Numerical Simulation of Hydrothermal Convection within Discretely Fractured Porous Media with Application to the Seafloor

by

Jianwen Yang

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of Physics
University of Toronto

© Copyright by Jianwen Yang 1997
The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-28093-4
Fluid circulation driven by thermal gradients is an important physical phenomenon characteristic of the subsurface of the earth. My thesis contributes to investigating the physical behaviour and computational techniques of heat transport and fluid flow in explicitly fractured porous media. Both analytical and numerical methods are developed to study problems related in particular to hydrothermal convective systems in the mid-ocean ridge environment.

An analytical solution of the heat transport process in a single fracture is first derived for the impermeable host rock. For general hydrothermal convective systems, a numerical algorithm is developed. The algorithm is validated by comparing numerical results with analytical solutions. It is indicated discrete fractures not only initiate and maintain hydrothermal convection but also greatly change an established convection pattern.

The effect of anisotropic permeability fields on hydrothermal convection is studied both analytically and numerically. A more useful form of the Rayleigh number is defined, which contains the geometric mean of the vertical and horizontal permeabilities. The critical value for the onset of stable convection is derived analytically. It is shown that anisotropy resists the initiation of hydrothermal convection.

An alternative theory is developed to explain the origin of small-scale seafloor heat-flow variations over a mid-ocean ridge flank where basement relief is not clear. It is indicated that inclusion of explicit fractures in upper oceanic crust promotes and main-
tains hydrothermal convection within the upper basalts. Consequently, the small-scale heat-flow variations are produced on the seafloor.

A 3-D hydrothermal system within the TAG-like sulfide mound is examined. A range of models with different physical parameters and boundary conditions is investigated. Numerical results reveal that a central high-temperature zone is coincident with a non-magnetic zone inferred from magnetic data. Upflow fluid motion is along the vertical high-permeability column surrounding a central pipe. The maximum fluid velocity in the latter is in the order of the observed values.
Acknowledgements

I would like to express my deep appreciation to my thesis supervisor, Professor R.N. Edwards, for his constant support and guidance in all aspects relating to and stemming from this work. His knowledge, dedication, kindness and sense of humour have made these years a most rewarding and enjoyable time.

Thanks are due to the members of my Ph.D supervisory committee, Professor G.W.K. Moore and Professor J.X. Mitrovica for their excellent advice and assistance. I also thank Professor G.F. West and Professor R.C. Bailey for valuable discussions related to this work.

I have benefited strongly from discussions on my research with Professor E.A. Sudicky of the Waterloo Centre for Groundwater Research.

I am extremely grateful to Mr. J.W. Molson of the Waterloo Centre for Groundwater Research for his assistance in designing and developing finite element software. I have benefited immensely from his code HEATFLOW.

I would like to thank graduate students in the Geophysics Division for their help and friendship, particularly G. Cairns, M. Jegen, and L. Konstantin with whom I have enjoyed a lot of great times.

The thesis might never have been finished without the support of my loving wife, Xing. She endured all the late nights and lost weekends with remarkable patience.

Financial support of the following groups is gratefully acknowledged: Natural Science and Engineering Research Council of Canada, the University of Toronto Open Doctoral Fellowship, and the Department of Physics at the University of Toronto.
## Contents

1 Hydrothermal Convection and the Mid-Ocean Ridge Environment 1
   1.1 Fundamental Physics and Governing Equations 2
   1.2 Simple Fracture Model 3
   1.3 Numerical Simulation Method 3
   1.4 Anisotropic Permeability Model 5
   1.5 Off-axis Seafloor Heat-flow Variations 5
   1.6 The TAG active Sulfide Mound 7

2 Fundamental Equations and Physics of Hydrothermal Convection System in Porous Media 9
   2.1 The Equation of Fluid Motion 9
   2.2 Conservation of Mass 11
   2.3 Conservation of Energy 11
   2.4 Supplemental Equations 12
   2.5 Rayleigh Number and Dimensional Analyses 13
   2.6 Discussions and Conclusions 14

3 Flow and Heat Transport in Fractured Porous Medium 15
   3.1 Introduction 15
   3.2 Flow Flux in a Single Fracture 16
   3.3 Mass Balance Equation in a Single Fracture 17
   3.4 Heat Transport Equation in a Single Fracture 19
   3.5 Heat Transport in Fractured Porous Medium: Analytical Solution for a Single Fracture 20
     3.5.1 A Simple Fracture-Matrix System 21
     3.5.2 Governing Equations 22
     3.5.3 General Transient Solution 23
     3.5.4 Steady State Solution 27
3.5.5 Some Calculation Examples ................................... 27
3.6 Discussions and Conclusions .................................... 28

4 Numerical Simulation Method of Hydrothermal Convection in Fractured Porous Medium 31
4.1 Introduction .................................................. 31
4.2 The Galerkin Finite Element Technique ......................... 32
4.3 Numerical Algorithm for Discretely Fractured Porous Medium .......................... 36
4.4 Validation of the Numerical Scheme ............................ 44
4.5 Discussions and Conclusions .................................. 48

5 Elementary Studies of Some Simplified Fractured Porous Media 59
5.1 Introduction .................................................. 59
5.2 An Unfractured Porous Medium ................................ 60
5.3 A Single Fractured Porous Medium .............................. 62
  5.3.1 Effect of the Fracture Aperture ................................ 62
  5.3.2 Effect of the Fracture Location ............................ 63
  5.3.3 Effect of the Grid Size of Elements ......................... 63
5.4 Super-critical Fractured Porous Medium ......................... 64
  5.4.1 Unfractured Porous Medium ................................ 64
  5.4.2 A Single Fractured Porous Medium ......................... 64
  5.4.3 Multiply Vertcally Fractured Porous Medium ................ 65
  5.4.4 Multiply Horizontally Fractured Porous Medium ............ 65
5.5 Discussions and Conclusions .................................. 65

6 Hydrothermal Convection in Anisotropic Permeable Media 84
6.1 Introduction .................................................. 84
6.2 Analytical Solution to the Onset of Hydrothermal Convection in 2-D Anisotropic Permeable Media ........................................ 86
6.3 Numerical Simulation Results .................................. 90
6.4 Discussions and Conclusions .................................. 92

7 Fracture-Induced Hydrothermal Convection in the Oceanic Crust and the Interpretation of Heat-Flow Data 99
7.1 Introduction .................................................. 100
7.2 The Crustal Hydrothermal Model ................................ 101
7.3 Numerical Simulation Results .................................. 102
7.4 Discussions and Conclusions ........................................... 105

8 Three-Dimensional Numerical Simulation of the Hydrothermal System 
within the TAG-like Sulfide Mounds .................................... 109
  8.1 Introduction .......................................................... 110
  8.2 A Simplified TAG-like Hydrothermal Model ................. 112
  8.3 Boundary and Initial Conditions .................................. 113
  8.4 Numerical Simulation Results ..................................... 114
  8.5 Discussions and Conclusions ....................................... 115

9 Summary of Original Contributions and Suggestions for Further Works 122
  9.1 Summary of Original Contributions ...................... 122
    9.1.1 Analytical Solutions ..................................... 122
    9.1.2 Numerical Solutions ..................................... 123
    9.1.3 Field-Scale Examples .................................. 123
    9.1.4 Software Development ................................ 124
  9.2 Suggestions for Further Work ................................ 124
# List of Figures

1.1 A comparison of ridge morphologies at different spreading rates ........ 2
1.2 A schematic model of TAG active sulfide mound .......................... 7
2.1 A schematic representation of the Darcy’s law .......................... 10
2.2 A homogeneous porous layer ........................................ 13
3.1 A schematic representation of a fractured porous medium ............. 15
3.2 The force balance on a layer of water in a 1-D horizontal fracture .... 16
3.3 Fracture-porous block geometry in one-dimensional case ............. 17
3.4 A simple fracture-matrix system ..................................... 21
3.5 The normalized temperature contours for the case 1: (a) \( t=1.0 \) day, (b) \( t=2.0 \) days, and (c) \( t=3.0 \) days. .............................. 29
3.6 The normalized temperature contours for the case 2: (a) \( t=0.5 \) day, (b) \( t=1.0 \) day, and (c) \( t=1.5 \) days. .............................. 30
4.1 (a) A 2-D fractured porous medium including a vertical fracture; (b) Discretion of the model by a 2-D equal-spaced grid; (c) The host porous medium; and (d) The discrete fracture. The numbers represent the nodal number of elements. The host medium and fracture share the nodes numbered by 26, 27, and 28. ........................................ 37
4.2 A simple fracture-matrix system ..................................... 45
4.3 The normalized temperature contours at the fluid velocity of \( 5 \times 10^{-3} \) m/s: (a), (b) and (c) are the analytical solutions; (d), (e) and (f) are the numerical solutions, corresponding to the time levels of 1, 2 and 3 days. .... 49
4.4 The normalized temperature contours at the fluid velocity of \( 10^{-2} \) m/s: (a), (b) and (c) are the analytical solutions; (d), (e) and (f) are the numerical solutions, corresponding to the time levels of 0.5, 1.0 and 1.5 days. .... 50
4.5 The normalized temperature distribution along the fracture. The solid lines denote analytical solution and the little open boxes represent numerical solution: (a) The fluid velocity is $5 \times 10^{-3}$ m/s; and (b) The fluid velocity is $1 \times 10^{-2}$ m/s. .... 51

4.6 A simple fracture-matrix system. .... 52

4.7 Comparison between the numerical solutions and the analytical solutions of the normalized contaminant concentration distribution along the fractures at different time levels. The aperture of each fracture is 0.1 mm. The fracture spacing is 10 m. Water velocity $v=0.1$ m/day. .... 53

4.8 Comparison between the numerical solutions and the analytical solutions of the normalized contaminant concentration distribution within the host rock matrix at the time of 500 days. The aperture of each fracture is 0.1 mm. The fracture spacing is 10 m. Water velocity $v=0.1$ m/day. .... 53

4.9 A 2-D test model. The host porous medium has a permeability of $10^{-10}$ m$^2$. The fracture has an aperture of 0.5 mm. .... 54

4.10 Numerical simulation results for the test model at time=100 days. .... 55

4.11 Numerical simulation results for the test model at time=200 days. .... 56

4.12 Numerical simulation results for the test model at time=300 days. .... 57

4.13 Temperature distribution along the fracture. The solid lines denote Waterloo code solution and the little open boxes represent my own code solution: (a) Time=100.0 days; (b) Time=200.0 days; and (c) Time=300.0 days. .... 58

5.1 A 2-D water-saturated unfractured porous medium. .... 60

5.2 Numerical simulation results for a 2-D water-saturated unfractured porous medium: (a) and (b)- the initial temperature and fluid velocity perturbations (maximum initial velocity is $3.35 \times 10^{-7}$ m/s); and (c) and (d)- the steady state temperature contours and fluid velocity field (maximum value is $1.53 \times 10^{-11}$ m/s). .... 61

5.3 A 2-D water-saturated fractured porous medium. The location and aperture of the vertical fracture are changeable. .... 62

5.4 Numerical simulation results at the steady state when the vertical fracture is located in the middle of the model. .... 67

5.5 Effect of the fracture's aperture on temperature contours. .... 68

5.6 Effect of the fracture's lateral location on numerical simulation results. .... 69

5.7 Effect of the fracture's lateral location on numerical simulation results. .... 70

5.8 Effect of the fracture's lateral location on numerical simulation results. .... 71
5.9 Effect of the fracture's lateral location on numerical simulation results.

5.10 Temperature contours at the steady state when the vertical fracture is located in the middle of the model and its aperture is 0.2 mm. The grid size is changeable.

5.11 Numerical simulation results at the steady state for a uniform porous medium with the Rayleigh number of 55. There are no explicit fractures: (a) temperature contours; and (b) fluid velocity field.

5.12 Initial perturbations: (a) Initial temperature distribution; and (b) Initial fluid velocity field.

5.13 Numerical simulation results at the steady state for a uniform porous medium with the Rayleigh number of 55. There are no explicit fractures: (a) temperature contours; and (b) fluid velocity field.

5.14 Initial perturbations: (a) Initial temperature distribution; and (b) Initial fluid velocity field.

5.15 Numerical simulation results at the steady state: (a) temperature contours; (b) fluid velocity field in porous medium; and (c) fluid velocity field in the fractures. The 0.2-mm-thick fracture is located in the centre.

5.16 Numerical simulation results at the steady state: (a) temperature contours; (b) fluid velocity field in porous medium; and (c) fluid velocity field in the fractures. The 0.2-mm-thick fracture is close to the right wall.

5.17 Numerical simulation results at the steady state. There are 20 vertical fractures: (a) temperature contours; (b) fluid velocity field in porous medium; and (c) fluid velocity field in the explicit fractures.

5.18 Numerical simulation results at the steady state. There are 20 vertical fractures: (a) temperature contours; (b) fluid velocity field in porous medium; and (c) fluid velocity field in the explicit fractures.

5.19 Numerical simulation results at the steady state. There are 10 horizontal fractures: (a) temperature contours; (b) fluid velocity field in porous medium; and (c) fluid velocity field in the explicit fractures.

5.20 Numerical simulation results at the steady state. There are 10 horizontal fractures: (a) temperature contours; (b) fluid velocity field in porous medium; and (c) fluid velocity field in the explicit fractures.

6.1 A 2-D anisotropic permeable model

6.2 The critical Rayleigh number increases with the anisotropic coefficient

6.3 Initial temperature and fluid velocity perturbations
6.4 Numerical simulation results at the steady state for the model 1
6.5 Temperature distributions at the steady state for the models 2 and 3. The
convective effect is now negligible.
6.6 (a) For the model 1, the normalized maximum fluid velocity increases
as time progresses, and finally reaches the steady state value. (b) For
the model 2, the normalized maximum fluid velocity decreases as time
progresses, and finally reaches zero at steady state. (c) For the model 3,
the normalized maximum fluid velocity decreases as time progresses, and
finally reaches zero at steady state.
6.7 Numerical simulation results at the steady state for the model 2 ($k_z/k_x =
100$): (a) temperature distribution; and (b) fluid velocity field. Note that
as $k_z/k_x$ increases the number of convection cells in the domain increases
6.8 Numerical simulation results at the steady state for the model 3 ($k_z/k_x =
0.01$): (a) temperature distribution; and (b) fluid velocity field. Note that
as $k_z/k_x$ decreases the number of convection cells in the domain decreases
7.1 Heat flow profile over the Juan de Fuca Ridge eastern flank
7.2 A 2-D water-saturated, sediment-sealed, layered model.
7.3 Numerical simulation results for the basic unfractured layered model.
7.4 The numerical simulation results at steady state for Model 1: (a) the
local-scale seafloor heat-flow variations; and (b) the local fluid flux per
unit length shown as arrows whose lengths are proportional to the flux
magnitude in a crustal cell or fracture. Also the temperature field shown
as the background gray scale.
7.5 The numerical simulation results at steady state for Model 2: (a) the local-
scale seafloor heat-flow variations; (b) the temperature field; and (c) the
fracture distribution.
7.6 A part of Figure 7.5 magnified illustrating: (a) the local-scale seafloor heat-
flow variations; (b) the temperature field; and (c) the integrated fluid flux
in the porous oceanic crust and fractures
8.1 Location of the TAG active sulfide mound.
8.2 A simplified TAG-like hydrothermal model
8.3 The steady state temperature distributions for the different thermal con-
ductivities of the TAG sulfide mound. The surface of the TAG mound is
assumed permeable.
8.4 The steady state fluid velocity patterns. The surface of the TAG mound is assumed permeable. ................................. 118
8.5 The steady state temperature distributions for the different thermal conductivities of the TAG sulfide mound. The surface of the TAG mound is assumed impermeable except for the top of the central column. .. 119
8.6 The steady state fluid velocity patterns. The surface of the TAG mound is assumed impermeable except for the top of the central column. .... 120
8.7 Sea surface and near-bottom magnetic field of the TAG sulfide mound. . 121
List of Tables

3.1 The physical parameters for the simple fracture-matrix system ........ 28
6.1 Parameters of the Models 1, 2, and 3 ........................................ 91
7.1 The parameters of the Models 1 and 2 ................................. 103
List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>thermal conductivity</td>
<td>([Jm^{-1}s^{-1}C^{-1}])</td>
</tr>
<tr>
<td>( k )</td>
<td>permeability</td>
<td>([m^2])</td>
</tr>
<tr>
<td>( k_f )</td>
<td>effective permeability of fracture</td>
<td>([m^2])</td>
</tr>
<tr>
<td>( K )</td>
<td>hydraulic conductivity</td>
<td>([ms^{-1}])</td>
</tr>
<tr>
<td>( \theta )</td>
<td>porosity of porous medium</td>
<td>([%])</td>
</tr>
<tr>
<td>( q )</td>
<td>Darcy flux</td>
<td>([ms^{-1}])</td>
</tr>
<tr>
<td>( v )</td>
<td>water velocity ((v = q/\theta))</td>
<td>([m/s])</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature</td>
<td>([^\circ C])</td>
</tr>
<tr>
<td>( p )</td>
<td>pressure</td>
<td>([kgm^{-1}s^{-2}])</td>
</tr>
<tr>
<td>( h )</td>
<td>water head</td>
<td>([m])</td>
</tr>
<tr>
<td>( t )</td>
<td>time</td>
<td>([s])</td>
</tr>
<tr>
<td>( x, y, z )</td>
<td>three-dimensional Cartesian coordinates</td>
<td>([m])</td>
</tr>
<tr>
<td>( 2b )</td>
<td>aperture of fracture</td>
<td>([m])</td>
</tr>
<tr>
<td>( L )</td>
<td>thickness of layer</td>
<td>([m])</td>
</tr>
<tr>
<td>( \rho )</td>
<td>density</td>
<td>([kgm^{-3}])</td>
</tr>
<tr>
<td>( c )</td>
<td>specific heat</td>
<td>([Jkg^{-1}C^{-1}])</td>
</tr>
<tr>
<td>( k_m )</td>
<td>thermal diffusivity</td>
<td>([m^2s^{-1}])</td>
</tr>
<tr>
<td>( \mu )</td>
<td>viscosity of water</td>
<td>([kgm^{-1}s^{-1}])</td>
</tr>
<tr>
<td>( \nu )</td>
<td>kinematic viscosity of water</td>
<td>([m^2s^{-1}])</td>
</tr>
<tr>
<td>( \alpha_v )</td>
<td>thermal expansion coefficient of water</td>
<td>([C^{-1}])</td>
</tr>
<tr>
<td>( g )</td>
<td>acceleration due to gravity</td>
<td>([ms^{-2}])</td>
</tr>
<tr>
<td>( R_a )</td>
<td>dimensionless Rayleigh number</td>
<td></td>
</tr>
<tr>
<td>( R_{ac} )</td>
<td>critical Rayleigh number</td>
<td></td>
</tr>
</tbody>
</table>

Subscripts

\( w \) water
\( r \) solid rock
\( m \) water-rock matrix
Chapter 1

Hydrothermal Convection and the Mid-Ocean Ridge Environment

My thesis is an investigation of hydrothermal convection in earth structures, with particular focus on the heat transport and fluid flow in discretely fractured porous media. I develop both analytical and numerical methods, and apply them to problems related in particular to fluid flow and heat transport in the mid-ocean ridge environment.

Mid-ocean ridges mark constructive plate margins where new oceanic lithosphere originates. Ridge morphology can be obtained by high resolution swath mapping technique such as SeaBeam. The gross morphology of ridges appears to be controlled by spreading rate. Typical morphologies are shown in Figure 1.1 [MacDonald, 1982]. At relatively slow rates of 10-50 mm/yr, such as the mid-Atlantic and Atlantic-Indian ridges, a prominent median rift is developed at the ridge axis. This is commonly 30-50 km wide and 1.5-3.0 km deep, and contains rugged topography. At intermediate rates of 50-90 mm/yr, such as at the Galapagos spreading center, the median rift is only 50-200 m deep and its topography relatively smooth. At rates of greater than 90 mm/yr, such as at the East Pacific Rise, no median rift is developed and the topography of the ridge crest is relatively smooth.

The subseafloor hydrothermal system, with seawater as the fluid medium, fractured porous oceanic crust as the solid medium, and a magma chamber associated with intrusive igneous rock as the heat source, is an extremely efficient mechanism for the exchange of heat and matter between seawater and oceanic crust. Nearly 25 % of Earth’s total heat flux, and approximately 33 % of the heat flux through the ocean floor, is transferred by hydrothermal advection [Williams and Von Herzen, 1974; Sclater et al., 1980; Stein
and Stein, 1994]. In addition, chemical and thermal exchanges between the ocean and oceanic crust in hydrothermal systems also play a major role in the genesis of many types of ore deposits. Ancient massive Fe-Cu-Zn±Pb±Ag sulfide deposits or “polymetallic sulfides”, now found and mined on land, were originally formed from submarine hot springs [Brimhall, 1991]. On modern oceanic ridge crests, hydrothermal activity is depositing polymetallic sulfides [Scott, 1985; Lowell and Rona, 1985; Rona et al., 1986]. Several massive sulfide deposits have been found in the East Pacific Rise (EPR), in the Juan de Fuca Ridge, and in the Mid-Atlantic Ridge.

1.1 Fundamental Physics and Governing Equations

An examination of any problem in ridge fluid flow requires first the setting up of the mathematical model. In Chapter 2, I summarize the fundamental equations governing conservation of mass, momentum, and energy, with special emphasis on their physical interpretations. The equations include the Darcy equation, and the mass and energy conservation equations. Constitutive equations are also provided, defining the temperature dependence of fluid density and viscosity.
1.2 Simple Fracture Model

Intense crustal fissuring and cracking is seen in the MOR environment. The horizontal velocity of crust, which increases from stationary at the ridge axis to the full spreading rate at the edge of the plate boundary zone, causes tensional stresses. These stresses and the rapid cooling of hot rock by seawater are responsible for this pervasive fracturing [MacDonald, 1982; Morgan, 1991]. At the mid-ocean ridge crest, fractures are open and mostly filled with seawater [Choukroune et al., 1984]. The network of fractures and cracks provides paths for cold seawater to penetrate the young, hot oceanic crust, setting up hydrothermal circulation. The behaviour of heat transport as well fluid flow in discrete fractures is poorly understood. Bodvarsson [1969] solved analytically the heat transport problem in a single fracture for the case of constant mass flow. Fluid entered the fracture with a temperature varying sinusoidally with time. Gringarten et al. [1975] derived an expression for the heat extracted from fractured hot dry rock, but ignored the conductive heat transport mechanism along the fracture axis.

In Chapter 3, I investigate the physical behaviour and governing equations of the flow and heat transport in a single fracture embedded in a host porous medium. For the special case of an impervious host rock, I derive an analytical expression of the temperature distribution along the fracture and inside the host medium subject to realistic boundary conditions and heat transport mechanism.

