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Design of Foundations Against Differential Settlement

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Abstract

The serviceability limit state (SLS) design of foundations typically proceeds by limiting the total settlement of individual foundations and thereby attempting to restrict the differential settlement between pairs of foundations. Due to the uncertain nature of the supporting ground, the magnitude of settlement and differential settlement are random. As it is often the differential settlement which governs serviceability, it is desirable to provide design requirements which suitably restrict differential settlements. This paper investigates, by Monte Carlo simulation, the distribution of the maximum differential settlement between pairs of foundations as a function of the spacing between foundations and the number of foundations – groups of 4, 9, or 16 foundations, arranged on a grid, are considered. The effects of the correlations between the equivalent stiffness of the ground under each foundation, as well as between the loads applied to the foundations, on the distribution of the maximum differential settlements and angular distortions are investigated. Ratios of resistance factor to resistance bias factor are presented that can be used to calibrate design requirements on the total settlement of individual foundations which also simultaneously achieve acceptable performance with respect to angular distortion.

Keywords: reliability-based design, differential settlement, angular distortion, total settlement, foundation design.

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Introduction

Whenever multiple foundations are used to support a structure, the potential for differential settlement between the foundations often governs the foundation design. In practice, the maximum differential settlement between pairs of foundations is usually unknown, and thus the design process is solely based on limiting the total settlement of individual foundations. Generally speaking, the maximum differential settlement is usually assumed to be some fraction, e.g. $3/4$ (French 1999), of the total settlement of individual foundations. However, it would be beneficial to characterize the distribution of maximum differential settlement given a maximum total settlement used in design.

Meyerhof (1953) provided some of the earliest requirements on differential settlement for structures, expressing the limitations in terms of angular distortion, defined as the differential settlement of a pair of foundations divided by the spacing between them. The paper by Skempton and MacDonald (1956) is widely referenced as the authority on the allowable settlement of buildings. Skempton and MacDonald also expressed limitations in terms of angular distortions. Grant et al. (1974) expanded on Skempton and MacDonald's work by considering a wider range of structures, but essentially came up with the same limitations on angular distortions. Other researchers have also contributed to the study of allowable movements in foundations and their effect on the supported buildings (Bjerrum 1963; Polshin and Tokar 1957; Burland and Wroth 1974; Wahls 1981).

The probabilistic analysis of foundation settlements has received some attention over the years. Zeitoun and Baker (1992) examined differential settlements between pairs of foundations in the problem of two beams acting on three supports. Roberts and Misra (2009) developed a reliability-based design approach for deep foundations which considered differential settlement

between a pair of deep foundations. More recently, Ahmed and Soubra (2013) predicted the probability of exceeding a tolerable differential settlement between two strip footings resting on a 2-D spatially variable elastic soil and found a worst-case correlation length. Schneider et al. (2015) also studied the differential settlement of a pair of strip footings founded on a spatially variable soil, and concluded that the use of a simple fraction of the total settlement (e.g. 75%) to predict differential settlement was overly conservative, particularly for sandy soils. Naghibi et. al (2016) also investigated the differential settlement between a pair of foundations and developed a distribution of the differential settlement which was validated by a Monte-Carlo simulation of a pair of foundations on a spatially random soil. Some studies have been undertaken to consider the maximum differential settlement between any pair of foundations in a group, e.g. Sheil and McCabe (2015), and Lin et al. (2015). However, to the authors' knowledge, none of these maximum differential settlement studies of foundations in a group have been probabilistic in nature.

In the current paper, the study of differential settlements is extended to a group of n_p foundations, as depicted in Figure 1, and the distribution of the maximum differential settlement and maximum angular distortion between the foundations, along with their relationships with the corresponding maximum total settlement of individual foundations, are studied. Values of n_p of 4, 9 and 16 are studied – considering up to 16 foundations covers many practical applications and is probably a reasonable range beyond which differential settlements lose importance. For example, the differential settlement between the first and last of 5 foundations arranged in a row would not likely govern the design – those two foundations would commonly be too widely separated, having three foundations between them.

The settlement analysis will be accomplished by modeling the ground as an equivalent elastic medium where the ground acts as a linear spring below each foundation relative to its design settlement. What this means is that the foundation is designed to yield a maximum tolerable settlement, e.g. 25 mm, under the factored design load, using whatever settlement predictions are available for the particular foundation type being considered (e.g. non-linear models for either shallow or deep foundations). The resulting foundation, of any type, is then assumed to “see” an equivalent mean spring stiffness equal to the mean ground resistance divided by the maximum tolerable design settlement (force/load). A more precise definition is provided below in the Section entitled “Generalized Estimation of Failure Probability”. The behaviour of the foundation around the maximum tolerable settlement is now assumed to be linearly elastic and spatial variations in load and elastic properties of the ground lead to differential settlements between foundations, as well as random variations in total settlement. It is the distribution of these total and differential settlements that are of interest in this paper, and design recommendations are made to limit maximum differential settlements and angular distortions.

In detail, a series of realizations of the random equivalent spring stiffness field under the foundations, along with realizations of the random loads acting on the foundations, were studied to extract the following quantities:

- i. the maximum differential settlement between any pair of foundations in the system
($\Delta_{\max}$),
- ii. the maximum total settlement of one of the foundations from the pair having the maximum differential settlement found in item i (δ_{Δ}),
- iii. the maximum angular distortion between any pair of foundations in the system ($\lambda_{\max}$),

- iv. the maximum total settlement of one of the foundations from the pair having the maximum angular distortion found in item iii (δ_{λ}).

Figure 1. Plan view of foundation locations.

The foundation group is assumed to be placed on a random spatially variable soil, which, as mentioned above, is represented by its equivalent stiffness properties. The total load, assumed to be a combination of dead and live loads, applied to each foundation is also assumed to be random and cross-correlated. Note that the stiffness of the supported structure is not directly modeled in this study, partly for simplicity, but also because every structure is different and it is hard to quantify that source of variability in a study of differential settlement. However, the assumption of a positive correlation between loads applied to each foundation does, to some extent, reflect the stiffness and load spreading capacity of the supported structure. For example, if a particularly high load is applied to one column above one foundation, the load will spread out, due to the stiffness of lateral structural elements, causing the loads felt in adjacent

foundations to also increase (hence a positive correlation). On the other hand, the negative correlation between loads and settlements, causing additional load shedding when a foundation settles excessively, is not captured by this model. The lack of the negative correlation between settlement and load is somewhat conservative, resulting in somewhat higher differential settlements than might be seen in reality. The overall effect of the stiffness of the supported structure is the subject of ongoing research. In the meantime, the results provided by this paper will be somewhat conservative.

Using Monte Carlo simulation, realizations of the equivalent spring stiffness under each foundation, along with the applied loads, are used to predict the settlement of each foundation, from which the maximum differential settlement is extracted. Each simulation involves the generation of spring stiffness and load values that are cross-correlated according to separation distance between foundations.

The paper is organized as follows: In the next section, a random field model is presented for a system of n_p foundations. The subsequent section discusses the corresponding simulation model. The methodology used is developed next and the results are then presented. An example of how this paper can be used to design foundations against excessive angular distortion is then provided. Conclusions and suggestions for future work are summarized in the final section.

Random Model

A random field $\tilde{X} = X(\tilde{x}_i)$, $i = 1, 2, \dots$ is a collection of random variables $X_1 = X(\tilde{x}_1), X_2 = X(\tilde{x}_2), \dots$, whose values are associated with each spatial location $\tilde{x}_i$. In this paper, $X(\tilde{x}_i)$ can represent either the random load or the random spring stiffness at location $\tilde{x}_i$.

Values in a random field are usually spatially correlated (e.g., Vanmarcke 1977 and 1984), and the spatial dependence in a field is characterized by the field correlation structure, which is commonly specified through a correlation function parameterized by correlation length, θ . In this paper, an isotropic exponentially decaying Markov correlation function is used, defined by

$$(1) \quad \rho_{ij} = \rho(\tau_{ij}) = \exp \left\{ \frac{-2|\tau_{ij}|}{\theta} \right\}$$

where τ_{ij} is the distance between any two points, X_i and X_j , in the field, and θ is the correlation length (Fenton and Griffiths 2008).

A lognormal distribution is commonly used for modeling engineering properties due to its non-negative nature and its simple relationship with the normal distribution. In particular, a lognormal random field can be easily produced through a simple transformation of a Gaussian random field. In general, if X is lognormal with mean and standard deviation μ_X and σ_X , then $\ln X$ is normal with parameters

$$(2) \quad \begin{aligned} \sigma_{\ln X}^2 &= \ln(1 + v_X^2) \\ \mu_{\ln X} &= \ln(\mu_X) - \frac{1}{2} \sigma_{\ln X}^2 \end{aligned}$$

where $v_X = \sigma_X / \mu_X$ is the coefficient of variation of X . In this research, both the load, F , and spring stiffness, K , are assumed to be lognormally distributed random variables. This implies that $\ln F$ and $\ln K$ are both normally distributed with parameters given by eq. (2) (where the subscript X is suitably replaced by either K or F). Furthermore, both load and spring stiffness are spatially varying random variables with an additional parameter being their correlation lengths, $\theta_{\ln F}$ or $\theta_{\ln K}$ respectively, replacing θ in eq. (1). The spring stiffness K would normally be derived from the equivalent elastic modulus of the soil (in the vicinity of the mean settlement) taking into account

the geometry of the foundation and the thickness of the supporting foundation layer (see, e.g., Poulos and Davis 1974; Fenton and Griffiths 2005).

