Canadian Geotechnical Journal

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Journal:	<i>Canadian Geotechnical Journal</i>
Manuscript ID	cgj-2020-0672.R1
Manuscript Type:	Article
Date Submitted by the Author:	27-Jan-2021
Complete List of Authors:	Smith, Edward; Monash University, Civil Engineering Bouazza, Abdelmalek; Monash University, Department of Civil Engineering King, Louis; FSG Geotechnics and Foundations
Keyword:	soil arching, embankments, finite element method, strain softening, nonlocal regularisation
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

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Numerical simulation of the progressive development of soil arching in column supported embankments

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Abstract

The soil arching mechanism is partly responsible for the transfer of stresses away from the soft subsoil and towards the relatively stiff column heads in column supported embankments. Experimental studies have shown that the load transfer resulting from soil arching evolves progressively with increasing subsoil settlement. Past numerical studies exploring soil arching in column supported embankments have typically not been able to capture this progressive development of load transfer. A series of improvements on past numerical studies are outlined which allow for improved simulation of the soil arching mechanism in column supported embankments. These improvements include implementation of a strain-softening constitutive model, non-local integral type regularisation and the application of the Arbitrary Lagrangian-Eulerian finite element method. The benefits of these improvements are observed through comparison of simulation results to recent experimental studies of column supported embankments. The comparison indicates that these techniques allow key aspects of soil arching kinematics and mechanics to be captured.

Keywords: soil arching, embankments, finite element method, strain softening, nonlocal regularisation

1. Introduction

Soil arching involves the transfer of pressure from a yielding support onto adjoining stationary supports and is regarded as one of the universal phenomenon encountered in soil mechanics (Terzaghi 1943). The study and application of soil arching theory to the design of Geosynthetic Reinforced Column Supported Embankments (GRCSEs) has increased considerably over the past two decades (Almeida et al. 2007; King et al. 2017a; van Eekelen et al. 2013; Zaeske 2001). GRCSEs are typically considered for soft soil sites, where adjacent infrastructure or constrained construction timelines limit other ground improvement methods' feasibility. Figure 1 illustrates how the installation of relatively stiff columns through soft soil and into a comparably strong end bearing material causes the embankment loads to concentrate onto the columns as the subsoil settles. The remaining load is borne by the subsoil and geosynthetic, with differential settlements between the columns and the subsoil leading to increased mobilisation of the geosynthetic reinforcement through membrane actions. In this way, the load supported by the subsoil is reduced, resulting in improved stability and serviceability outcomes for the structure.

The development of soil arching with the displacement of a yielding support was first studied in trapdoor experiments by Terzaghi (1936). It was observed that at small downwards movement of the trapdoor, there was a significant reduction in the vertical pressure on the trapdoor, with increasing sand density leading to more considerable load reductions. As the trapdoor displacement increased further, the load applied to the trapdoor began to recover until a residual state of arching was reached. These findings have been supported in subsequent studies by McNulty (1965), Ladanyi and Hoyaux (1969), and Tanaka and Sakai (1993).

Iglesia et al. (1999) developed the concept of the ground reaction curve (GRC) to characterize the support pressure imposed upon the trapdoor as it was displaced. This method was developed based on centrifuge modelling of trapdoor experiments and describes four phases of soil arching in a soil mass (Figure 2). The empirical equations from the Iglesia et al. (1999) GRC were extended to GRCSEs by King et al. (2017b) and used to describe the progressive soil-arching development in GRCSEs in a field case study in Melbourne, Australia (King et al. 2017a). New insight into the

kinematics of soil arching was also offered by the imaging of bench-scale column supported embankment tests by King et al. (2019b). For dense sand, two shear bands were seen to develop and propagate from the edge of each column head. One band extended above the columns at $55\text{--}65^\circ$ to the horizontal (type 1), whilst another extended away from the column at $25\text{--}35^\circ$ from the horizontal (type 2). The type 2 shear bands from one column head were seen to interact and converge with the radial extension of the type 1 shear bands from adjacent columns. The nature of the shear bands observed was akin to an inverted general bearing failure (Figure 3). This analogy assisted King et al. (2019b) in explaining the four phases of soil arching behaviour described by Iglesia et al. (1999):

- Initial arching – Shear strength within failure surfaces propagating from the column heads is mobilized.
- Maximum arching – Full length of the failure surfaces is undergoing dilation, average shear strength along the shear bands is a maximum.
- Load recovery – Much of the soil within the failure surfaces has experienced peak strength and is softening.
- Ultimate state – All soil within the failure surfaces is at, or approaching, critical state shearing.

For medium dense sand, no type 2 shear bands were seen to develop, and the shear banding was instead similar to a punching bearing failure. This behaviour is typical of loose granular soils or dense granular soils at great depth (Vesic 1973). The variation in soil strength and dilation with density and accumulating strain is fundamental to explaining the progressive development of soil arching.

This paper outlines a process, which allows reasonable simulation of several key aspects associated with the progressive development of soil arching. This paper also highlights limitations identified in previous studies which have attempted to simulate soil arching. A constitutive model is proposed, which addresses some of these shortcomings and is validated against experimental studies reported in the literature.

2. Background

Several studies have reported numerical simulation results of soil arching development in column supported embankments (Almeida et al. 2020; Ariyaratne et al. 2013; Badakhshan et al. 2020; Bhasi and Rajagopal 2015a, 2015b; Briançon et al. 2015; Chew et al. 2006; den Boogert 2011; Girout et al. 2014; Han and Gabr 2002; Han and Bhandari 2009; Hossain et al. 2006; Jenck et al. 2009; King 2017; Le Hello and Villard 2009; Liu et al. 2007; McGuire 2011; Nunez et al. 2012; Russell and Pierpoint 1997; Smith 2005; van der Peet 2014; van der Peet and van Eekelen 2014; Yu et al. 2016; Zhuang 2009; Zhuang et al. 2010; Zhuang et al. 2012). These studies have had varying levels of success in simulating a range of phenomena observed in laboratory and field studies of GRCSEs.

A critical behaviour of GRCSEs when simulating long-term performance is the progressive evolution of the arching mechanism with subsoil settlement, particularly the load recovery phase. Some studies (Almeida et al. 2020; Bhasi and Rajagopal 2015a, 2015b; Chew et al. 2006; Hossain et al. 2006; Liu et al. 2007; Smith 2005; Wachman et al. 2010) have calibrated their models to instrumented embankments; however, often the embankment geometry, soil conditions or monitoring duration were such that load recovery was not observed in the data reported. Where the load on the surface of the subsoil was presented (King 2017; Smith 2005; van der Peet and van Eekelen 2014; Zhuang 2009; Zhuang et al. 2012), the load recovery phase was not simulated either. Some studies, Chew et al. (2006) provided results at fixed points in time, without demonstrating how the arching mechanism evolved to that state (Chew et al. 2006). In other studies (Ariyaratne et al. 2013; Bhasi and Rajagopal 2015a, 2015b) discussions of the behaviour of the embankment over time were made; however, these studies did not consider the load recovery phase with continued subsoil settlement. It is also important to note that if the geosynthetic reinforcement is sufficiently stiff, or the subsoil is not soft, then the membrane actions and subsoil support may reach equilibrium with soil arching before load recovery can be observed. However, for all other cases, the failure to model the load recovery phase is attributed to two main issues which are discussed herein.