1.3 Numerical Simulation Method

Hydrothermal convective systems in the oceanic crust have generally been described as cellular convection (porous medium) models. The pertinent conservation equations are solved in a porous medium subject to conditions on the temperature and fluid flow on boundaries. Typically, the upper boundary is assumed to be at constant pressure (permeable) and isothermal; the lateral boundaries are assumed to be impermeable and adiabatic, and the lower boundary is assumed to be impermeable with either a fixed temperature or fixed heat flux. Often, only steady state solutions are derived. For instance, Fehn and Cathless [1979] and Fehn et al. [1983] used a finite-difference method to study the hydrothermal convection and to reproduce the hydrothermal field at the Galapagos spreading center. Gartline [1981] considered the role of sediment cover in sealing oceanic crust. Fisher et al. [1990] performed finite-element modellings for off-axis hydrothermal
circulation through the southern flank of the Costa Rica Rift around the DSDP/ODP Hole 504B. Fisher et al. [1994] modified their 1990 work by developing a refined model with curvilinear elements to improve the simulation of topography at the seafloor and within underlying sediment and basaltic layers. They were successful in duplicating the regional-scale seafloor heat-flow anomalies and other observations [Fisher and Becker, 1995]. Rosenberg and Spera [1990] and Rosenberg et al. [1993] completed numerical simulations of finite amplitude and steady convective fluid flow in layered and anisotropic media, and discussed the relationship between flow, temperature, and anisotropic permeability. Bessler et al. [1994] explored numerically the potential magnitude of fluid pressure and temperature differences at and beneath a sediment-basement interface in a sedimented-ridge setting at the Middle Valley. Recent numerical models have begun to consider three-dimensional circulation. For example, Travis et al. [1991] investigated fully 3-D simulations of deep hydrothermal circulation and the regional-scale flow and temperature pattern at mid-ocean ridges.

A serious drawback to the previous numerical models is the scale of resolution and the application of an isothermal upper boundary condition. They tend to address features of convective flow on a scale of kilometers but do not readily resolve small scale (~1-10 m) features such as discharge through faults and fractures that have high permeability. The isothermal upper boundary condition also precludes the modelling of high-temperature springs or black smokers which are common at the ridge-axial seafloor environments. Moreover, previous models have not investigated the hydrothermal convection in the fractured porous medium. They treated the rock matrix within each element as a continuous porous medium. When representing a fractured oceanic crust, fracture was treated simply as a zone consisting of several volume elements specified with high permeability. Discrete fractures have never been incorporated into the previous models explicitly yet. Furthermore, all previous models assumed that the fluid density is a linear function of temperature and the density variations only enter in the buoyancy term (i.e., the Boussinesq approximation) and the other fluid properties (e.g., viscosity) are all constant. In fact, the fluid viscosity is a very strong function of temperature.

In Chapter 4, I develop a numerical software to simulate hydrothermal circulation in the fractured oceanic crust. Each volume element is assigned a permeability, porosity, and thermal conductivity. Discrete fractures are included explicitly as planar structures connecting the mesh nodes. The method is distinctly different from previous studies, as it can treat not only the larger scale features of the porous media but also the smaller scale fractures. More reasonable boundary conditions are adopted, and a more realistic
temperature-dependent fluid density and fluid viscosity is included. The scheme is validated both by comparing numerical solutions with analytical solutions and by comparing numerical results derived from two separately-developed codes.

In Chapter 5, I apply the numerical algorithm to investigate elementary properties of some simplified fractured porous media. Computational results reveal that discrete fractures not only initiate hydrothermal convection but also can change an established pattern greatly.

### 1.4 Anisotropic Permeability Model

Of the parameters controlling the convective flow, permeability is the most critical. The permeability distribution in mid-ocean ridge environment is rarely isotropic and homogeneous, and it is likely highly anisotropic, at least in the upper oceanic crust, as suggested by the pervasive presence of discrete fractures at the sea floor and within boreholes [Pezard and Anderson, 1989; Pezard, 1990; Dick et al., 1991; Anderson and Zoback, 1982; Newmark et al., 1985a, 1985b; Morin et al., 1989; Agar, 1990]. Little effort has been made to investigate the effect of anisotropic permeability distribution on subseaﬂoor hydrothermal convection. The onset of hydrothermal convection in two dimensional anisotropic permeable media has been studied in the literature. However, the previous studies defined the Rayleigh number only in terms of the vertical permeability, and did not contain the horizontal permeability.

Two questions now arise, “Is the previous definition of the Rayleigh number reasonable?” “Is the horizontal permeability really insignificant?” In Chapter 6, I address these problems both analytically and numerically in detail. A more useful form of the Rayleigh number is defined, containing the geometric mean of the vertical and horizontal permeabilities. The critical value of this form of Rayleigh number is determined analytically and supported by results of numerical simulations.

### 1.5 Off-axis Seafloor Heat-flow Variations

On the ridge flank environments, systematic high and low heat-flow anomalies are commonly observed that strike parallel to structural grain. The cross-strike heat-flow variations commonly have a wavelength of 6 to 10 km, and are often correlated with seafloor
topography, with minima near topographic lows and maxima near topographic highs [Fisher et al., 1990; Fisher et al., 1994]. The spacing of many historical surveys was too great to resolve the much more subtle local-scale heat-flow variability. However, recent improvements in measurement technique and navigational control have enabled the non-aliassed delineation of the local-scale heat-flow variations. Site separations are typically a few hundred meters. Davis et al. [1992] obtained an interesting heat flow profile 5 Km long over the eastern Juan de Fuca ridge flank. The heat flow values are on average 250 mW/m² but there are superimposed variations of ± 30 mW/m² with a half-wavelength of about 600 m. Davis et al. [1995] interpreted the pattern as indicating low aspect ratio hydrothermal convection confined to the basement in a 600-m-thick layer of high permeability, $2 \times 10^{-12}$ m². However, there is no direct evidence to justify their model. The in-situ measurements of bulk permeability in DSDP and ODP basement holes have revealed that maximum permeability is about $10^{-13}$ m² in the upper few hundred metres ($\sim 200$ m) of igneous crust, and the value of $10^{-12}$ m² has never been obtained so far. The thickest permeable layer has been found in DSDP Hole 395A, and it has a thickness of 450 m. A thickness of 600 m is most likely overestimated for the uppermost permeable oceanic crust. Fisher and Becker [1995] argue that such a thick permeable layer may not necessary to explain the observations provided some basement relief is present. The model they developed requires an anisotropic layer only 180-200 m thick, mean permeability $10^{-13}$ m² with a relief of 20 m. The theory of Fisher and Becker [1995], while quite plausible in a general sense, may not be a valid explanation of this particular data set, because a seismic survey in the same area shows no obvious relief [Davis et al., 1992; Rohr, 1994]. Then, is there an alternative explanation?

The in-situ experiments of the Resistivity Logging and the Bore-hole Televiewer Imagery in DSDP/ODP Hole 504B reveal that the shallow oceanic crust is highly fractured [Pezard and Anderson, 1989; Pezard, 1990; Dick et al., 1991; Anderson and Zoback, 1982; Newmark et al., 1985a, 1985b; Morin et al., 1989; Agar, 1990]. It is inferred from the in-situ borehole electrical measurements [Pezard and Anderson, 1989; Pezard, 1990] that pillow lavas (Layer 2A) have a large amount of horizontal and vertical fracturing; the underlying flows and pillows (Layer 2B) contain mainly vertical fractures, which is believed to reflect the overall cooling of the lithosphere by hydrothermal fluids [Lister, 1972, 1974]; in transition zone and sheeted dikes (Layer 2C), the porosity is very low and the fracture distribution is again mostly vertical.

The questions now arise, "Could discrete fractures present in upper oceanic crust play a crucial role in initiating and maintaining subseaﬂoor convection?" "Could they cause
seafloor heat-flow variations in areas where basement relief is not a factor?" In Chapter 7, I propose an alternative explanation for the small-scale heat-flow variations, and indicate that fluid flow through fractures causes horizontal thermal gradients, initiates and maintains hydrothermal convection within the upper oceanic crust.

1.6 The TAG active Sulfide Mound

Direct observation of "hot springs" and "black smokers" leaves no doubt that active hydrothermal convection occurs at the axes of mid-ocean ridges. Sulfide-rich black smoker fluids typically have a temperature in the neighborhood of 350 °C; precipitates from black smoker discharge form sulfide chimneys and large sulfide deposits on the seafloor [Lowell, 1991]. High-temperature systems have been found at sites with a variety of tectonic settings and over a range of spreading rates [Lowell et al., 1995]. Sulfide ore deposits in ophiolite complexes are thought to be fossil remains of active high-temperature submarine systems [Richardson et al., 1987; Haymon et al., 1989].

![Figure 1.2: A schematic model of TAG active sulfide mound [after Rona and Speer, 1989]. Features are (1) black smokers venting at fast rates (1 m/s) from spire-shaped chimneys, (2) black smokers venting at slow rates from fractures, and (3) white and blue-white smokers venting at slow rates from onion-shaped chimneys.](image)

An immediate example of hydrothermal convection system in ridge crest environments is the Trans-Atlantic Geotraverse (TAG) active sulphide mound, which is located at 26°08'N and 44°49'W and situated in water depths between 3620 and 3670 m at the juncture between the floor and the east wall of the rift valley of the Mid-Atlantic Ridge.
As shown in Figure 1.2, the TAG active mound is about 200 m in diameter, 35 m high, and may contain 5 million tons of surface sulphide ore. Deep sea camera surveys and sampling from surface ship and submersible reveal that the exposed portion of the mound is composed of massive sulphides and sulphates. Indeed, the entire mound is probably constructed of massive sulphides precipitated from hydrothermal solution [Rona et al., 1993a; Rona et al., 1993b].

The TAG active hydrothermal system has become the subject of ongoing collaborative, multidisciplinary research due to its importance to ridge dynamics and ore-forming processes as well as to its potential economic significance [Rona et al., 1993a; Rona et al., 1993b]. Electromagnetic, heatflow, near-bottom temperature, fluid velocity and near-bottom magnetic data have been collected over the surface of the mound. In order to aid in the interpretation of these data sets, and ultimately to understand the internal character of the mound and the ore-forming processes, realistic estimates of the internal temperature and fluid velocity fields are therefore required.

In Chapter 8, I simulate the internal temperature and fluid velocity fields for the TAG-like sulfide mound. A range of models with different physical parameters and boundary conditions is investigated.
Chapter 2

Fundamental Equations and Physics of Hydrothermal Convection System in Porous Media

In this Chapter, fundamental equations are developed to describe the physical behaviour of hydrothermal circulation systems in porous media. They are the equation of fluid motion, the equation of mass conservation, and the equation of energy conservation. I assume that a porous medium's density, permeability and porosity are functions of space only, the medium is saturated by a single-component fluid (water), and thermal equilibrium exists between water and solid matrix.

2.1 The Equation of Fluid Motion

In 1856, Henry Darcy found that the flow through porous media is linearly proportional to the applied pressure gradient and inversely proportional to the viscosity of the fluid. As illustrated in Figure 2.1, for a one-dimensional geometry in which fluid is driven by the applied pressure gradient, the Darcy’s law takes the form

\[ q = -\frac{k (p_1 - p_0)}{\mu_w L} = -\frac{k}{\mu_w} \frac{dp}{dx}, \tag{2.1.1} \]

where \( q \) is the Darcy flux in the \( x \)-direction \( (ms^{-1}) \), \( k \) is the permeability \( (m^2) \), \( \mu_w \) is the dynamic viscosity of fluid \( (kg\ m^{-1}s^{-1}) \), \( p_0 \) is the pressure at the entrance to the section \( (Kgm^{-1}s^{-2}) \), \( p_1 \) is the pressure at the section exit, and \( L \) is the length of the section.
Figure 2.1: A schematic representation of the Darcy’s law.

The experimentally derived form of Darcy’s law was limited to one-dimensional flow. When the flow is three-dimensional, it can be generalized as [Bear, 1972]:

\[
q = -\frac{k}{\mu_w} \left[ \frac{\partial p}{\partial x} n_x + \frac{\partial p}{\partial y} n_y + (\frac{\partial p}{\partial z} + \rho_w g) n_z \right],
\]

where \( q \) is the Darcy flux vector with three components \((q_x, q_y, q_z)\) in \( x-, y-, \) and \( z-\) directions, \( \rho_w \) is the density of fluid \((kgm^{-3})\), \( g \) is the gravitational acceleration \((ms^{-2})\), and \( n_x, n_y, \) and \( n_z \) are three unit vectors in \( x-, y-, \) and \( z-\) directions, respectively.

Darcy’s law has a general validity for flow in earth materials if the flow is laminar and the scale of the porosity is smaller than the other characteristic dimensions.

The static pressure in (2.1.2) can be removed through the use of a reference hydraulic head \( h \) defined as [Frind, 1982]

\[
h = z + \frac{p}{\rho_0 g},
\]

where \( \rho_0 \) is a reference density of fluid, and \( z \) is the elevation above a datum \((m)\).

Under these rules, equation (2.1.2) has the form:

\[
q_x = -\frac{k\rho_0 g}{\mu_w} \frac{\partial h}{\partial x},
\]

\[
q_y = -\frac{k\rho_0 g}{\mu_w} \frac{\partial h}{\partial y},
\]

\[
q_z = -\frac{k\rho_0 g}{\mu_w} \frac{\partial h}{\partial z} - \frac{k\rho_w}{\mu_w} (\rho_w - \rho_0).
\]
Equation (2.1.4) can be also rearranged as

\[ q_x = -K \left[ \frac{\partial h}{\partial x} \right], \quad q_y = -K \left[ \frac{\partial h}{\partial y} \right], \quad q_z = -K \left[ \frac{\partial h}{\partial z} + \rho_{rel} \right]. \tag{2.1.5} \]

Where \( K \) is the hydraulic conductivity (m/s) defined by

\[ K = \frac{k\rho_0 g}{\nu_w}, \tag{2.1.6} \]

and \( \rho_{rel} \) is the relative density of fluid defined by

\[ \rho_{rel} = \frac{\rho_w}{\rho_0} - 1. \tag{2.1.7} \]

### 2.2 Conservation of Mass

If a fluid is incompressible and the porous medium is nondeformable, the mass conservation equation can be written as [Bear, 1972]:

\[ \frac{\partial (\rho_w q_x)}{\partial x} + \frac{\partial (\rho_w q_y)}{\partial y} + \frac{\partial (\rho_w q_z)}{\partial z} = 0. \tag{2.2.1} \]

In other words, the divergence of mass transport is equal to zero.

Equation (2.2.1) is a general mass conservation expression appropriate to the incompressible fluid and solid matrix. If I assume the fluid density variations caused by thermal expansion are so small that they are only essential to producing the buoyancy force but negligible in all other respects (known as the Boussinesq approximation), the above equation can be simplified as

\[ \frac{\partial (q_x)}{\partial x} + \frac{\partial (q_y)}{\partial y} + \frac{\partial (q_z)}{\partial z} = 0. \tag{2.2.2} \]

### 2.3 Conservation of Energy

If there exists a thermal equilibrium between water and solid matrix, the energy conservation equation can be expressed as [Bear, 1972]:

\[ \nabla \cdot (\lambda_m \nabla T) - \nabla \cdot (\rho_w c_w q T) - \rho_m c_m \frac{\partial T}{\partial t} = 0, \tag{2.3.1} \]
where, $\lambda_m = (1 - \theta) \lambda_r + \theta \lambda_w$ is the thermal conductivity of the porous matrix (the subscript $w$ refers to fluid and $r$ to solid rock), $\theta$ is the porosity, and $\rho_m c_m = (1 - \theta) \rho_r c_r + \theta \rho_w c_w$.

On the left hand side of (2.3.1), the first term represents the conductive heat transport associated with the molecular diffusion, the second term expresses the convective heat transport related to the actual movement of fluid, and the third term is the heat stored in unit volume during the unit time due to the variation in temperature.

### 2.4 Supplemental Equations

Darcy's law, and mass and energy conservation equations are not sufficient to describe a hydrothermal circulation system, and they have to be supplemented by the definition of the temperature dependence of fluid density and viscosity.

In all previous studies, the fluid viscosity $\mu_w$ is assumed a constant, whereas the fluid density $\rho_w$ is assumed to be a linear function of only temperature $T$ according to

$$\rho_w(T) = \rho_0 [1 - \alpha_v (T - T_0)],$$

(2.4.1)

where $\rho_0$ is the fluid density at reference temperature $T_0$, and $\alpha_v$ is the coefficient of fluid volume expansion ($^\circ C^{-1}$).

In fact, the temperature variation of fluid density and viscosity is well known [Weast, 1980], thus more realistic supplementary equations can be obtained using polynomial interpolation functions derived from a least squares algorithm [Molson et al., 1992]. The fitted polynomial for density is

$$\rho_w(T) = \rho_0 \cdot \{1.0 + [-0.435 \times 10^{-5} + 0.838 \times 10^{-8} \cdot (T - 4.0)] \cdot (T - 4.0)^2\},$$

(2.4.2)

while that for viscosity is expressed as

$$\mu_w(T) = 1.787 \times 10^{-3} \cdot \exp[-0.0254 + 0.443 \times 10^{-4} \cdot T] \cdot T].$$

(2.4.3)
2.5 Rayleigh Number and Dimensional Analyses

Consider free thermal convection in a homogeneous porous layer as shown in Figure 2.2. The model is water-saturated and has a thickness of \( L \). The top and the bottom are impermeable and have constant temperatures of \( T_0 \) and \( T_1 \) \( (T_1 > T_0) \), respectively.

When the Darcy's law and the Boussinesq approximation are used, and the fluid density is assumed to be a linear function of the temperature and the fluid viscosity is assumed constant, the above governing equations can be expressed in dimensionless form [Bear, 1972]. Accordingly, hydrothermal convection patterns are entirely controlled by a most important dimensionless parameter, the Rayleigh number defined as

\[
R_a = \frac{k g \alpha_0 \Delta T L}{k_m \nu},
\]

where \( k \) is the permeability of porous medium \( (m^2) \), \( \nu \) is the kinematic viscosity of fluid \( (m^2/s) \), \( k_m \) is the thermal diffusivity \( (m^2/s) \), and \( \Delta T = (T_1 - T_0) \) is the temperature difference between the upper and lower boundaries.

From the point of view of physics, the Rayleigh number represents the ratio of the buoyancy force promoting convection to the viscous stress impeding convection. Hydrothermal convection will not occur unless the Rayleigh number exceeds a certain critical value. For the above simple model, the critical Rayleigh number can be shown to be \( 4 \pi^2 \). The higher the Rayleigh number is, the more energetic the convection becomes.

It should be emphasized that due to the high non-linearity of hydrothermal convection systems, it is very hard to carry dimensional analysis further. As stated at the
beginning of this section, the dimensionless expressions of the governing equations and the Rayleigh number can be derived only under several assumptions. For more realistic hydrothermal convection systems, fluid density and viscosity are usually non-linear functions of the temperature, as shown in (2.4.2) and (2.4.3), and the Boussinesq approximation is probably also not true. Thus dimensional analysis is not always applicable, especially for high-temperature hydrothermal convection systems near mid-ocean ridge crests.

2.6 Discussions and Conclusions

The fundamental equations governing conservation of momentum, mass, and energy together with a number of constitutive equations for hydrothermal convection system, and their physical interpretations have been discussed in this Chapter. When the Boussinesq approximation is used, and the fluid density is a linear function of the temperature and the other fluid properties are constants, the hydrothermal convection patterns are entirely controlled by the Rayleigh number. For more realistic hydrothermal systems, however, dimensional treatments are not always applicable.
Chapter 3

Flow and Heat Transport in Fractured Porous Medium

3.1 Introduction

As emphasized in Chapter 1, discrete fractures are very common in the upper oceanic crust, and at the mid-ocean ridge crest, fractures are open and mostly filled with seawater [Choukroune et al., 1984]. However, the behaviour of heat transport as well as fluid flow in discrete fractures is still poorly understood. The objective of this Chapter is to present the basic physical processes and governing equations of the flow and heat transport in a fractured porous medium. An analytical solution for the heat transport in a single vertical fracture situated in an impervious host rock is also derived. This analytical solution will be used to check the numerical solutions described in Chapter 4.

![Figure 3.1: A schematic representation of a fractured porous medium.](image)

As shown in Figure 3.1, a fractured porous medium is defined as one material which
is composed of two parts: an network of discrete fractures, and a host porous medium. The host porous medium mainly contains granular pores and its physical properties are represented by an 'average' over the block. The fracture part is a 'discrete' feature of the medium and its effective permeability is dependent on the aperture.

3.2 Flow Flux in a Single Fracture

Turcotte and Schubert [1982] reported the water flow through a single ideal fracture driven by an applied pressure gradient, as shown in Figure 3.2.

![Figure 3.2: The force balance on a layer of water in a 1-D horizontal fracture with an applied pressure gradient. After Turcotte and Schubert, 1982.](image)

They show that water velocity is a parabola function of distance $y$ from the upper fracture wall. It is:

$$v = \frac{1}{2b} \frac{dp}{dx} (y^2 - 2by), \quad (3.2.1)$$

where $v$ is the water velocity, $2b$ is the fracture's aperture, and pressure gradient $dp/dx = (p_0 - p_1)/L$.

The mean fluid velocity in the fracture can be determined by calculating the following integral:

$$\bar{v} = \frac{1}{2b} \int_0^b \frac{1}{2b} \frac{dp}{dx} (y^2 - 2by)dy, \quad (3.2.2)$$

and it is

$$\bar{v} = \frac{(2b)^2 dp}{12 \mu_w \ dx}. \quad (3.2.3)$$
Chapter 3: Flow and Heat Transport in Fractured Porous Medium

Comparing Equation (3.2.3) with the Darcy's law (2.1.1), one can see that the fracture has an effective permeability of
\[ k_f = \frac{(2b)^2}{12}, \]  
which is only dependent on its aperture \(2b\).

Similarly, for the pipe-shaped fracture, its effective permeability is only dependent on its radius \(r\):
\[ k_f = \frac{r^2}{8}. \]  

In the following Chapters, I will use (3.2.4) and (3.2.5) to estimate the permeability of discrete fractures.

### 3.3 Mass Balance Equation in a Single Fracture

![Fracture-porous block geometry in one-dimensional case.](image)

Figure 3.3 illustrates a horizontal fracture with an aperture of \(2b\). I assume that the aperture \(2b\) is much smaller than the fracture's length, the local variation in the fracture thickness is negligible, the variation of fluid density across the fracture's width is negligible, and the hydraulic head (or pressure) is continuous across the interface between fracture and porous medium.

