**EXPERIMENTAL INVESTIGATION OF
TAYLOR BUBBLE ACCELERATION MECHANISM
IN SLUG FLOW**

by

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A thesis submitted in conformity with the requirements
for the degree of Master of Applied Science
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Nihil Sine DEO!

To my parents

ABSTRACT

Measurements of the total drag force on a solid Taylor bubble placed in a liquid flowing downward in a vertical tube have been performed. Variations in the drag force with the solid bubble displacement from the tube axis were studied in single bubble and dual bubble systems. The former arrangement included measurements for 7.5 cm and 15 cm long bubbles with normal and deformed shape noses. The latter experiment has been performed for different separation distances between the two 15 cm long, normal nose bubbles.

The experimental results showed a significant decrease in the drag force with bubble displacement from the tube axis for both 7.5 cm and 15 cm long bubbles (larger for the latter one). A comparison between the drag forces for the two bubble nose shapes showed lower values for the deformed nose than the normal nose bubble. The difference was obviously greater in the case of shorter bubbles.

In the two bubble system, the drag force measured on a trailing bubble at the center of the pipe decreased gradually for separation distances smaller than two or three pipe diameters, while outside this region it remained essentially constant. Further experiments to study the drag force variation with the bubble radial displacement for separation distances greater than two or three pipe diameters indicated similar values and variations as in single bubble experiments.

Experimental observations suggest that a new mechanism regarding Taylor bubble acceleration at separation distances smaller than a critical value should be considered in order to better understand the hydrodynamic phenomena preceding Taylor bubble coalescence.

ACKNOWLEDGMENTS

I would like to express my gratitude to **Professor Masahiro Kawaji** for his valuable guidance and active support throughout this research. His constant patience and encouragement were essential in completing this work.

I also appreciate the opportunity of attending in my first year at University of Toronto two courses taught by **Professor Olev Trass**. His lectures offered me a better understanding towards the link between theoretical and physical concepts.

I am very thankful to all colleagues, particularly to **Jason Xu**, in the Thermohydraulic Research Laboratory who offered unconditionally their help in different situations.

Any problem with the electronics was successfully solved with **Dan Tomchyson**'s help.

I want also to thank to my valuable friend, **Marzyieh Khatibzadeh**, for her generosity and trust.

Lastly, but most important, I would like to thank to my remarkable parents, father **Radu** and mother **Melania** for their support, patience and sacrifice.

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NOMENCLATURE

A	area [m ²]
a	parameter resulting from curve fitting
b	parameter resulting from curve fitting
c	coefficient
C	parameter entering the equation for bubble rise velocity in accelerating region
D	pipe diameter [m]
e	bubble eccentricity
Eo	Eotvos number = $\frac{\rho g D^2}{\sigma}$
f	friction factor
F	force [N]
Fr	Froude number = $\frac{U_{TB}}{\sqrt{gD}}$
g	gravitational acceleration constant [m/s ²]
h	gap distance [m]
j	superficial velocity [m/s]
k	dimensionless velocity
L	separation distance [m]
Mo	Morton number = $\frac{g\mu^4}{\rho\sigma^3}$
N _f	dimensionless number = $\frac{\rho g^{\frac{1}{2}} D^{\frac{1}{2}}}{\mu}$
P	pressure [Pa]
Q	volumetric flow rate [m ³ /s]
r	radial distance [m]
r ⁺	dimensionless distance from pipe centerline
R	pipe radius [m]
Re	Reynolds number

t	time [s]
U	rise velocity [m/s]
v	velocity [m/s]
x	axial distance [m]
x^+	dimensionless axial distance
y, z	Cartesian coordinates [m]

Greek Letters

δ	thickness of falling film [m]
Δ	change in
μ	viscosity [kg/ms]
ρ	density [kg/m ³]
σ	surface tension [N/m]
Γ	factor depending on the liquid mean velocity
τ	shear stress [N/m ²] bipolar coordinate [m]

Subscripts

app	apparent
avg	average
c	closed system
cross	cross sectional
D	drag
f	liquid film
fd	fully developed region
fD	entrance region considered in calculations
GS	gas slug
H	hydraulic

hy	hydraulic entrance region
LS	liquid slug
t	total
TB	Taylor bubble

1 INTRODUCTION

Slug flow is one of several two-phase flow regimes encountered in many practical situations, for example, in nuclear reactors during emergency core cooling, in geothermal power plants during the production of steam and water, boilers, condensers and chemical reactors where mass and heat transfer occurs between gas and liquid phases, and in production of hydrocarbons in wells and their transportation in pipelines.

To better understand and design equipment for these processes, numerous investigations have been undertaken over the years, encompassing experimental, numerical and theoretical approaches. The general two-phase flow problem is described by a system of nonlinear, partial differential equations resulting from the fundamental equations of fluid mechanics, namely: continuity, momentum and energy conservation together with a set of appropriate boundary conditions specified at gas-liquid interfaces, which are constantly moving free surfaces. The freely moving interface makes the analytical solution difficult to obtain even for simpler cases of slug flow such as a bullet-shaped gas bubble rising in a stagnant liquid. Most of the models, due to the limitations in experimental techniques and over-simplifications in the two-phase flow mechanisms considered, do not predict accurately the slug flow phenomena outside of a certain range of conditions. Besides these types of models, empirical approaches are also used to obtain correlations for global parameters such as void fraction and pressure drop. The shortcomings in this case are related to the restricted range of validity for the proposed correlations due to the lack of physical basis.

The flow structure and the characteristic features of the slug flow are highly related to the other three gas-liquid two-phase flow patterns existing in upward gas-liquid flow in a vertical pipe [Hewitt and Hall-Taylor, 1970; Taitel et al., 1980; Mishima and Ishii, 1984]. Subsequently, these regimes are presented in the order of increasing gas flow rate in fig.1.1 [Hewitt and Hall-Taylor, 1970]:

- . **Bubbly Flow:** The gas phase is fairly uniformly distributed as small, discrete bubbles in the continuous liquid phase.

