Canadian Geotechnical Journal

Seepage through a Circular Geomembrane Hole when covered by Fine-Grained Tailings under Filter Incompatible Conditions

Journal:	<i>Canadian Geotechnical Journal</i>
Manuscript ID	cgj-2020-0788.R2
Manuscript Type:	Article
Date Submitted by the Author:	20-May-2021
Complete List of Authors:	Fan, Jiying; Queen's Univ, Dept. of Civil Engineering Rowe, R. Kerry; Queen's University, Civil Engineering
Keyword:	Geomembrane, Hole, Leakage, Tailings, Seepage force
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

SCHOLARONE™
Manuscripts

Seepage through a Circular Geomembrane Hole when covered by Fine-Grained Tailings under Filter Incompatible Conditions

Jiying Fan¹ and R. Kerry Rowe^{2,*}

¹ Ph.D. Student, GeoEngineering Centre at Queen's-RMC, Dept. of Civil Engineering, Queen's Univ., Kingston, ON, Canada K7L 3N6, ORCID: 0000-0002-4632-1980, jiying.fan@queensu.ca

² Barrington Batchelor Distinguished University Professor, Canada Research Chair in Geotechnical and Geoenvironmental Engineering, GeoEngineering Centre at Queen's-RMC, Dept. of Civil Engineering, Queen's Univ., Kingston, ON, Canada K7L 3N6, ORCID: 0000-0002-1009-0447, kerry.rowe@queensu.ca

* Corresponding author

Abstract: Experiments are conducted to quantify leakage through a circular geomembrane (GMB) hole using two subgrades (underliners) not meeting modern filter criteria. Test results show that a 1.9- and 5.0-fold increase in hole diameter cause a 1.8- and 6.2-fold increase in leakage, respectively, whereas a 2-fold increase in GMB thickness results in an approximate 10% decrease in leakage. For a constant hydrostatic effective stress, a 2- and 3-fold increase in water head above the GMB lead to a 1.8- and 2.5-fold increase in leakage, respectively. Higher leakage arising from the intrusion of subgrade materials through the hole is quantified. A numerical model, considering the localized concentration of head loss around the hole and the consequent heterogeneity in hydraulic conductivity due to the seepage force, is presented. The percent of head loss within the hole and from the hole to any position above the GMB are independent of loading conditions and are only influenced by the GMB thickness and hole diameter. Finally, empirical equations for predicting leakage through a circular GMB hole in tailings storage applications are proposed. Compared with published equations that do not consider consolidation induced heterogeneity in hydraulic conductivity, the proposed equation (which does consider these factors) gives a more accurate prediction for the cases examined.

Keywords: Geosynthetics, Geomembrane; Hole; Leakage; Seepage force; Tailings.

1. Introduction

Geomembranes (GMB) have proven to be excellent seepage barriers for modern municipal solid waste (MSW) landfills, where the GMB is generally overlain by a high permeable layer and underlain by a lower permeable layer such as a compacted clay liner or geosynthetic clay liner (Rowe 2005, 2020; Rowe and AbdelRazek 2019; Yu and Rowe 2018). To date, leakage through GMB defects has been studied extensively for conditions applicable to the MSW landfills (e.g., Giroud and Bonaparte 1989a, 1989b; Giroud 1997; Rowe 1998, 2012, 2020). However, most Canadian waste, like that of other countries, is not municipal solid waste. Based on Statistics Canada (2012), both oil sand tailings and mine tailings contributed to 64.7% of solid waste in Canada, while MSW only contributed to 2.6%. With the increasing environmental concern about the migration of contaminants from tailings to the subgrade soil and groundwater, the mining industry is using GMB liners more frequently in recent years (Lupo and Morrison 2007).

Tailings storage facilities (TSFs) are engineered structures constructed to impound slurry, thickened, and paste tailings resulting from mineral processing activities. Geosynthetic liner systems for TSFs commonly comprise a single GMB liner even though the hydraulic head within the facility can exceed 100 m (Touze et al. 2008). This is because tailings may often have 40-70% fines content ($<75\ \mu\text{m}$), which can form a lower permeable layer at the base of TSFs above the liner. GMBs used in mining applications are most commonly either made from high-density polyethylene (HDPE) or linear low-density polyethylene (LLDPE) with a typical thickness of 1.0 to 2.5 mm

(Touze et al. 2008; Rowe et al. 2013). With these GMBs, the leakage (i.e., fluid flow under a hydraulic gradient) is effectively limited to flow through holes in the GMB that most commonly arise either during construction (including during placement of material over the GMB) or subsequently due to stress cracking (Abdelaal et al. 2014; Ewais et al. 2014; Rowe 2012, 2020). Given the size and remote location of TSFs, the subgrade soil beneath the GMB is often the in-situ soil or rock, where a filter-compatible subgrade (also referred to by some as an underliner) might not exist without significant earthworks (Lupo and Morrison 2007). The configuration of lower permeable tailings separated from a high permeable subgrade using a GMB liner in TSFs is distinctly different from that in MSW landfills, raising the question of the applicability of the seepage mechanism through a GMB hole for MSW landfills versus TSFs.

The leakage through GMB defects in TSFs had been considered over the past two decades. Giroud et al. (1997) proposed an analytical method to evaluate leakage through a GMB defect overlain and underlain by permeable media. This method assumed that the hydraulic conductivity for the soil above the GMB was homogeneous, and water head above the GMB was below the overlying soil surface. Rowe et al. (2017) tested leakage through a circular GMB hole using silty sand tailings with 12-45% fines content both under filter compatible and incompatible subgrades. Migration of fines from the tailings to the subgrade, piping failure under a filter incompatible subgrade, and no significant difference in leakage between 1- and 2-mm-thick GMB were observed. Brachman et al. (2017) studied leakage through a circular GMB hole using

axisymmetric finite element (FE) model (SEEP/W GeoStudio 2012) and found a localized concentration of head loss within a small region around the hole. According to the simulation, over 95% of the head loss occurred locally within a region of 10 times the hole radius ($10r$) from the hole centre, and about 75% of the losses occurred within $2r$ from the hole centre. Chou et al. (2018) examined leakage through a circular GMB hole using silty sand tailings with 28% fines content under filter incompatible conditions. Internal erosion and piping were observed at low stress. The piping-induced voids within the silty sand remained open due to arching at higher stresses, leading to much more significant leakage.

Due to the localized concentration of head loss around the hole, consolidation stress applied on the tailings within that region is not homogeneous because of the seepage force, resulting in a heterogeneous hydraulic conductivity, k , within the vicinity of the hole. Chou et al. (2020) performed hydraulic conductivity tests on small specimens retrieved near the GMB hole using a small rigid-wall permeameter. The tailings near the hole experienced a lower k relative to those away from the hole due to the reduced void ratio resulting from seepage-induced consolidation. Still, the localized decrease in k was less than an order of magnitude.

This paper describes a study on seepage through a hole in a GMB that is influenced by the heterogeneity in both the consolidation stress and k within tailings arising from the seepage force. Thus, the primary objective is to experimentally examine and numerically simulate the effect on leakage through a circular GMB hole in a simulated TSF for different: (1) hole diameters, (2) GMB thicknesses, (3) applied stresses and

water heads, and (4) subgrade (i.e., liner subgrade/foundation) grain-size distributions. The secondary objective is to investigate the impact of seepage force on the (1) consolidation stress, (2) hydraulic conductivity, and (3) hydraulic gradient within tailings. The final objective is to propose a leakage prediction method considering seepage-induced consolidation.

2. Experimental Investigation

2.1 Materials

A fine-grained tailings sample originating from a thickened copper-zinc operation was tested. The tailings had approximately 90% fines ($<75\ \mu\text{m}$), with 46% silt size ($2\text{--}20\ \mu\text{m}$) and 14% clay size ($<2\ \mu\text{m}$) particles. The specific gravity of the tailings varied between 3.6 and 3.7. The liquid limit and plasticity index of the tailings were 27% and 9%, respectively. According to the Unified Soil Classification System (ASTM D2487), the sample could be described as a low plasticity clay (CL). Additional tailings details may be found in Lopes et al. (2013). The grain-size distribution (GSD) of the tailings is shown in Fig. 1. Based on this GSD, the tailings were assessed to be internally stable (Kenny and Lau 1985, 1986; Wan and Fell 2008).

