ARITHMETIC DISABILITY OF ADULTS

by

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A thesis submitted in conformity with the requirements for the Degree of Doctor of Philosophy
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ABSTRACT

Arithmetic Disability of Adults
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The aim of this study was to investigate the numeracy skills of adults with arithmetic disability. The research involved comparisons among three groups: adults performing poorly in arithmetic (AD, n=24); adults matched for chronological age who did not experience difficulties in arithmetic (NCA, n=24); and a group of normally achieving students (NAL, n=21) who were functioning at the same arithmetic level as the AD adults. Comparing this latter group with the AD adults provided an opportunity to test a developmental-lag model of individual differences in arithmetic related skills.

To obtain a measure of their arithmetic skills, the participants were individually administered tests of computation, attitude regarding arithmetic, and reasoning in the domain of arithmetic. The findings suggested that AD adults were similar, both in computational skills and in terms of arithmetic related reasoning, to their younger counterparts of the same arithmetic level. Adults with an arithmetic disability as well as normally achieving children showed similar problems in solving arithmetic questions, whereas the error patterns of normally achieving adults were different. Adults with an arithmetic disability and normally achieving children showed lower self-confidence ratings before and after word-problem solving than the normally achieving adult group. The AD group reported significantly higher levels of math anxiety than the two comparison groups.

It was concluded that the group differences of arithmetic performance in this study support a weak developmental lag model in which arithmetic disabilities represent an incomplete maturation. Future research directions and educational implications for adults with arithmetic disability are discussed.
I would like to thank GOD for letting me see the light at the end of the tunnel called 'Ph.D. candidacy'. The encouragement and loving support of my family and friends (fortunately too many to mention each by name) made this process enjoyable and the breaks from it memorable.

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CHAPTER 1

INTRODUCTION

The Importance of Numeracy

With the advent of technology and the written word, literacy and numeracy skills have become critical to basic survival. Numeracy as defined by the Harper Dictionary of Modern Thought (1977:434) is "understanding of the scientific approach to study in terms of observation, measurement, assessment, experiment, and verification, and more specifically some mastery of interpretation of mathematical and statistical evidence". Comprehending and using numbers are crucial if one desires to survive the numerical information overload faced day-to-day. Competency in numeracy is essential in reaching one's full cognitive potential (Myers & Hammil, 1990). To succeed in many professions one needs to understand simple formulas, read graphs, as well as interpret statements about probability. This constituent suggests the importance of applied math in the contemporary world. Numeracy therefore can be seen as an important vehicle for adults, who all need to solve problems that involve numbers.

Although literacy, an important vehicle in today's society, has received much attention in the field of learning disabilities, there is relatively little known about existing disabilities in numeracy (Myers & Hammil, 1990). Not only has numeracy received minimal attention in the literature on learning disabilities (LD), but, of the research published to date, most have been studies
involving children only. Many factors have been investigated in terms of the overall arithmetic skills of children, including: working memory (Brainerd, 1983; Fletcher, 1985; Geary, Brown, & Samaranayake, 1991; Siegel & Linder, 1984; Swanson, 1993, 1994); visual-spatial abilities and short-term memory (Morrison & Siegel, 1991; Siegel & Feldman, 1983); basic fact computation (Garnett & Fleischner, 1983; Geary, Widaman, Little, & Cormier, 1986; Geary, Bow-Thomas, & Fan, 1993); cognitive difficulties (Cormier & Carlson, 1985; Ohlsson, Ernst, & Rees, 1992; Payne, & Squibb, 1990; Russell & Ginsburg, 1984); anxiety (Fisher, Allen, & Kose, 1996); execution of arithmetic problems (Ackerman, Anhalt, & Dykman, 1986; Geary, Cormier, Goggin, Estrada, & Lunn, 1993; Johnson & Layng, 1992; Donlan, 1993); problem-solving strategies (Cooney & Swanson, 1990; Cormier, Carlson, Das, 1990; Jitendra & Hoff, 1996; Meltzer, 1991); inferential skills and postfailure reflectivity (Shafrir, Siegel, & Chee, 1990); assessment procedures (Jordan, 1995; Siegel, 1989, 1991; Swanson, 1994), and neuropsychological assets and deficits (Rourke, 1993). Significantly, no study to date has investigated thoroughly the skills related to numeracy of adults with Arithmetic Disability (AD). In fact, most adults with learning difficulties in arithmetic related areas have been subject to benign neglect. Assessment services for adults with learning disabilities in the area of numeracy are scarce. Remedial assistance for such persons is also extremely limited (Sturomski, 1996).

This study was designed to investigate the similarities and differences of the cognitive and affective profiles of adults with arithmetic disability and two comparison groups, one matched on chronological-age and the other matched on arithmetic level. The purpose of this research study was to describe math learning disability in an adult population. More specifically, in this thesis the computational skills, attitude towards mathematics, and reasoning skills in the domain of mathematics were examined.
Definitional Issues of Arithmetic Disability

Arithmetic disability is a subtype of learning disability. Although a person who experiences arithmetic disability does not share the same social stigma as a person with reading disabilities, this disorder can still be quite debilitating in today's society (Greenbaum, Graham, & Scales, 1996). Undoubtedly, adults with arithmetic disability experience difficulties when asked to work with numbers. The extent of these difficulties, and how these difficulties are measured, however, are not the same among research studies in the field of learning disabilities.

Historically, different groups of investigators tended to focus on characterizing children with arithmetic disability in terms of their cognitive profile (Kirby & Becker, 1988; Russell & Ginsburg, 1984) and neuropsychological profile (Strang & Rourke, 1985; Rourke, 1993). Most of these studies, however, have not been replicated with adult populations.

Rourke and his associates were among the few investigators who attempted to document the functioning of adults with learning disabilities. They focused on the abnormal neurological functioning of adults with learning disabilities. Their findings suggest that there is a group of individuals with, what they termed, nonverbal learning disabilities, who have a distinct neurological, academic, and socio-emotional profile from individuals with verbal learning disabilities. Rourke (1985, 1989) hypothesized that the cause of nonverbal learning disabilities is the deviant development of the white matter of the right hemisphere. Rourke, Young, Strang, and Russell (1986) described eight adults with non-verbal learning disabilities. Although these individuals had average or above average verbal scores and lower than average math achievement, most
of them also had severe psychological difficulties in addition to their learning disability.

Another difficulty in researching arithmetic disability has been the fact that most studies of adults with learning disabilities failed to separate participants with solely arithmetic disability from those who had other types of disabilities as well. Furthermore, the methods used to identify the participants in studies are based on a variety of arithmetic screening tests, some of which may not be adequate.

Another frequently used mode of identifying people with arithmetic learning disability is based on average intelligence scores but discrepant achievement scores. Siegel and Linder (1984) and Siegel and Ryan (1984, 1988, 1989) categorized children with learning disability based on their academic performance on the Wide Range Achievement Test (WRAT). Their operational definition of arithmetic disability has been adopted in the present study. It incorporates intelligence scores in the average range; arithmetic achievement scores equal to or below the 25th percentile on the Arithmetic subtest and scores equal to or above the 30th percentile on the Reading subtest of the WRAT.

Although there are different views on definitional issues of arithmetic disability in the literature, a number of studies (e.g., Saito, 1992; Yaghoubzadeh, Geva, & Siegel, 1996) found supporting evidence for the above classification suggested by Siegel and Linder (1984). This form of classification of learning disabilities was also validated with a group of adults by Shafrir and Siegel (1994). Therefore, this particular operational definition of arithmetic disability was chosen to select participants for this study.
Literature Review

Poor arithmetic achievement has been correlated with a number of factors. Among these variables one finds reference to computation (e.g., Fleichner, Garnett, & Sheperd, 1980; Geary & Brown, 1991; Russell & Ginsburg, 1984), attitude (e.g., Bandura & Schunk, 1981; Dreger & Aiken, 1957; Metrick, 1987), and reasoning in the domain of numeracy (e.g., Garofalo & Lester, 1985; Polya, 1984; Stevens, 1932).

Computation

Computational skills refer to the ability to add, subtract, multiply, and divide. General knowledge of measurement, interpretation of tables and graphs, the meaning of fractions and percentages as well as probability and statistics all involve computation and are crucial to comprehend the vast amount of numerical information in today's society. Thus the assessment of these areas is essential.

In a study of computational skills, Geary and Brown (1991) investigated the speed of processing and strategy choice of elementary school children. They recorded strategies and solution times of 40 simple addition problems completed by gifted, normal, and mathematically disabled third- or fourth grade children. The participants' rate of verbal counting was also measured. Their findings suggest that the long-term memory organization of basic facts influences differences in computation performed by gifted, normally achieving, and mathematically disabled children. They reported that their gifted group had a verbal counting rate that was at an adult level and concluded that mastery of early numerical skills was a significant contributor to higher achievement on
solving addition problems. Evidence found by Fleischner, Garnett, and Sheperd (1980) also illustrated the slow and inaccurate processing of arithmetic questions exhibited by children with Arithmetic Disability. Russell and Ginsburg (1984) found converging evidence for the deficient computational skills of children with Arithmetic Disability compared with their age matched comparison group. They also illustrated that students with mathematics difficulty are similar to their younger peers in terms of multi-digit adding and subtracting.

In the last two decades, there has been an increasing interest in the investigation of the error production of individuals with different learning abilities. Most studies, however, have focused on children only. Cox (1975) was interested in identifying the error patterns of children on addition and subtraction questions of whole numbers as well as remediating such problems. She argued that the identification of systematic errors, "errors in at least three out of the five problems on a particular skill" (1975, p. 154) is critical because systematic errors can be remediated. Engelhardt (1977) used a qualitative method to analyze the computational errors of third- and sixth-grade students. Of the eight clusters of error types reported in that study, defective algorithm (applying the wrong procedure to solve a question) differentiated most students with low competency in math from the other participants. Strang and Rourke (1985) analyzed the errors of children with learning disability on the Arithmetic subtest of the WRAT. Their findings suggested that participants with specific arithmetic disability produced a larger number and wider variety of errors than participants with other types of learning disabilities.

Saito (1992) reported findings of error analysis of the arithmetic subtest of the WRAT-R. Her work explored the developmental lag hypothesis in arithmetic errors of children with Arithmetic Disability. She noted quantitative as well as qualitative differences of errors between children with Arithmetic
Disability and the normally achieving comparison groups. Szanto and Siegel (1994) also examined the developmental lag hypothesis by conducting an error analyses on the arithmetic subtest of the WRAT-R for three groups: 1) adults with arithmetic disability 2) adults matched on chronological age without arithmetic disability, and 3) adolescents matched on arithmetic level. The findings suggested that when making errors on the WRAT-R, the arithmetic disabled adults resembled the arithmetic level matched adolescents not the age matched comparison group, in terms of working with: 1) fractions, 2) multiplying three digit numbers, 3) percentage questions, 4) decimals, 5) algebraic questions. Adults with Arithmetic Disability of that study did not experience difficulties in adding, subtracting, multiplying, or dividing numbers having less than four digits. Most adults with Arithmetic Disability could also add and multiply common fractions such as 1/2 or 1/6.

The studies reviewed above provide evidence that the assessment of computational skills is paramount in the identification and remediation of individuals with learning disability. Most studies have concentrated on identifying and describing children with arithmetic disability and only one (Szanto & Siegel, 1994) investigated the computational skills of adults with arithmetic disability. In general, the literature reviewed thus far, reflected a major emphasis on arithmetic computation of adults, usually at the expense of investigating the individuals attitude toward working with numbers and reasoning skills. The following section of this literature review will describe studies on the relationship between attitude toward mathematics and performance in mathematics.
Attitude Toward Math

Most daily human activities are colored with emotion. Excitement, joy, fear, and anxiety are just some of the feelings one can experience in the course of a day. As a number of investigators argue (e.g., Aschcraft & Faust, 1994; Lazarus, 1984; Oatley, 1992), the experience of intense emotion can significantly influence one's cognitive functioning. An illustration of such an intense emotion which influences one's performance in the domain of numeracy is math anxiety. Math anxiety can be defined as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551). In math anxiety there is an interaction of cognitive and emotional processes (Cooper & Robinson, 1991; Dreger & Aiken, 1957; Frary & Ling, 1983;). The interaction between cognitive and emotional processes suggests the importance of studies examining math anxiety in arithmetic learning disability.

One of the earliest studies of math anxiety was conducted by Dreger and Aiken (1957). They examined the issue of anxiety associated with numbers in a college population and found that it related to achievement but not to intelligence levels of the participants. They also presented math anxiety as a different construct from general anxiety based on low correlations of a general anxiety scale scores with individual items from the number anxiety scale, results of cluster analysis which separated number anxiety from general anxiety, and significant difference in galvanic skin resistance between participants with general anxiety and number anxiety.

A study conducted by Adams and Holcomb (1986) found a negative correlation between anxiety and math performance among 92 students in an upper level college statistics course. Attitudes regarding arithmetic were
measured with the help of questionnaires in a study conducted by Frary and Ling (1983) on a population of 491 university students. They also found a negative correlation between anxiety and achievement as well as avoidance of math.