- **Slug Flow:** An increase in the gas flow rate leads to the coalescence of small bubbles into bullet shaped bubbles called gas slugs or Taylor bubbles which occupy most of the pipe's cross-sectional area. According to continuity, a thin liquid film will flow downward along the side of the rising Taylor bubble. Between the successive Taylor bubbles, a liquid slug containing small gas bubbles entrained from the tail of the preceding Taylor bubbles flows upward.
- **Churn Flow:** This is a highly disordered flow regime in which the continuity of the liquid slugs is destroyed as the gas slugs become narrower and irregular in shape. The vertical motion of the liquid becomes oscillatory.
- **Annular Flow:** Due to the increased gas flow rate, the core is entirely occupied by the upward moving gas which may contain entrained liquid droplets. Along the walls of the pipe, a wavy liquid film flows upward.

Fig.1.1 Flow Patterns in Vertical Flow (Hewitt and Hall-Taylor, 1970)

Even though the slug flow contains characteristics of the other three flow patterns, a complete study relating all the factors governing the motion of the two phases

in slug flow would be extremely difficult. Due to the development of new measurement techniques like flow visualization using video cameras or Photochromic Dye Activation which involves formation of traces in the flowing liquid using a laser [Kawaji et. al., 1993], new information regarding the instantaneous microscale phenomena can be obtained. Thus, a better understanding of the hydrodynamics of the falling liquid film and the local characteristics in the near wake and far wake regions in slug flow becomes possible.

From the recorded images of two accelerating and coalescing Taylor bubbles, it has been clearly observed that the penetration distance for the falling liquid film does not extend down to the trailing bubble [Ahmad, 1993], such that the acceleration of the trailing bubble is not due to the increased upward velocity of the liquid core. In this way, a possible explanation for the accelerating trailing bubble was rejected and another mechanism consistent with the existent experimental data has been proposed. From observations of the rising Taylor bubbles it was seen that the trailing bubble oscillated from side to side of the pipe wall, with a continuous change in the nose shape. This intensified as the bubble separation distance decreased. On the other hand, numerous two phase gas-liquid systems in unconfined media have been studied previously and the wall effects have been proven to exert an important influence on the hydrodynamics of the dispersed phase [Cheremisinoff, 1983].

Thus, a hypothesis for a new mechanism of trailing Taylor bubble acceleration based on the reduction of drag force in vertical slug flow due to bubble lateral displacement and bubble nose distortion was experimentally verified. Also, the process of bubble's random, alternate movements from one side of the pipe to the other, was explained based on the changes in the bubble tail slope. The aforementioned experimental and theoretical observations concerning certain aspects of slug flow are presented in the next Chapter. Due to the experimental difficulties expected when measurements are done for the gas-liquid system, an option to use a solid model of the Taylor bubble was adopted. A new test section was designed to provide all the required characteristics for studying the aforementioned aspects. The details of these features and also, the experimental procedures and data processing are presented in Chapter 3. The Chapter 4

includes experimental results obtained and discussions intended to explain the data according to the basic principles of fluid mechanics. Further analysis and information related to the skin friction force and drag coefficient variation with Reynolds number are presented and discussed in Chapter 5. A comparison between the real gas-liquid slug flow and the actual case studied - a solid-liquid system - and also, some new aspects of the phenomena preceding the bubble coalescence are included there. Conclusions and recommendations for further evidence supporting the postulated mechanism are finally given in Chapter 6 followed by the Appendices which provide more detailed data and information regarding certain aspects of this study.

2 LITERATURE SURVEY

This chapter briefly describes the main theoretical and experimental approaches on slug flow published previously as well as some of the aspects related to the dynamics of the Taylor bubble coalescence phenomena.

2.1 Description of Slug Flow

Slug flow is a two-phase flow characterized by large, bullet-shaped bubbles called gas slugs or Taylor bubbles, which bridge the cross-sectional area of a pipe. They rise with almost a constant velocity, U_{TB} , independent of their length, L_{TB} , as long as the bubble length equals or is greater than three tube radii [Zukoski, 1965]. The gas expansion phenomena may be neglected for short pipes. Also, no gas slugs at separation distances less than the minimum liquid slug length can precede the trailing bubble, because of coalescence phenomena.

Fig.2.1. Schematic of a Slug Unit

As the density and viscosity of a gas are much smaller than those of a liquid, it is assumed that the pressure difference between the nose and the tail of the Taylor bubble is small enough, such that we can consider the inner bubble surface as being isobaric. Experimental data confirm this [Akagawa and Sakaguchi, 1966].

The liquid ahead of the rising Taylor bubble is displaced, such that a thin falling liquid film is formed along the bubble under gravity force. The liquid film flows downward past the tail of the Taylor bubble and penetrates into the liquid slug. Due to a sudden increase in the static pressure, flow separation at the bubble tail occurs and also, due to penetration of the liquid film into the wake region, mixing vortices form. The liquid film velocity is length dependent as it is the result of a balance between gravity and friction. As the void fraction increases, the Taylor bubble becomes longer and the liquid film along the bubble accelerates in accordance with the continuity equation, entraining at the front of the liquid slug many small bubbles. In the absence of small bubbles, the flow is called ideal slug flow [Orell and Rembrandt, 1986].

When the separation distance between two successive Taylor bubbles is large enough, all bubbles have smoothly rounded heads and rise with a uniform velocity. The rate of adding liquid inflow from the falling liquid film ahead of the liquid slug equals the rate of liquid removal at the tail of the same liquid slug, thus the liquid slug length remains constant. In this case, the flow is termed stable slug flow. If the distance between two consecutive Taylor bubbles is less than a critical value, the trailing bubble will have a distorted nose, as it is traveling in the wake of the leading one [Nicklin, 1962, Moissis and Griffith, 1962]. Thus, it would accelerate and finally coalesce with the leading Taylor bubble. This is characteristic of the unstable slug flow.

2.2 Rise Velocity of Taylor Bubble

The rise velocity of Taylor bubbles is one of the important parameters in calculating some of the main characteristics of the slug flow, such as void fraction distribution and pressure drop.

A bubble rises through a liquid with a velocity which is the result of the forces acting on it: its own buoyancy and those due to the liquid inertia, viscosity and surface tension. The rise velocity of the Taylor bubble depends on the pipe diameter and inclination angle, the physical properties of gas and liquid phases (density, viscosity and surface tension), and the flow rates of the two phases.