The first issue pertains to the inability of the constitutive model used in these studies to simulate important soil behaviours. Typically an elastic, perfectly plastic with Mohr-Coulomb failure criterion model is adopted (Ariyaratne et al. 2013; Bhasi and Rajagopal 2015a, 2015b; Briançon et al. 2015; den Boogert 2011; Hossain et al. 2006; King 2017; Liu et al. 2007; McGuire 2011; Nunez et al. 2012; Russell and Pierpoint 1997; Smith 2005; van der Peet and van Eekelen 2014; Wachman et al. 2010; Yu et al. 2016; Zhuang 2009; Zhuang et al. 2012) or some hardening model is used in these studies (Almeida et al. 2020; den Boogert 2011; Jenck et al. 2009; King 2017; van der Peet and van Eekelen 2014). These models are unable to simulate strain softening and the associated variation in dilatancy with increasing plastic shear strains. These behaviours are often relied upon to explain the results of physical tests (King et al. 2019b).

The authors note the use of softening constitutive models in studies by McGuire (2011) which made use of a hyperbolic model (Duncan and Chang 1970) and Girout et al. (2014) which applied a hypoplastic model (Gudehus 1996; Kolymbas et al. 1995). Both of these numerical studies continued to over-estimate the extent of arching and therefore, under-estimate surface settlements. McGuire (2011) suggested that this resulted from other limitations within the constitutive model, whilst Girout et al. (2014) asserted that the inability to model the creep of the geosynthetic reinforcement was the primary issue. It should also be highlighted that neither McGuire (2011) nor Girout et al. (2014) discussed treatments for the mesh dependency, which often arises when softening is introduced into an analysis.

The second issue pertains to the simulation of insufficient subsoil settlement relative to the semi-rigid columns, particularly in studies using the finite element (FE) method or the finite difference (FD) method. These mesh-based techniques are susceptible to numerical instability, and accuracy issues as the mesh distorts in an attempt to capture behaviour at large displacements (de Borst and Vermeer 1984; Potts and Zdravkovic 1999). The GRC proposed by Iglesia et al. (1999) indicated that a normalised relative displacement of 3-5% (ratio of vertical displacement to equivalent clear spacing) is required for load recovery to commence. Frequently, the relative displacements achieved in numerical studies did not exceed this required displacement for load recovery to start (Briançon et al. 2015; Hossain et al. 2006; Liu et al. 2007; McGuire 2011; Smith 2005). Russell and

Pierpoint (1997), Girout et al. (2014) and King (2017) are some of the few authors which highlighted numerical stability issues in their simulations, particularly relating to the singularity point and mesh distortion around the edge of the column heads. However, this is probably a significant factor influencing the extent to which the subsoil settlement can be simulated in all studies.

Studies of soil arching in GRCSEs have also been undertaken using the DEM (Discrete Element Method) and SPH (Smooth Particle Hydrodynamics) which are mesh-free methods that are less susceptible to numerical errors and convergence problems which occur at large strains. Han and Bhandari (2009) investigated the impact of cyclic loading on the soil arching mechanism, and they argued that the DEM could easily accommodate yielding and strain softening behaviours. However, there is no evidence to suggest that these mechanisms developed during each of the load cycles to allow modelling of the load recovery phase of soil arching. Chew et al. (2006) and Le Hello and Villard (2009) used the DEM to simulate the granular material and FEM to simulate the geosynthetic reinforcement and were able to achieve larger relative displacements than the pure FEM approaches. However, the progressive development of the arching mechanism was not the focus of either study. Badakhshan et al. (2020) were able to demonstrate reasonable agreement to the experimental study reported by King et al. (2019b). However, their work did not expand the understanding of soil arching, and the load-displacement curve presented was not verified against experimental data. King (2017) compared FEM analysis to that using SPH and noted that whilst issues relating to mesh distortion and the singularity at the column edge were avoided in the SPH simulation; it was still unable to simulate the load recovery phase. King (2017) indicated that the lack of strain softening in the constitutive model adopted was a likely contributor to this result.

3. Improvements to the numerical approach

As noted earlier, a key limitation with many previous numerical studies into soil arching was the inability of the constitutive model adopted to simulate strain softening and the associated variation in soil dilatancy. For this study, a constitutive model featuring the following was implemented:

- Mohr-Coulomb failure criterion with non-associated plastic potential.

- A confining stress-dependent stiffness modulus which is consistent with the work by Janbu (1963). The implementation is similar to the stiffness modulus adopted in the Hardening Soil Model (Schanz et al. 1999).
- Linear variation in cohesion, friction angle and dilation angle over a user-defined range of equivalent deviatoric plastic strain (Figure 4). Such a method was proposed by Potts and Zdravkovic (1999) with a simplified version without hardening adopted by Potts et al. (1990), Troncone (2005) and Conte et al. (2013).
- Integral-type non-local regularisation to treat the mesh dependency which arises when adopting a softening constitutive model.

Another central concern with the use of mesh-based methods to simulate soil arching is the numerical stability issues that are encountered as the mesh distorts in an attempt to simulate sufficient subsoil settlement. In this section, techniques to treat this mesh distortion are also outlined.

3.1 Soil constitutive model

The relationship between the incremental stress tensor, $d\sigma_{ij}$, and the incremental strain tensor, $d\varepsilon_{ij}$, is as follows,

$$1. \quad d\sigma_{ij} = D_{ijkl}^{ep} d\varepsilon_{kl}$$

where D_{ijkl}^{ep} is the elasto-plastic stiffness tensor. The incremental strain tensor may be decomposed into elastic, $d\varepsilon_{ij}^e$, and plastic, $d\varepsilon_{ij}^p$, parts as follows,

$$2. \quad d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p$$

The elastic strain increment tensor can be described by Hooke's law,

$$3. \quad d\varepsilon_{ij}^e = [D^e]_{ijkl}^{-1} d\sigma_{kl} = \frac{ds_{ij}}{2G} + \frac{1}{9K} d\sigma_{mm} \delta_{ij}$$

where D_{ijkl}^e is the elastic stiffness tensor, ds_{ij} is the deviatoric stress increment, $G = \frac{E}{2(1+\nu)}$ is the

material shear modulus, $K = \frac{E}{3(1-2\nu)}$ is the material bulk modulus, and δ_{ij} is Kronecker's delta. E and

ν are the Young's modulus and Poisson's ratio, respectively. In this study, the confining stress-dependent stiffness modulus is given by,

$$4. \quad E = E_{ref} \left(\frac{c'_p \cot \phi'_p - \sigma'_3}{c'_p \cot \phi'_p - p_{ref}} \right)^m$$

where p_{ref} is the reference confining stress for measurement of E_{ref} and m controls the degree of stress dependence.

