Mathematics Teachers' Needs in Dynamic Geometric Computer Environments: In Search of Control

by

Douglas Emerson McDougall

A thesis submitted in conformity with the requirements for the degree of Doctor of Education
Graduate Department of Curriculum, Teaching & Learning
University of Toronto

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0-612-28123-X
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ABSTRACT

The study sought to understand the needs of experienced teachers who, for the first time, are teaching geometry in a computer-based exploratory environment rather than in the traditional environment of textbook, straight-edge and ruler. Insights into these needs were obtained through a qualitative case-study, in which data was collected by observation, as well as from interviews with teachers and students and from participant journal entries.

Analysis of the data showed that the four teachers participating in the study experienced an initial loss of control due to the new environment, in three categories: (1) Management control (they believed the new environment impaired their ability to maintain discipline), (2) Personal control (they were unable to determine their own expectations of the students and to assess students’ achievement), and (3) Professional control (they felt they no longer had all the answers).

As the teachers learned to use the new tools, however, they gained confidence in their ability to teach effectively with the new methods, and were even moved to reflect upon their previous teaching practices. Despite the
apparent lack of discipline, the absence of specific expectations, and the changes in their professional role, they came to recognize and accept that in the new exploratory environment the students were learning geometry and enjoying it.

The implication for teacher education is that preservice and inservice teachers should be given a mentor or coach to help them accept a temporary loss of control in the classroom as part of the change process. The teachers will emerge from this experience with increased confidence in mathematics and it will help teachers understand the issues present as they introduce dynamic geometric software packages into their classrooms. The implication for mathematics education is that students thrive in dynamic geometric software environments when teachers maintain control over the management of learning, over their own personal expectations, and over their role as a professional.
Acknowledgments

The preparation, design, writing, rewriting, organizing, and reorganizing of a thesis is done with many hands, heads, hearts and souls. I learned that the responsibility for completion was mine. However, a number of people have played important parts in my journey and, while I would like to mention them all by name, I will direct my comments to a few. Many others are also remembered in my thoughts.

I am indebted to Gila Hanna, my thesis supervisor, for her understanding of the trials and pitfalls of the doctoral journey, for guiding me when I needed direction and for giving me the freedom to explore when I needed it. She was always willing to listen to my thoughts, to encourage me and to provide me with numerous resources that helped me further my understanding of mathematics education. She is a model teacher, facilitator and friend.

I would also like to thank the members of my thesis committee. Lynn Davie provided me with guidance and encouragement. He had an unwavering confidence that I could complete this journey at my own rate. His support of graduate students has inspired me. Rina Cohen was equally supportive of my work and offered valuable and timely suggestions at critical points in my journey.

Derek Hodson was an inspiration to me. He read my work with a focused eye. He was my external reader at the Departmental Oral and continued providing valued assistance through the Final Senate Oral Examination. His piercing questions and insightful suggestions have made this a more precise and readable document. His open display of confidence and understanding helped me immensely through the last few months of my journey.

Bob McLean, my faculty advisor and friend, had confidence in my ability to complete this degree from the beginning. We worked together on a number of projects and Bob was particularly helpful in providing me with experiences using the Internet, Gophers, WWW, and computer-mediated communications. He completed the circle by participating in my Final Senate Oral Defense.

Merl Wahlstrom, the department chair, joined my circle at the Final Senate Oral Defense. His insightful questions and gentle manner were comforting. Bill Higginson, my external examiner, was inspiring. His thought-provoking questions and analogies gave me pause to think, reflect and make my own connections with recent research.
I thank Upper Canada College for their generous sabbatical leave program and Hamish Simpson, Headmaster, Upper Canada College Preparatory School, for his moral and emotional support and guidance. Thanks also to the Canadian Educational Standards Institute for providing me with a grant to help fund my research. Special thanks to Solette Gelberg, Executive Director, CESI for her encouragement and support and for being so thoughtful as to invite me to present my work to other independent school teachers.

There are a number of other people whose paths crossed mine at various stages of my work. Ron Ragsdale helped me conceptualize my initial questions. His early acceptance of my topic and his direction helped me find my way early. Lewis Berenson, a visiting professor from Israel, provided me with guidance at my thesis proposal hearing. His insight, questions and suggestions on the four cases helped me provide a richer context for the case studies. Les McLean provided me with timely resources and case study assistance and was always interested in my work. Shoshana Keiny, a visiting professor from Israel, shared her vision of education and shared her time and energy with me to formulate additional questions in my research. Thanks to Ann Kajander for her moral support and for providing me with valuable suggestions on the thesis.

I would like to thank my many friends and classmates at whose ideas, perceptions and discussions influenced my understandings of education and teacher change. I would like to single out two very close friends, Kathryn Davidson and Eileen Bragg, whose unending encouragement, support, assistance, and feedback helped shape this thesis and have been invaluable. They were part of every stage of my journey and made it a rewarding and exciting experience. I wish them both well as they continue their own thesis journeys.

I am indebted to Guy, Cathy, Karen, Mike and Simon, my Grade 8 teachers. They taught me about classrooms, teacher change, and the integration of computers into the mathematics curriculum. They are all risk-takers and have shared a part of their personal and professional life with me. Thank you for your interest and support of my work.

And finally, thanks to my wife Susan, for her continued support and encouragement. She endured the many late nights and busy weekends while I wrote and rewrote my thesis. She helped me keep my perspective and find strength through this long journey. She never lost faith and now it is my turn to thank her.
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Chapter One
Introduction

Seeing, rather than mere looking, requires an enlightened eye: this is true and as important in understanding and improving education as in creating a painting.

(Eisner, 1991, p. 1)

The purpose of the research described here was to determine the learning needs of mathematics teachers in an exploratory classroom environment. The choice of this topic stems from my personal-professional experience and interest in teacher education. As an instructor at a faculty of education, I have searched for ways to help teachers make changes in their teaching pedagogy. These experiences influenced the selection of my topic and the development of the initial questions for the study. This chapter outlines the formulation of questions, purpose of the study, my personal background, and the outline of the thesis.

1.1 Research Context

In my mathematics inservice program for middle school teachers, I have an assignment where teachers, in a collaborative group effort, prepare sample lesson plans on a topic from the Grades 7 through 10 curriculum. I demonstrate various introductory lessons, encourage discourse among the candidates, model cooperative and collaborative learning activities, and demonstrate various teaching strategies. By this time in the course, we have discussed constructivist perspectives, components of lessons, lesson planning including over 20 introductory and lesson activities that may be incorporated into a lesson plan,
the characteristics of the learner, and various roles for the teacher. Yet, when the pairs and trios present their model lessons to the class, teacher-directed, lecture-style lessons are the norm.

There are many reasons for using this lecture-style. These include teachers' lack of knowledge about the mathematics content and lack of confidence in the "new" teaching methods. They teach using techniques that are similar to those used when they were learning mathematics. Lecturing also allows teachers to better manage the classroom environment. How do we get teachers to change their teaching style?

According to the "new" teaching methods, the role of a mathematics teacher is to help students learn mathematics. Teachers have to create situations where students are 'psychologically safe', mathematical ideas are discussed, and students can work and think mathematically. Students will construct their own knowledge regardless of what the teacher does within the classroom. However, the teacher can influence which mathematical concepts are investigated.

Teachers have to listen to what students say about mathematical concepts. They should encourage discussion about mathematics in their classrooms. The teacher's role should be seen as one of facilitating the development of mathematical meaning within the mathematics classroom. However, this role may involve changing the way many teachers view mathematics teaching and learning.

An assumption for many new educational programs has been the premise that teachers will adapt to change and that we need only to instruct teachers on the nature of the new changes, be they curriculum, teaching techniques or assessment methods. However, in order to bring desirable changes to the system, we need to find out what is actually happening when teachers undertake changes in their teaching practice.
In the traditional classroom, the teacher's role has been one of "telling and describing", that is, teachers present ideas and directions to the class as a whole. If students encounter difficulty with the subject material, the students ask the teacher to confirm the correctness of their solution. In certain classrooms, the student may also ask for assistance from the other students in the class.

Romberg (1985) has pointed out that the job of teaching is to "assign lessons to a class of students, start and stop lessons according to some schedule, explain the rules and procedures of each lesson, judge the actions of students during the lesson, and maintain order and control throughout" (p. 5). Romberg believes that most teachers see the mathematics curriculum as something that needs to be covered, and only few teachers see student learning and understanding as the primary goal of mathematics education.

Management of the learning environment becomes an important issue for teachers trying to make changes in their teaching practice. The teaching role has been seen as one where the teacher controls the learning environment. That control can be restrictive: directing, ordering, telling, and demanding. How teachers use this control within their classroom will clearly influence the learning environment.

It is true that teachers make demands of students. Teachers tell students where to sit, when to listen, when to talk, when to work, when to move, and even when to 'learn'. Students are expected to listen to others, ask mathematical questions, complete their homework, and search for mathematical relationships.

Against this backdrop, teachers also expect students to think creatively, and independently--essentially to take responsibility for their own learning. Students can become confused about their role in the classroom as they try to cope with the controlled environment of the traditional classroom and the open exploration of the creative classroom. On one hand, students become passive
recipients of information by following the lead of the teacher. On the other hand, students are attempting to create alternate solutions to traditional problems.

The management of the learning environment must allow for students to construct their own knowledge and to take responsibility for their own learning. Students also need the freedom to discover, through exploration, different ways to build solutions. They need to spend time working with problems and searching for solutions. This process may be organized and recorded according to the predetermined plan of the teacher whose role is to facilitate the student's exploration (Burns, 1992). As such, it is important that teachers provide students with the opportunity to explore, analyze, and demonstrate their skills.

The use of the computer has been heralded as one teaching tool suitable to mathematics teachers to encourage the exploration of mathematics. The expanded use of computers in mathematics education may create a shifting of roles for teachers. Assuming that teachers are willing to utilize computers in their classrooms (NCTM, 1989, p. 67) to encourage the students to explore mathematical concepts, there is a need to investigate how this utilization can be implemented. Even though the National Council of Teachers of Mathematics (NCTM) Standards (1991) have provided the impetus to change the curriculum, teaching techniques, professional development methods, and assessment practices, there are many more factors to consider when teachers and curricula change. In fact, the interaction between computers, teachers and mathematics is complex (Noss, 1991).

Research in the area of teacher needs while teachers are in the process of reorganizing their pedagogical beliefs and related practices remains incomplete (Boufi, 1994). In a review of the proceedings of the Psychology of Mathematical
Education (PME) annual conferences from 1979 to 1991, Hoyles (1992) noted an increase in research that considers the teacher as an integral—and crucial—facet of learning mathematics. In 1979, all but three of the conference papers focused on student understanding of mathematical concepts. Over the next three years, several studies were published focusing on teachers' expectations and beliefs. By 1990, a large number of papers from a variety of perspectives examined teachers' beliefs and cognitions. This overview by Hoyles (1992) indicates that research has increasingly paid attention to teachers so that by 1992, teacher research was a major focus of research in mathematics education. Further, she argues that computers would assist researchers in exposing teachers beliefs, and that “computers can clarify and amplify the operation and influence of the social norms of the classroom and the belief systems within it” (Hoyles, 1992, p. 38).

Building on recent research on teacher change, this study investigated the use and implementation of geometric construction software to expose teacher beliefs. This study also examined the reactions of teachers as their students explore geometric constructions in a probing for understanding milieu.

1.2 Purpose of the Study

The purpose of this study is to determine learning needs of mathematics teachers and conditions that support teachers in meeting these needs in an exploratory computer-based classroom environment by dialoguing with four teachers and by studying the interaction between teachers and students as teachers attempt to break from tradition in their mode of teaching.
1.3 Statement of the Problem

In this study, I have investigated how best to assist teachers (who are not accustomed to utilizing computer exploratory activities as part of their teaching) to change their mode of teaching so as to allow their students to explore geometry, and make and test conjectures in class.

I have addressed this problem by describing and analyzing experiences of four mathematics teachers as they introduce geometric construction software into their classroom. Teachers were observed and interviewed to determine what they think they need to use exploratory activities in their classroom. The similarities and differences among the four case studies of the teacher’s perceptions of their needs are analyzed and recommendations for the implementation of exploratory modes of mathematics teaching are made.

The study also addressed the following questions:

1. How do teachers change their instructional strategies when they gain experience with the software?
2. In what ways does the role of the teacher change when teachers are placed in an exploratory computer-based environment?
3. What is the nature of the teacher’s intervention in the student’s learning within a computer environment?
4. What degree of confidence do teachers have in their own skills in teaching geometric constructions using a computer program?
5. Are teachers generally amenable to exploring mathematical patterns independently?
6. In what ways do teachers react to the exploration of geometric concepts in a mathematics classroom and how does teaching, using the software, fit into the teacher’s idea of exploration?
1.4 Researcher: Personal Background

According to Peshkin (1982), a strong relationship develops between the researcher's interests, the research method employed, and the claims ultimately made by the researcher. Therefore, because I have both collected and analyzed the data, it is important to inform the reader of my background, educational experiences, personal beliefs, interests, and possible biases. In addition, it is hoped that, by describing myself, the reader will gain some insights into my perspective. Thus, I can strive to separate my biases from my observations before making any claims about my findings.

My interest in teacher education and change stems from my sixteen years of teaching experience. I have worked in public secondary schools for eight years, in an independent elementary school for six years, and as a mathematics consultant for a large school board in Ontario for one year. I have taught students in the primary, junior, middle, and secondary school levels. I have also offered pre-service and in-service teacher courses at two Faculties of Education in Ontario.

As a department head of mathematics, mathematics consultant and an instructor of university-level teacher education courses in mathematics, I have gained first-hand knowledge about curriculum change. According to my personal experiences, the questions that teachers most frequently ask about change were: "When and how will I get the time to keep up with the technological and pedagogical developments?" In my role as a curriculum specialist, I interpreted their questions as "What kind of support will I be able to
get to make this change?” This led me to focus my research on determining the type of support that teachers need to help them make changes in their teaching practices.

I developed an interest in computer science in my first year of teaching. Even though my first teaching interest was mathematics, I was asked to teach computer science and to introduce a computer network to the school. After my second year of teaching, I started to teach computer education courses to elementary and secondary school teachers through the Continuing Education Program of a Faculty of Education. Because I worked in the same school district as the teachers who were enrolled in the course, I was able to visit these teachers after the course was completed. Through questioning, I discovered that the teachers did not receive sufficient administrative support from their schools to integrate computer applications into their teaching practice effectively. It was also apparent that few teachers were utilizing the computers in their own classrooms. I realized that, although the administrators of the school boards, the Ontario Ministry of Education, and the principals were suggesting that teachers should use computers in their classroom, little attention was given to how best to identify teacher needs for changing their practice to implement this expectation.

I moved to another city several years later to become the Head of the Mathematics Department of an elementary level independent school. My duties included providing professional development to teachers at the school. This task proved more difficult than in a public school system due to the isolation of the independent school teachers from other practicing teachers.

How could I encourage these teachers to look at alternative instructional strategies? I found that providing teaching materials and ongoing support brought better results than mandating a change in the curriculum and expecting
change to occur. I began to question myself further: "How can we best manage change within our schools? What support do teachers need to change their teaching practices? How is this change related to their beliefs and attitudes regarding pedagogy?"

I was teaching the middle school mathematics course to certified teachers, who upon completion of the course, would be qualified to teach Grades 7 to 10 mathematics in the Province of Ontario. Cooperative learning techniques were employed to assist teachers in understanding the concepts of the mathematics program and to develop strategies to teach mathematics concepts. I found that greater participation and discussion occurred when teachers collaborated with each other in presenting lessons, completing assignments, and discussing issues surrounding the teaching and learning of mathematics. This experience led me to the following questions: "Were there particular types of peer interactions that assisted teachers to change their teaching practices? Would teachers know how to identify and seek these types of interaction?"

I believe that teachers should know when to use technology in their classroom and how to use it appropriately. Therefore, as a teacher of Grade 8 mathematics and a strong believer in teacher growth, I centered my study on teacher change while using computer technology in the classroom.

Because of their availability and accessibility, I approached independent school teachers to participate in the study. As an independent-school teacher, I could develop the relationship that is necessary to observe interactions in the classroom without being seen as an "outsider." In this study, I have done observational research by being noncommittal, by offering advice when asked, and by providing whatever support the teacher deemed necessary.

The qualitative research method used in this study was selected based on the nature of the research questions. To understand the process of change and
the support the teachers might need requires immersing oneself into the “social environment” of the classroom. As noted by Rist (1983), the most powerful way of understanding human beings in their own environment is to “watch, talk, listen and participate with them” (p. 12).

1.5 Significance of the Study

This study takes the view that teacher education should be based on the learning needs of the individual teacher. I have designed the study with an emphasis on assisting the participants to identify their own learning needs. In this way, the learning needs of the teacher are defined by the individual.

This study provides personal accounts of the professional growth of four teachers. Firstly, these accounts may be useful to other teachers who may find themselves attempting to integrate computers into their classrooms. Secondly, the investigation may inform teacher educators and administrators about the personal/professional nature of teacher change and may contribute to the general discussion surrounding some initial assumptions and prerequisites for effective teacher change. By focusing on the learning needs of teachers as they attempt to break from traditional teaching methods, this study may contribute to a better understanding of teacher-directed change in mathematics education.

1.6 Limitations of the Study

Since the findings are based on four case studies of licensed teachers in four independent schools, they may not be generalizable to every teacher and every school. Furthermore, because of the small sample, this study will only present a picture of the practice of individuals who participated in the study.
Nevertheless, as argued in the method chapter, the findings may speak to other teachers in similar situations.

1.7 Plan of the Thesis

This thesis is organized as follows: Chapter Two examines how the concept of constructivism, the learning of geometry, computers in geometry education, teacher needs, teacher supports, and teacher change are dealt with in the literature. The literature review describes the factors that influence teacher change and the possible obstacles that may exist in mathematics education. The role of the teacher is explored, integrating teacher beliefs and the effects of teacher beliefs on teacher learning needs. The chapter also explores the use of dynamic geometric computer software and perspectives on teacher change within that environment.

Chapter Three gives a detailed account of the method of research, including participants, data gathering, setting and procedures. The results of a pilot study will provide the theoretical and practical support for the use of a case study research method. The issues of access and ethical considerations of the research instruments are discussed.

Chapter Four describes the four teachers, focusing on their perceived role of the teacher, their in-class experiences, their reflections, and the issue of teacher control within the classroom.

Chapter Five presents a discussion on the four case studies, an investigation of the major findings, and their relation with the current literature, as well as some suggestions for further research.
Chapter Two
Literature Review

2.1 Background

The purpose of this review is to examine literature related to teachers’ practice in mathematics teaching and the conditions necessary to support teacher change. It is a selective review based on the needs of teachers as they undergo change in a new environment.

This review brings together literature from diverse fields such as teacher education, teachers’ views of computer technology, theories of mathematics education, and teaching and learning through geometry computer software. This review provides a framework for recognizing, analyzing, and understanding the factors of teachers’ needs and supports.

2.2 Introduction

An important problem facing mathematics education is the implementation of recommendations contained in two National Council of Teachers of Mathematics documents (NCTM, 1989, 1991). These reform movement recommendations in mathematics education support the use of computers in the classroom. However, a critical problem is the translation of those recommendations into actual practice in schools.

The NCTM vision requires "significant and worthwhile changes" in teaching practice of mathematics teachers (Richardson, 1990; NCTM, 1991). Unlike other curriculum changes in mathematics, this change does not come
from within the mathematics community as a consequence of certain cultural
developments of the discipline, but as the consequence of the great changes in
the social and economic reality provoked by the impact of new information
technologies.

Research on the reactions of teachers facing curricula innovations by
which teachers reorganize their pedagogical practice and beliefs is still in its
infancy (Boufi, 1994; Bottino and Furinghetti, 1994). Introducing new types of
teaching methodology requires more than just telling the teachers to make
changes to their teaching practice. Teachers are constantly adjusting their
practice to take into account the context of schools and classrooms in which
learning takes place (Eisner, 1992). How, then, can teachers be assisted to make
changes in their classroom practice to support the expectations of the NCTM
vision statements?

There are many societal influences that affect teachers and their teaching
practice. The integration of the computer into our society, the beliefs and images
of teachers, and how teachers change influence mathematics teaching practices.
Computer technology is visible in almost every facet of our society. Its entry into
schools, however, has been slow and this is especially true for mathematics
classrooms (Kilpatrick and Davis, 1993). Many reasons exist for this lack of
integration into the mathematics curriculum: teachers' view of knowledge
acquisition (Hannafin & Freeman, 1995), lack of availability of computer
hardware (Becker, 1990), and teacher anxiety towards computers (Rosen & Weil,
1995; Berebitsky, 1985).

Classroom activities in which students are using computers to investigate
patterns or test conjectures make "heavy demands on teachers to work harder
keeping abreast of what students are doing" (Kilpatrick & Davis, 1993, p. 210).
This suggests that teachers will need to change their teaching practices although
there is little evidence on what changes will be necessary.

Teachers enter the teaching profession with images and assumptions about teaching based on their own experiences as students (Ball, 1988). Furthermore, Lampert and Ball (1990) have noted that neither methodology courses nor field experiences in classrooms have altered these images and beliefs of prospective teachers. Likewise, experienced teachers also have beliefs that influence their teaching and those beliefs are consistent with their classroom practices (Olsen & Singer, 1994). These beliefs will influence the types of assistance required by teachers making changes in their mode of teaching.

Successful teacher reform demands an understanding of what teachers know and how teachers change (Richardson, 1990; Shaw & Jakubowski, 1991; Wood, Cobb, & Yackel, 1991). Models have been developed to assist researchers to investigate teacher change (Simon, 1995; Underhill, 1991; Fullan, 1982; Fullan, 1991; Edwards, 1994b). These models may contribute to our understanding of what learning needs teachers have as they undertake a new teaching methodology.

The expanded use of computers in mathematics education can create a shifting of roles for the teacher. Whereas in the traditional classroom, the teacher’s role has been one of telling and describing to the class as a whole, Maher and Alston (1990) have recommended a shift in the teacher’s role to one of listening to students and questioning and probing them for understanding.

According to Burns (1992), students need the freedom to discover, through exploration, different ways to build solutions spontaneously. They also need to spend time working with problems and searching for solutions. Teachers can help students explore mathematics by providing materials, time and encouragement. Teachers should therefore become a facilitator to provide students with the opportunity to explore, analyze, and demonstrate their
problem solving skills.

Such a role is not that easy for the teacher:

... you have to give [the students] their freedom but you have to keep just enough control ... the staff member keeps abreast of where each group is at, decides just when the class is ready for each progress report, assists in making each discovery clear and precise, and at the end wraps things up, reveals the connections that may not yet have been seen, and ties the results in with any relevant mathematical lore, ... It is the sort of thing that Polya was famous for (Taylor, 1985, p. 1).

2.3 Constructivist Views of Learning and Teaching Mathematics

*Knowing, like touching, requires the organism to be active and to construct meaningful patterns out of experience.*

Eisner, 1994

A current theoretical view of teaching and learning, constructivism, suggests that learners construct their own knowledge through interaction with their environment, that knowledge is organized in networks that are increasingly more complex, that constructed knowledge is under a continual state of reorganization and restructuring, and that the construction of knowledge is partly a reflective activity (Noddings, 1990).

The widespread interest among mathematics education theorists, researchers, and practitioners has led to many different meanings for “constructivism.” Researchers are engaged in debates (Steffe & Gale, 1995) about whether knowledge development is fundamentally a social process or a cognitive process. Terms such as “radical constructivism” (von Glasersfeld, 1991) and “social constructivism” (Cobb, Wood and Yackel, 1990) are used in the literature to distinguish the various approaches taken by researchers.

A theory developed by Piaget and Inhelder (1967) claims that children
construct representations of space through the progressive organization of their
motor and internalized actions. The representation of space is constructed from
prior active manipulation of that environment. Their theory emphasizes the
personal role of the student in constructing their own knowledge. Some
constructivist theorists give priority to individual student’s sensory-motor and
conceptual activity and trace their work to Piaget (1970, 1980) or symbolic
interactionism (Blumer, 1969). The epistemological basis of the psychological
variant incorporates both the Piagetian notions of assimilation and
accommodation.

Von Glasersfeld (1992) uses the term knowledge in “Piaget’s adaptational
sense to refer to those sensory-motor and conceptual operations that have
proved viable in the knower’s experience” (p. 380). He develops his view of
learning as self-organization by clarifying the distinction that Piaget made
between two types of cognitive reorganization: empirical abstraction and
reflexive abstraction. He believes that empirical abstraction results in the
construction of a property of a physical object and reflexive abstraction is the
process of constructing mathematical and scientific concepts. These definitions
assume that the student is participating in cultural practices in which learning is
cognitive reorganization (construction).

Confrey (1990, 1991) sees constructivism as a theory that considers
knowledge and understanding to be the products of human experiences.
“Decentering, the ability to see a situation as perceived by another human being,
is attempted with the assumption that the construction of others ... have
integrity and sensibility within another’s framework” (1990, p. 108). This implies
that, from a constructivist perspective, it is legitimate to seek to understand the
symbols through which another individual constructs knowledge, as well as the
other’s method of signaling knowledge.
Edwards (1995), in outlining his view of constructivism, makes two assumptions. The first is that learners bring a series of beliefs and understanding to the learning process and then build upon this knowledge. The second is that knowledge can best be constructed within meaningful problem-solving contexts. Edwards, in reference to Vergnaud (1982), states that "knowledge emerges from problems to be solved and situations to be mastered" (p. 79).

Cobb (1994) believes that mathematical learning should be viewed as both a process of active individual construction and a process of inculturation into the mathematical practices of the wider society. The most important issue for him, then, is not whether the students are constructing knowledge but the nature or quality of the socially- and culturally-situated constructions.

Extensive research has focused on the "cognitive apprenticeship" (Collins, Brown & Newman, 1989) model, whereby learners acquire expertise by taking part in a particular community of practice. The focus on learning is on cognitive and metacognitive processes rather than the acquisition of physical skill (Browne & Ritchie, 1991). Teachers, in their role as facilitators, endeavour to "turn the attention" of students to the important features of the situation and engage in collaborative, critical inquiry to solve problems (Young, 1993).

Another theoretical viewpoint that learners actively construct their own knowledge through interaction with their social/cultural environment is called social constructivism. This middle ground integrates important constructs such as reflective abstraction with the tools and symbols which so interested Vygotsky (1978). This suggests that there is a sociocultural, as well as a cognitive grounding for constructivist theory in mathematics education (Edwards, 1994b).

Social constructivists link activity to participation in culturally organized practices and tend to assume that cognitive processes are subsumed by social and cultural processes. They adhere to Vygotsky’s (1979) contention that “the social
dimension of consciousness is primary in fact and time. The individual
dimension of consciousness is derivative and secondary” (p. 30).

Bauersfeld’s (1992) interactionism version of mathematics complements
von Glasersfeld’s radical constructivist psychological focus in that both theorists
view communication as a process of mutual adaptation wherein individuals
negotiate meanings by continually modifying their own interpretation.
Bauersfeld emphasizes that “learning is characterized by the subjective
reconstruction of societal means and models through negotiation of meaning in
social intervention” (1992, p. 39). This interventionist metaphor characterizes
negotiation as a process of mutual adaptation where teachers and students
establish expectations for each other’s activity and obligations for their own
activity (Cobb and Bauersfeld, 1995). Bauersfeld takes the classroom microculture
rather than the mathematical practices as his primary point of reference when he
speaks of negotiation.

Individual student’s mathematical activity is not only dependent upon
their own active construction of knowledge. Jaworski (1994) believes that, as
learners negotiate in this social environment, their individual perceptions come
closer to each other and the resulting interaction (intersubjectivity) is as if there
is common knowledge. Voigt (1992) and Cobb (1989), in support of Bauersfeld
(1992), propose that individual student’s mathematical activity and classroom
interaction are closely related.

Limited research has been undertaken in mathematics teacher education
that has focused on the development of theoretical frameworks for mathematics
pedagogy consistent with constructivism (Simon, 1995). Traditional views of
mathematics have normally been the main focus of research energy, effectively
diverting researchers from studying teachers who “had well-developed
constructivist perspectives and who understood and were implementing current
reform ideas” (Simon, 1995, p. 118).

Summary

Constructivist theory is prevalent in the NCTM vision statements. Although constructivism makes specific claims about the way in which students learn mathematics, it provides little guidance on how mathematics should be taught. The formulation of a mathematics pedagogy based on a constructivist view of learning is a considerable challenge and one that is only beginning to gain the attention of researchers.

The NCTM (1989, 1991) has developed mathematical standards thought to be consistent with a constructivist framework. The challenge facing mathematics educators today is how to introduce and implement this vision of mathematics into their own classroom activities. The new pedagogy suggested by these reforms implies a classroom environment different from the prevalent lecture-dominated classrooms of today (Goldsmith & Schifter, 1993). Therefore, based on the vision expressed in the NCTM documents, new teaching techniques and strategies are being developed to assist teachers in their delivery of meaningful mathematics.

2.4 The Learning and Teaching of Geometry

The pupil should use a hard sharp pencil. All drawings should be neat and accurate. An accuracy within one or two per cent can be obtained with ordinary instruments.

McDougall, 1920

Freudenthal (1973) described geometry as “the experience with and interpretation of, the space in which the child lives, breathes and moves.” It is
within this framework that this study explored the position of geometry within the mathematics curriculum and how teachers can provide an exploratory environment for their students. The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) stressed that students should be allowed to "discover relationships and develop spatial sense by constructing, drawing, visualizing, comparing, transforming, and classifying geometric figures" (p. 112). Geometry, according to the NCTM documents, is an integral component in mathematics study.

Why should we teach geometry? Usiskin (1995), in an article discussing geometry curricula, suggests three reasons for teaching geometry: it uniquely connects mathematics with the real physical world, it uniquely enables ideas from other areas of mathematics to be pictured and it nonuniquely provides an example of a mathematical system. From these relationships and connections with the physical world, we can make deductions (p. 160). As such, he concludes that the single most important reason for teaching geometry is that geometry provides an example of a mathematical system.

Mason (1991) posits that we learn geometry "to strengthen and help organize a sense of space; to educate awareness that there are certain geometrical facts [and] to gain direct control with the world through the mind" (p. 79). He suggests that being aware that there are facts in geometry that need to be learned and memorized is important and that these facts can be worked with and applied to various situations. Teachers and students need to explore these geometric facts so that the connections between facts and geometric relationships are exposed.

Geometric constructions can be used to provide experiences that deepen students' understanding of shapes and their properties (NCTM, 1989). However, studies in the United States have shown that teachers are not spending very
much class time working with geometric constructions. Schiddell and Ethington (1994) used the results of Second International Mathematics Study (SIMS) to describe the geometry curriculum of typical and enriched classrooms in the United States. They found that very little geometry was being taught at the Grade 8 level. Their investigation also showed that geometric constructions units received less than two class periods per year. This lack of classroom time to work on geometric constructions contributes to difficulties for students in later grades.

Geometric constructions have been part of the Canadian elementary school curricula for many years. A review of the first two geometry textbooks in Ontario, Canada (McDougall, 1920; Ontario Ministry of Education, 1939) identified geometric constructions in the Grade 8 curriculum. The Junior Mathematics-Grade Eight (Ontario Ministry of Education, 1939) textbook located geometric constructions within the Community Planning section. For each geometric construction, examples are given and students are asked to repeat the process for eight to ten similar exercises. The teachers in that era were expected to teach constructions to all students. This focus on geometric constructions in Ontario has been very consistent over the last 75 years. A preview of seven Grade 8 textbooks written for the Ontario and Canadian schools from 1972 to 1995 showed that there was little or no change in the constructions taught or the method espoused by teachers to teach these constructions.

Geometric constructions in secondary school textbooks have historically been treated the same way as the Grade 8 textbooks. That is, constructions are printed in the textbooks and the students are expected to reproduce the exact construction. From 1920 to 1992, textbooks have been designed to encourage teachers to teach and students to learn geometric constructions using a compass and straightedge, frequently without regard to meaningful investigations.
However, recent textbooks for Canadian schools have included explorations using paper folding and geometric software in geometric construction units.

The use of compass and straightedge tools for geometric constructions can entice teachers into believing that students are learning about geometric relationships. Schoenfeld (1985) studied a classroom in which students were working on a Grade 10 geometry constructions unit. The teacher encouraged students to memorize the constructions and to practice them. Schoenfeld found that students were able to memorize the constructions procedurally but they were unable to understand conceptually the relationships inherent in the constructions. The students felt that they had to construct the diagrams exactly as shown to them by the teacher. This emphasis on precision gave students the impression that accuracy was the primary criterion of judging a construction. When asked how students determined what relationships were inherent in the constructions, the teacher believed that students found the correct relationship by sheer luck.

How can students be better prepared to explore geometric constructions without relying on luck? Schwartz (1994) believes that students need to explore mathematics and to become critical, in a constructive way. Teachers, therefore, need to assist students to explore geometry and to communicate their findings.

Teachers and curriculum reformers should look at models of geometric structure to better understand student geometric learning. Van Hiele's model (1986) describes and defines, in terms of structure, the developmental stages through which students progress as they learn about geometry. According to this model, students progress through an invariant sequence of geometric understanding consisting of five levels: Visualization, Analysis, Informal Deduction, Formal Deduction, and Rigor. Progression from one level to the next depends on appropriate geometric experiences.
Not all mathematicians agree with the distinctiveness of the van Hiele model. Freudenthal suggests that there are many minute levels and, by implication, many more and complex levels than five distinct stages (cited in Gravemeijer, 1994). Freudenthal said that the principle of reflection, as described by Piaget (1980), brings out minute levels of geometric development that enables students to reinvent mathematics. Implications of both interpretations of how students learn geometry suggest that students must be given opportunities to investigate and make conjectures about the properties of, and relationships among, geometric figures (Talsma & Hersberger, 1990).

Parsons (1994), in his study of van Hiele's model, suggested that teachers' beliefs about geometry influence the crafting of lessons. He wrote that the beliefs of teachers were perhaps the most important influence on the learning of geometry by students.

An incongruency exists between the exploration of geometric relationships proposed by the NCTM Standards (1989) and actual practice in classrooms. Different teaching methods are needed to give students opportunities to explore geometric relationships.

2.4.1 Computers and Geometry

The use of computers in a geometric environment has been investigated by a number of researchers (e.g. Kilpatrick & Davis, 1993; Laborde, 1993; Noss, Hoyles, Healy & Hoelzl, 1994; Schwartz, 1994; Kajander, 1990, 1989). Computers, especially with their graphic capabilities, may facilitate the construction of geometric concepts (Clements & Battista, 1992, 1994). Since computers have been introduced into the teaching and learning of mathematics, several software packages have been developed aimed at improving learning of mathematics in
general and geometry in particular (Laborde, 1993).

One of the first software programs developed to investigate geometric relationships was LOGO, beginning with Papert in 1980. Many researchers have used LOGO with varying degrees of success in mathematics (Johnson-Gentile, Clements and Battista, 1994; Clements and Battista, 1994; Cohen, 1987; Hoyles, 1987; Hillel, 1986; Goldstein, 1985). Recently, Johnson-Gentile et al. (1994) used LOGO motion with two groups: one group used paper and pencil for the transformations while the other group used LOGO on a computer. They observed that the computer group developed better geometric thinking skills. Based on these and other results, it is important that we investigate the use of other software programs in the learning of geometric relationships.

A growing number of teachers have used the dynamic geometric software programs as the basis for geometric construction in place of a compass and straightedge. Dynamic geometric software allows the user to explore geometric properties and relationships and to manipulate images on the screen to investigate relationships and patterns in geometric constructions. This makes it possible for the user to make and explore conjectures about the generality of their observations (Chazan and Houde, 1989). Because students will construct adequate drawings with the computer, they can concentrate on the exploration and investigations of geometric relationships. This new focus on exploring conjectures can have the effect of bringing curiosity, inquiry, and research into the mathematics classroom (Schwartz, 1994).

The use of dynamic geometric software began in 1985 when Judah Schwartz and Michal Yerushalmy developed the Geometric Supposers (Schwartz and Yerushalmy, 1988). In their previous research on geometric constructions and proof, Yerushalmy et al. (1987) found that students had difficulty with problem posing and with inductive thinking. They felt that students would be
better inductive thinkers if they could manipulate the geometric figures on a computer screen. The Geometric Supposers software program was therefore developed to give students the opportunity to test conjectures quickly without the difficulties that arise when one uses a compass and straightedge. The Geometric Supposers program is a member of the first generation of geometric dynamic software.

Since 1987, two other programs, Cabri-geometrie and Geometer's Sketchpad, have been developed in addition to the Geometric superSupposer program. Laborde et al. (1988) developed Cabri-geometrie as a second generation program (presented at the ICME IV meeting in Budapest). The Geometer's Sketchpad program was developed by Nicholas Jackiw, in early 1990, also as a second generation of educational geometry software. These dynamic geometric software programs enable the student to perform constructions and to observe the changes while they manipulate geometric shapes on the computer screen. The transformation and scripting ability of these programs has broadened the scope of what can be done with geometric software.

There is a considerable amount of curriculum materials available to the dynamic geometric software user. A high school geometry textbook by Serra (1989) encourages the students to create their own geometric constructions and formulate the mathematics for the relationships that they discover. Students can work individually or in groups to do investigations and discover geometric properties. Students are encouraged to look for patterns and use inductive reasoning to make conjectures (Jackiw, 1992). Other manuals and textbooks (Serra, 1989; Bennett, 1996) have additional activities for students and teachers.

Researchers are asking important questions about what teachers can do to better integrate geometry into mathematics classrooms. In Linn and Pea (1994), Linn asked a key question about teachers in a geometric computer environment:
"What kind of support is really needed to create the kind of experimenting society where teachers really think that they can try out a curriculum, listen to what students have to say, make some adjustments and try it again?" (p. 12).

The call to investigate geometric computer tools has come from many researchers and reformers (Clements & Battista, 1994; Linn & Pea, 1994; ICMI Discussion Document, 1994; Davis, 1992; Chazan & Houde, 1989). The computer can provide an exploratory environment in which teachers and students can explore mathematics. It can also be used to investigate what needs teachers have to enable them to change their mode of delivery.

2.4.2 Implications for Teacher Change

Computers can be used to provide teachers with an environment where students can explore mathematics. Sheingold and Hadley (1990) reported that teachers using computers had radically changed their conceptions and methods in ways that could be interpreted as 'constructivist' - becoming more student-centred in terms of expectation and direction of learning. Prerequisites to reaching such a position were identified as resources, time and perseverance (p. 39).