Simulation Model

Various random field generation algorithms exist, of which the Covariance Matrix Decomposition (CMD, see e.g., Fenton and Griffiths 2008) method is employed in this research to provide realizations of the random load and spring stiffness fields. CMD is an exact method of producing realizations of a discrete random field (i.e., at the foundation locations). The method requires the mean, $\mu_{\ln X}$, and covariance matrix, $\underline{\underline{C}}$, having elements $C_{ij} = \rho_{ij} \sigma_{\ln X_i} \sigma_{\ln X_j}$, $i, j = 1, 2, \dots, n_p$. In detail, the elements of $\underline{\underline{C}}$ are the covariances between any pair of points in the field separated by distance τ_{ij} , where $\rho_{ij} = \rho(\tau_{ij})$ (see eq. 1). For a stationary random field, having spatially constant variance, the covariance matrix $\underline{\underline{C}}$ simplifies to being composed of the elements

$$C_{ij} = \begin{cases} \sigma_{\ln X}^2 & i = j \\ \sigma_{\ln X}^2 \rho_{ij} & i \neq j \end{cases} \quad (3)$$

As $\underline{\underline{C}}$ is a positive definite matrix, a properly correlated normally distributed (Gaussian) random field $G = G(x_i)$, $i = 1, 2, \dots, n_p$, where x_i is a point in the field, can be produced according to

$$G = \mu_{\ln X} + \underline{\underline{L}} \underline{\underline{Z}} \quad (4)$$

where $\underline{\underline{L}}$ is a lower triangular matrix satisfying $\underline{\underline{L}} \underline{\underline{L}}^T = \underline{\underline{C}}$ (obtained using a Cholesky Decomposition), and $\underline{\underline{Z}}$ is a vector of n_p independent standard normal random variables (mean zero, unit variance).

The lognormal random field, $\tilde{X}$, is obtained from the normal field, $\tilde{G}$, using the following transformation:

$$\tilde{X} = \exp\{\tilde{G}\}$$

Simulation Methodology

As mentioned previously, both the load and the soil's equivalent spring stiffness are assumed to be lognormally distributed. An individual foundation is subjected to random load F_i having mean μ_F and standard deviation σ_F . The foundation is also supported by an elastic soil with random equivalent spring stiffness K_i having mean μ_K and standard deviation σ_K . For each realization of F_i and K_i , $i = 1, 2, \dots, n_p$, where n_p is the number of foundations in a foundation group, the maximum differential settlement is defined as

$$\Delta_{\max} = \max_{i \neq j} |\delta_i - \delta_j|$$

where

$$\delta_i = \frac{F_i}{K_i}$$

is the settlement of foundation i . The maximum total settlement corresponding to the maximum differential settlement found in eq. (6) is defined as,

$$\delta_{\Delta} = \max\{\delta_i, \delta_j\}$$

where i and j are those foundation indices which yielded $\Delta_{\max}$ in eq. (6).

The maximum angular distortion is defined as

$$(9) \quad \lambda_{\max} = \max_{i \neq j} \left| \frac{\delta_i - \delta_j}{\tau_{ij}} \right|$$

where τ_{ij} is the distance between foundations i and j . Note that the pair of foundations leading to the maximum angular distortion may not necessarily be the same as those leading to the maximum differential settlement. For example, as shown in Figure 2, foundations 1 and 2 have the maximum angular distortion, whereas foundations 1 and 3 have the maximum differential settlement.

Figure 2: Illustration of the difference between $\Delta_{\max}$ and $\lambda_{\max}$

The maximum total settlement, δ_{λ} , corresponding to the maximum angular distortion found in eq. (9) is defined by eq. (8), except that i and j are those foundation indices which yielded $\lambda_{\max}$ in eq. (9).

The total settlement of an individual foundation, which is the ratio of load over spring stiffness as specified in eq. (7), is random and assumed to be lognormally distributed, thus,

$$(10) \quad \ln \delta_i = \ln \left(\frac{F_i}{K_i} \right) = \ln F_i - \ln K_i$$

is normally distributed with parameters:

$$(11) \quad \begin{aligned} \mu_{\ln \delta} &= \mu_{\ln F} - \mu_{\ln K} \\ \sigma_{\ln \delta}^2 &= \sigma_{\ln F}^2 + \sigma_{\ln K}^2 \end{aligned}$$

where independence between the random variables F_i and K_i (or $\ln F_i$ and $\ln K_i$) was assumed, as is reasonable, in order to compute $\sigma_{\ln \delta}^2$.

The simulation process is carried out as follows:

1. Two lognormal random fields, each of size n_p , are generated representing loads, F_i , and spring stiffnesses, K_i , associated with individual foundations in a foundation system arranged as depicted in Figure 1.
2. For each foundation, total settlement is calculated using eq. (7).
3. The maximum differential settlement, $\Delta_{\max}$, is determined for the foundation group using eq. (6), along with the corresponding maximum total settlement, δ_{Δ} , using eq. (8).
4. The maximum angular distortion, $\lambda_{\max}$, is determined for the foundation group using eq. (9), along with the corresponding maximum total settlement, δ_{λ} , using eq. (8).
5. Steps 1 through 4 are repeated n_{sim} times and the following quantities are estimated:

$\mu_{\Delta_{\max}}, \sigma_{\Delta_{\max}}, \mu_{\ln \delta, \Delta}, \sigma_{\ln \delta, \Delta}, \mu_{\lambda_{\max}}, \sigma_{\lambda_{\max}}, \mu_{\ln \delta, \lambda}$, and $\sigma_{\ln \delta, \lambda}$ each of which will be defined next.

The mean of $\Delta_{\max}$ is estimated using the usual method of moments,

$$(12) \quad \mu_{\Delta_{\max}} \approx \frac{1}{n_{sim}} \sum_{k=1}^{n_{sim}} \Delta_{\max, k}$$

where $\Delta_{\max, k}$ is the k^{th} realization of $\Delta_{\max}$. The mean of the maximum angular distortion, $\mu_{\lambda_{\max}}$, is calculated similarly.

The variance of $\Delta_{\max}$ is estimated as,

$$(13) \quad \sigma_{\Delta_{\max}}^2 \approx \frac{1}{n_{sim} - 1} \sum_{k=1}^{n_{sim}} (\Delta_{\max, k} - \mu_{\Delta_{\max}})^2$$

and similarly for the variance of the maximum angular distortion, $\sigma_{\lambda_{\max}}^2$.

The mean and variance of $\ln \delta_{\Delta}$ are estimated as follows:

$$\begin{aligned}
 \mu_{\ln \delta, \Delta} &\approx \frac{1}{n_{sim}} \sum_{k=1}^{n_{sim}} \ln \delta_{\Delta, k} \\
 \sigma_{\ln \delta, \Delta}^2 &\approx \frac{1}{n_{sim} - 1} \sum_{k=1}^{n_{sim}} (\ln \delta_{\Delta, k} - \mu_{\ln \delta, \Delta})^2
 \end{aligned}
 \tag{14}$$

where $\ln \delta_{\Delta, k}$ is the k^{th} realization of $\ln \delta_{\Delta}$. The mean and variance of $\ln \delta_{\lambda}$ are calculated similarly.

Regression Approximation to Simulation Results

Once the statistical parameters estimated in the previous section using simulation are derived, a regression can be performed to estimate these parameters without the need of a simulation in practice. In this section, a regression is fit to the results of simulations performed for all of the parameters listed in Table 1. The primes on dimensional quantities indicate values used in the simulation. Using the values listed in Table 1, the means and variances relating to foundation settlement will be approximated using regressions for various numbers of foundations, n_p , various coefficients of variation, v_K , and various standardized correlation lengths, $\theta'_{\ln K} / s'$. Note that the coefficient of variation of the total load, v_F , is assumed constant at 0.1. This assumption is based on coefficients of variation for live load of 0.27 and for dead load of 0.1 with a dead to live load ratio of 3, as shown by Naghibi and Fenton (2017). Note that $v_F = \frac{\sqrt{v_L^2 + v_D^2 R_{D/L}^2}}{1 + R_{D/L}}$ is quite insensitive to the dead to live load ratio, changing only slightly from 0.10 to 0.11 for dead to live load ratios going from 3 to 2.

Also, in this study, a standardized correlation length for the load of $\theta'_{\ln F} / s' = 2.0$ was assumed, which means that there is a reasonably strong correlation between loads on adjacent foundations. In that loads are distributed by the stiffness of the supported structure, a reasonably

strong correlation between foundation loads is believed to be appropriate. Although research into the best estimate of the actual correlation between foundation loads arising from a spatially variable supported structure, including effects of structural stiffness are ongoing, it is believed that the assumption made here is reasonable. As mentioned above, however, this model ignores the potential for negative correlation between the load on a foundation and its settlement, which thus is somewhat conservative.

Table 1. Input parameters used in the simulation.

Parameters	Values Considered
n_p	4, 9, 16
$\mu_{F'}$	1 kN
$\mu_{K'}$	1 kN/m
ν_F	0.1
$\nu = \nu_K$	0.2, 0.3, 0.4, 0.5
s'	1 m
$\theta'_{\ln F} / s'$	2.0
$\theta' = \theta'_{\ln K} / s'$	0.1 to 50.0 in 14 steps
n_{sim}	1,000,000

Note that unit values for the mean load and spring stiffness are used since the ground response is assumed to be linear around the design settlement. The results of this study can therefore be scaled to give results for any mean load and spring stiffness.