First, the influence of the liquid properties on the bubble rise velocity will be discussed. White and Beardmore [1962] have established, using dimensional analysis the main dimensionless groups which govern the motion of the bubbles in pipes: Eotvos

$$\left(\text{Eo} = \frac{\rho g D^2}{\sigma} \right), \text{Froude} \left(\text{Fr} = \frac{U_{TB}}{\sqrt{gD}} \right) \text{ and Morton} \left(\text{Mo} = \frac{g \mu^4}{\rho \sigma^3} \right) \text{ numbers, if gas viscosity}$$

and density are negligible compared to those of the liquid phase. They have plotted the Morton number versus Eo, with Fr as a parameter for different systems from their own data and also from those available in the literature, assigning regions in which the effect of some variables becomes unimportant. Depending on which and how many parameters control the propagation of the bubbles, one can have different regimes:

1. For inertia-controlled regime, the theoretical solution for a bubble rising in a stagnant column, may be found by assuming potential flow. Consequently, a general equation for the bubble rise velocity is given in the form:

$$U_{0,TB} = k \sqrt{g \cdot D} \quad (2.2.1)$$

where D is the tube diameter and k is a constant, actually the dimensionless velocity, i.e. the Froude number. By retaining three terms in the series expansion of the potential flow around a prescribed spherical front, it was found [Dumitrescu,1943] that $k=0.351$. Later, White and Beardmore [1962], Nicklin et al. [1962], and Zukoski [1966] confirmed this result experimentally. Also, the numerical simulation performed by Mao and Dukler [1991] led to the same results. Most of the practical cases, for which the property group,

$$N_r = \frac{\rho g^{\frac{1}{2}} D^{\frac{3}{2}}}{\mu}, \text{ is greater than 300 and } \text{Eo} > 100, \text{ fall in this region.}$$

The same type of analysis can be applied for the case in which the surface tension is not negligible and $N_r \rightarrow \infty$. Bendiksen [1985] has found that surface tension

monotonically reduces the rise velocity and proposed a correlation for the coefficient k in the rise velocity equation, which is valid for most practical cases.

Nickens and Yanitell [1987] used potential flow theory allowing also for surface tension through Kelvin-Laplace equation obtaining an expression for the Fr number as a function of Eo number, with a proportionality constant of 0.361 for large Eo numbers (negligible surface tension) and also a correct trend for lower values of Eo number as observed experimentally by Wallis [1962] and White and Beardmore [1962]. Also they have calculated the shape of the bubble nose by imposing a zero stream function on the surface of the bubble. Their conclusions were that as the surface tension increases, the nose becomes blunted which is related to a decrease in the rise velocity.

Harmathy [1960] studied the effect of surface tension on the rise velocity of different shape bubbles, including cylindrical ones in confined and unconfined media. But, he neglected the bubble viscosity effects which may have two main contributions: at small Eo numbers, modification in the circulation intensity inside the bubble which influences the shear stress at the gas bubble-liquid interface and at large Eo numbers, damping of the shape oscillations of the bubble, such that the drag force on the bubble is diminished.

Harmathy and Bretherton [1961] have assigned a limiting value for Eo number of 3.37, at which a bubble does not move in the column as the hydrostatic forces are, at least, completely balanced by surface forces.

3. The viscous regime is encountered when N_r is less than 2 and $Eo > 100$. It is characteristic of highly viscous fluids. The proposed correlation for the dimensionless bubble velocity, k , is:

$$k = 0.01 \cdot N_r \quad (2.2.2)$$

[White&Beardmore, 1962; Wallis,1969, p.287].

Reinelt [1987] has also solved equations for a Taylor bubble in creeping flow through a stagnant column.

Nickens and Yanitell [1987] considered small viscosity effects by replacing the actual tube diameter with an effective diameter which accounts for the thin boundary layer at the tube wall and then solving the potential flow problem.

4. For the general case when all the parameters are important, one can use either the graphical correlation of White and Beardmore or the equation proposed by Wallis:

$$k=0.345\left(1-e^{-0.01N_{fr}^{0.145}}\right)\left(1-e^{-(1.17-2.6k)}\right) \quad (2.2.3)$$

where m depends on N_{fr} . This equation can be reduced to the aforementioned simple forms for the inertial and viscous regimes. It also gives a good approximation for the intermediate range of N_{fr} , where the surface tension effects are negligible.

It is worthwhile to mention that for various regimes the bubble shape is quite different. In a highly viscous fluid, both the nose and the tail are rounded while for low viscosity the bubble tail is flat. There is no bubble shape factor in the previous analysis, as its shape is dependent on the groups already given.

For a bubble moving in a flowing liquid column, at steady state, the rise velocity is given by the sum between the bubble velocity in the fluid at rest and the contribution due to the liquid flow which is dependent upon the liquid mean velocity and the velocity profile:

$$U_{st} = U_{0,TB} + \Gamma \cdot U_m \quad (2.2.4)$$

$$U_m = U_{GS} + U_{LS} \quad (2.2.5)$$

where the Γ - factor depends on the mean liquid velocity, U_m - the mixture velocity given by the sum of gas and liquid superficial velocities [Nicklin, 1962]. For $Re > 8000$, Γ is 1.2 which is close to the ratio between the maximum to the average liquid velocity for fully developed turbulent flow. This means that the bubble motion is likely to be assisted by the liquid velocity at the center line.

If the Taylor bubble travels behind another bubble, its motion was found to depend exponentially on the separation distance, L , between the two bubbles,

$$U = U_{st} \left[1 + C \cdot \exp\left(\frac{-\beta \cdot L}{D}\right) \right] \quad (2.2.6)$$

where D is the column diameter, C and β are parameters which are determined experimentally [Moissis and Griffith, 1962; Clift et al., 1974; Hasanein et al., 1996] and apparently depend on the liquid properties. The data obtained by Hasanein et al. regarding the influence of the leading slug length showed systematically large

differences, a fact that might indicate that one single parameter for the undeveloped flow region is not enough.

For the inclined pipes, due to the transverse component of the buoyancy, the Taylor bubbles are distorted and axially off-centered thus the resistance to motion is diminished. The velocity of the bubbles increases in comparison with the vertical case, for inclination angles in the range of 0-45 degrees from the vertical axis, approximately [Zukoski, 1966], after which the velocity will decrease (for 45-90 degrees), probably due to the increased influence of surface tension and/or viscosity. Still, the interdependence between the angle of inclination and fluid's physical properties is not well understood.