However, these changes were not noticeable in all teachers in the study. Sheingold and Hadley also noted that teachers who were found to be especially adept at integrating the computer into their classroom practice had been teaching over 13 years, worked in schools with sufficient hardware and software, and had easy access to technical and educational assistance (p. 39). These elements are supported by Fullan's (1982, 1991) work on teacher change.

Clements and Battista (1994) summarized a number of studies that suggest that geometric computer environments can help develop students' thinking in geometry. According to these studies, students can make conjectures, evaluate
visual manifestations of those conjectures, and reformulate their thought (p. 188). Kilpatrick and Davis (1993), in their review of computers in mathematics education, found that if students use computers to test conjectures, then the demands on the teacher are increased and more effort is necessary by the teachers and students. Schoenfeld suggested that "we need to change the atmosphere in the classroom, to establish a different kind of classroom dynamic that would ultimately affect the student's habits of mind" (cited in Linn & Pea, 1994, p. 11).

Teachers play an important role in mathematics education. Their enthusiasm and interest can influence student interest and excitement for geometry (Mason, 1991). If teachers become uninterested or unimpressed with geometric relationships and facts, they tend to maintain a teacher-directed pedagogy. That is, they determine what questions are important to ask and what geometric facts are important to 'discover'. Use of dynamic geometric software programs can help teachers to develop or redevelop an enthusiasm for investigating geometric relationships.

Researchers have suggested that computers will change the way teachers teach. Pea (1987) stated that it is difficult to predict how the role of the teacher may change due to the increased use of technology. Yerushalmy (1987) called for a new type of teacher and a new type of teaching. She states that this new type of teaching must support and integrate exploration, inquiry, and ideas into the mathematics classroom. Furthermore, Schoenfeld, in questioning what researchers can do to investigate teachers in exploratory classrooms, asks "What makes the magic happen when it happens?" (cited in Linn & Pea, 1994, p. 11). Therefore, the challenge is to assist teachers in using the computer so that they can allow students to work in an open way and still be comfortable in managing the students, their classroom activities, and their time.

The emergent issue then becomes how can teachers change their mode of
teaching to help students gain control of their own mathematical learning and thinking with the help of technology? How can educators be encouraged to become experimenters in their own classrooms?

2.5 Teacher Change in Mathematics Education

Many researchers believe that educational reforms demand an understanding of the processes by which teachers change (Richardson, 1990; Fullan, 1991; Shaw & Jakubowski, 1991; Wood, Cobb & Yackel, 1991; Etchberger & Shaw, 1992; Edwards, 1994a). This process takes time (Fullan, 1982, 1991) and it is not easy for teachers to make changes in their practice.

Hart (1993) believes that mathematics teachers face a "difficult and complicated struggle" as they try to make changes in their practice (p. 189). She postulates that learning is a process that requires structures and organization in order to facilitate understanding. Can teachers use constructivist principles to change their teaching practices?

It is generally accepted that mathematics teaching practices have been narrow in focus and have been based on a "teacher-directed, transference of knowledge approach" (Rice, 1992). If it is advised that constructivist principles be applied to the learning environment of children, then it seems appropriate that the same principles may be adopted for teacher professional development and training.

There may be a connection between how students learn and how teachers learn (Edwards, 1994b). Underhill (1991) has described a model that may be used to determine how students learn. His model makes a number of assumptions about constructivist learning:
1. Cognitive conflict and curiosity are the two major mechanisms which motivate learners to learn.
2. Peer interaction is a major factor in producing cognitive conflict.
4. Reflection is the main factor which stimulates cognitive restructuring;
5. Items 1, 2, 3 and 4 are cyclical;
6. The cycle always occurs within and is informed by the learner's experience; and
7. This cycle empowers learners, i.e., puts them in control of their own learning (p. 230).

Underhill uses cycles in the process of constructive learning. Cognitive conflict and curiosity are primary motivating factors in the learning process. Since the process must occur within the individual's experiential field, any cognitive restructuring must eventually be followed by peer interaction, starting a new cycle. Perhaps the most powerful result of Underhill's cyclic model is that it yields a form of learner empowerment.

Edwards (1994a) has modified Underhill's model of the construction of knowledge in his model for teacher change. His model has four major elements: perturbation, change, interaction, and beliefs. Each element is connected to every other element through a process called reflection. Central to the reflection component, according to Edwards, is that one needs to develop metacognitive reflection on reflection. Skemp (1987) used the term reflective intelligence to describe this "reflection on reflection." Edward's model suggests that it is not possible for teachers to make a change in their practice through interaction with their peers without it first being funnelled through a perturbation or a change in their own beliefs.

Shaw and Jakubowski (1991) note the importance of perturbation, peer support and collaboration in teacher change. They believe that, in order for
substantive changes to occur, teachers may need to make a commitment to change. However, perturbations and commitment may not be sufficient to elicit change. Shaw and Jakubowski (1991) have further suggested that the teacher will need a vision of the change and that, without this vision, the teacher may not change. They also suggest that peer collaboration and support often aid in the construction of such visions. However, what are the supports that are needed to aid the teacher in the construction of that vision in a mathematics classroom?

Shaw, Davis, Sidani-Tabbaa and McCarty (1990) have suggested that, in addition to perturbation, there are five interrelated requisites necessary for change to occur: awareness of a need to change, commitment to change, vision, projection into that vision, and reflection. Using geometric constructions as an example, these six interrelated requisites are described with the following example. Suppose a mathematics teacher was teaching a curriculum unit on geometric constructions. The teacher may be dissatisfied (perturbation) with the amount of knowledge that the students have about geometric facts and relationships after completing the geometric constructions unit. The straightedge and compass constructions are inaccurate, and students feel that getting the "right" construction is based on luck and memorization of the method. The teacher is therefore aware of the need for change and commits to making necessary changes in his or her classroom.

By forming a commitment to change, the teacher may envision what this change will actually entail. The teacher may use cooperative learning techniques, or teach different constructions. Suppose, however, that the teacher decides to use computer technology as a tool to increase the student's knowledge of geometric facts and relationships. The teacher may visualize self and students in the computer lab investigating constructions in a new mode of teaching (projection into that vision).
The reflection stage is the crucial element for continued teacher change. Schon (1983) describes that when teachers become more reflective on their actions in the classroom, they begin to challenge the customs of the educational system and their own role as a teacher in the classroom. What actually happens when teachers attempt to make changes in their mode of teaching after they experience a perturbation? This question is best explored by investigating the various roles available to the mathematics teacher.

2.5.1 Changing Teacher Roles in Mathematics Education

Teachers play a central role in establishing the mathematical quality of the learning environment for students and in establishing norms for mathematical aspects of students' activity (Yackel and Cobb, 1996). This implies that the teacher does not take a passive role in the constructive perspective but plays a critical role as a representative of the mathematical community. Given this central role, what influences come to bear on the role of the teacher in mathematics education?

The role of the teacher in mathematics education is influenced by teachers' individual mathematical agenda. When teachers choose various situations for their classrooms, they make judgments about the relevance of the situation to their students, judge how well the task represents the concepts to be taught, and how likely the students are to "bump" into the appropriate mathematics in the course of investigating the problem (Lappan & Briars, 1995). Steffe (1990) writes about "provocations" caused by the teacher that induce students to experience perturbations. Therefore, teachers can provide exploratory activities in their classrooms that will challenge the current understandings of the students.

It is overly simplistic to suggest that we can leave students alone and they
will construct their own mathematical understandings. Much as it is useful to have students work problems and communicate their ideas to their classmates and their teacher, this does not seem sufficient in an unstructured classroom. Richards (1991) suggests that:

The teacher must design tasks and projects that stimulate students to ask questions, pose problems, and set goals. Students will not become active learners by accident, but by design, through the use of the plans that we structure to guide exploration and inquiry (p. 38).

The teacher's view of learning about mathematics and mathematics teaching clearly affects how teachers present the course material. The teacher needs to do more than just change the nature of the classroom task from teacher-directed to student-directed. Social constructivism implies that students need to communicate with each other. This communication could cause anxiety for teachers who feel that classrooms should be quiet, or that only one person should be talking at a time.

In the traditional classroom, the work of the teacher is to 'transmit' the knowledge, while the role of the student is to receive it (Romberg, 1992). The real role for the student is to meet teacher expectations as they regurgitate the content upon demand to pass through the system (Skemp, 1979). Students are expected to find the right answers and the teacher's role is to ensure that the student attains the right answer. However, Drucker (1957) suggests that the teacher should not worry about preparing questions that have a right answer, rather they should be concerned with "setting up the right problem" (p. 15).

The constructivist perspective challenges the traditional role of the teacher in which knowledge is transferred from the teacher to the student (Davis and Mason, 1989). The role of the teacher is to support, promote, encourage and in every way facilitate the creation of knowledge by the students (Romberg, 1992).
The *Professional Standards for Teaching Mathematics* (NCTM, 1991), envisions teachers as being proficient in:

- selecting mathematical tasks to engage student's interests and intellect;
- providing opportunities to deepen their understanding of the mathematics being studied and its applications;
- orchestrating classroom discourse in ways that promote the investigation and growth of mathematical ideas;
- using, and helping students use, technology and other tools to pursue mathematical investigations;
- seeking, and helping students seek, connections to previous and developing knowledge;
- guiding individual, small group, and whole-class work (p. 1).

Jaworski (1994) describes a 'teaching triad' as a means of characterizing the teaching process (Figure 1). The three domains of the triad are management of learning, sensitivity to students, and mathematical challenge. Management of learning contains the set of teaching strategies and beliefs about teaching which influences the classroom environment and the way lessons are taught. Sensitivity to students pertains to the "teacher-student relationship and the teacher's knowledge of individual students and influences the way in which the teacher interacts with, and challenges, students" (p. 108). Mathematical challenge arises from the way teachers offer mathematics to their students depending on individual needs and levels of powers" (p. 108). While these three domains are interrelated, Jaworski saw them as also being distinct.

![Figure 1. Jaworski's Teaching Triad](image-url)
This teaching triad focuses on three components that could be considered when studying teachers in the classroom. Superseding this triad is the teacher's perception of their role in the classroom. This role takes on different dimensions based on the subject, grade, beliefs, and practices of the teacher. The teacher has many needs that define their role as a teacher among which is maintaining a "psychologically safe" environment for their students (Cobb, Wood & Yackel, 1990). Being assured of this safe environment in a mathematics classroom, Cobb et al. (1990) argue, is one factor which encourages students to explain how they solve mathematical problems. It is within this safe environment that teachers and students thrive and learn.

But a safe environment is not the only characteristic of an effective classroom. The environment must be one in which students can explore mathematics. Burns (1992) suggested that students should spend time working with problems and searching for solutions and the teacher should facilitate this exploration. Further, Posamentier and Stepelman (1981) stated that teachers should provide students with the "opportunity to explore, analyze and demonstrate their skills" (p. 101).

From a constructivist viewpoint, we can never really know if the student has "learned" the material. Constructivists need to be aware that one student's interpretation may be very different from that of another student and that of the teacher. As mathematics education is about enculturation into mathematical practice, Cobb (1988) believes that the role of the teacher will change:

The teacher's role is not merely to convey to students information about mathematics. One of the teacher's primary responsibilities is to facilitate profound cognitive restructuring and conceptual reorganization (p. 89).
Therefore, the teacher can deliver curriculum content in a variety of roles. The role of the mediator acknowledges the teacher's role as apprenticing the learner into the community of learners (Brown, Collins & Duguid, 1989; Collins, Brown & Newman, 1989; Duffy & Jonassen, 1991; Choi & Hannafin, 1995). In this case, the community of learners are mathematicians and the teacher plays an active part in the student's learning as mediator or facilitator, as do other resources and classroom social relationships.

Another role for the teacher, as a mediator of student exploration, does not occur naturally. Cobb (1994) and Leont'ev (1981) both noted that mediating between student's personal meanings and culturally established mathematical meaning of wider society require the teacher to change their practice.

The teacher's understandings and beliefs about teaching and about the teacher's role are also challenged by the use of cooperative learning in the mathematics classroom. In many cases, teachers do not see the value of cooperative activities. Cooperative problem-solving activities create noise, which challenges the teacher's authority as classroom manager. Despite the evidence that cooperative learning activities lead to high-level creative thinking (Johnson, Johnson & Stanne, 1986), teachers rarely use these methods because they are considered disruptive to the basic order of the classroom.

Cooperative learning activities with the computer adds to the complexity of the role of the teacher. Johnson, Johnson, and Stanne's (1986) study of 74 Grade 8 students showed that "computer-assisted cooperative learning instruction promoted greater achievement, more successful problem solving and more task-related student-to-student interaction" (p. 39). This suggests that students should be placed in cooperative learning groups to maximize achievement when using computer-assisted learning. Teachers may want to have a computer-based exploratory classroom although they find that the
classroom management issue is not resolved.

How do teachers actually change their practice? Eisner (1992) found that teachers constantly adjust their practice to take into account the context of schools and classrooms in which learning takes place. This adjustment may not result in teacher change but rather in cosmetic changes that do not change the underlying belief structures.

To encourage teacher change, a better understanding of the reasons for change is required by the teachers and they must accept the changes as positive or necessary. If teachers make changes in their educational beliefs, goals, and behaviours, then these change will require considerable time, energy, thought, courage, and practice (Howson et al., 1981).

2.6 Teacher Learning Needs

During the past 20 years, there has been a significant increase in the research literature on in-service education and professional development. The work of Joyce and Showers (1980), Fullan (1982, 1991), Gursky (1986) and others have indicated that there are a number of factors influencing teacher change that have not been taken into consideration by those pressing for reform. Fullan (1982) has identified four factors that influence teacher change: 1) need, 2) clarity, 3) complexity and 4) quality and practicality of program (product quality).

Teachers frequently do not see a need for a change. Fullan suggests that implementation of change is more effective when relatively focused needs are identified. Clarity is an ongoing problem with the change process. Even when the participants agree that some kind of change is needed, the adopted change may not specify exactly what teachers should do differently. The complexity of the change refers to the difficulty and the extent of the change required of the
individual. The actual degree of change depends on the starting point of the individuals, thereby causing different changes for different people. The fourth factor is quality and practicality of the program. Programs with better materials and considered by teachers as being useful are adopted more readily. Change is a difficult, personal process that can be made easier when these four factors support the implementation of the change.

Both individual teacher characteristics and collegial factors play roles in determining the implementation of teacher change (Fullan, 1991). Some teachers, depending on their personalities, beliefs, previous experiences, and stage of career, are more likely to bring about successful implementation than others.

2.6.1 Teacher Beliefs: General Characteristics

Teachers enter the teaching profession with knowledge, skills, attitudes, values and beliefs based on previous experience. Teachers bring with them images and assumptions about teaching based on their own student experiences which affect how they teach (Ball, 1988).

In recent years, research on teacher beliefs has received increased attention (Hoyles, 1992). Teacher beliefs, both before and after teaching experiences, affect teaching styles and behaviour. Current teacher education programs have attempted to alter these beliefs by providing a combination of academic models of methodology courses and field experiences in classrooms. However, Lampert and Ball (1990) noted that neither of these models have altered the images, assumptions, and beliefs of prospective teachers.

Experienced teachers have beliefs that influence their teaching and those beliefs are consistent with their classroom practices (Olsen & Singer, 1994). These
beliefs can be justified by the degree to which they enable the teacher to deal with the realities of the classroom (Orton, 1996). Thompson (1984, 1992) investigated high school mathematics teachers' beliefs, and their classroom teaching and found that teacher beliefs, views, and preferences about their subject influence what they do in the classroom. However, in some cases, teacher behaviour is inconsistent with their professed beliefs, although the teachers were often unaware of the mismatch.

The teacher's predisposition to personal learning and their beliefs about how learners learn will influence their behaviours within the classroom and their personal view of their role within the classroom. If a teacher views the learner as a passive recipient of knowledge, they may be reluctant to adopt an open-ended, probing, or problem-solving approach in their mathematics classroom. This could lead to a reluctance to change the teaching methodologies that teachers use in their classrooms.

2.6.2 Teacher Beliefs: Mathematics Teaching and Learning

As the learning of mathematics requires the weaving together of different goals and ideas, and teachers bring with them "baggage" from their experiences as students, what are some of the characteristics of mathematics teachers that influence what they do? In many elementary schools, teachers are required to teach numerous subjects. In a typical Grade 7 or 8 classroom, the teacher may teach any combination of English, Science, History, Geography or Guidance. Although they may not enjoy teaching Mathematics, they agree to teach it. Many of these teachers may have themselves completed their last mathematics course in secondary school.

Berebitsky (1985) found that teachers of elementary school mathematics
have a low level of mathematical background. A number of potential pedagogical problems can exist in this situation. First, many teachers are not confident in their own mathematical ability. They have to rely on textbooks as an authority for mathematics. The mathematical concepts and the mode of presentation in textbooks are seldom questioned (Steffe, 1990).

Since some teachers may not have sufficient interest in mathematics, they will seldom explore mathematics on their own. Steffe (1990) found that teachers who do not explore mathematics are the ones who hold to the belief that mathematics is static, formal, and symbolic. These teachers believe that mathematics teaching consists of routine problems. To them, it is procedural knowledge that is important and their teaching focuses on the procedures rather than on insight and exploration.

The teacher's conception of knowledge acquisition is a major factor in forming beliefs about mathematics teaching and learning. These conceptions can be viewed as a continuum ranging from logical positivism (objectivism) to constructivism (Hannafin & Freeman, 1995). Some teachers tend to view knowledge acquisition from an objectivist perspective. For them, learning can be delivered to and received by the students. The role of the teacher and, hence, the educational system, is to transfer knowledge from outside the learner to within the learner (Driscoll, 1994). The teacher whose beliefs match the objectivist's view of knowledge acquisition would find lecturing a preferred method of teaching. However, some teachers view knowledge acquisition from a constructivist perspective. Knowledge, from this perspective, is constructed by the learner and therefore exists in each learner's mind and is uniquely shaped by the individual's experiences (Cobb, 1994; von Glasersfeld, 1989).

The traditional role of the teacher should now shift to one of "listening and questioning" and "probing for understanding" (Maher & Alston, 1990). The
role of the teacher needs to change. However, teachers tend not to respond favourably to suggestions to teach in an exploratory mode because these classrooms are perceived as difficult to control. Furthermore, it requires the teacher to tolerate uncertainty about what the students are learning (Cohen, 1989; Schoefeld & Verban, 1988).

The use of the word 'control' conjures up different images for each of us. It is an emotional word that can be used negatively to suggest that the teacher is not giving students any freedom to develop their own thoughts (Jaworski, 1994). It can also mean that the students take responsibility for their own learning. Classroom control is important for teachers and is used to influence the way students think and behave within the classroom. Teachers' use of their inherent control within the classroom will influence the type and form of activities that take place within the classroom. This control can be used to limit interaction between students by reducing the noise level to a minimum or nil and by insisting on individual work. However, as Jaworski found in one of her case studies, control can also create an environment in which mathematics thinking is fostered.

2.6.3 Teacher Views of Computer Technology

Although some teachers know that computers can improve some aspects of students learning, many of them are still apprehensive about using them. The major reasons for this apprehension are the teacher's view of knowledge acquisition, computer knowledge of the teacher, anxiety over computer use, and curriculum expectations.

Hannafin and Freeman (1995) examined the relationship between the teacher's view of knowledge acquisition and the amount of time spent on
computer-based instructional activities. They found that the teacher's view of knowledge acquisition was not associated with either (a) the likelihood they will use computers, or (b) the ways they use computers in instruction.

There is emerging evidence, however, that if sufficient hardware is available and if teachers were more knowledgeable about computers, they would use them (Becker, 1990). This knowledge should be distributed between technical knowledge about computers and educational knowledge about how and when to use computers and recognizing the difference between these two. Additionally, Romberg (1992) found that teachers need more traditional materials when using the computer.

Rosen and Weil (1995) reviewed nine studies of American elementary and secondary science and humanities teachers and found that computers are available in all schools but were not being used by many teachers. They suggested that many teachers are technophobic and that teachers are worried about dealing with actual computer hardware in the classroom, computer errors, and about learning to use computers. These concerns of teachers may be greater than simply worrying about the operations of computers within the classroom. A major obstacle to effective computer use in the classroom stems from the anxiety teachers have over using computers in education.

Teachers view the computer as more dehumanizing than does the general population (Rosen & Weil, 1995). Rosen and Weil (1995) outlined five variables to predict teacher computer anxiety: 1) level of computer skills, 2) principal’s support of computer use, 3) computer availability, 4) perceived mathematical ability and 5) formal computer training. They concluded that lack of experience with computers implies technophobia and that it was not the case that experience will eliminate the problem. They suggest that, for some teachers, psychological intervention may be necessary.
The idea of expanding exploration activities with students is not very popular amongst those teachers who believe that they need to get through the content by the end of the year. These teachers tend not to respond favourably to suggestions that they open up class time for exploration. They also find that classrooms using exploratory activities are difficult to manage and require them to tolerate uncertainty about what pupils are learning (Schoefeld & Verban, 1988; Cohen, 1989)

Kilpatrick and Davis (1993) advocate the use of computers because it allows students to use tools outside the classroom and it enables students to become actively involved in exploring mathematical ideas. As Gursky (1986) stated, significant changes in teacher beliefs and attitudes will only take place after changes in learning outcomes are evident. Increased student achievement may be enough to get teachers to explore mathematics using a computer environment.

Ponte (1990) suggests that the assumption that thinking about using computers within the mathematics classroom is "a good starting point for teachers to reflect on their own teaching practice" (p. 184). He realizes that:

although the computer may be introduced with little or no change in teacher's conceptions and teaching methods, their interest in making sensible use of this instrument and their disposition to learn new things, assume new classroom roles, and establish new teacher/student relationships creates a stimulating environment for general educational reflection (p. 184).

This reflection assists the teacher in making changes to their teaching practice (Schon, 1983).

It is difficult for some teachers to use computers so that students can explore mathematical relationships. Many teachers do use the computer in the
traditional teacher-directed model. That is, they prepare activities that have a single right answer and create an environment where the goal is to get the right answer. These environments may eliminate the possibility that students would do any exploration at all. A teacher who believes that students acquire knowledge only from outside the learner, regard learners as passive recipients of knowledge. These teachers are less willing to use computer software programs to facilitate these open-ended approaches to instruction. In fact, these teachers may be more successful with programs that develop low-order skills such as simple recall and basic comprehension than constructivist-oriented teachers (Hannafin and Freeman, 1995).

2.7 The Effects of Teacher Beliefs on Teacher Change

In the previous discussion, the role of the teacher was explored and the challenges that teacher anxiety poses led researchers to study the relationship between learning outcomes and teacher change. Gursky (1986) predicted that "significant changes in teacher beliefs and attitudes are likely to take place only after changes in learning outcomes are evidenced" (p. 7). He further states that change is a learning process for teachers that is developmental and primarily experimentally based. Therefore, teachers need to explore alternate teaching techniques to see that students can be more successful in a different teaching environment. Clark (1993) studied the roles and patterns of communication of teachers and students in Grade 7 and 8 classrooms. He was observing the teachers' transition from a transmission to an interactionist approach to teaching. He found that teachers did make changes in their beliefs and attitudes after observing an increase in student interaction.

Teacher change will not occur automatically. Steffe (1990) stated that the
two major obstacles to the improvement of mathematics teaching at the elementary level are the "authority" invested into the textbook and that teachers find it difficult to challenge existing teaching methods and strategies. He also found that teachers tended to see problems in mathematics textbooks as routine. But when teachers were themselves involved in exploratory activities outside the classroom, they tended to get their own students involved in exploratory activities. How do we get teachers to become more accepting of new teaching activities and to become explorers of mathematics?

Changing the modes of teaching to ones that include exploratory classrooms will not necessarily be sufficient to get students to become "explorers." Students are taught very early in their mathematical careers to search for a single solution and they know that they will receive grades based on this achievement. Both students, and parents, are led to believe that being able to successfully find one solution only to each question posed by the teacher will make students successful in their academic and professional careers. Somehow, the teacher has to convince the student that multiple solutions may exist in some cases and that exploration might lead to other solutions. The teacher must also convince the student of the value of this work for the future. However, if teachers are not confident in their own understanding of mathematics and the purposes of exploration, students may not accept the exploration for its 'own sake' argument.

Kilpatrick and Davis (1993) suggested that if we want to change educational goals, beliefs, and behaviours, then making the change requires considerable thought, time, courage, and practice. We need to combine assistance from experienced advisors with opportunities for collegial exchange among fellow innovators.

How can change be introduced to teachers so that they might be better
prepared for the change? Schwartz (1994), in his work on Geometric Supposers materials, suggests that it is important to disguise computer innovation in such a way that it does not appear to threaten the teacher. It is also important, Schwartz states, that the innovation does not appear to criticize the teacher’s previous methods or techniques. In fact, the innovation should be seen to augment the present activities rather than replace them. One way to ensure that this method is successful is to use materials that have relevance to the curriculum. The materials should include sections that allow the teacher to incorporate their previous materials easily.

Teachers need to be actively involved in the adoption of technologies and need to be supported by resources and advisory help (Aston, 1988). The importance of teachers being able to evaluate and reflect on their own experience critically in a meaningful and systematic way is reiterated by Persky (1990):

> When teachers engage with each other in ongoing reflection about their instructional use of technology, they are more likely to critically evaluate their practice and redesign instruction to better meet student needs and curriculum goals ... In order to support teacher development, administrators must put structures in place so teachers can communicate and collaborate on a regular basis (p. 37).

The challenge is to create an environment for the teacher so that the materials are familiar, the students are psychologically safe, the teacher is not overwhelmed by the computer environment, and the students have an opportunity to explore mathematics. Based on the characteristics stated earlier on teacher change, Linn and Pea (1994) asked how can we help teachers become experimenters in their own classrooms? More importantly, Pea asks an essential question in teacher change: "What does the teacher need to know to be able to pull off activities like this?" (cited in Linn & Pea, 1994, p. 10). In his partial response to his own question, Pea (noticing that Papert (1980) had some excellent
ideas about LOGO but different things happened in the classroom) suggests that the research questions we should ask with regard to dynamic geometric software should change after we see what goes on in classrooms (Linn & Pea, 1994).

2.8 Summary

Mathematics teaching can occur in many environments. The traditional classroom has teachers directing students to follow a series of activities, either teacher-directed or managed, with the students all working at the same pace. The teachers are expected to teach 30 students, maintain control and inspire the class to learn (Cuban, 1986). Based on the NCTM documents, teachers are encouraged to teach in a mode that includes exploration, inquiry, discovery, reflections and construction (NCTM, 1989). This mode has relevancy when using computers (Hoyles and Noss, 1992).

The teaching and learning of geometry allows students to explore their environment and to make and test conjectures. The use of dynamic geometric computer software allows the user to construct images to help the student think about spatial relationships. However, teacher beliefs about geometry, mathematics, and computers can stand in the way of teachers changing their mode of teaching.

There is a need to develop a careful understanding of the settings that encourage teachers to learn to use these new teaching environments and materials. There is a need to determine the real costs of teachers learning to teach geometry. There is also a need to empower teachers to create an experimenting environment in their classrooms. Linn and Pea (1994) asked an important question about teacher change in dynamic geometric computer environments: "What kind of support is really needed to create the kind of
experimenting society where teachers really think that they can try out a curriculum, listen to what students have to say, make some adjustments and try it again?" (p. 12). Teachers need to be observed in computer exploratory environments so that we can determine their learning needs so they can provide this educational experience to their students.
Chapter Three
Method

Several research questions were proposed for investigation in Chapter One. The primary questions are whether teachers change their mode of teaching at all and how they can be assisted in changing. This chapter elaborates the research approach and design of the study. The method and results of the pilot study, the procedures, data collection, and analysis for the main study are documented.

3.1 Research Approach and Design

The rationale for choosing one research design over another is directly related to the research questions. To actually "see" what is happening in a mathematics classroom, a researcher must become part of the classroom milieu. The arts and humanities have always found ways of describing, interpreting, and appraising the world (Eisner, 1991). It is the desire to describe and interpret our educational world that compels some researchers to use qualitative research methods.

Rist (1983) suggests that "qualitative research is appropriate for the articulation of the multiple ways in which people understand their world and react to it" (p. 16). In Rist's view, "qualitative research is well suited to the study of the implementation process" (1983, p. 23). He notes that unanticipated events occur in the study of social change and these are best investigated through qualitative research methods. Therefore, a qualitative research approach was used as the primary research tool in this study.
Bogdan and Biklen (1992) identify a number of common elements in qualitative research.

1. The researcher is the key instrument of the data collection, which takes place in a natural setting.
2. The data are descriptive and the focus is on the process rather than the product.
3. The data are analyzed to capture the richness of the environment and the meaning of the participant’s words from this analysis is of primary interest.

A case study approach was used. Qualitative researchers frequently attempt to strengthen the design of their studies through triangulation (Romberg, 1992). Romberg makes the point that failure to triangulate the data in order to strengthen the rigor of a case study is a common error. As such, the data were collected through three primary sources: semi-structured interviews, classroom observations, and journal entries by participants. More specifically, the data were gleaned from my field notes, transcripts of interviews with teachers, students and Head of School, a questionnaire, transcripts of classroom conversations between students and teachers, and participants’ journals.

A semi-structured interview technique was employed. That is, some questions were prepared before the interviews were posed to each participant. Additional questions were posed during the interview that were based on the answers given by the participant, the entries in their journals and my field notes. I wrote questions in the margins of my field notes that I felt would assist me in understanding the behaviour of the teacher. These additional questions guided the second, third, fourth and final teacher interviews. The strength of this method is that it forces the researcher to periodically review and interpret the data that have been previously collected.
I spent approximately three weeks with each teacher, observing their interactions with students and the computer software. Eisner (1991) suggests that what people do and say, and how they do and say it are very important in a qualitative research study. Therefore, while I was in each school, I participated in planning meetings, observing teachers and students in their classrooms, interviewing teachers and students, recording events as they happen using audio equipment, and writing down personal observations on paper.

To gain insight into how best to assist teachers change their mode of teaching and the supports they require to make these changes, I investigated the everyday experiences of the teachers and students as they worked with a geometric curriculum unit using a computer program. I observed the teachers and students in the computer lab and have attempted to provide a detailed description.

3.2 Pilot Study

A pilot project with one teacher was conducted during the month of September, 1995 to test the interview questions, the Geometer's Sketchpad software and to gather preliminary data for analysis before the major study. The setting, procedure, and results of the pilot study are outlined in this section.

3.2.1 Setting

A pilot study was conducted with a Grade 8 mathematics teacher at a boys-only independent school in a large metropolitan city in Ontario during the fall of 1995. The teacher was asked to be a participant in the pilot study after I gained permission from the Principal. The teacher, Guy, agreed to be part of the study.
Guy has been teaching for 32 years, the last eight in this school. He has taught Grade 8 for twenty years, Grade 7 for eleven years and Grade 6 for one year. He has taught in two other schools and spent one year on sabbatical in England. He has a B.A. degree, a M.Ed. degree and a Library Specialist certificate. He is qualified to teach in the primary, junior and intermediate levels of education.

During the 1995-96 school year, his teaching load consisted of two Grade 8 Mathematics classes and one Grade 8 English class. The teacher was the only Grade 8 mathematics teacher at the school. In addition to his regular teaching load, he performed duties as a Home Room teacher for Grade 8 students. There were six days in their school timetable cycle with 40 minute classes. The teacher had math with his two classes every day. He taught the English course two periods per day.

The computer room was located in the basement of a 120 year old building. The room had two small windows that could be opened to ventilate the room. There were twelve Power Macintosh computers, organized in a horseshoe shape, anchored by a laser printer at one end. Each computer had a folding chair in front of it and the room had twelve table and attached chair sets. A part-time computer teacher was responsible for maintaining the computer hardware, software and for teaching each class in the junior school for 30 minutes per week. The computer courses focused on word processing, data base management, and spreadsheets.

3.2.2 Procedure

The teacher was asked to teach his geometric construction unit, normally taught using compass and straight-edge tools, using the Geometer's Sketchpad.
computer program. The software was installed on the twelve computers and the teacher received a demonstration copy for his home Macintosh computer.

To collect data on the teacher's teaching practice and his work with the geometric construction software, I visited the school eighteen times during the months of September and October, 1995. Five of the visits were to interview the teacher and twelve of the visits were to observe the teacher and students in the computer lab. I also visited the Headmaster of the Junior School at the end of the in-class observations. All interviews and classroom visits were audiotaped and I made field notes of my observations. The teacher was asked to keep a daily journal to record his feelings, concerns, successes, failures and other teaching and learning experiences. I told him that I would read the journal at various points of the study and again at the end of the study.

A preliminary interview was held in September, 1995 to gather data on his teaching experience, his educational background, his typical day as a teacher in this school and his perceptions of his role as a teacher. Further questions focused on successful and non successful classroom activities, and the teachers who most influenced his teaching practice.

A second meeting was held the following week to review the curriculum unit and to determine which materials would be used with the students. The school's Grade 8 textbook, FM9, provided the basis upon which we designed the curriculum unit. Most of the computer activities were taken from the Exploring Geometry with The Geometer's Sketchpad (Bennett, 1996) workbook. Additional materials, designed by a math consultant with a local public school board, were used to complete the unit.

The third session was used to assist the teacher with the computer program. The teacher had a five-year old Macintosh computer at home that he used for word processing report card comments. He wanted to be taught how to
use the Geometer's Sketchpad software and how he might use it in his classroom. I taught him how to use the different software tools, the difference between "draw" and "construct" and the exploratory features of the program. Confidence in the ability to use the software is essential to using it to explore geometric relationships. On the fifth day of the project, the teacher wrote in his journal, "the work came a bit better, mainly because D.M. explained it, particularly the 'construction' choice on the menu bar" (teacher journal, September 25, 1995). Although the teacher told me that he understood the construction menu features, I observed that he was unable to use some of the tools until late in the study. I made a note to find out if teachers in the main study had similar difficulties using the software.

The fourth interview was held after the fourth day of classes. I looked at his journal for the first time. He had written about his frustrations with the software, his impressions of student learning, cooperative learning, and some of his perceived inadequacies with the software. But he did feel that his students were learning. When asked what he was learning with this software, the teacher replied:

I think that the biggest benefit to me is... the chance to get a computer and work on it... The boys seem to be learning what angles are, what construction means and what they are and I guess that's why they are here (interview, September 22, 1995).

The twelve classroom visits occurred over a period of three weeks. All mathematics classes were held in the computer lab, with the students working in pairs. The teacher assigned the pairs on the first day of the study. Although the teacher did not make reference to the pairing and I didn't ask questions about it, the teacher was disturbed by students working together. He thought that the students were noisy and not following instructions very well. However, he did
say that “although a type of ‘cooperative learning’ appears to be taking hold - for example, one student showed his partner, me and the two students next to him about the ‘shift’ key in measuring angles” (teacher’s journal, September 20, 1995).

After the tenth visit, I randomly chose two students from the class and asked them questions about their typical school day, their mathematics activities in class and in the computer room, on their impressions of the software and of mathematics in general. This audiotaped data provided me with another snapshot of the teacher’s practice and the prevalence of exploration in his classroom.

I videotaped the ninth and tenth class sessions. After review of the tapes, I realized that, other than observing the teacher moving from one student pair to another, little was gained from this mode of observation. Therefore, I decided not to continue this technique in the main study.

After the classroom visits, the Principal of the Junior School was interviewed and asked questions related to teacher development, support of the school for technological change and the philosophy of the school. The teacher was also interviewed at the end of the classroom visits to get his perceptions of the entire project.

The teacher assessed the students by asking the students, in pairs, to demonstrate one or more of the constructions on the eight and ninth day of the study. The teacher also administered a paper and pencil test on the day following the last computer lab session.

3.2.3 Pilot Study Results

The pilot study was designed to explore possible questions that could be asked to solicit teacher data, to practice interviewing techniques, to gain
experience taking field notes, and to work with the geometric software in a classroom setting. It was essential to determine if the data collection methods would reveal teacher's learning needs and the supports required to make changes in their mode of teaching.

As a journal can be a rich source of information, the teacher was asked to keep a journal and to write daily entries about one-half page in length. One weakness of the pilot study was my inability to convey to the teacher the importance of the journal to my analysis. In the main study, I reinforced the importance of regular reflection and entries in the journal.

Field notes were taken based on my observations of class sessions and my interaction with the teacher. Based on the results of the literature review and the pilot study, a number of factors and characteristics were identified as possibly contributing to how best to assist teachers to change their mode of teaching. The factors identified were:

- time - for teachers to reflect
  - amount of contact time with students
  - personal time for learning software and preparing lessons
- administrative support - time
  - financial support
  - “not to get in the way” approach
  - in-class assistance
- peer interaction and support
- confidence in mathematics, in teaching, using computers, in exploration
- anxiety - in mathematics, using computers
- knowledge about mathematics, geometry, computers, exploration
- management - classroom, control, cooperative learning
- motivation to change
- constraints to change
- teaching experience - years teaching, subjects taught, age of teacher
- competence - in teaching, mathematics, using computers, exploratory activities, asking questions
- tension between concepts (understanding) and procedures
- teacher style (teacher-directed, facilitator, mediator)
- reflective teacher
- expectations - teacher, principal, school, parental
- residual ideologies - tension between current ideologies and new teaching ideas
- experience in exploratory activities

Three major factors--teaching style, computer anxiety, and the role of the teacher--emerged from the pilot study. Thus, these factors are explored in this section.

Teaching Style

The teacher exhibited a structured approach to the learning environment. In his regular classroom, he began each lesson by taking up the homework, putting the new work on the board, and then assigning homework for that evening. It was evidenced that he needed to manage his class so that there was little noise in the computer room. He also expected his students to be very quiet. In his journal on the seventh day, he wrote:

I realized today that the noise level was higher than I am used to, probably because of the partnerships. I have to speak twice (at least) to get their attention - unusual for me (teacher's journal, September 27, 1995).

He stated at the beginning of the project, that he was concerned about the noise level and the lack of structure in the classroom, due to the nature of the experiment. As the project progressed, he appeared more comfortable with the noise in the last few classes. He was able to adapt to the new approach in this exploratory environment.

Computer Anxiety

The teacher has, for five years, owned a Macintosh computer that was
used for word processing. This year, his school had started to use computers to record comments on report cards. When asked how he felt about teaching mathematics with computers, he said:

Well, a little apprehensive. I cannot be a luddite. If you look at the Internet and modems and CD-ROM’s, things that I know little about, you realize that I have to get into this (initial teacher interview, September 11, 1995).