To perform the regression, a wide variety of possible functions were investigated, and the following exponential form was found to give very good agreement with simulation results

(15)
$$\mu_{\Delta_{\max}} = (a_{\mu_{\Delta}} + b_{\mu_{\Delta}} \nu) + (c_{\mu_{\Delta}} + d_{\mu_{\Delta}} \nu) \exp((e_{\mu_{\Delta}} + f_{\mu_{\Delta}} \nu) \theta')$$

which was fit to $\mu_{\Delta_{\max}}$ by the least squares method. Note that ν_K is replaced by ν and $\theta'_{\ln K} / s'$ is replaced by θ' for simplicity in eq. (15) and in the following. Similarly, the standard deviation,

$$(16) \quad \sigma_{\Delta_{\max}} = (a_{\sigma_{\Delta}} + b_{\sigma_{\Delta}} v) + (c_{\sigma_{\Delta}} + d_{\sigma_{\Delta}} v) \exp((e_{\sigma_{\Delta}} + f_{\sigma_{\Delta}} v) \theta')$$

was fit to $\sigma_{\Delta_{\max}}$. All 8 quantities, $\mu_{\Delta_{\max}}$, $\sigma_{\Delta_{\max}}$, $\mu_{\ln \delta, \Delta}$, $\sigma_{\ln \delta, \Delta}$, $\mu_{\lambda_{\max}}$, $\sigma_{\lambda_{\max}}$, $\mu_{\ln \delta, \lambda}$, $\sigma_{\ln \delta, \lambda}$ are predicted in the same fashion and the corresponding regressions are summarized in Appendix A. Note that the regressions for total settlement followed the same forms as eqs. (15) and (16) but only when the natural logarithm was used. The regressions well fit expressions involving differences between settlements but not so well the settlements themselves. However, the natural logarithm of the settlement (see eq. 10) resulted in a difference between two random variables which the regression again did a good job of fitting. So, for example, while the definitions of $\Delta_{\max}$ and $\lambda_{\max}$ (see eqs. 6 and 9) already involve differences, the definition of δ_i involves a quotient until the natural logarithm is taken, resulting in the difference shown in eq. (10).

Regression Estimates of Probability

The probability that the maximum differential settlement is higher than some threshold, t , can now be estimated using the lognormal distribution as follows:

$$(17) \quad P[\Delta_{\max} > t] = P[\ln \Delta_{\max} > \ln t] = 1 - \Phi\left(\frac{\ln t - \mu_{\ln \Delta_{\max}}}{\sigma_{\ln \Delta_{\max}}}\right)$$

where $\mu_{\ln \Delta_{\max}}$ and $\sigma_{\ln \Delta_{\max}}$ are obtained through the transformations of eq. (2) using the regressions given by eqs. (15) and (16) and listed in Appendix A. Similarly, the probability that the maximum angular distortion is higher than some threshold t can also be estimated using the lognormal distribution as follows:

275 (18)
$$P[\lambda_{\max} > t] = P[\ln \lambda_{\max} > \ln t] = 1 - \Phi \left(\frac{\ln t - \mu_{\ln \lambda_{\max}}}{\sigma_{\ln \lambda_{\max}}} \right)$$

276 where $\mu_{\ln \lambda_{\max}}$ and $\sigma_{\ln \lambda_{\max}}$ are obtained through the transformations of eq. (2) using the regressions
277 listed in Appendix A.

278 To demonstrate the accuracy of the regression predictions relative to the simulation results,
279 it is useful to compare the regression predictions of the means and standard deviations against
280 those estimated from the simulations. Figure 3 illustrates the agreement between regression
281 predictions and simulation estimates of $\mu_{\Delta_{\max}}, \sigma_{\Delta_{\max}}, \mu_{\ln \delta, \Delta}$, and $\sigma_{\ln \delta, \Delta}$. Similarly, Figure 4 depicts
282 the agreement for $\mu_{\lambda_{\max}}, \sigma_{\lambda_{\max}}, \mu_{\ln \delta, \lambda}$, and $\sigma_{\ln \delta, \lambda}$.

Figure 3. Agreement between regression predictions and simulation estimates of $\mu_{\Delta_{\max}}$ in a), $\sigma_{\Delta_{\max}}$ in b), $\mu_{\ln\delta,\Delta}$ in c), and $\sigma_{\ln\delta,\Delta}$ in d).

Figure 4. Agreement between regression predictions and simulation estimates of $\mu_{\lambda_{\max}}$ in a), $\sigma_{\lambda_{\max}}$ in b), $\mu_{\ln\delta,\lambda}$ in c), and $\sigma_{\ln\delta,\lambda}$ in d).

Since the simulations involve $n_{sim} = 1,000,000$ realizations, the standard deviation of a probability estimate is $\sqrt{p_f(1-p_f)/n_{sim}} \approx 0.001\sqrt{p_f}$ for small probability p_f (this is a well known error estimate, see, e.g., Devore, 2004). This means that if $p_f = 1 \times 10^{-4}$, then the standard deviation of its estimate is about 10^{-5} and therefore, $n_{sim} = 1,000,000$ can accurately resolve probabilities down to about 10^{-4} and reasonably accurately down to about 10^{-5} .

To demonstrate the agreement between maximum differential settlement exceedance probabilities given by the regression predictions and simulation, the probability $P[\Delta_{\max} > t]$ can be estimated both by the regression and by the simulation (eq. 17) and the pair of probability estimates can be plotted against each other. In Figure 5, these pairs of probabilities are plotted for $t = 0.05, 0.10, 0.15, \dots, 5.00$. The upper bound on t was selected to be approximately equal to the mean maximum differential settlement plus two standard deviations. For the worst case, when

302 $n_p = 16$, $\nu = 0.5$, $\theta' = 0.1$, the mean maximum differential settlement is $\mu_{\Delta_{\max}} = 2.4$ with standard
 303 deviation $\sigma_{\Delta_{\max}} = 0.94$, resulting in a max value of $2.4 + 2(0.94) = 4.3$, which was rounded up to
 304 5.00.

305
 306 Figure 5. Regression predicted, obtained via eq. (17), versus simulated probabilities, $P[\Delta_{\max} > t]$,
 307 for all cases listed in Table 1, and for $t = 0.05, 0.10, 0.15, \dots, 5.00$.

308
 309 Figure 5 illustrates very good agreement between regression predictions and simulation
 310 results since all points lie close to a line having slope 1. In more detail, 98% of the points in Figure
 311 5 have a relative error of less than 1.0, while 85% of the points have a relative error of less than
 312 0.5, and 65% of the points have a relative error of less than 0.2.

313 A similar plot to Figure 5, depicting the agreement between regression predicted (eq. 18)
 314 and simulated probabilities of maximum angular distortions, $P[\lambda_{\max} > t]$, for all cases listed in
 315 Table 1, and for $t = 0.05, 0.10, 0.15, \dots, 5.00$, is shown in Figure 6. Again, the agreement is seen
 316 to be very good. In more detail, 97% of the points in Figure 6 have a relative error of less than 1.0,

while 84% of the points have a relative error of less than 0.5, and 67% of the points have a relative error of less than 0.2.

The regression prediction is thus deemed to reasonably accurately predict the simulation results and so can be used instead.

Figure 6. Regression predicted, via eq. (18), versus simulated probability, $P[\lambda_{\max} > t]$, for all cases listed in Table 1 and for $t = 0.05, 0.10, 0.15, \dots, 5.00$.

Generalized Estimation of Failure Probability

The simulation results presented above were only for one specific case and it is thus desirable to be able to scale the results for more general use. The first step is to scale the correlation length. If θ' is the correlation length used in the simulation, and s' is the foundation spacing used in the simulation, then $\theta' = \theta_{\ln K} \frac{s'}{s}$ where $\theta_{\ln K}$ and s are the actual correlation length and foundation spacing. As mentioned above, the primes denote the scalable values used in the simulation.

The next step is to use the foundation proportioned by the Load and Resistance Factor Design (LRFD) method to determine the equivalent spring stiffness of the supporting ground around the design settlement. Let δ_{crit} be the design maximum acceptable total settlement of an individual foundation in the foundation system. Considering only dead and live loads, the LRFD involves proportioning the foundation so that at a settlement of δ_{crit} , the characteristic ground resistance, $\hat{R}_s$, satisfies

$$(19) \quad \Psi \varphi_{gs} \hat{R}_s \geq \alpha_L \hat{F}_L + \alpha_D \hat{F}_D$$

where Ψ is the failure consequence factor (see Canadian Standard Association 2019; Fenton et al. 2016), φ_{gs} is the geotechnical serviceability resistance factor, α_L and α_D are the live and dead load factors at the limit state considered in the supported structure, and $\hat{F}_L$ and $\hat{F}_D$ are the characteristic live and dead loads.

In the following, a typical failure consequence factor of $\Psi = 1$ will be assumed. Since the intent of the consequence factor is just to modify the target reliability, consequence factors other than 1 can be found simply by choosing different target reliabilities in the Section entitled “Settlement Design Recommendations” below and interpreting the resulting resistance factor as the total factor, $\Psi \varphi_{gs}$.

