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**The Mechanics of Arching in Induced Trench Construction – A New
Theoretical Formulation**

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Abstract

Full-scale experimental case studies have shown that the induced trench construction method effectively reduces the vertical earth load that is exerted on culverts installed beneath high embankments. Induced trench culverts are traditionally designed on the basis of Marston's theory; however, various theoretical shortcomings of this formulation have recently come to light. In this paper, a new induced trench theoretical formulation is presented. The proposed analytical model employs inclined shear planes within the embankment fill; such geometry is consistent with experimental findings reported in the literature, and leads to positive arching resulting from a reduction in vertical stress and an increase in horizontal stress (thus increasing the lateral earth pressure coefficient within the induced trench zone). A series of parametric studies are performed using finite element software. The proposed theoretical formulation is shown to be in good agreement with the numerical results, and correlations are developed to provide guidance in selecting the appropriate values of: 1) the induced trench lateral earth pressure coefficient, and 2) the height to the plane of equal settlement. Two instrumented full-scale induced trench case studies are discussed, and the proposed theoretical formulation is shown to produce results that are in good agreement with the experimental data.

Keywords: Induced Trench, Soil Arching, Analytical Study, Finite Element Modeling

Introduction

Reinforced concrete (rigid) culverts are commonly used in highway construction to provide adequate hydraulic capacity when traversing small watercourses and natural drainage channels. When rigid culverts are installed using the traditional positive projection technique, the earth load experienced by the culvert is greater than the overburden soil pressure as a result of negative arching that develops in the overlying fill. With increasing embankment height, earth loads become excessive and often exceed the structural capacity of standard precast concrete pipe sections at embankment heights of more than 10 meters (McGuigan and Valsangkar 2011a). In such circumstances, the induced trench construction method is a feasible design option that can be employed to reduce the earth loading.

The induced trench construction method involves installing a layer of compressible fill, such as hay, sawdust, or tire chips, within the embankment fill directly above the culvert. The presence of the compressible zone leads to differential settlement in the embankment fill, which mobilizes positive arching and reduces the earth load experienced by the culvert. The induced trench construction technique was originally proposed by Marston (1930) and was later developed by Spangler (Spangler 1950; Spangler and Handy 1973) for eventual implementation in the 1980 Concrete Pipe Design Manual (ACPA, 1980). Numerous successful induced trench installations are reported in the literature; Spangler (1958), Larsen (1962), Deen (1969), Taylor (1973), and Bacher et al. (1982) describe the performance of single induced trench culverts installed in the United States, while Sladen and Oswell (1988), McAfee and Valsangkar (2008), Parker et al. (2008), and Oshati et al. (2012) discuss induced trench case studies in Canada, Vaslestad and colleagues discuss the results of four induced trench case studies in Norway (Vaslestad et al., 1993; 2009; 2011; Vaslestad and Sayd, 2018), and Jean and Long (1989)

describe the performance of two induced trench arch structures in France. Twin-culverts have also been successfully installed using the induced trench technique, and two instrumented case studies are reported in the literature (McGuigan and Valsangkar 2011b; Bryden et al. 2020). In all cases, it is reported that the induced trench installations have performed well and that the vertical earth load was effectively reduced. Refer to Bryden et al. (2020) for an overview of more than 30 full-scale induced trench case studies reported in the literature.

Despite the apparent success of induced trench installations, a general sense of skepticism has developed in the industry as various researchers have expressed a lack of confidence with the underlying theory (Sladen and Oswell 1988; Scarino 2003; Handy 2004). Certain fundamental assumptions of the Marston-Spangler theory have been questioned, and, in addition to the theoretical shortcomings, general concerns have been expressed relating to the constructability of induced trench structures and their long-term performance (McGuigan and Valsangkar 2010). The induced trench construction method has consequently been omitted from recent North American design standards (ACPA 2011; CSA 2014; CSA 2019) and some jurisdictions have discontinued its use entirely (McGuigan et al. 2016).

The New Brunswick Department of Transportation and Infrastructure has constructed more than 100 induced trench culverts in recent years (McGuigan et al. 2016), and researchers at the University of New Brunswick (UNB) have been investigating the performance of induced trench culverts for nearly two decades. Instrumentation and monitoring programs have been undertaken at five full-scale induced trench structures (McAffee and Valsangkar 2005; Parker et al. 2008; McGuigan and Valsangkar 2011b; Oshati et al. 2012; Bryden et al. 2020), model-scale laboratory experiments have been performed with UNB's 1.6-meter radius geotechnical centrifuge (Bourque 2002; McLeod 2003; McGuigan and Valsangkar 2010; McGuigan and

Valsangkar 2011a), and experimental findings have been complemented by use of finite element analyses (Parker et al. 2008; McAfee and Valsangkar 2008; McGuigan and Valsangkar 2011b). The results of numerous experimental studies have shown that the induced trench construction method effectively reduces the culvert loading condition; however, current theoretical formulations and design techniques are subject to continued criticism.

In this paper, a new induced trench theoretical formulation is presented. The proposed analytical model employs inclined shear planes that are consistent with traditional soil mechanics principles, thus more accurately capturing the positive arching mechanism. Finite element modeling is performed to evaluate the failure planes and lateral earth pressures in the model; correlations are developed to provide guidance in selecting key model parameters, and the proposed formulation is validated with experimental data from two full-scale induced trench case studies.

Background

The traditional induced trench theory is founded on the formulation originally proposed by Marston (1930). The model was initially developed for predicting the vertical earth load on negative projection culverts, and was later extended to address induced trench installations (Clarke 1968). Marston's (1930) theory assumes that a compressible zone with width equal to the outside diameter of the culvert B_c is located above the culvert. It is further assumed that, as embankment construction progresses, differential settlement within the overlying fill leads to the development of vertical shear planes as shown in Figure 1. The shear planes are assumed to extend to a height H_e above the culvert crown; this elevation is termed the plane of equal settlement, which is defined as the location where the vertical stress is equal to the overburden

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$$V = C_i \gamma_s B_c^2 \quad (1a)$$

$$\sigma_v = C_i \gamma_s B_c \quad (1b)$$

$$C_i = \left[\frac{\left(e^{-2K \tan \phi \frac{H_e}{B_c}} - 1 \right)}{-2K \tan \phi} + \left(\frac{H}{B_c} - \frac{H_e}{B_c} \right) e^{-2K \tan \phi \frac{H_e}{B_c}} \right] \quad (1c)$$

119

120 Where C_i is the induced trench load coefficient, H is the total fill height above the culvert crown,
 121 and K is the lateral earth pressure coefficient assumed equal to Rankine's active earth pressure
 122 coefficient k_a as defined in Equation 2 (Bulson 1985).

$$k_a = \frac{1 - \sin \phi}{1 + \sin \phi} \quad (2)$$

123 It has been shown that, for the typical range of soil frictional properties encountered in practice,
 124 the product $K \tan \phi$ varies over the range of 0.13 to 0.19; it has therefore been recommended that
 125 $K \tan \phi$ be selected as 0.13 for analysis/design of induced trench culverts as to represent the case
 126 of minimal favorable friction (Spangler 1973).

127 The plane of equal settlement H_e is located by equating the total settlement of the interior
 128 prism with that of the exterior prisms. The exterior prisms are assumed to have a width B_c as
 129 shown in Figure 1, and the soil and compressible fill materials are assumed to deform following a
 130 linear elastic response both with elastic modulus E_s . By equating the total settlement of the
 131 interior and exterior prisms, the governing expression defined in Equation 3 is obtained; refer to
 132 Clarke (1968) for additional details regarding the derivation.

$$\begin{aligned}
& \frac{e^{-2K \tan \phi \frac{H'_e}{B_c}} - 1}{-2K \tan \phi} \left(\frac{H'}{B_c} - \frac{H'_e}{B_c} - \frac{1}{2K \tan \phi} \right) - \frac{H'_e}{B_c} \left[\frac{H'}{B_c} - \frac{1}{2} \left(\frac{H'_e}{B_c} \right) - \frac{1}{2K \tan \phi} \right] \\
& = \frac{2}{3} r_{sd} p' \left[\frac{e^{-2K \tan \phi \frac{H'_e}{B_c}} - 1}{-2K \tan \phi} + \left(\frac{H'}{B_c} - \frac{H'_e}{B_c} \right) e^{-2K \tan \phi \frac{H'_e}{B_c}} \right]
\end{aligned} \tag{3}$$

Where H'_e and H' are defined as the height to the plane of equal settlement and the ground surface, respectively, measured from the top of the compressible fill, and p' is the projection ratio as defined in Figure 1. The parameter r_{sd} is termed the settlement ratio, and represents the difference in settlement between the exterior and interior prisms at a height of $p'B_c$ above the culvert crown, divided by the deformation of the compressible fill. Note that the height to the plane of equal settlement H_e must be less than or equal to the total fill height H ; if, for a particular case, H_e is found to be greater than H , H_e must be replaced with H in Equation 1.

Due to the nature of the plane of equal settlement formula (Equation 3), no closed form solution exists and H_e must be evaluated numerically or by trial and error. Figure 2 displays the induced trench load coefficient C_i (i.e. the normalized vertical earth pressure) as a function of normalized fill height for $K \tan \phi = 0.13$ and $p' = 1.0$ for various r_{sd} values (where H_e has been evaluated numerically with Maple software).