Two subgrade materials, poorly graded sand (SP) and poorly graded gravel (GP) that did not satisfy modern filter criteria, were tested (Sherard et al. 1984a; Foster and Fell 2001). The SP and GP subgrades had the same uniformity coefficient of 4 (see Fig. 1). The diameter of the subgrade materials at which 15% by mass is finer, denoted as DF_{15} , was 0.85 mm for SP and 5.4 mm for GP. Based on the modern filter retention criteria, the tailings were assessed to be “excessive erosion” and “continuing erosion”

with the subgrades of SP and GP, respectively. The subgrade materials comprised angular to sub-angular particles from crushing stone with particle sizes larger than 1.18 mm (No. 16 sieve) and silica sand with particle sizes smaller than 1.18 mm. Hydraulic conductivities of SP and GP subgrades were estimated to be 3×10^{-3} and 0.1 m/s, respectively, based on the empirical correlation provided by Sherard et al. (1984b).

Two GMBs, a linear low-density polyethylene (LLDPE) with a thickness of 1 mm and high-density polyethylene (HDPE) with a thickness of 2 mm, were evaluated. The central circular hole diameters for the LLDPE GMB were 10 mm, 19 mm, and 50 mm, whereas, for the HDPE GMB, only a 10-mm-diameter hole was examined.

2.2 GLLS Tests

The test apparatus in this study was the same modified geosynthetic liner longevity simulator (GLLS) as that used in Brachman et al. (2017), which comprised a rigid cylindrical steel test cell with an inside diameter of 590 mm, and a height of 500 mm (see Fig. 2). An 8-mm-thick geocomposite drain (GCD) was first placed in the bottom of the cell. The 200-mm-thick subgrade was divided into several lifts. Care was taken to avoid segregation during placement. Each lift was tamped using a steel block to produce as flat a surface as possible. The dry density of these two subgrades was approximately 1900 kg/m³. After being fully saturated from the bottom, a layer of GMB with a central circular hole was placed. To prevent any preferential flow around the circumference of the GMB, a bentonite perimeter seal was introduced around the top and bottom of the GMB edges. This was followed by placing the fully saturated tailings with 66-71 % solids content (i.e., thickened and/or paste tailings) to simulate field

conditions (Lopes et al., 2013). The thickness of tailings after the test was close to 0.22 m. Friction along the inside surface of the cell was minimized by using two layers of 0.1-mm-thick polyethylene sheets, with a special lubricant between them, allowing the outer layer to slip with very little resistance during the consolidation of tailings.

After 24 hours of deposition and settlement of tailings, a sand leveling layer, wrapped in a separator geotextile, was used to fill the gap between the top of tailings and the rubber bladder. A prescribed stress path simulating the deposition of tailings and build-up of water head for the submerged tailings in a TSF was applied on top of the tailings in the GLLS cell (Table 1; Figs. 2 and 3). At any stage, there was a constant hydrostatic effective stress, σ_v' , an increase in both the pore pressure, u , and the total stress, σ_v , (e.g., Stage 2a to Stage 2; Stage 3a to Stage 3; Stage 4a to Stage 4) for tailings at the surface. Between adjacent stages, the pore pressure at tailings surface was held constant while the total and hydrostatic effective stresses increased (e.g., Stage 1 to Stage 2a; Stage 2 to Stage 3a; Stage 3 to Stage 4a). At each stage, total stress (σ_v) was applied by air pressure acting on the rubber bladder at a loading rate of 3-5 kPa/min, and tailings were consolidated as pore water seeped through the GMB hole. Once reaching the targeted pore pressure, the pore pressure was maintained at the tailings surface by injecting the pressurized water beneath the rubber bladder. Leakage was collected through a tube connected to the bottom port of the cell at the same elevation as the bottom of the GMB and was corrected to 20°C based on the temperature of the collected leakage. After reaching the steady leakage for at least five days, σ_v was slowly increased to the following loading stage and the process repeated.

2.3 Stress-Dependent Permeameter Tests

The hydraulic conductivity, k , of the tailings in a TSF influences the consolidation behavior of tailings and the seepage through any defect in the GMB, and k changes with ongoing deposition of tailings due to consolidation. A stress-dependent permeameter (Fig. 4) was used to conduct one-dimensional consolidation testing using incremental loading and conduct a k test after each consolidation stage.

Before the test, a saturated virgin specimen was placed in the cell in a loose state, taking care to minimize/avoid entrapped air. After assembling the system, the thickness of the specimen was recorded. A constant load was then applied to the specimen by hanging weights using a modified consolidation frame. Vertical displacement readings were measured as a function of time according to ASTM D2435 within ± 0.01 mm. During the consolidation test, valve 2 was closed and valve 3 opened (Fig. 4) so that water could drain from both the upper and lower porous stones. After 24 hours of consolidation, valve 2 was opened and valve 3 closed. A falling head test with a constant tailwater elevation was used to measure k of the specimen by introducing an initial hydraulic gradient of approximately 40. During the k test, the head loss across the specimen, deformations of the specimen, and temperature of the Deionized (DI) water were recorded simultaneously for approximately 24 hours. Subsequently, the load was increased and the process repeated.

The void ratio, e , corresponding to 100% primary consolidation was predicted using the log-time method. For most of the loading stages, 100% primary consolidation was achieved within 1 minute. Fig. 5 presents consolidation curves of four specimens.

For Specimens 2, 3, and 4, the curvature of the consolidation curves at lower consolidation stresses indicated the stress history those specimens experienced during both sampling and assembling the permeameter (i.e., the specimen in the cell was slightly compressed by hand to remove the entrapped air). The compression index, C_c , ranged between 0.24 and 0.30. Neglecting the stress history and assuming normally consolidated conditions, the dashed-line in Fig. 5 shows an empirical relationship between e and the consolidation stress (σ_v') based on the available data from four specimens.

$$e = -0.109 \ln \sigma_v' + 1.607 \quad (1)$$

The k after each loading stage was calculated according to ASTM D5856 and was corrected to 20°C based on the recorded temperature during permeation. A comparison was made of the k 's calculated using head losses at different time intervals during the same permeation test. The k was unaffected by the hydraulic gradient, which implied that the preferential flow along the interface between specimen and test cell was negligible. For the specimens tested, k ranged from 0.6×10^{-8} to 2.4×10^{-8} m/s. Consistent with the Kozeny-Carman equation (Taylor 1948), which is reported to be valid for any soil with k value in a range of 10^{-1} to 10^{-11} m/s (Chapuis and Aubertin 2003), the experimental results in Fig. 6 exhibited a relatively good linear correlation between k and $e^3/(1+e)$. The dashed line represents the empirical relationship between k and $e^3/(1+e)$ based on the available data from the specimens tested as follows:

$$k = \left(2.3 \times \frac{e^3}{1+e} - 0.127 \right) \times 10^{-8} \quad (2)$$

3. Numerical Model

In a TSF lined with a GMB, the tailings above the GMB provide resistance to the flow through the defect due to the relatively low k . The impact of subgrade on leakage through the hole was studied using FE axisymmetric model. Results revealed that the subgrade had a negligible effect on leakage if k of the subgrade was three orders of magnitude greater than that of the tailings. For the filter-incompatible conditions, k of the subgrade was at least five orders of magnitude greater than that of the tailings. Thus, the impact of subgrade on leakage and head loss was not considered in the following numerical model. FE results also depicted that the water head contours were ellipsoidal within a distance of two times the hole radius ($2r$) from the hole centre. Beyond $2r$, the contours were spherical and centered on the centre of the GMB hole. As a result, a seepage barrier region is defined herein as the region of $10r$ from the GMB hole centre since over 95% of head loss occurred within that region; this is the same as observed by Brachman et al. (2017).

The proposed model is based on the following assumptions: (1) the tailings were in perfect contact with the GMB, so that no preferential lateral flow along the interface between tailings and the GMB was considered; (2) the tailings were internally stable with the subgrade soil satisfying modern filter criteria, hence no fines migration and piping occurred, the variation of k within tailings resulted solely from the changing consolidation stress; (3) taking the datum to be at the upper surface of GMB (Fig. 7), the effect of elevation head within the seepage barrier region (i.e., $10r$ from the hole centre) was negligible relative to the pressure head so that water head was taken to be equal to the pressure head; and (4) the hole in the GMB was filled with tailings, and all

of the pressure head dissipated within the tailings (including that in the hole), such that the pressure head at the bottom of the hole was nil (i.e., atmospheric pressure).