In principle, flow transport within the fracture is three-dimensional. However, I can simplify the full 3-D model into a 2-D model in the plane of the fracture by integrating the general differential equation over the fracture's width, following Bear's steps [1993].
Within the fracture, the porosity \( \theta = 1 \), thus the Darcy flux \( q \) is equal to the mean fluid velocity \( \bar{v} \). Assuming no source or sink to be present, based on (2.2.1), the mass conservation equation at any point within the fracture is given by

\[
\frac{\partial}{\partial x} (\rho_w \bar{v}_x) + \frac{\partial}{\partial y} (\rho_w \bar{v}_y) + \frac{\partial}{\partial z} (\rho_w \bar{v}_z) = 0. \quad (3.3.1)
\]

Averaging this equation across the aperture \( 2b \) (normal to the fracture) yields an average mass balance equation in the single fracture; it is

\[
\int_{-b}^{b} \left[ \frac{\partial}{\partial x} (\rho_w \bar{v}_x) + \frac{\partial}{\partial y} (\rho_w \bar{v}_y) + \frac{\partial}{\partial z} (\rho_w \bar{v}_z) \right] dy = 0. \quad (3.3.2)
\]

Equation (3.3.2) can be further written as

\[
2b \left[ \frac{\partial \rho_w \bar{v}_x}{\partial x} + \frac{\partial \rho_w \bar{v}_y}{\partial y} \right] - (\rho_w \bar{v}_y) \big|_{y=-b} + (\rho_w \bar{v}_y) \big|_{y=b} = 0, \quad (3.3.3)
\]

where the last two terms on the left-hand side of (3.3.3) represent the normal components of the fluid leakage flux across the boundary interfaces \( (y = -b \) and \( y = b) \) which separate the fracture and the porous matrix. It is this leakage together with an assumed continuity in hydraulic head along the interface that provides the link between mass conservation equations in porous medium (2.2.1) and in the fracture (3.3.3).

Equation (3.3.3) is a general expression of the mass conservation equation in the fracture. Let's now consider a special case where the host matrix is impermeable for fluid flow. At the fracture-porous block interface, continuity of the fluid flux normal to the boundary requires that

\[
(\rho_w \bar{v}_y) \big|_{(y= -b, b)} = \theta (\rho_w v_{ypb}) \big|_{(y= -b, b)}, \quad (3.3.4)
\]

where \( \theta \) is the porosity of the porous medium, \( v_y \) and \( v_{ypb} \) are the normal components of fluid velocity in the fracture and porous block, respectively.

When the fracture is imbedded in an impervious rock, the fluid flow in the fracture takes place essentially parallel to the fracture. These leakage terms must vanish, thus (3.3.3) can be written as

\[
\left[ \frac{\partial (\rho_w \bar{v}_x)}{\partial x} + \frac{\partial (\rho_w \bar{v}_z)}{\partial z} \right] = 0, \quad (3.3.5)
\]

where \( \bar{v}_x \) and \( \bar{v}_z \) are the average fluid velocity components in \( x \) and \( z \) directions along the fracture, defined as
Chapter 3: Flow and Heat Transport in Fractured Porous Medium

\[ \dot{v}_x = \frac{1}{2b} \int_{-b}^{b} v_x dy, \]  
\[ \dot{v}_z = \frac{1}{2b} \int_{-b}^{b} v_z dy. \]  

Clearly, if one considers a 1-D pipe-shaped fracture along the \( x \) direction rather than the 2-D planar fracture shown in Figure 3.3, the mass conservation equation in the 1-D linear fracture has the form

\[ \frac{\partial (\rho_w \dot{v}_x)}{\partial x} = 0. \]  

In equation (3.3.8), the leakage term has been ignored.

**3.4 Heat Transport Equation in a Single Fracture**

Again following Bear's steps [1993], I here assume that the fluid is always instantaneously and thoroughly mixed across the fracture, thus thermal equilibrium exists between fluid and solid matrix, the variation in temperature across the fracture's aperture is negligible, and the temperature in the rock mass, immediately outside the fracture, is approximately equal to the temperature of the fracture.

Similarly, I average the general energy conservation equation at any point within the fracture (2.3.1) across the fracture aperture, referring to the geometry of Figure 3.3, to obtain

\[ \int_{-b}^{b} \left[ \nabla \cdot (\lambda_w \nabla T) - \nabla \cdot (\rho_w c_w v T) - \rho_w c_w \frac{\partial T}{\partial t} \right] dy = 0, \]  

where \( \lambda_w \) is the thermal conductivity of fluid.

Equation (3.4.1) can be rewritten as

\[ 2b \left[ \frac{\partial}{\partial x} (\lambda_w \frac{\partial T}{\partial x}) + \frac{\partial}{\partial z} (\lambda_w \frac{\partial T}{\partial z}) - \frac{\partial (\rho_w c_w \dot{v}_x T)}{\partial x} - \frac{\partial (\rho_w c_w \dot{v}_z T)}{\partial z} - \rho_w c_w \frac{\partial T}{\partial t} \right] + \]

\[ \left[ \lambda_w \frac{\partial T}{\partial y} - \rho_w c_w v_y T \right] \Bigr|_{y=b} - \left[ \lambda_w \frac{\partial T}{\partial y} - \rho_w c_w v_y T \right] \Bigr|_{y=-b} = 0, \]  

where the last two terms on the left-hand side of (3.4.2) represent influx heat-flow and efflux heat-flow through the two fracture walls due to fluid leakage and molecular diffusion. Also, these two terms together with an assumed continuity in temperature along the interface provide the link between energy conservation equations in the porous medium (2.3.8) and in the fracture (3.4.2).
Equation (3.4.2) is a general expression of the energy conservation principle in the fracture. Let's again consider a special case where the host matrix is impermeable for fluid flow.

When the fracture is imbedded in an impervious rock, the normal component of fluid flow in the fracture should be zero. Notice that when the fracture aperture is very small, temperature distribution is symmetric about the fracture,

\[ \left[ \lambda_w \frac{\partial T}{\partial y} \right]_{y=-b} = -\left[ \lambda_w \frac{\partial T}{\partial y} \right]_{y=b}. \]  

(3.4.3)

These two conditions together reduce (3.4.2) to

\[ \frac{\partial}{\partial x} (\lambda_w \frac{\partial T}{\partial x}) + \frac{\partial}{\partial z} (\lambda_w \frac{\partial T}{\partial z}) - \frac{\partial}{\partial x} (\rho_w c_w \bar{v}_x T) - \frac{\partial}{\partial z} (\rho_w c_w \bar{v}_z T) - \rho_w c_w \frac{\partial T}{\partial t} + \frac{\lambda_w}{b} \frac{\partial T}{\partial y} \big|_{y=b} = 0, \]  

(3.4.4)

where \( \bar{v}_x \) and \( \bar{v}_z \) are the average fluid velocity components in \( x \) and \( z \) directions along the fracture, defined in Equations (3.3.6) and (3.3.7).

Notice that

\[ \lambda_w \frac{\partial T}{\partial y} \big|_{y=b} = \lambda_m \frac{\partial T'}{\partial y} \big|_{y=b}, \]  

(3.4.5)

where \( T' \) and \( \lambda_m \) are respectively the temperature and thermal conductivity of the host rock.

Thus Equation (3.4.4) can be expressed as

\[ \frac{\partial T}{\partial t} + \frac{\partial (\bar{v}_x T)}{\partial x} + \frac{\partial (\bar{v}_z T)}{\partial z} - \frac{\partial}{\partial x} (\rho_w c_w \frac{\partial T}{\partial x}) - \frac{\partial}{\partial z} (\rho_w c_w \frac{\partial T}{\partial z}) - \rho_w c_w \frac{\partial T}{\partial t} - \lambda_m \frac{\partial T'}{\partial y} \big|_{y=b} = 0. \]  

(3.4.6)

For a 2-D problem on the x-y plane (where \( \partial T/\partial z = 0 \) and \( \bar{v}_z = 0 \)), it is

\[ \frac{\partial T}{\partial t} + \frac{\partial (\bar{v}_x T)}{\partial x} - \frac{\partial}{\partial x} (\rho_w c_w \frac{\partial T}{\partial x}) - \frac{\lambda_m}{\rho_w c_w b} \frac{\partial T'}{\partial y} \big|_{y=b} = 0. \]  

(3.4.7)

3.5 Heat Transport in Fractured Porous Medium: Analytical Solution for a Single Fracture

It is well known that fluids flowing through fractures in impermeable rock exchange heat with the surrounding medium. Bodvarsson [1969] solved the heat transport problem in a single fracture for the case of constant mass flow and the fluid entering the fracture with
Chapter 3: Flow and Heat Transport in Fractured Porous Medium

Figure 3.4: A simple fracture-matrix system.

a temperature that is a sinusoidal function of time. Gringarten et al. [1975] examined the heat extraction problem from fractured hot dry rock, but they ignored the conductive heat transport mechanism along the fracture.

In this section, I derive an analytical expression by using the Laplace transform for the heat transport in a single fracture which is situated in an impervious host rock. Tang et al. [1981] and Sudicky and Frind, [1982] employed the similar technique to investigate analytically the contaminant transport in fractured porous media.

The analytical solution presented here will be used to validate the numerical simulation technique described in Chapter 4.

3.5.1 A Simple Fracture-Matrix System

Consider the case of a thin rigid fracture situated in a saturated porous rock (Figure 3.4). The water velocity in the fracture $v$ is assumed constant, and a heat source of the constant temperature $T_0$ exists at the origin of the fracture. I assume that the width of the fracture $2b$ is much smaller than its length so that the variation in temperature across the fracture's aperture is negligible, the fluid is always instantaneously and thoroughly mixed across the fracture, thus thermal equilibrium exists between fluid and solid matrix, the permeability and the porosity of the porous matrix are negligible and heat transport in the matrix is mainly by molecular diffusion (i.e., conduction), heat transport along the fracture is much faster than transport within the matrix, and the temperature variation is so small that the fluid density $\rho_w$ is constant throughout the system.
These assumptions provide the basis for a one-dimensional representation of heat transport along the fracture itself and for taking the direction of heat flux in the porous matrix to be perpendicular to the fracture. This results in the simplification of the 2-D system to two orthogonal, coupled 1-D systems.

### 3.5.2 Governing Equations

The heat transport processes in the system of Figure 3.4 can be described by two coupled, one-dimensional equations, one for the fracture and one for the porous matrix. The coupling is provided by the continuity of the temperature and the horizontal heat flux across the interface.

Based on Equation (3.4.7), the differential equation of the heat transport for the fracture can be expressed as

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial z} - \frac{\lambda_w}{c_w \rho_w} \frac{\partial^2 T}{\partial z^2} - \frac{\lambda_m}{c_m \rho_m} \frac{\partial T'}{\partial x} \bigg|_{z=b} = 0 \quad (\infty \geq z \geq 0), \quad (3.5.1)$$

where $T$ is the temperature in the fracture, $T'$ is the temperature in the porous matrix, $z$ is coordinate along the fracture, and $v$ is water velocity in the fracture.

The differential equation for the porous matrix can be written as

$$\frac{\partial T'}{\partial t} - \frac{\lambda_m}{c_m \rho_m} \frac{\partial^2 T'}{\partial x^2} = 0 \quad (b \leq x \leq \infty), \quad (3.5.2)$$

where $x$ is coordinate perpendicular to the fracture, $\lambda_m$ is the thermal conductivity of the porous matrix, and $\rho_m$ and $c_m$ are respectively the density and the specific heat of the porous matrix ($\rho_m c_m = \theta \rho_w c_w + (1 - \theta) \rho_c c_c$).

The boundary and initial conditions for (3.5.1) are

$$T(0,t) = T_0, \quad (3.5.3a)$$

$$T(\infty,t) = 0, \quad (3.5.3b)$$

$$T(z,0) = 0, \quad (3.5.3c)$$

where $T_0$ is the source temperature.
Chapter 3: Flow and Heat Transport in Fractured Porous Medium

The boundary and initial conditions for (3.5.2) are

\[ T'(b, z, t) = T(z, t), \]  
\[ T'(\infty, z, t) = 0, \]  
\[ T'(x, z, 0) = 0. \]

Equation (3.5.4a) expresses the coupling of the porous matrix to the fracture.

### 3.5.3 General Transient Solution

Applying the Laplace transformation to (3.5.2) yields

\[ s\bar{T}' = \frac{\lambda_m}{c_n \rho_m} \frac{d^2 \bar{T}'}{dz^2}, \]

where \( \bar{T}' \) is the Laplace transformation of \( T' \), defined as

\[ \bar{T}'(z, s) = \int_0^\infty \exp(-st)T'(z, t)dt. \]

The only possible solution for (3.5.5) is of the form

\[ \bar{T}' = c'_1 \exp\left[-\left(\frac{c_m \rho_m}{\lambda_m} s\right)^{1/2}(x - b)\right], \]

where the constant \( c'_1 \) can be obtained by using (3.5.4a). Thus Equation (3.5.7) becomes

\[ \bar{T}' = \bar{T} \exp\left[-\left(\frac{c_m \rho_m}{\lambda_m} s\right)^{1/2}(x - b)\right]. \]

The gradient of \( \bar{T}' \) at the interface \( x = b \) is

\[ \frac{d\bar{T}'}{dx} \bigg|_{x=b} = -\bar{T}' \left(\frac{c_m \rho_m}{\lambda_m} s\right)^{1/2}. \]

Now let me apply the Laplace transform to Equation (3.5.1). I obtain

\[ s\bar{T} + \frac{d\bar{T}}{dz} = \frac{\lambda_m}{c_w \rho_w b} \frac{d\bar{T}'}{dx} \bigg|_{x=b} + \frac{\lambda_w}{c_w \rho_w} \frac{d^2 \bar{T}}{dz^2}. \]
Substituting (3.5.9) into (3.5.10) yields

$$
\frac{d^2 \tilde{T}}{dz^2} - \frac{v}{\lambda_w} \frac{d \tilde{T}}{dz} - \frac{c_w \rho_w}{\lambda_w} \left[ s + s^{1/2} / A \right] \tilde{T} = 0,
$$

(3.5.11)

where

$$
A = \frac{b c_w \rho_w}{(\lambda_m c_m \rho_m)^{1/2}}.
$$

(3.5.12)

If let

$$
X = \frac{\lambda_w}{c_w \rho_w},
$$

(3.5.13)

then equation (3.5.11) can be expressed as

$$
\frac{d^2 \tilde{T}}{dz^2} - \frac{v}{X} \frac{d \tilde{T}}{dz} - \frac{1}{X} \left[ s + s^{1/2} / A \right] \tilde{T} = 0.
$$

(3.5.14)

Equation (3.5.14) is a second-order ordinary differential equation which has a solution of the form

$$
\tilde{T} = c_1 \exp(\gamma z) + c_2 \exp(\gamma z),
$$

(3.5.15)

where $c_1$ and $c_2$ are undetermined constants and $r$ takes on the two forms

$$
r_{\pm} = \gamma \left\{ 1 \pm \sqrt{1 + \beta^2 \left[ s + s^{1/2} / A \right]} \right\}^{1/2},
$$

(3.5.16)

where

$$
\gamma = \frac{v}{2X}, \quad \text{and} \quad \beta^2 = \frac{4X}{v^2}.
$$

(3.5.17)

Because the temperature must be finite, the first term in (3.5.15) must vanish. I am thus left with

$$
\tilde{T} = c_2 \exp(\gamma z) \exp\left\{-\gamma z \left[ 1 + \beta^2 \left( s + s^{1/2} / A \right) \right]^{1/2}\right\}.
$$

(3.5.18)

The undetermined constant $c_2$ can be obtained by utilizing the boundary condition (3.5.3a). Applying the Laplace transform to this boundary condition yields

$$
\tilde{T}(0,s) = \frac{T_0}{s},
$$

(3.5.19)

which is seen to be the value of $c_2$. Equation (3.5.18) thus becomes

$$
\tilde{T} = \frac{T_0}{s} \exp(\gamma z) \exp\left\{-\gamma z \left[ 1 + \beta^2 \left( s + s^{1/2} / A \right) \right]^{1/2}\right\}.
$$

(3.5.20)
Chapter 3: Flow and Heat Transport in Fractured Porous Medium

Equation (3.5.20) must now be inverted. Based on the identity

$$\int_0^\infty \exp[-\xi^2 - \frac{\chi^2}{\xi^2}] d\xi = \frac{\pi^{1/2}}{2} \exp(-2\chi), \quad (3.5.21)$$

I can convert the exponential term in (3.5.20) into the integral form. Thus I have

$$\exp\left\{-\gamma z \left[1 + \beta^2 \left(s + s^{1/2}/A\right)^{1/2}\right]\right\} = \frac{2}{\pi^{1/2}} \int_0^\infty \exp\left\{-\xi^2 - \frac{\gamma^2 z^2}{4} \left[1 + \beta^2 \left(s + s^{1/2}/A\right)\right]\right\} d\xi. \quad (3.5.22)$$

Substituting (3.5.22) into (3.5.20) yields

$$\frac{T}{T_0} = \frac{2}{\pi^{1/2}} \exp(\gamma z) \int_0^\infty \exp\left[-\xi^2 - \frac{\gamma^2 z^2}{4} \right] \exp\left[-\frac{\gamma^2 z^2 \beta^2}{4\xi^2 A} \left(As + s^{1/2}\right)\right] / s \cdot d\xi. \quad (3.5.23)$$

Thus the original temperature $T$ will be given in terms of the inverse transform $L^{-1}$ as

$$\frac{T}{T_0} = \frac{2}{\pi^{1/2}} \exp(\gamma z) \int_0^\infty \exp\left[-\xi^2 - \frac{\gamma^2 z^2}{4} \right] L^{-1}\left\{\exp\left[-\frac{\gamma^2 z^2 \beta^2}{4\xi^2 A} (As + s^{1/2})\right] / s \right\} d\xi. \quad (3.5.24)$$

To evaluate the inverse transform, I make use of the following two identities:

$$L^{-1}\left\{\frac{\exp(-2a\sqrt{s})}{s}\right\} = \text{erfc}\left(\frac{a}{\sqrt{t}}\right), \quad (3.5.25a)$$

and

$$L^{-1}\left\{\exp(-sE)\tilde{f}(s)\right\} = f(t - E)U(t - E), \quad (3.5.25b)$$

where erfc is the complementary error function, defined as

$$\text{erfc}(x) = 1 - \frac{2}{\pi^{1/2}} \int_0^x \exp(-x^2) dx,$$

and

$$U = 0 \ (t < E) \quad \text{and} \quad U = 1 \ (t \geq E).$$

Based on (3.5.25), I have

$$L^{-1}\left\{\exp\left[-\frac{\gamma^2 z^2 \beta^2}{4\xi^2 A} s^{1/2}\right] / s\right\} = \text{erfc}\left(\frac{\gamma^2 z^2 \beta^2}{8\xi^2 A \sqrt{t}}\right)$$
and
\[ L^{-1}\left\{ \exp\left[-\frac{\gamma^2 s^2 \beta^2}{4\xi^2 A}(As + s^{1/2})\right]/s \right\} = U(t - \frac{\gamma^2 s^2 \beta^2}{4\xi^2}) \text{erfc}\left(\frac{\gamma^2 s^2 \beta^2}{8\xi^2 A\sqrt{t - \frac{\gamma^2 s^2 \beta^2}{4\xi^2}}}\right). \] (3.5.26)

Substituting (3.5.26) into (3.5.24) yields the final normalized temperature solution in the fracture as
\[ \frac{T}{T_o} = \frac{2}{\pi^{1/2}} \exp(\gamma z) \int_{-\infty}^{\infty} \exp\left[-\xi^2 - \frac{\gamma^2 s^2}{4\xi^2}\right] \text{erfc}\left(\frac{\gamma^2 s^2 \beta^2}{8\xi^2 A\sqrt{t - \frac{\gamma^2 s^2 \beta^2}{4\xi^2}}}\right) d\xi. \] (3.5.27)

I can also find the temperature distribution in the porous matrix. From equation (3.5.8), I have
\[ \frac{\hat{T'}}{T_o} = \frac{\hat{T}}{T_o} \exp\left[-\left(\frac{c_m \rho_m}{\lambda_m}\right)^{1/2}(x - b)\right]. \] (3.5.28)
From equation (3.5.23), I have
\[ \frac{\hat{T'}}{T_o} = \frac{2}{\pi^{1/2}} \exp(\gamma z) \int_{0}^{\infty} \exp\left[-\xi^2 - \frac{\gamma^2 s^2}{4\xi^2}\right] \exp\left[-\frac{\gamma^2 s^2 \beta^2}{4\xi^2 A}(As + s^{1/2})\right]/s. \]
\[ \exp\left[-\left(\frac{c_m \rho_m}{\lambda_m}\right)^{1/2}(x - b)\right] \cdot d\xi. \] (3.5.29)

Thus I have
\[ \frac{T'}{T_o} = \frac{2}{\pi^{1/2}} \exp(\gamma z) \int_{0}^{\infty} \exp\left[-\xi^2 - \frac{\gamma^2 s^2}{4\xi^2}\right]. \]
\[ L^{-1}\left\{ \exp\left[-\frac{\gamma^2 s^2 \beta^2}{4\xi^2 A} + \left(\frac{c_m \rho_m}{\lambda_m}\right)^{1/2}(x - b)s^{1/2}\right]/s \cdot \exp\left[-\frac{\gamma^2 s^2 \beta^2}{4\xi^2 s}\right] \right\} d\xi. \] (3.5.30)

Similarly, I can obtain the normalized temperature distribution in the host matrix as follows
\[ \frac{T'}{T_o} = \frac{2}{\pi^{1/2}} \exp(\gamma z) \int_{0}^{\infty} \exp\left[-\xi^2 - \frac{\gamma^2 s^2}{4\xi^2}\right] \text{erfc}\left(\frac{\gamma^2 s^2 \beta^2}{2\sqrt{t - \frac{\gamma^2 s^2 \beta^2}{4\xi^2}}}\right) d\xi, \] (3.5.31)
where
\[ Y = \frac{\gamma^2 s^2 \beta^2}{4\xi^2 A} + \left(\frac{c_m \rho_m}{\lambda_m}\right)^{1/2}(x - b). \] (3.5.32)
3.5.4 Steady State Solution

Equations (3.5.27) and (3.5.31) provide the general transient solutions for the temperature distribution in the fracture and the host matrix, respectively. Now let me investigate the steady state solution in the case of $t \to \infty$.

In equation (3.5.27), when $t \to \infty$, the lower limit of integration is zero, and the complementary function is unity. Based on the identity (3.5.21), I have

$$
\frac{T}{T_0} = \frac{2}{\pi^{1/2}} \exp(\gamma z) \int_0^\infty \exp \left[ -\xi^2 - \frac{\gamma^2 z^2}{4\xi^2} \right] d\xi
$$

$$
= \frac{2}{\pi^{1/2}} \exp(\gamma z) \frac{\pi^{1/2}}{2} \exp(-\gamma z) = 1. \quad (3.5.33)
$$

Similarly, I also have

$$
\frac{T'}{T_0} = 1. \quad (3.5.34)
$$

These results indicate that at the steady state ($t \to \infty$), the whole system will reach the constant temperature $T_0$ of the heat source.

3.5.5 Some Calculation Examples

To calculate the general transient solutions $T(z, t)$ and $T'(z, x, t)$, I have to evaluate the integrals (3.5.27) and (3.5.31). The integrands in these equations are fortunately well-behaved, taking on the form of a skewed bell-shaped curve. Thus evaluation of these integrals is trivial and can be done efficiently by the Gaussian quadrature method.