Combining Equations 1, 2 and 3 the stress increment can be obtained as follows,

$$5. \quad d\sigma_{ij} = D_{ijkl}^e (d\varepsilon_{kl} - d\varepsilon_{kl}^p)$$

The plastic strain increment tensor is calculated using the non-associated plastic flow rule,

$$6. \quad d\varepsilon_{ij}^p = d\lambda \frac{\partial g(\sigma_{mn}, \kappa)}{\partial \sigma_{ij}}$$

where $d\lambda$ is the increment of the plastic multiplier, g is the plastic potential function, and κ is a parameter controlling material hardening/softening.

The plastic multiplier for the constitutive model can be derived through enforcement of the consistency condition,

$$7. \quad df(\sigma_{mn}, \kappa) = \frac{\partial f(\sigma_{mn}, \kappa)}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f(\sigma_{mn}, \kappa)}{\partial \kappa} d\kappa = 0$$

where f is the yield function.

The Mohr-Coulomb yield surface is adopted in this study and expressed as follows,

$$8. \quad f(\sigma_{mn}, \kappa) = \alpha_\phi(\kappa) I_1 + \sqrt{J_2} - \beta_\phi(\kappa) c'(\kappa)$$

where, $I_1 = \sigma_{ij} \delta_{ij}$, $J_2 = \frac{1}{2} s_{ij} s_{ji}$, $J_3 = \frac{1}{3} s_{ij} s_{jk} s_{ki}$ are the first principal, second and third deviatoric stress invariants respectively, and $\theta = \frac{1}{3} \cos^{-1} \left(\frac{3\sqrt{3} J_3}{2 J_2^{3/2}} \right)$ is the Lode angle. $\alpha_\phi(\kappa)$ and $\beta_\phi(\kappa)$ are defined by,

$$9. \quad \alpha_\phi(\kappa) = \frac{2 \sin \phi'(\kappa)}{[3(1 - \sin \phi'(\kappa)) \sin \theta + \sqrt{3}(3 + \sin \phi'(\kappa)) \cos \theta]}$$

$$10. \quad \beta_\phi(\kappa) = \frac{6 \cos \phi'(\kappa)}{[3(1 - \sin \phi'(\kappa)) \sin \theta + \sqrt{3}(3 + \sin \phi'(\kappa)) \cos \theta]}$$

where $\phi'(\kappa)$, $c'(\kappa)$ are the effective friction angle and cohesion, respectively, as defined in Figure 4.

The hardening/softening parameter, κ , evolves with the deviatoric plastic strain as follows,

$$11. \quad d\kappa = \sqrt{\frac{1}{2} de_{ij}^p de_{ji}^p}$$

$$12. \quad \kappa = \int d\kappa$$

$$13. \quad de_{ij}^p = d\epsilon_{ij}^p - \frac{1}{3} d\epsilon_{kk}^p \delta_{ij}$$

The plastic potential function adopted is given by,

$$14. \quad g(\sigma_{mn}, \kappa) = \alpha_\psi(\kappa) I_1 + \sqrt{J_2} - \beta_\psi(\kappa) c'(\kappa)$$

where $\psi(\kappa)$ is the dilation angle as defined in Figure 4, and α_ψ and β_ψ are as for α_ϕ and β_ϕ with ψ substituted for ϕ' .

Combining Equations 5, 6, 7, 8, 13 and 14 allows for the plastic multiplier to be calculated as per Equation 15, which may be used to calculate the plastic strain increment in Equation 6 followed by the stress increment in Equation 5.

$$15. \quad d\lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl}^e d\epsilon_{kl}}{\frac{\partial f}{\partial \sigma_{ij}} D_{ijkl}^e \frac{\partial g}{\partial \sigma_{kl}} - \frac{\partial f}{\partial \kappa} \left[\frac{1}{2} \left(\frac{\partial g}{\partial \sigma_{ij}} - \frac{1}{3} \frac{\partial g}{\partial \sigma_{ij}} \delta_{ij} \right) \left(\frac{\partial g}{\partial \sigma_{ji}} - \frac{1}{3} \frac{\partial g}{\partial \sigma_{ji}} \delta_{ji} \right) \right]}$$

3.2 Singularity treatments

The shape of the Mohr-Coulomb yield surface presents issues for numerical implementation, primarily concerning the gradient discontinuity at the corners and the apex of the surface (Potts and Zdravkovic 1999). The treatment adopted at the corners involved replacing the yield and plastic potential functions with those for the Drucker-Prager criterion such that the dependence on the Lode angle was removed, and the required gradients could be evaluated objectively (Bui and Huynh 2011; Chen and Mizuno 1990; Zhao et al. 2019). In cases where the stress state went beyond the apex of the yield surface, the stresses were shifted to the apex of the yield surface (Zhao et al., 2019). For the constitutive model adopted in this study, an iterative process may be required during the apex correction given the dependence of the strength parameters on the plastic deviatoric strain.

3.3 Mesh dependency

A significant issue with the application of strain-softening constitutive models is that the results do not converge to the true solution as the problem discretization is refined (de Borst 1987). The cause of this mesh-dependency arises from the fact that classical softening models permit any width of strain localization to develop. Without a length-scale built into the constitutive model to constrain the localization dimensions, the numerical discretization of the problem takes on this role (Brinkgreve 1996). In the context of this study, the parameter κ which evolves with the plastic deviatoric strain (Equation 11), would be controlled by the mesh discretisation leading to softening and strain localisations which are likely incompatible with the typically observed behaviour of granular materials (e.g. the width of the strain localisation).

Several techniques are available to enhance the constitutive model such that the well-posedness of the boundary value problem is restored and a sensible solution can be found (Bažant and Jirásek 2002; Brinkgreve 1996; de Borst 1991). Integral-type non-local regularisation was the technique adopted in this study as the additional parameters required could be intuitively determined and significant changes to the constitutive model, and finite element code could be avoided.