Regarding the rise velocity dependence on the Taylor bubble length, Nicklin [1962] has done a very thorough study about how the pressure above and below the bubble might affect this parameter, concluding that the only increase in rise velocity with length is due to bubble expansion effects and there is no influence related to the bubble wake. A correction factor for bubble expansion in open systems can be applied, such that the rise velocity becomes:

$$U_{TB} = \left(1 - \rho g \frac{L_{TB}}{P}\right) U_{TB,c} \quad (2.2.7)$$

where P is the absolute pressure at the measuring station.

2.3 Effect of Nose Shape on Hydrodynamics

Most of the visual observations of Taylor bubbles in an unsteady slug flow regime in vertical tubes or in a steady regime in inclined pipes have a common feature: the bubble nose deformation which seems to be responsible in part for the bubble acceleration in the unsteady flow case and for the increased and constant rise velocity for the steady flow case.

Zukoski [1966] was among the first to notice that the shape of the nose plays an important role in the dynamics of the bubble. He studied experimentally the influence

of surface tension on the bubble rise velocity for tubes of different inclination angles. For horizontal (0°) and 45° degrees, there was a continuous increase in the rise velocity with decreasing surface tension, a fact attributed to the dramatic and continuous change of the radial curvature at the bubble vertex, while for the vertical case (90°) the change occurred for a certain range of surface tension, below which a constant plateau was observed due to the constant radius of the bubble vertex. Another remark made was that the bubble geometry depended on the level of turbulence upstream and the flow disturbances, the net effect being usually an increase in the rise velocity, due to fluctuations in liquid velocities as large as or larger than the rise velocity of the bubble.

Dumitrescu [1943] derived theoretically the Taylor bubble shape for an air-water system. He divided the shape profile into two regions, the nose region and the film region:

$$\begin{aligned} x^+ &= 0.75 \left[1 - \sqrt{1 - 1.778 r^{+2}} \right] & \text{when } x^+ \leq 0.5 \\ x^+ &= \frac{0.123}{(1 - r^+)^2} & \text{when } x^+ > 0.5 \end{aligned} \quad (2.3.1)$$

where x^+ is the normalized axial distance from the pipe center line as follows:

$$\begin{aligned} x^+ &= x/R \\ r^+ &= 1 - r/R \end{aligned} \quad (2.3.2)$$

The predicted shape agrees well with the experimental data [Mao and Dukler, 1991], even for an air-kerosene system. In contrast, the interfacial configuration for a trailing Taylor bubble in a dual Taylor bubble system was quite asymmetric and continuously changing [DeJesus, 1997]. Apparently, the liquid velocity profile ahead of the Taylor bubble nose significantly affected the shape, and thus the rise velocity of the trailing Taylor bubble. Furthermore, as the separation distance between two rising Taylor bubbles decreased, the shape of the trailing bubble became increasingly asymmetric. Previously, Collins et al. [1978] related the translational velocity of a Taylor bubble in developed slug flow to the value of the liquid velocity at the center-line, where the velocity attains its maximum. Also, Shemer and Barnea [1986] have observed random movements in the bubble nose

from one side of the pipe to another, in accordance with the recorded velocity profiles at the appropriate distance ahead of the bubble nose. Consequently, they correlated the rise velocity of a Taylor bubble with the maximum instantaneous liquid velocity upstream.

The deformation of the bubble nose in the undeveloped slug flow was recorded by Kagawa et al.[1982] in a vertical pipe, as presented in fig.5.5.2, in Section 5.5. It is clear that even in the far wake region of the leading bubble, the trailing Taylor bubble nose is displaced from the central vertical tube axis, on which it usually lies in developed slug flow configuration.

2.4 Hydrodynamics of Falling Liquid Film

The liquid film hydrodynamics is very complex and few details are available in the literature due to the difficulty in performing such detailed measurements. The velocity profile in the liquid film, the wall shear stress and the location of liquid film separation at the tail of the bubble are important to know for modelling slug flow and also, for a better understanding of the coalescence phenomena.

The forces involved in liquid film flow are: gravity, wall and interfacial shear stresses and inertia. Dumitrescu assumed the flow as being inviscid [1943], but his solution did not match the experimental data for film thickness obtained by Taylor and Davies [1950]. These results proved the divergence of the actual behavior from the potential flow theory. Brown [1965] proposed a model which retained the concept of potential flow in the bulk of the liquid film, but allowed for the effect of liquid viscosity, immediately near the wall. However, he supposed that the film is in equilibrium state and the flow is laminar. Fernandes et al. [1983] used the same idea but considered a turbulent liquid film to model the vertical slug flow. In their model, many constitutive equations were required to close the system of equations.

As most of the models considered fully developed slug flow, the inertia term in a momentum balance could be neglected. However, in order to calculate the film velocity one has to use appropriate relations for the interfacial stresses. The wall friction

factor is usually determined from single phase flow correlations [Fernandes et al., 1983, among others]. Assuming a fully developed velocity profile in the falling film and a constant thickness of the liquid film, Issa and Tang [1990] have determined the film velocity from a force balance and geometrical arguments, taking into account the friction at the pipe wall. The interfacial gas-liquid shear stress can hardly be predicted accurately. Considering that the pressure inside the Taylor bubble is constant, in most cases the interfacial gas-liquid shear has been neglected [Fernandes et al., 1983, Orell and Rembrand, 1986]. Still, the available literature offers some empirical equations for interfacial friction factor for flat interfaces.

The film thickness was found to reach an equilibrium value after a certain distance below the nose in Nicklin et al.'s experiments, in contrast with other results [Mao and Dukler, 1989] which indicated that usually, even for long bubbles, a constant film thickness is not attained. Also, Fabre and Line [1992] emphasized that the mean film thickness is underestimated if a fully-developed flow is assumed. This effect is amplified more for short bubbles.

The change in the flow direction encountered at the nose and the flow reversal at the tail of the Taylor bubble were revealed using wall shear stress probes. Mao and Dukler [1991] concluded that the change in flow direction at the nose of the bubble occurred a short distance below the nose vertex. Nakoryakov et al. [1986] used the wall shear stress probe to study the primary alteration of the liquid flow direction in the falling film around a Taylor bubble.

