THREE ESSAYS ON THE STATE

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
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Abstract

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The thesis formalizes the military origin of modern states and economies. It consists of three essays. Chapter four, the third essay, is the apex. Chapter two and three, the first and second essays, lay the ground for Chapter four.

Chapter two models an economy comprising a large number of agents interacting through both production and exchange and appropriation. The state, which monopolizes the means of coercion, centrally enforces a property rights regime and finances it through compulsory taxation. Complementarity between taxation and property rights policy is introduced and analyzed. A comparison is made between production with and without the state. Comparative statics are done with respect to changes in appropriation effectiveness to address Olson's argument (1982) that increased effectiveness in rent seeking will compromise economic performance.
Chapter three compares two alternative conflict models with interdependent sectors where the economy depends on the military to capture and secure a share of world resources and the military depends on the economy to pay for its expenditure. In one model, contestants set military expenditures equal to some proportion of production, and in the other model contestants set military expenditures equal to some absolute amount of production. The paper concludes that the proportionate approach results in a more intense conflict than the absolute approach and that both the scale factor in production and the mass factor in conflict are strategically important.

Chapter four analyzes the military origins of modern states and modern economies. The literature on state making and war making stresses that it was the competitive states system of Europe and its intensive warfare that led to the rise of modern states with bulky rationalized bureaucracy. Political economists with a statist perspective argue that the rise of modern economies is impossible without the rise of modern states and that both are driven mainly by geopolitical considerations. This paper formalizes these arguments and explains the civilization paradox: the war driven process of state making resulted in a declining share of military expenditures in government budget and national income. It also provides a geopolitical explanation for Wagner's law.
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CHAPTER 1
INTRODUCTION

Chapter 4, "A Model of War and State Making and the Civilianization Paradox", defines the thesis. It models the interaction between interstate rivalry and the choice of internal policy regimes. It focuses on the relationship between the unique competitive states system of Europe and the rise of modern European states and economies. This is a historical relationship widely observed but not yet analyzed formally. Chapter 4 analyzes the relationship between geopolitical and military considerations and the political and economic structure of states.

Chapter 4 is statist. It believes that the state plays a pivotal role in economic development. It views state building and economic development as twin aspects of modernization. It considers modernization as a deliberate decision by some states that felt the need for it. That need is essentially geopolitical and military in nature. It therefore views development and growth in the same way as Jones (1981, 1988) and Weiss and Hobson (1995).

Chapter 4 models the state in a holistic manner and believes that such holistic treatment of the state is critical for a true understanding of the state and its role in economic development. It models interstate rivalry and the military aspects of the state as well as taxation and public goods provision of the state. The economy relies on the state to capture and secure a share of world resources for it to function. Public goods provision by the state also enhances the production capacity of the economy. The state relies on taxation on economy to finance all state functions of taxation, public goods provision and defense. This results in interdependence between the state and the economy and interdependence between different policy branches of the state. The taxation administration and public goods provision branches and the defense sector also complement each other. They have a common infrastructure and positive externalities between them. That creates another type of policy interaction. The result of such interdependence and complementarity is that the choice of fiscal regime and economic organization has an impact on the war efficiency of the state in interstate rivalry.

2This concept of interdependence is first brought up by Dudley (1990).
3This concept of complementarity between policy branches is first brought up by Lane (1958, 1979).
States in conflict over resources choose optimal amount of military capacity given the military capacity of the rival state and their fiscal regimes. The result shows that under specific parameter values increasing economies of scale in conflict could lead to the rise of modern states and economies. Modern states and economies are more war efficient than their traditional predecessors. The exogenous shock of increasing economies of scale in conflict is the military revolution started in the 16th Century. The set of specific parameter values also predicts and explains the "civilization paradox" and Wagner's law. The civilization paradox is the empirical observation that the war driven process of state building ultimately resulted in a declining share of military expenditures in state budget and national income. In other words, the war driven process of state building ultimately resulted in the civilianization of the state.

Chapter 2 and 3 lay the ground for Chapter 4. They clear some technical and conceptual obstacles. Key concepts in Chapter 4 that have hitherto not been formally treated in economic literature are first analyzed rigorously in Chapter 2 and 3. Complementarity between different policy branches of state is first modeled in Chapter 2. The strategic implications of economies of scale in warfare in the context of a complete military-economy feedback loop is first analyzed in Chapter 3. These concepts are central to the understanding and modeling of the state in the third paper.

Chapter 2, "Endogenous Authoritarian Property Rights", analyzes taxation policy and property rights policy of the state in the context of a large number of market participants. It focuses on the impact of taxation and property rights policy on allocation of resources by the agents between productive and appropriative activities. With large number of market participants, self enforcement of property rights is unlikely. Property rights is a public good in this context. The property rights regime enforced by the state therefore plays a pivotal role in determining the performance of the economy.

Chapter 3, "Strategic Implications of Budgetary Approaches with Complete Military-Economy Feedback Loop", analyzes the impacts of budgetary approaches and production technology and conflict technology on the nature of interstate conflicts over resources. The two budgetary approaches analyzed are the absolute budgetary approach and the proportionate budgetary approach. The absolute budgetary approach is one in which the con-

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4 Refer to Parker (1996) for details.
5 The term is coined by Tilly (1992). The paradox is also observed by Weiss and Hobson (1995), Rasler and Thompson (1989) and Porter (1994).
6 This concept of economies of scale in conflict is first brought up by Hirschleifer (1989, 1991, 1995).
testants to resources set military expenditures equal to some absolute amount of resources captured or production from the resources captured. The proportionate budgetary approach is one in which the contestants to resources set military expenditures equal to some proportion of resources captured or production from resources captured. It sheds light on the role played by the economy as part of the war machine. It concludes that both production technology, together with its economies of scale in production, as well as conflict technology, together with its economies of scale in conflicts, have strategic implications. It also concludes that the proportionate budgetary approach leads to a more intense conflict than the absolute budgetary approach.

Chapter 3 complements Chapter 2 by analyzing issues of production and conflict in the case of small number of participants. The analysis on the strategic implications of the economies of scale in conflicts is drawn on heavily in Chapter 4 for analyzing the impact of interstate conflict on the choice of internal policy regime.
CHAPTER 2
ENDOGENOUS AUTHORITARIAN PROPERTY RIGHTS

The condition of man . . . is a condition of war of everyone against everyone.
Hobbes, Thomas. 1588-1679.
2.1. INTRODUCTION.

The purpose of this paper is to analyze the taxation and property rights policies of a net revenue maximizing state governing an economy with a large number of players, and the comparative statics of the system with respect to changes in taxation and property rights enforcement efficiency and appropriation effectiveness. Appropriation includes not only the familiar rent seeking but also attempts to profit from robbery, confiscatory redistribution and coercive encroachment.

We will analyze a property rights regime centrally created and enforced by state authority. Formal property rights theory has not yet modeled the role played by the state. This literature either deals with the effect of an exogenously determined property rights regime on the economy (Yang and Wills, 1990; Tornell, 1993) or with endogenous property rights creation and enforcement without a centralized authority (Hirshleifer, 1995; Bush, 1974; Skaperdas, 1992). As well, the papers on property rights determination without a centralized authority deal with the case of small number of players (Bush, 1974; Hirshleifer, 1991; Skaperdas, 1992). If the case of large numbers is analyzed, it is in the context of localized production without transactions among agents which provides effective opportunities for self delimitation and enforcement of property rights over resources (Hirshleifer, 1995). Such treatment is not representative of a market economy where impersonalized transactions involving large numbers of players are the norm. It is under such impersonalized market transactions and specialization of labor that appropriation is made easier. Under these conditions, private provision of property rights delimitation and enforcement is hampered due to the problem of collective action and the impersonalized and frequently non-repeated nature of market transactions. The role played by the state in solving such

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\(^7\)Marcouiller and Young (1995) modelled the relation between the state's taxation decision and its provision of order. However, they ignored the interactions among agents when modelling order. This paper tries to incorporate that.
problems therefore has an important effect on the economic performance of a country. We will model property rights in such context.

We will endogenize the property rights and taxation policies of the revenue maximizing state. We will also determine the agents' choice between production and appropriation. We model property rights as in Yang and Wills (1990), i.e., with its adequacy measured by the proportion of goods lost during transactions (transaction costs). The level of transaction costs is decided by the size of aggregate appropriation in the economy. We define the level of property rights specification and enforcement by the restitution rate, i.e., the proportion of appropriated income identified, recovered and returned to the owners by the law enforcing body of the state. This is a convenient measure of the effectiveness of the state in lowering the net return to appropriation and raising the net return to production. This definition simplifies the analysis of the combined effects of property rights and taxation policies on the economy. It also permits us to have a continuum of property rights regimes instead of the communal property-private property dichotomy. Since individual enforcement of property rights are assumed away in the model, when the restitution rate approaches zero, property rights become communal, as the fruit of individual productive effort is then free for everybody else to appropriate if they bother to put in the required effort. When the restitution rate approaches one, property rights are private. The state ensures that the individuals enjoy fully the rewards of their production by its enforcement of property rights: losses through appropriation are identified, recovered and returned to the owner. Of course, where on the continuum will the economy end up is the objective of analysis of this paper.

Taxation is treated as part of protection costs, as conceptualized by Lane (1958), which includes the price paid to the government for protection against theft (taxation) as well as losses through appropriation. Losses through taxation to the government and losses to the appropriators through appropriation are considered identical by producers. In other words, both appropriation and taxation are seen as encroachments on property rights by producers. The result of this formulation is that production with state enforcement of property rights and taxation will be larger (smaller) than production under anarchy if the restitution rate is larger (smaller) than the taxation rate. It is the relative rates of net marginal return of production and appropriation that decides the size of the gross output.

8Another approach of modelling is the common property-private property dichotomy. For example, see Tornell (1993).
This formulation brings us to the defining feature of the state: the capacity for compulsion (Levi, 1981; Stiglitz, 1989). The state may use this capacity either to curb appropriation, thus rendering protection to producers (enforcing property rights), or to force the producers to transfer resources to itself (taxation). The state is competing with appropriators for the income of the producers. Therefore, it has an incentive to raise the operational costs of the appropriators so as to reduce competition from them and increase profit.

Since both the enforcement of property rights and taxation policies depends on the compulsion capacity of the state, there exists a degree of complementarity between the two policies: many resources used by the state are dual in nature, serving both for taxation and enforcement of property rights (Lane, 1958). Comparative static analysis suggests that such complementarity between property rights enforcement and tax collection affects the state's policy behavior:

a.) the state's property rights enforcement will be suboptimal (too high for optimality) for maximizing gross output if there is little (large) complementarity between the fiscal and legal policy instruments;

b.) increased efficiency in property rights enforcement will increase (decrease) gross output if there is small (great) complementarity between taxation and property rights enforcement.

The rigidity or flexibility of the fiscal and property rights apparatus also has policy implications. Rigidity is measured by the rise in marginal administration costs associated with a rise in taxation or restitution rate, i.e., the convexity of the cost functions. For instance, increased efficiency in taxation might lead to a larger (smaller) gross output if better property rights enforcement could be achieved without (only with) much rise in costs, i.e., the legal apparatus is flexible (rigid).

Increased effectiveness in appropriation will reduce gross output, holding state policies constant. This is Olson's (1982) argument about the negative economic effects of interest groups rent seeking. However, in my model the taxation and restitution rates respond to such changes and have an impact on the gross output. Depending on the fiscal and legal capacity of the state, the total effect could result in raising, maintaining or lowering the gross output. If there is a rigid fiscal regime and a flexible property rights regime, the total effect would be to increase gross output, contrary to the Olson argument.

In section 2.2A, we lay out the model and its assumptions. In section 2.2B and
2.2C, we solve the maximization problem of the agents and the state. In section 2.2D, we analyze the comparative statics. In section 2.2E, we compare the policy packages of a net revenue maximizing state to that of a gross output maximizer. Section 2.3 is the conclusion.

2.2 THE MODEL.

2.2A. ASSUMPTIONS.

i.) The game:

This is an agency problem in which the principal, the state, maximizes its net tax revenue and the agents, i.e., the citizens, maximize their net personal income. The interactions between the state and the agents comprise a Stackelberg game: when engaging in production the agents are taxed by the state and when engaging in appropriation the agents face a penalty from the state which enforces property rights. The agents make their decisions simultaneously.

ii.) Assumptions:

a.) The state monopolizes the means of organized violence within the territory it controls. It is the only law making and enforcement agency in the defined territory. The state does not recruit its members from the population at large. Its members do not engage in production and exchange. They specialize in statecraft and earn their living through running the state. The state's only consideration is to make use of its taxation and property rights policies to maximize its revenue net of administration costs.

The administration costs are incurred for the enforcement of taxation and property rights policies and are denoted as:

\[ C(t, f), \text{ where } t \text{ is the taxation rate and } f \text{ is the restitution rate, a measure of property rights enforcement. Restitution is the proportion of lost and appropriated income that are identified and recovered by the state and returned to the owners.} \]

For comparative statics on changes in enforcement efficiency of property rights and taxation policies, we define shift parameters \( a \) and \( b \) where

---

9 For simplicity we assume fixed labour inputs, so net income represents net utility.
10 That is to say, the state employees are recruited from outside the economy. This simplification allows us to ignore the issue of endogenizing the size of the state and the economy. For the historical background of this type of state, see Oppenheimer (1975).
11 We assume that the level of inputs into the tax and property rights costs function are observable by the agents and hence there is no problem of commitment on the part of the state. That is to say, the state cannot announce a certain level of tax rate and restitution rate and later renge on them after the agents have made their decision on production and appropriation.
To estimate the average sampling error of the sample mean, we can use the Central Limit Theorem, which states that the sampling distribution of the mean approaches a normal distribution as the sample size increases, regardless of the shape of the population distribution.

In the context of the legal system, changes in the marginal cost of taxation and changes in property rights can represent changes in a representative function. If changes in property rights lead to an increase in the marginal cost of taxation, this can result in a decrease in the overall efficiency of the system.

Thus, changes in property rights can be analyzed using the following equation:

\[ f_q + r + (f'q)C = (q, a, f'q)C \]

where:
- \( f_q \) represents the initial cost function.
- \( r \) is a constant representing other fixed costs.
- \( f'q \) represents the derivative of the cost function with respect to property rights.
- \( C \) is the cost of taxation.

This equation can be used to analyze the impact of changes in property rights on the overall cost of taxation and to determine the optimal level of property rights.
either through production and exchange or through appropriating the income of others.\textsuperscript{17} Income acquired through production and exchange is \( P(l_i) \) and income acquired through appropriation is \( S(1 - l_i) \sum_{j \neq i}^n P(l_j) \), where \( n \) is the total number of agents in the economy and \( S(1 - l_i) \) is the proportion of production of other \((n - 1)\) individuals that are appropriated by individual \( i \). (Note: \( l_i \) refers to labor input for production and \( (1 - l_i) \) refers to labor input for appropriation.) Total gross income is therefore

\[
P(l_i) + S(1 - l_i) \sum_{j \neq i}^n P(l_j). \tag{1}
\]

We assume \( P' = \frac{\partial P(l_i)}{\partial l_i} > 0 \) for \( 0 \leq l_i \leq 1, P'' = \frac{\partial^2 P(l_i)}{\partial l_i^2} < 0, P''' = 0 \) and \( P(0) = 0 \). For comparative statics purposes, we define shift parameter \( \alpha \) where \( P(l_i) = Q(l_i) + \alpha l_i \) and \( Q \) is an underlying production function. Thus changes in \( \alpha \) represent ceteris paribus changes in the marginal productivity of labor.

We assume \( S(0) = 0, S(1 - l_i) = S \times (1 - l_i) \), i.e., \( \frac{\partial S(1 - l_i)}{\partial (1 - l_i)} = S > 0 \), where \( S \) is a constant and therefore \( \frac{\partial^2 S(1 - l_i)}{\partial (1 - l_i)^2} = 0 \).

c.) The loss sustained by individual \( i \) due to appropriation of others is

\[
\sum_{j \neq i}^n S(1 - l_j) P(l_i).
\]

Total income of agent \( i \) net of transaction costs is:

\[
\left(1 - \sum_{j \neq i}^n S(1 - l_j)\right) P(l_i) + S(1 - l_i) \sum_{j \neq i}^n P(l_j). \tag{2}
\]

Total income of the economy is:

\[
\sum_{i}^n \left(\left(1 - \sum_{j \neq i}^n S(1 - l_j)\right) P(l_i) + S(1 - l_i) \sum_{j \neq i}^n P(l_j)\right) = \sum_{i}^n P(l_i). \tag{3}
\]

In aggregate, appropriation does not create wealth. It is a mere transfer. It reduces the return to production and the incentive to produce. For notational simplicity, we denote \( \sum_{i}^n P(l_i) \) as \( Y \).\textsuperscript{19}

d.) We assume that individuals cannot negotiate among themselves a property rights regime and effectively enforce it due to the following reasons:

i. Property rights enforcement is a public good. The individuals choose to free-ride except in the case of the most localized or personalized transactions. For impersonalized transactions, which is the hallmark of market economy and the concern of the present model, of welfare.

\textsuperscript{17}Since labour does not enter directly into the utility function, there are no income effects, and we can derive unambiguous comparative statics of labour supply.

\textsuperscript{18}Transaction costs refer to losses through appropriation by others.

\textsuperscript{19}The crucial assumption is that appropriation, while diverting resources away from production, does not cause damages or injuries.
private enforcement of property rights is non-existent or inadequate due to the public good problem.\textsuperscript{20}

ii. The state does not allow the individuals to enforce law and order themselves since that could mean devolving the state power.\textsuperscript{21} The state chooses to monopolize legitimate organized violence.

iii. There are economies of scale in property rights enforcement. Property rights enforcement can be most effectively done by a monopoly firm, i.e., the state. It will be very costly for the individuals to negotiate a property rights regime and enforce it through the market mechanism. Even when a privately enforced property rights regime is created, it is more costly than the authoritarian law and order and will not survive. \textsuperscript{22}

e.) When the taxation and property rights policies are taken into account, the total net income of agent $i$ is:

$$
(1-t-(1-f)\sum_{j \neq i} S(1-l_j))P(l_i) + (1-f)S(1-l_i)\sum_{j \neq i} P(l_j).
$$

Through taxation and restitution, the state determines the incentive structure in the economy and the size of production and appropriation.\textsuperscript{23} When engaging in appropriation, agents face a penalty from the state which enforces property rights. The state identifies lost and appropriated income, recovers it and returns it to the owner. When engaging in production, an agent faces taxation of the state.

When the restitution rate approaches zero, the state does not enforce property rights. Since individuals choose not to, or are prevented from enforcing their own property rights, the fruit of an agent's production becomes effectively communal property: it is there for other agents to appropriate if they invest the required effort. When the restitution rate approaches one, we have the private property rights system well enforced. In between the two extremes of communal and private property rights systems is the continuum of partly communal and partly private property rights system with the component of private (communal) property rights system larger when the restitution rate is higher (lower).

The community or the state does not dictate or enforce communal property rights

\textsuperscript{20}See Olson (1971) for the logic behind the difficulty of a privately enforced property rights regime. For historical studies of pre-state private creation and enforcement of law and order, see Benson (1990) and Friedman (1979).

\textsuperscript{21}For a study on the possibility and benefits of private property rights regimes under a constitutional state, see Benson (1990).

\textsuperscript{22}For a glance at the competition between property rights regimes privately enforced and the authoritarian property rights regime backed by the state, see Benson (1990).

\textsuperscript{23}By the above formulation, we assume the agents cannot appropriate from the state or rebel against it. They can only try to evade taxation or property rights enforcement by the state.
in this case. However, as agents do not provide their own enforcement of property rights due to the reasons elaborated in assumption d.), resources they put into market transactions and the resulting rewards are de facto in public domain, or are communal property. A good example is the capital put into money and capital market by myriad investors. If the public authority does not enforce property rights rules, these investments are in public domain for appropriation by activities such as insider trading.24

Before we proceed, we look at the following entities:

Total income of the economy equals total net income of agents plus gross revenue of the state:

\[ Y = \sum_{i} (1 - t - (1 - f)) \sum_{j \neq i} S (1 - l_j) P (l_i) + \sum_{i} (1 - f) S (1 - l_i) \sum_{j \neq i} P (l_j) + t \sum_{i} P (l_i) \]
\[ = \sum_{i} (1 - t) P (l_i) + t \sum_{i} P (l_i) \]
\[ = \sum_{i} P (l_i). \]  

(5)

Net revenue of the state is:

\[ t \sum_{i} P (l_i) - C (t, f). \]

(6)

Social surplus equals the total net income of agents plus net revenue of the state:

\[ \sum_{i} (1 - t - (1 - f)) \sum_{j \neq i} S (1 - l_j) P (l_i) + \sum_{i} (1 - f) S (1 - l_i) \sum_{j \neq i} P (l_j) + t \sum_{i} P (l_i) - C (t, f) \]
\[ = Y - C (t, f). \]  

(7)

\( t \) and \( f \) are common knowledge. The agents make use of this common knowledge and information about \( t \) and \( f \) to accurately forecast the level of transaction costs in the economy.

g.) There is no external, informal or self-sufficient sector. Secession from the market economy is impossible.

2.2B. AGENT S REACTION FUNCTION.

Given that all other \( n - 1 \) identical individuals have made the optimal labor decision, denoted by \( \bar{l} \), individual \( i \) solves:

\[ \max_{l_i} \left( 1 - t - (1 - f) (n - 1) S (1 - l) \right) P (l_i) + (1 - f) S (1 - l_i) (n - 1) P (\bar{l}) \]  

(4)

PROPOSITION 1:

Depending on the relative efficiency of production and appropriation, and the relative magnitude of taxation and restitution rate as well as the degree of market complexity.

24Another good example is intellectual property.
there are three possible types of equilibria: full production where all resources of agents are put into productive activities, zero production where all resources of agents are put into appropriation and an interior solution where agents divide their resources between production and appropriation.

Proof: Appendix.

With a centralized property rights enforcement regime (the state), zero production is not possible as it would result in zero gross revenue and net losses for the state. When there is full production in anarchy, establishment of state authority with its property rights regime could not increase gross output. It might lead to a lower output. It will definitely lead to a lower net income for the agents (as the state will enforce a positive tax rate) and a lower social surplus (as measured by gross output minus administration costs).

A combination of high (low) value of \( n, s \) and \( t \) and low (high) value of \( \alpha \) and \( f \) places the economy on or closer to no (full) production equilibrium. The sustainability or efficiency of anarchy depends on factors such as the relative efficiency of production and appropriation and, the degree of market complexity. Therefore, the answer to the question of how strong a government should be, or is there a need for a government at all depends on above factors as well.

