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Internal erosion: critical hydraulic gradient in 1-D vertical seepage and its relation to soil gradation

by

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Abstract

A hydromechanical envelope that governs the onset of soil erosion by seepage-induced internal instability is examined with reference to theoretical and laboratory stress-gradient paths for one-dimensional upward and downward seepage flow. The method is validated from permeameter test results on glass ballotini. Using the theory, analysis of a database of 22 gradations establishes a semi-empirical relation between attributes of a grain size distribution curve and a stress-reduction factor governing potential for internal erosion. The semi-empirical relation provides a screening-tool for susceptibility of a soil to seepage-induced internal instability.

Key words:

internal erosion, critical gradient, vertical seepage, hydromechanical envelope, hydromechanical path

Introduction

The onset of seepage-induced internal stability is characterized with reference both to effective stress in the finer fraction of the grain size distribution, and also to hydraulic gradient causing seepage within the soil. From permeameter tests with upward flow and no surface load, Skempton and Brogan (1994) found the critical hydraulic gradient i_{cr} (at the end of path Q_0 to Q_u , in Fig. 1) for onset of erosion in an internally unstable soil to be smaller than the Terzaghi (1943) theoretical value i_c (at the end of path Q_0 to Q_s , in Fig. 1) for onset of fluidization or piping-by-heave, and proposed $i_{cr} = \alpha i_c = \alpha \gamma' / \gamma_w$, where α is the Skempton-Brogan stress-reduction factor ($0 \leq \alpha \leq 1$).

Li and Fannin (2012) extended the α concept to upward flow in soil with surface load (path P_0 to P_u in Fig. 1). The resulting hydromechanical envelope unifies the fluidization of an internally stable soil with the segregation-piping of an internally unstable soil, for which the location in stress-gradient space is given by:

$$i = \frac{\alpha}{1 - 0.5\alpha} (\bar{\sigma}'_{vm} + 0.5 \frac{\gamma'}{\gamma_w}) \quad (1)$$

where, $\bar{\sigma}'_{vm} = \sigma'_{vm} / (\gamma_w \Delta z)$ = normalized mean vertical effective stress; σ'_{vm} = mean vertical effective stress in upward flow (see Fig. 2b); Δz = layer or specimen thickness; γ' = submerged unit weight of soil; γ_w = unit weight of water.

The case of upward flow in these prior laboratory studies is just one of many encountered where concern exists for internal erosion. For example, the direction of seepage flow exiting the core material of a large zoned embankment dam, into a transition or filter zone of significantly greater hydraulic conductivity, is predominantly downward. In downward

flow tests with no surface load, Ronnqvist et al. (2017) and Douglas et al. (2019) report erosion of embankment dam materials at gradients in the range 2 to 10. The maximum gradient to yield internal instability in downward flow is relatively larger than that for upward flow. The finding is generally consistent with results for embankment dam soils in permeameter tests with surface load (Moffat and Fannin, 2011), from which it was inferred experimentally that the hydromechanical envelope is independent of seepage direction.

In this paper we propose a method to extend the Li and Fannin (2012) analytical approach to the case of downward flow and, from application of the method to a database of permeameter results, examine the relation between grain size distribution and the stress-reduction factor α .

Method for the critical seepage gradient

Relative to the hydrostatic condition (Fig. 2a), the mean vertical effective stress in a soil element that is subject to one-dimensional seepage flow increases with downward flow and decreases with upward flow (Fig. 2b). Li and Fannin (2012) established the critical hydraulic gradient in upward flow is given by:

$$i_{cu} = \alpha(\bar{\sigma}'_{vm0} + 0.5 \frac{\gamma'}{\gamma_w}) \quad (2)$$

where, $\bar{\sigma}'_{vm0} = \sigma'_{vm0}/(\gamma_w \Delta z)$ = initial normalized mean vertical effective stress; σ'_{vm0} = initial mean vertical effective stress.

