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Role of $f(G)$ Gravity in the Study of Non-Static Complex Systems

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Abstract

The concept of complexity for dynamical spherically symmetric dissipative self-gravitating configuration [1] is generalized in the scenario of modified Gauss-Bonnet gravity. For this purpose, a spherically symmetric fluid with locally anisotropic, dissipative, and non-dissipative configuration is considered. We choose the same complexity factor for the structure as we did for the static case, while we consider the homologous condition for the simplest pattern of evolution. In this approach, we formulate structure scalars that demonstrate the essential properties of the system. A fluid distribution that fulfills the vanishing complexity constraint and proceeds homologously corresponds to isotropic, geodesic, homogeneous, and shear-free fluid. In the dissipative case, the fluid is still geodesic but it is shearing, and there is a wide range of solutions. In the last, the stability of vanishing complexity is examined.

Keywords: Self-gravitating system; Anisotropy; Hydrodynamics.

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1 Introduction

There are many research articles in which the definition of complexity has been proposed by using different approaches [2–7]. Lopez-Ruiz and collaborators [8,9] have established this prior work. In terms of general relativity, a new definition of complexity for various self-gravitating fluids [1, 10, 11] was established a few years ago. The conception of this new work of complexity is extended to modified theories [12–19]. According to a new definition of complexity, a fluid configuration that is spherically symmetric and static is based on the idea that minimal complex systems resemble the isotropic and homogeneous fluid distribution. As a result, we set the complexity factor to zero for such a type of distribution. Keeping previous ideas in mind, complexity is defined by the contribution of local pressure anisotropy, inhomogeneity, and energy density. These terms are combined in the scalar function Y_{TF} , which is obtained from the orthogonal splitting of the Riemann tensor and used to analyze the degree of complexity.

It is important to mention here that the complexity factor defined in this way vanishes not only for above mentioned simple configuration, but also when two terms anisotropic pressure and density inhomogeneity appears in its definition and cancel each other. As a result, vanishing complexity could refer to a variety of systems [8]. Lloyd and Pagels [20] analyzed the complexity factor which is useful for all physical systems and also discussed self-gravitating structures in an apparent state. The complexity of non-linear dynamical systems was introduced by Crutchfield and Young [21]. They presented a method for calculating the complexity of a system using entropy and dimension, and also a mechanism for recreating the basic equations of a system. Shamir et. al., [22] examined wormhole solutions in the presence of modified theories by using noncommutative spacetime. Malik [23] and his collaborators discussed the Noether symmetry approach with Bianchi Type 1 geometry in $f(R, \varphi, \chi)$ theory.

Herrera et al. [24] investigated the effects of density inhomogeneity and local anisotropy on the spherical collapse of matter distribution. In terms of structure scalars, Herrera et al. [25] proposed all possible solutions of Einstein field equations for static spherically symmetric cases. Yousaf and Bhatti [26], Yousaf [13, 27, 28] also Yousaf et al. [29] explored these results in modified theories for different cosmos models. For static fluid distributions, the complexity factor is described [10] and also it is defined for dynamical self-gravitating configurations [1]. In this article, we extend this work of the

complexity factor for modified theories such as modified Gauss-Bonnet (GB) gravity.

We have to solve two different problems for time dependent system. On the one hand, we take the simplest system and analyze the pattern of evolution for complexity, and on the other hand, we extend the definition of complexity of the system for dissipative fluids which are dependent on time. Here we assume two possible cases of evaluation for the simplest pattern which are homologous evolution and homogeneous expansion. A geodesic fluid is formed by imposing $Y_{TF} = 0$ and homologous evolution. Furthermore, we say a fluid is shear-free with isotropic pressure and homogeneous energy density if it is time-dependent and non-dissipative. Also, we can establish a unique solution with the lowest complexity factor when homologous constraints and homogeneous expansion interact. For the general dissipative condition, the fluid is shearing, and there are numerous solutions. Finally, we concentrate on stability based on vanishing complexity conditions and physical variables that can produce deviations.

The following is a summary of the paper's structure. The formalism, general equations, and variables essential for our discussion are presented in the next section. The complexity factor is established in part III, and the simplest pattern of evolution (homologous and homogeneous expansion circumstances) is described in section IV. In sections V and VI, the fundamental implications of the vanishing complexity constraints and homogeneous evolution are examined. In section VII, the stability of the vanishing complexity factor is explored, and in the final section, all of the acquired results and remaining issues are presented. Lastly, the appendix include some important equations.

2 Field Equations of $f(\mathbb{G})$ Theory

In this section, we derive field equation for modified GB gravity which one of the modified gravity theories introduced in the literature. In order to see details about modified gravity please see [30–42]. The GB theory was firstly introduced by [43]. The action for $f(\mathbb{G})$ gravity is defined as [44, 45]

$$S = \int \left(l_m + \frac{f(\mathbb{G}) + R}{k} \right) \sqrt{-g} d^4x. \quad (1)$$

Here the Lagrangian density of matter distribution, the coupling constant and the determinant of the metric tensor are expressed with l_m , k and g , respectively. The field equations for $f(\mathbb{G})$ gravity are derived from the variation of this action with respect to the metric tensor as

$$R_{\gamma\nu} + \frac{1}{2}g_{\gamma\nu}R = 8\pi T_{\gamma\nu}, \quad \gamma, \nu = 0, 1, 2, 3, \quad (2)$$

where $T_{\gamma\nu}$ is the energy-momentum tensor (EMT) whose expression is given as

$$T_{\gamma\nu} = T_{\gamma\nu}^{(m)} + T_{\gamma\nu}^{(\mathbb{G})}, \quad (3)$$

here matter part of EMT is described as $T_{\gamma\nu}^{(m)}$ and modified part due to effective terms of GB gravity is expressed with $T_{\gamma\nu}^{(\mathbb{G})}$. The matter part of EMT for dynamical spherically symmetric fluid distribution is given by

$$T_{\gamma\nu}^{(m)} = (\rho + P_{\perp})V_{\gamma}V_{\nu} + P_{\perp}g_{\gamma\nu} + (P_r - P_{\perp})\chi_{\gamma}\chi_{\nu} + q_{\gamma}V_{\nu} + V_{\gamma}q_{\nu}, \quad (4)$$

where ρ , $P_{\perp}$, P_r , q^{γ} , V^{γ} and χ^{γ} represent energy density, tangential pressure, radial pressure, heat flux, four velocity and four vector, respectively. Under co-moving condition, these vectors obey

$$V^{\gamma}V_{\gamma} = -1, \quad V^{\gamma}q_{\gamma} = 0, \quad \chi^{\gamma}\chi_{\gamma} = 1, \quad \chi^{\gamma}V_{\gamma} = 0. \quad (5)$$

