EQUITY, EFFICIENCY AND THE SECOND BEST IN DYNAMIC POLICY ANALYSIS

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy, Graduate Department of Economics, in the University of Toronto.

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0-612-27722-4
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ABSTRACT

A general framework for dynamic policy analysis in the second best is presented. The framework is characterized by a parsimonious social welfare functional, an independent positive model of the economy, second-best constraints on lump-sum redistribution, market failures, and inter-regional trade equilibrium.

Chapter one reviews the literature regarding market-based vs. social discounting in cost-benefit analysis. The major arguments in favour of market discounting are analyzed and found to be inappropriate in the context of the second best. Alternative techniques to account for the opportunity cost of public finance and ensure the efficiency and feasibility of policy are reviewed.

Chapter two is concerned with the choice of a dynamic social welfare functional. The familiar isoelastic form is adopted with a number of changes designed to make it conform better with common principles of equity, particularly anonymity. Two decision rules are derived from the isoelastic welfare functional for use in cost-benefit analysis: one a standard welfare change measure, and the other the present value of dollar benefits which relies upon
the social rate of discount. Since both decision rules require information on the distribution of a project's net benefits, consistency of policy analysis requires the use of a common analytical framework, rather than a common social rate of discount.

Chapter three demonstrates the application of the second-best framework by estimating the optimal control of greenhouse gas emissions in a four-region model of the world. To date, most economic research on policies to control global warming has been limited to internalizing the greenhouse externality in a positive model of the economy. In contrast, chapter three determines the optimal policy by constrained welfare maximization. Control paths of emissions and equivalent carbon taxes (permit prices) are estimated, as well as paths of side payments (permit allocations) to compensate poorer regions for their abatement efforts. The effect of the second-best framework is to give much higher levels of emissions abatement than most previous research. Emissions reductions are on the order of four times higher than results from positive models, and equivalent carbon taxes are at least fourteen times higher.
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INTRODUCTION

The appropriate context for dynamic policy analysis is the second best, no less so than for static analysis. Nevertheless, many empirical researchers have continued to prefer the simplicity afforded by first-best rules. In dynamic problems, the first best entails, among other things, using the marginal product of capital as the discount rate in cost-benefit analysis. This state of affairs persists despite the progress that has been achieved at the theoretical level in developing a second-best, social rate of discount.\footnote{See especially Bradford (1975) and Sen (1982).}

Dynamic policy adds an intergenerational aspect to the familiar issues which define the second best — market failures, imperfections and distributional inequities. The question of whether there exists significant inequities in the distribution of wealth between generations has caused a great deal of debate. Many argue that the present generation is saving too little, while to others this argument seems incongruous in light of the prospect of continued consumption growth arising from technical change. Indeed, the topic of dynamic inefficiency raises the possibility that the present generation might be saving too much.

At their root, such disagreements over the existence of intergenerational inequities probably arise from a lack of clarity about the meaning of the term. Unfortunately the common definition of equity as fairness does not adequately express the intended implication of an underlying social welfare functional. In the context of a welfare functional, a distribution is equitable if the corresponding market (Pareto) equilibrium is also the welfare optimum; otherwise, the distribution is "inequitable". According to this definition, both those who argue that the present generation saves too little and those who argue that it saves too much are arguing that an intergenerational inequity exists. A more precise term would be
"suboptimal". However, the use of equity (or inequity) has the advantage of distinguishing clearly the issue of distribution from that of efficiency. Moreover, it is conventional. Therefore, it is maintained in the present study.

The linking of equity to an explicit social welfare functional makes clear the strong likelihood that intergenerational inequities do exist. Of particular significance is the fact that an inequity can exist to the detriment of a group even if the group enjoys higher consumption or utility than others. Such is the case for social welfare functions which define optimality in terms of marginal conditions. For example, under utilitarian welfare, an intergenerational inequity exists to the detriment of future generations if the marginal product of capital is higher than the social rate of discount. In contrast, a welfare function which defines optimality in terms of utility levels rather than marginal conditions would probably result in a different assessment. The best-known example of this variety is Rawls' maximin rule.

An equilibrium value of the marginal product of capital that is higher than the social discount rate can be explained in a number of ways. Perhaps the most common explanation is the simultaneous taxation of investment income and principal. A more fundamental explanation is the existence of differences between the social welfare functional and the process which generates the market equilibrium. According to this view, individuals consent to a social welfare function while behaving contrary to it in their private decisions. There are several explanations in the literature why people might behave in this manner. One is that the social welfare functional is agreed upon in a hypothetical original position, whereas personal decisions are made "after the game is under way". Another explanation is that there is a public-goods aspect to private savings, and therefore the level of private savings is sub-optimal. A third explanation — one that will figure very prominently in this dissertation

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2Of course, this form of taxation is also Pareto inefficient.
— is that individuals must discount the future more heavily than the planner because they are uncertain about their time of death, whereas the planner has an infinite life.\textsuperscript{3}

Any one of these explanations provides a reasonable argument for the existence of a gap between the marginal product of capital and the social discount rate. Therefore, it seems only prudent to account for the possibility of intergenerational inequities when conducting dynamic policy analysis. More precisely, it seems likely that society currently saves too little — at least under a utilitarian welfare objective — especially in the matter of natural resources and environmental policy.

A further complication of dynamic policy analysis is the interaction of the various elements of second-best. For example, intertemporal externalities, in which the costs and benefits of certain activities accrue in different periods, also have an impact on intra-generational distribution if the costs and benefits are allocated differently within generations. Perhaps the most prominent case of such an intertemporal externality is the likely existence of a global warming effect. In this scenario, the emission of greenhouse gases from the burning of fossil fuels causes changes in the global climate over the long term. These changes in turn are expected to cause damage to many economic sectors, especially agriculture, as well altering the natural environment and endangering human health. These costs will accrue to future generations, while the benefits of burning fossil fuels accrue to the present generation in the form of present income. The externality has an intra-generational effect, in addition to the more obvious inter-generational effect, because the costs are expected to fall hardest on the poorest regions of the world, whereas the benefits to date have accrued mostly to the rich industrial countries. This interaction of

\textsuperscript{3}Boadway and Bruce (1984) provide an excellent overview of the literature on social welfare functions. For references on the specific points mentioned above, see pp. 179-82 and 317-20.
inter-generational and intra-generational effects with market failure calls for a very judicious formulation of second-best policy.

The task at hand, then, is to develop a general framework for dynamic policy analysis in the second best which is flexible enough to account for a number of interdependent policy problems. Such a framework would represent an advance over current tools, particularly in the determination of appropriate policies to deal with market failures or imperfections in the context of multi-dimensional inequities in the distribution of wealth. The framework presented in the following chapters will be characterized by:

- a parsimonious social welfare functional, with no intergenerational discounting of utility, in keeping with the anonymity axiom of social choice theory;
- a positive model of the economy which is independent of the social welfare functional;
- the explicit modelling of the constraints which are expected to exist on lump-sum or other forms of redistribution;
- modelling of the relevant market imperfections or failures;

In this framework, the equity of policy choices will be ensured by maximization of the social welfare functional, subject to appropriate constraints. The specification of an independent model representing the positive economy is necessary due to the strong likelihood of distributional inequities arising from a divergence between the social welfare functional and the process governing the positive economy. One of the key constraints on the maximization of the social welfare functional is that the equilibrium conditions of the positive model must be satisfied. In this way, the analyst accounts for both the effects of

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*One is reminded that in the so-called optimal growth model associated with Cass and Koopmans (and somewhat spuriously with Ramsey), the social welfare functional and the positive objective functional are one and the same, namely an infinite sum (integral) of discounted utility. Upon reflection, it is clear that this objective would probably never win acceptance as a social welfare functional. Perhaps the model should be renamed the positive growth model.
policy on general equilibrium and the effect of general equilibrium on the feasibility of policy. Finally, efficiency requires, among other things, accounting for the opportunity cost of public finance.

Considered in isolation, none of the elements of this framework is new. However, the combination of all the elements in the dynamic context entails a level of computational sophistication which is only now becoming possible for researchers who rely on personal computers to solve numerical optimization problems. It is hoped that the framework that is presented in the following three chapters will provide a useful template for analysts who wish to find relevant answers — that is, answers in the context of second best — to dynamic policy problems.

Chapter one reviews the literature on the question of market-based vs. social discounting in cost-benefit analysis. The major arguments in favour of market discounting are presented and found to be inappropriate in the context of the second best. Alternative techniques to account for the opportunity cost of public finance and to ensure the efficiency and feasibility of policy are reviewed.

Chapter two is concerned with the choice of a dynamic social welfare functional. The familiar isoelastic form is adopted with a number of changes designed to make it conform better with common principles of equity, particularly the principle of anonymity. Two decision rules are derived from the isoelastic welfare functional for use in cost-benefit analysis, one a standard welfare change measure and the other the present value of dollar benefits which relies upon the social rate of discount. Since both decision rules require information on the distribution of a project's net benefits, consistency of policy analysis requires the use of a common analytical framework, rather than a common social rate of discount as often thought. The chapter concludes with some useful suggestions concerning the
choice of data for the determination of parameter values.

Chapter three demonstrates the application of the full policy framework by estimating the optimal control of greenhouse gas emissions in a four-region model of the world. Among other features, this model is characterized by:

- intra-generational distributional inequities, in the form of different per capita income levels between the regions;
- an independent general equilibrium process, reflecting inter-generational inequities due to the divergence of the welfare functional and the general equilibrium process;
- limited compensation flows from rich to poor regions;
- efficient abatement of greenhouse gas emissions.

To date, most economic research to control global warming has been limited to internalizing the greenhouse externality in a positive model of the economy. Among the most prominent examples of this line of research are Nordhaus (1994) and Nordhaus and Yang (1996). In contrast, chapter three estimates socially optimal policies in the context of the second best. Control paths of greenhouse gas emissions and equivalent carbon taxes (permit prices) are estimated, as well as paths of side payments (allocations of tradeable permits) to compensate poorer regions for their abatement efforts. For the sake of comparison, the climate-economy model of Nordhaus and Yang (1996) is used, with modifications to accommodate the second-best framework. The effect of this approach is to give significantly higher initial levels of emissions reduction and taxation than most previous research. For example, in chapter three, the most conservative estimate of the optimal initial value of a carbon tax is $110 (U.S.) per ton. If the incidence of this tax were to fall entirely on consumers, the resulting increase in the price of gasoline would be approximately 20 percent. The most aggressive strategy in chapter three results in estimates twice as high. In contrast,
Nordhaus and Yang (1996) estimate an optimal initial tax in a positive model of the economy at just under $6 per ton of carbon, which is equivalent to a 1.3 percent increase in the price of gasoline. Clearly, the difference between first-best and second-best policy analysis is significant, at least in the matter of greenhouse warming.
1.1 Introduction

The debate on the correct choice of discount rate in cost-benefit analysis has continued since the 1950's without achieving a consensus. In its broadest outline, the debate is characterized by two opposing points of view, one advocating discounting by the market rate of interest or some variant of it, and the other advocating discounting by a social rate not directly related to the market rate of interest. The practice of discounting with the market rate is supported by three distinct arguments in the literature. The first is a relatively straightforward opportunity-cost argument, based upon a concern for the displacement of private capital by the financing of public projects. The second argument in favour of market discounting is that allocative and distributional concerns can be separated by virtue of the second theorem of welfare economics. The third argument in favour of market discounting concerns the possible non-convergence of the present value expression of a project under low rates of social discount and an infinite horizon. The practice of discounting with a social rate of discount is supported by the existence of market imperfections and constraints on the government's ability to implement lump sum redistribution. In the presence of such imperfections and constraints, the allocations and prices which prevail in the uncontrolled economy, including the interest rate, are not consistent with the optimization of social welfare. Therefore, cost-benefit analysis becomes an exercise in second-best policy making (indeed n-th best), in which the discount rate must be derived directly from a social welfare function (SWF).

A moment's reflection is sufficient to confirm that market imperfections, failures and
constraints are in fact ubiquitous. Therefore, it seems prudent to heed the warning of those who argue in favour of a social rate of discount and conduct cost-benefit analysis on the basis of an explicit social welfare functional. Nonetheless, the arguments made in favour of market discounting raise important issues concerning opportunity cost, efficiency, and non-convergence which must not be ignored. This chapter investigates these issues in the context of second-best cost-benefit analysis. The first section of the chapter summarizes the three main arguments for market discounting which were mentioned above. The second section demonstrates the shortcomings of these arguments. Finally, the third section of the chapter reviews appropriate techniques to account for opportunity cost, efficiency and non-convergence in the context of a social welfare functional. In brief, the displacement rationale for market discounting is found to lack generality, even in the context of the first best; the new welfare economics is not valid in the context of the second best, and the non-convergence rationale entails subordinating one’s concern for equity to the exigencies of the mathematics. Instead of market discounting, opportunity cost can be accounted for with explicit cost items or by marking up direct costs with a shadow price of capital. Efficiency must be understood in terms of constrained welfare maximization and can only be ensured by comparing the entire set of possible projects. Finally, non-convergence is best addressed with von Weizsacker’s overtaking criterion. Because these methods are independent of the choice of the discount rate, they are certainly appropriate for use in conjunction with a social rate of discount. Indeed, one of the major causes of the failure to achieve a consensus regarding the rate of discount is the belief that all issues can be addressed simultaneously by the correct choice of the discount rate. In contrast, by developing separate techniques for distinct issues, the concept of the discount rate is unburdened and allowed to perform its essential function, which is to reflect the social assessment of intertemporal tradeoffs.
1.2 Three arguments for market discounting

The displacement argument in favour of market discounting is simple and well-known. The market rate of interest is assumed to be the true opportunity cost of capital. Therefore, the use of a relatively lower social rate of discount results in losses for society, because it leads to the undertaking of public projects which divert capital from private projects with higher rates of return.

The argument for market discounting based upon the second theorem of welfare economics presents a different face to what is essentially the same concern, namely opportunity cost and efficiency. By virtue of the two fundamental welfare theorems, it is possible under ideal circumstances to separate the issue of allocative efficiency from that of equity, provided that the initial distribution of income and wealth is optimal or that the government can make lump-sum transfers to make it optimal. In this view, the proper concern of cost-benefit analysis is allocative efficiency, since equity is either already satisfied or will be taken care of separately. Allocative efficiency is characterized by the Pareto principle, which is embodied in the Hicks-Kaldor compensation criterion. According to the compensation criterion, a project should be approved if the gainers can afford to compensate the losers and still be better off. Future costs and benefits are discounted by the market rate of interest because it is the rate at which the gainers can transfer benefits into the future, in the form of savings, and losers can transfer costs into the future, in the form of debt or reduced savings. Any other rate is dominated by the market rate because aggregate gains are maximized with it, and these gains can be used to compensate losers. Therefore, the market rate is Pareto efficient. In practice, it is not important whether compensation is actually paid, according to this line of reasoning, since in theory the government is simultaneously adjusting the distribution of income and wealth to achieve equity. Howarth and Norgaard
Page (1988) and Freeman (1977) invoke the compensation criterion to justify using the market rate of interest as the discount rate.

Finally, the non-convergence argument in favour of market discounting provides a warning against using a social discount rate that is too low. The problem is that a rate that is too low will result in a non-convergent expression of the present value of a project if the project is also characterized by an infinite horizon. Non-convergent present values make comparison of projects almost impossible, although the sign of the expression can often still be determined. However, if the present value does not have a limit, then not even its sign can be determined. Heal (1993) uses this argument to refute Ramsey’s famous injunction against utility discounting.\(^5\) With utility discounting thus defended, the first-order necessary condition of the Ramsey-Cass-Koopmans optimal growth model can be invoked to justify discounting with the market rate of interest. The necessary condition is \(\delta + \alpha g = r\), where the entire left-hand side is the consumption discount rate, \(\delta\) is the utility discount rate, and \(r\) is the market rate of interest. Since the consumption discount rate (i.e. left-hand side) equals the market interest rate in this model, one may as well use the interest rate as calculate the consumption discount rate directly. Thus, an argument in defence of utility discounting (positive \(\delta\)) becomes an argument for discounting by the market rate of interest.\(^6\)

1.3 The case against market discounting

None of the three arguments summarized above provides a sound basis for choosing the rate of discount. There are three flaws with the displacement argument for market

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\(^5\)Ramsey (1928) describes utility discounting as "a practice which is ethically indefensible and arises merely from the weakness of the imagination" (p.543).

\(^6\)Nordhaus (1994, 124) invokes the necessary conditions of the optimal growth model to justify market discounting. For a description of the model, see Blanchard and Fischer (1988, 39-41).
discounting. First, Bradford (1975) has shown that the argument is only valid in the unlikely event that reinvestment out of the returns of the public project is identical to the reinvestment that would result from the displaced private project(s). Unless this condition is met, the market rate of interest does not reflect the opportunity cost of public funds. The second flaw of the displacement rationale is that it implies that public projects are automatically approved whenever a positive cost-benefit evaluation is obtained, without any regard for ranking projects in order of the size of benefit. While governments may at times operate in this fashion, this fact does not provide an argument against a social rate of discount but against evaluating projects in isolation from one another. Finally, the third flaw of the displacement rationale is that displacement may in fact be insignificant in an open economy with a high degree of capital mobility. As Lind (1990) observes, foreign capital disconnects the tight relationship between domestic savings, private investment and government finance. Although new public spending in one jurisdiction must displace spending somewhere, the assumption of no domestic displacement may be reasonable for small projects.

Regarding the second argument in favour of market discounting, which is based upon the new welfare economics, the existence of market imperfections and the inability of government to effect redistribution in a lump-sum fashion means that the preconditions for the neat separation of allocative efficiency and redistribution are not met, and therefore the justification for using the Hicks-Kaldor compensation criterion is lost. In the real economy, it is highly unlikely that the market equilibrium is Pareto efficient, let alone that it coincides with the social optimum. But even if it were Pareto efficient, the usefulness of Pareto efficiency as a welfare criterion is diminished if the government cannot implement lump-sum

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7The fact that the government could get the distribution right in the future is not a valid argument for using the current rate of interest now, because current market prices, including the interest rate, are dependent upon the distribution.
transfers to move society to the social optimum. What is required is a more general notion of efficiency than the Pareto principle, one that is defined in terms of the social optimum rather than the status quo. Obviously, welfare maximization is the relevant concept, where the maximization is subject to the various imperfections and constraints in the economy.