The total head loss, H , through the tailings, is

$$H > \Delta h_1 + \Delta h_2 + \Delta h_3 + \dots + \Delta h_n \quad (3)$$

where Δh_1 is the head loss within the tailings present in the hole; and Δh_j ($n \geq j > 1$) is the head loss through tailings overlying the GMB between two adjacent head contours within the seepage barrier region (see Fig. 7).

In a homogeneously permeable medium, Darcy's Law defines a simple proportionality relationship between the rate of flow (Q) and hydraulic gradient (i). The cross-sectional area of soil mass across which the flow occurs (A) and the hydraulic conductivity of the medium (k) in Darcy's Law are both unchanged. However, leakage through the GMB defect is a three-dimensional problem with the progressively reducing A and k as flow converges on the hole (Fig. 7). Therefore, Darcy's Law is modified herein to consider the varying A and k within a flow tube within the seepage barrier region and is akin to a pipe filled with soil initially under uniform density but where the diameter of the pipe is decreasing, and hence the flow per unit area is increasing and the seepage drag forces increasing giving rise to a decrease in hydraulic conductivity as the end (hole) is approached. The leakage Q is equal to the flow through the tailings present within the hole, which is also equal to the flow through each water head contour in the tailings overlying the GMB:

$$Q = k_1 \frac{\Delta h_1}{L_1} A_1 = k_2 \frac{\Delta h_2}{L_2} A_2 = \dots = k_n \frac{\Delta h_n}{L_n} A_n \quad (4)$$

where k_1 is the hydraulic conductivity within the hole, which is the mean value between

k within the lower surface of the hole, denoted as k_{1L} , and within the upper surface of the hole, denoted as k_{1U} ; k_j ($n \geq j > 1$) is the hydraulic conductivity of the tailings overlying the GMB at each predefined water head contour; A_1 is the area of the circular GMB hole; A_j ($n \geq j > 1$) is the surface area of the ellipsoidal or sphere contour; t is the thickness of GMB; L_j ($n \geq j > 1$) is the length of the flow path between two adjacent water head contours, which is equal to the vertical distance directly above the hole centre between two adjacent water head contours (see Fig. 7). The consolidation stress σ_v' , and the consequent k within the seepage barrier region are influenced by the total stress σ_v , and the cumulative head loss from this location to the bottom of the GMB hole. For k_{1L} , $\sigma_{v1L}' = \sigma_v$; for k_{1U} , $\sigma_{v1U}' = \sigma_v - \Delta h_1 \gamma_w$; for k_j ($n \geq j > 1$), $\sigma_{vj}' = \sigma_v - (\Delta h_1 + \dots + \Delta h_j) \gamma_w$. The corresponding k at different locations can then be predicted based on Eqs. 1 and 2.

Given the hole dimension, the total stress σ_v , and the pore pressure u , applied on the intact GMB surface out of the seepage barrier region, then Q , distributions of k and Δh within the seepage barrier region can be obtained by solving Eq. 4 based on Eqs. 1, 2, and 3. The flow chart in Fig. 8 provides a method by gradually increasing Δh_1 from a minimal value (e.g., $\Delta h_1 = 0.01H$ as an initial value, and increases by 0.01 m after each trial) satisfying Eq. 3 until it reaches a threshold value (denoted as $\Delta h_{1critical} + 0.01$) that the head loss within the seepage barrier region exceeds H . Then Δh_1 is taken as $\Delta h_{1critical}$ to solve Eq. 4 since almost all the head loss occurs within the seepage barrier region. In the numerical model (Fig. 7), tailings within the seepage barrier region (i.e., $10r$ from hole centre) were divided into 26 parts, i.e., tailings present within the hole and 25 parts overlying the GMB separated according to the shapes of water head contours; A_j ($n \geq j$

≥ 1) is independent of loading stages and remains constant for a given test with a specific hole diameter and GMB thickness; L_j ($n \geq j > 1$) is constant and equal to $0.4r$ (i.e., $L_j = 10r/25$). An example MATLAB program implementing the model for the case with $2r = 10$ mm and $t = 1$ mm is given in Appendix A.

4. Leakage

At each loading stage, leakage in the GLLS slowly came to equilibrium. Table 2 summarizes the measured and the predicted leakage using the previously described numerical model for the test with hole diameter (i.e., $2r$) of 10 mm, GMB thickness (t) of 1 mm, and SP subgrade. Noting that, at the elevation of the GMB, the highest hydraulic conductivity, k_{upper} , will be out of the seepage barrier region and is controlled by the hydrostatic effective stress for tailings on the intact GMB surface, while the lowest hydraulic conductivity, k_{lower} , is in the tailings at the bottom of the GMB hole and is controlled by an effective stress which is equal to the total stress at that location because the pore pressure is zero. Thus, the lower and upper bound leakages in Table 2 can be calculated using FE axisymmetric model by assuming that the tailings were homogeneous and k was predicted based on σ_v (lower bound) and σ_v' (upper bound) in the tailings just above the intact GMB surface, respectively. In contrast, the proposed numerical model considers the variable hydraulic conductivity as a function of the effective stress, which changes with spatial distance from the hole.

At the same loading stage, leakage predicted using the proposed numerical model (Figs. 7 and 8) was close to the measured leakage to better than 4% (Table 2). In comparison, the measured leakage was approximately 5% greater than the lower bound

FE leakage and 40% below the upper bound FE leakage. The better performance of the proposed model than FE with either an upper or lower bound k arises because the proposed model considers the variation in k calculated within the seepage barrier region of $10r$ from the hole centre as indicated in Fig. 9. To simulate the varying k , one would have to divide the tailings in a FE model according to the water head contours in Fig. 7 and assign an appropriate k within each radial zone. The results of such an analysis using k varying as shown in Fig. 9a are given in Table 2 (FE Variable k) are on average 20% above the observed values and equal to about the average of the upper and lower bound values.

When the experiments were performed for a range of hole diameters (i.e., $2r$) and two GMB thicknesses, the leakage predicted using the proposed simple numerical model was in good agreement with the measured values (within about 5%) for all cases examined (Table 2).

4.1 Impact of Hole Diameter on Leakage

For a 1-mm-thick GMB at the same loading stage, a 1.9-fold increase in hole diameter from 10 to 19 mm caused a 1.8-fold increase in both the measured and the predicted leakage (Table 2), while a 5-fold increase in hole diameter from 10 to 50 mm resulted in a 6.2-fold increase in leakage. Thus, the leakage increased slightly more than linearly with the hole diameter and non-linearly with the hole area (i.e., πr^2). This result occurs because of the reconfiguration in both water head contours and Δh_j ($n \geq j \geq 1$) within the seepage barrier region as a response to the varying hole size, as the flow converged on the circular hole (as discussed in more detail subsequently).

4.2 Impact of GMB Thickness on Leakage

At the same loading stage, a 2-fold increase in GMB thickness from 1 to 2 mm resulted in an approximately 10% decrease in both the observed and the predicted leakage through a 10-mm-diameter GMB hole (Table 2). This variation arises because of the change in Δh_j ($n \geq j \geq 1$) due to the increasing GMB thickness, as explained later. Another impact arising from the increasing GMB thickness was a more extended time needed to reach the steady leakage for the test with 2-mm-thick GMB, especially at the higher loading stages.

4.3 Impact of Water Head on Leakage

A 3-fold increase in water head going from Stage 2a to Stage 2 caused an approximately 2.5-fold increase in leakage (Table 2). A 2-fold increase in water head going from Stage 3a to Stage 3 and from Stage 4a to Stage 4 both resulted in an approximately 1.8-fold increase in leakage (Table 2). The hydrostatic effective stress in tailings remote from the hole remained constant during the same loading stages (e.g., Stage 2a and Stage 2), however, the leakage was not proportional to the water head.

Seepage-induced consolidation is a reactive force arising from the hydraulic resistance of tailings to the water flow, with a total magnitude of around $\gamma_w H$ within the seepage barrier region. Higher seepage-induced consolidation and lower k within the seepage barrier region arose when increasing the water head, H , resulting in a less significant leakage increase than in H .

