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Optimal design of an adaptive robust controller using a multi-objective artificial bee colony algorithm for an inverted pendulum system

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Abstract

In this paper, a multi-objective artificial bee colony (MOABC) optimization algorithm is utilized to improve the performance of an adaptive robust control technique. This approach is implemented on an inverted pendulum system. More precisely, the proposed controller is a combination of a decoupled sliding mode controller (DSMC) and adaption laws based on the gradient descent approach. In order to achieve the optimum control operation, the MOABC, as a novel meta-heuristic method simulated from the smart foraging activity of honey bee groups, is employed to optimize the coefficients of the suggested controller. In this regard, the objective functions are determined as the integral time of the absolute value of the pole angle and cart position errors. Finally, the time responses of the system states and control effort are presented to prove the effectiveness and feasibility of the suggested strategy compared to other contemporary studies referenced in the paper.

Keywords: Adaptive robust control; Decoupled sliding mode control; Gradient descent approach; Adaptation laws; Multi-objective artificial bee colony algorithm.

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1. Introduction

The inverted pendulum system as an unstable under-actuated nonlinear system is a favorite benchmark model to evaluate diverse control approaches. This complex high-order plant is able to provide a stunning representation of the capabilities of feedback systems such as robots, satellites, and so on. In this regard, several studies have been recently conducted on control strategies, such as fuzzy control, nonlinear control, adaptive control etc., that can stabilize this complicated system (Lo and Kuo 1998; Hung and Chung 2007; Sahnehsaraei et al. 2014; Mahmoodabadi et al. 2020; Mahmoodabadi et al. 2013; Bayramoglu and Komurcugil 2014; Mahmoodabadi and Khoobroo Haghbayan 2020; Mahmoodabadi et al. 2018). For instance, Huang et al. (2019) used a mixture of the high-order disturbance observer and sliding mode control for the stabilization of a mobile wheeled inverted pendulum system. Rigatos et al. (2020) investigated a nonlinear optimal control scheme to balance and control the position and speed of a wheeled inverted pendulum system. Patra et al. (2020) designed a fractional-order proportional-integral-derivative controller for the inverted pendulum to stabilize the cart position and the pole angle. Tang et al. (2020) employed the proportional-integral-derivative compensator subject to a linear quadratic regulator to control a rotary inverted pendulum model. Su et al. (2018) applied an event-triggered fuzzy control method to stabilize a fourth-order nonlinear inverted pendulum system. Eventually, Mahmoodabadi and Haghbayan (2020) designed a mixture of sliding mode control and feedback linearization methods to stabilize an under-actuated type of inverted pendulum plants.

On the other hand, the sliding mode controller is an impressive robust control that can handle the uncertain systems, effectively (Slotine and Li 1991; Hadipour Lakmesari et al. 2020). Although the sliding mode control is known as an efficient technique to control nonlinear systems, it includes some issues to stabilize under-actuated systems. Therefore, this approach was upgraded to the decoupled sliding mode control in order to stabilize the under-actuated plants. Recently, many researchers have applied this strategy in various fields of sciences and technologies. For example, Mahmoodabadi et al. (2019) proposed a robust self-tuning decoupled sliding mode controller for a highly nonlinear inverted pendulum system and optimized its parameters via a multi-objective genetic algorithm. Haghighi and Mobayen (2018) employed an adaptive super-twisting decoupled terminal sliding mode control technique for some fourth-order systems. In a similar way, Coban and Ata (2017) employed this strategy to stabilize an inverted pendulum on a cart. Finally, Jahanshahi (2018) designed a robust adaptive controller subject to supervisory decoupled sliding mode technique to provide a suitable treatment process for the HIV infection model.

Besides, the adaptive methods, as one of the application approaches in the area of expertise of automatic control, are broadly utilized when the parameters of the controlled systems change or are initially uncertain. In fact, these schemes are able to modify the control parameters according to the system conditions (Ioannou and Sun 2012; Bellman 2015) utilizing estimation strategies such as the gradient descent method (Zeng et al. 2019; Patel et al. 2020). Some of the most successful applications of the gradient descent method to adapt the controller parameters could be mentioned as follows. Luo et al. (2016) developed a data-based policy dynamic programming algorithm using a gradient descent scheme to design an adaptive optimal control method. Na et al. (2017) studied an adaptive control scheme for vehicle active suspensions employing the gradient descent approach. Agrawal et al. (2019) proposed a special type of the PID controllers based on the gradient descent method and a multi-layer neural network scheme to regulate the liquid level in a nonlinear tank system. Furthermore, in the field of combination of the adaptive schemes with the robust controllers, Chen et al. (2016) applied an adaptive robust control

scheme to cope with nonlinearities existing in multilateral teleoperation systems. Babaghasabha et al. (2016) considered this strategy for fully constrained cable robots and analyzed the stability of the overall closed-loop system. In a similar way, Hu et al. (2016) presented an adaptive robust controller to remove the chaos effects on the performance of a permanent-magnet synchronous motor system.