The compensation criterion would still seem to provide a minimal level of equity in the second best if compensation is actually paid, since no one is actually made worse off under this rule. However, without an optimal initial distribution, the compensation criterion implies a preference for the status quo which is not supported by the social welfare function. And indeed it is well-known that, for this reason, the compensation principle provides only a partial ordering of projects or social states. It cannot rank projects which entail permanent losers in addition to winners. In contrast, the compensation principle without compensation is wholly indefensible in the absence of an optimal initial distribution. In general, if compensation is actually paid to future generations, in the form of increased savings, then the market rate of interest is the relevant technology for maximizing the returns to this savings, assuming it is the true opportunity cost of capital. But if compensation is not paid, then the interest rate is not relevant to the project under conditions of second best, except insofar as it relates to the value of displaced private investment.

A flaw of both the new welfare economics and the displacement rationale is that the market rate of interest is assumed to be the true opportunity cost of capital, when in fact this assumption is probably not satisfied. Figure 1.1 is used to show why not. The left panel of Figure 1.1 represents private sector investment opportunities, with the projects lined up along the x-axis in descending order of their rate of return. Similarly, the right panel represents

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8See Boadway and Bruce (1984, 96-102) for a discussion of the compensation criterion.

9The comparison of the compensation criterion with and without compensation is due to d'Arge et al. (1982).
public sector projects. Private projects are denoted \( x \), and public projects \( y \). The market rate of interest or marginal product of private capital is represented as \( r \), and corresponding to it there is a marginal private project, \( x \). Similarly, there is a public project \( y \), whose rate of return is equal to the market rate of interest. The social rate of discount is represented by \( \rho \), and, for the sake of demonstration, it is assumed to be less than \( r \). The private and public projects with returns equal to the social discount rate are denoted \( x_s \) and \( y_s \).\(^{10}\) By assumption, all private projects with return greater than \( r \) have been undertaken by private agents but none with return lower than \( r \). Consequently, conventional wisdom holds that \( r \) is the opportunity cost of capital. However, in order to know the true opportunity cost of capital, it is also necessary to know the pattern of public investments. Obviously, the true opportunity cost is \( r \) only if all public projects to the left of \( y \), have been undertaken. But in general there is no reason to expect this condition to be met. It is only due to the large number of private agents, all of them searching for new opportunities, that all intra-marginal projects in the private sector are undertaken. By contrast, there are typically only two or three levels of government in any jurisdiction and therefore only that many bureaucracies dedicated to searching out and evaluating public investment opportunities. Moreover, governments are certainly not as quick as private entrepreneurs at converting research into projects. Therefore, it is almost certain that the pattern of public investment will have many gaps along the horizontal axis in Figure 1.1. In particular, there is no reason to assume that all projects to the left of \( y \), have been undertaken. Therefore, \( r \) is unlikely to be the true opportunity cost of capital. Rather, some value greater than \( r \), corresponding to an as-yet undiscovered public project to the left of \( y_r \), is likely the true opportunity cost of capital. Yet

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\(^{10}\)Figure 1.1 is drawn as if the profiles of private and public sector projects were independent of each other. In fact, the rate of return of a given project is usually highly dependent upon the choice of other projects, both public and private. Fortunately, the assumption of independence does not invalidate the subsequent discussion, while simplifying the presentation considerably.
by definition this value is unknown. Fortunately, it is not necessary to know the true opportunity cost of capital in order to maximize welfare conditional upon known information. For this task, only the cost of displaced (or stimulated) private investment and a discount rate that expresses the social assessment of intertemporal tradeoffs are needed. The proper treatment of displaced (stimulated) private investment will be discussed momentarily. As for the social assessment of intertemporal tradeoffs — i.e. the social discount rate — this value can only be derived from a social welfare functional.

Finally, the third argument in favour of market discounting, namely that a low social discount rate can result in a non-convergent present value, is also unpersuasive. The problem is that this argument requires sacrificing the ethical attributes of the social discount rate to the mathematical exigencies of an infinite horizon. It would be much better to find a way to adapt the mathematics to the ethics. Such an approach has been developed and will be discussed momentarily.

1.4 Opportunity cost, efficiency and non-convergence

Despite the failure of the arguments in favour of market discounting, they nonetheless raise important issues concerning opportunity cost, efficiency and feasibility which must be addressed when discounting with a social rate. Three rules or techniques have been developed in the literature which are sufficient to address these concerns: one to account for the displacement of private capital, one to ensure welfare maximization, and one to deal with non-convergence.

Bradford (1975) shows that the correct technique for dealing with the displacement (or stimulus) of private capital is to account for these items in the stream of costs and benefits of the project rather than through the discount rate. These new cost and benefit items can be
modeled explicitly if the analyst is using a general equilibrium framework. For example, in mixed economy models, households’ budget constraints account for the effects of government borrowing and taxation on private investment. Alternatively, if the analyst is using a partial equilibrium framework, displacement can be accounted for by using either Bradford (1975) or Lind’s (1982) shadow value of capital to mark up the direct costs and benefits. Discounting then proceeds with the social rate, derived from the social welfare functional.\textsuperscript{11,12}

The means by which the analyst ensures welfare maximization in cost-benefit analysis depends upon whether the government faces any constraint in its discretion to tax and spend. If it does not, then a positive cost-benefit evaluation is all that is needed for a project to go ahead, since under these circumstances the government undertakes all projects with a return greater than the social discount rate, in both the public and private sectors, in order to maximize welfare. On the other hand, if there is a constraint on the government’s discretion, then in order to maximize welfare a project must be among the subset of projects with the highest evaluations which exhaust the spending constraint.\textsuperscript{13} Of course, if the government fails to check for welfare maximization in this fashion, then it may indeed be misled into approving a lower-return project at the expense of a higher-return project. But the problem in this case is not the social rate of discount but the failure to check for maximization. While

\textsuperscript{11}Neither Bradford nor Lind advocate a social rate of discount. Rather, they use the consumer’s discount rate (equal to the after-tax market rate of interest), as distinct from the marginal product of capital (before-tax market rate). Lind acknowledges that the shadow value of capital can be constructed with a social rate.

\textsuperscript{12}Lind’s shadow value of capital, while more complete than Bradford’s, is often undefined as a result of the infinite horizon used in its construction. To deal with this problem, he resorts to some rather arbitrary restrictions on variables in order to get a finite value. A better approach would be to impute the costs and benefits of displaced and stimulated capital as they accrue, even in a partial equilibrium analysis, and then confront the issue of convergence all at once by means of the overtaking criterion, which will be discussed shortly. A technique for the imputation of costs and benefits can be inferred from the construction of the shadow price. Cline (1992, 270-4) mistakenly attributes the divergent tendency of Lind’s shadow price to double counting of displaced private capital, but there is no double counting.

\textsuperscript{13}Boadway and Bruce (1984, 295).
the coordination of projects in this fashion is certainly more costly than a one-shot approach in which projects are evaluated in isolation, the absence of a reliable one-shot approach makes it unavoidable.

The rules for welfare maximization are not affected if the government is constrained to stay out of the private sector, for it is sufficient that the rules be applied to the subset of projects which the government is permitted. However, one consequence of being constrained from entering the private sector is that the government may undertake public projects with lower returns than the marginal private project (after accounting for displaced capital) even while maximizing welfare. The apparent contradiction of this statement is explained by the fact that the welfare maximization in this case is conditional upon the constraint barring the government from the private sector. Since the government cannot undertake private projects, no attainable welfare is lost. Indeed, insisting that the rate of discount be equal to the market rate in this case would result in an unnecessary welfare loss. Figure 1.1 illustrates the point. It is socially optimal that all projects up to \( x_p \) and \( y_p \) be undertaken. Therefore, if the government is prohibited from investing in the private sector, then projects between \( x_r \) and \( x_p \) will not be undertaken, resulting in a loss of welfare. A further loss is incurred if the market rate is used to discount public projects, because then projects between \( y_r \) and \( y_p \) will also not be undertaken. Clearly, under these circumstances it is desirable that investment should be asymmetric between the two sectors (assuming all displacement costs are treated properly), with the lower social rate of discount used in the public sector.

The final difficulty when discounting with a social rate, the possibility of non-convergence of the present value expression, is addressed by using the overtaking

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14In practice, governments are not usually prohibited by legislation from investing in the private sector, but the politicization of government ventures may have the same effect, by reducing the returns below the level which private agents can earn in the same activity.
criterion developed by von Weizsacker (1965). Developed in the context of optimal planning, the overtaking criterion is adapted here to cost-benefit analysis. Let the dollar value of the aggregate net benefits of a project at time \( t \) be represented as \( Z(t) \). Then the project can be represented as the infinite sequence \( \{ Z(t) \}_{t=0}^{\infty} \). Further, let the present discounted value of the project up to some arbitrary horizon, \( T \), be denoted \( Z(0,T) \). Also, define \( D(t) \) to be the social discount factor at time \( t \). Then \( Z(0,T) = \int_0^T D(t) Z(t) \, dt \). The overtaking criterion can now be defined for the cost-benefit context.

**Overtaking criterion:** A project \( \{ Z_A(t) \}_{t=0}^{\infty} \) overtakes another project \( \{ Z_B(t) \}_{t=0}^{\infty} \) if there exists some \( T^* \) such that, for all \( T \geq T^* \), \( Z_A(0,T) - Z_B(0,T) > 0 \).

In words, this condition states that project A is considered better than project B if there exists a finite \( T^* \) such that the present discounted value of project A at time \( T^* \) exceeds that of project B, and the inequality remains in the same direction for all \( T \geq T^* \). A project is best if it overtakes all other alternatives under consideration. Society is considered to be indifferent between two projects if the overtaking criterion cannot be satisfied for the pair. Thus, the overtaking criterion gives a complete decision rule for ranking any set of projects.

The overtaking criterion is relatively simple in the cost-benefit context because the analyst is given only a finite set of projects to evaluate. Given such a finite set, it is sufficient to make pairwise comparisons in order to obtain a complete and transitive ordering of all the projects in the set. In contrast, optimal planning with the overtaking criterion is not
as simple, since in this case the analyst must find the optimal policy from the entire feasible set, which usually has an unlimited number of elements. Von Weizsacker (1965), Gale (1967) and McFadden (1967) develop analytical solutions for a number of control models using the overtaking criterion. In general, analytical solutions are much more difficult to obtain with the overtaking criterion than with a convergent objective. However, most empirical models are too complex for analytical solutions anyway. The standard approach in empirical control problems is to choose a finite horizon sufficiently large that the solutions for early periods are insensitive to further increases in the horizon. The fact that this approach does not yield the entire program over an infinite horizon is not important from the policy perspective, since, in an uncertain environment, the exercise is to be re-done periodically anyway, as new information becomes available. Thus, the policy choices in the near-term are always the correct ones, even if the long-term values are not.

While the last two sentences smack of dynamic inconsistency, in fact such a concern is misplaced. In a certainty context, the planner or policy-making agency makes a commitment regarding the policy choices in the near term, i.e. those that are on the turnpike of the infinite planning exercise, but not about those further out. Assuming the policy-making apparatus is dynamically consistent, any repetition of the exercise in a subsequent period will not change the policy choices which were previously on the turnpike. In an uncertainty context with missing markets (e.g. missing insurance markets), the economy has a sequential structure. In each period, the planning exercise is updated to reflect the new information that has become available — i.e. the state of the world that has been revealed for that period — and the policy choices are adjusted accordingly. Indeed, all agents adjust their choices in light of the new information. In this world, the planner cannot make a certainty commitment about future policy choices but only a commitment concerning the expected value of policy.
Subsequent realizations of policy which differ from the expected values reflect the resolution of uncertainty but not dynamic inconsistency of the policy-making apparatus. Of course, policy choices of a putty-clay nature, such as the building of physical infrastructure, cannot be changed as easily as other announced policies. In contrast, taxation can be changed easily in light of the sequential resolution of uncertainty. Of course, in an uncertainty context with complete markets, all policies and all private decisions are made at the outset. The planner makes commitments regarding policy choices contingent on states of the world, and private agents buy and sell claims to contingent commodities. In this world, the sequential resolution of uncertainty does not alter the planner’s announced policy choices.\textsuperscript{15}

\textsuperscript{15}See Starrett (1988, 100-03) for a discussion of sequential economies in the presence of missing markets.
CHAPTER 2
EQUITY AND OVERLAPPING GENERATIONS
IN WELFARE-BASED DECISION RULES

2.1 Introduction

Most treatments of intergenerational equity in public economics to date have focused on one of three topics: the social rate of discount in cost-benefit analysis, the Rawlsian maximin objective, or sustainability of per capita consumption paths. Alternatively, the constant-elasticity-of-substitution or isoelastic form of welfare provides a more general context for considering intergenerational equity, with utilitarianism and maximin represented as special cases.¹⁶

Usually, when used in a dynamic setting, the isoelastic welfare functional has instantaneous utilities as its arguments. Unfortunately, this practice provides a poor representation of intergenerational equity, because it suggests an unwarranted concern for an individual’s prospects from moment to moment. From the perspective of equity, it would be better to choose a single value to represent the total lifetime well-being of each individual and use these values as the arguments of the social welfare functional. Blanchard’s (1985) calculation of expected lifetime utility provides an ideal candidate for such a value. Calvo and Obstfeld (1988) incorporate this value into a utilitarian welfare structure, and Marini and Scaramozzino (1995) adapt Calvo and Obstfeld’s framework to include environmental externalities and population growth. The present chapter extends Calvo and Obstfeld’s work by incorporating expected lifetime utility into an isoelastic welfare structure, and then derives two cost-benefit decision rules.

In addition to a more equitable accounting of individuals, the use of expected lifetime utility

¹⁶See Boadway and Bruce (1984, 141-2) for an introduction to isoelastic welfare.
utilities as the arguments of social welfare has the benefit of introducing overlapping
generations (OLG) into the model. The use of overlapping generations permits a distinction
to be made between personal time preference and the social discounting of generations.
Personal time preference is used in the calculation of individuals' expected lifetime utilities,
while generational discounting reflects the planner's preferences regarding the aggregation of
utilities in social welfare. Given a concern for intergenerational equity, the logical choice of
the generational discount rate would appear to be zero. This choice follows from the
anonymity axiom of social choice theory and is intuitive. The perennial concern that a low
rate of discount will result in an inefficient use of resources is misplaced and has been
addressed elsewhere.

The first decision rule presented in the chapter gives the value of a project in terms of
welfare, while the second is the familiar discounted dollar value, obtained by means of the
social rate of discount for aggregate consumption. Both decision rules are shown to
require rather detailed information on the distribution of project net benefits and lifetime
consumption paths. As a result, the search for a universal social discount rate which can be
applied to aggregate benefits without any consideration of distribution must be abandoned as
futile. Instead, consistency of policy analysis can be assured by the adoption of a common
welfare framework and positive model of the economy. The isoelastic OLG model presented
here must be considered a strong candidate for the welfare side of such a framework, given
the appeal of its underlying ethical properties.

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17 Burton (1993) points out that this separation of personal and generational discount rates resolves much of the confusion which has prevailed on the topic of discounting.

18 See especially Bradford (1975) and Lind (1982). In brief, the opportunity cost of public finance is best accounted for by means of adjustments to the stream of costs and benefits rather than the choice of discount rate.

19 Henceforth this discount rate will be referred to as the aggregate social rate of discount.
The chapter is divided into four sections. The first section develops the isoelastic welfare functional with Blanchard's expected lifetime utilities. The second and third sections derive the decision rules for cost-benefit analysis. Since both rules are shown to depend critically on distributional information, it is a matter of indifference which one the analyst uses. The fourth section discusses the implementation of the framework, with some suggestions regarding definitions of variables and potential data sources for the parameters.

2.2 Isoelastic social welfare and expected lifetime utility

It is assumed initially that individuals of the same generation have the same lifetime income and therefore the same expected lifetime utility. Differential income levels within generations will be introduced later, by means of a type variable. With this assumption, the isoelastic SWF is

\[
W(0) = \int_{-n}^{\infty} \Psi(v) \frac{U(v)^{1-\rho}}{1-\rho} \, dv.
\]

(2.1)

\(W(0)\) is welfare at time 0; \(v\) is the generational index, denoting "vintage"; \(U(v)\) is the expected lifetime utility of an individual of vintage \(v\) (i.e. born at time \(v\)); \(\rho\) is the inverse of the elasticity of substitution of expected lifetime utilities along the social indifference curves; \(\Psi(v)\) is the size of the cohort born at time \(v\); and \(n\) is the maximum age attainable by any individual. Note that the lifetime utilities of those born before the initial period but still alive after it are included in the SWF.

The expected lifetime utilities of different generations are treated symmetrically in (2.1). This feature is justified on the basis of equity, by the anonymity axiom of social choice theory. Simply stated, the anonymity axiom requires that a preference ordering such as the SWF in (2.1) ignore the identities of individuals in aggregating their preferences. Without an
anonymity axiom, isoelastic welfare could be written as

\[ W(0) = \int_{-h}^{\infty} \Psi(v) e^{-\alpha v} \frac{U(v)^{1-\rho}}{1-\rho} \, dv, \]

where \( \alpha \) represents a generational discount rate. A generational discount rate of zero is implicit in (2.1).²⁰

The isoelastic SWF takes on a number of familiar forms depending on the value of \( \rho \). When \( \rho = 0 \), it is the classical utilitarian. The limit as \( \rho \to 1 \) is the Bernoulli-Nash (linear in the logarithms). And as \( \rho \to \infty \), it is the Rawlsian maximin form. These cases are characterized by the shape of the social indifference curves shown in Figure 2.1. Because it is a function (i.e. the inverse) of the elasticity of substitution along these indifference curves, \( \rho \) reflects society’s willingness to trade-off utility between individuals. A greater value of \( \rho \) reflects greater social aversion to inequality. Therefore, \( \rho \) may be considered an equity parameter, chosen by consensus or some other political means.

The structure of expected lifetime utility which is adopted here is similar to Blanchard’s (1985) model of uncertain lifetimes. The individual’s instantaneous utility is represented as \( u(g(t), c(v,t)) \) where \( g(t) \) is the vector of public-goods flows at time \( t \) and \( c(v,t) \) is the vector of private consumption at time \( t \) of the generation born in period \( v \).