Let me assume that the fracture-matrix system has the following parameters as shown in Table 3.1.

Figure 3.5 illustrates the normalized temperature contours for the case 1 at different time levels of 1.0, 2.0 and 3.0 days. Figure 3.6 shows the temperature contours for the case 2 at different time levels of 0.5, 1.0 and 1.5 days. The fracture is also indicated on the diagram. From these results, it can be seen that fluid flowing through a fracture in impermeable rock exchanges a great amount of heat with the surrounding medium. As fluid transports along the fracture, it warms the fracture itself, as well as the host matrix due to the heat releasing from the fracture by the diffusion mechanism. As time progresses, both the fracture and host medium become warmer. Comparing Figure 3.5
Table 3.1: The physical parameters for the simple fracture-matrix system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2b$</td>
<td>1 mm</td>
</tr>
<tr>
<td>$c_m$</td>
<td>800 J/kg°C</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>2650 kg/m³</td>
</tr>
<tr>
<td>$c_w$</td>
<td>4174 J/kg°C</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>1000 kg/m³</td>
</tr>
<tr>
<td>$\lambda_m$</td>
<td>2.0 J/m/s°C</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>0.5 J/m/s°C</td>
</tr>
</tbody>
</table>

$v = 5 \text{ mm/s}$ (Case 1)
$v = 1 \text{ cm/s}$ (Case 2)

with 3.6, it can also be seen that a larger fluid velocity results in the quicker increase in temperature of the solution domain. Due to the fluid motion, the convective heat transport along the vertical fracture is faster than the conductive heat transport perpendicular to the fracture. That is the reason why the temperature contours have the triangle-like feature with the fracture as its symmetrical axis. The larger the fluid velocity, the shaper the ‘triangle’ becomes.

### 3.6 Discussions and Conclusions

In this Chapter, the fundamental mechanism of heat and flow transport in the fractured porous medium has been presented. It has been indicated that fracture permeability is only dependent on its aperture $2b$, defined as $(2b)^2/12$. For the pipe-shaped fracture, it is only dependent on its radius $r$, defined as $r^2/8$. The mass and energy conservation equations have been derived by integrating the general governing equations across the aperture of fracture.

An analytical expression for the heat transport in a single fracture situated in an impermeable host rock has been derived by using the Laplace transformation technique. The calculation results have indicated that aqueous fluids flowing through fractures in impermeable rock indeed exchange a great amount of heat with the surrounding medium. The analytical solution will be used to validate the numerical simulation technique described in Chapter 4.
Figure 3.5: The normalized temperature contours for the case 1: (a) $t=1.0$ day, (b) $t=2.0$ days, and (c) $t=3.0$ days. The contour intervals are from 0.1 to 1.0.
Figure 3.6: The normalized temperature contours for the case 2: (a) \( t=0.5 \) day, (b) \( t=1.0 \) day, and (c) \( t=1.5 \) days. The contour intervals are from 0.1 to 1.0.
Chapter 4

Numerical Simulation Method of Hydrothermal Convection in Fractured Porous Medium

4.1 Introduction

In Chapter 3, I described the basic physical processes and mathematical equations governing the flow and heat transport in a fractured porous medium. I also derived an analytical solution of the heat transport for a special case in which a single vertical fracture is situated in an impervious host rock. For the general case, unfortunately, no analytical solution exists, and the problem must be tackled numerically.

Previous numerical models [Fehn and Cathles, 1979; Fehn et al., 1983; Gartling, 1981; Fisher et al., 1990; Fisher et al., 1994; Fisher and Becker, 1995; Rosenberg and Spera, 1990; Rosenberg et al., 1993; Bessler et al., 1994; Davis et al., 1995] treated the rock matrix within each element as a continuous porous medium. A fracture was treated simply as a zone consisting of several volume elements specified with higher permeability. Fractures have not been incorporated into the previous models explicitly yet. The in-situ measurements indicated that the fracture aperture in the upper oceanic crust ranges from 0.1 mm to several mm [Pezard and Anderson, 1989; Pezard, 1990; Dick et al., 1991; Anderson et al., 1982; Newmark et al., 1985a; 1985b; Morin et al., 1989; Agar, 1990]. The previous studies could not deal with so narrow fractures. Replacement of a fracture by several volume elements actually averages its effect, which may give rise to quite different results. The following sections will indicate that the explicit fractures, even if their aperture is very small, not only initiate hydrothermal convection but also
can greatly change an established pattern.

In this chapter, I first introduce the well-known Galerkin finite element method, then develop a numerical algorithm to simulate hydrothermal convection in a discretely fractured porous medium. The scheme can represent simultaneously both the larger and smaller scale features of the medium.

### 4.2 The Galerkin Finite Element Technique

Mass and energy conservation equations (2.2.1) and (2.3.1) discussed in Chapter 2 form a transient, nonlinear system, coupled through Darcy's equation (2.1.5). Nonlinearities exist in the temperature-dependent fluid density and viscosity (2.4.2) and (2.4.3). It is almost impossible to derive an analytical solution to these governing equations, even for a 1-D model. Numerical solutions are therefore sought by the finite element method.

In the finite element method with linear elements, a complex function is always approximated by means of a simple interpolation function, defined in terms of nodal values. The key to the method is a minimization principle which allows us to solve for the nodal values in such a way that the numerical error is minimized on average over the domain. To generate the algebraic equations of the unknown nodal values, I apply the Weighted Residual Method. Suppose I have a partial differential equation of the form:

\[ \Lambda(u) = 0, \]

where \( \Lambda \) is a Cartesian differential operator. Let a trial solution be expressed in the form:

\[ u(x, y, z) \approx \hat{u}(x, y, z) = \sum_{j=1}^{n} u_j N_j(x, y, z), \]

where \( u_j \) are nodal values, \( N_j(x, y, z) \) are the basis functions, and \( n \) is the total number of nodes in the domain.

Substituting the trial solution into the original differential equation (4.2.1) yields

\[ \Lambda(\hat{u}(x, y, z)) = R(x, y, z) \neq 0. \]

The non-zero residual \( R(x, y, z) \) on the right-hand side expresses the error due to the approximate representation.
According to the theory of weighted residuals, I can minimize $R(x, y, z)$ on the average over the domain by satisfying a set of weighted residual equations, which are:

$$\int_V R(x, y, z)W_i(x, y, z)dV = 0, \quad i = 1, 2, \ldots, n,$$

(4.2.4)

where $V$ designates the solution domain, and $W_i(x, y, z)$ are a set of $n$ weighting functions corresponding to the $n$ nodes.

In the general weighted residual method, the weighting functions are independently chosen, and many forms of weighting functions are possible. A special variant of weighted residuals is the Galerkin Method, in which the weighting functions $W_i$ are chosen to be identical to the basis function $N_i$. This choice has some advantages, for example, the coefficient matrix for the flow equation is symmetrical. The Galerkin Method has been well proven in groundwater circulation and heat transport [e.g., Huyakorn and Pinder, 1983].

By setting the integral to zero in (4.2.4), the weighted residual equation effectively forces the residual to zero since the integral of weighting function is always non-zero. But because the integral equation will also be satisfied if $R$ were non-zero locally in such a way that the error would average out to zero over the domain, the error is forced to zero globally, that is, over the domain as a whole. By writing one weighted residual equation for each node, we require that the error everywhere is forced to zero on the average.

In order to complete the solution, we substitute for the residual $R(x, y, z)$ in the Galerkin equation to obtain:

$$\int_V \Lambda(\bar{u}(x, y, z))N_i(x, y, z)dV = 0, \quad i = 1, 2, \ldots, n.$$  

(4.2.5)

We can further substitute the trial solution (4.2.2) into the Galerkin equation to obtain:

$$\int_V \{\Lambda(\sum_{j=1}^{n} u_jN_j(x, y, z))\}N_i(x, y, z)dV = 0, \quad i = 1, 2, \ldots, n.$$  

(4.2.6)

Equation (4.2.6) gives us $n$ equations in $n$ unknowns, which can be solved for the unknown nodal values $u_j$.

Now let us investigate the Galerkin element solutions of the mass and energy conservation equations (2.2.1) and (2.3.1). The solution domain is divided into a network of
finite elements. Again, the trial solutions are expressed as

$$\hat{h}(x, y, z, t) = \sum_{j=1}^{n} N_j(x, y, z) h_j(t), \quad (4.2.7)$$

and

$$\hat{T}(x, y, z, t) = \sum_{j=1}^{n} N_j(x, y, z) T_j(t), \quad (4.2.8)$$

where $\hat{h}$ is the approximate hydraulic head, $\hat{T}$ is the approximate temperature, $N_j$ is the basis function associated with node $j$, $h_j$ is the hydraulic head at node $j$, and $T_j$ is the temperature at node $j$.

The Galerkin method of weighted residuals is then used to transform these governing equations into integral equations of the form

$$\int_V N_i \Lambda_1(\hat{h}) dV = 0, \quad (4.2.9)$$

$$\int_V N_i \Lambda_2(\hat{T}) dV = 0, \quad (4.2.10)$$

where $\Lambda_1$ and $\Lambda_2$ represent the differential operators of the left-hand side of equations (2.2.1) and (2.3.1), and $V$ is the volume of the solution domain.

Since $\hat{h}$ and $\hat{T}$ are expressed by linear basis functions, their second derivatives will be zero. In order to obtain a non-trivial solution, the second derivatives must therefore be eliminated through integration by parts using Green's theorem.

The integrals of the terms involving $N_j$ and their first-order derivatives over each element can be performed using the analytical influence coefficient method for rectangular elements [Huyakorn et al., 1986]. Summing the elemental contributions, we obtain

$$[M_{ij}^h](dh_i/dt) + [S_{ij}^h]h_j + [F_i^h] = 0, \quad (4.2.11)$$

$$[M_{ij}^T](dT_i/dt) + [S_{ij}^T]T_j + [F_i^T] = 0, \quad (4.2.12)$$

where $i, j = 1, 2, \cdots, n$ ($n$ is the total number of nodes in the system).

In (4.2.11) and (4.2.12), the coefficient matrices $[M]$, $[S]$, and $[F]$ can be obtained from the textbooks [e.g., Huyakorn and Pinder, 1983]. The above equations are coupled through the convective terms and the temperature-dependent variations of fluid density and viscosity.
Approximating the temporal derivatives in (4.2.11) and (4.2.12) by a weighted first-order finite difference approach, these equations can be rewritten as

\[
\left[ \gamma S^h_{ij} + \frac{M^h_{ij}}{\Delta t} \right]^{t+\Delta t} h_j^{t+\Delta t} = -\gamma F^h_i^{t+\Delta t} - (1 - \gamma) F^h_i^t + \frac{M^h_{ij}}{\Delta t} - (1 - \gamma) S^h_{ij}^t h_j^t, \quad (4.2.13)
\]

\[
\left[ \gamma S^T_{ij} + \frac{M^T_{ij}}{\Delta t} \right]^{t+\Delta t} T_j^{t+\Delta t} = -\gamma F^T_i^{t+\Delta t} - (1 - \gamma) F^T_i^t + \frac{M^T_{ij}}{\Delta t} - (1 - \gamma) S^T_{ij}^t T_j^t, \quad (4.2.14)
\]

where \( \gamma \) is the time-weighting factor, ranging from 0.5 to 1.0, and \( \Delta t \) is the size of time step. In particular, \( \gamma = 1 \) and \( \gamma = 0.5 \) correspond to the fully implicit and Crank-Nicholson time scheme, respectively.

Equations (4.2.13) and (4.2.14) form a system of 2n, non-linear simultaneous equations, which is solved by updating the fluid properties at each time-stepping cycle and by using the Picard iterative technique [Huyakorn and Pinder, 1983]. I first update fluid density and viscosity using the latest temperature \( T^l \) as defined in (2.4.2) and (2.4.3), and determine the head distribution \( h^{l+1} \) based on the updated fluid properties, by solving the matrix equation (4.2.13), where \( l \) is the level of iteration. I then determine the Darcy flux distribution \( q^{l+1} \) based on Darcy's law (2.1.5) using \( h^{l+1} \), and evaluate temperature distribution \( T^{l+1} \), by solving the matrix equation (4.2.14). Finally, I need to investigate whether the convergence criteria

\[
\max \left| h_j^{l+1} - h_j^l \right| \leq \delta h
\]

\[
\max \left| T_j^{l+1} - T_j^l \right| \leq \delta T
\]

are satisfied. If not, \( h \) and \( T \) must be updated and these steps are repeated. \( \delta h \) and \( \delta T \) are the convergence criteria for the flow and heat transport equations, respectively.

In designing the finite element grid, the primary considerations are the grid Peclet and Courant criteria, defined as [Daus et al., 1985]:

\[
P_x = \frac{v_x \Delta x}{k_m} \leq 2, \quad P_y = \frac{v_y \Delta y}{k_m} \leq 2, \quad P_z = \frac{v_z \Delta z}{k_m} \leq 2, \quad (4.2.15)
\]

and

\[
C_x = \frac{v_x \Delta t}{\Delta x} \leq 1, \quad C_y = \frac{v_y \Delta t}{\Delta y} \leq 1, \quad C_z = \frac{v_z \Delta t}{\Delta z} \leq 1, \quad (4.2.16)
\]

where \( k_m \) is thermal diffusivity of the porous medium, \( \Delta x, \Delta y, \) and \( \Delta z \) are grid spacings in three directions, and \( \Delta t \) is time step.
These criteria should be used to determine the grid spacings and time steps prior to numerical calculations for any given models.

4.3 Numerical Algorithm for Discretely Fractured Porous Medium

The numerical technique presented below has been validated extensively and applied successfully in the simulation of groundwater movement and contaminant transport in fractured porous media [Sudicky and McLaren, 1992]. Here I extend its application to the simulation of hydrothermal circulation in discretely fractured porous medium, subject to the assumptions that fracture aperture is much smaller than the size of elements, and temperature and head are continuous across the interface between the fracture and the host medium.

The fracture network is included explicitly by superimposing a number of planar elements representing individual fracture segments onto the sides of the volume elements. These elements are assigned a permeability of \((2b)^2/12\), based on the equation (3.2.4) in Chapter 3. By arranging the planar and volume elements in this manner, the continuity of temperature and head at the fracture-matrix interface is automatically satisfied because each element type shares common nodes along the interface. Moreover, upon element assembly, the fluid and heat exchange fluxes across the interface are accounted for naturally such that explicit calculation of the normal components of the fluid leakage flux in (3.3.3) and the influx heat-flow and efflux heat-flow in (3.4.2) is unnecessary.

Let me use a 2-D model to explain the details of the numerical technique. In 2-D cases, the planar fractures are infinitely long in the strike direction so appear as lines in a cross-sectional view.

Figure 4.1(a) shows a 2-D fractured porous medium in which the bold line represents a vertical discrete fracture with an aperture of \(2b\). The 2-D solution domain is discretized by a 2-D equal-spaced rectangular grid, with 10 elements in x-direction and 5 elements in z-direction, and there are totally 66 nodes, as shown in Figure 4.1(b). The vertical fracture is incorporated onto the sides of the rectangular elements, and it shares the nodes 26, 27, and 28 with the host porous medium. The fractured model is then decomposed into the porous medium part (Figure 4.1(c)) and the vertical fracture part (Figure 4.1(d)), maintaining the same discretion as in Figure 4.1(b).
Figure 4.1: (a) A 2-D fractured porous medium including a vertical fracture; (b) Discretion of the model by a 2-D equal-spaced grid; (c) The host porous medium; and (d) The discrete fracture. The numbers represent the nodal number of elements. The host medium and fracture share the nodes numbered by 26, 27, and 28.
To demonstrate the fundamental principle of the numerical scheme, let me only consider the 2-D fluid flow equation.

If assume that the head $h$ is continuous across the interface between the fracture and the host medium, then the fluid flow equations can be obtained for both the porous medium and the fracture by substituting the Darcy’s law (2.1.5) into the mass conservation equation (2.2.1).

\[
\frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} + K_{z_{rel}} \right) = \begin{cases} 0 & \text{for the regions outside the fracture} \\ \gamma_{\text{gain}} & \text{for the regions containing the fracture} \end{cases} 
\]

and, for the particular case of a vertical fracture

\[
\frac{\partial}{\partial z} \left( K_f \frac{\partial h}{\partial z} + K_{f_{rel}} \right) = -\gamma_{\text{loss}},
\]

where, $K_x$ and $K_z$ are the hydraulic conductivities in $x$- and $z$-directions, $K_f$ is the fracture hydraulic conductivity, defined as $K_f = (2b)^2 g/12 \nu$, $\gamma_{\text{gain}}$ is a function of position, has the dimension of fluid flux per unit area, and represents the fluid gain of the host medium from the fracture, and $-\gamma_{\text{loss}}$ represents the fluid loss of the fracture to the host medium.

To approximate the flow equations by the Galerkin finite element method, I discretize the head $h$ using a trial function of the form

\[
\hat{h}(x, z) = \sum_{j=1}^{66} h_j N_j^2(x, z), \quad \text{for the porous medium} \tag{4.3.2a}
\]

and

\[
\hat{h}(z) = \sum_{j=28}^{28} h_j N_j^1(z), \quad \text{for the vertical fracture} \tag{4.3.2b}
\]

where $N_j^2(x, z)$ and $N_j^1(z)$ are respectively 1-D and 2-D basis functions, and $h_j$ are nodal values of $h$.

Let the cell size be $L_x$ and $L_z$, and the aperture of the fracture be $2b$. Then the fluid gain of a 2-D cell containing the fracture should equal to the fluid loss of a linear fracture element, and because I can assume that I can work with average values of $\gamma$ over the cross-section area of a cell (or fracture element) without significant loss of accuracy on larger scales, they have the form:

\[
\Gamma = L_x L_z \gamma_{\text{gain}}, \quad \text{or} \quad \gamma_{\text{gain}} = \Gamma / L_x L_z, \tag{4.3.3a}
\]
and

\[-\Gamma = -2bL_z \gamma_{loss}, \quad \text{or} \quad \gamma_{loss} = -\Gamma / 2bL_z. \quad (4.3.3b)\]

Where \( \Gamma \) is assumed constant over the elements.

The formal calculation of the fluid balance, as presented below, has to take into consideration of all cells which touch the fracture.

Substituting (4.3.3a) and (4.3.2a) into (4.3.1a), substituting (4.3.3b) and (4.3.2b) into (4.3.1b), and applying the Galerkin residual theory, I have

\[
\int_{A} \left\{ \frac{\partial}{\partial x} \left( K_x \frac{\partial \hat{h}}{\partial x} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial \hat{h}}{\partial z} + K_z \rho_{rel} \right) \right\} N_i^2(x, z) dx dz \quad (i = 1, 2, \ldots, 66)
\]

\[
= \int_{A} \left\{ \frac{\Gamma}{L_z L_z} \right\} N_i^2(x, z) dx dz \quad (i = 26, 27, 28) \quad (4.3.4a)
\]

and

\[
\int_{L_A} \left\{ \frac{\partial}{\partial z} \left( K_f \frac{\partial \hat{h}}{\partial z} + K_f \rho_{rel} \right) \right\} N_i^1(z) dz = -\int_{L_A} \left\{ \frac{\Gamma}{2bL_z} \right\} N_i^1(z) dz \quad (i = 26, 27, 28), \quad (4.3.4b)
\]

where \( A \) is the total area of the porous medium, \( L_A \) is the total length of the vertical fracture, and \( (i = 26, 27, 28) \) denotes that the vertical fracture shares the nodes 26, 27, and 28 with the host porous medium.

Equation (4.3.4b) can be rewritten as

\[
(2b) \int_{L_A} \left\{ \frac{\partial}{\partial z} \left( K_f \frac{\partial \hat{h}}{\partial z} + K_f \rho_{rel} \right) \right\} N_i^1(z) dz = -\int_{L_A} \left\{ \frac{\Gamma}{L_z} \right\} N_i^1(z) dz \quad (i = 26, 27, 28). \quad (4.3.4c)
\]

Adding (4.3.4a) to (4.3.4c) yields a set of 66 equations, indexed by \( i \)

\[
\int_{A} \left\{ \frac{\partial}{\partial x} \left( K_x \frac{\partial \hat{h}}{\partial x} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial \hat{h}}{\partial z} + K_z \rho_{rel} \right) \right\} N_i^2(x, z) dx dz \quad (i = 1, 2, \ldots, 66) \quad (4.3.4d)
\]

\[
+ \quad (2b) \int_{L_A} \left\{ \frac{\partial}{\partial z} \left( K_f \frac{\partial \hat{h}}{\partial z} + K_f \rho_{rel} \right) \right\} N_i^1(z) dz \quad (i = 26, 27, 28) \quad (4.3.4e)
\]

\[
= \int_{A} \left\{ \frac{\Gamma}{L_z L_z} \right\} N_i^2(x, z) dx dz \quad (i = 26, 27, 28) \quad (4.3.4f)
\]
Now let me prove that terms (4.3.4f) and (4.3.4g) cancel each other. For the 2-D rectangular element as shown in Figure 4.1c, \(i\) and \(j = 1, 2, 3, 4\). The basis functions can be expressed as [Huyakorn and Pinder, 1983]:

\[
N_1^2(x, z) = 1 - x/L_x - z/L_z + xz/L_xL_z, \quad (4.3.5a)
\]

\[
N_2^2(x, z) = x/L_x - xz/L_xL_z, \quad (4.3.5b)
\]

\[
N_3^2(x, z) = xz/L_xL_z, \quad (4.3.5c)
\]

\[
N_4^2(x, z) = z/L_z - xz/L_xL_z. \quad (4.3.5d)
\]

For the 1-D linear element as shown in Figure 4.1d, \(i = 1, 2\). The basis functions are:

\[
N_1^1(z) = 1 - z/L_z \quad \text{and} \quad N_2^1(z) = z/L_z. \quad (4.3.6)
\]

By direct integration of (4.3.4f) and (4.3.4g), based on (4.3.5) and (4.3.6), I have

\[
\int_{A} \left\{ \frac{\Gamma}{L_xL_z} \right\} N_i^2(x, z) dx dz \quad (i = 26, 27, 28) = \begin{bmatrix} \frac{\Gamma_I/2}{\Gamma_I/2} \\ \frac{\Gamma_{II}/2}{\Gamma_{II}/2} \end{bmatrix}, \quad (4.3.7a)
\]

and

\[
- \int_{L_A} \left\{ \frac{\Gamma}{L_z} \right\} N_i^1(z) dz \quad (i = 26, 27, 28) = \begin{bmatrix} -\frac{\Gamma_I/2}{\Gamma_I/2} \\ -\frac{\Gamma_{II}/2}{\Gamma_{II}/2} \end{bmatrix}, \quad (4.3.7b)
\]

where \(\Gamma_I\) and \(\Gamma_{II}\) denote the fluid gain or loss over two linear fracture elements, respectively. Note that the fracture consists of two elements, as shown in Figure 4.1(d).