We will concentrate our analysis on the interior equilibrium with identical agents.\(^{25}\)

By assumption of identical agents and an interior solution, we write the FOC as:

\[
(1 - t - (1 - f) (n - 1) (1 - l) S) P' - (1 - f) S (n - 1) P = 0. \tag{8}
\]

We dropped the bar sign over \( l \) for simplicity.

Solving the problem of all agents simultaneously, we establish (as shown in the appendix):\(^{26}\)

i. \( l_s < 0, P_s = P'l_s < 0, Y_s = nP_s < 0. \)

ii. \( l_\alpha < 0, P_\alpha = P'l_\alpha + l > 0, Y_\alpha = nP_\alpha > 0. \)

There is a difference between the impact of reduced efficiency in appropriation and the impact of increased efficiency in production on the labor decision of agents and total income of the economy.

Reduced efficiency in appropriation reduces the marginal return from appropriation by oneself and increases productive effort directly (the positive direct effect). It also

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\(^{25}\)The interior equilibrium is unique. For proof, see appendix.

\(^{26}\)For the details of above comparative statics, see appendix. The results in v to ix have a lot to do with the diminishing marginal productivity of labor.
reduces the appropriation of others, lowers the level of transaction costs and therefore increases marginal return from production by oneself and increases productive effort (the positive indirect effect). The total effect is an increase in productive effort, i.e., $P_s < 0$.

Increase in production efficiency affects output in two ways:

a.) holding the level of individual productive effort constant, it increases output directly.

b.) through its impact on the level of productive effort of agents which can be subdivided into two:

1. It increases the marginal return from production by oneself and increases productive effort directly (the positive direct effect).

2. It increases the marginal return from appropriation by oneself as others are producing more and therefore reduces productive effort (the negative indirect effect).

The total effect is an increase in productive effort.

iii. $l_t < 0, P_t = P'l_t < 0, Y_t = nP_t < 0$.

Since we have fixed the labor supply and therefore there is no substitution between labor and leisure, the result $P_t < 0$ is caused by the transfer of resources away from production into appropriation as the return from production is reduced by increased taxation.

iv. $l_f > 0, P_f = P'l_f > 0, Y_f = nP_f > 0$.

The result $P_f > 0$ is in accordance with the argument in the property rights literature that better enforcement of property rights increases production. It is caused by the transfer of resources away from appropriation into production as the return to appropriation is reduced and the return to production is enhanced by increased restitution.

v. $P_{ts} < 0, Y_{ts} = nP_{ts} < 0$.

vi. $P_{fs} > 0, Y_{fs} = nP_{fs} > 0$.

Increased efficiency in appropriation increases the impact of taxation and property rights policy on the economy.

vii. $P_{at} < 0, Y_{at} = nP_{at} < 0$.

Increase in taxation rate increases the impact of taxation policy on the economy.

viii. $P_{ff} < 0, Y_{ff} = nP_{ff} < 0$.

Increase in restitution rate reduces the impact of property rights policy on the economy.

ix. $P_{tf} > 0, Y_{tf} = nP_{tf} > 0$. 
Increase in taxation rate increases the impact of property rights policy and an increase in restitution rate reduces the impact of taxation policy.

PROPOSITION 2:
The establishment of state authority will increase (decrease) gross product only if the state enforces a restitution rate higher (lower) than taxation rate.

PROOF:
From (7), we know that for the interior solution case, production takes place at the point where:
$$\frac{(1-t)}{(1-f)(n-1)} = (1 - l) + \frac{P}{P^e}$$ for centralized property rights enforcement and
$$\frac{1}{(n-1)} = (1 - l) + \frac{P}{P^e}$$ for anarchy.
Differentiating the right hand side with respect to $l$, we have
$$\frac{d(1-t)}{dl} = \frac{-P''P}{(P')^2} > 0 :$$ The right hand side is monotonically positively related to $l$ and $P$. Therefore, only when
$$\frac{(1-t)}{(1-f)} > 1$$ will production with the state be greater than production in anarchy. Q.E.D.

The state alters the rates of net return to production and appropriation by taxation and property rights policies. It is these rates of net return that determine the allocation of agents' labor endowment between production and appropriation. When taxation rate exceeds (is lower than) the restitution rate, the net returns from production under a state falls short of (exceeds) that under anarchy. The establishment of state authority therefore will increase (decrease) gross product if the state enforces a restitution rate higher (lower) than taxation rate.

We will now proceed to the problem of the state.

2.2C. THE STATE'S DECISION.

The state solves:
$$\max_{t,f} tnP(t,f) - C(t,f)$$
subject to maximization of income by the agents.

First order conditions are:
\begin{align*}
\text{wrt } t : \quad & tnR_t + nP - C_t = 0. \\
\text{wrt } f : \quad & tnP_f - C_f = 0.
\end{align*}

Since $tnP_f = C_f > 0$, we have $f > 0$.
(This is due to the assumption that $\lim_{f \to 0} C_f = 0$ and the interior equilibrium assumption of $l > 0$. In case of a very large $C_f$ even when the restitution rate approaches
zero, we might have zero restitution rate chosen by the state. Assuming \( \lim_{r \to 0} C_r = 0 \), the taxation rate of such a regime will be positive if production is positive with zero restitution rate: the state collects taxes from the economy with no services in return. The taxation rate will be zero if production at zero restitution rate is nil; the business of government is not sufficiently profitable to justify setting up a state in this case.)

There will be an increase in state net revenue due to the increased production caused by the reduced appropriation which in turn is due to the increased property rights enforcement. The state is competing with the appropriators for tax revenue. The more that is appropriated, the less there is to tax. Hence, the state finds it in its interest to enforce a positive level of property rights.\(^{27}\)

Law and order, or the property rights regime, is a public good. The economy consists of a large number of individuals who are incapable of providing for their own collective goods of law and order due to the problem of collective action as argued by Olson (1971). The state is the all encompassing interest group (as its membership is universal) with the incentive (due to tax revenue) to provide law and order to the economy. Other less encompassing interest groups might lack the incentive to do so. At this point, it is illuminating to note that the state will not overtax production to zero, irrespective of the cost efficiency in taxation. Such, however, is not the case for the atomistic appropriators whose efforts to appropriate might results in a corner solution with zero production (see PROPOSITION 1).\(^{28}\)

We will now look at the key factors that determine restitution and taxation rates.

**2.2D. COMPARATIVE STATICS.**

We will examine the effects of changes in efficiency in taxation and property rights specification and enforcement, as well as the changes in appropriation effectiveness on the taxation rate, restitution rate and gross product.

States do differ in their cost efficiencies in enforcing property rights and taxation policies. Some states have a cheaper and better supply of recruits for the fiscal and legal bureaucracy. For instance, the establishment of Manchurian rule over China supplied the Chinese empire with a new source of cheap and efficient tax collectors and law and order

\(^{27}\)Compare the way the state raises its revenue by reducing return to appropriators to the argument put forth by Salop and Scheffman (1983).

\(^{28}\)The reason for inability of the appropriators to further their collective interests is well expounded by Olson (1971, 1982).
enforcers in the form of the semi-nomadic-semi-agrarian Manchurian banner men that the previous Ming Dynasty lacked. Differences in public finance and jurisprudential know how and practices also affect the cost efficiency of the state in enforcing policies. A good example is the rediscovery of the Roman law and its impacts on the states of Europe since Renaissance. Besides, regime legitimacy and popularity of policies and monitoring technology could also affect the cost efficiency of carrying out taxation and property rights policies. Comparative statics i. to vi. are about the impacts of such factors.

i. \( t_a < 0. \)

ii. \( f_a < 0. \)

Increased inefficiency in taxation lowers the taxation rate. It also reduces the restitution rate. Law and order enforcement is the input of the state into the production of the economy while taxation rate is its share in the economy. A smaller share, i.e., a lower taxation rate due to the increased inefficiency in taxation, results in a lesser incentive for the state to enforce property rights. The outcome is a lower restitution rate. The complementarity between taxation and property rights enforcement in cost structure is likely to reinforce this effect.

iii. \( Y_a : \) The sign of \( Y_a \) depends on the value of \( C_{ff} \). \( Y_a \) is negative for zero or small \( C_{ff} \) and positive for large \( C_{ff} \).

The effect of a reduced efficiency in taxation on gross output depends on \( C_{ff} \). Reduced efficiency in taxation lowers both the taxation and restitution rates. While lower taxation increases gross output, lower property rights enforcement reduces gross output. The net effect depends on the relative magnitude of direct adjustment in taxation rate and that of indirect adjustment in property rights enforcement. For large (small) \( C_{ff} \), where the indirect change in the restitution rate is small (large), increased inefficiency in taxation will raise (reduce) gross output as the positive effect on gross output due to direct changes in taxation overwhelms (is overwhelmed by) the negative effects of indirect changes in the restitution rate. In other words, when the state becomes more efficient in collecting taxes, the effect on gross output depends on how easy it is to further increase the restitution rate. If that is easy (difficult), increased efficiency in taxation will raise (lower) gross output.

For large \( C_{ff} \), as \( t_a < 0 \) and \( Y_a > 0 \), net of tax income of agents has increased.

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29 For a good analysis on the impact of availability of cheap and efficient Manchurian banner men as a source of policy enforcers on the Chinese empire, see Huang (1988).
30 It facilitated the emergence and consolidation of centralized nation states.
31 For the details of the comparative statics in this section, see appendix.
Agents are better (worse) off with increased taxation inefficiency (efficiency) of the state. The case for small $C_{ff}$ is ambiguous.

**PROPOSITION 3:**

Increased efficiency in taxation will increase (decrease) gross output if there is a flexible (rigid) property rights regime.

Proof: Appendix.

Comparing $Y_a$ with $P_s < 0$, we note that increased ability by the government to extract resources from the producers might increase gross output while increased effectiveness in appropriation to do so will definitely lower gross product (holding $t$ and $f$ constant). The state can extract resources from both the producers (taxation) and the appropriators (restitution). So long as the state compensates its increased taxation by higher restitution rate, production need not be lower.

iv. $t_b < 0$.

v. $f_b < 0$.

Increased inefficiency in property rights enforcement reduces the restitution rate directly and the taxation rate indirectly.

A higher level of property rights enforcement due to the increased efficiency in restitution raises production and gives the state a greater incentive to acquire a larger share. The result is a higher taxation rate.

vi. $Y_b : Y_b < 0$ when $|C_{tf}|$ is small and $Y_b > 0$ when $|C_{tf}|$ is large.

The net effect of reduced efficiency in property rights enforcement on gross output depends on $|C_{tf}|$. When $|C_{tf}|$ is small (large), the indirect reduction in taxation rate is small (large). In this case, reduced efficiency in enforcement of property rights reduces (increases) gross output as the negative effect of direct reduction in property rights enforcement level overwhelms (is overwhelmed by) the positive effect of indirect reduction in taxation rate.

For large $|C_{tf}|$, as $t_b < 0$ and $Y_b > 0$, net of tax income of agents has increased. Agents are better (worse) off with increased enforcement inefficiency (efficiency) in property rights of the state. The case for small $|C_{tf}|$ is ambiguous.

**PROPOSITION 4:**

If complementarity in administration of taxation and property rights policies is important (unimportant), increased efficiency in property rights enforcement reduces (increases) gross output.
Proof: Appendix.

Increased efficiency in appropriation could be the result of technological changes or a changed political and social climate more conducive to interest groups rent seeking activities, as argued by Olson (1982). Comparative statics vii. to ix. are about the impacts of such changes.

vii. $t_s : t_s$ is positive for small $C_{ff}$ and negative for large $C_{ff}$.

As improvement in appropriation effectiveness increases the negative impacts of fiscal policy, its direct effect is to lower the taxation rate. However, as it also increases the positive impacts of property rights policy and increases the restitution rate, its indirect effect is to increase the taxation rate.

When $C_{ff}$ is small (large), the positive indirect effect on taxation rate is large (small) and it dominates (is dominated by) the negative indirect effect.

viii. $f_s : f_s$ is positively related to $t$ and $C_{tt}$ and negatively related to $|C_{tf}|$.

As improvement in appropriation effectiveness increases the positive impacts of property rights policy, its direct effect is to raise the restitution rate. However, as it also increases the negative impacts of fiscal policy and decreases the taxation rate, its indirect effect is to decrease the restitution rate.

When $C_{tt}$ is small (large), the negative indirect effect on the restitution rate is large (small) and it dominates (is dominated by) the positive direct effect.

For larger $t$, the state has a larger stake in the economy and therefore is more likely to raise the restitution rate in response to an increase in appropriation effectiveness. Hence, $f_s$ is larger for a larger $t$.

A larger $|C_{tf}|$ indicates a greater complementarity between fiscal and legal policies. That means a larger indirect effect and hence $f_s$ is negatively related to $|C_{tf}|$.

ix. $\frac{dY}{ds} = n (Pt_s + Pf_s + P_s) = n \left( \frac{dP}{ds} \right)$.

We will apply the model to discuss Olson’s (1982) argument on the negative economic impacts of interest groups rent seeking.

The effect of policy adjustments on the economy due to the improved appropriation effectiveness (i.e., the sign and value of $Pt_s + Pf_s$) depends critically on $C_{ff}$ and $C_{tt}$. This impact is besides the reduction in output holding taxation rate and restitution rate constant ($P_s < 0$). The total impact is $\frac{dP}{ds} = Pt_s + Pf_s + P_s$. Olson’s argument (1982) that society with a prolonged period of peace and stability is more conducive to interest groups activities
(i.e., a larger $s$) and therefore economic performance is more likely to be compromised refers only to $P_s$, i.e., he ignored the impacts of policy response of the state. The sign of the total impact is undetermined. We will only analyze four extreme cases:

a. $C_{ft} = \infty$ and $C_{tt} = \infty$: no adjustment in both $t$ and $f$. We have $t_s = 0$ and $f_s = 0$ and therefore $P_t t_s + P_f f_s = 0$ and $\frac{dP}{dt} = P_s < 0$. All the following have been reduced: gross output, net of tax income of agents, net revenue of the state and social surplus.

b. $C_{ft} = 0$ and $C_{tt} = 0$: large adjustment in both $t$ and $f$. We have $P_t t_s + P_f f_s < 0$ and therefore $\frac{dP}{dt} = P_t t_s + P_f f_s + P_s < 0$. All the following have been reduced: gross output, net of tax income of agents, net revenue of the state and social surplus.

c. $C_{ft} = 0$ and $C_{tt} = \infty$: large adjustment in $f$ and no adjustment in $t$. We have $t_s = 0$ and $f_s > 0$ (since the indirect effect is zero) and therefore $P_t t_s + P_f f_s = P_f f_s > 0$ and $\frac{dP}{dt} = P_f f_s + P_s > 0$. Gross output and net of tax income of agents have increased while net revenue of state is reduced. The impact on social surplus is ambiguous.

d. $C_{ft} = \infty$ and $C_{tt} = 0$: large adjustment in $t$ and no adjustment in $f$. We have $t_s < 0$ (as the indirect effect is zero) and $f_s = 0$ and therefore $P_t t_s + P_f f_s = P_t t_s > 0$ and $\frac{dP}{dt} = P_t t_s + P_s < 0$. Gross output and net revenue of state is reduced. The impact on net of tax income of agents and social surplus is ambiguous.

In case c, Olson's argument does not hold. Due to the enhanced property rights enforcement by the state prompted by the increased effectiveness in appropriation, the gross output actually increases.$^{32}$

**PROPOSITION 5:**

For state with a rigid fiscal regime and a flexible property rights regime, Olson's (1982) argument that increased effectiveness in rent seeking will compromise economic performance does not hold. Enhanced property rights enforcement by the state more than compensates the negative impacts of increased effectiveness in rent seeking. Gross output and net income of agents actually increase.

Proof: Appendix.

**2.2E. COMPARING WITH GROSS OUTPUT MAXIMIZER.**

For comparing the policy package of the revenue maximizing state to that of the gross output maximizer, we ask the question: in which direction will the taxation and

$^{32}$Olson (1982) also stressed that interest groups might capture state apparatus in their rent seeking activities and erode the coercive capacity of the state to result in negative economic impacts. We ignore this issue here.
restitution rates change when the revenue maximizing state faces a stricter constraint that requires it to guarantee a larger minimum level of gross output? The rationale for such consideration, in the words of Lane (1942), is that for the net tax revenue, "the amount collected is not limited by the shape of the demand curve, since protection is a necessity, but it may be restrained by the danger that too high a tribute will stimulate attempts to break the monopoly, i.e., will attract invaders, stimulate smuggling, or provoke insurrection". In other words, the revenue maximizing state might have to maintain a minimum level of gross output to preempt potential competition from within and beyond the boundary.

The constrained revenue maximizing state solves:
\[
\max_{t,f} tnP(t,f) - C(t,f) + \theta \left( \bar{Y} - nP(t,f) \right)
\]  
subject to the maximization of agents.
\( \bar{Y} \) is the targeted level of gross output.

Comparing the two policy packages, we have the following:

PROPOSITION 6:

With a differentiated administrative structure (i.e., small \( |C_{ff}| \)), the restitution rate of the gross output maximizer is always larger than that of the revenue maximizer. The tax rate of the gross output maximizer will be larger (smaller) than that of the revenue maximizing state if the restitution rate could (not) be increased without much rise in costs. That is, \( f_y > 0 \), and \( t_y < 0 \) for large \( C_{ff} \) and \( t_y > 0 \) for small \( C_{ff} \).

For large \( |C_{ff}| \), both the gross output maximizing restitution rate and taxation rate are lower than those of the revenue maximizer. The reduction in restitution rate will be small while the reduction in taxation rate will be large.

Proof: Appendix.

With two policy instruments (\( t \) and \( f \)), the state can achieve a higher gross product through the following policy combinations:

1.) an increase in the restitution rate and a reduction in the taxation rate;

2.) an increase in the restitution rate that has a greater positive impact on the gross output coupled with an increase in the taxation rate that has a lesser negative impact on the gross output; and

3.) a reduction in the restitution rate that has a smaller negative impact on the gross output together with a reduction in taxation rate that has a greater positive impact on the gross output.
In all three cases, net revenue of state has reduced. In case one and case three, we know that net of tax income of agents has increased.

For states with differentiated administrative structure (i.e., small $|C_{tf}|$), proposition 3 ruled out the third option and the state chooses among the first and second options depending on the cost structure of property rights enforcement.

When it is easy (costly) to increase enforcement of property rights for achieving the targeted level of higher gross output, i.e., a small (large) $C_{ff}$, the state will choose a policy package of a large (small) increase in property rights enforcement and an increase (a reduction) in taxation rate. Such a package will achieve the targeted level of higher gross output at least cost in terms of the revenue forgone.

For states with an undifferentiated administrative structure (i.e., large $|C_{tf}|$), a required increase in gross output results in a reduction in both the taxation and the restitution rates, with the reduced taxation rate having a larger positive impact on gross output than the negative impact of the reduced restitution rate. That is, option 3.

To summarize, a state with an undifferentiated administrative structure down sizes its apparatus when confronted with a demand for rising gross output while states with a differentiated administrative structure enlarge the legal apparatus and might down size or enlarge the fiscal apparatus.

2.3. CONCLUSION.

The model emphasizes that there is a continuum of property rights regimes for an economy with complex market transaction network, with the two extremes being communal property rights (or absence of enforcement of private property rights) and private property rights. The sustainability of communal property rights system without state authority depends on factors such as production efficiency, appropriation effectiveness and the degree of impersonalization of market transactions.

With a monopoly state, the determination of property rights system depends critically on the administrative cost structure and efficiency of the state. The rigidity or flexibility of the fiscal and property rights apparatus of the state, and the degree of complementarity between the two apparatus, affect the way the state responds to changes in the following: efficiency in taxation and property rights enforcement, appropriation effectiveness and the minimum level of gross output the state has to maintained. By treating taxation as part of the protection costs, which include also losses through appropriation,
the model raises the constitutional issue of quis custodiet ipsos custodes: who will constrain the coercive state that constrains appropriation and inhibits free riding in property rights enforcement by compulsory taxation? While research on the private creation and enforcement of property rights has greatly improved our understanding on the topic (Skaperdas (1992), Hirshleifer (1995)), the role of the state in the determination of property rights has not been adequately analyzed. This paper hopes to help fill that gap by its emphasis on the administrative cost structure of the state, the interaction among large numbers of agents in the economy and, the simultaneous determination of property rights and taxation policies.

Finally, I would outline two possible extensions:

i.) In the present model, secession from the market economy is impossible, i.e., the number of agents is exogenous. It will be fruitful and more realistic to endogenize the number of agents in the economy by allowing the agents to choose between self sufficiency and participation in the market economy.

ii.) The coercive capacity of the state manifests itself not only in the taxation and enforcement of a property rights regime. It is also shown in the defense capacity of state. Defense is to the state what property rights is to citizens: it concerns the ownership and control of resources in the international arena. Therefore, a logical extension of the model would be to incorporate the analysis of the defense policy.
Appendix.

2.2B. Agents' Reaction Function.

There are $n$ identical agents in the economy. Total amount of labor endowment of each agent is 1.

Production function $P(l)$ has the following characteristics: $P' > 0$ for $0 \leq l \leq 1$, $P'' < 0$; $P''' = 0$ and the appropriation $S(1-l)$ function has the following characteristics:

\[
\frac{\partial S(1-l)}{\partial (1-l)} = S, S' = 0, \text{i.e., } S(1-l) = S \times (1-l) \text{ where } S \text{ is a constant.}
\]

For simplicity, we denote the gross domestic product of the economy as $Y(t, f) \equiv nP(t, f)$.

Given that all other $n-1$ identical agents have made their optimal labor decision, individual $i$, solves:

\[
\max_l \left(1 - t - (1 - f)(n - 1)S \left(1 - \frac{l}{f} \right) \right) P(l) + (1 - f)S(1 - l) \left(1 - l \right) P \left(\frac{l}{f} \right).
\]

By assumption of identical agents and an interior solution, we can write the FOC as:

\[
(1 - t - (1 - f)(n - 1)(1 - l) S) P' - (1 - f) S(1 - l) P = 0.
\]

We dropped the bar sign over $l$ for simplicity.

There is an interior solution only under certain conditions. This leads us to:

**Proof for proposition 1:**

When $(P'(1))(1 - t) \geq (1 - f)(n - 1)(P(1)) S$, or \(\frac{(P'(1))(1 - t)}{(P(1))(1 - f)(n - 1)} \geq S\), where

\[
\frac{(P'(1))}{(P(1))} < 1,
\]

we have full production, i.e., $l = 1$.