Non-local regularisation methods provide a means of relating the local strain at a point to the strains in the surrounding area (Summersgill et al. 2017). In the case of integral-type techniques, this is achieved by replacing local state variables within the constitutive model with their weighted average. The weighting function proposed by Galavi and Schweiger (2010) was adopted in this study as it avoids the concentration of strains at the centre of strain localizations which occurs when a Gaussian function is used whilst only requiring one additional parameter to be defined as opposed to two for the function proposed by Brinkgreve (1996). The weighting function $w_0(\mathbf{x}, \boldsymbol{\zeta})$ is defined in terms of the length scale l , the position of a point of interest $\mathbf{x}$, and some other point $\boldsymbol{\zeta}$ for which the influence at $\mathbf{x}$ is to be calculated. The function is expressed as follows,

$$16. \quad w_0(\mathbf{x}, \boldsymbol{\zeta}) = \frac{\|\mathbf{x} - \boldsymbol{\zeta}\|}{l} \exp\left(-\frac{\|\mathbf{x} - \boldsymbol{\zeta}\|^2}{l^2}\right)$$

The averaging process should not modify a uniform field, and so the weighting function is often normalized as follows,

$$17. \quad w(\mathbf{x}, \boldsymbol{\zeta}) = \frac{w_0(\mathbf{x}, \boldsymbol{\zeta})}{\int_V w_0(\mathbf{x}, \boldsymbol{\zeta}) d\boldsymbol{\zeta}}$$

where V is the volume of the region subject to regularisation. A fully non-local softening model is based on non-local stresses and strains; however, it is often convenient to consider only a particular state variable as non-local and the remaining stresses and strains as local (Bažant and Lin 1988). In this study the local state variable κ_L^k was subject to regularisation with the non-local value, $\kappa_{NL,n}^k$, at point k in increment n obtained as follows,

$$18. \quad \kappa_{NL,n}^k = \sum_{a=0}^N w(\mathbf{x}^k, \mathbf{x}^a) \kappa_{L,n}^a$$

where N is the total number of points to perform regularisation over. The yield function (Equation 8), plastic potential function (Equation 14) and associated derivatives are modified accordingly.

For the weight function outlined in Equation 16, the width of the strain localisation measured in simulations has been found to be roughly equal to the length scale, l (Galavi and Schweiger 2010; Manica et al. 2018). In this study, l is set equal to $15d_{50}$ (i.e. fifteen times the median particle diameter) which is in the range for the typical width of strain localisations in granular material (Alshibli and Sture 1999; Galavi and Schweiger 2010; Mühlhaus and Vardoulakis 1987; Roscoe 1970; Vardoulakis and Graf 1985). In this way, non-local regularisation allows the simulated softening behaviour to be made compatible with the observed behaviour of strain localisations in granular materials, as opposed to being controlled by the mesh discretisation.

Introduction of integral-type non-local regularisation converts the consistency condition into an integro-differential equation which must be solved to obtain the plastic multiplier (Brinkgreve 1996; de Borst et al. 1993). The approach used in this study is based on work by Strömberg and Ristinmaa (1996). Strömberg and Ristinmaa (1996) proposed to decouple the consistency condition at each material point to give point-wise relations for determination of the plastic multiplier (as per Equation 15). The non-local state variables and yield function are then evaluated. An iterative procedure ensues until the non-local yield function is satisfied. The extent to which the decoupling assumption is in error, and therefore, the number of iterations required is dependent on the degree of non-locality and the size of the increment taken.

3.4 Implementation of the constitutive model

All analysis presented in this study was undertaken using Abaqus (2019) Explicit. The algorithms, subroutines and features used in the implementation of the constitutive model are described in the Supplementary Data (Figure S1). The constitutive model was implemented using the UMAT interface and then made accessible as a VUMAT using the interface proposed by Bienen et al. (2014).

Several simulations were undertaken to verify the implementation of the constitutive model. Of particular interest are the biaxial simulations used to test the effectiveness of the non-local regularisation technique. The boundary conditions and material parameters adopted for the simulations are presented in the Supplementary Data (Figure S2 and Table S1 respectively). Simulations were run for several mesh gradings (Supplementary Data Table S2) using (a) no regularisation (b) non-local regularisation and (c) non-local regularisation with one iteration on the non-local yield function. The load-displacement curves for each case are presented in Figure 5. It is seen that the simulation without regularisation is mesh dependent and that application of non-local regularisation removes this dependency and allows for convergence to an objective solution. It is also seen that permitting only one iteration on the non-local yield function did not notably hinder the performance of the regularisation implementation. This is likely a result of the small-time steps used in an explicit FE algorithm such as that employed in Abaqus/Explicit. Given the time-savings advertised by limiting the iterations, all further simulations were undertaken in this manner.

3.5 Mesh distortion treatments

In Lagrangian FE formulations, the finite elements are attached to and deform with the material (Di et al. 2007; Gadala and Wang 1998). If the material deforms severely, so do the elements and the susceptibility to numerical error or premature termination increases (Potts and Zdravkovic 1999). This is a principal issue with past numerical studies into soil arching as the column head penetrates the embankment material. In contrast, a Eulerian FE formulation allows the mesh to remain stationary whilst the material flows through the elements (Benson 1989; Gadala and Wang 1998). This approach avoids any mesh distortion; however, it can lead to issues when modelling solid

materials, as the stresses can become diffused as the material flows between elements, as well as introducing difficulties in modelling the free-surface boundaries (Di et al. 2007; Qiu et al. 2011).

The Arbitrary Lagrangian-Eulerian (ALE) FE formulations combine the benefits of both the Lagrangian and Eulerian formulations. In an ALE FE formulation, the mesh is associated with a reference computational domain which can move arbitrarily and independently of the material, thus avoiding issues relating to severe mesh distortion (Di et al. 2007; Gadala and Wang 1998). The ALE formulation available within Abaqus/Explicit uses an operator-split algorithm involving a Lagrangian phase where-in loads are applied, and the mesh deforms, followed by an Eulerian phase where-in the nodal positions are adjusted to remove mesh distortion and the solution advected to this new mesh (Dassault Systèmes 2019; Di et al. 2007).

4. Model setup

The range of experiments undertaken and available measurements from the 1g bench-scale study by King et al. (2019b) are briefly summarised in Table Table 1. The high-resolution X-Ray computed tomography (CT) imaging taken during the experiment provided information regarding the soil kinematics for validation against the numerical simulations.

4.1 Soil parameters

The parameters adopted for the UWA Superfine sand are presented in Table Table 2 based on triaxial testing over a range of confining stresses, σ'_3 , presented by Chow et al. (2019) and direct shear testing at low normal stress (8.2 kPa) by King (2019). The soil cohesion was assumed to be negligible, and therefore all associated softening parameters were disregarded. It should also be noted that the preparation of medium dense sand samples may have led to variability in the associated parameters.