4.4 Impact of Subgrade Coarseness on Leakage

An important assumption for the proposed numerical model is that the GMB hole was

filled with fully consolidated tailings. This was applicable for the cases with SP subgrade, as shown in Figs. 10a and 10b. For the tests with GP subgrade, there were some subgrade gravel particles intruding through the hole (see Figs. 10c and 10d). On the one hand, this intrusion might weaken the seepage barrier effect within the hole since tailings might not be fully consolidated, resulting in a greater leakage; on the other hand, the hole area decreased due to the impermeable gravel particle, leading to a lesser leakage. The combined effect on the measured leakage was an increase (Table 3) relative to the corresponding cases using the SP subgrade (Table 2), especially when the applied stress was relatively small (e.g., Stages 1, 2a, and 2), showing that the predominant impact arising from the intrusion of subgrade materials through the hole was to weaken the seepage barrier effect. For example, intrusion caused a 2-fold increase in leakage through smaller holes (i.e., 10 mm and 19 mm) and a 1.2-fold increase through a 50-mm-diameter hole at Stage 1. With the increasing loading, the difference between the measured leakage using the coarser GP subgrade (Table 3) and the finer SP subgrade (Table 2) decreased, reflecting the reducing impact of the subgrade intrusion on leakage.

To simulate the greater leakage induced by the subgrade intrusion through the hole, the numerical model in Fig. 7 was further modified by assuming $\Delta h_1 = 0$ due to a much higher k within the hole compared to the k above the GMB. This assumption essentially negates GMB thickness. The pressure head for the tailings within the upper surface of the hole was taken as nil. Leakage calculated using the modified model is presented in Table 3. At Stages 2, 3a and 3, the calculated leakage was close to the measured leakage

for most of the cases, while the calculated leakage was lesser than the measured leakage at Stages 1 and 2a and was greater than the measured leakage at Stage 4a. A possible explanation for this discrepancy is that tailings directly above the hole (i.e., Δh_2 area in Fig. 7) was not fully consolidated because of the intrusion of gravel during the initial loading stages, resulting in a much greater measured leakage at Stages 1 and 2a; with the increasing loading Stages to 2, 3a and 3, tailings above the hole was fully consolidated, and the hole was filled with gravel particles and loose tailings, so that the measured leakage was close to the predicted leakage; Δh_1 was no longer negligible for Stage 4a because tailings within the hole had consolidated and k within the hole decreased to a value close to the corresponding cases using the finer SP subgrade, leading to a similar leakage at Stage 4a.

4.5 Migration of Tailings Slurry

After the experiment, a footprint of tailings, spreading radially outward from the hole at/just below the lower interface between the GMB and subgrade for a distance of about $2r$, was observed for all the tests (see Figs. 11a and 11c), leaving a thin layer of tailings on the subgrade surface. A slight vertical migration of tailings slurry within 40 mm below the subgrade surface was observed only for the test with a 50-mm-diameter hole and GP subgrade (see Fig. 11d). The migration of tailings here was different from the piping-induced silty sand erosion observed by Chou et al. (2018).

It is helpful to break up the process of tailings movement through a GMB hole into three phases: migration during the sub-aqueous deposition (Phase I), erosion during the initial consolidation (Phase II), and erosion during piping (Phase III). The difference

between Phases II and III lies in the soil skeleton. At Phase II, the soil skeleton has not entirely formed. Once the soil is eroded, its position will be taken over by the adjacent soil. At Phase III, the soil skeleton has formed, once the soil is eroded, its position will become an open void due to the arching effect; open voids will be connected together and become open channels with the continuing erosion, resulting in a much higher leakage. The most common means of identifying that piping and internal erosion have occurred is to monitor the leakage (Fell et al. 2003). It is inferred that piping did not occur for any of the tests reported in this study because no abnormally high leakage was detected. The movement of tailings through the GMB hole detected here is hypothesized to have predominantly occurred during the placement of tailings slurry (Phase I) and during initial consolidation (Phase II), given the absence of a notable change in leakage that could be attributed to piping and a qualitatively small amount of particle migration observed.

5. Head Loss within the Hole

5.1 Spatial Distributions of Hydraulic Conductivity and Hydraulic Gradient

The spatial distributions of hydraulic conductivity (k) and hydraulic gradient (i) within the seepage barrier region assumed for the proposed numerical model are shown in Figs. 9 and 12, respectively. Both k and i were influenced by the distance from the hole centre. k first experienced a notable increase within $2r$ from the hole centre, then gradually approached a value of k_{upper} with the consolidation stress close to $\sigma_v' = \sigma_v - H\gamma_w$ (see Fig. 9); correspondingly, there was a sharp decrease in i within $2r$, then gradually reduced to a value less than 10 for some cases (see Fig. 12). This is because the majority

of head loss (i.e., more than 80%) occurred within a region of $2r$ from the hole centre in these experiments; both k and i were therefore varied within a relatively larger range due to the bigger impact from the seepage force than tailings beyond $2r$.

At the same loading stage, i within the hole and the consequent Δh_1 decreased with the increasing hole diameter (see Fig. 12). For a thicker GMB with $t = 2$ mm, the curves were slightly below the corresponding curves for $t = 1$ mm (see the dashed lines in Fig. 12a). However, Δh_1 increased with the increasing t since a 2-fold increase in thickness from 1 to 2 mm resulted in a 10% decrease in i within the hole. Despite a slight discrepancy at the boundary of the seepage barrier region, the distribution of i between adjacent loading stages almost coincided if having the same u (see Fig. 12), meaning that Δh_j ($n \geq j \geq 1$) remained constant at the same H . Further analysis showed that percent of head loss within the hole (i.e., $\Delta h_1/H$) and between adjacent water head contours (i.e., $\Delta h_2/H, \dots, \Delta h_n/H$) were constant for a given test at different loading stages and were only influenced by the geometric properties of the hole (i.e., GMB thickness and hole diameter).

5.2 Modelling of Percent of Head Loss within the Hole

Since $\Delta h_1/H$ was independent of the loading stage for a given test where the intrusion of subgrade materials through the hole was absent, the model was used to examine how $\Delta h_1/H$ varied as the hole diameter increased from 10 to 500 mm and the GMB thickness from 0.5 to 3 mm (Fig. 13). There is a general trend of decreasing $\Delta h_1/H$ with increasing r and decreasing t . The variation of $\Delta h_1/H$ arising from r was more prominent for r less than 25 mm, and the curves gradually flattened as r increased from 50 to 250 mm. For

example, as r increased from 5 to 25 mm, $\Delta h_1/H$ decreased significantly from 21.2% to 5.5% for $t = 1$ mm and from 49.2% to 15.4% for $t = 3$ mm, whereas increasing r from 50 to 250 mm, $\Delta h_1/H$ gradually decreased from 2.9% to 0.6% for $t = 1$ mm and from 8.3% to 1.8% for $t = 3$ mm. Therefore, using a thicker GMB with a smaller hole strengthened the seepage barrier effect within the hole, leading to a higher $\Delta h_1/H$.

5.3 Predicting Leakage through a Circular GMB Hole below Fine-Grained Tailings

The numerical results for $\Delta h_1/H$ in Fig. 13 were further summarized by the empirical Eq. 5 for $5 \text{ mm} \leq r \leq 25 \text{ mm}$ and by Eq. 6 for $25 \text{ mm} < r \leq 250 \text{ mm}$. Leakage through a circular GMB hole with no subgrade intrusion can be predicted using Darcy's laminar flow through the hole by Eqs. 7 and 8, depending on the hole size.

for $0.005 \text{ m} \leq r \leq 0.025 \text{ m}$

$$\frac{\Delta h_1}{H} = 19.06t^{1.295}r^{0.097\ln t - 0.169} \quad (5)$$

for $0.025 \text{ m} < r \leq 0.25 \text{ m}$

$$\frac{\Delta h_1}{H} = 2.34t^{1.062}r^{0.034\ln t - 0.744} \quad (6)$$

for $0.005 \text{ m} \leq r \leq 0.025 \text{ m}$

$$Q = 59.87k_1Ht^{0.295}r^{0.097\ln t + 1.831} \quad (7)$$

for $0.025 \text{ m} < r \leq 0.25 \text{ m}$

$$Q = 7.35k_1Ht^{0.062}r^{0.034\ln t + 1.256} \quad (8)$$

where Q is in m^3/s ; k is in m/s ; H , t and r are in m. According to Eqs. 7 and 8, the impact of hole geometry on leakage is reflected by parameters t and r ; the impact of loading condition on leakage is illustrated by parameters k_1 and H . Also, k_1 is affected by the

hole geometry and the resultant Δh_1 , since k_1 is the average value between k at the lower and upper surfaces of the hole. To accurately predict the leakage through the circular GMB hole overlain by tailings, Δh_1 should first be predicted based on Eqs. 5 and 6, k_{1L} , k_{1U} , and the resultant k_1 can then be calculated according to σ_v and Δh_1 .