When $(P'(0))(1 - t - (1 - f)(n - 1) S) \leq 0$, or \(\frac{(1-t)}{(1-f)(n-1)} \leq S\), we have zero production, i.e., $l = 0$.

For the interior case of \(\frac{(1-t)}{(1-f)(n-1)} > S \geq \frac{(P'(1))(1-t)}{(P(1))(1-f)(n-1)}\), we have agents spending their endowed resources in both production and appropriation, i.e., $0 < l < 1$.

Q.E.D.

**Proof for** \(\frac{(P'(1))}{(P(1))} < 1\):

Since $P''$ is a constant, we have $P' = P'' dl = P'' l + \alpha$. Then $P = \int P'dl = \int (P'' l + \alpha) dl = \frac{1}{2} P'' l^2 + \alpha l + \eta$. By assuming that $(P(0)) = 0$, we have $\eta = 0.33$. Therefore, $P(1) = \frac{1}{2} P'' + \alpha$ and $P'(1) = P'' + \alpha$. That gives us \(\frac{(P'(1))}{(P(1))} = \frac{P'' + \alpha}{\frac{1}{2} P'' + \alpha} < 1\).

Q.E.D.

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23 This simplification does not change any result.
Proof for uniqueness of the interior equilibrium:

The slope of the reaction function of individual $i$ with respect to the labor decision of other individuals is:

$$\left(\frac{1}{n-1}\right) \frac{\partial l_i}{\partial l} = -\left(\frac{1}{n-1}\right) \frac{(1-f)(n-1)S \left(P'(l) - P'(\bar{l})\right)}{P'' \left(1-t-(1-f)(n-1) \left(1-\bar{l}\right) S\right)}.$$

The reaction function has negatively slope for $l_i > \bar{l}$, zero slope for $l_i = \bar{l}$ and positive slope for $l_i < \bar{l}$.

The second derivative of the reaction function is:

$$\left(\frac{1}{n-1}\right) \frac{\partial^2 l_i}{\partial l^2} = \left(\frac{1}{n-1}\right) \left(\frac{1}{P'' \left(1-t-(1-f)(n-1) \left(1-\bar{l}\right) S\right)}\right) \left(P'' \left(1-f\right) \left(n-1\right) S\right)$$

$$\times \left(P'' \left(1-t-(1-f) \left(n-1\right) \left(1-\bar{l}\right) S\right) + (1-f) \left(n-1\right) S \left(P' \left(l\right) - P' \left(\bar{l}\right)\right)\right).$$

At symmetrical equilibrium with $l_i = \bar{l}$, the second derivative is positive.

The interior equilibrium is therefore unique.

Q.E.D.

Solving the problem of all agents simultaneously, we have:

$$l_\alpha = -\frac{(1-t-(1-f)(n-1)(1-\bar{l})S + (1-f)(n-1)(1-\bar{l})S)}{P''(1-t-(1-f)(n-1)(1-\bar{l})S)} > 0.$$

$$P_\alpha = P'l_\alpha + l = \frac{-\left(P''(1-t-(1-f)(n-1)(1-\bar{l})S + (1-f)(n-1)(1-\bar{l})S\right) + l > 0, \text{ since } P > (P')(l).$$

$$l_s = \frac{(1-f)(n-1)(1-t-f)(n-1)(1-\bar{l})S}{P''(1-t-(1-f)(n-1)(1-\bar{l})S)} < 0.$$

$$P_s = P'l_s = \frac{P''(1-t-(1-f)(n-1)(1-\bar{l})S)}{P''(1-t-(1-f)(n-1)(1-\bar{l})S)} < 0.$$

$$l_t = \frac{P''(1-t-(1-f)(n-1)(1-\bar{l})S)}{P''(1-t-(1-f)(n-1)(1-\bar{l})S)} < 0.$$

$$P_t = P'l_t = \frac{-\left((1-t-(1-f)(n-1)(1-\bar{l})S + (1-f)(n-1)(1-\bar{l})S\right)}{P''(1-t-(1-f)(n-1)(1-\bar{l})S)} > 0.$$

$$l_f = \frac{-\left((1-t-(1-f)(n-1)(1-\bar{l})S + (1-f)(n-1)(1-\bar{l})S\right)}{P''(1-t-(1-f)(n-1)(1-\bar{l})S)} > 0.$$

$$P_f = P'l_f = \frac{1}{P''(1-t-(1-f)(n-1)(1-\bar{l})S)} < 0.$$

$$P_{fs} = \frac{1}{P''(1-t-(1-f)(n-1)(1-\bar{l})S)} < 0.$$

$$P_{tf} = \frac{(P')(l-1)S (n-1) (1-\bar{l}) S}{P''(1-t-(1-f)(n-1)(1-\bar{l})S)} > 0.$$

$$P_{lf} = \frac{(P')(1-t-(1-f)(n-1)(1-\bar{l})S)}{P''(1-t-(1-f)(n-1)(1-\bar{l})S)} > 0.$$
2.2D. Comparative Statics.

The state solves the following problem:

\[ \max_{t,f} tnP(t, f) - C(t, f). \]  \hspace{1cm} (6)

subject to maximization of the agents.

FOC:

wrt \( t \): \( n(tP_t + P) - C_t = 0. \)  \hspace{1cm} (9)

wrt \( f \): \( n(tP_f) - C_f = 0. \)  \hspace{1cm} (10)

Second order condition requires that the Jacobian

\[ |J| = \begin{vmatrix} (n(tP_t + 2P_t) - C_t) & (n(tP_t + P_f) - C_t) \\ (n(tP_f + P_f) - C_f) & (n(tP_f) - C_{ff}) \end{vmatrix} > 0. \]

Using implicit function rule and Cramer's rule, we have:

i. \( t_a = \frac{1}{|J|} (n(tP_f) - C_{ff}) < 0. \)

ii. \( f_a = -\frac{1}{|J|} (n(tP_f + P_f) - C_{tf}) < 0. \)

Proof for proposition 3:

iii. \( Y_a \equiv nP_a = nPt_a + nP_ff_a \)

\[ = \frac{1}{|J|} \left( nP_t \left( n(tP_f) - C_{ff} \right) - nP_f \left( n(tP_t + P_f) - C_{tf} \right) \right) \]

\[ = \frac{1}{|J|} \left( nP_t \left( P_{ff} - P_f P_t \right) - nP_t C_{ff} + nP_f C_{tf} - n^2 P_f^2 \right), \]

where \( P_{ff} - P_f P_t = \)

\[ \left( \frac{P^2(1-t-(1-f)(n-1)(1-l))}{(n-1)(1-l)S} \right)^3 \times \{(P_n)^2(n-1)SP_t f (P_n)^2 \}

\times (1 - t - (1 - f) (n - 1) (1 - l) S) < 0. \]

\( Y_a \) is negative for a small \( C_{ff} \) and positive for a large \( C_{ff} \).

Q.E.D.

iv. \( t_b = -\frac{1}{|J|} (n(tP_t + P_f) - C_{tf}) < 0. \)

v. \( f_b = \frac{1}{|J|} (n(tP_t + 2P_t) - C_u) < 0. \)

Proof for proposition 4:

vi. \( Y_b \equiv nP_b = nPt_b + nP_ff_b \)

\[ = \frac{1}{|J|} \left( -nP_t \left( n(tP_f + P_f) - C_{tf} \right) + nP_f \left( n(tP_t + 2P_t) - C_u \right) \right) \]
\[ \frac{1}{r} \left( t(n)^2 (P_t P_{tt} - P_{tt} P_t) - nP_tC_{tt} + nP_tC_{tf} + (n)^2 P_t P_f \right), \]

where \( P_t P_{tt} - P_{tt} P_t = \left( \frac{P}{(1-t)(1-f)(1-l)(1-S)} \right)^3 \left( - (n-1) S P (P')^3 P'' \right) < 0. \]

\( Y_5 > 0 \) for large absolute value of \( C_{tt} \) and \( Y_5 < 0 \) for small absolute value of \( C_{tf} \).

Q.E.D.

vii. \( t_s = \frac{1}{2r} \left( (tnP_{ts} + nP_s) (n \ (tP_{tf} - C_{tf}) \right) \\
+ \frac{1}{4r} (tnP_{ts}) (n \ (tP_{tf} + P_t - C_{tf}), \)

where the first term is the negative direct effect and the second term is the positive indirect effect.

\[ t_s = \frac{1}{2r} \left( (tn)^2 \left( P_{tf} P_{fs} - P_{ts} P_{ff} \right) \\
+ \frac{1}{4r} (tn^2) \left( P_{tf} P_{fs} - P_{ts} P_{ff} \right) \\
- \frac{1}{4r} (C_{tf}) (tnP_{fs}) \\
+ \frac{1}{4r} (C_{tf}) (tnP_{ts} + nP_s), \]

where \( (P_{tf} P_{fs} - P_{ts} P_{ff}) = \)

\[ \left( \frac{P}{(1-t)(1-f)(n-1)(1-l)(1-S)} \right)^3 \times \left\{ 2 (P')^2 (n-1)^2 S P \left( (1-t) (1-f) (n-1) (1-l) S \right) \right. \]

\[ \left. - ((n-1) S)^2 P (P')^2 (1-t) (1-f) (n-1) (1-S) S \right\} > 0 \]

and \( (P_{tf} P_{fs} - P_{ts} P_{ff}) = \)

\[ \left( \frac{P}{(1-t)(1-f)(n-1)(1-l)(1-S)} \right)^3 \times \left\{ (P')^2 (n-1)^2 ((1-t) (1-f) (n-1) (1-l) S) S \right\} > 0. \]

\( t_s \) is positive for small \( C_{tf} \) and negative for large \( C_{tf} \).

viii. \( f_s = \frac{1}{4r} \left( tnP_{ts} \right) \left( (n \ (tP_{tt} + 2P_t) - C_{tt}) \right) \\
+ \frac{1}{4r} (tnP_{ts} + nP_s) (n \ (tP_{tf} + P_f) - C_{tf}), \)

where the first term is the positive direct effect and the second term the negative indirect effect.

\[ f_s = \frac{1}{2r} \left( (tn)^2 \left( P_{ts} P_{tf} - P_{fs} P_{tt} \right) \\
+ \frac{1}{4r} (tn^2) \left( P_{ts} P_{tf} + P_{ts} P_{tf} - 2P_t P_{fs} \right) \\
+ \frac{1}{4r} (n)^2 P_{tf} P_s \\
- \frac{1}{4r} (C_{tf}) (tnP_{ts} + nP_s) \\
+ \frac{1}{4r} (C_{tt}) (tnP_{fs}) \right), \]

where \( (P_{ts} P_{tf} - P_{fs} P_{tt}) = \)
\[
\frac{(t \ln P_f)^2}{(P^* (1 - t - (1 - f)(n - 1)(1 - f) S))^4}
\times \left\{ 2 \left( P^* \right)^2 (n - 1)^2 ((1 - l) P^* + P) (1 - f) SP
\right.
\left. - (P^*)^3 (n - 1)^2 SP (1 - f) (S_{l} - (1 - l)) \right\} > 0
\]

and \((P_t P_f + P_f P_s - 2 P_i P_{sf}) = 0\).

\(f_s\) is positively related to size of \(t\) and \(C_{tt}\) and negatively related to absolute size of \(C_{tt}\).

Q.E.D.

Proof for proposition 5:

\(\frac{\partial f}{\partial s} = n (P_t s + P_f s + P_s) = n \left( \frac{dP}{ds} \right)\).

We will check the sign of \((P_t s + P_f s + P_s)\).

Proof for \(P_t s + P_f s < 0\) when \(C_{tt} = 0\) and \(C_{tt} = 0\):

\(P_t s + P_f s = \frac{1}{P^* (n - 1)^2 [P_t s - P_s P_f] (P_s) + (P_t s - P_f P_s) (P_f)]\)

+ \(\frac{1}{P^* (n - 1)^2 [P_t s - P_s P_f] (P_s) + (P_t s - P_f P_s) (P_f)]\)

+ \(\frac{1}{P^* (n - 1)^2 [P_t s - P_s P_f] (P_s) + (P_t s - P_f P_s) (P_f)]\)

where \([P_t s - P_s P_f] (P_s) + (P_t s - P_f P_s) (P_f) = 0\),

\((P_t s P_f + P_f s - 2 P_i P_{sf}) (P_f) = 0\) and,

\([P_t s - P_s P_f] (P_s) = \frac{(P^* (1 - t - (1 - f)(n - 1)(1 - f) S))^4}{(P^* (1 - t - (1 - f)(n - 1)(1 - f) S))^4}
\times \left\{ 2 \left( P^* \right)^2 (n - 1)^2 ((1 - l) P^* + P) (1 - f) SP
\right.\]

\left. - (P^*)^3 (n - 1)^2 SP (1 - f) (S_{l} - (1 - l)) \right\} < 0.

When \(C_{tt} = 0\) and \(C_{tt} = 0\), we have \(P_t s + P_f s < 0\).

Proof for \(P_f s + P_s > 0\) when \(C_{tt} = \infty\) and \(C_{tt} = 0\):

When \(C_{tt} = \infty\), we have the taxation rate fixed. We denote it as \(\tilde{t}\). We are left

with only one first order condition to work with:

\(n (t P_f) - C_f = 0\).

Solving with implicit function rule, we have:

\(f_s = \frac{\tilde{t} n P_t s}{\tilde{t} n P_f s} > 0\).

\(\frac{dP}{ds} = P_f s + P_s = \frac{1}{P_f} (-P_f P_s + P_s P_f) > 0\),

where \((-P_f P_s + P_s P_f) = \)
\[
\left( \frac{P'}{P'\left(1-t\left(1-f\right)\left(n-1\right)\left(1-S\right)\right)} \right)^2 \left( -P'PS \left(1-f\right) \left(n-1\right)^2 \left(1-t\right)P' + P \right) < 0.
\]

**Proof for** \(Pt + Ps < 0\) **when** \(C_{tt} = 0\) **and** \(C_{ff} = \infty\):

When \(C_{ff} = \infty\), we have the restitution rate fixed. We are left with only one first order condition to work with:

\[n \left(tP_t + P\right) - C_t = 0.\]

Solving with implicit function rule, we have:

\[t_s = -\frac{n(tP_{tt} + P_s)}{n(tP_{tt} + 2P_t)} < 0.\]

\[\frac{dP}{ds} = P_t t_s + P_s \]

\[= \frac{1}{(tP_{tt} + 2P_t)} \left( -P_t \left(tP_{tt} + P_s\right) + P_s \left(tP_{tt} + 2P_t\right) \right) \]

\[= \frac{1}{(tP_{tt} + 2P_t)} \left( t \left(P_s P_{tt} - P_t P_{tt}\right) + P_s P_t \right) < 0,\]

where \(P_s P_{tt} - P_t P_{tt} = \)

\[\frac{P''}{P''\left(1-t\left(1-f\right)\left(n-1\right)\left(1-S\right)\right)} \left( (P')^3 \left(1-f\right) \left(n-1\right)P \right) > 0.\]

Q.E.D.

**Proof for proposition 6:**

For comparison of the policy package between that of the national income maximizer and the revenue maximizer, we solve the following problem:

\[
\max_{t,f} tnP(t,f) - C(t,f) + \theta \left[ \bar{Y} -nP(t,f) \right],
\]

where \(\bar{Y}\) is the targeted level of gross output.

FOC:

\[Z_t : tnP_t + nP - C_t + \theta \left(-nP_t\right) = 0.\]

\[Z_f : tnP_f - C_f + \theta \left(-nP_f\right) = 0.\]

\[Z_{\theta} : Y -nP(t,f) = 0.\]

By the set up of the problem, \(\theta < 0.\)

For comparative statics purposes, we calculate the followings:

\[Z_{tt} : 2nP_t + tnP_{tt} - C_{tt} + \theta \left(-nP_{tt}\right) < 0,\]

\[Z_{ff} : tnP_{ff} - C_{ff} + \theta \left(-nP_{ff}\right) < 0,\]

\[Z_{tf} : tnP_{tf} + nP_f - C_{tf} + \theta \left(-nP_{tf}\right) > 0,\]

\[Z_{\theta t} : -nP_t > 0,\]

\[Z_{\theta f} : -nP_f < 0.\]

and the Hessian
\[
\begin{vmatrix}
0 & Z_{\theta t} & Z_{\theta f} \\
Z_{\theta t} & Z_{tt} & Z_{tf} \\
Z_{\theta f} & Z_{tf} & Z_{ff}
\end{vmatrix} > 0.
\]

With above information, we have:

\[
t_y = \frac{Z_{\theta t}}{|H|} - \frac{Z_{\theta f}}{|H|} = \frac{1}{|H|} \left( \left( \left( n \right)^2 P_f^2 + (\theta - t) \left( n \right)^2 \left( P_t P_{tt} - P_f P_{tf} \right) + n P_t C_{tt} - n P_f C_{tf} \right) \right)
\]

For the case where \( C_{tt} \) is of a small absolute value, \( t_y \) is positive for small \( C_{ff} \) and negative for large \( C_{ff} \).

For the case where \( C_{tf} \) is of a large absolute value, \( t_y \) is negative if \( C_{ff} \) is large.

\[
f_y = \frac{Z_{tt}}{|H|} - \frac{Z_{tf}}{|H|} = \frac{1}{|H|} \left( \left( \left( n \right)^2 P_t^2 + (\theta - t) \left( n \right)^2 \left( P_t P_{tt} - P_f P_{tf} \right) - n P_t C_{tt} + n P_f C_{tf} \right) \right).
\]

For the case where \( C_{tf} \) is of a small absolute value, \( f_y > 0 \).

For the case where \( C_{tf} \) is of a large absolute value, \( f_y < 0 \) if \( C_{tt} \) is small.

Therefore, if \( C_{tf} \) is of a large absolute value and we have \( f_y < 0 \) and \( t_y < 0 \), \( C_{ff} \) has to be large and \( C_{tt} \) has to be small, which means the reduction in restitution rate is small while the reduction in tax rate is large.

Q.E.D.
CHAPTER 3

STRATEGIC IMPLICATIONS OF BUDGETARY APPROACHES
WITH COMPLETE MILITARY-ECONOMY FEEDBACK LOOP

Money is the vital nerve of war.
Colbert, Jean-Baptiste. 1619-1683.

3.1. Introduction

Kennedy (1987) observes that victory in great power contests goes to those powers with greater economic might. He is not alone in recognizing the importance of the strength of the economy in deciding the outcome of contests. Morgenthau (1954) for instance includes economic might as one determinant of a country’s status in international conflict. The feedback from the economy to the military has also long been recognized in economic literature. Dudley (1990) analyzes a model where the military depends on the fiscal bureaucracy and the economy for expenses and the bureaucracy and the economy depends on the military for security. He concludes that a technological shock emanating from either sector will have impacts on both sectors.34

This feedback from the economy to the military is however not analyzed in existing conflict literature. Hirshleifer (1995) discusses the impact of conflict technology and production technology on the outcome of conflicts over resources.35 He assumes that contestants directly use resources captured as inputs into conflicts and the residual resources are for production. There is no feedback from the economy to the military. Given the conflict and production technology, the size of the military sector therefore determines the amount of total resources captured, and the amount of resources available for production and the actual level of production. The objective of the players is then to maximize the amount of net resources available for the economy. He finds that conflict technology, especially the mass factor, has an impact on the nature of conflict while production technology, including the scale factor, has no impact.36

34 This "feedback loop" is also mentioned by McNeill (1982) and Porter (1994).
35 Hirshleifer (1991) has a similar and simpler model.
36 The mass factor in conflict technology (Hirshleifer, 1989 and 1995) measures the relative advantage or disadvantage that a larger player has over his smaller rival, analogous to the scale factor in production which measures the relative advantage or disadvantage that a larger firm has over his smaller rival.
Skaperdas (1992) and Neary (1994) analyze a situation where there are both elements of cooperation and conflict. They assume however that there are constant returns to scale in production. Dixit (1987) discusses the impact of conflict technology on the nature of strategic behavior. He assumes a fixed prize to be contested among the players. There is no production in his model. Analytically, assuming constant returns to scale in production and assuming a fixed prize to be contested among the players is the same. The feedback from the economy to the military and the impact of the scale factor of production on conflict is therefore not analyzed in these papers.

This assumption of no feedback from the economy to the military is both restrictive and unrealistic. With feedback from the economy to the military and the other way round, guns and butter are not merely substitutes for each other. They also complement each other and the performance of one depends on that of the other. Production technology and the scale factor will then have strategic importance. We will allow for feedback from the economy to the military in our models to complete the "feedback loop" mentioned by Dudley (1990).

Hirshleifer (1995) assumes that contestants set military expenditure equal to some proportion of total resources captured. He finds that the mass factor in conflict has to be smaller than one (that is, there are diseconomies of scale in conflict) in order for an anarchic system to have an interior and stable solution. If instead the contestants set military expenditure equal to some absolute amount of resources, this restriction is unnecessary. We call the budgeting approach adopted by Hirshleifer (1995) the proportionate budgetary approach. We term the other budgeting approach where contestants set their fighting effort equal to some absolute level of resources or production the absolute budgetary approach. Hirshleifer (1988), Skaperdas (1992), Neary (1994) and Dixit (1987) use the absolute budgetary approach. The impact of this difference in budget approaches has not been analyzed. This paper will analyze the strategic implications of this difference in budgetary approaches.

We will develop two models: one with the proportionate budgetary approach and the other with the absolute budgetary approach. We will allow for the complete feedback

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37 By anarchy, Hirshleifer (1995) refers to the case where there is more than one contestant for resources.

38 An example is post war Japan which sets its military budget below one per cent of GDP by constitutional requirement.

39 For example, England in the 18th and 19th century set her naval strength to be at least the equal of the combination of the next two most powerful naval powers.
loop. With complete feedback loop, the economy is part of the war machine. Both the scale factor and the mass factor are therefore strategically important. In the proportionate budgetary game, a contestant that has a larger proportion of his production devoted to military expenditures has a dual advantage in equilibrium: he controls a larger share of resources and he devotes a larger proportion of production from this larger share of resources to military purposes. The proportionate budgetary approach therefore leads to a more intense conflict than the absolute budgetary approach.

The case of no feedback from economy to military, as in Hirshleifer (1995) and Dixit (1987), is mathematically a special case of the more general model. When there is constant return to scale in production, the mathematical solution to the proportionate budgetary model is the same as that of the Hirshleifer (1995) model. When there is constant return to scale in production and contestants use the absolute budgetary approach, the mathematical solution is the same as that of the Dixit (1987) model.