Lazarus (1973), based on personal observation and anecdotal evidence, documented the importance of investigating math anxiety. He reported on an extreme form of math anxiety, math phobia, which he defined as an "irrational and imperative dread of mathematics" (p. 16). He emphasized the need for more attention to this condition among adults. In a study conducted by Quilter and Harper (1988), 147 college educated adults reported their feelings related to mathematics. A randomly selected sub-group of 15, who reported negative attitudes toward math, also offered detailed information about their past and present difficulties related to numeracy in an interview. The participants of this study indicated three major areas that blocked their learning in this subject: conceptual problems, affective connotations, and their learning environment.

One's attitude or opinion can be inferred not only from emotions associated with the subject but also from reported self-confidence rating. Educational psychologists have been interested in the relationship of self-efficacy — one's personal judgment of capability to perform in a given area — on achievement (Bandura, 1989; Vadhan & Stander, 1993). Bandura and Schunk (1981) found a positive relationship between mathematical self-efficacy and performance among elementary school children. Schunk (1985) examined the role of self-efficacy during classroom learning and stressed the significance of reward contingencies in improving both self-efficacy and achievement in academic learning. The role of confidence regarding math was also investigated by Reyes (1984). This study offered suggestions for success in interpreting numerical information by emphasizing not only content knowledge but also a positive attitude regarding math. The impact of self-efficacy in the domain of
math was also examined by Cooper and Robinson (1991). The level of self-efficacy was assessed by asking 290 undergraduate students of the applied sciences to indicate their confidence in their ability to complete advanced math courses. These investigators suggested that by altering one's performance expectations, anxiety can be lowered and performance in math increased.

Although studying mathematics is believed to be largely concerned with cognitive factors, the research cited above indicates that the individual's feelings and self-confidence related to the subject are also regarded as worthy of assessment. There are both practical and emotional problems that may result from arithmetic disabilities in adults. Routine tasks such as planning purchases, balancing checkbooks, or calculating the amount of a tip one should leave at a restaurant, all become difficult, if not impossible, to perform accurately. Metrick (1987) illustrated with a case study that "By adolescence, emotional issues are often thoroughly entangled with math learning problems, making it critical to address both" (p. 75). A negative attitude toward math could constitute increased levels of frustration with numbers, test anxiety, and in extreme cases, math phobia (Hagen, Kamberelis, & Segal, 1991). However, if math anxiety can be reduced, then better math performance may result (Dodd, 1992). This might lead to greater availability of career opportunities for adults with arithmetic disability. Unfortunately, the literature does not include studies that examined the math attitude of an adult population with learning disability. It is argued that data related to attitudes toward arithmetic would be valuable for designing remediation programs in math for adults with arithmetic disability.
Reasoning In Math

As a number of researchers have indicated, there is a need for empirical investigations in the area of arithmetic disability among adults (Garnett, 1987; Morrison & Siegel, 1991). In order to aid those individuals who have problems in the domain of numeracy, measurements of competency in this area should not be solely product-oriented. More specifically, assessment of areas other than computation also need to be explored. Reasoning skills in arithmetic is a factor that merits greater attention in the assessment of arithmetic disability.

In one of the earliest studies of reasoning in math, Stevens (1932) investigated the correlation among silent reading, ability in basic arithmetic, general intelligence, and solving reasoning problems of the Stanford Arithmetic Test. A significant positive relationship was found between basic arithmetic and reasoning skills in arithmetic problem solving in that study. However, arithmetic problem solving was not significantly related to reading ability or intelligence.

In a review, Garofalo and Lester (1985) outlined the importance of metacognition and metacognitive monitoring in mathematical performance. They employed Flavell’s (1976) definition of metacognition: "one's knowledge and active monitoring of one's own cognitive processes and products" (p. 232). These investigators concluded that the systematic study of one's explicit knowledge of problem solving in math needs to be examined in order to learn more about the regulatory behaviors of problem solving in this domain. Meltzer (1989) examined the characteristics of high school students with learning disability in the context of problem solving and metacognitive strategies. Her findings indicated that students with learning disability used different problem solving strategies on a written problem solving task than normally achieving
students of the same age. These findings, however, cannot be generalized to the current investigation because it is not clear whether or not they had tested participants with specific arithmetic disability.

Lampert (1990) and Lampert, Rittenhouse, and Crumbaugh (1994) documented a style of teaching mathematics that has the potential to enhance the abilities of students to communicate in the domain of numeracy. They encouraged students to openly discuss how they approached problems in class. The students, like most others in a math class, started by answering questions about how they figured out an answer with phrases such as 'I just know' or 'I just thought it' or 'I don't know how to figure it out' but by modeling and encouragement they soon discussed their own way of thinking about a problem and obtaining an answer. The emphasis of these discussions was not whether the answer was correct or not. This teaching method emphasized, instead, the enrichment of vocabulary of the students, so they would have 'the words' to talk about what mental processes led them to a particular answer. Thus, their teaching routinely included modeling of thinking about thinking in the domain of numeracy. Although the effect of this method was not empirically investigated, an increase of mathematical discussion among the students was reported. They exchanged hypotheses and strategies related to problem solving more frequently. Unfortunately, this method of teaching was not implemented with special populations.

A growing list of researchers maintain that a major contributor to the pattern of poor performance in mathematics is the over-emphasis on correct answers rather than on cognitive processes such as reasoning skills (Firestone, 1990; Lampert, 1990; Polya, 1984; Skemp, 1987). Contrary to the notion that all mathematics produces exact right answers, mathematics seems to be a process of organizing information into categories (Tobias, 1984). In fact, getting the right
answer to a problem does not necessarily mean that one has grasped the full significance of that problem. The level of understanding of the problem-solver can be inferred from the reasoning or strategies the individual chose to employ (Dodd, 1992). Therefore, the individual's reasoning skills in the domain of numeracy were chosen to be an integral part of the assessment of arithmetic disability in this study.

Design

C.A.R.: A model of assessment

Task selection for the present multidimensional investigation was guided by previous empirical research (Christopher et al., 1989; Connelly, 1985; Slate, Jones, Graham, & Bower, 1994) and theory development (Siegel, 1988b, 1989) in the psychology of learning disability. In this study, numeracy was considered a vehicle of effective communication in the 20th century. Numeracy as a vehicle of the information highway shares some functional characteristics with the most common vehicle of the 20th century: the car. The three letters in the word 'car' can be associated with three integral parts of numeracy: C = computation, A = attitude, and R = reasoning (see Figure 1).

Exploring the same analogy, the following similarities emerged. Just like using a well functioning car, using well developed numeracy skills can speed up movement and help one to carry the load one needs to transport. Much like a car, numeracy can be seen as useful means to get from point A to point B on the numerically overloaded information highway. A car has different parts that function together and if an integral part is damaged most other parts are also affected. Similarly, a breakdown in computation (Geary & Brown, 1991; Russell
& Ginsburg, 1984), a negative attitude (Adams & Holcomb, 1986; Bandura & Schunk, 1981), faulty reasoning (Garofalo & Lester, 1985; Littlefield & Rieser, 1993) or a combination of any of these factors can and do hinder one's ability to understand and interpret numerical information. The extent of problems with the integral parts of numeracy may warrant a diagnosis in the domain of learning disability, called arithmetic disability. It is argued that the above described assessment model, C.A.R., is useful in studying adults with AD because it can aid in identifying the particular source or sources of difficulty and thus have direct application for remediation.

Accordingly, in this study, the following three major areas related to numeracy abilities were investigated: computational skills, attitude, and reasoning. Computational skills of the participants were assessed both in the written as well as the oral form. Siegel (1988b), stressed the significance of examining error patterns of individuals with learning disabilities and pointed to the lack of such investigations in the literature. This critical aspect of the examination of differences of error patterns was addressed by an error analysis on the responses of the participants to the written computation task. As Morrison and Siegel (1991) noted "... errors are thought to be especially informative in revealing factors relevant to the psychological and learning processes underlying performance" (p. 200). Thus, differences in error patterns on the measures administered to the subjects were considered indicators of differences in information processing. Attitudes towards math were measured by a questionnaire in which subjects reported their past experiences related to numeracy and their self-confidence rating regarding solving math problems. The level of reasoning ability was inferred from the problem solving skills of the subjects as well as from the strategies they reported.
Arithmetic disability: Delay or deficit?

The current study examined the developmental lag versus deficit issue in the domain of numeracy. This design is frequently used in the reading disability literature with children (e.g., Bradley & Bryant, 1978; Felton & Wood, 1992; Guthrie, 1973; Olson, Wise, Conners, Rack, & Fulker, 1989; Stanovich, Nathan, & Vala-Rossi, 1986; Stanovich, Nathan, & Zolman, 1988; Stanovich & Siegel, 1994). The question of delay or deviance in the study of learning disability concerns whether a group of individuals with learning disability display similar or different cognitive patterns to a group of younger individuals without learning disability, who function at the same achievement level. Although developmental-lag models have rarely been used in studies of arithmetic, it is argued that this type of model is helpful as a framework to understand learning disability in the area of not only literacy but also numeracy. One way to address the question of whether adults with arithmetic disability are characterized by differences in developmental sequence is by examining cognitive profiles of individuals in an arithmetic-level matched design. If the examined cognitive skills are similar for the adults with arithmetic disability and for the children without disabilities who are at the same arithmetic level, then the developmental-lag hypothesis is supported. However, significant differences between the performance of the two groups would be considered evidence for a deficit, signaling different developmental paths (Guthrie, 1973; Kulak, 1993; Stanovich & Siegel, 1994).

The same analogy as utilized in the reading disability literature was applied in the present study. In this arithmetic level match design an older group of participants with arithmetic disability was compared with two comparison groups: a group of Normally achieving Chronological-Age matched
adults group (NCA) and a younger group of Normally achieving participants who were matched on Arithmetic Level (NAL) with the group of adults with Arithmetic Disability. This design provided a method to investigate whether adults with Arithmetic Disability experience a delay or deficit in terms of understanding and application of arithmetic.

Research Questions

Interestingly, no comparable study has included comparisons of adults with arithmetic disability with chronological-age matched adults and arithmetic level matched children without learning disability. Furthermore, no previous study using the arithmetic level matched design examined a wide range of concepts related to numeracy, such as computation, attitude and reasoning in an adult population with arithmetic disability.

The purpose of this research was to characterize arithmetic learning disability in an adult population in terms of computation, attitude, and reasoning. There were two main research questions, both, designed to enrich the documented knowledge of arithmetic learning disability.

1. How do adults with and without arithmetic disability differ in computation skills and attitude toward numeracy? What are the processes involved in math problem solving for adults with arithmetic learning disability; what is unique about the information-processing characteristics of these individuals when compared to their cohort group?

2. Do adults with arithmetic disability and children who perform at the same arithmetic level without arithmetic disability differ in computation and problem solving skills and attitude toward numeracy? More precisely, is
there evidence of deficit or delay in the three integral parts of numeracy: computation, attitude, and reasoning.

Hypotheses

The hypotheses were formulated based on the findings of the limited literature on arithmetic disability of adults and based on studies using the developmental-lag framework in the reading disability literature.

1. It was hypothesized that adults with arithmetic disability progress through similar stages of cognitive development as normally achieving adults of the same age, but would differ in their rate of development and the level of cognitive development reached. Quantitative differences were expected in terms of the computational mistakes of the group of adults with arithmetic disability and the adult comparison group. More precisely, it was hypothesized that computation would cause difficulties to both adults with arithmetic disability and normally achieving adults, but the adults with arithmetic disability were expected to produce a greater number of errors. Furthermore, adults with arithmetic learning disability were expected to exhibit deficient reasoning skills, have less effective strategies for problem solving, and have a negative attitude toward math.

2. It was hypothesized that the patterns of cognitive skills exhibited by adults with arithmetic disability as a group would be similar to those exhibited by the arithmetic level comparison group on tests of computation and problem solving. Similar error patterns of the adults with arithmetic disability and the normally achieving arithmetic level matched groups would be taken as indication of a developmental delay of adults with arithmetic disability.
The pattern of errors on the written arithmetic test exhibited by adults with arithmetic disability as a group was not expected to differ qualitatively or quantitatively from those displayed by the arithmetic level comparison group. That is, the adults with arithmetic disability and the normally achieving arithmetic level matched groups were expected to make similar types of errors (no qualitative difference) as well as similar number of errors (no quantitative difference). The difference between these two groups regarding their attitude toward math was expected to be significant. That is, math anxiety was hypothesized to be an important factor for adults with arithmetic disability but not for the participants in the arithmetic level matched comparison group.
CHAPTER 2

METHOD

Participants

Participants in this study were selected from volunteers of a research project at the Ontario Institute for Studies in Education at the University of Toronto. An estimate of the participants' level of intelligence was measured by an abbreviated version of the Wechsler Adult Intelligence Scale-Revised (WAIS-R; Wechsler, 1981) or Wechsler Intelligence Scale for Children (WISC-III; Wechsler, 1994), depending on the age of the individual. To obtain an estimated IQ score, participants were administered the Block Design and Vocabulary subtests of the WAIS-R or WISC-III. The Block Design, a timed subtest which is part of the Performance Scale of the Wechsler Intelligence tests. Participants are given white and red blocks to reproduce designs that are drawn. The Vocabulary subtest is part of the Verbal Scale of the Wechsler Intelligence tests. Here participants are asked to define high and low frequency words. Individuals, who obtained an estimated IQ score (Tellegen & Briggs, 1967) of 80 or higher, were included in the present study.

