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Irreversible Thermodynamics at Surfaces: Inverse Edelstein Effect for Topological Insulator

Wayne M. Saslow^{1,*}

¹ *Texas A&M University, College Station, Texas, 77843, U.S.A.*

Abstract in English: We employ Onsager's Irreversible Thermodynamics to study the Inverse Edelstein effect (IEE) for a non-magnetic material (NM) adjacent to a topological insulator (TI) with a strong spin-orbit interaction. The TI surface state region is treated as quasi two-dimensional (2d). For the IEE, the source is a 3d spin flux incident from the NM that converts, at the NM/TI interface, to a quasi-2d charge current in the TI. For the Edelstein Effect (EE), the source is a quasi-2d charge flux incident from the TI that converts, at the interface, to a three-dimensional (3d) spin flux in the NM. For strong spin-orbit coupling, as considered here, when the 3d spin flux crosses to the 2d TI, the quasi-2d charge current is produced along with a quasi-2d spin accumulation. (For weak spin-orbit coupling, production of charge current and of spin accumulation are distinct processes.) We compute the associated rates of heating.

Resumé en Français: Nous utilisons la thermodynamique irréversible d'Onsager pour étudier l'effet Inverse Edelstein (IEE) pour un matériau non-magnétique (NM) adjacent à un isolant topologique (TI) avec une forte interaction spin-orbite. La région de l'état de surface TI est traitée comme quasi bi-dimensionnelle (2d). Pour l'IEE, la source est un flux de spin 3d incident du NM qui se transforme, à l'interface NM/TI, en un courant de charge quasi-2D dans le TI. Pour l'effet Edelstein (EE), la source est un flux de charge quasi-2D incident du TI qui convertit, à l'interface, en un flux de spin tridimensionnel (3d) dans le NM. Pour un fort couplage spin-orbite, comme considéré ici, lorsque le flux de spin 3d passe au 2d TI, le courant de charge quasi-2d est produit avec une accumulation de spin quasi-2D. (Pour un couplage spin-orbite faible, la production de courant de charge et d'accumulation de spin sont des processus distincts.) Nous calculons les taux de chauffage associés.

I. INTRODUCTION

For systems with strong enough spin-orbit interaction, the spin and space variables couple significantly. Under the correct circumstances, a charge flux can induce a spin flux – the Edelstein Effect (EE) – and a spin flux can induce a charge flux – the Inverse Edelstein Effect (IEE).[1 and 2] Fig. 1 illustrates the EE for a Topological Insulator (TI) with a spin-orbit interaction, that is adjacent to a non-magnet (NM). Solid arrows indicate the flow direction and dashed arrows indicate the spin accumulation, a non-equilibrium magnetization.[3–6] Fig. 2 illustrates the IEE, which we study in this work. Unlike what happens for conserved fluxes, like heat and charge, for non-conserved fluxes these are not inverse processes. The purpose of the present work is to employ Onsager's irreversible thermodynamics to provide a macroscopic description of these phenomena.

In this version of the EE (Fig. 1), the source is the quasi-2d charge flux j_{2y} (in A/m) along y , generated in the TI, with accompanying quasi-2d spin accumulation $\vec{m}_2$ (in A) along x . At the TI/NM interface j_{2y} converts to an upward flowing (z) 3d spin flux $\vec{J}_z$ in the NM, with spin polarization $\vec{m}$ in the same x direction as $\vec{m}_2$. (The Latin index indicates real space and the vector indicates spin space.) The spin flux in turn enters an adjacent ferromagnet FM, where it is detected. Fig. 1 depicts only the 3d spin flux in the NM.

In the IEE (Fig. 2) the flow direction solid arrows are the reverse of those in Fig. 1. Thus, for the IEE, in the

FIG. 1. Edelstein Effect (EE) for a TI (treated as quasi-2d) adjacent to a non-magnet (NM) and a ferromagnet (FM). In the TI a quasi-2d charge flux j_{2i} (in A/m) is driven along $i = y$; j_{2i} is the source. The spin-orbit interaction gives the TI a 2d spin polarization $\vec{m}_2$ (in A) along x (or along $-x$, depending on the sign of the spin-orbit interaction.) At the TI/NM interface this produces in the NM a 3d spin flux $\vec{J}_z$ (in J/m²) with spin-polarization $\vec{m}$ (in A/m) along $\vec{m}_2$.

FM one produces the downward flowing source $\vec{J}_z$, a 3d spin flux, which enters the NM. At the NM/TI interface the 3d spin flux $\vec{J}_z$ converts to 2d charge flux j_{2y} in the TI. This is related to Fig. 1 by both time-reversal and a 180° rotation in spin space about the $\hat{z}$ axis.