3.11. The models.

We will use the ratio form of conflict technology function proposed by Hirshleifer (1989). This, besides facilitating comparison with the Hirshleifer (1995) model, makes interpretation of the mass factor easier and more intuitive. In the case of two contestants, the conflict technology function is:

\[ P = \frac{F_1}{F_1 + F_2} \]

\( P \) is the share of world resources captured and controlled by contestant 1, \( F_1 \) is the amount of resources invested in conflict by contestant 1, \( F_2 \) is the amount of resources invested in conflict by contestant 2 and \( m \) is the military decisiveness or mass parameter. A larger \( m \) means there are greater economies of scale in conflict and a larger military force can more effectively seize and control a larger share of world resources.

Total amount of world resources to be contested is normalized to one.

The production function takes the form:

\[ Y_1 = A_1 P^h \quad \text{and} \quad Y_2 = A_2 (1-P)^h, \]

where \( h \) is the scale factor in production.

The goal of each contestant is to maximize its profits.

\[ R^1 = A_1 P^h - c_1 F_1 \] is the profits of contestant one and

\[ R^2 = A_2 (1-P)^h - c_2 F_2 \] is the profits of contestant two.

\( A_1 \) and \( A_2 \) measures the production efficiency of contestants 1 and 2.
c_1 and c_2 are contestant 1 and 2's unit cost of transforming production into fighting effort. They measure the logistical efficiency of the contestants.

We include the opportunity costs of the statesmen-soldiers-contestants in F_1 and F_2.

The above could be rewritten as \( \pi^1 = \frac{c_1}{A_1} = P^h - \frac{\alpha}{A_1} F_1 \) and \( \pi^2 = \frac{c_2}{A_2} = (1 - P)^h - \frac{\alpha}{A_2} F_2 \) without changing the results that we are interested in. \( \frac{\alpha}{A_1} \) and \( \frac{\alpha}{A_2} \) are the costs of converting raw resources under one's control into military capacity. They determine the war efficiency of the contestants. Given the complete feedback loop from the military to the economy and from the economy to the military, war efficiency is decided by both the production efficiency and the logistical efficiency.

For notational simplicity, we choose the unit of \( F \) such that \( \frac{\alpha}{A_1} = a \) and \( \frac{\alpha}{A_2} = 1 \). That gives us:

\[ \pi^1 = P^h - a F_1 \] and \( \pi^2 = (1 - P)^h - F_2 \).

Depending on the way the contestant sets his budget, we have the following two versions of the problem:

a) Contestant one solves \( \max_{f_1} \pi^1 = (1 - f_1) P^h \), where \( f_1 = \frac{\alpha}{P^h} \) and contestant two solves \( \max_{f_2} \pi^2 = (1 - f_2) P^h \), where \( f_2 = \frac{\alpha}{(1 - P)^h} \). The contestants set the military budgets as some proportion of production. We call this the proportionate budgetary approach.

b) Contestant one solves \( \max_{F_1} \pi^1 = P^h - a F_1 \) and contestant two solves \( \max_{F_2} \pi^2 = (1 - P)^h - F_2 \). The contestant sets the military budget as some absolute amount of production. We call this the absolute budgetary approach.

We will solve these problems in turn.

3.2. Proportionate budgetary approach.

We will first derive the equilibrium success ratio, following Hirshleifer (1995, p. 33). We allow however for feedback from the economy to the military and asymmetry in war efficiency.

\[ \frac{P}{1 - P} = \left( \frac{f_1}{f_2} \right)^m = \left( \frac{\frac{\alpha}{P^h}}{\frac{\alpha}{(1 - P)^h}} \right)^m = \left( \frac{\alpha}{\alpha} \right)^m \cdot \frac{P^h}{1 - P} \]

This reduces to

\[ f_1^m P^{h+m-1} = a^m f_2^m (1 - P)^{h+m-1} \cdot \]

So, finally, equilibrium success ratio is

\[ \frac{P}{1 - P} = \left( \frac{f_1}{f_2} \right)^{m.} \]

For simplicity, let \( \theta \equiv \frac{m}{1 - h_m} \).
This gives us
\[ P = \frac{\frac{f_i}{f_i^*}}{f_i^* + \frac{\sigma f_i^*}{f_i^*}}. \]

We will explain and analyze the various games under the proportionate budgetary approach in this section. We will also compare the results with Hirshleifer (1995). In Section 3, we will analyze the various games under the absolute budgetary approach and compare them with those derived here.

3.21. The symmetrical proportionate budgetary game.

We have \( a = 1 \).\(^{40}\)

Contestant one solves
\[
\max \pi^1 = (1 - f_1) \left( \frac{f^*_1}{f^*_1 + f^*_2} \right)^h.
\]

First order condition is
\[
\pi^1 = -\left( \frac{f^*_1}{f^*_1 + f^*_2} \right)^h + (1 - f_1) h\theta \left( \frac{f^*_1}{f^*_1 + f^*_2} \right)^h \left( \frac{f^*_2}{f^*_1 + f^*_2} \right) f^*_1 = 0.
\]

This reduces to
\[
-1 + (1 - f_1) h\theta \left( \frac{f^*_2}{f^*_1 + f^*_2} \right) f^*_1 = 0.
\]

At a symmetrical equilibrium,
\[
(1 - f_1) h\theta \left( \frac{1}{2} \right) = f.
\]

That gives us
\[
f = \frac{hm}{2(1-hm)+hm} \quad \text{and} \quad F = \left( \frac{hm}{2(1-hm)+hm} \right) 2^{-h}.\(^{41}\)
\]

We have \( f < 1 \) if \( hm < 1 \), \( \lim_{hm \to 1} f = 1 \), \( f > 1 \) if \( hm > 1 \). An interior solution exists if and only if \( hm < 1 \), that is, if \( 0 < \theta < \infty \). For non-negative profits condition, we need \( hm \leq 1 \). When \( h = 1 \), the model reduces to the symmetrical game in Hirshleifer (1995, p. 34-36) with \( b = 1 \).

For convenience, we name \( hm \) the aggregate scale factor.

The profit of the contestant at symmetrical Nash equilibrium is
\[ \pi = (1 - f) 2^{-h} = \left( \frac{2(1-hm)}{2(1-hm)+hm} \right) 2^{-h}. \]

Checking for the second order condition, strategic substitute or complement relationship and strategic stability condition at symmetrical equilibrium, we have Diagram 1.\(^{42}\)

For our purposes, it is important to note that

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40 This could be that the two contestants are equally efficient in both production and logistics or that one contestant has an edge over its rival in logistical (production) efficiency but suffers an equal degree of production (logistical) inefficiency that just results in equal total war efficiency.

41 The mathematical details of this and later sections are in appendix.

42 See also appendix 3.21 for details.
a.) with \( hm < 2 \) we have strategic stability.

b.) with \( hm \leq 1 \), we have non-negative profits for both contestants, the second order condition is satisfied, the military expenditures of both contestants are strategic complements and there is strategic stability.

We have a stable interior solution and strategic complementarity for \( hm < 1 \).

Comparing this with Hirshleifer (1995), it is clear that with the feedback from the economy to the military, the scale factor has an impact on strategic outcomes. The economy is part of the war machine. The claim by Hirshleifer (1995, p. 33) that "for an interior stable equilibrium, the decisiveness parameter must lie in the range \( 0 < m < 1 \)" should be modified to "for an interior stable equilibrium, the value of the aggregate scale factor must lie in the range \( 0 < hm < 1 \)." Economies (diseconomies) in the military aspects could be compensated by diseconomies (economies) in the economic arena.

3.22. Endogenous number of contestants with proportionate budgetary approach.

If there are \( N \) contestants, the representative \( i \) contestant solves

\[
\max_{f_i} \pi_i = (1 - f_i) \left( \frac{f_i^o}{f_i^o + \sum_{j \neq i} f_j^o} \right)^h.
\]

When \( h = 1 \) the model here is the same as the symmetrical game for endogenous number of contestants in Hirshleifer (1995) with \( b = 1 \).

First order condition is:

\[
\pi_i^* = -\left( \frac{f_i^o}{f_i^o + \sum_{j \neq i} f_j^o} \right) + (1 - f_i) h \theta \left( \frac{f_i^o}{f_i^o + \sum_{j \neq i} f_j^o} \right)^h \left( \frac{\sum_{j \neq i} f_j^o}{f_i^o + \sum_{j \neq i} f_j^o} \right) f_i^{* - i} = 0.
\]

This reduces to

\[
-1 + (1 - f_i) h \theta \left( \frac{\sum_{j \neq i} f_j^o}{f_i^o + \sum_{j \neq i} f_j^o} \right) f_i^{* - i} = 0.
\]

At a symmetrical equilibrium,

\[
(1 - f) h \theta \left( \frac{N-1}{N} \right) = f.
\]

That gives us

\[
f = \frac{(N-1)hm}{N(1-hm)+(N-1)hm}
\]

and

\[
f < 1 \text{ if } hm < 1, \quad \lim_{hm \to 1} f = 1 \text{ and } f > 1 \text{ if } hm > 1.
\]

For an interior solution, we need \( hm < 1 \). For non-negative profits condition, we
For proportionate budgetary approach, it is technically impossible to deter entry of new contestants. Since there is free entry, for \( hm < 1 \) the number of contestants would be infinite and for \( hm = 1 \) the number of contestants would be indeterminate. For \( hm > 1 \), the equilibrium number of contestants is one.

3.23. The asymmetrical proportionate budgetary game.

The asymmetry is a sum of the differences in production and logistical efficiency. A country that has slight edge over its rival in logistical efficiency but somehow suffers a greater production inefficiency will have a higher cost of converting raw resources into military capacity than its rival. The former U.S.S.R. is a good example: its great ability in building a powerful military machine for cold war purposes was more than counterbalanced by its stark inefficiency in economic arena. On the other hand, some countries might have a greater efficiency in economic aspects that was more than offset by even greater disadvantages in military aspects and therefore also suffer from a greater cost for converting raw resources into military capacity. Good examples are the agrarian world during the age of cavalry from A.D. 200—1500: despite their economic superiority versus their nomadic rivals, lack of horse power rendered the agrarian peoples vulnerable to attacks from the steppe. Of course, there is also the case where one contestant is more efficient than the other in both production and logistical aspects.

We normalize the conversion costs for contestant 2 as one. We denote the conversion costs for contestant 1 as \( a \), where \( a > 1 \).

Contestant 1 solves
\[
\max_{f_1} (1 - f_1) P^h,
\]
where \( f_1 = \frac{aP_2}{P_1} \).

Contestant 2 solves
\[
\max_{f_2} (1 - f_2)(1 - P)^h,
\]
where \( f_2 = \frac{P_2}{(1-P)^2} \).

The first order condition for contestant 1 is
\[
\pi_1^1 : \left[-1 + (1 - f_1) (h \theta) \left( \frac{a^2 f_2}{f_1^2 + a^2 f_2^2} \right) f_1^{-1} \right] \left( \frac{f_1^2}{f_1^2 + a^2 f_2^2} \right)^h = 0.
\]

---

43For a symmetrical proportionate game, this condition is invariant with respect to the number of contestants.

44For details of the technical impossibility of deploying entry deterrence strategy, see appendix 3.22.

45For details, see Keegan (1993) and McNeill(1982).
The First Order Condition for contestant 2 is
\[ \pi_2 = \left[ -1 + (1 - f_2) (h\theta) \left( \frac{f_1}{R^2 + \alpha f_2} \right) a^2 f_2^{-1} \right] \left( \frac{e^{\theta} f_2}{R^2 + \alpha e^{\theta} f_2} \right)^h = 0. \]
Only when \( f_1 = f_2 \) can both equations be satisfied.\(^\text{46}\)

Given that \( f_1 = f_2 \), we have:
\[ P = \left( \frac{1}{1 + \alpha e^\theta} \right), \quad (1 - P) = \left( \frac{e^\theta}{1 + \alpha e^\theta} \right) \text{ and } \frac{1 - P}{P} = a \frac{m}{1 - a}. \]

\( P \) is negatively related to \( a \). As \( hm \) tends to 1, \( P \) tends to zero. In other words, for \( hm \geq 1 \), the disadvantaged contestant has disappeared due to more intensive conflicts.

We also have:
\[ f = \frac{hm a e^\theta}{1 - hm a e^\theta} \text{ where } \]
\[ f < 1 \text{ if } hm < 1, \]
\[ f = 1 \text{ if } hm = 1 \text{ and } \]
\[ f > 1 \text{ if } hm > 1. \]

The requirement for an interior solution is the same as the symmetrical case: \( hm < 1. \) Since \( f_1 = f_2 \), both contestants have zero profits when \( hm = 1. \) At that point however, the disadvantaged contestant has disappeared since for \( hm = 1, P = 0. \) For a sustainable asymmetrical game with the disadvantaged player capturing a positive share of the resources and both players having at least zero profits, we need \( hm < 1. \) For \( hm \geq 1 \), there is only one player, as the contestant with higher conversion costs do not capture any positive share of resources at equilibrium.

3.3. Absolute budgetary approach.

In this section we will analyze the various games with the absolute budgetary approach and compare the results with those that we derived under the proportionate approach.

3.31. The symmetrical absolute budgetary game.

We will now solve the symmetrical game with absolute budgetary approach.

Contestant 1 solves
\[ \max_{F_1} \left( \frac{F_1^m}{F_1^m + c_2^m} \right)^h - F_1. \]
First order condition is
\[ \pi_1 = hm \left( \frac{F_1^m}{F_1^m + c_2^m} \right)^h \left( \frac{F_1^m}{F_1^m + c_2^m} \right) F_1^{h-1} - 1 = 0. \]

\(^{46}\)This is in contradiction to what Hirshleifer (1995, p. 39-40) claims to the effect that the more efficient contestant (with the lower conversion costs) will have a higher proportionate military effort. Hirshleifer relies on simulations, and it is not clear that he derived the equilibrium success ratio for the asymmetrical case.
Evaluating at the symmetrical Nash equilibrium, we have
\[ F = hm2^{-1}(h+1) \] and \[ f = hm^{-1}. \]
Both the absolute amount and proportion of resources devoted to fighting are lower under the absolute budgetary approach.

The absolute amount of fighting effort under absolute budgetary approach is
\[ \left( \frac{hm}{2} \right) \left( 2^{-h} \right), \] which is less than the absolute amount of fighting effort under proportionate budgetary approach, \( \left( \frac{hm}{2-hm} \right) \left( 2^{-h} \right). \)

Proportionate amount of fighting effort under absolute budgetary approach is
\[ \left( \frac{hm}{2} \right), \] which is less than the proportionate amount of fighting effort under proportionate budgetary approach, \( \left( \frac{hm}{2-hm} \right). \)

At symmetrical equilibrium, \( \frac{dF}{dm} > 0 \) and \( \frac{df}{dm} > 0. \)
\[ \pi^1 = \pi^2 = 2^{-h} - hm2^{-(h+1)} = 2^{-h} \left( 1 - hm2^{-1} \right). \]

We need \( 1 - hm2^{-1} \geq 0 \) or \( hm \leq 2 \) for non-negative profits for the contestants.
For \( hm > 2 \), conflict is so intense that both contestants set so high a military expenditure such that both have negative profits. This cannot be sustained. For an equilibrium, one must exit the game.

Profits of contestants under absolute budgetary approach are greater than profits of contestants under proportionate budgetary approach:
\[ 2^{-h} \left( 1 - \frac{hm}{2} \right) > 2^{-h} \left( 1 - \frac{hm}{2-hm} \right). \]

Evaluating at symmetrical Nash equilibrium, we have the slope of the reaction function:
\[ \frac{dF}{dF_x} = \frac{m(h-1)}{2(h-1)m2^{-(h-1)}}. \]

Within the region of stable interior solution, depending on the value of the scale factor, the military expenditures of both contestants could be either strategic complements or substitutes. They are strategic complements for \( h < 1 \) and strategic substitutes for \( h > 1 \).

For \( 1 > (h-1)m \), there is strategic stability. When \( h \leq 1 \), the system will be stable irrespective of the value of \( m \). When there are no economies of scale in production, the contestants have no incentive to capture a larger and larger share of resources. Even if it is advantageous to have a larger fighting force (the value of mass factor is greater than one), the incentive to do so is not there. The result is strategic stability whatever the value of mass factor.\(^{47}\)

\(^{47}\)See the section on asymmetrical absolute budgetary game for another perspective on the stability condition.
A summary of the game is in diagram 2.48

Comparing with the proportionate budgetary approach, we note the following differences:

i.) For the absolute budgetary game, within the region of stable interior solution, the military expenditure of contestants are strategic complements for \( h < 1 \) and strategic substitutes for \( h > 1 \).

For the proportionate budgetary game, within the region of stable interior solution, the military expenditures of contestants are strategic complements.

ii.) For the absolute budgetary game, strategic stability is attained for \( 1 > (h - 1) m \). When there is constant or decreasing returns to scale in production, i.e., \( h \leq 1 \), the mass factor of conflict technology has no impact on the strategic stability of the system. The system is stable irrespective of the value of \( m \).

For the proportionate budgetary game, there is strategic stability for \( hm < 2 \).

iii.) For the absolute budgetary game, we need \( hm < 2 \) for at least zero profits condition.

For the proportionate budgetary game, we need \( hm < 1 \) for at least zero profits condition.

iv.) The symmetrical game with absolute budgeting approach has a larger set of interior and stable solutions then the game with proportionate budgetary approach. Fighting effort in both absolute and proportionate terms is smaller than that under the proportionate budgetary approach. Profits are larger than those under the proportionate budgetary approach.49

3.32 Endogenous Number of Contestants with absolute budgetary approach.

If there are \( N \) contestants, the typical contestant \( i \) solves

\[
\max_{F_i} \left( \frac{F_i}{F_i + \sum_{j \neq i}^N F_j} \right)^h - F_i.
\]

First order condition is:

\[
hm \left( \frac{F_i}{F_i + \sum_{j \neq i}^N F_j} \right)^h \left( \frac{\sum_{j \neq i}^N F_j}{F_i + \sum_{j \neq i}^N F_j} \right)^{h-1} - 1 = 0.
\]

48See also appendix 3.31.
49For details, refer to appendix 3.21 and 3.31 and diagram 1 and 2.
Given the assumption of identical contestants, the first order condition can be simplified to:

\[ hmN^{-a} (N - 1) = F. \]

The profits of the representative contestant are:

\[ \pi = N^{-h} - F \]

\[ = N^{-h}(1 - hmN^{-1} (N - 1)). \]

Setting \( \pi = 0 \) for zero profits condition, we have:

\[ N = \frac{hm}{hm-1}. \]

When the aggregate scale factor is equal to or greater than two, there is only one contestant in equilibrium. When the aggregate scale factor is smaller than two but larger than one, the number of contestants is larger than one but finite. As the aggregate scale factor reduces in value, the number of contestant increases. When the aggregate scale factor approaches one from above, the number of contestants approaches infinity. When the aggregate scale factor is equal to or smaller than one, the number of contestants is infinite. This accords with the intuition we get from the entry deterrence game: when the aggregate scale factor is equal to or smaller than one, it is impossible to deter entry of new contestants.50 A good example is the simultaneous collapse of ancient empires all over the world during A.D. 200—400. Military technology that caused ascendancy of cavalry relative to infantry reduced economies of scale in application of force as cavalry relies less on superiority of number to win battles. Old empires disintegrated as a result and in their place came myriad tiny states or state-like force wielding organizations.51

3.33. Asymmetrical absolute budgetary game.

We denote the conversion costs for the disadvantaged contestant 1 as \( a \), where \( a > 1 \).

Contestant 1 solves

\[
\max_{F_1} \left( \frac{F_1^m}{F_1^m + F_2^m} \right)^h - aF_1. 
\]

FOC is

\[ hm \left( \frac{F_1^m}{F_1^m + F_2^m} \right)^h \left( \frac{F_1^m}{F_1^m + F_2^m} \right) F_1^{-1} - a = 0. \]

Contestant 2 solves

\[
\max_{F_2} \frac{F_2^m}{F_1^m + F_2^m} - F_2. 
\]

50For the technical visibility of deploying entry deterrence strategy under various parameters values, see appendix 3.32.

FOC is:

\[
hm \left( \frac{r_1}{r_1 + F_2} \right)^{h} \left( \frac{r_2}{r_2 + F_2} \right) F_2^{-1} - 1 = 0.
\]

From above, we have:

\[
F_1 = \left( \frac{a}{1} \right)^{\left( \frac{1}{1 + \frac{m}{1 + m}} \right)} F_2 \quad \text{and}
\]

\[
P = \frac{1}{1 + \frac{m}{1 + \frac{m}{1 + m}}}, \quad 1 - P = a \left( \frac{1 + \frac{m}{1 + m}}{1 + \frac{m}{1 + m}} \right) \quad \text{and} \quad \frac{1 - P}{P} = a \left( \frac{1 + \frac{m}{1 + m}}{1 + \frac{m}{1 + m}} \right).
\]

When \(m(h - 1)\) tends to 1 and \(\left( \frac{1}{1 + \frac{m}{1 + m}} \right)\) tends to infinity, we have \(F_2\) tends to zero. For \(m(h - 1) \geq 1\) and therefore \(\left( \frac{1}{1 + \frac{m}{1 + m}} \right) \leq 0\), the higher cost contestant has disappeared due to his disadvantage in converting resources under his control into military capacity.

The condition for the viability of a higher conversion cost contestant is the same as that for the strategic stability condition in the symmetrical game:

\[m(h - 1) < 1.\]

We have

\[
F_1 = \frac{hm}{a} \left( \frac{1}{1 + \frac{m}{1 + m(h - 1)}} \right)^h \left( \frac{a \left( 1 + \frac{m}{1 + m(h - 1)} \right)}{1 + \frac{m}{1 + m(h - 1)}} \right)
\]

\[
\pi^1 = \left( \frac{1}{1 + \frac{m}{1 + m(h - 1)}} \right)^h \left[ 1 - hm \left( \frac{a \left( 1 + \frac{m}{1 + m(h - 1)} \right)}{1 + \frac{m}{1 + m(h - 1)}} \right) \right].
\]

Since \(\left( \frac{a \left( 1 + \frac{m}{1 + m(h - 1)} \right)}{1 + \frac{m}{1 + m(h - 1)}} \right)^h > \frac{1}{2}\), we need \(hm < 2\) for \(\pi^1 > 0\). Only when \(hm \leq\)

\[
\left( \frac{a \left( 1 + \frac{m}{1 + m(h - 1)} \right)}{1 + \frac{m}{1 + m(h - 1)}} \right)^{-1} < 2 \text{ will the disadvantaged player has at least zero profit condition.}
\]

Since \(1 > \left( \frac{a \left( 1 + \frac{m}{1 + m(h - 1)} \right)}{1 + \frac{m}{1 + m(h - 1)}} \right)^{-1}\), at \(\pi^1 = 0\) we have \(hm > 1\).