We can also express the matter EMT in this form

$$T_{\gamma\nu}^{(m)} = \rho V_{\gamma}V_{\nu} + \Pi_{\gamma\nu} + Ph_{\gamma\nu} + q(V_{\gamma}\chi_{\nu} + \chi_{\gamma}V_{\nu}), \quad (6)$$

where

$$\begin{aligned} P &= \frac{1}{3}(P_r + 2P_{\perp}), \quad h_{\gamma\nu} = g_{\gamma\nu} + V_{\gamma}V_{\nu}, \\ \Pi_{\gamma\nu} &= \Pi(\chi_{\gamma}\chi_{\nu} - \frac{1}{3}h_{\gamma\nu}), \quad \Pi = -(P_{\perp} - P_r). \end{aligned} \quad (7)$$

As we know that we have assumed the spherically symmetric space-time. We further suppose that it is coupled with the locally anisotropic heat radiating matter content. Thus we describe the fluid under consideration given by the expression (4) where it has been explicitly assumed that there are only two principal stresses unequal (P_r , P_t). Accordingly, the anisotropic tensor only

depends on one scalar function (Π). We can define 4-velocity and 4-vector in components form as

$$V^\gamma = \left(\frac{1}{e^{\frac{\gamma}{2}}}, 0, 0, 0 \right), \quad \chi^\gamma = \left(0, \frac{1}{e^{\frac{\gamma}{2}}}, 0, 0 \right). \quad (8)$$

These components satisfy the following conditions

$$V^\gamma = C^{-1} \delta_0^\gamma, \quad q^\gamma = q A^{-1} \delta_1^\gamma, \quad \chi^\gamma = A^{-1} \delta_1^\gamma. \quad (9)$$

The expression of EMT for GB gravity is given as

$$\begin{aligned} T_{\gamma\nu}^{(\mathbb{G})} = & \frac{1}{k} \left[(4R_{\gamma\xi} R_\nu^\xi - 2R R_{\gamma\nu} - 2R_{\gamma\xi\delta\eta} R_\nu^{\xi\delta\eta} + 4R_{\gamma\xi\nu\eta} R^{\xi\eta}) f_{\mathbb{G}} \right. \\ & + \frac{1}{2} g_{\gamma\nu} f(\mathbb{G}) - 2R g_{\gamma\nu} \nabla^2 f_{\mathbb{G}} + 2R \nabla_\gamma \nabla_\nu f_{\mathbb{G}} - 4R_\gamma^\xi \nabla_\nu \nabla_\xi f_{\mathbb{G}} \\ & - 4R_\nu^\xi \nabla_\gamma \nabla_\xi f_{\mathbb{G}} + 4R_{\gamma\nu} \nabla^2 f_{\mathbb{G}} + 4g_{\gamma\nu} R^{\xi\eta} \nabla_\xi \nabla_\eta f_{\mathbb{G}} \\ & \left. - 4R_{\gamma\xi\nu\eta} \nabla^\xi \nabla^\eta f_{\mathbb{G}} \right]. \end{aligned} \quad (10)$$

Here, $f_{\mathbb{G}} = \frac{df(\mathbb{G})}{d\mathbb{G}}$ and $\nabla^2 = \nabla^\gamma \nabla_\gamma$ is d' Alembert operator and the line element for spherically symmetric dissipated matter configuration is given by

$$ds^2 = -C^2 dt^2 + A^2 dr^2 + B^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (11)$$

It is important to mention here that the shear viscosity could be consumed into the components (P_r and $P_\perp$) of the collapsing fluid, so specifically, it is not included in the system. Similarly, dissipation could be absorbed in ρ , q and P_r , so we do not specifically insert in the free streaming approximation. The Maxwell equations are formulated as

$$G_{\gamma\nu} = 8\pi T_{\gamma\nu}. \quad (12)$$

The components of the EMT (nonzero) are described as

$$T_{00}^{(m)} = \rho C^2, \quad T_{01}^{(m)} = -qCA, \quad T_{11}^{(m)} = P_r A^2, \quad T_{22}^{(m)} = T_{33}^{(m)} = P_\perp C^2. \quad (13)$$

By using Eqs.(2)-(13) the field equations are found as under

$$8\pi(\rho C^2 - T_{00}^{(\mathbb{G})}) = \left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \frac{\dot{B}}{B} - \left(\frac{C}{A} \right)^2 \left[2\frac{B''}{B} + \left(\frac{B'}{B} \right)^2 - 2\frac{A'B'}{AB} \right]$$

$$- \left(\frac{A}{B} \right)^2 \Big], \quad (14)$$

$$8\pi(T_{01}^{(\mathbb{G})} + qCA) = -2 \left(\frac{B'\dot{A}}{BA} + \frac{C'\dot{B}}{CB} - \frac{\dot{B}'}{B} \right), \quad (15)$$

$$8\pi(P_r A^2 - T_{11}^{(\mathbb{G})}) = \left(\frac{A}{C} \right)^2 \left[\left(2\frac{\dot{C}}{C} - \frac{\dot{B}}{B} \right) \frac{\dot{B}}{B} - 2\frac{\ddot{B}}{B} - \left(\frac{C}{B} \right)^2 \right] \\ + \left(2\frac{C'}{C} + \frac{B'}{B} \right) \frac{B'}{B}, \quad (16)$$