The choice of a TI is, in part, because the strong spin-orbit interaction that occurs for heavy metals (e.g. Pt)

FIG. 2. Inverse Edelstein Effect (IEE) for a TI surface (treated as quasi-2d) adjacent to a non-magnet (NM) and a ferromagnet (FM). From the FM a 3d spin flux $\vec{J}_z$ (in J/m^2) with spin accumulation $\vec{m}$, with direction determined by the FI magnetization, is driven into the NM and then into the top of the TI surface layer; $\vec{J}_z$ is the source. From there to the bottom of the quasi-2d TI surface layer, $\vec{J}_z$ converts to a quasi-2d spin accumulation $\vec{m}_2$ (in A), and a quasi-2d charge flux j_{2i} (in A/m) with direction determined by the spin-orbit interaction.

also causes decay of the spin flux. This limits the sample thickness and produces uncertainty in how much of the sample is affected.

Fig. 2 represents the IEE geometry of Ref. [7], which employed Fe as FM, Ag as NM, and α -Sn as TI. The Fe was driven into ferromagnetic resonance and a transverse voltage was measured across the (TI) Sn. It has long been known that the FM (Fe) sends a 3d spin flux $\vec{J}_z$ from the Fe to the adjacent NM (Ag),[8] but the specific spin flux generation mechanism is not relevant for the present work.[9] The IEE can be detected via a voltage difference in the TI along x ; in the EE, detecting the spin flux in the NM produced by an electric current often requires optical, rather than electrical, methods.[10]

A. Motivation

Zhang and Fert have analyzed the IEE effect for a TI with a “massless” Fermi surface (one with energy proportional to wavevector) and a strong spin-orbit coupling that is associated with a single chirality.[11] They take spin-polarized electrons incident from the NM to transmit into chiral electronic states of the TI, and then within the TI scatter to equilibrate their charge current. Using the 2d kinetic equation for the TI with steady-state 3d input distribution from the NM, they obtain the steady-state output distribution function in the TI.

This work considers the same IEE for the same geometry and source as Ref. [11] but uses Onsager’s theory of irreversible thermodynamics. This provides a straightforward formalism depending only on the symmetry of the problem, and independent of the specific

decay processes.[12–14]

In irreversible thermodynamics for bulk materials, *gradients* in the intensive thermodynamic variables, like temperature T , ordinary chemical potential μ , and spin chemical potential $\vec{\mu}$, drive the corresponding fluxes of heat, charge, and spin, which tend to equilibrate the system. On the other hand, for flow across surfaces, in irreversible thermodynamics *differences* in the intensive thermodynamic variables drive the corresponding fluxes.[8, 15–17] Not all dissipative equilibration processes are caused by fluxes. In a uniform magnetic system, if the magnetization $\vec{M}$ is misaligned in a field $\vec{H}$, then there is a thermodynamic force $\vec{M} \times \vec{H}$ that drives $\vec{M}$ to equilibrium without any flux.

Decay of 2d spin accumulation introduces a 2d spin accumulation relaxation time τ_{TI} that appears in the theory in the same way as does Zhang and Fert’s τ_p . Transport of $\vec{J}_i$ across the TI/NM interface introduces an interface relaxation time τ_i , which appears in the theory in the same way as does Zhang and Fert’s τ_t . We compute the rate of heat production dQ/dt associated with each of these relaxation times.

Following Ref. [7], for the TI we assume a “massless” Fermi surface and a spin-orbit interaction strong enough that, for each charge carrier, the spin and momentum are “spin-momentum locked” normal to one another; and that both are normal to the plane of the TI (either due to local anisotropy at the surface or to the demagnetization field of the induced spin accumulation). Moreover, as in Fig. 1, we consider the spin polarization of the spin flux to be normal to the flow direction.

If the spin polarization of $\vec{J}_i$ is rotated by 90° to point along y , then the 2d charge current rotates to point along $-x$. We ignore the third spin polarization direction, which produces a negligible charge current in the TI.[18 and 19]

B. Outline

Sect. II provides background for interpreting the EE and IEE. For non-experts, it discusses units; some authors employ spin-polarized number flux and some use spin-flux; both are related to the magnetization flux that appears in the equation of motion for $\vec{m}$. This work specifies the full dimensionality of all quantities employed.

In both the 3d NM and the quasi-2d TI the non-equilibrium magnetization $\vec{m}$ called the *spin accumulation* is due to excitations. Sect. III present the irreversible thermodynamics at surfaces. Sect. IV relates, in the language of irreversible thermodynamics, the pre-“spintronics” effective field $\vec{h}^*$ (in A/m) acting on $\vec{m}$ [8, 15, and 16] to the more modern spin chemical potential $\vec{\mu}$ (in J).[20] The change in the local equilibrium magnetization $\vec{M}$, due to spin state orientations rather than excitations, has a distinct effective field $\vec{H}^*$. [8 and 20] Fig. 1 of Ref. [21] illustrates, for a conducting ferro-

magnet, the meaning of both $\vec{m}$ and $\vec{M}$.