The line demarcating the zero profit condition for the disadvantaged contestant lies above \(hm = 1\) and below \(hm = 2\). The line for at least a positive share for the disadvantaged contestant for the absolute budgetary approach is \(m(h - 1) < 1\). It lies above \(hm = 1\) as well.

For the proportionate budgetary approach, only when \(hm < 1\) will the disadvantaged contestant have at least zero profit and a positive share of captured resources. The asymmetrical game under the proportionate budgetary approach has a smaller set of sustainable equilibria which the disadvantaged contestant having at least zero profit and a positive share of resources than under the absolute budgetary approach.

\[\text{52 See section 3.31.}\]
The ratio of the share of resources of the advantaged to disadvantaged contestants under proportionate budgetary approach is \( \frac{R_P}{R_P} = a\left(\frac{m}{1-hm}\right) \) and under absolute budgetary approach is \( \frac{R_P}{R_P} = a\left(\frac{m}{1-hm}\right) \). The ratio is larger under the proportionate budgetary approach.

The ratio of proportionate military efforts of the two contestants is:
\[
\frac{I_1}{I_2} = a\left(\frac{m}{1+h}\right) = a\left(\frac{m}{1-h}\right).
\]

We have the following subcases:

a.) For \( \frac{m}{m+(1-hm)} > 1 \), we have the disadvantaged contestant fights proportionately harder.

b.) For \( \frac{m}{m+(1-hm)} = 1 \), we have the disadvantaged contestant fights proportionately as hard as the advantaged contestant, although he still invest less in fighting absolutely since he controls a smaller share of resources.

c.) For \( \frac{m}{m+(1-hm)} < 1 \), we have the disadvantaged contestant fights proportionately less harder.

The region where under asymmetrical proportionate budgetary game the disadvantaged contestant still survives, that is where \( hm < 1 \), belongs to subcase c. Unlike the proportionate game, where both contestants have equal proportionate fighting effort in this region, under the absolute game the disadvantaged player fights proportionately less harder. This is the region where the scale and mass factors are smaller. Under the absolute budgetary game, which is a less intensive conflict game, the disadvantaged player chooses to shirk. For larger mass and scale factors, the disadvantaged player in the absolute game might fight as hard (subcase b.) or harder (subcase a.) than the advantaged player. This two subcases however lies in regions \( (hm = 1 \text{ and } hm > 1) \) where under the proportionate game the disadvantaged player does not survive.

The absolute budgetary approach, to conclude, results in a less intense asymmetrical conflict between the contestants than the proportionate budgetary approach.
3.4. Conclusion.

With complete feedback loop from the military to the economy and from the economy to the military, the economy becomes part of the war machine. Both the scale factor in production and the mass factor in conflict therefore have strategic implications. They jointly determine the nature of conflict and the sustainable number of contestants. Diseconomies of scale in one technology to some extent may be compensated by economies of scale in the other technology. All important lines in diagram 1 and 2 slope downward. For instance, it is well known that Oriental civilizations based on river basin agriculture developed large centralized empires due to the need for large scale treatment of flood control and irrigation management. This is the so-called "hydraulic society". In effect, a large value for the production scale factor led to a large natural monopoly state.53

The value of the mass factor and the scale factor also jointly decide the viability of the disadvantaged player. Given asymmetry in war efficiency, the disadvantaged contestant could survive under small scale and mass factors but would opt out of the game or be wiped out by the advantaged rival under larger scale and mass factors.

The collapse of the Soviet Union is a good illustration. Reasons commonly cited for her collapse, such as minority problems, rigidity caused by central planning, backward technology and geopolitical disadvantage as a land-locked country, explain the asymmetry in war efficiency between U.S.S.R. and her Western rivals. They however do not explain why U.S.S.R. managed to survive and do well in her earlier history but failed to cope with rivalry with the Western powers in later part of her history. The U.S.S.R., being a disadvantaged contestant, controlled a smaller share of total world resources than the Western camp. This shortcoming would not be fatal if the scale factor is not too large. Since economic integration with the free world was ruled out due to the cold war confrontation, as the scale factor grew in size, the disadvantage escalated and the U.S.S.R. ran into financial and economic difficulty. Ultimately, the insolvent Soviet Union was dismantled by her CEOs.54

The way the contestants set their military budget is strategically important. There

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53 For a very good discussion on the impact of military and information technology on the size of states, refer to Dudley (1991).

54 There are two major reasons to suspect that the scale factor for the whole economy has grown in size since WWII. The first is the emergence of many global and regional free trade associations and economic unions, such as European Community, Association of Southeast Asian Nations and North American Free Trade Area. The second is the failure of self sufficient import substitution strategies and the success of export oriented economies.
are three essential points of difference between the two approaches:

a.) Under symmetrical Nash equilibrium, fighting effort in both absolute and proportionate terms is larger under proportionate budgetary approach than under absolute budgetary approach. Profits are smaller under the proportionate budgetary approach than under the absolute budgetary approach.

b.) Endogenous number of contestants.

With the proportionate budgetary approach, when the aggregate scale factor is smaller than one, the number of contestants is infinite. When the aggregate scale factor is equal to one, the number of contestants is indeterminate. When the aggregate scale factor is larger than one, there is only one contestant.

With the absolute budgetary approach, when the aggregate scale factor is equal to or smaller than one, the number of contestants is infinite. When the aggregate scale factor is smaller than two but larger than one, the number of contestants is negatively related to the aggregate scale factor and larger than one but finite. When the aggregate scale factor is equal to or greater than two, there is only one contestant.

c.) The set of interior and stable equilibria for the symmetrical game and the set of sustainable asymmetrical equilibria where the disadvantaged contestant have at least zero profit and a positive share are both smaller under the proportionate budgetary approach than under the absolute budgetary approach.

d.) It is technically impossible to adopt entry deterrence strategy under the proportionate budgetary approach whatever the value of the mass factor and the scale factor. For the absolute budgetary approach, entry deterrence is technical feasible only if the aggregate scale factor is larger than one.

In the proportionate approach, a contestant having a larger share of resources devoted to fighting has a dual advantage at equilibrium: he has captured a larger share of resources and, he has devoted a larger part of captured resources to fighting. By tying the size of the military budget to the size of resources captured and the size of the economy that built on the captured resources, the size of the military is determined by both the proportion of resources or production to be devoted to conflict and the size of the captured resources or production resulted from it. The size of resources captured therefore has dual importance. It decides the gross production or revenue from conflict. It also in part determined the size of the military budget. In contrast, in the absolute budgetary approach, size of resources captured decides only the gross production or revenue from conflict but not the size of
the military budget. The outcome of conflict therefore is of greater importance in the proportionate budgetary game than the absolute budgetary game. The result is that the proportionate budgetary approach has a more intense contest than the absolute budgetary approach in which the contestants set a fixed military budget not tying to the size of the captured resources or the size of the economy that built on the captured resources.
Diagram 1: The symmetrical proportionate budgetary game.
Note for Diagram 1: The symmetrical proportionate budgetary game.

Line \( a : hm = 1. \)

Line \( b : hm = 2. \)

Line \( c : (h - 1) m = 2. \)

Line \( d : (h - 2) m = 2. \)

Area 1: Military expenditures of contestants are strategic complements. There is strategic stability. Second order condition is satisfied. Profits of contestants are positive.

Area 2: Military expenditures of contestants are strategic complements. There is strategic stability. Second order condition is violated. Profits of contestants are negative.

Area 3: Military expenditures of contestants are strategic complements. There is strategic instability. The second order condition is satisfied. Profits of contestants are negative.

Area 4: Military expenditures of contestants are strategic substitutes. There is strategic instability. Second order condition is violated. Profits of contestants are negative.

Area 5: Military expenditures of contestants are strategic substitutes. There is strategic stability. Second order condition is violated. Profits of contestants are negative.
Diagram 2: The symmetrical absolute budgetary game.
Note for Diagram 2: The symmetrical absolute budgetary game.

Line a: \( hm = 2 \).
Line b: \( h = 1 \).
Line c: \( (h - 1)m = 1 \).
Line d: \( (h - 1)m = 2 \).

Area 1: Military expenditures of contestants are strategic complements. There is strategic stability. Second order condition is satisfied. Profits of contestants are positive.

Area 2: Military expenditures of contestants are strategic substitutes. There is strategic stability. Second order condition is satisfied. Profits of contestants are positive.

Area 3: Military expenditures of contestants are strategic substitutes. There is strategic instability. The second order condition is satisfied. Profits of contestants are positive.

Area 4: Military expenditures of contestants are strategic complements. There is strategic stability. Second order condition is satisfied. Profits of contestants are negative.

Area 5: Military expenditures of contestants are strategic substitutes. There is strategic stability. Second order condition is satisfied. Profits of contestants are negative.

Area 6: Military expenditures of contestants are strategic substitutes. There is strategic instability. Second order condition is satisfied. Profits of contestants are negative.

Area 7: Military expenditures of contestants are strategic complements. There is strategic instability. Second order condition is violated. Profits of contestants are negative.
Appendixes.

3.21. Symmetrical proportionate budgetary game.

Contestant one solves
\[ \max \pi^1 = (1 - f_1) \left( \frac{f_1^2}{f_1^2 + f_2^2} \right)^h. \]

First order condition is:
\[ \pi^1 = -\left( \frac{f_1^2}{f_1^2 + f_2^2} \right)^h + (1 - f_1) h \theta \left( \frac{f_1^2}{f_1^2 + f_2^2} \right)^h \left( \frac{f_2^2}{f_1^2 + f_2^2} \right) f_1^{-1} = 0. \]

This reduces to
\[ -1 + (1 - f_1) h \theta \left( \frac{f_1^2}{f_1^2 + f_2^2} \right) f_1^{-1} = 0. \]

At a symmetrical equilibrium,
\[ (1 - f) h \theta \left( \frac{1}{2} \right) = f. \]

That gives us
\[ f = \frac{hm}{2(1-hm)+hm} \quad \text{and} \quad F = \left( \frac{hm}{2(1-hm)+hm} \right)^{2-h}. \]

From above, we know that \( f < 1 \) if \( hm < 1 \), \( \lim_{hm \to 1} f = 1 \), \( f > 1 \) if \( hm > 1 \).

An interior solution exists if and only if \( hm < 1 \).

Profit at a symmetrical Nash equilibrium is
\[ \pi = (1 - f)^{2-h} = \left( \frac{2(1-hm)}{2(1-hm)+hm} \right)^{2-h}. \]

\[ \pi_{11} = -h \theta^2 \left( f_1^{-1} - 1 \right) \left( \frac{f_1^2}{f_1^2 + f_2^2} \right) \left( \frac{f_2^2}{f_1^2 + f_2^2} \right) f_1^{-1} - h \theta \left( \frac{f_2^2}{f_1^2 + f_2^2} \right) f_1^{-2}. \]

Evaluating at a symmetrical equilibrium,
\[ \pi_{11} = \left(-1\right) \left( \frac{2(1-hm)+hm}{2(1-hm)} \right) \left( \frac{m+2(1-hm)+hm}{hm} \right) \]
\[ = \left(-1\right) \left( \frac{2-hm}{2(1-hm)} \right) \left( \frac{m+2-hm}{hm} \right). \]

We have the sign of \( \pi_{11} \) for the following parameter values:
\[ \pi_{11} < 0 \quad \text{for} \quad hm \leq 1, \]
\[ \pi_{11} > 0 \quad \text{for} \quad 1 < hm < 2, \]
\[ \pi_{11} = 0 \quad \text{for} \quad hm = 2, \]
\[ \pi_{11} < 0 \quad \text{for} \quad (h-1)m < 2 < hm, \]
\[ \pi_{11} = 0 \quad \text{for} \quad (h-1)m = 2 < hm, \]
\[ \pi_{11} > 0 \quad \text{for} \quad 2 < (h-1)m < hm. \]

The cross derivative is:
\[ \pi_{12} = h \theta^2 \left( f_1^{-1} - 1 \right) \left( \frac{f_1^2}{f_1^2 + f_2^2} \right) \left( \frac{f_2^2}{f_1^2 + f_2^2} \right) f_2^{-1}. \]

Evaluating at a symmetrical equilibrium,
\[ \pi_{12} = \left( \frac{2(1-hm)+hm}{2(1-hm)} \right) \left( \frac{1}{h} \right). \]
\[ = \left( \frac{2-hm}{2(1-hm)} \right) \left( \frac{1}{h} \right) \]
\[ \pi_{12} > 0 \text{ for } hm \leq 1, \]
\[ \pi_{12} < 0 \text{ for } 1 < hm < 2, \]
\[ \pi_{12} = 0 \text{ for } hm = 2, \]
\[ \pi_{12} > 0 \text{ for } 2 < hm. \]

The slope of the reaction function is
\[ \frac{\partial f_1}{\partial f_2} = -\frac{\pi_{12}}{\pi_{11}} \]
\[ = 2 + m - hm. \]

\( \frac{\partial f_1}{\partial f_2} \) is positive (military efforts for contestants are strategic complements) for \( f < 1 \) (\( hm < 1 \)).

Checking for strategic stability condition at a symmetrical equilibrium:
\[ (\pi_{11})^2 - (\pi_{12})^2 = \]
\[ \left[ (-1) \left( \frac{2(1-hm)+hm}{2(1-hm)} \right) \left( \frac{m+2(1-hm)+hm}{hm} \right) \right]^2 - \left[ \left( \frac{2(1-hm)+hm}{2(1-hm)} \right) \left( \frac{1}{h} \right) \right]^2 \]
\[ = \left( \frac{2(1-hm)+hm}{2(1-hm)hm} \right)^2 \left[ \left( \frac{m+2(1-hm)+hm}{hm} \right)^2 - 1 \right] \]
\[ = \left( \frac{2-hm}{2(1-hm)hm} \right)^2 \left[ 4(1-hm) + 2m(2-hm) + h^2m^2 \right]. \]
\[ = \left( \frac{2-hm}{2(1-hm)hm} \right)^2 (2 - (h - 2)m)(2 - hm). \]

For strategic stability, we need above to be positive.

For \( 0 < h < 2 \), we have the following subcases:

i. \( (2 - (h - 2)m) < 0 \) and \( 2 - hm < 0 \).

Given that \( h < 2 \), the condition that \( (2 - (h - 2)m) < 0 \) cannot be satisfied.

ii. \( (2 - hm) > 0 \) and \( (2 - (h - 2)m) > 0 \).

Since \( h < 2 \), the condition that \( (2 - (h - 2)m) > 0 \) is satisfied automatically.

There is strategic stability if \( 2 > hm \).

For \( 2 \leq h \), we have the following subcases:

i. \( (2 - hm) < 0 \) and \( (2 - (h - 2)m) < 0 \).

When \( 2 < m(h - 2) \) both inequalities are satisfied and there is strategic stability.

ii. \( (2 - hm) > 0 \) and \( (2 - (h - 2)m) > 0 \).

When \( hm < 2 \) both inequalities are satisfied and there is strategic stability.

The summary of this section is in Diagram 1.

3.22. Entry deterrence with proportionate budgetary approach.

We will analyze the technical feasibility of an entry deterrence strategy. Contestant 2 is the incumbent monopoly state who has to decide whether to deter or accommodate the
entry of a potential rival, contestant 1.

Contestant one solves
\[
\max_{f_1} \pi_1 = (1 - f_1) \left( \frac{f_1^h}{f_1^h + f_2^h} \right)^h.
\]
First order condition is:
\[
\pi_1^1 = - \left( \frac{f_1^h}{f_1^h + f_2^h} \right)^h + (1 - f_1) h \theta \left( \frac{f_1^h}{f_1^h + f_2^h} \right)^h \left( \frac{f_2^h}{f_1^h + f_2^h} \right) f_1^{-1} = 0.
\]
This reduces to
\[-1 + (1 - f_1) h \theta \left( \frac{f_2^h}{f_1^h + f_2^h} \right) f_1^{-1} = 0.
\]
Second derivative is
\[
\pi_{11}^1 = -h \theta^2 \left( f_1^{-1} - 1 \right) \left( \frac{f_1^h}{f_1^h + f_2^h} \right) \left( \frac{f_2^h}{f_1^h + f_2^h} \right) f_1^{-1} - h \theta \left( \frac{f_2^h}{f_1^h + f_2^h} \right) f_1^{-2}.
\]
That gives
\[
\pi_{11}^1 = \left[ -\theta \left( 1 - f_1 \right) \left( \frac{f_1^h}{f_1^h + f_2^h} \right) - 1 \right] \left( \frac{f_2^h}{f_1^h + f_2^h} \right) h \theta f_1^{-2}.
\]
Contestant two sets \( f_2 \) such that
\[
\pi_1^1 = 0,
\]
\[
\pi_{11}^1 < 0
\]
and
\[
(1 - f_1) \leq 0.
\]
If \((1 - f_1) = 0\), then \(\pi_1^1 < 0\). Entry deterrence is infeasible irrespective of the value of \(h m\).

If \((1 - f_1) < 0\) and \(h m > 1\), while it is possible that \(\pi_1^1 = 0\), we however have \(\pi_{11}^1 > 0\). Entry deterrence is infeasible.

If \((1 - f_1) < 0\) and \(h m = 1\), we have \(\pi_1^1 = -\infty\) and \(\pi_{11}^1 = \infty\). Entry deterrence is infeasible.

If \((1 - f_1) < 0\) and \(h m < 1\), then \(\pi_1^1 < 0\). Entry deterrence is infeasible.

To conclude, entry deterrence strategy is infeasible with the proportionate budgetary approach.

3.23. Asymmetrical proportionate budgetary game.

we normalize the conversion costs for contestant 2 as one. We denote the conversion costs for contestant 1 as \(a\), where \(a > 1\).

Contestant 1 solves
\[
\max_{f_1} (1 - f_1) P^h,
\]
where \(f_1 = \frac{a f_1}{P^h}\).

Contestant 2 solves
\[
\max_{f_2} (1 - f_2) (1 - P)^h,
\]
where \(f_2 = \frac{f_2}{(1 - P)^h} \).

The First Order Condition for contestant 1 is
\[
\pi_1 = -\left( \frac{f_1^h}{1 + \alpha f_2^h} \right) + (1 - f_1) (h\theta) \left( \frac{f_1^h}{1 + \alpha f_2^h} \right) f_1^{-1} = 0.
\]
\[-1 + (1 - f_1) (h\theta) \left( \frac{f_1^h}{1 + \alpha f_2^h} \right) f_1^{-1} = 0.
\]

The First Order Condition for contestant 2 is
\[
\pi_2 = -\left( \frac{f_2^h}{1 + \alpha f_2^h} \right) + (1 - f_2) (h\theta) \left( \frac{f_2^h}{1 + \alpha f_2^h} \right) f_2^{-1} = 0.
\]
\[-1 + (1 - f_2) (h\theta) \left( \frac{f_2^h}{1 + \alpha f_2^h} \right) f_2^{-1} = 0.
\]

\(\pi_1\) can be rewritten as
\[
(h\theta) \left( \frac{f_1^h}{1 + \alpha f_2^h} \right) = \frac{f_1}{1 - f_1}
\]
and
\(\pi_2\) can be rewritten as
\[
(h\theta) \left( \frac{f_2^h}{1 + \alpha f_2^h} \right) = \frac{f_2}{1 - f_2}.
\]

It is apparent that only when \(f_1 = f_2\) can both equations be satisfied.

Given that \(f_1 = f_2\), from both FOC we have
\[-1 + (1 - f) (h\theta) \left( \frac{\alpha}{1 + \alpha} \right) f^{-1} = 0
\]
and \(P = \left( \frac{1}{1 + \alpha} \right)\) and \((1 - P) = \left( \frac{\alpha}{1 + \alpha} \right)\) and \(1 - P = a \frac{m}{1 - hm}\).

\(P\) is negatively related to \(a\). As \(hm\) tends to 1, \(P\) tends to zero. In other words, for \(hm \geq 1\), the disadvantaged contestant disappeared.

We also have
\[
f = \frac{h\theta \left( \frac{\alpha}{1 + \alpha} \right)}{1 + h\theta \left( \frac{\alpha}{1 + \alpha} \right)}.
\]

\(f\) is positively related to \(a\).

Above could be expressed as
\[
f = \frac{hm \left( \frac{\alpha}{1 + \alpha} \right)}{1 - hm + hm \left( \frac{\alpha}{1 + \alpha} \right)} = \frac{hma}{1 - hm + \alpha}.
\]

\(f < 1\) if \(hm < 1\),
\(f = 1\) if \(hm = 1\) and
\(f > 1\) if \(hm > 1\).

The requirement for an interior solution is the same as the symmetrical case.

Since \(f_1 = f_2\), both contestants face the zero profit condition when \(hm = 1\). At that point however, the disadvantaged contestant has disappeared since for \(hm = 1, P = 0\). Therefore, for a sustainable asymmetrical game, we need \(hm < 1\).
3.31. Symmetrical absolute budgetary game.

Contestant 1 solves
\[ \max_{F_1} \left( \frac{F_1^m}{F_1^m + F_2^m} \right)^h - F_1. \]

First order condition is:
\[ \pi_1^1 = hm \left( \frac{F_1^m}{F_1^m + F_2^m} \right)^h \left( \frac{F_1^m}{F_1^m + F_2^m} \right) F_1^{-1} - 1 = 0. \]

Evaluating at the symmetrical equilibrium, we have
\[ F = hm 2^{-(h+1)} \text{ and } f = hm 2^{-1}. \]
\[ \pi_1^1 = \pi_2^2 = 2^{-h} - hm 2^{-(h+1)} = 2^{-h} \left( 1 - hm 2^{-1} \right). \]

We need \(1 - hm 2^{-1} \geq 0\) or \(hm \leq 2\) for zero or positive profits for the contestants.

For \(hm > 2\), conflict is so intense that both contestants set so high a military expenditure such that both have negative profits. One must exit the game.