He was concerned about his lack of computer skills and knowledge. However, this was not the first time that he felt that he knew less than the students in an educational field. He believed that his previous experience teaching French assisted him in accepting that he did not need to know more about the computer than his students. Further evidence that he was able to reduce his anxiety over computers emerged from his journal:

At times it's fun to be floundering and one of the boys comes up and helps me; it's even more fun when I can help them—maybe I'm not a LUDDITE after all (teachers journal, October 5, 1995).

The change in his level of computer anxiety was significant to the study.

Role of the Teacher

This teacher uses “Socratic” methods as his mode of teaching. He noted on the second day of the study that he was “still in a 'Socratic' mode” (teacher’s journal, September 19, 1995). His questions to the class at the beginning of the study confirmed that his preferred style was to ask questions and then summarize the student’s answers. His view of the role of the teacher changed throughout the study. In response to my question, “what do you see as the role of the teacher?,” he said:
Well, I don’t see the role of the teacher as a facilitator, one who facilitates methods in which the children are natural learners . . . , I see the role of the teacher, particularly this year, to provide the skills necessary for my boys to take next year's courses (initial teacher interview, September 11, 1995).

However, through observation, I noted that he was beginning to take on the role of a facilitator in the classroom. He also noticed that he was taking on a different role and recorded it in his journal, "I have become simply, a facilitator - this bugs me as a professional, I must admit, but modern 'purists' would, I’m sure, applaud" (teacher journal, October 2, 1995). He seemed bothered by becoming a facilitator. He felt that his role as a teacher was to direct the lesson and to be the authority in the classroom. He did note, however, that his mentor at another school board acted as a facilitator and that perhaps it was an important role for the teacher in some cases.

I have selected these three factors to illustrate some of the changes that were taking place in the pilot study. The other factors listed above were also evident but did not contribute significantly to the purpose of the pilot study. For each of the factors, data were collected from at least three of the following sources: transcripts of interviews with teachers, students, and the Head of the school, field notes, audio-taped transcripts of the class interactions, and the teacher's journal.

Based on the results of the pilot study, I introduced a questionnaire to the main study to elicit more information about the teacher’s views about mathematics, learning mathematics, and teaching mathematics. These questions provided discussion themes for the second and subsequent interviews.
3.3 Main Study

3.3.1 Sample

The target population was Grade 8 teachers working at independent schools in Ontario, within a 150 km radius of the university. I selected four independent schools which had computer labs, one boys-only school, one girls-only school, and two co-ed schools.

The four teachers were a male teacher in a boys-only school, a female teacher in a girls-only school and two other teachers (one male and one female) from two co-ed schools. These four teachers volunteered to work on this project, with approval of the appropriate Head of School. To ensure confidentiality, the names of the teachers and the schools have been changed in this thesis.

3.3.2 Data Collection

I visited the teachers before the first in-class session, outlined the scope of the project and my expectations, set a timetable with the times and dates of all meetings. Data were collected between November 1995 and March 1996.

The collection of data was ongoing throughout the study. I observed teachers and students using a variety of data collection tools. I identified myself as a researcher and did not instruct the teachers on how to teach this unit but I told them that I was available for help should they need it. I did, however, assist them in selecting appropriate curriculum materials to match their current curriculum unit on geometric constructions by providing them with a teacher's manual of Geometer's Sketchpad worksheets (Bennett, 1996) and worksheets prepared by other teachers.
I used observation techniques such as audio-taping classroom discussions and recording observations in hand-written field notes. The teachers were interviewed at least four times during the study: twice before the first classroom session, at the midpoint of the in-class sessions, and at the conclusion of the classroom visits. A follow-up interview was held after they had a chance to read the transcripts.

A questionnaire (Edwards, 1994b) on teacher beliefs and attitudes was completed by each teacher before the class sessions to provide me with additional information. The teachers' answers to this questionnaire (Appendix A) were used to investigate similarities and differences among teacher journal entries, interview answers and teacher behaviour in the classroom. Additional questions were asked during interview sessions based on the responses to the questionnaire and on my field notes.

In addition, three students in each class were probed about their interest, attitudes and feelings in the areas of mathematics courses, content of this course, computers, teachers of mathematics, geometry, the software, their freedom to explore, and their anxiety about mathematics.

Each Head of School was interviewed. Questions focused on the types of supports teachers receive in the school in the areas of computer hardware for their classroom, computer training, teacher professional development opportunities, and teacher change.

3.3.3 Coding the Data

All participants and school names have been changed to ensure confidentiality. Full anonymity was assured on the consent forms (Appendix I). All interview recordings were transcribed and the transcripts were reviewed.
against the recordings to ensure accuracy of the data. Participants checked the transcripts and all four confirmed that the contents were accurate.

The data set consisted of my field notes of classroom activities and teacher behaviour; transcripts of audiotaped classroom discourse; transcripts of teacher, student, and Head of School interviews; and journal entries written by teacher participants.

I created a set of codes based on the findings of the pilot study and my initial reading of the transcripts. The data were decomposed into sentences, identifying specific incidents and coding each sentence into as many categories as possible. The data were initially sorted according to the following categories: experience, supports, interests, roles, and curriculum. Each of these categories were subsequently subdivided into more specific dimensions. The new dimensions were: experience (personal, professional, and environmental), supports (administrative, peer, personal, and environmental), interests, role (teacher, administrative, students, parents, and Head of Mathematics Department, Head of School), and curriculum (mathematics, geometry, planning).

In addition to coding data according to how they fit the meanings derived from the interviews and the field notes, the data were coded by participant name, date and document source such as interview, field notes, journal, and classroom audiotapes. The data were also coded according to base data of the participants such as gender, age, experience teaching, experience teaching mathematics, and computer experience both at home and at school.
3.3.4 Data Analysis

The contents of the transcripts were analyzed using a systematic approach (Glaser & Strass, 1967) to identify which factors contribute to the learning needs of teachers. The goal was to discover the relevant categories and the relationships among those categories (Strauss and Corbin, 1990).

A qualitative data analysis computer program, called NUD*IST (Non-numerical Unstructured Data Indexing, Searching and Theorizing), was used to assemble and manage documents and transcripts, create and manage codes for thinking about the data, index segments of the data, and to search for words and phrases in the text of the documents. The advantages of using computer software for managing large sets of data are numerous. Coding using manual methods can become mundane and one challenge is to make it rapid and automatic. The use of the computer facilitated this exploration, one that would be time-consuming and likely impossible to do using manual coding and analysis methods.

Using the software, the data were labelled and categorized using the initial codes described in the previous section. These codes were organized into a conceptual tree, where the leaves represent categories or nodes and the branches represent the relationships or connections between the nodes. These nodes could be merged, collapsed or reorganized through manipulation of the software.

Using frequency counts on words and phrases, the data was further coded into sub-categories. Each sub-category code was attached by a branch from the category node. As connections were made between categories and sub-categories, some of the codes were merged with other sub-categories. Since I could code every sentence as often as I wished, changing and adding codes was easily done on the computer. The software supported the frequent rearranging of text while
maintaining a history of the routes of exploration.

As common themes began to emerge from the participants' words and phrases, additional coding and reorganization occurred, thus generating theoretical constructions that were used to develop theory and to identifying the links between categories. This process continued until all levels of the nodes had been analyzed. These categories and their resultant patterns provided the theoretical connections between the data and the findings. As I reorganized the data, I maintained electronic memos of my findings. This helped me make additional linkages between data as the analysis progressed.

Moreover, when two or more case studies are conducted concurrently, these cases need to be analyzed separately. Once the interpretation for the cases had progressed to a point where each case appeared unique, a cross-case analysis took place. I found that, to capture the essence of the cross-case analysis, I needed to create a series of matrices as suggested by Miles and Huberman (1984). These matrices assisted me in the reconceptualization of the data into the organization displayed in this thesis.
Chapter 4
The Case Studies

4.1 Introduction

This chapter provides an analysis of four teachers using Geometer's Sketchpad, a dynamic geometric software program, to teach geometric constructions to their Grade Eight students. These case studies record the teachers' experiences as they change their form of delivery of a geometry construction unit.

Within each case, I will describe the school, provide some biographical information about the teacher, their philosophy of teaching mathematics through a vignette or two, and characteristics that I think portray the teacher in the computer classroom. A summary is provided at the end of each case, highlighting some of the most interesting characteristics of the teacher.

4.2 The Case of Cathy

The 1995-96 school year was Cathy's seventeenth year of teaching. She works in an all-girls independent school in a large metropolitan city in Canada. Cathy had experience teaching in a government school in the Bahamas and eleven years at another all-girls, independent school in this same city before beginning to teach at St. Francis School six years ago. The school is situated in an exclusive residential area, with architecture that blends into the surroundings. The building itself dates back to the turn of the century.

St. Francis School, is organized around two divisions. The junior division
contains Grades K to Grade Six while the senior division starts at Grade Seven. There are 255 students in the Junior School and 535 students in the Senior School. The school has a new Head of School, replacing the former one who retired in 1995 after 14 years of service.

Cathy attended St. Francis School as a student. She went to a large, prestigious university in the city and studied in a General Science program for two years and then went to teacher's college, specializing in Physical Education. She taught Physical Education and Geography, eventually becoming a Mathematics teacher:

For two years I taught in the Bahamas and I taught a Grade 3 class so I taught everything but Phys. Ed. and Music. I taught at another independent girls school in this city as a Phys. Ed. and Geography teacher. Then I became Phys. Ed. and Math [teacher] and then became mostly a Math teacher. I was head of Phys Ed. [at the other independent school] for several years . . . At this school, I teach Math [to] Grade Seven, Eight, Nine and Ten [students] (interview, Feb. 5, 1996).

Cathy worked in real estate in Canada for three years before arriving at St. Francis School. When I asked Cathy why she left teaching at an all-girls school to work at the real estate office, she said:

A lot of reasons. The politics of the school. Thirteen of us out of a staff of thirty left the same year. I had had it. I was bored. And I think I was bored in the gym which is why I was doing more and more Math but I was still frustrated. It was politics (interview, Feb. 5, 1996).

Her reasons for returning to the teaching profession were personal as well:

. . . I missed [teaching] and I really wanted to work with the principal who was here. I had for a long time (interview, Feb. 5, 1996).

Math is Cathy's favourite subject. She likes the directness of Mathematics "because I'm comfortable with it, because I was good at it [and] because it's so
specific . . . " (interview, Feb. 5, 1996). She doesn't have any formal training in Mathematics but she has a high interest in Mathematics. When asked if she investigates different patterns outside of the classroom setting, she replied in the affirmative. She involves herself in other professional development activities by reading books, Math magazines:

I do a lot of reading . . . in . . . Mathematical teachers' books and . . . [I try] to find ways of integrating computers into all of my courses. And just any book I can get my hand on that might have some things in it that are appropriate (interview, Feb. 5, 1996).

Cathy teaches geometric constructions to her Grade Eight students every year using the compass and straight-edge tools. She finds certain classroom materials and books written for teachers to be excellent resources for this topic. But she does not restrict herself to using the textbook as the only resource:

We have teachers' manuals which [we use]. The Grade Seven teachers' manual is excellent but the Grade Eight teachers' manual is lousy . . . sometimes I've even gone back to the Grade Seven teachers' manual and looked at it. So I get a lot of ideas for how to teach the lesson [in Grade Eight from the manuals]. As I said the Grade Seven one's excellent. And then as you do it with the Grade Seven [students] you think 'oh, that's neat.' You could do that, sort of a different variation of that with [the Grade Eight students]. And also reading these Mathematics Teacher magazines (interview, Feb. 23, 1996).

She strongly believes that geometry is very important. On February 23, 1996, the following exchange took place while discussing the role of geometry:

Q: Where do you see the role of geometry in Mathematics? Why do we teach geometry?
Cathy: Oh, especially for girls it's so important, just visual perceptions and problem solving as well, but just spatial concepts that girls have such difficulty with.
Q: Why do girls have these difficulties?
Cathy: Well, they say it's . . . that physiologically [girls are] different and when we go to school in the very beginning boys aren't ready to read, girls are. Boys are directed to use gross motor skills, to be in
the gym, to be on teams, to fight even and that spatial awareness is
developed in boys just from fighting.

I asked Cathy if we need to teach girls to fight. Cathy didn’t go that far but she
thinks that:

We need to teach the girls how to use their body and to use space
and that’s sort of the key to geometry. So we help the boys along by
helping them learn how to read. We never give the girls a chance
to explore space (interview, Feb. 23, 1996).

Cathy believes that geometry plays a big role in life:

I think geometry is a different way, it’s problem solving and it uses
different skills to solve the problems. I think that [geometry’s] role
in life is in . . . helping us solve problems and looking at things [in]
different ways. In geometry it’s . . . like algebra, it’s just a completely
different branch. And, of course, there are [aspects of] geometry that
come into our life, . . . practical things like Pythagorus and . . .
parallel lines (interview, Feb. 23, 1996).

Cathy’s computer experience is extensive. She has worked on both
Windows and Macintosh platforms and has had a computer at home for the past
five years. She uses the computer at school daily for various activities such as:

making up all my assignments [and] my tests. I use e-mail [and]
Internet. Part of my job here in the last few years has been to help
teachers integrate computers into their curriculum [and] into their
classroom. So I teach teachers and I teach [their] classes so I’m using
the computers a lot with kids (interview, Feb. 5, 1996).

She is an independent learner who has taught herself much of what she knows
about computers:

I took three years off teaching before I came to this school. I retired
from teaching and I managed a friend’s real estate office. I’ve got
my real estate license but I’ve never sold. My job was to manage the
office and before I came they got an IBM computer. No one in the
office knew how to use it so I sat down and figured out how to
wordprocess and put the offers and everything onto it so that,
instead of taking five hours to type up an offer, it took five minutes
because all the clauses were already there (interview, Feb.. 5, 1996).
She learned how to use the Macintosh computer on the job at this school and is now a computer science teacher:

Then I came here, [I] was offered a job teaching Math and computers and I said, 'no, I can't do the computers' and the head of the department said that's not a problem. And I wasn't sure I could do the computers here - a) it was Mac, b) I didn't really know much more than how to word process. [Nonetheless] through watching, . . . listening and figuring it out I learned how to use Macintosh Basic (interview, Feb. 5, 1996).

This reinforces her approach to education is learning by doing.

4.2.1 Cathy’s Reflection Upon Her Classroom Experiences

During my observations of her work, Cathy demonstrated a propensity to continuously reflect on what was transpiring in her classroom. The breadth of these reflections spans Cathy's teaching and her professional development activities. In Cathy's first journal entry on February 5, 1996, she predicted the impact this project would have on her:

I know that this is an excellent opportunity for me and for my school. I know that it will make me learn [to use] the software and to integrate it into the various curricula. This will help [make] it happen [as well] in other classes and grades . . . I know that [in] other classes and grades, [students] will use it later because I will feel comfortable and will be able to pass on my 'expertise'. It is the first 'specific' software that I have used, unlike Excel or Word. It is also exciting to be part of a study with an opportunity to have feedback and research from other schools.

Her reflective activity was evident in our interview discussions and throughout her journal entries. One of the early concerns in her reflection involved preparing and organizing the materials and skills needed to start the unit:

I think that I will learn a lot and that it will be interesting. I know that I have a lot of work to do: to learn the software, to prepare my
Grade Eight class, to write a cover letter for the agreement, to collect the agreements and to prepare myself and prepare lessons (journal, Feb. 5, 1996).

She stated that she was nervous and excited about the project. In the following exchange recorded on February 13, 1996, Cathy again reiterates her desire to get started:

Q: How are you feeling about the project now that you've done a little bit more work.
Cathy: Oh I'm really excited.
Q: Oh good, still?
Cathy: [I am] nervous.
Q: Nervous?
Cathy: Really nervous.
Q: From what?
Cathy: . . . I don't know why but I keep thinking of all these little things I have to do, just [to] get everything set and ready to go . . . I'm not petrified. I guess I'm excited on one hand and a little bit nervous on the other.

I asked her if she had done anything new like this before and she stated that she felt this way when she first started teaching computers: "... when I came here and started teaching computers . . . I didn't have a clue because I had never done it before" (interview, Feb. 13, 1996). She suggested that nervous was the wrong word but rather that she was anxious about it. She thought that it was the same kind of nervousness teachers exhibit on the first day of school each year.

On the day before the first in-class session, Cathy wrote about her nervousness prior to the students arriving in class:

I made notes for my [introduction]. I guess that I am beginning to [speculate]. But I know that it doesn't matter how long this first lesson (or any of them) takes. I played with the software for a while to make sure that I really feel comfortable. I guess that I am more nervous than I thought. I went to the lab early to set up the overhead and to make sure that the computers [in which the Geometer's Sketchpad was loaded] were [marked with stickers] (journal, Feb. 14, 1996).
Then, once the class started, Cathy felt uncomfortable:

I really was nervous when I first started and I was put off my stride by the two senior girls who would not leave quickly (journal, Feb. 15, 1996).

Her nervousness seemed to disappear quickly and, shortly thereafter, she appeared more at ease.

Cathy did not refer to her nervousness again. She agreed that she was not nervous after the first day. In our interview midway through the study, she said that she felt more comfortable with the software. I asked her if she needed me in the classroom now and she replied:

no, because . . . I feel really comfortable with the software itself. I know how to handle it and I know way more than [the students] (interview, Feb. 23, 1996).

Interestingly, Cathy's journal was rich in reflection about students. She was concerned about the students selecting appropriate partners:

I gave them an opportunity to pick partners for this work. I am worried about two of them in particular. I gave them three choices, hoping that I could find appropriate pairs to circumvent a potential 'event' [of mismatched partners]. However, I couldn't come up with a solution. I spoke to their English teacher who does a lot of computer work with them and together we decided to leave the two of them as partners. I will speak to the class next Wednesday about letting them pick their own partners, but I will speak to these two separately (journal, Feb. 6, 1996).

She talked to other teachers, trying to gather more information about these same two students. She finally decided to let the two students work together.

Good planning was also taking place before teaching individual lessons and she was making mental notes, trying to anticipate what was going to happen in each class. For Cathy, being prepared is paramount to her role as a teacher.
She learns from her experiences, tries to anticipate what might happen within the classroom and prepares strategies to handle these situations. Evidence of this mental activity can be found throughout my interview notes and Cathy's journal. When I asked her to describe a classroom activity that went poorly, she replied:

I would think my poorest lessons are when a) I'm not organized or b) I don't feel comfortable with it. And I can't think of an activity per se although I'm sure that I'll dream about it tonight. The only thing would be an activity that I had not organized well enough [or] I hadn't thought ahead [or] I hadn't thought 'okay, what's going to happen if they don't all have scissors or what's going to happen if there's not enough string'. So it's been my lack of organization or preparation [that produces a poor lesson]. And that's typical. Any lesson you're not ready for is going to be a disaster (interview, Feb. 5, 1996).

Cathy points out that journal writing, while a valuable vehicle for reflection, is often underused due to its time-consuming nature:

If I were writing a journal . . . every day doing a journal, it would be great except that it's really time-consuming so it would be a balance. . . [Even without making entries in my journal], I'm still thinking but it's the actual writing it down that becomes time-consuming (interview, Feb. 13, 1996)

Did the journal writing serve as a catalyst to her thinking? Cathy replied:

Oh absolutely . . . I'll also wake up in the morning and say 'I forgot to say that in my journal'. So because it's not for me [but it was done for the project], I feel an obligation to do it well whereas if it were just for me I wouldn't wake up and say 'oh I forgot to write that in my journal entry.' I'd just think about it. But I think a journal is really helpful.

Her attention to detail and her reflective activities clearly are key factors in Cathy's daily planning and classroom activities.
4.2.2 Cathy as an Organizer in the Classroom

 Particularly prominent in Cathy's teaching practice were the routines she had established. As with many other teachers conducting a traditional classroom, these routines follow a four-phase process. The first phase consists of activities that involve attendance taking, establishing the necessary decorum, getting books opened to the appropriate page, and materials displayed. The second phase includes a review of the homework, answering questions, solving problems related to the homework, or conducting an introductory activity. The prepared lesson is the focus of the third phase. In many cases, the teacher presents a new topic or demonstrates some Mathematical algorithm or problem-solving technique. Finally, the students are assigned homework that may be started during class.

 While Cathy's classroom activities ordinarily typify the four phase process, her style and organization depends on the content she is teaching. A student's description of what transpires in an ordinary class meeting confirmed the above analysis of Cathy's utilization of class time. When asked what she does in a typical Math class, she replied:

 We usually spend about twenty minutes at the beginning of the class [taking up the homework and in] every class we learn something new . . . then she gives us our homework and we spend the rest of our class doing our homework and if we have any left, we do it at home (interview, student D, Mar. 1, 1996).

 I asked Cathy if she considered herself a structured teacher:

 No . . . I don't think I [am]. I've been teaching for too long to remember what they taught me in teachers' college and I thought that [teacher's college] was useless anyway. But I think that if I saw someone teaching something and I liked the way they taught . . . I would do it the same way when I was teaching it (interview, Feb. 13, 1996).
Cathy seems to distinguish between the structured activities of a traditional classroom and her own teaching practice. This approach follows from her conviction that to duplicate another teacher's pedagogical practices would make her feel confined. Further evidence of this belief can be found in her response to my question on whether it would be helpful for her to spend time watching another teacher use the software:

... I think that [it] would help all of us do anything new to see someone else doing it but on the same hand I don't think it's necessary [to watch another teacher]... I think that [there is] going to be more exploring [for] me and the kids, rather than [me telling the students] 'this is the way'. I would have felt really structured [if this was] the way I have got to [teach it] (interview, Feb. 13, 1996).

Cathy perceives an unstructured teacher as one who is able to control her own activities. For example, in this project, I met with each of the four teachers to create a package of worksheets from Exploring Geometry with The Geometer's Sketchpad (Bennett, 1996). In Cathy's case, we followed the topics outlined in the Grade Eight textbook used by the school. I copied the worksheets for the first class and suggested that I would return two days later with the entire package photocopied for each student. On the first day, Cathy presented me and the class with an activity sheet that she had designed (See Appendix C for Cathy's worksheets). She did not like the format or the organization of the activity page suggested in the manuals, and therefore, she made the effort to prepare her own:

I know that there are worksheets for the actual lessons that you are going to provide for me. I made up an Introduction worksheet for the class basing it on Tutorial 1 from the Workshop Guide. I will try it out on myself tomorrow to see how long it takes (journal, Feb. 8, 1996).

She continued to create her own activity sheets throughout the study. Midway through the study, Cathy made the following entry in her journal:
I came in early to work on today's worksheet. I am not completely happy with the ones that you gave me. I also want to add some things: complete the challenge from yesterday and take it a bit further; answer L's question about the specific length of line; add a few more skills like perpendicular lines and changing the preferences. I also want to add a lesson on exterior angles of a triangle because it is in the homework (journal, Feb. 15, 1996).

When asked how often she designs her own activity worksheets, she indicated that "When I taught computers, . . . I [designed all my own] worksheets" (interview, Feb. 23, 1996). She also designs worksheets for her Mathematics class sessions. Cathy creates worksheets because she does not like to rely on the textbook for her lessons:

I taught the Grade Eleven [data processing] and I taught Grade Seven [Mathematics], so that is . . . the way I taught computers because there's no textbook [in the Grade Eleven data processing class]. I don't even use the textbook [Mathematics]. I use the textbook for homework and I don't use it in my lesson necessarily. I might or might not depending on what the textbook says . . . I am basically textbook driven in the sense that that's where the curricula comes from (interview, Feb. 23, 1996).

Cathy designed a new worksheet for every class session. She would introduce other related activities, blending measurement activities into the study of geometric relationships. On one occasion, she used Geometer's Sketchpad software to develop the relationship between pi and the circumference of the circle. This introductory activity demonstrated Cathy's increasingly sophisticated understanding of the potential uses of the dynamic geometric software.

Cathy believes that effective classroom structure requires good organization. She needs to maintain this sense of underpinning structure to be successful. An entry in her journal supports this relationship:

Overall, I think that the experience [using Geometer's Sketchpad] for the girls is very good - and when I have myself and the sheets completely organized and perfect, then it will be excellent. There is
so much potential for [the girls] to explore further than we normally would and [an opportunity] to see other relationships in geometry. There is also great potential for other grades to use this [program as well] (interview, Feb. 15, 1996).

4.2.3 Disposition to Taken-as-Shared Meanings

Cathy's classroom practice adhered, in the main, to the elements of Wood, Cobb and Yackel's (1991) definition of taken-as-shared meaning in the negotiation of mathematical meanings. Her role as a teacher more closely resembled that of a facilitator than of a transmitter of knowledge. Cathy described her role in the negotiation of meanings in mathematics in a conversation:

Q: Let's say you wanted the definition of something in your class [which a student is to] construct or draw. How do you get it? How do you and the class eventually get to that point where you have a definition?
Cathy: I ask them what they think it means. The other day, [we needed to determine] what does vertically opposite mean? Is there a [familiar] word there? So I would write the word down the 'vertex'.
Q: How often would you give them the definition?
Cathy: ... Very little. The other day ... we had [discussed] the depreciation of a car ... I made them do it on the graph to see what happens if [the car] depreciates 10% every year and it levels out and looks like it's worth nothing and how long it would take to be worth almost nothing. So then we looked at the graph and I said, 'What do you see here?' This is called 'slope.' So if I can get them to talk about how slopes can vary and one of them used the word rise and run or something like that. So I said, 'Go home and ask your parents what the parts of a step are called' ... They came back to school and none of them had done it. So I said, 'Go home tonight and ask'. And they replied: 'Well, why don't you just tell us instead of our parents telling us' (interview, Feb. 23, 1996).

She believes in asking questions where other people are the 'experts', either the students themselves or their parents. When asked why she asks students
questions, she replied:

Because I think they know the answers and . . . they are feeling comfortable with it . . . their peers . . . answering the questions instead of my asking the question and answering it. And it frustrated them the other day when I wouldn't answer a question and they said, 'What if my mother doesn't know?' I said, 'Well find out if your mother knows. Take this question home.' And they said, 'Well, you can just tell us.' 'No, I don't want to just tell you' (interview, Feb. 5).

Cathy maintains that were she to give the students the answer readily, the students would forget it as soon as she gives it to them:

Because if I tell them [the answer], they can forget it as fast as I've told them . . . If they can think for two seconds and use their brain and maybe they can see the relationship, then that relationship will stay with them if they understand it rather than my just telling them (interview, Feb. 23, 1996).

The students were also aware that the teacher was not the transmitter of knowledge. One student described an example in class outlining how definitions are developed in class:

Q: How do you determine definitions in your class? Say you're going to come up with the definition of a word. Does your teacher tell you? How does it work?
D: Let me think of one. She'll say 'what does the unit price mean?' . . . She'll write everything down [we say] but then she'll circle the right one.
Q: When you have a question for her does she just give you the answer?
D: No. She writes [everything we say] on the board. Everyone gets so mad because we spend our class time figuring out one question (interview, student D, Mar. 1, 1996).

Cathy realizes that this method may leave students confused or stuck. She agreed with the statement that "students should never leave class feeling confused or stuck" (questionnaire, q. 18). This is what she had to say regarding a
student's feeling confused when she leaves a class without an answer:

Well, . . . I give them something to go away with and think about. I know what you're trying to say [in the questionnaire] but I think that I could argue that I somewhat disagree with that statement because I sometimes think that they should go home with a problem and think about it. But confused in the negative sense of the word, no (interview, Mar. 26, 1996).

On Feb. 5, 1995, Cathy elaborated on whether it was okay for students to be a little confused or frustrated in mathematics:

Oh absolutely. I was trying to teach them slope but I wanted them to come up with . . . 'rise' and 'run' and they couldn't come up with it. So I said, 'Go home and ask what the parts of the stair are called'. Of course they're Grade Eight and they all came back and not one person had asked. Well go home tonight and ask. It was to do with the depreciation of a car and I wanted to show them, they could already see what it was doing but I wanted them to have some language to use with it because when I introduce slopes in Grade Ten they look at me like I'm from Mars . . . so I thought if I can do it in Grade Eight maybe my Grade Ten [students] will understand . . . why [we use slope] (interview, Feb. 5, 1996)

Cathy didn't need to be the mathematics authority in her classroom. When I asked if she needed to be the expert on every mathematical relationship, topic or concept, she replied:

No, it doesn't bother me at all . . . It would bother me if I taught something wrong and then had to come in the next day and say, 'Oops, yesterday I made a mistake' because they would have learned [something incorrect] whereas just making a mistake on a test or numbering wrong or writing the wrong answer on the board, it doesn't bother me at all. I mean I'm not perfect and they know that (interview, Feb. 23, 1996).

While she knew that she was not perfect, she worried about the possible reactions of students when she made a mistake. She said that it didn't happen very often but "... it would be embarrassing but I'm not afraid to say I made a
mistake” (interview, Feb. 23, 1996).

4.2.4 Cathy’s View of the Role of the Teacher

There are many roles that a teacher could play in the classroom. These include a management role, a teaching role and a social worker role. Cathy as an educator presents a blending of these three roles in an appropriate balance. As a teacher, Cathy sees herself with many responsibilities. In this particular project, she feels she had the special task of getting the students to learn how to use this software:

I think my role is to load the software and then create a lesson or a process for which they can explore. So step-by-step instructions, as well as [leading the student to] discover what the computer can do in Mathematics (interview, Mar. 26, 1996).

How teachers learn is closely related to how they think students learn, according to Cathy. Cathy has indicated that she likes to engage in exploratory mathematics activities outside the classroom. She thinks that this has something to do with how she herself learns. Here is her description of how she learns:

By exploring stuff, by looking . . . lecturing me is not the way I learn, and so I just like to explore things and that’s the way I learned how to use computers myself. Just by doing it and on a . . . need to know basis . . . that’s the way I taught [this unit] (interview, Feb. 23, 1996).

Cathy had a different approach when answering questions from individual students. She believed that students need to explore geometric relationships. She was willing to ask questions to lead the students to explore and discover the relationships but she seldom gives away the answers (field notes, Feb. 15, 1996). The following exchange recorded from the February 15, 1996 class session illustrates Cathy’s method of getting her students to think:
Student: Why do the angles stay the same [in an equilateral triangle]?
Cathy: {No answer, eyes raised}
Student: Oh, because the radius stay the same and only the line lengths change. How do I write that?
Cathy: {No answer}
Student: That's OK. I will figure it out (class lesson, Feb. 15, 1996).

4.2.5 Student Learning Responsibilities

I observed that the students took increasingly greater responsibility for their own learning. They worked independently, to a point where Cathy appeared to have little to do. When I asked her if she minded not being busy, she stated:

I love that role. Yes, I mean it's boring sometimes but then I'll just step back and . . . [say] 'what's the big picture here?' and take it in as a scene rather than a group of individuals (interview, Feb. 23, 1996).

When compared to her role in the regular Mathematics classroom, Cathy realized that her role changed slightly:

I think there's sometimes in the ordinary Math classroom where I'm again not needed and I think sometimes that I could just leave them alone and they could teach themselves . . . there are times when they're working or when they're doing something where I'm not needed [in the computer room] . . . I'm much more the focus of the class when I'm in the [mathematics] classroom than in the computer room (interview, Feb. 23, 1996).

Cathy has shown great excitement over this project. When asked before the start of the in-class sessions how she felt about the project, she replied:

I am so excited. I think if it's approached the right way they'll be excited . . . they love computers and to think that they don't have to use their protractor and their compass and it's something different for them. And they did a lot of geometry with me last year in Grade Seven . . . I think because they like computers, because they've had
so much computer this year that they’re really enjoying it. I think it’s a good challenge for them (interview, Feb. 5, 1996).

When asked how the students like the software, she replied:

I think they really liked it. I think they liked not having to use pencil and paper and compass and all that stuff. The last day, unfortunately, was the day before a holiday but it worked well and I was impressed by that . . . (interview, Mar. 26, 1996).

Cathy noted that the students enjoyed not having to use the compass and straightedge tools as a major reason why the students liked to use the software.

Success was important to Cathy and she experienced it in her work and with the students. There were a few surprises:

There were other people in the class who are very bright but can get very bored [and they] were really into it. Student A doesn’t always do very well [but she] succeeded in this project. Another girl who is usually very good wasn’t very good on software but she really learned a lot. I found out when we talked about it but yesterday she had all the answers but she struggled to do it on the computer . . . she was the one that said she didn’t want to do it . . . But there were other children in the class that I saw who made other changes (interview, Mar. 26, 1996).

Student S was one student that Cathy noticed early and frequently as having a great deal of success. When I asked about S’s development over the study, Cathy replied:

It’s something new. She’s never learned that way before so she never had a fear of [it] . . . she has really low self-esteem and in the classroom situation she’s the same every day. She fails regularly but as soon as she was given an opportunity to see other things on the screen, she changed. Maybe she sees things better visually (interview, Mar. 26, 1996).

S’s growth was significant and worthwhile, according to Cathy. Cathy had many positive things to say about S:
Well, I was actually talking to her last year's teacher and I said, 'Did you teach [student] S last year?' and she said, 'Yeah, I did'. Well, S, right now, is the best as far as [being] persistent, motivated, interested. I [asked her previous homeroom teacher about S] and she said, 'That's amazing' because last year she couldn't care less about anything. You couldn't have gotten her interested in anything ever. So just to see her [increased] interest and that she cares [is exciting] (interview, Mar. 26, 1996).

S's success was not limited to the geometry classroom. Cathy outlined S's success as being more global:

... every time I see [student] S [I notice] ... her confidence in herself throughout the whole school now. Her whole personality changed (interview, Mar. 26, 1996).

The success that student S and her classmates were having with this program affected Cathy:

I was excited for her and for myself too. It was great. And it shows that there's value in the program and there are certain topics that the computer can help to teach. Once I learn from my mistakes then they can all benefit from this program (interview, Mar. 26, 1996).

4.2.6 Control Needs of the Teacher

Cathy's needs for control were minimal. She is confident in her ability to maintain a learning environment. She integrates her work through her activities and the taken-as-shared meanings with her students. She maintains that the students should listen and pay attention to her:

I want them to pay attention when I'm talking. The difference in the computer room is that when, for instance, at the beginning or at the end or when I'm speaking to the whole group, I expect them to be paying attention. Unfortunately, some kids don't need eye contact to pay attention. Although I find that very difficult sometimes if I think they're doing something else. We all know we could be doing something. But when they're working at the
computer that's distracting (interview, Feb. 23, 1996).

She always felt in control although many things were happening in the classroom at the same time. My field notes supported Cathy's contention that she was in control. Early in the project, she felt that she lost some degree of control when some girls, who were not in her class, would not leave her computer lab when she was ready to teach the class.

Cathy was not concerned that she was not the focus of attention. She realized that being in control was not reflected by being the centre of attention.

4.2.7 Control over the Technology

Cathy felt very much in control of the technology. The school computer technician assisted her by loading the software onto the computers. Since she had forgotten to ask him to put a copy on the server, she decided to do it herself. She labeled the computers and had the software loaded on every second computer to facilitate the cooperative group pairings.

She spent time working with the software:

I spent about an hour and a half using Geometer's Sketchpad. I used the Workshop Guide. I read the Intro. (page 2-3) and then did Tutorials 1 and 2. This was a lot of fun and the exercises seemed logical and sufficient for me to begin to feel comfortable with the software (journal, Feb. 7, 1996).

Cathy was quite proud of her work at this point:

Kathleen, another Math teacher, came in when I was finishing up the tutorials and wanted to see 'how it worked.' She was very impressed, partly because I made it look easy and remembered some of the more 'impressive' things to show her (journal, Feb. 7, 1996).

By the first day, Cathy was ready for the students:

I got the computers all organized in the lab. I labeled them so that they will know which computer had the software. I installed it on
the server and on a 10th computer (9th student one) as you suggested. I made sure that it was in the Applications folder of each computer and I made the window only show the software as you did down in my office (journal, Feb. 13, 1996).

She was confident in her work with the software but she always wanted to know more:

. . . I'm not very comfortable myself with the software. I mean it's not difficult once you're over the steps, once you have it. So I feel very comfortable with that and as I go through each lesson I do it myself so I feel comfortable with that. Yeah, it's just little details of the software that would make it more meaningful (interview, Feb. 23, 1996).

4.2.8 Summary

The case of Cathy demonstrates that the need for control over the teaching environment is based on a personal philosophy towards instruction. Cathy's control over her environment required her to be organized and to develop her own classroom activities. Cathy permitted her students to explore the software and share in the formulation of definitions and the discovery of relationships.

Cathy is a reflective practitioner who, through written and mental practice, makes adjustments to her pedagogical techniques before, during and after each lesson. She is an organized teacher who believes that students becoming active, independent learners is a very important goal.
4.3 The Case of Karen

The 1995-96 school year was Karen's eleventh year of teaching. Karen works in a Grade K-12 independent school in a large metropolitan city. The school is a co-educational environment, attracting students from various areas of the city.

The school is organized along two levels: a lower school and an upper school. The lower school, with students from age 2 to age 12 (Grade Seven), and the upper school, with the students in Grade Eight to Twelve, are located in the same building. Each of the schools is headed by its own Director of Academics. A Headmaster oversees the operation of the entire school.

Karen is trained and licensed to teach Physical Education, a subject that occupies one half of her current teaching assignment. Karen holds a Bachelor of Physical Education degree from a large, prestigious university located in the same city. She also holds a Bachelor of Education degree from a small university in the same province. She is also licensed to teach Physical Education to students in Grades Four to Twelve and Science to students in Grades Four to Ten.

When I asked Karen why she wanted to be a teacher, she indicated that she didn't know that she was going to be a teacher until she was in university:

I picked something that I was good at . . . I tried to find the easiest way out that I possibly could and the fastest . . . I didn't know at that time when I was in undergraduate that I was going to be a Phys. Ed. teacher. I didn't make the connection until I graduated and applied for Teachers' College . . . So [I didn't say] 'I've got nothing to do so [I will] go to Teacher's College' . . . It was like a natural thing . . . I always wanted to be a teacher and so it was just like the next progression (interview, January 9, 1996)

She believed that pursuing a Physical Education program would satisfy her interest in learning about biology. However, she has recently expressed an
interest in other vocations. She wanted to be a midwife because the human body is a great interest to her.

Karen does not have any formal training in Mathematics. Her background is in Physical Education and Science. She was asked to teach Mathematics because, in her opinion, "I think that [Mathematics] goes hand-in-hand with the Phys. Ed. I guess they assume that if you do Phys. Ed. then you know Math" (interview, January 9, 1996) and she feels that she is not concerned about that image because she feels that she can handle the course "I'm good in Math. I find Math easy up to a certain level (interview, January 9, 1996)." She did show some concern about the amount of geometry in the Grade 8 course and especially about the geometry unit on circles: "That may be a bit difficult because . . . when I went to school, we didn't do it . . ." (interview, January 9, 1996).