Individuals are uncertain about the length of life they will enjoy. Therefore length of life is represented as a random variable, \( h^* \), with a continuous and at-least-twice-differentiable cumulative distribution function, \( F(h^*) \), and a related probability density function, \( f(h^*) \). \( h^* \) is bounded from above by \( \bar{h} \), the maximum feasible age. At every age, \( h \), the individual is faced with an instantaneous death rate, \( p(h) \), which gives the probability that she will die at

²⁰See Boadway and Bruce (1984, 143-61) for a discussion of the complete set of social choice axioms needed to generate isoelastic welfare.
Figure 2.1
that age. The probability that an individual is still alive at age \( h \) is \( 1 - F(h) \), which is equivalent to \( \exp\left\{- \int_0^h p(s) \, ds\right\} \), or \( e^{-ph} \) when \( p(h) \) is constant.\(^{21}\) For notational convenience, \( p(h) \) will be assumed to be constant in the remainder of the chapter.

The expected lifetime utility of an individual born at time \( v \) is defined to be

\[
U(v) = \int_v^{v+h} \left\{ \int_v^t u[g(s), c(v,s)] e^{-\theta (t-v)} \, ds \right\} f(t-v) \, dt ,
\]

which, after integration by parts, becomes

\[
U(v) = \int_v^{v+h} u[g(t), c(v,t)] \left[ 1 - F(t-v) \right] e^{-\theta (t-v)} \, dt .
\]

All of the variables in this expression have already been defined, with the exception of \( \theta \), which is the individual’s pure rate of time preference.\(^{22}\) Substituting for \( 1 - F(t-v) \), the expression becomes

\[
(2.2) \quad U(v) = \int_v^{v+h} u[g(t), c(v,t)] e^{-(\theta + p)(t-v)} \, dt .
\]

In words, expected lifetime utility is equal to the integral of discounted instantaneous utility,

\(^{21}\)Proof (Calvo and Obstfeld, 1988): The instantaneous death rate, \( p(h) \), is calculated from the distribution of \( h^* \) as \( p(h) = f(h)/[1-F(h)] \). Because \( f(h)=F'(h) \) and \( f(0)=0 \), this expression can be rearranged and solved as a differential equation for \( 1-F(h) \), yielding the result.

\(^{22}\)Having controlled for uncertainty about the individual’s time of death, it is not entirely clear why she would still have a pure rate of time preference. Yet the literature on life-cycle planning under uncertainty includes a separate pure rate of time preference. Perhaps the state of the individual’s health is liable to decline with age and thus may affect her ability to derive utility from consumption. In this case, \( \theta \) controls for uncertainty about the future state of the individual’s health in the same way that \( p \) controls for uncertainty about time of death.
where the discounting arises from the individual’s pure rate of time preference and from uncertainty about her length of life.

In reality, most individuals also include altruism toward parents and children in their calculations of lifetime utility. This issue is important in the construction of positive models of household savings, since bequests are significant in the real world. However, positive measures of altruism are redundant in the SWF, and may even conflict with it, since the SWF already expresses the social or normative consensus on how the utilities of different generations ought to be aggregated. More precisely, private assessments of the optimal degree of altruism are only consistent with the social assessment if they are equal to it. Therefore, the inclusion of a private assessment of altruism in the SWF is either redundant or in conflict with the SWF. The redundancy of private assessments when the private assessments are all identical can be demonstrated relatively easily in the case of utilitarian welfare. Using identical additive terms to represent altruism in the calculation of each agent’s expected lifetime utility, the result is just an ordinal scaling of a SWF in which altruism is not included. When non-identical private assessments are used, the underlying ethical content of the SWF is no longer clear. Indeed, the inclusion of non-identical private altruism in the calculation of lifetime utilities violates the spirit, if not the letter, of the anonymity axiom, because in this case a given level of consumption is given greater weight in the SWF if it is enjoyed by someone whose family has a high level of altruism than if it is enjoyed by someone whose family has a low level of altruism. The letter of the anonymity axiom is violated if the axiom is re-defined in terms of consumption levels rather than utilities. In this case, anonymity requires that any permutation of a vector of consumption levels among individuals leaves welfare unchanged. Defining anonymity in this manner requires that all individuals be characterized by the same utility specification, which rules out non-identical
private assessments of altruism. For this reason, altruism is not included in the definition of lifetime utilities shown above. Note that the traditional specifications of dynamic SWF's, in which the arguments are instantaneous utilities, also rule out the inclusion of private assessments of altruism.

2.3 Welfare-change decision rule

The total value of a public project in terms of welfare is given by the differential change in welfare which results from it. However, as it is written, the SWF in (2.1) is non-convergent, and this non-convergence may also carry over to the differential. Fortunately, Von Weizsacker's overtaking criterion can be used to deal with this problem. Since the overtaking criterion does not require calculation of the total value of a project but only the value accruing within a finite horizon, T, provided T > T*, it is acceptable to work with a finite version of (2.1); i.e.

$$W_T(0) = \int_{-h}^{T} \Psi(v) \frac{U(v)^{1-\rho}}{1-\rho} dv .$$

The welfare change resulting from a project during the arbitrary period [0,T] is

$$dW_T(0) = \int_{-h}^{T} \Psi(v) U(v)^{-\rho} dU(v) dv .$$

From the definition of expected lifetime utility, dU(v) is calculated as

$$dU(v) = \int_{v}^{v-h} du(v,t) e^{-(\theta + \rho)(t-v)} dt .$$

After substituting the expression for dU(v) into dW_T(0), the notation for vintage, v, is converted into age, h, where h = t-v. Then the order of integration is switched, yielding a
measure of project value integrated over time instead of generations:

\[
dW_T(0) = \int_0^{T-h} \left\{ \int_0^{\bar{h}} \Psi(t-h) U(t-h)^{-\rho} \, du(h,t) \, e^{-{(\theta + \rho)h}} \, dh \right\} \, dt \\
+ \int_0^{T-h} \left\{ \int_0^{\bar{h}} \Psi(t-h) U(t-h)^{-\rho} \, du(h,t) \, e^{-{(\theta + \rho)h}} \, dh \right\} \, dt \\
+ \int_{T-h}^{T} \left\{ \int_{T-h}^{\bar{h}} \Psi(t-h) U(t-h)^{-\rho} \, du(h,t) \, e^{-{(\theta + \rho)h}} \, dh \right\} \, dt.
\]

The first term on the right-hand side of this expression can be dropped, since du(h,t) = 0 for \( t < 0 \). The last term can also be dropped, without loss of generality, since \( T \) is an arbitrary value anyway. Therefore, the revised welfare measure of the project, during the period \([0,T]\), is represented as

\[(2.3) \quad dW_T(0) = \int_0^{T-h} \left\{ \int_0^{\bar{h}} \Psi(t-h) U(t-h)^{-\rho} \, du(h,t) \, e^{-{(\theta + \rho)h}} \, dh \right\} \, dt.\]

Defining the dynamics of population growth makes it possible to convert cohort size at birth, \( \Psi(t-h) \), into current population in (2.3). Following Buiter (1988), population dynamics consists of an instantaneous birth rate, \( \beta(t) \), the death rate, \( p(t) \), which has already been discussed, and a total growth rate, \( n(t) = \beta(t) - p(t) \). All three will be assumed to be constant, so the time subscripts will be dropped henceforth. The population at time 0 is normalized to 1, so that the total population at time \( t \) is \( e^n \). While each individual is uncertain about her length of life, the size of each cohort is assumed to be large enough that there is no uncertainty about the size of the total population nor about the size of each cohort as it ages. Specifically, the proportion of a cohort that is still alive at age \( h \) is just equal to the probability that any one individual is still alive at age \( h \), i.e. \( e^{-nh} \). Therefore, the size of the cohort of age \( h \) that is still alive at time \( t \), which will be denoted \( P(h,t) \), is given by
\( P(h, t) = \Psi(t-h)e^{\phi h} \), which is the size of the cohort at birth times the proportion that is still alive at age \( h \). Now the size of the cohort at birth, \( \Psi(t-h) \), is equal to \( \beta e^{\phi(t-h)} \), the instantaneous birth rate times the total population at time \( t-h \). Then, after substitution and rearrangement, \( P(h, t) \) is found to be equal to \( e^{\phi h}\beta e^{-\phi h} \). Since \( e^{\phi} \) is the size of the total population, \( \beta e^{\phi h} \) must be the proportion of total population that is of age \( h \). This fact will prove useful later.

Making use of the population dynamics, the current cohort size, \( P(h, t) \), can be used to eliminate the size at birth, \( \Psi(t-h) \), and the discount factor \( e^{\phi h} \) from (2.3), since it is known from the population dynamics that \( P(h, t) = \Psi(t-h) e^{-\phi h} \). With this change, (2.3) becomes

\[
(2.4) \quad dW_T(0) = \int_0^T \left\{ \int_0^{\tilde{h}} P(h, t) U(t-h)^{-\rho} du(h, t) e^{-\theta h} \right\} dt.
\]

Then substitute \( e^{\phi h}\beta e^{-\phi h} \) for \( P(h, t) \) and move the population growth factor, \( e^{\phi} \), outside the braces. With these changes the expression becomes

\[
(2.4') \quad dW_T(0) = \int_0^T e^{\phi t} \left\{ \int_0^{\tilde{h}} \beta e^{-\beta h} U(t-h)^{-\rho} du(h, t) e^{-\theta h} \right\} dt.
\]

In each of (2.3), (2.4) and (2.4'), the expression in braces is a different interpretation of the static welfare change of a project. In (2.3), it is the expected welfare change accruing at time \( t \), calculated from the perspective of time 0, and using the individuals’ discounting parameters, \( \theta + \rho \), and the welfare weights given by \( U(t-h)^{-\rho} \). In (2.4), the expression in braces is the actual or ex post welfare change accruing at time \( t \), where each individual’s instantaneous utility is discounted for age by the personal rate of time preference, \( \theta \), and
weighted by $U(t-h)^\gamma$. In (2.4'), the expression in braces is again the ex post welfare change at time $t$, but this time expressed in per capita terms and then multiplied by the growth factor, $e^{\alpha t}$, to give the total value. Note that, in aggregating at a given time, all three of these expressions weight individuals according to their personal discounting factor, $e^{-\beta h}$, and the social welfare weight, $U(t-h)^\gamma$, unlike the standard Benthamite welfare function in which individuals are aggregated on an unweighted basis. Either one of (2.3), (2.4) or (2.4') is suitable for applied work, depending upon the nature of the data available. The remainder of the chapter is based upon (2.4).

(2.4) is a suitable decision rule for comparing projects and policies in terms of their impacts on welfare. Boadway and Bruce (1984, 271-79) develop a similar approach, in which the utility changes accruing to individuals — the $du(h,t)$'s — are measured by the equivalent changes in the individuals' expenditure functions, assuming a fixed set of reference prices (i.e. changes in money metric utility). Alternatively, utility benefits can be converted into dollar terms by normalizing them by the recipient’s marginal utility of consumption. Denote this quantity $u_c'(h,t)$, and the dollar benefits of the project accruing to the individual $B(h,t)$. Then the welfare change measure of the project can be written as

---

23The equivalence of ex ante and ex post interpretations is due to the lack of aggregate uncertainty about population dynamics. This characterization of population dynamics is empirically valid in a context of large cohorts and the correct expectation of the death rate, $p$.

24It was demonstrated earlier that $\beta e^{-\beta h}$ is the proportion of the total population that is of age $h$.

25Calvo and Obstfeld (1988) show that discounting the individuals' welfare-changes back to their date of birth rather than to time $0$ is necessary in order to ensure the time consistency of the decision rule. Of course, as shown here, it also arises from the definition of expected lifetime utility.

26Strictly speaking, this marginal utility is a gradient, since instantaneous utility is defined over a vector of consumption goods. Assume for convenience that consumption can also be represented by an index number. Then consider $u_c'(h,t)$ to be defined in terms of this consumption index and therefore a scalar value.
This measure becomes somewhat simpler in the special case when benefits are uncorrelated with age (and income), for then the average level of benefits can be substituted for the individual levels, i.e. \( B(t)/P(t) \) substituted for each \( B(h,t) \) where \( B(t) \) is the aggregate benefits of the project at time \( t \), measured in dollars. In this case, the welfare change of the project becomes

\[
(2.6) \quad dW_t(0) = \int_0^\tau \left\{ \int_0^h P(h,t) U(t-h)^{-\rho} u_c'(h,t) B(h,t) e^{-\theta h} dh \right\} dt.
\]

\( \kappa(t) = \frac{1}{P(t)} \int_0^h P(h,t) U(t-h)^{-\rho} u_c'(h,t) e^{-\theta h} dh. \)

\( \kappa(t) \) is the average of the marginal welfare of private consumption and does not depend upon the distribution of project benefits.\(^{27}\) The analyst may wish to convert the welfare value of the project into dollar terms by dividing through (2.6) by \( \kappa(0) \). If she does, then \( \kappa(t)/\kappa(0) \) is analogous to a discount factor. However, it is not necessary to make this normalization, and it would appear to add an unnecessary complication to the model.

Thus far it has been assumed that all individuals of the same age have the same income. It is a simple matter now to account for different incomes with the introduction of a

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\(^{27}\)This special case can also be demonstrated by Starrett's (1988, 148-49) decomposition of welfare benefits into pure efficiency and distributional terms. His welfare weights are defined analogously to \( \kappa \), and his distributional term vanishes when the weights are uncorrelated with individual benefits.
type variable, which will be denoted by $\gamma$. For simplicity, $\gamma$ is assumed to satisfy the condition $0 \leq \gamma \leq 1$, but any normalization would be acceptable since $\gamma$ is a label rather than a quantity or value. Type is assigned to each individual randomly by nature and is characterized by a probability density function, $\mu(\gamma)$, which describes the ex ante probability that an individual will be a particular type before she is born. After birth, her type is known. Now, $P(h,\gamma,t)$ is used to denote the size of the cohort of age $h$ and type $\gamma$, which is still alive at time $t$. The size of each cohort is assumed to be large enough that $\mu(\gamma)$ can be used to determine $P(h,\gamma,t)$ by the formula $P(h,\gamma,t) = \mu(\gamma)P(h,t)$. Other variables are also differentiated by income type. Therefore, the decision rule is rewritten finally as

$$
\begin{aligned}
(2.6) \quad dW_{T}(0) &= \int_{0}^{\tau} \left\{ \int_{0}^{h} \int_{0}^{1} P(h,t) \mu(\gamma) U(t-h,\gamma) - \lambda \ u_{\gamma}(h,\gamma,t) B(h,\gamma,t) e^{-\theta h} \ d\gamma \ dh \right\} \ dt.
\end{aligned}
$$

This version of the decision rule is just a generalization of (2.5). The expression in braces is the static welfare-change of a project, measured across income and age groups.

2.4 Aggregate social rate of discount

An alternative to the welfare-change decision rule shown in (2.6) is the traditional discounted sum of dollar benefits. The key variable in this approach is the social rate of discount for aggregate consumption. This variable is defined as the negative of the slope of the social indifference curve in aggregate consumption space; i.e.

$$
(2.7) \quad d_{t} = -\frac{dC_{t-1}}{dC_{t}} \bigg|_{W} - 1
$$

$$
= \frac{\partial W}{\partial C_{t}} - 1
$$
in discrete time, or
\[
d(t) = - \frac{d}{dt} \left[ \frac{\partial W}{\partial C(t)} \right]
\]
in continuous time.\(^{28}\)

Of course, marginal welfare at time \(t\), \(\partial W/\partial C(t)\), depends upon how the extra unit of aggregate consumption is distributed. Once again, ignore income differences that are not related to age. Then let \(\sigma_c(h,t)\) denote the share of the marginal unit devoted to the private consumption of individuals of age \(h\), at time \(t\), and \(\sigma_g(t)\) the share devoted to the provision of public goods. (The shares devoted to private consumption may be thought of as income transfers by the government.)

The social discount rate can now be calculated for the isoelastic SWF developed above. The derivative of welfare with respect to aggregate consumption is

\[
(2.8) \quad \frac{\partial^2 W_T(0)}{\partial C(t)} = \int_0^h P(h,t) U(t-h)^{-\rho} \left[ u_c'(h,t) \sigma_c(t) + u_g'(h,t) \sigma_g(t) \right] e^{-\beta h} \, dh.
\]

Once again assume the marginal utilities are scalar valued (see fn.26). In discrete time, the integral operator in (2.8) is replaced by summation notation, and the calculation of the social discount rate proceeds in a straightforward though tedious manner, following (2.7). The continuous time formulation is rather more difficult to derive, as it requires the time derivative of marginal welfare, which is clearly very complicated. For convenience, it will be assumed that \(P(h,t), U(t-h)\) and \(\sigma_c(h,t)\) are invariant with respect to time and that all of the marginal unit is allocated to individuals, i.e. \(\sigma_g(t)=0\). Then

\footnote{Sen (1982).}
It is common to assume an isoelastic form for instantaneous utility. Ignoring public goods for the moment, the isoelastic form is

\[ u[C(h,t)] = \frac{C(h,t)^{1-\eta} - 1}{1-\eta} \]

where \( C(h,t) \) is the aggregate consumption index of an individual of age \( h \), at time \( t \), and \( \eta \) is both the elasticity of marginal utility and the inverse of the elasticity of substitution between consumption in successive periods. Using this form, it can be shown that

\[ u_c'' \dot{c} = -\eta u_c' \frac{\dot{c}}{c} . \]

Making this substitution in (2.9) and dividing through by (2.8) gives the aggregate social discount rate in continuous time:

\[
(2.10) \quad d(t) = \eta \frac{\int_0^h \int P(h) U(h)^{-\rho} u_c'(h,t) \frac{\dot{c}(h,t)}{c(h,t)} \sigma(h) e^{-\theta h} \, dh}{\int_0^h \int P(h) U(h)^{-\rho} u_c'(h,t) \sigma(h) e^{-\theta h} \, dh} .
\]

If all groups enjoy the same rate of growth of consumption, then this expression simplifies to \( \eta \dot{c}/c \), where \( \dot{c}/c \) is the common growth rate. This version of the discount rate appears frequently in the literature accompanied by a pure rate of time preference, \( \delta \), and with \( \dot{c}/c \) interpreted as the growth rate of per capita consumption; e.g. \( \delta + \eta \dot{c}/c \). In the overlapping generations model developed here, a pure rate of time preference is equivalent to a generational weight. As mentioned above, the generational weight is zero in (2.1), by

\[ ^{39} \text{For example, see Dasgupta (1982), Boadway and Bruce (1984, 320-21) and Cline (1992, 249-55).} \]
virtue of the anonymity axiom. Cline (1992, 251-55) surveys the literature and finds the value of \( \eta \) between 1 and 1.5 and per capita consumption growth between 1 and 2 percent, based on postwar U.S. data. Taken together, these values result in an estimate of the aggregate discount rate, \( \eta \frac{c}{c} \), between 1 and 3 percent.