$$8\pi(P_\perp B^2 - T_{22}^{(\mathbb{G})}) = \left(\frac{B}{A} \right)^2 \left[\frac{C''}{C} + \frac{B''}{B} - \frac{C'}{C} \left(\frac{A'}{A} - \frac{B'}{B} \right) - \frac{A'B'}{AB} \right] \\ - \left(\frac{B}{C} \right)^2 \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\dot{C}}{C} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) + \frac{\dot{A}\dot{B}}{AB} \right], \quad (17)$$

here dot and prime represent the derivative with respect to t and r , respectively. Now we define the acceleration and expansion in 4D, which are denoted respectively by a_γ and Θ , as

$$a_\gamma = V_{\gamma;\nu} V^\nu, \quad \Theta = V_{;\gamma}^\gamma, \quad (18)$$

and $\sigma_{\gamma\nu}$ is its shear whose expression is given by

$$\sigma_{\gamma\nu} = V_{(\gamma;\nu)} + a_{(\gamma} V_{\nu)} - \frac{1}{3} \Theta h_{\gamma\nu}. \quad (19)$$

With the help of Eq.(9) and (18) we get the following form for acceleration

$$a_1 = \frac{C'}{C}, \quad a = \sqrt{a^\gamma a_\gamma} = \frac{C'}{CA}, \quad (20)$$

and expansion becomes as

$$\Theta = \frac{1}{C} \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right). \quad (21)$$

From Eq.(9) and (19) the nonzero components of shear are

$$\sigma_{11} = \frac{2}{3} A^2 \sigma, \quad \sigma_{22} = \frac{\sigma_{33}}{\sin^2 \theta} = -\frac{1}{3} B^2 \sigma, \quad (22)$$

and scalar of shear is expressed as

$$\sigma^{\gamma\nu} \sigma_{\gamma\nu} = \frac{2}{3} \sigma^2, \quad (23)$$

where

$$\sigma = \frac{1}{C} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right). \quad (24)$$

Equation (15) gets the following form by using Eqs.(21) and (24) as

$$4\pi A q^{(eff)} = \frac{1}{3}(\Theta - \sigma)' - \sigma \frac{B'}{B}. \quad (25)$$

Misner and Sharp [46] introduced the mass function, which can be represented as

$$m = \frac{B}{2} \left[1 + \left(\frac{\dot{B}}{C} \right)^2 - \left(\frac{B'}{A} \right)^2 \right]. \quad (26)$$

The proper time derivative D_T is introduced as

$$D_T = \frac{1}{C} \frac{\partial}{\partial t}, \quad U = D_T B < 0, \quad (27)$$

where U is defined as the velocity of collapsing fluid which is equal to the derivative of the areal radius with proper time. By using the velocity, mass function can be expressed as

$$E = \frac{C'}{A} = \left(1 + U^2 - \frac{2m}{B} \right)^{1/2}. \quad (28)$$

By putting the value of E in Eq.(25), it appears as

$$4\pi q^{(eff)} = E \left[\frac{1}{3} D_B (\Theta - \sigma) - \frac{\sigma}{B} \right], \quad (29)$$

here D_B is proper radial derivative with expression

$$D_B = \frac{1}{B'} \frac{\partial}{\partial r}. \quad (30)$$

Using field equations with proper time derivative and proper radial derivative, we get the expressions for mass as

$$D_T m = -4\pi (P_r^{(eff)} U + q^{(eff)} E) B^2, \quad (31)$$

$$D_B m = 4\pi \left(\rho^{(eff)} + \frac{q^{(eff)} U}{E} \right) B^2. \quad (32)$$

After some calculation we get the final equation for mass function as

$$\frac{3m}{B^3} = 4\pi \rho^{(eff)} - \frac{4\pi}{B^3} \int_0^r B^3 \left(D_B \rho^{(eff)} - 3q^{(eff)} \frac{U}{BE} \right) B' dr. \quad (33)$$

2.1 Structure Scalars

In this section we just recall the concept of structure scalars, these scalars are very helpful in examining the complexity factor, these scalars are formulated in GR by Herrera et al. [47]. Magnetic part of the Weyl tensor in a spherically symmetric case vanishes, so we discuss just about the electric part which is defined as

$$E_{\gamma\nu} = \mathcal{C}_{\gamma\lambda\nu\xi} V^\lambda V^\xi, \quad \lambda, \xi = 0, 1, 2, 3. \quad (34)$$

Nontrivial components of the above equation are given as

$$E_{11} = \frac{2}{3}A^2\mathcal{E}, \quad E_{22} = -\frac{1}{3}B^2\mathcal{E}, \quad E_{33} = E_{22}\sin^2\theta, \quad (35)$$

where

$$\mathcal{C}_{\gamma\nu\lambda\xi} = (g_{\gamma\nu\mu\sigma}g_{\lambda\xi\tau\chi} - \eta_{\gamma\nu\mu\sigma}\eta_{\lambda\xi\tau\chi})V^\mu V^\tau E^{\sigma\chi}, \quad \mu, \sigma, \tau, \chi = 0, 1, 2, 3, \quad (36)$$

here $\eta_{\gamma\nu\mu\sigma}$ shows the Levi-Civita tensor and $g_{\gamma\nu\mu\sigma} = g_{\gamma\mu}g_{\nu\sigma} - g_{\gamma\sigma}g_{\nu\mu}$. Also, $E^{\gamma\nu}$ is expressed as

$$E^{\gamma\nu} = \mathcal{E} \left(\frac{1}{3}h^{\gamma\nu} + \chi^\gamma\chi^\nu \right), \quad (37)$$

where

$$\begin{aligned} \mathcal{E} &= \frac{1}{2C^2} \left[\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} - \left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right) \left(\frac{\dot{C}}{C} + \frac{\dot{B}}{B} \right) \right] - \frac{1}{2B^2} \\ &+ \frac{1}{2A^2} \left[\frac{C''}{C} - \frac{B''}{B} + \left(\frac{A'}{A} + \frac{B'}{B} \right) \left(\frac{B'}{B} - \frac{C'}{C} \right) \right]. \end{aligned} \quad (38)$$