Sect. V relates, for the TI, the 2d charge flux $\vec{j}_2$ and its associated 2d spin accumulation $\vec{m}_2$, in the limit of strong spin-orbit coupling. Sect. VI relates $\vec{m}_2$ and the 3d spin flux $\vec{J}_z$ (incident from z) that decays into $\vec{m}_2$. Sect. VII relates $\vec{j}_2$ and $\vec{J}_z$, whose quasi-2d nature causes it to go to zero at the interface of the TI with its bulk. At this point our τ_{TI} is identified with τ_p of Ref. [11]. Sect. VIII introduces spin chemical potentials $\tilde{\mu}_{\text{TI}}$ in the TI and $\tilde{\mu}_{\text{NM}}$ in the NM (a spin excitation supporting material). Sect. IX applies irreversible thermodynamics to relate $\tilde{\mu}_{\text{TM}}$ and $\tilde{\mu}_{\text{NM}}$ to the spin flux $\vec{J}_z$ crossing the TI/NM interface. Sect. X provides a brief summary.

Throughout this work we review and compare different physical units, because the literature contains the proportional but distinct densities magnetization (density of μ_B), spin density (density of $\hbar$), and spin polarization density (units of number density), and their corresponding fluxes.[22] An Appendix discusses the generation of spin fluxes in an FM using ferromagnetic resonance (FMR). Using the language of irreversible thermodynamics we discuss spin pumping from a FM and spin transfer torque to an FM, for both the linear ac response and the quadratic dc response.

II. SOME MEASURED LENGTHS

The 2d charge flux j_{2y} has physical units of (charge)·(2d number density)·(velocity), or A/m, and the 3d spin flux $\vec{J}_z$ has physical units of (angular momentum)·(3d number density)·(velocity), or J/m². Two ratios appear in the literature, which we define in physical units; some works use other units.[23]

The IEE considers the ratio of the output 2d number flux j_2/e (in 1/m-s) to its input, the 3d polarization flux at the TI/NM interface $J_{z,\text{TI}}/(\hbar/2)$ per electron (in 1/m²-s). This gives the length[7]

$$\lambda_{\text{IEE}} = \frac{\hbar}{2e} \left| \frac{j_2}{J_{z,\text{TI}}} \right|. \quad (1)$$

In Fig. 1, the TI/NM interface is the third horizontal line from the top.

The EE considers the ratio of the output $J_z/(\hbar/2)$ at the TI/FM interface, to its input, j_2/e in the TI. This gives the inverse length[24]

$$q_{\text{EE}} = \frac{2e}{\hbar} \left| \frac{J_{z,\text{FM}}}{j_2} \right|. \quad (2)$$

Because of the different initial conditions for the two effects, there is no simple relation between λ_{IEE} and q_{EE} . The present work considers only λ_{IEE} .

III. IRREVERSIBLE THERMODYNAMICS AT SURFACES

Onsager's irreversible thermodynamics is primarily known for using flux-flux correlation functions to obtain transport or decay coefficients, and for using correlation function symmetry properties to obtain the symmetry properties of decay or transport coefficients.[12–14]

However, more basic and less technical than these early forms of the fluctuation-dissipation theorem, Onsager's methods show that the flux of a conserved *extensive* thermodynamic quantity is proportional to the *gradient* of the associated *intensive* quantity, leading to diffusion. They also show that, for a non-conserved extensive thermodynamic quantity, its source is proportional to the associated intensive quantity, which leads to decay.

The corresponding transport coefficients and source decay coefficients are subject to a sign constraint arising from the non-negative nature of the rate of entropy production. For systems not far from equilibrium, this gives that the entropy flux (heat flux/temperature) is proportional to the gradient of temperature, and flows from hot to cold. Further, charge flux $\vec{j}$ of electrons is proportional to the gradient of the scalar electrochemical potential $\tilde{\mu} = \mu - eV$, with μ is the chemical potential and V is the voltage; in terms of number density the number flux flows from high $\tilde{\mu}$ to low $\tilde{\mu}$. Likewise, spin flux $\vec{J}_i$ is proportional to the gradient of the spin vector potential $\vec{\mu}$, and flows from high $\vec{\mu}$ to low $\vec{\mu}$.

