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Experimental Investigation and Numerical Modeling of Piezoelectric Bender Element Motion and Wave Propagation Analysis in Soils

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Experimental Investigation and Numerical Modeling of
Piezoelectric Bender Element Motion and Wave Propagation
Analysis in Soils

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Abstract

The bender element (BE) test has been widely used for the dynamic characterization of soil specimens at low-shear strain levels. However, the actual behavior of the BE inside a soil specimen remains unknown. Thus, the current ASTM standard does not consider the interference of compressional and shear waves in BE testing, which can lead to significant errors in the evaluation of shear wave velocities. The main objective of this paper is to present a numerical model of the BE system to better understand the response of the BEs inside a soil sample. The model is calibrated, verified, and then used to demonstrate the importance of taking into consideration the interaction between compressional and shear waves for the correct interpretation of BE measurements. The model successfully captured the measured vibrations of the BE in air as well as inside transparent soils. More importantly, the numerical simulations provide a new understating of the significant interactions of P-waves and S-waves especially in clay soils. Thus, the proposed coupled piezoelectric and solid mechanics model can be used to study the soil-BE interaction so that sound recommendations can be given to improve the interpretation of BE tests in different soils.

Keywords: Bender element test, laser vibrometer, wave propagation, solid mechanics, piezoelectric, finite element method

1. Introduction

The dynamic soil properties play an important role in the design of earthquake-resistant structures and foundations. The bender element (BE) test and resonant column (RC) test are the most popular methods used for the evaluation of dynamic soil properties such as the shear wave velocity at low shear strains. The RC test is used to determine the resonant frequency of a soil column, which is related to the shear wave velocity and shear modulus. However, the RC test is time-consuming and costly in comparison to the BE test. The BE utilizes piezo-ceramic materials for the conversion of an electrical signal into mechanical energy. Two bender elements are placed at the two ends of the soil specimen in which one BE is used to introduce a mechanical impulse and the other one is used to receive the propagating pulse (normally in mV). The BE generates not only S-waves in the direction of their plane but also P-waves in the direction normal to their plane. The P-waves reflected from the cell walls can interfere with the generated S-waves (Lee and Santamarina 2005).

The behavior of the BE has been studied both numerically and experimentally in the literature. For example, Lee and Santamarina (2005) showed that the P-wave reflected from the cell wall in the BE test arrives to the BE receiver earlier than the direct S-wave. In practice, the interpretation of the BE test results requires the consideration of the geometry of the specimen, such as the radius-to-height ratio (Lee and Santamarina 2005). It is found that the resonant frequency of the BE embedded within the soil specimen depends on the stiffness of the BE, soil stiffness, and stress level in the soil (Camacho et al. 2015). The BE has been also used in a triaxial apparatus for the measurement of anisotropy of fine-grained soils (Jovičić and Coop 1998); it has shown that the BE is effective in measuring the inherent anisotropy resulting from the plastic strain history. Youn et al. (2008) compared the BE test

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34 with the resonant column test as well as the torsional shear test in sands; the results showed
35 that the values of the shear wave velocity determined from the BE test correspond well with
36 the resonant column test as well as the torsional test in dry conditions (Youn et al. 2008).
37 In saturated conditions, however, the current interpretation methods for BE measurements
38 tends to overestimate the shear wave velocity in comparison to the two other tests (resonant
39 column test and the torsional test), which is likely due to the interaction between the S- and
40 P-waves (Youn et al. 2008). The stiffness of unsaturated soils was tested with the BE and
41 suction-controlled resonant column tests by Hoyos et al. (2015). A good agreement between
42 the BE and resonant column tests was reported in terms of the measurement in stiffness
43 of silty sand under a suction-controlled condition. Gu et al. (2015) compared the results
44 of the BE test with the RC test and the cyclic torsional shear test at various confining
45 pressures, densities, and degrees of saturation. It was found that the travel time, under
46 saturated conditions, obtained from the BE test is considerably smaller than that from the
47 RC test due to the dispersion of the S-wave. However, this result could be attributed to the
48 strong participation of P-waves observed in BE tests in saturated media. Recommendations
49 in terms of the selection of the impulse frequency is given to reduce the subjectivity in
50 interpreting the arrival time of S waves (Camacho et al. 2012); a single sinusoidal pulse is
51 recommended and the travel length-to-wavelength ratio should be high enough to reduce
52 the near-field effect.