Karen's lack of formal training in mathematics surfaced in our discussions about teaching this unit. She avoided teaching the geometric constructions units in the past.

. . . I don't think it's something that they get a chance to do a lot of because it's always that subject that kind of gets by the wayside because there's not enough time in a year. We'll do it in the last two weeks of June and, nobody cares about it anyway . . . (interview, January 9, 1996).

Karen has used the geometry unit as an end of year filler. She doesn't believe that the geometry concepts are important:

I think that they will really like [the software] actually because I think it's a lot different than what they're used to doing -- straight, regular math (interview, January 9, 1996).

This attitude towards geometry seems to stem from Karen's experiences in high school:

My formal teaching is high school. I didn't take math in university
so when you’re in high school, you do what has to be done and then these are the things that get left out. I know what a coordinate is and I know what all these things are but to construct something, I’d have to sit down and learn it (interview, planning meeting, January 9, 1996).

Certainly, these high school experiences provide a set of experiences for the teacher of middle school mathematics. However, it is interesting to note that Karen is pursuing a similar pattern by leaving the construction unit until the end of the year.

Karen is one of four mathematics teachers in the Upper School. She teaches all three of the Grade 8 classes, while the Head of the Mathematics Department teaches the senior level classes. The Headmaster and another teacher each teach one class of mathematics at the Grade Nine level.

Karen’s computer experience was limited. She had a computer at home that she shared with her family. Her husband was the ‘computer expert’ in the family and he helped her with her computerized report cards three times per year. Karen felt strongly that she needed her husband to help her with computer problems. She noted in her response to my question about the stress she was having at home that "[I] don’t know what I’m doing because I can’t even run the program on the computer at home and, of course, my husband wasn’t at home to figure it out for me" (interview, January 25, 1996).

4.3.1 Structured Teacher

Karen favours a "traditional" teaching style. She starts each session by taking attendance, introducing the topic, handing out worksheets and having the students work on those worksheets. Karen admitted that she liked to have a structured classroom. Her description of her regular class lesson structure reinforced this view:
[The students] would come [into the classroom], . . . pick up their drill sheets and they write down their numbers . . . they do their drill for ten minutes and then, [as] I don't take up homework, they mark it at the back of the book and then I take up any questions they might have. We do a problem of the day . . . and then I teach a formal lesson . . . they have time at the end [of the period] to do their homework. I teach two lessons in the 70 minutes (interview, January 25, 1996).

The students confirm her structured teaching practices:

Well, normally at the beginning of the class we take a drill on division of three numbers with the numbers from 1 to 9 and there's 80 questions and we get 10 minutes to see how many we can complete. And then we have a problem of the day to do and then she normally writes a note on the board that we have to copy down because we're learning something new. And then we do homework (interview, Student E, January 18, 1996).

A second student describes a similar scenario:

Well, we sit down and [during] the first fifteen or twenty minutes we do a drill and then we do the problem of the day and then she gives us a lesson and then we start our homework (interview, Student M, January 18, 1996).

These teacher-directed classroom activities are in conflict with Karen's strong agreement with the statement "average mathematics students, with a little guidance, should be able to discover the basic ideas of mathematics for themselves" (questionnaire, q. 13). Her structured approach was recorded in my field notes:

This is the fourth day and the same patterns continues. Karen starts the class by talking about administrative concerns and then questions the students about what they were working on in class the previous day. The students work throughout the period, with the teacher circulating among each pair of students until . . . the bell rings (field notes, January 18, 1996).

Karen's planning reflects her views on classroom management and
routine. In her daily planning book, she writes:

... everything [down] because I can't remember anything ... At the beginning of the year, I do more planning and write specifics [in my book]. I don't write formal lesson plans but I write what I want to write on the board and what I want to say ... towards the end of the term, like November, December, I'm already into it. I know what's happening and I don't really make long notes. I just write, 'this is the topic. This is the homework.' That's about it (interview, January 9, 1996).

Karen does not reuse her lesson plans but she reuses her assignments, quizzes and tests. However, Karen does rewrite her exams every year.

I [re]use assignments from year to year: Even tests, I don't [rewrite] . . . [However] I go back to the exams and I look at them and I upgrade it. I make [the exams] different and better . . . but little quizzes and tests -- I don't change them (interview, Jan. 9, 1996).

In analyzing Karen, I began to sense a link between the level of control that she sought in the classroom and the degree of teacher-directed learning experiences that she was accustomed to provide in her teaching style. I noticed that she became very tense at various times throughout the project, further supporting this link. For example, after the third class session, she found that working at home with the software was helpful:

The first two days were rough and today was fine. It was great. I felt very comfortable and I was able to do the pre-work at home and have success and so the stress just went away (interview, January 16, 1996).

When asked if her more relaxed disposition in the classroom was related to her increased facility with the geometric software, she replied in the affirmative. However, through further questioning, Karen attributed her feeling of being more relaxed in the classroom to having acquired a higher level of control in her class management. Her increased confidence in the use of the
software and her increased knowledge of geometry made her feel in charge: "I need to feel in control." This need for control is a factor that influences Karen's work with the geometry unit.

4.3.2 The Teacher's Anecdotal Records

Karen found it difficult to reflect on her teaching practice in written form. Despite numerous requests and suggestions, Karen recorded only two entries in her journal over the three weeks of the geometry unit. Initially I attributed her lack of journal entries to her not having found the time to reflect on what was happening in the classroom. However, once I reviewed the interview transcripts, I found that this was not the case.

Karen felt that she didn't need to write anything down except on those two occasions. She wrote: "I guess I only wrote when I felt that I had something to say and that was the first time which was the preamble to everything and then I wrote one more other time and that was it" (interview, January 25, 1996). The first entry was at the beginning of the project where she described her rising anxiety level:

I think I already told you how the anxiety level and everything was quite high and I had spoken to you on the phone. I think prior to that saying I was just panicking because I didn't know what I was doing and didn't know what I was going to be . . . Yes, panic had set in (interview, Jan. 26, 1996).

This panic was not relieved quickly that evening:

Don't know what I'm doing because I can't even run the program on the computer at home and, of course, my husband wasn't at home to figure it out for me. So I was just going blind into the situation (interview, January 25, 1995).

She felt that the situation had become worse because she: "didn't look at it over
the holidays and I didn't care to look at it over the holidays" (interview, January 25, 1996). In fact, she didn't start to look at the computer software until the start of the in-class sessions, other than our two introductory sessions before the Christmas holidays. This situation was significant enough for Karen to make an entry in her journal, and to refer to it two other times in other interviews.

The second entry in her journal also referred to a significant experience. She wrote about her most recent experience working with the software at home: "I tried the perpendicular sheet at home and it worked well . . . I felt that [I was] beginning to have control. I felt good about that so I would write that down" (interview, January 25, 1996). In this instance, she felt in control again. What happened over those two weeks that led Karen to she should make another entry in her journal?

Karen didn't think that it was important to record everyday events and feelings. She stated that when she thought about writing something down: "I thought, 'what am I going to say?' I feel just fine, like a normal math class. Everything's honky dory so, 'let's just go'" (interview, January 25, 1996). She didn't believe that writing things down was a technique of reflection for her. She felt that she would rather think about it:

It was just a regular day. I had no anxiety here or there. I felt in control and everything was fine. But on this particular day . . . I felt good and wrote something because it was the first time since the beginning that I actually felt good. I thought that would be the time to write something down (interview, January 25, 1996).

It was at this point, through her writing of the second journal entry, that Karen felt she made the transition from being 'out of control' to being 'in control'.

When I asked her to try to pinpoint the kinds of things that prevented her from maintaining control at that particular time, she said that she was concerned about the overall program and some of the computer problems but she felt that
the latter source of difficulty could be overcome or, alternately, she could get someone's help with the computer software. Her biggest fear concerned the Mathematics content itself. More specifically, she said that the areas that she didn't feel that she had control over was "the math, like constructing . . . and being able to answer their questions" (interview, January 25, 1996). Her comments reinforced her concern about her level of mathematical background and her ability to master the geometric topics.

4.3.3 Transition to Taken-as-Shared Meanings

Wood, Cobb, and Yackel (1991) suggest that an alternate to teacher imposed meanings would be a negotiation of taken-as-shared meanings. Using this approach, the mathematics classroom might be transformed into a mathematical community where the students and the teacher share in the responsibility of defining relationships in Mathematics. Simon and Schifter (1991) describe several characteristics of such classrooms. They suggest that the teacher is no longer a transmitter of knowledge but becomes a facilitator of learning. Under this approach, the teacher avoids commenting on the correctness of particular student answers, relying instead on the students in the classroom to shape the form of the answers. In this way, the teacher is no longer the arbitrator of mathematical validity as that role is passed on to all members of the mathematical community that is formed within the classroom.

My initial observations of Karen indicated that she was the one to decide what counts as a correct response within the classroom. Although she did not respond to students' answers with comments such as "correct" and "good," she directed the students to eventually give her the answers she wanted.

At least two things occurred on my third visit that caused me to focus on the role of meaning negotiation within Karen's classroom throughout the study.
On the second visit, I had observed that Karen's class began with a discussion about a field trip planned for later that week. The skate-a-thon discussion consumed ten minutes of the class time while the students were told what to do, what to bring and what their responsibilities were to be on this trip. Then the students asked questions and the teacher responded and clarified the meaning of her previous directions to all questions.

When discussions turned to the software program, the teacher began to ask the students specific questions: "What have you learned about the program? How do you measure angles? What did we learn about triangles?" In each case, the teacher directed the student to a particular answer she had in mind by asking increasingly focused questions. Over the course of the computer sessions, she became more receptive to incorrect or partial answers. She encouraged the students to continue to expand on their answers until she was satisfied with their answers. Students were not encouraged to react or comment on the answers given by other students. The notion of developing individual and community meanings was not evident.

The tension between answering student questions and working in a taken-as-shared meaning with students was evident in Karen's teaching practice. She strongly agreed with the statement "teachers should not necessarily answer students questions but should let them puzzle things out themselves" (questionnaire, q. 19). She didn't believe that students should be told the answers but they should puzzle them out for themselves. "Yes, I let the students puzzle out problems all the time" (interview, January 25, 1996). Students support this notion that the teacher doesn't answer their questions:

Well, if there's questions like 'is this right?' She won't tell you whether it's right or not, but she explains things to us very well (interview, Student E, January 18, 1996).
This also supports Karen's comments about what she says to them at the start of the year about asking questions:

I do a lot more of 'putting it on them' as opposed to me answering. And I tell them that at the beginning of the year - I don't answer any questions. I don't tell you whether you're right or wrong on anything, that kind of thing. So I get like the brighter math students to say, 'okay, what did you come up with? and they'll explain it and I'll say, 'okay, yeah, that's right' or 'yeah, that's good, can you explain that to everybody else?' kind of thing (interview, January 16, 1996).

Karen expanded on this notion when asked if she believes it is important for students to explain their answers:

Q: Do you believe it's important for students to be able to explain their answers?
Karen: Yes.
Q: Why?
Karen: Because they have to have conversations with other people and be able to explain things too. We can't just assume that we know what they're talking about. They have to be able to use the correct terminology and make themselves very clear so that they can be understood by other people (interview, January 16, 1996).

When asked what specific methods she used to get the students to answer questions and explain their answers, Karen said:

... I do a lot of terminology work. I write a lot of definitions on the board and I expect them [to use them] when they talk to me. Just like in the class here ... I get them to use the words that they're supposed to be using (interview, January 9, 1996).

This entry reveals that since Karen anticipates students giving full answers, she provides the appropriate terminology she expects them to use. This was supported by my observations that "she insisted on answers that were understandable and mathematically correct" (field notes, January 16, 1996).

In a later conversation, Karen takes this form of teacher-directed
questioning further towards taking full control over the format of the answers that she expects. While describing the desirable amount of mathematics background of the teacher, Karen proposed that the role of the teacher was to "guide the students with questions" (interview, January 25, 1996). Karen's perception of control over the questions and the answers made her comfortable that her students were learning mathematics.

Karen's tight rein over the questions to elicit the 'correct' answer sometimes clashed with the occasional phenomenon of students discovering alternate solutions or approaches for solving the problems at hand. Karen herself supports the notion that there is more than one way to get a right answer (questionnaire, q. 5). When asked to describe how she takes into account that notion in her classroom, she stated:

Well, it mostly comes up in the word problem of the day because they work on an individual basis . . . [and] there are different methods of obtaining the correct answer. So once . . . they come up with the answer, I will say, 'is there another way to get this? Is there a faster way or a short cut' . . . then somebody will put up their hand, [and say] 'well, you could have done it like this'. [I will say] 'Yes, you could have done it like that. Is there another way?' [another student may say] 'Yes, there's another way.' Sometimes we have students who do the extra math classes and they'll get into algebraic type expressions and I don't get into that because it's just way above [most of the other student's] head. So I'll say, 'yes, that is another way but we won't even explore it because it's just not necessary'. But [we will consider] the trial and error method and then they'll have some systematic way of trying to figure it out and that's it.

When I do my math lessons sometimes somebody will point out, 'oh, you could do this or could we take this short cut or could we do this?' . . . there are different ways to do it. I generally teach to the middle or slightly below the middle . . . I sometimes teach the short cut if I know that they'll need the short cut later on in math. Otherwise, I don't really teach a short cut unless they figure it out themselves and then I say, ' . . . use that [method]' (interview, January 25, 1996).
On the other hand, there were occasions when Karen did instigate a process of negotiation in her classroom. For example, when describing a lesson on order of operations, Karen showed some interest in taken-as-shared meanings:

Let's say [as] an example, I've taught my lesson [and] . . . especially with order of operations, we have three different answers in the classroom, this kind of thing. So I don't say what the correct answer is. I'll say, 'isn't that interesting. We have this answer and we have this answer and this answer. I wonder which answer is correct. How many people think it's this answer? Well, isn't that interesting' (interview, January 25, 1996).

However, if Karen felt that a lesson was getting out of hand, then she would "just work it out, step by step, and then it'll be like a let's see who was right type of thing" (interview, January 25, 1996). In other words, she would revert to her previous teacher-centred instructional practice.

Although she continued to direct student's questions until the end of the study, she did demonstrate some change in this area. Her questions became more open-ended and on a number of occasions she left questions unanswered. I observed that students started to generate their own questions by the end of the fourth day. When students began to ask questions about what they should find, Karen replied "You are ahead of yourself. Explore and you may find a relationship" (field notes, January 22, 1996).

This indicated, for me, the beginning of the transfer of learner control from the teacher to the student. The teacher's habit of not answering questions but redirecting questions so that the students would arrive at the "teachers" correct answer was modified in this instance. The implication of this, I think, is quite powerful. The students had expected some direction from the teacher and they didn't receive it. As a result, the students began, in earnest, to explore on their own and to develop their own understandings. This is a radical departure
from Karen's past experience.

4.3.4 Karen's View of the Role of the Teacher

Central to many of the indicators of change was the teacher’s role in the classroom. My first impression was that Karen's underlying philosophy of teaching was the transmission of knowledge through formal instruction. Her questioning techniques and structured classroom indicated that her role was clear: 'I am here to teach'.

The role of me as a teacher I think foremost is to teach, have the kids learn something while they're here. And then secondly, is to hope that they have a good time doing it. And I try and get close to my kids in a certain way. I don't befriend them but I try very hard to find out about them and talk to them in the hall. I'm always in the hall, 'how are you doing?' 'How was your weekend?'. Because it's such a small school it's very close knit group and I know them quite well. And as you see, they knock on the door, they want to use my phone, they come in and talk. But foremost I'm here to teach. We're not here to have a good time or solve your [social] problems. I'm not a social worker (interview, January 9, 1996).

Her view of her role as a teacher was further explained when she described the characteristics of a good math teacher:

[Someone] who is very knowledgeable in mathematics and who can take the kids [who need it] the extra mile . . . A good math teacher is also someone who is very patient with the students who are really having difficulty with certain concepts because Math is an area that some kids really don't get [it] . . . Well, I shouldn't say you either get it or you don't, there are some that can work hard at it and get it. But it's something that is very black and white for some kids and you have to be patient with the ones that just don't see the concepts . . . sometimes you just can't help it, . . . these kids just don't understand it (interview, January 16, 1996).

Karen was very concerned about my impressions of her role as a teacher. She was concerned that I might be evaluating or judging her work. "I find the
scariest thing is I don't want the researcher to think I'm stupid" (journal, January 9, 1996). At the end of the project, she answered my question about whether it bothered her that I was in the room all the time, she replied "No." "Why not," I probed. "I don't know. It just didn't bother me." After additional questions in this area, she proposed that it didn't bother her because:

We also spoke before and we got to know each other a little bit. I guess if you would have just come in and sat down and I didn't . . . know you or what your objective [was] then maybe I would have been bothered (interview, January 25, 1996).

Karen was concerned, however, with her lack of formal training in mathematics. She felt that she was a good teacher but showed concern about not being a mathematician. "I know the material and I can go back and forth, inside out. I can answer anybody's questions. I can do anything. But I'm not a mathematician and I don't know it all and I think that's part of my problem" (interview, January 16, 1996). She wasn't sure how important it was to be a mathematician:

I don't know. Not necessarily. I don't think so but that's just me. I'm a perfectionist and so I guess somewhere inside of me I feel that in order to be excellent I would have to know it all and I don't. And secondly, me as a teacher, I'm not always the most helpful to all the students which again would be the perfect thing and I'm not that. So therefore, I am not an excellent teacher (interview, January 16, 1996).

She continued to gain confidence in her description of herself:

So I know deep down that I'm fine and I'm . . . good. I don't know if I would go that far but I don't get . . . the kids saying, 'oh, you're excellent'. They're not going to say that to me anyway because they're teenagers . . . I do my job and I do my job but is that great or is that just you're okay? (interview, January 16, 1996).

Karen's role as a math teacher was closely tied to her role as a Physical
Education teacher: "I would say I'm a pretty good Phys. Ed. teacher . . . I would say I'm a better Phys. ed. teacher than I am a math teacher" (interview, January 16, 1996). Knowledge in Physical Education are propelled here to suggest that:

Because I know more and I'm very knowledgeable in the area and I also am very skilled in the sports that we do. I can do any type of skill that comes up in a class and not look like an uncoordinated Phys. ed. teacher. So the kids put a lot of pressure on me all the time, 'okay, Miss, take the shot'. Okay, I took the shot and then they shut up . . . of course, I don't always make it but they know that I'm good (interview, January 16, 1996).

This passage is a reflection of Karen's need to base her evaluation on what can be observed or concretely measured. Asked whether students know that she is knowledgeable in mathematics, Karen stated:

I think so. I fumble a couple of times on a math problem. But I think maybe it's more my knowing that I'm fumbling as opposed to them knowing that I'm fumbling because, again, after teaching so long you can cover up things so well (interview, January 16, 1996).

Karen also felt that she lacked computer knowledge. Her students recognized that their teacher was inexperienced in the use of the software. When asked whether he thought less of her because her skill level in the use the computer program, Student E stated:

Not at all because I think she's learned like a lot since you've been here because I find that now I can ask her questions and she'll know the answers but beforehand it was something new (interview, January 18, 1996).

As a follow-up question, I asked "how do you think she felt not knowing the program and still trying to teach you?" Student E replied, "Sort of like she wasn't as much in control maybe because she didn't know what was going to happen or what was happening" (interview, January 18, 1996).

His fellow classmate had other comments on the role of the teacher
during this project:

Q: Do you have any other comments on the project or mathematics?
M: Well, I enjoyed it all. It was a different experience. I found that it would have been better if our teacher would have known a bit more about it because she is learning also.
Q: And if she knew more how would that change it?
M: Because if we had trouble with something she wouldn't have to sit there and try it all by herself, like she could probably teach it to us.
Q: Did you think anything different of her because she didn't know how to do it?
M: No because we were in the same situation.
Q: Was there ever a time when you knew more than she did?
M: Yeah probably.
Q: How did you feel about that?
M: It felt different knowing more than the teacher did.
Q: Do you think in order for a teacher to be good at teaching they have to be very good at the subject?
M: They have to know a bit about, they don't have to be a genius at it, but they should know what they're trying to teach and what they are teaching.

4.3.5 Control Needs of the Teacher

My first observation of her practice left me with the impression that it was very important that she maintain strict control over her teaching environment. This impression was reinforced when I asked Karen to describe a classroom activity that went poorly. She described a situation where students played a trivial pursuit-type game where "it was very loud and it wasn't good. I didn't like it" (interview, January 9, 1996). When I asked her to expand on what criteria she uses to determine whether a lesson went well or not, she replied, "the amount of talking in the class, how much control I feel over the class dictates whether it went well or not." This notion of control seemed unimportant at the time but, through further discussions, it emerged as a major influential factor to Karen's activities and behaviour in the project.
The loss of control over the classroom environment was reported by Karen to both the students and the teacher. The control levels have been identified in earlier passages. The teacher felt comfortable and very interested in the project at the beginning of the study "... the Director of Studies or the Head of the School approached me and said that somebody was interested [in a project using computers] and I said 'sure'. I'm very easy going and whoever wants to come in and do whatever is fine with me" (interview, January 25, 1996).

Karen's control levels dropped around the first day of the classroom visits:

And then the only other one was at the beginning of the whole session where I think I already told you how the anxiety level and everything was quite high. I had spoken to you on the phone, I think prior to that too. I was just panicking because I didn't know what I was doing and didn't know what I was going to be [doing] (interview, January 25, 1996).

Apparently, Karen felt that she regained control after the third day and recorded it in her journal as a turning point. In her interview with me, she made the following comments on this issue:

I wrote yesterday [in the journal] because I actually experienced something different. Up until yesterday ... I couldn't manage anything on my own and do any of the work so I tried the perpendicular bisector at home and no problem. It was great. I followed all the instructions and I investigated. I even did the [more explorations] part and it worked out just fine. So I felt [as if] I was having control over the class and I felt good that today I can go into class and know what I'm talking about and give instructions and actually help them out. So that was the actual turn around (interview, January 16, 1996).

And later:

I tried the perpendicular sheet at home and it worked well and so I felt that therefore [I was] beginning to have control and so I thought okay, I felt good about that so I would write that down (interview, January 25, 1996).
Karen's need for control was further evident in the design of her class sessions. Initially, Karen wanted me to attend all three of her Grade 8 classes although I was only gathering data from one class. She felt that she could have better control over the software if I was in attendance. However, Karen's need for control diminished during the study until she reached a point where she told me that I "didn't need to stay for all three classes" (field notes, January 22, 1996). I made additional notes on Karen's apparent increased confidence in her sense of control and how she expressed this in terms of giving students opportunities to explore:

I noticed an increase in confidence as the sixth day arrived. She gave full class descriptions on preferences and other tools. She asked me if we could change the order of the activities so that the students would see the SSS congruency relationship (field notes, January 22, 1996).

My field notes record further evidence that Karen felt very much in control of her teaching practice by the end of the study. At the beginning of the study, Karen handed out the worksheets one at a time and all students had to work at the same pace (See Appendix B for the worksheets). Many of the students finished early and were told to "do some additional investigations" (class tapes, January 12, 1996). Karen wanted to make sure that every student started each page at the same time. Some students were not happy with this format, suggesting that "this was boring" (field notes, January 18, 1996) when asked about the pace of the worksheets. Students were able to complete the work very quickly and looked forward to some additional challenges. However, by the end of the study, Karen was allowing students to work at their own pace by making the sheets available to the students upon request. Most students worked faster than anticipated by Karen.
4.3.6 Control over the Technology

There were a number of occasions when Karen felt that she was not in control of the technology. Two days prior to the start of the classroom visits, I received a frantic message on my answering machine asking me to call Karen. She was trying to work at home with the computer and she was unable to run the program: “panic has set in — [I] don’t know what I’m doing - [I] can’t even run the program on my computer at home” (journal, January 9, 1996). Her situation became more intense.

Well, first of all, I couldn’t boot the thing up on my computer at all. I couldn’t run it. So of course I cried and then my husband came home and he followed the instructions just like I followed the instructions . . . he had to go to a meeting on Sunday so of course I couldn’t boot it. Even after he showed me, I still couldn’t run it on the computer so I just read through the stuff and then I wrote down some notes about how I was feeling because you asked me to write it down so that I wouldn’t forget (interview, January 9, 1996).

Karen’s concern over the technology did not appear to be caused by her fear of the technology but rather the lack of pre-planning. Her problems, as identified in her January 9, 1996 interview, stems from her beginning to explore the software late in the process. She suggested that she doesn’t plan too far ahead:

The panic definitely set in on the weekend because I’m a last minute person. I never do anything in advance and of course here it is last minute and I don’t know what I’m doing and I’m not going to be prepared. I didn’t know what I was supposed to be teaching and then I wrote probably when it’s all over it won’t be as bad as I thought because generally speaking that’s how it usually works out (interview, January 9, 1996).
4.3.7 Summary

The case of Karen demonstrates that the perceived need for control over the teaching environment is an important issue in teacher change. Karen's perception of the role as a teacher created a tension between having a structured classroom where the meanings are teacher-directed and a flexible classroom where taken-as-shared meanings are formulated.

The variation of the teacher's need for and actual control over their teaching environment emerged from the data. Karen's transition from a traditional, structured and controlled environment to lack of control in the new environment and, finally, to maintaining a new type of control will be explored in the next chapter.
4.4 The Case of Simon

The 1995-96 school year was Simon's third year of teaching. Simon teaches in a coeducational school of approximately 550 students and 48 teachers. The school is located in a suburb of a large metropolitan city, surrounded by open fields on one side and a housing development on the other.

Maryvale School, like most independent schools, is competing against other schools in the district for students. Students must complete a written test and undertake an interview to be selected to attend the school. At the time of this study, there was a small waiting list for each of the entry years.

The school has four divisions: primary, junior, middle and senior. Each division is supervised by a director. There is also a Director of Academics responsible for schedules of students, teachers, classrooms and exams as well as maintaining the vertical integration of the curriculum. A Headmaster oversees the operation of the entire school.

Simon holds a Bachelor of Mathematics degree from a large, well-known Mathematics Faculty in Canada and a Bachelor of Education degree from another large institution in Ontario. He has earned an Honours Specialist qualification in Mathematics, the highest standing offered in the province of Ontario. He teaches Mathematics and Computer Science.

Simon had always been very good in Mathematics and choosing to teach followed naturally for him. When I asked him why he wanted to become a teacher, he replied:

Probably a combination of the fact that I like to work with kids and I was pretty good at Math. Those were the two things that I think led me into [teaching] (interview, Jan. 29, 1996).

He felt that he was influenced by his high school Grade Nine, Eleven and
Calculus teacher:

Probably . . . somebody that taught me in Grade 9 and 11 and also taught me Calculus . . . He wasn't trained to be a math teacher. He was a geography teacher by trade but he ended up teaching this [senior] Calculus [class]. So it was a bit of a struggle for him but I thought he did a good job of it . . . it made me think I'm pretty good at Math and maybe, with a lot of effort, I could be a good teacher. He seemed to be a good teacher, not so great at math, and he made it through (interview, Jan. 29, 1996).

Simon thought that it was an advantage for the teacher to be good in Mathematics and Simon believed that he was good in Mathematics. Simon felt that he was influenced by his senior high school teachers:

Q: . . . Do you remember the mathematics teachers you had when you were in elementary school or secondary school?
Simon: Yes.
Q: What is it that you remember about them?
Simon: Mostly I remember their different personalities because I didn't have a lot of difficulty with mathematics so I focused on that.
Q: And they were quite different in their personalities that you remember?
Simon: Yes, some of them were very strict and hard. Others were really easy-going, like, it didn't seem that Mathematics made them fit into a particular mold.
Q: How much do you think these people influenced your teaching style?
Simon: I would say a fair amount, especially in high school . . . and during my upper years of high school, I would say I patterned myself somewhat after the teachers I had for those years.
Q: Was there a single individual or was it the collection of five or six people you may have had?
Simon: Really the collection, I would say (interview, Jan. 29, 1996).

4.4.1 Simon as a Beginning Teacher

Simon is a relatively new teacher. He graduated from teacher's college three years before my visits. He has attained high standing in his formal education and is looking forward to his teaching career. He is a member of the
professional mathematics organizations for independent school mathematics teachers and provincial mathematics teachers. He attends mathematics conferences annually and tries to keep up by reading professional articles:

... I try to read articles from the Mathematics Teacher and various other publications when I get the time (interview, Nov. 20, 1995).

4.4.2 Simon as a Structured Teacher

Simon, like Karen, favours a "traditional" didactic teaching style. He prefers to discuss the homework, "teach" the lesson and assign homework. His students have recognized this pattern:

We just get assigned pages. First we learn about it by notes on the board and then he gives us the homework assignment and we're supposed to do it. The next day we take up the homework (interview, Student J, Feb. 6, 1996).

Student P added more details: "We usually take up the homework and we do a lesson and then we get homework and we have to do it" (interview, Student P, Feb. 6, 1996).

The Head of Mathematics at the school is responsible to make sure that every course is appropriate for the grade level. The textbook used by Maryvale School has been approved for use in schools by the Ontario Ministry of Education. The textbook has become the curriculum at Maryvale School. Simon agreed that the curriculum is based on the textbook. The text book was changed recently but Simon notes that the practice of using the textbook as the curriculum still exists:

We changed to a new textbook. When I got here we were using the Journeys in Math 8 [textbook] and the curriculum was essentially based on the textbook ... [we are] still following a textbook though it's a different one. It's not that different with the content (interview, Jan. 29, 1996).
Simon felt that, although he was responsible for directing the activities in his classroom, students needed to take ownership of their work. At our introductory interview when asked to describe an activity that went well, he replied:

Well, I used an activity one day from the textbook . . . it was [actually] a series of activities building towards the formula for circumference of a circle. So [the students] looked at perimeters of equilateral triangles, squares, regular pentagons, regular hexagons, and so on and saw the relationship between the perimeter and either the diagonal or the altitude (interview, Nov. 20, 1995).

When asked why it was particularly well done, he said:

I think what worked well was that the students were doing something and weren't just being told that something was true. They actually got to find out that something was true and they felt . . . they had an ownership of it because of that (interview, Nov. 20, 1995).

Simon likes to get his students involved in the lesson. He feels that this is important, for otherwise some students will 'sit in the weeds' and not do the work. Simon is not always sure that students working in cooperative groups learn as much as students do in whole class discussions:

Well, I try to make sure that [students are answering questions] when I'm working with them on an individual basis. I go around and give every kid a question and see what they can do and not let the ones with their hands down sit there and never answer a question . . . I haven't done as much cooperative group work as I would have wanted to (interview, Jan. 29, 1996).

Simon has had very little experience with cooperative learning activities. He has not had any training in cooperative learning techniques at this school but he did have some experiences in his practice teaching sessions:

[I had] a little bit when I was a co-op student . . . and since I've been
here I haven't had any training [or] professional development on it but I've had some resources handed to me, and so on, with suggestions of what I could do [with the resources] (interview, Jan. 29, 1996).

He has been provided with activities by other teachers, but, with no training, he has not had an opportunity to become comfortable with these teaching techniques.

Simon was not particularly concerned about the cooperative group dyads for this study:

... There might be a few particular students that I might be keeping an extra eye on to make sure they are not just letting the other person carry the load for them and they're not learning (interview, Nov. 20, 1995).

He was planning to let the students select the pairs and then "check to make sure that there are no problem pairs" (interview, Nov. 20, 1996). He has some experience with students selecting their own pairs in his classroom and describes his method of determining the pairings:

Well, what I do is, at the beginning of the year, I let them take any seat they want and then, if within the first few weeks I see that there are going to be problems, then there might be a little bit of a shuffle to restructure ... they get the feeling that they got to sit where they wanted, they could sit near their friends, but they know that if they're causing problems they're going to get moved (interview, Nov. 20, 1995).

Simon maintains a daily planning book. His planning techniques are similar to those Karen used in her first few years of teaching. Simon uses his daily planner in many ways:

Q: How do you plan your day when you're ready to put together activities? Do you record ... it in a book?
Simon: Right. I use my daily planner to make sure that I know what it is I'm going to be teaching the next day and always make
sure that I've got any photocopying or whatever I need done beforehand, hopefully the day before, so I don't have to rush in the morning.

Q: Do you create actual lesson plans or do you record in your book what it is you are going to do?
Simon: What I usually do is record in the book what I'm going to do and then, depending on the complexity of it, may or may not create a separate lesson plan (interview, Nov. 20, 1995).

As a new teacher, Simon maintains more detailed lessons plans than Karen. He creates full notes for many lessons and records what he is going to do:

[I record] the order of things . . . what needs taken up. Even if it's written right [on the previous page], I rewrite it because I don't like to be flipping back and forth thinking, 'Where is that? What was yesterday's homework? There it is. Here's what we have to take up. Here's what we're going to do.' The homework [is recorded on today's page] (interview, Jan. 24, 1996).

Now in his third year of teaching, Simon records more in his daily lesson guide than he did in his first year as a teacher. He found that, as he became more experienced, he needed to write more in his notes:

Q: Have you [written this much in your planning book] from your first year teaching?
Simon: No.
Q: Did you do more [in the beginning] or less?
Simon: Less.
Q: So it's as you become more experienced you found that you need to write more down?
Simon: I don't know if I wrote more down or not . . . what I used to do is I'd have this is what we're going to do and then somewhere else I [also] had what it was that we were going to do. And now more often, . . . [and] if I have something that I'm pretty solid on [then] I don't think I'll need [to write down more information] (interview, Jan. 24, 1996).
4.4.3 Simon's Attitude Toward Negotiating Meaning

Simon felt uncomfortable with adopting taken-as-shared meanings within his teaching environment and his understanding of a teacher's role in knowing the answers to students' questions. He was not sure that teachers needed to have all of the answers in Mathematics (questionnaire, q. 15). Therefore, Simon has developed strategies that assist him to answer questions from the students. When asked what teachers should do if they do not know the answers, Simon replied:

Ask the student to explore the answer and try to find out the answer for himself and come back and report on it and let the teacher . . . and . . . the rest of the students know what the answer is (interview, Jan. 29, 1996).

Simon was somewhat ambiguous about whether a teacher should tell students he doesn't know the answers:

Not necessarily. [The teacher] might just ask them, or say that's a good question. Why don't you look further into that? I think that's the cliché way of doing it but I don't know . . . whether to say that you don't know the answer (interview, Jan. 29, 1996).

Simon adds that, on occasions, he has admitted to not knowing an answer: "I'm sure I must have [told them that I made a mistake] but I don't remember a specific occasion (interview, Jan. 29, 1996)."

In the process of teacher-student interactions, once the student gives an answer, Simon may reword the answer using his own terminology:

I think it tends to go that way . . . I'll ask for information and students will describe things in their own words and I certainly won't tell them that's wrong, especially if it's exactly the right idea . . . I'll just put it in my words or the words that they can expect to see in a textbook because I think, in reality, they are going to see something in a textbook and, if it doesn't match what they think [it should be then] they might think [that] they are wrong. I'm just
trying to connect [what they say with my interpretation of what] they are going to see when they read on their own (interview, Feb. 15, 1996).

Simon does answer most of his students' mathematics questions in a ratio of "probably 3 to 1 [or] 4 to 1" (interview, Feb. 15, 1996). He believes that: "in [a Mathematics] class maybe more interaction or more redirection towards another student [will occur than in a computer class]" (interview, Feb. 15, 1996). I asked Simon to define the reasons for the difference:

I think most of the questions I got in the lab were about the software . . . I think when I was asked about the software, I have a tendency to say 'okay, you need to do this, this and this' and maybe that's not the right thing to do but I'm not sure [what is the right thing to do] (interview, Feb. 15, 1996).

Simon is not entirely comfortable with a situation in which he, the teacher, may not be able to answer a student's question. On the first day of the in-class activities, Simon wrote:

The idea of teaching Mathematics [using the computer program] was a bit different for me, and I couldn't answer all of the students questions about the software (journal, Jan. 23, 1996).

He normally expects to answer all of the questions asked by his Grade 8 students: "Usually, yes. 99% of the time I expect to be able to answer their questions" (interview, Jan. 24, 1996). This is a different situation for Simon. Feeling that he was not totally in control of the questions, he felt somewhat out of control of the learning environment. This lack of control led to Simon to question whether his students were learning anything:

. . . I'm not sure how much they're learning. I can't gauge that yet. So it's a little bit unsettling for me because it's not the normal environment for math, even though I teach Computer Science. When I'm [teaching a] Computer Science [class], I'm in that frame of mind. When I'm [teaching a] Math class, it's a different kind of
approach ... (interview, Jan. 24, 1996).

Simon does not mind assigning questions to students that they may not be able to answer:

I'll send them home with homework questions I don't think they [are] able to answer . . . I know that we're going to discuss the next day . . . I don't necessarily ask them during class and then say, 'We're out of time, go home' . . . they might find when they're working on their own that they come up to something they can't do or they don't think they can do (interview, Jan. 29, 1996).

Simon is less sure about whether it is OK for students to leave the class confused about a problem:

I think that can be a source of motivation for some but for others it can be a source of frustration. So I think you need to know the particular kid and know whether it's going to send them home in tears or whether it's going to make them work all that much harder to try and figure out the answer (interview, Jan. 29, 1996).

Simon doesn't investigate mathematics on his own. He finds that he doesn't have time: "Not very often. I don't find I have a lot of time. The time I have that's free time I don't want to think much about mathematics" (interview, Jan. 29, 1996). He did, however, spend time investigating mathematics when he was in high school.

He wasn't sure about the relationship between how he learns and how the students learn. He suggested that students learn through experience and require some background in counting:

I think [students learn] a lot by experience and just being exposed to a lot of different concepts and . . . to the world and there is some . . . memorization and rote and they have to have a background in counting, [that is] being able to count and various things as they grow up. And if they miss out on some of that stuff, the tendency - I think - is that they are weaker Math students . . . I'm not sure whether there are absolutely natural Math students or whether we
think they are naturals because they catch on to everything as they go along (interview, Jan. 29, 1996).

He believed that calculators can be useful only if the students can do mathematical computation:

... I think [students] need to be able to add, subtract, multiply and divide and be able to do that mentally and on paper and they need to be able to do it in a variety of ways. They need to be able to do it with the calculator too. And in our middle school we don't let them use calculators in Grade 7, especially on tests. In Grade 8, not very much either. Not until they get into Grade 9. So I think, again, if the ground work is already laid, if we knew that at Grade 6, they all were aces at mental math and ... computation, then we could say 'alright now we can do some extended exploration using the calculator.' But when I'm not sure or we're not sure whether they can even do $5 \times 12$ on a piece of paper or hopefully they can figure it out in their head, I don't want to hand a calculator over to them (interview, Jan. 29, 1996).