Substituting (4.3.7a) and (4.3.7b) into (4.3.4d), (4.3.4e), (4.3.4f) and (4.3.4g), I have

\[
\int_{A} \left\{ \frac{\partial}{\partial x} \left( K_x \frac{\partial \hat{h}}{\partial x} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial \hat{h}}{\partial z} + K_z \rho_{rel} \right) \right\} N_i^2(x, z) dz dx \quad (i = 1, 2, \ldots, 66)
\]

\[
+ \quad (2b) \int_{L_A} \left\{ \frac{\partial}{\partial z} \left( K_f \frac{\partial \hat{h}}{\partial z} + K_f \rho_{rel} \right) \right\} N_i^1(z) dz \quad (i = 26, 27, 28)
\]

\[= 0. \quad (4.3.8)\]
Based on the Green's law, (4.3.8) can be written as
\[
\sum_{j=1}^{66} h_j \int_A \left\{ K_z \frac{\partial N_i^2}{\partial x} \frac{\partial N_j^2}{\partial z} + K_z \frac{\partial N_i^2}{\partial z} \frac{\partial N_j^2}{\partial x} \right\} dx dz + \int_A K_z \rho_{rel} \frac{\partial N_i^2}{\partial z} dx dz \quad (i = 1, 2, \ldots, 66)
\]
\[+ \sum_{j=26}^{28} h_j (2b) \int_{L_A} K_f \frac{\partial N_i^1}{\partial z} \frac{\partial N_j^1}{\partial x} dz + (2b) \int_{L_A} K_f \rho_{rel} \frac{\partial N_i^1}{\partial z} dz \quad (i = 26, 27, 28)
\]
\[= 0. \tag{4.3.9}
\]

This equation can be expressed in the form of matrix
\[
\sum_{j=1}^{66} h_j M_{ij}(i = 1, 2, \ldots, 66) + \sum_{j=26}^{28} h_j M_{ij}''(i = 26, 27, 28)
\]
\[= F_i'(i = 1, 2, \ldots, 66) + F_i''(i = 26, 27, 28). \tag{4.3.10}
\]

Where
\[
M_{ij} = \int_A \left\{ K_z \frac{\partial N_i^2}{\partial x} \frac{\partial N_j^2}{\partial z} + K_z \frac{\partial N_i^2}{\partial z} \frac{\partial N_j^2}{\partial x} \right\} dx dz = \sum_{e=1}^{50} M_{ij}^e, \tag{4.3.11a}
\]
\[
F_i' = - \int_A K_z \rho_{rel} \frac{\partial N_i^2}{\partial z} dx dz = - \sum_{e=1}^{50} F_i^e'. \tag{4.3.11b}
\]
\[
M_{ij}'' = \int_{L_A} (2b) K_f \frac{\partial N_i^1}{\partial z} \frac{\partial N_j^1}{\partial x} dz = \sum_{e=1}^{2} M_{ij}''^e, \tag{4.3.11c}
\]
and
\[
F_i'' = - \int_{L_A} (2b) K_f \rho_{rel} \frac{\partial N_i^1}{\partial z} dz = - \sum_{e=1}^{2} F_i''. \tag{4.3.11d}
\]

where $M_{ij}^e$, $F_i^e'$, $M_{ij}''^e$, and $F_i''^e$ are the coefficient matrixes of an individual element.

These matrixes can be written as
\[
M_{ij}^e = \int_{A_e} \left\{ K_z \frac{\partial N_i^2}{\partial x} \frac{\partial N_j^2}{\partial z} + K_z \frac{\partial N_i^2}{\partial z} \frac{\partial N_j^2}{\partial x} \right\} dx dz
\approx (K_z) \int_{A_e} \frac{\partial N_i^2}{\partial x} \frac{\partial N_j^2}{\partial z} dx dz + (K_z) \int_{A_e} \frac{\partial N_i^2}{\partial z} \frac{\partial N_j^2}{\partial z} dx dz, \tag{4.3.12a}
\]
\[
F_i'^e = \int_{A_e} K_z \rho_{rel} \frac{\partial N_i^2}{\partial z} dx dz
\approx (K_z \rho_{rel}) \int_{A_e} \frac{\partial N_i^2}{\partial z} dx dz, \tag{4.3.12b}
\]
\[ M_{ij}^e = \int_{L_z} (2b)K_f \frac{\partial N_1^1}{\partial z} \frac{\partial N_1^1}{\partial z} dz \]
\[ \approx ((2b)K_f) \int_{L_z} \frac{\partial N_1^1}{\partial z} \frac{\partial N_1^1}{\partial z} dz, \quad (4.3.12c) \]
and
\[ F_{i}^e = \int_{L_z} (2b)K_f \rho_{rel} \frac{\partial N_i^1}{\partial z} dz \]
\[ \approx ((2b)K_f \rho_{rel}) \int_{L_z} \frac{\partial N_i^1}{\partial z} dz, \quad (4.3.12d) \]

where, \( \langle K_x \rangle \), \( \langle K_z \rangle \) and \( \langle K_z \rho_{rel} \rangle \) are values of \( K_x \), \( K_z \) and \( K_z \rho_{rel} \) at the 2-D element centroid, and \( A_e \) is the area of an individual 2-D element. \( ((2b)K_f) \) and \( ((2b)K_f \rho_{rel}) \) are the values of \( (2b)K_f \) and \( (2b)K_f \rho_{rel} \) at the 1-D element centroid.

Based on (4.3.5), the coefficient matrixes \( M_{ij}^e \) and \( F_{i}^e \) can be determined easily by direct integration of (4.3.12a) and (4.3.12b), and they are equal to

\[ M_{ij}^e = \langle K_x \rangle \frac{L_z}{L_z} [M^{xx}]^e / 6.0 + \langle K_z \rangle L_z \frac{L_z}{L_z} [M^{zz}]^e / 6.0, \quad (4.3.13a) \]
and

\[ F_{i}^e = \frac{L_z}{2} \langle K_z \rho_{rel} \rangle [F_z]^e, \quad (4.3.13b) \]

where,

\[
[M^{xx}]^e = \begin{bmatrix}
2 & -2 & -1 & 1 \\
-2 & 2 & 1 & -1 \\
-1 & 1 & 2 & -2 \\
1 & -1 & -2 & 2
\end{bmatrix},
\]

\[
[M^{zz}]^e = \begin{bmatrix}
2 & 1 & -1 & -2 \\
1 & 2 & -2 & -1 \\
-1 & -2 & 2 & 1 \\
-2 & -1 & 1 & 2
\end{bmatrix},
\]

and

\[
[F_z]^e = \begin{bmatrix}
-1 \\
-1 \\
1 \\
1
\end{bmatrix}.
\]

Substituting (4.3.13) into (4.3.11a) and (4.3.11b), I can obtain the coefficient matrix
Chapter 4: Simulation of Hydrothermal Convection in Fractured Porous Medium

$M'_{ij}$ and the right vector $F'_i$ as follows:

$$M'_{ij} = \begin{bmatrix}
M'_{i,1} & \cdots & M'_{i,26} & M'_{i,27} & M'_{i,28} & \cdots & M'_{i,66} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
M'_{26,1} & \cdots & M'_{26,26} & M'_{26,27} & M'_{26,28} & \cdots & M'_{26,66} \\
M'_{27,1} & \cdots & M'_{27,26} & M'_{27,27} & M'_{27,28} & \cdots & M'_{27,66} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
M'_{66,1} & \cdots & M'_{66,26} & M'_{66,27} & M'_{66,28} & \cdots & M'_{66,66}
\end{bmatrix}, \quad (4.3.14a)$$

and

$$F'_i = \begin{bmatrix} F'_1 \\ \vdots \\ F'_{26} \\ F'_{27} \\ \vdots \\ F'_{66} \end{bmatrix}. \quad (4.3.14b)$$

Similarly, based on (4.3.6), the coefficient matrixes $M''_{ij}$ and $F''_i$ can be determined by direct integration of (4.3.12c) and (4.3.12d), and they are equal to

$$M''_{ij} = \frac{(2b)K_f}{L_z} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad (4.3.15a)$$

and

$$F''_i = ((2b)K_f \rho_d) \begin{bmatrix} -1 \\ 1 \end{bmatrix}. \quad (4.3.15b)$$

Substituting (4.3.15) into (4.3.11c) and (4.3.11d) yields the matrices $M''_{ij}$ and $F''_i$ as follows:

$$M''_{ij} = \begin{bmatrix} M''_{26,26} & M''_{26,27} & M''_{26,28} \\ M''_{27,26} & M''_{27,27} & M''_{27,28} \\ M''_{28,26} & M''_{28,27} & M''_{28,28} \end{bmatrix}, \quad (4.3.16a)$$

and

$$F''_i = \begin{bmatrix} F''_{26} \\ F''_{27} \\ F''_{28} \end{bmatrix}. \quad (4.3.16b)$$

Substituting (4.3.14) and (4.3.16) into (4.3.10) produces the final matrix equation for
Solving this equation gives rise to the final solution of \( h \) for the fractured porous medium as a whole. The similar procedures can be also applied to solve the temperature distribution for the discretely fractured porous medium.

For a 3-D fractured porous medium, there are two kinds of discrete fractures: one is the 1-D line (or pipe) fracture, the other is the 2-D planar fractures. The fundamental principle of the numerical solutions is the same.

### 4.4 Validation of the Numerical Scheme

I have elected to validate the numerical scheme against the analytical expression derived in Chapter 3. As illustrated in Figure 4.2, a 10 m long vertical fracture is included in the middle of a water-saturated 2-D model of 10 m \( \times \) 10 m. The fracture is given an aperture of 1 mm. The permeability and porosity of the host medium surrounding the fracture are assigned such small values that the host rock can be considered as impermeable and the fluid velocity along the fracture is constant.

The top boundary is fixed temperature at 0 °C. All other boundaries are assumed to be adiabatic except for the bottom of the fracture at which temperature is assigned to a constant value of \( T_0 \). Both the lower and upper boundaries are assumed to be permeable for fluid flow. The side boundaries are assumed impermeable. On the upper and lower boundaries, the constant head is chosen in such a way that the magnitude of the fluid velocity in the fracture is equal to \( 5 \times 10^{-3} \) m/s and \( 10^{-2} \) m/s, respectively. The fluid velocities in the host matrix can, for practical purposes, be considered to be zero. Both the initial temperature and fluid velocity are assumed to be equal to zero all over the
The simulation domain is discretized by a 2-D equal-spaced grid, and the number of elements in each of x- and z-directions is 20. The fracture-matrix system is given the following thermal transport parameters: $c_m = 800 \text{ J/kg/°C}$, $\rho_m = 2650 \text{ kg/m}^3$, $c_w = 4174 \text{ J/kg/°C}$, $\rho_w = 1000 \text{ kg/m}^3$, $\lambda_m = 2.0 \text{ J/m/s/°C}$, $\lambda_w = 0.5 \text{ J/m/s/°C}$, and $\theta = 0.001$.

When the fluid velocity along the fracture is equal to $5 \times 10^{-3} \text{ m/s}$, the normalized temperature distributions within the solution domain at different time levels of 1.0, 2.0 and 3.0 days are illustrated in Figure 4.3. Analytical solutions are shown in Figures 4.3(a), 4.3(b) and 4.3(c); whereas numerical solutions are shown in Figures 4.3(d), 4.3(e) and 4.3(f). It can be seen that the numerical simulation results are fairly close to the analytical solutions at all times. When the fluid velocity is equal to $10^{-2} \text{ m/s}$, the normalized temperature contours at different time levels of 0.5, 1.0 and 1.5 days are shown in Figure 4.4. Clearly, the numerical solutions (Figure 4.4(d), 4.4(e) and 4.4(f)) are again in a good agreement with the analytical solutions (Figures 4.4(a), 4.4(b) and 4.4(c)).

To quantify the difference between numerical solution and analytical solution, I plot the normalized temperature profile along the vertical fracture, as shown in Figure 4.5(a and b), where the solid lines denote the analytical solution and the little open boxes...
Chapter 4: Simulation of Hydrothermal Convection in Fractured Porous Medium

represent the numerical solution. I calculate the relative error between analytical and numerical solutions at each node point of the fracture, and print out the maximum value for each time level, as shown in Figure 4.5. It can be seen that the maximum relative error varies from 7.1% to 15.7%. Although the numerical solutions are quite close to the analytical solutions in an overal sense, the difference between them is still visible. I believe that the boundary conditions applied in numerical simulation may be the major reason. In doing the numerical simulation, I assume that the lower boundary is adiabatic except for the bottom of the fracture at which temperature is assigned to a constant value of $T_0$. The lateral boundaries are also assumed adiabatic. Thus heat cannot be lost through the lower and lateral boundaries. However, in deriving the analytical solution, I just fix the temperature $T_0$ at the bottom of the fracture and in infinitely far away it is zero without assuming any other boundary conditions, thus heat can be lost through the lower boundary. That is the reason why near the lower boundary the numerical solutions are globally slightly larger than the analytical solutions. On the other hand, on the upper boundary I fix the numerical solution at 0 °C; however only in infinitely far away the analytical solution becomes 0 °C. That is the reason why near the upper boundary the numerical solutions are globally slightly smaller than the analytical solutions.

Sudicky and Frind [1982] derived an analytical solution for the contaminant transport in a system of parallel fractures embedded in a porous rock matrix, as shown in Figure 4.6. The water velocity in each fracture is constant, a contaminant source with a constant concentration $C_0$ exists at the origin of each fracture, and the permeability of the host rock matrix is very low. Under these assumptions, they obtained an analytical expression for the concentration distribution along the fractures and within the host medium.

From a point of view of physics, heat transport is similar to the contaminant transport. Both systems have conductive (molecular diffusion) and convective (advective) transport mechanisms. With minor modification, the current finite element code can also solve contaminant transport in a fractured porous medium.

Thus, I apply their analytical solutions [Sudicky and Frind, 1982] to verify our numerical solutions for the contaminant transport problem. The test model is shown in Figure 4.6. Each fracture has an aperture of 0.1 mm, and the fracture spacing is 10 m. Both the lower and upper boundaries are assumed to be permeable for fluid flow. The initial contaminant concentration and fluid velocity are assumed to be zero. On the upper and lower boundaries, the constant head is chosen in such a way that the magnitude of the fluid velocity in each fracture is equal to 0.1 m/day. The permeability of the host rock
matrix is so small that the fluid velocity can be considered to be zero.

The analytical solutions are evaluated on the basis of the code CRAFLUSH developed by Sudicky in 1992, and the numerical solutions are computed by using our current software. Contaminant concentration distributions along the fractures and inside the host rock matrix are shown in Figure 4.7 and Figure 4.8, respectively. Clearly, the numerical solutions are in a very good agreement with the analytical solutions. The relative error between them is less than 2.0%.

In the test examples stated above, I have assumed that the host matrix is impermeable for fluid circulation. In order to check the validation of the numerical scheme for the permeable host medium, I developed independently a 2-D finite-element code. I adopt intentionally the fully implicit backward difference time stepping scheme, which is different from the Leismann time-weighting scheme used in the code HEATFLOW [Leismann and Frind, 1989], to approximate the time derivative.

The test model is a 2-D water-saturated porous medium of $10\text{ m} \times 10\text{ m}$, as illustrated in Figure 4.9. Permeability, thermal conductivity, and porosity are respectively assumed to be $10^{-10} \text{ m}^2$, $3.0 \text{ J/m/s/}^\circ\text{C}$, and 5%. All four boundaries are assumed impermeable to fluid flow. The temperature on the top and bottom boundaries is fixed at 0 °C and 10 °C, respectively. The lateral walls are adiabatic. The bold line denotes a discrete fracture, having an aperture of 0.5 mm. The solution domain is discretized by a 2-D equal-spaced grid, and the number of elements in x- and z-direction is 9. The initial temperature distribution is assumed to vary linearly with depth. The initial fluid velocity is zero over the whole solution domain.

Results for this test model are shown in Figures 4.10, 4.11 and 4.12, corresponding to the times of 100, 200 and 300 days. It can be seen that the numerical simulation results derived from two different computer programmes are very consistent. To quantify the difference between two numerical solutions, I plot the temperature distribution along the vertical fracture, as shown in Figure 4.13(a, b and c), where the solid lines denote the Waterloo code solution and the little open boxes represent my own code solution. I calculate the relative error between them at each node point of the fracture, and print out the maximum value for each time level, as shown in Figure 4.13. It can be seen that the maximum relative error varies from 1.3% to 2.8%.

The numerical scheme has been also extensively verified in the Waterloo Centre for Groundwater Research by comparing the numerical results with the in-situ experimental
data for the aquifer thermal energy storage application [e.g. see Molson et al., 1992].

4.5 Discussions and Conclusions

In this Chapter, a unique numerical simulation technique has been developed to simulate the hydrothermal circulation in discretely fractured porous media. Each volume element is assigned a permeability, porosity, and thermal conductivity. Fractures are included explicitly as planar structures connecting the mesh nodes. The method is different from all previous studies, and can represent simultaneously both the larger and smaller scale features of the medium. The numerical scheme has been validated by comparing the numerical solution with the analytical solutions for a simple model in which the host medium is impermeable. When the host medium is permeable, the scheme is validated by comparing the numerical results derived from two separately-developed codes. Together with the verification efforts made in Waterloo, we believe that the numerical scheme is correct and effective in simulating hydrothermal convection system within discretely fractured porous media.
Figure 4.3: The normalized temperature contours at the fluid velocity of $5 \times 10^{-3}$ m/s: (a), (b) and (c) are the analytical solutions; (d), (e) and (f) are the numerical solutions, corresponding to the time levels of 1, 2 and 3 days. The contour intervals are from 0.1 to 1.0.
Figure 4.4: The normalized temperature contours at the fluid velocity of $10^{-2}$ m/s: (a), (b) and (c) are the analytical solutions; (d), (e) and (f) are the numerical solutions, corresponding to the time levels of 0.5, 1.0 and 1.5 days. The contour intervals are from 0.1 to 1.0.
Figure 4.5: The normalized temperature distribution along the fracture. The solid lines denote analytical solution and the little open boxes represent numerical solution: (a) The fluid velocity is $5 \times 10^{-8}$ m/s, and for the times of 1.0, 2.0 and 3.0 days, the maximum relative error between analytical and numerical solutions is 15.7%, 13.4% and 10%, respectively. (b) The fluid velocity is $1 \times 10^{-2}$ m/s, and and for the times of 0.5, 1.0 and 1.5 days, the maximum relative error between analytical and numerical solutions is 10.4%, 11.4% and 7.1%, respectively.
Figure 4.6: A simple fracture-matrix system. The water velocity in each fracture is constant. A contaminant source with a constant concentration $C_0$ exists at the origin of each fracture. The permeability of the host rock matrix is very low.
Chapter 4: Simulation of Hydrothermal Convection in Fractured Porous Medium

Figure 4.7: Comparison between the numerical solutions and the analytical solutions of the normalized contaminant concentration distribution along the fractures at different time levels. The aperture of each fracture is 0.1 mm. The fracture spacing is 10 m. Water velocity \( v = 0.1 \) m/day. The relative error is less than 2.0 %.

Figure 4.8: Comparison between the numerical solutions and the analytical solutions of the normalized contaminant concentration distribution within the host rock matrix at the time of 500 days. The aperture of each fracture is 0.1 mm. The fracture spacing is 10 m. Water velocity \( v = 0.1 \) m/day. The relative error is less than 2.0 %.
Figure 4.9: A 2-D test model. The host porous medium has a permeability of $10^{-10}$ m$^2$. The fracture has an aperture of 0.5 mm. The system is given the following parameters: $c_m = 800$ J/kg/°C, $\rho_m = 2630$ kg/m$^3$, $c_w = 4174$ J/kg/°C, $\lambda_m = 3.0$ J/m/s/°C, $\lambda_w = 0.5$ J/m/s/°C, and $\theta = 0.05$. 
Figure 4.10: Numerical simulation results for the test model at time=100 days: (a), (b) and (c) are based on the code HEATFLOW; (d), (e) and (f) are based on my own developed code. (a) and (d) are temperature contours; (b) and (e) are fluid velocity fields in host porous medium (maximum value is $2.6 \times 10^{-6}$ m/s); (c) and (f) are fluid velocity fields along the fracture (maximum value is $5.0 \times 10^{-6}$ m/s).
Figure 4.11: Numerical simulation results for the test model at time=200 days: (a), (b) and (c) are based on the code HEATFLOW; (d), (e) and (f) are based on my own developed code. (a) and (d) are temperature contours; (b) and (e) are fluid velocity fields in host porous medium (maximum value is $7.5 \times 10^{-6}$ m/s); (c) and (f) are fluid velocity fields along the fracture (maximum value is $1.5 \times 10^{-5}$ m/s).
Figure 4.12: Numerical simulation results for the test model at time=300 days: (a), (b) and (c) are based on the code HEATFLOW; (d), (e) and (f) are based on my own developed code. (a) and (d) are temperature contours; (b) and (e) are fluid velocity fields in host porous medium (maximum value is $1.3 \times 10^{-5}$ m/s); (c) and (f) are fluid velocity fields along the fracture (maximum value is $2.5 \times 10^{-5}$ m/s).
Figure 4.13: Temperature distribution along the fracture. The solid lines denote Waterloo code solution and the little open boxes represent my own code solution: (a) Time=100.0 days, and the maximum relative error is 1.3%. (b) Time=200.0 days, and the maximum relative error is 2.8%. (c) Time=300.0 days, and the maximum relative error is 1.8%.
Chapter 5

Elementary Studies of Some Simplified Fractured Porous Media

5.1 Introduction

In Chapter 4, I developed a numerical algorithm to simulate hydrothermal convection in discretely fractured porous media. Now let me apply the scheme to some theoretical models.

As described in Chapter 2, the Rayleigh number is a most important dimensionless parameter controlling hydrothermal convection patterns in uniform media. When the Rayleigh number is less than a certain critical value, hydrothermal convection may be initiated by introducing an initial perturbation but cannot be maintained, and the initial perturbation dies down as time progresses. There are several problems in hydrothermal convection in the earth for which the Rayleigh number is close to this critical value.

This chapter focuses on hydrothermal convection in media which, on average, are sub-critical but which have very different permeabilities at different parts due to presence of fractures. I show that the existence of even a single fracture can initiate and maintain free convection in a medium which has an averaged Rayleigh number below the critical value for convection to occur. Average in this sense means using the rules similar to those used to sum the resistors in parallel or in series in electronic networks [Mckibbin and Tyvand, 1984]. Further, convection patterns produced in a closed system are very strongly dependent on the location of the single fracture. It follows that the extrapolation of rules derived for the averaged medium, such as the definition of a single Rayleigh number to describe the process of convection, may be misleading. The vigour of hydrothermal
convection is underestimated.