53 It is known that the transmitter response of the BE inside the soil specimen is very
54 different from the input electrical signal. The BE response is not well represented by a
55 cantilever beam as typically assumed in the literature (Karl et al. 2003; Lee and Santamarina
56 2005; Zhou et al. 2007). Hence, to investigate the disparity between the input signal and
57 the actual transmitter response, Irfan et al. (2019) used a new BE test setup using a laser
58 vibrometer. They used a transparent granular soil specimen such that the real transmitter
59 response can be measured by the laser vibrometer. The BE was inserted in the water, sucrose

60 solutions, mineral oils, and air to account for the effects of viscosity and density of the fluid
61 medium and for the calibration of the BE transducer. The important findings from such
62 experimental tests were: 1) the input square and step function pulses excite higher modes
63 in comparison to the sine function pulses; 2) the fluid density dominates the transmitter
64 response more than the viscosity of fluid; 3) there are P-wave components generated in
65 typical BE tests.

66 Several methods have been developed in the past to interpret the signal obtained through
67 the BE test. The first group is to calculate the shear wave velocity based on the first arrival
68 time of S-wave, which includes the start-start method, peak-peak method, cross correlation
69 method and cross power method. The cross correlation method is used to measure the degree
70 of correlation of input and output signals (Viggiani and Atkinson 1995) and the cross power
71 method measures the correlation of input and output signal in frequency domain. However,
72 the arrival-time based methods usually results in subjective and inaccurate interpretation of
73 the shear wave velocity. Currently, it is still unclear which method provides the most reliable
74 results (Gu et al. 2015). An automatic shear wave velocity estimation method was developed
75 by Finas et al. (2016) by applying the Akaike information criteria, which is effective for a
76 signal-to-noise ratio (SNR) smaller than 4. This inherent complexity in the analysis of BE
77 tests is clarified in this paper by the use of calibrated numerical simulations.

78 The behaviour of the BE has been numerically studied in the literature. Arulnathan et al.
79 (1998) used the finite element (FE) method to illustrate that the interpretation of the BE
80 signals based on the cross correlation between the input and output signals is misleading due
81 to the effects of wave interference at the boundaries, the phase lag between the mechanical
82 energy and electrical input and multi-dimensional wave travel issues. A two-dimensional
83 (2D) discrete element method was used by O'Donovan et al. (2012) to study the response of
84 an idealised granular material in the BE test. The particle velocity data was used to show
85 the propagation of a central S-wave accompanied by P-waves moving along the sides of the

soil specimen. A 2D finite element model was used by Ingale et al. (2020) to study the effect of soil types and frequency on BE measurements. It was shown that the FE analysis is consistent with the S-wave velocity obtained through the peak-to-peak method. Arroyo et al. (2006) studied the effect of sample size on BE measurements by means of 3D finite element modeling. It was assumed that the displacement at the tip of the BE transmitter has the same displacement as the input voltage. However, this boundary condition assumed by Arroyo et al. (2006), Arulnathan et al. (1998), Hardy et al. (2002), Jovičić et al. (1996) does not consider the piezoelectric behavior of the BE transmitter and the disparity between the input signal and the actual transmitter response.

An analytical model of the piezoelectric BE motion was developed by Zhou et al. (2007) based on the first-order shear deformation theory by assuming a single rotation angle. An analytical modelling approach was also developed based on the beam theory under the quasi-static equilibrium condition, which can be used for the optimized design of piezoelectric bending actuators (Dunsch and Breguet 2007). A close-form 3D piezoelectric model was developed by Rabbani et al. (2019) to investigate the free vibration of triclinic piezoelectric hollow cylinder using the transfer matrix method and the state space method.

Despite the above-mentioned efforts, the actual behaviour of the BE inside the soil still remains unknown. Currently, there is no standard interpretation of the BE measurements due to the complex wave interaction introduced by the BE within the soil specimens. The accurate interpretation of BE measurements and wave propagation analysis is impossible without knowing the motion of the BE inside the soil.