Unfortunately, the assumption of the same rate of growth of consumption for all ages and income types cannot be justified. Furthermore, the assumptions of \( P(h,t) \) and \( U(t-h) \) invariant with respect to time and \( \sigma_q(t) = 0 \) lack generality, although the time invariance of \( \sigma_q(h,t) \) is probably reasonable for most projects. Even if these latter assumptions could be justified, the resulting expression for the discount rate, (2.10) in continuous time or (2.7) in discrete time, is a rather complicated function of personal consumption levels and the distribution of project costs and benefits. Thus any hope of a simple social discount rate, free of distributional concerns, is dashed. Nonetheless, \( \eta \frac{c}{c} \) may be useful in giving a rough idea of the gap between values commonly used in cost-benefit analysis and an equitable social rate of discount. In comparison with Cline's estimate of \( \eta \frac{c}{c} \) between 1 and 3 percent, the U.S. government's Office of Management and Budget has mandated, since 1972, a real discount rate for government projects of 10 percent.\(^{30}\)

Because the aggregate social rate of discount changes as the distributional characteristics of projects change, the search for a universal rate which is applicable to all projects must be abandoned as futile — indeed undesirable. Lind (1982) argues that a single rate is desirable because it minimizes the opportunity for manipulation of results on a project by project basis. However, this view confounds the planner's preferences with the physical descriptions of the projects. In effect, it requires that analysts ignore the distributional

\(^{30}\)See Cline (1992, 263-65) and Hartman (1990) for background on U.S. government policy. The author is not familiar with the policies of Canadian governments and their agencies. The impressiveness of the gap between \( \eta \frac{c}{c} \) and the OMB's rate is mitigated by the fact that the use of the social discount rate requires adjusting the direct costs of the project for the displacement effect of public finance, whereas the OMB's policy does not.
characteristics of projects. A better approach is to use the same welfare framework, such as (2.6), for all projects, with the same value of the equity parameter, $\rho$. This approach eliminates any opportunity for manipulation, by fixing the planner's preferences, while also requiring analysts to specify the distributional effects of projects. Of course, the choice of the equity parameter is no small matter. But if researchers conduct sensitivity analyses, then governments can choose their programs by choosing a value for $\rho$.

2.5 Implementation

Implementation of either the welfare change measure of a project or the aggregate social rate of discount requires knowledge of the scalar quantities $\bar{h}$, $\theta$, and $\rho$, the distribution of income type, $\mu(\gamma)$, the variables $P(h,t)$, $U(t-h,\gamma)$, $u_c'(h,\gamma,t)$ and $B(h,\gamma,t)$. Furthermore, breaking down expected lifetime utility, $U(t-h,\gamma)$, into its components indicates that the analyst must also know the value of the death rate, $p$, and the specification of instantaneous utility, $u(h,\gamma,t)$. A number of these variables and parameters are readily observed from existing data or empirical work, while some others require further development or clarification before they can be estimated empirically. Of the first type are $\bar{h}$, $P(h,t)$, $\mu(\gamma)$, $\theta$ and $p$. The quantities requiring further development before they can be estimated are $B(h,\gamma,t)$ and $u(h,\gamma,t)$. Finally, the equity parameter, $\rho$, can only be determined politically. The obvious approach for the analyst is to report results for a wide range of values of $\rho$.

$\bar{h}$, $P(h,t)$, and $p$ all refer to demographics, while $\mu(\gamma)$ may be estimated from data on income distribution. In practice, the values will have to be broken down into discrete units, such as income deciles for $\mu(\gamma)$. The choice of $p$, the death rate, is somewhat more problematic than the others. If $p$ is considered to vary with age, then actuarial data can be used. However, the use of variable death rates complicates the analysis tremendously. For
the sake of simplicity, it is preferable to assume a constant value of \( p \). In his model of "perpetual youth", Blanchard (1985) indicates that \( p \) is the inverse of life expectancy. This definition provides one method for estimating \( p \). Unfortunately, using a life expectancy of 75 years — a value which is typical of men in most developed countries — means that the proportion of each cohort still alive at age 90, i.e. \( e^{-90p} \) where \( p=1/75 \), is equal to 30 percent. Clearly, the assumption of constant \( p \) is very far from a true representation of mortality. In fact, adult mortality is low and relatively constant in developed countries until about age 60, after which it increases dramatically.\(^{31}\) A rough way to approximate this time profile of mortality is to combine a constant \( p \) with a judicious choice of \( \bar{h} \). In this approach, mortality is constant until the cohort reaches the maximum age, \( \bar{h} \), at which time everyone still alive dies immediately. \( p \) and \( \bar{h} \) can be chosen so as to approximate the true mortality profile as closely as possible.

\( \theta \) can be estimated directly or inferred from an appropriate model of the economy. For example, the equation which characterizes the evolution of aggregate consumption in Blanchard (1985), assuming an elasticity of marginal utility of 1, is

\[
(2.11) \quad \dot{C}(t) = [\tau(t) - \delta_{K} + \alpha - \theta + n(t)] \cdot C(t) - [p + n(t) + \alpha] \cdot [p + \theta] \cdot K(t),
\]

where \( C(t) \) is aggregate consumption, \( \tau(t) \) is the marginal product of capital, \( \delta_{K} \) is the depreciation rate of capital, \( \alpha \) is the rate of decline of individual labour income over time (simulating retirement), \( \theta \) of course is the personal rate of time preference, \( n(t) \) is the population growth rate, \( p \) is the constant death rate, and \( K(t) \) is the aggregate capital

Now, define \( m(t) \) as the growth rate of consumption; i.e. \( m(t) = \frac{\dot{C}(t)}{C(t)} \) or 
\[
m(t) = \frac{[C(t + 1) - C(t)]}{C(t)}
\]
in discrete time. Then, dividing both sides of (2.11) by \( C(t) \) and making the substitution for \( m(t) \), (2.11) can be rearranged to give an expression for \( \theta \):

\[
(2.12) \quad \theta = \frac{\left[ r(t) - \delta_K + \alpha + n(t) - m(t) \right] C(t) - p \left[ p + n(t) + \alpha \right] K(t)}{C(t) + \left[ p + n(t) + \alpha \right] K(t)}
\]

If this model describes the economy reasonably well, then (2.12) holds at every moment in time. The analyst can pick a time for which she has data and evaluate \( \theta \). In practice, all of the variables and parameters on the right-hand side of (2.12) are straightforward with the exception of \( \alpha \). The analyst can try different values of \( \alpha \) to determine whether \( \theta \) is sensitive to it. If so, and failing a value for \( \alpha \), the only other solution is to obtain a direct estimate of \( \theta \).

\( B(h, \gamma, t) \), the value of benefits accruing to an individual of age \( h \), type \( \gamma \), is in theory part of the design of the project, but in practice the analyst may only have an estimate of aggregate benefits, \( B(t) \). Estimation of the \( B(h, \gamma, t) \)'s can proceed by constructing a subjective probability function, \( v(h, \gamma) \), to represent the expected distribution of aggregate benefits across age and income types. Then actual benefits can be represented as

\[
B(h, \gamma, t) = v(h, \gamma) B(t) - \epsilon(h, \gamma, t)
\]

where \( \epsilon(h, \gamma, t) \) is a random component with expected value of 0. Starrett (1988) and Boadway and Bruce (1984) provide overviews of techniques for estimating aggregate net benefits, \( B(t) \), including hedonic methods, capitalization, and other econometric methods based upon aggregate data. Of course, the estimates of the \( B(t) \)'s must also include allowances for the displacement (and possibly stimulus) of private capital formation due to the distortionary impact of public financing. These allowances can be

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32 This equation differs from Blanchard's version in its inclusion of population growth. See Marini and Scaramozzino (1995) and Buiter (1988) for treatment of population growth.
estimated directly or by adjusting the measure of net benefits by means of a shadow value of capital. Lind (1982) provides a discussion of the shadow value of capital.

Estimation of the individuals' instantaneous utility, $u(h, \gamma, t)$, requires an assumption regarding its functional form. For convenience, the same functional form is usually assumed for all individuals. It is also convenient to assume that utility is weakly separable between public and private consumption and that both types of consumption can be represented by an index number; i.e., $u[g(t), c(h, t, \gamma)] = G(t) \tilde{u}[C(h, t, \gamma)]$. $G(t)$ is the index of public goods at time $t$, and $\tilde{u}[.]$ is the sub-utility function defined over the private goods index. Since utility is ratio-scale in the case of isoelastic welfare, the analyst is free to normalize $G(t)$ in any way. The simplest normalization is in terms of the time-0 value. Then $G(t)$ can be represented in terms of the growth in the level of public goods since time 0. Thus, defining $\omega(t)$ as the growth rate of public goods at time $t$, $G(t) = \exp \int_0^t \omega(s) \, ds$ or $e^{\omega t}$ if $\omega$ is constant. The analyst must predict the value of $\omega(t)$ over time.

Regarding the sub-utility function, $\tilde{u}[.]$, it seems reasonable to use the isoelastic form defined above, since there is empirical evidence that people's utility over private consumption is isoelastic. Therefore, the sub-utility function is represented as

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33Alternatively, the analyst may use equivalence scales to account for systematic differences between individuals. See Boadway and Bruce (1984, 274-75) on this topic. Note that if the anonymity axiom is defined in terms of consumption levels instead of utility, as discussed in section one, then the same utility specification must be used for all individuals. Thus, in addition to being rather convenient, the use of a common specification can also be interpreted as having some ethical content.

34See Boadway and Bruce (1984, 143-161) for a discussion of the invariance requirements of isoelastic welfare.
where \( C(h,t,\gamma) \) is the aggregate consumption bundle and \( \eta \) is the elasticity of marginal utility and the inverse of the elasticity of substitution between consumption in successive periods. \( \eta \) is frequently estimated to be near 1, in which case the sub-utility function reduces to \( \ln C(h,\gamma,t) \).\(^{35}\) In addition to \( \eta \), the analyst must have estimates of the \( C(h,t,\gamma) \)'s.

With the definition of \( u(h,\gamma,t) \) in hand, it is now possible to estimate expected lifetime utility as:

\[
U(t-h, \gamma) = \int_{t-h}^{t-h-\hat{h}} G(t) \frac{C(t-h,\gamma,t)^{1-\eta}}{1-\eta} e^{-(\theta+p)(s-(t-h))} ds
\]

\[
= \int_{t-h}^{t-h-\hat{h}} e^{os} \ln C(t-h,\gamma,t) e^{-(\theta+p)(s-(t-h))} ds \quad \text{when } \eta = 1 .
\]

2.6 Conclusion

The two decision rules presented above — the welfare change measure and the aggregate social rate of discount — are derived from an isoelastic welfare functional defined over individuals' expected lifetime utilities. The use of expected lifetime utilities as the arguments of the functional is preferable to the more common practice of using instantaneous utilities, because the planner is concerned with the total, lifetime well-being of an individual rather than her prospects at each and every moment. In addition, the use of lifetime utilities permits an overlapping generational structure.

The isoelastic specification is a very flexible form which subsumes many other

\(^{35}\)Use l'Hôpital's rule to get this result. Cline (1992,251-55) surveys the empirical results on \( \eta \). See also Blanchard and Fischer (1989, p.44).
familiar forms as special cases. Although it requires some rather strong axiomatic assumptions, these assumptions are only marginally stronger than those required for utilitarianism, upon which standard cost-benefit analysis is based. Moreover, the association of intergenerational equity with the anonymity axiom has intuitive appeal. Disagreements will arise of course over the value of the equity or elasticity parameter, but this choice is expected to be political.

The aggregate social rate of discount has been shown to depend critically upon the distribution of project net benefits, and therefore the search for a universal discount rate which has occupied economists for so long must be abandoned. Rather, consistency in project analysis can be achieved by adopting the same welfare functional, as well as using the same economy model to predict the evolution of consumption and distribution, for all projects. It is a matter of indifference whether the analyst uses the welfare-change decision rule or the social discount method, since they both are derived from the same welfare functional and therefore give the same ordinal ranking of projects.

Despite the impossibility of finding a universal social discount rate and comparing it with values typically used in project analysis, it is still almost certain that traditional practices based on market discounting have biased decisions against the interests of future generations. This conclusion is based upon the implicit non-zero pure rate of time preference or generational discounting inherent in market interest rates. While most theoretical models of policy analysis are agnostic about the value of generational discounting,\textsuperscript{36} the invocation of the anonymity axiom in this chapter results in a choice of zero for generational discounting. Consequently, it would be surprising if this model did not give greater weight to the interests of future generations than market discounting does.

\footnote{\textsuperscript{36}See for example Calvo and Obstfeld (1988).}
3.1 Introduction

The emergence of global warming on the international agenda has tested the limits of conventional policy analysis, due to the multifarious complexities of the issue. The Intergovernmental Panel on Climate Change (IPCC 1996) estimates that the global mean surface temperature has increased between 0.3 and 0.6 degrees Celsius during the past 100 years and that it is unlikely that this increase has been due to natural causes only. Specifically, it is conjectured that emissions of the so-called greenhouse gases (GHG's) — primarily carbon dioxide, methane and nitrous oxide, from the burning of fossil fuels, land-use changes and agriculture — have contributed to this increase, and that, if unchecked, these emissions will contribute to further warming. While this hypothesis remains subject to debate, it is anticipated that the consequences of such warming, if it were to occur, would include potentially large negative impacts on economic activities, as well as significant changes in patterns of weather and precipitation, the level of water tables, flora and fauna, and a significant rise in sea level. The problem is further complicated by the long time lags involved. Perturbations in the atmospheric concentration of carbon dioxide, the principal greenhouse gas, have an expected lifetime of more than a century, and therefore the impacts of global warming are expected to be long-lasting.

For the purpose of policy formulation, global warming involves at least five distinct issues. First obviously there is an intertemporal externality. The benefits of emitting greenhouse gases accrue from the use of energy at the time the emissions are made, whereas the costs associated with the resultant warming persist for several generations. Second and
third are the problems of inter and intra-generational inequity in the distribution of wealth. Intra-generational inequity reflects the obvious differences in the standard of living among individuals and countries. The differences among countries are particularly relevant since the costs of global warming are expected to fall most heavily on the poorest countries while a disproportionate share of the benefits of fuel consumption accrue to the richest countries. Inter-generational inequity, while perhaps less obvious, is equally important. Inter-generational inequity (for that matter any inequity) results whenever the process which generates the market equilibrium is something other than the social welfare functional. As discussed previously, perhaps the most compelling explanation for such a distinction between the market and social welfare is that individuals are expected to discount the future more heavily than the planner, due to uncertainty about time of death. The planner, on the other hand, is blessed with infinite (albeit hypothetical) life. The fourth element of the policy context of global warming concerns international trade and investment flows. Since global warming is international in scope, policies to address it have the potential to have significant impacts upon the existing patterns of trade and investment. Indeed, at present, this issue is probably the biggest concern of policy makers in connection with global warming. Finally, the fifth element of the policy context of global warming is uncertainty. Not only are the underlying assumptions about future economic performance uncertain, but the nature of global warming is still subject to debate in the scientific community. Therefore, to be complete, any policy prescription must take these uncertainties into consideration.

Most economic studies of global warming to date can be separated into three categories: first, estimates of abatement costs and benefits; second, cost-benefit analyses of specific policies, such as a cutback of GHG emissions to 1990 levels; and third, exercises to determine the optimal control of emissions abatement, either in static equilibrium or dynamic

Despite the progress of recent years, much work remains to be done. Of particular note, little effort has been made to cast greenhouse policy in the context of the second best. Rather, with the exception of Cline (1992), virtually all the literature has sought to develop greenhouse control policies in positive models of the economy. This positive approach asks the question: if a market could be created for the services provided by the atmosphere, what price would be consistent with the economy’s assessment of intertemporal tradeoffs? This approach does not adequately address the important issues of inter and intra-generational distributional inequities which are among the key elements of the second best and which, as discussed, are especially relevant to global warming.

In contrast to the positive approach, the present study seeks greenhouse control policies which maximize a social welfare functional, subject to the necessary second-best constraints. Among other features, the model which will be used is characterized by:

- a normative social welfare functional;
- a positive general equilibrium process;
- disaggregation of the world into four regions;
- intra-generational distributional inequities, in the form of different per capita income levels between the regions;
- efficient abatement of greenhouse emissions, achieved through the implementation of either carbon taxes or tradeable emission permits;
- limited side payments from rich to poor regions, to compensate poor regions for losses due to their abatement efforts.

For the sake of comparison with a positive approach to greenhouse policy, the economy-climate model of Nordhaus and Yang (1996) is used, with modifications for the second best framework. Although this model embodies a number of improbable assumptions, including exogenous technological change, the total absence of no-regrets opportunities for emissions abatement, and top-down estimates of abatement costs, it nonetheless represents the state-of-the-art, at least in the economics literature, in combining dynamic control modelling with a regional representation of the economy. Since a regional representation is necessary to operationalize distributional inequities, this model is the logical choice for comparing second-best and first-best welfare frameworks. Further research is anticipated to modify some of these more unfortunate assumptions.

Not surprisingly, the effect of this second-best policy formulation is to give levels of control of GHG emissions which are much higher than those estimated in the positive models. For example, the control rate of emissions obtained under various scenarios in the present study is typically four times as high as that of Nordhaus and Yang (1996), with equivalent rates of carbon taxes anywhere from 14 to 26 times higher. The low range of these estimates translates into an initial tax of approximately $110 per ton of carbon, which is equivalent to an increase in the retail price of gasoline of approximately 20 percent, assuming the incidence of the tax falls entirely on consumers.

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37In the climate-change literature, the label "top-down" refers to the standard econometric approach of estimating functions on the basis of past observations, where the data are often highly aggregated. In contrast, "bottom-up" denotes estimates based on engineering projections of the plausible state of technology in the future.
The chapter is divided into five sections, denoted 3.2 through 3.6. Section 3.2 describes a variety of second-best scenarios which are used to estimate optimal greenhouse policies. Section 3.3 provides details of the climate-economy model, the social welfare functional, the treatment of opportunity cost, and the general equilibrium process. Section 3.4 describes the welfare maximization process, including the necessary constraints. Section 3.5 gives the results for the different second-best scenarios under utilitarian social welfare. Section 3.6 describes the effect on policy of welfare specifications which place greater emphasis on equity than the utilitarian.