The Wide Range Achievement Test-3 (WRAT-3, Wilkinson, 1993) is one of the most widely used instruments to estimate academic achievement. All three subtests, reading single words, spelling single words, and solving written arithmetic problems, were administered to each subject. To rule out the interference of reading disabilities and other challenges to problem solving in
only those participants who scored at or above the 30th percentile on the Reading subtest of the WRAT-3 were included in the present study. To be considered arithmetic disabled, an individual was required to have a score less than or equal to the 25th percentile on the Arithmetic subtest of the WRAT-3. To be included in the normally achieving group, the participants were required to have a score more than or equal to the 30th percentile on all three subtests of the WRAT-3.

Participants were in three groups. The target group included adults with arithmetic disability (n=24: 12 females, 12 males). There were two comparison groups in this study, one matched for age and one matched for arithmetic grade level. The chronological age level matched group included normally achieving adults (n=24: 12 females, 12 males). To be included in the normally achieving arithmetic level match group (n=21: 14 females, 7 males), children were required to perform at the same level on the arithmetic subtest of the WRAT-3 as members of the group of adults with arithmetic disability. The children selected based on the above criteria were in grades five or six at the time of the data collection phase of this study.

Participants did not report physical or emotional disturbances. The background information (estimated IQ, achievement scores, and age) of the three groups is presented in Table 1.

Tasks

In the present study, samples of computational arithmetic, attitude toward mathematics, and reasoning skills in the domain of mathematics were obtained from the participants.
Table 1

Background Characteristics of the Arithmetic Disabled Adults, Children Matched for Arithmetic Level, and Adults Matched for Age Level

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Arithmetic Disabled Adults (n=24)</th>
<th>Children Matched for Arithmetic Level (n=21)</th>
<th>Adults Matched for Chronological Age Level (n=24)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (yrs)</td>
<td>30.88\textsuperscript{a} (11.81)</td>
<td>10.95\textsuperscript{b} (1.02)</td>
<td>27.67\textsuperscript{a} (7.40)</td>
<td>.001</td>
</tr>
<tr>
<td>range</td>
<td>18-62</td>
<td>9-12</td>
<td>18-44</td>
<td></td>
</tr>
<tr>
<td>Estimated IQ</td>
<td>104.43 (7.57)</td>
<td>109.38 (12.95)</td>
<td>109.17 (9.12)</td>
<td>ns</td>
</tr>
<tr>
<td>range</td>
<td>91-117</td>
<td>91-129</td>
<td>91-122</td>
<td></td>
</tr>
<tr>
<td>Reading\textsuperscript{*}</td>
<td>70.75 (13.15)</td>
<td>75.24 (19.33)</td>
<td>76.67 (13.24)</td>
<td>ns</td>
</tr>
<tr>
<td>range</td>
<td>50-88</td>
<td>39-99</td>
<td>39-94</td>
<td></td>
</tr>
<tr>
<td>Spelling\textsuperscript{*}</td>
<td>43.67\textsuperscript{a} (25.20)</td>
<td>69.47\textsuperscript{b} (23.25)</td>
<td>65.79\textsuperscript{b} (19.53)</td>
<td>.001</td>
</tr>
<tr>
<td>range</td>
<td>3-86</td>
<td>30-98</td>
<td>30-95</td>
<td></td>
</tr>
<tr>
<td>Arithmetic\textsuperscript{*}</td>
<td>14.96\textsuperscript{a} (6.73)</td>
<td>67.57\textsuperscript{b} (17.94)</td>
<td>61.71\textsuperscript{b} (20.18)</td>
<td>.001</td>
</tr>
<tr>
<td>range</td>
<td>4-23</td>
<td>42-99</td>
<td>32-97</td>
<td></td>
</tr>
</tbody>
</table>

Notes. 1) * percentile scores; 2) means with different superscripts are significantly different from each other.
Computational Arithmetic

Computational skills were measured in three ways. Participants were presented with a time limited written computational test: the arithmetic subscale of the WRAT-3, the Interpreting Data subtest of the Key-Math-R, a mental computational test without time limit, and a written problem solving test without time limit. The following is a detailed description of these instruments.

The arithmetic subtest of the WRAT-3 is a written test of computational skills. It includes 40 questions; the participants are required to complete as many questions as possible in the time limit of 15 minutes. There are 7 types of questions on this test:

- addition (Q#: 1,2,6,10,12 n=5),
- subtraction (Q#: 3,4,5,7,14,19 n=6),
- multiplication (Q#: 8,9,13,15,22,23,32 n=7),
- division (Q#: 11,17,20 n=3) of real numbers,
- fractions (Q#: 18,21,24,25,26,27,31,33,36,37 n=10),
- decimal numbers (Q#: 16,28,35 n=3), and
- algebraic equations (Q#: 29,30,34,38,39,40 n=6).

In general, items on this test become progressively more difficult to answer. In addition to scoring the participants' responses according to the guidelines provided by Wilkinson (1993), responses were also coded so the pattern of errors could be studied. A summary of the four-level decision tree of possible responses to the Arithmetic questions on the WRAT-3 are presented in Figure 2.

**Level 1** In general, a response on this arithmetic test either did or did not receive credit. A point was awarded when the answer was correct. However, if credit was not given, the response was analyzed further, and placed into a Level 2 category.

**Level 2** Responses that did not receive credit were examined and placed in one of the following two mutually exclusive categories:
Figure 2. Flowchart of the four-level error analyses of the Arithmetic subtest of the WRAT-3.

Response

LEVEL1
- Credit
- No Credit

LEVEL2
- Blank
- Error

LEVEL3
- Miscalculation
- Procedure
- Combination

LEVEL4
- Number fact
- Wrong operation
- Precision
- Faulty operation
- Guess
1) Blank: there was no evidence of an attempt to answer the question.

2) Error: the question had an incorrect solution e.g.,
   Q#36: answer = 4.5 instead of -3.5,
   Q#8: answer = 2 instead of 8.

**Level 3** For the purpose of the error analysis this previous category, where the question had an incorrect solution, was examined. Three categories were formed: miscalculation, procedural problems, and the combination of miscalculation and procedural problems. Essentially, an erroneous response can be due to miscalculation e.g.,
   Q#1: answer = 2 instead of 3;
   Q#29: answer = 21 instead of 21.2.

or procedural problems e.g.,
   Q#32: answer = 4 instead of 0.45,
   Q#31: answer = 52.5 instead of 0.525.

These two categories, however, were not mutually exclusive. Some of the errors therefore were classified into a third group where a combination of both miscalculation and procedural problems were apparent e.g.,
   Q#31: answer = 52.01 instead of 0.525,
   Q#28: answer = 37.20 instead of 245.

**Level 4** The above mentioned two main groups of errors, miscalculation and procedural problems, were subdivided as follows:

**Miscalculations** were of two kinds:

1) Number fact error: the correct operation was applied, however, there was a computational mistake e.g.,
Q#20:  answer = 158  instead of 161R2;  
Q#16:  answer = 61.01  instead of 57.01.

2) Precision error: the correct operation was applied, and there was no evidence of a number fact error, however, the answer was not exact, e.g.,
Q#20:  answer = 161  instead of 161R2;  
Q#33:  answer = 37%  instead of 37.5%.

Procedural problems were further divided into three categories:

1) Wrong operation: the wrong operation was applied, e.g.,
Q#37:  answer = $9 \frac{7}{8}$  instead of $23 \frac{1}{4}$;  
Q#32:  answer = 4  instead of 45.

2) Faulty operation: The appropriate operation was applied in the wrong way, e.g.,
Q#37:  answer = $18 \frac{7}{8}$  instead of $23 \frac{1}{4}$.  
Q#14:  answer = 23  instead of 17.

3) Guess: There was no evidence of steps in calculating the answer, yet there was a response that did not fit any of the above categories, e.g.,
Q#37:  answer = 20  instead of $23 \frac{1}{4}$;  
Q#23:  answer = 3%  instead of 75.
Responses to the Arithmetic questions on the WRAT-3 were coded according to this four-level analyses so the pattern of errors of the three groups of participants could be studied. The reliability of this error classification was conducted on a subsample of 50 answers by a scorer who was unaware of the status of the participants. A reliability score was calculated by dividing the number of agreements by the total number of items and multiplying the value by 100. The resulting calculation indicated the overall interrater reliability to be 88%. The Kappa correlation (Cohen, 1960) between the two sets of scores was also calculated. The reliability index obtained through this method also indicated a high degree of inter scorer reliability ($r=.85, p<.001$). Disagreements were resolved by discussion.

The 'Interpreting Data' subtest of the application section of the Canadian edition of KeyMath Revised: A Diagnostic Inventory of Essential Mathematics (Key-Math-R) (Connolly, 1991), is an individually administered oral achievement test. It measures one's understanding and application of mathematical concepts and skills. In the 'Interpreting Data' subtest, the participants were exposed to six questions in each of three domains. In the first domain the items related to the interpretation of figures and tables. For example, "This chart shows the animals entered in a pet show. In all, how many rabbits and fish were in the show?". Items in the second domain required subjects to interpret graphs. For example, "This graph shows how many cars and trucks were sold by companies A, B, and C in the month of April. In all, how many trucks did these companies sell?" The third and most abstract domain tested the subjects' skills in probability and statistics. For example, "The six sides of this block are numbered one, two, three, four, five, and six. What is the chance, expressed as a fraction, that on one toss an even number will be rolled?" The test items are accompanied by a colored picture representation of the text. The items
are read to the subjects who are then required to answer verbally without a time limit.

An arithmetic written problem-solving test was constructed for this study, consisting of six word-problems (see appendix A). Word-problems were selected from the Key-Math-R (Connolly, 1991). The six word-problems that were selected required the manipulation of numbers and an understanding of the following mathematical concepts: addition, subtraction, multiplication, division. Participants were allowed to work on the six word problems for as long as they needed up to 60 minutes. In terms of categorizing the answers, the focus was on the process the subjects used as well as on whether the answer was correct. More precisely, partial marks were awarded for partially correct answers. Participants could receive 0, 1, or 2 points for each of the six questions. Two points were awarded for accurate answers. One point was awarded for an inaccurate answer that contained the correct procedure but was inaccurate due to computational error. Inaccurate answers which contained a procedural error were awarded zero points. Appendix B, illustrated with examples describes the evaluation process of the accuracy of the participants' answers to the word problems.

**Attitude and Confidence Regarding Mathematics**

Attitude toward mathematics was measured by the Mathematics Anxiety subscale of the Fennema-Sherman Mathematics Scale (1976). This particular self-report measures the feeling of anxiety related to mathematics. The scale includes six positively stated and six negatively stated sentences. Split-half reliability was reported to be .89. Participants were required to indicate whether they agree or disagree with these statements on a five point likert-type scale.
In order to include in the assessment a rating of the subjects' level of confidence regarding mathematics a method discussed by Lucangeli, Galderisi, and Cornoldi (1995) was used. Participants were required to read the word-problems and prior to their attempt to solve the problems they were asked to rate on a five point scale how confident they were about being able to solve the word-problem correctly. The scale ranged from "I am absolutely sure that I will solve this problem correctly" to "I know that I will not be able to solve this problem". After solving each word-problem, they were asked to indicate how confident they were that their solution to the word-problem was correct. The possible answers on this scale ranged from "I am absolutely sure that I have done it correctly" to "I know that I got it wrong" (see appendix A).

**Reasoning Skills in Math**

In addition to studying the pattern of errors, the cognitive process of error-production was also investigated. In order to determine how participants reasoned to reach their answers, they were asked to provide a detailed written description of their solution following the completion of the word-problems. Their strategy used to solve each problem was classified by the experimenter based on whether the procedures used were explained fully, partially with minor flaws, partially with major flaws, or not at all (see Table 2).

After the description of the strategies used to solve the word problems, participants were required to write down what errors other people may make in solving questions of that kind. The responses were classified according to the type of error described. Participants who responded to this question described mistakes related to computation, procedures, attitude, and understanding of the word problem (see Appendix C).
Table 2. Examples of scoring of the strategies applied to the Problem solving task

**Instructions:** Reflect on how you have solved the problem, and describe as best as you can the strategies you used to solve the problem.

**Question:** A truck goes 3.1 kilometers on a liter of diesel fuel. How many liters will it need to travel the 960 kilometers to Regina?