Second derivative is:
\[ \pi_{11}^1 = hm \left( \frac{F_1^m}{F_1^m + F_2^m} \right)^h \left( \frac{F_1^m}{F_1^m + F_2^m} \right) F_1^{-2} \left[ hm \left( \frac{F_1^m}{F_1^m + F_2^m} \right) - 1 - m \left( \frac{F_1^m}{F_1^m + F_2^m} \right) \right]. \]

Evaluating at the symmetrical equilibrium, we have:
\[ \pi_{11}^1 = 2^{-/(h+1)} F^{-2} hm \left( h - 1 \right) m 2^{-1} - 1. \]

For \((h - 1) m < 2\), we have \(\pi_{11}^1 < 0\).
\[ \pi_{12}^1 = hm^2 \left( \frac{F_1^m}{F_1^m + F_2^m} \right)^h \left( \frac{F_1^m}{F_1^m + F_2^m} \right) F_1^{-1} F_2^{-1} \left[ \left( \frac{F_1^m}{F_1^m + F_2^m} \right) - h \left( \frac{F_1^m}{F_1^m + F_2^m} \right) \right]. \]

Evaluating at the symmetrical equilibrium, we have:
\[ \pi_{12}^1 = 2^{-(h+2)} F^{-2} hm^2 \left( 1 - h \right). \]

Evaluating at a symmetrical equilibrium, we have the slope of the reaction function:
\[ \frac{\partial F_1}{\partial F_2} = \frac{m(h-1)}{2(h-1)m 2^{-1} - 1}. \]

For strategic stability, we check the sign of
\[ (\pi_{11}^1)^2 - (\pi_{12}^1)^2. \]

Above will be positive if \(1 > (h - 1) m\).

3.32. Entry deterrence with absolute budgetary approach.

We will analyze the technical feasibility of adopting an entry deterrence strategy with absolute budgetary approach.

Contestant 1 solves
\[ \max_{F_1} \left( \frac{F_1^m}{F_1^m + F_2^m} \right)^h - F_1. \]

First order condition is:
\[ \pi_1^1 = hm \left( \frac{F_1^m}{F_1^m + F_2^m} \right)^h \left( \frac{F_1^m}{F_1^m + F_2^m} \right) F_1^{-1} - 1 = 0. \]

Second and cross derivatives are:
\[
\pi_{11} = hm \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right)^{h} \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) F_1^{-2} \left[ hm \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) - 1 - m \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) \right].
\]
\[
\pi_{12} = hm^2 \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right)^{h} \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) F_1^{-1} F_2^{-1} \left[ \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) - h \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) \right].
\]

Contestant 2 sets \( F_2 \) such that
\[
hm \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right)^{h} \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) F_1^{-1} - 1 = 0 \quad \text{and} \quad \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right)^{h} - F_1 = 0.
\]

From the two constraints, we have \( \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) = \left( \frac{1}{hm} \right) \), \( \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) = \left( \frac{hm-1}{hm} \right) \) and

\[
F_1 = \left( \frac{hm-1}{hm} \right)^{h}. \quad \text{Solving for } F_2 \text{ gives:}
\]
\[
F_2 = (hm)^{-h} \left( hm - 1 \right)^{h-1/m}.
\]

Since contestant 1's entry is deterred, we have:
\[
\pi^2 = 1 - (hm)^{-h} \left( hm - 1 \right)^{h-1/m}.
\]

From above, we have \( hm \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) = 1. \) That gives us:
\[
\pi_{11} = -m \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right)^{1-h} < 0.
\]

The second order condition for contestant 1 is satisfied.

Since if contestant 1 enters the contest, we have \( F_1 > 0. \) That implies \( \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) < 1. \) We therefore arrived at the following restriction on the value of the scale and mass factors for a viable entry deterrence strategy:
\[
hm > 1.
\]

3.33. Asymmetrical absolute budgetary game.

We normalize the conversion costs for contestant 2 as one. We denote the conversion costs for contestant 1 as \( a, \) where \( a > 1. \)

Contestant 1 solves
\[
\max_{F_1} \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right)^{h} - aF_1.
\]

First order condition is:
\[
hm \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right)^{h} \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) F_1^{-1} - a = 0.
\]

Contestant 2 solves
\[
\max_{F_2} \frac{F_1^{m}}{F_1^{m} + F_2^{m}} - F_2.
\]

First order condition is:
\[
hm \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right)^{h} \left( \frac{F_1^{m}}{F_1^{m} + F_2^{m}} \right) F_2^{-1} - 1 = 0.
\]

From above, we have
\[
F_1 = \left( \frac{1}{a} \right)^{\left( 1+\frac{1}{m(1-h)} \right)} F_2.
\]

Depending on the value of the scale factor in production, we have:
a. For \( h = 1, \frac{F_1}{F_2} = \left( \frac{1}{a} \right) \): For constant return to scale in production, the ratio of investment in conflict between the advantaged player to disadvantaged player is equal to the asymmetry in conversion costs.

b. For \( h < 1, \frac{F_1}{F_2} > \left( \frac{1}{a} \right) \): Decreasing return to scale in production reduces the difference between \( F_1 \) and \( F_2 \), i.e., it reduces the impact of differences in conversion costs on the choice of fighting capacity.

c. For \( h > 1, \left( \frac{1}{a} \right) > \frac{F_1}{F_2} \): Increasing return to scale in production magnifies difference between \( F_1 \) and \( F_2 \), i.e., it increases the impact of differences in conversion costs on the choice of fighting capacity.

From above, we have \( P = \frac{1}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \), \( 1 - P = \frac{a \left( \frac{1 + m (1 - h)}{1 + m (1 - h)} \right)}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \), and \( \frac{1 - P}{P} = a \left( \frac{1 + m (1 - h)}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right) \).

When \( m (h - 1) \) tends to 1 and \( \left( \frac{m}{1 + m (1 - h)} \right) \) tends to infinity, we have \( \frac{F_1}{F_2} \) tends to zero. For \( m (h - 1) \geq 1 \) and therefore \( \left( \frac{a}{\frac{1 + m (1 - h)}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)}} \right) \leq 0 \), the higher costs contestant has disappeared due to the disadvantage.

The condition for the viability of a higher conversion costs contestant is the same as that for the strategic stability condition in the symmetrical game:

\( m (h - 1) < 1 \).

From \( P = \frac{1}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \) and the first order condition of contestant 1, we have

\[
F_1 = \frac{hm}{a} \left( \frac{1}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right)^h \left( a \left( \frac{1 + m (1 - h)}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right) \right) \quad \text{and} \\
f_1 = \frac{hm}{a} \left( a \left( \frac{1 + m (1 - h)}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right) \right)
\]

The profit of contestant 1 is

\[
\pi^1 = \left( \frac{1}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right)^h - a \frac{hm}{a} \left( \frac{1}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right)^h \left( a \left( \frac{1 + m (1 - h)}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right) \right) = \left( \frac{1}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right)^h \left[ 1 - hm \left( a \left( \frac{1 + m (1 - h)}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right) \right) \right] \]
\]

Since \( \left( a \left( \frac{1 + m (1 - h)}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right) \right) > \frac{1}{2} \), we need \( hm < 2 \) for \( \pi^1 \geq 0 \). In other words, only when \( hm \leq \left( a \left( \frac{1 + m (1 - h)}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right) \right)^{-1} < 2 \) will the disadvantaged player has at least zero profit.

Since \( 1 > \left( a \left( \frac{1 + m (1 - h)}{1 + a \left( \frac{m}{1 + m (1 - h)} \right)} \right) \right) \), at \( \pi^1 = 0 \) we have \( hm > 1 \).

The line demarcating the zero profit condition for the disadvantaged contestant lies
above $hm = 1$ and below $hm = 2$. The line for at least a positive share for the disadvantaged contestant for the absolute budgetary approach is $m(h-1) < 1$. It lies above $hm = 1$ as well.

From above, we have

$$F_2 = hm \left( \frac{a^{\frac{m}{1+m(1-h)}}}{1+a^{\frac{m}{1+m(1-h)}}} \right)^h \left( \frac{1}{1+a^{\frac{m}{1+m(1-h)}}} \right)$$

and

$$f_2 = hm \left( \frac{1}{1+a^{\frac{m}{1+m(1-h)}}} \right).$$

The ratio between the fight efforts of the two contestant is

$$\frac{f_1}{f_2} = a^{\frac{m}{1+m(1-h)}} - 1.$$ 

For $\frac{m}{1+m(1-h)} > 1$, we have the disadvantaged contestant fights proportionately harder.

For $\frac{m}{1+m(1-h)} = 1$, we have the disadvantaged contestant fights proportionately as hard as the advantaged contestant, although he still invest less in fighting absolutely since he control a smaller share of resources.

For $\frac{m}{1+m(1-h)} < 1$, we have the disadvantaged contestant fights proportionately less harder.
CHAPTER 4
A MODEL OF WAR AND STATE MAKING
AND THE CIVILIANIZATION PARADOX

Fukoku, Kyohei.
Rich country, strong soldiery.

4.1. Introduction.

The war and state making theory argues that it is the expectation and preparation for war, the actual conduct and experience of war and the post war reconstruction and reorganization that drive the state making process (Tilly, 1975, 1992; Rasler and Thompson, 1989; Duffy, 1980; Porter, 1994). The pioneer of the literature, Tilly (1975, p. 42.), put it this way: “war made the state, and the state made war.” Tilly (1975, p.73) summarized the European experience in state making and war making as: “The formation of standing armies provided the largest single incentive to extraction and the largest single means of state coercion over the long run of European state-making. Recurrently we find a chain of causation running from (1) change or expansion in land armies to (2) new efforts to extract resources from the subject population to (3) the development of new bureaucracy and administrative innovations to (4) resistance from the subject population to (5) renewed coercion to (6) durable increases in the bulk and extractiveness of the state.” It hypothesizes a relationship between the scale of international conflict and the choice of state fiscal apparatus and military capacity. The initial spark to this chain of events was the series of innovations in military technology that increased the economies of scale in war from the Sixteenth Century. This is the so called military revolution (Dudley, 1991; Duffy, 1980; Keegan, 1993; McNeill, 1982; Parker, 1996; Tilly, 1992).

Tilly (1992), who studied the millenary European war and state making experience, observed the paradox that “The state-transforming processes we have surveyed produced a surprising result: civilianization of government. The result is surprising because the expansion of military force drove the processes of state formation.” (Tilly, 1992; p. 122.) The civilianization of the war and state making process are supported by the following budgetary trends:
a.) "...... expenditure on non-military activities grew even faster than military expenditure. ...... Non-military activity and expenditure captured a larger and larger part of governmental attention. ......, non-military activities are ballooning so fast that military expenditure declined as a share of most state budgets, despite the great expansion of those budgets." (Tilly, 1992; p. 122-123.)

b.) "...... civilian production eventually grew quickly enough to outstrip military expansion, with the result that military expenditures declined as a share of national income. ...... Eventually, indeed, national income rose faster than military expenditure." (Tilly, 1992; p. 122-124.)

The above two points together with the well-known facts of a rising government share in national income as a result of war and state making effort (Tilly, 1992; Rasler and Thompson, 1989; Webber and Wildavsky, 1986; Porter, 1994; Weiss and Hobson, 1995) are the three budgetary stylized facts observed in the European experience of war and state making. In existing literature, so far there is no formal model to capture and explain them all. This paper will present such a model.

Concomitant with the rise of modern European states was the emergence of modern European economies. Jones (1981, 1988) and (Porter, 1994; Weiss and Hobson, 1995) argued that it was the unique competitive states system of Europe that resulted in her superior economic performance and subsequent industrialization. This paper will formalize this statist argument of geopolitically driven economic development and link it to the war and state making theory. This paper will show in a formal model how the military revolution has ushered in the bureaucratic revolution and the industrial revolution.

The literature discussing the causes of growth of government (Peacock and Wiseman, 1967; Bird, 1971; Thomas, 1984; Breton, 1989) did not make use of the insights gained from the research of the literature on property rights and conflicts. Rasler and Thompson (1989) and Porter (1994) argued that it was mainly the preparation for war, the actual conduct of war and the experience of war that caused states to involve extensively in the economy and install welfare programs. This paper will formalize and analyze the military origin of the interventionist welfare states. It will present a formal derivation of the Wag-

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55Tilly (1992) has given only 19th and 20th Century figures to illustrate this point. Historical statistics for earlier periods are scanty and unreliable. For details, refer to Flora (1987) and Mitchell (1975).

56The figures presented by Tilly (1992) better describe the rise of modern states and industrial economies in Europe than the case of the early starter England. For supporting figures and detailed historical analysis of the role of geopolitical pressure in the rise of English modern state and industrial economy in 18th and 19th Century, refer to Weiss and Hobson (1985).
ner's law and give a geopolitical interpretation to it. It will do so by endogenizing variables such as military expenditure, total state budget and size of economy in a model of interstate conflict.\textsuperscript{57}

This paper is an extension of the literature on property rights and conflicts (Hirshleifer, 1989, 1991 and 1995; Dixit, 1987; Skaperdas, 1992; Tullock, 1974). In the existing property rights and conflicts literature, conflicts and cooperation between contestants are analyzed. The contestants are however portrayed as monolithic entities without agency problems or hierarchy. The links between external conflicts and internal policy of contestants, such as taxation and public goods provision are therefore ignored. Such simplicity restricted the application of the literature to issues of great concern to economists and other social scientists. These issues includes the causes of growth of government, the rise of the welfare states, the geopolitical and military origins of modern states and modern economies and the role of the state in economic development. This paper will bridge the gap by analyzing the strategic behavior of states in conflict with fiscal regime of the states endogenized.

The key concept of the model is the increasing economies of scale in warfare (or the mass factor).\textsuperscript{58} A larger mass factor enhances the relative advantage of the bigger contestant. The mass factor is an aggregate technological parameter and does not refer to any specific technological improvement in the weaponry or the auxiliary system. Examples of increases in mass factor are: the emergence of large standing army and navy in the 16th and 17th Century due to the military use of fire power and the emergence of the citizen mass army from the French Revolution due to the political impact of nationalism. From 1500 onwards, armed forces and military budget grew in size, testifying to increases in the mass factor.\textsuperscript{59}

The pivotal mechanism of the model is the complementarity between the fiscal and military sectors of the state. This complementarity arises because of shared infrastructure or positive externalities between the two apparatus. Lane (1958) first emphasized the concept of complementarity between the fiscal and martial branches of the state. He did

\textsuperscript{57}A high tax rate and a high level of public goods provision allow the state to better penetrate, organize and mobilize the economy for its purposes. This is the hallmark of a modern state. A high level of state involvement in the economy is on the other hand the hallmark of an industrial economy. Refer to Porter (1994), Peacock and Wiseman (1967) and Webber and Wildavsky (1986) for detailed treatment.

\textsuperscript{58}For an original discussion of the economies of scale in warfare, refer to Hirshleifer (1989, 1991, 1995).

not, however, provide a formal model to draw out fully the implications. This paper will formalize the concept of complementarity between the fiscal apparatus and the military establishment and apply it to the theory of war and state making.

By formalizing the link between interstate conflict and the choice of state structure and economic organization, the paper shows that if the rise of European modern states and economies was war driven, then during modernization as the modern state and the modern economy metamorphosed from their traditional predecessors, we should observe Wagner's law and the civilianization paradox. War is an ultimate test of fiscal strength and economic might, especially large scale warfare. The civilianization of the state and the increased involvement of the state in the economy increase the war efficiency of the state.

The model in conclusion explains a link between the rise of the West in global economic and military eminence and its relation to the competitive states system of Europe. It also deepens our understanding of the modern state, especially in its role as a war machine and our understanding of the role played by the modern economy as an auxiliary system of this war machine.

4.2. The Model.

We will analyze a symmetrical game of two states contesting for resources for expository simplicity. Extension to more players is straightforward and does not add any insight.60

We assume there is interdependence between the fiscal apparatus and the military sector. The fiscal bureaucracy relies on the military to capture and secure a share of world resources for the economy to use in production. The fiscal apparatus then provides public goods to raise the productivity of the economy and taxes the economy. The military relies on the fiscal apparatus to pay for its expenditure. The result of such interdependence is that the efficiency of the fiscal bureaucracy, the economy and the military all have an impact on the outcome of international contests. The choice of the state fiscal structure then has a strategic importance. The nature of international conflict therefore will affect the choice of the fiscal structure.61

Economic agents, i.e., the tax paying citizens of states, are assumed to be dis-

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60 The number of major powers in the European balance of power game remained fairly constant from medieval era to modern age. Only the small marginal states declined in numbers. We therefore use the exogenous players approach since our focus is the institutional aspect of war and state making.

61 For details on the concept of interdependence between the fiscal bureaucracy and the military, see Dudley (1990). On the strategic implications of such interdependence, see Chapter 3.
tributed evenly on the territory or world to be contested among the states. We normalized
the size of the territory to be contested as one. Each agent has a unit of labor endowment
and combines it with the public goods provided by the state to produce. The return from
production depends positively on the level of provision of public goods.

The goal of each state is to maximize profits. We assume that the states use the
absolute budgetary approach where contestants set military expenditures equal to some
absolute amount of resources. For computational simplicity, we make the choice of fiscal
regimes a discrete one:

A state either chooses $t^h$, a high tax rate or $t^l$, a low tax rate and $g^h$, a high
level of public goods provision or $g^l$, a low level of public goods provision. A higher level
of public goods provision will result in a higher productivity for the private sector. The
superscript $l$ denotes a low tax rate and low public goods provision and $h$ denotes a high tax
rate and a high public goods provision. $A^h$ is the productivity parameter of a high public
goods regime and $A^l$ is the productivity parameter of a low public goods regime. We have
$A^h > A^l$, $t^h > t^l$ and $g^h > g^l$.

We assume for simplicity:

i. $\left(1 - t^l\right) A^l = w$ when the state chooses $t^l$ and $g^l$;
ii. $\left(1 - t^h\right) A^l < w$ when the state chooses $t^h$ and $g^l$;
iii. $\left(1 - t^h\right) A^h > w$ when the state chooses $t^h$ and $g^h$ and
iv. $\left(1 - t^l\right) A^h > w$ when the state chooses $t^l$ and $g^h$.

$w$ is the subsistence wage. That rules out policy combination two. We also assume
that with a low tax rate, the state cannot afford to provide a high level of public goods.
That rules out policy combination four. The choice is therefore between policy combination
one, a low tax and low public goods regime and policy combination three, a high tax and
high public goods regime.

---

62 By having exogenous number of contestants and assuming constant amount of resources, we ignore
the fact that the European great powers greatly augmented the amount of resources they controlled by
incorporating weaker states in Europe and elsewhere into the great powers' territory or sphere of influence.
Being a static model, addition of resources due to population growth, investment and advances in knowledge
are assumed away.

63 We abstract from the issue of the distortionary effect of taxation.

64 The alternative is the proportionate budgetary approach, where contestants set military expenditures
equal to some proportion of total state budget or national income. For our purpose here, the absolute and
proportionate budgetary approaches yield essentially the same results. For a detailed treatment of the two
budgetary approaches, see Chapter 3.

65 We assume that with a low tax rate, the state cannot afford to provide a high level of public goods. See
below.
We define public goods as those either for the direct consumption of the civilian population, such as the public health care system, or those that enhance the productivity of the economy, such as infrastructure. We exclude those purely for defense purposes, that is, the military expenditure. The provision of public goods however has impacts on the defense capacity of the state. A good communication and transportation system, for instance, enhances both production and defense. Another example is that a good education system produces both a well trained work force for the economy and a well trained population that could better support a total war effort in either their civilian or military role. This creates a complementarity between the fiscal and military sectors.

Due to complementarity between the military and the fiscal apparatus of the state, we have the profit of a state choosing high tax rate and high public goods provision as

$$\pi(t^h, g^h, b) = t^h \left(A^h - \beta^h - \frac{g^h}{n_h} + g^h\theta^h\right) P - \left(1 - t^h\gamma^h - g^h\delta^h\right) b$$

and the profit of a state choosing low tax rate and low public goods provision as

$$\pi(t', g', b) = t' \left(A' - \beta' - \frac{g'}{n'} + g'\theta'\right) P - \left(1 - t'\gamma' - g'\delta'\right) b.$$  

$P$ is the share of world resources or territory controlled by the state given the defense capacity of the state, $b$, the defense capacity of the rival state, and the conflict technology function. We adopt the ratio form of conflict technology function as defined by Hirshleifer (1989) where $P_1 = \frac{m_1}{m_1 + m_2}$ and the subscript denoting the identity of the contestant. $m$ is the mass or scale factor in conflict, measuring the advantage a larger player has over his smaller rival.

In the above equations, the cost of running a high (low) fiscal capacity structure is $t^h\beta^h + g^h - t^h g^h \theta^h (t'\beta' + g' - t' g' \theta') \text{ per unit territory.}$ $t^h\beta^h (t'\beta')$ is the cost of taxation in a high (low) fiscal regime. $g^h (g')$ is the cost of public goods provision in a high (low) fiscal regime. $\theta^h (\theta')$ denotes the degree of positive externalities between taxation and public goods provision in a high (low) fiscal regime. The average and marginal cost of supplying an additional unit of fighting force under the high (low) fiscal regime is $1 - t^h\gamma^h - g^h\delta^h (1 - t'\gamma' - g'\delta').$ $\gamma^h (\gamma')$ denotes the degree of positive externalities between taxation and defense in a high (low) fiscal regime. $\delta^h (\delta')$ denotes the degree of positive externalities between public goods provision and defense in a high (low) fiscal regime.

$\gamma^h (\gamma')$ signifies the importance of monitoring and surveying civilian population and economy to the defense effort. Increased importance of intelligence collection and the control of flow of people, goods and services and information across the boundary increases the size of $\gamma^h (\gamma').$ The customs service and the inland revenue department therefore share
a larger complementarity with the defense sector. $\delta^h (\delta^t)$ signifies the importance of public goods provision to defense effort. Reduced opportunity for espionage, sabotage and finding sympathetic allies within the country by the enemy due to better law and order conditions, for instance, enhances defense effort. A better public health care system enhances defense effort in by supplying healthier soldiers to the front line and maintaining a healthier workforce at the home base for total defense effort. With emergence of total war, i.e., increased civilian participation in war, such as through economic warfare, involvement in civil defense, etc., the value of $\gamma^h (\gamma^t)$ and $\delta^h (\delta^t)$ become larger.

A state that monitors the economy better and has a larger public infrastructure has a lower logistical cost. It is more efficient in converting revenue in its hand into military effort.