As the scalar function X_{TF} and Y_{TF} comes from the orthogonal splitting of the Riemann tensor, so let us defines tensor as follows

$$Y_{\gamma\nu} = R_{\gamma\lambda\nu\xi} u^\lambda u^\xi, \quad (39)$$

$$Z_{\gamma\nu} = {}^*R_{\gamma\lambda\nu\xi} u^\lambda u^\xi = \frac{1}{2}\eta_{\gamma\lambda\alpha\nu}R_{\nu\xi}^{\alpha\lambda} u^\lambda u^\xi, \quad (40)$$

$$X_{\gamma\nu} = {}^*R_{\gamma\lambda\nu\xi}^* u^\lambda u^\xi = \frac{1}{2}\eta_{\gamma\nu}^{\alpha\lambda}R_{\alpha\lambda\xi}^* u^\lambda u^\xi, \quad (41)$$

where $R_{\gamma\nu\lambda\xi}^* = \frac{1}{2}\eta_{\alpha\nu\lambda\xi}R_{\gamma\nu}^{\alpha\lambda}$. With the help of above results these tensors $X_{\gamma\nu}$, $Y_{\gamma\nu}$ and $Z_{\gamma\nu}$ are examined as,

$$X_{\gamma\nu} = \frac{1}{3}X_T h_{\gamma\nu} + X_{TF} \left(\chi_\gamma\chi_\nu - \frac{1}{3}h_{\gamma\nu} \right), \quad (42)$$

$$Y_{\gamma\nu} = \frac{1}{3}Y_T h_{\gamma\nu} + Y_{TF} \left(\chi_\gamma \chi_\nu - \frac{1}{3}h_{\gamma\nu} \right), \quad (43)$$

$$Z_{\gamma\nu} = \frac{1}{3}Z_T h_{\gamma\nu} + Z_{TF} \left(\chi_\gamma \chi_\nu - \frac{1}{3}h_{\gamma\nu} \right). \quad (44)$$

Trace and trace-free parts for these scalar functions are calculated as

$$X_T = 8\pi\rho^{(eff)}, \quad (45)$$

$$X_{TF} = -4\pi\Pi^{(eff)} - \epsilon, \quad (46)$$

$$Y_T = 4\pi(\rho^{(eff)} + 3P_r^{(eff)} - 2\Pi^{(eff)}), \quad (47)$$

and

$$Y_{TF} = \epsilon - 4\pi\Pi^{(eff)}. \quad (48)$$

As we known that we have assumed the spherically symmetric space-time. Therefore, the magnetic part of the Weyl tensor turns out to be zero. Thus we have calculated only the electric part of the Weyl tensor and thus the scalar Y_{TF} from it. By using Eq.(26), (38) and field equations, we attain mass function as

$$\frac{3m}{B^3} = 4\pi(\rho^{(eff)} - \Pi^{(eff)}) - \epsilon. \quad (49)$$

Equations (33) and (48) with the help of above equation gives the value for scalar function Y_{TF} as

$$Y_{TF} = -8\pi\Pi^{(eff)} + \frac{4\pi}{B^3} \int_0^r B^3 \left(D_B \rho^{(eff)} - 3q^{(eff)} \frac{U}{BE} \right) B' dr. \quad (50)$$

X_{TF} also gets the following form

$$X_{TF} = -\frac{4\pi}{B^3} \int_0^r B^3 \left(D_B \rho^{(eff)} - 3q^{(eff)} \frac{U}{BE} \right) B' dr. \quad (51)$$

For the scalar function and inhomogeneity energy density, a differential equation could be expressed as [48]

$$(X_{TF} + 4\pi\rho^{(eff)})' = -X_{TF} \frac{3B'}{B} + 4\pi q^{(eff)} A(\Theta - \sigma). \quad (52)$$

For the non-dissipative case, above equation gets the following form

$$X_{TF} = 0 \Leftrightarrow (\rho^{(eff)})' = 0. \quad (53)$$

For general dissipative case Eq.(52) becomes as

$$X_{TF} = 0 \Leftrightarrow (\rho^{(eff)})' = q^{(eff)} A(\Theta - \sigma) = q^{(eff)} A \frac{3\dot{B}}{B}. \quad (54)$$

This results describes the importance of X_{TF} in the study of energy density inhomogeneity of the system.

2.2 Exterior Spacetime and Junction Conditions

By considering bounded fluid distributions, the Darmois conditions must have to satisfy by the system. We suppose the Vaidya line element for outside boundary, whose expression is given as

$$ds^2 = - \left[1 - \frac{2M(v)}{r} \right] dv^2 - 2drdv + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (55)$$

where v and $M(v)$ denotes the retarded time and total mass, respectively. On the surface $r_\Sigma = r = \text{constant}$, by combining the complete non-adiabatic sphere and Vaidya space-time, demands

$$m(t, r) \approx M(v), \quad (56)$$

and

$$\begin{aligned} 2 \left(\frac{\dot{B}'}{B} - \frac{\dot{A}B'}{AB} - \frac{\dot{B}C'}{BC} \right) &\approx -\frac{A}{C} \left[2 \frac{\ddot{B}}{B} - \left(2 \frac{\dot{C}}{C} - \frac{\dot{B}}{B} \right) \frac{\dot{B}}{B} \right] \\ &+ \frac{C}{A} \left[\left(2 \frac{C'}{C} + \frac{B'}{B} \right) \frac{B'}{B} - \left(\frac{A}{B} \right)^2 \right]. \end{aligned} \quad (57)$$

On the comparison Eqs.(57) with Eqs.(15) and (16), we attains

$$q^{(eff)} \approx P_r^{(eff)}. \quad (58)$$

Hence the matching of Eqs.(11) and (55) on the boundary implies (56) and (58).