In this paper, a piezoelectric-solid mechanics model is proposed to study the BE motion in different media. The model is validated using the BE motion in the air, transparent soil, and Ottawa sand monitored by a laser vibrometer device. The estimation of the soil parameters such as the shear wave velocity and damping ratio of the Ottawa sand using the piezoelectric-solid mechanics model developed in this paper is then compared with indepen-

112 dent experimental data obtained via the conventional RC test. Finally, the propagation of
113 P- and S-waves within a soil specimen due to the BE motion is thoroughly studied and the
114 suitability of empirical methods in estimating the arrival time of the S-wave is discussed.

115 **2. Methodology and Experimental Setup**

116 In this work, a FE model of a BE-soil sample is calibrated and verified using the laser
117 vibrometer measurements on a) BE in air, b) BE in transparent soil, and c) BE in Ot-
118 tawa sand specimen. Then, the results are independently verified using resonant column
119 measurements. The detailed procedure is summarized in Figure 1.

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Figure 1: Flowchart of the detailed procedures for the calibration and verification of the BE motion as well as wave analysis within a soil specimen.

2.1. Experimental Setup

A general schematic of the experimental setup is illustrated in Figure 2. The soil specimen is vibrated at the bottom via a piezoelectric BE. The generated P wave and S wave contribute to the overall displacement at the top surface of the soil specimen. The BE motion is then studied through the laser vibrometer readings as well as a piezoelectric-solid mechanics FE modeling using two main soil sample configurations: a) transparent soil to evaluate the BE response and b) an Ottawa sand specimen to evaluate the surface response induced by the

127 BE.

Figure 2: Configuration of the BE test and the laser vibrometer apparatus

128 The BE response under a sinusoidal electrical impulse is monitored in the air through a
 129 laser vibrometer apparatus. Based on the response of the BE, a coupled piezoelectrical-solid
 130 mechanics finite element model is calibrated so that the numerical prediction fits with the
 131 laser vibrometer measurements. The calibrated model in the air is then verified against the
 132 laser vibrometer readings in a transparent soil. In the end, the calibrated model is used to
 133 predict independent vibrations induced by the BE motion within an Ottawa sand specimen
 134 under a confining pressure of 100 kPa. The soil parameters determined through trial and
 135 error from the FE model are then compared with the values obtained from a conventional
 136 RC test.

137 2.2. *Experimental Tests*

138 In the experimental BE tests shown in Figure 2, a function generator (model HP-33120A)
139 generates the desired input voltage signal applied to the BE transmitter through the steel
140 base. The signal is monitored by the oscilloscope (HP-54645A). The BE transforms the
141 electrical energy into the mechanical energy, which then applies an ultrasonic impulse to the
142 soil specimen. The BE transmitter used in this test protrudes 6 mm into the soil specimen
143 and has a 14 mm by 1.0 mm cross section. The laser vibrometer (polytec, 2013) measures
144 the displacement at a single point.

145 The measurements from the Ottawa sand specimen are used for the further verification
146 of the proposed numerical model. The soil sample is 7.0 cm in diameter and 14 cm in height.
147 The density of the dry sand is 1,848 kg/m³. The soil specimen is slightly compacted and
148 covered with a latex membrane to hold the sand in place. A 100 kPa vacuum pressure is
149 applied at the bottom of the sand specimen. The vibration introduced by the BE transmitter
150 was captured through the laser vibrometer on the wall of the soil specimen membrane, as
151 shown in Figure 3. The measurements along the specimen are taken for every 1 cm. A
152 reflective adhesive tape is applied to the membrane wall to improve the signal intensity.
153 The rubber membrane has a negligible effect on the displacement measurement as it is very
154 thin (0.2 mm thick).

Figure 3: Vibration measurements through the laser vibrometer for the Ottawa sand specimen

155 Furthermore, the resonant column test is performed based on the ASTM standard ASTM
 156 (2007) for verification of the BE measurements at 100 kPa. The built-in source in the
 157 spectrum analyzer (HP-35670A) is used to apply a sinusoidal sweep input voltage. Due to
 158 the limited power in the spectrum analyzer, the power amplifier (Bogen GS-250) is used
 159 to amplify the input voltage. Such input current introduces the vibration of the magnets,
 160 which in turn induce a torsional excitation in the soil sample. The response of the specimen
 161 is recorded in terms of acceleration via the accelerometers (PCB353A78 and PCB 353B65)
 162 mounted on the driving plate. The current in the coils and the acceleration are amplified and
 163 filtered (200 Hz low pass) using a filter amplifier (KrohnHite 3384) before being processed
 164 by the spectrum analyzer for the transfer function calculations. The spectrum analyzer
 165 calculates the transfer function in real time. The resonance frequency and damping ratio of
 166 the soil specimen are computed from the transfer function.