One of the most important issues in the process of the controller design is the determination of the proper values for the constant parameters. Applying the meta-heuristic approaches, such as Genetic Algorithm (GA) (Kramer 2017), Particle Swarm Optimization (PSO) (Clerc 2010), Ant Colony Optimization (ACO) (Dorigo 2006), Team Game Algorithm (TGA) (Mahmoodabadi et al. 2018), Artificial Bee Colony (ABC) (Karaboga 2006) etc., could be an appropriate way to solve this problem. The ABC algorithm as one of the recent swarm intelligence optimization techniques was initially introduced by Karaboga (2006) and inspired by the group behavior of bees. This algorithm has displayed superior performance in comparison with other well-known intelligence optimizations. Due to its accuracy and fast performance, the algorithm has become an effective optimization approach to solve a wide range of single-objective and multi-objective optimization problems. For instance, Xie et al. (2019) considered a combination of a programming model and a multi-objective ABC algorithm for energy efficient distributed job shop scheduling. Pawar and Khalkar (2020) proposed a strategy for multi-objective optimization of wire-electric discharge machining based on the MOABC. Zhang et al. (2017) used this algorithm for parallel batch-processing machine scheduling in order to minimize dyeing processes cost. Luo (2017) introduced an especial type of the MOABC algorithm to solve multi-objective and many-objective optimization problems. Xue et al. (2018) introduced a self-adaptive artificial bee colony algorithm for global optimization of some benchmark functions. Dereli and Koker (2020) used the ABC approach to solve the inverse kinematics of a seven-degrees-of-freedom robotic arm. Eventually, Nayyar et al. (2020) presented a modified ABC algorithm to analyze the robot path planning problem. This research work develops authors' previous works (Sahnehsaraei et al. 2014; Mahmoodabadi et al. 2013; 2018; 2019; 2020) by presenting an optimal adaptive robust control method based on the gradient descent method and the multi-objective artificial bee colony. To this aim, the DSMC is combined with the gradient descent approach to handle the inverted pendulum system and provide a suitable performance, acceptable stability, and reliability. To select the parameters of the controller, the multi-objective artificial bee colony optimization algorithm is implemented in regard to two proper design criteria. Finally, the proposed adaptive robust controller optimized via the MOABC is applied for the inverted pendulum system subjected to challenging external disturbances in order to verify its effectiveness and feasibility.

Figure 1. Configuration of the considered inverted pendulum system.

The rest of this content is structured as follows. After this brief introduction, the inverted pendulum system as well as its dynamical equations are described in Section 2. Section 3 introduces the suggested control system which is based on the DSCM and adaptation laws. Section 4 describes the MOABC algorithm as well as the related flowchart. Consequently, Section 5 explains how to apply the adaptive DSMC controller on the inverted pendulum system and employ the MOABC optimization process on its parameters. Eventually, the paper is summarized in Section 6.

2. Inverted pendulum system modeling

In this section, the model of the considered inverted pendulum as depicted in Figure 1, which is an under-actuated, nonlinear, unstable and high order system, is explained. This study considers stabilization of the inverted pendulum system, as one of the most famous benchmarks, to evaluate the introduced control method. It is worth noting that the system is a fourth-order system containing two second-order subsystems in the noncanonical form with states $[x_1, x_2]^T$ and $[x_3, x_4]^T$. In more details, the system state vector is $x = [x_1, x_2, x_3, x_4]^T$ comprising $x_1 = \theta$ the angle of the pole with respect to the vertical axis, $x_2 = \dot{\theta}$ the angular velocity of the pole, $x_3 = x$ the position of the cart and $x_4 = \dot{x}$ the velocity of the cart. The system dynamical equations can be written as follows.

$$\dot{x}_1 = x_2 \quad (1)$$

$$\dot{x}_2 = f_1 + b_1 u + d \quad (2)$$

$$\dot{x}_3 = x_4 \quad (3)$$

$$\dot{x}_4 = f_2 + b_2 u + d \quad (4)$$

where, d represents the uncertainty imposed on the system that $|d| \leq d_{\max}$, and other parameters are defined as below.

$$f_1 = \frac{(M + m)g \sin x_1 - mL \sin x_1 \cos x_1 x_2^2}{L \left(\frac{4}{3}(M + m) - m \cos^2 x_1 \right)} \quad (5)$$

$$f_2 = \frac{mg \sin x_1 \cos x_1 - \frac{4}{3}mL x_2^2 \sin x_1}{\frac{4}{3}(M + m) - m \cos^2 x_1} \quad (6)$$

$$b_1 = \frac{\cos x_1}{L \left(\frac{4}{3}(M + m) - m \cos^2 x_1 \right)} \quad (7)$$

$$b_2 = \frac{\frac{4}{3}}{\frac{4}{3}(M + m) - m \cos^2 x_1} \quad (8)$$

Besides, m , M , L and g respectively represent the masses of the pole and cart, the half-length of the pole, and the gravity acceleration.