Extending the same analytical approach to the case of downward seepage flow, the mean vertical effective stress is given by:

$$\sigma'_{vm} = \sigma'_{vm0} + 0.5i\gamma_w\Delta z$$

and in a non-dimensional form:

$$\bar{\sigma}'_{vm} = \bar{\sigma}'_{vm0} + 0.5i \quad (3)$$

Substituting Eq. 3 into to Eq. 1, the critical hydraulic gradient in downward flow is given by:

$$i_{cd} = \frac{\alpha}{1-\alpha}(\bar{\sigma}'_{vm0} + 0.5\frac{\gamma'}{\gamma_w}) \quad (4)$$

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The change in normalized mean vertical effective stress that occurs in the presence of upward or downward seepage flow is represented schematically by the two respective hydromechanical paths in $\bar{\sigma}'_{vm} - i$ space (Fig. 1). Consider the initial hydrostatic condition ($i = 0$) yielding, for example, point (P_0) at $\bar{\sigma}'_{vm0}$ on the origin of the hydromechanical path. For downward flow, mean vertical effective stress increases with increasing hydraulic gradient (from point P_0 to P_d), and the path follows a theoretical slope of $\Delta i / \Delta \bar{\sigma}'_{vm} = 2$.

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For the unique case of self-weight loading only, $\bar{\sigma}'_{vm0} = 0.5\gamma' / \gamma_w$, and the critical hydraulic gradient in downward flow may be expressed as:

$$i_{cd} = \frac{\alpha}{1-\alpha} \frac{\gamma'}{\gamma_w} \quad (5)$$

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104 Validation of the method

Li (2008) performed permeameter tests with upward and downward flow, respectively, on a gap-gradation F7 comprised of medium sand-sized glass ballotini with a 30% finer fraction content of fine sand-sized ballotini. The test specimen was reconstituted by slurry

deposition and consolidated with surface loading to $\sigma'_{t0} = 150$ kPa (see Fig. 2a), after which multi-stage seepage was imposed through increments of hydraulic gradient whilst the flow control system acted to maintain a constant value of $\sigma'_{t0} = 150$ kPa (see Fig. 2b). The variation of seepage velocity with hydraulic gradient, for tests F7-150-U (upward flow) and F7-150-D (downward flow), exhibits a linear relation at relatively low values of gradient (see Fig. 3) that corresponds to a hydraulic conductivity of 1.25×10^{-2} cm/s. The onset of internal instability is associated with a disproportionate increase in seepage velocity, and companion observations of eroded finer fraction and strong piping. From inspection of Fig. 3, $i_{cu} < i_{cd}$, and thus the critical seepage condition varies with direction of flow.

Hydromechanical paths are depicted for both tests in Fig. 4. Each path terminates at the onset of internal instability (see Fig. 3), from which a hydromechanical envelope to instability is established in stress-gradient space. The envelope yields a back-calculated value of $\alpha = 0.07 \pm 0.01$ (from Eq. 1). The slope of the hydromechanical path, calculated using the initial and final points (see insert to Fig. 4), is -1.9 for test F7-150-U and 2.0 for test F7-150-D: the experimentally-derived values are found in close agreement with the theoretical value of 2 (see Fig. 1).

Discussion

Critical gradient with upward and downward flow

For a given value of $\bar{\sigma}'_{vm0}$, permeameter testing with upward or downward flow yields a different hydromechanical path (see Fig. 1), and a different critical hydraulic gradient

(points P_u and P_d respectively, in Fig. 1). For the case of an internally stable soil with $\alpha = 1$, $i_{cd} = \infty$ (Eq. 5). A marginally unstable soil with for example $\alpha = 0.9$ yields $i_{cd} = 9$: assuming for simplicity that $\gamma' = \gamma_w$, then permeameter testing under self-weight loading only would, in theory, give $i_{cu} = 0.9$ and $i_{cd} = 9$. Accordingly, it seems reasonable that for most internally unstable soils, a maximum hydraulic gradient of 10 would be sufficient to trigger the onset of instability in downward flow tests without surface loading of the test specimen, an observation that is consistent with the laboratory results of Ronnqvist et al. (2017) and Douglas et al. (2019).

Application of the method

Li (2008) compiled a database on the onset of internal instability from four permeameter studies (see Fig. 5). Upward flow only tests are reported for 18 gradations (Skempton and Brogan, 1994; Liu, 2005; Li, 2008; Moffat and Fannin, 2011), and downward flow only tests for 2 gradations (T5 from Moffat and Fannin, 2011; F8 from Li, 2008). The database also includes 2 gradations tested with upward and downward flow respectively (including the results for F7 given in Figs. 3 and 4; T0 from Moffat and Fannin, 2011). A total of 13 gradations (Skempton and Brogan, 1994; Liu, 2005; HF01 from Li, 2008) were tested without surface loading of the test specimen ($\sigma'_{to} = 0$, see Fig. 2) and 9 gradations (Li, 2008; Moffat and Fannin, 2011) with surface loading, in a database of 45 permeameter tests. Experimentally-derived values of α were back-calculated for the 22 gradations (see Fig. 5) using Eq. 1 for tests with surface load (see for example, Fig. 4), else Eq. 2 for upward flow tests without surface load (see for example, Skempton and Brogan, 1994).