3 The Complexity Factor

In this article, we discuss two additional issues for the term that evaluates the complexity of the system. The complexity factor is dependent on anisotropic pressure, inhomogeneous energy density, and dissipative variables in earlier work [10], now we shall examine the complexity along with GB terms for the dynamical systems. In order to understand the pattern of evaluation of the non-static complex system, we shall assume that the homologous evolution model.

4 Pattern of Evolution

After the formation of the complexity of the system for the fluid configuration, the next step is to determine the simplest pattern of evolution. We can identify two evolutionary processes that can be called the simplest based on entirely basic ideas, and they are homogenous expansion ($\Theta' = 0$) and homologous evolution. In the non-dissipative scenario, both types of evolution require each other, as we'll see shortly. The reasoning described in the following part allows us to assume homologous evolution as the simplest model in the generalized, dissipative condition.

4.1 Homologous Evolution

For the homologous evolution, Eq.(25) appears as

$$D_B \left(\frac{U}{B} \right) = \frac{4\pi}{E} q^{(eff)} + \frac{\sigma}{B}. \quad (59)$$

By applying integration to the above equation, we get

$$U = a(t)B + B \int_0^r \left(\frac{4\pi}{E} q^{(eff)} + \frac{\sigma}{B} \right) B' dr, \quad (60)$$

by putting the value of integration function a above equation becomes

$$U = \frac{U_\Sigma}{B_\Sigma} B - B \int_r^{r_\Sigma} \left(\frac{4\pi}{E} q^{(eff)} + \frac{\sigma}{B} \right) B' dr. \quad (61)$$

We get $U = B$ from preceding equations (60) and (61), which is a typical feature of homologous evolution [49–51]. This can happen if two terms of

integral cancel each other or if the fluid is non-dissipative and shear-free. For two areal radii B_I and B_{II} , we have concentric shells as

$$\frac{B_I}{B_{II}} = \text{constant}. \quad (62)$$

The equation above obviously indicates that throughout the evolution of the fluid distribution, the evolution pattern corresponding with the homologous criteria is the simplest (one of them). Therefore, for the homologous evaluation

$$U = a(t)B, \quad a(t) \equiv \frac{U_\Sigma}{B_\Sigma}. \quad (63)$$

As a result, R is a separable function, thus

$$B = B_1(t)B_2(r). \quad (64)$$

Finally, by using of Eq.(62) and (64) in Eq.(28), homologous condition becomes

$$\frac{4\pi}{B'} Aq^{(eff)} + \frac{\sigma}{B} = 0. \quad (65)$$

4.2 Homogeneous Expansion

A homogenous expansion is another evolution pattern that could be classified as “simple”. Equation (28) provides

$$4\pi q^{(eff)} = -\frac{B'}{A} \left[\frac{1}{3} D_B(\sigma) + \frac{\sigma}{B} \right]. \quad (66)$$

By applying the homologous condition in the homogeneous expansion, we obtain the results $D_B(\sigma) = 0$, which implies that $\sigma = 0$, means dissipation is not present. The above equation tells us if the expansion scalar is homogenous and the fluid is shear-free, it is totally obvious that the fluid is nondissipative. The fluid is also homogeneous in this situation, as shown by Eq.(61).

5 Some Kinematical Considerations

The homologous condition that was described in Eq.(65) can be represented as

$$4\pi Aq^{(eff)} = -\frac{B'\sigma}{B}. \quad (67)$$

By putting the above value into Eq.(29), it becomes

$$(\Theta - \sigma)' = 0. \quad (68)$$

By replacing the values of Θ and σ , it appears as

$$(\Theta - \sigma)' = \left(\frac{3\dot{B}}{CB} \right)' = 0. \quad (69)$$

With the help of Eq.(64), we obtain

$$C' = 0. \quad (70)$$

As a result of this, it is assumed that the fluid is geodesic. We can also put $C = 1$ without losing generality by reparametrizing the coordinate r . However, the contrary is also valid, i.e., the geodesic criteria requires homologous fluid. Certainly, we get $C = 1$ from the geodesic condition, stating

$$\Theta - \sigma = \frac{3\dot{B}}{B}. \quad (71)$$

We get $(\Theta - \sigma)' = 0$ for $B \sim r$ by evaluating the above equation close to the center. We find Eq.(71) close to the center by taking multiple derivatives with respect to r

$$\frac{\partial^n (\Theta - \sigma)}{\partial r^n} = 0, \quad (72)$$

for $n > 0$. As a result, the geodesic and homologous conditions implicate each other. Our this result is well-consistent with [1].

The homologous condition in the nondissipative case requires that the fluid is shearfree as well as is geodesic. The shearfree condition requires the homologous condition in this nondissipative case. In the non-dissipative case, the homogeneous expansion with Eq.(66) implies

$$\frac{\sigma'}{\sigma} = -\frac{3B'}{B}, \quad (73)$$

after simplification it becomes as

$$\sigma = \frac{g(t)}{B^3}, \quad (74)$$

here $g(t)$ is an arbitrary function of integration. As at the center, B becomes zero when $r = 0$, this implies that $g(t) = 0$, therefore $\sigma = 0$. Also if $\sigma = 0$ in Eq.(29), we get $\Theta' = 0$. Finally for the non-dissipative case

$$\sigma = 0 \Leftrightarrow U \sim B \Leftrightarrow \Theta' = 0. \quad (75)$$

The homogeneous expansion condition and the homologous condition imply each other in the nondissipative scenario. Our this result is well-consistent with [1].

It is worth noting that, as demonstrated by [52], the scalar function $Y_{TF} = 0$ iff a shear-free geodesic fluid remains geodesic and shear-free during its evolution. As a result, if $Y_{TF} = 0$ and the fluid is geodesic, a system that initiates its evolution from the beginning will remain shearfree. This is just another argument in favor of using $Y_{TF} = 0$ as the complexity factor.