3. Adaptive robust controller

The sliding mode control is known as a nonlinear control approach with extraordinary features of robustness, accuracy and easy tuning (Slotine 1991; Hadipour Lakmesari et al. 2020; Mahmoodabadi et al. 2019; Haghighi and Mobayen 2018). Although it is an efficient scheme to handle nonlinear systems with uncertainties, it has not a suitable performance to stabilize under-actuated ones. In contrast with the SMC approach, the DSMC is proper to handle systems with the non-canonical forms such as the inverted

pendulum system. More precisely, the significant idea behind the DSMC is to design a control strategy so that some subsystems are simultaneously handled based on just an input item. Moreover, the DSMC, as a particular class of robust control methods for non-linear under-actuated systems, is used to guarantee the sliding condition and modify the dynamics of the system with a discontinuous control signal. In other words, one of the major reasons to present this control technique is to slide the states of subsystems from initial conditions to sliding surfaces. Accordingly, in this paper, as an alternative, the decoupled sliding mode control method is applied to regulate the states of the considered under-actuated inverted pendulum system. In order to design the DSMC, in the first step, sliding surface functions should be defined. Afterward, a discontinuous sliding term and an equivalent controller would be determined based on the states of the dynamical equations. To reach this goal, sliding surfaces S_1 and S_2 would be organized as follows.

$$S_1(x) = x_2 - x_{2d} + \lambda_1(x_1 - x_{1d} - z) \quad (9)$$

$$S_2(x) = x_4 - x_{4d} + \lambda_2(x_3 - x_{3d}) \quad (10)$$

that

$$z = z_u \text{sat}\left(\frac{S_2(x)}{\Phi_z}\right) \quad (11)$$

where, x_{1d} , x_{2d} , x_{3d} and x_{4d} denote the desired states so that $e_1 = x_1 - x_{1d}$, $e_2 = x_2 - x_{2d}$, $e_3 = x_3 - x_{3d}$ and $e_4 = x_4 - x_{4d}$ are correspondingly considered as the system errors, λ_1 and λ_2 are strictly positive constants, and Φ_z denotes the width of a boundary layer for S_2 . Moreover, z transfers S_2 to S_1 , and z_u has a positive value. In order to prove the stability of the system and design the related controller, Lyapunov function V is formulated as follows, with respect to sliding condition $\frac{1}{2} \frac{d}{dt} S_1^2 \leq -\eta |S_1|$ in which η is a positive constant.

Figure 2. Block diagram of the suggested optimum adaptive robust control approach.

$$V = \frac{1}{2} S_1^2(x) \quad (12)$$

If the derivative of the Lyapunov function satisfies the following equation, the stability of the system would be guaranteed based on the Lyapunov theorem.

$$\dot{V} \leq -\eta |S_1(x)| \quad (13)$$

Besides, if the control effort is regarded as below, Equation (13) holds, and $\dot{V}$ will be negative definite.

$$u = \hat{u} - K_1 \text{sat}\left(\frac{S_1(x) b_1(x)}{\Phi_1}\right) \quad (14)$$

that

$$\hat{u} = -\frac{1}{b_1(x)} (f_1(x) - \lambda_1 x_1 - \lambda_2 x_2 - \lambda_3 x_3 - \lambda_4 x_4) \quad (15)$$

Accordingly, the control scheme can be described in this way. When the system starts from arbitrary initial conditions, then control effort u leads the sliding surface to converge to zero. This causes $x_4 = x_{4d}$ and $x_2 = x_{2d} + z$ as well as $x_1 = x_{1d}$ and $x_3 = x_{3d}$. Consequently, based on Equation (11), intermediary variable z converges to zero, and the system errors along with their derivatives exponentially tend to zero until the control objectives are reached at a finite time.

Next, using the chain derivative rule and the gradient descent method as an appropriate adaptation law, the decoupled sliding mode control gains are updated by the following equations (Mahmoodabadi et al. 2017).

$$\dot{\Phi}_1 = -\gamma_1 \frac{\partial S_1}{\partial \Phi_1} = -\gamma_1 \frac{\partial S_1}{\partial u} \frac{\partial u}{\partial \Phi_1} \quad (16)$$

$$\dot{K}_1 = -\gamma_2 \frac{\partial S_1}{\partial K_1} = -\gamma_2 \frac{\partial S_1}{\partial u} \frac{\partial u}{\partial K_1} \quad (17)$$

where, γ_1 and γ_2 represent the constant parameters having positive values. In order to determine the appropriate values for control gains λ_1 , λ_2 , Φ_z , z_u , γ_1 and γ_2 , the multi-objective optimization algorithm based on the artificial bee colony would be employed regarding the conflict objective functions. Schematically, the structure of the proposed optimal adaptive decoupled sliding mode controller is shown in Figure 2.