Thus Onsager's theory provides the theoretical basis for what might otherwise be considered the mere phenomenological facts that the heat current is proportional to the temperature gradient, and that the electric current is proportional to the voltage gradient. However, it also constrains the signs of the diagonal coefficients, and constrains the magnitudes of the off-diagonal coefficients (as in thermoelectricity).

Johnson and Silsbee gave what we believe to be the first irreversible thermodynamics discussion of transport across interfaces, of heat, charge, and spin.[16] More recently, the non-conserved charge-weighted electron-hole number flux at a semiconductor-vacuum interface, first described with Shockley's recombination velocity, has been placed within the framework of irreversible thermodynamics.[25 and 26]

The generalization to flow across an interface is as follows. The entropy flux is proportional to the *difference* in temperatures, and flows from hot to cold. The coefficient of proportionality is known as the thermal boundary resistance, or the Kapitza resistance. Further, charge flow across an interface is proportional to the difference in the $\tilde{\mu}$, with coefficient of proportionality known as the electrical boundary resistance. Likewise, we will show that spin flux $\vec{J}_i$ across an interface is proportional to the differences in the $\vec{\mu}$'s, with the coefficient of proportionality amounting to a spin current boundary resistance.

IV. SPIN ACCUMULATION EFFECTIVE FIELD AND SPIN CHEMICAL POTENTIAL

We now relate the spin accumulation effective field $\vec{h}^*$ to the spin chemical potential $\vec{\mu}$. We take the electron to have effective mass m_e , and g-factor $g \approx 2$, so the intrinsic electron magnetic moment (in A-m²) is

$$|\mu_e| = g\mu_B \approx \frac{e\hbar}{m_e}, \quad \mu_B \equiv \frac{e\hbar}{2m_e}. \quad (3)$$

Let $\vec{n}_2$ be the excess per unit area of occupied spin states, with $n_2 \equiv |\vec{n}_2|$, and let $\tilde{n}_2$ be the 2d number density. (For a fully polarized system $n_2 = \tilde{n}_2$.) Then $\vec{m}_2$ and $\vec{n}_2$ are related by

$$\vec{m}_2 = -|\mu_e|\vec{n}_2 = -\frac{e\hbar}{m_e}\vec{n}_2. \quad (4)$$

Now note that the 2d magnetic energy density ε_2 (in J/m²=T·A) can be written either as:

- (a) $-\vec{m}_2 \cdot (\mu_0 \vec{h}^*)$, where $\vec{m}_2$ is in A (not A/m as in 3d) and $\vec{h}^*$ is in A/m, or;
- (b) $\vec{\mu} \cdot \vec{n}_2$, where $\vec{\mu}$ is in J and the 2d-vector polarization density $\vec{n}_2$ is in 1/m². Thus

$$\varepsilon_2 = -\vec{m}_2 \cdot (\mu_0 \vec{h}^*) \equiv \vec{\mu} \cdot \vec{n}_2. \quad (5)$$

Thus $\vec{m}_2$ flows from low to high $\vec{h}^*$, preferring large $\vec{h}^*$, because of the minus sign in (5). However, $\vec{n}_2$ and the associated spin density flow from high to low $\vec{\mu}$.

Comparison of (4) and (5) relates $\vec{\mu}$ and $\vec{h}^*$ by

$$\vec{\mu} = \frac{e\hbar}{m_e}(\mu_0 \vec{h}^*) = 2\mu_B(\mu_0 \vec{h}^*). \quad (6)$$

Hence $\vec{m}_2$ and $\vec{h}^*$ indeed are equivalent to $\vec{n}_2$ and $\vec{\mu}$.

By (3), per electron, the magnetic moment has magnitude $\mu_e = 2\mu_B = e\hbar/m_e$. Since the spin angular momentum has magnitude $|\vec{s}| = \hbar/2$, their ratio is $\mu_e/|\vec{s}| = 2e/m_e$. Hence the spin accumulation and spin density, and spin accumulation flux $\vec{J}_{mi}$ and spin flux $\vec{J}_i$, are in that ratio:

$$\vec{m}_2 = -\frac{2e}{m_e}\vec{s}_2, \quad \vec{J}_{mi} = -\frac{2e}{m_e}\vec{J}_i. \quad (7)$$

Use of local states having $\vec{s} = (\hbar/2)\vec{\sigma}$ in a single band with 2d density $\tilde{n}_2$ gives a result to be employed shortly:

$$\vec{m}_2 = -\frac{e\hbar}{m_e}\tilde{n}_2\langle\vec{\sigma}\rangle, \quad (8)$$

V. FOR STRONG SPIN-ORBIT TI'S, SPIN ACCUMULATION IMPLIES CHARGE FLUX AND VICE-VERSA

We assume that the topological insulator has a Hamiltonian that favors electron spin $\vec{s}$ normal to its wavevector $\vec{k}$. Edelstein[1] considers *weak* spin-orbit coupling characterized by the Rashba parameter α . With

wavevector $\vec{k} \equiv \vec{p}/\hbar$, plane normal $\hat{z}$, and Pauli spin operator $\vec{\sigma}$ the Rashba spin-orbit interaction Hamiltonian is[27]

$$H_{SO} = \alpha(\vec{k} \times \hat{z}) \cdot \vec{\sigma}, \quad (9)$$

where α is in units of J·m. For $\alpha > 0$ this favors $\vec{\sigma} \times \vec{k}$ along $-\hat{z}$, as in Fig. 1 of Ref. [11] and in our Fig. 1).