<table>
<thead>
<tr>
<th>Description</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fully explained procedures used</strong></td>
<td>3</td>
</tr>
<tr>
<td>'I &quot;sifted&quot; out the pertinent information. I imagined a liter of fuel lasting a 3.1 km piece of the Regina trip. I wanted to know how many &quot;pieces&quot; would be needed to make up the trip to Regina, i.e. I divided the trip into pieces.'</td>
<td></td>
</tr>
<tr>
<td>'Tried to divide the 960 kms by 3.1 liters to see the amount of fuel it will need for the trip'</td>
<td></td>
</tr>
<tr>
<td>'Regina has nothing to do with it, divide # kms by 3.1 L'</td>
<td></td>
</tr>
<tr>
<td>'Write down known information, write down relationship between quantities, solve, and then do division: 960:3.1'</td>
<td></td>
</tr>
<tr>
<td><strong>Incomplete explanation or minor errors in reasoning</strong></td>
<td>2</td>
</tr>
<tr>
<td>'Divided 960 by 3'</td>
<td></td>
</tr>
<tr>
<td>'Just divided and divided'</td>
<td></td>
</tr>
<tr>
<td>'I used a formula to solve the problem'</td>
<td></td>
</tr>
<tr>
<td>'I tried to write it as a math question, break down question to make sense of it'</td>
<td></td>
</tr>
<tr>
<td><strong>Exhibited major flaws in reasoning used to select procedures</strong></td>
<td>1</td>
</tr>
<tr>
<td>'Multiply the liter of gas by kilometer'</td>
<td></td>
</tr>
<tr>
<td>'I read the problem and tried to solve it, I multiplied it'</td>
<td></td>
</tr>
<tr>
<td>'I tried to divide 3.1 by 960 moving the decimal over one'</td>
<td></td>
</tr>
<tr>
<td>'Dividing the kilometers of fuel by the mileage'</td>
<td></td>
</tr>
<tr>
<td><strong>Did not explain procedures used</strong></td>
<td>0</td>
</tr>
<tr>
<td>'thinking of steps'</td>
<td></td>
</tr>
<tr>
<td>'I don't know'</td>
<td></td>
</tr>
<tr>
<td>blank</td>
<td></td>
</tr>
</tbody>
</table>
The reliability on the above described scoring was conducted on a subsample of 36 answers by a scorer who was unaware of the status of the participants. A reliability score was calculated by dividing the number of agreements by the total number of items (36) and multiplying the value by 100. The resulting calculation indicated the overall interrater reliability to be 89% for the description of strategies and 90.8% for the 'error production of others' question. Inconsistencies were resolved by discussion.

The Kappa correlation between the two sets of scores was also calculated. The reliability index obtained through this method also indicated a high degree of inter scorer reliability both for the description of strategies ($r=.85, p<.001$) as well as the 'error production of others' question ($r=.87, p<.001$).

Procedure

Each subject was administered the same assessment battery individually and in a quiet room. The entire battery was administered in one session, which lasted two to three hours. The tests were administered in the following order. First, Vocabulary and Block Design, the two subscales of the Wechsler Subscales were administered. Next, participants completed the Math Anxiety questionnaire. This was followed by the three subtests of the WRAT-3: Reading, Spelling, and Arithmetic. After completing the Interpreting Data subtest of the Key-Math-R, the participants were offered to take a break. Testing resumed after approximately 15 minutes with the word problem solving task.
CHAPTER 3

RESULTS

Computational Skills

The analyses on the three tests of computation: the arithmetic subtest of the WRAT3, the Interpreting Data subtest of the Key-Math-R, and the problem solving exercise will be presented for each of these three tests separately.

A one-way ANOVA revealed (see Table 3) that there was a significant difference among the three groups on the mean percentile scores of the Arithmetic subtest of the WRAT-3 ($F(2,66) = 75.83, p < .001$). To investigate specific group differences, the Scheffé test was used in all subsequent analyses of the present study. As expected this post-hoc analysis indicated that the AD group scored significantly lower than the two comparison groups ($p < .05$). The difference in the mean scores between the two comparison groups was not statistically significant. This finding is obviously a consequence of the definition used and validates it.

A one-way ANOVA revealed that there was a significant difference among the three groups on the mean percentile scores of the spelling subtest of the WRAT-3 ($F(2,66) = 8.53, p < .001$). Post-hoc analyses indicated that the AD group scored significantly lower than the two comparison groups ($p < .05$). The difference in the mean scores between the two comparison groups was not statistically significant. A one-way ANOVA revealed that there was no significant difference among the three groups on the mean percentile scores of the reading subtest of the WRAT-3 ($F(2,66) = 1.61, p < .21$). (The means and
Table 3

Percentile means and standard deviations on the WRAT-3 for the Arithmetic Disabled Adults, Children Matched for Arithmetic Level, and Adults Matched for Age Level

<table>
<thead>
<tr>
<th>Arithmetic Disabled Adults (n=24)</th>
<th>Children Matched for Arithmetic Level (n=21)</th>
<th>Adults Matched for Chronological Age Level (n=24)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} ) (SD)</td>
<td>( \bar{x} ) (SD)</td>
<td>( \bar{x} ) (SD)</td>
<td></td>
</tr>
<tr>
<td>Arithmetic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(percentiles)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.95(^a) (6.73)</td>
<td>67.57(^b) (17.93)</td>
<td>61.71(^b) (20.18)</td>
<td>.001</td>
</tr>
<tr>
<td>Error Analyses (percent scores)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.62(^a) (5.12)</td>
<td>48.69(^a) (6.40)</td>
<td>72.39(^b) (9.71)</td>
<td>.001</td>
</tr>
<tr>
<td>No Credit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>49.37(^a) (5.12)</td>
<td>51.30(^a) (6.40)</td>
<td>27.60(^b) (9.71)</td>
<td>.001</td>
</tr>
</tbody>
</table>

**Note.** Means with different superscripts are significantly different from each other.
<table>
<thead>
<tr>
<th></th>
<th>Arithmetic Disabled Adults</th>
<th>Children Matched for Arithmetic Level</th>
<th>Adults Matched for Chronological Age Level</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{x}$ (SD)</td>
<td>$\bar{x}$ (SD)</td>
<td>$\bar{x}$ (SD)</td>
<td></td>
</tr>
<tr>
<td>Incorrect</td>
<td>36.27&lt;sup&gt;a&lt;/sup&gt; (17.66)</td>
<td>38.34&lt;sup&gt;a&lt;/sup&gt; (25.57)</td>
<td>65.30&lt;sup&gt;b&lt;/sup&gt; (20.90)</td>
<td>.001</td>
</tr>
<tr>
<td>Omission</td>
<td>63.72&lt;sup&gt;a&lt;/sup&gt; (17.67)</td>
<td>61.65&lt;sup&gt;a&lt;/sup&gt; (25.57)</td>
<td>34.69&lt;sup&gt;b&lt;/sup&gt; (20.90)</td>
<td>.001</td>
</tr>
<tr>
<td>Error Analyses</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Level 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miscalculation</td>
<td>45.49 (19.73)</td>
<td>31.16 (27.36)</td>
<td>43.57 (20.41)</td>
<td>ns</td>
</tr>
<tr>
<td>Procedural Error</td>
<td>51.86 (19.43)</td>
<td>60.29 (26.13)</td>
<td>56.04 (20.53)</td>
<td>ns</td>
</tr>
<tr>
<td>Combination</td>
<td>2.53&lt;sup&gt;a&lt;/sup&gt; (8.49)</td>
<td>8.53&lt;sup&gt;b&lt;/sup&gt; (10.41)</td>
<td>0.38&lt;sup&gt;c&lt;/sup&gt; (1.85)</td>
<td>.005</td>
</tr>
<tr>
<td>Types of Procedural Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wrong operation</td>
<td>32.41 (33.02)</td>
<td>15.19 (21.36)</td>
<td>23.82 (22.57)</td>
<td>ns</td>
</tr>
<tr>
<td>Faulty operation</td>
<td>38.67&lt;sup&gt;a&lt;/sup&gt; (32.09)</td>
<td>44.12&lt;sup&gt;a&lt;/sup&gt; (31.56)</td>
<td>67.27&lt;sup&gt;b&lt;/sup&gt; (27.07)</td>
<td>.005</td>
</tr>
<tr>
<td>Guess</td>
<td>28.91 (32.86)</td>
<td>40.67&lt;sup&gt;a&lt;/sup&gt; (34.89)</td>
<td>8.89&lt;sup&gt;b&lt;/sup&gt; (19.00)</td>
<td>.005</td>
</tr>
</tbody>
</table>

Note. Means with different superscripts are significantly different from each other. (continued)
TABLE 3 (continued)

<table>
<thead>
<tr>
<th>Types of Miscalculation</th>
<th>Arithmetic Disabled Adults</th>
<th>Children Matched for Arithmetic Level</th>
<th>Adults Matched for Chronological Age Level</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number fact error</td>
<td>84.89 (17.76)</td>
<td>95.58a (13.21)</td>
<td>75.21b (27.42)</td>
<td>.05</td>
</tr>
<tr>
<td>Precision error</td>
<td>15.79 (17.86)</td>
<td>4.41a (13.21)</td>
<td>24.78b (27.42)</td>
<td>.05</td>
</tr>
</tbody>
</table>

Note. Means with different superscripts are significantly different from each other.

standard deviations on the spelling and reading subtests for the three groups are shown in Table 1.)

To investigate whether the AD group was deficient in arithmetic or was at a developmentally similar level to the normally achieving arithmetic level matched group, an error analysis was performed on the Arithmetic subtest of the WRAT-3. The error analysis involved two steps. First, a four-level error analysis was performed on this 40 item computation test. Next, the interaction of the types of items and style of response was investigated.

The following is a description of the outcome of the four-level error analyses. At the first-level of the error analyses responses were scored based on whether they received credit or not. A one-way ANOVA revealed that there was a significant difference among the three groups on the mean percent of the
correct responses produced ($F(2,66) = 74.88, p < .001$). As expected, it was found that the number of correct responses produced by the NCA group was significantly higher than the number of correct responses produced by the AD group and the arithmetic level matched group. These last two groups, however, were not significantly different from each other ($p > .05$).

At the second-level of the error analysis, the groups were compared in terms of their response style to the questions for which they did not receive credit. These responses were categorized as incorrect, or omitted. The relative frequency of these two types of responses was calculated for each individual in the present study by dividing the count of incorrect answers or omissions by the number of answers that did not receive credit. A one-way ANOVA revealed that there was a significant difference among the three groups on the mean percent of the incorrect responses produced ($F(2,66) = 13.36, p < .001$) as well as omissions ($F(2,66) = 13.36, p < .001$) on the Arithmetic subtest of the WRAT3. Post-hoc analyses indicated that the AD and NAL groups had significantly lower percentage of errors but higher percentage omissions than the NCA comparison group ($p < .05$). The differences in the mean percentages of incorrect responses and omitted answers between the AD and the arithmetic level comparison groups were not statistically significant.

At the third-level of the error analysis, the groups were compared in terms of the type of incorrect responses they produced. The incorrect responses of the participants in the three groups were categorized as miscalculation, procedural error, or combination of miscalculation and procedural error. The relative frequency for each type of error was expressed as a percent by dividing the count of each type of error by the total number of incorrect responses the individual made. Analysis of variance (see Table 3) revealed that the percentage of the total errors due to miscalculation was not significantly different for the three groups.
Differences were also not significant regarding the percentage of total errors due to procedural mistakes. However, there was a group effect on the errors classified as containing both computational and procedural mistakes ($F(2,66) = 6.84, p < .005$); the arithmetic level matched group had a higher percentage of this type of error than the other two groups.

At the fourth-level of the error analysis, the groups were compared in terms of the type of procedural errors and miscalculations they produced. First, the procedural errors of the participants in the three groups were categorized as wrong operation, faulty operation, or guessed answer. The relative frequency for each type of procedural error was expressed as a percent by dividing the count of each type of procedural error by the total number of procedural errors the individual produced (see Table 3). Analysis of variance revealed that the percentage of wrong operations was not significantly different for the three groups. However, the mean procedural errors classified as 'faulty operation' were significantly different for the three groups ($F(2,65) = 5.97, p < .005$). The chronological-age level matched group had a higher percentage of this type of error than the other two groups, as indicated by the Scheffé post-hoc test. The ANOVA for the procedural errors classified as 'guessed answers' was also statistically significant ($F(2,65) = 6.64, p < .005$). The significant difference was between the comparison groups. In this case, participants in the arithmetic level matched group guessed significantly more frequently than those in the chronological-age level matched group.

Next, the miscalculations of the participants in the three groups were categorized as number fact errors or precision errors. The relative frequency for both types of miscalculation errors was expressed as a percent by dividing the count of each type of miscalculation error by the total number of 'miscalculation errors' the individual produced (see Table 3). Analysis of variance revealed that
the percentage of the 'number fact' error was statistically significant ($F(2,60) = 4.65, p < .05$). The significant difference was between the comparison groups. In this case, participants in the arithmetic level matched group had significantly more numer fact errors than those in the chronological-age level matched group. The findings were also significantly different in the mean percentage of 'precision errors' for the three groups ($F(2,59) = 4.59, p < .05$): the arithmetic level matched group had a significantly lower percentage of this type of error than the chronological-age level matched group as indicated by the Scheffé post-hoc test.

Next, the interaction of the types of questions and style of responses was investigated. Questions were categorized in the following way: addition ($n=5$), subtraction ($n=6$), multiplication ($n=6$), division ($n=3$), fractions ($n=10$), decimal numbers ($n=3$), and complex algebraic equations ($n=6$). The groups were compared in terms of the type of questions they had a tendency to omit, respond correctly, or had an error on (see Table 4).

Over 90% of the participants responded correctly to addition and subtraction questions. These two types of questions did not distinguish among the three groups of participants, therefore these questions were eliminated from further analyses.