4. Artificial bee colony algorithm and multi-objective optimization

The ABC is known as one of the strongest swarm intelligence algorithms which is inspired by the social behavior of a group of bees. In this algorithm, a colony of the artificial bees includes three types of them, i.e. scout, employed and onlooker bees, that have their specific responsibility. In fact, the onlooker bees wait on the dance area to make a decision for selection of a food source. Employed bees are responsible for exploiting the food sources, and scout bees perform random searches. In addition, half population of the colony constitutes employed bees and the other half consists of onlooker bees. The number of food sources around the hive is exactly equal to the number of employed bees, and each employed bee is allocated to one food source.

In other words, the position of a food source denotes a feasible solution for the optimization problem. Indeed, an employed bee looks for a new food source using the following equation (Karaboga 2005).

$$v_{ij} = \zeta_{ij} + rand(-1,1)(\zeta_{ij} - \zeta_{kj}) \quad (18)$$

where, $j \in [1, 2, \dots, D]$ and $k \in [1, 2, \dots, SN]$ are randomly generated integer numbers, while SN is the number of the food sources, and D denotes the flying direction. Further, an onlooker bee selects a food source based on its related probability calculated via the following equation.

$$p_i = \frac{fit_i}{\sum_{n=1}^{SN} fit_n} \quad (19)$$

where, fit_i as the fitness value of solution i is calculated using Equation (20).

$$fit_i = \begin{cases} \frac{1}{1+f_i} & f_i \geq 0 \\ 1+|f_i| & f_i < 0 \end{cases} \quad (20)$$

where, f_i is the cost function related to food source i . If the employed bees are not able to improve their solutions after a certain number of cycles, they will abandon the food sources and become the scouts. When the locations of all food sources are specified, the next cycle of the artificial bee colony algorithm will start. The procedure of obtaining the optimum solutions by this algorithm could be summarized as follows (Karaboga 2005).

- A. Randomly generate the initial population of the algorithm.
- B. Assign the employed and onlooker bees on the food sources, and allocate the scouts to the new food sources.
- C. Iterate step B until a stopping criterion is satisfied.

On the other hand, a great number of real-life problems require the simultaneous optimization of several objective functions. Hence, multi-objective optimization algorithms should be devised in order to overcome and solve these problems. Similar to other multi-objective intelligence algorithms, the considered approach based on the artificial bee colony stores the non-dominated solutions found at each iteration into an external archive. Moreover, in order to compute the new positions of the employed and onlooker bees, the non-dominated solutions are utilized for random selection of the food sources. Furthermore, the members of the archive are randomly assigned to determine the locations of the scout bees. Figure 3 briefly explains the introduced multi-objective artificial bee colony algorithm as a flowchart.

5. Optimum adaptive robust control for the inverted pendulum system

In order to design the adaptive robust controller for the inverted pendulum system, the sliding surfaces demonstrated in Equations (9) and (10) are rewritten regarding the zero desired values as follows.

$$S_1 = \lambda_1(x_1 - z) + x_2 \quad (21)$$

$$S_2 = \lambda_2 x_3 + x_4 \quad (22)$$

The time derivatives of intermediate variable z and first sliding surface S_1 could be calculated as follows.

$$\mathbf{\dot{S}} = \begin{cases} \frac{z_u}{\Phi_z} [\lambda_2 x_4 + f_2 + b_2 u + d] & \left| \frac{S_2}{\Phi_z} \right| \leq 1 \\ 0 & \left| \frac{S_2}{\Phi_z} \right| > 1 \end{cases} \quad (23)$$

Figure 3. Flowchart of the introduced multi-objective artificial bee colony algorithm.

$$\mathcal{S}_1 = \begin{cases} \lambda_1 x_2 - \lambda_1 \frac{z_u}{\Phi_z} [c_2 x_4 + f_2 + b_2 u + d] + f_1 + b_1 u + d & \left| \frac{S_2}{\Phi_z} \right| \leq 1 \\ \lambda_1 x_2 + f_1 + b_1 u + d & \left| \frac{S_2}{\Phi_z} \right| > 1 \end{cases} \quad (24)$$

Regarding $\mathcal{S}_1 = 0$, the equivalent control effort is obtained as follows.

$$\hat{u} = \begin{cases} \frac{\Phi_z}{\lambda_1 z_u b_2 - b_1 \Phi_z} [\lambda_1 x_2 - \lambda_1 \frac{z_u}{\Phi_z} [\lambda_2 x_4 + f_2 + d] + f_1 + d] & \left| \frac{S_2}{\Phi_z} \right| \leq 1 \\ -\frac{1}{b_1} (\lambda_1 x_2 + f_1 + d) & \left| \frac{S_2}{\Phi_z} \right| > 1 \end{cases} \quad (25)$$