Taking eq. (19) at the equality leads to the required characteristic ground resistance,

$$(20) \quad \hat{R}_s = \frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\varphi_{gs}}$$

which is the characteristic ground resistance (using characteristic ground properties) at the point where the foundation has settled by δ_{crit} (note that the computation of $\hat{R}_s$ may very well involve a non-linear ground response model). The actual mean ground resistance, μ_R , is related to $\hat{R}_s$

through a model bias factor, k_R , which depends on the accuracy of the settlement prediction model:

$$(21) \quad k_R = \frac{\mu_R}{\hat{R}_s} \Rightarrow \mu_R = k_R \hat{R}_s = \left(\frac{k_R}{\phi_{gs}} \right) (\alpha_L \hat{F}_L + \alpha_D \hat{F}_D)$$

Since this mean resistance occurs at settlement δ_{crit} , the corresponding equivalent mean spring stiffness of the ground at that settlement is

$$(22) \quad \mu_K = \frac{\mu_R}{\delta_{crit}} = \left(\frac{k_R}{\phi_{gs}} \right) \left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\delta_{crit}} \right)$$

To compute the total settlement failure probability, $P[\delta_{max} > \delta_{crit}]$, the simulation-based results presented earlier must be scaled. Note that δ_{max} can be either δ_Δ or δ_λ , depending on which of differential settlement or angular distortion is of more interest (in most cases δ_λ would be of more interest). Consider first the fact that the distribution of

$$(23) \quad \delta' = \frac{F'}{K'}$$

was derived using simulation parameters $\mu_{K'} = 1$ and $\mu_{F'} = 1$. If the actual $\mu_F = 2\mu_{F'}$, then all settlement realizations double. In other words, the actual (random) settlement can be predicted from the simulation model as follows,

$$(24) \quad \delta = \left(\frac{\mu_F}{\mu_{F'}} \right) \left(\frac{\mu_{K'}}{\mu_K} \right) \frac{F'}{K'}$$

and so

$$(25) \quad P[\delta_{\max} > \delta_{\text{crit}}] = P\left[\left(\frac{\mu_F}{\mu_{F'}}\right)\left(\frac{\mu_{K'}}{\mu_K}\right)\frac{F'}{K'} > \delta_{\text{crit}}\right] = P\left[\delta'_{\max} > \left(\frac{\mu_{F'}}{\mu_F}\right)\left(\frac{\mu_K}{\mu_{K'}}\right)\delta_{\text{crit}}\right]$$

Now the simulation-based regression predictions for the distribution of $\delta'_{\max}$ can be used. The overall scaling factor for δ_{crit} is

$$(26) \quad \left(\frac{\mu_{F'}}{\mu_F}\right)\left(\frac{\mu_K}{\mu_{K'}}\right) = \left(\frac{k_R}{\varphi_{gs}}\right)\left(\frac{\mu_{F'}}{\mu_F}\right)\left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\mu_{K'} \delta_{\text{crit}}}\right)$$

This scaling factor can, in turn, be used to find the φ_{gs} value required to acceptably limit total settlement as follows:

$$(27) \quad \begin{aligned} P[\delta_{\max} > \delta_{\text{crit}}] &= P\left[\delta'_{\max} > \left(\frac{k_R}{\varphi_{gs}}\right)\left(\frac{\mu_{F'}}{\mu_F}\right)\left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\mu_{K'}}\right)\right] \\ &= P\left[\ln \delta'_{\max} > \ln\left\{\left(\frac{k_R}{\varphi_{gs}}\right)\left(\frac{\mu_{F'}}{\mu_F}\right)\left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\mu_{K'}}\right)\right\}\right] \\ &= \Phi\left[-\frac{\ln\left\{\left(\frac{k_R}{\varphi_{gs}}\right)\left(\frac{\mu_{F'}}{\mu_F}\right)\left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\mu_{K'}}\right)\right\} - \mu_{\ln \delta, \lambda}}{\sigma_{\ln \delta, \lambda}}\right] = \Phi(-\beta_{\text{SLS}}) \end{aligned}$$

This equation can be solved for φ_{gs} in terms of β_{SLS} . Since k_R is generally unknown, the entire term (φ_{gs} / k_R) will be solved for here:

$$(28) \quad \frac{\varphi_{gs}}{k_R} = \left(\frac{\mu_{F'}}{\mu_F}\right)\left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\mu_{K'} \exp\{\mu_{\ln \delta, \lambda} + \sigma_{\ln \delta, \lambda} \beta_{\text{SLS}}\}}\right)$$

The term (φ_{gs} / k_R) was preferred to (k_R / φ_{gs}) since it more directly expresses the required resistance factor. Note that in eq. (28) $\mu_{\ln \delta, \Delta}$ and $\sigma_{\ln \delta, \Delta}$ could be used in place of $\mu_{\ln \delta, \lambda}$ and $\sigma_{\ln \delta, \lambda}$,

depending on whether angular distortion or differential settlement are of more importance in the overall design requirements.

In a similar fashion, in order to compute the probabilities relating to angular distortion, the only thing that needs to change is the scale factor. The scale factor for λ_{crit} is

$$(29) \quad \left(\frac{\mu_{F'}}{\mu_F} \right) \left(\frac{s}{s'} \right) \left(\frac{\mu_K}{\mu_{K'}} \right) = \left(\frac{\mu_{F'}}{\mu_F} \right) \left(\frac{s}{s'} \right) \left(\frac{k_R}{\varphi_{gs}} \right) \left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\mu_{K'} \delta_{\text{crit}}} \right)$$

The probability $P[\lambda_{\text{max}} > \lambda_{\text{crit}}]$ can now be computed from the simulation results as follows:

$$(30) \quad \begin{aligned} P[\lambda_{\text{max}} > \lambda_{\text{crit}}] &= P \left[\lambda'_{\text{max}} > \left(\frac{\mu_{F'}}{\mu_F} \right) \left(\frac{s}{s'} \right) \left(\frac{k_R}{\varphi_{gs}} \right) \left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\mu_{K'} \delta_{\text{crit}}} \right) \lambda_{\text{crit}} \right] \\ &= P \left[\ln \lambda'_{\text{max}} > \ln \left\{ \left(\frac{\mu_{F'}}{\mu_F} \right) \left(\frac{s}{s'} \right) \left(\frac{k_R}{\varphi_{gs}} \right) \left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\mu_{K'} \delta_{\text{crit}}} \right) \lambda_{\text{crit}} \right\} \right] \\ &= \Phi \left(- \frac{\ln \left\{ \left(\frac{\mu_{F'}}{\mu_F} \right) \left(\frac{s}{s'} \right) \left(\frac{k_R}{\varphi_{gs}} \right) \left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\mu_{K'} \delta_{\text{crit}}} \right) \lambda_{\text{crit}} \right\} - \mu_{\ln \lambda_{\text{max}}}}{\sigma_{\ln \lambda_{\text{max}}}} \right) = \Phi(-\beta_{\text{SLS}}) \end{aligned}$$

where s is the actual foundation spacing and $s' = 1$ is the foundation spacing used in the simulation.

If the probability in eq. (30) is to not exceed $\Phi(-\beta_{\text{SLS}})$, then the required (φ_{gs} / k_R) becomes

$$(31) \quad \frac{\varphi_{gs}}{k_R} = \left(\frac{\mu_{F'}}{\mu_F} \right) \left(\frac{s}{s'} \right) \left(\frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\mu_{K'} \exp \{ \mu_{\ln \lambda_{\text{max}}} + \sigma_{\ln \lambda_{\text{max}}} \beta_{\text{SLS}} \}} \right)$$

An entirely similar analysis could be done for settlement problems which induce ULS failure in the supported structure simply by exchanging β_{SLS} with β_{ULS} . Note that the foundation design will also change, since ULS factored loads and limiting angular distortions differ from those used

in SLS. Note further that settlement of the ground is still being considered here, so that φ_{gs} is still deemed to be appropriate for the ground. However, because of the induced ULS conditions in the supported structure, ULS factored loads should be applied to the ground.

The entire analysis given above can be repeated for $P[\Delta_{\max} > \Delta_{\text{crit}}]$ to determine the (φ_{gs} / k_R) factor required to acceptably limit the differential settlement. However, the CFEM does not give limits on differential settlement, just on angular distortion, and so the probability $P[\Delta_{\max} > \Delta_{\text{crit}}]$ will not be pursued further here.

Example

To illustrate how the results presented in this paper can be used in a design context, consider the following example. Suppose a 9-foundation system is arranged on a square grid with center-to-center separation distance of $s = 5$ m, each subjected to a mean total load $\mu_F = 200$ kN with coefficient of variation of $\nu_F = 0.1$. Suppose further that the supporting soil has equivalent spring stiffness with coefficient of variation of $\nu_K = 0.3$. Assume that the correlation lengths of the spring stiffness and load fields are $\theta_{\ln K} = 15$ m and $\theta_{\ln F} = 10$ m respectively.

Using the usual geotechnical settlement limitation, it is desired to design each foundation so that its total settlement only exceeds $\delta_{\text{crit}} = 0.025$ m (=25 mm) with acceptably low probability. Given the values listed above, the Load and Resistance Factor Design (LRFD) of each foundation proceeds by computing their characteristic live loads as $\hat{F}_L = \mu_L / k_L$ and dead loads as $\hat{F}_D = \mu_D / k_D$ where $k_L = 0.9$ and $k_D = 1.05$ are the live and dead load bias factors (Fenton et al. 2016) and μ_L and μ_D are the mean live and dead loads. If the ratio of mean dead to live loads is

assumed to be $R_{D/L} = \mu_D / \mu_L = 3.0$, so that $\mu_D = 3\mu_L$, then $\mu_F = \mu_L + \mu_D = \mu_L + 3\mu_L = 4\mu_L$ means that $\mu_L = 50 \text{ kN}$ and $\mu_D = 150 \text{ kN}$. In turn, the characteristic live and dead loads are $\hat{F}_L = 50 / 0.9 = 55.6 \text{ kN}$ and $\hat{F}_D = 150 / 1.05 = 142.9 \text{ kN}$. Assuming again that only live and dead loads act on the foundation, the LRFD of each foundation involves specifying the foundation geometry such that $\Psi \phi_{gs} \hat{R}_s \geq \alpha_L \hat{F}_L + \alpha_D \hat{F}_D$. According to the CHBDC (2019), the resistance factor, ϕ_{gs} , for typical understanding is $\phi_{gs} = 0.8$ and the consequence factor for typical failure consequences is $\Psi = 1.0$. The National Building Code of Canada (NBCC 2015) does not specify geotechnical resistance factors, therefore, the CHBDC values are used here. For bridges, the load factors are $\alpha_L = 0.9$ and $\alpha_D = 1.0$ at SLS and $\alpha_L = 1.7$ and $\alpha_D = 1.2$ at ULS (Table 3.1 in CHBDC 2019). For buildings, the National Building Code of Canada (NBCC) specifies load factors of $\alpha_L = \alpha_D = 1.0$ at SLS and $\alpha_L = 1.5$ and $\alpha_D = 1.25$ at ULS (NBCC 2015).