The grain size distribution of the 22 gradations in the database was evaluated with reference to the Kenney and Lau (1985, 1986) shape parameter $(H/F)_{\min}$, where F is the percentage passing at the grain size D and H is the increment of percentage passing over a designated grain size interval of D to $4D$. The relation of $(H/F)_{\min}$ with α (see Fig. 5a) appears generally good and yields a coefficient of correlation $r \approx 0.65$, a finding that lends confidence to the use of the method. The scatter that is evident in some of the data points is tentatively attributed to the influence of factors other than grain size distribution alone, such as grain shape and void ratio. Informed by this insight, consideration was thus given to the micro-structure of each reconstituted test specimen in the database using the Li and Fannin (2013) simplified Kovacs' (1981) criterion, presented here as the geometric index d'_{85}/O_{50} , where d'_{85} is the 85th percentile of grain diameter in the finer fraction and O_{50} is the average pore diameter of the coarser fraction. The assumed linear relation of d'_{85}/O_{50} with α (see Fig. 5b) given by $\alpha = 3.85 (d'_{85}/O_{50}) - 0.616$, yields $r \approx 0.8$ and an improvement over that obtained using $(H/F)_{\min}$. The upper bound value of $d'_{85}/O_{50} = 0.42$ between stable and unstable gradations at $\alpha = 1$ was determined empirically by Li and Fannin (2013). The lower bound value of $d'_{85}/O_{50} = 0.16$ at $\alpha = 0$ is inferred with reference to the F8 gradation (Li, 2008). The improvement is attributed to the d'_{85}/O_{50} index accounting for the effect of not only grain size distribution, but also grain shape and porosity of the coarser fraction. The finding implies the stress-reduction present in the finer fraction of an internally unstable gradation, quantified by α from the method, correlates well with attributes of soil micro-structure quantified by d'_{85}/O_{50} , and is independent of flow direction and boundary stress on a rigid-wall permeameter test specimen.

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178 **Concluding remarks**

179 A hydromechanical envelope that governs the onset of soil erosion by seepage-induced
180 internal instability is described with reference to stress-gradient paths for one-dimensional
181 upward and downward seepage flow in $\bar{\sigma}'_{vm} - i$ space. The method is validated from
182 permeameter test results on glass ballotini (Li, 2008). For most internally unstable soils, a
183 hydraulic gradient of less than 10 is demonstrated theoretically to trigger the onset of
184 instability, in downward flow tests without surface loading of the test specimen. Analysis
185 of a database of 22 gradations establishes a semi-empirical relation between a geometric
186 index d'_{85}/O_{50} and a stress-reduction factor α from the method. The resulting semi-
187 empirical $d'_{85}/O_{50} - \alpha$ relation provides a screening-tool for susceptibility of a soil to
188 seepage-induced internal instability that accounts for micro-structure of the test specimen
189 and is independent of flow direction and test condition. The method also proves that a
190 simple upward flow test, with no surcharge of the test specimen, is sufficient to determine
191 a value of stress-reduction factor α that is generally representative of the soil.

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236 List of Notation

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238 i_{cd} critical hydraulic gradient for an internally unstable material in a downward flow239 i_{cu} critical hydraulic gradient for an internally unstable material in an upward flow240 σ'_x vertical effective stress, $x = v_m, v_{m0}$.241 $\bar{\sigma}'_x$ $\bar{\sigma}'_x = \frac{\sigma'_x}{\gamma_w \Delta z}$ normalized mean vertical effective stress, $x = v_m, v_{m0}$.242 Δz the thickness of specimen243 γ' submerged unit weight of the soil specimen244 γ_w unit weight of water245 D grain size246 F percentage passing at grain size D 247 H increment of percentage passing between grain size D and $4D$ 248 $(H/F)_{\min}$ geometrical index from Kenney and Lau (1985, 86)249 d'_{85} 85th percentile of grain diameter in the finer fraction250 r coefficient of correlation251 O_{50} average pore diameter of the coarser fraction

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Figure 4 Hydromechanical path in stress-gradient space for gradation F7 (a) generalized relation and (b) detailed insert

Figure 5 Variation of α with (a) shape parameter $(H/F)_{\min}$ and (b) geometric index d'_{85}/O_{50} (open symbols: upward flow tests; closed symbols: downward flow tests; half-closed symbols: upward flow and downward flow tests)

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