The following is a description of the differences and similarities of the three groups of participants on the other 5 types of questions (multiplication, division, fractions, decimal numbers, and complex algebraic equations). The relative frequency of correct responses was expressed as a percent by dividing the count of correct responses on each type of question by the total number of questions in the category for each individual.
Table 4

Means and standard deviations by Type of Question on the WRAT-3 for the
Arithmetic Disabled Adults, Children Matched for Arithmetic Level, and Adults
Matched for Age Level

<table>
<thead>
<tr>
<th>Type of Question</th>
<th>Arithmetic Disabled Adults (n=24)</th>
<th>Children Matched for Arithmetic Level (n=21)</th>
<th>Adults Matched for Chronological Age Level (n=24)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% correct</td>
<td>63.09&lt;sup&gt;a&lt;/sup&gt; (13.91)</td>
<td>60.54&lt;sup&gt;a&lt;/sup&gt; (14.91)</td>
<td>82.73&lt;sup&gt;b&lt;/sup&gt; (11.89)</td>
<td>.001</td>
</tr>
<tr>
<td>% incorrect</td>
<td>20.83 (15.17)</td>
<td>21.08 (16.04)</td>
<td>14.88 (10.72)</td>
<td>ns</td>
</tr>
<tr>
<td>% omitted</td>
<td>16.07&lt;sup&gt;a&lt;/sup&gt; (16.07)</td>
<td>18.36&lt;sup&gt;a&lt;/sup&gt; (11.19)</td>
<td>2.38&lt;sup&gt;b&lt;/sup&gt; (5.43)</td>
<td>.001</td>
</tr>
<tr>
<td>Division</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% correct</td>
<td>52.72&lt;sup&gt;a&lt;/sup&gt; (23.90)</td>
<td>73.01&lt;sup&gt;b&lt;/sup&gt; (29.09)</td>
<td>83.33&lt;sup&gt;b&lt;/sup&gt; (17.02)</td>
<td>.001</td>
</tr>
<tr>
<td>% incorrect</td>
<td>31.94&lt;sup&gt;a&lt;/sup&gt; (26.88)</td>
<td>14.28&lt;sup&gt;b&lt;/sup&gt; (19.92)</td>
<td>15.27&lt;sup&gt;b&lt;/sup&gt; (16.96)</td>
<td>.01</td>
</tr>
<tr>
<td>% omitted</td>
<td>15.27&lt;sup&gt;a&lt;/sup&gt; (25.96)</td>
<td>12.69&lt;sup&gt;a&lt;/sup&gt; (22.30)</td>
<td>1.38&lt;sup&gt;b&lt;/sup&gt; (6.80)</td>
<td>.05</td>
</tr>
</tbody>
</table>

Note. Means with different superscripts are significantly different from each other.
<table>
<thead>
<tr>
<th>Type of Question</th>
<th>Arithmetic Disabled Adults (n=24)</th>
<th>Children Matched for Arithmetic Level (n=21)</th>
<th>Adults Matched for Chronological Age Level (n=24)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fractions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% correct</td>
<td>29.19&lt;sup&gt;a&lt;/sup&gt; (9.74)</td>
<td>20.0&lt;sup&gt;a&lt;/sup&gt; (13.03)</td>
<td>60.41&lt;sup&gt;b&lt;/sup&gt; (19.44)</td>
<td>.001</td>
</tr>
<tr>
<td></td>
<td>24.58</td>
<td>30.95</td>
<td>30.0</td>
<td>ns</td>
</tr>
<tr>
<td></td>
<td>46.25&lt;sup&gt;a&lt;/sup&gt; (22.99)</td>
<td>49.04&lt;sup&gt;a&lt;/sup&gt; (26.43)</td>
<td>9.58&lt;sup&gt;b&lt;/sup&gt; (13.66)</td>
<td>.001</td>
</tr>
<tr>
<td>Decimals</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% correct</td>
<td>31.94&lt;sup&gt;a&lt;/sup&gt; (11.95)</td>
<td>38.09&lt;sup&gt;a&lt;/sup&gt; (15.93)</td>
<td>56.94&lt;sup&gt;b&lt;/sup&gt; (23.01)</td>
<td>.001</td>
</tr>
<tr>
<td></td>
<td>15.27</td>
<td>26.98</td>
<td>30.55</td>
<td>ns</td>
</tr>
<tr>
<td></td>
<td>52.77&lt;sup&gt;a&lt;/sup&gt; (25.85)</td>
<td>34.92&lt;sup&gt;b&lt;/sup&gt; (19.65)</td>
<td>12.5&lt;sup&gt;c&lt;/sup&gt; (16.48)</td>
<td>.001</td>
</tr>
<tr>
<td>Complex Questions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% correct</td>
<td>2.08&lt;sup&gt;a&lt;/sup&gt; (5.63)</td>
<td>0.79&lt;sup&gt;a&lt;/sup&gt; (3.63)</td>
<td>38.89&lt;sup&gt;b&lt;/sup&gt; (26.31)</td>
<td>.001</td>
</tr>
<tr>
<td></td>
<td>22.22</td>
<td>23.01</td>
<td>17.36</td>
<td>ns</td>
</tr>
<tr>
<td></td>
<td>75.69&lt;sup&gt;a&lt;/sup&gt; (18.37)</td>
<td>76.19&lt;sup&gt;a&lt;/sup&gt; (29.14)</td>
<td>43.75&lt;sup&gt;b&lt;/sup&gt; (26.83)</td>
<td>.001</td>
</tr>
</tbody>
</table>

Note. Means with different superscripts are significantly different from each other.
One-way ANOVAs revealed that there were significant differences among the three groups on the mean relative frequency of correct responses on the following types of questions of the WRAT3: multiplication ($F(2,66) = 18.65, p < .001$), division ($F(2,66) = 10.38, p < .001$), fractions ($F(2,66) = 47.91, p < .001$), decimals ($F(2,66) = 13.00, p < .001$), and complex algebraic equations ($F(2,66) = 42.81, p < .001$). Post-hoc analyses indicated that the adults matched for chronological age produced significantly more correct responses on multiplication questions, fractions, decimal numbers, and complex algebraic equations than the AD and NAL groups ($p < .05$). The difference between the AD and NAL groups was not statistically significant on those types of questions. However, there is one exception to this otherwise uniform pattern. Post-hoc analysis indicated that on the division questions, the AD adults produced significantly fewer correct responses than the two comparison groups ($p < .05$). The difference between the comparison groups were not statistically significant on the division questions.

The relative frequency of incorrect responses was expressed as a percent by dividing the count of incorrect responses on each type of question by the total number of items in the category that each individual attempted. One-way ANOVAs revealed that there was a significant difference among the three groups on the mean relative frequency of incorrect responses only. This main effect was obtained on the division questions ($F(2,66) = 4.87, p < .01$), the other types of questions of the WRAT3 did not elicit a significantly different pattern of incorrect responses from the three groups. Post-hoc analyses indicated that on the division questions, the AD adults produced significantly more incorrect responses than the two comparison groups ($p < .05$). The difference between the comparison groups was not statistically significant on the division questions.

The relative frequency of omitted responses was expressed as a percent by dividing the number of omitted responses on each type of question by the total
number of questions in the category for each individual. One-way ANOVAs revealed that there were significant differences among the three groups on the mean relative frequency of omitted responses on the following types of questions of the WRAT3: multiplication ($E(2,66) = 12.66, p < .001$), fractions ($E(2,66) = 24.57, p < .001$), decimals ($E(2,66) = 21.95, p < .001$), and complex algebraic equations ($E(2,66) = 12.94, p < .001$). Post-hoc analyses indicated that the adults matched for chronological age omitted a significantly fewer number of multiplication questions, fractions, and complex algebraic equations than the AD and NAL groups ($p < .05$). The difference between the AD and NAL groups were not statistically significant on those types of questions. However, post-hoc analysis indicated that on the decimal questions, the AD adults omitted significantly more responses than the two comparison groups ($p < .05$). The difference between the NCA and NAL groups was also statistically significant. The NAL group omitted significantly more responses than the NCA group ($p < .05$) on the division questions.

One-way ANOVA revealed that there was a significant difference among the groups on the mean percentile scores of the Interpreting Data subtest of the KEY-MATH-R ($E(2,66) = 32.13, p < .001$). Post-hoc analyses indicated that the AD group scored significantly lower than the two comparison groups ($p < .05$). The difference between the mean scores of the two comparison groups were not statistically significant. The findings for the three groups on the Interpreting Data subtest of the Key-Math-R are shown in Table 5.
Table 5

**Means, standard deviations, and range on the Key-Math-R for the Arithmetic Disabled Adults, Children Matched for Arithmetic Level, and Adults Matched for Age Level**

<table>
<thead>
<tr>
<th>Arithmetic Disabled Adults (n=24)</th>
<th>Children Matched for Arithmetic Level (n=21)</th>
<th>Adults Matched for Chronological Age Level (n=24)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{x}) (SD)</td>
<td>(\bar{x}) (SD)</td>
<td>(\bar{x}) (SD)</td>
<td></td>
</tr>
<tr>
<td>35.21a (12.33)</td>
<td>70.86b (23.12)</td>
<td>75.0b (19.83)</td>
<td>.001</td>
</tr>
<tr>
<td>range</td>
<td>16-63</td>
<td>37-99</td>
<td>40-100</td>
</tr>
</tbody>
</table>

**Error Analyses**
(raw scores)

<table>
<thead>
<tr>
<th>Charts &amp; Tables</th>
<th>Graphs</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.38a (0.65)</td>
<td>3.92a (1.32)</td>
<td>3.62a (1.06)</td>
</tr>
<tr>
<td>(4-6)</td>
<td>(1-6)</td>
<td>(1-5)</td>
</tr>
<tr>
<td>range</td>
<td>range</td>
<td>range</td>
</tr>
<tr>
<td>4.6</td>
<td>1.6</td>
<td>3.6</td>
</tr>
<tr>
<td>(0.64)</td>
<td>(0.78)</td>
<td>(0.78)</td>
</tr>
</tbody>
</table>

**Note.** Means with different superscripts are significantly different from each other.
To investigate whether the AD group was qualitatively different in interpreting numerical data or was at a developmentally similar level as the normally achieving arithmetic level matched group an item analyses was performed. The questions on the Interpreting Data subtest of the Key-Math-R were divided into three increasingly more abstract categories. The easiest questions required participants to calculate answers based on tables and charts, the next level of questions were based on graphs, and the most abstract questions had to do with probability and statistics.

One factor ANOVA revealed a significant effect of group on interpreting charts and tables ($F(2,66) = 14.46, p < .001$); the arithmetic level matched group scored lower than the two adult groups ($p < .05$). There was a significant group effect on interpreting graphs ($F(2,66) = 13.08, p < .001$), as well as on answering probability questions ($F(2,66) = 28.96, p < .001$). However, on the last two categories, the normally achieving adult group scored higher than the AD group and the arithmetic level matched group ($p < .05$). The latter two groups did not differ statistically.

A one-way ANOVA revealed that there was a significant difference among the groups on the mean scores of the problem solving questions selected from the Problem Solving subtest of the KEY-MATH-R ($F(2,65) = 10.96, p < .001$). Post-hoc analyses indicated that the AD group and the arithmetic level matched groups scored significantly lower than the chronological-age level matched group did ($p < .05$). The difference between the mean scores of the AD and arithmetic level matched groups did not reach statistical significance.

A one-way ANOVA revealed that there was a significant difference among the groups on the mean time required to work on the problem solving questions ($F(2,61) = 4.03, p < .05$). Post-hoc analyses indicated that the AD group took a significantly longer time to complete this task than the chronological-age
level matched group ($p < .05$). The difference between the mean scores of the AD and arithmetic level matched groups did not reach statistical significance. Results of the three groups on the word problem set are presented in Table 6.

Table 6

Means, standard deviations, and range on the Problem Solving Results for the Arithmetic Disabled Adults, Children Matched for Arithmetic Level, and Adults Matched for Age Level

<table>
<thead>
<tr>
<th></th>
<th>Arithmetic Disabled Adults (n=24)</th>
<th>Children Matched for Arithmetic Level (n=21)</th>
<th>Adults Matched for Chronological Age Level (n=24)</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accuracy</strong></td>
<td>7.70$^a$ (2.80)</td>
<td>6.19$^a$ (2.80)</td>
<td>9.71$^b$ (1.94)</td>
<td>.001</td>
</tr>
<tr>
<td>(max=12)</td>
<td>range 2-12</td>
<td>range 2-11</td>
<td>range 5-12</td>
<td></td>
</tr>
<tr>
<td><strong>Completion Time</strong></td>
<td>44.26$^a$ (12.04)</td>
<td>36.90 (12.56)</td>
<td>33.62$^b$ (12.39)</td>
<td>.05</td>
</tr>
<tr>
<td>(max=60 min)</td>
<td>range 19-60</td>
<td>range 15-60</td>
<td>range 12-60</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* Means with different superscripts are significantly different from each other.
Attitude Toward Math

Attitude toward math was measured by a self-report on math anxiety and the pre-and post-confidence rating on the word problem task. The means, standard deviations, and range for the two measures assessing attitude toward math are presented in Table 7.

Anxiety

A one-way analysis of variance revealed a significant group effect on math anxiety scores ($F(2,65) = 15.19, p < .001$). Post-hoc analysis indicated that the AD group reported a significantly higher level of math anxiety ($p < .05$) than the two comparison groups.

Confidence Rating

A one-way analysis of variance indicated a significant effect for group on both pre-confidence ($F(2,65) = 9.1, p < .001$), and post-confidence ($F(2,66) = 7.13, p < .005$). In both cases, the results of post-hoc analyses indicated that the AD and arithmetic level matched groups reported significantly less confidence in solving the math problems than the chronological-age level matched group ($p < .05$) but the AD and NAL groups did not differ from each other.