In the following models, the choice of fracture aperture is based on the experimental results [Pezard, 1990]; whereas other parameters (e.g., model size, permeability, thermal conductivity, and temperature contrast) are specified in such a way that the calculated Rayleigh number is very close to the critical value. The simulation results are essential to understand some fundamental characteristics of the temperature and fluid velocity patterns in fractured porous media. Other larger scale examples are given in Chapters 7 and 8.

5.2 An Unfractured Porous Medium

First consider a 2-D water-saturated unfractured porous medium with a dimension of 10 m × 20 m, as illustrated in Figure 5.1. The simulation domain is discretized by a 2-D equal-spaced grid. There are 40 and 20 elements in x- and z-directions, respectively. All four boundaries are assumed impermeable. The temperatures of the upper and lower boundaries are fixed 0 °C and 10 °C, respectively. The side boundaries are assumed adiabatic. Permeability, thermal conductivity and porosity of the model are assigned to $10^{-10}$ m², 5.0 J/s/°C and 5 %, respectively.

Based on the formula (2.5.1), the calculated Rayleigh number corresponding to these physical parameters is equal to 39, smaller than the critical value of $4\pi^2$. If I introduce an initial temperature perturbation and fluid velocity field with a maximum value of $3.35 \times 10^{-7}$ m/s, as illustrated in Figure 5.2 (a) and (b), then the initial temperature/fluid
perturbation is damped as time progresses, and the strength of thermal convection becomes weaker and weaker towards the final steady state. The final temperature contours are a set of horizontal lines, demonstrating no significant convective effect, and the maximum fluid velocity now has a value of $1.53 \times 10^{-11}$ m/s, as shown in Figure 5.2 (c) and (d), respectively. The fluid velocity field shown in Figure 5.2(d) is at the numerical 'noise' level, which cannot produce a significant effect on temperature distribution shown in Figure 5.2(c). This process is by definition sub-critical.

Figure 5.2: Numerical simulation results for a 2-D water-saturated unfractured porous medium: (a) and (b) - the initial temperature and fluid velocity perturbations (maximum initial velocity is $3.35 \times 10^{-7}$ m/s); and (c) and (d) - the steady state temperature contours and fluid velocity field (maximum value is $1.53 \times 10^{-11}$ m/s, i.e. at the numerical 'noise' level).
5.3 A Single Fractured Porous Medium

Now introduce a vertical fracture rising from the bottom to the top of the cell, as shown in Figure 5.3, keeping other conditions intact. The initial temperature distribution varies linearly with depth, and the initial fluid velocity is zero over the whole solution domain.

![Figure 5.3: A 2-D fractured porous medium. The location and aperture of the vertical fracture are changeable.](image)

When the fracture is located in the centre and its aperture is 0.2 mm, the averaged vertical and horizontal permeabilities \([\text{Mckibbin and Tyvand, 1984}]\) are \(1.00033 \times 10^{-10} \text{ m}^{-2}\) and \(10^{-10} \text{ m}^{2}\), respectively. Thus the calculated Rayleigh number is still below the critical value. The steady state numerical simulation results are shown in Figure 5.4. It can be seen that hydrothermal convection has developed in the host porous medium. Along the vertical fracture, fluid moves upwards due to the buoyancy force; the corresponding flow in the host porous medium near the vertical fracture is an upwelling zone. The isotherms are deformed from parallel lines into lines which are convex to the upper boundary. The presence of even a single fracture can therefore initiate and maintain free convection. The Rayleigh number on average is smaller than the critical value but, clearly, locally large enough for convection to be sustained.

5.3.1 Effect of the Fracture Aperture

I now vary the fracture aperture from 0.3 mm to 0.5 mm when it is located at the centre. The computed steady state temperature contours are shown in Figure 5.5. The curvature of isothermal lines increases with increasing the fracture aperture. The larger the fracture
aperture, the greater the effect of the fracture on the convection system. The patterns of fluid velocity fields in both the vertical fracture and the host porous medium are similar to those shown in Figure 5.4(b and c). However, corresponding to the aperture of 0.3, 0.4 and 0.5 mm, the maximum values of fluid velocities are increased to $1.8 \times 10^{-5}$, $2.4 \times 10^{-5}$ and $3.0 \times 10^{-5}$ m/s in the host porous medium, and $1.7 \times 10^{-4}$, $4.0 \times 10^{-4}$ and $7.1 \times 10^{-4}$ m/s along the vertical fracture, respectively.

5.3.2 Effect of the Fracture Location

I here fix the fracture aperture at 0.2 mm, but vary its horizontal location with the off-centre distance of 2.5 m, 5.0 m, 7.5 m and 9.5 m. The numerical simulation results at the steady state are shown in Figures 5.6, 5.7, 5.8 and 5.9, respectively. Again, hydrothermal convection has developed in the host porous medium due to the fluid movement along the fracture. No matter where the vertical fracture is located, a common feature is that fluid always moves upwards along the fracture and the upwelling zone is always near it. When the fracture is located in the middle (see Figures 5.4 and 5.6), the upwelling zone is in the central part with the downwelling zone close to the lateral boundaries. When the fracture is close to the right wall (see Figures 5.8 and 5.9), the upwelling zone is close to the right boundary with the downwelling zone located in the middle. The isotherms in the central region are deformed from convex lines to the upper boundary (see Figures 5.4 and 5.6) into concave lines (see Figures 5.8 and 5.9). A particularly interesting phenomenon can be seen in Figure 5.7 where the fracture has an off-centre distance of 5 m. The left side host medium of the fracture is not large enough in space to contain two convective cells; whereas the right side host medium is too narrow to develop a complete convective cell. Thus only one convective cell is developed in the whole solution domain. No matter where the fracture is located, in the middle, or close to the lateral boundary, there are always two convective cells developed in the porous medium: the cell containing the fracture is dominant, and the other is relatively weak. Convection patterns produced in a closed system are very strongly dependent on the location of the fracture.

5.3.3 Effect of the Grid Size of Elements

I again assume the fracture is located in the centre and has an aperture of 0.2 mm. Now let me change the grid size of the elements to see whether the scaling effect is important or not. When the grid size increases from 0.4 m to 0.667 m, while the other conditions
are kept the same, the temperature contours are illustrated in Figure 5.10(a and b). It can be seen that the variations in the grid size do not produce a significant effect on the numerical solutions. The relative error between Figure 5.10a and Figure 5.10b can be calculated for each common node, and its averaged value is 2.87%. The fluid velocity fields for these two models are also similar, and they are very close to those shown in Figure 5.4(b) and 5.4(c).

5.4 Super-critical Fractured Porous Medium

Next let me investigate how explicit fractures change established convection patterns.

5.4.1 Unfractured Porous Medium

Assume there are no explicit fractures, but the thermal conductivity is decreased to 3.6 J/s/°C. The Rayleigh number is 55, larger than the critical value of \(4\pi^2\). Hydrothermal convection is sustainable. The steady state numerical simulation results are dependent upon the pattern of initial perturbations. The unfractured porous medium has two possible solutions. Figure 5.11 and Figure 13 show the steady state temperature distribution and fluid velocity field, respectively corresponding to the initial perturbations illustrate in Figure 5.12 and Figure 5.14. It can be seen that two convection cells have been developed in the porous medium. The aspect ratio of the two convection cells is 1.

5.4.2 A Single Fractured Porous Medium

Assume there is a 0.2-mm-thick vertical fracture rising from the bottom to the top, as shown in Figure 5.3. The initial temperature distribution varies linearly with depth, and the initial fluid velocity is zero over the whole solution domain. When the fracture is located in the centre and close to the right wall, the steady state numerical results are shown in Figures 5.15 and 5.16, respectively. It can be seen that temperature contours and convection pattern in the latter just reverse those in the former. The location of a single vertical fracture can also change significantly established convection patterns for the super-critical media.
5.4.3 Multiply Vertically Fractured Porous Medium

Now assume there exist 20 vertical fractures spaced uniformly. Each fracture has an aperture of 0.2 mm, extending from the bottom to the top. Other parameters are kept the same as above. Again the numerical results are dependent on the starting model. When the initial perturbation has the feature of Figure 5.12, the steady state numerical simulation results are shown in Figure 5.17. When the initial perturbation has the feature of Figure 5.14, the steady state numerical simulation results are shown in Figure 5.18. It can be seen that the temperature contours seem to have been compressed in the horizontal direction. Three thin and narrow convection cells have been formed in the host porous medium. The aspect ratio of the convection cells is about 0.7. This is not surprising because the existence of the vertical fractures leads to less hydraulic resistance in vertical direction, so fluid flows more easily in the vertical direction than the horizontal direction.

5.4.4 Multiply Horizontally Fractured Porous Medium

Let me then assume there are 10 1-mm-thick horizontal fractures spaced uniformly but no any vertical fractures. All other conditions are kept the same. Again the numerical results are dependent on the starting model. When the initial perturbation has the feature of Figure 5.12, the steady state numerical simulation results are shown in Figure 5.19. When the initial perturbation has the feature of Figure 5.14, the steady state numerical simulation results are shown in Figure 5.20. Now the isothermal lines seem to have been stretched along the horizontal direction. Only one convection cell has been formed in the host porous medium. The shape of the convection cell now becomes long and flat with an aspect ratio of about 2.0. This is also reasonable because the horizontal fractures lead to less hydraulic resistance for fluid motion.

5.5 Discussions and Conclusions

In this Chapter, the numerical scheme developed in Chapter 4 has been used to investigate both the sub-critical and super-critical hydrothermal convection and the controlling factor of discrete fractures, for some highly theoretical models.

Numerical simulation results have indicated that explicit fractures play an important role in controlling temperature distribution and fluid velocity fields of hydrothermal con-
vection systems. The extrapolation of rules derived for uniform media may be misleading for estimating the vigour of hydrothermal convection in discretely fractured porous media. Explicit fractures can induce and maintain the hydrothermal convection even if the Rayleigh number is less than its critical value associated with a non-fractured medium. The larger the fracture's aperture, the greater its effect. Convection patterns produced in a closed system are very strongly dependent on the location of a single vertical fracture. Furthermore, explicit fractures can significantly change an established convection pattern. Temperature contours are compressed along the horizontal direction and convection cells become thin and narrow when there exists a set of vertical fractures; on the other hand, temperature contours are stretched along the horizontal direction and convection cells become flat and long when there exists a set of horizontal fractures.
Figure 5.4: Numerical simulation results at the steady state when the vertical fracture is located in the middle of the model and its aperture is 0.2 mm: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $6.5 \times 10^{-6}$ m/s); and (c) fluid velocity field in the vertical fracture (the maximum value is $3.8 \times 10^{-5}$ m/s).
Figure 5.5: Effect of the fracture’s aperture on temperature contours: (a) 0.3 mm; (b) 0.4 mm; and (c) 0.5 mm. The fracture is located in the centre of the model.
Figure 5.6: Effect of the fracture's lateral location on numerical simulation results at the steady state: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $6.5 \times 10^{-6} \text{ m/s}$); and (c) fluid velocity field in the vertical fracture (the maximum value is $3.8 \times 10^{-5} \text{ m/s}$). The fracture's aperture is 0.2 mm.
Figure 5.7: Effect of the fracture's lateral location on numerical simulation results at the steady state: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $6.5 \times 10^{-6} \, m/s$); and (c) fluid velocity field in the vertical fracture (the maximum value is $3.8 \times 10^{-5} \, m/s$). The fracture's aperture is 0.2 mm.
Figure 5.8: Effect of the fracture's lateral location on numerical simulation results at the steady state: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $6.5 \times 10^{-6} \, m/s$); and (c) fluid velocity field in the vertical fracture (the maximum value is $3.8 \times 10^{-5} \, m/s$). The fracture's aperture is 0.2 mm.
Figure 5.9: Effect of the fracture's lateral location on numerical simulation results at the steady state: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $6.5 \times 10^{-6}$ m/s); and (c) fluid velocity field in the vertical fracture (the maximum value is $3.8 \times 10^{-5}$ m/s). The fracture's aperture is 0.2 mm.
Figure 5.10: Temperature contours at the steady state when the vertical fracture is located in the middle of the model and its aperture is 0.2 mm. The grid size is changeable: (a) 0.4 m; and (b) 0.667 m. The averaged relative error between (a) and (b) is 2.87 %.
Figure 5.11: Numerical simulation results at the steady state for a uniform porous medium with the Rayleigh number of 55, corresponding to the starting model in Figure 5.12. There are no explicit fractures: (a) temperature contours; and (b) fluid velocity field (the maximum value is $4 \times 10^{-6} \text{ m/s}$).
Figure 5.12: Initial perturbations: (a) Initial temperature distribution; and (b) Initial fluid velocity field (the maximum value is $2.0 \times 10^{-6}$ m/s).
Figure 5.13: Numerical simulation results at the steady state for a uniform porous medium with the Rayleigh number of 55, corresponding to the starting model in Figure 5.14. There are no explicit fractures: (a) temperature contours; and (b) fluid velocity field (the maximum value is $4 \times 10^{-6} \text{ m/s}$).
Figure 5.14: Initial perturbations: (a) Initial temperature distribution; and (b) Initial fluid velocity field (the maximum value is $2.0 \times 10^{-6} \text{ m/s}$).
Figure 5.15: Numerical simulation results at the steady state: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $8.2 \times 10^{-6} \text{ m/s}$); and (c) fluid velocity field in the fractures (the maximum value is $3.8 \times 10^{-5} \text{ m/s}$). The averaged Rayleigh number is 55. The 0.2-mm-thick fracture is located in the centre.
Figure 5.16: Numerical simulation results at the steady state: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $8.2 \times 10^{-6}$ m/s); and (c) fluid velocity field in the fractures (the maximum value is $3.8 \times 10^{-5}$ m/s). The averaged Rayleigh number is 55. The 0.2-mm-thick fracture is close to the right wall.
Figure 5.17: Numerical simulation results at the steady state corresponding to the initial perturbation in Figure 5.12. There are 20 vertical fractures: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $4.3 \times 10^{-6} \text{ m/s}$); and (c) fluid velocity field in the explicit fractures (the maximum value is $1.3 \times 10^{-5} \text{ m/s}$).
Figure 5.18: Numerical simulation results at the steady state corresponding to the initial perturbation in Figure 5.14. There are 20 vertical fractures: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $4.3 \times 10^{-6} \text{ m/s}$); and (c) fluid velocity field in the explicit fractures (the maximum value is $1.3 \times 10^{-5} \text{ m/s}$).
Figure 5.19: The numerical simulation results at the steady state corresponding to the initial perturbation in Figure 5.12. There are 10 horizontal fractures: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $5 \times 10^{-6} \text{ m/s}$); and (c) fluid velocity field in the explicit fractures (the maximum value is $3.6 \times 10^{-4} \text{ m/s}$).
Figure 5.20: The numerical simulation results at the steady state corresponding to the initial perturbation in Figure 5.14. There are 10 horizontal fractures: (a) temperature contours; (b) fluid velocity field in porous medium (the maximum value is $5 \times 10^{-6}$ m/s); and (c) fluid velocity field in the explicit fractures (the maximum value is $3.6 \times 10^{-4}$ m/s).
Chapter 6

Hydrothermal Convection in Anisotropic Permeable Media

6.1 Introduction

The permeability distribution in the mid-ocean ridge environment is rarely isotropic and homogeneous, and it is likely highly anisotropic, at least in the upper oceanic crust, as suggested by the pervasive presence of discrete fractures at the sea floor and within boreholes. Obviously, the permeability along fractures is higher than that perpendicular to fractures. It is therefore reasonable to investigate the effect of anisotropic permeability on subseafloor hydrothermal circulation systems. In this Chapter, I look at the basic physics, in particular the critical parameter values for stable convection to occur.

Horton and Rogers [1945] and Lapwood [1948] examined the onset of convective instability in a homogeneous water-saturated porous layer heated from below. The basic model has been generalized to accommodate a number of complicating effects: Beck [1972] and Zebib and Kassoy [1976] investigated the effects of lateral boundaries; Lowell and Shyu [1978], Murphy [1979], and Kassoy and Cotte [1985] examined stability in a tall thin slab representing a fault zone; Lowell [1979] derived the condition for the onset of convection in a long, deep, but narrow, vertical fault zone with anisotropic permeability; Kvernvoeld and Tyvand [1979] investigated the convection currents in anisotropic porous media, and reported results on steady-state convection; Rosenberg and Spera [1990, 1993] performed 2-D numerical calculation of finite amplitude and steady convective fluid flow in layered and anisotropic porous media simulating the mid-ocean ridge hydrothermal system, and discussed numerically the relationship between flow, temperature, and anisotropic
The onset of hydrothermal convection in two-dimensional anisotropic permeable media has been studied [e.g., Wooding, 1976; Kvernvold and Tyvand, 1979]. The previous studies defined the Rayleigh number only in terms of the vertical permeability $k_z$. It was expressed as

$$R_a = \frac{k_x g \sigma \Delta TL}{k_m \nu},$$

(6.1.1)

and did not contain the horizontal permeability $k_x$. A Rayleigh number $R_a$ should be able to reflect the strength of convection. In other words, the bigger the $R_a$, the stronger the convection should be, and vice versa. However, a definition which depends only on one component of permeability cannot do this. The larger Rayleigh number does not necessarily mean the stronger convection when $k_x$ is much less than $k_z$, and the smaller Rayleigh number does not absolutely imply the weaker convection when $k_x$ is much larger than $k_z$.

The previous studies did however evaluate a condition for criticality. It is

$$R_{ac} = \pi^2 \left[ \frac{1}{k_x} \right]^{1/2} + 1^2.$$

(6.1.2)

Now, both $k_x$ and $k_z$ appear, and the critical condition on the parameters is $R_a = R_{ac}$, or

$$\frac{g \sigma \Delta TL}{k_m \nu} = \pi^2 \left[ \frac{1}{k_x} + \frac{1}{k_z} + \frac{2}{\sqrt{k_x k_z}} \right].$$

(6.1.3)

The critical condition as described is misleading. Based on equation (6.1.2), when $k_z$ is less than $k_x$, the critical Rayleigh number $R_{ac}$ is less than $4\pi^2$ of an isotropic medium, which may cause people think that a 2-D anisotropic permeable medium has a less resistance for convection to develop; whereas when $k_z$ is larger than $k_x$, the critical value is larger than an isotropic medium’s value, which may let people think that a 2-D anisotropic permeable medium has a larger resistance for convection to develop. This paradoxical phenomenon is originated from ignoring the horizontal permeability in the previous definition of the Rayleigh number.

I offer in this Chapter an alternative definition of the Rayleigh number, which contains the geometric mean of the vertical and horizontal permeabilities. The critical Rayleigh number for the onset of convection is determined analytically and supported by results of numerical simulations. I discover that the anisotropy in permeability always resists the initiation of thermal convection. Some numerical results for the super-critical hydrothermal convection in 2-D anisotropic media are also provided.
6.2 Analytical Solution to the Onset of Hydrothermal Convection in 2-D Anisotropic Permeable Media

An analytical solution for the onset of hydrothermal convection in 2-D isotropic porous medium was originally derived by Horton and Rogers [1945] and Lapwood [1948] as I have indicated. In this section, I follow the style of Turcotte and Schubert [1982], but consider a 2-D water-saturated anisotropic porous medium. As illustrated in Figure 6.1, the horizontal and vertical permeabilities are \( k_x \) and \( k_z \), respectively. The upper boundary is maintained at temperature \( T_0 \), and the lower boundary is kept at temperature \( T_1 \), where \( T_1 > T_0 \). The model has a thickness of \( L \).

As stated in Chapter 2, the governing equations for the fluid flow and thermal transport problem may be written with the Boussinesq, Darcy flow, and negligible inertia approximations as:

\[
\begin{align*}
\mathbf{q} &= -\frac{k}{\mu_w} \left[ \nabla p - \rho_w g \mathbf{n} \right]; \\
\nabla \cdot \mathbf{q} &= 0; \\
\lambda_m \nabla^2 T - c_w \rho_w \nabla \cdot (\mathbf{q} T) &= c_m \rho_m \frac{\partial T}{\partial t}; \\
\rho_w &= \rho_0 - \rho_0 \alpha_w (T - T_0).
\end{align*}
\]

Let me introduce a new pressure variable

\[
P = p - \rho_0 g z.
\]
Using Equations (6.2.1), (6.2.4), and (6.2.5), I obtain

\[ q = -\frac{k}{\mu_w} [\nabla P + \alpha_v \rho_0 g T n]; \tag{6.2.6} \]

\[ \nabla \cdot q = 0; \tag{6.2.7} \]

\[ \lambda_m \nabla^2 T - c_w \rho_w \nabla \cdot (q T) = c_m \rho_m \frac{\partial T}{\partial t}. \tag{6.2.8} \]

Before convection takes place, the temperature distribution is controlled by the conductive mechanism of heat transport, and it is

\[ T_c = T_0 + \left( \frac{T_1 - T_0}{L} \right) z. \tag{6.2.9} \]

At the onset of convection the temperature difference \( T' \equiv T - T_c \) is arbitrarily small. The \( x \) and \( z \) components of the Darcy velocity \( q_x, q_z \) are similarly infinitesimal when motion first initiates.

The energy conservation Equation (6.2.8) may be written in terms of \( T' \) as

\[ \lambda_m \left( \frac{\partial^2 T'}{\partial z^2} + \frac{\partial^2 T'}{\partial z^2} \right) - c_w \rho_w \left( q_x \frac{\partial T'}{\partial x} + q_z \frac{\partial T'}{\partial z} \right) - c_w \rho_w q_z \frac{(T_1 - T_0)}{L} = c_m \rho_m \frac{\partial T'}{\partial t}. \tag{6.2.10} \]

Since \( T', q_x, \) and \( q_z \) are small quantities, the nonlinear terms \( q_x \partial T'/\partial x \) and \( q_z \partial T'/\partial z \) on the left side of Equation (6.2.10) are all negligible. Thus the governing equations for the 2-D anisotropic permeability medium can be expressed as

\[ q_x = -\frac{k_z}{\mu_w} \frac{\partial P'}{\partial x}; \tag{6.2.11} \]

\[ q_z = -\frac{k_z}{\mu_w} \left( \frac{\partial P'}{\partial z} + \alpha_v \rho_0 g T' \right); \tag{6.2.12} \]

\[ \frac{\partial q_x}{\partial x} + \frac{\partial q_z}{\partial z} = 0; \tag{6.2.13} \]

and

\[ c_m \rho_m \frac{\partial T'}{\partial t} + c_w \rho_w q_z \frac{(T_1 - T_0)}{L} = \lambda_m \left( \frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial z^2} \right). \tag{6.2.14} \]

The top and the bottom boundaries are both isothermal and impermeable, thus these equations must be solved subject to the boundary condition \( q_z = T' = 0 \) at \( z = 0, L \).