Figure 2: Marston's Induced Trench Load Coefficient C_i as a function of normalized fill height (for $K \tan \phi = 0.13$ and $p' = 1.0$)

Marston's theory was adopted on an international scale (Scarino 2003). Design guidelines containing charts similar to that shown in Figure 2 (with recommendations in selecting r_{sd}) were implemented in the 1980 Concrete Pipe Design Manual (ACPA 1980), and a large number of induced trench structures were successfully constructed (McAffee and Valsangkar 2005). However, skepticism relating to the induced trench construction method subsequently developed in the industry as various theoretical shortcomings of Marston's theory came to light.

1. The shear planes in Marston's theory are assumed to extend vertically from the compressible fill boundary to the plane of equal settlement; however, such failure geometry is not realistic and is assumed only to simplify the analysis (Sladen and Oswell 1988). There is no experimental data supporting the assumption of vertical shear planes. Terzaghi (1943) acknowledges that such shear planes are inclined, and instrumented full-scale induced trench case studies have shown that differential settlements extend beyond

the idealized vertical column (Bryden et al. 2020). In recent years, a centrifuge testing program conducted at the University of New Brunswick involved experimental observation of the settlement field above induced trench culverts; the results of these centrifuge tests revealed that settlement contours above the compressible zone slope outwards at an angle in the range of 25-35 degrees from vertical (McAffee, 2005; McAfee and Valsangkar, 2005; McAfee et al., 2011).

2. Rankine's active earth pressure coefficient k_a is a ratio of principle stresses, which, by definition, must act on a plane of zero friction (Handy 1985). Marston's theory assumes $K = k_a$ when evaluating friction along the vertical shear planes, which is a fundamental contradiction (Handy 2004).
3. The induced trench load formula (Equation 1a or 1b) and the plane of equal settlement formula (Equation 3) are adapted from the negative projection scenario (Clarke 1968; Bulson 1985). Marston's induced trench theory is therefore derived on the basis that: i) the compressible fill has a stiffness equal to that of the embankment fill, ii) the compressible fill has a unit weight equal to that of the embankment fill, and iii) shearing support is provided to the interior column along its entire length – including in the region $p'B_c$ above the culvert crown which contains compressible fill. None of these assumptions are directly transferable to induced trench installations.
4. The plane of equal settlement formula (Equation 3) is derived on the assumption that the exterior column widths are equal to B_c , which may not accurately represent the zone of arching. However, the resulting impact on H_e is overshadowed by the dominant and somewhat ambiguous nature of the settlement ratio parameter r_{sd} , which has been subject to criticism in its own regard; Sladen and Oswell (1988) have stated that: “the basis of the

settlement ratio approach is essentially a combination of informed intuition and empiricism.”

5. Marston’s theory assumes that the width of the induced trench zone is equal to the outside diameter of the culvert B_c . In practice, the compressible zone need not be of width identical to that of the culvert, and McGuigan and Valsangkar (2010) have suggested an optimal compressible zone width of $1.2B_c$ based on the results of laboratory testing and numerical analysis.

The various concerns relating to Marston’s theory identified in the literature and described herein contributed to the omission of the induced trench construction method from modern North American design standards (ACPA 2011; CSA 2019). However, some jurisdiction, including the New Brunswick Department of Transportation and Infrastructure (NBDTI), have continued to implement the induced trench construction method and Marston’s theory with success (McGuigan et al. 2016). It must be acknowledged that Marston’s theory and the resulting design procedures were calibrated with a large amount of experimental data (through r_{sd}), and as such, the structures constructed with this design approach are reported to have performed well (Sladen and Oswell 1988) and there appear to be no indication of pipe failures reported in the literature (Scarino 2004). It is likely that the use of lower-bound input parameters and the apparent conservative nature of the theory have further contributed to its general success (McAffee and Valsangkar 2005).

The internal positive arching and load-transfer mechanisms, which are fundamental to the induced trench design methodology, have received attention by researchers in recent years. Kang et al. (2007; 2008) used finite element software to optimize the compressible zone geometry and

to investigate the down-drag forces on the sides of induced trench box culverts. McGuigan and Valsangkar (2010) performed extensive centrifuge testing and numerical analyses investigating the loading conditions of induced trench box culverts, while Turan et al. (2013) report the loading conditions for a full-scale test embankment constructed with an induced trench. Meguid and colleagues performed 1-g model scale laboratory testing and extensive numerical analyses to investigate the loading conditions of induced trench box culverts constructed with expanded polystyrene (EPS) geofoam (Meguid et al., 2017a; Meguid et al., 2017b; Meguid and Hussein, 2017). Additional reduced-scale model tests have been used to evaluate the effectiveness of EPS geofoam in induced trench construction (Al-Naddaf et al., 2019; Ma et al., 2019), and the feasibility of using thin geocomposite material in place of the traditional compressible zone has also been explored (Plácido and Portelinha, 2019). Ni et al. (2018) performed numerical analyses assessing the effectiveness of tire derived aggregate (TDA) in induced trench construction, while Mahgoub and El Naggar (2019) used two full-scale test-sites to investigate the benefits of installing TDA above pre-existing buried structures. The various induced trench studies reported in the literature and described herein are in agreement that installation of a compressible inclusion (i.e. induced trench construction) is effective in reducing the vertical earth load exerted on an underlying culvert.

It has been well-documented that use of the induced trench construction method effectively reduces the culvert loading conditions; however, despite the latest advances in this field, the need for a more rational induced trench theory that more accurately represents the soil arching mechanism is evident. A few researchers have made progress in this regard. Krynine (1945) suggested the use of a modified lateral earth pressure coefficient k_w , which was derived on the basis of Mohr's circle for vertical shear planes and is defined in Equation 4. Krynine's interpretation was commended as a "noteworthy contribution" (Terzaghi 1945); however,

subsequent evaluations in relation to experimental testing revealed that the predicted frictional forces were too high (Handy 2004).

$$k_w = \frac{1 - \sin^2 \phi}{1 + \sin^2 \phi} \quad (4)$$

Qin et al. (2017) recently developed a modified induced trench theory whereby Marston's theory was adjusted to incorporate: 1) a parabolic vertical stress distribution within the interior column, and 2) use of a simplified settlement compatibility approach in evaluating H_c . When compared to Marston's theory, Qin et al.'s (2017) method is shown to more closely estimate field measurements; however, many of the theoretical shortcomings identified in the literature and described herein remain unaddressed. As such, a new theoretical formulation for induced trench culverts has been developed and is described in this paper.

Proposed Induced Trench Theory

A culvert with outside diameter B_c constructed with the induced trench construction method is initially installed in the positive projecting condition. Once the embankment fill has reached a pre-established elevation (usually a height of roughly B_c above the culvert crown), a trench with vertical walls having a depth t and width B_o is excavated along the culvert length. A zone of embankment fill with thickness H_c of 0.3 to 0.8 meters is generally left in-place above the culvert for practical/constructability purposes (Bryden et al. 2020). The excavated trench is then filled with compressible material, and typical embankment construction procedures are resumed until the final embankment height (H' above the compressible fill) has been reached.

When the first lift of embankment material is placed above the compressible fill, significant settlement in the induced trench zone occurs; such settlement extends beyond the induced trench boundary due to instability in the adjacent soil, and shear planes develop in the

adjacent soil at an angle α measured from the vertical as shown in Figure 3. As embankment construction progresses, the shear planes are assumed to propagate along a linear trajectory until the plane of equal settlement has been reached at a height H'_e above the compressible fill. The width of the induced trench zone $B(z)$ is therefore a function of depth z as defined in Equation 5a, and reaches the limiting value of B_l at the plane of equal settlement (corresponding to $z = 0$) as defined in Equation 5b.

$$B(z) = B_0 + 2(H'_e - z) \tan \alpha \quad (5a)$$

$$B_l = B_0 + 2H'_e \tan \alpha \quad (5b)$$

Figure 3: Schematic Diagram of Proposed Induced Trench Theory

Neglecting any potential shear support provided along the vertical boundaries of the compressible fill, the vertical earth pressure at the culvert crown σ_v may be computed as:

$$\sigma_v = \gamma_s H_c + \gamma_{cf} t + \sigma_{v(cf)} \quad (6)$$

Where γ_s and γ_{cf} represent the unit weight of the embankment fill and compressible fill, respectively, and $\sigma_{v(cf)}$ is the vertical stress at the top of the compressible fill including the effects of positive arching. The vertical stress at the top of the compressible fill $\sigma_{v(cf)}$ is evaluated by vertical static equilibrium of a differential element dz . Figure 4a shows the external stresses acting on an induced trench differential element, where τ_f and σ_{nf} represent the shear and normal stresses acting on the shear planes, respectively.

(a)

(b)

Figure 4: Induced Trench Differential Element Displaying a) the External Stresses, and b) the Resultant Vertical Forces Including Body Force W

The shear stress acting on the shear plane τ_f may be defined by the Mohr-Coulomb failure criterion described in Equation 7 for cohesionless embankment fill.

$$\tau_f = \sigma_{nf} \tan \phi \quad (7)$$

By evaluating the vertical and horizontal components of the stresses acting on the shear surfaces at angle α , the resultant vertical stress τ_0 acting over the elemental height dz may be expressed as:

$$\tau_0 = \sigma_x \tan(\phi + \alpha) \quad (8)$$

Where σ_x represents the horizontal earth pressure. Assuming that the vertical earth pressure is uniform across the width of the induced trench zone $B(z)$ and that the horizontal earth pressure is related to the elemental vertical earth pressure σ_z by a constant of proportionality K_{IT} , the horizontal earth pressure may be expressed as:

$$\sigma_x = K_{IT} \sigma_z = K_{IT} \frac{V(z)}{B(z)} \quad (9)$$

Where $V(z)$ is the resultant vertical load acting on an induced trench element at depth z .