427 SLS Design

First looking at the situation where foundation settlement may lead to SLS conditions in the supported structure, the corresponding LRFD equation requires that the characteristic ground resistance satisfies

$$(32) \quad \hat{R}_s = \frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\Psi \phi_{gs}} = \frac{(1.0)(55.6) + (1.0)(142.9)}{(1.0)(0.8)} = 248.1$$

where $\phi_{gs} = 0.8$ is as specified in the CHBDC.

Thus, using a resistance bias factor of $k_R = 1.1$ (Fenton et al. 2016), the mean ground resistance is $\mu_R = k_R \hat{R}_s = (248.1)(1.1) = 272.9 \text{ kN}$ corresponding to a total design settlement of 0.025 m. Note that the $k_R = 1.1$ value used by Fenton et al. (2016) was actually estimated for ULS

design problem (bearing capacity) . The k_R value at SLS depends very much on the bias of the settlement prediction model and may be significantly larger than 1.1 (large values of k_R at SLS imply overprediction of settlement by the model). While the determination of k_R is beyond the scope of this paper and a topic of future research, the results here will be presented as a function of k_R . Using $\mu_R = 272.9 \text{ kN}$ at 0.025 m settlement, the design equivalent mean spring stiffness of the ground is $\mu_K = 272.9 / 0.025 = 10,916.4 \text{ kN/m}$.

The CFEM (CGS 2006) specifies allowable angular distortions of about $1/500$ for cracking of walls and partitions, which will be deemed here to be an SLS condition in the supported structure.

In order to use the regression results to find the failure probability $P[\lambda_{\max} > \lambda_{\text{crit}}]$, the maximum acceptable angular distortion, λ_{crit} , must be scaled by a factor of (see eq. 29):

$$(33) \quad \frac{\mu_{F'}}{\mu_F} \times \frac{s}{s'} \times \frac{\mu_K}{\mu_{K'}} = \left(\frac{1}{200} \times \frac{5}{1} \times \frac{10,916.4}{1} \right) = 272.9$$

In addition, a correlation length of $\theta' = \theta_{\ln K} \frac{s'}{s} = (15) \frac{1}{5} = 3$ must be used to calculate the mean and standard deviation of $\ln \lambda_{\max}$, according to the regressions specified in Appendix A.

Using equations in Appendix A, the values of $a_{\mu_\lambda}, b_{\mu_\lambda}, c_{\mu_\lambda}, d_{\mu_\lambda}, e_{\mu_\lambda}, f_{\mu_\lambda}$ can now be calculated for $n_p = 9$ as follows,

$$a_{\mu_{\lambda}} = -0.000285 (9)^2 + 0.009448 (9) + 0.052734 = 0.114681$$

$$b_{\mu_{\lambda}} = -0.001695 (9)^2 + 0.059201 (9) + 0.282023 = 0.677537$$

$$c_{\mu_{\lambda}} = 0.000613 (9)^2 - 0.028244 (9) - 0.050726 = -0.255269$$

$$d_{\mu_{\lambda}} = -0.006800 (9)^2 + 0.272000 (9) + 0.999800 = 2.897000$$

$$e_{\mu_{\lambda}} = -0.000112 (9)^2 + 0.002692 (9) - 0.212580 = -0.197424$$

$$f_{\mu_{\lambda}} = 0.000225 (9)^2 - 0.004599 (9) + 0.100125 = 0.076959$$

from which $\mu_{\lambda_{\max}}$ is calculated for $v_K = 0.3$ and $\theta' = 3$ to be,

$$\begin{aligned} \mu_{\lambda_{\max}} &= (a_{\mu_{\lambda}} + b_{\mu_{\lambda}} v) + (c_{\mu_{\lambda}} + d_{\mu_{\lambda}} v) \exp((e_{\mu_{\lambda}} + f_{\mu_{\lambda}} v) \theta') \\ &= (0.114681 + 0.677537(0.3)) \\ &\quad + (-0.255269 + 2.897000(0.3)) \exp((-0.197424 + 0.076959(0.3))(3.0)) \\ &= 0.681781 \end{aligned}$$

Similarly, for the standard deviation $\sigma_{\lambda_{\max}}$,

$$a_{\sigma_{\lambda}} = 0.000237 (9)^2 - 0.007832 (9) + 0.023678 = -0.027613$$

$$b_{\sigma_{\lambda}} = -0.001004 (9)^2 + 0.034336 (9) + 0.327626 = 0.555326$$

$$c_{\sigma_{\lambda}} = 0.000245 (9)^2 - 0.009199 (9) - 0.100417 = -0.163363$$

$$d_{\sigma_{\lambda}} = -0.000750 (9)^2 + 0.026750 (9) + 1.084000 = 1.264000$$

$$e_{\sigma_{\lambda}} = 0.000035 (9)^2 - 0.001138 (9) - 0.211311 = -0.218718$$

$$f_{\sigma_{\lambda}} = -0.000336 (9)^2 + 0.012854 (9) + 0.126169 = 0.214639$$

so that

$$\begin{aligned} \sigma_{\lambda_{\max}} &= (a_{\sigma_{\lambda}} + b_{\sigma_{\lambda}} v) + (c_{\sigma_{\lambda}} + d_{\sigma_{\lambda}} v) \exp((e_{\sigma_{\lambda}} + f_{\sigma_{\lambda}} v) \theta') \\ &= (-0.027613 + 0.555326(0.3)) \\ &\quad + (-0.163363 + 1.264000(0.3)) \exp((-0.218718 + 0.214639(0.3))(3)) \\ &= 0.274834 \end{aligned}$$

The mean and standard deviation of $\lambda_{\max}$ can then be converted to ln-space, using eq. (2),

$$\sigma_{\ln \lambda_{\max}}^2 = \ln \left(1 + \nu_{\lambda_{\max}}^2 \right) = \ln \left(1 + \frac{0.274834^2}{0.681781^2} \right) = 0.150572$$

$$\begin{aligned} \mu_{\ln \lambda_{\max}} &= \ln \left(\mu_{\lambda_{\max}} \right) - \frac{1}{2} \sigma_{\ln \lambda_{\max}}^2 \\ &= \ln(0.681781) - \frac{1}{2} \times 0.150572 = -0.458333 \end{aligned}$$

At SLS, we are interested in finding (for $\lambda_{\text{crit}} = 1/500$)

$$\begin{aligned} P[\lambda'_{\max} > 272.9 \lambda_{\text{crit}}] &= \Phi \left(-\frac{\ln(0.5458) - \mu_{\ln \lambda_{\max}}}{\sigma_{\ln \lambda_{\max}}} \right) \\ &= \Phi \left(-\frac{\ln(0.5458) - (-0.458333)}{\sqrt{0.150572}} \right) \\ &= \Phi(0.3793) = 0.6478 \end{aligned}$$

The simulation-based estimate of this probability (using $n_{\text{sim}} = 1,000,000$ realizations) is 0.617, which is very good agreement with that predicted above. In either case, the probability of exceeding the limiting angular distortion specified by CFEM for SLS conditions seems excessively high. This means either that the resistance factor, $\phi_{gs} = 0.8$, used in this SLS design may be too high or that the settlement prediction models are much more conservative than assumed here (i.e., that k_R is much higher than its assumed value of 1.1).

The maximum total settlement corresponding to the maximum angular distortion has the following probability of exceeding the original single foundation total settlement design limit (see eq. 25):

$$\begin{aligned}
P[\delta_{\max} > \delta_{\text{crit}}] &= P\left[\delta'_{\max} > \left(\frac{\mu_{F'}}{\mu_F}\right)\left(\frac{\mu_K}{\mu_{K'}}\right)\delta_{\text{crit}}\right] \\
&= P\left[\delta'_{\max} > \left(\frac{1}{200}\right)\left(\frac{10916.4}{1}\right)(0.025)\right] \\
&= P[\ln \delta'_{\max} > \ln(1.3645)] = \Phi\left(-\frac{\ln(1.3645) - \mu_{\ln \delta, \lambda}}{\sigma_{\ln \delta, \lambda}}\right) \\
&= \Phi\left(-\frac{\ln(1.3645) - (0.382221)}{0.242524}\right) = \Phi(0.2945) = 0.6158
\end{aligned}$$

where $\mu_{\ln \delta, \lambda}$ and $\sigma_{\ln \delta, \lambda}$ are obtained from the regressions listed in Appendix A. The simulation-based estimate of this probability is 0.598, which is in excellent agreement with the above prediction. However, the probability of exceeding δ_{crit} estimated by either method also seems to be quite excessive.