Recently, DeJesus et al. [1996] have obtained detailed qualitative and quantitative information on the flow field along Taylor bubbles using the Photochromic Dye Activation technique for different bubble length. Liquid film velocity profiles for 60 mm and 115 mm length Taylor bubbles were similar. The liquid at the bubble interface moves faster than the liquid near the wall, a fact that proves a negligible shear force exists at the gas-liquid interface. The boundary layer developed at the pipe wall near the bubble tail, reached approximately 53% of the film thickness for the 60 mm bubble and about 85-90% for the 115 mm length bubble. These results indicate that the falling film flow is still undeveloped for both cases. An analytical prediction of falling film velocity using

different correlations and comparison to experimental data indicated that only the Bernoulli equation can describe the flow within a short distance (up to 3 cm) below the nose. The other correlations having the assumption of developed flow (i.e. constant film thickness) gave predictions not sufficiently close to the experimental profiles. Also, the use of the PDA technique allowed determination of how the film penetrating into the liquid slug dissipated. The film penetrated farthest for the shortest Taylor bubble and was relatively constant for Taylor bubbles greater than or equal to 60 mm in length. The film penetration distance ranged from 1 to 1.5 tube diameters, and the values were nearly the same for a solitary Taylor bubble and a trailing bubble.

2.5 Wake Region

When penetrating into the front of the liquid slug, the liquid film undergoes flow reversal due to the adverse pressure gradient and, as a result, vortices and intense mixing characterize this region. The size and the pattern of flow in the wake depend on the liquid properties and on the tube diameter.

In real slug flow, small gas bubbles entrained from the tail of the Taylor bubble experience numerous coalescence and collision phenomena, thus enhancing the turbulence level. Thus, due to the complexity of the flow and the limitations in the available measurement techniques, the wake region has been studied only qualitatively.

Moissis and Griffith [1962] have measured the velocity profiles in the wake of a solid Taylor bubble placed in a downward flowing column using a total head probe. Despite the dynamic differences due to the stationary pipe walls with respect to the solid bubble and the no-slip boundary conditions for the solid Taylor bubble which does not represent the gas surface, the velocity profiles obtained showed the characteristics of a wake as expected.

Nakoryakov and Kashynski [1986] used an electrochemical method to determine the local void fraction and liquid velocity profile and also, fluctuating

components of the liquid velocity behind Taylor bubbles in upward gas-liquid slug flow. They described the region immediately after the gas slug as being characterized by a low value of the wall shear stress - the toroidal eddy region beyond the gas slug tail.

As previously mentioned, Shemer and Barnea [1989] measured velocity profiles in the wake of a leading Taylor bubble using a hydrogen bubble technique and related the relative rise velocity of the trailing bubble to these profiles.

Analyzing photographs of the slug wake developed in a 2 inch I.D. tube, Campos and Carvalho [1988] classified the wake as axisymmetric for lower Reynolds numbers ($Re < 180$) and oscillatory for higher values. They have measured the dye concentration in the wake of different length bubbles by injecting a gas slug to travel in a column (I.D.=19mm) half full with liquid containing a tracer dye, half with pure liquid, and observed that the flow in the wake dominated the process of mixing. Nonetheless, the extent of mixing in their case was similar for all bubble lengths studied.

Unlike the aforementioned techniques, a nonintrusive PDA method has been used by DeJesus et al. [1996] to collect information regarding the microstructure of the flow field in the near and far wake of a Taylor bubble, in both the near-wall and far-wall regions. When entering the suddenly enlarged region behind the tail of the Taylor bubble, the liquid film retains a large axial velocity at least one pipe diameter below the tail. For bubbles of different length (60 mm and 115 mm) the flow pattern and magnitude of the flow were described as follows: a counterclockwise vortex occurring directly below the tail, spreading of the falling liquid film before losing its identity, and the presence of small vortices beyond 20 mm from the Taylor bubble tail. Accordingly, the component of radial velocity becomes significantly higher compared to the other Taylor bubble region. The far-wake region was characterized by the presence of multiple small vortices visible up to four pipe diameters behind the Taylor bubble tail [DeJesus, 1997].

2.6 Minimum Stable Slug Length

The stable length of the liquid slug is important to know for modelling purposes. Experiments suggest that this length does not depend on gas and liquid flow rates or fluid properties. The minimum stable slug length is a concept introduced for the first time by Dukler and Hubbard [1975], thought to be equal to the distance necessary for a fully developed velocity profile to be established in the liquid slug.

This length has been predicted by two similar approaches. Taitel et al. [1980] suggested that this concept is related to the distance necessary to re-establish a fully developed velocity profile at the back region of the liquid slug after the boundary layer has been destroyed in the mixing region. They simulated the processes of destruction and re-establishment of the boundary layer using a jet entering a large reservoir. They found a 5% decay in the jet velocity over 16 pipe diameters, which represents the minimum stable slug length. One of the model restrictions is the fact that viscous effects near the solid wall are overlooked. On the other hand, Dukler et al. [1985] used the concepts of boundary layer mixing and recovery in the liquid slug region. Their model considered that the liquid film entering the slug is entrained in strong mixing vortices at the front, thus the boundary layer is destroyed. In the next region, the boundary layer starts to rebuild at the wall, until a fully developed velocity profile is achieved. In this approach, the maximum velocity increases with the distance from the tail of the Taylor bubble, contrary to experimental observations. This might be due to the differences in the dynamics of the liquid-solid wall compared to the gas-liquid interface which changes the velocity profile shape: symmetric for flow between walls and asymmetrically curved in the other case. The predicted mixing length was 3-5 pipe diameters.

By relating the rise velocity of a Taylor bubble to the maximum liquid velocity in front of it, and also considering the fact that the maximum velocity decreases with the distance from the leading bubble tail due to the decay of the velocity fluctuations, Shemer and Barnea [1986] reduced the prediction of the minimum stable

slug length to an estimate of the extent of the wake region behind the bubble. They obtained a stable slug length of 20 pipe diameters for the vertical slug flow.

3 EXPERIMENTAL

An important component of this study was the design and construction of a small test-section which offers the possibility of studying the effect of displacement of a solid Taylor bubble from the tube axis on drag force as well as making modifications such that reasonable symmetrical drag force data could be obtained. The following sections contain detailed descriptions of the experimental apparatus and procedures.