3.2 Policy scenarios

Six policy scenarios will be compared in this study. The first is the path of unconstrained emissions which society is following currently.\textsuperscript{38} The remaining five scenarios are variations of a cooperative international regime in which all countries participate, such as envisioned in the UN Framework Convention on Climate Change.

The many important issues attendant upon how such cooperation is to be secured and enforced are not treated in this study. Given the assumptions of the Nordhaus-Yang model, particularly the absence of no-regrets opportunities for abatement and the absence of any feedback between policy and technological change, all of the five scenarios result in costs for the present generation, in terms of foregone consumption, in at least one of the regions. The presence of these costs would seem to make agreement on emissions control unlikely on the basis of self-interest alone. However, recent research has identified many opportunities for

\textsuperscript{38}In fact, the implementation of existing fuel taxes achieves some degree of emissions control, especially in Europe and Japan where such taxes are highest. However, the impact of existing taxes is small compared with the scenarios which will be considered in this study. For simplicity, therefore, the present regime will be referred to as the unconstrained equilibrium.
no-regrets reductions in GHG's, and the feedback of policy to technological innovation is likely to make the costs to the present generation much lower than the estimates arrived at below.\textsuperscript{39}

The five control scenarios fall into two categories: first, those that achieve control through a combination of carbon taxes and compensation payments from rich to poor nations; and second, those that involve tradeable emissions permits. In general, a tax and compensation regime can be designed to replicate any permit regime. However, as each regime seems particularly well suited to different arrangements of control and transfers, a number of each type was considered. Three of the five scenarios are tax regimes; two involve permits.

The first tax regime is Nordhaus and Yang's cooperative scenario, which internalizes the greenhouse externality in a positive model of the economy. This regime does not involve maximization of a normative social welfare functional, and no transfer payments are made between countries. In effect, the regime addresses the question: if a market could be created for the services of the atmosphere, what price would be consistent with the existing market equilibrium? The regime is referred to below as "RICE cooperative" (Regional Integrated model of the Climate and the Economy).

The second tax regime estimates the optimal path of carbon taxes and inter-country transfers by maximizing a normative social welfare functional. The amount of compensation a country can receive in this scenario is limited to the cost of its abatement effort. Total compensation received by recipient countries is equal to the total given by donors in each period. This compensation regime was chosen because it reflects a particular principle of equity concerning the sharing of costs and benefits associated with GHG emissions control.

\textsuperscript{39}Palmer et al. (1995) and Porter and Van der Linde (1995).
Since the costs of greenhouse gas emissions are expected to fall disproportionately upon developing countries while the benefits to date have accrued disproportionately to the rich industrial countries, it seems only reasonable to pay for the costs of abatement from a common global fund financed primarily by the industrial countries. At the same time, the limitation of compensation to abatement costs seems realistic, since it is unlikely that the broader issue of distribution will be resolved any time soon and certainly not prior to dealing with global warming. This compensation regime is also realistic in the sense that many developing countries are currently pressing for similar arrangements in the negotiations under the UN Framework Convention on Climate Change. Finally, the efficiency of the carbon taxes is ensured by constraining them to be equal across all countries in each period. If this constraint is not imposed, the differences in the welfare weights assigned to the countries, arising from their differences in per capita consumption, result in different tax rates in each country, which is Pareto inefficient.\footnote{If different countries faced different carbon taxes, then the marginal cost of emissions abatement would also be different among countries. This result holds because countries undertake emissions abatement to the point where the marginal cost of abatement just equals the carbon tax. But if the marginal cost of abatement were different among countries, then high-marginal-cost countries could be made better off, without making other countries worse off, by relaxing abatement in these countries and having them pay for an equivalent amount of abatement to be undertaken in countries with lower carbon taxes.} In the remainder of the chapter, this regime is referred to as "SW (Social Welfare maximization) tax and transfers".

The third tax regime estimates the optimal path of carbon taxes, by maximizing the social welfare functional, but does not provide for inter-country transfers. This regime is also equivalent to a permit regime without trading. The regime is referred to as "SW no transfers".

The first permit regime which will be considered maximizes the social welfare functional with respect to the initial allocation of permits, subject to equilibrium in the market for permits. Equilibrium in the market for permits is characterized by a single world
price and no excess demand. Each country adjusts its abatement level so that its marginal cost of abatement is equal to the permit price. Consequently, countries with low abatement cost functions undertake more abatement, and countries with higher cost functions abate less and purchase relatively more permits. Inter-country transfers are achieved through permit sales and purchases. Unlike SW tax and transfers, in permit regimes the amount of compensation or net transfers a country receives is not limited by the amount of abatement it undertakes but rather is determined by the initial allocation of permits, the permit price, and the number of permits the country sells. Therefore, permit regimes in general provide an opportunity for limited direct redistribution, in addition to transfers which offset domestic abatement costs. In this particular case, the initial allocation of permits is made, at the optimum, to the poorest country(ies) in order to maximize the welfare value of the resultant transfers. This regime is referred to as "SW optimal permits".

While the optimal permit regime is desirable from the perspective of welfare maximization, there are at least two features which make it problematic. The first feature concerns the potential for perverse incentives. Because this regime concentrates the initial allocation of permits among the poorest regions, it may create a welfare trap for poor countries, in which these countries become worse off if they improve their situation too much. The second problematic feature highlights the shortcoming of an overly simplistic view of equity. Should it be assumed that every poor country is poor due to forces outside its control? Or to what extent are many poor countries poor due to bad policies, bad leadership, or perhaps corruption? Those which improve their lot through hard work, honest government and patient investment, and in so doing make themselves ineligible for an initial allocation of permits, may reasonably argue that there is no equity in rewarding the fruits of ineptitude.

For these two reasons, a second permit regime will be estimated with a fixed initial
allocation of permits. Specifically, in this regime permits are allocated to a country in accordance with its share of world population. This allocation conforms with a particular notion of equity in which all individuals are entitled to an equal share in the global atmospheric commons. This allocation has widespread appeal among leaders of developing countries and has been proposed in the Framework negotiations. The regime will be referred to below as "SW per capita permits".\(^4\)

3.3 The model

3.3. i Climate-economy model

The climate-economy model is shown in Appendix 3.A. It is identical to the model of Nordhaus and Yang (1996), with the exception of the country groupings or regions and the compensation regimes discussed above.\(^4\) The economy of each region is modeled as a standard dynamic optimization problem for a representative agent. Output is divided in each period between consumption, gross domestic investment, foreign investment (net exports), and inter-country transfers (compensation payments for abatement or for carbon permits). Regions accumulate domestic capital and balances (positive or negative) of net foreign assets.

Greenhouse warming affects the economy in the following manner. In each period, a country’s production is characterized by an emissions/output ratio, \(\sigma(t)\). This ratio is exogenous and declines over time, reflecting improvements in the efficiency of energy technology and the diffusion of new capital. However, even while the emission/output ratio declines, total emissions increase due to an increase in the capital stock and total output.

\(^4\) Another important regime involves an initial allocation of permits that corresponds to actual use, i.e. with no net purchases or sales. As mentioned above, this regime is equivalent to "SW no transfers", so in fact it is already included. This regime was advocated by the Bush administration during the negotiations leading up to the Framework Convention, in 1992, whereas most developing countries favoured a per capita allocation of permits.

\(^4\) The climate dynamics are explained in Nordhaus (1994, pp.11-21), which uses an aggregate version of the model.
Emissions add to the stock of GHG's in the atmosphere, and this stock dissipates very slowly, by mixing into the oceans among other things. New emissions are made continuously, with the net effect that the total atmospheric stock of GHG's grows. This stock of gases contributes to the control of global mean temperature by regulating the passage of heat from the Earth's surface into space, a process called radiative forcing. Increases in the stock of GHG’s increases radiative forcing, which translates into increased global mean temperature. The economic impact of the increase in global mean temperature, and its concomitant environmental changes, is modelled as a reduction in total factor productivity, by means of the greenhouse damage factor, \( D(t) \), and the output scaling factor, \( \Omega(t) \). The sensitivity of climate models is typically represented by the long-run or equilibrium impact on mean atmospheric temperature of a doubling in the concentration of CO\(_2\)-equivalent. In Nordhaus and Yang’s model, this value is 2.91 degrees C.

Nordhaus and Yang (1996) divide the world into six regions: the United States (USA), Japan (JPN), the European Community (EEC), the former Soviet Union (FSU), China (CHN), and the rest of the world (ROW). The greater complexity of the second-best policy developed here has required reducing the number of regions to four.\(^4\) For this purpose, it was deemed appropriate to combine the United States, Japan and the European Community (UJE) as these three have similar living standards. For most variables, this combination only required adding quantities together; e.g. population, output and capital. In the case of growth rates of exogenous variables (e.g. growth of population, total factor productivity, emission/output ratio), a weighted average was employed, with the weights corresponding to either the share of total initial output or total final output of the three, as

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\(^4\) The optimization algorithm could not handle six regions, given the complexities of the model and the second-best policy constraints.
appropriate.\textsuperscript{44}

Despite their relative similarity in income, the U.S., Japan and Europe are quite different in terms of their per capita consumption of fossil fuels and their resultant emissions of GHG's. Therefore, a complete policy analysis will require separate control policies for each region. Solving a second best policy model for all six regions is the subject of ongoing research. In addition, it would be desirable to obtain more separation among the Rest of the World category. At present, this category contains high-income countries (Canada, Australia), middle-income countries, the newly-industrialized tigers of southeast Asia, as well as developing countries and the poorest of the poor. A systematic classification according to income and fuel-use characteristics would be desirable (although almost certainly too complex for numerical optimization).

In addition to the emissions/output ratio, \( \sigma_j(t) \), exogenous quantities also include labour, \( L_j(t) \), and total factor productivity, \( A_j(t) \). The evolution of these exogenous variables is characterized by positive exponential growth rates, \( g_j^\sigma(t) \), \( g_j^L(t) \) and \( g_j^A(t) \), which decline asymptotically to 0. The initial values of \( \sigma_j(t) \), \( L_j(t) \), \( A_j(t) \), \( g_j^\sigma(t) \), \( g_j^L(t) \) and \( g_j^A(t) \) are shown in Appendix 3.A. The determination of the rate of decline of the growth rates is rather involved and is described in the command files of Nordhaus and Yang (1996). The end result of these calculations is that \( \sigma_j(t) \), \( L_j(t) \) and \( A_j(t) \) reach steady states by the year 2200.

3.3.ii Social welfare functional

The social welfare functional employed is a discrete-time version of the isoelastic form developed in chapter two. Social welfare is defined over the expected lifetime utilities of representative agents of each region, born in each period. More precisely,

\textsuperscript{44}Details are contained in the program command files, which are available from the author upon request.
\[(3.1) \quad W(T) = \sum_{j=1}^{N} \sum_{t=0}^{T} \Psi_j(t) \frac{U_j(t)}{1 - \rho} \]

where \(W(T)\) is social welfare defined over a planning interval \([1,T]\), \(j\) indexes the regions, \(\Psi_j(t)\) is the size of the cohort born in country \(j\) during period \(t\), \(U_j(t)\) is expected lifetime utility, and \(\rho\) is the inverse of the elasticity of substitution between agents — i.e. the planner's equity parameter. As discussed in chapter two, there is no generational discounting, in keeping with the anonymity principle of equity. The special case of \(\rho=0\) gives the utilitarian form of social welfare; the limit of \(\rho\to1\) the Bernoulli-Nash form (linear in the logarithms), and \(\rho\to\infty\) the maxi-min.

Time is measured in decades. Individuals are assumed to have a maximum lifetime of 80 years, in other words eight planning periods. With these assumptions, expected lifetime utility is defined in the following manner:

\[U_j(t) = \sum_{s=t}^{t+7} \ln c_j(s) e^{-(\theta - p_j)(s-t)\rho} \]

where \(c_j(s)\) is per capita consumption, \(p_j\) is the (constant) death rate for country \(j\), and \(\theta\) is the personal pure rate of time preference. One of the appealing features of this model is that it is possible to account for the different demographic characteristics of countries through the choice of \(p_j\). However, to be credible, any treatment of the differing demographic characteristics among countries must account for the fact that these characteristics are endogenous. For example, the death rate is a function of national income, among other things. Such a treatment is beyond the scope of the present study. Rather, a common value of the death rate has been assumed. Specifically, the sum \(\theta + p_j\) is assumed to be equal to 0.03, which is Nordhaus and Yang's assumption concerning the utility discount rate.

Equation (3.1) has an endpoint problem, since some of the \(U_j(t)\)'s overlap the initial
period, \( t=1 \), and some overlap the terminal period, \( T \). It is for this reason that the time index in (3.1) begins at -6. An individual born in \( t = -6 \) will be in her last period of life in the initial planning period, \( t=1 \). Values of \( c_j(s) \) for \( s \in [-6,0] \) and \( s>T \) are exogenous to the model. In the actual runs, \( c_j(0) \) was fixed at 80 percent of first-period per capita income, and the \( c_j(s) \)'s for earlier periods were projected back from \( c_j(0) \) by means of a geometric rate of decline of 1 percent per annum. For \( s>T \), the \( c_j(s) \)'s are fixed at the "no-control" market equilibrium value for \( T \) (since the no-control model does not suffer the end-point problem). This approximation is reasonable, since the values close to \( T \) are not "on the turnpike" of the infinite problem anyway. As for the \( c_j(s) \)'s in \([-6,0]\), it is true that greater precision would be afforded by using historical data. However, the back projection described above is not unreasonable.

The values of cohort size used in (3.1), \( \Psi_j(t) \), are calculated from the expression

\[
(3.2) \quad L_j(t) = \Psi_j(t) + \sum_{h=1}^{7} \Psi_j(t-h) e^{-10hP} ,
\]

which states that current population, \( L_j(t) \), is composed of the newborn generation, \( \Psi_j(t) \), plus the remaining members of earlier generations.\(^{45}\) It is assumed that cohort size reaches a steady state in 2130, which is consistent with achieving steady-state population in 2200. Then, cohorts prior to 2130 are determined by backwards induction of (3.2).

3.3.iii Opportunity cost of public finance

The opportunity cost of public finance is accounted for in the model by the induced changes in consumption and investment which result from the various policies. It is not

\(^{45}\)In chapter two, it was shown that the exponential expression \( e^{-10P} \) gives the proportion still alive of a generation born \( h \) periods earlier.
necessary to include a pure rate of time preference in the social welfare functional in order to reflect the market rate of interest, as frequently argued. If such a rate is included, then it must be understood and justified as reflecting the intertemporal preferences of the planner, not the opportunity cost of private capital. It is hard to imagine what principle of equity such a rate is meant to satisfy. Alternatively, a positive model of the economy, such as the optimal growth model associated with Cass and Koopmans, does include a pure rate of time preference in the objective functional, in order to reflect the actual inter-temporal tradeoffs made by the market economy. The key distinction, then, is between a normative social welfare functional and a positive model based upon an objective which is intended to replicate the market economy.

In a context of limited public funds, it is possible that a policy arrived at by maximizing a social welfare functional in a second-best framework may be dominated by another public policy. Therefore, in order to ensure the best use of its funds, the government must compare all of the projects on its menu.

3.3.iv Market (general equilibrium) process

The market or general equilibrium process used in this study is the optimization of a Negishi objective functional, following the practice of Nordhaus and Yang (1996).\(^4\) The Negishi technique provides an alternative to the usual approach of modelling general equilibrium with a set of excess demand functions. In brief, the underlying concept is that there is always some objective functional for which the market equilibrium is the optimal point. Negishi (1960) shows that under very general conditions this objective is a weighted

\(^4\)Manne and Rutherford (1994) have labelled this objective the Negishi global welfare functional. This invocation of welfare is unfortunate as it suggests something normative. In the present study, the positive nature of the Negishi technique will be emphasized by referring instead to the Negishi objective functional.
sum of the utilities of individual agents, where each weight is inversely proportional to the individual's marginal utility of income. This weighting scheme gives greater weight to richer individuals than to poor, which is consistent with market equilibrium.

In the model developed by Nordhaus and Yang, each region is characterized by an infinitely-lived representative agent consuming the per capita level of consumption. The Negishi objective functional is

\[
(3.3) \quad \sum_{j=1}^{J} \sum_{i=1}^{T} \phi_j(t) L_j(t) \frac{\ln c_j(t)}{(1 - \delta_u)^{t-1}}
\]

where \( \phi \) is the Negishi weight, \( \delta_u \) is the utility discount rate, which is assumed to be 0.03, \( L_j(t) \) is population (equal to the labour force), \( c_j(t) \) per capita consumption, and \( \ln c_j(t) \) the instantaneous utility of region \( j \) in period \( t \). Given the correct values of the Negishi weights, general equilibrium is the solution which maximizes (3.3) subject to the structural equations of the economy-climate model shown in Appendix 3.A.

Finding the correct values of the Negishi weights is accomplished in an iterative fashion. First, the problem is solved using arbitrary values of the weights, then the weights are updated as described in Nordhaus and Yang (1996, p.747) or Manne and Rutherford (1994, pp.199-200). Alternatively, in a single-good model such as the present one, it is also possible to impose the optimality condition of 0 for all net foreign asset balances in the terminal period, and then solve for the corresponding equilibrium Negishi weights.\(^{47}\)

The solution of the Negishi problem, and therefore the market equilibrium, including inter-regional trade and investment flows, will depend of course upon the values of the policy variables, which in this case are the carbon taxes and inter-regional transfers or the carbon permit allocations. In turn, the solution of the social welfare maximization, which determines

the optimal value of the policy variables, must also satisfy a Negishi equilibrium. Thus, in principle, the second-best optimum is the result of an iterative process in which the market achieves equilibrium conditional upon a set of policy variables, and the planner then adjusts the policy variables conditional upon the given market equilibrium. In principle, this process repeats until a fixed point is achieved (assuming one exists). In practice, the modelling of this process is complicated by the difficulty of finding a convergent path toward a fixed point. If the step length used to update the policy variables at each new iteration is too large, the process might easily diverge. On the other hand, one cannot know at the outset how large is too large, and therefore one risks the danger, by being too cautious in the choice of step length, of inflating computational time tremendously.