ULS Design

The CFEM (CGS 2006) specifies a limiting angular distortion of about 1/200 to avoid damage in the supported structure, which will be considered here to be a ULS condition within the supported structure. In this case, ULS load factors $\alpha_L = 1.5$ and $\alpha_D = 1.25$ (NBCC 2015) will be used for the design load. Since the structural damage is induced by a settlement condition in the ground, the resistance factor used for the ground (involving settlement parameter uncertainties) will still be ϕ_{gs} . The LRFD now gives a characteristic resistance of the designed foundation of

$$(34) \quad \hat{R}_s = \frac{\alpha_L \hat{F}_L + \alpha_D \hat{F}_D}{\Psi \phi_{gs}} = \frac{(1.5)(55.6) + (1.25)(142.9)}{(1.0)(0.8)} = 327.5$$

Again, using a resistance bias factor of $k_R = 1.1$ (e.g., Fenton et al. 2016, which is almost certainly an underestimate of the actual settlement bias factor), the mean ground resistance that the

foundation needs to achieve for ULS performance is $\mu_R = k_R \hat{R}_s = (327.5)(1.1) = 360.3 \text{ kN}$, which still must correspond to a total settlement of 0.025 m at an acceptable level of reliability. This in turn means that the design mean spring stiffness of the ground must now be $\mu_K = 360.3 / 0.025 = 14,411.4 \text{ kN/m}$.

In order to use the simulation-based regressions, the maximum acceptable angular distortion, λ_{crit} , needs to be scaled by a factor of (see eq. 29):

$$(35) \quad \frac{\mu_{F'}}{\mu_F} \times \frac{s}{s'} \times \frac{\mu_K}{\mu_{K'}} = \left(\frac{1}{200} \times \frac{5}{1} \times \frac{14,411.4}{1} \right) = 360.3$$

The regression factors and the statistics of λ_{max} remain as for SLS conditions, so that the probability of exceeding $\lambda_{\text{crit}} = 1/200$ becomes

$$(36) \quad \begin{aligned} P[\lambda_{\text{max}} > 360.3 \lambda_{\text{crit}}] &= \Phi \left(-\frac{\ln(1.8015) - \mu_{\ln \lambda_{\text{max}}}}{\sigma_{\ln \lambda_{\text{max}}}} \right) \\ &= \Phi \left(-\frac{\ln(1.8015) - (-0.458333)}{\sqrt{0.150572}} \right) = \Phi(-2.6981) = 0.0035 \end{aligned}$$

The simulation-based estimate of this probability is 0.0017, which is in reasonably good agreement, with the regression estimate being somewhat conservative. For both estimates, these probabilities are still somewhat high for ULS design (the lifetime target for an individual foundation should be somewhat less than 0.001), suggesting again that φ_{gs} should be reduced or that k_R is actually higher than the 1.1 used in this example.

The maximum total settlement corresponding to the maximum angular distortion now has the following probability of exceeding the original design limit (see eq. 25):

$$\begin{aligned}
 P[\delta_{\max} > \delta_{\text{crit}}] &= P\left[\delta'_{\max} > \left(\frac{\mu_{F'}}{\mu_F}\right)\left(\frac{\mu_K}{\mu_{K'}}\right)\delta_{\text{crit}}\right] = P\left[\delta'_{\max} > \left(\frac{1}{200}\right)\left(\frac{14411.4}{1}\right)(0.025)\right] \\
 &= P[\ln \delta'_{\max} > \ln(1.801)] = \Phi\left(-\frac{\ln(1.801) - \mu_{\ln \delta, \lambda}}{\sigma_{\ln \delta, \lambda}}\right) \\
 &= \Phi\left(-\frac{\ln(1.801) - (0.382221)}{0.242524}\right) = \Phi(-0.8499) = 0.1977
 \end{aligned}$$

where $\mu_{\ln \delta, \lambda}$ and $\sigma_{\ln \delta, \lambda}$ are obtained from the regressions listed in Appendix A. The simulation-based estimate of this probability is 0.193, which is in very close agreement with the above prediction. However, similarly to the SLS design case, the probability of exceeding δ_{crit} for the ULS design case seems to be too high.

If the target lifetime reliability at ULS is approximately 1/1000 (corresponding to a reliability index of about 3.0), then the foundation in this example is not quite safe enough for ULS design with respect to angular distortion. Similarly, the approximately 60% angular distortion failure probability at SLS seems excessively high in practice, again suggesting that settlement prediction calculations used in practice are conservative. Recall, however, that the ability of the structure to redistribute load due to its own stiffness has only been partially captured in this model, so that the failure probabilities presented here are likely to be somewhat higher than for a real structure.

Settlement Design Recommendations

The previous example suggests that current resistance factors may not be sufficient to provide the desired reliability indices for SLS and ULS design of foundations against total and differential settlement. It should be emphasized that one significant uncertainty in this analysis is what the value of the resistance bias factor, k_R , actually is. The geotechnical community is of the opinion that typical settlement predictions are quite conservative, implying a larger value of k_R

than used herein, which was based on ground bearing capacity. It is thus of interest to investigate what the (φ_{gs}/k_R) value must be in order to achieve desired differential settlement reliability. Considering the reliability against excessive angular distortions, eq. (31) provides the required (φ_{gs}/k_R) factor. For the same conditions used in the above example, i.e. for $n_p = 9$ and $\theta = \theta_{\ln K}/s = 3$, Figures 7 and 8 illustrate the required (φ_{gs}/k_R) factor for SLS and ULS designs. For ULS problems, β_{ULS} is substituted for β_{SLS} in eq. (31) and the load factors become $\alpha_L = 1.5$ and $\alpha_D = 1.25$ for the NBCC (2015).

As mentioned above, the serviceability resistance factor for geotechnical design, φ_{gs} , is used for both SLS and ULS performance of the supported structure simply because foundation settlement is still deemed to be a serviceability condition in the ground (involving settlement parameter uncertainties).

Figure 7. Required (φ_{gs}/k_R) factors for SLS design as a function of the target β_{SLS} for $n_p = 9$ and $\theta = \theta_{\ln K}/s = 3$.

Figure 8. Required (ϕ_{gs} / k_R) factors for ULS design as a function of the target β_{ULS} for $n_p = 9$ and $\theta = \theta_{lnK} / s = 3$.

For SLS conditions in the supported structure, if the target $\beta_{SLS} = 2$, when $v_K = 0.3$, then the required $\phi_{gs} / k_R = 0.29$. This value is much lower than commonly assumed for SLS conditions which suggests that settlement predictions are actually quite conservative (k_R is large).

For ULS design, the load factors are greater than 1 and the critical angular distortion is larger than for SLS design so that the probability of failure is smaller, all else being the same. As a result, the resistance factor ratio is expected to be higher than that required for SLS design, as observed in Figure 8, where, assuming a target $\beta_{ULS} = 3$ and $v_K = 0.3$, the required $\phi_{gs} / k_R = 0.65$.

For completeness, Figure 8 is repeated with load factors from CHBDC (CSA S6-19 2019), which are $\alpha_L = 1.7$ and $\alpha_D = 1.2$. The required ϕ_{gs} / k_R values are virtually identical to those required for the NBCC. It appears, therefore, that under the conditions investigated here, the same results can be used for both codes.

Figure 9. Required (ϕ_{gs} / k_R) factors for ULS design as a function of the target β_{ULS} for $n_p = 9$ and $\theta = \theta_{\ln K} / s = 3$ using load factors from CHBDC (CSA S6-19 2019).

Conclusions

In this paper, the distribution of the pair-wise maximum differential settlement of a foundation group is investigated and regressions are developed as functions of the degree of variability, spatial correlation, and number of foundations. The regression predictions are derived for the means and standard deviations of total settlement, differential settlement, and angular distortion, which are calibrated using a simulation with $\frac{\mu_{F'}}{\mu_{K'}} = 1$, $v_F = 0.1$, $\frac{\theta'_{\ln F}}{s'} = 2$, and $s' = 1$.

The results can be scaled to other foundation conditions in order to estimate ULS and SLS structural failure probabilities, as demonstrated in the section entitled “Generalized Estimation of Failure Probabilities”. The application of these scaling factors is illustrated by an example using $\phi_{gs} = 0.8$ and $k_R = 1.1$. The example computes probabilities of SLS and ULS failure of the

supported structure due to excessive angular distortion between foundations using the regressions provided in Appendix A. The failure probability predictions are in good agreement with the simulation, however, the probabilities are generally too high to be acceptable, suggesting either that the geotechnical resistance factor must be reduced or that the resistance bias factor associated with the settlement prediction is considerably larger than 1.1.

The results of this paper assist in establishing probabilistic settlement design requirements which allow for acceptable target reliabilities against excessive differential settlement or angular distortion between foundations when individual foundations are designed against excessive total settlement. In other words, the distribution of differential settlement or angular distortion is used in this paper to derive required resistance factor to resistance bias ratios which can then be used, in an LRFD framework, to design individual foundations against prescribed maximum settlements while still achieving acceptable reliability against differential settlement.

Perhaps the surprising outcome of the settlement design recommendations is that the (ϕ_{gs} / k_R) factor seems to be lower than commonly used in practice, if k_R is close to 1 (i.e. the settlement model is unbiased). The resistance factor used for settlement design is generally much closer to 1. The discrepancy between the resistance factor found in this paper and that used in practice is probably due to k_R actually being much higher than 1 (i.e. the settlement model is very conservative). The actual value of k_R in settlement design is a topic of ongoing research. In any case, if the designer has high confidence in the accuracy of their settlement prediction model (i.e. $k_R \approx 1$), the results of Figures 7 and 8 can be used directly to provide the required geotechnical resistance factor, ϕ_{gs} , in the LRFD framework. Note that Figures 7 and 8 are derived assuming

that the angular distortion is of more importance in the SLS and ULS performance of the supported structure.