167 3. Theoretical Background

168 The motion of the BE in different media considering the converse piezoelectric effect can
 169 be numerically simulated using a piezoelectric-solid mechanics theoretical framework. Also,
 170 the soil is considered as a solid, isotropic, and homogeneous medium. This framework can
 171 be then implemented in a FE numerical tool for further analyses. In this section, we will
 172 briefly review the assumptions and field equations. The FE modeling is then discussed.

173 3.1. Kinematic Assumptions

174 The linearized form of the Green-Lagrange strain tensor, ε_{ij} , for infinitesimal deforma-
 175 tions of solid media (BE and soil) and electrical field E_i^f are described, respectively, as
 176 follows Matin et al. (2010).

$$\begin{aligned}\varepsilon_{ij} &= \frac{1}{2}(u_{i,j} + u_{j,i}) \\ E_i^f &= -\phi_{,i}\end{aligned}\tag{1}$$

177 where $u_{i,j}$ represents $\frac{\partial u_i}{\partial x_j}$; u_i represents the displacement vector components of the solid
 178 medium in each direction and x_j represents the coordinates; E_i^f denotes the electrical field
 179 vector; ϕ is the electric potential.

180 3.2. Constitutive Models

181 The constitutive models that describe the stress-strain and electrical displacement-field
 182 relationships are defined as:

$$\begin{aligned}\sigma_{ij} &= C_{ijkl} \varepsilon_{kl} - e_{kij} E_k \\ D_i &= e_{ikl} \varepsilon_{kl} + \epsilon_{ik} E_k\end{aligned}\tag{2}$$

183 where σ_{ij} is the stress tensor; e_{kij} and ϵ_{ik} denote the piezoelectric tensor and dielectric
 184 permittivity tensor, respectively. D_i represents the electrical displacement. C_{ijkl} is the

fourth-order linear elastic stiffness tensor described in Equation 3 for isotropic materials Liu et al. (2020), Mase et al. (2009):

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \quad (3)$$

where $C_{11} = C_{33} = \frac{E'(1-\nu)}{(1+\nu)(1-2\nu)}$; $C_{12} = C_{13} = \frac{E'\nu}{(1+\nu)(1-2\nu)}$; $C_{44} = C_{55} = C_{66} = \frac{E'}{2(1+\nu)}$; E' and μ are Young's modulus and Poisson's ratio, respectively. It is worth mentioning that the stress-strain relationship for soils can be written as $\sigma_{ij} = C_{ijkl} \varepsilon_{kl}$. Also, the piezoelectric tensor e_{ijk} (coulomb/m²) is written as Myers et al. (2010):

$$e = \begin{pmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{pmatrix} \quad (4)$$

Piezoelectric materials have the ability to produce an electrical voltage with an applied load; vice versa, motions are generated if an electric field is applied. Such phenomena is described through the piezoelectric tensor.

3.3. Conservation Laws

Conservation of the linear momentum for a solid medium (BE and soil) is written as:

$$\sigma_{ij,j} = \rho \ddot{u}_i \quad (5)$$

where ρ is the bulk density.

Gauss's law is used to describe the conversation of charge in the piezoelectric BE:

$$D_{i,i} = 0. \quad (6)$$

198 3.4. Field Equations

199 The governing equations for the BE can be written in terms of the displacement vector
200 u_i as well as the electric field vector E_i^f as:

$$\begin{aligned} \mu u_{i,jj} + (\lambda + \mu) u_{j,ji} - e_{kij,j} E_k^f &= \rho \ddot{u}_i \\ e_{ikl,i} \varepsilon_{kl} + \epsilon_{ik,i} E_k^f &= 0. \end{aligned} \quad (7)$$

201 It should be noted the field equation 7 is obtained through the conservation of momentum
202 and the conservation of charge, respectively. The coupled field equations are then solved
203 simultaneously due to the coupling tensor e_{ikl} , which represents the inherent properties of
204 the piezoelectric materials.