Fortunately, an alternative exists to this iterative process for finding second-best social optima. After first finding the correct Negishi weights of the no-control market equilibrium, i.e. the Negishi equilibrium with the policy variables set to zero, subsequent Negishi equilibria for non-zero values of the policy variables are approximated by constraining the social welfare maximization by the first-order conditions of the no-control Negishi problem. In this manner, two optimizations are combined in one. The dual variables of the Negishi problem are now treated as primal variables in the constrained welfare maximization. This approach has the benefit of throwing out candidate solutions for social welfare optimization which are not also approximate Negishi equilibria. However, it also has the disadvantage of complicating the welfare optimization with additional constraints. Indeed, it is due to the addition of the Negishi constraints that the number of regions in the model must be reduced from six to four. Moreover, the result obtained with this approach is only an

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4Work is currently underway to determine how this technique can be successfully applied to the six-region case. One possibility is to use the solution from the four-region model as the basis for choosing the starting point of the search algorithm in the six-region model. This approach holds promise since it should enable one to get quite close to the optimal solution at the outset. The choice of starting point is usually critical to the success of optimization algorithms.
approximation to a true Negishi/ welfare-maximization fixed point, since the values of the Negishi weights remain fixed at those of the no-control equilibrium. Nonetheless, this approximation may be justified in light of constraints on time and computing resources. The first-order conditions of the Negishi problem are shown in Appendix 3.B.49

3.4 Policy formulation

In the tax scenarios for controlling greenhouse warming (RICE cooperative, SW tax and transfers, SW no transfers), the policy instruments are the control rates of GHG emissions, \( \mu_i(t) \), and the inter-regional compensation levels, \( \Gamma_j(t) \). Compensation may be positive or negative, depending upon whether a region is a recipient (positive) or a donor (negative). In the permit scenarios (SW optimal permits, SW per capita permits), the policy instruments are the initial allocations of permits, the \( \tilde{P} \)’s. For each of the welfare maximization scenarios (SW tax and transfers, SW no transfers, SW optimal permits, SW per capita permits), the optimal program is determined by maximizing the social welfare functional with respect to the policy variables as well as with respect to output, consumption, investment, and net exports, subject to the constraints of the climate-economy model and the Negishi first-order conditions. This process is carried out for different values of the equity parameter, \( \rho \). In the present study, values of \( \rho \) in the range of \([0,20]\) were tried. In the case of RICE cooperative, the solution is determined by maximizing the Negishi objective functional with respect to the policy variables, as well as output, consumption, etc., subject to the constraints of the climate-economy model. The solution to the RICE cooperative scenario is determined without reference to the social welfare functional because it represents

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49Only those first-order conditions which are not also structural equations in the climate-economy model are shown. The structural equations are already taken into account, as they are constraints in the social welfare maximization as well as the Negishi problem.
a positive model of GHG control rather than a normative model.

Despite appearances, the carbon tax rates are actually implicit rather than explicit in the tax scenarios. The emissions control rate, \( \mu(t) \), is the control variable and determines the marginal cost of abatement in each region and period, according to A.17. (The centre element of A.17 is the marginal cost of abatement.) The marginal cost of abatement is constrained to be the same in all regions at any given period, so that the policy is Pareto efficient. Of course, in the certainty context, a quantity-constraint policy such as this one is just the dual of a tax policy. In the equivalent tax policy, the planner would choose a single tax rate for all regions in each period. Private agents would reduce their emissions until their marginal costs of abatement were equated with the tax rate and then pay the tax on their remaining emissions. In this manner, the marginal cost of abatement would again be equalized across regions. It is clear, therefore, that the equivalent tax rate in the dual tax policy can be determined from the quantity-constraint policy by looking at the marginal cost of abatement. This equivalence is shown in A.17. Although it is assumed throughout that GHG policies will be implemented using taxes, the policies are modelled as quantity-constraint problems, and then the equivalent tax rates are read from the marginal cost values.

One aspect of the tax approach that is not implicit in the quantity-constraint approach is the so-called double dividend of tax recycling. When revenues from the carbon tax are used to replace revenues previously raised by distortionary taxation, it may be expected that the change in taxes alone will result in improved welfare, owing to the reduction in deadweight loss associated with the distortionary taxation. This improvement in welfare can be translated into an equivalent increase in income, by means of equivalent variation. Nordhaus (1994, 120-21) considers this issue by assuming that revenue from the carbon tax
replaces revenue from distortionary taxes with a marginal deadweight loss of 0.3, i.e. 30 cents of deadweight loss for every dollar of revenue. Therefore, every dollar raised by the carbon tax results ceteris paribus in an equivalent income benefit of 30 cents. Not surprisingly, this assumption results in marked increases in the optimal values of carbon taxes, in all periods. This line of research was not pursued in Nordhaus and Yang (1996), and neither is it pursued here. There is much debate at present about whether the carbon tax is not also distortionary. See for example Goulder (1995). Of course, any attempt to arrive at definitive policy recommendations on the control of GHG's will have to address this issue.

In contrast to the tax scenarios, the modelling of the permit scenarios is explicit rather than implicit. The planner chooses the initial allocation of permits and enforces the regime. Competition in the market for permits ensures a single price in every period. Agents reduce their emissions until their marginal cost of abatement has risen to the level of the permit price, after which they purchase additional permits to cover their emissions if they are short, or sell permits if they have a surplus. In terms of the modelling, a uniform permit price is imposed, in accordance with A.17, which equates the permit price with the marginal cost of abatement. Equilibrium in the permit market is ensured by A.20, which states that the total demand for permits (total emissions) is equal to the total supply. Note that in this framework, the control rate of emissions, \( \mu_j(t) \), is endogenous. Finally, the marginal cost of abatement is derived from the total cost, which is equal to

\[
TC_j(t) \frac{A_j(t) K_j(t) \gamma L_j(t)^{1-\gamma}}{(1 + a_{ij} T(t)^{\lambda_j})}.
\]

The results presented in section 3.5 are based upon the "best-guess" or expected values of initial variables and parameters. Of course, a complete analysis of global warming
must take into account the substantial uncertainties which characterize many of these quantities, particularly on the environmental side of the model. Such a treatment is beyond the scope of the present study. However, an excellent method for dealing with these uncertainties has been developed by Nordhaus (1994). In this method, approximate probability distributions are constructed to characterize the major uncertainties and then used to arrive at five representative states of the world. A Monte Carlo simulation is run for each scenario to determine the optimal control policy assuming the planner is an expected welfare maximizer. In general, Nordhaus finds that the effect of accounting for uncertainty in this manner is to give more aggressive initial control policies than the certainty-equivalent case. In other words, in addition to its certainty-equivalent value in reducing greenhouse damages, emissions abatement is also desirable for spreading greenhouse risk across generations and regions.

Regarding inter-regional trade and investment, Nordhaus and Yang reluctantly opt for a no-trade equilibrium, and the same approach is followed here. Runs of the unrestricted market equilibrium, using the Negishi technique with no greenhouse controls, result in interregional flows far beyond the levels actually observed. Upon reflection, this discrepancy between the predicted and observed levels of trade is not surprising as it reflects the model's attempt to equalize the significant differences in rates of return on investment which exist between regions, especially between the highly capitalized industrial countries and the developing world. Nordhaus and Yang experiment with quantity constraints on net imports and foreign asset balances. However, this approach does not provide much useful information about the impact of greenhouse policy on trade, since trade flows in this case are dominated
by the choice of the quantity constraints. Compared with the shortcomings of both the unrestricted and the quantity constrained models, the no-trade equilibrium has at least simplicity to recommend it. The no-trade equilibrium is modeled by constraining $BT_j(t)$, $NFA_j(t)$, and $\pi_j(t)$ to be equal to 0 and dropping equations B.2 and B.6 from the Negishi first-order conditions.

Nordhaus and Yang hypothesize that the inter-regional differences in the rate of return on investment in the no-trade model (or in any quantity-constrained model for that matter) may reflect a preference for investment in domestic human capital, which is not highly mobile. While this argument has some merit, it also seems likely that these differences may reflect overoptimism by the authors in their assessment of the efficiency of developing-country economies. Indeed, the persistence of such differentials in the rates of return suggests that the market’s expectation of total factor productivity, $A_j(t)$, is much lower for developing countries or regions than the values estimated by Nordhaus and Yang (1996). Alternatively, the differentials may reflect a premium demanded for undiversifiable risk. Such risk can be modelled explicitly when uncertainty is taken into account. Either of these two alternatives provides a potential way to model a realistic trade equilibrium without resorting to quantity constraints. More attention will be paid to this problem in future research.

The time horizon used for the maximizations is 25 periods, in accordance with Nordhaus and Yang, where each period represents a decade. Results are reported for the first 11 periods (1990 to 2100). In the RICE cooperative scenario of Nordhaus and Yang, which is based upon the Negishi market equilibrium, the results in the first 11 periods are

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Nordhaus and Yang reject quantity constraints for a different reason, namely that they result in different levels of the carbon tax in each region. This result is due to the fact that the authors do not impose a separate constraint to force the taxes to be equal.
insensitive to increases in the horizon. Therefore, these results can be viewed as being "on the turnpike" of the desired infinite problem. Unfortunately, this property does not carry over to the welfare maximization scenarios (SW tax and transfers, SW optimal permits, SW per capita permits, SW no transfers) due to the much lower time discounting implicit in the social welfare functional compared with the Negishi objective functional. Therefore, the results reported for these scenarios would change if a longer horizon were employed. Further research is planned to address this problem. Nonetheless, the results reported provide an indication of the direction the results take when welfare maximization is undertaken, compared with the positive framework of Nordhaus and Yang.

3.5 Results: $\rho = 0$

This section presents the results of the second-best optimization runs for the five greenhouse policy scenarios described in section 3.2, assuming a value of the equity parameter of 0 (utilitarian social welfare). The numerical optimizations are performed with the GAMS software (General Algebraic Modelling System) developed by Brooke, Kendrick and Meeraus (1992). The results are presented in terms of the following variables: social welfare, GHG emissions and emissions abatement, global mean temperature, greenhouse damages, carbon taxes and permit prices, compensation flows, aggregate goods output, and consumption.

The ranking of the five control scenarios in order of increasing social welfare, assuming an equity parameter of 0, is: RICE cooperative, SW no transfers, SW per capita permits, SW tax and transfers, and finally SW optimal permits. Not surprisingly, greater

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51 For information about the software, contact the GAMS Development Corporation, 1217 Potomac Street NW, Washington D.C., 20007, email: webmaster@gams.com.
inter-country compensation correlates with greater social welfare. The average rate of emissions abatement also increases in the same order initially, reaching a peak in SW tax and transfers and then declining somewhat in SW optimal permits. As with emissions abatement, so goes the carbon tax/permit price.

The rates of emissions abatement for the five control scenarios are shown in Figures 3.1 through 3.5. Emissions abatement, \( \mu(t) \), is measured as the percentage reduction in GHG emissions compared with the baseline "no-control" scenario. In the RICE cooperative scenario, shown in Figure 3.1, the initial rates of abatement, which are implemented in the year 2000, are between 7 percent for UJE and 15.5 percent for China. These rates then rise gradually to a range of 10 and 18 percent by the mid-century, and tend to level out by the end of the century. In SW no transfers, shown in Figure 3.2, the initial rates are somewhat higher, between 12.4 percent for UJE and 26 percent for China and level off by the end of the century between 26.9 percent and 39.2 percent. The remaining three scenarios show a different pattern, starting out with much higher levels of control initially, rising to a peak in the second or third decade of the century, and then stabilizing by the end of the century. In SW per capita permits, shown in Figure 3.3, the range of abatement rates is between 29.1 percent (UJE) and 61.2 percent (China) initially, increases to 34.3 percent and 70.5 percent in 2010, and then levels out to 25.6 and 37.4 percent by 2100. SW tax and transfers, shown in Figure 3.4, has the highest levels of abatement control. The range is between 40 percent (UJE) and 84 percent (China) initially and then rises to a peak between 48.6 and 99.9 percent in 2010. Subsequently, China and FSU experience a secular decline, while ROW and UJE decline slightly in 2020 and then rise slightly thereafter until the end of the century. Figure 3.5 shows the levels of emissions abatement for SW optimal permits. The range begins between 32.4 percent (UJE) and 68.1 percent (China) in 2000, dips slightly in 2010,
percent reduction from baseline

Figure 3.2: Emissions Abatement SW No Transfers

CHN + PSU ROW UWE
Figure 3.4
Emissions Abatement, SW Tax & Transfers

percent reduction from baseline

0 10 20 30 40 50 60 70 80 90 100

2000  '20  '40  '60  '80  2100

□ CHN  + FSU  ◇ ROW  △ UJE
Figure 3.5: Emissions Abatement SW Optimal Permits

Percent reduction from baseline
rises to a peak in 2020 between 41 and 82 percent, and finally converges toward the end of the century between 38.9 and 56.8 percent.

The high levels of abatement recommended in the last three scenarios must be interpreted with caution. Of particular note, the 99.9 percent reduction of emissions required for China in the decade beginning in 2010, in SW tax and transfers (Figure 3.4), seems particularly suspicious. On the technical level, this result is explained by the fact that the abatement cost functions in the Nordhaus and Yang model do not tend to infinity as the control rate approaches unity (i.e. 100 percent reduction of emissions) but rather are finite-valued at this point. In theory, then, it is possible to achieve the complete elimination of GHG emissions from the economy at finite cost, presumably by substituting GHG-free energy sources for fossil fuels. In reality, it is probably unlikely that China could divorce itself from fossil fuels so soon, except at catastrophic cost. At the very least, it would seem prudent to require a more explicit discussion of alternative fuel sources, including a GHG-free backstop technology, before accepting these numbers with any degree of confidence.

This situation highlights the weakness of top-down or econometric estimates of abatement cost functions: because they are fitted to observations that vary at most by a few percentage points from the status quo, extrapolation of such functions becomes increasingly unreliable the further away one goes from the status quo. Further research is required to incorporate bottom-up estimates of abatement costs, i.e. estimates based upon engineering projects of plausible trends in technology.

It is noteworthy that, although SW optimal permits has the highest level of welfare, it has a lower level of abatement than SW tax and transfers. The explanation for this state of affairs is found in the compensation numbers. Because SW optimal permits allows greater compensation and concentrates it on the poorest region(s), it does not need to rely as heavily
on the abatement of the greenhouse damages to aid these regions. The fact that China has the highest rates of abatement in all the scenarios, followed in descending order by FSU, ROW and UJE, reflects the lower marginal cost of abatement associated with it in this model. For, in an efficient policy, abatement is undertaken first where it is cheapest. In practice, the relatively low marginal costs of abatement associated with China and the FSU in Nordhaus and Yang’s model can be interpreted as reflecting the relatively higher dependence on coal-fired electricity generation in these regions. Conversely, the relatively high marginal cost of abatement in UJE can be interpreted as reflecting greater efficiency in the market for energy in the United States, Japan and Europe, as well as newer technology.

Figure 3.6 shows the differences in global aggregate emissions between the scenarios. Higher rates of abatement result in lower aggregate emissions obviously. All of the maximization scenarios result in the stabilization of GHG emissions at or below the 1990 level — 5.96 billion tons of carbon-equivalent — during the first decade of the century, and those that include compensation payments (SW tax and transfers, SW per capita permits, SW optimal permits) maintain this stabilization past 2020.

Figure 3.7 shows the accumulation of GHG’s in the atmosphere, in terms of equivalent tons of carbon. Initially, the differences in emissions between the scenarios do not have much of an effect on atmospheric concentration, due in part to the large stock of GHG’s already committed by past emissions and by natural processes. However, over the course of several decades, the differences in emissions become apparent, especially as income continues to grow. By the end of the century, the most aggressive control scenario, SW tax and transfers, results in a reduction in atmospheric concentration of 29 percent.

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52The emissions shown are from fuel consumption only. Other GHG emissions result from changes in land use, such as deforestation, and from agriculture. In Nordhaus and Yang’s climate-economy model, these other emissions are exogenous. Equation A.9 describes the time profile of these exogenous emissions.
Figure 3.7
Atmospheric Concentration of GHG's

- □ no control
- + RICE cooperative
- ◊ SW: tax & transfers
- △ SW: no transfers
- × SW: optimal permits
- ▽ SW: p.c. permits
compared with the baseline scenario of no control, and the scale of this reduction continues to increase beyond the end of the century (not shown).

Obviously, the differences among the scenarios in the atmospheric concentration of GHG's are reflected in differences in the severity of greenhouse warming. Figure 3.8 shows the resultant changes in global mean temperature from pre-industrial times. As with the concentration of GHG's, the differences between the scenarios in terms of temperature are not large early in the century, due to the extent of warming already committed by past emissions. Nonetheless, by the end of the century, the most aggressive abatement regime results in a reduction in global warming of approximately 0.8 degrees Celsius, compared with the no-control baseline, and the scale of this reduction is expected to continue to increase beyond the end of the century.

Figure 3.9 shows the total annual damages attributable to global warming, in terms of lost output. These damages are calculated by the expression

\[ A_j(t) K_j(t)^\gamma L_j(t)^{1-\gamma} \left[ 1 - \frac{1}{1 + D_j(t)} \right] , \]

which gives potential output less actual output (gross of abatement costs). The same comments apply here as above. In particular, by the end of the century, the most aggressive abatement regime results in an annual reduction of damages of almost 1 percent of world output (0.96 percent), compared with the no-control baseline, and this reduction is expected to increase beyond the end of the century. In dollar terms, this reduction in damages in 2100 is on the order of $2 trillion (1990 U.S.).

As discussed in section 3.4, the carbon tax or permit price is equal to the marginal cost of emissions abatement. Therefore regimes with higher rates of abatement also have higher taxes or permit prices. Figure 3.10 shows these values in $U.S. (1990) per ton of
Figure 3.9
Total Damage due to Greenhouse Warming

percent of world output (per annum)
carbon equivalent. The initial values range from the very low to the very high. The regimes that do not pay compensation (RICE cooperative and SW no transfers) are bunched together at the low end of the initial range, showing $7.65 and $21.95 per ton of carbon respectively. Both regimes show smooth growth in the tax rate throughout the century, with RICE cooperative growing to $33.11 per ton of carbon in 2100 and SW no transfers growing much more to $144.48. In contrast, SW tax and transfers shows the highest levels of taxation of all five regimes. The initial value for SW tax and transfers is $199.98 per ton, and the tax continues to rise thereafter. By 2100, the tax has risen to $566.08, although its rate of increase has slowed. SW optimal permits and SW per capita permits show initial permit prices between the two extremes: $134.51 and $109.80 respectively. SW optimal permits jumps to $251.12 in 2020 and finishes the century at a stable rate of $291.59. SW per capita permits rises somewhat from its initial price in 2010 and then returns to its long-run equilibrium level of $132 per ton of carbon. In general, these differences in the levels of carbon taxes reflect differences among the regimes in the extent of abatement. SW tax and transfers has the highest levels of abatement, and this fact is reflected in the very high levels of the carbon tax associated with it. Conversely, RICE cooperative has the lowest levels of abatement and therefore also the lowest levels of the carbon tax.53

In order to put these values of taxes and permit prices in perspective, it is worthwhile to convert them into equivalent increases in the retail price of gasoline. Using a conversion factor of 0.003 tons of carbon per U.S. gallon of gas, or equivalently $7.93 \times 10^{-4}$ tons per litre,54 a permit price of $109.80 U.S. per ton of carbon (the initial value of SW per capita

53In the present model, the relationship between the carbon tax or permit price and the amount of abatement is known with certainty. In reality, the marginal cost of abatement is uncertain in advance, and therefore the relationship between the carbon tax or permit price and the amount of abatement is also uncertain. This issue can be treated explicitly when uncertainty is incorporated into the model. See for example Nordhaus (1994).