Based on Eqs. 7 and 8, leakage through the circular GMB hole overlain by tailings is not simply proportional to the hole area or inversely proportional to the GMB thickness. There is a simultaneous change in $\Delta h_1/H$ with the variation of hole geometry (i.e., t and r). Also, although increasing H increases Q due to a more significant hydraulic gradient, the increase in Q is offset to a certain extent by a lower k_1 arising from the higher seepage-induced consolidation within the seepage barrier region. This is why the increase in the measured leakage was less significant than the increase in water head.

6. Discussion

Comparison among the predicted leakage using Eqs. 7 and 8 (referred to as the Fan-Rowe equation), Wissa-Fuleihan equation (Wissa and Fuleihan 1992; Badu and Giroud 2018), Bernoulli's equation, Bonaparte equation (Bonaparte et al. 1989), Giroud equation (Giroud et al. 1997), Rowe-Booker equation (Rowe and Booker 2000), and modified Rowe-Booker equation (derived from Rowe-Booker equation and Darcy's equation) was made. Parts of results are present in Table 4.

The Bonaparte equation and the subsequent Giroud equation which superseded the Bonaparte equation both considered the influence of the k of the medium overlying the GMB on leakage through the defect in a GMB underlain by a highly permeable medium.

However, the critical assumption in these equations is that the “*permeable medium that overlies the geomembrane is thick enough to completely contain the liquid. Therefore, the liquid head away from the geomembrane defect is less than the thickness of the permeable medium*”. In many tailings’ applications, as in these experiments, there is water above the tailings and as a consequence the assumption made in developing these equations is not satisfied and strictly speaking they should not be used. Leakage calculations made using Bonaparte and Giroud equation, were orders of magnitude too high, illustrating that they were not appropriate for the situation being examined. The analytical equation by Giroud et al. (1997) defined a parameter called the liquid supply rate to maintain a stable phreatic surface, that parameter was not shown to influence the leakage significantly.

The Wissa-Fuleihan equation divided tailings into two parts, tailings present within the GMB hole and tailings overlying the GMB. Leakage through the tailings present within the GMB hole was expressed using Darcy’s equation, leakage in the tailings overlying the GMB was expressed by Forchheimer’s equation (Forchheimer 1930) for an infinitely deep tailings layer. The Rowe-Booker equation was derived by Rowe and Booker (2000) as a generalized three-dimensional solution for leakage through a circular hole in a GMB that is in perfect contact with soil (Rowe et al. 2004). The Rowe-Booker equation reduces to the Forchheimer’s equation for a very thick layer compared to the hole size and tends to the one-dimensional solution for a very thin layer (e.g. in the hole). The modified Rowe-Booker equation is given by:

$$Q = \Omega \xi H / (\Omega + \xi) \quad (9a)$$

$$\Omega = \{4 + [2.455 + 0.685 \tanh(0.6 \ln (r/T))] r/T\} rk_a \quad (9b)$$

$$\zeta = \pi r^2 k_b / t \quad (9c)$$

where r is the radius of the hole, T is the thickness of the tailings, H is the water head above the GMB, k_a is a representative of effective stress at just above the surface of the GMB and k_b is a representative of $\sigma_v = \sigma_{vb}'$ in the tailings at the bottom of the hole.

The predicted leakage using the Rowe-Booker equation which can be obtained with a simple one-line equation gives the best agreement with the observed leakage except for the Fan-Rowe equation which was derived by considering a range of stresses and k values. The modified Rowe-Booker equation and the Wissa-Fuleihan equations give very similar values in this case, where the tailings are much thicker than the hole radius. This would not be true for a larger hole. For example, if the hole radius was increased to 0.05 m and all the other parameters were kept the same, the Wissa-Fuleihan equation for Stage 1 with predicted leakage of 17.7 L/week compared to 19.5 L/week for the modified Rowe-Booker equation, for Stage 4 the comparison would be 97.3 L/week for the Wissa-Fuleihan equation and 107.7 L/week for the modified Rowe-Booker equation, and the difference would become more significant with increasing hole size relative to tailings thickness. This difference would be important early in the tailings placement and would decrease with increasing tailings thickness assuming the hole size will remain constant. As expected, the predicted leakage by Bonaparte et al. (1989) and Giroud et al. (1997) were substantially more than that predicted because they are not meant for this application and should not be used outside their published range of applicability. Bernoulli's equation studies the free flow through an orifice plate where

the plate is located between two infinitely pervious media. Consequently, the predicted leakage by Bernoulli's equation gave the highest values, overestimating leakage by five orders of magnitude.

The predicted leakage by Wissa-Fuleihan or modified Rowe-Booker equation was around 30 % higher than the test result. The Rowe-Booker equation was 15-20% above the test results and the Fan-Rowe equation was 4-10% above the observed values. The Fan-Rowe equation gave the best results because it was obtained by considering the heterogeneity in k due to seepage drag forces and consequent consolidation, whereas the other solutions only used two (Wissa-Fuleihan and Modified Rowe-Booker equations) or one (Rowe-Booker equation) k value.

The aforementioned empirical Eqs. 7 and 8 on leakage prediction in tailings storage applications are restricted to the circular hole condition. Field experience and physical experiments show that GMB defects can be circular and non-circular (Gilson-Beck 2019; Rowe et al. 2013). According to Bernoulli's equation, the discharge coefficient varies between 0.6 and 0.85 (Giroud and Bonaparte 1989a). The consolidation stress on tailings considered in the numerical model arises from both the hydrostatic effective stress and the seepage force. There is a smooth transition on the seepage-induced consolidation from 0 at the boundary of the seepage barrier region (i.e., $10r$ from hole centre) to approximately $\gamma_w H$ within the lower surface of the hole. However, inconsistency about this smooth transition may arise from: (1) intrusion of the subgrade material through the hole, (2) heterogeneity of the overlying tailings (e.g., segregation during the subaqueous deposition for silty sand tailings), (3) arching around the hole

for a thicker GMB with a smaller hole size, (4) an engineered low permeability liner beneath the GMB (e.g., compacted clay liner or geosynthetic clay liner) and the consequent notable head loss through the subgrade, and (5) drainage system overlying the GMB.

GLLS tests validated the numerical model (Fig. 7) with GMB thickness of both 1 and 2 mm and a minimum r of 5 mm. For $r < 5$ mm, $\Delta h_1/H$ increases exponentially with the reducing r . Therefore, there is a threshold r at a given t , denoted as r_0 ; for $r < r_0$, $\Delta h_1/H$ is 100%, meaning that the seepage barrier effect and the consequent head loss only occur within the hole. Based on Eq. 5, r_0 is calculated to be 0.86 mm for $t = 1$ mm and 1.38 mm for $t = 2$ mm. Such small holes can be described as pinholes. However, if arching occurs above the pinhole, then tailings within the hole, if any, are probably in the at-rest condition since no stress is applied on those tailings. In this case, the GMB hole can be considered a pipe, where Poiseuille's equation is applicable.