We impose the restrictions that:

\[
(t^h \beta^h + g^h - t^h g^h \theta^h) > 0 \quad \text{and} \quad (t^t \beta^t + g^t - t^t g^t \theta^t) > 0, \quad \text{i.e., despite complementarity between taxation and public goods provision, it still costs resources to set up the fiscal regime;}
\]

\[
(g^h - t^h g^h \theta^h) > 0 \quad \text{and} \quad (g^t - t^t g^t \theta^t) > 0, \quad \text{as well as} \quad (t^h \beta^h - t^h g^h \theta^h) > 0 \quad \text{and} \quad (t^t \beta^t - t^t g^t \theta^t) > 0, \quad \text{i.e., complementarity with taxation does not totally eliminate the costs of public goods provision and vice versa;}
\]

\[
(1 - t^h \gamma^h - g^h \delta^h) > 0 \quad \text{and} \quad (1 - t^t \gamma^t - g^t \delta^t) > 0, \quad \text{i.e., complementarity between the military and fiscal apparatus of the state do not totally eliminate the costs of running a defense sector.}
\]

We have the following notational simplification for convenience:

\[
r^t \equiv t^t \left( A^t - \beta^t - \frac{g^t}{\theta^t} + g^t \theta^t \right),
\]

\[
r^h \equiv t^h \left( A^h - \beta^h - \frac{g^h}{\theta^h} + g^h \theta^h \right),
\]

\[
\epsilon^t \equiv \left( 1 - t^t \gamma^t - g^t \delta^t \right),
\]

\[
\epsilon^h \equiv \left( 1 - t^h \gamma^h - g^h \delta^h \right),
\]

\[
l \equiv \frac{t^t}{\delta^t} \quad \text{and}
\]

\[
h \equiv \frac{t^h}{\delta^h}.
\]

$r^t$ and $r^h$ are the net revenue accruing to the state (less expenses of the fiscal bureaucracy but not yet deducting the military expenditure) from administering a unit of captured resources. $\epsilon^t$ and $\epsilon^h$ are the average costs of providing a unit of military capacity given the positive externalities from the fiscal bureaucracy. For convenience, we call $r^t$ and $r^h$ net civilian revenue and $\epsilon^t$ and $\epsilon^h$ net military costs. $r^t$ could be greater than, equal to or
smaller than \( r^A \) and \( e' \) could be greater than, equal to or smaller than \( e^A \). The net civilian revenue measures the profits accrue to the state given the fiscal regime when there is no military effort at all. This is decided by production efficiency which depends on the level of public goods provision, and fiscal efficiency in taxation and public goods provisions. The net military costs measure the logistical efficiency of the state in converting net civilian revenue into defense capacity, given the fiscal regime. The logistical efficiency is affected by the degree of complementarity between the fiscal bureaucracy and the military apparatus.

The states will first make a decision on the fiscal regime, i.e., they choose either \( r' \) and \( e' \) or \( r^A \) and \( e^A \). After they have chosen the fiscal structure, they decide on the defense capacity. The states simultaneously choose their military capacity. They also simultaneously choose their fiscal regime.

The war and state making theory asserts that the European state building process was war driven. It was the increasing scale in warfare that led to the growth in size and roles of government, especially in its fiscal capacity. For the war and state making theory to hold, we need to have a model where the states choose low fiscal capacity structure when the mass factor is small and choose a high fiscal capacity structure when the mass factor is large. The model must also predict and explain the civilianization paradox and Wagner’s law.

### 4.21. The Game

A low fiscal capacity state 1 solves the following problem for the decision on its military capacity:

\[
\max_{b_1} \pi_1^l = r' P - e' b_1.
\]

FOC is

\[
r' m P (1 - P) b_1^{r-1} - e' = 0.
\]

A high fiscal capacity state 2 solves the following problem for the decision on its military capacity:

\[
\max_{b_2} \pi_2^h = r^h (1 - P) - e^h b_2.
\]

FOC is

\[
r^h m P (1 - P) b_2^{-1} - e^h = 0.
\]

For comparison with military capacity in an asymmetrical situation, we calculate military capacity at the low fiscal capacity and high fiscal capacity symmetrical equilibria:
Military expenditures are positively related to the size of the mass factor. They also depend on the types of fiscal regime. \( l \) measures the war efficiency of the low fiscal capacity regime and \( h \) measures the war efficiency of the high fiscal capacity regime. War efficiency measures the amount of military capacity that could be produced from a unit of captured resources given the fiscal regime. War efficiency is the net civilian revenue divided by the net military cost. It is determined by production efficiency, fiscal efficiency and logistical efficiency. Alternatively we could speak of the conversion cost which is the net military cost divided by the net civilian revenue: \( \frac{1}{r} \) is the amount of captured resources required to produce a unit of military capacity for the low fiscal capacity state and \( \frac{1}{h} \) is the amount of captured resources required to produce a unit of military capacity for the high fiscal capacity state.

At two states’ symmetrical equilibrium, we need \( m \leq 2 \) for \( \pi \geq 0 \).

At asymmetrical situation, whereby state one chooses low fiscal capacity and state two chooses high fiscal capacity, we have the military capacity of state one and two as follows:

\[
\begin{align*}
\beta_1 (l, h) &= \frac{r^2}{c_m} m \pi (1 - \pi) = l m \pi (1 - \pi), \\
\beta_2 (l, h) &= \frac{r^2}{c_h} m \pi (1 - \pi) = h m \pi (1 - \pi).
\end{align*}
\]

The ratio between military capacity of state one and two is

\[
\frac{\beta_1}{\beta_2} = \left( \frac{h}{l} \right).
\]

That gives us:

\[
\beta_1 (l, h) = \left( \frac{h}{l} \right) \beta_2 (l, h).
\]

When a low fiscal capacity state has a lower (higher) conversion cost than a high fiscal capacity state, i.e., \( l > h \) (\( l < h \)), we have \( \beta_1 (l, h) > \beta_2 (l, h) \) (\( \beta_1 (l, h) < \beta_2 (l, h) \)). When they have equal conversion cost, we have \( l = h \) and \( \beta_1 (l, h) = \beta_2 (l, h) \).

From above, we have:

\[
\begin{align*}
P(l, h) &= \frac{m}{m+h}, \\
1 - P(l, h) &= \frac{h}{m+h}, \\
\pi_1 (l, h) &= r^l P (1 - m (1 - P))
\end{align*}
\]
\[ \pi_2 (l, h) = r^h (1 - P) (1 - mP) \, . \]

The war and state making theory asserts that the states choose low fiscal capacity structure when the mass factor is small and choose a high fiscal capacity structure when the mass factor is large.

**PROPOSITION 1:**

If and only if \( r^l > r^h, e^l > e^h \) and \( h > l \) will the war and state making theory hold.

Proof: Appendix.

If however \( r^l > r^h \), then the high fiscal capacity structure of the modern state is very expensive. The increase in tax revenue due to greater extraction efficiency and greater production efficiency does not compensate for the higher costs of taxation and public goods provision. If \( e^l > e^h \) and \( h > l \), the high fiscal capacity bureaucracy however generates a higher degree of positive externalities on the defense sector and together with its greater efficiency in production and taxation, it is more war efficient. The low fiscal capacity regime therefore generates a larger net civilian revenue while the high fiscal capacity regime is more war efficient. When the mass factor is small and international conflict is on a smaller scale, states choose to have a low fiscal capacity as that gives them a larger profit. If however the mass factor of conflict is large and international conflict is on a larger scale, states choose to have a high fiscal capacity regime as it is more war efficient. The critical level of \( m \) that such a transformation in fiscal regime takes place depends on the value of \( r^l, r^h, e^l, e^h \) and \( h > l \). Refer to Diagram 3 for a summary.

The state faces two issues: the need to war with other states for a share of resources and the task of generating revenue from the resources captured. The state looks at the relative revenue and logistical efficiency of the high fiscal capacity and low fiscal capacity regimes and the scale of interstate conflicts to decide the choice of state structure. Given the values of \( r^l, r^h, e^l, e^h, h \) and \( l \), there is a critical level or range for the value of mass factor below which the states will choose the low fiscal capacity structure and above which they will choose the high fiscal capacity structure. That critical value of mass factor depends positively on \( r^l \) and negatively on \( r^h \). When the traditional form of state structure and economy generates a large net civilian revenue for state, there needs to be greater economies of scale in conflict to prompt the state to choose a high fiscal capacity regime and a modern economy. There is less incentive to modernize. When the high fiscal capacity regime and the modern economy generates a larger net civilian revenue for the state, the critical level
of economies of scale in conflict that will prompt the state to choose a high fiscal capacity regime is lower. There is more incentive to modernize.

That critical value of mass factor depends negatively on $e^h$ and positively on $e^l$. When the traditional form of state structure and economy have a greater logistical efficiency, there needs to be a greater economies of scale in conflict to prompt the state to choose a high fiscal capacity regime and a modern economy. When the high fiscal capacity regime and the modern economy have a larger logistical efficiency, the critical level of economies of scale in conflict that will prompt the state to choose a high fiscal capacity regime is lower.

For $m > 2$, there is only a natural monopoly state in equilibrium. The natural monopoly states will choose the low fiscal capacity state. A natural monopoly state sets $b = 0$ as there is no rival state to contest resources. Given that $r^l > r^h$, $e^l > e^h$ and $l > h$, the natural monopoly state will therefore choose a low fiscal capacity structure as defense capacity is not important.66

PROPOSITION 2:

For mass factor greater than two, there is only a natural monopoly state in equilibrium and it chooses the low fiscal capacity structure.

4.3. The civilianization paradox and Wagner’s law.

We will now derive the budgetary implications of a state transiting from a low fiscal capacity regime with a traditional economy to a high fiscal capacity regime with a modern economy after an exogenous technological shock of increasing mass factor has realized.67

PROPOSITION 3:

There is a smaller share of military expenditures in state budget in the high fiscal capacity state than in the low fiscal capacity state.

PROOF:

The share of military expenditures in state budget in the high fiscal capacity state is:

$$\frac{e^h y^h}{r^h \left( p^h + \frac{e^h}{e^h} - g^h \phi^h \right) p + e^h y^h}.$$ 

66The Aztec and Inca empires before the arrival of the Spaniards are best examples of natural monopoly states. In their world, there is no serious rival to their empires and therefore little need for defence capacity. For details, refer to Parker (1996).

67The impacts on the fiscal structure of the state were slower to be realized as institutions evolved very slowly.
The share of military expenditures in state budget in the low fiscal capacity state

\[ \frac{e^l L}{t^l \left( b^l + \frac{e^l}{A^l} - g^l \theta^l \right) P + e^l L} . \]

Given \( r^l > r^h \), \( e^l > e^h \) and \( A^h > A^l \), we have:

\[ \frac{e^l L}{t^l \left( b^l + \frac{e^l}{A^l} - g^l \theta^l \right) P + e^l L} > \frac{r^l m^2 - 2}{(t^l A^l - r^l) 2^{-1} + r^l m^2 - 2} > \frac{r^h m^2 - 2}{(t^h A^h - r^h) 2^{-1} + r^h m^2 - 2} = \frac{e^h L}{t^h \left( b^h + \frac{e^h}{A^h} - g^h \theta^h \right) P + e^h L} . \]

Q.E.D.

The share of civilian expenditures in total budget has gone up.

PROPOSITION 4:

The ratio of military expenditures over national income has decreased.

PROOF:

The share of military expenditures in national income in the high fiscal capacity state is: \( \frac{e^h L}{A^h P} \).

The share of military expenditures in national income in the low fiscal capacity state is: \( \frac{e^l L}{A^l P} \).

Given \( r^l > r^h \), \( e^l > e^h \) and \( A^h > A^l \), we have:

\[ \frac{e^l L}{A^l P} = \frac{r^l m^2 - 2}{A^l 2^{-1}} > \frac{r^h m^2 - 2}{A^h 2^{-1}} = \frac{e^h L}{A^h P} . \]

Q.E.D.

The civilian economy has grown relative to the military.

Proposition 3 and 4 are what Tilly (1992) referred to as the civilianization paradox of the war and state making process.

PROPOSITION 5:

The share of public sector in national income is higher in the modern economy than in the traditional economy.

PROOF:

The share of public sector in national income in the high fiscal capacity state is:

\[ \frac{t^h \left( b^h + \frac{e^h}{A^h} - g^h \theta^h \right) P + e^h L}{A^h P} . \]

The share of public sector in national income in the low fiscal capacity state is:

\[ \frac{t^l \left( b^l + \frac{e^l}{A^l} - g^l \theta^l \right) P + e^l L}{A^l P} . \]

Given \( r^l > r^h \), \( e^l > e^h \) and \( A^h > A^l \), we have:

\[ \frac{t^h \left( b^h + \frac{e^h}{A^h} - g^h \theta^h \right) P + e^h L}{A^h P} = \frac{(t^h A^h - r^h) (1 - m^2 - 1)}{A^h 2^{-1}} > \frac{(t^l A^l - r^l) (1 - m^2 - 1)}{A^l 2^{-1}} = \frac{t^l \left( b^l + \frac{e^l}{A^l} - g^l \theta^l \right) P + e^l L}{A^l P} . \]
Q.E.D.

The growth of the public sector was faster than the growth of the economy: this is Wagner's law. Together with proposition 4 we know that the share of civilian public expenditures in national income has increased.

In sum, if the war and state making theory holds, the civilianization paradox and Wagner's law will be observed as modern states and modern economies metamorphose from their traditional predecessors.

4.4. Digressions on asymmetry.

There are conceptually two types of asymmetry: the asymmetry in war efficiency between states that is independent of their choice of fiscal regimes and the asymmetry in war efficiency between states pertaining to their choice of fiscal regimes. We term the first type of asymmetry the institutional independent asymmetry and the second type of asymmetry the institutional dependent asymmetry.

4.4.1. Institutional independent asymmetry.

State 2 is the disadvantaged state with a lower war efficiency. Such disadvantage for instance could be the result of a geopolitical composition that lacks strategic depth. Choice of fiscal regimes and therefore the structure of the economy will not change this disadvantage. State 1 is the more war efficient state. We have $c > 1$ measuring the greater war inefficiency that state 2 suffers.

A low fiscal capacity state 1 solves the following problem for the decision on its military effort:

$$\max_{b_1} \pi_1^f = r^f P - e^f b_1.$$  
A high fiscal capacity state 1 solves the following problem for the decision on its military capacity:

$$\max_{b_1} \pi_1^h = r^h P - e^h b_1.$$  
A low fiscal capacity state 2 solves the following problem for the decision on its military capacity:

$$\max_{b_2} \pi_2^l = r^l (1 - P) - c e^l b_2.$$  
A high fiscal capacity state 2 solves the following problem for the decision on its military capacity:

$$\max_{b_2} \pi_2^h = r^h (1 - P) - c e^h b_2.$$
Mathematical detail and examples of equilibria generated by numerical methods are in appendix 2.

After examining a very large number of examples, we arrived at the following proposition:

**PROPOSITION 6:**

When under the set of parameters such that asymmetry in institutional independent war efficiency resulted in a sustainable asymmetrical equilibrium (where both states have non-negative profits and a positive share of resources) with different choices of fiscal capacity, the more favored state will choose a low fiscal capacity regime while the disadvantaged state will choose a high fiscal capacity regime.

This probably explains why in history it is often the initially disadvantaged fringe states that first created a modern state and modern economy, instead of their more favored counterparts.

Examples:

i.) It was England that first created the modern state and the modern economy instead of the more geopolitical secured continental land mass states such as France, Germany or Russia.  

ii.) It was nineteenth century Japan that successfully created a modern state instead of the much more powerful and secure China. Compared with Japan, China responded lethargically to Western challenges.

Both England and Japan are island states without strategic depth of defense that land mass continental powers enjoy. They for instance cannot trade space for time in a war like Russia and China. Such geopolitical vulnerability prompted both states to strive hard in fiscal and military aspects. They therefore created respectively the first modern state and industrial economy in history and the first non-Western modern state and industrial economy.

4.42. Institutional dependent asymmetry.

We examine four pure types of institutional dependent asymmetry. We will not analyze those cases where the institutional dependent asymmetry is a combination of two or more types of those examined here as that will not add much insight. We assume that

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68 Refer to Rasler and Thompson (1989) and Tilly (1992) for detailed historical analysis on the different path of state making of European nations.

69 For a comparative historical analysis, refer to Weiss and Hobson (1995).
despite the asymmetry the restriction that $r^t > r^h$, $d^t > e^h$ and $h > i$ still hold.70

a.) Both states having same values of $e^t, r^h$ and $e^h$ but state 1 has a larger $r^t$:

$r^t_1 > r^t_2$
$e^t_1 = e^t_2$
$r^h_1 = r^h_2$
$e^h_1 = e^h_2$.

Traditional state and economy suits state 1 more than state 2. State 1 for instance has abundant agricultural resources that state 2 lacks. State 2 has a stronger urge to install modern state and modern economy. Numerical examples are in Table 3.

It was England that first created the modern state instead of Portugal, Spain, France or Netherlands. With a much larger net civilian revenue under a low fiscal capacity state and traditional economy from successful agricultural, colonial, commercial and financial activities, Portugal, Spain, France and Netherlands had less incentive to adopt a high fiscal capacity structure and a industrialized economy.71

b.) Both states having same values of $r^t, r^h$ and $e^h$ but state 1 has a larger $e^t$:

$r^t_1 = r^t_2$
$e^t_1 > e^t_2$
$r^h_1 = r^h_2$
$e^h_1 = e^h_2$.

Traditional state and economy is logistically more efficient for state 2 than state 1. State 2 for instance is ruled by a nomadic horseman elite whose form of statecraft and military practice will be totally out of place in a modern state and economy. State 1 has a stronger urge to install modern state and modern economy. Numerical examples are in Table 4.

The state elite of Ottoman empire and Ching China for instance were very reluctant to initiate modernization as they could not fit into the modern way of warfare, politics and economic activities.72

c.) Both states having same values of $r^t, e^t$ and $e^h$ but state 1 has a larger $r^h$:

70If these restrictions do not hold for a particular state, then the war and state making argument fails for the state in question. Increased scale in warfare will not lead to political and economic modernization for that state unless there is a fundamental change in the composition of the state. For details, refer to appendix A.1.

71Refer to Rasler and Thompson (1989) and Tilly (1992) for detailed historical analysis on the different path of state making of European nations.

The modern state is less costly or a modern economy is more productive for state 1 than state 2. State 1 for instance is a small homogeneous country with most goods and services flowing through a few easily monitored and taxed sea ports. State 2 is a large heterogeneous land mass country. The cost of running a modern state and economy will be larger for state 2 than state 1. State 1 has a stronger urge to modernize. Numerical examples are in Table 5.

For the large continental powers such as France, the German empire (or the later Austria-Hungary Empire) and Russia, the costs of maintaining a high fiscal capacity regime with all its taxation administration and public goods provisions over a large and heterogeneous land mass are enormous. That resulted in a much smaller net civilian revenue for the high fiscal capacity regime. These states therefore modernized much later. For England, her dependence on international trade and her island geography means that taxes could be easily collected in a few ports. England is also small and homogeneous. That lowers the costs of a fiscal capacity administration. England therefore modernized earlier.73

This comparison between England and the large European continental powers also applies to the different path of modernization between Japan and China as well.

Both states having same values of $r'$, $e'$ and $r^h$ but state 1 has a larger $e^h$:

- $r'_1 = r'_2$
- $e'_1 = e'_2$
- $r^h_1 > r^h_2$
- $e^h_1 > e^h_2$.

Industrialization lowers net military cost for state 2 more than state 1. State 2 for instance is a homogeneous nation state while state 1 is a multinational empire where the majority of the population do not identify with the state. Industrialization will logistically benefit state 2 more than state 1 as it is more difficult to organize and mobilize a heterogeneous civilian population that do not identify with the state for defense purposes. State 2 has a stronger urge to modernize. Numerical examples are in Table 6.

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73Refer to Rasler and Thompson (1989) and Tilly (1992) for detailed historical analysis on the different path of state making of European nations.
Multinational empires such as Austria Hungary empire and Ottoman empire ultimately failed to modernize.

The results we derived here confirm the intuition we have from section 2 that state looks at the extraction and logistical efficiency of different fiscal regimes and scale of interstate conflict to decide the choice of fiscal structure.

4.5. Applications and conclusion.

War is not merely a contest of military valor and equipment. It is also a test of political organization, fiscal strength and economic might. This is especially so for large scale warfare. Having a greater production and taxation capacity and a greater logistical efficiency, the modern state is the most efficient and gigantic war machine mankind has ever invented. To be efficient in wars, the modern state however was compelled to enlarge its role in the economy to boost production, both for a higher level of revenue extraction and for the reason that there is greater mobilization of civilian resources in large scale international conflicts. There arose the civilianization paradox as observed by Tilly (1992) and Wagner's law. It is the civilianization of the state and its enlarged role in the economy that makes it more war efficient.

The civilianization paradox is also observed by others such as Rasler and Thompson (1989), Porter (1994) and Weiss and Hobson (1995). Porter (1994) in fact observed that it was essentially the expectation, preparation and experience of wars that led states to enlarge their role in the economy in general and their role in welfare provision in particular. Total warfare for instance not only requires a healthy population and contented workers for war effort, it also needs a population that reproduces abundantly to replenish the immense human losses the war generates. Modern states are therefore both warfare states and welfare states.

Given its myriad roles and massive bureaucracy, the modern state is very expensive. Despite a larger production and a higher tax rate, it still generates a smaller net civilian revenue. If there is no intensive and large scale international conflict, it will not be adopted. That explains why it was in Europe that modern states first emerged. From A.D. 1200 onwards, only Europe unquestionably operated with a competitive states system with rivals about equal in military and economic might engaged in the balance of power game. 

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74 For details, refer to Porter (1994).
76 Europe has operated a competitive states system for over a millenium. In that warring environment,
"The Chairman, to the extent that the views expressed in this document are reflected in the policies of the European Union, is seized of the issues raised in this document. The Commissioner for Economic and Monetary Affairs and the Internal Market expresses his view on the subject matter of this report and the conclusions of the panel. The report was adopted on 9th May 1991."
productive economy that they created and a larger share of a more productive economy that they taxed, the result is still a smaller net civilian revenue than that of the traditional state with a low fiscal capacity and a traditional economy. If there is no intensive and large scale international conflicts, the modern state will not be adopted and the modern economy will therefore not emerge.

In Europe, geopolitical pressure of a competitive state system dictated the adoption of the modern state. The modern economy, despite the high costs of operation, was adopted because it is part of the modern state war machine. It has a greater production capacity and a greater logistical efficiency. It also allows the state to extract a larger tax revenue. It is in a nutshell, an expensive military auxiliary of the modern state. Being an expensive military auxiliary, it therefore was to be found in a warring environment like Europe where there were great need for it. It was not to be found in a peaceful natural monopoly state system or a hegemonic state system without intensive and large scale warfare. The modern states, together with the modern economy, therefore first appeared in the warring states environment of Europe.