Figure 4b displays the vertical forces acting on the induced trench differential element; static vertical equilibrium (including the body force W) leads to the governing differential equation defined in Equation 10.

$$\frac{dV(z)}{dz} + \left[2 \frac{K_{IT}}{B(z)} \tan(\phi + \alpha) \right] V(z) = \gamma_s B(z) \quad (10)$$

The general solution of Equation 10 is:

$$V(z) = \frac{\gamma_s B(z)^2}{2 \tan \alpha (\psi - 2)} + CB(z)^\psi \quad (11)$$

Where ψ is defined in Equation 12, and C is an arbitrary constant of integration.

$$\psi = K_{IT} \frac{\tan(\phi + \alpha)}{\tan \alpha} \quad (12)$$

The integration constant C is evaluated by imposing a boundary condition. At the plane of equal settlement ($z = 0$), the vertical earth pressure must be equal to the overburden soil pressure; this leads to the boundary condition defined in Equation 13.

$$V(0) = \gamma_s(H' - H'_e)B_1 \quad (13)$$

Where B_1 is the width of the induced trench zone at $z = 0$ (as defined in Equation 5b). The particular solution of Equation 10 is therefore:

$$V(z) = \gamma_s B(z)^2 \left[\frac{1 - \left(\frac{B_1}{B(z)}\right)^{2-\psi}}{2 \tan \alpha (\psi - 2)} + \left(\frac{H' - H'_e}{B_1}\right) \left(\frac{B_1}{B(z)}\right)^{2-\psi} \right] \quad (14)$$

When Equation 14 is evaluated at $z = H'_e$, the resulting expression represents the total vertical load on the compressible fill $V_{(cf)}$ as defined in Equation 15a. The equivalent vertical stress on the compressible fill $\sigma_{v(cf)}$ may therefore be evaluated with Equation 15b. Note that the height of the plane of equal settlement H'_e must always be less than or equal to the total fill height H' ; that is, H'_e must be replaced with H' if H'_e is found to be greater than H' .

$$V_{(cf)} = \gamma_s B_0^2 \left[\frac{1 - \left(\frac{B_1}{B_0}\right)^{2-\psi}}{2 \tan \alpha (\psi - 2)} + \left(\frac{H' - H'_e}{B_1}\right) \left(\frac{B_1}{B_0}\right)^{2-\psi} \right] \quad (15a)$$

$$\sigma_{v(cf)} = \gamma_s B_0 \left[\frac{1 - \left(\frac{B_1}{B_0}\right)^{2-\psi}}{2 \tan \alpha (\psi - 2)} + \left(\frac{H' - H'_e}{B_1}\right) \left(\frac{B_1}{B_0}\right)^{2-\psi} \right] = \frac{V_{(cf)}}{B_0} \quad (15b)$$

Based on the proposed theoretical formulation, Equations 15a and 15b are the governing expressions for the vertical load and pressure acting on the compressible zone, respectively. Two interesting features relating to the general structure of the proposed expressions are summarized as follows:

1. If the shear planes are assumed to extend vertically by substituting $\alpha = 0$ in Equation 15b, the expression is reduced to Marston's theory (i.e. Equation 1b) except for the fact that H , H_e , K , and B_c have been replaced with H' , H'_e , K_{IT} , and B_0 , respectively. This result is obtained by incorporation of the limit defined in Equation 16.

$$\lim_{\alpha \rightarrow 0} \left[\left(\frac{B_1}{B_0} \right)^{2-\psi} \right] = e^{-2K_{IT} \tan \phi \frac{H'_e}{B_0}} \quad (16)$$

2. If the orientation of the failure planes α and the lateral earth pressure coefficient K_{IT} are assumed to adhere to Rankine's theory of active earth pressure ($\alpha = 45^\circ - \phi/2$; $K_{IT} = k_a$), ψ is equal to 1 and Equation 15b reduces to the overburden soil pressure $\gamma_s H'$; that is, soil arching cannot be accommodated with Rankine's earth pressure theory.

Shear Plane Geometry and Lateral Earth Pressure in the Induced Trench Zone

Significant settlements occur when the first lift of embankment fill is placed above the compressible zone; such settlements are presumed to be greatest at the induced trench centerline and to decrease as one approaches the boundary of the induced trench zone. The resulting inward rotation of the overlying embankment fill leads to: 1) instability within the adjacent soil, and 2) lateral confinement within the induced trench zone. Conceptually, such rotation at the induced trench boundary is analogous to rotation of a retaining wall; this suggests that failure planes within the adjacent soil will extend at an angle α from vertical. Upon placement of each successive lift of embankment fill, the width of the yielding base (i.e. the induced trench zone) increases proportional to the shear plane inclination, and such phenomenon continues throughout construction until the plane of equal settlement has been reached. This concept is portrayed graphically in Figure 5.

Figure 5: Conceptual Diagram of Embankment Deformation Upon Placing a) Lift 1, and b) Lift 2 Above the Compressible Zone

By use of digital photography, particle image velocimetry, and close-range photogrammetry during centrifuge testing of model-scale induced trench culverts, McAfee and colleagues observed that settlement contours (near the compressible zone) slope outwards at an angle in the range of 25-35 degrees from vertical (McAfee 2005; McAfee et al., 2011). McAfee (2005) used poorly graded sand with a relative density of 69% as embankment fill during centrifuge testing, and it is reported that such material has a friction angle ϕ in the range

of 35° to 37° at a relative density of 52% (Bourque, 2002). These experimental observations imply that the sliding surface can be reasonably approximated as $\alpha = 45^\circ - \phi/2$; for simplicity and to maintain consistency with traditional soil mechanics principles, such sliding surface ($\alpha = 45^\circ - \phi/2$) is employed within the proposed theoretical formulation.

Based on the presumed failure geometry, additional insights are also obtained regarding the induced trench lateral earth pressure coefficient; settlement in the induced trench zone causes: 1) a transfer of vertical stress to the adjacent soil by shear, and 2) an increased state of lateral confinement. Settlement in the induced trench zone (i.e. arching) therefore leads to a reduction in vertical stress and an increase in lateral stress, which has the net result of increasing the lateral earth pressure coefficient in the induced trench zone. Since the ratio of horizontal stress to vertical stress is presumed to depend on both vertical stress reduction and lateral confinement, the actual lateral earth pressure coefficient within the induced trench zone (i.e. between the plane of equal settlement and compressible fill) is likely to increase with depth. Note, however, that the theoretical formulation described herein is derived on the basis that the lateral earth pressure coefficient K_{IT} is constant throughout the induced trench zone; an average representative value must therefore be employed during analysis.

Comparison with Existing Induced Trench Theories

Figure 6 displays the normalized vertical stress at the top of the compressible fill ($\sigma_{v(cf)}/\gamma_s B_0$) as a function of normalized embankment height (H'/B_0) based on the induced trench theoretical formulation described herein (for $\phi = 30^\circ$). The curves shown in Figure 6 are computed with Equation 15b for various values of the induced trench lateral earth pressure coefficient K_{IT} for the case in which the plane of equal settlement coincides with the ground

surface ($H_e' = H'$). Note that for $K_{IT} = k_a$, the vertical stress is simply equal to the overburden soil pressure; however, as K_{IT} is increased, the vertical stress is reduced as a result of positive arching. Equivalent curves that are representative of Marston's theory and Qin et al.'s (2017) theory are also shown in Figure 6 for comparison; note that such curves have been generated with Equation 1b by employing $K \tan \phi = 0.13$ and by substituting σ_v , H_e , and B_c with $\sigma_{v(cf)}$, H_e' , and B_0 , respectively, while also incorporating a stress distribution factor $a = 2$ in plotting Qin et al.'s result (refer to Qin et al. 2017 for additional details). As shown in Figure 6, Marston's theory and Qin et al.'s theory are in relatively good agreement with the proposed theoretical formulation for the case in which $K_{IT} = 1.6k_a$ to $1.8k_a$.

Figure 6: Vertical Stress Acting on the Induced Trench Compressible Fill Computed with the Proposed Theory vs. Existing Theories (for $H_e' = H'$; $\phi = 30^\circ$)

Based on the theoretical formulation presented in this paper, the vertical stress on the culvert crown may then be obtained with Equation 6. Figure 7 displays the vertical stress on the culvert crown computed with the proposed theoretical formulation, along with that predicted by Marston's theory and Qin et al.'s theory, for the case in which the plane of equal settlement

coincides with the ground surface ($H_e' = H'$). In generating the relationships defined in Figure 7, it has been assumed that the soil cover H_c is equal to $0.2B_0$, that the compressible fill thickness t is equal to $0.8B_0$, and that the unit weight of the compressible fill is equal to $0.1\gamma_s$. When comparing the various theoretical models in Figure 7, one must recall that the existing induced trench formulations (Marston's theory and Qin et al.'s theory) are simply extensions of the negative projection scenario, and are therefore developed on the basis that: 1) the unit weight of the compressible fill is equal to that of the overlying soil, and 2) that shear resistance is provided over the full height H_e . The proposed theoretical formulation is observed to more accurately capture the various fill materials, therefore providing a more realistic representation of the induced trench configurations employed in practice.