7. Summary and Conclusions

This paper has explored seepage through a circular GMB hole due to the localized concentration of head loss near the hole. The mechanical and hydraulic properties of the fine-grained tailings tested were studied using a stress-dependent permeameter. A numerical model, considering seepage-induced consolidation and the consequent heterogeneity in hydraulic conductivity k , was validated by the coupled physical and hydraulic experiments. The effect of hole diameter, GMB thickness, water head, and subgrade grain-size on leakage was discussed in detail. The k , hydraulic gradient (i), and percent of head loss within the seepage barrier region were illustrated. Finally,

empirical equations for predicting leakage through a circular GMB hole were proposed and compared with the published equations. For the specific conditions and materials examined, the following conclusions were reached:

1. The k for the fine-grained tailings tested was linearly correlated with $e^3/(1+e)$.
2. Leakage was not proportional to the hole area (i.e., πr^2) or inversely proportional to the GMB thickness (t). Based on the experiments, leakage increased almost linearly with hole diameter, while a 2-fold increase in GMB thickness from 1 to 2 mm resulted in an approximate 10% decrease in leakage.
3. Both the total stress (σ_v) and water head (H) on the intact GMB surface had an impact on the leakage. Keeping the hydrostatic effective stress ($\sigma_v - \gamma_w H$) constant, a 2- and 3-fold increase in H caused a 1.8- and 2.5-fold increase in leakage, respectively. The increase in leakage was less than the increase in H due to a greater seepage-induced consolidation within the seepage barrier region arising from a higher H .
4. Within the seepage barrier region, k decreased and i increased as the distance to the GMB hole decreased. Percent of head loss within the hole and from the hole to any position above the GMB remained constant for a given test at different loading stages and were only influenced by the geometric properties of the hole (i.e., GMB thickness and hole diameter).
5. Both a thicker GMB and a smaller hole diameter strengthened the seepage

barrier effect within the hole, leading to a higher percentage of head loss within the hole.

6. The intrusion of the subgrade materials through the hole weakened the seepage barrier effect within the hole, leading to a higher leakage, especially at low consolidation stresses. With the increasing stress, the hole was progressively filled both with the subgrade materials and the consolidated tailings, and the impact of intrusion from the subgrade decreased.
7. No piping was observed for the fine-grained tailings tested with two filter-incompatible subgrades. Only a small amount of tailings migrated through the GMB hole during both the placement of tailings slurry and the initial consolidation.

The equation developed in this paper provide the best agreement between the observed and calculated leakage through a circular hole below tailings and underlain by a highly permeable material (within about 10% of observed values). The Rowe-Booker equation (Rowe and Booker 2000; Rowe et al. 2004) gave results within about 20%, the modified Rowe-Booker equation and Wissa-Fuleihan equation (Wissa and Fuleihan 1992; Badu and Giroud 2018) gave results within about 30% of the observed values, all were far better than other published equations in the literature. Equations developed for landfill and similar applications (e.g. Bonaparte et al. 1989; Giroud et al. 1997) should not be used for GMBs below tailings.

Acknowledgements

The research reported in this paper was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) under Strategic Grant STPGP 521237–18. The tailings tested were obtained and shipped to Queen’s for testing by Golder Associates. The value of discussions with Dr. Frank Barone, Mr. Roy Lopez, Mr. Harvey MacLeod, and Dr. Prabeen Joshi is gratefully acknowledged.

References

- Abdelaal, F. B., Rowe, R. K., and Brachman, R. W. I. 2014. Brittle rupture of an aged HPDE geomembrane at local gravel indentations under simulated field conditions. *Geosynthetics International*, 21(1), 1-23.
- ASTM. 2015. Standard Practice for Measurement of Hydraulic Conductivity of Porous Material Using a Rigid-Wall, Compaction-Mold Permeameter. *ASTM D5856*, West Conshohocken, Pa.
- ASTM. 2011. Standard Practice for One-Dimensional Consolidation Properties of Soils Using Incremental Loading. *ASTM D2345*, West Conshohocken, Pa.
- ASTM. 2017. Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification System). *ASTM D2487*, West Conshohocken, Pa.
- Badu-Tweneboah, K., and Giroud, J. P. 2018. Discussion of “Leakage through Holes in Geomembranes below Saturated Tailings” by R. Kerry Rowe, Prabeen Joshi, RWI Brachman, and H. McLeod. *Journal of Geotechnical and Geoenvironmental Engineering*, 144(4), 07018001.
- Bonaparte, R., Giroud, J.P. and Gross, B.A. 1989. “Rates of Leakage through Landfill Liners”, *Proceedings of Geosynthetics '89*, Vol. 1, IFAI, San Diego, CA, USA,

February 1989, pp. 18-29.

Brachman, R. W. I., Joshi, P., and Rowe, R. K. 2017. A new laboratory apparatus for measuring leakage through geomembrane holes beneath mine tailings. *Canadian Geotechnical Journal*, 54(2), 147-157.

Chapuis, R. P., and Aubertin, M. 2003. On the use of the Kozeny Carman equation to predict the hydraulic conductivity of soils. *Canadian Geotechnical Journal*, 40(3), 616-628.

Chou, Y. C., Rowe, R. K., and Brachman, R. W. 2018. Erosion of silty sand tailings through a geomembrane defect under filter incompatible conditions. *Canadian Geotechnical Journal*, 55(11), 1564-1576.

Chou, Y. C., Rowe, R. K., and Brachman, R. W. 2020. Leakage through a hole in a geomembrane beneath a fine-grained tailings. *Canadian Geotechnical Journal* (under review).

Ewais, A. M. R., Rowe, R. K., Brachman, R. W. I., and Arnepalli, D. N. 2014. Service life of a high-density polyethylene geomembrane under simulated landfill conditions at 85° C. *Journal of Geotechnical and Geoenvironmental Engineering*, 140(11), 04014060.

Fell, R., Wan, C. F., Cyganiewicz, J., and Foster, M. 2003. Time for development of internal erosion and piping in embankment dams. *Journal of Geotechnical and Geoenvironmental Engineering*, 129(4), 307-314.

Forchheimer, P. 1930. *Hydraulik*, 3rd Ed., B.G. Teubner (Leipzig), Berlin, 596.

Foster, M., and Fell, R. 2001. Assessing embankment dam filters that do not satisfy

- design criteria. *Journal of Geotechnical and Geoenvironmental Engineering*, 127(5), 398-407.
- Gilson-Beck, A. 2019. Controlling leakage through installed geomembranes using electrical leak location. *Geotextiles and Geomembranes*, 47(5), 697-710.
- Giroud, J. P. 1997. Equations for calculating the rate of liquid migration through composite liners due to geomembrane defects. *Geosynthetics International*, 4(3-4), 335-348.
- Giroud, J. P., and Bonaparte, R. 1989a. Leakage through liners constructed with geomembranes—part I. Geomembrane liners. *Geotextiles and Geomembranes*, 8(1), 27-67.
- Giroud, J. P., and Bonaparte, R. 1989b. Leakage through liners constructed with geomembranes—Part II. Composite liners. *Geotextiles and Geomembranes*, 8(2), 71-111.
- Giroud, J. P., Khire, M. V., and Soderman, K. L. 1997. Liquid migration through defects in a geomembrane overlain and underlain by permeable media. *Geosynthetics International*, 4(3-4), 293-321.
- Kenney, T. C., and Lau, D. 1985. Internal stability of granular filters. *Canadian Geotechnical Journal*, 22(2), 215-225.
- Kenney, T. C., and Lau, D. 1986. Internal stability of granular filters: Reply. *Canadian Geotechnical Journal*, 23(3), 420-423.
- Lopes, R., Bahia, R., Jefferies, M., and Oliveira, M. 2013. Paste deposition over an existing subaqueous slurry deposit of high sulphide content tailings—the Neves Corvo

- experience. In *Proceedings of the 16th International Seminar on Paste and Thickened Tailings*, Australian Centre for Geomechanics, Perth.
- Lupo, J. F., and Morrison, K. F. 2007. Geosynthetic design and construction approaches in the mining industry. *Geotextiles and Geomembranes*, 25(2), 96-108.
- Rowe, R. K. 1998. Geosynthetics and the minimization of contaminant migration through barrier systems beneath solid waste. In *Proceedings 6th International Conference on Geosynthetics*, Atlanta 1, 27–103.
- Rowe, R. K. 2005. Long-term performance of contaminant barrier systems. *Geotechnique*, 55(9), 631-678.
- Rowe, R. K. 2012. Short and long-term leakage through composite liners. *Canadian Geotechnical Journal*, 49(2), 141-169.
- Rowe, R. K. 2020. Protecting the Environment with Geosynthetics: 53rd Karl Terzaghi Lecture. *Journal of Geotechnical and Geoenvironmental Engineering*, 146(9), 04020081.
- Rowe, R. K., and AbdelRazek, A. Y. 2019. Effect of interface transmissivity and hydraulic conductivity on contaminant migration through composite liners with wrinkles or failed seams. *Canadian Geotechnical Journal*, 56(11), 1650-1667.
- Rowe, R. K. and Booker, J. R. 2000. Theoretical solutions for calculating leakage through composite liner systems. Developments in theoretical geomechanics: *Proceedings of The John Booker Memorial Symposium*, Sydney, pp. 580–602.
- Rowe, R. K., Brachman, R. W. I., Irfan, H., Smith, M. E., and Thiel, R. 2013. Effect of underliner on geomembrane strains in heap leach applications. *Geotextiles and*

Geomembranes, 40, 37-47.