3.1 Test Section

The test section design had to fulfill the following requirements:

1. A highly precise transversing mechanism for the bullet-shaped solid Taylor bubble;
2. A strong guide rod and smoothly moving solid bubble system such that the vibrations due to the liquid flow be minimized;
3. Good sealing such that no leakage occurs.

A sketch of the main block of test-section is presented in fig.3.1.1 and the whole assembly, with component parts identified by numbers, in fig.3.1.2. Figures 3.1.3 to 3.1.9 include detailed drawings of these constitutive elements of the big block, with the corresponding numbers. All dimensions are given in inches.

The main body made of lucite, contains three pieces:

-the central piece, no.1 in fig.3.1.1 has four threaded holes (no.2), shown in the same figure. The upper holes are used to install the micrometer heads with acrylic seats (pieces no.4 and no.6), respectively. The micrometer heads are installed in the opposite ends to allow forward and backward movements. Each of them contacts a horizontal steel rod (pieces no.10 and no.11, respectively) threaded into a brass cross (no.7). The lower holes in fig.3.1.1 are made to allow the insertion of a second horizontal steel rod (no.14) through the same brass cross (no.7) providing rigidity to the assembly and preventing vibrations due to flow inside the tube. This rod is inserted and sealed using a lucite seat (no.12). The bottom of the cross can slide along the aforementioned steel rod when the

micrometer heads (no.5) are used. In this way, the bubble (no.13), sliding on a vertical steel rod (no.9) threaded into the top of the cross, has a precisely determined lateral position at any time;

Fig.3.1.1. Main Block of the Lucite Test-Section

Fig. 3.1.2. Scheme of Lucite Test- Section with Main Components

Fig.3.1.3. Lucite Micrometer Head Seat - Right-Hand Side

Piece No. 4

Fig.3.1.4. Horizontal Steel Rod - Right-Hand Side

Piece No.11

Fig.3.1.5. Lucite Micrometer Head Seat - Left-Hand Side

Piece No. 6

Fig.3.1.6. Horizontal Steel Rod - Left-Hand Side

Piece No.10

Fig.3.1.7. Lucite Seat for Horizontal Rod

Piece No. 12

Fig. 3.1.8. Vertical Guide Rod - Piece no.9

Fig.3.1.9. Brass Cross - Piece No.7

-the top and bottom parts (no.3) form the flanges for the central part, (no.1), and are machined to increase the surface contact area such that leaks are prevented. For the same reason O-ring seals have been used (see fig.3.1.1 for location).

This cylindrical piece has been installed on a 3.03 m high, 2.56 cm I.D. Pyrex tube, at about two meters from the liquid inlet at the top, such that the flow is completely developed in the plastic bubble region. The penetration depth for the tube inside the main body was long enough to provide good alignment of the three pieces.

It is worthwhile to mention that special care was exercised in the positioning - parallel or perpendicular-of the component elements with respect to each other. Still, a gap between the bubble and the tube walls was extremely small, approximately 1.2 mm when the bubble was centered. Also, the insertion of the lucite block required cutting the tube, thus a joint of certain flexibility appeared. But this joint led to a problem where perfect bubble alignment with respect to the tube wall was difficult to obtain.

Both normal and deformed nose Taylor bubbles were modeled and used in the present experiments because it is known that when a Taylor bubble travels in the wake of another bubble the nose becomes distorted and the bubble swings laterally as it rises upward. The normal nose bubble was shaped as a spherical head connected to a cylindrical body of the same diameter, D_{TB} , 23.3mm. The bubble diameter has been chosen such that a liquid film thickness of about 1.2 mm would be obtained between the bubble and column walls, which matches the experimentally measured values for the gas-liquid system in the same tube [DeJesus, 1997]. The bubble consisted of two parts - head and body- which can be threaded together. The latter had a variable length such that two different length bubbles could be studied, 7.5 and 15 cm, respectively. A sketch of this bubble is shown in fig.3.1.10-a.

Even if an exact shape of a deformed bubble could be visualized and drawn, it was impossible to machine. So, an approximate shape of the deformed bubble close enough to the real case has been used, as shown in fig.3.1.10 - b. The lower piece of the bubble had a “layered cake” structure such that it would fit nicely into the upper piece and create a flush side surface. Also the surfaces of the plastic bubbles were thoroughly polished to reduce the friction drag as much as possible.

3.2 Test Loop for Drag Force Measurements

As shown in fig.3.2.1, a plastic Taylor bubble was suspended from a load-cell and placed in a downward flowing liquid stream in a vertical tube of 25.6 mm inner diameter. Using the micrometer head system the bubble was transversely moved from the tube center towards the left and the right walls to measure any change in the drag force. Kerosene was symmetrically fed into a liquid inlet tank of 20.3 cm diameter and 30.5 cm height, open to the atmosphere, installed atop the vertical tube, such that the fluctuations in flow due to the centrifugal pump would be damped. At the liquid entrance in the vertical pipe, a cylindrical honeycomb piece was installed inside the pipe, to assure a uniform flow inside the tube and to break up the characteristic vortices which might form at the entrance. After flowing over the plastic Taylor bubble and through the outlet section the kerosene was passed through a rotameter, preceded by a globe valve which controlled the flow rates inside the vertical tube. Then, the liquid was drained into a collector tank, provided with a cold water cooling system and connected to the kerosene storage drum from where it was re-circulated through the test loop.

The whole test section was aligned vertically by using a plumb with an accuracy of less than about 2 mm over the 3 m length, with a smaller error for the direction in which the micrometers were transversed. The load-cell was installed on a perfectly horizontal plastic board placed above the liquid inlet tank.

The sensor - bubble alignment was important as off-centered loads might lead to erroneous responses, but in this case the small bubble displacement (at the most 1.2 mm) was basically insignificant for more than 2.3 m distance between the bubble and sensor.

The tension-compression load-cell ELJ-0.5N used for drag force measurements has a sensitivity of 115.8 mV/FS with a 10 V excitation and has built in a thermocompensation module which makes it usable for a range of temperatures (0-60)^oC. A 9V DC battery was used as a power supply for the sensor, which means that except for the slightly reduced sensitivity (104.23 mV/FS), all other specifications have remained the same. The analog output of the load-cell was converted to a digital signal through a

data acquisition board DAS-8 (calibrated in advance) and an EXP16 multiplexer board with a gain of 50 necessary to increase the signal level up to the input range for DAS-8. LABTECH NOTEBOOK was the software utilized to collect data with a sampling frequency of 300 Hz in a personal computer with a 80486-66 CPU. The time-averaged value for the drag force was obtained from the data collected for a period of 30 s.