By putting $\Theta' = 0$, Eq.(29) appears as

$$\sigma' + \frac{3\sigma B'}{B} + 12\pi q^{(eff)} A = 0. \quad (76)$$

The solution of the above equation is

$$\sigma = -\frac{12\pi \int_0^r B^3 q^{(eff)} A dr}{B^3}. \quad (77)$$

The homologous condition contradicts the above formulation until we make an assumption $q^{(eff)} = 0 = \sigma$. It tells us about the difficulties of applying both homogeneous and homologous expansion criteria simultaneously in the existence of dissipation along with the GR terms.

6 Sone Dynamical Considerations

Though in general dissipative case, the fluid is geodesic for the homologous condition. So with this condition Eq.(111) turns out to be

$$D_T U = -\frac{m}{B^2} - 4\pi P_r^{(eff)} B. \quad (78)$$

By using scalar function Y_{TF} in above equation, we get

$$3\frac{D_T U}{B} = -4\pi(\rho^{(eff)} + 3P_r^{(eff)} - 2\Pi^{(eff)}) + Y_{TF}. \quad (79)$$

Now with the help of mass function, GB field equations and Eq.(38) give

$$4\pi(\rho^{(eff)} + 3P_r^{(eff)} - 2\Pi^{(eff)}) = -\frac{2\ddot{B}}{B} - \frac{\ddot{A}}{A}. \quad (80)$$

With the help of Eq.(27), we can write

$$3\frac{D_T U}{B} = \frac{3\ddot{B}}{B}. \quad (81)$$

By putting the value of above equation in Eq.(79), we attain

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} = Y_{TF}. \quad (82)$$

As we suppose that fluid is homologous, then by using Eq.(63) in Eq.(79), we have

$$3\left(\dot{a}(t) + a(t)\frac{\dot{B}}{B}\right) = -4\pi(\rho^{(eff)} + 3P_r^{(eff)} - 2\Pi^{(eff)}) + Y_{TF}. \quad (83)$$

For the no-complex system (that can be obtained by putting zero value to the complexity factor), the integration of Eq.(82) yields

$$A = B_1(t) \left[b_1(r) \int \left(\frac{1}{B_1^2(t)} e^{-\int \frac{XB_1(t)}{B_1(t)} dt} dt \right) dt + b_2(r) \right], \quad (84)$$

here $b_1(r)$ and $b_2(r)$ are functions of integration. Above equation can easily gets the following form

$$A = B_1(t)B_2'(r) \left[\tilde{b}_1(r) \int \left(\frac{1}{B_1^2(t)} e^{-\int \frac{XB_1(t)}{B_1(t)} dt} dt \right) dt + \tilde{b}_2(r) \right], \quad (85)$$

the values of $b_1(r)$ and $b_2(r)$ are $B_2'(r)\tilde{b}_1(r)$ and $B_2'(r)\tilde{b}_2(r)$, respectively. By defining the new variable, we get

$$Z = \tilde{b}_1(r) \int \left(\frac{1}{B_1^2(t)} e^{-\int \frac{XB_1(t)}{B_1(t)} dt} dt \right) dt + \tilde{b}_2(r), \quad (86)$$

so

$$A = ZB', \quad (87)$$

where $B' = B_1(t)B_2'(r)$.

6.1 The non-dissipative case

If the fluid is non-dissipative, then by using the homologous condition and Eq.(24) in Eq.(75), we have

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} = 0 \Rightarrow Y_{TF} = 0. \quad (88)$$

Also fluid is shear-free, so we obtain the conditions with Eq.(24) and (84) as

$$b_1(r) = 0 \Rightarrow A = B_1(t)b_2(r) = B_1(t)B'_2(r)\tilde{b}_2(r). \quad (89)$$

By using the conditions of above equation and reparametrizing of r , we obtain $Z = 1$ which implies that $A = B'$ [53]. We can rewrite field equations as

$$4\pi q^{(eff)} = -\frac{\dot{Z}}{Z^2 B}, \quad (90)$$

$$8\pi(P_r^{(eff)} - P_\perp^{(eff)}) = \frac{\dot{Z}\dot{B}}{ZB} + \frac{1}{Z^2 B^2} \left(\frac{Z'B}{ZB'} + 1 - Z^2 \right). \quad (91)$$

Above expression gives $P_r^{(eff)} - P_\perp^{(eff)} = \Pi = 0$ for the value of $Z = 1$, also by using vanishing complexity condition, we attain $\rho' = 0$.

We already have an idea that the nondissipative and shearfree fluid with isotropic pressure is dust with vanishing Weyl tensor and homogenous energy density [47, 54]. This type of system, of course, provides the most basic configuration. This is the only system that evolves vanishing complexity condition and homologously. It is important to mention here that homogeneous and homologous expansion imply each other in nondissipative case.

6.2 The dissipative case

For the dissipative case, we use the Eq.(24) and (88) and get

$$\dot{\sigma} = -Y_{TF} + \left(\frac{\dot{B}}{B} \right)^2 - \left(\frac{\dot{A}}{A} \right)^2. \quad (92)$$

Now by taking the time derivative of Eq.(67) then above equation becomes

$$Y_{TF} \frac{B'}{B} = 4\pi A q^{(eff)} \left(\frac{\dot{q}^{(eff)}}{q^{(eff)}} + 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right), \quad (93)$$

by applying the vanishing complexity condition, we have

$$q^{(eff)} = \frac{f(r)}{A^2 B}, \quad (94)$$

also

$$\dot{q}^{(eff)} = -q^{(eff)}(\Theta + \sigma). \quad (95)$$

The general approaches provided in [55–58] can be used to find solutions of this type. The absence of transitory occurrences (stationary state), can now be considered the simplest dissipative regime in a dissipative process [59,60]. Assuming that the state is stationary then we obtain

$$q^{(eff)} = -\frac{\kappa T'}{A}, \quad (96)$$

by putting the value of Eq.(94) in above equation, it becomes

$$T' = -\frac{f(r)}{\kappa AB}. \quad (97)$$

At this stage, however, neither the assumption of the relaxation time vanishes as a symbol of minimal complexity in the dissipative regime nor the existence of accurate solutions of this type can be supported further.