Further, following Ref. [11] we consider *strong* spin-orbit coupling, so that saturation of the effect occurs. Then there are two bands labeled by helicity (rather than spin), where as noted $\alpha > 0$ gives $\vec{\sigma} \times \vec{k}$ along $-\hat{z}$. In a topological insulator there need be no spin-orbit coupling; in that case we show that $\alpha = v_F/\hbar$.

Now take $\tilde{n}_2$ as the *total* 2d number density of a single band; thus $\tilde{n}_2 \neq |\vec{n}_2|$. Also, assume low enough temperature that only the lower energy band is occupied. Then

$$\tilde{n}_2 = \frac{1}{(2\pi)^2}(\pi k_F^2) = \frac{k_F^2}{4\pi}. \quad (10)$$

With 2d wavevector k_F the energy band splitting is $2\alpha k_F$.

The 2d charge flux $\vec{j}_2$ (which we want to be along $\hat{y}$) is the product of the charge $-e$, $\tilde{n}_2$, and the average of the velocity operator, which here is

$$\vec{v} = \frac{\partial H_{SO}}{\partial \vec{p}} = \frac{1}{\hbar} \frac{\partial H_{SO}}{\partial \vec{k}} = \frac{\alpha}{\hbar} \hat{z} \times \vec{\sigma}. \quad (11)$$

As a consequence,

$$v_F = \frac{\alpha}{\hbar}, \quad (12)$$

which is independent of momentum $\vec{p}$, as for a relativistic particle. Also, the excited state energies have the form $\varepsilon = v_F p$, just as for relativistic particles. Dispersion of this form can occur for either a topological insulator or for a system with quadratic dispersion and effective mass m_e , that is dominated by the linear dispersion. This will occur for $v_F k_F \gg (\hbar k_F)^2/m_e$, or $\alpha \gg (\hbar)^2 k_F/m_e$.

By (8), (10), (11), and (12),

$$\vec{j}_2 = -e\tilde{n}_2\langle\vec{v}\rangle = -e\tilde{n}_2 v_F \hat{z} \times \langle\vec{\sigma}\rangle = \frac{\alpha}{m_e} \hat{z} \times \vec{m}_2. \quad (13)$$

Figs. 1 and 2 are consistent with (13).

From (13) it follows that

$$\vec{m}_2 = -\frac{m_e}{\alpha} \hat{z} \times \vec{j}_2. \quad (14)$$

Eq. (13) implies, for a 2d topological insulator with normal $\hat{z}$, that any 2d spin accumulation $\vec{m}_2$ and 2d charge flux $\vec{j}_2$ have proportional magnitudes, that both are in the plane, and that they are normal to one another. Correspondingly, Eq. (14) implies that a 2d charge flux $\vec{j}_2$ is associated with a 2d spin accumulation.

For an incident spin flux with spin accumulation along the flow direction $\hat{z}$ it is possible to magnetize the TI along $\hat{z}$. However, each of the 2d carriers in the TI accomplishes this by magnetizing slightly along $\hat{z}$ in the

same way, with no correlation to their momentum. As a consequence, none of the 2d states develops an additional charge flux. Therefore, for this spin polarization we expect no charge flux in the TI.[11]

VI. A 3D SPIN FLUX CAN DRIVE A 2D SPIN ACCUMULATION AND VICE-VERSA

In the experiment of Ref. [7] a 3d spin (i.e. spin accumulation) flux flows downward into the TI. This spin flux, which we write as $\vec{J}_z$, might be due, for example, to spin-pumping or to the spin Seebeck effect.[9] To relate flux of spin ($\hbar/2$ per electron) to flux of spin accumulation (a magnetic moment $-e\hbar/m_e$ per electron), we employ (7).