The triaxial testing presented by Chow et al. (2019) was undertaken on samples with 70 mm diameter and 150 mm height. The samples were prepared by pluviation, either isotropically or anisotropically consolidated ($K_0 = 0.5$), and then subjected to drained compressive loading.

Elastic parameters were established through calibration of numerical simulations to triaxial testing described by Chow et al. (2019) with E_{ref} allowed to vary with relative density (Figure 6 and Supplementary Data Table S3). The peak strength and dilation parameters adopted in the triaxial simulations were obtained from the experimental values established by Chow et al. (2019). Strength and dilation parameters for the bench scale testing were adopted from the direct shear testing reported by King (2019). The critical state parameters adopted differ from those used by King (2019) owing to recent low-density triaxial tests detailed in Chow et al. (2019).

Softening parameters were also established through calibration to the triaxial tests illustrated in Figure 6. The parameters were then mapped to the bench-scale experiments using the state parameter concept described by Been and Jefferies (1985). The relationship between the softening parameters and the state parameter is illustrated in Figure 7.

4.2 Analysis approach

Numerical simulations were conducted on a unit cell fraction of the bench-scale embankment by exploiting planes of symmetry under the assumption that the apparatus sidewall boundary effects could be ignored (Figure 8). This unit cell approach has also been adopted in past studies by Zhuang et al. (2012), Girout et al. (2014), (King 2019), Almeida et al. (2020) and Badakhshan et al. (2020) among others. The geometry was further simplified by approximating the unit cell using an axisymmetric analysis. To maintain the Area Replacement Ratio (i.e. the ratio of column head area to the unit cell area, ARR) of the unit cell the radius, $r_{c,ax}$, was adjusted when undertaking axisymmetric simulations as follows,

$$r_{c,ax} = \sqrt{\frac{\text{Unit Cell Area}}{\pi}} = \begin{cases} \sqrt{\frac{s^2}{\pi}}, & \text{Square layout} \\ \sqrt{\frac{\sqrt{3}s^2}{2\pi}}, & \text{Triangular layout} \end{cases}$$

where s is the column centre-to-centre spacing.

This approach has been applied extensively in the literature and found to produce results consistent with a 3D analysis (Girout et al. 2014; Han and Gabr 2002; Jenck et al. 2009; Plaut and Filz 2010; Smith and Filz 2007; Smith 2005).

The following also applied to each simulation:

- Only the lower 40 mm of the soil were simulated, the remaining 60 mm were applied as surcharge owing to observations of shear localization extent in the experimental results.
- Downwards movement of the mechanical tray was simulated by displacing the column upwards.
- The column underwent constant upwards acceleration over a 0.1-second duration before being maintained at a constant velocity of 5 mm/s for a further 0.9 seconds. This was found to limit the kinetic energy to maintain a quasi-static analysis whilst also minimizing model run time and associated errors (Fan et al. 2018).
- The column was simulated as a discrete rigid body due to its significantly greater stiffness relative to the soil.
- Based on the mesh convergence analysis discussed in the Supplementary Data (Section 2.3), the soil was meshed using a seed size of 1 mm with axisymmetric CAX4R (Axisymmetric, 4 node, reduced integration) elements.
- Based on the mesh distortion analysis discussed in the Supplementary Data (Section 2.2), the Arbitrary Lagrangian-Eulerian functionality available in Abaqus/Explicit was applied using default settings.
- King (2019) established that the column-soil interface for the experimental apparatus was rough such that the resistance mobilised should be close to the shear strength of the soil. A sensitivity analysis on the interface roughness in numerical simulations found the column-soil roughness to have a negligible influence on results for friction coefficients ranging from $0.25 \tan \phi_p'$ to $\tan \phi_p'$. Given these findings, all simulations were undertaken using a column-soil interface friction coefficient equal to $\tan \phi_p'$.

5. Simulation results

King et al. (2019b) undertook imaging at increments of approximately 0.5 mm mechanical tray displacement. The results from these experiments were presented as incremental displacement and strain fields between pairs of images, obtained by digital volume correlation. The methods

outlined by King et al. (2019a) were used to produce results in terms of three key metrics: normalized vertical displacement (NVD), maximum natural shear strain ($\gamma_{max,log}$) and volumetric strain (ε_v).

19. $NVD = u_z / \delta_{sp}$
Where u_z is the vertical displacement field and δ_{sp} is the settlement plate displacement
20. $\gamma_{max,log} = \log_e (1 + \varepsilon_1) - \log_e (1 + \varepsilon_3)$
Where ε_1 is the major principal strain and ε_3 the minor principal strain.
- . $\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$
Where ε_1 , ε_2 and ε_3 are the principal strains.

The same fields were generated based on the output of the numerical simulations. The orientation of the output plane is illustrated in Figure 9. NVD versus elevation above the base (z) were also produced for points vertically aligned with the ‘above’ and ‘amid’ locations. These locations were selected as the differential settlement between them is expected to be the maximum differential settlement at a given height in the sand (King et al. 2019b). Comparing the convergence of the plots from the two locations allows for the critical height (i.e., the height at which a plane of equal settlement develops) to be established.

The results of simulations are presented for the triangular column layout, results of simulations for the square column layout offered similar insights into the performance of the numerical simulations and were therefore excluded for brevity. NVD versus depth plots for medium dense and dense sands are presented in Figures 10 and 11, respectively. The $\gamma_{max,log}$ and ε_v fields for medium dense sand are shown in Figures 12 and 13, respectively, and similarly dense sand in Figures 14 and 15, respectively.

At smaller tray settlements, the medium dense sand (Figure 10) simulation output typically demonstrated greater discrepancy with the experimental results below the convergence height than the dense sand (Figure 11). The ‘above’ simulation results in Figure 10 illustrate over-estimation of the NVD below the convergence height of up to 0.3 at any given elevation. This result is likely associated with discrepancies in the adopted dilation angle (under-estimation) which prevented sufficient dilation of the material above the column head. Therefore, it increased settlements at the ‘above’ location compared to the experiment. The parameters adopted in the study are from low normal stress direct

shear tests (8.2 kPa) detailed in King (2019), and it should be noted that establishing the strength and dilatancy parameters for the soil at such low pressures is relatively difficult. The experimental values could contain some degree of error, and this error is likely exacerbated as the soil density decreased.

The height at which the convergence of the ‘above’ and ‘amid’ NVD occurred is generally similar between the experimental and numerical results (Figures 10 and 11). The comparison is observed to be more favourable as the tray settlement increased. This observation is likely associated with the accuracy of the adopted dilation angle, with the influence of the adopted value decreasing as the dilation angle softened with increased tray settlement.