205 Similarly, the field equation governing the propagation of stress waves into the soil spec-
206 imen due to the piezoelectric BE motion can be written as

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} = \rho \ddot{u}_i. \quad (8)$$

207 3.5. Finite Element Modeling and Boundary Conditions

208 The 2D FE method is used to solve the field equations described above via COMSOL
209 Multiphysics (COMSOL 2007). The triangular element type is used for the analysis. The
210 direct solver is used instead of an iterative solver due to its robust nature (Liu et al. 2019).
211 There are 5,145 elements with an average quality of 0.8592 (length to width ratio). Based on
212 a extensive mesh sensitivity analysis, it is found that the numerical results are no longer sen-
213 sitive to the mesh size if a finer mesh is used. With such settings in COMSOL Multiphysics,
214 the relative error can be controlled to an acceptable tolerance. The mesh distribution in the
215 soil specimen and BE is shown in Figure 4.

216 The BE having a height of 6 mm, a length of 14 mm, and a width of 1.0 mm is shaped

Figure 4: Mesh distribution for the BE test

as a cantilever beam and contains two layers of piezoelectric ceramic plates with a metal plate in the middle. If the poling direction of these two layers of piezoelectric elements is in the same direction, it would be called parallel and if the poling direction is in opposite directions, it would be called series type. The parallel type needs a low voltage to work, but the series type needs, for the same application, twice the voltage magnitude to work (Zhou et al. 2008; Mulmi et al. 2008). The parallel BE is used as a transmitter and the series type is used as a receiver (Zhou et al. 2008; Mulmi et al. 2008). The Ottawa sand specimen has a width of 70 mm and a height of 140 mm.

A sinusoidal impulse voltage with a frequency of 9 kHz, which is close to the resonant frequency of the BE used in our tests, is applied to the BE (plus sign in Figure 4). A fixed

boundary condition is applied to the top and bottom of the soil specimen. A fixed boundary condition is also applied to one end of the BE transmitter (lower end) and receiver (upper end). The remaining boundaries are considered as a free surface (zero stress) to allow the reflection of stress waves. The interface between the BE and surrounding soil is modeled by meeting the continuity conditions. The initial displacement and velocity of the BE and surrounding soils are set to be zero. The components of the coupling tensor in Equation 4 has the following values: $e_{31} = -5.35 \text{ C/m}^2$; $e_{15} = 15.78 \text{ C/m}^2$ and $e_{33} = 12.29 \text{ C/m}^2$ (Myers et al. 2010). The density of the piezoelectric ceramic plates used in the BE structure is $7,870 \text{ kg/m}^3$. The mechanical properties of the piezoelectric ceramic plates and the metal plate used in the BE structure, transparent soil and Ottawa sand are obtained through the calibration procedure by trial and error as presented in Section 4.1 and Section 4.2.

4. Results and Discussion

4.1. BE Motion Calibration and Verification

The laser pointer is concentrated at a point with a height of 5 mm shown in Figure 2. The comparison of the BE motion in the air between the experimental measurements and numerical predictions in time and frequency domains is shown in Figures 5a and 5b, respectively. As can be seen in Figure 5, the numerical results using the calibrated piezoelectric-solid mechanics model are in good agreement with the experimental measurements.

The mechanical properties of the BE are calibrated through trial and error to best fit the numerical predictions with the laser measurements. The best matching between the numerical results and the laser measurements was achieved when Young's modulus and Poisson's ratio of the piezoelectric ceramic plates are 65 GPa and 0.3, respectively; Young's modulus of the metal plate used in the BE structure is 243 GPa; and, the damping ratio for the piezoelectric ceramic plates and the metal plate used in the BE structure is 2.5%.

(a) time domain

(b) frequency domain

Figure 5: Comparison between the experimental measurements and numerical results for the BE motion in the air in (a) time domain, (b) frequency domain.

251 The calibrated FE model is further verified through the experimental BE tests on the
 252 transparent soil (made of silica and oil) where the BE motion was monitored via the laser

253 vibrometer. Under the same impulse voltage, the comparison between the numerical FE
254 results and the experimental measurements of the BE motion are shown in Figure 6 in time
255 and frequency domains.