With the 3d spin accumulation $\vec{m}$ subject to an effective field $\vec{h}^*$ and gyromagnetic ratio $-\gamma$ with $\gamma \approx e/m_e > 0$, $\vec{m}$ satisfies[20 and 22]

$$\partial_t \vec{m} + \partial_i \vec{J}_{mi} = -\gamma \vec{m} \times \vec{h}^* - \frac{\vec{m}}{\tau_m}. \quad (15)$$

Here τ_m is the spin accumulation relaxation time. Both $\vec{m}$ and $\vec{h}^*$ are non-equilibrium terms and thus in the linear approximation to disequilibrium their product may be neglected. We now apply this to the TI, with spin accumulation flux $\vec{J}_{mz}$ at $z = a$ and zero at $z = 0$.

We now integrate (15) over the TI thickness along z (at this point we employ a quasi-2d picture), and apply the above boundary conditions for the TI. With $\tau_m \equiv \tau_{TI}$, (15) becomes

$$\partial_t \vec{m}_2 = -\vec{J}_{mz} - \frac{\vec{m}_2}{\tau_{TI}}. \quad (16)$$

In the steady state, use of (7) yields

$$\vec{0} = \frac{2e}{m_e} \vec{J}_z - \frac{\vec{m}_2}{\tau_{TI}}. \quad (17)$$

Both magnetic vectors $\vec{m}_2$ and $\vec{h}^*$ are zero in equilibrium. Taking them to be proportional, and using $\vec{h}_2^*$ for $\vec{h}^*$, yields

$$\vec{h}_2^* = \frac{\vec{m}_2}{\chi_2}, \quad (18)$$

where the 2d spin accumulation susceptibility χ_2 has units of length (m). Hence the spin precession term $\vec{m}_2 \times \vec{h}_2^*$ is not merely small, it is zero.

Thus (17) gives the result that, for a 3d spin flux $\vec{J}_z$ only at the top 2d surface $z = a$,

$$\vec{m}_2 = \frac{2e\tau_{TI}}{m_e} \vec{J}_z. \quad (19)$$

Because of the assumed strong spin-orbit interaction, the TI lattice provides $\vec{m}_2$ at a constant rate (an inverse Einstein de-Haas effect), which leaves by a spin flux from the top surface.

Associated with the 2d spin accumulation $\vec{m}_2$ is a rate of heat production per area of $\mu_0 \vec{h}_2^* \cdot (\vec{m}_2/\tau_{TI})$. When integrated over the full cross-sectional area A , this gives a total rate of heat production

$$\frac{dQ_{m2}}{dt} = \frac{\mu_0 \vec{m}_2}{\chi_2} \cdot \frac{\vec{m}_2}{\tau_{TI}} A = \frac{\mu_0 |\vec{m}_2|^2}{\chi_2 \tau_{TI}} A. \quad (20)$$

In 3d one has $|\vec{m}|^2/\tau_{TI}$, with the same dimensionality as $|\vec{m}_2/\chi_2|^2(\chi_2/\tau_{TI})$. The term $(\chi_2 \tau_{TI})^{-1}$ has dimensionality of velocity, as expected for a surface rate.

VII. FOR TI'S AN ENTERING OR LEAVING 3D SPIN FLUX IMPLIES A 2D CHARGE FLUX AND VICE-VERSA

In this section we assume that a spin flux normal to the plane of the topological insulator can enter or leave it. A later section applies irreversible thermodynamics to relate such a spin current to a difference in (vector) spin chemical potentials.

Use of (13) and (19) gives

$$\vec{J}_2 = \frac{\alpha}{m_e} \hat{z} \times \vec{m}_2 = 2 \frac{e\alpha\tau_{TI}}{\hbar m_e} \hat{z} \times \vec{J}_z. \quad (21)$$

Applying (21) to the IEE, as in Fig. 2, the spin part (vector) of the source $\vec{J}_z$ is along $\hat{x}$, but the flow direction has a negative sign, so (21) gives $\vec{J}_2$ along $-\hat{y}$.

Correspondingly, one may think of the 2d charge flux $\vec{J}_2$ as the source, which by (21) yields a 3d spin accumulation flux

$$\vec{J}_z = \frac{\hbar m_e}{2e\alpha\tau_{TI}} \vec{J}_2 \times \hat{z}. \quad (22)$$

Applying (22) to the EE, as in Fig. 1, the source $\vec{J}_2$ is along $\hat{y}$, with accompanying spin accumulation $\vec{m}_2$ along $\hat{x}$. Since $\vec{J}_z$ must flow upward (positive flow direction), (22) gives $\vec{m}$ along $\hat{y}$.