Fig.3.2.1. Test Loop for Drag Force Measurements

3.3 Experiments

Two types of experiments have been conducted for the conditions described below:

- single bubble experiments that consisted of measurements of the drag force due to the liquid flowing over the suspended plastic bubble, for different lateral bubble displacement values. The same procedure was followed to obtain data for six different

flow rates and two different bubble lengths: 7.5 cm and 15 cm, and, also, for two different nose bubble shapes: normal and deformed.

- two-bubble experiments for which a system of leading and trailing bubbles has been used. Two types of runs have been performed: first, the drag force on the trailing bubble has been recorded for different bubble separation distances, in the range 0-9D, with the bubble located at the central position, and, second, the drag force profiles as a function of lateral bubble displacement have been obtained for three different bubble separation distances: 3D, 5D and 7D.

Table 3.2.1 shows the test conditions including the liquid flow rate, corresponding liquid superficial velocities and liquid Reynolds numbers based on the tube diameter.

Table 3.2.1. Summary of Experimental Flow Rates and Corresponding Parameter Values

Liquid flow rate (l/min)	4.593	5.064	6.124	7.066	8.244	9.068
Liquid superficial velocity (cm/s)	14.872	16.397	19.83	22.88	26.69	29.36
Liquid Reynolds No.	2203	2429	2937	3389	3954	4349

The range of the liquid flow rates was selected to cover the experimental bubble rise velocities obtained for an air-kerosene system: 17-18 cm/s for one Taylor bubble rising in a stagnant liquid column and in the range of 18-24 cm/s for a trailing bubble traveling in the wake of another bubble. The flow rate range tested was larger than the limits mentioned for the actual bubble rise velocity.

3.4 Data Reduction

The drag force values were obtained in mV. A FORTRAN program was written and used to calculate the time-averaged force, expressed in N.

Further procedures regarding data reduction and calculations are described in Chapter 4.

4 RESULTS AND DISCUSSIONS

The experimental work was performed to study the influence on the drag force of the radial position and length of the bubble, the shape of the nose and the leading bubble wake on the trailing bubble. These will be discussed in the subsequent sections of this chapter.

4.1 Effect of Radial Displacement of Bubble

For six different liquid flow rates, the drag force was measured on a 15 cm long plastic bubble for incremental displacement of the bubble from the tube axis. The results, presented in figures 4.1.1 to 4.1.6 clearly show a decrease in the measured drag force with the bubble eccentricity. The dimensionless eccentricity is obtained by dividing the displacement of the bubble axis from the tube axis by the difference between the bubble and pipe radii.

In all figures the variation in the drag force is characterized by a gradual and significant decrease with eccentricity. When the bubble is placed off-centered, the flow in the small gap region is subjected to greater viscous friction effects, such that more mass of liquid will be directed toward the larger gap region. So, there should be an increase in the liquid flow rate for a constant axial pressure gradient, or in the case of a constant flow rate, a decrease in the axial pressure drop, see Appendix 2. Also, comparing the centered and off-centered bubbles, the wake volumes would be different. It is known that for a sphere placed in a free stream, as the Reynolds number increases, the separation point on the surface of the sphere moves toward the rear, the wake width decreases and the sphere experiences less form drag. A similar explanation can be used in the present case. The separation point is fixed at the edge of the plastic bubble tail and when the bubble is off-centered the size of the wake becomes smaller, and at the same time loses its symmetry, as depicted in fig.4.1.7.

Fig.4.1.1 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 6 in Length Bubble (Liquid Reynolds No.=2203)

Fig.4.1.2 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 7.5 cm Length Bubble (Liquid Reynolds No.=2428)

Fig.4.1.3 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 15 cm Length Bubble (Liquid Reynolds No.=2937)

Fig.4.1.4 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 15 cm Length Bubble (Liquid Reynolds No.=3389)

Fig.4.1.5 Drag Force and Coefficient Variation with Radial Displacement for a Normal Nose, 15 cm Length Bubble (Liquid Reynolds No.=3954)

Fig.4.1.6 Drag Force and Coefficient Variation with Radial Displacement for a Normal Nose, 15 cm Length Bubble (Liquid Reynolds No.=4349)

Fig.4.1.7. Different Wake Sizes for Centered and Off-Centered Taylor Bubbles

According to the data, the total drag force variation from the center toward the wall does not seem to change much with the liquid Reynolds number, even though an increase in the intensity of mixing in the wake is expected for the highest Re number used, i.e. for $Re=4349$. This might be explained by the fact that just when the flow transition is taking place in the annular space (at Re number $\cong 3800$, equivalent film $Re_f \cong 2000$), the flow pattern in the wake might change.

The data show reasonably good symmetry, namely a Gaussian-type distribution with a peak at or near the pipe axis (zero eccentricity). The slight off-centering of the peak force from the central position is probably due to the difficulties in aligning the bubble axis perfectly vertical and parallel with the tube wall. As a consequence, positioning the tail of the bubble at the center may not always mean that the nose of the bubble was also centered. Due to the non-systematic asymmetry in the drag force profiles, a general trend of the maximum drag force location could not be detected.

Another cause for asymmetry may be the imperfections in the symmetry of the flow field. To support this, measurements were also performed by moving the bubble laterally at different azimuthal angles ($0, \pm 60, 90$ degrees) and different profiles of the

drag force, although more or less symmetric, have been obtained. The two causes mentioned might also explain the different values obtained in the same run for the extreme positions of the bubble: left and right. Still, from figure 4.1.8 which depicts the maximum and minimum values as well as the drag force recorded at zero eccentricity, it can be seen that for most of the cases the maximum value was obtained almost at the center.

For some of the figures, the drag force values at extreme eccentricities (1 and -1) have been discarded if they were much higher than the values near the wall because when the bubble was pushed toward the wall, extra tension was built in the wire and the real value of the drag force could not be detected.