(a) Time domain

(b) Frequency domain

Figure 6: Comparison between the experimental measurements and numerical results for the BE motion in the transparent soil in (a) time domain, (b) frequency domain.

The mechanical properties of the transparent soil used in the FE analysis are calibrated by trial and error. It is found that the speeds of the P wave and S wave in the transparent soil studied in this test are approximately 1,200 m/s and 15 m/s, respectively. The equivalent damping ratio of the transparent soil including the visco-elastic effect is assumed as 0.3. This damping value is high because it is representing not only the damping of the soil, but also the effect of the added mass of the transparent soil to the response of the BE (Irfan et al. 2019). The very low shear wave velocity derived numerically is due to the fact that the confining pressure is practically zero in this experiment. The high value of the P-wave velocity is generated because of the saturated conditions. The calibrated piezoelectric-solid mechanics model for the BE motion is still able to capture the motion of the BE in the transparent soil.

4.2. BE Motion in Ottawa Sand

The BE motion can be directly monitored in the air and transparent soil through the laser vibrometer since the laser light can penetrate into these media. It is not, however, the case for the BE test performed on real soils. Therefore, the displacement at the sides of the soil specimen is monitored instead of the BE itself. The setup for this test can be seen in Figure 3. The original laser measurements at the elevations of 2.5 cm (trace 0) to 13.5 cm (trace 12), with an interval of 1 cm, are shown in Figure 7. This signal is contaminated with the higher resonant modes of the BE motion (Lee and Santamarina 2005). Since the applied voltage signal is 9 kHz, the components above 15 kHz are removed through the wavelet synchrosqueezed transform (Daubechies et al. 2011). The components below 15 kHz are obtained through the inverse wavelet synchrosqueezed transform.

Figure 7: Original displacement measurement along Ottawa sand specimen using the laser vibrometer

Based on the calibrated piezoelectric-solid mechanics model for the BE, the soil parameters are then modified through trial and error to match the filtered laser vibrometer measurements. For example, the comparison between numerical and experimental displacements is shown in Figure 8. The best fitting was achieved by using the shear wave velocity of 240 m/s, compression wave velocity of 380 m/s (equivalent to a Young's modulus of 249 MPa and a Poisson's ratio of 0.17), and a damping ratio of 1% for the Ottawa sand. The determined P and S wave velocities can also be verified from the original laser measurements. A constant slope (400 m/s slope) is also clearly visualized in the original displacement measurements, as labeled in Figure 7. The determined P wave velocity (380 m/s) was relatively close to the P wave velocity (400 m/s) visualized in the original laser measurements. Furthermore, the dispersion curves were also computed using the numerical and measured displacement data.

289 Figure 9 shows the comparison between the measured and numerical dispersion curves for
290 both symmetric and antisymmetric modes. The symmetric modes, also called longitudinal
291 modes, are generated due to the wave propagation in the longitudinal direction. On the
292 other hand, the antisymmetric modes are generated because of the wave propagation in the
293 transverse direction (Graff 1991).

294 The numerical predictions showed a reasonable agreement with the laser measurements
295 in terms of the distribution of dispersion curves. Therefore, the P wave and S wave veloc-
296 ities for the Ottawa sand are 380 m/s and 240 m/s, which is verified through comparison
297 in displacement measurements and dispersion curves. The RC test was also performed to
298 validate the shear wave velocity and damping ratio of the Ottawa sand under various confin-
299 ing pressures. Under a confining pressure of 100 kPa, the shear wave velocity and damping
300 ratio are found to be 263 m/s and 1.06%, which are consistent with the values used in the
301 numerical model through trial and error.

Figure 8: Comparison between the numerical results and experimental displacement measured at a distance of 5.5 and 7.5 cm from the bottom

Figure 9: Comparison between the numerical and experimental dispersion curves

302 4.3. Wave Analysis within Soil Specimen

303 The wave propagation is further analyzed through the displacement distribution within
304 the Ottawa sand specimen. The transmitter started the generation of mechanical energy
305 around $t = 0.31$ ms. This reference value is shown by *Trigger Point* in Figure 10.