Reasoning In Math

The strategies used to solve each word problem were classified by the experimenter based on whether the procedures used were explained fully, partially with minor flaws, partially with major flaws, or not at all. The more
Table 7

Means, standard deviations, and range on the Attitude Toward Math for the
Arithmetic Disabled Adults, Children Matched for Arithmetic Level, and Adults
Matched for Age Level

<table>
<thead>
<tr>
<th></th>
<th>Arithmetic Disabled Adults (n=24)</th>
<th>Children Matched for Arithmetic Level (n=21)</th>
<th>Adults Matched for Chronological Age Level (n=24)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{x}$ (SD)</td>
<td>$\bar{x}$ (SD)</td>
<td>$\bar{x}$ (SD)</td>
<td></td>
</tr>
<tr>
<td>Raw Scores</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Math Anxiety</td>
<td>26.61$^a$ (9.80)</td>
<td>45.33$^b$ (9.80)</td>
<td>37.25$^b$ (13.67)</td>
<td>.001</td>
</tr>
<tr>
<td>range</td>
<td>12-53</td>
<td>18-57</td>
<td>12-60</td>
<td></td>
</tr>
<tr>
<td>Pre-confidence</td>
<td>14.70$^a$ (5.03)</td>
<td>16.0$^a$ (4.96)</td>
<td>20.04$^b$ (3.33)</td>
<td>.001</td>
</tr>
<tr>
<td>range</td>
<td>6-21</td>
<td>4-24</td>
<td>14-24</td>
<td></td>
</tr>
<tr>
<td>Post-confidence</td>
<td>15.67$^a$ (6.20)</td>
<td>15.86$^a$ (5.10)</td>
<td>20.62$^b$ (3.69)</td>
<td>.005</td>
</tr>
<tr>
<td>range</td>
<td>1-24</td>
<td>6-24</td>
<td>10-24</td>
<td></td>
</tr>
</tbody>
</table>

Note. Means with different superscripts are significantly different from each other.
complete the explanation was, the more points were awarded for the answer (see Table 2 for examples of scoring). The results on the strategies the participants used to answer the word-problems are shown in Table 8.

A one-way ANOVA revealed that there was a significant difference among the three groups on the mean raw scores of explaining the procedures they used to solve the word-problems ($F(2,65) = 5.00, p < .05$). Post-hoc analyses indicated that the AD and NAL groups had significantly lower scores than the chronological-age level comparison group ($p < .05$). The difference in the mean scores of explaining the procedures used to solve the word-problems between the AD and the arithmetic level comparison groups was not statistically significant.

After the description of the strategies used to solve the word problems, participants were required to write down what errors other people might make in solving questions of that kind. The responses were classified according to the type of error described by the participants. Participants who responded to this question described mistakes related to computation, procedures, attitude, and understanding of the word problem. Table 8 summarizes these data. The frequency of these four types of responses was calculated for each individual by dividing the count of computation, procedures, attitude, or understanding error by the number of total responses to this question (Appendix C includes examples of the different responses).

A one-way ANOVA revealed that there was a significant difference among the three groups on the mean number of responses produced ($F(2,65) = 3.50, p < .05$) as well as omissions ($F(2,65) = 5.81, p < .005$) on this question. Post-hoc analyses indicated that the AD group had a significantly fewer number of responses to this hypothetical question than the NCA group and the NAL groups.
(p < .05). However, the differences in the mean scores of total responses and omitted answers between the AD and the arithmetic level comparison groups were not statistically significant.

Table 8

Means and standard deviations on the Problem Solving Task for the Arithmetic Disabled Adults, Children Matched for Arithmetic Level, and Adults Matched for Age Level

<table>
<thead>
<tr>
<th></th>
<th>Arithmetic Disabled Adults (n=24)</th>
<th>Children Matched for Arithmetic Level (n=21)</th>
<th>Adults Matched for Chronological Age Level (n=24)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Scores</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean (SD)</td>
<td>Mean (SD)</td>
<td>Mean (SD)</td>
<td></td>
</tr>
<tr>
<td>Strategy (max=18)</td>
<td>9.00(^a) (4.17)</td>
<td>8.76(^a) (4.44)</td>
<td>12.16(^b) (3.96)</td>
<td>.05</td>
</tr>
<tr>
<td>Total Number of Possible Errors Described (max=18)</td>
<td>7.57(^a) (5.18)</td>
<td>10.19 (5.55)</td>
<td>11.25(^b) (3.89)</td>
<td>.05</td>
</tr>
<tr>
<td>Total Number of No Responses (max=6)</td>
<td>1.74(^a) (2.09)</td>
<td>1.09 (1.37)</td>
<td>0.25(^b) (0.74)</td>
<td>.005</td>
</tr>
</tbody>
</table>

Note. Means with different superscripts are significantly different from each other. (continued)
TABLE 8 (continued)

<table>
<thead>
<tr>
<th></th>
<th>Arithmetic Disabled Adults (n=24)</th>
<th>Children Matched for Arithmetic Level (n=21)</th>
<th>Adults Matched for Chronological Age Level (n=24)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\bar{x}) (SD)</td>
<td>(\bar{x}) (SD)</td>
<td>(\bar{x}) (SD)</td>
<td></td>
</tr>
<tr>
<td>Raw Scores</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type of Possible Errors Described (percent scores)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computational</td>
<td>25.66 (32.7)</td>
<td>18.71 (19.15)</td>
<td>24.98 (15.89)</td>
<td>ns</td>
</tr>
<tr>
<td>Procedural</td>
<td>35.36 (24.78)</td>
<td>39.25 (26.37)</td>
<td>39.53 (16.51)</td>
<td>ns</td>
</tr>
<tr>
<td>Attitude</td>
<td>2.47 (6.44)</td>
<td>6.21 (12.5)</td>
<td>2.31 (7.08)</td>
<td>ns</td>
</tr>
<tr>
<td>Understanding</td>
<td>36.51 (21.2)</td>
<td>35.84 (19.28)</td>
<td>33.19 (15.71)</td>
<td>ns</td>
</tr>
</tbody>
</table>

Note. Means with different superscripts are significantly different from each other.
CHAPTER 4

DISCUSSION

In this study the similarities and differences of aspects of the cognitive and affective profiles of adults with arithmetic disability and two comparison groups, one matched on chronological-age and the other matched on arithmetic level were investigated. The purpose of this study was to describe arithmetic learning disability in an adult population by employing the following definition of arithmetic disability: arithmetic performance below the 25th percentile and reading performance above the 30th percentile on the WRAT-3 (Siegel & Linder, 1984). More specifically this multidimensional investigation was designed to answer the following questions:

1. How do adults with and without arithmetic disability differ in computation skills and attitude toward numeracy? What are the processes involved in math problem solving for adults with arithmetic learning disability; what is unique about the information-processing characteristics of these individuals when compared to the comparison groups?

2. Do adults with arithmetic disability and children who perform at the same arithmetic level without arithmetic disability differ in computation and problem solving skills and attitude toward numeracy? More precisely, is there evidence of deficit or delay in the three integral parts of numeracy: computation, attitude, and reasoning.

The similarities and differences in computational skills are addressed first in the discussion. Next, the similarities and differences in attitudes among the
three groups are examined. These are followed by the comparison of the three
groups in terms of their reasoning skills. Next, a modification of the
developmental lag framework is presented. The implications of the findings are
linked to possible designs for remediation programs for individuals with
arithmetic disability. Prior to the conclusions drawn from this research, the
limitations of the study are outlined in terms of the characteristics of the
participants, the assessment model, and the design utilized.

Similarities and Differences in Computational Skills of the Three Groups

The error analyses performed in this study provided a detailed assessment
of the computational skills of adults with arithmetic disability. The response
pattern on the computational problems of the WRAT-3 and the KeyMath-R most
of the time indicated that in contrast to the normally achieving adults, the AD
adults exhibited lower performance, whereas the arithmetic level matched group
exhibited similar skills to the AD group. The discussion of these findings are
presented in the following two sections. First, written computation is discussed by
examining the results of the error analyses on the WRAT-3 questions. Next, the
mental computational abilities of the three groups are discussed, based on the
findings from the 'Interpreting Data' subtest of the Key-Math-R.

Written Computation

The response pattern on the computational problems of the WRAT-3 was
examined in this research. Differences in error patterns on the measures
administered to the subjects were considered as indicators of differences in information processing (Morrison & Siegel, 1991).

In the present study a hierarchical four-level error analysis was used. This type of error analyses has several advantages compared with previous methods of arithmetic error analyses. First, it is transferable, that is it can be used with any other test of arithmetic. Its particular diagnostic significance is that it can be used to determine where in the hierarchy a disruption in learning has occurred and what type of breakdown it is. This is crucial if one needs to develop relevant remediation plans. Furthermore, the different levels of the hierarchy can be used to indicate strengths as well as weaknesses of individuals or groups of individuals in computational skills.

At the first level of analysis one can establish whether the answer to the question was correct or not. This surface level assessment of answers is common. In most math classes educators stop the process of evaluation after the first level: an answer is either correct or it is not. The findings of this study indicate that it is not enough to merely consider the obvious, one must delve deeper from the surface manifestation to locate the root of the problem. As expected, adults with arithmetic disability made a larger number of errors than the normally achieving adults. It is possible that the group of adults with arithmetic disability would have been able to have fewer errors and complete more questions if more time would have been available for them to process the arithmetic problems. The issue regarding completion time could be answered by a study designed to examine the difference in performance on the same arithmetic test with and without time limit. It is also possible that adults with arithmetic disability had enough time for completion but (as many of them handed in the test prior to the 15 minutes allowed for completion) wanted to avoid spending more time on this activity.
The second level of analysis helped to establish whether failing to receive credit was due to an error or to lack of response. The findings indicated that the group of adults with arithmetic disability and the normally achieving arithmetic level matched group were very similar, both making more errors and omitting more responses than the normally achieving chronological age matched group. Failure to answer a question can be due to lack of knowledge, lack of time, or attitudinal factors. It is possible that a combination of all three factors influenced the performance of the participants in this study. Further investigation is needed to explore these issues.

The importance of the third level of analysis is the detection of the source of errors. There was no main effect obtained for miscalculation, or procedural errors. However, the combination of these two types of errors was significantly higher for the normally achieving arithmetic level matched group than for the other two groups. This finding seems to indicate that children in the arithmetic level matched group had more complex difficulties in calculation than either of the adult groups.

The fourth level of analysis was designed to detect different types of miscalculation and procedural errors. At this level the observed differences were in terms of faulty operation, guess, and precision errors. The normally achieving chronological age matched group had a higher relative frequency of 'faulty operation' type of procedural errors compared to the other two groups (AD and NAL). This indicates that the normally achieving adults were more inclined to apply the appropriate operation required even if their procedure was not exactly correct.

Guessed responses included no evidence of steps in calculating the answer and such answers did not fit any other error categories. The normally achieving
arithmetic level matched group used guessing more as a procedure to obtain an answer than the arithmetic disabled or chronological age matched groups. This reflects the fact that children were more likely to provide responses whether they understood the questions or not.

In terms of the types of miscalculation errors the differences detected were between the comparison groups. The adult comparison group seemed to have less number fact errors but more precision errors than the normally achieving arithmetic level matched group. This is perhaps due to the fact that adults had more experience with number facts and use estimation and calculators more frequently than children and have a tendency to stop a division question after the first or second decimal of the answer has been reached as opposed to continuing.

In summary, although the chronological age matched group had fewer errors than the other two groups, there were more similarities among the three groups than differences in terms of the type of errors they made on the WRAT-3. Therefore, a system of errors that would clearly differentiate the three groups was not obtained at this level of the analyses.

Another way of establishing a pattern of errors applied in the current study was the examination of the number of errors on the different types of questions. The findings on computational problems of the WRAT-3 indicated that, in contrast to the normally achieving adults, the adults with arithmetic disability experienced most of their difficulties in working with fractions, decimals, division, and questions that involved complex algebra. In fact, the two types of questions that they did not perform significantly lower on were simple addition and subtraction questions. This finding indicates that adults with arithmetic disability do not have difficulty with basic manipulation of whole numbers. Another important finding revealed through this item analysis, which converges with
findings of Yaghoubzadeh, Geva, and Siegel (1996), was that the adults with arithmetic disability and the normally achieving arithmetic level matched children lost credit on the WRAT-3 primarily because they failed to respond to questions as opposed to committing errors in their responses. The only exception to this pattern was in the case of division problems. On division questions, adults with arithmetic disability made more mistakes than the other two groups. Division, one of the four fundamental operations required in computation, requires an understanding of the place value, which is basic to many mathematical operations (Sutaria, 1985). The high number of errors of adults with arithmetic disability was possibly due to their poor understanding of place value and other complexities of performing division.

The findings of this study suggested that the errors committed by the arithmetic disabled adults on the WRAT-3 mostly resembled those of the arithmetic level matched children as opposed to their age matched peers on most of the questions. Although all three groups made errors due to miscalculation, reflecting difficulty with recalling number facts, adults with arithmetic disability and the normally achieving arithmetic level matched children displayed severe difficulties in working with fractions, decimals, division, and questions that involved complex algebra. These findings are similar to an investigation of the arithmetic errors on the WRAT-R (Szanto & Siegel, 1994): both adults with arithmetic disability and an arithmetic level matched group showed similar difficulties in solving written arithmetic problems, whereas the error patterns of normally achieving adults were different. In terms of the type of mistakes they tended to make on written arithmetic calculation, adults with arithmetic disability were similar to children who perform at the same arithmetic level. The conclusions of other studies converge with the present findings by indicating that
children with arithmetic disability perform at a similar level on some math tasks as younger children without arithmetic disability who are matched on arithmetic level (e.g., Saito, 1992). However, there were a number of differences between the adult groups and the normally achieving arithmetic level matched group. For instance, the normally achieving arithmetic level matched group had a higher relative frequency of complex calculation mistakes that included procedural errors as well as miscalculations. This group of children was also more likely to guess their responses than the adults groups of this study.