Rowe, R. K., Joshi, P., Brachman, R. W. I., and McLeod, H. 2017. Leakage through holes in geomembranes below saturated tailings. *Journal of Geotechnical and Geoenvironmental Engineering*, 143(2), 04016099.

Rowe, R. K., Quigley, R. M., Brachman, R. W. I. and Booker, J. R. 2004. *Barrier systems for waste disposal facilities*. London: Taylor & Francis (E & FN Spon).

Sherard, J. L., Dunnigan, L. P., and Talbot, J. R. 1984a. Filters for silts and clays. *Journal of Geotechnical Engineering*, 110(6), 701-718.

Sherard, J. L., Dunnigan, L. P., and Talbot, J. R. 1984b. Basic properties of sand and gravel filters. *Journal of Geotechnical Engineering*, 110(6), 684-700.

Taylor, D. 1948. *Fundamentals of soil mechanics*. Chapman and Hall, Limited.; New York.

Touze-Foltz, N., Lupo, J., and Barroso, M. 2008. Geoenvironmental applications of geosynthetics. *In Proceedings 4th European Geosynthetics Conference*, Edinburgh, U.K.

Wan, C. F., and Fell, R. 2008. Assessing the potential of internal instability and suffusion in embankment dams and their foundations. *Journal of Geotechnical and Geoenvironmental Engineering*, 134(3), 401-407.

Wissa, A. E. Z., and Fuleihan, N. F. 1992. Design and reclamation of phosphogypsum disposal sites. *AIChE Spring National Meeting, Advances in Phosphate Fertilizer Technology*, New Orleans, LA, 20.

Yu, Y., and Rowe, R. K. 2018. Development of geomembrane strains in waste

containment facility liners with waste settlement. *Geotextiles and Geomembranes*, 46(2), 226-242.

Draft

Appendix

This Appendix provides the MATLAB code for predicting leakage through the GMB hole with $2r = 10$ mm and $t = 1$ mm, for the fine-grained tailings present in this paper. The relationships between the consolidation stress and void ratio, and between void ratio and hydraulic conductivity for the tested fine-grained tailings based on Eqs. (1 and 2), respectively, have been programmed inside. The predefined water head contours are independent of stress condition and soil type, and only impacted by the GMB hole size. *% in the following codes mean comments, which are not implemented in the code.*

% The following steps are to input the total pressure and pore pressure on the intact GMB surface.

totalpressure=input('enter total pressure on the intact GMB surface (kPa): ');

porepressure=input('enter pore pressure on the intact GMB surface (kPa): ');

g=9.81; *% Acceleration due to gravity.*

% e_0 and k_0 are the void ratio and hydraulic conductivity within the lower surface of the hole, respectively.

$e_0 = -0.109 \cdot \log(\text{totalpressure}) + 1.6068$; *% Based on Empirical Eq. (1).*

$k_0 = 2.3 \cdot 10^{-8} \cdot (e_0)^3 / (1 + e_0) - 1.27 \cdot 10^{-9}$; *% Based on Empirical Eq. (2).*

% The following steps are to define the water head contours within the seepage barrier region. $A(1)$ is the hole area; $A(2)$, $A(3)$, and $A(4)$ are the surface areas of the ellipsoidal water head contours; $A(ii)$ for $ii > 4$ is the surface area of the spherical water head contour.

for ii=1:26

if ii<5

$A(1) = \pi \cdot 0.005^2$;

```

A(2)=108.6052e-6;

A(3)=167.1425e-6;

A(4)=265.3235e-6;

else

    A(ii)=2*pi*(2*(ii-1))^2/1000^2;

end

end

% The following steps are to solve Eq. (4) with the boundary condition of Eq. (3). h(1) is
the head loss within the hole, the initial assigned value is 0.01*porepressure/g, and
increases by 0.01m for each loop. e(1) and k(1) are the void ratio and hydraulic
conductivity within the upper surface of the hole, respectively. H is the consecutive head
loss from the hole to the boundary of the seepage barrier region.
for h=0.01*porepressure/g:0.01:0.5*porepressure/g
    h(1)=h;

    e(1)=-0.109*log(totalpressure-h(1)*g)+1.6068;

    k(1)=2.3*10^(-8)*(e(1))^3/(1+e(1))-1.27*10^(-9);

    q(1)=A(1)*h(1)*(k(1)+k_0)/2; % Leakage through the hole.

    H=h(1);

    for ii=2:26

        iii=ii-1;

        for jj=2:-0.01:0

            h(ii)=jj*h(iii);

            e(ii)=-0.109*log(totalpressure-H*g-h(ii)*g)+1.6068;

```

$$k(ii)=2.3*10^{(-8)}*(e(ii))^3/(1+e(ii))-1.27*10^{(-9)};$$

$$q(ii)=A(ii)*h(ii)*k(ii)/2;$$

if $\text{abs}((q(1)-q(ii))/q(1)) < 0.01$ % Over 99% accuracy for predicting leakage

through each water head contour.

break;

end

end

$$H=H+h(ii);$$

end

if $H < \text{porepressure}/g$

else

break;

end

end

% After the aforementioned iterations, the head loss within the hole used to predict leakage in the following steps is set to be $h(1)-0.01$, where $h(1)$ is the head loss within the hole when $H \geq \text{porepressure}/g$, 0.01 is the increment of head loss within the hole for each iteration.

$$h(1)=h(1)-0.01;$$

$$e(1)=-0.109*\log(\text{totalpressure}-h(1)*g)+1.6068;$$

$$k(1)=2.3*10^{(-8)}*(e(1))^3/(1+e(1))-1.27*10^{(-9)};$$

$$q(1)=A(1)*h(1)*(k(1)+k_0)/2;$$

$$H=h(1);$$

```

for ii=2:26

    iii=ii-1;

    for jj=2:-0.01:0

        h(ii)=jj*h(iii);

        e(ii)=-0.109*log(totalpressure-H*g-h(ii)*g)+1.6068;

        k(ii)=2.3*10^(-8)*(e(ii))^3/(1+e(ii))-1.27*10^(-9);

        q(ii)=A(ii)*h(ii)*k(ii)/2;

        if abs((q(1)-q(ii))/q(1))<0.01

            break;

        end

    end

    H=H+h(ii);

End

% The following steps are to output the leakage through the GMB hole and the percent of
head loss within the seepage barrier region.

leakage=(k(1)+k_0)/2*h(1)*pi*0.005^2/0.001*24*60*60*7*1000;

fprintf('leakage (L/week)= %f\n', leakage);

fprintf('percent of head loss within the seepage barrier region

= %f\n',(H*g)/porepressure);

```

Table Captions

Table 1. Applied stress path on top of the tailings in leakage experiment

Table 2. Leakage (Liters/week corrected to 20°C) comparison among SEEP/W axisymmetric model, numerical model, and GLLS test results with SP subgrade (r = hole radius)

Table 3. Leakage (Liters/week corrected to 20°C) comparison between numerical model and GLLS test results for 1-mm-thick GMB, GP subgrade, and assuming head loss within the hole $\Delta h_1 = 0$ in the model due to the intrusion of the subgrade materials (r = hole radius)

Table 4. Leakage (Liters/week) comparison among the test leakage and the predicted leakage using the existing empirical equations ($2r = 10$ mm, $t = 1$ mm, SP subgrade)

Draft

Table 1. Applied stress path on top of the tailings in leakage experiment

Stage	Total stress σ_v (kPa)	Pore pressure u (kPa)	Effective stress σ_v' (kPa)
1	66	60	6
2a	90	60	30
2	210	180	30
3a	270	180	90
3	450	360	90
4a	540	360	180
4	900	720	180