Figures 10 and 11 also demonstrate a gradual increase in incremental NVD values with increasing tray settlement which is captured by both experimental results and numerical simulations. King et al. (2019b) explain this observation by noting that as the material in the shear bands approaches the critical state there is less soil dilatancy, leaving only the columns to compensate for the volume change associated with tray settlement. Therefore, as the tray settlement continued, the NVD above the critical height approached $1 - ARR$, such that the settling soil volume at the tray elevation equalled the settling soil volume above the critical height. The numerical simulation results indicate that the softening constitutive model was able to capture this reduction in dilatancy with reasonable accuracy.

The general shape of the simulated failure surfaces compares favourably to the King et al. (2019b) experimental results, including the failure surface height and the similarity with an inverted bearing failure (Figures 12 to 15). The dense sand (Figures 14 and 15) demonstrated a more pronounced type 2 shear band, and type 1-2 shear band interaction, which is consistent with observations made by King et al. (2019b). This is likely a result of the increased strength and dilatancy of the dense sand allowing it to influence material at a greater distance from the column head compared to the medium dense sand (Figures 12 and 13).

For dense sand simulations, the non-local regularisation had a notable effect. It is seen that introduction of non-local regularisation reduced excessive softening due to over-concentration of strains within the strain-localizations to bring the results more in line with experimental outcomes. As

an example, the volumetric strain fields for dense sand (Figure 15) demonstrated that without regularisation, the soil in the shearing zone had ceased dilating by the end of the simulation. This result is inconsistent with the experimental results and is rectified by the introduction of non-local regularisation. The benefit of the non-local regularisation is also observed in the NVD plots for dense sand, where it is seen that simulations without regularisation tended to have a higher NVD resulting from over-softening of the dilation angle (Figure 11). In contrast, for medium dense sand simulations, the introduction of non-local regularisation had a negligible impact on results (Figures 10, 12 and 13). This is likely explained by the smaller difference between the peak and critical state parameters for medium dense sand which reduced the magnitude of the softening experienced and therefore reduced the need for regularisation.

6. Additional experimental studies

Whilst the study undertaken by King et al. (2019b) offered important insights into soil arching kinematics; it also had two principal limitations. Firstly, the experiments were conducted at 1g and therefore did not replicate the stress gradients expected for a field-scale embankment. Secondly, there was no load data available to allow for verification of the mechanics simulated by numerical models. Therefore, additional experimental studies were required to compare with numerical simulations and verify that the numerical model was able to capture the mechanics of soil arching.

The McGuire (2011) study involved bench-scale experiments of both single and multi-column supported embankments which are summarised in Table Table 3. For single-column experiments, surface profile measurements and load-displacement data are available for select tests. For multi-column experiments, surface profile measurements are available. Numerical simulations of these experiments were most useful for verifying the ability to capture soil kinematics through surface displacements and the strain-softening mechanism through column load measurements.

The Fagundes et al. (2015) study involved 20g centrifuge modelling of multi-column embankments with column load and differential settlement measurements taken, which are summarised in Table Table 4. The availability of load-displacement data with multiple columns and increased stress gradients allows the verification of the numerical model to capture soil arching

mechanics at prototype scale stress gradients. However, it is noted that the embankment material used in these physical model tests was not sufficiently dense (relative density of 57%) to replicate the behaviour of the dense granular material typically placed above column heads. Given that this material was prepared in a medium dense state, it is unlikely to exhibit appreciable strain softening. As such, verification was limited to the initial portion of the load-displacement curve. Further, the presentation of differential settlement data as opposed to total settlement data restricted any assessment on arching kinematics. Numerical simulations of these experiments were most useful for verifying the ability to capture the initial portion of the load-displacement curve at prototype scale stress gradients.

A range of experiment geometries from both McGuire (2011) and Fagundes et al. (2015) were simulated, of which only a selection of the results are presented. The conclusions drawn from other simulations are consistent with those presented in this paper, and therefore these results are not included for the sake of brevity.

6.1 Bench-scale testing by McGuire (2011)

The McGuire (2011) study was undertaken on Light Castle Sand, the parameters for which were adopted based on the extensive literature review by McGuire (2011). The softening parameters were calibrated using a similar process to that outlined in Section 4.1 and are listed in the Supplementary Data (Table S5).

Analysis proceeded in the same manner as that described in Section 4.2 with the following modifications:

- For single column analysis, the columns underwent constant upwards acceleration over a 0.1-second duration before being maintained at a constant velocity of approximately 4 mm/s for a further 4.9 seconds.
- For multi-column analysis, the columns underwent constant upwards acceleration over a 0.1-second duration before being maintained at a constant velocity of approximately 2 mm/s for a further 9.9 seconds.

- For multi-column analysis, the soil was also partitioned at a depth of $0.7(s - d)$ where d is the column head diameter. The mesh below the partition was kept uniform whilst the mesh above the partition was graded coarser toward the surface. The height of $0.7(s - d)$ was selected as it was found to be a reasonable upper limit on the height of shear bands into the embankment material for the King et al. (2019b) experiments. In cases where strain localisations extended beyond $0.7(s - d)$, a higher partition was adopted.

Column load and surface elevation data for the 76.2mm single-column experiment with 165.1 mm soil depth and 18.7 kPa surcharge is presented in Figure 16. The surface deformation was well captured by the simulation, and the general trends of the experimental data are apparent in the numerical results. The authors note that the simulated column load is slightly less stiff and achieved a lower residual column load compared to the McGuire (2011) experiments. These variations are likely associated with the adopted stiffness and critical state parameters. The soil parameters adopted gave a generally good fit across the range of experiments conducted by McGuire (2011). Discrepancies in the adopted values could have been introduced through variances in the sand batch used in laboratory tests to establish the parameters and that used in the embankment experiments, as well as variation in the achieved initial density between embankment experiments.

The results of the McGuire (2011) multi-column experiments are presented in terms of a surface to base settlement ratio (SBR), which were synthesised as discussed in the Supplementary Data (Section 3). The McGuire (2011) results are compared to the simulation outputs in Figure 17 for a case with a sample height less than the critical height and in Figure 18 for a case with a sample height greater than the critical height. The surface deformation illustrated in Figure 17 generally demonstrates good agreement between the numerical simulation and the experimental results, although Figure 17a demonstrates under-estimated surface settlements at the extremity of the unit cell whilst Figure 17c, 17d and 17e demonstrate under-estimated surface settlements above the column. These discrepancies are likely associated with an over-estimated dilation angle. The simulation results presented in Figure 18 demonstrate a generally flat surface profile, consistent with an embankment taller than the critical height.

6.2 Centrifuge testing by Fagundes et al. (2015)

The Fagundes et al. (2015) study was undertaken on Hostun Sand, the parameters for which were adopted based on work by Girout et al. (2014) and are generally consistent with parameters suggested by Benz (2007) and those adopted in the study by Almeida et al. (2020). The softening parameters were calibrated using a similar process to that outlined in Section 4.1 and are listed in the Supplementary Data (Table S6).