Needless to say that the final form of the control force is as $u = \hat{u} - K \text{sat}(\frac{S_1 b_1}{\Phi_1})$. Based on the explained gradient descent mechanism and the chain derivative rule, the adaptive form of control gain Φ could be represented as follows.

$$\Phi = \begin{cases} -\gamma_1 \frac{\partial S_1 \Phi}{\partial \Phi_1} = -\gamma_1 \frac{\partial S_1 \Phi}{\partial u} \frac{\partial u}{\partial \Phi_1} = -\gamma_1 (S_1 \frac{\partial \Phi}{\partial u} + \Phi \frac{\partial S_1}{\partial u}) \frac{\partial u}{\partial \Phi_1} & \left| \frac{S_2}{\Phi_z} \right| \leq 1 \\ -\gamma_1 \frac{\partial S_1 \Phi}{\partial \Phi_1} = -\gamma_1 \frac{\partial S_1 \Phi}{\partial u} \frac{\partial u}{\partial \Phi_1} = -\gamma_1 (S_1 \frac{\partial \Phi}{\partial u} + \Phi \frac{\partial S_1}{\partial u}) \frac{\partial u}{\partial \Phi_1} & \left| \frac{S_2}{\Phi_z} \right| > 1 \end{cases} \quad (26)$$

Because $\frac{\partial u}{\partial \Phi_1} = \frac{-K S_1 b_1}{\Phi_1^2}$, $\frac{\partial S_1}{\partial u} = 0$, and

$$S_1 \frac{\partial \Phi}{\partial u} = \begin{cases} S_1 (-\lambda_1 \frac{z_u}{\Phi_z} b_2 + b_1) & \left| \frac{S_2}{\Phi_z} \right| \leq 1 \\ S_1 b_1 & \left| \frac{S_2}{\Phi_z} \right| > 1 \end{cases} \quad (27)$$

$$\text{Hence, } \Phi = \begin{cases} -\gamma_1 [K b_1 (-\lambda_1 \frac{z_u}{\Phi_z} b_2 + b_1) (\frac{S_1}{\Phi_1})^2] & \left| \frac{S_2}{\Phi_z} \right| \leq 1 \\ -\gamma_1 K (\frac{S_1 b_1}{\Phi_1})^2 & \left| \frac{S_2}{\Phi_z} \right| > 1 \end{cases} \quad (28)$$

Accordingly, with respect to $\frac{\partial u}{\partial K} = -\text{sat}(\frac{S_1 b_1}{\Phi_1})$ and Equation (22) for the second sliding surface, the adaptive formulation of controller gain K could be illustrated as follows.

$$K = -\gamma_2 \frac{\partial S_2 \Phi}{\partial K} = -\gamma_2 \frac{\partial S_2 \Phi}{\partial u} \frac{\partial u}{\partial K} = -\gamma_2 (S_2 \frac{\partial \Phi}{\partial u} + \Phi \frac{\partial S_2}{\partial u}) \frac{\partial u}{\partial K} = \gamma_2 b_2 S_2 \text{sat}(\frac{S_1 b_1}{\Phi_1}) \quad (29)$$

where, γ_1 and γ_2 are positive constant parameters. Besides, the optimum values of constant parameters $\gamma_1, \gamma_2, \lambda_1, \lambda_2, \Phi_z, z_u$ as well as the initial values of controller gains Φ and K would be found by means of the MOABC algorithm proposed in Section 4. For all cases, the initial population and the maximum number of iterations are considered as 100 and 300, respectively. In addition, two following objective functions are regarded to minimize the integral of the time absolute errors (ITAE) of θ and x .

$$f_1 = ITAE(x_1) = ITAE(\theta) = \int t |e_1(t)| dt = \int t |\theta| dt \quad (30)$$

$$f_2 = ITAE(x_3) = ITAE(x) = \int t |e_3(t)| dt = \int t |x| dt \quad (31)$$

5.1 Example 1

In this example, the proposed optimum adaptive robust controller is implemented on the inverted pendulum system subjected to disturbance d , as shown in Figure 4. The physical parameters of the system are illustrated in Table 1. It should be mentioned that the initial values of the system states are regarded as $\theta = 45^\circ$, $\dot{\theta} = 0$, $x = 0.5$, and $\dot{x} = 0$, and the friction force is neglected. Further, the control input is bounded at $-50\text{ N} \leq u \leq 50\text{ N}$.

Figure 4. External disturbance exerted on the inverted pendulum system in Example 1.

Table 1. Physical parameters of the inverted pendulum system considered in Example 1.

Parameters	Description	Value
m	The mass of the pole	0.2212kg
L	The half-length of the pole	0.24m
M	The mass of the cart	0.5164kg
g	The gravity acceleration	9.8 m/s ²

Figure 5. Pareto front achieved by the multi-objective ABC algorithm for the integral time of the absolute value of the pole angle error $ITAE(\theta)$ and the cart position error $ITAE(x)$ for Example 1.