Figure 7: Vertical Stress Acting on the Culvert Crown Computed with the Proposed Theory vs. Existing Theories (for $H_e' = H'$; $\phi = 30^\circ$; $H_c = 0.2B_0$; $t = 0.8B_0$; $\gamma_{cf} = 0.1\gamma_s$)

For one to compute the vertical stress acting on the crown of an induced trench culvert with the proposed theoretical formulation (i.e. Equation 6), it is necessary to first identify: 1) the induced trench lateral earth pressure coefficient K_{IT} , and 2) the height to the plane of equal

settlement H_e' . To further investigate such parameters (and the positive arching mechanism in general), a series of induced trench configurations are evaluated numerically using finite element software.

Numerical Modeling

The performance of an individual circular culvert installed using the induced trench construction method is simulated with Abaqus/CAE finite element software (Dassault Systemes 2017). Two-dimensional modeling space is used to generate a cross-section of the culvert (with outside diameter B_c), which is overlain by a compressible zone with width B_o and thickness t located a distance H_c above the culvert crown. Three soil regions are defined in the model: 1) structural-fill adjacent to the culvert and extending a distance H_c above/below the culvert, 2) native soil located beneath the culvert/structural-fill, and 3) embankment-fill located above the culvert/structural-fill and extending to a height H' above the compressible zone.

Figure 8: Model Zones and Boundary Conditions for Finite Element Analysis

Due to symmetry about the vertical culvert-diameter, only half of the domain is modeled for computational efficiency. The boundary conditions are identified in Figure 8; the base boundary, side boundary, top boundary, and symmetric boundary are defined mechanically as being restrained in the horizontal/vertical directions, restrained in the horizontal direction only, unrestrained, and symmetric about the vertical plane, respectively. The soil regions are extended horizontally a distance of $15B_c$ from the culvert centerline to eliminate boundary effects. A tie-constraint is imposed at the soil-culvert boundary, thus ensuring that a perfect bond is maintained at the interface without slippage.

The geometric parameters B_c , H_c , t , B_0 , and H' for preliminary analysis are selected as 1.0 meters, 0.1 meters, 0.65 meters, 1.2 meters, and 15.0 meters, respectively. Concrete and compressible fill materials are assumed to deform following a linear-elastic response, while Mohr-Coulomb plasticity (cohesionless) is assigned for the soil regions; the mechanical parameters used for preliminary analysis are defined in Table 1. Note that such geometric/mechanical parameters have been selected based on typical properties reported in the literature for induced trench studies (McGuigan and Valsangkar 2011b; McGuigan et al., 2016; Bryden et al. 2020).

Table 1: Material Properties for Preliminary Finite Element Analysis

Material Type	Density (kg/m^3)	Elastic Modulus (MPa)	Poisson Ratio	Friction Angle ($^\circ$)
Concrete (Culvert)	2500	35×10^3	0.20	---
Native Soil	2100	20	0.25	40
Structural Fill	2200	15	0.25	40
Embankment Fill	2200	15	0.25	30
Compressible Fill	60	1	0.25	---

Note: cohesion = 0.1 kPa and dilation angle = 0.1° for all models with Mohr-Coulomb plasticity.

The model domain was discretized into approximately 30,000 elements, where the soil and culvert regions were discretized with an approximate global element sizes of 0.10 meters and 0.05 meters, respectively, and 4-node bilinear plane-strain quadrilateral elements (CPE4) were assigned to all mesh regions. Care was taken during mesh development to ensure that elemental aspect ratios of approximately 1.0 were maintained as much as possible, and model convergence was verified via mesh sensitivity analysis. Figure 9 displays the mesh geometry near the culvert.

Figure 9: Model Geometry and Finite Element Mesh in the Culvert Vicinity

To simulate construction procedures employed in practice, the embankment was constructed sequentially within Abaqus software using a series of steps. In the initial step, all regions other than the native soil region were removed from the model and the active region (i.e. the native soil) was brought to initial equilibrium under a uniform gravitational acceleration of -9.81 m/s² in the vertical direction. The culvert/structural-fill regions were then introduced in the second step, which were then-too subjected to a vertical gravitational acceleration of -9.81 m/s². The compressible-fill layer (including the adjacent embankment-fill) was then introduced in the third step following a similar procedure, and the overlying embankment fill was subsequently

installed in 15 sequential steps (with each step adding a uniform lift 1.0 meters in thickness to the active model domain). The modeling sequence therefore involved a total of 18 steps, with static equilibrium being reached at the completion of each step.

Results of Preliminary Analysis

The vertical and horizontal stresses within the induced trench model domain upon completion of the final construction stage are displayed graphically in Figures 10a and 10b, respectively. Note that the culvert region has been omitted to improve the resolution of stress contours displayed in the soil regions. As expected, the stress contours in Figures 10a and 10b are indicative of positive arching; the vertical stress in the region above the compressible zone is observed to be less than that of the free-field, while the horizontal stress in the region above the compressible zone is observed to be greater than that of the free-field.

Figure 10: Abaqus Result Displaying a) Vertical, and b) Horizontal Stress Contours at Completion of the Final Construction Stage (for the Preliminary Induced Trench Configuration Defined in Table 1).

The stress contours displayed in Figures 10a and 10b suggest the presence of shear planes within the embankment fill above the compressible zone. Figure 11 displays the equivalent plastic strain contours within the model domain, which clearly display shear planes sloping outwards at an angle of approximately 30 degrees. Recall that the proposed theoretical formulation is founded on the assumption that shear planes slope outwards at an angle $\alpha = 45^\circ - \phi/2$, which is consistent with the results of this numerical analysis (since $\phi = 30^\circ$ as defined in Table 1).

Figure 11: Abaqus Result Displaying Equivalent Plastic Strain Contours (at Completion of the Final Construction Stage; for the Preliminary Induced Trench Configuration Defined in Table 1).

The primary purpose of the present numerical study is to evaluate the induced trench lateral earth pressure coefficient K_{IT} and the plane of equal settlement H_e' for implementation in the proposed induced trench theoretical formulation. As such, the state of stress within the embankment-fill in the region directly above the compressible-fill is of particular interest. Figures 12a, 12b, 12c, and 12d display the vertical stress σ_z , vertical stress ratio $\sigma_{z(ro)}$, horizontal stress σ_x , and lateral earth pressure coefficient K , respectively, as a function of height above the compressible fill h for the induced trench configuration defined in Table 1 (at the completion of the final construction stage). Note that the vertical stress ratio $\sigma_{z(ro)}$ is defined as the ratio of vertical stress σ_z to free-field vertical stress $\sigma_{z(ff)}$, while the lateral earth pressure coefficient K is defined as the ratio of horizontal stress σ_x to vertical stress σ_z . The stress conditions shown in Figures 12a, 12b, 12c, and 12d have been extracted from the Abaqus result at the symmetric boundary between the top of the compressible fill and the ground surface (i.e. along Line A as defined in Figures 10a and 10b).

Figure 12: a) Vertical Stress, b) Vertical Stress Ratio, c) Horizontal Stress, and d) Lateral Earth Pressure Coefficient as a Function of Height Above the Compressible Fill (at Completion of the Final Construction Stage; for the Preliminary Induced Trench Configuration Defined in Table 1).

As shown in Figure 12a, the vertical stress increases linearly with depth near the ground surface at a rate equivalent to that of the free-field vertical stress (i.e. the overburden soil pressure). However, the presence of the compressible zone leads to a reduction in vertical stress in the vicinity of the compressible fill. Recall that, by definition, the plane of equal settlement corresponds to the elevation at which the vertical stress is equal to the overburden soil pressure; for practical purposes, the plane of equal settlement is assumed to correspond to the elevation at which the vertical stress is within 5% of the free-field vertical stress. Figure 12b displays the vertical stress ratio as a function of fill height, and the plane of equal settlement H_e' is shown to be located 4.0 meters above the compressible fill.

Figures 12a and 12c display a reduction in vertical stress and an increase in horizontal stress, respectively, for the vertical domain between the top of the compressible fill and the plane of equal settlement. Such stress characteristics are indicative of positive arching and have the net result of increasing the lateral earth pressure coefficient K as shown in Figure 12d. It is worth noting that the jagged nature of the horizontal stress profile displayed in Figure 12c is a direct result of the modeling sequence, as static equilibrium is achieved at the completion of each step during embankment construction (constructed in 1-meter lifts). Recall that the induced trench lateral earth pressure coefficient K_{IT} for implementation in the proposed theoretical formulation corresponds to an average representative value over the vertical domain H_e' ; since the plane of equal settlement has been identified at $H_e' = 4.0$ meters, the induced trench lateral earth pressure coefficient is observed to be $K_{IT} = 0.526$ for this configuration.

By implementing $H_e' = 4.0$ meters and $K_{IT} = 0.526$ within the proposed theoretical formulation (Equation 15b) for the preliminary induced trench configuration defined in Table 1, the vertical stress at the top of the compressible fill is computed to be $\sigma_{v(cf)} = 145.6$ kPa. Note that

this result is in good agreement with the Abaqus numerical result, which is shown to yield a vertical stress of 148.3 kPa at the top of the compressible zone (Figure 12a). The induced trench theoretical formulation described in this paper is therefore shown to produce results which are in good agreement with those obtained via finite element analysis, provided that H_e' and K_{IT} are selected appropriately. To further evaluate the relationship between the induced trench model parameters and the values of H_e' and K_{IT} , a series of parametric studies are performed.