Analysis proceeded in the same manner as that outlined in Section 4.2 with the following modifications:

- Column displacements were limited to achieve a maximum normalised relative displacement of 15%. Normalised relative displacement was given by δ_{sp}/b' where the equivalent clear spacing (b') was calculated as $2r_{c,ax} - d$.
- The column underwent constant upwards acceleration over a 0.1-second duration before being maintained at a constant velocity of 40 mm/s until the required displacement was achieved.
- The soil was partitioned at a depth of $0.7(s - d)$ in a similar manner to numerical simulations of the McGuire (2011) multi-column experiments.

- Numerical simulations were undertaken at prototype scale, consistent with the studies by Girout et al. (2014) and Almeida et al. (2020).

Fagundes et al. (2015) presented results in terms of column efficacy and differential settlement. Column efficacy is a measure of the load supported by a column relative to the total unit cell load. Differential settlement is calculated as the difference in the settlement at the embankment surface between the 'above' and 'amid' locations (Figure 9). The results of the numerical simulations for configuration 2 (Table Table 4) are presented in Figure 19.

The results indicate a generally good agreement between the experimental and numerical results. The column efficacy is well predicted and where differential settlements were apparent in the experimental results, this behaviour is captured by the numerical simulations. It is noted that agreement between the experimental and simulated differential settlements for the 3.2 m tall embankment after a normalised relative displacement of ~9%. Given that only differential settlement data is available (as opposed to the total settlements) it is difficult to diagnose the cause of this discrepancy, however, it should be recognised that relative displacements beyond ~10% are unlikely to be realised in practice. For the range of relative displacements of interest to practicing engineers, the differential settlements from numerical simulations are in good agreement with the Fagundes et al. (2015) experimental results.

Over the range of normalized displacements simulated, only shallow embankments are observed to demonstrate any appreciable load recovery. The experimental data presented by Fagundes et al. (2015) demonstrated some reduction in column load for taller embankments; however, this reduction commenced at normalized relative displacements >15% which is well beyond the expected values based on the GRC proposed by King et al. (2017a) (3-5%). This lack of load recovery evident in tests with taller embankments is likely a result of the material being prepared in a medium dense state.

The observation that only shallow embankments with heights less than the critical height experienced load recovery would suggest that the load recovery observed in the Fagundes et al. (2015) experiments is a result of failure surfaces intercepting, or interacting with, the embankment surface due to the shallow embankment height as opposed to load recovery due to appreciable strain

softening. This conclusion is supported by additional simulations run using the linear elastic, perfectly plastic with the Mohr-Coulomb failure criterion model available within Abaqus (2019). The results demonstrate that although a perfectly plastic constitutive model is adopted, column load reduction is still predicted for shorter embankments (Supplementary Data Figure S6).

Model tests with embankment material prepared at higher relative densities are likely required to observe load recovery due to strain softening. As a result, whilst the column efficacy predicted is encouraging, these simulations do not validate the ability of the model setup to simulate the load recovery, which typically occurs in column supported embankments.

7. Conclusion

A process for undertaking finite element simulations of soil arching experiments has been outlined.

The following were implemented to address issues with previous numerical studies:

- A constitutive model has been proposed and implemented which allows simulation of strain softening and the associated variation in dilation which is essential to simulate the progressive development of soil arching, particularly post-peak arching stresses.
- A non-local integral-type regularisation technique has been applied to address issues relating to mesh dependency.
- The ALE implementation available with Abaqus/Explicit has been applied to treat issues relating to mesh distortion.

These improvements have allowed the simulation of a number of key aspects of the soil arching mechanism as captured in experiments by King et al. (2019b), McGuire (2011) and Fagundes et al. (2015). Some discrepancies were observed between the numerical simulations and experimental results which were attributed to the adopted soil parameters. An onerous back-analysis process could be pursued to establish case specific soil parameters; however, this was not the purpose of this study and would not add value to the conclusions drawn. The study was focused on demonstrating how an updated numerical approach was able to capture key aspects of soil arching behaviour using soil parameters established from laboratory experiments. Further experimental studies are required to

allow verification of the model on dense granular materials which demonstrate load recovery at prototype scale stress conditions.

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Acknowledgements

The authors acknowledge the Australian Government Research Training Program Scholarship provided to the first author. The authors would also like to thank Associate Professor Ha H. Bui, Monash University, for his guidance relating to constitutive modelling and the time in reviewing this manuscript. In addition, the guidance provided by Associate Professor Britta Bienen, University of Western Australia, on the intricacies of Abaqus is gratefully acknowledged.

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TABLES

Table 1 Summary of experiments conducted by King et al. (2019b)

Column Layout	Square	Triangular
Column diameter	15.0 mm	12.6 mm
Column spacing	45.0 mm	40.0 mm
Soil depth	100 mm	
Surcharge ^a	6 kPa	
Soil density	Medium Dense, Dense	
Surface deformation	None	
Column load	None	
Imaging	Monochromatic Synchrotron X-Ray CT	

^a Applied using a steel plate on the soil surface

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Table 2 UWA Superfine sand parameters adopted for simulations

Parameter	Symbol	Medium Dense	Dense
Minimum density	ρ_{min} (kg/m ³)	1497	
Maximum density	ρ_{max} (kg/m ³)	1774	
Relative density	D_r (%)	63.0	94.5
Density	ρ (kg/m ³)	1660	1756
Median particle diameter	d_{50} (mm)	0.18	
Poisson's ratio	ν (-)	0.2	
Reference modulus	E_{ref} (MPa)	33	40
Reference pressure	p_{ref} (kPa)	100	
Power law	m (-)	0.65	
Initial friction angle	ϕ'_i (°)	31.9	
Peak friction angle	ϕ'_p (°)	40	46
Critical state friction angle	ϕ'_{cs} (°)	31.9	
Initial dilation angle	ψ'_i (°)	9	14
Peak dilation angle	ψ'_p (°)	9	14
Slope of critical state line	λ	0.009	
Critical state void ratio at $p' = 1 \text{ kPa}$	Γ	0.764	
Initial state parameter ^a	ξ	-0.14	-0.23
Deviatoric plastic strain to reach peak state	$\kappa_{\phi_{p1}}$ (-)	0.01	0.01
Deviatoric plastic strain to commence softening	$\kappa_{\phi_{p2}}$ (-)	0.04	0.035
Deviatoric plastic strain to reach critical state	$\kappa_{\phi_{cs}}$ (-)	0.175	0.25
At rest earth pressure coefficient ^b	K_0	$1 - \sin(\phi'_p)$	