At the onset of convection, \( \partial / \partial t = 0 \) [Turcotte and Schubert, 1982]. Thus Equation
(6.2.14) can be written as

\[ c_w \rho_w q_z \left( \frac{T_1 - T_0}{L} \right) = \lambda_m \left( \frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial z^2} \right). \]  

(6.2.15)

The pressure perturbation \( P' \) can be eliminated from the above equations by differentiating Equation (6.2.11) with respect to \( z \) and Equation (6.2.12) with respect to \( x \) and subtracting. I obtain

\[ k_z \frac{\partial q_z}{\partial z} - k_x \frac{\partial q_z}{\partial x} = \frac{k_x k_z \alpha \rho g}{\mu_m} \frac{\partial T'}{\partial x}. \]  

(6.2.16)

I can eliminate \( q_z \) between Equations (6.2.16) and (6.2.13) by the same procedure of cross differentiation and subtraction to get

\[ k_x \frac{\partial^2 q_z}{\partial x^2} + k_z \frac{\partial^2 q_z}{\partial z^2} = -\frac{k_x k_z \alpha \rho g}{\mu_m} \frac{\partial^2 T'}{\partial x^2}. \]  

(6.2.17)

The component \( q_z \) can be obtained by solving Equation (6.2.15), and it is

\[ q_z = \lambda_m \left( \frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial z^2} \right) \frac{L}{(T_1 - T_0)c_w \rho_w}. \]  

(6.2.18)

Substituting Equation (6.2.18) into Equation (6.2.17) yields a single equation for \( T' \) in the form as follows

\[ k_x \frac{\partial^4 T'}{\partial x^4} + k_z \frac{\partial^4 T'}{\partial z^4} + (k_x + k_z) \frac{\partial^4 T'}{\partial x^2 \partial z^2} = -\frac{k_x k_z \alpha \rho g (T_1 - T_0)}{\nu L \kappa_m} \frac{\partial^2 T'}{\partial z^2}. \]  

(6.2.19)

The boundary conditions must also be written in terms of \( T' \). Since \( T' = 0 \) on \( z = 0, L, \) \( \partial^2 T'/\partial x^2 \) is also zero on these boundaries. With \( q_z = 0 \) and \( \partial^2 T'/\partial x^2 = 0 \) on \( z = 0, L, \) Equation (6.2.15) gives \( \partial^2 T'/\partial z^2 = 0 \) on the boundaries. Thus, the complete set of boundary conditions for the fourth-order differential Equation (6.2.19) is \( T' = 0 \) and \( \partial^2 T'/\partial z^2 = 0 \) on \( z = 0, L. \)

The elementary solution of \( T' \) can be immediately obtained, it is

\[ T' = T_a \sin \frac{\pi z}{L} \sin \frac{2\pi x}{\lambda'}. \]  

(6.2.20)

where \( T_a \) is the amplitude of the temperature perturbation and \( \lambda' \) is its wavelength. Indeed, this solution satisfies both the differential equation (6.2.19) and the boundary conditions.
Chapter 6: Hydrothermal Convection in Anisotropic Permeable Media

Substituting (6.2.20) into (6.2.19) yields

\[ \frac{1}{\sqrt{k_x k_z}} \left\{ \frac{k_x \left( \frac{2 \pi L}{\lambda'} \right)^4 + k_z \pi^4 + (k_x + k_z) \pi^2 \left( \frac{2 \pi L}{\lambda'} \right)^2}{\left( \frac{2 \pi L}{\lambda'} \right)^2} \right\} = \frac{\sqrt{k_x k_z \alpha \eta g (T_1 - T_0) L}}{\nu \kappa_m}. \] (6.2.21)

The dimensionless combination of parameters on the right hand side of (6.2.21) is now defined as the Rayleigh number for the 2-D anisotropic permeability model. It is

\[ R_a = \frac{\sqrt{k_x k_z \alpha \eta g \Delta T L}}{\nu \kappa_m}, \quad (\Delta T = T_1 - T_0). \] (6.2.22)

Note the appearance of the geometric mean of vertical and horizontal permeabilities, \( \sqrt{k_x k_z} \), as a measure of averaged permeability of the anisotropic medium. Thus, Equation (6.2.21) now becomes

\[ \frac{1}{\sqrt{k_x k_z}} \left\{ \frac{k_x \left( \frac{2 \pi L}{\lambda'} \right)^4 + k_z \pi^4 + (k_x + k_z) \pi^2 \left( \frac{2 \pi L}{\lambda'} \right)^2}{\left( \frac{2 \pi L}{\lambda'} \right)^2} \right\} = R_a = R_{ac}. \] (6.2.23)

The Rayleigh numbers given in Equation (6.2.23) are the critical Rayleigh numbers \( R_{ac} \) for the onset of convection with wavelength \( \lambda' \). Obviously, \( R_{ac} \) depends strongly on the value of \( 2 \pi L / \lambda' \). There exits a minimum value of \( R_{ac} \) at which thermal convection can take place. The value of wavelength corresponding to minimum \( R_{ac} \) can be obtained by differentiating the left side of Equation (6.2.23) with respect to \( 2 \pi L / \lambda' \) and setting the result equal to zero. In this way, I obtain

\[ \lambda' = 2L \left( \frac{k_x}{k_z} \right)^{1/4}. \] (6.2.24)

The minimum value of critical Rayleigh number \( R_{ac} \) can be derived by substituting Equation (6.2.24) into Equation (6.2.23), and the result is

\[ \text{Minimum} \ (R_{ac}) = 2\pi^2 + \frac{k_x + k_z}{\sqrt{k_x k_z}} \pi^2. \] (6.2.25)

It can be easily proved that the following expression is always true

\[ \text{Minimum} \ (R_{ac}) \geq 4\pi^2. \] (6.2.26)

Two points can be immediately drawn from Equations (6.2.25) and (6.2.26):

1. When the 2-D model is isotropic in permeability (i.e., \( k_x = k_z \)), the critical
Chapter 6: Hydrothermal Convection in Anisotropic Permeable Media

Rayleigh number $R_{ac} = 4\pi^2$, which is exactly as same as previously published results.

(2) When the 2-D model is anisotropic in permeability (i.e., $k_x \neq k_z$), the critical Rayleigh number $R_{ac} > 4\pi^2$. In other words, the anisotropic systems have a higher critical Rayleigh number than the isotropic systems. Furthermore, the larger the anisotropic coefficient ($\sqrt{k_x/k_z}$ or $\sqrt{k_z/k_x}$), the higher the critical Rayleigh number $R_{ac}$, as illustrated in Figure 6.2. That is to say, the anisotropy in permeability resists the initiation of thermal convection.

I also derive the critical condition on the parameters by letting (6.2.22) equal to (6.2.25). It is

$$\frac{g\alpha v\Delta T L}{k_m \nu} = \pi^2 \left[ \frac{1}{k_z} + \frac{1}{k_x} + \frac{2}{\sqrt{k_z k_x}} \right],$$

which is the same as the previous expression (6.1.3).

6.3 Numerical Simulation Results

Now let me employ the numerical simulation results of the finite element method to confirm the above observations. The details of the numerical simulation techniques are given in Chapter 4. Here, I consider two kinds of 2-D models: one is isotropic in permeability, and the other is anisotropic with an anisotropic coefficient of 10. The ratio of model thickness to model width is fixed at 0.5. Other model parameters are given in Table 6.1.
Table 6.1: Parameters of the Models 1, 2, and 3.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ (m)</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$T_1 - T_0$ (°C)</td>
<td>90.0</td>
<td>90.0</td>
<td>90.0</td>
</tr>
<tr>
<td>$\alpha_v$ (°C)</td>
<td>$8 \times 10^{-4}$</td>
<td>$8 \times 10^{-4}$</td>
<td>$8 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\nu$ (m²/s)</td>
<td>$1.787 \times 10^{-6}$</td>
<td>$1.787 \times 10^{-6}$</td>
<td>$1.787 \times 10^{-6}$</td>
</tr>
<tr>
<td>$k_m$ (m²/s)</td>
<td>$7.187 \times 10^{-7}$</td>
<td>$7.187 \times 10^{-7}$</td>
<td>$7.187 \times 10^{-7}$</td>
</tr>
<tr>
<td>$k_x$ (m²)</td>
<td>$1.8 \times 10^{-10}$</td>
<td>$1.8 \times 10^{-11}$</td>
<td>$1.8 \times 10^{-9}$</td>
</tr>
<tr>
<td>$k_z$ (m²)</td>
<td>$1.8 \times 10^{-10}$</td>
<td>$1.8 \times 10^{-9}$</td>
<td>$1.8 \times 10^{-11}$</td>
</tr>
<tr>
<td>$R_{ac}$</td>
<td>40.0</td>
<td>119.0</td>
<td>119.0</td>
</tr>
<tr>
<td>$R_a$</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Obviously, these three models have the exactly same geometric shape as well as the averaged permeability ($k_{mean} = 1.8 \times 10^{-10}$ m²). Now assuming that these three models have the exactly same initial temperature perturbation and fluid velocity field, as shown in Figure 6.3.

For Model 1, since the Rayleigh number $R_a = 100$ is larger than the critical Rayleigh number $R_{ac} = 40$, the thermal convection is supposed to happen. The numerical results certainly confirm that the thermal convection occurs. As illustrated in Figure 6.6 (a), the maximum fluid velocity over the simulation domain increases from the initial value $v_{ini}$, through the higher time levels ($v/v_{ini} = 1.45$ at $t=0.3$ days, $v/v_{ini} = 2.35$ at $t=0.5$ day, and $v/v_{ini} = 3.55$ at $t=0.7$ day), to the steady state case ($v/v_{ini} = 4.2$), which implies that the initial temperature/flow perturbation has been amplified, and as time progresses, the convection strength becomes stronger and stronger, and finally reaches the steady state. Figure 6.4 (a and b) shows the temperature distribution and flow field at the steady state. Obviously, there are three convection cells in the domain.

For Model 2, since the Rayleigh number $R_a = 100$ is smaller than the critical Rayleigh number $R_{ac} = 119$, I predict that the thermal convection will not take place. The numerical results indeed prove that the model inhibits the initiation of thermal convection. As shown in Figure 6.6 (b), the maximum fluid velocity decreases from the initial value $v_{ini}$, through the higher time levels ($v/v_{ini} = 0.24$ at $t=0.5$ day, $v/v_{ini} = 0.16$ at $t=1.0$ day, $v/v_{ini} = 9.7 \times 10^{-4}$ at $t=10.0$ days, and $v/v_{ini} = 2.12 \times 10^{-5}$ at $t=50.0$ days), to the steady state case ($v/v_{ini} = 0.0$), which means that the initial temperature/flow motion perturbation has been inhibited. As time progresses, the strength of thermal convection becomes weaker and weaker, and finally reaches the steady state without any
convection. Figure 6.5 shows the temperature distribution at the steady state. Clearly, the initial temperature perturbation shown in Figure 6.3 has been flattened, and the temperature contours now are a set of perfectly horizontal lines, which means that the conductive mechanism of heat transport is dominant.

Model 3 is similar to Model 2 with the anisotropy reversed, I expect that thermal convection will not happen due to \( R_a = 100 < R_{ac} = 119 \). The numerical results again confirm our prediction. The maximum fluid velocity also decreases down to zero from the initial value to the steady state limit, as illustrated in Figure 6.6 (c). The temperature contours with respect to the steady state are exactly as same as what is shown in Figure 6.5.

Finally, let me investigate the super-critical hydrothermal convection. Assume that the temperature contrast \( T_1 - T_0 \) is increased up to 200 °C, and other conditions are kept the same as shown in Table 6.1 for the models 2 and 3. Now the Rayleigh number \( R_a = 220 \) is larger than the critical Rayleigh number \( R_{ac} = 119 \), thus the super-critical convection can be maintained.

Figures 6.7 and 6.8 illustrate the steady state temperature distribution and fluid velocity fields for the model 2 and model 3, respectively. Obviously, when the ratio of the vertical permeability to the horizontal permeability \( k_z/k_x \) decreases from 100 (Figure 6.7) to 0.01 (Figure 6.8), the shape of the convection cells changes from the thin and narrow features to the flat and long features. The tendency for the width of convection cells to increase as \( k_z/k_x \) decreases is not surprising because permeability is greater, and therefore there is less hydraulic resistance to flow, in the horizontal direction.

In general, convection cells are long and flat when horizontal permeability is greater than vertical permeability and thin and narrow when vertical permeability is greater than horizontal permeability.

### 6.4 Discussions and Conclusions

In this Chapter, both analytical and numerical techniques have been used to show the onset of convective stability in a 2-D anisotropic permeable structure. A more useful form of the Rayleigh number has been defined, containing the geometric mean of vertical and horizontal permeabilities. The critical value of this form Rayleigh number has been derived analytically, and it is higher than that for isotropic models. The larger
the anisotropic coefficient, the higher the critical Rayleigh number. In other words, the anisotropy in permeability resists the initiation of thermal convection if other parameters are kept exactly the same. Numerical simulation results have confirmed the above conclusions.

In the case that the Rayleigh number is larger than its critical value (i.e., the supercritical convection), simulation results have indicated that when horizontal permeability is greater than vertical permeability convection cells become long and flat; when vertical permeability is greater than horizontal permeability convection cells become thin and narrow. A similar conclusion has been drawn when there exist multiple horizontal or vertical fractures in host porous medium, as stated in Chapter 5.
Figure 6.3: Initial temperature and fluid velocity perturbations for the models 1, 2, and 3: (a) temperature distribution; and (b) fluid velocity field (the maximum initial velocity over the domain is $v_{mi} = 3.1 \times 10^{-5} \text{ m/s}$).
Figure 6.4: Numerical simulation results at the steady state for the model 1: (a) the temperature distribution; and (b) the fluid velocity field. The maximum fluid velocity is now up to $1.3 \times 10^{-4} \text{ m/s}$.

Figure 6.5: Temperature distributions at the steady state for the models 2 and 3. The convective effect is now negligible.
Figure 6.6: (a) For the model 1, the normalized maximum fluid velocity increases as time progresses, and finally reaches the steady state value. (b) For the model 2, the normalized maximum fluid velocity decreases as time progresses, and finally reaches zero at steady state. (c) For the model 3, the normalized maximum fluid velocity decreases as time progresses, and finally reaches zero at steady state.
Figure 6.7: Numerical simulation results at the steady state for the model 2 ($k_z/k_x = 100$): (a) temperature distribution; and (b) fluid velocity field. Note that as $k_z/k_x$ increases the number of convection cells in the domain increases.
Figure 6.8: Numerical simulation results at the steady state for the model 3 ($k_z/k_x = 0.01$): (a) temperature distribution; and (b) fluid velocity field. Note that as $k_z/k_x$ decreases the number of convection cells in the domain decreases.
Chapter 7

Fracture-Induced Hydrothermal Convection in the Oceanic Crust and the Interpretation of Heat-Flow Data

Small-scale seafloor heat-flow variations with a characteristic wavelength of about 1200 m have been observed on a profile over a sediment sealed ridge flank. Two theories have been advanced to explain them - low aspect ratio hydrothermal convection in a highly permeable basement layer 600 m thick, or high aspect ratio convection in an anisotropic permeable layer 200 m thick, induced by 20 m topographic variations with a 1000 m half wavelength. Neither theory is totally satisfactory. The first requires a very thick highly permeable zone while the absence of any resolvable basement relief on a complementary seismic survey limits the credibility of the second.

The objective of this Chapter is to develop an alternative theory to explain the origin of the small-scale seafloor heat-flow variations on the sediment-sealed ridge flanks where basement relief is not clear. The finite element scheme developed in Chapter 4 is used to address this problem. Numerical simulation results show that convection in a layered model with uniform physical parameters similar to those observed in DSDP/ODP Hole 504B cannot be maintained. The inclusion of fractures in the upper oceanic crust promotes convection. Fluid flow through fractures causes horizontal thermal gradients, initiates and maintains sub-critical convection within the upper basalts. The predicted heat flow variation is comparable to the observed data.
7.1 Introduction

In the mid-ocean ridge flanks, systematic high and low heat-flow anomalies that strike parallel to structural grain are commonly observed. The cross-strike heat-flow variations commonly have a wavelength of 6 to 10 km, and are often correlated with seafloor topography, with minima near topographic lows and maxima near topographic highs [Fisher et al., 1990; Fisher et al., 1994]. The spacing of many historical surveys is too great to resolve the much more subtle local-scale heat-flow variability. However, recent improvements in measurement technique and navigational control have enabled the non-aliased delineation of the local-scale heat-flow variations through detailed surveys with a site separation of only a few hundred meters. Davis et al. [1992] obtained an interesting heat flow profile 5 km long over the eastern Juan de Fuca ridge flank. The heat flow values are on average 250 mW/m² but there are superimposed variations of ± 30 mW/m² with an half-wavelength of about 600 m, as shown in Figure 7.1.

![Figure 7.1: Heat flow profile plotted above seismic line 90-8b over 1.2 m.yr. old crust of the Juan de Fuca Ridge eastern flank [after Davis et al., 1992]. Note the variations in heat flow with an half wavelength of about 600 m with an essentially flat-lying seafloor. Higher heat flow values to the east of this line correlate with an apparent shallowing of basement.](image)

Davis et al. [1995] interpreted the pattern as indicating low aspect ratio hydrothermal convection confined to the basement in a 600-m-thick layer of high permeability,
Figure 7.2: A 2-D water-saturated, sealed, layered model showing the bulk permeability \( k \). The average thermal conductivity and porosity are 2.0 J/m·s·°C and 5% throughout. The subdomain outlined is the plotted region for Figures 7.4 and 7.6.

\( 2 \times 10^{-12} \) m\(^2\). However, there is no direct evidence to justify their model. The in-situ measurements of bulk permeability in DSDP/ODP basement holes have revealed that maximum permeability is about \( 10^{-13} \) m\(^2\) in the upper few hundred metres (about 200 m) of igneous crust. The value of \( 2 \times 10^{-12} \) m\(^2\) has never been obtained to date. The thickest permeable layer has been found in DSDP Hole 395A, and it has a thickness of 450 m [Becker, 1990]. A thickness of 600 m is most likely overestimated for the upper most permeable oceanic crust. Fisher and Becker [1995] argue that such a thick permeable layer may not be necessary to explain the observations provided some basement relief is present. The model they developed requires an anisotropic layer only 180-200 m thick, mean permeability \( 10^{-13} \) m\(^2\) with a relief of 20 m. The theory of Fisher and Becker [1995], while quite plausible in a general sense, may not be a valid explanation of this particular data set, because a seismic survey in the same area shows no obvious relief [Davis et al., 1992; Rohr, 1994].

Could discrete fractures play a similar causative role? In this Chapter, I demonstrate that fractures within the upper oceanic crust can initiate and maintain sub-critical convection and cause local-scale seafloor heat-flow variations in areas where basement relief is not a factor.

### 7.2 The Crustal Hydrothermal Model

The layered crustal model which contains the hydrothermal system is shown in Figure 7.2. The solution domain extends 2 km horizontally and 1 km vertically.
Chapter 7: Fracture-Induced Hydrothermal Convection

The physical properties are based on the \textit{in-situ} measurements in DSDP/ODP Hole 504B [Becker, 1989]. The upper boundary, maintained at 0 °C, is at the seafloor. It is assumed sealed by the low permeability of the subjacent sediment. The lower boundary, maintained at 100 °C, is also assumed impermeable since it lies within the low permeability sheeted dikes. The side boundaries are assumed impermeable and adiabatic.

As stressed in Chapter 1, the resistivity log of DSDP/ODP Hole 504B reveals that the shallow basalts are highly fractured [Pezard and Anderson, 1989; Pezard, 1990]. The pillow lavas in Layer 2A are extensively fractured both horizontally and vertically, while the flows and pillows in Layer 2B contain mainly vertical fractures. There is no information about the fracture-spacing which will be assumed random.

When fractures are added to any layer, the permeability of the corresponding host rock is decreased so as to maintain the same bulk average values. The fractured porous medium, as a whole, is made up of alternating layers with very different permeabilities. For these types of materials, the effective average horizontal and vertical permeabilities can be estimated [McKibbin and Tyvand, 1984]. The bulk permeability is then assigned the geometric mean of the horizontal and vertical permeabilities, which is in turn constrained to equal to the \textit{in-situ} measurement values shown in the non-fractured model (Figure 7.2).

The solution domain shown in Figure 7.2 is discretized by a 2-D equal-spaced grid, and there are 200 and 100 elements in x and z directions respectively. The initial temperature distribution is assumed to vary linearly with depth. The initial fluid velocity is assumed to be zero over the whole solution domain. The simulation proceeds until the temperature and pressure reach steady state.

7.3 Numerical Simulation Results

The numerical scheme developed in Chapter 4 now is used to do simulations. Consider first the properties of the basic unfractured layered model (Figure 7.2). The hydrothermal convection is sub-critical, since the Rayleigh number for any layer, computed from the given parameters, is below the critical value. I will demonstrate that convection may be initiated but cannot be maintained. To do so, introduce an initial seafloor heat-flow, temperature and fluid perturbations (maximum initial velocity is $1.4 \times 10^{-8}$ m/s) as illustrated in Figure 7.3 (a, b and c). As expected, the initial perturbations are damped as time progresses, and the strength of fluid convection becomes weaker and weaker.
towards a final steady state where heat conduction is dominant (maximum velocity is $5.32 \times 10^{-13}$ m/s). Naturally, no significant heat-flow variations are seen on the seafloor (refer to Figure 7.3 (d, e, and f)).

Consider next two modifications in which fractures are included explicitly, a simple case to illustrate the basic physics, and a more realistic example. In Model 1, a single vertical fracture traverses layers 2A and 2B. Layer 2A also includes a single 2-km-long horizontal fracture. In Model 2, Layers 2A and 2B are fractured extensively (Figure 7.5c). There are 100 vertical fractures extending from the bottom of Layer 2B to the top of Layer 2A. Layer 2A has 10 2-km-long horizontal fractures. The fractures are spaced randomly in both the vertical and horizontal directions. The minimum fracture-spacing is 10 m, the size of the grid elements. Other relevant parameters are given in Table 7.1.

<table>
<thead>
<tr>
<th></th>
<th>Model 1 Layers</th>
<th>Model 2 Layers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2A</td>
<td>2B</td>
</tr>
<tr>
<td>Permeability of host medium (m$^2$)</td>
<td>$9 \times 10^{-14}$</td>
<td>$4.8 \times 10^{-16}$</td>
</tr>
<tr>
<td>Aperture of vertical fractures (mm)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Aperture of horizontal fractures (mm)</td>
<td>0.3</td>
<td>none</td>
</tr>
</tbody>
</table>

Table 7.1: The parameters of the Models 1 and 2.