7 Stability of No-Complex Systems

We can examine the evolution for a scalar function X_{TF} that is given in the Eq.(112), which can be further classified as

$$\begin{aligned} & -\frac{3\dot{B}}{B}Y_{TF} - \dot{Y}_{TF} - 8\pi\dot{\Pi}^{(eff)} - 4\pi\left(\rho^{(eff)} + P_r^{(eff)}\right)\sigma - 16\pi\Pi^{(eff)}\frac{\dot{B}}{B} \\ & - 4\pi\left[Z + \frac{1}{A}\left(q^{(eff)'} - \frac{q^{(eff)}B'}{B}\right)\right] = 0. \end{aligned} \quad (98)$$

For the non-dissipative case, the above equation gets the form

$$-\dot{Y}_{TF} - 8\pi\dot{\Pi}^{(eff)} - 4\pi Z = 0. \quad (99)$$

Now taking the time derivative of Eq.(50) and analyzing it at $t = 0$, we obtain $(\dot{\rho})' = 0$. Again by taking the time derivative of Eq.(98), and replacing $Y_{TF} = 0$, the only compatible solution is

$$-\ddot{Y}_{TF} - 8\pi\ddot{\Pi}^{(eff)} - 4\pi Z + 8\pi\dot{\Pi}^{(eff)}\frac{\dot{B}}{B} + 12\pi Z\frac{\dot{B}}{B} = 0. \quad (100)$$

Above equation with the second time derivative of Eq.(50) gives

$$2\dot{\Pi}^{(eff)}\dot{B} - ZB + 3Z\dot{B} = \frac{1}{B^2} \int_0^r (\ddot{\rho}^{(eff)})' dr. \quad (101)$$

Generally, at the starting point for the dissipative system, we have

$$\begin{aligned} & - \dot{Y}_{TF} - 8\pi\dot{\Pi}^{(eff)} - 4\pi\left(\rho^{(eff)} + P_r^{(eff)}\right)\sigma - 16\pi\Pi^{(eff)}\frac{\dot{B}}{B} \\ & - 4\pi\left[Z + \frac{1}{A}\left(q^{(eff)'} - \frac{q^{(eff)}B'}{B}\right)\right] = 0. \end{aligned} \quad (102)$$

Here, we only want to discuss that the effective terms of GB gravity also effects the stability of the vanishing complexity condition.

8 Concluding Remarks

The complexity of dynamical spherically symmetric fluid configuration in modified GB gravity has been explored. To do so, we discussed two different (but connected) characteristics of the complexity concept when interacting with a dynamical fluid. Firstly, we investigated the issue of determining the complexity for the fluid itself, secondly, we examined the evolution pattern of the fluid distribution for the degree of complexity in GB gravity.

The scalar function Y_{TF} has been chosen as a complexity factor of the fluid. We choose it because it is the same as for the static case, by limiting this case to static conditions, we obtain the previous results. The homologous condition indicates that Y_{TF} becomes zero in the nondissipative case.

Later, we examined the evolution pattern for the complexity. In the pattern of evolution, we discussed homogenous expansion and homologous conditions. In the dissipative case, fluid is geodesic, and it represents the simplest pattern of evolution. We obtain the vanishing complexity condition for the non-dissipative case, it means that evolution has already the simplest fluid configuration, and it gives us simplest and unique solution for the system.

Next, we discussed the stability for the scalar function $Y_{TF} = 0$. As long as the pressure remains isotropic, it seems that stability will propagate in time for the nondissipative case. The issue is significantly more complex in the dissipative case, because due to dissipative values, the system may deviate from the condition $Y_{TF} = 0$. All of our results support GR results [1].

Data availability statement

All data generated or analyzed during this study are included in this published article (and its supplementary information files).

Appendix A

The extra curvature terms due to GB gravity as found as under

$$T_{00}^{(G)} = \frac{C^2}{2} \left(Gf_G - f \right) - W_1 f_G'' - W_2 f_G' - W_3 \dot{f}_G, \quad (103)$$

$$T_{01}^{(G)} = -W_4 \dot{f}_G - W_5 f_G' - W_6 \ddot{f}_G, \quad (104)$$

$$T_{11}^{(G)} = -\frac{A^2}{2} \left(Gf_G - f \right) - W_7 \dot{f}_G - W_8 f_G' - W_9 \ddot{f}_G, \quad (105)$$

$$T_{22}^{(G)} = -\frac{B^2}{2} \left(Gf_G - f \right) - W_{10} \ddot{f}_G - W_{11} f_G'' - W_{12} \dot{f}_G \\ - W_{13} f_G' - W_{14} \ddot{f}_G, \quad (106)$$

where

$$W_1 = \frac{4}{A^4 B^2} \left(B'^2 C^2 - C^2 A^2 - \dot{B}^2 A^2 \right),$$

$$W_2 = \frac{4}{C A^5 B^2} \left(A' A^2 C \dot{B}^2 + 2 B' B'' A C^3 - 3 B'^2 A' C^3 + A' A^2 C^3 - 2 B' \dot{B} A^2 C \dot{A} \right),$$

$$W_3 = \frac{4}{C^2 A^3 B^2} \left(3 \dot{A} A^2 \dot{B}^2 - B'^2 \dot{A} C^2 + A^2 \dot{A} C^2 - 2 B'' C^2 A \dot{B} + 2 \dot{B} B' A' C^2 \right),$$

$$W_4 = \frac{4}{A^2 C^3 B^2} \left(2 \dot{B} B' C A \dot{A} - C' C^2 B'^2 - 2 A^2 \dot{B} C \dot{B}' + 3 C' A^2 \dot{B}^2 + C' A^2 C^2 \right),$$