54Cline (1992, p.7).
Figure 3.10
Carbon Tax/ Permit Price

$U.S. (1990)$ per ton carbon

- RICE cooperative
- SW: tax & transfers
- SW: no transfers
- SW: optimal permits
- SW: p.c. permits
permits) is equivalent to 8.7 cents per litre and 32.9 cents per gallon of gasoline. These values represent an approximate increase of 20 percent in the price of regular gasoline in Canada and a 23 percent increase in the U.S. price. A tax of $199.98 per ton of carbon (the initial value of SW tax and transfers) is equivalent to 15.9 cents per litre and 60.0 cents per gallon of gas; i.e. a 37 percent increase in the Canadian price and a 42.6 percent increase in the U.S. price. In contrast, the initial value of the tax under RICE cooperative, $7.65 per ton of carbon, is equivalent to 0.6 cents per litre and 2.3 cents per U.S. gallon of gas, which represents a 1.4 percent increase in the Canadian price and a 1.6 percent increase in the U.S. price.

Figures 3.11 to 3.14 show the pattern of compensation flows under the three scenarios that include compensation. SW optimal permits has the highest levels of total compensation, followed by SW tax and transfers and SW per capita permits. Figure 3.11 shows total compensation as a percentage of world output. In addition to differences in the levels of total compensation, the three compensation scenarios also exhibit different arrangements concerning donors and recipients. However, one common thread among the scenarios is that UJE — the richest region — always pays. In SW tax and transfers, shown in Figure 3.12, China, FSU and ROW all receive compensation from UJE. In SW optimal permits, shown in Figure 3.13, UJE, FSU and ROW all pay compensation to China, in the form of permit purchases. Finally, in SW per capita permits, shown in Figure 3.14, UJE and FSU — the first and second richest — make net purchases of permits from China and ROW until 2090, after which China also purchases permits from ROW.

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55 On February 20, 1997, a typical retail price of regular gasoline was 43 cents U.S. per litre in Toronto and $1.41 U.S. per gallon in Rochester, New York. The Canadian price has been converted into U.S. dollars with an exchange rate of 0.7356 $U.S./$C.

56 Total compensation is measured as the sum of contributions by donor countries, or equivalently the sum of receipts by recipient countries.
$U.S. trillion (1990)

Figure 3.12
Compensation, SW Tax & Transfers
Figure 3.13
Compensation, SW Optimal Permits

$U.S. trillion (1990)$

CHN  FSU  ROW  UJE
The relative costs and benefits of the different scenarios to the present generation(s) — i.e. to those currently alive — are revealed in the impacts on output and consumption over the course of the next century. Figure 3.15 shows the changes in global output as a percent of the no-control baseline during this period. With the exception of RICE cooperative, all regimes show a sustained reduction in global output which in most cases lasts beyond 2100 (not shown). Not surprisingly, the amount of output loss during this period is correlated directly with the level of emissions abatement undertaken. SW no transfers shows only a small loss of output, which becomes a gain after 2090. SW per capita permits experiences its maximum loss of 1.36 percent in the decade after 2020, after which the loss declines steadily to 0.24 percent at the end of the century and becomes a gain shortly thereafter (not shown). SW optimal permits bottoms out in 2040 with a loss of 2.20 percent, after which it improves. Finally, SW tax and transfers sustains the largest and most sustained losses in output, with a maximum value of 2.38 percent in 2060, and experiences the slowest recovery. Eventually, all regimes exhibit an increase in aggregate output compared with the no-control baseline, as the sacrifices of early abatement efforts translate into reduced greenhouse damages. However, in most cases, these increases do not occur until after 2100.

Despite the losses in aggregate output under most regimes during the next century, some regions do gain in terms of consumption during this period. Figures 3.16 to 3.20 show the distribution of gains and losses in terms of changes in consumption from the no-control baseline. Figure 3.16 shows the case of RICE cooperative. In this regime, UJE is the only region to experience some early benefits (though small). This result is not surprising, since UJE undertakes the least abatement (in proportionate terms) and makes no transfers to other regions. Under these circumstances, the effect on consumption of early losses in output due to emissions abatement is mitigated by reducing the amount of capital investment. This

86
Figure 3.16
Consumption Change, RICE cooperative

percent change from baseline

□ CHN  + FSU  ◇ ROW  △ UJE
reduction in investment results in the one-time loss of consumption in 2010 but is offset after
2020 by reductions in greenhouse damages which follow from the earlier abatement efforts.
The other regions all experience initial reductions in consumption in RICE cooperative.
However, with the exception of the decade beginning in 2010, the changes experienced by all
of the regions are quite small, for the most part falling within 0.3 percent of the baseline
over the course of the century. After 2030, the trend is upward for all regions, and after
2080, all regions are experiencing gains in consumption.

The pattern of consumption changes for SW no transfers, shown in Figure 3.17, is
similar to RICE cooperative except that the tendency in this case is for somewhat greater
losses and slower recovery. UJE and ROW show very similar behaviour for the two regimes.
In contrast, FSU and China show greater initial losses in the current regime which then
increase for most of the century and only begin to improve in the last decade. In 2100, FSU
is still experiencing a 0.40 percent loss in consumption compared with baseline and China a
0.88 percent loss. UJE and ROW in contrast show gains after 2070.

It comes as no surprise that the truly large changes in consumption over the next
century are exhibited by the regimes which involve compensation payments, since these are
also the regimes that have the highest levels of abatement control. This positive relationship
between compensation and abatement indicates that higher levels of abatement are in effect
purchased from the poor by the rich. This state of affairs is not surprising, since the poorer
regions of the world have the lowest marginal costs of abatement in this model but are also
the least able to afford it. In fact, the rich, industrial countries emit more GHG’s in the
aggregate than poor countries, and therefore provide the largest opportunity for aggregate
abatement. However, because poor countries have lower marginal abatement costs initially,
and because rich and poor pay the same tax or permit price in this model, the poor in effect
percent change from baseline

Figure 3.17
Consumption Change, SW No Transfers
go first in reducing emissions, before the rich begin to undertake abatement with their higher marginal costs. A second explanation of the positive relationship between compensation and abatement is the greater welfare weight placed on poor countries by the planner. The more compensation paid to the poor, the more their welfare weights are reduced, and therefore the more willing is the planner to trade off present output in the form of abatement effort for future reductions in greenhouse damages.

Figure 3.18 shows the consumption changes for SW tax and transfers. All of the recipients of compensation in this regime (China, FSU, ROW) experience gains in consumption initially, while UJE experiences growing losses throughout the century. The gains of China, FSU and ROW are reduced to small losses by 2020, after which their consumption levels rise slowly for the remainder of the century. ROW experiences sustained gains after 2040. The initial changes range from a 3.43 percent gain for China to a 0.45 percent loss for UJE. By the end of the century, ROW enjoys a gain of 1.2 percent. China and FSU are unchanged from the baseline, and UJE experiences a rather large loss compared with the baseline of 8.36 percent. The divergence between UJE and the other regions reflects the fact that UJE pays compensation to all regions in this scenario.

The particular pattern exhibited by CHN, FSU and ROW in Figure 3.18, of moderate to large gains in the first two periods, relative to baseline consumption, followed by a period of relative losses after 2010, indicates the extent to which greenhouse and distributional objectives are combined in this policy scenario. In absolute terms, consumption is lowest in the initial period and grows in each successive period, despite the relative losses after 2010. Consequently, the planner places the greatest welfare weight on initial consumption, and it pays to beggar the future by increasing initial consumption at the expense of capital formation. This sacrifice of capital is the cause of the relative drop in consumption after
percent change from baseline

Figure 3.18
Consumption Change, SW Tax & Transfers
2010. The drop is offset eventually by reduced greenhouse damages. Thus, in addition to controlling greenhouse damages, the tax and transfer policy also provides a means to influence the inter-temporal distribution of consumption. In contrast, UJE builds up its capital stock over the course of the century, at the expense of consumption, in order to provide the basis for compensation transfers to the poorer regions.

Consumption change in SW optimal permits, shown in Figure 3.19, shows a rather different configuration of winners and losers than SW tax and transfers. In this regime, China is the only sustained winner over the century, although ROW experiences very slight gains until 2010. China's gain in consumption grows quickly to a staggering 84 percent of baseline in the decade following 2020. This gain declines subsequently, although it is still large at 35.5 percent at the end of the century. The biggest loser in this regime in terms of baseline percentage is FSU, which sustains a consumption loss of approximately 10 percent throughout the century. ROW and UJE experience smaller percentage losses in most periods. However, although FSU sustain the biggest loss in terms of baseline percentage, UJE is still the biggest loser in absolute dollar terms.

The divergence between China and all other regions, in SW optimal permits, reflects the allocation of all initial permits to China after 2010. This concentration of the initial permits with one region maximizes social welfare by ensuring that all transfers (net purchases) go to the poorest region, in this case China. The policy has a large redistributational element, as the poorest region receives more in transfers than the cost of its abatement efforts, a fact reflected in the soaring consumption change figures for China. Unfortunately, as discussed in section 3.2, the policy may also create perverse incentives for the beneficiary, since too much improvement in economic performance could lead to the loss

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57 Before 2010, the initial permits all go to ROW.
of the "worst-off" status and with it the loss of the initial permits and attendant transfers. For these reasons, SW optimal permits is probably not a desirable regime, despite the fact that it has the highest social welfare value of the five regimes tested.\textsuperscript{58}

Consumption changes under the preferred permit regime, SW per capita permits, are shown in Figure 3.20. As with the data on compensation flows (Figure 3.14), the clear winner under this regime in terms of consumption change over the next century is ROW. From an initial gain of 1.42 percent, compared with the baseline consumption under no controls, ROW dips to a slight loss in 2020, after which it experiences sustained although small gains. China experiences much larger gains than ROW initially, but experiences losses after 2040 and these losses continue beyond the end of the century (not shown). China’s peak consumption gain is 8.48 percent, which comes between 2000 and 2010, and its loss in 2100 is 1.59 percent. FSU experiences significant losses over the century, although not as large as in SW optimal permits. FSU bottoms out with a loss of 7.36 percent of baseline in 2010 and improves thereafter, with a loss of 1.99 percent by 2100. UJE also bottoms out in 2010, although with a much smaller relative loss than FSU of 1.5 percent, and finishes the century with a small loss of 0.35 percent.

A more succinct summary of consumption gains and losses experienced by the present generation is provided by the present discounted values of the changes under the various scenarios, compared with the baseline case of no control. This information is reported in Table 3.1. For this purpose, the personal consumption discount rate is used, which varies over time and among the regions. This choice of the discount rate reflects the after-tax rate

\textsuperscript{58}The emergence of China as the worst-off agent in this model and therefore the object of some special policy attention does not seem credible in light of its recent dynamic growth. This situation can be attributed to two shortcomings of the model. First, GDP values have been translated into U.S. dollars using market exchange rates rather than purchasing power parities. Second, the catch-all region ROW lumps together the world's poorest countries with middle-income countries, the newly industrialized tigers of southeast Asia, and even some members of the OECD (e.g. Canada and Australia). Future research is planned to address these problems.
percent change from baseline

Consumption Change, SW P.C. Permits

Figure 3.20
of return available to individuals and is therefore appropriate for calculating present values from the perspective of private agents.\textsuperscript{59} The negative sign of the present values for RICE cooperative and SW no transfers for all regions, shown in Table 3.1, indicates that no one currently alive would favour these two scenarios unless motivated by altruism for future generations. The present values for SW tax and transfers are again negative for UJE but positive for the other regions. Under SW optimal permits and SW per capita permits, the present values are negative for UJE and FSU and positive for CHN and ROW. These patterns of positive and negative values reflect the direction of compensation flows or net permit purchases discussed above.

The conflicting interests of the present generation, which are reflected in the different signs of the present values reported in Table 3.1, highlight the difficulties in winning political support for any aggressive policy of emissions abatement, despite the fact that the more aggressive scenarios give greater social welfare. Policies which contain redistributational elements usually entail a certain amount of conflict. However, the present task is complicated by the fact that the contemplated redistributions are not just between contemporary groups, which can fight among each other, but also from the current generation to those as yet unborn, who are not present to press their case. Furthermore, in the richest countries, represented by UJE in Table 3.1, the present generation loses in all five of the scenarios tested. This fact is particularly significant since none of the more aggressive regimes can be implemented without the richest countries agreeing to make compensation payments to the poor. Nonetheless, it is possible that the altruism of the present generation is sufficient to

\textsuperscript{59}The particular formula used for this rate, in constructing Table 3.1, is $0.03 + g$, where 0.03 is the private pure rate of time preference and $g$ is the annual growth rate of per capita consumption, which is endogenous. The choice was made to use the growth rates from the no-control scenario. The average value of the discount rate among the four regions, using this formula, starts at 4.72 percent in the first period, peaks in the second period at 6.80 percent, then falls gradually to 4.18 percent by 2100. These values are comparable to the average, post-war, real, after-tax returns on long-term U.S. government bonds (Lind 1982).
Table 3.1

Present Value of Per Capita Consumption Changes

<table>
<thead>
<tr>
<th>SW per capita per mits</th>
<th>SW optimal permits</th>
<th>SW tax &amp; transfers</th>
<th>SW no transfers</th>
<th>Rice cooperative</th>
<th>ROW</th>
<th>CHN</th>
<th>UE</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.48</td>
<td>227.1</td>
<td>-1490.88</td>
<td>-1494.27</td>
<td>-1922</td>
<td>313.4</td>
<td>122.9</td>
<td>-2177.1</td>
</tr>
<tr>
<td>14.93</td>
<td>169.35</td>
<td>108.10</td>
<td>2663.69</td>
<td>306.93</td>
<td>799.89</td>
<td>20.60</td>
<td>-3566.74</td>
</tr>
<tr>
<td>5.95</td>
<td>67.50</td>
<td>14.59</td>
<td>359.77</td>
<td>46.79</td>
<td>121.43</td>
<td>31.82</td>
<td>-5508.70</td>
</tr>
<tr>
<td>-1.68</td>
<td>-179.0</td>
<td>-2.81</td>
<td>-69.14</td>
<td>-6.80</td>
<td>-15.90</td>
<td>-0.22</td>
<td>-38.76</td>
</tr>
<tr>
<td>-1.10</td>
<td>-124.3</td>
<td>-0.76</td>
<td>-18.99</td>
<td>-2.87</td>
<td>-7.48</td>
<td>-0.04</td>
<td>-6.51</td>
</tr>
<tr>
<td>$US (1990)</td>
<td>% (a)</td>
<td>$US (1990)</td>
<td>% (a)</td>
<td>$US (1990)</td>
<td>% (a)</td>
<td>$US (1990)</td>
<td>% (a)</td>
</tr>
</tbody>
</table>

(a): The present value expressed as a percentage of first-period per capita consumption under no control.
enable a supportive coalition to be formed. In any case, the existence of significant no-regrets opportunities for abatement and endogenous technological change, neither of which has been modelled here, suggests that the actual costs to the present generation will be much lower than estimated here.

3.6 Variation of the equity parameter

At first pass, it might be expected that increasing the equity parameter in (3.1) would favour the present generation by lowering the level of optimal abatement control in the early periods. This expectation is based upon the observation that living standards increase over time in Nordhaus and Yang's model, due to the combined effect of capital accumulation and growth of total factor productivity. As a result of these increases, an increase in the planner's concern for equity must translate into increased concern for the welfare of the present generation, because it is the poorest.

This line of reasoning echoes the distinction made in the introduction between welfare functionals that define the optimum in terms of marginal conditions and those that define the optimum in terms of relative levels. When $\rho=0$, (3.1) is utilitarian and defines the welfare optimum entirely in terms of marginal conditions. In this case, a distributional inequity is likely to exist to the detriment of future generations, despite their higher living standards, because in effect their living standards are not high enough. On the other hand, as $\rho$ is increased, the levels of expected lifetime utility play an increasingly larger role in the definition of the welfare optimum. In the limit, as $\rho\to\infty$, the planner pays no attention to marginal conditions and is concerned only with the level of well-being of the worst-off agent, which in this case is the present generation.

In fact, expectations concerning the impact of higher values of $\rho$ are complicated by
the existence of intra-generational inequities. While each region is indeed poorest in the first period, some are obviously poorer than others. In the uncontrolled market equilibrium, the ranking of the regions in decreasing order of per capita consumption in the first period is UJE, FSU, ROW, CHN. Consequently, as $\rho$ is increased, the planner becomes more concerned with the first-period consumption of the poorest region, China, rather than first-period consumption in general. But since general transfers for the express purpose of equalizing consumption are not allowed in the second best context used here, the only way the planner can help China in the first period is to increase the initial abatement efforts of all countries. This policy reduces greenhouse damages in the future, compared with less aggressive regimes, and the reduced damages allow China to dedicate more of its output in the first period to consumption and less to capital accumulation. In short, because future damages are decreased, China does not need to save as much and can consume more now. The increased abatement costs which fall to China as a result of this policy do not affect it as long as these costs are covered by compensation from richer regions. Therefore, the actual effect of increasing $\rho$ in a regionally disaggregated, second-best world is likely to be an increase, rather than a decrease, in early abatement efforts.