306 The S wave velocity determined by the BE is relatively higher than the value obtained
307 by the standard RC test (Fam et al. 2002). The S wave velocity in the BE test is commonly
308 calculated as travel distance divided by the S wave arrival time. However, the arrival time of
309 the S wave is normally empirically selected around the first peak in the output signal. Based
310 on the determined S wave velocity from previous discussion, the exact S wave arrival time
311 can be located around 0.53 ms relative to the beginning of impulse (as labeled by Trigger
312 Point in Figure 10). The measured response in BE testing is given in terms of voltage
313 (Volts) while the numerical response is given by displacement (nm). These two signals are
314 not expected to be identical as the received signal does not have units of displacement,
315 velocity, or acceleration. The comparison between the measured and numerically calculated
316 signal at the BE receiver location is shown in Figure 10. A reasonable agreement between
317 the numerical results and experimental data is achieved. The S wave arrival time is normally
318 selected shortly before the peak in the BE receiver signal. However, it is shown that the
319 S wave arrival time is actually affected by the interaction with the P-wave. Therefore, the
320 proposed numerical model can be used to improve the interpretation of the effects of P-waves
321 on BE tests results.

Figure 10: Comparison between the experimental data and numerical results at the location of the BE receiver (this figure shows a relative comparison; the BE signals are not linearly related to the vibration measurements in terms of displacement, velocity, or acceleration).

The horizontal displacement contours corresponding to the travel times equal to one up to five excitation periods of BE are shown in Figure 11. The positive and negative displacements are shown in blue and red colors, respectively. A full wavelength is defined by a positive (red) and negative displacement (blue) wavefronts. For the dominant frequency of 8.7 kHz for the BE vibration in Ottawa sand, the S and P waves' wavelengths can be calculated as $\lambda_s = 2.8$ cm and $\lambda_p = 4.4$ cm, respectively. Two wavelengths are identified due to the different propagation speeds of P and S waves. In the BE test, the generation of the S wave mode (in terms of its amplitude) is much stronger than that of the P wave mode. Only a half wavelength (blue front) was identified for the P wave mode due to its weaker generation in the BE test. Furthermore, P wave attenuates with travelling distance and the identification of its full wavelength becomes impractical. Therefore, only a half wavelength of the P wave mode is labeled in Figure 11. After the first period (1T), the reflection of P

and S waves can be visualized clearly. The separation between the P wave and S wavefronts increased from the 2nd and 3rd periods. The P wavefront arrives at the BE receiver tip sometime between the 3rd and 4th periods. Similarly, the S wavefront arrives at the BE receiver tip sometime between the 4th and 5th periods.

Figure 11: Horizontal displacement contour in Ottawa sand (video 1 in the Supplementary materials)

Based on previous studies (e.g., (Khan et al. 2020; Leong et al. 2005)), it is recommended that the wave path length to wavelength ratio is at least 3.33. In our BE measurement, as shown in Figure 10, the wavelength is 2.8 cm and the length of the sample is 14 cm. Thus, the length to wavelength ratio is 5, which is bigger than the recommended 3.33. However, the interference of the P wave still exists, and these effects of P-wave reflections were not included in previous recommendations.

4.4. Numerical Study of the Effect of Different Poisson's Ratios (Loose Sand, Soft Clay)

The wave propagation is also investigated in two different soil specimens using the piezoelectric-solid mechanics FE model. In Case 1, the P and S wave velocities are 120 m/s and 69.3 m/s (equivalent to a Young's modulus of 22 MPa and a Poisson's ratio of 0.25) to simulate a loose sand specimen. In Case 2, a soft clay soil sample with a P wave velocity of 120 m/s and an S wave velocity of 35.2 m/s (equivalent to a Young's modulus of

350 6.7 MPa and a Poisson's ratio of 0.45) is studied. The horizontal displacements at the BE
351 receiver location are shown in Figure 12 for Case 1 and Case 2. The exact P wave travel
352 time is 1.07 ms relative to the beginning of the impulse as labeled by Trigger Point. In Case
353 1, the exact S wave travel time is 1.85 ms relative to the beginning of the impulse. Similarly,
354 in Case 2, the S wave travel time is 3.63 ms.