**Mental Computation**

When tested on a task that involved interpreting charts and tables, adults with arithmetic disability did not differ from their age matched cohort group. Also, as a group they performed better than the arithmetic level matched group. A potential explanation for these results is that over the years adults with arithmetic disability have had more exposure to charts and tables in their day-to-day activities than children. It is also possible that the children were not yet taught in school how to interpret data from charts and tables.

The ability to understand graphs and probability is important if one is to make practical implications of the knowledge in a variety of situations. The performance of the group of adults with arithmetic disability and the arithmetic level matched comparison group was virtually identical on the interpretation of graphs and probability questions but different from the normally achieving adult group. These results suggest that adults with arithmetic disability have considerable difficulties with interpreting abstract mathematical information
even if they are allowed to process questions with a generous time limit (up to 60 minutes). It is also possible that they had less experience with such a task.

Overall, the differences between the group of adults with arithmetic disability and the chronological age matched group were found to be quantitative in nature. That is, adults with arithmetic disability committed more errors on two of the three subscales of the Interpreting Data subtest of the KeyMath-R than the normally achieving adults. The performance of adults with arithmetic disability was not distinguishable from that of the younger individuals performing at the same arithmetic level on graphs and probability questions but was higher on reading charts and tables. This is perhaps due to the nature of the material presented in the three categories of items on the Interpreting Data subtest of the Key-Math-R. That is, charts and tables are not as cognitively demanding because the information conveyed is more structured and requires less interpretation than the graphs and probability questions.

Similarities and Differences in Attitude Among the Groups

An often empirically ignored, yet central issue regarding arithmetic disability, is the attitude one has toward numeracy. In the present study the participants' attitudes toward math was investigated by self-reports on math anxiety and confidence. The relationships in the data on attitude toward math for the adults with arithmetic disability displayed significant difference from those for the comparison groups in terms of anxiety, but some interesting similarities were apparent in terms of confidence rating.
Math anxiety

It was hypothesized that adults with arithmetic disability would report a higher level of math anxiety than participants in the comparison groups. A statistically significant main effect was obtained. Unlike the responses of the adults with arithmetic disability on the math anxiety questionnaire, the responses of the chronological-age and the arithmetic level matched groups did not indicate high levels of math anxiety. The relatively high anxiety reported by adults with arithmetic disability suggests that this factor may play an important role in their low level of performance on tasks that involve the manipulation and interpretation of numbers. This is consistent with the relationship of math anxiety and achievement found by a number of researchers (Dreger & Aiken, 1957; Frary & Ling, 1983; Quilter & Harper, 1988). This emotional component of arithmetic disability may influence the choices one makes regarding math. For example, it is possible that adults with arithmetic disability omitted many responses on the WRAT-3 not because they did not have enough time, but because they did not want to perform an activity which they associated with anxiety. This may be the effect of having a sense of low self-efficacy in math. If it is, it would support Schunk's (1989) assumption that those who "hold a low self-efficacy for accomplishing a task may attempt to avoid it, whereas those who believe they are capable should participate more eagerly."(p. 14).

Based on the findings of this study, it is recommended that, math anxiety should be a condition that is routinely examined in an assessment for remediation of arithmetic disability. Studies involving intervention procedures to reduce
anxiety might reveal more about the effect of this factor on performance in the domain of mathematics.

Confidence rating

The findings related to the effect of self-confidence on solving word problems supported the general consensus in the literature: beliefs about one's abilities may affect one's achievement (e.g., Bandura & Schunk, 1981; Meltzer, 1991). The adults in the chronological-age level matched group reported significantly higher pre- and post-confidence related to the word problem task than the group of adults with arithmetic disability the normally achieving arithmetic level matched group. The low self-confidence rating of the group of adults with arithmetic disability was expected. The low self-confidence rating of the arithmetic level matched children is possibly due to their lack of exposure to some of the problems they were required to solve. Most of the children of the normally achieving arithmetic level matched group were in grades 5 or 6 and they might not have been taught how to solve the problems presented to them or did not have enough experience with problem solving in general, which presumably influenced their confidence rating. Adults with arithmetic disability also reported significantly lower confidence for solving word problems than their age mates. These results, however, cannot be explained by lack of academic exposure, since all the adult participants of this study held a high school diploma or equivalent. It is more likely that their lack of success with math, due to their disability, has supported an entrenched negative view of themselves in this domain.

Individuals with low perceived competency and poor performance in math lack the positive feeling of success in this area. The absence of such feeling may
lead to lack of practice, low self-efficacy, and possibly to the development of math phobia. As such, potentially there is a cyclical process at work: weak skills in numeracy lead to low self-efficacy and, in turn, low self-efficacy leads to weak numeracy skills.

The findings of this study indicate that performance on tests of mathematics is not based just on cognitive factors. This research indicates that the individual's feelings and self-confidence related to the subject are also involved. There are a number of adults with arithmetic disability whose learning disability is coupled with negative attitudes regarding numbers. A negative attitude toward math could constitute increased levels of frustration with numbers, and an abandonment of situations where manipulation of numbers is required (Hagen, Kamberelis, & Segal, 1991). However, if math anxiety can be reduced, then better math performance may result (Dodd, 1992). This might lead to greater availability of career opportunities for adults with arithmetic disability. Therefore, it is important to develop a greater understanding of the role of anxiety in the identification and remediation programs for this population. The findings of this study add to the body of knowledge about the nature of arithmetic disability of adults and call attention to the difference in the attitude toward math that exists between individuals with and without arithmetic disability.

Comparison of the Three Groups on Reasoning Skills

Problem solving in math integrates several processes. It requires the individual to integrate basic math skills in applied situations. Problem solving capability, then, is the highest goal to achieve in learning mathematics and results
from the integration of basic skills, metacognitive skills, and previous experience one had with the subject.

To complete the written word problems of this study participants needed to read and understand what the problem stated, apply math skills, as well as use written language skills to express the answer and to list their strategies. Although the word problems were approximately at the grade 5 level, in terms of basic computational skills, the group of adults with arithmetic disability performed significantly more poorly than the normally achieving adult group. Since there were no time constraints, and all the facts and data were available to adults with arithmetic disability it is possible that they could not retrieve and organize the information in a written form to help them reach a logical solution. Not only were the solutions of the group of adults with arithmetic disability and the chronological age matched group different, but so were their strategy descriptions.

The strategies one uses for thinking in a problem solving situation are controlled by one's internal processing (Flavell, 1976). The participants' knowledge of their own cognitive processes were examined by asking them to write down the strategies they used to solve the word problems. The performance patterns of adults with arithmetic disability and the arithmetic-level groups showed virtually the same level of skills in problem solving.

There may be several explanations for these findings. If the adults with arithmetic disability and the normally achieving arithmetic level matched participants had deficits in the reading process of decoding or comprehension, the word problems would not have been correctly interpreted. However, significant differences in reading processes were not shown to exist among the three groups of this study. A more likely explanation of the low performance on the word problem task would be the observed low basic math skills of adults with arithmetic
disability. Low skills in computation, coexisting with poor metacognitive skills have the potential to depress performance on word problem solving.

The written expression of reasoning skills in the domain of mathematics involves content knowledge of math as well as written language skills. It can be argued that the low scores on the problem solving task indicated that adults with arithmetic disability had difficulties expressing the strategies they used in solving the word-problems. More specifically, it is possible that adults with arithmetic disability did not have adequate language skills to express their thoughts on how they solved the word-problems. A third possibility is that both poor metacognition and written expression skills contributed to the difficulties of adults with arithmetic disability, which in turn depressed the quality of their responses to the open-ended questions regarding how they completed the word-problems.

Based on this study conclusions cannot be drawn regarding the cause of the low scores of adults with arithmetic disability on the task of describing their strategies used to solve the word problems. Whether the deficit lies in metacognition or written expression needs to be investigated in the future.

The adults with arithmetic disability produced significantly fewer responses than the normally achieving adult group to the question that required them to hypothesize about the type of errors others would have made on the same word problem. However, the type of errors listed by the three groups of participants did not differ significantly. This suggests that although adults with arithmetic disability, in general, are aware of the potential difficulties one might be facing during word problem solving, they are not able to describe as many as members of the comparison groups.

In summary, when their performance was compared with the performance of chronological-age matched comparisons, adults with arithmetic disability were
poorer at describing the strategies they used to solve word problems as well as on hypothesizing about the types of difficulties others may have with solving such problems. These results add to current views about arithmetic disability of adults by providing information on the marked differences of the reasoning skills of adults with and without arithmetic disability. The following factors are the most likely contributors of the low word problem scores: lower levels of basic computational skills, lower self-efficacy, and poorer metacognitive skills. As a whole, the findings indicate that the information processing of adults with arithmetic disability is different from their normally achieving chronological age mates and very similar to the information processing of children at the same arithmetic level.

A Modification of the Developmental Lag Framework

The findings of this study indicate that adults with arithmetic disability seemed to have developed unevenly in some areas of numeracy and consequently show marked underachievement. Although there seemed to be a similar developmental path of the adults with arithmetic disability and the normally achieving arithmetic level matched group in some of the computational and reasoning skills examined, there were a number of differences observed between adults with arithmetic disability and the normally achieving arithmetic level matched group. For instance, the normally achieving arithmetic level matched group had a higher relative frequency of complex calculation mistakes that included procedural errors as well as miscalculations. This group of children was
also more likely to guess their responses than the adult groups of this study. In addition, adults with arithmetic disability reported a significantly higher level of math anxiety than both the comparison groups. The examination of the number of errors the groups had on the different types of questions indicated that on division questions the adults with arithmetic disability made more mistakes than the other two groups. This finding indicates that there may be a specific deficit of adults with arithmetic disability in comparison with the arithmetic level matched children in terms of division questions. This potential deficit reflects a poor understanding of place values - a basic and crucial concept in mathematics of every-day events.

The findings of this study suggest that the developmental path in numeracy for adults with arithmetic disability differs from that of normal adults. Normal development in numeracy would allow an adult to perform written and mental computation without great difficulty. Such adults would be expected to have sufficient metacognitive skills to solve word problems of an elementary level but not be expected to have a negative attitude toward numbers. Compared to normal development, a developmental lag concerns slowness experienced by individuals in reaching anticipated points in development which are considered essential if basic skills like understanding and working with whole numbers, fractions, decimals are to be learned.

The developmental lag explanation had been used in the reading disability literature by a number of researchers (e.g., Olson et al., 1989; Stanovich et al., 1988). Although the developmental lag framework seems useful in examining arithmetic disability, certain changes are necessary to examine the academic skills of adults within this framework. Noticeably, the age of the participants in other studies using the developmental lag model and the current study are markedly
different. Traditionally, researchers examining learning disability through a developmental lag framework included only children in their different ability groups. Therefore, the mean chronological age difference between their target group and comparison groups were markedly lower than in the present study. A delay of a few years in terms of academic skills can be considered a strong form of the developmental delay. However, as Coltheart (1987) notes, regarding dyslexia, one's reading age several years behind one's chronological age may indicate more than just a developmental delay. He considered such a pattern as indication of an abnormally slow development.

Based on Coltheart's (1987) argument, the significantly lower level of math skills (on average a delay of approximately 20 years) in the case of adults with arithmetic disability in this study can be explained by only a weak form of developmental lag. Since most of the participants with arithmetic disability were at least in their 30s, 40s, 50s, they would be considered to lag 20, 30, or even 40 years behind their arithmetic level matches. The developmental lag framework, therefore, does not seem meaningful in its traditional form when one examines adults with arithmetic disability since there is no indication from growth rate that adults with arithmetic disability would ever catch up to their chronological age mates. Thus, as it was suggested by Coltheart (1987) regarding dyslexia, it is recommended that unless the arithmetic level match group is just a few years behind (chronologically) the target group, the field of arithmetic disability abandon the traditional form of developmental lag explanation of the condition of arithmetic learning disability. More precisely, it is suggested that researchers differentiate between degrees of developmental delay. Integrating the evidence for delay and deviance in this study, it seems more rational to view adults with
arithmetic disability to have incomplete development in arithmetic and exhibiting a weak developmental delay.

Whether or not one describes arithmetic disability in terms of a delay or deficit, the fact remains that these adults experience a variety of mathematics related disturbances which are evident in their computation skills, attitude, as well as reasoning skills. These difficulties can be and should be remediated. The following section contains implications of the findings in this study regarding remediation and offers some directions as to how to help adults with arithmetic disability.

Implications for Remediation

Computation

The error analyses indicated that the group of adults with arithmetic disability had difficulties with computation. The ability to calculate is an important skill to possess because it enhances numerical problem solving. To accommodate those with arithmetic disability some remediation programs provide them with calculators to increase their accuracy and speed. This, however, on its own is not sufficient when adults with arithmetic disability face word problems in a math class or in real life situation. Another common remedial procedure is to provide extra time to complete tasks related to numeracy. The findings of this study indicate that performance on tests without time limit does not solve the problem either. For example, there was no time limit on the KEY-MATH and the group of adults with arithmetic disability still performed more poorly than the chronological age matched group. Therefore, it is argued
that common remedial procedures such as extra time would not be sufficient to aid adults with arithmetic disability to complete problems related to graphs and probability questions.