Influence of the Induced Trench Relative Stiffness Parameter E_{ro}

The degree of positive arching which develops above an induced trench culvert is dependent on the relative stiffness of the induced trench zone compared to that of the adjacent soil. If the inner zone and outer zone are defined as the vertical domain between the bottom of the culvert and the top of the compressible fill at the culvert centerline and within the adjacent soil, respectively, the equivalent vertical stiffness of such domains (assuming zero interaction) can be computed with the expressions defined in Equations 17a and 17b. Note that the expressions defined in Equations 17a and 17b employ the weighted-harmonic averaging method, which is applicable for systems composed of springs in series.

$$E_{IN} = \frac{t + H_c + B_c}{\frac{t}{E_{cf}} + \frac{H_c}{E_{sf}} + \frac{B_c}{E_c}} \quad (17a)$$

$$E_{OUT} = \frac{t + H_c + B_c}{\frac{t}{E_{ef}} + \frac{H_c}{E_{sf}} + \frac{B_c}{E_{sf}}} \quad (17b)$$

Where E_{cf} , E_{ef} , and E_{sf} are defined as the elastic moduli of the compressible-fill, embankment-fill, and structural-fill, respectively, and E_c is the equivalent stiffness of the culvert.

In order for positive arching to develop during induced trench construction, the stiffness of the inner zone E_{IN} must be less than that of the outer zone E_{OUT} . Assuming that the equivalent

stiffness of the culvert E_c is significantly greater than that of the soil materials, the induced trench relative stiffness parameter E_{ro} can be computed as shown in Equation 18. Note that, based on this definition, positive arching will be developed for induced trench structures with $0 < E_{ro} < 1$.

$$E_{ro} = \frac{E_{IN}}{E_{OUT}} = \frac{\frac{t}{E_{ef}} + \frac{H_c}{E_{sf}} + \frac{B_c}{E_{sf}}}{\frac{t}{E_{cf}} + \frac{H_c}{E_{sf}}} \quad (18)$$

A series of independent induced trench configurations are simulated using Abaqus finite element software to investigate the influence of the induced trench relative stiffness parameter E_{ro} on the degree of positive arching. Two model geometries are employed for the current analysis ($t = 0.65$ meters and $t = 1.0$ meters) and the elastic modulus of the compressible fill E_{cf} is varied from 1 to 5 MPa for each geometric configuration as defined in Table 2. Note that all other geometric/mechanical properties and modeling procedures employed for these configurations remain unchanged and are equivalent to those used for preliminary analysis (defined in Table 1).

Table 2: Stiffness Characteristics, Abaqus Finite Element Results, and Theoretical Results for Induced Trench Configurations with Variation in Relative Stiffness Parameter E_{ro} (Set 1 & Set 2)

Configuration Description	Relative Stiff. Parameter, E_{ro}	Abaqus Numerical Results			Theoretical Results			Percent Difference in $\sigma_{v(cf)}$
		H_e' (m)	K_{IT}	$\sigma_{v(cf)}$ (kPa)	H_e' (m)	K_{IT}	$\sigma_{v(cf)}$ (kPa)	
Set 1: $t = 0.65$ m								
$E_{cf} = 1$ MPa	0.194	4.0	0.526	148.3	3.9	0.518	152.0	2.5%
$E_{cf} = 2$ MPa	0.377	2.7	0.476	205.1	3.0	0.483	186.5	9.1%
$E_{cf} = 3$ MPa	0.549	1.9	0.445	241.6	2.2	0.449	224.4	7.1%
$E_{cf} = 4$ MPa	0.712	1.3	0.418	268.0	1.4	0.418	263.5	1.7%
$E_{cf} = 5$ MPa	0.867	0.9	0.392	283.9	0.6	0.388	299.6	5.5%
Set 2: $t = 1.00$ m								
$E_{cf} = 1$ MPa	0.150	4.3	0.559	131.7	4.1	0.526	144.7	9.9%
$E_{cf} = 2$ MPa	0.295	3.2	0.506	180.5	3.4	0.498	170.2	5.7%
$E_{cf} = 3$ MPa	0.434	2.4	0.474	216.3	2.7	0.472	198.5	8.2%
$E_{cf} = 4$ MPa	0.568	1.8	0.450	243.0	2.1	0.446	228.8	5.8%
$E_{cf} = 5$ MPa	0.697	1.4	0.426	263.8	1.5	0.421	259.8	1.5%

Note: H_e' and K_{IT} for the theoretical result are computed with the approximate correlations defined in Equations 19 and 20, respectively.

The resulting stress characteristics along Line A (defined in Figure 10) for the five induced trench configurations with $t = 0.65$ meters (set 1) defined in Table 2 are shown in Figures 13a, 13b, 13c, and 13d, which display the vertical stress σ_z , vertical stress ratio $\sigma_{z(ro)}$, horizontal stress σ_x , and lateral earth pressure coefficient K , respectively, as a function of height above the compressible fill h (at the completion of the final construction stage). As shown in Figures 13a and 13c, the degree of positive arching (i.e. the reduction in vertical stress and the increase in horizontal stress) is observed to decrease as the elastic modulus of the compressible fill E_{cf} is increased. Such reduction in arching has the net result of reducing the distance to the plane of equal settlement H_e' as shown in Figure 13b, while also reducing the lateral earth pressure coefficient K over the vertical domain $0 \leq h \leq H_e'$ as shown in Figure 13d. Assuming that the plane of equal settlement H_e' corresponds to the elevation h at which the vertical stress is within 5% of the overburden soil pressure, H_e' is shown to be 4.0 meters, 2.7 meters, 1.9 meters, 1.3 meters, and 0.9 meters for set 1 configurations having E_{cf} equal to 1 MPa, 2 MPa, 3 MPa, 4 MPa and 5 MPa, respectively. The induced trench lateral earth pressure coefficient K_{IT} for set 1 configurations is shown to range from 0.526 to 0.392; Table 2 contains a summary of the Abaqus results including the height to the plane of equal settlement H_e' , the induced trench lateral earth pressure coefficient K_{IT} , and the vertical stress at the top of the compressible fill $\sigma_{v(cf)}$ for each configuration. Note that similar numerical modeling results are reported in Table 2 for the configurations having $t = 1.0$ meters (set 2).

Figure 13: a) Vertical Stress, b) Vertical Stress Ratio, c) Horizontal Stress, and d) Lateral Earth Pressure Coefficient as a Function of Height Above the Compressible Fill (at Completion of the Final Construction Stage; for Set 1 Induced Trench Configuration Defined in Table 2).

Based on the results of this numerical study, the plane of equal settlement H_e' (normalized to the induced trench width B_0) is plotted against the induced trench relative stiffness parameter E_{ro} in Figure 14 for the configurations defined in Table 2. It is shown that the results of both geometric configurations adhere to a similar relation, thus suggesting that correlations in E_{ro} are representative of induced trench configurations spanning a range of geometric and mechanical properties. The results of this numerical study do not differ significantly from a linear representation as shown in Figure 14; the linear expression defined in Equation 19 is therefore deemed to be a suitable approximation. Note that this correlation yields $H_e' = 0$ for $E_{ro} = 1.0$, thus retaining the characteristic of zero arching for the case in which the inner and outer zones are equivalent in stiffness.

$$H_e' = 4B_0(1 - E_{ro}) \tag{19}$$

Figure 14: Normalized Plane of Equal Settlement Location vs. Induced Trench Relative Stiffness Parameter E_{ro} Results from Abaqus Numerical Study

The induced trench lateral earth pressure coefficient K_{IT} (normalized to the failure plane inclination $\tan\alpha$) is plotted against the induced trench relative stiffness parameter E_{ro} in Figure 15 for the configurations defined in Table 2. As shown in Figure 15, the relationship between K_{IT} and E_{ro} can also be approximated using a linear correlation with reasonable accuracy; the resulting linear approximation for this relationship is defined in Equation 20.

$$K_{IT} = \tan(\alpha) \left(\frac{25}{26} - \frac{1}{3} E_{ro} \right) \quad (20)$$

Figure 15: Normalized Lateral Earth Pressure Coefficient vs. Induced Trench Relative Stiffness E_{ro} from Abaqus Numerical Study

The location of the plane of equal settlement H_e' and the induced trench lateral earth pressure coefficient K_{IT} as calculated with Equations 19 and 20, respectively, are shown in Table 2 for each respective configuration. The vertical stress at the top of the compressible fill $\sigma_{v(cf)}$ is then computed with the proposed induced trench theoretical formulation (Equation 15b) for each

configuration; the result in all cases is observed to be within 10% of that obtained via finite element analysis, which is deemed acceptable in terms of model validation.

Influence of Compressible Fill Width B_0

The proposed induced trench theoretical formulation suggests that the vertical stress acting on the compressible zone $\sigma_{v(cf)}$ is directly related to the width of the compressible zone B_0 as shown in Equation 15b. To evaluate such relation, along with the influence of B_0 on the location of the plane of equal settlement H_e' and the induced trench lateral earth pressure coefficient K_{IT} , a series of induced trench configurations (set 3) are simulated using Abaqus finite element software. The width of the compressible zone B_0 is varied from 1.0 meter to 2.0 meters amongst the set 3 configurations as indicated in Table 3, while all other properties and modeling procedures remain equivalent to those used for preliminary analysis (as described previously and defined in Table 1).