Draft

Table 2. Leakage (Liters/week corrected to 20°C) comparison among Finite Element axisymmetric model, proposed numerical model, and GLLS test results with SP subgrade (r = hole radius)

1-mm-thick GMB											2-mm-thick GMB	
2r = 10 mm							2r = 19 mm		2r = 50 mm		2r = 10 mm	
Finite Element (FE)				Proposed Numerical model ^d	Observed in GLLS tests ^e		Proposed Numerical model ^d	Observed GLLS test	Proposed Numerical model ^d	Observed GLLS test	Proposed Numerical model ^d	Observed GLLS test
Stage	Lower bound k^a	Upper bound k^b	Variable k^c									
1	0.9	1.5	1.2	1.0	0.9	0.9	1.9	1.6	6.2	6.3	0.9	0.9
2a	0.8	1.1	1.0	0.8	0.8	0.8	1.6	1.5	5.3	5.6	0.8	0.8
2	2.0	3.2	2.5	2.2	2.0	2.2	4.2	4.0	13.7	13.6	1.9	2.0
3a	1.8	2.5	2.2	1.9	1.9	1.9	3.7	3.4	11.9	11.5	1.7	1.7
3	3.1	5.0	4.0	3.4		3.3	6.6	6.2	21.5	20.4	3.0	3.2
4a	2.9	4.1	3.5	3.0		3.0	5.9	5.4	19.4	17.2	2.7	2.9
4	4.8	8.2	6.4	5.4		5.2						

Notes: ^a FE (SEEP/W) calculation assuming constant k based on $\sigma_v = \sigma_{vb}$ in the tailings at the bottom of the hole. ^b FE calculation assuming constant k based on σ_v just above GMB surface away from the hole. ^c FE calculation with variable k near the hole (k is the same as in Fig. 9a) and resulting prediction is approximately equal to the average of upper and lower bounds. ^d From the proposed simple numerical model (Fig. 7). ^e Duplicate tests.

Table 3. Leakage (Liters/week corrected to 20°C) comparison between numerical model and GLLS test results for 1-mm-thick GMB, GP subgrade, and assuming head loss within the hole $\Delta h_1 = 0$ in the model due to the intrusion of the subgrade materials (r = hole radius)

Stage	$2r = 10$ mm		$2r = 19$ mm		$2r = 50$ mm	
	Numerical model	GLLS test	Numerical model	GLLS test	Numerical model	GLLS test
1	1.3	1.9	2.4	3.1	6.5	7.7
2a	1.2	1.6	2.1	2.6	5.6	6.2
2	3.0	3.3	5.3	6.3	14.3	14.6
3a	2.6	2.5	4.6	4.6	12.3	12.1
3	4.7	4.2	8.3	8.0	22.4	21.2
4a	4.1	3.3	7.4	6.8	20.0	17.8

Table 4. Leakage (Liters/week) comparison among the test leakage and the predicted leakage using the existing empirical equations ($2r = 10$ mm, $t = 1$ mm, SP subgrade)

Stage	GLLS test	Fan-Rowe equation ^a	Wissa-Fuleihan equation ^b	Rowe-Booker equation ^c	Modified Rowe-Booker equation ^d
1	0.9	1.0	1.3	1.1	1.3
2a	0.8	0.8	1.0	1.1	1.0
2	2.2	2.2	2.8	2.5	2.9
3a	1.9	1.9	2.3	2.3	2.3
3	3.3	3.4	4.4	3.9	4.4
4a	3.0	3.0	3.7	3.6	3.8
4	5.2	5.4	7.1	6.0	7.1

Notes: ^a The proposed Eq. 7 in this paper. ^b The equation was first published by Wissa and Fuleihan (1992) and referred to as the W-F equation by Badu and Giroud (2018). ^c Homogeneous tailings using the Rowe and Booker equation (Rowe and Booker 2000) and k based on $\sigma_v = \sigma_{vb}'$ in the tailings at the bottom of the hole. ^d Tailings with k_a above the hole based on σ_v' at just above the GMB surface away from the hole (Upper bound k used in FE analysis) and k_b in the hole based on $\sigma_v = \sigma_{vb}'$ in the tailings at the bottom of the hole (lower bound k used in FE analysis of Table 2) as in the Wissa-Fuleihan equation but using the modified Rowe-Booker equation (Eq. 9).

Figure Captions

Fig. 1. Gradations of tailings and the subgrade materials

Fig. 2. Cross section through a modified geosynthetic liner longevity simulator (GLLS) test cell

Fig. 3. Applied stress path on top of the tailings in GLLS experiment

Fig. 4. Schematic of the stress-dependent permeameter

Fig. 5. Consolidation curves for tailings specimens and the empirical equation assuming normally consolidated conditions

Fig. 6. Results of hydraulic conductivity tests (corrected to 20°C) and the proposed empirical equation

Fig. 7. Cross section of water head contours in the numerical model

Fig. 8. Flow chart of the proposed numerical model used to calculate leakage, head loss, and hydraulic conductivity within the seepage barrier region (i.e., $10r$ from the hole centre)

Fig. 9. Distribution of hydraulic conductivity directly above the centre of hole with radius r for the test conditions (GMB thickness $t = 1$ mm, SP subgrade): (a) $2r = 10$ mm; (b) $2r = 19$ mm; (c) $2r = 50$ mm.

Fig. 10. Seepage barrier within the 1-mm-thick GMB hole ($r =$ hole radius): (a) $2r = 10$ mm, SP subgrade; (b) $2r = 50$ mm, SP subgrade; (c) $2r = 10$ mm, GP subgrade; (d) $2r = 50$ mm, GP subgrade.

Fig. 11. Subgrade after exhumation ($r =$ hole radius): (a) & (b) $2r = 19$ mm, SP subgrade; (c) & (d) $2r = 50$ mm, GP subgrade.

Fig. 12. Distribution of hydraulic gradient directly above the centre of hole with radius r for the test conditions (GMB thickness $t = 1$ mm, SP subgrade): (a) $2r = 10$ mm (dashed lines are for $t = 2$ mm); (b) $2r = 19$ mm; (c) $2r = 50$ mm.

Fig. 13. Percent of head loss within the hole ($\Delta h_1/H$) as a function of hole radius r and GMB thickness t

Fig. 1. Gradations of tailings and the subgrade materials

Fig. 2. Cross section through a modified geosynthetic liner longevity simulator (GLLS) test cell

Draft

Fig. 3. Applied stress path on top of the tailings in GLLS experiment

Fig. 4. Schematic of the stress-dependent permeameter

Fig. 5. Consolidation curves for tailings specimens and the empirical equation assuming normally consolidated conditions

Fig. 6. Results of hydraulic conductivity tests (corrected to 20°C) and the proposed empirical equation

Fig. 7. Cross section of water head contours in the numerical model

Fig. 8. Flow chart of the proposed numerical model used to calculate leakage, head loss, and hydraulic conductivity within the seepage barrier region (i.e., $10r$ from the hole centre)

(a)

(b)

(c)

Fig. 9. Distribution of hydraulic conductivity directly above the centre of hole with radius r for the test conditions (GMB thickness $t = 1$ mm, SP subgrade): (a) $2r = 10$ mm; (b) $2r = 19$ mm; (c) $2r = 50$ mm.

Fig. 10. Seepage barrier within the 1-mm-thick GMB hole (r = hole radius): (a) $2r = 10$ mm, SP subgrade; (b) $2r = 50$ mm, SP subgrade; (c) $2r = 10$ mm, GP subgrade; (d) $2r = 50$ mm, GP subgrade.

Fig. 11. Subgrade after exhumation (r = hole radius): (a) & (b) $2r = 19$ mm, SP subgrade; (c) & (d) $2r = 50$ mm, GP subgrade.

Fig. 12. Distribution of hydraulic gradient directly above the centre of hole with radius r for the test conditions (GMB thickness $t = 1$ mm, SP subgrade): (a) $2r = 10$ mm (dashed lines are for $t = 2$ mm); (b) $2r = 19$ mm; (c) $2r = 50$ mm.

Fig. 13. Percent of head loss within the hole ($\Delta h_1/H$) as a function of hole radius r and GMB thickness t