More important than the mechanical skill or speed of calculation is to understand why a particular arithmetic process is used to solve a particular problem. Without the ability to reason a calculator is just an anxiety producing item in a problem solving situation and extra time just prolongs the experience of math anxiety for adults with arithmetic disability. The reasons behind the mechanical aspects of arithmetic should be stressed and the connections among the procedures (e.g., the relationship between addition and multiplication; the similarities and differences of expressing an answer in a fraction or decimal form) should be pointed out in remediation programs. Adults with arithmetic disability would need tutoring with a focus on comprehension of arithmetic procedures. Such tutoring should facilitate the basic understanding of concepts on which they could build a mathematical frame of reference.

Reasoning skills

The essence of problem solving in mathematics can be viewed as an exercise in the process of systematic inquiry in general (Polya, 1984). A very important goal of any remediation program designed for adults with arithmetic disability should be to strengthen their metacognitive thinking foundation:

" Probably the most fundamental skill one needs to acquire in order to survive in the world is the ability to solve problems. How one begins to approach a problem, proceeds through it, and arrives at a logical solution is dependent on the process of esquire one is able to initiate in problem
solving. While the process or the mode is likely to vary with each different situation and situational demand, the important point is that it is basic to problem solving. In other words, one cannot begin to solve problems without some forethought and well-planned strategies." (Sutaria, 1985:356).

There is some indication in the literature (Garofalo & Lester, 1985; Meltzer, 1985) that teaching metacognitive and communicative aspects of numeracy skills to adults with arithmetic disability might enhance their reasoning skills. This type of instruction entails teaching how to reflect on, talk about, and actively monitor knowledge in the realm of mathematics. As Lampert (1990) described, math related dialogue among students is not only possible but is also beneficial from a knowledge building point of view. It involves the participants in the task of reflection at the same time as actively thinking - skills that has to be developed with practice. The findings of Cormier and his colleagues (1990) also emphasize the value of overt verbalization in complex cognitive activity, such as problem solving.

To form a strong base for learning, individuals need success at learning. The purpose of remedial mathematics education should be to teach adults with arithmetic disability to understand and generalize mathematical procedures, so they could apply them in their day-to-day activities. If mathematics is used as a vehicle to learn about practical issues in numeracy, then it has value for everyone even beyond the formal school years (Firestone, 1990). By active learning individuals can articulate their understanding of concepts and assumptions. This method allows for the expression of personal views. These views, then, can be challenged and confronted by another perspective presented by an instructor. This method also has the potential to facilitate the development of critical thinking related to mathematics (Lampert, 1990). Integrating talking about math into the
teaching process would allow the instructor to find out what is blocking the learning process of adults with arithmetic disability. Through discussing such misconceptions, adults with arithmetic disability would be able to fit concepts into their existing body of knowledge, and thus, actively construct their own cognitive representation of concepts. This method would increase their metacognitive skills. Open discussion about the processes involved in thinking about thinking in math would also have the potential to lower their level of anxiety related to numbers (Tobias, 1978). By learning to learn math with the help of metacognitive skills, it is possible to argue that the gap should decrease between fractions, decimals, algebra or word problem solving and interpreting numerical statements needed to make common financial decisions, such as buying or leasing a desired item or the value of paying off long overdue debts on credit cards. Even if the topic discussed during a session is not immediately relevant to the learner's life, it should be treated as a worthwhile exercise of one's cognitive capacity or as a concept that would be developed further in the future: "all mathematics is practical mathematics - application is sometimes merely a matter of time." (Eves, 1990, p. 2646).

To enhance the numeracy skills of adults with learning disability, it is important to integrate the different aspects of numeracy. It is argued that the assessment model C.A.R. proposed in this study is also useful in guiding the process of remediation. The synthesis of computation, attitude, and reasoning skills should be the goal of remediation in the domain of numeracy for adults with arithmetic disability.
Limitations of the Study

Although the research presented in this paper has begun to answer some of the questions related to assessing and remediating adults with learning disabilities, the way in which computation, attitude, and reasoning contribute to arithmetic learning disability, is far from clear, particularly among adults who are otherwise academically well-qualified. Therefore, conclusions drawn from the data are only tentative. The following three general areas need to be investigated further in relation to assessing adults with an arithmetic learning disability: 1) information gathered on the characteristics of the participants; 2) the assessment model utilized; 3) the design of the investigation. Each of these issues is discussed in turn.

Characteristics of the participants

The sample size of the three groups in this study was small. However, even with a small sample size the data revealed interesting trends for further investigation with a larger number of participants.

Including data on the social economic status of the participants would enrich the current body of literature available on adults with arithmetic disabilities by establishing some of the non-academic correlates of arithmetic disability of adults. Such information may be helpful for those who design remediation programs for adults with arithmetic disability.
Information regarding the participants' backgrounds in terms of educational history in the domain of numeracy would have been useful to interpret some of the findings of the present study. For instance, more information regarding the participants' previous experience with math could have provided information on why the mean math anxiety score of the group of adults with arithmetic disability was significantly higher than that of the normally achieving arithmetic level matched group and the chronological age matched group. Therefore, it is recommended that future investigations include data on the number of math related courses and when these courses were taken by the participants, as well as data on how well they performed in these courses and on remediation received in the past.

The assessment model

Although the examination of the participants' C.A.R. revealed several aspects of the differences between adults with and without an arithmetic disability, there is a lack of empirical evidence of this method as a superior method of assessment. Testing the reasoning skills related to word problem solving need to be developed further. It was not clear whether the lower spelling scores of the arithmetic disability group or their poorly developed metacognitive skills caused difficulties in the written expression of strategies related to word problems for adults with arithmetic disability. These components would be important to differentiate and explore in order to gain insight into the processes involved in math problem solving for adults with arithmetic disability. To investigate the cognitive strategies used by individuals with arithmetic disability, it would be valuable to conduct short interviews with the subjects after the completion of the
arithmetic problems. To go beyond the mere documentation of observable errors and to determine how subjects reached their answers, they should be asked to provide a detailed verbal description of their solution in a structured interview. If necessary, questioning and prompting should be utilized to facilitate the conversation so as to reveal the different features of thought processes involved with problem solving. During the interview the focus should be on the process the participants followed as opposed to whether the answer is correct.

Further studies will also be needed to determine the functioning on other cognitive processing correlates than computation, attitude, and reasoning of adults with arithmetic disability. For instance, limited working memory resources may have contributed to the apparent low cognitive profile of the group of adults with arithmetic disability (Geary et al., 1991; Yaghoubzadeh, Geva, & Siegel, 1996).

The design

Overall, the lack of statistical difference between the arithmetic disability and the arithmetic level matched groups was consistent on most of the tasks in the present study. According to Stanovich et al. (1986), such a uniform support of the developmental lag hypothesis is noteworthy in a multivariate study "when a large number of statistical tests are run on a set of variables, some spurious significant differences could well appear, and this tendency works against the lag hypothesis" (Stanovich et al., 1986, p. 278). However, failing to reject the null hypothesis does not prove that it is true. The best that can be stated is that there is not enough evidence one way or the other. A power analysis indicated that with a moderate effect size of 0.4 the power was calculated to be 0.64. The fact that the power was
less than 0.8 suggests that the risk of Type II error is high (Cohen, 1992) in this study. To reduce the risk of making a Type II error, the sample size per group could be increased in future investigations.

A limitation of the developmental lag design described by Kulak (1993) may apply in the present study. She called attention to the fact that if qualitative differences in performance were not observed between groups then there are two possible explanations. It may be that the data supports the developmental delay hypothesis or alternatively, that the test administered did not appropriately measure the performance in the required area of study. Kulak's argument concerns the validity of the tests used in this study. Since the tests were frequently used standardized assessment tools designed to investigate aspects of mathematics it is more likely that the results support a form of the developmental delay rather than indicate that the instruments were not adequate in measuring the performance of the participants.

There is a need for controlled and carefully planned experimental research studies in this area of learning disability. An empirical study, designed with the above suggestions in mind, could address the practical issue of how to link the teaching-learning process with pedagogy and curriculum development in the context of continuing education of adults with arithmetic disability. Research that utilizes experimental treatments will further the current understanding of the interactive relationship among computation, attitude, and reasoning skills of adults with arithmetic disability. The findings of such a study could be generalized and thus would allow the development of more personalized and involving remediation programs -- which in turn might take a direct approach to correct or accommodate existing problems of adults with arithmetic disability.
Conclusion

This multidimensional investigation was designed to describe arithmetic learning disability in an adult population. A developmental lag model of individual differences in arithmetic related skills was investigated by comparing the target group with two comparison groups, one matched on chronological age and another matched on arithmetic level.

In summary, the math related cognitive performance profiles of the adults with arithmetic disability and the arithmetic level matched groups were very similar. The participants with arithmetic disability resembled children who performed at the same level in arithmetic. They had similar levels of mental and written computational skills, relatively low self-confidence related to solving word problems, and a marked difficulty in communicating in writing the strategies they used to solve the word problems.

The findings of this study suggest a necessary modification of the developmental lag model as it relates to adults with arithmetic. Since the adults with arithmetic disability were on average twenty years older than the normally achieving arithmetic level matched group, a traditional or strong form of the developmental lag model should not be applied in this case. It is proposed that similar pattern of performance of adults with learning disability, as presented in this thesis, should be discussed within a weak developmental lag model instead.

On the basis of these data, it appears that there are at least 5 important features of arithmetic learning disability among adults. These include difficulties in: 1) working with fractions, percentage, and decimal questions, 2) performing division, and solving algebraic questions, 3) interpreting abstract information such as graphs and probability questions, 4) having poorly developed metacognitive
skills in the arithmetic domain, and 5) having a negative attitude toward mathematics. Adults with arithmetic disability did not experience difficulties in adding, subtracting, multiplying whole numbers. It is concluded that since adults with arithmetic disability exhibit difficulties other than just computational problems the assessment and remediation should reflect their needs by including procedures related to math anxiety as well as metacognitive skills.

This study demonstrated that even within a relatively small sample of participants, there are important group differences between adults with and without arithmetic disability. As well, it highlighted some of the similarities between adults with arithmetic disability and normally achieving arithmetic level matched children. Given these findings, the present study makes two important contributions to the existing literature. Adults with arithmetic disability seem to progress through similar stages of cognitive development as adults without learning disability but differ in their rate of development and the highest level reached; math anxiety and poorly developed metacognitive skills may play a significant role in the condition of arithmetic disability.
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APPENDICES
APPENDIX A

Read carefully this problem without trying to solve it.

3. A truck goes 3.1 kilometres on a litre of diesel fuel. How many litres will it need to travel the 960 kilometres to Regina?

How sure are you that you will solve the above problem correctly?

a) absolutely sure that I can do it correctly
b) quite sure that I can do it correctly
c) unsure, I don’t know whether I will do it right or wrong

d) very unsure, and I think that probably I will not succeed

e) you know that you will not be able to get it right

Now, solve the problem and include all the steps you used!
Now that you have calculated the answer, indicate how sure you are in solving it correctly.

a) I am absolutely sure that I have done it correctly
b) I am quite sure that I have done it correctly
c) I am uncertain, and I don't know whether I have done it right or wrong
d) I am very sure, and I believe that I have most probably got it wrong
e) I know that I got it wrong

Why are you so sure?

_________________________________________________________________

_________________________________________________________________

Reflect on how you have solved the problem, and describe as best as you can the strategies you used to solve the problem.

_________________________________________________________________

_________________________________________________________________

_________________________________________________________________

Try to write down what errors people may make in solving problems of this type.

1  __________________________________________

2  __________________________________________

3  __________________________________________
Appendix B

Scoring of the accuracy for the Problem solving task (KEY-MATH-R)

There were 6 short problem solving questions presented in increasing level of difficulty. The accuracy was evaluated in the following way:

100% accurate answer

2 points

For example:

Question: A truck goes 3.1 kilometers on a liter of diesel fuel. How many liters will it need to travel the 960 kilometers to Regina?

Answer: $960 / 3.1 = 309.67$

Inaccurate answer due to computational error

1 point

For example:

Question: A truck goes 3.1 kilometers on a liter of diesel fuel. How many liters will it need to travel the 960 kilometers to Regina?

Answer: $960 / 3.1 = 311$

Inaccurate answer due to procedural error, or no answer

0 point

For example:

Question: A truck goes 3.1 kilometers on a liter of diesel fuel. How many liters will it need to travel the 960 kilometers to Regina?

Answer: $960 \times 3.1 = 2976$
Appendix C

Scoring for the error production question on the Problem solving task (KEY-MATH-R)

Instruction: "Write down what errors people may make in solving a question of this type".

The answers were categorized in the following way:

**Computational mistake**
'Arithmetic error', 'make a mistake in long division'

**Procedural problem**
'Multiply when you need to divide', 'guessing', 'not knowing which formula to use'

**Indication of attitude/anxiety**
'get overwhelmed', 'anxiety', 'question is too hard', 'rushed', 'give-up',

**Not understanding question**
'do not get it', 'not reading question properly'

**No response**
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