Finally, it must be noted that one of the goals of this paper is to provide a means to calibrate existing LRFD code provisions for use by practitioners. However, another possibility which will become open to practitioners in the future is to estimate the failure probability of their specific design directly through Monte Carlo simulations. Although this latter approach is definitely on the horizon, it will be a few years yet before random finite element models (RFEM) are sufficiently advanced to probabilistically model both the ground, the supported structure, and their interactions. Nevertheless, research along these lines is well underway.

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661

662 **List of Abbreviations and Symbols Used**

663 CHBDC Canadian Highway Bridge Design Code

664 CFEM Canadian Foundation Engineering Manual

665 CGS Canadian Geotechnical Society

666 CMD Covariance Matrix Decomposition

667 LRFD Load and Resistance Factor Design

668 NBCC National Building Code of Canada

669 ULS Ultimate Limit State

670 SLS Serviceability Limit State

671 a, b, c, d, e, f regression coefficients of subscripted quantities

672 $\tilde{C}$ covariance matrix

673 C_{ij} covariance matrix element at row i and column j

674 F true total load (random)

675 F' true total load (random) used in the simulation

676 F_i true total load (random) acting on the i^{th} foundation

677 $\hat{F}_D$ characteristic dead load

678 $\hat{F}_L$ characteristic live load

679 G normally distributed random field

680	G_i	element i of the normally distributed random field
681	K	soil's spring stiffness (random)
682	K_i	soil's spring stiffness (random) under the i^{th} foundation
683	K'	soil's spring stiffness (random) used in the simulation
684	k_D	dead load bias factor ($= \mu_D / \hat{F}_D$)
685	k_L	dead load bias factor ($= \mu_L / \hat{F}_L$)
686	k_R	resistance bias factor ($= \mu_R / \hat{R}_s$)
687	$\underline{\underline{L}}$	lower triangular matrix satisfying $\underline{\underline{L}}\underline{\underline{L}}^T = \underline{\underline{C}}$
688	n_p	number of foundations
689	n_{sim}	number of simulations
690	p_f	failure probability
691	$P[.]$	probability operator
692	$R_{D/L}$	mean dead to live load ratio (μ_D / μ_L)
693	$\hat{R}_s$	characteristic resistance at SLS of the ground
694	s	center-to-center foundation spacing
695	s'	center-to-center foundation spacing used in the simulation
696	t	differential settlement threshold
697	ν	short form for ν_K in regression equations
698	ν_K	spring stiffness coefficient of variation ($= \sigma_K / \mu_K$)

- 699 v_F total load coefficient of variation ($= \sigma_F / \mu_F$)
- 700 v_L live load coefficient of variation
- 701 v_D dead load coefficient of variation
- 702 v_X coefficient of variation of the random field X ($= \sigma_X / \mu_X$)
- 703 $v_{\lambda_{\max}}$ coefficient of variation of the random field X ($= \sigma_{\lambda_{\max}} / \mu_{\lambda_{\max}}$)
- 704 $\tilde{x}_i$ spatial coordinate of the i^{th} point in the field
- 705 $\tilde{X}$ random field
- 706 $X_i = X(\tilde{x}_i)$ the random field value at location $\tilde{x}_i$
- 707 $\tilde{Z}$ vector of independent standard normal random variables
- 708 α_L live load factor
- 709 α_D dead load factor
- 710 β_{ULS} reliability index at ULS
- 711 β_{SLS} reliability index at SLS
- 712 δ_i foundation settlement as ratio of load to spring stiffness ($\delta_i = F_i / K_i$)
- 713 δ_{Δ} maximum total settlement corresponding to $\Delta_{\max}$
- 714 δ_{λ} maximum total settlement corresponding to $\lambda_{\max}$
- 715 $\delta_{\Delta,k}$ k^{th} realization of δ_{Δ}
- 716 δ' foundation settlement predicted by the simulation

- 717 $\delta'_{\max}$ value of $\delta_{\max}$ used in the simulation
- 718 $\delta_{\max}$ either δ_{Δ} or δ_{λ} , depending on which is of more interest
- 719 δ_{crit} maximum acceptable design total settlement of a single foundation
- 720 $\Delta_{\max}$ maximum differential settlement in a foundation group
- 721 Δ_{crit} maximum acceptable differential settlement of a pair of foundations
- 722 $\Delta_{\max,k}$ k^{th} realization of $\Delta_{\max}$
- 723 ϕ_{gs} geotechnical resistance factor at SLS in the ground
- 724 λ_{crit} maximum acceptable design angular distortion between foundations
- 725 $\lambda_{\max}$ maximum angular distortion observed in a foundation group
- 726 $\lambda'_{\max}$ maximum angular distortion observed in the simulation
- 727 μ_K mean spring stiffness
- 728 $\mu_{K'}$ mean spring stiffness used in the simulation
- 729 μ_D mean dead load
- 730 μ_F mean total load
- 731 $\mu_{F'}$ mean total load used in the simulation
- 732 μ_L mean live load
- 733 μ_R mean resistance
- 734 μ_X mean of the random variable X

- 735 $\mu_{\lambda_{\max}}$ mean maximum angular distortion
- 736 $\mu_{\Delta_{\max}}$ mean maximum differential settlement
- 737 $\mu_{\ln K}$ mean of the ln-spring stiffness
- 738 $\mu_{\ln F}$ mean of the ln-foundation total load
- 739 $\mu_{\ln X}$ mean of the random variable $\ln X$
- 740 $\mu_{\ln \delta}$ mean of the ln-total foundation settlement
- 741 $\mu_{\ln \Delta_{\max}}$ mean maximum ln-differential settlement
- 742 $\mu_{\ln \lambda_{\max}}$ mean maximum ln-angular distortion
- 743 $\mu_{\ln \delta, \Delta}$ mean maximum ln-total settlement corresponding to $\Delta_{\max}$
- 744 $\mu_{\ln \delta, \lambda}$ mean maximum ln-total settlement corresponding to $\lambda_{\max}$
- 745 θ general isotropic correlation length and short form for $\theta_{\ln K} / s$ in the regression equations
- 746 θ' correlation length used in the simulation
- 747 $\theta_{\ln K}$ isotropic correlation length of the ln-spring stiffness field
- 748 $\theta'_{\ln K}$ isotropic correlation length of the ln-spring stiffness field used in the simulation
- 749 $\theta_{\ln F}$ isotropic correlation length of the ln-load field
- 750 $\theta'_{\ln F}$ isotropic correlation length of the ln-load field used in the simulation
- 751 ρ correlation function
- 752 $\rho_{ij} = \rho(\tau_{ij})$ correlation coefficient between X at two points

- 753 σ_K standard deviation of spring stiffness
- 754 σ_F standard deviation of total load
- 755 σ_X standard deviation of the random variable X
- 756 $\sigma_{\lambda_{\max}}$ standard deviation of maximum angular distortion
- 757 $\sigma_{\Delta_{\max}}$ standard deviation of maximum differential settlement
- 758 $\sigma_{\ln K}$ standard deviation of ln-spring stiffness
- 759 $\sigma_{\ln F}$ standard deviation of the ln-foundation total load
- 760 $\sigma_{\ln X}$ standard deviation of the random variable $\ln X$
- 761 $\sigma_{\ln \delta}$ standard deviation of the ln-total foundation settlement
- 762 $\sigma_{\ln \Delta_{\max}}$ standard deviation of the maximum ln-differential settlement
- 763 $\sigma_{\ln \lambda_{\max}}$ standard deviation of the maximum ln-angular distortion
- 764 $\sigma_{\ln \delta, \Delta}$ standard deviation of the maximum ln-total settlement corresponding to $\Delta_{\max}$
- 765 $\sigma_{\ln \delta, \lambda}$ standard deviation of the maximum ln-total settlement corresponding to $\lambda_{\max}$
- 766 τ_{ij} distance between i^{th} and j^{th} points in the field
- 767 $\Phi(\cdot)$ standard normal cumulative distribution function
- 768 Ψ failure consequence factor
- 769
- 770 **Appendix A**
- 771 Regression predictions for $\mu_{\Delta_{\max}}, \sigma_{\Delta_{\max}}, \mu_{\ln \delta, \Delta}, \sigma_{\ln \delta, \Delta}, \mu_{\lambda_{\max}}, \sigma_{\lambda_{\max}}, \mu_{\ln \delta, \lambda}, \sigma_{\ln \delta, \lambda}$.