$$W_5 = \frac{4}{A^3 C^2 B^2} \left(\dot{B}^2 A^2 \dot{A} - 2 C' C A B' \dot{B} - 3 C^2 \dot{A} B'^2 + 2 B' C^2 \dot{B}' A + \dot{A} A^2 C^2 \right),$$

$$W_6 = \frac{4}{A^2 C^2 B^2} \left(C^2 B'^2 - A^2 \dot{B}^2 - A^2 C^2 \right),$$

$$W_7 = \frac{4}{A^2 C^5 B^2} \left(2 \dot{B} B' C' C^2 A^2 - C^2 \dot{C} A^2 B'^2 - 2 C A^4 \dot{B} \ddot{B} + 3 \dot{C} A^4 \dot{B}^2 + \dot{C} C^2 A^2 B^2 \right),$$

$$W_8 = \frac{4}{A^2 C^3 B^2} \left(\dot{B}^2 C' A^2 - 2 \dot{C} A^2 B' \dot{B} - 3 C' C^2 B'^2 + 2 C A^2 B' \ddot{B} + C' C^2 A^2 \right),$$

$$\begin{aligned}
W_9 &= \frac{4}{A^2 C^4 B^2} \left(B'^2 C^2 A^2 - \dot{B}^2 A^4 - C^2 A^4 \right), \\
W_{10} &= \frac{4}{C^4 A^3} \left(B'' C^2 A B - \dot{A} \dot{B} A^2 B - B B' A' C^2 \right), \\
W_{11} &= \frac{4}{C^3 A^4} \left(B \ddot{B} C A^2 - C' C^2 B' B - B \dot{B} \dot{C} A^2 \right), \\
W_{12} &= \frac{4}{A^3 C^5} \left(\dot{C} C^2 B' B A' - \dot{A} C^2 B' B C' - C A^2 \dot{A} B \ddot{B} - \dot{C} C^2 A B'' B + 2 C' C^2 A B \dot{B}' \right. \\
&\quad \left. - C A^2 \ddot{A} B \dot{B} + C'' C^2 A B \dot{B} - 2 C C'^2 A B \dot{B} + 3 \dot{C} \dot{A} A^2 B \dot{B} - C' C^2 A' B \dot{B} \right), \\
W_{13} &= \frac{4}{C^3 A^5} \left(\dot{C} A^2 \dot{B} B A' - \dot{A} A^2 \dot{B} B C' - C A^2 A' B \ddot{B} - C' C^2 A B'' B + 2 C A^2 \dot{A} B \dot{B}' \right. \\
&\quad \left. + C A^2 \ddot{A} B' - C'' C^2 A B B' - 2 C \dot{A}^2 A B B' + 3 C' A' C^2 B B' - \dot{C} \dot{A} A^2 B' B \right), \\
W_{14} &= \frac{2}{C^3 A^2} \left(B \dot{A} C B' + B \dot{B} C' A - B \dot{B}' C A \right).
\end{aligned}$$

Appendix B

By using Eq.(14), we obtain nontrivial equations for the Bianchi identities as

$$\begin{aligned}
Z = T_{;\nu}^{\gamma\nu} V_{\gamma} &= -\frac{1}{C} \left[\dot{\rho}^{(eff)} + (\rho^{(eff)} + P_r^{(eff)}) \frac{\dot{A}}{A} + 2(\rho^{(eff)} + P_{\perp}^{(eff)}) \frac{\dot{B}}{B} \right] \\
&\quad - \frac{1}{A} \left[q^{(eff)'} + 2q^{(eff)} \frac{(CB)'}{CB} \right] = 0, \tag{107}
\end{aligned}$$

$$\begin{aligned}
T_{;\nu}^{\gamma\nu} \chi_{\gamma} &= \frac{1}{A} \left[P_r^{(eff)'} + (\rho^{(eff)} + P_r^{(eff)}) \frac{C'}{C} + 2(P_r^{(eff)} - P_{\perp}^{(eff)}) \frac{B'}{B} \right] \\
&\quad + \frac{1}{C} \left[\dot{q}^{(eff)} + 2q^{(eff)} \left(\frac{\dot{B}}{B} + \frac{\dot{A}}{A} \right) \right] = 0. \tag{108}
\end{aligned}$$

With the help of radial, proper time derivatives, Eq.(20), (21) and (28), above equations get the following form

$$D_T \rho^{(eff)} + \frac{1}{3} \left(3\rho^{(eff)} + P_r^{(eff)} + 2P_{\perp}^{(eff)} \right) \Theta + \frac{2}{3} \left(P_r^{(eff)} - P_{\perp}^{(eff)} \right) \sigma$$

$$+ ED_B q^{(eff)} + 2q^{(eff)} \left(a + \frac{E}{B} \right) = 0, \quad (109)$$

$$\begin{aligned} & D_T q^{(eff)} + \frac{2}{3} q^{(eff)} (2\Theta + \sigma) + ED_B P_r^{(eff)} + \left(\rho^{(eff)} + P_r^{(eff)} \right) a \\ & + \left(P_r^{(eff)} - P_{\perp}^{(eff)} \right) \frac{E}{B} = 0. \end{aligned} \quad (110)$$

Above equation can be more simplified by using mass function, Eq.(16) and (28), then it becomes

$$D_T U = -\frac{m}{B^2} - 4\pi P_r^{(eff)} B + Ea. \quad (111)$$

With the help of Eq.(108) and [47,48], we can obtain the scalar function X_{TF} as

$$\begin{aligned} & (4\pi \rho^{(eff)} + X_{TF}) + \frac{1}{3} (2X_{TF} - Y_{TF} + X_T + Y_T) (\Theta - \sigma) C \\ & + 12\pi q^{(eff)} \frac{CB'}{AB} = 0. \end{aligned} \quad (112)$$

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