Figure 7.4 shows the simulation results for Model 1. The fluid is driven by buoyancy force upward along the vertical fracture (Figure 7.4b) with a maximum velocity of 0.00011 m/s. Local-scale horizontal thermal gradients are generated in Layer 2A, which cause sub-critical convection within that layer to be initiated and maintained. The convecting fluid has a maximum velocity of $5 \times 10^{-9}$ m/s. A second weaker convection cell spans both Layers 2A and 2B and replenishes the fluid in the vertical fracture in the lower region. The arrows in Figure 7.4b represent local estimates of the fluid flux per unit length both in the fracture and in the surrounding medium. The arrow display supports the divergence free nature of the flow everywhere. The peak of the seafloor heat-flow (Figure 7.4a) is above the vertical fracture. Doubling the distance to the lateral
Figure 7.3: Numerical simulation results for the basic unfractured layered model: (a), (b) and (c) - the initial seafloor heat-flow variations, and temperature and fluid velocity perturbations (maximum initial velocity is $1.4 \times 10^{-8}$ m/s); and (d), (e) and (f) - the steady state seafloor heat-flow variations, and temperature contours and fluid velocity field (maximum value is $5.32 \times 10^{-13}$ m/s).
Chapter 7: Fracture-Induced Hydrothermal Convection

boundaries does not alters the flow pattern in the central region.

Figures 7.5 and 7.6 illustrate the simulation results for Model 2. Strong vertical fluid motion caused by buoyancy force along vertical fractures (Figure 7.6c) produces sufficiently high horizontal thermal gradients in Layer 2B (Figures 7.5b and 7.6b) to drive convection in Layer 2A. The maximum fluid velocity both in the discrete fractures and porous medium is $1.0 \times 10^{-4}$ m/s and $1.6 \times 10^{-8}$ m/s, respectively. The aspect ratio of the convection cells ranges from 1.7 to 2 (Figure 7.6c). The wavelength of the seafloor heat-flow variations is about 800 m (Figures 7.5a and 7.6a). It is related to the product of the aspect ratio and the thickness of layer 2A. Doubling the width of this model changes the flow only in detail near the lateral boundaries.

7.4 Discussions and Conclusions

In this Chapter, an alternative theory has been proposed to explain the origin of the seafloor heat-flow variations on mid-ocean ridge flanks. The numerical simulation results have indicated that neither a thick highly permeable layer, nor basement relief, nor sediment-thickness variations are required to explain local-scale seafloor heat-flow anomalies. Discrete fractures, widely existing in oceanic crustal material, can serve a similar role as basement topography in initiating and maintaining low aspect ratio sub-critical convection in porous pillow lavas.

Without including explicit fractures in the upper basalts, sub-critical convection within hole 504B-type oceanic crust cannot be maintained, and no obvious seafloor heat-flow variations can be produced. When fractures are introduced explicitly, fluid can flow upwards along the vertical fractures, which in turn gives rise to local-scale horizontal thermal gradients. As a result, sub-critical convection within the upper basalts can be initiated and maintained, and local-scale seafloor heat-flow variations can be induced.
Figure 7.4: The numerical simulation results at steady state for Model 1: (a) the local-scale seafloor heat-flow variations; and (b) the local fluid flux per unit length shown as arrows whose lengths are proportional to the flux magnitude in a crustal cell or fracture. Also the temperature field shown as the background gray scale.
Figure 7.5: The numerical simulation results at steady state for Model 2: (a) the local-scale seafloor heat-flow variations; (b) the temperature field; and (c) the fracture distribution.
Figure 7.6: A part of Figure 7.5 magnified illustrating: (a) the local-scale seafloor heat-flow variations; (b) the temperature field; and (c) the integrated fluid flux in the porous oceanic crust and fractures.
Chapter 8

Three-Dimensional Numerical Simulation of the Hydrothermal System within the TAG-like Sulfide Mounds

The currently active TAG sulfide mound has become a focus of attention due to its importance to ridge dynamics and ore-forming processes. Electromagnetic, heat-flow, fluid velocity, temperature and near-bottom magnetic data have been collected over its surface on the seafloor. The interpretation of these data sets and ultimate understanding of the internal character of the mound and the ore-forming chemistry require realistic estimates of the internal temperature and fluid velocity field.

The objective of this Chapter is to develop the internal temperature and fluid velocity fields for TAG-like mounds. A range of models with permeable and impermeable upper boundaries and different thermal conductivities is investigated. The solutions have three common features: a central high-temperature zone which coincides with a non-magnetic zone inferred from magnetic data; fluid upflow along the vertical high-permeability column, surrounding a central pipe; and a maximum fluid velocity of the order of the observed maximum value at the central black smokers. Thermal conductivity variations strongly influence temperature distribution but not the fluid velocity pattern.
8.1 Introduction

Ancient massive Fe-Cu-Zn±Pb±Ag sulfide deposits or "polymetallic sulfides", now found and mined on land, were originally formed from submarine hot springs [Brimhall, 1991]. On modern oceanic ridge crests, hydrothermal activity is depositing polymetallic sulfides [Scott, 1985; Lowell and Rona, 1985; Rona et al., 1986]. Several massive sulfide deposits have been found in the East Pacific Rise (EPR), in the Juan de Fuca Ridge, and in the Mid-Atlantic Ridge. These modern seafloor polymetallic sulfides can be regarded as natural laboratories for observing and measuring ore-forming processes. This knowledge may ultimately result in greater efficiency in finding ancient massive sulfide deposits on land.

An immediate example of the modern seafloor massive sulfide deposits is the Trans-Atlantic Geotraverse (TAG) active sulphide mound, which is located at 26°08'N and 44°49'W and situated in water depths between 3620 and 3670 m at the juncture between the floor and the east wall of the rift valley of the Mid-Atlantic Ridge [Rona et al., 1986; Rona and Speer, 1989] (Figure 8.1). The currently active TAG mound first formed about 40-50,000 years ago and has had intermittent pulsed high-temperature activity every 5000-6000 years over the past 20,000 years. It is believed that this periodicity may reflect renewed magmatic activity at the ridge axis. A common magma source is suggested by the presence of a midcrustal seismic low-velocity zone indicating a largely solid but still hot intrusion at a subbottom depth of 3 km beneath the adjacent axial high of the rift valley [Kong et al., 1992], and this hot magma intrusion is believed to be the heat source of the TAG hydrothermal system.

The TAG active mound is about 200 m in diameter, 35 m high, and may contain 5 million tons of surface sulphide ore (as shown in Figure 1.2). Deep sea camera surveys and sampling from surface ship and submersible reveal that the exposed portion of the mound is composed of massive sulphides and sulfates. Indeed, the entire mound is probably constructed of massive sulphides precipitated from hydrothermal solution [Rona et al., 1993a, 1993b]. The TAG active mound is the largest seafloor sulfide deposit discovered so far, and it is comparable in size and mineral composition with some ancient massive sulfide deposits found and mined on land. The Archean Noranda massive sulfide mound of eastern Canada, for instance, resembles the TAG mound in size, shape, and mineral composition from the center to the edge of the mound.

The TAG active hydrothermal system has become the subject of ongoing collabo-
Figure 8.1: Location of the TAG active sulfide mound.

rative, multidisciplinary research since the discovery of ongoing high-temperature hy-
drothermal activity [Rona et al., 1986] due to its importance to ridge dynamics and
ore-forming processes as well as to its potential economic significance. Electromagnetic,
heatflow, near-bottom temperature, fluid velocity and near-bottom magnetic data have
been collected using the submersibles over the surface of the mound. In order to aid in the
interpretation of these data sets, and ultimately to understand the internal character of
the mound and the ore-forming processes, realistic estimates of the internal temperature
and fluid velocity fields are required. Electrical conductivity depends on ion mobility
in pore fluids, and is a strong function of temperature. The conductivity of sea water
increases 10 fold for a 200 °C increase in temperature. Remanent magnetization is un-
stable above the Curie point temperature of 150 ~ 200 °C for the young mid-ocean ridge
basalts. Thus a knowledge of possible internal temperature distributions is also essential
for geo-electrical and geo-magnetic studies.

Until recently, mid-ocean ridges were viewed mainly as two-dimensional features
[Lowell and Burnell, 1991; Davis et al., 1995; Fisher and Becker, 1995]. Little effort
has been made to derive a flow and heat geometry based on the three-dimensional ther-
mal regime and permeability field at mid-ocean ridges. One previous study [Travis et
al., 1991] considered fully 3-D simulations of deep hydrothermal circulation at mid-ocean
ridges and investigated the regional-scale flow and temperature pattern. The study ap-
plied an isothermal upper boundary condition, which precluded the modeling of high-
temperature springs or black smokers which are common at the ridge-axial sea floor environments. In addition, discrete fractures, commonly observed at the seafloor and within boreholes, have not been incorporated explicitly into the models.

In this Chapter, the numerical scheme developed in Chapter 4 is used to simulate the internal 3-D temperature and fluid velocity fields within the TAG sulfide mound, adding highly permeable "short circuits" to simulate a discrete conduit. I compute the temperature and fluid velocity patterns for a range of possible active TAG models with different thermal conductivities and permeable and impermeable upper boundaries.

8.2 A Simplified TAG-like Hydrothermal Model

Consider a cubic model (200 m × 200 m × 200 m) inserted into a uniform and flat oceanic crust, as shown in Figure 8.2. A 3-D simulation domain (400 m × 400 m × 400 m) encloses the model. Its upper boundary is coincident with the seafloor. A vertical column is included with a cross-sectional area of 20 m × 20 m, starting at a depth of 400 m and rising through the body to the seafloor. This column, being initiated by the intersection of active axis-parallel marginal and axis-transverse fault systems [Rona et al., 1993], has a significantly higher permeability than the host oceanic crust and forms an obvious preferential fluid flow path. In the very center of the column, there is a vertical pipe which simulates a conduit within the TAG mound [Rona et al., 1993a, 1993b]. The simulation domain is discretized by a 3-D equal-spaced grid. There are 40 elements in each of x-, y-, and z-directions.
8.3 Boundary and Initial Conditions

The numerical simulation technique has been described in detail in Chapter 4. A simple vertical pipe has been incorporated explicitly by superimposing a number of line elements representing individual pipe segments onto the nodes in the centre of the domain. The pipe is assigned a permeability of $r^2/8$, where $r$ is the radius of the pipe [Lacombe et al., 1995]. By arranging the line and volume elements in this manner, the continuity of temperature and pressure at the interface between the pipe and host medium is automatically satisfied.

In marine environments, especially at the ridge-axial sites, temperature on the seafloor is not necessarily constant. At the interface between the mound and the seawater, I assume a convective heat flux which is linearly proportional to the temperature gradient across the surface boundary layer, following Carslaw and Jaeger [1959]. This Cauchy-type boundary condition may be written as:

$$\lambda_m \frac{\partial T}{\partial z} = (\gamma + q_c \rho \nu)(T_w - T_s), \quad (8.3.1)$$

where $\lambda_m$ is the thermal conductivity of medium near seafloor, $\gamma$ is the linear heat transfer coefficient across the seafloor, $T_w$ is the overlyed seawater temperature, $T_s$ is the

Figure 8.2: A simplified TAG-like hydrothermal model.
(unknown) surface temperature within oceanic crust, and \( q \) is the surface recharge flux.

On the lower boundary, the temperature field is fixed at 400 °C at the bottom of the vertical high-permeability column, and everywhere else fixed at 50 °C. I assume for most models that both the upper and lower boundaries are permeable to fluid flow, justified by the high permeability of the shallower basalts [Becker, 1990]. The sides are assumed impermeable and adiabatic. The initial temperature distribution varies linearly with depth, and the initial fluid velocity is zero over the whole solution domain. The simulation proceeds until the temperature and pressure reach the steady state.

The permeability of the vertical column is assigned a value of \( 10^{-12} \) m\(^2\), one order higher than the surrounding crust's permeability of \( 10^{-13} \) m\(^2\) [Becker, 1990]. The host crust has a thermal conductivity of 3.0 J/s/°C and porosity is given as 10% everywhere in the solution domain.

It is relatively hard to constrain the thermal conductivity of the TAG sulfide mound due to the absence of any published results. The parameter varies from 3.5 to 20 J/s/°C based on its possible extremes derived from extrapolating known thermal conductivities of pyrite, anhydrite, and mixtures of these, to high temperature [Tivey and McDuff, 1990].

### 8.4 Numerical Simulation Results

The steady state temperature distribution on a central x-z section at \( y=200 \) m is illustrated in Figure 8.3 (a, b, and c) for different thermal conductivities of 3.5, 8.5, and 20 J/s/°C. Notice a high-temperature zone, centered by the vertical column and having a diameter of about 100 m, which surrounds the central core. The zone is consistent with the presence of a stockwork zone or alteration pipes in ophiolite suites [Scott, 1985]. The temperature within the zone is sufficient to impede the normal crustal magnetization and generate a detectable magnetic anomaly since the young mid-ocean ridge basalt has a relatively low Curie point temperature (150 ~ 200 °C). Tivey et al. [1993] interpreted a non-magnetic zone from a recent near-bottom magnetic survey, see Figure 8.7. The calculated temperature distribution over the upper boundary (seafloor) is also similar to the observed temperature data on the surface of the TAG mound [Rona et al., 1989], see Figure 1.2. At the center, the temperature is high (up to 360 °C), and it decreases rapidly towards the edge of the mound. The larger the thermal conductivity of the TAG mound becomes, the farther the temperature contours inside the mound expand outwards from
the central column.

The fluid velocity fields in the central pipe and in the host porous medium are nearly independent of the thermal conductivity. Fluid flows upwards from the hot lower boundary to the colder upper boundary under influence of the vertical buoyancy force, as illustrated in Figure 8.4. The fluid flows much faster along the vertical column and the central pipe than in the host medium. The maximum fluid velocity in the porous medium is 0.2 mm/s. The maximum value in the central pipe depends on its radius. It is 1.2 m/s for a radius of 1 mm, which is of the order of the observed maximum velocity of 1 m/s at the central black smokers [Rona et al., 1989], see Figure 1.2. Increasing the radius of the pipe results in even bigger fluid velocity, but its impact on the steady state temperature distribution is not significant.

In the models described above, the top boundary is assumed permeable. It is possible that the settling of sulfide particles from the hydrothermal plume may form a metalliferous layer on the surface of the active mound [Rona et al., 1989] and partially seal the top boundary. Let me suppose that the upper boundary is impermeable except for the top of the vertical column. The steady state temperature distribution on a central x-z section at y=200 m is illustrated in Figure 8.5 (a, b, and c) for different thermal conductivities of 3.5, 8.5, and 20 J/s/°C. Again, a high-temperature zone has formed beneath the seafloor. But notice the temperature contours display the characteristic mushroom-shaped pattern near the seafloor. The larger the thermal conductivity of the TAG mound becomes, the farther the temperature contours inside the mound expand outwards from the central column.

The fluid velocity fields in the central pipe and in the host porous medium are again nearly independent of the thermal conductivity. As illustrated in Figure 8.6 (a, and b), fluid flows up along the vertical column and the pipe and erupts into seawater at its top. Beside the vertical column, fluid reflects back from the impermeable seafloor to the bottom through the permeable host crust and forms convective cells. The maximum fluid velocity in the porous medium and the pipe is 0.12 mm/s and 0.66 m/s, respectively.

8.5 Discussions and Conclusions

In this Chapter, the numerical simulation technique developed in Chapter 4 has been employed to investigate the internal temperature and fluid velocity patterns within TAG-like sulfide mounds. A range of models with permeable and impermeable upper boundaries
and different thermal conductivities has been investigated. Simulated temperature distributions are comparable to the observed pattern and can explain the non-magnetic zone interpreted from a near-bottom magnetic survey. The fluid moves up along the vertical column and the central pipe and erupts into seawater in a manner similar with the black smokers of the TAG mound which are venting from the central chimney and mixed with seawater. The maximum fluid velocity in the pipe is in the order of the observed value in the central black smokers. The fluid flows up the central permeable zone are high enough to form a hot isothermal core, about which the temperature distribution is controlled primarily by thermal conductivity. The central pipe is of negligible importance to the overall thermal and flow structure of the mound, despite its presumably vital role in creating locally a black smoker mineral deposit.
Figure 8.3: The steady state temperature distributions (°C) on the x-z section at y=200 m for the different thermal conductivities of the TAG sulfide mound: (a) 3.5 J/s/°C; (b) 8.5 J/s/°C; and (c) 20 J/s/°C. The surface of the TAG mound is assumed permeable.
Figure 8.4: The steady state fluid velocity patterns: (a) the fluid velocity field in the porous medium on the x-z section at y=200 m with a maximum value of 0.2 mm/s; and (b) the fluid velocity field in the central pipe with a maximum value of 1.2 m/s. The surface of the TAG mound is assumed permeable.
Figure 8.5: The steady state temperature distributions (°C) on the x-z section at y=200 m for the different thermal conductivities of the TAG sulfide mound: (a) 3.5 J/s/°C; (b) 8.5 J/s/°C; and (c) 20 J/s/°C. The surface of the TAG mound is assumed impermeable except for the top of the central column.
Figure 8.6: The steady state fluid velocity patterns: (a) the fluid velocity field in the porous medium on the x-z section at y=200 m with a maximum value of 0.12 mm/s; and (b) the fluid velocity field in the central pipe with a maximum value of 0.66 m/s. The surface of the TAG mound is assumed impermeable except for the top of the central column.
Figure 8.7: Cartoon showing the relationship between the sea surface and near-bottom magnetic anomaly signature and the simplified vertical structure of a hydrothermal vent system [after Tivey et al., 1993]. In the upper two figures, the magnetic field profiles due to the narrow non-magnetic pipe and the broad area of demagnetization at depth are plotted for two different observation levels, altitude of 100 m, i.e. near-bottom, and altitude of 4 km, i.e. sea surface. The profiles are along the magnetic meridian with a field inclination of 45°; north is to the right. The narrow upflow zone (diameter 100 m) consists of highly altered and non-magnetic rock and produces a large-amplitude, short-wavelength near-bottom magnetic anomaly but relatively little effect at the sea surface. The deeper demagnetized zone forms a halo around a heat source such as a late-stage intrusion and thins the magnetic source on a broader kilometer scale resulting in a long-wavelength anomaly that produces the majority of the magnetic signal at the sea surface but only contributes to the regional gradient in the near-bottom survey.
Chapter 9

Summary of Original Contributions and Suggestions for Further Works

9.1 Summary of Original Contributions

My thesis mainly contributes to investigating the physical behaviour and computational techniques of heat transport and fluid flow in explicitly fractured porous media, particularly in the mid-ocean ridge environment. I have developed both analytical and numerical solutions for these kinds of systems. Parts of Chapters 7 and 8 have been published [Yang et al., 1996a; 1996b]. Parts of the thesis have been presented in the international conferences and workshops [Yang et al., 1996c; 1995a; 1995b; 1993].

9.1.1 Analytical Solutions

I have derived an analytical expression of the temperature distribution along the fracture and inside the host medium for the special case of an impervious host rock by using the Laplace transformation technique. Not only has this solution provided some of fundamental properties of hydrothermal convection in fractured rock, but also it has served as a criterion to validate the numercial simulation scheme for general hydrothermal convection in fractured porous media.

I have derived an analytical expression for the onset of hydrothermal stability in 2-D anistropic permeable media. A more useful form of the Rayleigh number has be defined, containing the geometric mean of vertical and horizontal permeabilities. A conclusion that the anisotropy in permeability resists the initiation of thermal convection has been
drawn and has been confirmed by numerical simulation results.

9.1.2 Numerical Solutions

I have developed a numerical software to simulate 2-D and 3-D hydrothermal circulation in discretely fractured anisotropic porous media. Each volume element is assigned a permeability, porosity, and thermal conductivity. Discrete fractures are included explicitly as planar structures connecting the mesh nodes. This algorithm is different from previous studies, and can represent simultaneously both the larger and smaller scale features of the medium.

Numerical simulation results for some simplified fractured porous media have indicated that explicit fractures play an important role in controlling temperature distribution and fluid velocity fields of hydrothermal convection systems. The extrapolation of rules derived for uniform media may be misleading for estimating the vigour of hydrothermal convection in discretely fractured porous media. Explicit fractures can induce and maintain the hydrothermal convection even if the Rayleigh number is less than its critical value; furthermore, they can significantly change an established convection pattern.

9.1.3 Field-Scale Examples

The numerical simulation code has been used to investigate some important yet unsolved hydrothermal convection phenomena encountered in mid-ocean ridges and other areas.

I have developed an alternative theory to explain the origin of the seafloor heat-flow variations on mid-ocean ridge flanks where basement relief is not clear. Numerical simulation results have indicated that the inclusion of fractures in upper oceanic crust promotes convection. Fluid flow through fractures causes horizontal thermal gradients, initiates and maintains sub-critical convection within the upper basalts. The predicted heat flow variation is comparable to the observed data. The theory has been received with great interest by marine geophysicists and geologists.

I have investigated the internal temperature and fluid velocity patterns within TAG-like sulfide mounds. Simulated temperature distributions are comparable to the observed pattern and can explain the non-magnetic zone interpreted from a near-bottom magnetic survey. The fluid pattern is also comparable to the observed behaviour of the
smokers in the active TAG sulfide mound.

9.1.4 Software Development

I have developed 1-D and 2-D finite-element software to solve the time-dependent coupled fluid flow and heat transport in fractured anisotropic porous media.

I have developed 2-D finite-element software to solve the time-dependent coupled groundwater flow and heat transport associated with the nuclear fuel waste contained in the hypothesized vault excavated in fractured anisotropic porous media.

I have modified the code HEATFLOW, in cooperation with J. W. Molson and E. A. Sudicky at the Waterloo Centre for Groundwater Research, to solve the 3-D time-dependent coupled fluid flow and heat transport in fractured anisotropic porous oceanic crust.

I have modified the code HEATFLOW, in cooperation with J. W. Molson and E. A. Sudicky, to solve the 3-D time-dependent coupled groundwater flow and heat transport associated with the nuclear fuel waste contained in the hypothesized vault excavated in fractured anisotropic porous media.

9.2 Suggestions for Further Work

Many chapters of my thesis have just opened the doors to some new interesting directions, but not closed them yet. Although some major conclusions have been made, extending studies in these directions may still be needed. The further works should focus on quantifying the effect of discrete fractures on hydrothermal convection by running different models with a range of parameters.

The numerical simulation studies on subseafloor hydrothermal systems are still in its infancy, and a number of critical problems remain to be solved in the future:

I. Developing numerical methods to solve two-phase (seawater and steam) flow in seawater systems.

II. Developing numerical methods to incorporate chemical and biological processes.

III. Developing numerical methods to investigate the temporal evolution of high-temperature
hydrothermal systems at mid-ocean ridge crests, where the permeability is not only a function of space, but also a function of time, due to the time-dependent thermoelastic and chemical effects.
REFERENCES


References


Gartling, D. K., Finite element analysis of thermal convection in deep ocean sediments,
References


References


Molson, J. W., and E. O. Frind, HEATFLOW: density-dependent flow and thermal
energy transport model in three dimensions, Waterloo Centre for Groundwater Research, 1993.


Tang, D. H., E. O. Frind, and E. A. Sudicky, Contaminant transport in fractured porous


IMAGE EVALUATION
TEST TARGET (QA-3)

1.0  2.0  2.5
1.1  2.0  1.8
1.25 1.4  1.6

1.0  1.8  1.25
1.1  2.0  1.4
1.25 1.6  1.4

150mm  6"