Table 2. Objective functions and optimum values of the design variables for three points determined in the Pareto front of Example 1.

		Point A	Point B	Point C
Design variables	γ_1	13.1652	12.3618	11.8899
	γ_2	63.7975	66.8592	66.5827
	λ_1	4.0409	3.4061	3.3169
	λ_2	0.6758	0.8206	1.1326
	Φ_z	27.4614	27.4129	27.4202
	z_u	12.1020	14.7029	15.2755
	$\Phi_1(0)$	201.1229	201.5650	201.6823
	$K(0)$	367.4334	375.5198	373.8103
Objective functions	$ITAE(\theta)$	41.901	43.0299	47.6301
	$ITAE(x)$	1.6372	1.2731	1.1458

In this example, using the proposed optimization algorithm, the Pareto front indicated in Figure 5 is obtained based on the objective functions introduced in Equations (30) and (31). Furthermore, Figures 6 and 7 show the comparison between the time responses of the adaptive control gains for three optimum points A, B and C. As shown in Figure 8, the suggested controller successes in bringing the cart to setpoint 0 from initial position 0.5 m in about 3s. During this process, the optimal adaptive robust controller manages to bring the pendulum angle to setpoint 0 from the initial position 45° in about 2.9s and keeps it in upright position as depicted in Figure 9. Further, the changes of the control efforts for the three optimum points during 1s are demonstrated in Figure 10.

Figure 6. Comparison between adaptive control gains K for three optimum points A, B and C in Example 1.

Figure 7. Comparison between adaptive control gains Φ for three optimum points A, B and C in Example 1.

Figure 8. Time responses of the cart position obtained using the proposed optimal adaptive robust controller for Example 1.

Figure 9. Time responses of the pole angle obtained using the proposed optimal adaptive robust controller for Example 1.

Figure 10. Control efforts obtained using the proposed optimal adaptive robust controller for Example 1.

5.2 Example 2

In this example, the performance of the suggested control strategy is compared with those presented by Lo and Kuo (1998), Hung and Chung (2007), Sahnehsaraei et al. (2014) and Mahmoodabadi et al. (2020). Hence, in order to have a fair comparison, the constant parameters of the inverted pendulum system are set as those reported in these references which are also illustrated in Table 3. Accordingly, the initial values of the system states are considered as $\theta = -60^\circ$, $\dot{\theta} = 0$, $x = 0$, and $\dot{x} = 0$. Moreover, the control input is bounded at $-60N \leq u \leq 60N$. Regarding these values and using the proposed optimization algorithm, the Pareto front illustrated in Figure 11 is achieved with regard to the objective functions introduced in Equations (30) and (31). It is obvious from this figure that all the optimum design points in the Pareto front are non-dominated solutions. In other words, the related design variables of the Pareto front depicted in Figure 10 are the best feasible design points. Additionally, the figure clearly shows that selecting a better value for each objective function in a Pareto front gives rise to a worse value for another one. In other words, all presented points in the Pareto front are non-dominated and the designer could select each of them regarding the problem conditions. In this example, in order to compare the better time responses of the system outputs obtained by the proposed strategy and the methods reported by Lo and Kuo (1998), Hung and Chung (2007), Sahnehsaraei et al. (2014) and Mahmoodabadi et al. (2020), points A, B and C are chosen from the Pareto front. The corresponding design variables and objective functions are displayed in Table 4.

Table 3. Physical parameters of the inverted pendulum system considered in Example 2.

Parameters	Description	Value
m	The mass of the pole	0.05kg
L	The half-length of the pole	0.5m
M	The mass of the cart	1kg
g	The gravity acceleration	9.8 m/s ²
d _{max}	The upper bound of the uncertainty	0.0873

Figure 11. Pareto front achieved by the multi-objective ABC algorithm for the integral time of the absolute value of the pole angle error $ITAE(\theta)$ and the cart position error $ITAE(x)$ for Example 2.

Table 4. Objective functions and optimum values of the design variables for three points determined in the Pareto front of Example 2.

		Point A	Point B	Point C
Design variables	γ_1	25.18044	25.2353	25.1927
	γ_2	6.83785	6.7284	6.7609
	λ_1	2.00651	1.9992	2.0014
	λ_2	0.3816	0.6632	0.6585
	Φ_z	10.2769	10.2712	10.2747
	z_u	1.61915	1.7199	1.6803
	$\Phi_1(0)$	10.5484	10.5332	10.5322
	$K(0)$	23.54055	14.4686	18.3902
Objective functions	$ITAE(\theta)$	135.4621	158.5296	204.0219
	$ITAE(x)$	48.7483	26.72488	21.3621

Figure 12. Comparison between adaptive control gains K for three optimum points A, B and C in Example 2.