$\mu_{\Delta_{\max}} = (a_{\mu_{\Delta}} + b_{\mu_{\Delta}} v) + (c_{\mu_{\Delta}} + d_{\mu_{\Delta}} v) \exp((e_{\mu_{\Delta}} + f_{\mu_{\Delta}} v) \theta')$	$a_{\mu_{\Delta}} = -0.000423 n_p^2 + 0.014048 n_p + 0.042738$ $b_{\mu_{\Delta}} = -0.002529 n_p^2 + 0.100435 n_p + 0.173423$ $c_{\mu_{\Delta}} = 0.000662 n_p^2 - 0.027442 n_p - 0.058020$ $d_{\mu_{\Delta}} = -0.008140 n_p^2 + 0.295226 n_p + 1.006343$ $e_{\mu_{\Delta}} = -0.000410 n_p^2 + 0.013976 n_p - 0.243037$ $f_{\mu_{\Delta}} = 0.000305 n_p^2 - 0.010195 n_p + 0.117549$
$\sigma_{\Delta_{\max}} = (a_{\sigma_{\Delta}} + b_{\sigma_{\Delta}} v) + (c_{\sigma_{\Delta}} + d_{\sigma_{\Delta}} v) \exp((e_{\sigma_{\Delta}} + f_{\sigma_{\Delta}} v) \theta')$	$a_{\sigma_{\Delta}} = 0.000554 n_p^2 - 0.010912 n_p + 0.030769$ $b_{\sigma_{\Delta}} = -0.003303 n_p^2 + 0.072094 n_p + 0.247166$ $c_{\sigma_{\Delta}} = -0.000101 n_p^2 - 0.005711 n_p - 0.108246$ $d_{\sigma_{\Delta}} = 0.001567 n_p^2 - 0.009167 n_p + 1.190600$ $e_{\sigma_{\Delta}} = -0.000781 n_p^2 + 0.026259 n_p - 0.284031$ $f_{\sigma_{\Delta}} = 0.000388 n_p^2 - 0.015202 n_p + 0.209803$
$\mu_{\ln \delta, \Delta} = (a_{\mu_{\delta, \Delta}} + b_{\mu_{\delta, \Delta}} v) + (c_{\mu_{\delta, \Delta}} + d_{\mu_{\delta, \Delta}} v) \exp((e_{\mu_{\delta, \Delta}} + f_{\mu_{\delta, \Delta}} v) \theta')$	$a_{\mu_{\delta, \Delta}} = -0.000296 n_p^2 + 0.010624 n_p - 0.021851$ $b_{\mu_{\delta, \Delta}} = -0.000794 n_p^2 + 0.030623 n_p + 0.356914$ $c_{\mu_{\delta, \Delta}} = 0.000117 n_p^2 - 0.004015 n_p - 0.011441$ $d_{\mu_{\delta, \Delta}} = -0.003044 n_p^2 + 0.100673 n_p + 0.420514$ $e_{\mu_{\delta, \Delta}} = -0.000418 n_p^2 + 0.014032 n_p - 0.243543$ $f_{\mu_{\delta, \Delta}} = 0.000249 n_p^2 - 0.007732 n_p + 0.104916$
$\sigma_{\ln \delta, \Delta} = (a_{\sigma_{\delta, \Delta}} + b_{\sigma_{\delta, \Delta}} v) + (c_{\sigma_{\delta, \Delta}} + d_{\sigma_{\delta, \Delta}} v) \exp((e_{\sigma_{\delta, \Delta}} + f_{\sigma_{\delta, \Delta}} v) \theta')$	$a_{\sigma_{\delta, \Delta}} = 0.000048 n_p^2 - 0.001612 n_p + 0.043994$ $b_{\sigma_{\delta, \Delta}} = 0.000017 n_p^2 - 0.001080 n_p + 0.870149$ $c_{\sigma_{\delta, \Delta}} = -0.000009 n_p^2 + 0.000141 n_p + 0.002143$ $d_{\sigma_{\delta, \Delta}} = 0.000973 n_p^2 - 0.030407 n_p - 0.176037$ $e_{\sigma_{\delta, \Delta}} = -0.000560 n_p^2 + 0.019000 n_p - 0.325140$ $f_{\sigma_{\delta, \Delta}} = -0.000019 n_p^2 + 0.000404 n_p - 0.015368$

$\mu_{\lambda_{\max}} = (a_{\mu_{\lambda}} + b_{\mu_{\lambda}} v) + (c_{\mu_{\lambda}} + d_{\mu_{\lambda}} v) \exp((e_{\mu_{\lambda}} + f_{\mu_{\lambda}} v) \theta')$	$a_{\mu_{\lambda}} = -0.000285 n_p^2 + 0.009448 n_p + 0.052734$ $b_{\mu_{\lambda}} = -0.001695 n_p^2 + 0.059201 n_p + 0.282023$ $c_{\mu_{\lambda}} = 0.000613 n_p^2 - 0.028244 n_p - 0.050726$ $d_{\mu_{\lambda}} = -0.006800 n_p^2 + 0.272000 n_p + 0.999800$ $e_{\mu_{\lambda}} = -0.000112 n_p^2 + 0.002692 n_p - 0.212580$ $f_{\mu_{\lambda}} = 0.000225 n_p^2 - 0.004599 n_p + 0.100125$
$\sigma_{\lambda_{\max}} = (a_{\sigma_{\lambda}} + b_{\sigma_{\lambda}} v) + (c_{\sigma_{\lambda}} + d_{\sigma_{\lambda}} v) \exp((e_{\sigma_{\lambda}} + f_{\sigma_{\lambda}} v) \theta')$	$a_{\sigma_{\lambda}} = 0.000237 n_p^2 - 0.007832 n_p + 0.023678$ $b_{\sigma_{\lambda}} = -0.001004 n_p^2 + 0.034336 n_p + 0.327626$ $c_{\sigma_{\lambda}} = 0.000245 n_p^2 - 0.009199 n_p - 0.100417$ $d_{\sigma_{\lambda}} = -0.000750 n_p^2 + 0.026750 n_p + 1.084000$ $e_{\sigma_{\lambda}} = 0.000035 n_p^2 - 0.001138 n_p - 0.211311$ $f_{\sigma_{\lambda}} = -0.000336 n_p^2 + 0.012854 n_p + 0.126169$
$\mu_{\ln \delta, \lambda} = (a_{\mu_{\delta, \lambda}} + b_{\mu_{\delta, \lambda}} v) + (c_{\mu_{\delta, \lambda}} + d_{\mu_{\delta, \lambda}} v) \exp((e_{\mu_{\delta, \lambda}} + f_{\mu_{\delta, \lambda}} v) \theta')$	$a_{\mu_{\delta, \lambda}} = -0.000210 n_p^2 + 0.007270 n_p - 0.012449$ $b_{\mu_{\delta, \lambda}} = -0.000569 n_p^2 + 0.023061 n_p + 0.378766$ $c_{\mu_{\delta, \lambda}} = 0.000206 n_p^2 - 0.006414 n_p - 0.005043$ $d_{\mu_{\delta, \lambda}} = -0.003240 n_p^2 + 0.110137 n_p + 0.385189$ $e_{\mu_{\delta, \lambda}} = -0.000168 n_p^2 + 0.006608 n_p - 0.222140$ $f_{\mu_{\delta, \lambda}} = -0.000059 n_p^2 + 0.002446 n_p + 0.073216$
$\sigma_{\ln \delta, \lambda} = (a_{\sigma_{\delta, \lambda}} + b_{\sigma_{\delta, \lambda}} v) + (c_{\sigma_{\delta, \lambda}} + d_{\sigma_{\delta, \lambda}} v) \exp((e_{\sigma_{\delta, \lambda}} + f_{\sigma_{\delta, \lambda}} v) \theta')$	$a_{\sigma_{\delta, \lambda}} = 0.000037 n_p^2 - 0.000990 n_p + 0.041999$ $b_{\sigma_{\delta, \lambda}} = 0.000025 n_p^2 - 0.001362 n_p + 0.871251$ $c_{\sigma_{\delta, \lambda}} = -0.000066 n_p^2 + 0.002357 n_p - 0.005008$ $d_{\sigma_{\delta, \lambda}} = 0.000758 n_p^2 - 0.026669 n_p - 0.182546$ $e_{\sigma_{\delta, \lambda}} = -0.000689 n_p^2 + 0.019261 n_p - 0.323314$ $f_{\sigma_{\delta, \lambda}} = 0.000326 n_p^2 - 0.003827 n_p - 0.007236$

772

773 **Figure Captions**

774 Figure 1. Plan view of foundation locations.

775 Figure 2. Illustration of the difference between $\Delta_{\max}$ and $\lambda_{\max}$

776 Figure 3. Agreement between regression predictions and simulation estimates of $\mu_{\Delta_{\max}}$ in a), $\sigma_{\Delta_{\max}}$
 777 in b), $\mu_{\ln \delta, \Delta}$ in c), and $\sigma_{\ln \delta, \Delta}$ in d).

778 Figure 4. Agreement between regression predictions and simulation estimates of $\mu_{\lambda_{\max}}$ in a),
 779 $\sigma_{\lambda_{\max}}$ in b), $\mu_{\ln \delta, \lambda}$ in c), and $\sigma_{\ln \delta, \lambda}$ in d).

780 Figure 5. Regression predicted, obtained via eq. (17), versus simulated probabilities, $P[\Delta_{\max} > t]$,
 781 for all cases listed in Table 1, and for $t = 0.05, 0.10, 0.15, \dots, 5.00$.

782 Figure 6. Regression predicted, obtained via eq. (18), versus simulated probabilities, $P[\lambda_{\max} > t]$,
 783 for all cases listed in Table 1, and for $t = 0.05, 0.10, 0.15, \dots, 5.00$.

784 Figure 7. Required (ϕ_{gs} / k_R) factors for SLS design as a function of the target β_{SLS} for $n_p = 9$ and
 785 $\theta = \theta_{\ln K} / s = 3$.

786 Figure 8. Required (ϕ_{gs} / k_R) factors for ULS design as a function of the target β_{ULS} for $n_p = 9$
 787 and $\theta = \theta_{\ln K} / s = 3$.

788 Figure 9. Required (ϕ_{gs} / k_R) factors for ULS design as a function of the target β_{ULS} for $n_p = 9$
 789 and $\theta = \theta_{\ln K} / s = 3$ using load factors from CHBDC (CSA S6-19, 2019).

790

791 Table Captions

792 Table 1. Input parameters used in the simulation.