4.4.4 Simon's View of the Role of the Teacher

I began to better understand Simon's perception of the function of the teacher when I asked him to describe a lesson that did not go too well:

I usually look at the student response or their attitude towards [the lesson] and my own perception of what went on - whether ... the level of noise was appropriate ... Sometimes I don't want just [any] noise [and] sometimes there needs to be some noise in the class ... I think it was [a good class if] there weren't any discipline problems or behaviour problems ... The ultimate goal is if they're learning math (interview, Nov. 20, 1995).

For the most part, Simon felt that having no discipline or behaviour problems were prime indicators of a successful lesson. For many new teachers, good classroom behaviour contributes to a feeling of being in control of the class. Simon believes, however, that there is more to being a good teacher than good classroom control. I asked him to define a good teacher:
I think you can have a lot of different kinds of math teachers who might all be good math teachers. I don't know if you can say you have to have this and this and this but some of the things they might have are good questioning ability, good ability to motivate students with interesting problems and challenges. I think that knowledge of the subject is helpful (interview, Jan. 29, 1996).

Simon recognized that teaching is a very complex endeavour. When I asked Simon if being good at problem solving was sufficient to make someone a good teacher, he replied:

...I don't think [being a good problem solver] for sure makes you a good teacher. I don't think you can say 'here's a math genius'...we've got professors at university who are really strong at math obviously but they're not the greatest lecturers because they're not focused on that. They're worried about their research...I think [being a good problem solver] has something to do with it but I don't know how much (interview, Jan. 29, 1996).

Simon has not settled on a definition of the role of the teacher in the classroom. He believes that knowledge of the subject is helpful and an ability to motivate students is also important.

Simon thinks that he should be more innovative in his classroom. When asked about his definition of innovative activities, Simon said:

maybe a discovery lesson or something that I don't do every day like [having] students teach each other [or] working on a jigsaw[cooperative learning activity] (interview, Nov. 20, 1995).

However, not every new activity was successful. I asked Simon to describe something that did not go well in his classroom. His response:

Yes, I can think of...a few occasions [or] situations where I maybe wasn't as well prepared as I should have been especially if I was thinking of doing some sort of what I consider innovative activity, and perhaps I didn't plan it out or think of all the different things that could happen, and then something unexpected happened and
Simon eventually changed his definition of what should be the role of a teacher. At the beginning of the study, Simon believed that his role was that of a deliverer of information:

I would say [that my role is] a deliverer of information . . . I'm not sure if I'm using the right word, mediator, a go-between between the information that . . . the students need to know, and giving [students] that information (interview, Nov. 20, 1995).

Later he came to believe that teaching in a Socratic manner defines the teacher's role:

In the classroom I was more often a teacher . . . [teaching in a] Socratic lesson type of situation a lot of the time (interview, Feb. 15, 1996).

The introduction of the software program into the classroom brought about a welcome change of pace for Simon and his students:

I'd use it again because I think it worked and it also makes a nice change for both the students and for me. I think math, for a lot of students, can be a little hum-drum. You're always in the classroom. You're always doing similar kinds of things even though you might be working on activities one day [and a] Socratic lesson on another day. . . . It tends to be fairly similar from day to day and [the software] makes a nice change and I think [the students] can get a lot out of it (interview, Feb. 15, 1996).

Simon's perception of the role of the teacher changed from his initial view of a mediator, that is, a 'go-between' the information and giving that information to students, to that of a facilitator: "I think it was even more so than usual a role of facilitator, a guide rather than teacher, teaching the students" (interview, Feb. 15, 1996).
4.4.5 Simon’s Successful Experiences with the Program

Simon’s overall reaction to the experience was positive. When asked how he thought the project went, he replied:

I think it went pretty well. Certainly, because it was the first time that I’ve done something like that, there were a few rough spots. I didn’t know what to expect every day and how the kids were going to react every day. I think in general it went pretty well (interview, Feb. 15, 1996).

Did Simon think that the students learned from their experience with Geometer’s Sketchpad? He replied:

Well, I think a lot of them found out some things that they wouldn’t have found out in the regular classroom . . . I would have just gone through the normal set of constructions and groups of information . . . they did some explorations and found out things that were different than even what I was asking them to find out and they also were able to go deeper into some topics. . . . For example, we never would have talked about inscribed angles and central angles . . . in a normal Grade 8 classroom. But, with the computer, it was pretty easy for them to look at that (interview, Feb. 15, 1996).

Simon realized that his students learned things that they would not learn using compass and straight-edge. Furthermore, his students were able to delve deeper into some of the topics than would have been the case if they only had the ordinary geometry tools at their disposal.

He also felt that the students were successful in their study of the geometry unit:

[The project] was mainly positive. A couple of students who are not really comfortable using the computers . . . tended to be the ones who weren’t good at math either [and who] were about equally uncomfortable in [the computer lab] as they usually are doing their math. But there were a few [students] that aren’t great at math but when they went into the computer lab [they felt successful] . . . there were at least as [many of them who] really liked using the software
so I think they benefited . . . I don't think there was any losses (interview, Feb. 15, 1996).

But did they learn more or less than those using compass and protractors in last year's Grade 8 curriculum unit? He replied: "It'd be hard to say for sure but I would certainly think it was at least as much and maybe more, probably more for some of them" (interview, Feb. 15, 1996).

Simon felt that his students learned more than just geometric relationships with this project:

Well I think that in addition to learning a lot of the things that they normally would have . . . like [the relationship of the] sum of the angles of a triangle, how to bisect an angle, some terminology, perpendicular bisector . . . that they also learned how to use a new piece of software and saw that they can do diagrams and constructions very accurately with that software so it goes beyond just mathematics. They're learning how to apply it and how to use a piece of technology which is good. I think that's the direction we should be heading (interview, Feb. 15, 1996).

Simon did not think that all students learned the same things. He felt some students were more successful than others. Simon described the possible characteristics that made some students more successful than others:

I think the ones who are usually very precise and very conscientious learned what I expected them to [learn] which was what they were supposed to get out of the handouts or the activities which were on the review sheets . . . I think some of the others might have missed a few of those points but learned some other things along the way. I can't quantify it because I didn't ask them any questions about it for sure but I have the feeling that a lot of them figured out some other things just in fooling around with the software that otherwise they probably wouldn't have. If they were sitting in a math class not paying attention to something they probably wouldn't have been finding out something else about math (interview, Feb. 15, 1996).

I asked Simon if his experiences with this software would change anything
in his classroom. He replied: "I think I probably will. I think it will make me shift towards . . . student-centered learning . . . more than. . . right now" (interview, Feb. 15, 1996). He further clarified this by saying that he would make some adjustments to the curriculum unit for the next class. He wanted them to spend more time exploring:

I think I would right off the bat say to the students that once they had felt that they had covered everything that I'd asked them to do that they should explore [and] look at all the different menus and just play around with different things. The explorations at the bottom of the sheets were also good and I don't think a lot of them had enough time to get to those [activities] so maybe a little bit more time [on the explorations would have been helpful (interview, Feb. 15, 1996).

4.4.6 Control over the Technology

Simon felt very comfortable with the technology and he did not need technical assistance with the software. He "toyed around with [the software] a bit at home and during some free time" (interview, Feb. 15, 1996). When Simon needed help, he consulted the on-line help or the printed manuals. In like manner, Simon learned to use various software programs:

Usually the programs I encounter [are] like word processors, databases, spreadsheets . . . the geometry software was a bit different, but generally the stuff that I have learned and have needs to learn ... It's hard to explain . . . [I have been] using [the computer] for so long that I don't know if I can really describe how [how I learned every program]... (interview, Feb. 15, 1996).

That is not to say that Simon finds everything about computers easy:

. . . I've tried, on a few occasions, to learn a new programming language and that's been . . . difficult even though I have a Math degree and I went through a lot of Computer Science courses and did a fair amount of programming and learned how to use specific languages. When I tried to learn a few new languages [on my own], that has been more difficult (interview, Feb. 15, 1996).
Simon did feel uncomfortable early in the study. He thought that Geometer's Sketchpad was similar to other software programs with which he was familiar. Simon did not completely understand the technical elements of the software nor the potential uses of the software. He felt that I could have helped him by "spending a one hour session . . . looking at how to use the software" (interview, Feb. 15, 1996). I asked him why we did not spend that time looking at the software:

Simon: Probably no time.
Q: On your part?
Simon: On my part, yes.
Q: Could part of the reason have been that you felt that it was just another computer program and you knew it pretty well and your confidence in your ability in computers that, 'Oh, I've used lots of computer programs so I can probably do that'.
Simon: Yes, it certainly might have been.
Q: Did it surprise you when you found out that it was a little different?
Simon: To some extent, yes. [I] had to measure angles and a few things like that but . . . the interface was [also] different (interview, Feb. 15, 1996).

Although Simon has training in the use of computers and computer programming, he was surprised by the interface of this software program. He realized that many of the features of the Geometer's Sketchpad program were different from his computer experiences and that he would like to have spent more time learning the software and working with the worksheets (See Appendix B for worksheets).

Simon did not have problems managing the computer lab. He has experience as a computer science teacher and the computer lab was a normal teaching environment for him:

[I had no] major problems. I'm used to being in the computer lab as a computer science teacher so it was a similar kind of experience where there's more talk and so on between [students] than they...
usually would be in the classroom but that is . . . normal for a computer lab situation (interview, Feb. 15, 1996).

4.4.7 Maintaining Classroom Control through Evaluation

Teachers motivate students in different ways. Simon used evaluation as a means of motivating for his students. Initially, Simon said that he was using evaluation to find out whether the students had learned the material. He discussed his evaluation program:

. . . we have quizzes fairly regularly and tests at the end of each unit so that's where I look [for evidence of learning]. Still . . . in that way I'm a bit of an old school kind of teacher in that that's where I look ultimately to see if they've really learned the material (interview, Nov. 20, 1995).

He evaluates his students by using quizzes and tests. The students also write exams twice a year:

. . . we have fairly regular quizzes and unit tests. That's a fairly . . . major chunk of our mark. Homework, daily homework checked, some assignments once in a while, portfolios, doing assignments and putting them in portfolios, checking their portfolios and grading them as well [are all evaluation techniques that I use] (interview, Jan. 29, 1996).

He has even ventured into portfolios, a method of collecting materials that may be passed to the subsequent teacher: "Well, they do some portfolio activities. I've got a stack of portfolios over here" (interview, Nov. 20, 1995).

This is the first year of using the portfolios and the school has not made a decision about what they will do with the portfolios at the end of the school year:

This is the first year we tried so I'm not sure whether we're going to pass those on to the Grade 9 teachers or not. We probably will but I don't know how that's going to affect their mark (interview, Jan. 29, 1996).
Simon was concerned that the students were not learning anything. He thought that students should be learning and he would evaluate them to find out what they were learning. He also felt that he needed to do something to get them to be serious about this work. Simon handed out an assignment to get the students to use the compass and straight-edge to determine the measurement of some angles. I asked him what he was thinking about when he assigned it:

... I was thinking... they're coming into class and I don't think all of them are taking it that seriously and I need to make them realize that... this is still [important]... Okay, what was I thinking about? I was thinking about the assignment... I needed to let them know that... this is still a unit of our course and it was still worth something and I wanted to generate some marks... They don't have the software at home so I didn't want them to do something completely different from what they did. I wanted them to look at the sheet on angle bisectors and use that as their method. And I wasn't saying, 'Okay here, you can look it up in your textbook but here's your circle tool, it's a compass this time. It's the same as what's on the computer but... you can use your hand to draw the circles.'... I didn't want them to do arcs instead of circles or anything like that, just do the same thing but this is the only way I could have them do it at home that I could think of. [They] don't have much access to the lab outside of class time (interview, Jan. 29, 1996).

One way for Simon to get his students to become better investigators was to make it part of their evaluation:

Q: ... The question was how can we encourage [the students] to do [the work]?
Simon: I was talking about the specific investigations on those sheets?
Q: Yes.
Simon: Probably make it worth a mark.
Q: So make it worth some part of their evaluation?
Simon: Yes.

He continued to express concerns over the learning environment. He wrote in his journal that he was uncomfortable the first day. When asked why
he felt uncomfortable, he replied:

I think it was a little bit me sort of giving up the control over lessons and not being sure whether the information was going to get across to them or not . . . giving [students the] . . . activities where they were to do certain things and . . . explore as well. Hopefully they were going to get information but I wasn't confident that they were getting it (interview, Feb. 15, 1996).

He felt more comfortable later in the project but he remained concerned whether the students could stay focused: "I don't think more; probably equal to or a little bit less, but they weren't out of control as compared to normal" (interview, Feb. 15, 1996). Simon believes that a certain amount of control is needed in his classroom: "Probably medium [control]. I give them a little bit of freedom to chat quietly amongst themselves" (interview, Feb. 15, 1996).

4.4.8 Summary

At the start of this study, Simon viewed the teacher's role of teaching as a transmitter of information. After his work with the software, he saw a need for the teacher to become a facilitator whose job was to guide the students.

Simon was becoming a more reflective teacher. He was aware that there were tensions between his interest in providing a taken-as-shared-meaning classroom environment and his more traditional approach to teaching.
4.5 The Case of Mike

Mike teaches in a boys-only independent school in a large metropolitan city in Canada. The school has 550 boys, ranging from Grade Three to Grade Twelve. The 1995-96 school year was Mike's eighteenth year of teaching and his fifth year at Stevenson College. Prior to moving to Stevenson College, he taught thirteen years at an all-girls independent school in the same city. He has been teaching Grade Eight Mathematics courses for the past fourteen years.

The school is located along a major north-south traffic corridor. The building is placed in a forested area beside a ravine. Numerous additions to the original building has created a multi-level teaching area. The location, trees and ravine contribute to making this an attractive school.

The student population is drawn from various schools in the community and from the greater metropolitan area. Each prospective student and family must visit the campus to take an interview and the students write an entrance test in both Mathematics and English. The school is so popular, it cannot accommodate all the applicants so a waiting list is generated every year.

The school is divided into three divisions: junior, middle and upper division. Each division is supervised by a Director and there is a Director of Studies for the school. The Headmaster is responsible for the operation of the school.

Mike has a three year Bachelor of Arts degree from a large, prestigious university and a Bachelor of Education degree from a teacher's college. He started a masters degree in between those two degrees but did not complete it as he went into the workforce first:

I did three years at [university and I] was all set to go back to fourth year . . . and at the last minute switched and did my fourth year at
[another university] ... in geography. I did one year of graduate school ... [and] made the mistake of thinking that I could then go on to teachers’ college and finish my Masters thesis ... that, of course, was a great mistake because they kept me so busy in teacher’s college [that] I didn't have any extra time (interview, Feb. 6, 1996).

He is licensed to teach Geography and Mathematics at the Middle and Secondary School levels. He has also taken some Physical Education courses but he is unsure of his actual teaching qualifications in this subject:

I guess about the only other courses I could take that would be teacher-related would be Phys. Ed. ... I can't remember what the qualification level is to teach Phys. Ed. ... I do have some Phys. Ed. [teaching] background [but] that was quite a few years ago (interview, Feb. 6, 1996).

Mike had always wanted to be a teacher. When asked why he wanted to be a teacher, Mike replied:

... it was something that I... always wanted to do. I don't know why. There are other aspects of my life that had a lot of teaching in it. I began instructing skiing at a fairly young age so, by the time I became a professional teacher, I had been a professional ski instructor for ... four or five years. ... In the late '70's, getting a job in the teaching profession was tough and even if you did get one, your chances of being kept on [were slim] ... (interview, Feb. 6, 1996).

For a short time, Mike was in business before he became a teacher: "About a year. So not very long but [I was in business for] a year and a half" (interview, Feb. 6, 1996). He had some experience as a ski instructor and he enjoyed that experience: "I taught [skiing] for seventeen years [and] was an assistant ski school director at [a ski hill] for the last ... six or seven years of that seventeen years" (interview, Feb. 6, 1996).

Mike had another reason to become a teacher -- he felt that he could do a better job than some of his elementary school teachers. When I asked him if any
of his elementary school teachers influenced his work as a teacher, he replied:

Elementary? I can’t really put any specific things to it. I would imagine they probably did because all of my life when I can remember sitting in school when I was younger, I guess being as cocky as I was, [I thought that] I could probably do a better job than this person . . . I can think back and remember some very good teachers that I have had in the past but, as a direct influence, [they were] probably not [significant] (interview, Feb. 6, 1996).

Mike had a great interest in theatre and has produced many shows for Stevenson College. During this study, he was set and design manager for a community theatre production. He felt that his theatre work contributes to his exploration of Mathematics outside of the classroom:

One of the things that I’m very heavily involved in is community theatre and set design so I’m . . . using Math all the time. As a matter of fact, my kids get rather annoyed at me sometimes because I constantly go on about it. I do remember a specific time [I used Math in the theatre]. It was about 2:00 a.m. in the morning and I’m really under a deadline of trying to get a set done . . . I had to make an arch . . . and I needed to know how high [the arch] was . . . I [was] sitting there thinking ‘how am I going to figure out the radius of this circle’ and all of a sudden it dawned on me, and I honestly remember at 2:00 a.m. in the morning saying, ‘This is just like question 5 on page whatever’ . . . I had just taken up the identical question in school that day [and] I figured it all out . . . [I] took out the old string and pencil . . . and it worked like a charm. So I went back in the class the next day and said, ‘Guess what?’ I had another situation earlier this year [where] I needed to know the size of a spot for a particular light. You can buy lights in various degree sizes and I knew the distance that I had to throw. It was a special effect I had to do for this last stage . . . I went out to the car and got my calculator, came back and did a little trig [calculation and] I figured it out and I said, ‘Okay, I need a 34° lens in order to get this size’. [When I was in school] the next day, . . . [I said] to the kids in my Grade 12 class, ‘Okay, here’s the problem. I can buy a light either in [various] degree sizes . . . what size would I have to buy in order to project an image this big on the wall from ten feet away?’ So they plugged it in and they came up with the right size (interview, Feb. 6, 1996).
Mike brings activities and problems to the classroom from his theatre experiences. He feels that it is important for him to demonstrate to his students that he is still learning and that there is a place for Mathematics in the Arts.

Mike tries to keep current in his subject area through a number of professional development activities:

[I try to] read . . . magazines, The Math Teacher, and other things. I try to attend conferences whenever possible. I would probably say in the last eighteen years of teaching I have probably been to the OAME [Ontario Association for Mathematics Educators] at least a dozen times out of those eighteen. The independent schools have their own [Mathematics organization], ISOMA, is now a recognized chapter [of OAME]. They have conferences every year that I try to get to. I was just at the one this past year. So doing things like the Geometer's Sketchpad [are professional development activities] (interview, Feb. 6, 1996).

Mike feels that his Department Head is another source of ideas and materials for his classroom: "My Department Head is wonderful, absolutely wonderful to work with. [He is] very, very supportive" (interview, Feb. 6, 1996). Mike feels that the Department Head is very approachable and provides guidance in many areas: "A great sense of autonomy yet overall guidance, very approachable for ideas but he lives by the ultimate that says these are my opinions but it's your course and whatever your decision you reach is it's what you're dealing with. So I [find his approach] very, very nice" (interview, Feb. 6, 1996). In fact, it was his Department Head who introduced the idea of participating in this project:

[He] is always . . . very supportive of ideas that we want to do. He was the first one that introduced the idea of you coming into the school . . . He didn't push. He didn't come to me and say, 'I want you to do this' or 'I've arranged to do this'. He approached me and said, 'There is a teacher that wants to come in and do this. What do you think?' And so I'm glad I did obviously. But again, he's very supportive. He has already been speaking with the Director of Curriculum and the person in charge of timetabling and also
speaking to the Head of Computers to make sure that the Grade Eight's will have an opportunity next year to get some computer time. So he's very helpful in that regard (interview, Apr. 4, 1996).

Mike feels that he was brought to Stevenson College to improve the Grade Eight curriculum:

... One of the reasons I was hired here is that originally all Grade Eight's used to take the Grade Nine Math program... [The school] decided to... teach the Grade Eight's the regular Grade Eight program. The small problem with that is that... the Grade Seven [teacher]... used to teach a beefed up Grade Seven course in order to get the kids ready for Grade Nine. [He] hasn't changed much of his course yet my mandate was to not teach the Grade Nine [course]. So we... developed a program that [was] a reinforcement year to really nail down the skill level of the kids... it also allows [us] an opportunity to do a lot of different things with the kids that you probably wouldn't ordinarily do. I do a problem solving unit with them. I get an opportunity to do Mira Math. So we do explore some different stuff that I probably wouldn't get the opportunity to do if it was a regular Grade Eight (interview, Feb. 6, 1996).

The current Grade Eight program provides Mike with an opportunity to do different things with his students. He includes a special problem solving unit and teaches the geometric construction unit using Miras. (A Mira is a transparent, red plastic manipulative, shaped like the letter H, used for transformational and image geometry.)

4.5.1 Structured Teacher

Mike provides structure to his classroom by offering teacher-directed activities. He believes that teachers should direct the activities at the middle school level. However, Mike is flexible in the topics that he teaches and how he teaches them. The Grade Eight students are being provided with a reinforcement program to 'nail down the skill level of the kids'.

The Grade Eight program is taught using both a Grade Eight and a Grade
Nine book. Mike works with these students by first testing them and then provides a program to suit their needs:

... when I get them in Grade 8 their skills are pretty good ... I give them an evaluation test [to find out what they remember] ... from Grade Seven ... I test them on a [number of Mathematical concepts] and some kids are quite good. There are a lot of kids that obviously have forgotten a lot of the basic concepts ... I review a lot of the ideas and then I'll extend it that one and two steps (interview, Feb. 6, 1996).

He also provides extension topics to the Grade Nine level:

We use ... [a] general level Grade Nine Math book so it gives me the opportunity [to] review and [extend the topics]. So it does go up into a lot of Grade Nine topics but not in quite as much detail as an advanced level textbook would do (interview, Feb. 6, 1996).

Mike structures his Mathematics classroom activities in a traditional manner. He starts his class by taking up the homework, teaching a lesson and assigning the homework. His students agree that this is his usual format: "[He] takes up last night's homework. [He] writes us a new note and then assigns us work" (interview, Student J, Mar. 4, 1996). Student R was a little more specific on what happens in the Mathematics class: "He'll probably starts off by taking up the homework from last night and then we will discuss the next lesson, [he will] teach it, and then we'll do the [homework] " (interview, Student R, Mar. 4, 1996).

Mike believes that he is a well-organized teacher. He prepares for his classes by making lesson plans on his computer. He believes that the teacher can make mistakes and he is not afraid to admit them to the students: "No, not at all ... I'm laughing because I make them quite frequently ... I mean teachers are human" (interview, Mar. 7, 1996). He did not always react to his mistakes in such a gentle manner. He has changed how he treats mistakes for various reasons:
Maybe because I made so many of them . . . I'm the type of teacher that really likes being prepared going into a class and this unit put me behind the eight ball a number of times because [I was] trying to keep that one step ahead of the students . . . there were a couple of lessons when, in all honesty, they were like thrown together very, very quickly so [it was] quite easy to make typos and stuff like that (interview, Mar. 7, 1996).

His normal reaction when he declares that he has made a mistake is to learn from it. His students have differing reactions to his mistakes:

I think [the students are] fine with [the mistakes]. Where they get really mad is if you do something on a test where they've already done the question or something like that and you have to go back and say 'can you change this to whatever.' But what is really good is [when] a student can identify it as an error. I think that's a learning experience in itself to say this can't be correct. [For example, they may say] 'you can't have a circle going through this or this or a line going from wherever to wherever because it just doesn't work' (interview, Mar. 7, 1996).

That is not to say that Mike has all the answers. He simply believes that it is okay for the teacher not to have all the answers:

Absolutely. There are lots of times in class [when this happens] . . . I still, after eighteen years of teaching, get questions that I don't know the answers to but I'm willing to try looking for the answers with the students. It would be boring if I had all the answers (interview, Feb. 6, 1996).

Mike likes to plan his classes and maintain a certain amount of control. He recognizes that things will be different from what he planned yet he informs his students of his mistakes and how they can all learn from them.

Mike also felt that it was important to be prepared for class. He had a busy term and he felt rushed at certain times during the project. However, he was able to find time to create his own worksheets, rather than use the ones we selected from the Geometer's Sketchpad teacher's manual (See Appendix D for
worksheets). Mike felt that working through the worksheets before class was important for him to feel comfortable with the unit:

Definitely going through the sheets [and] the activities on the computer before going into the class is obviously critical and I made a point of trying to do that. There's the odd time I may not have had a worksheet done . . . I used the worksheet from the workbook with . . . some minor alterations written into it. So I really think that there has to be some focus for the students [and] some directions to follow so working on worksheets I think and having those prepared beforehand are absolutely critical (interview, Apr. 4, 1996).

4.5.2 Shared Meanings through Student Discussion

Mike uses his classroom to create an environment where students can talk to each other about Mathematics. He wanted his students to share with each other their ideas and thoughts on Mathematics. Students sharing ideas with each other will increase the noise level in a classroom. However, he allows a natural process whereby students work quietly on some days and make noise on others:

. . . there are days [when students] work . . . quietly, plugging away at things. Yet there are other days when they get really talking and comparing answers and those are the days that I feel good about because I've got these kids . . . discussing things . . . I would consider good things happening in classrooms [when there is noise] (interview, Feb. 6, 1996).

When I asked if there were restrictions on the type of sounds permitted in the classroom, Mike replied:

As long as I'm not teaching a lesson, [there can be noise]. That's my criteria. If I'm up teaching or taking things up then I insist that they do pay attention but I encourage or try to encourage, especially in the upper grades, communication amongst them [such as] working with a partner (interview, Feb. 6, 1996).
I noticed early in the study that there was an increased level of noise in Mike's computer classroom. I also noticed that the computer room noise decreased after the third day. I asked him if there was a pattern to the amount of noise in the classroom over the course of the unit, he replied:

What I found very interesting was that the noise level in the computer class dropped off quite considerably after about the third lesson but there was a lot of noise at the beginning. I guess an excitement of a new thing. But the same holds true for the Miras too. I found that the noise level of that activity also drops when the kids finally get down to more of the nitty gritty of the activities and they start feeling more comfortable with the tool they concentrate more on the Mathematics (interview, Mar. 7, 1996).

I noticed that many of the students did not stop their work when Mike was talking to them. Many of the students didn't even look up towards Mike when he tried to get their attention. When I asked Mike if it bothered him that students were not looking at him when he was trying to get their attention, he replied:

. . . [It bothered me] at some times. I think a lot of it depends on what it was I was saying. . . . if it was a very, very key fundamental or if I was giving a general instruction, asking them to do something then yes [I need their attention] but if it was a 'you might want to try doing this or have you thought of doing this' [type of situation] then no, it wasn't that critical that everybody be paying attention (interview, Apr. 4, 1996).

Most of the noise came from students asking questions and sharing conjectures with each other in their groups of two. Mike was concerned at the beginning about the students being paired because he felt that there were some problems with cooperative learning. He was not sure if students working in cooperative groups could learn just as well as from whole class instruction (questionnaire, q. 23):

. . . I think a lot of [learning] is individualistic [and] that some
students really benefit from a cooperative learning environment . . . but I think, on the other hand, I think there's a fair number of students who get suckered into [thinking] 'I'm going to let everybody else in the group do the work for me'. So that's why I'm kind of caught between [what] I would like to believe -- that the cooperative group setting is beneficial to everyone but experience has taught me that that's not always the way and we quite often get complaints from students coming in and saying, 'So and so's not pulling their weight' (interview, Feb. 6, 1996).

This dichotomy made it difficult for Mike to insist that every student work with a partner. Mike did realize that there were benefits to having the students working in pairs:

There were some kids . . . that felt better wanting to do it by themselves because they got impatient waiting to hand over the mouse . . . I soon realized [that] to get everybody on their own computer just increases the number of questions and the number of stations that [I] have to keep track of which would be very problematic even with just a class size of twenty-three . . . I noticed a couple of boys who, for one reason or another, tended to fragment off and start doing their work on their own which I didn't have any major problems with because I think in some it was a response to them beginning to understand it a bit more and . . . really liking this unit. And perhaps their partners were goofing around or didn't share the same interest that the boy that fragmented off might have had (interview, Mar. 7, 1996).

Did Mike think that it was important for the students to work together? Overall, Mike demonstrated a commitment to wanting his students to work collaboratively in pairs: "I still think working together is the best [way of] sharing ideas, absolutely" (interview, Mar. 7, 1996) while others should be able to work on their own. Mike was impressed, however, by the degree of cooperation that he saw in his classroom:

Mike: "Yes, yes. It was actually kind of funny watching the boys [and how] they really got almost in sync with one another [so that while] one person may handle the mouse and the other person handled the shift button . . ."
Q: They didn't have to talk about it.
Mike: They didn't even talk. They knew what they were doing and it just became very quick. So that was kind of funny that [they developed] that relationship . . . (interview, Mar. 7, 1996).

Mike believes that conversation in the classroom is important. Mike does encourage his students to share their answers in class:

I'll ask the boys to describe something or explain how they got something and they'll stand up and explain to the other boys. I didn't do it in this unit. I don't know why. I guess we were just busy doing stuff in the class but, [when] I go through this a second time, . . . that's probably one of the changes that I would make . . . I would be trying to get a bit more dialogue going between the students (interview, Mar. 7, 1996).

He thinks that student conversations help students learn:

. . . One of the things that I . . . tried to do and I guess one of the skills that you'd pick up as a teacher is to anticipate problems with certain answers. Just as an example, if we're talking about the orthocentre falling inside or outside the triangle and we agree that the orthocentre falls outside when the triangle is obtuse I would make a point of making sure that the person who gave me that statement defines an obtuse triangle because otherwise the other boys say 'okay, I know, okay, it's obtuse.' But if they don't know what obtuse means and I guess that's a skill that teachers pick up and you . . . say 'okay, if you [the student] don't understand obtuse you've got problems' (interview, Mar. 7, 1996).

The students have seen Mike using this technique while trying to get students to share definitions with each other. When asked how definitions are formed within the classroom setting, a student replied: "He usually asks us if we know and if we don't know he'll write it on the board" (interview, Student J, Mar. 4, 1996). He uses a similar technique for getting students to help each other answer questions:

So quite often I use the technique that if somebody comes up and I have to explain a question [then] I'll explain it to them. But if another student comes up and asks me the same question, I won't
answer the question but I will get the first student I had to explain to the kids. So it's a technique that I use quite often. And that I think works quite well (interview, Feb. 6, 1996).

Mike extends his encouragement of students sharing their work to the computer lab. He believes that students should be making their own conjectures. When I asked him to tell me how important it was for students to arrive at their own conjectures, he replied:

I think [it is] quite important. I think that is one of the backbones of this whole unit otherwise . . . I could have just written all the definitions down on a piece of paper, handed it out and say here, memorize this and you’re going to be tested on it in two days because that’s really what it comes down to but there’s no fun in that. There’s no learning in that. There’s no appreciation of the definitions. So I think it’s very important that they make conjectures (interview, Mar. 7, 1996).

Mike felt that his students did get better at making conjectures: "yes, I thought they got better" (interview, Mar. 7, 1996).

4.5.3 Mike’s Exploration in Mathematics

Mike believes that students should be exploring Mathematics. His work with Miras has guided him in his work with Geometer’s Sketchpad. He was continuing to use the Miras as the tool for exploration of geometric constructions and relationships with his other two Grade Eight Mathematics classes. He realized that the activities in the Mira Mathematics unit were teacher-directed:

. . . the first few lessons [of the Mira unit] . . . [I] tend to be fairly teacher-directed [because] . . . the activities are set out . . . [and are] fairly clear . . . [In the Geometric Sketchpad project, I] asked [the students] to do a task but [I] didn’t necessarily tell you how to do the task (interview, Mar. 7, 1996).

When he started the geometry unit using the Geometer’s Sketchpad

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program, he was not sure what would happen:

[In] the first lessons I wasn't sure what to expect of the kids. I haven't had them in the computer room before . . . the error that I made was probably being too teacher-directed in my approach . . . [Next time] I would definitely let the kids have a lot more free time to just explore because they were a lot more creative and computer comfortable than I had anticipated. So instead of . . . saying . . . 'this is the circle tool', it would have been a lot better . . . to let them play for ten or fifteen minutes and then come back and say . . . 'what did you find out about this particular tool?' Now I'm not convinced that would work in every situation but certainly in a situation where the students had a fairly good feel for the computers already (interview, Mar. 7, 1996).

He expressed concern about how well the students would be able to read the instructions for his computer worksheets. However, he stated that the Miras were more difficult to use than the computer software:

I actually found the instructions easier on the computers because a lot of it is built into the menus . . . I found that giving instructions and writing up the sheets for the computer-based activities were a lot easier to write up and to deal with than the Mira-based activities because the Mira doesn't have a menu. The Mira doesn't have English associated with it. You still have to be able to manipulate the Mira in such a way so when you say . . . 'I want to draw a perpendicular bisector' you can use words like select the segment, find the midpoint, go to instruct and it's very task specific whereas with the Mira it's harder to sort of couch the terminology . . . I find [that the students] struggled a little bit more with the [Mira] tool (interview, Mar. 7, 1996).

He believed that the Mira could be confusing to some students because of the possibility of multiple images: ". . . the problem is [that the students] see quite often multiple images and they're not sure which line goes where . . . the biggest problem with the Mira is [that] the students aren't sure which reflection they're supposed to map onto what point or what line" (interview, Mar. 7, 1996).

Mike did realize that good handouts were important. By Day Three of the study, he started to create more specific handouts:
... I ... started realizing the need for better and more specific handouts ... [on] Day 3 ... after Day 3, the handouts and the kids got to know what to expect in the class (interview, Mar. 7, 1996)

4.5.4 Mike's Reflections on Mathematics Education

From the beginning of the project, Mike showed real interest in working with the computer software. He was really excited about the project, particularly the video that came with the software that shows teachers using the software:

I'm excited particularly after watching the video. I thought the video was outstanding. I'm excited at the fact that you've been flexible enough that [we have] dovetailed it very nicely with my other Mira Math activity that I do so I'm excited to see the comparison about the two. The kids seem to be excited too. I walked into class [and] I mentioned [our meeting] to them, and I walked in today and a lot of them were saying, 'Is today the day we're going into the computer lab?' So they're ready for it (interview, Feb. 6, 1996).

He wanted to maintain a program similar to his Mira activities and found that the students were also excited. I asked him if he thought that he would change once he goes into the computer room, he replied:

It's going to be interesting. I don't know. I have to admit that one of the things that I've been a little bit hesitant about and I guess it's this phobia that I've been involved with, is I feel very comfortable on a Macintosh and if it was a Macintosh lab I'd be in there like hands down but I just don't feel comfortable with the IBM yet ... When I first changed schools, ... you're getting your feet wet and the first couple of years is survival techniques so now that I'm feeling much more comfortable here. I'm glad for this opportunity to finally get off my backside and get into using the computer a lot. So I think that's a very good, it's going to be good for me (interview, Feb. 6, 1996).

Mike thought that he would like to have someone in the classroom to assist him with the software. More importantly, Mike knows that, although he
is unsure of how to work with the computer in his classroom, the learning experience would be good for both the students and himself:

Probably helping me understand the software a little better, although there has to be . . . something to be said about not knowing the software that well so I'm experimenting with the students. I think that there's . . . a plus to have somebody in the classroom that knows how to do things all the time . . . [it is also] a real learning experience to say I'm not sure how to do this so let's try this and I think it's beneficial to the kids to understand that maybe the teachers don't always have everything at their fingertips all the time (interview, Feb. 6, 1996).

4.5.5 Mike's View of the Role of the Teacher

Mike sees the role of the teacher as being a continuum -- from provider of information to a guide who enables students to become self-directed learners. He believes that a teacher should be a "provider of information" to the lower grade students and a guide to the upper grade students. He sees the role of the teacher changing to one where there are fewer teacher-directed activities as the students get older:

I think a lot of it depends on the grade level that you're [teaching]. I guess at the lower level, I see the teacher more as a provider . . . in the higher grades, [the teacher is] . . . less of a provider and more of a guide . . . [the teacher should] try to wean off more and try to get the kids . . . thinking on their own [more often] so it's less, spoon-fed isn't the right word, . . . less teacher-directed as you go up higher . . . like I said, it depends on the grade (interview, Feb. 6, 1996).

For Mike, fewer teacher-directed activities means that the teacher spends more time guiding the students rather than imposing a tight rein on their activities.

Mike further describes his vision of the role of a teacher in his response to my question about the characteristics of a classroom activity that went well:

I guess for a Grade Eight class . . . if you can spend your time accomplishing what you have . . . set out to do going into the class
[then] things go well . . . if your timing is right for the lesson and you're not feeling rushed, those are always your good classes. The days when the kids aren't wandering . . . mind-wise, you are [not] losing their attention, are [considered] your good classes. So obviously you try to interject humour when you can. You do try to be different. Math is sometimes awkward . . . to vary the pace a little bit because, personally, I feel taking up of homework is critical so I do try to spend a fair bit of time on that. With the Grade Eight's, one of the things I'm excited about is that, in the past, one of the things that does tend to go very, very well is the use of Mira Math in construction because it gives the kids manipulatives, something hands-on to work with so it's quite a bit different than the more standardized type of formula [and] manipulation . . . I know the class is good . . . when the kids are talking to one another (interview, Feb. 6, 1996).

Mike believes that there are many indicators of a productive classroom activity: goals are consistently met, evidence of good planning, students are attentive in a relaxed atmosphere. To Mike, these characteristics are present when good things are happening in the classroom.

Mike noticed that his role changed while using the Geometer's Sketchpad program. He had to change his role from being teacher-directed to student-directed:

[I] became a lot less teacher-focused and a lot more student-focused. You had to be a lot more . . . specific in your task because you can't always just get around to everybody at once so it wasn't a lesson, it wasn't the type of thing I found that I originally thought I could take students through step-by-step but then found that they would have to work more at their own individual pace . . . therefore, the lessons or the activities of the day had to be very clearly laid out to allow students to proceed at a pace that they are comfortable with (interview, Mar. 7, 1996).

He realized that students would not be working at the same pace. But could Mike permit his students to proceed at their own pace?:

Oh absolutely . . . [It] would have been very difficult . . . to have tried, as I originally did in my first lesson or two, to try saying 'okay,
this is the activity and this is how we're going to proceed.' That obviously had to be adjusted. So yes, I did make some changes (interview, Mar. 7, 1996).