Figure 13. Comparison between adaptive control gains Φ for three optimum points A, B and C in Example 2.

Figure 14. Time responses of the cart position obtained using the proposed optimal adaptive robust controller and the methods introduced by Lo and Kuo (1998), Hung and Chung (2007), Sahnehsaraei et al. (2014) and Mahmoodabadi et al. (2020) for Example 2.

Figure 15. Time responses of the pole angle obtained using the proposed optimal adaptive robust controller and the methods introduced by Lo and Kuo (1998), Hung and Chung (2007), Sahnehsaraei et al. (2014) and Mahmoodabadi et al. (2020) for Example 2.

The time histories of the adaptive control gains for three optimum points are compared in Figures 12 and 13. With respect to the optimum values obtained in Table 4, Figures 14 and 15 respectively compare the time responses of the cart position and the pole angle acquired by the proposed method and the decoupled fuzzy sliding-mode control (Lo and Kuo 1998), decoupled sliding-mode with fuzzy-neural network control (Hung and Chung 2007), approximate feedback linearization and sliding mode control optimized by particle swarm optimization and genetic algorithms (Sahnehsaraei et al. 2014) and decoupled fuzzy

sliding mode control optimized by artificial bee colony and bacterial foraging algorithms (Mahmoodabadi et al. 2020). It is observable from Figure 14 that the proposed scheme successes to bring the cart to the desired position in a shorter period of time in comparison with the other methods. As shown in this figure, the presented controller by Hung and Chung (2007) could not bring the cart to the desired point ($0m$), and the settling time values for the methods introduced by Lo and Kuo (1998), Sahnehsaraei et al. (2014) and Mahmoodabadi et al. (2020) are respectively $t_s = 15s, 27s$ and $10s$. However, the suggested controller by this research (point C) is able to reduce it to about $5s$. While the settling time is improved considerably (respectively 66.6%, 100%, 81% and 50% reduction compared to the decoupled fuzzy sliding-mode control, decoupled sliding-mode with fuzzy-neural network control, approximate feedback linearization and sliding mode control optimized by particle swarm optimization and genetic algorithms and decoupled fuzzy sliding mode control optimized by artificial bee colony and bacterial foraging algorithms, the overshoot value also reaches an acceptable value ($M_p = 6.2$) in comparison to values of these methods that are respectively 11.3, 7.8, 6.4 and 6. It means that the suggested controller presents an efficient method with respect to control the cart position. Furthermore, Figure 15 indicates that the proposed controller has a better settling time in comparison to the decoupled fuzzy sliding-mode control (Lo and Kuo 1998), approximate feedback linearization and sliding mode control optimized by particle swarm optimization and genetic algorithms (Sahnehsaraei et al. 2014) and decoupled fuzzy sliding mode control optimized by artificial bee colony and bacterial foraging algorithms (Mahmoodabadi et al. 2020); however, it has a higher overshoot value. Eventually, Figure 16 denotes that the control input limitation is satisfied and does not exceed the limitation of the maximum absolute value of the control effort. Moreover, this figure shows that the control effort related to the recommended technique like approximate feedback linearization and sliding mode control scheme quickly reaches zero compared to the decoupled fuzzy sliding-mode control method. Meanwhile, two other published studies did not present the time histories of the control effort behavior.

Figure 16. Control efforts obtained using the proposed optimal adaptive robust controller and the methods introduced by Lo and Kuo (1998) and Sahnehsaraei et al. (2014) for Example 2.

Table 5. Physical parameters of the inverted pendulum system considered in Example 3.

Parameters	Description	Value
m	The mass of the pole	0.05kg
L	The half-length of the pole	0.5m
M	The mass of the cart	2kg
g	The gravity acceleration	9.8 m/s ²

Figure 17. Pareto front achieved by the multi-objective ABC algorithm for the integral time of the absolute value of the pole angle error $ITAE(\theta)$ and the cart position error $ITAE(x)$ for Example 3.

Table 6. Objective functions and optimum values of the design variables for point A in the Pareto front of Example 3.

		Point A	Point B	Point C
Design variables	γ_1	4.7096	4.7055	4.6781
	γ_2	10.9887	11.8604	14.5960
	λ_1	2.7483	2.7482	2.7473
	λ_2	1.5075	1.5638	1.8678
	Φ_z	8.5633	8.6403	8.5735
	z_u	3.7339	3.7347	3.7169
	$\Phi_1(0)$	3.1382	3.1451	6.5324
	$K(0)$	115.4118	110.9972	95.8142
Objective functions	$ITAE(\theta)$	39.6468	40.1781	41.3445
	$ITAE(x)$	1.0037	0.9779	0.8729

5.3 Example 3

Finally, in this example, to prove the superiority of the proposed control strategy, it is compared with Pareto optimal decoupled sliding mode control (Mahmoodabadi et al. 2013), time-varying sliding-coefficient-based decoupled terminal sliding-mode control (Bayramoglu and Komurcugil 2014) and optimal adaptive hybrid control (Mahmoodabadi and Khoobroo Haghbayan 2020). Therefore, in order to have a fair comparison, the constant parameters of the inverted pendulum system are set as those reported in those references which are also illustrated in Table 5. Accordingly, the initial values of the system states are considered as $\theta = 30^\circ$, $\dot{\theta} = 0$, $x = 0$, and $\dot{x} = 0$. Moreover, the control input is limited at $-150\text{ N} \leq u \leq 150\text{ N}$.