^a Evaluated 15 mm above the mechanical tray^b Proposed by Jaky (1944)(as described by Hayat (1992)) and commonly adopted in practice (Girout et al. 2014; McGuire 2011; Michalowski 2005; Schanz et al. 1999)

Table 3 Summary of experiments conducted by McGuire (2011)

Experiment	Single-Column	Multi-Column
Number of tests	63	120
Column diameter	19.1 - 76.2 mm	19.1 - 50.8 mm
Column spacing	N/A	88.9 – 177.8 mm
Soil height	30.5 – 261 mm	58.4 – 251 mm
Surcharge	11.0-18.7 kPa	N/A
Relative density	73 - 105%	73 – 105%
Surface deformation	As surface elevation	As surface settlement
Column load	76.2 mm column only	None
Imaging	None	None

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Table 4 Summary of experiments conducted by Fagundes et al. (2015)

Configuration	1	2	3	4
Column diameter (m)	0.50	1.00	1.00	1.00
Column spacing (m)	2.00	4.00	2.82	2.00
Embankment height (m)	1.8 – 7.2 ^a	1.0 – 7.2	1.0 – 7.2	1.0 – 7.2
ARR (%)	4.91	4.91	9.88	19.63
Surface deformation	As differential settlement			
Column load	Yes			
Imaging	None			

^a Two tests undertaken at 1.8 m to check repeatability

FIGURE CAPTIONS

Figure 1 Illustration of typical components in a GRCSE

Figure 2 Ground reaction curve as proposed by Iglesia et al. (1999)

Figure 3 Shear band types identified during King et al. (2019b) experiments

Figure 4 Variation of c' , ϕ' and ψ with deviatoric plastic strain after Potts and Zdravkovic (1999)

Figure 5 Platen pressure plots for biaxial verification simulations with (a) no regularisation (b) non-local regularisation (c) non-local regularisation with only 1 iteration on the non-local yield function

Figure 6 Experimental triaxial tests presented by Chow et al. (2019) compared to numerical simulations for (a) Isotropically consolidated dense material with σ'_3 of 50, 100 and 200kPa (b) Isotropically consolidated loose material with σ'_3 of 50, 100 and 200kPa (c) $K_0 = 0.5$ consolidated material with σ'_3 of 25, 50 and 500kPa

Figure 7 Softening parameters established for UWA Superfine Sand over a range of state parameters

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Figure 9 Plan showing orientation of the output planes at 'amid' and 'above' locations for analysis of King et al. (2019b) experiments

Figure 10 NVD versus depth for triangular column layout with medium dense sand for δ_{sp} (a) 0.9 to 1.4 mm (b) 1.9 to 2.4 mm (c) 2.9 to 3.4 mm

Figure 11 NVD versus depth for triangular column layout with dense sand for δ_{sp} (a) 0.9 to 1.4 mm (b) 2.0 to 2.5 mm (c) 3.0 to 3.5 mm

Figure 12 $\gamma_{max,log}$ fields for triangular column layout with medium dense sand (a) King et al. (2019b) results, (b) simulations without regularisation and (c) simulations with non-local regularisation

Figure 13 ϵ_v fields for triangular column layout with medium dense sand (a) King et al. (2019b) results, (b) simulations without regularisation and (c) simulations with non-local regularisation

Figure 14 $\gamma_{max,log}$ fields for triangular column layout with dense sand (a) King et al. (2019b) results, (b) simulations without regularisation (c) simulations with non-local regularisation

Figure 15 ϵ_v fields for triangular column layout with dense (a) King et al. (2019b) results, (b) simulations without regularisation (c) simulations with non-local regularisation

Figure 16 Simulation output compared to McGuire (2011) results for $d = 76.2$ mm single-column penetrating into 165.1 mm of sand prepared at 87% relative density with 18.7 kPa surcharge showing (a) column stress normalised by initial column stress (b) surface elevation at δ_{sp} of 6.35 mm and 12.7 mm

Figure 17 SBR simulation data compared to McGuire (2011) results for multiple $d = 50.8$ mm columns with $s = 177.8$ mm into 116.8 mm of sand prepared at 87% relative density at δ_{sp} of (a) 2.8 mm (b) 5.6 mm (c) 8.1 mm (d) 10.9 mm (e) 13.7 mm (f) 16.5 mm

Figure 18 SBR simulation data compared to McGuire (2011) results for multiple $d = 50.8$ mm columns with $s = 88.9$ mm penetrating into 121.9 mm of sand prepared at 87% relative density at δ_{sp} of 10.2 mm

Figure 19 Fagundes et al. (2015) results compared with simulation output for column configuration 2 with various embankment heights (H) showing (a) column efficacy (b) differential settlement

MATERIALS

LOADS

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▲ 200 elements ● 450 elements ◆ 800 elements ■ 1250 elements
(a)

▲ 200 elements ● 450 elements ◆ 800 elements ■ 1250 elements
(b)

▲ 200 elements ● 450 elements ◆ 800 elements ■ 1250 elements
(c)

Figure 6: Experimental triaxial tests presented by Chow et al. (2019) compared to numerical simulations for (a) Isotropically consolidated dense material with σ'_3 of 50, 100 and 200kPa (b) Isotropically consolidated loose material with σ'_3 of 50, 100 and 200kPa (c) $K_0 = 0.5$ consolidated material with σ'_3 of 25, 50 and 500kPa

Figure 7: Softening parameters established for UWA Superfine Sand over a range of state parameters

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Figure 9: Plan showing orientation of the output planes at 'amid' and 'above' locations for analysis of King et al. (2019b) experiments

(a)

(b)

(c)

— Experimental data-Above — Experimental data-Amid
 TL-MD-AX-Above - No regularisation TL-MD-AX-Amid - No regularisation
 --- TL-MD-AX-Above - With regularisation --- TL-MD-AX-Amid - With regularisation

Figure 10: Comparison of experimental results by King et al. (2019b) to numerical simulations showing NVD versus elevation above tray for triangular column layout with medium dense sand for δ_{sp} (a) 0.9 to 1.4 mm (b) 1.9 to 2.4 mm (c) 2.9 to 3.4 mm

(a)

(b)

(c)

Figure 11: Comparison of experimental results by King et al. (2019b) to numerical simulations showing NVD versus elevation above tray for triangular column layout with dense sand for δ_{sp} (a) 0.9 to 1.4 mm (b) 2.0 to 2.5 mm (c) 3.0 to 3.5 mm

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Figure 13: ε_v fields for triangular column layout with medium dense sand (a) King et al. (2019b) results, (b) simulations without regularisation and (c) simulations with non-local regularisation

Figure 14: $\gamma_{max,log}$ fields for triangular column layout with dense sand (a) King et al. (2019b) results, (b) simulations without regularisation (c) simulations with non-local regularisation

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