Table 3: Abaqus Finite Element Results and Theoretical Results for Induced Trench Configurations with Variation in Compressible Zone Width B_0 (Set 3)

Configuration Description	Abaqus Numerical Results			Theoretical Results			Percent Difference in $\sigma_{v(cf)}$
	H_e' (m)	K_{IT}	$\sigma_{v(cf)}$ (kPa)	H_e' (m)	K_{IT}	$\sigma_{v(cf)}$ (kPa)	
Set 3:							
$B_0 = 1.0$ m	3.6	0.527	142.4	3.2	0.518	149.5	5.0%
$B_0 = 1.2$ m	4.0	0.526	148.3	3.9	0.518	152.0	2.5%
$B_0 = 1.4$ m	4.4	0.522	154.0	4.5	0.518	154.5	0.3%
$B_0 = 1.6$ m	4.8	0.519	160.2	5.2	0.518	156.9	2.0%
$B_0 = 1.8$ m	5.1	0.517	166.9	5.8	0.518	159.4	4.5%
$B_0 = 2.0$ m	5.4	0.515	171.1	6.4	0.518	161.9	5.4%

The numerical results for each set 3 configuration, including: the location of the plane of equal settlement H_e' , the induced trench lateral earth pressure coefficient K_{IT} , and the vertical

stress on the compressible zone $\sigma_{v(cf)}$, are summarized in Table 3. The elevation of the plane of equal settlement H_e' is observed to increase with increasing B_0 at a rate proportional to the width of the compressible zone B_0 , while the induced trench lateral earth pressure coefficient K_{IT} is observed to remain essentially unaffected by variations in B_0 . The numerical results also reveal that the vertical stress acting on the compressible zone $\sigma_{v(cf)}$ increases with increasing B_0 , which is in agreement with the proposed theoretical relationship defined in Equation 15b.

For each set 3 configuration defined in Table 3, the location of the plane of equal settlement H_e' and the induced trench lateral earth pressure coefficient K_{IT} is computed with the approximate correlation defined in Equation 19 and 20, respectively. The vertical stress acting on the compressible fill $\sigma_{v(cf)}$ is then calculated with the proposed theoretical formulation defined in Equation 15b, with all results being summarized in Table 3. The computed vertical stresses are observed to be within approximately 5% of those obtained by numerical modeling in all cases, which provides a further sense of validation for the proposed theoretical formulation and the general suitability of the suggested correlations for H_e' and K_{IT} .

Influence of Embankment Fill Friction Angle ϕ

Based on the theoretical formulation described herein, the embankment fill friction angle ϕ influences not only the shear stress acting on the shear planes but also the geometry of the failure wedge; as the friction angle increases, the inclination of the failure planes α also increase (i.e. approaching vertical). To investigate the influence of the friction angle ϕ on the induced trench arching mechanism, a series of induced trench configurations (set 4) are evaluated with Abaqus finite element software as defined in Table 4. The embankment fill friction angle ϕ is varied from 28 to 38 degrees amongst the configurations defined in Table 4; note that all other

geometric/mechanical properties and modeling procedures remain unchanged and are equivalent to those used for preliminary analysis (defined in Table 1).

Table 4: Abaqus Finite Element Results and Theoretical Results for Induced Trench Configurations with Variation in Embankment Fill Friction Angle ϕ (Set 4)

Configuration Description	Failure Plane Inclination, $\tan\alpha$	Abaqus Numerical Results			Theoretical Results			Percent Difference in $\sigma_{v(cf)}$
		H_e' (m)	K_{IT}	$\sigma_{v(cf)}$ (kPa)	H_e' (m)	K_{IT}	$\sigma_{v(cf)}$ (kPa)	
Set 4:								
$\phi = 28^\circ$	0.601	3.6	0.555	155.9	3.9	0.539	162.5	4.2%
$\phi = 30^\circ$	0.577	4.0	0.526	148.3	3.9	0.518	152.0	2.5%
$\phi = 32^\circ$	0.554	4.3	0.492	137.4	3.9	0.497	142.0	3.4%
$\phi = 34^\circ$	0.532	4.1	0.476	130.5	3.9	0.477	132.6	1.6%
$\phi = 36^\circ$	0.510	3.8	0.467	121.1	3.9	0.457	123.7	2.2%
$\phi = 38^\circ$	0.488	3.6	0.461	116.2	3.9	0.437	115.3	0.8%

The numerical results for set 4 configurations are summarized in Table 4. It is shown that the plane of equal settlement H_e' is not significantly influenced by variations in friction angle ϕ ; for set 4 configurations, H_e' does not differ by more than 10% from the median value as shown in Table 4. However, the induced trench lateral earth pressure coefficient K_{IT} is observed to decrease with increasing friction angle ϕ at a rate proportional to the failure plane inclination $\tan\alpha$, and the vertical stress on the compressible zone $\sigma_{v(cf)}$ is also shown to decrease with increasing friction angle ϕ .

The location of the plane of equal settlement H_e' and the induced trench lateral earth pressure coefficient K_{IT} are calculated for each set 4 configuration using the suggested correlations defined Equations 19 and 20, respectively, and the results are summarized in Table 4 (note that $E_{ro} = 0.194$ for all set 4 configurations). The vertical stress on the compressible fill $\sigma_{v(cf)}$ is then calculated with the proposed theoretical formulation (Equation 15b), and the result in all cases is observed to be within 5% of the numerical result.

Influence of Embankment Height H'

The induced trench configurations evaluated thus far have had an embankment height H' equal to 15 meters, which in all cases is shown to be greater than the height to the plane of equal settlement H_e' ; such induced trench structures are said to be in a state of 'incomplete projection' (i.e. the plane of equal settlement falls within the embankment fill; $H' > H_e'$). In some cases, however, the embankment height H' may be less than the height to the plane of equal settlement H_e' ; such induced trench structures are said to be in a state of 'complete projection' (i.e. the plane of equal settlement H_e' is hypothetical in nature as it falls above the surface of the embankment fill; $H' \leq H_e'$). To investigate the influence of the embankment height H' on the induced trench arching mechanism, a series of induced trench configurations are evaluated with Abaqus finite element software (set 5) as defined in Table 5. The embankment height H' is varied from 1 to 15 meters amongst the configurations defined in Table 5, while all other properties and numerical modeling procedures remain unchanged and are equivalent to those used for preliminary analysis defined previously.

Figures 16a, 16b, 16c, and 16d display the distribution of vertical stress σ_z , vertical stress ratio $\sigma_{z(ro)}$, horizontal stress σ_x , and lateral earth pressure coefficient K , respectively, acting along Line A (defined in Figure 10a) for set 5 configurations defined in Table 5. It is shown in Figure 16a that the vertical stress is reduced in the vicinity of the compressible zone for all embankment heights H' , while Figure 16b shows that the relative reduction in vertical stress remains essentially unaffected for embankment heights H' greater than 4.0 meters. It is also shown that the plane of equal settlement H_e' (corresponding to $0.95\sigma_{z(ff)}$) remains relatively unaffected and approaches 4.0 meters as H' is increased. Similar observations are reported in terms of horizontal stresses; it is shown in Figure 16c that the horizontal stress is increased in the vicinity of the

compressible zone for all embankment heights H' , while the lateral earth pressure coefficient K remains essentially unaffected for embankment heights H' greater than 4.0 meters. It is worth noting that the jagged nature of the horizontal stress distribution shown in Figure 12c, which is a result of the construction modeling sequence (as described previously), is more pronounced at lower embankment heights.

Table 5: Abaqus Finite Element Results and Theoretical Results for Induced Trench Configurations with Variation in Embankment Height H' (Set 5)

Configuration Description	Numerical Results			Theoretical Results			Percent Difference in $\sigma_{v(cf)}$
	K_{IT}	$\sigma_{v(cf)}$ (kPa)	H_e' (m)	h_k (m)	K_{IT}	$\sigma_{v(cf)}$ (kPa)	
Set 5:							
$H' = 1.0$ m	0.970	12.8	3.9	0.75	0.786	13.4	4.9%
$H' = 2.0$ m	0.805	23.4	3.9	1.50	0.700	22.8	2.5%
$H' = 3.0$ m	0.687	33.5	3.9	2.25	0.627	33.0	1.4%
$H' = 4.0$ m	0.612	43.3	3.9	3.0	0.569	44.1	1.9%
$H' = 6.0$ m	0.553	62.7	3.9	3.9	0.518	69.7	11.2%
$H' = 8.0$ m	0.538	82.0	3.9	3.9	0.518	88.0	7.3%
$H' = 10.0$ m	0.532	101.0	3.9	3.9	0.518	106.3	5.2%
$H' = 15.0$ m	0.526	148.3	3.9	3.9	0.518	152.0	2.5%

Table 5 contains the Abaqus numerical results for set 5 configurations, including the induced trench lateral earth pressure coefficient K_{IT} and the vertical stress acting on the compressible zone $\sigma_{v(cf)}$. The numerical K_{IT} values shown in Table 5 are representative of the average K between the compressible zone ($h = 0$) and the numerical plane of equal settlement ($h = H_e' = 4.0$ m). Note that, for configurations in which H' is less than H_e' , K_{IT} is obtained as the average K value between the compressible zone ($h = 0$) and a height 0.5 meters below the ground surface H' ; such vertical domain is employed to eliminate the influence of irrational K values reported in the vicinity of the ground surface.

Figure 16: a) Vertical Stress, b) Vertical Stress Ratio, c) Horizontal Stress, and d) Lateral Earth Pressure Coefficient as a Function of Height Above the Compressible Fill (at Completion of the Final Construction Stage; for Set 5 Configuration Defined in Table 5).

Based on the approximate correlations for H_e' and K_{IT} defined in Equations 19 and 20, H_e' and K_{IT} for set 5 configurations are evaluated as 3.9 meters and 0.518, respectively (since $E_{ro} = 0.194$). Such parameter values are in good agreement with the numerical results for set 5 configurations in the 'incomplete projection' condition (those with $H > H_e'$); however, such parameter values are not representative of configurations in the 'complete projection' condition (those with $H' \leq H_e'$), which warrants additional considerations.

