam travelling through the layer k is q_k . This beam travelling through layer k interacts with canopy
 629 components within that layer with a probability r_k . The probability to generate a point in the layer k is:

$$p_k = r_k q_k$$

630 The probability q_k is the probability that the beam passed through each previous layer without interacting with
 631 canopy components.

$$\begin{aligned} q_k &= \mathcal{P}(\bar{R}_{k+1} \cap \bar{R}_{k+2} \cap \dots \cap \bar{R}_n) \\ &= (1 - r_{k+1}) \times (1 - r_{k+2}) \times \dots \times (1 - r_n) \\ &= \prod_{i=k+1}^n (1 - r_i) \end{aligned}$$

632 Then,

$$p_k = r_k \prod_{i=k+1}^n (1 - r_i)$$

633 The case where $k = n$ is a particular case which does not follow the rule because the formula does not make
 634 sense:

$$p_n = r_n \prod_{i=n+1}^n (1 - r_i)$$

635 Adding a virtual layer $n + 1$ with a probability r_{n+1} of interaction of 0 solves the issue adding a neutral element
 636 into the product:

$$p_k = r_k \prod_{i=k+1}^{n+1} (1 - r_i)$$