To remain traditional or to modernize is the result of rational calculation by the state. When deciding to adopt the traditional or the modern state and economy, states look at the scale of interstate conflicts, their geopolitical conditions and the revenue and logistical efficiency of the traditional and the modern state and economy. The first modern state and economy in Europe (England) and the first modern state and economy in Asia (Japan) are both island states that suffer geopolitical disadvantage and have a low operation cost of running a modern state and economy. They are more prone to adopt modern state structure and modern economic organization by both institutional dependent and independent criteria. The late starters or non starters, such as Russia and China, are large continental size states that are geopolitically secure and suffer an enormous cost of running a modern state structure. They are more prone to adopt traditional state structure and economic organization.

The state, just like an individual, might shirk if conditions permit it to do so. In the case of a natural monopoly state where there is no competition from rival states or in

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81 Due to immense threat from two rival states in Northern China, Sung China involved extensively in the economy and extracted a very large share to complement and support the gigantic defence sector. The economy prospered with elaborate internal and external trading networks. There was intensive growth. Such miraculous performance was not repeated in the powerful and secured Ming and Ching periods. For details, refer to Jones (1981, 1988).

82 Refer to Weiss and Hobson (1995) for a comparative historical analysis.
the case of a state geopolitical secured from severe external threat, it shirks and chooses the low fiscal capacity structure and enjoy a larger net civilian revenue. Such shirking in fiscal capacity is permitted by a secure geopolitical environment. A traditional state structure and a traditional economy is the result of such geopolitical-fiscal shirking. States that do not enjoy such privilege and find themselves in the midst of severe and large scale international conflicts are compelled by geopolitical considerations to choose a high fiscal capacity structure. A modern state structure and a modern economy is the result of such geopolitical pressure.

The asymmetry in warring capacity between the modernized Western states and the traditional states of other older civilizations when they first met was also explained by the model. As modern states are more war efficient, the modernized Western states therefore expanded at the expense of traditional states in other parts of the world. This fact could be better appreciated by acknowledging that even if we control for military technological asymmetry between different states, the different state structure still has an impact on the performance of states in interstate conflicts. For instance, a Ching China that modernized its military sector but not its fiscal and other political structure was defeated continuously by a modernized Japan and the Western powers up until the collapse of the dynasty. Another good example was the encounter between the European colonial powers and the Indian princely states in the 17th and 18th century. Despite a parity in military technology and martial tradition between the two sides when they first encountered each other in the 17th century, the Indian powers ultimately lost out due to a more inefficient state organization in general and an inefficient fiscal structure in particular.

This paper, in conclusion, formalizes and explains Wagner's law and the civilianization paradox in the European war and state making process, which have been well observed by scholars in the field (Tilly, 1992; Rasler and Thompson, 1989; Webber and Wildavsky; Porter, 1994; Weiss and Hobson, 1995; Peacock and Wiseman, 1967; Bird, 1971; Thomas, 1984; Breton, 1989). It provides a formal explanation for the reasoning made by historians about the possible link between the competitive states system of Europe and her industrial revolution (Jones, 1981, 1988; Kennedy, 1987). It shows that civilianization of the state improved war efficiency. It therefore enhances our understanding of the modern

---

83 For a discussion of the Chinese modernization experience, see Huang (1988).
84 For a discussion on how different state organizations affected military performance in India during early phase of European colonialisation, see Nadarajah (1992).
state, especially in the role as a war machine and the relationship between the state and the economy, particularly the role the latter has as a military auxiliary to the former.
Diagram 3: The mass factor and choice of fiscal regime.
Note for Diagram 3: The mass factor and choice of fiscal regime.

$m$ (the horizontal axis) : the mass factor.

$l$ (the vertical axis) : $\frac{I}{A}$.

$h$ (midway on the vertical axis) : $\frac{h}{A}$.

$(l,l)$ : in equilibrium, both states opt for the low fiscal capacity regime.

$(h,h)$ : in equilibrium, both states opt for the high fiscal capacity regime.

$(l,l)$ and $(h,h)$ : multiple equilibria where the two contesting states could both opt for the low fiscal capacity regime or they could both opt for the high fiscal capacity regime.

$(l)$ : the natural monopoly state opt for the low fiscal capacity regime.
Appendixes

A.1 Proof for proposition one:

A low fiscal capacity state 1 solves the following problem for the decision on its military capacity:

\[ \max_{b_1} \pi_1^{b_1} = r'P - e' b_1. \]

FOC is

\[ r'mP (1 - P) b_1^{-1} - e' = 0. \]

A high fiscal capacity state two solves the following problem for the decision on its military capacity:

\[ \max_{b_2} \pi_2^{b_2} = r^h (1 - P) - e^h b_2. \]

FOC is

\[ r^h mP (1 - P) b_2^{-1} - e^h = 0. \]

For comparison with military capacity at asymmetrical situation, we calculate military capacity at the low fiscal capacity and high fiscal capacity symmetrical equilibria:

\[ b_1^1 (l, l) = \frac{r'}{e'} m 2^{-2} = lm 2^{-2}, \]

\[ \pi_1^1 (l, l) = r'^{-1} (1 - m \left( \frac{1}{2} \right)), \]

\[ b_2^2 (h, h) = \frac{r^h}{e^h} m 2^{-2} = hm 2^{-2}, \]

\[ \pi_2^2 (h, h) = r^h 2^{-1} \left( 1 - m \left( \frac{1}{2} \right) \right). \]

At two states symmetrical equilibrium, we need \( m \leq 2 \) for \( \pi \geq 0 \).

At asymmetrical situation, whereby state one chooses low fiscal capacity and state two chooses high fiscal capacity, we have the military capacity of state one and two as follows:

\[ b_1^1 (l, h) = \frac{r'}{e'} mP (1 - P) = lmP (1 - P), \]

\[ b_2^2 (l, h) = \frac{r^h}{e^h} mP (1 - P) = hmP (1 - P). \]

The ratio between military capacity of state one and two is:

\[ \frac{b_1^1}{b_2^2} = \left( \frac{1}{n} \right). \]

That gives us:

\[ b_1^1 (l, h) = \left( \frac{1}{n} \right) b_2^2 (l, h). \]

From above, we have:

\[ P (l, h) = \frac{m}{m + h}, \]

\[ 1 - P (l, h) = \frac{h}{m + h}. \]
\[ \pi_1 (l, h) = r^l P (1 - m (1 - P)) \] and
\[ \pi_2 (l, h) = r^h (1 - P) (1 - mP) . \]

We will now look at how the values of \( e', e^h, r^l, r^h \) and \( m \) decide the choice of state fiscal capacity.

i.) \( r^l < r^h \).

There are three following sub cases:

a.) \( l < h \).

We have \( e' \) could be larger than, equal to or smaller than \( e^h \).

That leads to \( b_1^l (l, h) < b_2^l (l, h), \ P (l, h) < \frac{1}{2} < 1 - P (l, h) \) and
\[ \pi_1 (l, h) = r^l (P (l, h)) (1 - m (1 - P (l, h))) < \pi_1 (h, h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \] and
\[ \pi_2 (l, h) = r^h (1 - P (l, h)) (1 - mP (l, h)) > \pi_2 (l, l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) . \]

The Nash equilibrium is for both states to choose high fiscal capacity \((h, h)\) regardless of the value of the mass factor.

b.) \( l = h \).

We have \( e' < e^h \).

That leads to \( b_1^l (l, h) = b_2^l (l, h), \ P (l, h) = \frac{1}{2} = 1 - P (l, h) \) and
\[ \pi_1 (l, h) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) < \pi_1 (h, h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \] and
\[ \pi_2 (l, h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) > \pi_2 (l, l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) . \]

The Nash equilibrium is for both states to choose high fiscal capacity \((h, h)\) regardless of the value of the mass factor.

c.) \( l > h \).

We have \( e' < e^h \).

That leads to \( b_1^l (l, h) > b_2^l (l, h), \ P (l, h) > \frac{1}{2} > 1 - P (l, h) \) and
\[ \pi_1 (l, h) = r^l P (l, h) (1 - m (1 - P (l, h))) > \pi_1 (h, h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \] and
\[ \pi_2 (l, h) = r^h (1 - P (l, h)) (1 - mP (l, h)) < \pi_2 (l, l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \] for high \( m \) and
\[ \pi_1 (l, h) = r^l P (l, h) (1 - m (1 - P (l, h))) < \pi_1 (h, h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \] and
\[ \pi_2 (l, h) = r^h (1 - P (l, h)) (1 - mP (l, h)) > \pi_2 (l, l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \] for small \( m \).

The Nash equilibrium is for both states to choose low fiscal capacity \((l, l)\) if the mass factor is large and for both states to choose high fiscal capacity if the mass factor is small, \((h, h)\).

ii.) \( r^l = r^h \).

We have three sub cases.
a.) \( h > l \).
That gives us \( e^l > e^h \).
That leads to \( b_1^l (l,h) < b_2^l (l,h) \), \( P (l,h) < \frac{1}{2} < 1 - P (l,h) \) and
\[ \pi_1 (l,h) = r^l (P (l,h)) (1 - m (1 - P (l,h))) < \pi_1 (h,h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \]
\[ \pi_2 (l,h) = r^h (1 - P (l,h)) (1 - mP (l,h)) > \pi_2 (l,l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \].
The Nash equilibrium is for both states to choose high fiscal capacity \((h,h)\) regardless of the value of the mass factor.

b.) \( h = l \).
That gives us \( e^l = e^h \).
\[ \pi_1 (l,h) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) = \pi_1 (h,h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \]
\[ \pi_2 (l,h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) = \pi_2 (l,l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \].
The states are indifferent between the two types of fiscal structure regardless of the value of the mass factor.

c.) \( h < l \).
That gives us \( e^l < e^h \).
That leads to \( b_1^l (l,h) > b_2^l (l,h) \), \( P (l,h) > \frac{1}{2} > 1 - P (l,h) \) and
\[ \pi_1 (l,h) = r^l P (l,h) (1 - m (1 - P (l,h))) > \pi_1 (h,h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \]
\[ \pi_2 (l,h) = r^h (1 - P (l,h)) (1 - mP (l,h)) < \pi_2 (l,l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \].
The Nash equilibrium is for both states to choose low fiscal capacity \((l,l)\) regardless of the value of the mass factor.

iii.) \( r^l > r^h \).
There are three sub cases.

a.) \( l = h \).
That gives us \( e^l > e^h \) and \( b_1^l (l,h) = b_2^l (l,h) \), \( P (l,h) = \frac{1}{2} = 1 - P (l,h) \) and
\[ \pi_1 (l,h) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) > \pi_2 (h,h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \]
\[ \pi_2 (l,h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) < \pi_1 (l,l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \].
The Nash equilibrium is for both states to choose low fiscal capacity regardless of the value of the mass factor.

b.) \( l > h \).
We have \( e^l \) could be larger than, equal to or smaller than \( e^h \).
For this case, as \( b_1^l > b_2^l \) and \( P (l,h) > \frac{1}{2} > 1 - P (l,h) \), we have
\[ \pi_1 (l,h) = r^l (P (l,h)) (1 - m (1 - P (l,h))) > \pi_2 (h,h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \] and
\[ \pi_2(l, h) = r^h (1 - P(l, h))(1 - mP(l, h)) < \pi_1(l, l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \]

The Nash equilibrium is for both states to choose low fiscal capacity regardless of the value of the mass factor.

c.) \( l < h \).

This gives us \( c^l > c^h \).

As \( b_1^1(l, h) < b_2^1(l, h) \) and \( P(l, h) < \frac{1}{2} < 1 - P(l, h) \), we have

\[ \pi_1(l, h) = r^l (P(l, h))(1 - m(1 - P(l, h))) > \pi_2(h, h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \]

\[ \pi_2(l, h) = r^h (1 - P(l, h))(1 - mP(l, h)) < \pi_1(l, l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \] for small value of \( m \).

\[ \pi_1(l, h) = r^l (P(l, h))(1 - m(1 - P(l, h))) < \pi_2(h, h) = r^h \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \]

\[ \pi_2(l, h) = r^h (1 - P(l, h))(1 - mP(l, h)) > \pi_1(l, l) = r^l \left( \frac{1}{2} \right) (1 - \frac{m}{2}) \] for large value of \( m \).

For \( r^l > r^h \) and \( \frac{c^l}{c^h} > \frac{c^h}{c^l} > 1 \), the Nash equilibrium is for both states to choose low fiscal capacity if the mass factor is small and for both states to choose high fiscal capacity if the mass factor is large.

In sum, if and only if \( r^l > r^h \), \( c^l > c^h \) and \( l < h \) will the war and state making theory hold.

Q.E.D.
A.2 Institutional independent asymmetry.

State 2 is the disadvantaged state with a lower war efficiency. \( C > 1 \).

A low fiscal capacity state 1 solves the following problem for the decision on its military effort:

\[
\max_{\pi_1^l} \pi_1^l = r^l P - \epsilon^l b_1.
\]

FOC is:

\[
r^l mP (1 - P) b_1^{-1} - \epsilon^l = 0.
\]

\[
b_1^l = \frac{r^l}{\epsilon^l} mP (1 - P).
\]

A high fiscal capacity state 1 solves the following problem for the decision on its military capacity:

\[
\max_{\pi_1^h} \pi_1^h = r^h P - \epsilon^h b_1.
\]

FOC is:

\[
r^h mP (1 - P) b_1^{-1} - \epsilon^h = 0.
\]

\[
b_1^h = \frac{r^h}{\epsilon^h} mP (1 - P).
\]

A low fiscal capacity state 2 solves the following problem for the decision on its military capacity:

\[
\max_{\pi_2^l} \pi_2^l = r^l (1 - P) - \epsilon^l b_2.
\]

FOC is:

\[
r^l mP (1 - P) b_2^{-1} - \epsilon^l = 0.
\]

\[
b_2^l = \frac{r^l}{\epsilon^l} mP (1 - P).
\]

A high fiscal capacity state 2 solves the following problem for the decision on its military capacity:

\[
\max_{\pi_2^h} \pi_2^h = r^h (1 - P) - \epsilon^h b_2.
\]

FOC is:

\[
r^h mP (1 - P) b_2^{-1} - \epsilon^h = 0.
\]

\[
b_2^h = \frac{r^h}{\epsilon^h} mP (1 - P).
\]

In above, \( c \geq 1 \).

When both states choose a low fiscal capacity, we have:

\[
\frac{b_1^l (l\bar{L})}{b_2^l (l\bar{L})} = c.
\]

When both states choose a high fiscal capacity, we have:

\[
\frac{b_1^h (h\bar{A})}{b_2^h (h\bar{A})} = c.
\]

When state 1 chooses a low fiscal capacity and state 2 chooses a high fiscal capacity, we have:
\[
\frac{\delta^1(l,h)}{\delta^2(l,h)} = c \left( \frac{1}{h} \right).
\]

When state 1 chooses a high fiscal capacity and state 2 chooses a low fiscal capacity, we have:

\[
\frac{\delta^1(h,l)}{\delta^2(h,l)} = c \left( \frac{h}{l} \right).
\]

We there have:

\[
P(l, l) = P(h, h) = \frac{c^m}{c^m + 1},
\]

\[
P(l, h) = \frac{c^m h^m}{c^m h^m + h^m},
\]

\[
P(h, l) = \frac{c^m h^m}{c^m h^m + h^m}.
\]

The profits of state 1 and 2 under the various strategic configurations are:

\[
\pi_1(h, l) = r^h \left( P(h, h) (1 - m (1 - P(h, l))) \right)
\]

\[
= r^h \left( \frac{c^m h^m}{c^m h^m + h^m} \right) \left( 1 - m \left( \frac{1}{c^m + 1} \right) \right).
\]

\[
\pi_1(l, l) = r^l \left( P(l, l) (1 - m (1 - P(l, l))) \right)
\]

\[
= r^l \left( \frac{c^m}{c^m + 1} \right) \left( 1 - m \left( \frac{1}{c^m + 1} \right) \right).
\]

\[
\pi_1(l, h) = r^l \left( P(l, h) (1 - m (1 - P(l, h))) \right)
\]

\[
= r^l \left( \frac{c^m h^m}{c^m h^m + h^m} \right) \left( 1 - m \left( \frac{h^m}{c^m h^m + h^m} \right) \right).
\]

\[
\pi_2(h, h) = r^h \left( P(h, h) (1 - m (1 - P(h, h))) \right)
\]

\[
= r^h \left( \frac{c^m h^m}{c^m h^m + h^m} \right) \left( 1 - m \left( \frac{h^m}{c^m h^m + h^m} \right) \right).
\]

\[
\pi_2(l, l) = r^l \left( P(l, l) (1 - m (1 - P(l, l))) \right)
\]

\[
= r^l \left( \frac{1}{c^m + 1} \right) \left( 1 - m \left( \frac{1}{c^m + 1} \right) \right).
\]

\[
\pi_2(l, h) = r^l \left( P(l, h) (1 - m (1 - P(l, h))) \right)
\]

\[
= r^l \left( \frac{c^m h^m}{c^m h^m + h^m} \right) \left( 1 - m \left( \frac{c^m h^m}{c^m h^m + h^m} \right) \right).
\]

\[
\pi_2(h, h) = r^h \left( P(h, h) (1 - m (1 - P(h, h))) \right)
\]

\[
= r^h \left( \frac{1}{c^m + 1} \right) \left( 1 - m \left( \frac{1}{c^m + 1} \right) \right).
\]

When both states choose same type of fiscal capacity, we need \( m < 2 \) for non-negative profits condition for the disadvantaged state 2. As the asymmetry increases, we need smaller \( m \) for non-negative profits condition.\(^{85}\)

We use numerical methods to investigate the equilibria. Examples are in table 1 and 2.

\(^{85}\)For details, refer to chapter 3.
A.3 Tables for digressions on asymmetry.

Notations for table 1, 2, 3, 4, 5 and 6.

In the following tables, we have:

i.) \( nil \) means no Nash equilibrium;

ii.) \((l, l)\) means both states choose low fiscal capacity at Nash equilibrium.

iii.) \((h, h)\) means both states choose high fiscal capacity at Nash equilibrium.

iv.) \((l, h)\) means state 1 chooses low fiscal capacity and state 2 chooses high fiscal capacity.

v.) \((h, l)\) means state 1 chooses high fiscal capacity and state 2 chooses low fiscal capacity.

Asymmetry in fiscal capacity is printed in bold.

Table 1: institutional independent asymmetry.

We do comparative statics with respect to \( c \) (the institutional independent asymmetric parameter) in this table.

\[ \pi_2 < 0 \] means that profits of state 2, the disadvantaged state, is negative at Nash equilibrium and therefore only state 1 will survive, as a monopoly and choosing a low fiscal capacity.

\[ c^l = 0.9, r^l = 1.8, l = 2; c^h = 0.3, r^h = 0.75, h = 2.5; m = 1.5. \]

\[
\begin{array}{cccccccccc}
  c & 1 & 1.3 & 1.6 & 1.9 & 2.2 & 2.5 & 2.8 & 3.1 & 3.4 & 3.7 \\
  NE & (l, l) & (l, h) & (l, h) & (l, h) & \pi_2 < 0 & \pi_2 < 0 & \pi_2 < 0 & \pi_2 < 0 & \pi_2 < 0 & \pi_2 < 0 \\
\end{array}
\]

Table 2: institutional independent asymmetry.

We do comparative statics with respect to \( m \) (the mass factor) in this table.

\[ \pi_2 < 0 \] means that profits of state 2, the disadvantaged state, is negative at Nash equilibrium and therefore only state 1 will survive, as a monopoly and choosing a low fiscal capacity.

\[ c^l = 0.9, r^l = 3.6, l = 4; c^h = 0.3, r^h = 1.8, h = 6; c = 1.5. \]

\[
\begin{array}{cccccccccc}
  m & 0.8 & 0.9 & 1 & 1.1 & 1.2 & 1.3 & 1.4 & 1.5 & 1.6 & 1.7 \\
  NE & (l, l) & (l, l) & (l, l) & (l, l) & (l, h) & (l, h) & (h, h) & (h, h) & \pi_2 < 0 & \pi_2 < 0 \\
\end{array}
\]
Table 3: institutional dependent asymmetry ($r_1^I > r_2^I$).
We do comparative statics with respect to $m$ (the mass factor) in this table.
\[ e' = 0.9, r_1^I = 2.16, r_2^I = 1.8, l_1 = 2.4, l_2 = 2, e^h = 0.3, r^h = 1, h = 3.33. \]

<table>
<thead>
<tr>
<th>$m$</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N.E.$</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>(l,h)</td>
<td>(l,h)</td>
<td>(l,h)</td>
<td>(h,h)</td>
<td>(h,h)</td>
</tr>
</tbody>
</table>

Table 4: institutional dependent asymmetry ($e_1^I > e_2^I$).
We do comparative statics with respect to $m$ (the mass factor) in this table.
\[ e_1^I = 1.35, e_2^I = 0.9, r^I = 1.8, l_1 = 1.33, l_2 = 2, e^h = 0.3, r^h = 1, h = 3.33. \]

<table>
<thead>
<tr>
<th>$m$</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N.E.$</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>(h,l)</td>
<td>(h,l)</td>
<td>(h,h)</td>
<td>(h,h)</td>
<td>(h,h)</td>
<td>(h,h)</td>
</tr>
</tbody>
</table>

Table 5: institutional dependent asymmetry ($r_1^h > r_2^h$).
We do comparative statics with respect to $m$ (the mass factor) in this table.
\[ e' = 0.9, r^I = 1.8, l = 2, e^h = 0.3, r_1^h = 1.2, r_2^h = 1, h_1 = 4, h_2 = 3.33. \]

<table>
<thead>
<tr>
<th>$m$</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N.E.$</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>nil</td>
<td>(h,l)</td>
<td>(h,h)</td>
<td>(h,h)</td>
<td>(h,h)</td>
<td>(h,h)</td>
</tr>
</tbody>
</table>

Table 6: institutional dependent asymmetry ($e_1^h > e_2^h$).
We do comparative statics with respect to $m$ (the mass factor) in this table.
\[ e_1^I = 0.9, r^I = 1.8, l = 2, e_1^h = 0.45, e_2^h = 0.3, r^h = 1, h_1 = 2.22, h_2 = 3.33. \]

<table>
<thead>
<tr>
<th>$m$</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N.E.$</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>(l,l)</td>
<td>(l,h)</td>
<td>(l,h)</td>
<td>(h,h)</td>
<td>(h,h)</td>
</tr>
</tbody>
</table>
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