Values of the equity parameter in the range of 0 to 20 were tried for various regimes but most resulted in no changes from the initial runs which were based upon a value of 0 (utilitarian case). However, one case did produce a significant departure, namely a value of 10 in SW tax and transfers. The results of this case confirmed the expectations discussed above concerning emissions abatement, consumption and investment. These results are shown in Figures 3.21 through 3.23. Figure 3.21 shows the increase in carbon taxes when $\rho = 10$. Figure 3.22 confirms the expected increase in the initial per capita consumption level of China, and also of FSU and ROW, as well as a rather marked decrease in the consumption
Carbon Taxes, Equity, Comparison

Equity param = 0 + Equity param = 10

$U.S. (1990) per ton carbon
level of UJE over the course of the century, compared with the baseline case of $\rho = 0$. Finally, Figure 3.23 shows the changes in investment which are necessary to pay for the increased initial consumption of China, FSU and ROW. With the exception of the decade following 2000, UJE experiences increased investment, compared with the baseline case of $\rho = 0$. These increases perform two functions. First, they compensate for the negative impact of higher future abatement efforts on domestic output. Second, they provide the basis for higher compensation payments to China, FSU and ROW. As for these regions, all three of them experience reduced investment over the course of the century, compared with the baseline. These reductions pay for higher consumption in the initial period, and the subsequent reductions in output due to the smaller capital stocks are offset partly by increased compensation received from UJE. Overall, then, the effect of increasing the equity parameter, $\rho$, in the social welfare functional is to increase the aggressiveness of greenhouse abatement efforts.

3.7 Conclusion

From the sometimes overwhelming mass of details presented in this chapter, a clear story emerges concerning greenhouse policy in the second best. This story consists of four major themes. First, the lower implicit discount rate of a normative social welfare functional results in more aggressive abatement of GHG's than in studies such as Nordhaus and Yang (1996) which seek to internalize the greenhouse externality in a positive model of the economy. Second, a positive relationship exists between abatement effort and compensation payments from rich to poor. In effect, the rich purchase abatement from the poor, as the poor have lower marginal costs of abatement, for various reasons, and yet are least able to pay for it. Moreover, the planner places greater welfare weight upon the consumption of the
percent change from baseline

Consumption Change, Equity Parameter 10

Figure 3.22
Figure 3.23
Investment Change, Equity Parameter 10

percent change from baseline

CHN + FSY ROW UJE

10

2000 20 40 60 80 100
poor, and therefore is more willing to trade off present output, in the form of abatement costs, for future reductions in greenhouse damages when the poor are better compensated.

Third, second-best greenhouse control policy provides an opportunity to mitigate distributional inequities somewhat by influencing the inter-temporal and inter-regional distribution of consumption and income. For example, the combination of transfer payments and future reductions in greenhouse damages permits an improvement in the initial consumption levels of poor regions at the expense of their domestic saving. Fourth, increasing the emphasis on equity, by increasing the equity parameter, $\rho$, in the social welfare functional, results in more aggressive abatement of GHG's. Most of the results presented in this chapter have been for utilitarian welfare, which corresponds to an equity parameter of 0. Even in this case, the initial levels of optimal emissions reduction are significantly higher than in the positive treatment of Nordhaus and Yang (1996). Making the welfare functional "more Rawlsian" by increasing $\rho$ increases the levels of optimal emissions reduction.

Obviously there remain many shortcomings in the second-best framework used in the present study. Six areas have been identified for priority attention. First, the time horizon must be increased to ensure that results for the desired number of initial periods are on the turnpike of the infinite problem. Second, the model must be extended to include uncertainty. Third, a credible treatment of inter-regional trade and investment must be developed. Fourth, better estimates of damage and cost functions are required, especially for China, the former Soviet Union, and other non-OECD countries. Fifth, the regional

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60Preliminary work on this topic for SW tax and transfers shows that increasing the horizon from $T=25$ to $T=30$ increases carbon taxes approximately 7 percent in the decade 2000-2010 and total compensation flows almost 11 percent. These findings suggest that the results presented in sections 3.5 and 3.6 should be interpreted as lower bounds on the values of carbon taxes, emissions abatement and transfers in the infinite problem. Thus, the case for aggressive abatement is even stronger than indicated above.
groupings should be reconstructed to better reflect differences in income and fuel-use characteristics. Sixth, national incomes should be converted into a common currency by means of purchasing power parities, rather than market exchange rates. In addition to these priority areas, two other topics present themselves as opportunities for further improvement. First, the opportunity to account for changing demographic characteristics has not yet been fully realized. As a first step, it would be desirable to have different estimates of the death rate, \( p \), for each region. Second, a more exhaustive investigation of the effect of variations in the equity parameter, \( \rho \), would also be desirable.

As a consequence of these shortcomings, the results on emissions control presented above can only be interpreted as qualitative indicators of the effect of second-best policy, in comparison with results obtained in first-best or positive models. They certainly must not be interpreted as specific recommendations. Nonetheless, the drastic increase in the levels of optimal emissions control, compared with the results of Nordhaus and Yang (1996) and others, is indicative of the likely direction of future research and therefore provides some support for moderate immediate action to control emissions.

Finally, it must be repeated that, although the policies estimated above represent the optimal levels of greenhouse control \textit{ceteris paribus} for a variety of second-best scenarios, they may not provide the best use overall of limited government revenues (assuming the policies have a negative impact on revenues). In order to ensure the best use of revenues, the governments involved must compare the results of these policies with all other available options. One thing is certain however: if these policies are dominated, it will be by a public project and not by displaced private investment. For, as discussed in section 3.3, displaced private investment has already been treated as a cost in estimating these policies.
Appendix 3.A

Climate-Economy Model

Definitions

Indices

\[ t \quad = \quad \text{time} \]
\[ j \quad = \quad \text{country or region} \]

Endogenous variables

\[ C_j(t) \quad = \quad \text{aggregate consumption} \]
\[ c_j(t) \quad = \quad \text{per capita consumption} \]
\[ Y_j(t) \quad = \quad \text{available gross output} \]
\[ K_j(t) \quad = \quad \text{capital stock} \]
\[ I_j(t) \quad = \quad \text{gross investment} \]
\[ R(t) \quad = \quad \text{net return on foreign asset balances} \]
\[ NFA_j(t) \quad = \quad \text{net foreign asset holdings, country j (positive if assets, negative if liabilities)} \]
\[ BT_j(t) \quad = \quad \text{balance of trade} \]
\[ EF_j(t) \quad = \quad \text{GHG emissions from fuel use (million tons CO}_2\text{)} \]
\[ EL_j(t) \quad = \quad \text{GHG emissions, land use changes (m. tons CO}_2\text{)} \]
\[ E(t) \quad = \quad \text{global emissions of GHG’s (CO}_2\text{ and CFC’s only)} \]
\[ M(t) \quad = \quad \text{mass of greenhouse gases in the atmosphere} \]
\[ F(t) \quad = \quad \text{radiative forcing due to greenhouse gases (Watts/m}^2\text{ per decade)} \]
\[ T(t) \quad = \quad \text{change in mean atmospheric temperature since pre-industrial times} \]

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61 The climate-economy model is based upon Nordhaus and Yang (1996) and Nordhaus (1994), with changes discussed in chapter three.
\( T^*(t) = \) change in mean deep-ocean temperature since pre-industrial times

\( D_j(t) = \) greenhouse damage factor

\( T C_j(t) = \) cost of emissions reduction (proportional loss of output)

\( \Omega_j(t) = \) output scaling factor; accounts for losses due to greenhouse damage and emissions control

\( T X(t) = \) carbon tax (1990 $U.S. per ton of carbon equivalent)

\( P(t) = \) equilibrium price of carbon permits

**Exogenous Variables**

\( \sigma_j(t) = \) greenhouse gas emissions/output ratio

\( L_j(t) = \) labour supply; equal to population

\( A_j(t) = \) total factor productivity

\( g_j^\sigma(t) = \) growth rate of \( \sigma_j(t) \)

\( g_j^L(t) = \) growth rate of \( L_j(t) \)

\( g_j^A(t) = \) growth rate of \( A_j(t) \)

\( O(t) = \) forcings of "other" greenhouse gases (i.e. not \( CO_2 \), CFC's)

**Control Variables**

\( \mu_j(t) = \) emissions reduction (proportion of uncontrolled emissions)

\( \Gamma_j(t) = \) transfer received in tax-and-transfer scenario (positive for recipients, negative for donors)

\( PT_j(t) = \) allocation of permits to country \( j \) at time \( t \)

**Parameters**

\( \delta_k = \) rate of depreciation of the capital stock

\( \delta_r = \) decline rate of emissions from deforestation

\( \delta_M = \) rate of transfer of greenhouse gases from atmosphere to deep ocean
$$\delta_u = \text{utility discount rate in the Negishi objective functional}$$

$$\gamma = \text{elasticity of output with respect to capital}$$

$$\beta = \text{marginal atmospheric retention ratio of greenhouse gases}$$

$$C_1, C_3, C_4 = \text{thermal coefficients}$$

$$\lambda = \text{thermal feedback parameter}$$

$$a_{ij}, a_{nj} = \text{damage function parameters}$$

$$b_{ij}, b_{nj} = \text{abatement cost parameters}$$

**National economies**

Capital accumulation (domestic):

(A.1) $$K_j(t) = (1 - 10 \delta_K)K_j(t - 1) + 10I_j(t - 1)$$

Production function:

(A.2) $$Y_j(t) = \Omega_j(t)A_j(t)K_j(t)^\gamma L_j(t)^{1-\gamma}$$

Net return on foreign asset balances (weighted average of regional marginal products of capital):

(A.3) $$R(t) = \frac{\sum_j \gamma Y_j(t)}{\sum_j K_j(t)} - \delta_K$$

Allocation of aggregate output:

(A.4) $$Y_j(t) = C_j(t) + I_j(t) + BT_j(t) - \Gamma_j(t) \quad \text{(tax regimes)}$$

or

$$Y_j(t) = C_j(t) + I_j(t) + BT_j(t) + \left[ EF_j(t) - \bar{P}T_j(t) \right] P(t) \quad \text{(permit regimes)}$$

Net foreign asset accumulation:

(A.5) $$NFA_j(t) = [1 + 10R(t - 1)]NFA_j(t - 1) + 10BT_j(t - 1)$$
Global trade balance:

(A.6) \[ \sum_{j=1}^{J} BT_j(t) = 0 \]

Transversality condition:

(A.7) \[ \delta_u K_j(T) \leq I_j(T) \]

**Climate-economy interface**

Emissions from fuel consumption:

(A.8) \[ EF_j(t) = [1 - \mu_j(t)] \sigma_j(t) A_j(t) K_j(t)^\gamma L_j(t)^{1-\gamma} \]

Global aggregate emissions (fuel consumption plus changes in land use, i.e. deforestation):

(A.9) \[ E(t) = \sum_{j=1}^{J} \left[ EF_j(t) + EL_j(1)(1 - \delta_T)^{t-1} \right] \]

Global GHG accumulation:

(A.10) \[ M(t) - 590 = 10 \beta E(t) + (1 - \delta_M) [M(t-1) - 590] \]

Radiative forcing:

(A.11) \[ F(t) = 4.1 \left[ \ln(M(t)/590)/\ln2 \right] + O(t) \]

Atmospheric warming:

(A.12) \[ T(t) = T(t-1) + C_1 \left[ F(t) - \lambda T(t-1) - C_3 (T(t-1) - T^*(t-1)) \right] \]

Deep-ocean warming:

(A.13) \[ T^*(t) = T^*(t-1) + C_4 (T(t-1) - T^*(t-1)) \]

**Impacts of greenhouse warming and policy**

Damages from warming:

(A.14) \[ D_j(t) = a_{ij} T(t)^{2i} \]
Total cost of abatement (proportion of output forgone as a function of the emissions control rate):

(A.15) \[ TC_j(t) = b_{ij} \mu_j(t)^{b_{ij}} \]

Output scaling factor:

(A.16) \[ \Omega_j(t) = \frac{1 - TC_j(t)}{1 - D_j(t)} \]

Marginal cost of abatement (1990 $U.S. per ton of carbon-equivalent abated):

(A.17) \[ TX(t) = \frac{b_{ij} b_{2j} \mu_j(t)^{b_{2j}-1}}{\left[ 1 - a_{ij} T(t)^{b_i} \right] \sigma_j(t)} = P(t) \]

Compensation constraint in tax-and-transfer scenario (compensation received is no greater than abatement cost incurred):

(A.18) \[ \Gamma_j(t) \leq TC_j(t) \left[ A_j(t) K_j(t) \right] L_j(t)^{1-\gamma} \]

Global compensation balance in tax-and-transfer scenario:

(A.19) \[ \sum_{j=1}^{J} \Gamma_j(t) = 0 \]

Permit market clearance:

(A.20) \[ \sum_j \tilde{P}T_j(t) = \sum_i EF_j(t) \]

**Parameter values**

(Vector elements correspond to UJE, CHN, FSU and ROW respectively.)

\[ \delta_k = 0.065 \text{ (per year)} \]

\[ \delta_T = 0.1 \]

\[ \delta_m = 0.0833 \text{ (per decade)} \]

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52 Parameter values and initial values of variables are taken from Nordhaus (1994) and the GAMS command files of Nordhaus and Yang (1996). The combination of values for the USA, EEC and Japan into single values for UJE follows the averaging principles discussed in section 3.3.1. Details are contained in the GAMS program command files, which are available from the author upon request.
\[ \delta_u = 0.03 \]
\[ \gamma = 0.25 \]
\[ \beta = 0.64 \]
\[ C_1 = 0.226 \left( \frac{\degree C \cdot m^2 \cdot \text{decade}}{\text{Watt}} \right) \]
\[ C_3 = 0.44 \left( \frac{\text{Watts}}{\degree C \cdot m^2 \cdot \text{decade}} \right) \]
\[ C_4 = 0.02 \]
\[ \lambda = 1.41 \left( \frac{\text{Watts}}{\degree C \cdot m^2 \cdot \text{decade}} \right) \]
\[ a_1 = [0.01148, 0.01523, 0.00857, 0.02093] \]
\[ a_2 = [1.5, 1.5, 1.5, 1.5] \]
\[ b_1 = [0.05718, 0.15, 0.15, 0.10] \]
\[ b_2 = [2.887, 2.887, 2.887, 2.887] \]

**Variables: initial values**

(All values from 1990, except where noted.)

\[ L(1) = [740.41, 1133.68, 289.32, 3102.69] \text{ (millions)} \]
\[ K(1) = [41140.47, 1025.79, 2281.90, 9842.22] \text{ (bil. $U.S.$)} \]
\[ Y(1) = [15224.9, 370.02, 855.21, 4628.62] \text{ (bil. $U.S.$)} \]
\[ A(1) = [7.53, 0.335, 1.76, 1.12] \]
\[ \sigma(1) = [1.658, 18.080, 12.465, 3.673] \text{ (thous. tons / bil. $U.S.$)} \]
\[ g^s(1) = [-12.58, -14.92, -17.03, -8.95] \text{ (per cent per decade)} \]
\[ g^k(1) = [4.02, 10.19, 5.96, 17.21] \text{ (per cent per decade)} \]
\[ g^w(1) = [14.72, 36.12, 18.23, 27.76] \text{ (per cent per decade)} \]
\[ M(1) = 688.4 \text{ (billion tons CO}_2\text{ equivalent, carbon weight, 1965)} \]
\[ EF(1) = [25.24, 6.69, 10.66, 17.0] \text{ (million tons CO}_2\text{ equivalent)} \]

\[ EL(1) = [0.1, 1.36, 0.173] \text{ (million tons CO}_2\text{ equivalent)} \]

\[ T(1) = 0.5 \text{ (degrees C, 1965)} \]

\[ T^*(1) = 0.1 \text{ (degrees C, 1965)} \]
Appendix 3.B

Negishi First-order Conditions

\[(B.1) \quad 10\pi_j^K(t+1) = \frac{\phi_j(t) \cdot L_j(t)}{(1 + d)^{t-1}} C_j(t)\]

\[(B.2) \quad 10\pi_j^K(t+1) = 10\pi_j^N(t+1) + \pi_j^B(t)\]

\[(B.3) \quad \pi_j^Y(t) = 10\left[\pi_j^K(t+1) + \gamma \frac{\Sigma_j \pi_j^N(t+1) NFA_j(t)}{\Sigma_j K_j(t)}\right]\]

\[(B.4) \quad \pi_j^{EF}(t) = 10\beta \pi_j^M(t+1) \quad \text{(tax regimes)}\]

\[\quad \text{OR}\]

\[\quad = 10\left[\beta \pi_j^M(t+1) + \pi_j^K(t+1) P(t)\right] \quad \text{(permit regimes)}\]

\[(B.5) \quad \pi_j^K(t+1) - \pi_j^K(t) = 10\pi_j^K(t+1) \delta_\pi - \pi_j^Y(t) \frac{Y_j(t)}{K_j(t)}\]

\[\quad - 10\left[\Sigma_j \pi_j^N(t+1) NFA_j(t)\right] \frac{dR(t)}{dK_j(t)} + \pi_j^{EF}(t) \frac{\gamma_{EF}(t)}{K_j(t)}\]

\[(B.6) \quad \pi_j^N(t+1) - \pi_j^N(t) = -10\pi_j^N(t+1) R(t)\]

\[(B.7) \quad \pi_j^M(t+1) - \pi_j^M(t) = \pi_j^M(t+1) \delta_\pi - \pi_j^T(t+1) \frac{4.1 C_1}{\ln 2} \frac{1}{M(t)}\]

\[(B.8) \quad \pi_j^T(t+1) - \pi_j^T(t) = \pi_j^T(t+1) C_1 (\lambda + C_3) - \pi_j^T(t+1) C_4\]

\[\quad - \Sigma_j \pi_j^Y(t) Y_j(t) \frac{a_{ij} a_{2j} T(t)^{\alpha_{ij} - 1}}{1 + a_{ij} T(t)^{\alpha_{ij}}}\]
(B.9) \( \pi^*(t + 1) - \pi^*(t) = \pi^*(t + 1) C_1 - \pi^T(t + 1) C_1 C_3 \)

**Definitions**

\( \pi^K(t) = \) present value dual variable of capital in Negishi optimization

\( \pi^Y(t) = \) present value dual variable of output in Negishi optimization

\( \pi^{EF}(t) = \) present value dual variable of fuel emissions in Negishi optimization

\( \pi^N(t) = \) present value dual variable of net foreign assets in Negishi optimization

\( \pi^B(t) = \) present value dual variable of global trade balance in Negishi optimization

\( \pi^M(t) = \) present value dual variable of atmospheric carbon in Negishi optimization

\( \pi^T(t) = \) present value dual variable of atmospheric mean temperature in Negishi optimization

\( \pi^*(t) = \) present value dual variable of deep-ocean mean temperature in Negishi optimization
BIBLIOGRAPHY


[thesis]