Eq. (22) implies that the ratio of the spin flux at the TI/NM interface to the charge flux in the TI is the same whether the spin flux enters or leaves the TI. By (1) it gives the length

$$\lambda_{IEE} = \frac{\hbar |\vec{J}_2|}{2e |\vec{J}_z|} = \frac{\alpha\tau_{TI}}{m_e}. \quad (23)$$

With $v_F = \alpha/\hbar$, comparison with (ZF.13), or $\lambda_{IEE} = v_F \tau_p$, gives $\tau_{TI} = \tau_p$. That is, Zhang and Fert's relaxation time τ_p , associated with electron scattering in the TI, is the same as our τ_{TI} .

VIII. SPIN CHEMICAL POTENTIALS ARE PROPORTIONAL TO 3D SPIN FLUX

We now rephrase the above results in terms of spin chemical potentials. Recall that in irreversible thermodynamics the difference in spin chemical potentials across

the interface drives the spin accumulation flux across the interface. For clarity we use $\vec{\mu}_{\text{TI}} \equiv \vec{\mu}_2$ and $\vec{\mu}_{\text{NM}} \equiv \vec{\mu}_3$.

In SI units, a 3d spin accumulation satisfies

$$\vec{m}_3 = \chi_3 \vec{h}_3^*, \quad (24)$$

where χ_3 is dimensionless in SI units.

By (18) and (6), it follows that

$$\vec{m}_2 = \chi_2 \vec{h}_2^* = \frac{\chi_2}{2\mu_0\mu_B} \vec{\mu}_{\text{TI}}. \quad (25)$$

Here $\vec{\mu}_{\text{TI}}$ refers to the TI/NM interface. The coefficient of $\vec{\mu}_{\text{TI}}$ has units of A/J=1/Wb, or inverse flux. Combining (25) with (19) relates the spin flux (entering or leaving the TI) to the spin potential at the top of the TI:

$$\vec{J}_z = \frac{m_e}{4e} \frac{\chi_2}{\mu_0\mu_B\tau_{\text{TI}}} \vec{\mu}_{\text{TI}}. \quad (26)$$

Thus an excess $\vec{\mu}_{\text{TI}}$ drives a 3d spin flux out of the TI.

IX. IRREVERSIBLE THERMODYNAMICS AT AN INTERFACE

We now apply Onsager's irreversible thermodynamics to the interface. The spin flux across the interface is driven by the difference in spin chemical potentials,[8] or

$$\vec{J}_z = \Sigma_s (\vec{\mu}_{\text{NM}} - \vec{\mu}_{\text{TI}}). \quad (27)$$

The surface spin flux conductivity Σ_s has units of (J/m²)/(A/m)=J/(A·m). Just as the flux of scalar number n goes from high to low scalar chemical potential energy μ , so the flux of vector number $\vec{n}$ and spin $\vec{s}$ and fluxes go from high to low vector chemical potential energy $\vec{\mu}$.

From the standpoint of units, comparison of (26) and (27) enables us to introduce an interface relaxation time τ_I via

$$\Sigma_s \equiv \frac{m_e}{e} \frac{\chi_2}{\mu_0\mu_B\tau_I}. \quad (28)$$

Then (27) and (28) give

$$\vec{J}_z = \frac{m_e}{e} \frac{\chi_2}{\mu_0\mu_B\tau_I} (\vec{\mu}_{\text{NM}} - \vec{\mu}_{\text{TI}}). \quad (29)$$

By (26) one finds, successively,

$$\frac{1}{4\tau_{\text{TI}}} \vec{\mu}_{\text{TI}} = \frac{1}{\tau_I} (\vec{\mu}_{\text{NM}} - \vec{\mu}_{\text{TI}}), \quad (30)$$

$$(4\tau_{\text{TI}} + \tau_I) \vec{\mu}_{\text{TI}} = 4\tau_{\text{TI}} \vec{\mu}_{\text{NM}}, \quad (31)$$

$$\vec{J}_z = \frac{m_e}{e} \frac{\chi_2}{\mu_0\mu_B(4\tau_{\text{TI}} + \tau_I)} \vec{\mu}_{\text{NM}}. \quad (32)$$

For spin chemical potential we employ units of chemical potential energy, whereas Zhang and Fert[11] use voltage units. Then (32) has the same structure as (12) of Zhang and Fert, where their $e\vec{\mu}$ is our $\vec{\mu}_{\text{NM}}$ and their τ_p is our τ_{TI} , and our τ_I (interface) is their τ_t (tunneling).

If $\vec{J}_z$ is known, then $\vec{\mu}_{\text{TI}}$ is given by (26), and $\vec{\mu}_{\text{NM}}$ by (32). Hence all of the various quantities appearing in the theory can be determined.

The corresponding rate of heating is as follows. Charge flow is driven across an interface by the voltage difference ΔV , so the associated heating rate per unit area, given by their product, has units of (A/m²)·(J/C)=J/s·m². Correspondingly, a 3d spin accumulation flux $(-2e/m_e)\vec{J}_z$ across an interface is driven by the thermodynamic "force" $\Delta\vec{h}^* = \Delta\vec{\mu}/(2\mu_0\mu_B)$.