Recall that, by definition, the induced trench lateral earth pressure coefficient K_{IT} is defined as the average representative K over the induced trench zone. For culverts in the 'incomplete projection' condition, such domain corresponds to $0 < h < H_e'$, and the induced trench lateral earth pressure coefficient $K_{IT(inc)}$ may be approximated using the correlation shown in Figure 15 and defined in Equation 21.

$$K_{IT(inc)} = \tan(\alpha) \left(\frac{25}{26} - \frac{1}{3} E_{ro} \right) \quad (21)$$

It is shown in Figure 16d that the lateral earth pressure coefficient K follows a similar distribution regardless of embankment height H' ; K is shown to be greatest in the vicinity of the compressible zone, and to decrease at a non-linear rate with increasing height h . Assuming that the lateral earth pressure coefficient K increases following a parabolic trajectory from a value k_a at a height of $h = H_e'$ to a value k_{arch} at a height of $h = 0$, k_{arch} can be computed with the expression shown in Equation 22.

$$k_{arch} = 3K_{IT(inc)} - 2k_a \quad (22)$$

Where k_{arch} represents the effective lateral earth pressure coefficient in the induced trench zone at the top of the compressible zone. For the induced trench configurations defined in Table 5, $K_{IT(inc)} = 0.518$ from Equation 21 and $k_{arch} = 0.887$ from Equation 22; the corresponding parabolic

approximation for the distribution of lateral earth pressure coefficient K for these configurations is displayed graphically in Figure 17.

Figure 17: Parabolic Approximation for the Distribution of Lateral Earth Pressure Coefficient K in the Induced Trench Zone ($0 < h < H_e'$)

For culverts installed in the ‘complete projection’ condition, the vertical domain of the induced trench zone is restricted to the ground surface. Based on the assumed parabolic K distribution described herein, the induced trench lateral earth pressure coefficient for culverts in the ‘complete projection’ condition $K_{IT(com)}$ can be obtained as the area under the K -distribution (over some representative vertical domain h_k) divided by the effective vertical domain h_k , as shown in Figure 17 and defined in Equation 23.

$$K_{IT(com)} = k_a + \left(\frac{k_{arch} - k_a}{3} \right) \left(\frac{H_e'}{h_k} - \frac{(H_e' - h_k)^3}{h_k H_e'^2} \right) \quad (23)$$

Where h_k is the effective vertical domain used in calculating $K_{IT(inc)}$, which is recommended to be taken as $0.75H' \leq H_e'$ (to restrict the influence of K in the vicinity of the ground surface). Note that for the limiting case in which h_k is equal to H_e' , Equation 23 yields $K_{IT(com)} = K_{IT(inc)}$.

The induced trench lateral earth pressure coefficients K_{IT} computed with the proposed approximate correlation for set 5 configurations are summarized in Table 5, where $K_{IT} = K_{IT(inc)}$ as defined in Equation 21 for configurations with $h_k \geq H_e'$, and $K_{IT} = K_{IT(com)}$ as defined in Equation 23 for configurations with $h_k < H_e'$ (where $h_k = 0.75H' \leq H_e'$ as indicated in Table 5). Note that for transition cases in which $0.75H' < H_e' < H'$, the ‘complete projection’ condition is employed (i.e. $K_{IT} = K_{IT(com)}$). The vertical stress on the compressible fill $\sigma_{v(cf)}$ is then calculated with the proposed theoretical formulation (Equation 15b), where the condition $H_e' \leq H'$ is invoked for configurations in the ‘complete projection’ condition; the result in all cases is observed to be within approximately 10% of the finite element result as shown in Table 5.

Model Summary

The induced trench theoretical formulation presented in this paper is shown to produce results that are in good agreement with those obtained by finite element analysis, provided that the location of the plane of equal settlement H_e' and the induced trench lateral earth pressure coefficient K_{IT} are selected appropriately. A series of parametric studies are performed using Abaqus finite element software to investigate the influence of certain key model parameters, including: the thickness of the compressible zone t , the stiffness of the compressible material E_{cf} , the width of the compressible zone B_0 , the friction angle of the embankment fill ϕ , and the height of the embankment fill H' . Based on the numerical results, approximate correlations are developed to provide guidance in selecting the appropriate values of H_e' and K_{IT} . It is shown that

by implementation of the approximate correlations for H_e' and K_{IT} described herein, the proposed theoretical formulation yields results that are within approximately 10% of those obtained by use of advanced finite element modeling. Figure 18 contains a flow chart depicting the recommended sequence of analysis to incorporate the suggested parameter correlations within the proposed induced trench theoretical formulation.

Figure 18: Flow Chart Depicting the Recommended Sequence of Analysis for Implementation of the Suggested Parameter Correlations within the Proposed Theoretical Formulation

798 **Validation**

799 To further evaluate the effectiveness of the proposed induced trench theoretical
800 formulation, the predicted stresses are compared with experimental data collected from two full-
801 scale induced trench case studies. The structures of interest include induced trench twin culverts
802 constructed at Manzer Brook and Black River in 2009 and 2018, respectively. An overview of
803 the pertinent construction details, research instruments, and recorded earth pressures are
804 described herein; refer to McGuigan and Valsangkar (2011b) and Bryden et al. (2020) for
805 comprehensive descriptions of these case studies.

806

807 *Overview of Induced Trench Case Studies at Manzer Brook and Black River*

808 The Manzer Brook structure consists of twin 4.48-meter outside-diameter reinforced
809 concrete culverts (133 meters in length) constructed with a clear spacing S of 2.84 meters beneath
810 21.7 meters of fill. Well-graded pit run gravel was used for backfill around the structures and the
811 overlying embankment was constructed with material comprised predominantly of rock-fill and
812 excavated glacial till. During embankment construction, a compressible zone (consisting of
813 sawdust) was installed 0.72 meters above the structures with a width of 14.2 meters and a
814 thickness of 3.0 meters. A total of 24 earth pressure cells were installed around the circumference
815 of the Manzer Brook culverts during construction, including two pressure cells for the purpose of
816 monitoring vertical stresses at the culvert crowns.

817 The Black River structure consists of twin 3.6-meter outside-diameter reinforced concrete
818 culverts constructed beneath 15.3 meters of fill. The culverts measure 124.5 meters in length and
819 were installed with a clear spacing of 2.0 meters. Backfill around the structures and extending to
820 a height 0.3 meters above the culverts consisted of structural fill (75mm minus crushed rock),

while the remaining embankment fill material was composed of local sandstone fill. During construction, a single compressible zone (consisting of loose hay) with a width and thickness of 11.0 meters and 2.3 meters, respectively, was installed above the culverts. Instrumentation was installed at the Black River site during construction to monitor the culvert earth pressures, embankment deformations, and groundwater elevations by use of eight earth pressure cells (PC), two ShapeArrays (SAAX) / Settlement Plates (SP), and two piezometers (PZ), respectively. Note that two pressure cells were installed at the culvert crowns to monitor the vertical loading conditions. Figure 19 displays a general schematic of an induced trench twin culvert and is representative of both Manzer Brook and Black River structures.

Figure 19: Generalized Cross-Section of the Induced Trench Twin Culvert Structures at Manzer Brook and Black River, with Pressure Cells of Interest Denoted by PC

For the purpose of the current discussion, the earth pressures recorded at the culvert crowns are of particular interest. The crown pressure cells at both sites, identified by PC in Figure 19, consist of Geokon model 4800 vibrating wire earth pressure cells with an operating range and accuracy of 700 kPa and ± 0.7 kPa, respectively (Geokon 2018). Each crown pressure cell was installed approximately 100 mm above the culvert crown on compact bedding sand and was overlain by approximately 100 mm of compact bedding sand. Care was taken during backfilling/compaction to provide a local stiffness consistent with that of the surrounding soils. The sand/cell enclosures at the Black River site were encased in nonwoven geotextile to prevent the migration of fines.

Upon completion of embankment construction at Manzer Brook, the vertical stress at the two crown pressure cells were observed to be 303 kPa and 307 kPa, which correspond to approximately 67% of the free-field vertical stress for $\gamma_s = 21$ kN/m³. The vertical earth pressure recorded by the two crown cells at the Black River site were 162 kPa and 146 kPa; the average vertical stress measured at the culvert crown was therefore 154 kPa, which corresponds to 48% of the free-field vertical stress at a depth of 15.3 meters (for $\gamma_s = 21$ kN/m³).

Theoretical Result vs. Experimental Result

The relevant geometric parameters pertaining to the Manzer Brook and Black River structures, along with the mechanical properties for the various fill materials, are summarized in Table 6. UNB has performed extensive laboratory testing to evaluate the properties of typical fill materials employed on various projects; the properties of compressible fill have been assigned based on laboratory test results reported by McAfee and Valsangkar (2004), where the elastic modulus E_{cf} is selected based on one-dimensional compressibility results for woodchips over the

load range of 50 to 300 kPa (Manzer Brook) and for hay over the load range of 50 to 150 kPa (for Black River). The mechanical properties of embankment fill and structural fill are representative of highway embankments constructed with compact granular fill, and are consistent with values reported by McGuigan and Valsangkar (2011b) for the Manzer Brook structure.