Mike decided to address the issue of students working at their own pace early in the project. Since he was creating the activities himself, he felt that the handouts became critical to the success of this project: "The handouts became critical. Great care had to be taken in the wording of the handout so that the students could proceed on their own at their own pace" (interview, Mar. 7, 1996).

By the end of the study, Mike had redefined his view of the role of a teacher. He started the study believing that the role of the teacher was to direct the lessons and the pace of the lessons. However, when I asked him to describe the role of the teacher when using Geometer's Sketchpad, he replied:

I found the role of the teacher actually changed throughout the [project]. I found, . . . in error almost, that I became too teacher-concentrated or too teacher-centred . . . at the beginning. But [I very quickly became] student-directed . . . as I went on. So the role of the teacher basically was obviously to prepare the worksheets that the students were working on but to try to get the students to work more on their own and draw their own conclusions at the end. So it became more of a support and guidance person rather than a teacher-directed [person] (interview, Apr. 4, 1996).

This revelation corresponded with Mike's view of the role of a senior level teacher rather than a middle school teacher. Did Mike think that this was a difficult change to make?

No. Because I have had other units that are like that too. I've done cooperative group work, problem solving [and] other topics. The Mira unit itself is basically the same . . . situation (interview, Apr. 4, 1996).

He believed that having experienced a change in a similar situation contributed to a better understanding of the changes that can occur in teaching.
Mike’s students noticed a difference between the teacher’s role in the Mathematics classroom and the computer lab:

Here he’ll show us how to do it and then we’ll go out and do it and then maybe learn something as we go along . . . we wouldn’t do that in our classroom. We [would] . . . do the exercise that he tells us to do but here he’ll tell us to do something, we’ll go do it and then [if we] mess up but [then] figure out, you’ll learn something (interview, Student R, Mar. 4, 1996).

Student R felt that he had learned something with this program and that it was fine to 'mess up' and then figure something out on his own or with his partner. Student R agreed that he liked to learn this way. Another student in the class felt that the teacher was teaching differently: "Yes a little. He just hands us the sheet to do and we spend the whole class doing that" (interview, Student A, Mar. 4, 1996). He finds this section interesting and would like to continue using the software. But he did notice his teacher is teaching differently in the computer lab:

... [the teacher] has changed. [In the computer room], he . . . gives us the page and says . . . do it and he’ll be around to help . . . In class he’ll be teaching but if you have a question he’ll be around to help (interview, Student A, Mar. 4, 1996).

Mike's transition to becoming a guide was evident by Mike's interviews, his students observations, and my own observations of what happened in the classroom.

Mike was very comfortable with his role in the classroom and working in new areas. I asked him to describe what characteristics he needed to work in a project like this:

Well I've been teaching for eighteen years [which] obviously . . . helps. So [I am] comfortable with [my] role as a teacher . . . My classroom environment for the most part is a little bit different. When I'm teaching a lesson, I'm much more insistent on quiet and
non-talking. There are times when a work period will be again a very quiet work period but there are other times, depending upon the actual nature of the topic, where I encourage working with somebody else or communicating. Now in the higher grades my work periods are almost all a talking environment because I find there are just as many kids in the classroom that probably are just as good at teaching a topic to somebody as myself and therefore I much encourage dialogue amongst my students. Again, under a working environment as opposed to a teaching environment. And since most of the computer stuff was a dialogue type activity and a learning activity [rather than] me standing at the front of the room teaching. My past experience as a teacher I found the talking fine (interview, Apr. 4, 1996).

He realized that a teacher needs to be flexible and be able to 'let go of the reins':

. . . I think that you have to in this environment [and] have to let go of the reins a fair bit. I think it would have been a totally wrong approach to every day have gone in there and turn on the computer overhead and say, 'Okay, class now do this. Has everybody got this? Okay, now plug in this and do this.' If that's the type of teacher that you are, I don't think this type of program would work. That's how I tried doing it day one and I said, 'Whoa, this is not right' and [I] rarely used the overhead and maybe I went too far that way . . . There were probably times when I should have maybe used the computer overhead a little bit more and sort of given a little bit more instructions rather than try to run around and get to everybody during the class (interview, Apr. 4, 1996).

4.5.6 Students' Learning Experiences

Mike felt that he would make changes in any subsequent work using Geometer's Sketchpad. Given more time, he would make changes to his handouts and student organization:

. . . there are a lot of things that I would change . . . Little minor changes to the handouts . . . For example, when we do talk about the orthocentre falling inside or outside, whether it's an acute or obtuse triangle, one thing I never thought of when I was doing the handouts was I actually got them to display the angles while they're working it so they can actually see the change from an acute to an obtuse . . . I found that pieces of paper came stuffed in pockets [and] whatever else . . . next time, I would be more insistent that students
bring a binder or something to the computer lab with them so they can keep those things in file. The other thing I would probably do is have . . . some place on the sheet that the teacher can initial to indicate that that activity has been completed correctly by the boy. That's one thing I found I was a little bit out of touch with sometimes . . . the kids were getting the handouts and I was able to get around to see a fair number of the boys in the class (interview, Mar. 7, 1996).

He felt that he was a little out of touch with what the students were doing and learning:

. . . I felt a little out of touch as to whether or not they were actually getting the work done on the handouts, like actually writing down comments, etc. I guess that'll surface when I give the test (interview, Mar. 7, 1996).

Mike felt that the students were learning about the program while working on his activity sheets. He had different degrees of attention and interest by the students. He felt that the brighter students may have been more successful:

. . . I think that the students who I felt really got into it tended to be the brighter students. They were the ones who weren't afraid to take some chances, to try to push the investigations a little bit further and do things that were beyond maybe what I asked for such as measuring angles or measuring length . . . just pushing it a little bit further. The weaker students tended to follow directly . . . specific questions. I did find however, a number of students who, the weaker students, the strugglers, the hard to get enthused students, tended to really like this section and do quite well on it. I can think of a number of students who seemed to really enjoy this unit and it was nice to see them get involved (interview, Apr. 4, 1996).

Mike was surprised by some of the work and interests of his weaker students. He was really impressed by one boy in particular who displays some initiative:

Mike: Yes, Student A [is] one of them which I believe is one of the boys that you interviewed. I was very impressed with him
specifically. He is one boy that definitely came to mind.

Q: What did he do differently?
Mike: He stayed on task basically. He was a boy who is a bit of a daydreamer in class, hard to get enthused about Mathematics and get him . . . the incentive. He seemed to have a lot more incentive to learn in the computer classroom than he did in the regular classroom (interview, Apr. 4, 1996).

Mike has also seen great improvement in the days following the study:

Actually, ironically [Student A] has been very, very good since getting back from the March break. Now I don't know if that's a function of him getting a little bit hooked on the Mathematics or third term [when it is] time to buckle down on the work that we're doing now . . . integers tend to be fairly mechanical which some of the boys just like doing the number crunching. It's hard to say but there definitely is a difference in [student] A now than prior to doing this unit (interview, Apr. 4, 1996).

Mike felt that all students were able to learn something from the program.

He thinks that they went beyond the investigative skills:

Well I guess on the surface they learned how to use the program and the properties that I was trying to teach them on triangles. Beyond the superficial, . . . is they learned investigative skills. What I tried [to do] with the sheets as they went on in the unit was to become less specific in my directions and [provide] a more . . . investigative approach so hopefully investigative skills improved. There would have been an element of cooperation in this in the fact that they were working with a partner so they would have to learn how to cooperate with another partner [and] share ideas (interview, Apr. 4, 1996).

Mike also felt that he could have been better trained for the software:

I guess maybe a little bit more training on the actual program itself before going into the unit. I felt that I was flying blind a little bit but in some ways that was kind of good because I was discovering things along with the kids . . . there were times when I felt that I wasn't quite on top of things as I'd like to be because I wasn't sure how things would necessarily work (interview, Mar. 7, 1996).

This was his first time to use the computer with an entire classroom of students.
He wanted a system that would allow him to see more students more effectively:

... I will set up ... a station at my desk and get kids to come to me rather than circulate. I don’t use that all the time but what I find is that students are quite often me next, me next, me next yet if they sort of do a q-line at a desk then that cuts down on that but that obviously we couldn’t do that in this computer lab. [I was] forced to make the decisions to go around and say okay, who’s next, etc. So maybe being a little bit more aware of that beforehand may have helped (interview, Mar. 7, 1996).

Mike held the pragmatic view that almost every new activity would lead to some discomfort. When asked if he has been in a situation where you are not exactly sure of all the details about what you are about to teach, Mike replied:

... quite often ... any time you teach a whole new course you are [unsure of the details]. One thing that comes to mind is we used to take all the Grade Eight [students] up to [a camp] every year ... [Last year], I did this trig unit up there where I had to get the kids to measure things and do all sorts of things. I had no idea how that was going to work but you give it your best shot and you say, well I anticipate this is how the kids are going to react (interview, Mar. 7, 1996).

As Mike says, change is a 'part of teaching'.

Mike felt that his students were becoming better at making conjectures about Mathematics and that they could get even better at exploring:

... For instance, when the boys were finished their activities and they had a few minutes left over in class I would tend to throw out to them very quick things where I wouldn’t give very specific instructions at all such as joining two mid-points of a triangle and just leave them with that ... [another time] I [told them to] connect the four midpoints in a quadrilateral and ... leave them [to] explore [it] (interview, Mar. 7, 1996).

4.5.7 Learning Needs for Other Colleagues

Mike enjoyed the project and believed that Geometer's Sketchpad is a
powerful program. When asked what he thought of the project, he replied:

I was very pleased. I enjoyed it. I thought the program was extremely powerful, very easy to learn and I found that the students picked up on it quite easily. And I found that they . . . in relation to the Mira they found it less cumbersome to use than the Mira. And I think probably the greatest differences . . . in thinking about it is that when they're using the Mira if they know to get a certain centre they have to draw a perpendicular bisector to a line the Mira doesn't have any written instructions on it. They have to know how to manipulate the Mira to create a perpendicular bisector. Yet with the computer it actually says perpendicular bisector. They use under the menus words such as midpoint, [and] perpendicular (interview, Apr. 4, 1996).

He had no trouble using the program or the worksheets:

. . . Every single topic that I wanted to cover that I did cover in the Mira unit was very easily done [using] Geometer's Sketchpad and the workbook that goes with the Geometer's Sketchpad essentially had an outline for each of the topics I wanted to do. Even though I wanted to do them on worksheets, it was nice to have those as a support (interview, Apr. 4, 1996).

Mike did have some suggestions on what he would change if he was going to use Geometer's Sketchpad with another class:

I like the program the way it was set up so I would probably keep it roughly the same. The changes that I would make would be probably . . . have it more of a discovery rather than the teacher-centred activity. I would probably insist that the students had a separate binder or small folder or something they can keep all the handouts in. I was finding a lot of students were losing their handouts or forgetting to bring them to class because I'd handed out in the computer lab yet the Math binder was back in the classroom . . . Each activity would also probably have a place on the sheet for a teacher initial so that I could . . . keep track of the fact that the students were doing the activities and more importantly I think the writing down something rather than just doing the activity and not writing things down. So those are some of the main things (interview, Apr. 4, 1996).

Mike indicated that his colleagues would use this program based on his
conversations with them:

Oh absolutely. As a matter of fact there have been a number of colleagues who have already expressed an interest and, you know, I've been getting excited and I've been pulling people to my desk and showing them [the program] (interview, Apr. 4, 1996).

Mike thought that there were some useful preliminary activities that he might employ to inservice his colleagues in using this program effectively in their classrooms:

I guess basically to sit down with them and show them some of the basics. The videotape that you lent to me at the beginning was very helpful. That got me really going on it. I said, 'Oh wow, this really is powerful'. So again, I can see this being used well beyond the Grade Eight level because of the potential of it. There are a lot of things especially in circle geometry that is good for reinforcing a lot of the ideas in even the Grade Eleven and Grade Twelve course (interview, Apr. 4, 1996).

Mike also believed that other supporting roles were needed:

Obviously the cooperation of the computer people [was necessary]. [A computer teacher] has been excellent [because] he was able to take his Grade Ten class . . . down to the lower lab . . . Fortunately, the timetabling worked out [well]. Obviously, for next year, that is something that we would know about at the beginning of the year and we would be able to hopefully schedule something well in advance rather than just hoping that we could get into the classroom. But [the computer teacher] was very cooperative that way . . . the technical support person at the school . . . very good about getting the program loaded onto the network in time for the students. So yes, there's been lots of help and support (interview, Apr. 4, 1996).

4.5.8 Summary

Mike has changed his view of the role of the teacher from that of a "provider of information" to that of a guide. He believes that Middle School students should have the same opportunities to explore and share experiences in
mathematics as those currently afforded his senior students.

Mike had allowed his students to take control of their own learning. This new class of 'explorers' and 'experts' became the focus of Mike's use of the computer software. There is a strong sense of exploration and discovery in Mike's teaching practice.
Table 1: A Cross-Case Summary of the Four Teachers

<table>
<thead>
<tr>
<th></th>
<th>Cathy</th>
<th>Karen</th>
<th>Simon</th>
<th>Mike</th>
</tr>
</thead>
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<tr>
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<td>3</td>
<td>14</td>
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<td>extensive</td>
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<td>beginning</td>
<td>progressing</td>
</tr>
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<td>Facilitator</td>
<td>Transmitter of Knowledge</td>
<td>Provider of Information</td>
<td>Provider of Information</td>
</tr>
<tr>
<td><strong>Perceived Role of the Teacher - Post-study</strong></td>
<td>Facilitator</td>
<td>Transmitter of Knowledge</td>
<td>Facilitator</td>
<td>Student-focused Guide</td>
</tr>
<tr>
<td><strong>Control Needs - Pre-study</strong></td>
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<td>high</td>
<td>moderate</td>
<td>low</td>
</tr>
<tr>
<td><strong>Control Needs - Post-study</strong></td>
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<td>moderate</td>
<td>moderate</td>
<td>low</td>
</tr>
<tr>
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<td>increased own proficiency</td>
<td>evaluation of students</td>
<td>designing worksheets</td>
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Chapter 5
Discussion and Interpretations of Findings

5.1 Introduction

This discussion will be organized around four major points. The first will be a reexamination of the original research questions posed in Chapter 1. Next, the major findings will be identified and linked with the current literature. This will be followed by a discussion of the implications that derive from these findings. Finally, suggestions for further research will be explored.

5.2 The Research Questions

The study focused on the research questions posed in Chapter 1. Those questions were:

1. How do teachers change their instructional strategies when they gain experience with the software?
2. In what ways does the role of the teacher change when teachers are placed in an exploratory computer-based environment?
3. What is the nature of the teacher's intervention in the student's learning within a computer environment?
4. What degree of confidence do teachers have in their own skills in teaching geometric constructions using a computer program?
5. Are teachers generally amenable to exploring mathematical patterns independently?
6. In what ways do teachers react to the exploration of geometric concepts in a mathematics classroom and how does teaching, using the software, fit into the teacher's idea of exploration?

7. How high are the teacher's management needs of the learning environment?

Each of these questions will be considered in light of the case studies that were developed in the previous chapter.

5.2.1 Discussion on Each Research Question

1. How do teachers change their instructional strategies when they gain experience with the software?

Each teacher demonstrated various instructional strategies when using the computer software in their classroom. Cathy introduced the project by having her students gather around an overhead projector so that they could observe some of the features of the program and how they might use some of the tools. She then distributed a worksheet that allowed the students to practice with the tools. Cathy created her own worksheets, partly due to the open-endness of the proposed worksheets, and also to control the pace of investigations by her students. She continued to hand out worksheets, one at a time, until the fifth day. As she gained confidence in her students, her approach to empowering them to take increasing control of their own learning was to allow them to work at their own pace.

Cathy also encouraged her students to do more exploration work in geometry. She posed questions to solicit further interaction between the pairs of
students to facilitate additional learning. She consolidated their learning by asking questions at the beginning of most classes and encouraging the students to share their findings with each other.

Initially, Karen felt uncomfortable with both the geometric concepts and the technology. She felt that she needed to maintain a tight rein over the environment as a defense against her lack of knowledge in these areas. As her confidence level increased, due to her own success in completing worksheets quickly and correctly, she began to let the students work more independently. That is, she began to encourage students to explore their own conjectures. In fact, she spent many minutes sitting with the students and exploring with them at the computer.

Karen asked questions at the beginning of each class about the software and encouraged her students to ask her questions about the new geometric relationships that they found. As her confidence grew, she encouraged students to ask each other questions about the software, the emerging conjectures, and geometric relationships. These questions became more open-ended and indicated that the students were moving from technical 'how do I do this?' skill type questions to 'what happens if' higher order cognitive type questions. Although this teaching strategy made it difficult for Karen to answer all the questions herself, it allowed her to share the learning responsibility with students.

By the end of the study, Karen had shifted from a teaching strategy whereby all students work at the same pace to one where each pair of students was encouraged to work at their own pace and to begin the next worksheet as they felt comfortable with their investigations. Although Karen felt occasionally overwhelmed with the variety of questions that students posed, she developed a strategy to encourage the students to explore their own questions, both
individually and in pairs.

Simon, like Karen, kept a tight rein on the learning environment. He summarized the previous day's work on a whiteboard or an overhead computer template to help keep the students up-to-date on their investigations. He tried to involve the students in these discussions by asking them questions and leading them to his predetermined answers through additional questions. The students may not have been focusing on the geometric relationships being discussed in class but rather on duplicating the teacher's 'correct' answers.

Simon had to work in two different computer labs in this study. In the library, the students were on the opposite sides of a long table, facing each other. Simon stood at one end of the two rows and used the white board to summarize the previous day's lesson. He was unable to demonstrate the software, which was his custom so he found himself talking about the investigations. In the senior computer lab, he used a computer overhead template and was able to demonstrate the software to the students. In this venue, he spent most of his time demonstrating the actual constructions rather than exploring the geometric relationships that emerged from the investigations. He felt more comfortable focusing on the concrete aspects of the geometric constructions than on the underlying conceptual framework. He continued this instructional strategy throughout the study although he suggested on numerous occasions that he should get the students to talk more about their work.

Simon felt that to be in control of the environment, he needed to be in control of the questions and the discussion issues that students posed. He, therefore, insisted that all students work at the same pace, using the worksheets as his guiding factor. Although he recognized that some students were completing their worksheets very quickly, he continued to use the worksheets as a way of placing boundaries around student questions. On the last two days of
the study, as Simon became more comfortable with the software, and his own understanding of this investigative environment, he allowed the students to work at their own pace.

Mike, in a somewhat different orientation, approached the learning environment with an exploratory framework. His experience with the Mira instruments may have allowed him to be flexible with his instructional strategies. On the first day, Mike used a computer overhead projector template to demonstrate some features of the program. He was not concerned that every student looked at him during his presentation because he felt that they could do more than one thing at a time. During the second class, Mike had students share their findings with each other and only used the overhead on one other occasion.

Mike, like Cathy, designed his own worksheets based primarily on the worksheets used by Karen and Simon. Unlike Simon, Mike wanted to provide his students with additional instructions and to encourage the students to make their own conjectures. He was concerned about the students working in pairs because he saw learning as being essentially an individual activity. He was also concerned about students being required to work in pairs so he permitted some students to work alone. He was quite comfortable with this arrangement. He also recognized that by increasing the number of student contacts, he increased his own workload and the subsequent number of times that he must visit and assist each group. This issue was not entirely resolved although Mike felt that a potential compromise was to encourage the students to visit him at the front of the class. Even this solution had its problems in that it could potentially lead to long lines of students waiting for help. He also noticed that he was more effective in getting pairs of students to collaborate and help each other with their investigations if they stayed at their computers.
Mike encouraged his students to answer their own questions. He frequently felt that his role in the computer lab was that of a facilitator or guide. He wanted to teach the students by rephrasing questions for his students and supporting them in doing further investigations. He would interrupt the class to outline a problem discovered by one of his students and invite others to investigate the problem. Mike could be found at any computer, investigating with his students, posing new questions or simply doing his own investigation triggered by something he saw on a computer screen or a question posed by a student. Mike enjoyed investigating with the students and shared his investigations with the students.

Mike followed the same initial strategy employed by the other three teachers in the study, by handing out only one worksheet at a time, insisting that all students work at the same pace. By the end of the study, as I observed with the other three teachers, Mike allowed the students to work on the worksheets at their own pace.

2. In what way does the role of the teacher change when placed in an exploratory computer-based classroom environment?

The evidence clearly shows that all four teachers changed in both their perceptions of the role of the teacher in the classroom and in their individual behaviour in the classroom. There were only subtle changes in the perception of the role of the teacher by both Cathy and Karen, who were, interestingly, at opposite ends of the continuum. Cathy felt that the role of the teacher both at the beginning and end of the study should be that of a facilitator. She believed that teachers should provide an environment where the students can explore. She created her own worksheets so students could be more focused in their
explorations and be more successful in the investigation of their conjectures. She believed in "learning by doing" and wanted her students to have this kind of environment when they are learning mathematics. Karen, in contrast, maintained a constant view of teacher as that of a "transmitter of knowledge". This view may be traced to her training as a Physical Education teacher where much of the learning is skill-based. She was concerned about her own lack of understanding of mathematics, but worked hard at relating to the students in a positive manner. Karen maintains that, while she does not have to be the authority of mathematics in her classroom, she realizes that she does not know all the answers and that she has an interest in gaining more expertise.

Both Mike and Simon made changes in their perception of the role of the teacher. Simon seems to have made the transition from the role of a 'deliverer of information' to that of a facilitator. He initially believed that the teacher should create an environment with no discipline or behaviour problems and that good classroom discipline contributes to being in control of a classroom. By the end of the study, Simon felt that he could be more innovative in his classroom and that he may introduce additional cooperative learning experiences into his lessons. I observed Simon's behaviour in the class change from "deliverer of information" to a facilitator of learning through a variety of teaching strategies and the creation of a learning community.

Mike's perception of the role of the teacher was more complicated. He believed that the teacher should be more of a provider of information to middle school students--where students still require a mastery of basic math skills--and more of a guide to secondary school students. The role of the teacher, as he saw it, depended on the age of the student. As the students matured, fewer teacher-directed activities were required. Learner control is thus a function of age and entry skill level. Mike noticed, however, that his role became more complex as
he became more student-focused when he used the geometric software. He found that he could not take the students through the material 'step-by-step' but that they had to have opportunities to explore and learn at their own individual pace. He became quite comfortable with students working in this fashion.

Mike was also aware of how his own behaviour changed throughout the study. His initial style of being very teacher-directed modified moderately to one that was more student-centred. He believed that the role of the teacher is to develop activities to guide students, but to also allow students to work on their own and to draw their own conclusions. Mike felt comfortable 'letting go of the reins' and take on this new role as a teacher.

3. What is the nature of the teacher's intervention in the student's learning within a computer environment?

The interaction between teacher and student was the focus of this question. The specific changes in Cathy's and Mike's work included designing their own worksheets, eventually allowing students to work at their own pace and sharing ideas with their students. Cathy and Mike had moved towards a taken-as-shared meaning culture within their classroom. Simon and Karen recognized the tension between giving definitions and sharing meaning with their students while not necessarily encouraging the development of this form of learning community.

Cathy showed the greatest change in the transition towards taken-as-shared meanings with her students. She realized that she did not have to be the authority on each and every topic and concept. She allowed the students to start to take a bigger role in ascertaining geometric relationships. By the end of the study, she was encouraging her students to share their ideas within each pair and
between groups.

The source of the transition to the negotiation of shared meanings is more difficult to pin down. The primary source for Cathy was her maintenance of an investigative classroom environment that allowed the students to share their work on a regular basis. Cathy's experience working with girls assisted her in opening the lines of communication between the pairs and encouraging the students to work together. The secondary source of the transition may be the use of cooperative learning in the classroom. Since the activities designed by Cathy encouraged discussion, Cathy could spend time directing her students to discuss their findings with other pairs or with the entire class. The design of the activities that Cathy prepared contributed to the negotiation of shared meanings within the classroom.

Mike demonstrated similar changes towards negotiation of meaning within his classroom. For Mike, this change was perhaps unconscious at first, but it did take hold in his practice. Mike began to notice the students' questioning change and their increased sharing of ideas and comments. Simon was less impacted by the negotiation of shared meanings in his classroom. He was not writing or talking about his observation of student interaction. Simon, in fact, was not convinced that his students were learning anything. He was unable to have students articulate their understanding of the concepts to his satisfaction.

4. What degree of confidence do teachers have in their own skills in teaching geometric constructions using a computer program?

The degree of confidence that each teacher brings to the study appears to be based on their comfort level with the computer, geometric concepts, and
mathematical experience. The four teachers observed in this study differed in many of these factors. Cathy, for example, had extensive computer and mathematical experience, had taught for eleven years in mathematics classrooms, from Grades seven to twelve, and she enjoyed investigating mathematics and geometric relationships. She had also participated in professional development activities such as reading mathematics teachers’ books and journals, was very comfortable with her knowledge of mathematics and, therefore, felt confident in her skills in teaching geometric constructions using a computer program.

Similarly, Mike had experience in working with computers and exploring geometric relationships. He had created a Mira mathematics unit where students use a plastic, transparent manipulative while using reflections as a technique to discover geometric relationships. He brought fourteen years of experience teaching mathematics at the Grade eight to ten level as well as experiences from his theatre interest to bear on his students’ mathematical experiences. He was confident in his own skills to use this geometric software in his classroom. He, like Cathy, developed his own worksheets, further supporting his confidence in his own skills to provide an enriching environment.

Simon, as a graduate of a major mathematics undergraduate degree program, felt very confident in his computer and mathematics ability. He felt comfortable with his skills in the geometric environment but had some difficulties allowing his students to explore in this environment because of his perceived role of the teacher.

Karen had little computer experience and limited mathematics experience. Both factors contributed to Karen’s initial lack of confidence in her skills to teach geometric constructions using the geometric software. Her confidence level hit a
low during the week prior to the study when she was unable to get the software to work at home. She felt that her skills in geometry were poor because of her lack of training in this area in elementary and secondary school. She realized that most teachers avoid doing geometry altogether or teach it during the last few weeks of the school year.

Karen has been teaching mathematics for three years. The combination of her limited computer experience and minimal experience in geometric relationships contributed highly to Karen's initial concerns and nervousness in using this software. A change in Karen's perception of her skills occurred in the second week. She had some experience with some worksheets and, as the computer software became more familiar to her, she felt that she was in control of the classroom and the software. She continued to have lingering doubts about the geometric relationships until the third week. At that time, she felt that she could answer students' questions and that she would need to investigate additional geometric relationships in the future. Karen became more confident in her skills using the software and realized that personal investigations in geometry will contribute to a better understanding and confidence in geometry.

5. Are teachers generally amenable to exploring mathematical patterns independently?

Mathematics explorations provide the underpinning for the use of dynamic geometric software. Users of the software investigate conjectures by manipulating images on the screen and by exploring additional situations through the use of the moveable points, lines, and circles. Both Cathy and Mike had extensive experience and interest in investigating mathematics outside of the classroom. Cathy, as an avid reader of mathematics books and journals,
created her own worksheets to develop the same interest in her students. Mike's interest in geometry and the theatre provided him with a rich investigative style.

Although Simon had an undergraduate degree in mathematics, he found he did not have time to investigate mathematics. Being a new teacher in an independent school proved a time-consuming job. He did, however, spend time investigating mathematics when he was a student in high school.

Karen did not investigate mathematics outside of the classroom, because she did not have time to investigate mathematics despite her having an interest in the subject. She introduced new ideas into her classroom from workshops and she plans to continue to integrate new ideas as time permits.

For the most part, all four teachers were open to new ideas. They each agreed to participate in the project and had a willingness to learn. The degree to which they investigate mathematics is not directly related to their previous knowledge of mathematics.

6. In what ways do teachers react to the exploration of geometric concepts in a mathematics classroom and how does teaching, using the software, fit into the teacher's idea of exploration?

The most significant reaction to this environment was through the different methods employed to regain 'control' over the learning environment. The teachers had various reactions when placed in an unstructured learning environment. Cathy and Mike both focused their first lesson on the tools and the possibilities for investigation with the students. They designed their own worksheets to create a more organized approach to the investigations. However, in both cases, they did not restrict the breadth of investigation. In fact, the worksheets provided the supportive environment for the students and guided
the students into looking at other related investigations. On the other hand, Karen used her personal experience to feel more comfortable in this environment. She worked at the assignments until she felt in control of the investigations and, through that increased confidence, allowed the students to take increased control of their own learning. Simon was concerned about whether the students were learning and assigned homework to ensure that some learning was taking place. He used evaluation as a method of regaining the control of the classroom that he thought was lost as a result of the open-ended software.

7. How high are the teacher's management needs of the learning environment?

This question proved to be the most significant question of the study. Cathy felt comfortable in her role as a teacher, with the software and with her interactions with the students. She felt somewhat concerned at the beginning of the first class when some lingering students disrupted her flow but that was quickly dispelled and she flourished for the remainder of the study. She was comfortable with the pairings, the student-to-student interactions, and the results of their investigations.

Karen was less comfortable with the computer environment. She did, however, maintain a structured environment with the students. This structure was compromised early in the study. Karen did not feel comfortable with the software, which contributed to her feeling that she did not have control of the classroom. Her students were learning the software faster than she was and they were asking questions with greater detail than she could answer. She felt tense throughout this early part, mostly due to her perceived lack of management.
control over the classroom. However, Karen did feel that she regained this management control. She attributed this change to her increased success with the software. For Karen, control over the environment is directly related to her control over the software. Her need for control over the learning environment was essential to her success in the project.

Simon required moderate control over his environment. He permitted his students to talk during their investigations; his management needs were directly tied to discipline and behavioural concerns. As the need for discipline rose, Simon needed to maintain a tighter rein on his students. This control was reached through stricter evaluation techniques and through separating the student dyads in his regular classroom.

Mike, like Cathy, had low management needs in his classroom. He encouraged the students to discuss mathematics and to share their conjectures and geometric relationships. He did not mind noise in the classroom. In fact, he did not mind students talking while he was giving a few students additional investigative ideas. Mike attributed his low control needs to his eighteen years of experience as a teacher. His confidence level appears inversely proportional to his low management needs.

5.3 Major Findings

The evidence clearly supports the conclusion that the four teachers, to an extent, allowed their students to direct their own learning with the geometric software. Particularly noticeable was the manner in which each of the four teachers changed in their perception of what they thought was happening in the classroom. The major findings can be summarized as follows:
The newness of the materials influences the management of the instructional time and requires additional planning and preparation time for the teacher.

Mathematics teachers need to continually explore mathematical concepts and ideas to be better prepared for different learning situations. Not only do mathematics teachers not have all of the answers, it is important to recognize that they will never have all the answers. After using the computer software, the teachers recognized that they did not need to know the answers and they began to develop strategies to deal with these situations.

Mathematics teachers need to reflect on their teaching to make the required adjustments to teaching style and mathematical content.

The emerging role of the teacher, from a provider of information to facilitator, is unresolved for some of the teachers. In activity-based classrooms, the role of the teacher is paramount to the success of the program.

Teacher control can be organized into three categories: management, personal and professional.

The implications of the findings will be discussed and, where possible, placed within the context of existing research on mathematics teacher change.

Management of Time

It has been suggested that successful change requires an investment in time (Fullan, 1982, 1991; Schon, 1983). This lack of time can appear in various forms. For Simon and Karen, the lack of time was evident through their lack of
planning full lessons. Lesson planning takes time, and, as teachers who are new to the teaching of mathematics, both teachers did not have time to reflect on their teaching practice. They were therefore unable to spend enough time to acquaint themselves with the potential uses of the software and how it may be integrated into their own teaching practice. Quite possibly, teachers involved in innovative computer software programs need time to work out the innovations for themselves.

This lack of time for reflection, planning, and investigating mathematics has implications for curriculum designers and textbook authors. As is evident from the literature and these four cases, the textbook, for many teachers is the curriculum. The examples shown, the problems posed, and the exercises listed become the content for the Grade 8 mathematics course. To encourage teachers to use dynamic geometric software, it is worth the time, energy, and cost to provide instruction on the software and opportunities for discussion and investigation of both its philosophy and features. These contexts will assist the teacher in modeling the investigative approach and identify possible problems before teachers are faced with the implementation of this powerful tool.

Teacher Exploration of Mathematical Ideas

Students need the freedom to discover, through exploration, different ways to make conjectures and to build solutions. They need to spend time working with problems and searching for solutions. This time may be spent developing conjectures and pursuing possible alternatives to gain a better understanding of the problem and its associated characteristics. Teachers can provide an environment to assist students in making these discoveries. This process should be organized through the assistance of a teacher whose role is to
facilitate student's exploration (Burns, 1992).

Mathematical explorations are not restricted to students. Mathematics teachers should also become mathematics students. They must be challenged at their level of mathematical understanding, allowing them to increase their mathematical learning (Simon & Shifter, 1991). These explorations, given appropriate reflective activities by the teacher, will offer opportunities for developing increased depth of understanding that can be modeled and shared with the students.

Geometry is one mathematical field where exploration can be introduced to students. However, teacher beliefs about geometry influence the crafting of lessons (Parsons, 1994). If teachers believe that geometry is just another topic that can be left to the last few weeks of June, then their lessons may reflect a procedural focus. Bauersfeld (1992) proposes that the “objective of mathematics education is not that students produce correct solutions to mathematical problems but that they do it by insightful and by reasonable thinking” (p. 10). How can we encourage teachers to provide this type of learning environment for their students?

Teachers need to explore geometric relationships themselves to be better prepared to teach geometric relationships. Recent writings in the field of teacher education have suggested that this process must begin within the individual's experimental field (Underhill, 1991; Edwards, 1994b; Shaw & Jakubowski, 1991). Cathy and Mike are investigators of mathematical relationships while Karen and Simon have engaged in this activity less often.

Steffe (1990) suggests that teachers who do not explore mathematics are the ones who hold to the belief that mathematics is static, formal, and symbolic. It is this very belief that we should try to change so that teachers can experience the beauty of mathematics.
The implications for teacher investigations in mathematics are numerous. Teachers may need to begin or continue to engage in mathematical investigations to better anticipate the questions posed by their students and to recognize the interrelationships between investigations and conjectures. Many teachers have not engaged in these investigations and therefore cannot help their students develop reasonable conjectures and explore possible avenues of investigations. Teacher education programs that include mathematical investigations and conjecturing experiences within their preservice and inservice programs may lead to more productive mathematics learning environments.

Role of Reflection in Teacher Change

The role of reflection has been investigated to a small extent in the discussion of the four teachers. This reflective process supports the suggestions made by researchers on the impact of reflection on teacher change (Schon, 1983; Shaw and Jakubowski, 1991; Shulman; 1986; Wood, Cobb and Yackel, 1991).

The implication for those teachers interested in effecting change in their teaching practice is clear. In the present study, both Mike and Cathy provided rich journal entries as they reflected on their work. They both encouraged exploration within their classroom and they made adjustments to their worksheets as they recognized possible learning situations emerging. Teachers need to reflect on the impact the innovation is having on their teaching practice. This is not to say that all four teachers were not thinking about their teaching practice. It appears that the actual writing and reflecting on what is written further assists the teacher in reconstructing their own knowledge about teaching and learning.
Schon (1983) suggests that when teachers become more reflective on their actions within the classroom environment, they begin to challenge their roles as teachers. These role changes are necessary if teachers are going to be successful in the probing for understanding mathematical environment. The preservice and inservice education community, therefore, has a responsibility to introduce activities that will allow teachers to reflect on their own work using journals or other written forms. These activities may form the basis of a teaching portfolio that includes written reflections, mathematical explorations, and critical expositions on teacher case studies. It is through these vehicles that we encourage teachers to become reflective practitioners.

Teachers should also form groups to discuss mathematical explorations and to share their journal writings. These groups can also discuss teacher case studies and share teaching strategies helpful in exploratory classroom situations.

Role of the Teacher

The perception of the teacher of their role in the learning and teaching process is important. Maher and Alson (1990), among many, have suggested that the role of the teacher change from one of 'telling and describing' to one of 'listening and questioning and probing for understanding'. Teachers will need to make a shift in how they perceive their role in the classroom. This supports Kilpatrick and Davis (1993) suggestion that the demands on mathematics teachers will increase and more effort is required by both the teacher and the student.

To what extent is teacher modeling an effective teaching strategy? The previous discussion on teacher exploration contributes to our understanding of the role of the mathematics teacher. If teachers involve themselves in the
exploration of mathematical relationships, their students will benefit from the
teacher's openness to new ideas. If these explorations take place in front of
students, the teacher will demonstrate that exploring mathematics is an essential
part of a mathematical community.

As teachers change from teacher-centred classrooms to student-centred
classrooms, students may become self-motivated and may become more responsible for their learning. Keller (1996) advocates that, in this environment, teachers will find a new freedom to learn along with their students. Teachers become active participants by providing computer instruction, modelling, coaching and scaffolding while reshaping their own learning (Choi & Hannafin, 1995). This "cognitive apprenticeship" frees the teacher to serve in the role of mentor/coach rather than lecturer/provider (Young, 1993).

However, not all teachers feel comfortable in this learning environment. In the traditional classroom, the teacher is the authority for mathematics. In these classrooms, the teacher frequently takes up the homework, 'teaches' the lesson, assigns homework and may have time to permit the students to begin their homework. This traditional situation "is organized, routine, controlled and predictable--unlikely environment for the creation of knowledge" (Romberg, 1992, p. 765). In this situation, teachers feel comfortable in their role and, therefore, can maintain control over their teaching environment. More specifically, if teachers know the mathematics content, they have a better opportunity to control the questions posed within the classroom.

Within an exploratory classroom, as was experienced by some of the teachers in the present study, teachers may feel a lack of control over their environment. Mason and Pimm (1986) call this loss of control the 'sacrifice of overt control'. Although Mason and Pimm were making reference to teachers who allow their students to talk within the classroom, a similar situation occurs
within an exploratory classroom. Teachers may be called upon to answer questions which they cannot answer and which make them feel that their seat of authority is being taken away (Gibbs and Orton, 1994).

Etchberger and Shaw (1992) describe a continuum for teachers' roles in teaching mathematics. They suggest that the teacher's role ranges from "dispenser of knowledge" to the teacher as "provider of situations and information." At the start of the study, Karen, Simon, and Mike believed that teachers should be deliverers or providers of information. By the end of the study, Simon and Mike’s perceived role of the teacher had moved to the other end of the continuum while Karen’s perception made some movement in that direction. According to Etchberger and Shaw, reflection generated powerful transformations.

Pea (1987) suggests that we cannot predict how the role of the teacher may change as the use of technology increases. From this study, we can suggest that teachers will have to face issues concerning their control of the learning environment.