The associated heating rate per unit area is given by their product, and again has units of (A/m²)·(J/C)=J/(s·m²). When integrated over the full cross-sectional area A , the net rate of heating can be determined.

Using (29) to replace $|\vec{\mu}_{\text{NM}} - \vec{\mu}_{\text{TI}}|$ by $|\vec{J}_z|$ gives

$$\frac{dQ_{J2}}{dt} = \frac{2e}{m_e} |\vec{J}_z| \frac{|\vec{\mu}_{\text{NM}} - \vec{\mu}_{\text{TI}}|}{\mu_B} A = |\vec{\mu}_{\text{NM}} - \vec{\mu}_{\text{TI}}|^2 \frac{\chi_2 A}{\mu_0\mu_B^2\tau_I}. \quad (33)$$

As expected, the relaxation time τ_I is in the denominator: there is a rate of heating driven by a difference in spin chemical potentials. For the rate of entropy production per unit area one takes (33) divided by the temperature T .

X. SUMMARY AND DISCUSSION

The present work applied Onsager's irreversible thermodynamics to obtain a macroscopic description of the Edelstein Effect – where a 3d spin flux incident on a topological insulator produces a 2d charge flux and a 2d spin accumulation – and of the Inverse Edelstein Effect.

We considered two heating rates proportional to the area of the TI: one from decay of its 2d spin accumulation $\vec{m}_2$, given by (20); and one from transport of the 3d spin flux across the TI/NM interface, given by (33). Each process has its own relaxation time.

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* waslaw@tamu.edu

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Appendix: Ferromagnet in Resonance as Generator

The spin current in the inverse Edelstein effect (IEE) may be produced by ferromagnetic resonance (FMR) at frequency f of an FM adjacent to the NM. This is depicted in Fig. 2.

This Appendix discusses spin-pumping with irreversible thermodynamics, where in the resonance-driven ferromagnet, the spin chemical potential $\vec{\mu}_{\text{FM}}$ is the driving term. We consider a thin layer of FM, with nearly uniform magnetic response.

Fig. 2 takes the FM equilibrium magnetization $\vec{M}$ along $\hat{x}$. In FMR the ac FM magnetization $d\vec{M}$ is nor-

mal to $\hat{x}$; on transmission into the NM this gives an ac spin accumulation $\vec{m}$ and ac spin current polarized normal to $\hat{x}$ (not depicted). To second order in $d\vec{M}$ the dc magnetization of the FM decreases, causing dc spin accumulation (excitations) magnetically polarized along $-\hat{x}$ to flow from FM to NM. This is equivalent to a dc spin accumulation along $\hat{x}$ flowing to FM from NM.

In the language of spin-pumping, and adjusting for units, eq.(14) of Ref. [11] has the same mathematical structure as[29]

$$\vec{J}_z = g_I(\frac{\hbar}{2}\hat{M} \times \partial_t \hat{M} - \vec{\mu}_{\text{NM}}), \quad (\text{A.1})$$

where g_I is an interface conductance in units of m^{-2} and the time-average gives a dc flux. The first term is driven from the FM and the second is backflow into the FM.

In the language of irreversible thermodynamics we use the spin chemical potential $\vec{\mu}$ rather than the first term in the parenthesis of (A.1). (Some previous works use $\vec{h}^*$ rather than $\vec{\mu} = \mu_B \mu_0 \vec{h}^*$ [8, 15, 16, and 20].) One then

has

$$\vec{J}_z = g_I(\vec{\mu}_{\text{FM}} - \vec{\mu}_{\text{NM}}). \quad (\text{A.2})$$

We now consider the dc $\vec{\mu}_{\text{FM}}$.

With $h_{\text{rf},0}$ the maximum rf field amplitude, the maximum magnetization tipping of $\hat{M}$ in the FM is the first-order term

$$|d\hat{M}|_0 = \frac{\gamma\mu_0|h_{\text{rf},0}|}{\omega}. \quad (\text{A.3})$$

The time-average decrease in M is the second-order term

$$dM = -\frac{M}{2}|d\hat{M}_0|^2. \quad (\text{A.4})$$

Then, for the FM one has[20]

$$\vec{\mu}_{\text{FM}} = -\frac{\gamma\hbar}{2}\mu_0 \frac{dM}{\chi_{||}}\hat{M}. \quad (\text{A.5})$$

As noted, this depletion causes dc magnetization along $-\hat{M}$ to flow from the FM into the NM. After passing through the NM (with some absorption), by (27) this spin accumulation flux then is driven into the TI.