The vertical stress acting at the crown of the Manzer Brook and Black River structures is computed with the proposed induced trench theoretical formulation following the analysis procedure described in Figure 18. All results are summarized in Table 6. It is found that both structures are in a state of ‘complete projection’ since the plane of equal settlement falls above the physical embankment surface. $K_{IT(com)}$ is evaluated with Equation 23 (where $h_k = 0.75H'$), and Equation 15b (where $H_e' = H'$) is used to compute the vertical stress at the top of the compressible fill $\sigma_{v(cf)}$. The vertical stress acting at the culvert crown σ_v is then computed with Equation 6; the vertical stress on the Black River culverts is computed to be 158.2 kPa, which falls within the range of experimentally recorded earth pressures (146 kPa to 162 kPa), and differs by just 2.7% from the average experimental value of 154 kPa. The vertical stress on the Manzer Brook culverts is computed to be 279.3 kPa, which is approximately 8% less than the experimental stresses recorded in the field. The proposed induced trench theoretical formulation is therefore shown to produce results that are in good agreement with the experimental data.

Table 6: Properties of the Manzer Brook and Black River Induced Trench Structures, and Summary of the Theoretical Results in Comparison to the Experimental Data

Parameter	Manzer Brook	Black River
Geometric Properties:		
Total Fill Height, H	21.7 m	15.3 m
Culvert Diameter, B_c	4.48 m	3.6 m
Compressible Fill Height, t	3.0 m	2.3 m
Compressible Fill Width, B_0	14.2 m	11.0 m
Culvert Cover, H_c	0.72 m	0.3 m
Fill Height Above Compressible Fill, H'	17.98 m	12.7 m
Mechanical Properties:		
Embankment Fill Unit Weight, γ_s	21 kN/m ³	21 kN/m ³
Embankment Fill Stiffness, E_{ef}	15 MPa	15 MPa
Embankment Fill Friction Angle, ϕ	30°	30°
Structural Fill Stiffness, E_{sf}	15 MPa	15 MPa
Compressible Fill Unit Weight, γ_{cf}	2.6 kN/m ³	0.5 kN/m ³
Compressible Fill Stiffness, E_{cf}	2.5 MPa	1.0 MPa
Theoretical Result:		
Relative Stiffness Parameter, E_{ro}	0.438	0.178
Plane of Equal Settlement, H_e'	31.9 m	36.2 m
Incomplete Lat. Earth Press. Coef., $K_{IT(inc)}$	0.471	0.521
Projection Condition	Complete	Complete
Arching Lat. Earth Press. Coef., k_{arch}	0.746	0.896
Effective Height, h_k	13.5 m	9.5 m
Complete Lat. Earth Press. Coef., $K_{IT(com)}$	0.596	0.761
Stress on Compressible Fill, $\sigma_{v(cf)}$	256.3 kPa	150.8 kPa
Vertical Stress on Culvert, σ_v	279.3 kPa	158.2 kPa
Experimental Result (Average):		
Vertical Stress on Culvert, σ_v	305 kPa	154 kPa
Percent Difference in σ_v :		
Theory vs. Experiment	8.4%	2.7%

Conclusions

In this paper, the authors have proposed a new theoretical formulation for evaluating the vertical earth pressure acting on induced trench culverts. The proposed formulation is derived on the basis that shear planes develop within the embankment fill during construction, and that such shear planes are inclined at an angle $\alpha = 45^\circ - \phi/2$. The basic assumption of inclined shear planes is consistent with recent experimental observations from centrifuge tests (McAffee et al. 2011)

and full-scale field studies (Bryden et al. 2020). Based on the proposed geometry, differential settlement within the embankment fill leads to positive arching as a result of: 1) a transfer of vertical load from the induced trench zone to the adjacent soil by shear, and 2) an increased state of lateral confinement due to the tapered geometry of the failure wedge. Such arching phenomenon causes a reduction in vertical stress and an increase in horizontal stress within the induced trench zone, which has the net result of increasing the induced trench lateral earth pressure coefficient. In comparison to the existing induced trench models, the proposed formulation is shown to provide a more realistic representation of the induced trench configurations employed in practice.

Advanced finite element analyses have been performed with Abaqus software and are described herein; the proposed induced trench theory is shown to produce results that are in good agreement with the numerical results, provided that the induced trench lateral earth pressure coefficient K_{IT} and the height to the plane of equal settlement H_e' are appropriately selected. A series of parametric studies are subsequently performed, and approximate correlations are developed to provide guidance in selecting K_{IT} and H_e' in terms of the induced trench relative stiffness parameter E_{ro} . By implementing the suggested parameter correlations within the proposed induced trench model, the theoretical results are shown to be within approximately 10% of the numerical (finite element) results for the various parameter ranges evaluated herein.

Two instrumented full-scale induced trench structures located at Manzer Brook and Black River are described herein for experimental validation. The Manzer Brook structure consists of twin 4.5-meter outside diameter culverts constructed beneath 21.7 meters of fill, while the Black River structure consists of twin 3.6-meter outside diameter culverts constructed beneath 15.3 meters of fill; earth pressure cells were installed during construction at both sites, and the average

vertical stress on the culvert crown was recorded as 305 kPa and 154 kPa, respectively. By utilizing the proposed theoretical model, the vertical stress on each of these structures is predicted to be 279.3 kPa (Manzer Brook) and 158.2 kPa (Black River). Such results are in good agreement with the experimental data and are within 10% of the average experimental results. The proposed induced trench theoretical formulation is therefore shown to accurately predict the vertical stress acting on these two full-scale field installations.

The induced trench theoretical formulation described in this paper is shown to produce results that are in agreement with finite element analyses, centrifuge modeling, and recent field case studies, provided that the model parameters K_{IT} and H_e' are selected appropriately. Recommendations are contained herein to provide some guidance in selecting K_{IT} and H_e' ; however, the reader is cautioned that such recommendations are developed from numerical studies of limited extent. Appropriate values of K_{IT} and H_e' must be selected for evaluation of other induced trench configurations. The results of parametric studies contained herein further suggest that the degree of positive arching is dependent on the mechanical properties of backfill materials, with the stiffness of the compressible material being of particular influence; it is of utmost importance that representative values are selected during evaluation of induced trench structures.

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List of Symbols

- a = Qin et al.'s parabolic stress distribution factor;
 $B(z)$ = Width of the induced trench zone as a function of depth z ;
 B_c = Outside diameter of the culvert;
 B_0 = Width of the compressible fill;
 C = Arbitrary constant of integration;
 C_i = Induced trench load coefficient (Marston's theory);
 d_c = Vertical deformation of the culvert (Marston's theory);
 E_{IN} = Equivalent vertical stiffness of the inner zone;
 E_{OUT} = Equivalent vertical stiffness of the outer zone;

E_c	=	Equivalent stiffness of the culvert;
E_{cf}	=	Elastic modulus of compressible fill;
E_{ef}	=	Elastic modulus of the embankment fill;
E_{ro}	=	Induced trench relative stiffness parameter;
E_s	=	Elastic modulus of soil/compressible fill (Marston's theory);
E_{sf}	=	Elastic modulus of structural fill;
H	=	Height to ground surface from top of culvert;
H'	=	Height to ground surface from top of compressible fill;
H_c	=	Thickness of soil cover between culvert crown and compressible zone;
H_e	=	Height to plane of equal settlement from top of culvert;
H_e'	=	Height to plane of equal settlement from top of compressible fill;
h	=	Height above the top of the compressible fill;
h_k	=	Effective vertical domain for calculating $K_{IT(com)}$;
K	=	Lateral earth pressure coefficient;
K_{IT}	=	Induced trench lateral earth pressure coefficient;
$K_{IT(com)}$	=	Induced trench lateral earth pressure coefficient for complete projection case;
$K_{IT(inc)}$	=	Induced trench lateral earth pressure coefficient for incomplete projection case;
k_a	=	Rankine's active earth pressure coefficient;
k_{arch}	=	Effective lateral earth pressure coefficient in the induced trench zone at $h = 0$;
k_w	=	Krynine's modified lateral earth pressure coefficient;
p'	=	Ratio of the compressible fill height to B_c (Marston's Theory);
p	=	Ratio of the culvert projection above original ground to B_c (Marston's Theory);
r_{sd}	=	Settlement ratio (Marston's theory);
S	=	Clear Spacing Between Twin Culverts;
S_d	=	Settlement of fill materials between the culvert crown and the top of the compressible fill (Marston's theory);
S_f	=	Settlement of the culvert invert (Marston's theory);
S_g	=	Settlement of the material adjacent to the culvert/compressible fill (Marston's theory);
t	=	Thickness of the compressible fill;
V	=	Vertical earth load acting on the culvert crown (Marston's theory);
$V(z)$	=	Resultant vertical load acting on an induced trench element at depth z ;
$V_{(cf)}$	=	Resultant vertical load on the compressible fill;
W	=	Body force of embankment fill;
z	=	Depth below plane of equal settlement;
α	=	Inclination of shear planes;
γ_{cf}	=	Unit weight of the compressible fill;
γ_s	=	Unit weight of embankment fill;
σ_{nf}	=	Normal stress acting on the shear planes;
σ_v	=	Vertical stress acting on the culvert crown;
$\sigma_{v(cf)}$	=	Vertical stress at the top of the compressible fill;
σ_x	=	Horizontal stress;
$\sigma_{x(fff)}$	=	Free-field horizontal stress;
σ_z	=	Vertical stress;
$\sigma_{z(fff)}$	=	Free-field vertical stress;
$\sigma_{z(ro)}$	=	Vertical stress ratio;
τ_f	=	Shear stress acting on the shear planes;

- τ_0 = Resultant vertical stress acting over elemental height dz ;
 ϕ = Internal friction angle of embankment fill;
 ψ = Parameter defined in Equation 12.

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