Figure 12: Horizontal displacement at the receiver location for Case 1 (Poisson's ratio = 0.25, loose sand, video 2 in the Supplementary materials) and Case 2 (Poisson's ratio =0.45, soft clay, video 3 in the Supplementary materials)

355 The signal obtained in Case 2 is more contaminated by the P wave because of its slower
356 S wave velocity. The components located between the first P wave arrival time (1.37 ms)

and first S wave arrival time (3.94 ms) are due to the P wave reflection from the sides of the soil specimen. This can be captured by the horizontal displacement contour at various wave peaks before the S-wave arrival, as illustrated in Figure 12. The horizontal displacement contour at various time (normalized in terms of the period) is shown in Figure 14. In this case, the selection of the S wave arrival time is rather challenging and unpredictable based on the empirical methods (e.g., start-start method and peak-peak method). However, the real arrival time of the S wave is not closely near to the largest amplitude, as shown in Figure 12.

Figure 13: Horizontal displacement of BE transmitter numerically predicted at a height of 5 mm for Case 1 (Poisson's ratio = 0.25, loose sand) and Case 2 (Poisson's ratio = 0.45, soft clay)

The horizontal displacement of the BE transmitter is shown in Figure 13 for both Case 1 and Case 2. From Figure 13, it is found that the BE transmitter vibrates with the dominant frequency of 7.8 kHz for the BE vibration in Case 1. Corresponding, the S and P waves' wavelengths can be calculated as $\lambda_s = 0.89$ cm and $\lambda_p = 1.5$ cm, respectively. In Case 2, the dominant frequency of the BE motion is 7.5 kHz and the S and P waves' wavelengths can

370 be calculated as $\lambda_s = 0.47$ cm and $\lambda_p = 1.6$ cm, respectively. The displacement contours at
371 different times (normalized in terms of period) are shown in Figure 14 and Figure 15 for Case
372 1 and Case 2, respectively. It is confirmed from Figure 14 and Figure 15 that the reflected P
373 waves from the sides of the specimen can arrive at the BE receiver location faster than the
374 shear wavefront. Therefore, the wave interactions of P and S waves with soil boundaries can
375 largely increase the complexity of the selection of the S wave arrival time. The traditional
376 empirical methods ((e.g., start-start method and peak-peak method)) cannot accommodate
377 the complex nature of wave interaction and may result in misleading predictions of the S
378 wave velocity.

Figure 14: Horizontal displacement contour in low-stiffness clay for Case 2 (Poisson’s ratio =0.45, soft clay, video 3 in the Supplementary materials)

Figure 15: Horizontal displacement contour in low-stiffness sand for Case 1 (Poisson's ratio = 0.25, loose sand, video 2 in the Supplementary materials)

5. Conclusion

In this paper, the response of different media to a BE motion is thoroughly studied via a piezoelectric-solid mechanics FE model as well as experimental tests. The numerical results are compared with the motion of the BE in the air, transparent soil, as well as the Ottawa sand captured by a laser vibrometer. It is concluded that:

- The proposed piezoelectric-mechanical model captures the motion of the BE with sufficient accuracy in the air, transparent soil, as well as the Ottawa sand. The best agreement was achieved for the BE motion in the air.
- The numerical response obtained by the proposed model is consistent with the laser vibrometer measurement at the sides of the Ottawa sand specimen. Furthermore, the numerical predictions show a reasonable agreement with the laser measurements in terms of the distribution of dispersion curves for both symmetric and antisymmetric modes.

- 392 • A reasonable agreement between the numerical BE response and experimental BE
393 measurement is achieved at the receiver location. The shear wave velocity and damping
394 ratio obtained through the proposed model are consistent with the ones obtained by
395 the resonant column test.
- 396 • The proposed numerical method shows that there is a significant P-wave/S-wave in-
397 teraction that demonstrates why the empirical methods for the selection of S-waves
398 in BE testing could be incorrect depending on the different parameters that affect the
399 participation of P-waves.
- 400 • The proposed piezoelectric-solid mechanics model can be used to study the complex
401 wave interactions, which significantly improves the interpretation of the effects of P-
402 waves on BE test results. The proposed model clearly show that the interpretation of
403 BE measurements in clays could be more challenging because of the strong participa-
404 tion of P-waves on the response of the BE.

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