In this example, in order to better display the time responses and avoid disarrangement in comparative figures, only point A is chosen from the Pareto front. Besides, the corresponding design variables and objective functions are illustrated in Table 6.

Figure 18. Comparison between adaptive control gains K for three optimum points A, B and C in Example 3.

Figure 19. Comparison between adaptive control gains Φ for three optimum points A, B and C in Example 3.

Figure 20. Time responses of the cart position obtained using the proposed optimal adaptive robust controller and the methods introduced by Mahmoodabadi et al. (2013), Bayramoglu and Komurcugil (2014), and Mahmoodabadi and Khoobroo Haghbayan (2020) for Example 3.

Figure 21. Time responses of the pole angle obtained using the proposed optimal adaptive robust controller and the methods introduced by Mahmoodabadi et al. (2013), Bayramoglu and Komurcugil (2014), and Mahmoodabadi and Khoobroo Haghbayan (2020) for Example 3.

In this example, the effectiveness of the suggested optimal adaptive robust controller is more firmly proved during a different comparison with approaches introduced by Mahmoodabadi et al. (2013), Bayramoglu and Komurcugil (2014) and Mahmoodabadi and Khoobroo (2020). Accordingly, Figures 18 and 19 respectively depicted the comparison between adaptive control gains K and Φ over time. Figure 20 strongly verifies the effectiveness of the suggested controller compared to other published studies. In fact, although the proposed controller brings the cart to the desired position $x=0m$ in about $t_s = 1.1s$, the settling time of Pareto optimal decoupled sliding mode control, time-varying sliding-coefficient-based decoupled terminal sliding-mode control and optimal adaptive hybrid control are respectively 14s, 14s and 2.2s. Besides, the suggested controller has the minimum undershoot $M_p = -0.8$ in comparison with $M_p = -1$, -4.5 and -4.5 respectively found by Mahmoodabadi et al. (2013), Bayramoglu and Komurcugil (2014), and Mahmoodabadi and Khoobroo Haghbayan (2020). These mean 92.14 %, 50 %, 50% reduction in the settling time as well as 82%, 82% and 20 % reduction in the undershoot in comparison with Pareto optimal decoupled sliding mode control, time-varying sliding-coefficient-based decoupled terminal sliding-mode control and optimal adaptive hybrid control. In other words, for stabilizing the cart position, the suggested method has not only a considerable reduction of the settling time, but also a significant superiority over other approaches with regard to the overshoot. Further, in this example, the proposed controller demonstrates the better settling time to handle the pole angle so that it is $t_s = 1.8s$ in comparison to $t_s = 5.5$, 5.5 and $3.1s$ correspondingly belong to Mahmoodabadi et al. (2013), Bayramoglu and Komurcugil (2014), and Mahmoodabadi and Khoobroo Haghbayan (2020), as shown in Figure 21. It means that the suggested controller succeeds to decrease the settling time by 67.2%, compared to Pareto optimal decoupled sliding mode control, time-varying sliding-coefficient-based decoupled terminal sliding-mode control and optimal adaptive hybrid control.

6. Conclusions

In this research, an optimal adaptive robust control scheme was introduced and applied for stabilizing the inverted pendulum system. In order to reach this goal, two sliding surfaces were designed for the system states and connected by means of an intermediate variable. Accordingly, the gradient descent was utilized to adapt the DSMC parameters, and the multi-objective ABC algorithm was implemented to optimize the controller gains. The conflicting objective functions are regarded as the integral time of the absolute value of the pole angle and cart position errors that should be minimized, simultaneously. Finally, the obtained results were conducted to prove the superiority of the suggested strategy in comparison with those of several references given in the literature review. Although the suggested approach is favorable, there are several developments that can be still made.

- 1- Fuzzy systems can be designed in order to tune the controller gains.
- 2- Other multi-objective optimization algorithms can be applied to find the Pareto optimum fronts related to the conflicting objective functions.
- 3- The introduced scenario can be applied for stabilizing other higher-order dynamical systems.
- 4- Fractional order calculus may be utilized to design fractional order sliding surfaces.
- 5- Linear/nonlinear model predictive control approaches can be applied for stabilization of the inverted pendulum system.

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