Teacher Control

The issue of control permeated the entire study. What were the teachers controlling? How did this desire for control affect their teaching? Each teacher had a number of objectives for their lessons. Teachers wanted students to learn the geometric relationships and they wanted to be able to evaluate how well their students grasped the concepts. Jaworski (1994) recognized the issue of control in her research. In the present study, how the teacher reacted to this control issue contributed to their perceived degree of success of the program.

Teaching strategies are closely linked to classroom management strategies
(Keller, 1996). Teachers in Keller's study noted that students are more on task and self-managed in a computer classroom than in the regular mathematics classroom. Teachers also noticed that there was an increased noise level in the computer lab. These changes may not be directly linked to the use of the computer but as a result of other changes that were made to accommodate their use. Teachers took more interest in their students' successes and allowed the students the freedom to explore learning materials. These changes were enacted by teachers 'letting go of the reins'.

All four teachers in the present study faced the issue of control. Karen, Simon, and Mike made explicit reference to their temporary loss of control while Cathy made implicit reference to this issue. Of the four teachers, Karen's experience was most significant and will be used to develop a new understanding of the issue of teacher control.

Karen perceived a significant loss of control early in the study. She felt uncomfortable with the software and felt that she was not in control of the learning environment. Karen's students, although they were conditioned to ask and expect Karen to answer their questions, recognized this loss of control. The students began to take responsibility for their own learning. They began to share conjectures and to help each other develop the skills necessary to explore geometric relationships using the software. Karen regained her sense of control by the third day of the study through her successful interaction with the software the previous evening. By this time, however, her students were feeling comfortable with the software and helping each other. By the sixth day, Karen regained her confidence in her ability to teach geometric constructions using the software. Both Karen and her students were transformed by the experience. The students felt in control of their own learning and Karen regained her sense of control over the learning environment and her confidence as a teacher.
Karen's experience is similar to the experiences Frobisher (1994) noticed happening to teachers when a problem-centred classroom diverges from the traditional model. The sense of insecurity that teachers experienced using the computer software is consistent with the experiences of the teachers in a problem-based mathematics classroom.

The issue of teacher control can be viewed from three perspectives: management control of the learning environment, personal control, and professional control.

Management Control

The most significant effect on the teaching pedagogy of the teachers involved in this study relates to their interaction with their students. The teachers were, to various degrees, inculturated into the traditional teaching paradigm where the teacher structures the classroom so that the teachers can be the authority. Berebitsky (1985) found that elementary school mathematics teachers have a low level of mathematical background. There are a number of problems inherent in this situation. Teachers are not confident in their mathematical ability and, therefore, the textbooks are taken as the authority for mathematics. Steffe (1990) suggests that the mathematical concepts and how they are taught seldom get questioned.

Teachers having low level of mathematics background or those who depend on the textbook as if it were the curriculum, tend not to respond favourably to suggestions that they teach in an exploratory mode. They perceive this environment as being too difficult to control and that it requires the teacher to tolerate uncertainty about what the students are learning (Schoefeld & Verban, 1988; Cohen, 1989).
The role of the teacher in mathematics education also influences the control mechanisms the teacher places on the classroom environment. When teachers choose various situations for their classroom, they make judgments about the relevance of the situation to their students and how likely the students are to "bump" into the appropriate mathematics in the course of investigating the problem (Lappan & Briars, 1992). These activities will vary depending on the level of control and the tolerance level a teacher has within the classrooms. These levels of control and tolerance levels may restrict the use of cooperative/collaborative learning activities used by the teacher. Johnson et al. (1986) found that students working in cooperative learning groups had increased achievement within computer-based environments. Even with these findings, teachers may choose to have students work one to one with a computer simply to minimize the noise. In so doing, the teacher may inadvertently lessen the opportunities for students' discourse and shared meanings within a small mathematical community.

Personal Control

Teaching can be an isolated activity. The teacher is expected to teach around 30 students, maintain control, and inspire the class to learn (Cuban, 1986). Compound this problem by introducing a computer software tool and tensions develop between the teacher's perception of their role as a facilitator within the classroom and their personal control needs for perceived control of the learning environment.

These personal control needs are expressed in many forms. The need for mutual trust within the learning environment between the students and the teacher, the need of being the authority within the mathematics classroom and
the ability to freely admit mistakes are within this category. How a teacher perceives herself within the classroom and how the teacher reacts to personal rather than professional change has an impact on the degree to which change is accepted by the teacher.

Karen provides an interesting case for the importance of personal control within a middle school mathematics teacher's practice. Her initial difficulties with the software were documented earlier. While she had some concerns over the content of the geometry program, she was most concerned with her personal control over her environment. She wanted to maintain a personal presence in the classroom and expects respect from her students. She felt that this respect was synonymous with her personal control within the classroom. Karen made some changes in her teaching practice. These changes, however, were closely related to her feeling comfortable with and in control of her personal acceptance of the need to share the authority of mathematics. She also felt comfortable to share her lack of complete understanding of the software with her students. It was at this point that she began to recognize that her control over the teaching environment increased.

Simon, as a new teacher, had similar hopes in the classroom. He wanted students to recognize him as an individual. He was not concerned about making mistakes but he was unable to freely inform his class that mistakes were part of life. Simon also needed to maintain a degree of control over his personal life. He does not spend much time investigating mathematics outside of the classroom. Simon's willingness to share his 'authority' with his students began the change process. He realized that he didn't have to know everything and that sharing knowledge with the students actually allowed his personal control to increase.

Mike and Cathy appear to be very comfortable with their role both in and
out of the classroom. They have a personal interest in investigating mathematics and freely admit to their students that they make mistakes. They do not need to be the centre of attention and, perhaps as they have both taught for a number of years, felt confidence in their abilities to make changes within their teaching practice without creating a loss of personal control over their environment.

The issue of personal control is important to teachers new to the profession and those new to teaching mathematics. The need to be the centre of attention and to be the mathematics authority in the classroom does influence how a teacher reacts to change within their classroom. Students benefit from seeing teachers as evolving, learning members of the mathematics community. Rather than providing students with information and then determining if they have captured the concepts, knowledge and skills, teachers will need to become a part of a learning community and act as a model and a participant.

Professional Control

Prospective teachers enter a profession steeped in tradition and history. As a profession, teachers are well regarded in some communities and not in others and may experience some trepidation about their role within the community. Within the independent school system, teachers are usually well regarded for their hard work and dedication to the profession.

All four of the teachers in this study agree that there are many roles for the teacher within the classroom. They agree that being good in mathematics is important but not essential. The ability to motivate students is a key factor, according to Simon, while Cathy believes that teachers should ask questions to encourage students to explore mathematics. Both Cathy and Simon, by the end
of the study, saw the teacher's role as that of a facilitator while Mike used the word guide to describe his role in a more student-focused classroom environment. Karen continued to believe that her role was to 'teach'. That is, she should provide an environment where she is the transmitter of knowledge to the students. In each case, the perceived role of the teacher dictated the types of questions posed, the distribution of the worksheets, and the interaction between teachers and students.

How a teacher perceives the role of the teacher will contribute to the type and degree of control used in the classroom. A teacher who believes that the teacher should be a facilitator will naturally maintain a different form of control over the classroom. A facilitator will have less difficulty with open-ended activities and will invite questions from the class that will be different in scope and depth than from a teacher who believes that students need to be told what to learn and under what conditions. The transmission-type teacher will be less likely to open the students to new questions and interaction, the building blocks of a mathematical community.

Wood et al. (1991) describe taken-as-shared meanings as one component of a mathematical community. The sharing of definitions, questions, answers, and explorations contribute to the sense of building knowledge within a community. Although the community cannot have 'knowledge', the members of the community can construct their own knowledge through the social interaction with the other members of that community. This study provides further evidence that teachers can build a mathematical community within a computer-based exploratory environment that will assist students to have the freedom to explore mathematics and to share their findings with the classroom community.

Teacher control as a professional is a reality in middle school mathematics teaching. Teacher educators can assist teachers to maintain a level of control
over their professional lives by providing them with the tools to be mathematical explorers. Teachers need to be placed in learning environments where they can explore mathematics, interact with their peers through discussion and case studies, and work with dynamic computer environments. These dynamic computer environments provide an environment where teachers and students can interact and share their conjectures and findings with each other. Teacher educators should provide opportunities within their curriculum for teacher exploration in these computer-based tools.

5.4 Implications for Further Research

In most mathematics classrooms, the teacher is the central figure. It has been proposed that teachers transform their classrooms into taken-as-shared meaning communities where the teacher and students work together to develop a mathematical community. However, this study has suggested that this transition is not an easy one. Thus, we must ask new research questions: do the students move the locus of authority from the teacher to the computer? That is, do the students "believe" the computer software results are infallible? How could the students find out if their investigations were correct? Do the students have the autonomy to believe that they are able to generate expertise or is the computer the final authority?

The present study suggests that teachers should become explorers of geometric relationships. Their students were invited to make conjectures about the investigations provided by the teacher. Further research should focus on the students' conjectures and to the role of proof in these investigations. One research question could be: do the students' have a conviction that they have proven a relationship? That is, does the relationship work with every case and is
it a valid proof?

What is the student's view of proof and conviction? Does the computer make it better? Does the conviction created by the computer make proof useless? These three questions follow the premise that teachers should provide activities that allow the students to construct their own knowledge about mathematics, and in this case study, geometric relationships. The topics which immediately follow the constructions unit involve geometric proofs. Achieving a better understanding about how students view proof and conviction will assist the teacher in designing or finding activities, preferably computer activities, that would further student understanding about proof and conviction.

While some studies of change in mathematics teacher's locus of control have covered more than one year, few if any have encompassed the 3 to 5 years that Fullan (1992, 1991) suggests are necessary for the change process. There is a need for such longitudinal studies. Therefore, further research should be conducted to determine the transferability of some of the changes which occurred in this study in the computer classroom to the mathematics classroom. Do teachers move further towards their role as a facilitator in the classroom? Do teachers create time to investigate mathematics on their own?

Finally, teacher education programs should include activities that place teachers in learning environments where they can explore mathematics, interact with their peers through discussion and case studies, and to work with dynamic computer environments. These dynamic computer environments provide an environment where teachers and students can interact and share their conjectures and findings with each other. Educators of teachers should provide opportunities within their curriculum for teacher exploration using these computer-based tools. Moreover, research could be conducted to develop a better understanding of how the inservice and preservice programs provide potential
and current teachers with insight on creating a mathematics learning community within their classrooms.

5.5 Suggestions to Teachers in Making the Shift in Control

From this study, a number of suggestions can be made to teachers, consultants, teacher educators, and other researchers which may assist them in implementing a geometric exploratory classroom using a dynamic geometric software program. In point-form, the following items would be most helpful and necessary for teachers attempting to make this shift:

- the availability of a mentor. Having a coach available for technical and mathematical content support would be helpful.
- that sufficient time be provided for teachers embarking on this process.
- a valuing of the process by the administration of the school, the district officials, and the teachers themselves. In this study, the department heads and Heads of the Schools were very supportive of the project.
- reflection on the activity through journals and meetings with other teachers.
- a willingness to focus on the shift in control. These teachers participated freely and willingly in this project. What would have happened if every teacher had to be involved in the program as is normally the case in curriculum reform?
- specifically, teachers need to consider their noise tolerance level, class structure and the role of cooperative/collaborative activities in the mathematics classroom, the shift in control to the students, and ways still to feel 'in control' as a facilitator as well as how effective student
evaluation will be handled.

In considering these issues, teacher educators, teachers, researchers, and Heads of Schools can begin to consider the personal and professional cost of making wholesale curriculum changes. The need to feel in control of one's life and/or career is a basic survival need that must be satisfied if we wish to improve teaching practice. This study has permitted me to investigate the needs teachers have when they replace one set of physical tools with a set of electronic tools.
References


Science, San Jose University.


ICMI. (1994). What is research in mathematics education and what are the results (Discussion paper). College Park, Maryland: ICMI.


APPENDIX A: BELIEF QUESTIONNAIRE
Appendix A

Beliefs about Mathematics

Instructions: Indicate your agreement or disagreement with each statement below by circling the number that best reflects your belief.

<table>
<thead>
<tr>
<th>I. Your views about mathematics</th>
<th>Strongly Agree</th>
<th>Not Sure</th>
<th>Strongly Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I can handle basic math, but I don't have the kind of mind needed to do advanced mathematics.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. A lot of things in math must simply be accepted as true and remembered; there aren't really explanations for them.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Math helps you to think better.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. To be good at mathematics, you need to have a kind of &quot;mathematical mind&quot;.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. There is more than one right way to get the right answer in mathematics.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Mathematics is not just a bag of tricks.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>II. Learning mathematics</th>
<th>Strongly Agree</th>
<th>Not Sure</th>
<th>Strongly Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. For students to get better at math, they need to practice a lot.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. If middle school students use calculators, they won't learn the math they need to know.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. If students get into arguments about ideas and procedures in math class, it can interfere with their learning of mathematics.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. In learning math, students must master topics and skills at one level before going on.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. Once students can reason abstractly, the use of models and other visual aids becomes less necessary.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. How you get an answer is as important as whether the answer is right or wrong.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. Average mathematics students, with a little guidance, should be able to discover the basic ideas of mathematics for themselves.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14. The teacher should consistently use activities which require original thinking.</td>
<td>1 2 3 4 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
III Teaching Mathematics

15. If a student asks a question in math, the teacher should know the answer. 1 2 3 4 5

16. Being personally good at mathematical problem solving has little to do with being a good math teacher. 1 2 3 4 5

17. Basic computational skill and a lot of patience are sufficient for teaching middle school mathematics. 1 2 3 4 5

18. Students should never leave math class feeling confused or stuck. 1 2 3 4 5

19. Teachers should not necessarily answer student's questions but should let them puzzle things out themselves. 1 2 3 4 5

20. The most important issue is not whether the answer to any math problem is correct, but whether students can explain their answers. 1 2 3 4 5

21. Teachers should follow the math textbook that is used in their school. 1 2 3 4 5

22. Teachers should spend most of their class period explaining how to work specific problems. 1 2 3 4 5

23. Students working in cooperative groups can learn just as well as from whole class instruction. 1 2 3 4 5

IV Estimate the percentage of total class time you spend on the following: (0% - 100%).

☐ activities using manipulative models
☐ calculator/computer activities
☐ students working together in small groups
☐ going over homework assignments
☐ student explanations of their work
☐ students working independently at their seats
☐ teacher explanations of new material
☐ guided practice activities
APPENDIX B: WORKSHEETS FOR KAREN AND SIMON
The Freehand Tools

The dot icon is the point tool. Click in the sketchpad and create several points.

The circle icon is a compass for creating circles. Click the circle tool, then press and drag in the sketchpad to create a circle. The circle comes with two points: the circle's centre and a point defining the circle's radius (this point is referred to as a control point).

The arrow icon (at the top of the toolbox) is the selection/translate tool. Position the tip of the arrow over one of the points, press, and drag the point. Drag the other point. Drag the circle itself.

Assignment:

1. How do you change the location of the circle's centre? What do you notice?
2. Is there another way to change the circle's radius?
3. How do you move the circle without changing its radius?
4. Create this figure using the circle tool.

Notice that the centre of the second circle is the control point of the first circle. (And the control point of the second circle is the centre of the first circle.

5. Use the selection tool (the arrow) to move the circles (without changing its radius) to verify that your construction is correct.

If the figure "falls apart" on a drag, the control points and centres are not coincident. Recall, you can use the Edit menu to Undo any mistakes.
**LESSON 1 PAGE 3**

**CONSTRUCT AN EQUILATERAL TRIANGLE**

**Assignment (continued):**

The diagonal segment icon is the straightedge tool for constructing segments, rays, or lines.
Press the straightedge tool and hold down the mouse button. When you hold down the mouse button, segment, ray, and line icons pop out to the right. Drag to the right one space and release to select the segment tool. Drag two spaces to select the ray tool to create rays or three spaces to select the line tool to create full lines.

Click the segment tool and press one of the circle centres. Drag to the other circle centre to connect them with a segment. Recall that the small black squares on it indicate that it's selected.

The hand icon is the label/text tool.
Move the hand pointer in your sketch so that the finger is right over a point. (The hand will darken when correctly positioned.) Click the point. The point's label is displayed. To change the label, double click the label. To hide the label, click the object again.

6. Use the segment tool and the circles previously constructed to create an equilateral triangle. Label your triangle with vertices A, B, and C.

**Present your findings:**

**Illustrate your construction with a captioned diagram.**
(This should be done in your notebook.)

**Note:** You are only allowed to use a straight edge and compass.

**Save your diagram.** Use the file menu and save as "lesson1" on your disk. (Be sure not to save on the hard drive.)

7. Create a design using GSP.
Calculate

This command allows you to calculate arithmetic expressions using the values of selected measurements. When the measured objects change, the measurements and calculations change accordingly.

1. Construct any triangle ABC using the segment tool. Measure all interior angles.

2. Use the Calculate command from the Measure menu to find the sum of all three interior angles. Highlight the three angles you wish to sum by Shift-Clicking. Notice that you must choose the measurement(s) you want from the pop-up menu and the math operators from the calculator.

3. The Tabulate command from the Measure menu allows you to present the data in table form. Highlight the items you wish to put in this form by Shift-Clicking and then select the tabulate command.
Calculated Page 2

4. When the measured objects change, the measurements and calculations change accordingly and you may use the Add Entry command from the Measure menu to display different calculations.

\[
\begin{align*}
\text{Angle(ABC)} &= 34^\circ \\
\text{Angle(ACB)} &= 44^\circ \\
\text{Angle(CBA)} &= 102^\circ \\
\text{Angle(CBA)} + \text{Angle(ACB)} + \text{Angle(BAC)} &= 180.00^\circ
\end{align*}
\]

5. To change a label, double click on the label with the Text Tool (the finger). To change the look of the table, select the table, then use Flip Direction command in the Measure menu.

\[
\begin{align*}
\text{Angle(BAC)} &= 27^\circ \\
\text{Angle(ACB)} &= 96^\circ \\
\text{Angle(CBA)} &= 57^\circ \\
\text{Sum} &= 180.00^\circ
\end{align*}
\]

<table>
<thead>
<tr>
<th>Angle(BAC)</th>
<th>Angle(ACB)</th>
<th>Angle(CBA)</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.33</td>
<td>55.35</td>
<td>85.34</td>
<td>180.00</td>
</tr>
<tr>
<td>34.03</td>
<td>44.36</td>
<td>101.67</td>
<td>180.00</td>
</tr>
<tr>
<td>27.05</td>
<td>95.75</td>
<td>57.22</td>
<td>180.00</td>
</tr>
</tbody>
</table>
**Lesson 2**

**Investigation of Angles**

**Homework:**

1. **Define:**
   a) Equilateral Triangle (two definitions?)
   b) Circle
   c) Perpendicular
   d) Angle

2. **Present your findings:**
   Illustrate your conjecture(s) with a captioned diagram.

3. **Worksheet**
   Save your diagram. Use the file menu and save as “lesson2” on your disk.

**Assignment:**

1. Use your diagram from lesson 1. Drag your diagram. If it falls apart your construction is incorrect.
2. Record which vertex does not change the diagram on a drag. Explain your answer.
3. Record which segment(s) does not change the diagram on a drag. Explain your answer.
4. Discuss your findings with your partner.
5. Use the commands under the measure menu to
   a) Measure all three angles of your triangle.
   b) Measure all three segments of your triangle.
   What do you notice about your answers to a) and b)?
6. Use the construct menu to construct a line that is perpendicular to one side of the triangle and that passes through the intersection of the other two sides.
7. Measure various angles and various side lengths. State your conjecture(s). Be sure to drag your figure to test your conjecture(s).
Construction: Duplicating an Angle

In this activity, you’ll learn how to duplicate a given angle. The method described is equivalent to the method you would use with a compass and straightedge. You might want to follow the first few steps then try to figure out the rest on your own. This construction is a building block for many other, more complex constructions. You may want to record and save a script for duplicating an angle.

Sketch

**Step 1:** Construct rays \( \overline{AB} \) and \( \overline{AC} \). (This is your given angle.)

**Step 2:** Construct \( \overline{DE} \), one side of new angle.

**Step 3:** Construct circle \( \overline{AF} \) and \( \overline{AF} \) with \( F \) on \( \overline{AB} \).

**Step 4:** Construct \( \overline{FG} \), where \( G \) is the point of intersection of the circle and \( \overline{AC} \).

**Step 5:** Construct a circle with center \( D \) and radius \( \overline{AF} \).

**Step 6:** Construct \( \overline{HJ} \), the point of intersection of this circle with \( \overline{DE} \).

**Step 7:** Construct a circle with center \( H \) and radius \( \overline{FG} \).

**Step 8:** Construct \( \overline{DJ} \), where \( J \) is the point of intersection of these two circles.

**Step 9:** If you wish, hide the circles, segments, and points \( H, F, \) and \( G \).

Investigate

Move points \( A, B, C, D, \) or \( E \). Do the angles remain congruent? Confirm that they’re congruent by measuring them. When you try to drag \( J \), why doesn’t \( \angle JDH \) change?

Present Your Findings

Discuss your construction with your partner or group. To present your findings you could create a commented script that duplicates an angle, explaining why it works.

Explore More

1. Construct two unconnected segments and an angle. Now construct a triangle by duplicating the angle and the two sides on the sides of the angle. Is there more than one way to do it? Are all the triangles you can construct this way congruent?

2. The construction described in this activity duplicates angles only in a counterclockwise direction. (If you move your original angle past 180°, your duplicate will still have equal measure, but will have a different orientation.) Come up with a construction for duplicating an angle in the clockwise direction.
Investigation: Properties of Parallel Lines

In this investigation you'll discover relationships among the angles formed when parallel lines are intersected by a third line called a transversal.

Sketch

*Step 1:* Construct \( \overline{AB} \) and point \( C \), not on \( \overline{AB} \).
*Step 2:* Construct a line parallel to \( \overline{AB} \), through \( C \).
*Step 3:* Construct \( \overline{CA} \) and points \( D, E, F, G, \) and \( H \) as shown.
*Step 4:* Measure the eight angles in your figure.

Investigate

Drag point \( A \) or \( B \) and watch which angles stay equal. (Be careful not to change the point order on your lines—that will change the angles Sketchpad measures!) Make a table in your sketch to keep track of these angles' measures as the transversal is changed. In the chart below, one example of each type of angle pair is given. Fill in the chart with other angle pairs of that type, then state what relationship, if any, you observe between the angles in a pair. Note: there are more than two pairs of one of these types. Can you identify which type has more than two pairs?

<table>
<thead>
<tr>
<th>Angle Type</th>
<th>Pair 1</th>
<th>Pair 2</th>
<th>Relationship?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding</td>
<td>( \angle FCE ) and ( \angle CAB )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternate Interior</td>
<td>( \angle ECA ) and ( \angle CAG )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternate Exterior</td>
<td>( \angle FCE ) and ( \angle HAG )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consecutive Interior</td>
<td>( \angle ECA ) and ( \angle BAC )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consecutive Exterior</td>
<td>( \angle FCD ) and ( \angle HAG )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conjecture: Write your conjectures below.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Present Your Findings

Discuss your results with your partner or group. To present your findings you could print a captioned sketch showing parallel lines and the measures of the angles.

Explore More

What about the converses of the above conjectures? Start out with two lines that are not quite parallel. Intersect them with a transversal and measure corresponding angles. Now move a line until the angles are equal. Measure the slopes of the lines. If their slopes are equal, then they're parallel. Write your findings and conjectures on the back of this sheet.
Investigation: Perpendicular Bisectors

In this activity, you’ll use only Sketchpad’s freehand tools to construct and investigate properties of perpendicular bisectors.

Sketch

Step 1: Construct $\overline{AB}$.

Step 2: Construct circles $\overline{AB}$ and $\overline{BA}$.

Step 3: Construct $\overline{CD}$, where $C$ and $D$ are the points of intersection of the circles.

Step 4: Construct $E$, the point of intersection of $\overline{AB}$ and $\overline{CD}$.

Step 5: Hide the circles.

Investigate

Line $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$. Move points $A$ and $B$. What’s special about point $E$? Can you come up with a shortcut for constructing a perpendicular bisector using the Construct menu? Construct a point $F$ on $\overline{CD}$. Measure the distances $FA$ and $FB$. Move point $F$ up and down the line. What can you say about any point on a segment’s perpendicular bisector?

Conjecture: Write your conjectures below.

Present Your Findings

Discuss your findings with your partner or group. To present your findings you could:

1. Create a script for constructing a perpendicular bisector with comments explaining why your construction works.
2. Print a captioned sketch with measures illustrating your conjectures.

Explore More

1. Construct the perpendicular bisectors of the three sides of a triangle. Investigate their point(s) of intersection. Can you construct a circle that circumscribes the triangle?

2. Construct $\overline{AC}$ and $\overline{AF}$. Mark your perpendicular bisector as mirror and reflect $A$, $\overline{AC}$ and $\overline{AF}$ across it. Where is $A'$ (the reflection of $A$) located? How do the triangles formed by this reflection help explain why $C$ and $F$ are equidistant from $A$ and $B$?
Construction: Angle Bisectors

You can bisect an angle automatically with Sketchpad's Construct menu. But an angle bisector is not difficult to construct using only freehand tools. In this activity, you'll bisect an angle the way Euclid did it, then you'll investigate properties of angle bisectors.

Sketch

Step 1: Construct $\overline{AB}$ and $\overline{AC}$.
Step 2: Construct circle $AD$, with $D$ on $\overline{AB}$.
Step 3: Construct circle $DE$, where $E$ is the intersection of $\overline{AC}$ and circle $AD$.
Step 4: Construct circle $ED$.
Step 5: Construct $\overline{AF}$, where $F$ is the intersection of circles $DE$ and $ED$ that is farthest from $A$.
Step 6: Hide the circles and points $D$, $E$, and $F$.
Step 7: Construct a point $G$ on the angle bisector.

Investigate

Drag point $B$ or $C$. Does $\overline{AG}$ continue to bisect the angle? Measure the distances from $G$ to $\overline{AB}$ and $G$ to $\overline{AC}$. Move point $G$ along the angle bisector. What do you notice about the distances to the sides of the angle?

Conjecture: Write your conjectures below.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Present Your Findings

Discuss your findings with your partner or group. To present your findings you could:

1. Create a script to construct an angle bisector and comment it to explain why it works and describe your conjectures.
2. Print a captioned sketch with measures illustrating your conjectures.

Explore More

You'll notice that your angle bisector construction behaves strangely if you open up your angle past $180^\circ$. This behavior is different than the behavior of the angle bisector Sketchpad constructs when you choose Angle Bisector in the Construct menu.

Experiment with the Construct menu Angle Bisector, then see if you can construct one that behaves that way. Hint: you'll use a midpoint.
Lesson 9

Angles in a Circle

Assignment:

1. Construct circle AB. (see diagram below)
2. Construct segments AC and AD to create a central angle CAD.
3. Construct EC and ED, where E is a point on the circle, to create inscribed angle CED.

Central Angles

An angle with its vertex at the centre of a circle is called a central angle.

Inscribed Angles

An angle whose sides are chords of a circle and whose vertex is on the circle is called an inscribed angle.

Present your findings:

Illustrate your conjectures with a captioned diagram.

4. Measure the central and inscribed angles.
5. Move points C and D. Conjectures?
6. Move point E. Conjecture?

Save as "lesson9"

7. Use the construct menu to construct the angle bisector of \( \angle CED \).
8. Where this bisector meets the circle, label the point of intersection F.
9. Construct and measure \( \angle CFD \).
10. Measure the arc length from C to D.
11. When \( \angle CED \) becomes greater than 90°, what happens to the arc length from C to D?
12. Move E all around the circle. What happens?
Lesson 8

Chords in a Circle

Assignment:

1. Construct a circle with a quadrilateral in it.
   (as shown below)

   Chords in a Circle
   Each of the four sides of the quadrilateral is a chord of the circle (a segment with endpoints on a circle).

2. Measure the lengths of these chords and the arcs intercepting them.

3. Measure the angles of the inscribed quadrilateral.

4. Move the vertices around the circle and look for relationships among your measurements.

5. State your conjectures.

   Save as "lesson8"

6. Can you find a way to use chords to determine the centre of a circle?
   (Check your result by hiding the centre, figure out where it is, then use the Show All Hidden command to see if you were right.)
Demonstration: Triangle Congruence—SSS?

In this demonstration you'll work with a sample sketch to see how many different triangles can be created given the lengths of the three sides.

Sketch

**Step 1:** Open the sketch: SSS (Mac) or 3triangl\congrnc\ss.gsp (Windows).
You'll see two broken triangles and three separate segments.

**Step 2:** Drag different parts in the broken triangles. Note that you can't change the lengths of the sides of these "triangles."

**Step 3:** Drag the two points labeled B in the broken triangles so that they coincide, forming triangles.

**Step 4:** See if you can construct two different shaped triangles in this way.

**Step 5:** Change the lengths of one or more of the "given" sides (the free segments below the triangles) and try the experiment again.

Investigate

Could you form triangles with different sizes or shapes given the three sides? If you were given two triangles with three pairs of congruent sides, would that be enough information to determine that the triangles were congruent?

**Conjecture:** Write your conjectures below.

---

**Explore More**

1. Change the three given segments so that you can no longer connect the points B to create a triangle. Under what conditions will the points B not reach each other?

2. Open a new sketch and see if you can create this demonstration yourself. For a hint, show all the hiddens in the demonstration sample sketch. The command Circle By Center+Radius was used to construct congruent segments.
**Demonstration: Triangle Congruence—SAS?**

In this demonstration you'll work with a sample sketch to see if you can make two non-congruent triangles given two sides and the angle between them.

**Sketch**

**Step 1:** Open the sketch: SAS (Mac) or 3triangle\congruence\sas.gsp (Windows). You'll see two broken triangles and two separate segments and an angle.

**Step 2:** Drag different parts of the broken triangles. You'll find that some lengths and angles can't be changed because they're constrained by the "givens."

**Step 3:** Drag endpoint B (drag) of BC in one broken triangle so that the points B overlap, forming a triangle.

**Step 4:** With the second broken triangle, see if you can form a triangle with a different size and/or shape.

**Step 5:** Change the given sides and/or angle and try the experiment again.

**Investigate**

Could you form non-congruent triangles given two sides and the angle between them? If you were given two triangles with two pairs of corresponding sides and the angles between them congruent, would that be enough information to determine that the triangles were congruent?

**Conjecture:** Write a conjecture below.

---

**Explore More**

See if you can create this demo sketch yourself. Use the script Duplicate Angle CW (Mac) or 1lineang\dupangcw.gss (Windows) or your own utility script to duplicate angles and use the Construct menu command Circle By Center+Radius to duplicate segments.
Demonstration: Triangle Congruence—ASA?

In this demonstration you’ll work with a sample sketch to see if you can create two different triangles given two angles and the side between them.

Sketch

*Step 1:* Open the sample sketch ASA (Mac) or 3triangl\congrmce\asa.gsp (Windows). You’ll see two broken triangles and two separate angles and a segment.

*Step 2:* Drag different parts of the broken triangles. You’ll find that some lengths and angles can’t be changed because they’re constrained by the "givens."

*Step 3:* Drag the points C in one broken triangle so that they overlap, forming a triangle.

*Step 4:* See if you can connect points C in the other broken triangle to create a second triangle, not congruent to the first.

*Step 5:* Change the given sides and/or angle and try the experiment again.

Investigate

Could you form non-congruent triangles given two angles and the side between them? If you were given two triangles with two pairs of corresponding angles and the sides between them congruent, would that be enough information to determine that the triangles were congruent?

Conjecture: Write a conjecture.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

Explore More

Just for kicks, see if you can make the two points C overlap by changing the givens instead of dragging the points themselves.
In this demonstration you'll work with a sample sketch to see how many different triangles you can create given the three angles.

**Sketch**

**Step 1:** Open the sketch: AAA (Mac) or \texttt{3triangle\cngrmc\aaa.gsp} (Windows). You'll see a triangle and three separate angles.

**Step 2:** Drag one of the points on the sides of the separate angles, \( \angle A \) or \( \angle B \). Notice that \( \angle C \) changes as you do this. You can't directly change its measure because it was constructed so that the measures of the three given angles add up to 180\(^\circ\).

**Step 3:** Drag point \( A \) or point \( B \) in the triangle. Again, you can't drag point \( C \) in the triangle, because \( \angle C \) is determined by \( \angle A \) and \( \angle B \).

**Investigate**

Using three given angles, is it possible to make two triangles that are not congruent? If you were given two triangles with three pairs of congruent angles, would that be enough information to guarantee that the triangles were congruent?

**Conjecture:** Write a conjecture.

---

**Explore More**

1. Mark some point as center and dilate \( \triangle ABC \) using the Transform menu. Your two triangles are not congruent. Have their angles changed?

2. Change the given angles \( \angle A \) or \( \angle B \) so that \( \angle C \) disappears. What happened to \( \angle C \)? When is it impossible for two particular angles to be in the same triangle?
Demonstration: Triangle Congruence—SSA?

In this demonstration you’ll work with a sample sketch to see how many different triangles can be created given two sides and an angle not between them.

Sketch

Step 1:  Open the sketch SSA (Mac) or 3triangle\congruence\ssa.gsp (Windows). You’ll see two broken triangles and two separate segments and an angle.

Step 2:  Drag different parts of the broken triangles. You’ll find that some lengths and angles can’t be changed because they’re “given.”

Step 3:  Drag the points B in one broken triangle so that they overlap, forming a triangle.

Step 4:  See if you can connect the points B in the other triangle so that the two triangles are not congruent.

Step 5:  Change the given sides and/or angle and try the experiment again.

Investigate

How many different triangles can you form given two sides and the angle not between them? Do two sides and an angle not between them uniquely determine a triangle? If you were given two triangles with two pairs of corresponding sides and the angles not between them congruent, would that be enough information to determine that the triangles were congruent?

Conjecture:  Write your conjectures below.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Explore More

By changing the given sides and angle, you’ll find that in some cases you can’t create any triangle and with some combinations you can create only one triangle. Under what conditions can you create only one triangle given two sides and an angle not between them?
APPENDIX C: WORKSHEETS FOR CATHY
INTRODUCTION

Part A

1. Use the Segment tool and construct a triangle.

2. Use the Selection arrow and drag different vertices and sides of the triangle.

3. Use the Text tool (hand icon) to label one of the vertices. [Move the hand so that the tip of the finger is centred over it. The hand will reverse colour. Click on the vertex. This should give you a capital letter. When you open a new sketch, it starts from A.]

4. Move that letter around near the point. [Press on it and drag.]

5. Change the letter to X and change its font. [Double click on it and type X in the dialog box. Click on Style... and then change the font to Times. Use return, return.]

6. Add labels the other vertices. Hide the labels. [Click on the points again.]

7. Use File, New sketch to get a clear screen.

Part B

1. Use the Segment tool to construct a segment.

2. Select the Circle tool. Press on one of the segment’s endpoints and drag until the pointer [one control point of circle] is directly over the other endpoint. Use the Selection arrow to drag each point to confirm that the segment’s endpoints really define the circle. [The whole circle and its radius should all move together.]

3. Use the Segment tool to construct a second radius. [Start at the centre, drag, and release when the pointer is positioned anywhere on the circle.]

4. Connect the endpoints of these radii to complete the triangle.

5. What special kind of triangle is it? ___________ How do you know? ___________
   Use the Selection arrow to drag the different vertices of the triangle. Does it always stay isosceles? See how each point behaves differently to change the size and shape of the triangle. Which point won’t change the length of the legs (equal sides) of the triangle when you drag it? Why? ________________

6. Hide the circle. [Use the Selection arrow and click on the circumference of the circle. You will see 4 black squares around the circle to show it is selected. Use Display, Hide circle.] Now drag the vertices of the triangle. See that even with the circle hidden, it still acts in the geometry of your construction. The hidden circle continues to make it isosceles.
7. Use Edit, Select all and the delete key to clear your screen.

**Part C**

1. Press the Segment tool and hold down the mouse button. You will see segment, ray and line icons pop out to the right. Create rays and lines and segments in your sketch.

2. Label the endpoints of one of the segments.

3. Find the length of the segment. [Select the segment by clicking on it (not the endpoints) with the Selection arrow. Use Measure, Length.]

4. Repeat #2 and 3 with another segment.

5. Change the length of one segment. [Drag on one of the endpoints.]

6. Press the option key and use Edit, Undo ..., Edit, Undo all to clear your screen.

**Part D**

1. Use the Circle tool to construct a circle.

2. Use the Segment tool (not ray or line) to construct a segment by pressing somewhere outside the circle, then dragging and releasing on the point (circle) on the circumference. Move the circle. What happens? Move one end of the segment. What happens? Move the other end of the segment. What happens? Why?________


4. Use Edit, Select all and the delete key to clear your screen.

**Part E**

1. Make a triangle. Label the vertices.

2. Measure one of the angles. [Click on one vertex, use the shift key and then click on the other 2 vertices. Use Measure, Angle.] What angle did you measure? ______

3. Measure the other angles. How can you tell what angle you have measured? ______

4. Dragging each of the vertices. What happens to the measurements? ______

**Challenge**

Construct an equilateral triangle using 2 circles and the segment tool.
GRADE 8 GEOMETER'S SKETCHPAD

HOW TO

TOOLS

Selection (arrow icon) - to select an object
Point (dot icon) - to draw a point
Straightedge (segment icon) - to draw a segment defined by 2 points
Straightedge menu - pops out to the right to draw ray or line defined by 2 points
Compass (circle icon) - to draw a circle described by 2 points: centre and control
Text (hand icon) - to add labels, to create a box and write inside

TASKS

Labels

To label a point, use text tool. Move the hand so that the tip of the finger is centered over the point and click on it.

To change the label (either the name or the font, etc.), double click on the label.

To hide the label, click on the point again.

Measure

To measure length, select the line segment (2 black squares are shown) and use Measure, Length.

To measure an angle, click on 3 vertices in order while holding down the shift key. The middle letter is the one that is being measured. Use Measure, Angle.

To measure the slope of a line, select it (2 black squares are shown) and use Measure, Slope.

Calculate

To add, subtract, etc., use Measure, Calculate and then click on the measures in order using the appropriate signs from the keypad.

Parallel/Perpendicular Lines

To construct either of these, select the point and the line while holding down the shift key. Use Construct menu.
Others

To hide a circle, select it (4 black squares are shown) and use Display, Hide circle.

To display the hidden circle, use Display, Show all hidden.

To automatically label lines, select Display, Preferences and click in the box beside Points.

To find the midpoint of a line, select the line (2 black squares are shown) and use Construct, Point at Midpoint.

To create a table, highlight the selected data that you want in table form. Use Measure, Tabulate.

To undo actions, use Edit, Undo. This will keep going backwards undoing things one at a time.

To undo everything, use option key and Edit, Undo, Edit, Undo.

To select everything, use Edit, Select all.

To get a new sketch, use File, New sketch.
GRADE 8 GEOMETER'S SKETCHPAD

ART: Daisy

A daisy design is a simple design that can be created using only a compass. From the basic daisy, you can create more complex designs based on a regular hexagon.

1. Construct circle AB.

2. Construct circle BA. (Make sure that they are 'glued'.)

3. From the points of intersection of these circles, continue constructing circles to existing points. (All these circles should have equal radii.)

4. Points labelled here show one possible order for constructing these circles: CB, DC, ED, then FG and GF. Construct FG instead of FE to avoid having 3 intersecting circles without an intersection point. If your last circle refuses to be constructed, you're probably releasing the mouse on the intersection of 3 circles. If so, select 2 circles and construct their intersection with the Construct menu. Then use this point to construct your final circle.

5. You might want to remove the labels.

6. At this point, you may wish to use the Segment tool to add some lines to your design.

7. You could construct circle and polygon interiors and experiment with shading, but you can probably get better results by printing out the basic design and adding colour by hand.
ART: Hexagons and Stars

1. The 6 points of the daisy define 6 vertices of a regular hexagon. You can use these points as the basis for a hexagon. To make a regular hexagon, use the Segment tool to connect the outside points of the daisy.

2. A 6-pointed star can be made by connecting alternate points to create intersecting equilateral triangles.

3. Midpoints of lines can be used to create more complex designs.

4. Once you have all the lines and polygon interiors that you want, you can hide unneeded points. You probably don't want to hide your original 2 points though as you can use these points to manipulate the figure.

5. Experiment - see what you can create.
LESSON 1: Sum of the Interior Angles of a Triangle (ASTT)

Warm-up

1. Construct an equilateral triangle using 2 circles and the segment tool. This was yesterday's challenge. Make sure that the circles are 'glued'.

2. Measure all 3 sides. [Select tool; Measure, Length] What do you notice? ________________

3. Measure all 3 angles. [Select all 3 vertices, Measure, Angle] What do you notice? ________________

4. What are your 2 conjectures about equilateral triangles? ________________

5. Hide both circles. [Select them. Use Display, Hide circle.]

6. Move the various vertices of the triangle to change its shape. What happens to the lengths of the sides? ________________ Why? ________________

7. Use the Text tool and make a box under your triangle. Write your conjecture in the box, starting with 'Equilateral triangles have...'

8. Put the circles back in. [Use Display, Show all hidden.]

9. Use the Text tool to give your page a title and add your names. Use Print preview to view your page and then print 2 copies of it.

Lesson

1. Use the Segment tool to construct a triangle. Use the Text tool to label the vertices.

2. Measure each angle.

3. Find the sum of all 3 interior angles. [Use Measure, Calculate. Use the Selection arrow to click on one of the angle measurements, use the + sign, then click on the 2nd angle, use + and then click on the third. Press return.] What is the sum? ________________

4. As you manipulate the triangle (move the vertices around), what happens to the sum of the angle measures in all of the different triangles? ________________

5. What is your conjecture about the Sum of the Interior Angles of a Triangle (ASTT)? ________________

6. Write your conjecture in a box. Start 'The sum of the interior angles of a triangle...'

7. Put the data in table form. [Use the Selection arrow. Highlight the first angle measurement and using the shift key, highlight the other 2 measurements and the sum. Use Measure, Tabulate.] This data will not change as you manipulate the triangle. Try it.

8. Give your page a title and add your names. Use Print preview. Print 2 copies of it.
LESSON 2: Exterior Angles of a Triangle

Warmup

1. Change the units to inches and add labels for points. [Use Display, Preferences. Open menu of cm. Click on box for Autoshow labels.]

2. Make a line segment. Make it exactly 2 inches long. [Drag on 1 end of it.]

3. Draw a point off that line.

4. Construct a perpendicular from the point to the line. [Select the point, use the shift key and select the line. Use Construct, Perpendicular.] Label the point of intersection.

5. Measure the angles $\angle ADC$ and $\angle BDC$. What does this prove? ______________

6. Write your conjecture about perpendicular lines on the sketch.

7. Give your page a title and your names. Use Print preview and then print 2 copies.

Lesson

The exterior angle of a triangle is formed when one of the sides is extended. An exterior angle lies outside a triangle.

1. Construct ray $\overrightarrow{AB}$.

2. Construct $\overline{AC}$ and $\overline{CB}$ to create $\triangle ABC$.

3. Construct D on $\overline{AB}$, outside of the triangle.

4. Measure the exterior angle $\angle CBD$.

5. Measure the remote interior angles $\angle ACB$ and $\angle CAB$.

6. Find the sum of the 2 remote interior angles. [Use Measure, Calculate.] How does this sum relate to the exterior angle’s measurement?__________________________

7. What is your conjecture about the Exterior Angles of a Triangle? ______________

8. Write your conjecture on the sketch. Start with 'The exterior angles of a triangle...”

9. Give your page a title and your names. Use Print preview and then print 2 copies.

Homework

Text page 295 #1-6
GRADE 8 GEOMETER'S SKETCHPAD

LESSON 3: Angles Formed by Intersecting Lines (VOA) and (SAT)

Warm-up

1. What was your conjecture about the exterior angles of a triangle? ________________

2. Can you explain why this conjecture is true? The reason has to do with the conjecture about the sum of the angles in a triangle (ASS).

3. Write a proof that explains the exterior angle conjecture. ________________

Lesson

When 2 lines intersect, they form 4 angles whose vertices are the point of intersection.

1. Use Display, Preferences to automatically label lines.

2. Construct $\overline{AB}$ and $\overline{CD}$ so that they intersect. Add a point at their intersection.

3. Measure all 4 angles. What relationships do you notice? ________________

4. Drag the endpoints. What happens to the relationships among the angles? ________________

5. How do you describe the angles that are equal? ________________

   They are called vertically opposite angles.

6. What relationship do you see between the pairs of unequal angles? ________________

   Use Measure, Calculate to add and test your conjecture. These angles that share one side and whose other sides form a line are called supplementary angles.

7. Summarize your conjectures in 2 boxes. Start 'Vertically opposite angles...' and 'Supplementary angles...'

8. Give your page a title and your names. Use Print preview. Print 2 copies.


10. If you knew the value of 1 angle out of 4, could you find the other 3 angles? ________________

Homework

1. Text page 299 #1-2

2. Draw a compass rose showing all 8 points: N, NE, E, SE, S, SW, W, NW. Label the points. What angles are vertically opposite? ________________

   What is the measure of each angle on the compass rose? ________________
LESSON 4: Properties of Parallel Lines (Z), (F), (C)

Warm-up

1. Last class you investigated intersecting lines. Do you remember your 2 conjectures?
2. If you know the measure of 1 angle, can you calculate the other 3 angles? ________
3. If 3 lines intersect in a single point, how many angles are formed? ________
4. Draw 3 lines that intersect at one point using the Segment tool. How many angles do you have to know in order to find out the rest? ________

Lesson

1. Construct AB and a point C, not on AB.
2. Construct a line parallel to AB, through C. [Select AB and C. Use Construct, Parallel.]
3. Construct CA. Add points D, E, F, G and H. Make the diagram exactly as shown below.

4. Measure the 8 angles in your figure. Move the measurements of each angle into the angles as shown above.
5. Drag point A or C and watch which angles stay equal.
6. Fill in the following table:

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Pair 1</th>
<th>Pair 2</th>
<th>Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>&lt;DCAand&lt;CAB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F (right side)</td>
<td>&lt;FCEand&lt;CAB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F (left side)</td>
<td>&lt;FCDand&lt;CAG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>&lt;ECAand&lt;CAB</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Write your conjectures on the sketch. Title it. Add your names. Print 2 copies.

Homework

1. Text page 301 #2-3.
GRADE 8 GEOMETER'S SKETCHPAD

LESSON 5: Angle Bisectors

Warm-up

1. Let's explore the converse (opposite) of the conjectures about parallel lines from Lesson #4. Start with 2 lines that are not quite parallel. Intersect them with a transversal (diagonal line) and measure the angles in the Z patterns. Now move a line until the pairs of angles are equal.

2. Measure the slope of the lines. [Select the line and use Measure, Slope.] If their slopes are equal, then the lines are parallel.

3. What did you find? _______ What is your conjecture? __________________

Lesson

You can bisect an angle automatically with the Construct menu but we are going to construct an angle bisector using freehand tools.

1. Construct \( \overline{AB} \) and \( \overline{AC} \). Construct circle \( AD \), with \( D \) on \( \overline{AB} \).

2. Construct circle \( DE \), \( E \) is the intersection of \( \overline{AC} \) and circle \( AD \). Construct circle \( ED \).

3. Construct \( \overline{AF} \), with \( F \) as the intersection of circles \( DE \) and \( ED \).

4. Hide the circles and points \( D \), \( E \) and \( F \). Construct a point \( G \) on the angle bisector.

5. Measure \( \angle CAG \) and \( \angle GAB \). Are they equal? _______ What does this prove? _______

6. Drag point \( B \) or \( C \). Does \( \overline{AG} \) continue to bisect the angle? _______

7. Measure the distances from \( G \) to \( \overline{AB} \) and \( G \) to \( \overline{AC} \). Move \( G \) along the angle bisector. What happens to the distances to the sides of the angle? ________________________________

8. What are your conjectures? ___________ Explain why it works? ___________

9. Write your conjecture on the sketch. Title it. Add your names. Print 2 copies.
**LESSON 6: Perpendicular Bisectors**

**Warm-up**
1. Construct an angle bisector as you did yesterday. Hide the circles.
2. Construct another angle, select the 3 points and use the Construct menu to bisect it.
3. Compare what happens to each angle bisector as you make the angles more than 180°.

**Lesson**

You can perpendicularly bisect a line automatically with the Construct menu but we are going to construct a perpendicular bisector using freehand tools.

1. Construct \( \overline{AB} \). Construct circles \( \overline{AB} \) and \( \overline{BA} \).
2. Construct \( \overline{CD} \), where \( C \) and \( D \) are the points of intersection of the circles.
3. Construct \( E \), the point of intersection of \( \overline{AB} \) and \( \overline{CD} \).
4. Hide the circles.

   ![Diagram of points A, B, C, D, E](image)

5. Line \( \overline{CD} \) is the perpendicular bisector of \( \overline{AB} \). What does that mean? ______

6. Measure \( \angle \text{AEC} \) and \( \angle \text{BEC} \). What is their measurement? ______

7. Move points \( A \) and \( B \). What is special about point \( E \)? ______ Measure \( \text{AE} \) and \( \text{EB} \). Are the distances equal? ______

8. Construct a point \( F \) on \( \overline{CD} \). Measure the distances \( \text{FA} \) and \( \text{FB} \). What is true? ______

9. Move point \( F \) up and down the line. What can you say about any point on a segment's perpendicular bisector? ______

10. Write your conjecture on the sketch. Title it. Print 2 copies.

11. Why does this construction work? ______
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LESSON 7: Chords in a Circle

Warm-up

1. Construct the perpendicular bisectors of the 3 sides of a triangle. [Use the menu.]
2. Investigate their point of intersection. Drag the vertices. What happens? ________
3. Construct a circle that circumscribes the triangle (centre of the circle is the point of intersection and the vertices are on the circumference of the circle).
4. Title it 'Circumcircle of a Triangle'. Add your names. Print 2 copies.

Lesson

A chord of a circle is a line segment with endpoints on a circle.

1. Construct a circle and a quadrilateral inscribed in it as shown below. Each of the 4 sides of the quadrilateral is a chord of the circle.

2. Measure the lengths of the chords.
3. Measure the interior angles of the inscribed quadrilateral. Move the measurements inside their respective angles.
4. Move the vertices around the circle and look for relationships among the measurements.
5. Can you find a way to use the chords to determine the centre of the circle? Check your result by hiding the centre, figure out where it is and then use Display, Show All Hidden to see if you were right. [Use midpoints and perpendicular bisectors.]
6. What are your 3 conjectures? ____________________________________________

7. Write your conjectures on the sketch. Title it, including your names. Print 2 copies.
8. Construct a circle with radius of 2.6 cm.
9. Construct a circle with diameter of 5.01 cm.
LESSON 8: Angles in a Circle

Warm-up

1. Construct a circle AB. Construct the radius and diameter of this circle.

2. Measure the circumference(C). Measure the radius(r) and the diameter(d)

3. Calculate the ratio of the circumference to the diameter. (Divide C by d.)

4. Drag point B to change the diameter. What happens to the diameter? ________
What familiar number is this ratio? ________

5. Measure the area of the circle. Calculate the ratio of the area to the radius$^2$(r$^2$) __

Lesson

An angle whose sides are radii and with its vertex at the centre of the circle is called a central angle. An angle whose sides are chords of the circle and whose vertex is on the circle is called an inscribed angle.

1. Construct a circle AB.

2. Construct AC and AD to create the central angle <CAD.

3. Construct AD and ED, where E is a point on the circle, to create inscribed angle <CED.

4. Measure <CAD and <CED.

5. Move C or D, to change <CED but keep it acute. What is your conjecture? ________

6. Move E to find various inscribed angles on the same arc. What is your conjecture? ________

7. Write your conjectures on the sketch. Title it. Print 2 copies.

8. What happens when you move B to make the circle bigger or smaller? ________

9. What happens to central angle <CAD when inscribed angle <CED is 90°? ________

10. Construct another circle AB. Construct its diameter CB. Construct CD and BD, where D is a point on the circumference. What can you say about any angle inscribed on a semi-circle? ________________
GRADE 8 GEOMETER'S SKETCHPAD

ASSIGNMENT

You have learned how to construct several things using The Geometer's Sketchpad. For example, using freehand tools only, you constructed an isosceles triangle and an equilateral triangle, in Lesson #5, you bisected an angle and in Lesson #6, you right bisected a straight line. You were given the steps for these constructions. Refer to these lessons when necessary. Notice the use of circles with equal radii.

(a) Now you are to make 2 constructions using only freehand tools with your partner:

- 1- construct a perpendicular to a given line from a point off the line

- 2- construct a perpendicular to a given line at a point that is on the line

(b) Do them as separate sketches with respective titles and your names.

(c) Measure the 2 angles that prove that it is a perpendicular. Move the measurements into the angles as you did in Lesson #4.

(c) Then write the steps you used for the constructions. Look at the lessons that you have done as a pattern for you. You may use the text tool on the sketch or do a separate Word document.

Due:
GRADE 8 GEOMETER'S SKETCHPAD

ENRICHMENT

Scripts - Making your own
1. Start a new sketch. Use Display, Preferences to turn off Autoshow labels.
2. Choose File, New script. Size the windows so you can see both script and sketch.
3. Record the construction of a triangle. [Click REC button in script window. Construct a triangle in the sketch window.]
4. Create midpoints. [Use Edit, Select all segments. Use Construct, Point at midpoint.]
5. Stop recording. [Click the STOP button in script window.]
6. Play back the script. [Select the midpoints. Click PLAY button in script.]
7. Play it again.
8. Use FAST to play it again.
9. Create another 3 points unrelated to the triangle, select them and play the script.
10. Create another 3 points, select them and use STEP button, step by step.
11. Create 4 points. Record a script to create a 4-sided figure. Stop recording. Create and select another 4 points and play the script.
12. Create another 4 points but select them in a different order. Play the script.

Scripts - Investigating others
1. Open a new sketch.
2. Use File, Open, Sample scripts, Art, Tumbling blocks.
3. The Given list tells you what to create and then select them. [Point A, Point B]
4. Play the script.
5. Try other sample scripts.
Animation

1. In a new sketch, draw a circle and a line segment with one endpoint on the circle.

2. Animate the segment's endpoint around the circle. [Select the circle and the segment's endpoint on the circle. Use Display, Animate. Click on the Animate button.]

3. Press and hold the mouse button to stop the animation.

4. Create an animation button. [Select the circle and segment's endpoint on the circle. Use Edit, Action, Animation. Click on the Animate button.]

5. Double-click on your Animate button to start the animation. Click anywhere to stop.

6. Create some original animations.

Tracing

1. Open a new sketch. Construct a point.

2. Trace this point. [Select it. Use Display, Trace point. Drag the point around the sketch and observe the locus. When you release the mouse, the path becomes a continuous line.]

3. Use the trace feature and animate one circle's centre around the circumference of another. [Draw a circle dragging straight down so that the point that defines the radius is directly below the centre of the circle. Construct a new point anywhere on the circle. Select this point and then the other point on the circle. Construct a second circle using Construct, Circle by centre+point. With the second circle still selected, use Display, Trace circle. Select the centre of the second circle and the first circle itself. Hold down Option and use Display, Animate.]

4. Click anywhere to stop this animation.

5. Use File, Open, Presentation sketches, Bogus Cosmos. It was saved with the Animate button selected so that it starts as soon as you open it. Also open Life's highway in the same Presentation sketches.
GRADE 8 GEOMETER'S SKETCHPAD

LESSON 9: Pythagorean Theorem

Warm-up

1. Construct a circle AB. Construct AB. Construct CA with point C on the circumference. Construct CB and then hide the circle.

2. Measure the sides and the angles. What kind of triangle is it? ________________

3. What are your 2 conjectures about this kind of triangle? _______________________

4. Write your conjectures on the sketch. Title it and print 2 copies.

Lesson

The hypotenuse of a right angled triangle is the longest side and opposite the right angle.

1. Construct AB. Select it and label it 'a'.

2. Construct a perpendicular to AB at A. [Select A and the line and use Construct menu.]

3. Construct a point C on the perpendicular line above A. Construct a line segment from C to A. Hide the perpendicular line. Select CA and label it 'b'.

4. Construct CB, the hypotenuse, to complete the triangle. Select CB and label it 'c'.

5. Measure <CAB. Put its measure in the angle.

6. Measure the lengths of the sides.

7. Calculate the square of AB (a) and the square of CA (b). Add the squares together.

8. Calculate the square of CB (c). How does this compare to the sum above? __________

9. Move C and observe any changes.

10. What is your conjecture? ____________________________________________

11. Rewrite this conjecture using a, b and c in a formula. _______________________

12. Write your conjecture and the formula on the sketch. Title it. Print 2 copies.

Homework

Text page 305 #2-3 and pages 306-307 #1-4
LESSON 10: Constructing Special Angles

Warmup

1. Bisect 2 of the angles of a triangle. [Use the construct menu.]

2. Use their point of intersection to construct a circle that touches the sides of the triangle, that is, it inscribes the triangle.

3. Construct a line segment from the 3rd vertex (that is not bisected) through the centre to the other side of the triangle. Does this line bisect this angle? _______
   Measure to prove it.

4. Title this sketch and print 2 copies.

Lesson - 60°, 30°, 15°

1. Construct an equilateral triangle using 2 circles and a line segment.

2. Hide one of the line segments. Measure the angle that is left. ____

3. Construct an angle bisector by using freehand tools. Measure the angles. _________

4. Bisect one of these angles using freehand tools. Measure the angles. ________

5. Title the sketch. Print 2 copies.

Lesson - 90°, 45°, 22.5°

1. Construct a right angle using freehand tools. Measure the angle.________

2. Construct an angle bisector using freehand tools. Measure the angles.__________

3. Bisect one of these angles using freehand tools. Measure the angles. ________
   (Use Display, Preferences...Angle units, Degrees, Precision to tenths from units)

4. Title the sketch. Print 2 copies.

Lesson - other combinations

1. Construct the following angles using freehand tools only.
   (a) 120°   (b) 225°   (c) 300°   (d) 240°   (e) 105°   (f) 75°

2. Do each as a separate sketch. Show the calculations you used to decide what angles to construct.

3. Title the sketches. Print 2 copies.
You can bisect an angle automatically with Sketchpad's Construct menu. But an angle bisector is not difficult to construct using only freetools. In this activity, you'll bisect an angle the way Euclid did it, then you'll investigate properties of angle bisectors.

Sketch

Step 1: Construct $\overline{AB}$ and $\overline{AC}$.
Step 2: Construct circle $AD$, with $D$ on $\overline{AB}$.
Step 3: Construct circle $DE$, where $E$ is the intersection of $\overline{AC}$ and circle $AD$.
Step 4: Construct circle $ED$.
Step 5: Construct $\overline{AF}$, where $F$ is the intersection of circles $DE$ and $ED$ that is farthest from $A$.
Step 6: Hide the circles and points $D$, $E$, and $F$.
Step 7: Construct a point $G$ on the angle bisector.

Investigate

Drag point $B$ or $C$. Does $\overline{AG}$ continue to bisect the angle? Measure the distances from $G$ to $\overline{AB}$ and $G$ to $\overline{AC}$. Move point $G$ along the angle bisector. What do you notice about the distances to the sides of the angle?

Conjecture: Write your conjectures below.

Present Your Findings

Discuss your findings with your partner or group. To present your findings you could:

1. Create a script to construct an angle bisector and comment it to explain why it works and describe your conjectures.
2. Print a captioned sketch with measures illustrating your conjectures.

Explore More

You'll notice that your angle bisector construction behaves strangely if you open up your angle past 180°. This behavior is different than the behavior of the angle bisector Sketchpad constructs when you choose Angle Bisector in the Construct menu. Experiment with the Construct menu Angle Bisector, then see if you can construct one that behaves that way. Hint: you'll use a midpoint.
Investigation: Circumscribing a Triangle

In this investigation you'll discover properties of perpendicular bisectors in a triangle. You'll also learn how to construct a circle that passes through each vertex of a triangle. You might want to record a script of your construction so that you can demonstrate your investigation.

Sketch

**Step 1:** Construct triangle $ABC$.

**Step 2:** Construct the midpoints, $F$, $E$, and $D$, of sides $AB$, $BC$, and $CA$.

**Step 3:** Construct lines perpendicular to $AB$ through $F$ and perpendicular to $BC$ through $E$. Construct the point of intersection, $G$, of these lines.

**Step 4:** Construct a line perpendicular to $CA$ through $D$.

**Step 5:** Construct a circle with center $G$ and radius endpoint $A$.

Investigate

When you constructed the third perpendicular bisector did it pass through the point of intersection of the other two perpendicular bisectors? Drag any vertex or side of your triangle. Do the three bisectors always intersect in a single point? Does the circle $GA$ always pass through vertices $B$ and $C$? What does this tell you about the distances from $G$ to $A$, $B$, and $C$? Notice that as you move a vertex, $G$ may fall inside, on, or outside the triangle. Measure $\angle ABC$ and drag $B$ to investigate these three cases.

The point of concurrency of the three perpendicular bisectors is called the circumcenter.

**Conjecture:** Write your conjectures below. (Use the back of this sheet if necessary.)

Present Your Findings

Discuss your results with your partner or group. To present your findings you could:

1. Add comments to your script then play or step through it to explain to someone how you arrived at your conjectures.

2. Add captions to your sketch, print it out, and write an explanation of your investigation and conjectures.

Explore More

See if you can circumscribe other shapes besides triangles. Describe what you try and include any additional conjectures you come up with on the back of this sheet.
CHORDS AND CIRCLES

You should use your label tool for this exercise.

1. Draw a circle with center A. (and control point B on the circle.)

2. By selecting the circle construct the point C on the circle.

3. Repeat step 2 for the point D.
   
   You may move either one of these points along the circle if you wish.

4. Construct the line segment CD. This segment is called a "chord" of the circle.

5. Construct the perpendicular bisector of CD.
   Hint: You need to construct the midpoint E first.
   What conclusion can you draw?

6. Repeat steps 2 - 5 for another chord FG. Does your previous conclusion still hold true?

7. Select one of the endpoints of one of the chords. Drag its location around the circle.
   Does your conclusion still hold?

8. Summarize your findings.

The above is one of the most fundamental properties of circles.
THE CENTROID OF A TRIANGLE.

1. Draw Δ ABC.

2. Construct midpoints on each of the three sides of the triangle.

3. A median is defined as the line segment from a vertex of a triangle to the midpoint of the opposite side. Construct two medians and construct their point of intersection. Now construct the third median. What conclusion can you draw?

4. The point of intersection of the medians of a triangle is called the "centroid" of the triangle and is often referred to as the balance point of a circle. Select one of the vertices and transform the size of the triangle. Does the median ever fall outside the triangle?

5. There are some other interesting properties of the centroid. Measure each of the six distances; first from the vertex to the centroid and then from the centroid to the midpoint of the opposite side for each median. Again transform the shape and size of the triangle. What conclusion can you draw?

6. To help understand why the centroid is called the balance point of the circle, select any one of the vertices, the centroid and the midpoint on the vertex's adjacent side and construct a Polygon Interior. Measure the area of this triangle and repeat this process for each of the remaining five triangles. Again change the size and shape of the triangle. What conclusion can you draw? Do you now know why it is called the balance point?
THE INCIRCLE OF A TRIANGLE

1. Construct \( \triangle ABC \).

2. Construct the angle bisector of any two of the angles and construct the point of intersection of these two rays. Construct also, the point of intersection \( D \) of these two angle bisectors.

3. Construct the third angle bisector. What conclusion can you draw?

This point of intersection is called the "incenter" of the circle.

4. Construct a perpendicular \( \overline{DE} \) from the incenter to any side of \( \triangle ABC \).

5. Construct a circle with center \( D \) and passing through the point \( E \). What conclusion can you draw?

This circle is called the ___________ of the triangle.

Why did you have to draw a perpendicular? Hint: draw the remaining perpendiculars and compare their lengths.

6. Transform the shape of your triangle and record any observations. Does the incenter ever fall outside the triangle?
1. Using the Vertices or the segments tools, draw an ACUTE \( \triangle ABC \). (Use the label/text tool to label the vertices)

2. Construct a line passing through \( B \) and perpendicular to the side \( AC \).

3. By selecting the perpendicular line and the line \( AC \), create the point of intersection "D".

4. By selecting "B" and "D", construct line segment BD.

5. Hide the perpendicular line. (you must select it by clicking outside the triangle)

6. The segment BD is called ________________

7. Construct the remaining two altitudes. What conclusion can you draw?

8. Select any two altitudes and create the point of intersection. This point is called the "ORTHOCENTER" of the triangle.

9. Click on any one of the vertices and drag the point such that the triangle changes size. (Keep the triangle acute for now) Try it again using another vertex. What conclusion can you draw?

10. What happens when you drag one of the vertices such that the triangle becomes obtuse? (i.e. has an angle greater than 90°)

11. What conclusion can you draw if you place the orthocenter on top of one of the vertices?

In the next lesson you will learn how to deal with the orthocenter of obtuse triangles.
1. Create an obtuse \( \Delta ABC \). (one with an angle greater than 90°) Label the triangle!

2. Select any one of the vertices and the side of the triangle opposite that vertex. By using the "construct" menu, construct a perpendicular line.

3. Repeat this with the other two vertices. You will notice that sometimes the perpendicular line may fall inside the triangle and at other times outside the triangle.

4. Select any two of the perpendicular lines and create the point of intersection. Label this point as well. This point is also called the ____________________.

5. Click on one of the vertices and transform the size of the triangle. What conclusions can you draw?

6. Again place the orthocenter on top of one of the vertices. What conclusion can you draw?

7. Transform the triangle such that it now becomes an acute triangle. What conclusion can you draw?

8. Make overall conclusions regarding how to find an orthocenter and where the orthocenter falls with respect to the type of triangle you have.
AN EXTENSION OF CIRCUMCIRCLES

1. Construct Δ ABC.

2. **Construct** (using Point On Object) points D, E and F on the sides BC, AC and AB respectively.

3. Construct the circumcircles of Δ AFE, Δ CDE and Δ BDF. What conclusions can you draw? Hint: you may wish to hide your lines.

4. Transform the size and shape of your triangle and also move the points on each side along the sides. Does this effect your above observations?
THE EULER LINE

This is a very busy exercise so you may wish to use different colours for the various lines of construction for each part of the activity.

1. Construct a large acute $\triangle ABC$.

2. Construct $D$, $E$ and $F$ the midpoints of $BC$, $AC$ and $AB$ respectively.

3. Construct $\triangle DEF$

4. Construct $P$, the orthocentre of $\triangle ABC$.

5. Construct $Q$, the centroid of $\triangle ABC$.

6. Construct $R$, the circumcenter of $\triangle ABC$.

7. Construct $S$, the circumcenter of $\triangle DEF$.

8. If you have done all your constructions correctly, these four points should be co-linear. This line is called the "EULER" line.

Choose any two of these points and construct the EULER line.
APPENDIX E: CATHY’S EVALUATION TEST
1. Determine the missing measures:

(a) \[ \begin{align*}
\angle x &= \ldots \\
\angle y &= \ldots 
\end{align*} \]

(b) \[ \begin{align*}
\angle a &= \ldots 
\angle b &= \ldots 
\angle c &= \ldots 
\end{align*} \]

(c) \[ \begin{align*}
\angle x &= \ldots 
\angle y &= \ldots 
\angle z &= \ldots 
\angle w &= \ldots 
\angle a &= \ldots 
\angle b &= \ldots 
\angle c &= \ldots 
\angle d &= \ldots 
\angle e &= \ldots 
\angle f &= \ldots 
\angle g &= \ldots 
\angle h &= \ldots 
\angle i &= \ldots 
\angle j &= \ldots 
\angle k &= \ldots 
\angle l &= \ldots 
\angle m &= \ldots 
\angle n &= \ldots 
\angle o &= \ldots 
\angle p &= \ldots 
\angle q &= \ldots 
\angle r &= \ldots 
\angle s &= \ldots 
\angle t &= \ldots 
\angle u &= \ldots 
\angle v &= \ldots 
\angle w &= \ldots 
\angle x &= \ldots 
\angle y &= \ldots 
\angle z &= \ldots 
\angle aa &= \ldots 
\angle bb &= \ldots 
\angle cc &= \ldots 
\angle dd &= \ldots 
\angle ee &= \ldots 
\end{align*} \]
APPENDIX F: KAREN'S EVALUATION TEST
1. In the diagram,
   A) Name an inscribed angle. 
   B) What is the centre of the circle?
   C) If $\angle ABC$ is $64^\circ$, what is the size of $\angle AOB$?
   D) If you drag vertex $C$ along the circumference to halfway between $B$ and $C$ at present, the size of $\angle ACB$ would?

2. $\overline{AB}$ is parallel to $\overline{CD}$, $\overline{EF}$ is the transversal. $E$ is on $\overline{AB}$, and $F$ is on $\overline{CD}$.
   A) What is the alternate angle to $\angle BEF$?
   B) State the opposite angle to $\angle DFH$
   C) If $\angle GEB$ is $85^\circ$, then $\angle AEF$ is?
   D) If $\angle GEB$ is $85^\circ$, the $\angle GAE$ is?

3. If $\overline{BD}$ bisects $\angle ABC$ and $\angle ABC$ is $68^\circ$, then $\angle ABD$ is

4. A) When you move the vertices around the circle, what was the relationship among your measurements?
   B) State a chord of this circle.
   C) State a radius of this circle.
5. Find the value of the unknown angles.

\[ \begin{align*}
  x &= \\
  y &= \\
  z &= 
\end{align*} \]

6. In one sentence, explain how you would measure an angle using the computer programme.

7. \( AB \) is the perpendicular bisector of \( 
\overline{CD} \), and meets \( 
\overline{CD} \) at \( E \). What does "perpendicular bisector" mean.

8. Refer to question 7. Take any point on \( AB \). With respect to points \( C \) and \( D \), what can you conclude?

9. What is an equilateral triangle?

10. State 3 methods used to prove that two triangles are congruent.
11. State 2 methods that are not used to prove two triangles congruent.

12. What is unique about the methods, used in question #10, that make them possible methods for proving triangles congruent.

13. If two triangles are congruent then what is true about the two triangles?

BONUS:

TOTAL = 32

Write a mathematical proof to show that these two triangles are congruent.
APPENDIX G: SIMON'S EVALUATION TEST
SIMON'S REVIEW SHEETS
Grade 8 Mathematics - Geometry Test

Name: ____________

1. Use the diagram on the right to answer the following questions:

   (a) Name an inscribed angle in the diagram.

   ________

   [3]

   (b) If \( \angle ACB = 64^\circ \), what is the size of \( \angle AOB \)?

   ________

   (c) Suppose that you used the Geometer's Sketchpad to drag point C along the circumference of the circle and placed it halfway between where B and C are now. What would be the new size of \( \angle ACB \)?

   ________

2. Line segment AB is parallel to CD and GH is a transversal. E and F are the points of intersection as shown in the diagram.

   \( \angle GEB = 73^\circ \).

   (a) What is the size of \( \angle GEA \)?

   ________

   [3]

   (b) What is the size of \( \angle CFH \)?

   ________

   (c) What is the sum of all 8 angles in the diagram?

   ________

3. (a) If line segment BD is an angle bisector of \( \angle ABC \), and \( \angle ABC = 46^\circ \), what is the size of \( \angle DBC \)?

   ________

   [2]

   (b) If \( \angle BCD = 118^\circ \), what is the size of \( \angle BDC \)?

   ________
4. (a) State a chord of the circle shown.

(b) Name two opposite angles of the quadrilateral.

(c) What is the relationship between these opposite angles?

(d) What is the sum of the 4 angles of the quadrilateral in the diagram?

5. Explain what the term “perpendicular bisector” means (two properties).

6. Suppose that line segment AB is a perpendicular bisector of line segment CD. Choose any point on AB. With respect to this point and the points C and D, what can you conclude? (Hint: draw a diagram!)

7. When using paper and pencil to do geometric constructions, two useful tools are a straightedge (ruler) and a compass.

What electronic tool in Geometer’s Sketchpad can be used in place of:

(a) a straightedge?

(b) a compass?
8. In one sentence, explain how you would measure an angle in the Geometer’s Sketchpad program.

9. In Geometer’s Sketchpad, Alexis draws a circle and a line segment which crosses the circle at two points. Explain how she could ACCURATELY find these two points using the program.

10. (a) Two triangles are congruent if all of the corresponding ________

and all of the corresponding ________ are equal.

(b) State two of the methods you saw in class that could be used to prove that two triangles are congruent.

(c) State one of the methods you saw in class that could NOT be used to prove that two triangles are congruent.

11. (a) What is an equilateral triangle?

(b) What is the measure of each angle of an equilateral triangle?
BONUS

12. Explain how you could use Geometer's Sketchpad to construct an equilateral triangle (a diagram might be helpful).

[2]
What have we seen so far?

1. The sum of the angles of a triangle equals 180°.
   \[ \angle ABC + \angle BCA + \angle CAB = 180° \]

2. An \textit{ANGLE BISECTOR} is a ray which cuts an angle exactly in half. You need to know how to construct an angle bisector using the Geometer's Sketchpad software.
   - ray AF bisects \( \angle CAB \)
   - Therefore \( \angle CAF = \angle BAF \)

3. A \textit{PERPENDICULAR BISECTOR} is a line which has two special properties:
   
   (i) it cuts a line segment exactly in half (BISECTOR)
   (ii) it makes a 90° angle with the line segment (PERPENDICULAR)

   You need to know how to construct a perpendicular bisector using the Geometer's Sketchpad software.
   - line CD is a perpendicular bisector of ray AB

4. A line which cuts across two \textit{PARALLEL} lines is called a transversal. The eight angles created are special because they only have two different measurements and these two measurements add up to 180°. 

   - \( AB \) is parallel to \( CD \), so
   - \( \angle AGE = \angle BGH = \angle CHG = \angle DHF \)
   - \( \angle BGE = \angle ACH = \angle DHG = \angle CHF \)

   and \( \angle AGE - \angle BGE = 180° \) (lots of other pairs add up to 180° as well).
APPENDIX H: MIKE'S EVALUATION TEST
1. Answer the following as true (T) or false (F).
   a) the circumcenter is the balance point of the triangle ___.
   b) the orthocenter can fall outside the triangle ___.
   c) a median joins a vertex and the midpoint of the opposite side ___.
   d) the incenter is equal distances from all three vertices ___.
   e) the incenter is equal distances from all three sides ___.
   f) the centroid can never fall outside the triangle ___.

2. Fill in the blanks with the appropriate response.
   a) the ______________ is the point of intersection of the three altitudes of the triangle.
   b) the centroid is the point of intersection of the three ______________ of the triangle.
   c) before constructing an incircle one must ____________________________
      to one of the sides in order to determine the ______________ of the incircle.
   d) a circumcenter is the point of intersection of the three ______________ of the triangle.
   e) the ______________ is the point of intersection of the three angle bisectors of the
      triangle.
   f) if the circumcenter falls outside the triangle then the triangle is ______________.

3. In your own words define the following:
   a) median
   b) circumcircle
APPENDIX I: CONSENT FORMS
Teacher Agreement Form

I, ________________________, consent to serve as a subject in the research study entitled: A Study on the Freedom of Teachers and Students to Explore in a Geometric Environment.

The nature and general purpose of the research procedure has been explained to me by Douglas E. McDougall, B. Math, B. Ed, M. Ed. The researcher is authorized to proceed on the understanding that I may withdraw from the study at any time, without reason.

I understand that neither the teacher nor the students are placed under any risk and that reasonable safeguards have been taken to minimize any potential but unknown problems.

Witness ______________________ Signed ______________________
(subject)

Date ______________________

Please sign 2 copies.
Student Agreement Form

I authorize ____________________________ as a subject in the research study

entitled: A Study on the Freedom of Teachers and Students to Explore in a
Geometric Environment.

The project is an investigation into the use of computer software to teach the geometric
constructions unit of the Grade 8 mathematics course. The research study will be conducted by
Douglas E. McDougall, B. Math, B. Ed, M. Ed. The investigator is authorized to proceed on
the understanding that I may withdraw my child from the study at any time, without reason.

I understand that neither the teacher nor the students are placed under any risk and that
reasonable safeguards have been taken to minimize any potential but unknown problems.

Signed ____________________________

(parent/guardian)

Date ____________________________

Please sign 2 copies.
Head of School
Agreement Form

I, ____________________________, consent to serve as a subject in the research study entitled: Mathematics Teachers’ Needs in a Computer-Based Exploratory Classroom.

The nature and general purpose of the research procedure has been explained to me by Douglas E. McDougall, B. Math, B. Ed, M. Ed. The researcher is authorized to proceed on the understanding that I may withdraw from the study at any time, without reason.

I understand that neither the teacher nor the students are placed under any risk and that reasonable safeguards have been taken to minimize any potential but unknown problems.

Witness ____________________________ Signed ____________________________

(subject)

Date ____________________________

Please sign 2 copies.