For a 15 cm length bubble, measurement of drag force variation with bubble lateral displacement was repeated under the same flow conditions to study the reproducibility of the results. The highest difference in the measured values was approximately 10%. However, the magnitude of reduction in the drag force for the total radial displacement of the bubble remained almost the same.

Fig.4.1.8 Extreme Values of the Drag Force for a Centered and Off-centered 15 cm Length, Normal Nose Bubble

4.2. Influence of Bubble Length

The same type of experiments as in the previous section have been performed for the 7.5 cm long bubble for the same range of flow rates. In figures 4.2.1 to 4.2.6, it can be seen that all of the previous observations related to the drag force variation as a function of the bubble's radial displacement are again applicable here. The next two plots, figs. 4.2.7 and 4.2.8 show the distribution of the drag force over the whole range of eccentricity, for both 7.5 and 15 cm length bubbles, for the minimum and maximum flow rates, respectively.

As expected, the longer bubble experiences greater drag at the same liquid flow rate, due to the increased skin friction and form drag. Also, at the same flow rate the drag variation for the longer bubble is greater than that for the shorter bubbles. The increase in the form drag might be explained from momentum balance arguments. For the larger bubble, the velocity profile in the liquid film entering the wake with a suddenly enlarged flow area below the tail, is more developed, so due to the greater mixing in the wake, the drag will increase. Of course, one can argue that if the velocity profiles are fully developed for both bubbles, the form drag should be the same no matter the length (after the skin friction influence has been removed). Thus, it is possible that the velocity profiles are not fully developed, at least for the shorter bubble, although the hydrodynamic entry length calculations reveal a smaller value than the length of the shortest bubble, as one can see from Table A5.1, Appendix 5. Additional experiments with even longer bubbles might reveal if this is true.

Still, the literature related to the flow over cylinders and other oblong bodies reports an increase in the drag coefficient and drag force with the aspect ratio, L_B/D_B . For example for two cylinders with aspect ratios of 6 and 3, respectively, (as in actual experiment) placed in an axial flow in a pipe, the ratio of the drag coefficients is 1.18 [Idelchik, 1994]. Although the flow over the actual plastic bubbles is characterized by smaller values of the drag coefficients, an attempt to estimate the ratio of the drag forces experienced by the two bubbles leads us to a result very close to that of the cylinders, as shown below:

Table 4.2.1. Ratios of Form Drag Forces for Two Different Length Bubbles at
Different Liquid Film Reynolds Numbers

Re_f	1153.3	1271.6	1537.7	1774.3	2070.1	2277.0
$(F_D)_{15cm}/(F_D)_{7.5cm}$	1.188	1.141	1.288	1.463	1.488	1.334
$(F_{fD})_{15cm}/(F_{fD})_{7.5cm}$	1.154	1.106	1.260	1.443	1.469	1.312

where F_D is the form drag when the skin friction subtracted from the total drag was calculated assuming the flow fully-developed,

F_{fD} is the form drag when the skin friction subtracted from the total drag was estimated also for the entrance region.

Fig.4.2.1 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 7.5 cm Length Bubble (Liquid Reynolds No=2203)

Fig.4.2.2 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 7.5 cm Length Bubble (Liquid Reynolds No.=2429)

Fig.4.2.3 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 7.5 cm Length Bubble (Liquid Reynolds No.=2937)

Fig.4.2.4 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 7.5 cm Length Bubble (Liquid Reynolds No.=3389)

Fig.4.2.5 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 7.5 cm Length Bubble (Liquid Reynolds No.=3954)

Fig.4.2.6 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 7.5 cm Length Bubble (Liquid Reynolds No.=4349)

Fig.4.2.7 Drag Force Comparison between 7.5 cm and 15 cm Length Bubbles at Liquid Flow Rate 4.593 l/min

Fig.4.2.8 Drag Force Comparison between 7.5 cm and 15 cm Length Bubbles at Liquid Flow Rate 9.068 l/min

4.3 Influence of the Bubble Nose Shape

Experiments identical to those described in the previous sections have been performed with 7.5 cm and 15 cm long bubbles with a deformed nose shape. Figure 4.3.1 presents the deformed nose of a Taylor bubble seen from two different, perpendicular angles. Cross section A shows the bubble deformation placed in the plane A-A' which is, at the same time the plane in which the bubble is displaced from its central location towards the tube walls. Cross section B shows the same bubble in plane B-B', which is perpendicular to plane A-A'. Note that the deformation cannot be seen from this angle.

Fig.4.3.1 Deformed Bubble Orientation

Figures 4.3.2 to 4.3.7 show the drag force variations with lateral displacement for a 7.5 cm length bubble with a deformed nose, at three different flow rates, and comparison of the drag forces for the normal and deformed nose shapes at the same flow rate. It is obvious that the deformed bubble experiences a much smaller drag in all cases. The reduction seen ranges from slightly less than 20% to about 30%. With increasing liquid Reynolds number, this reduction from normal nose value seems to decrease. It is clear that the reduction in the total drag must be due to the significant decrease in pressure losses. For the short Taylor bubbles it is known that when their nose becomes distorted and prolonged, they accelerate.

Fig.4.3.2 Drag Force and Coefficient Variations with Radial Displacement for a Deformed Nose, 7.5 cm Length Bubble (Liquid Reynolds No.=2937)

Fig.4.3.3 Drag Force Variation with radial Displacement for a Normal and Deformed Nose, 7.5 cm Length Bubble

Fig.4.3.4 Drag Force and Coefficient Variations with Radial Displacement for a Deformed Nose, 7.5 cm Length Bubble (Liquid Reynolds No.=3389)

Fig.4.3.5 Drag Force Variation with Bubble Radial Displacement for a Normal and Deformed Nose, 7.5 cm Length Bubble

Fig.4.3.6 Drag Force and Coefficient Variations with Radial Displacement for a Deformed Nose, 7.5 cm Length Bubble (Liquid Reynolds No.=3954)

Fig.4.3.7 Drag Force Variation with Bubble Radial Displacement for a Normal and Deformed Nose, 7.5 cm Length Bubble

Figures 4.3.8 to 4.3.15 present similar variations in drag force for the 15 cm length bubble, and comparisons between the normal and deformed nose. For most of the figures the bubble position was as in fig.4.3.1- a, that means with deformation placed in the bubble displacement plane. Consequently asymmetric profiles have been obtained. An exception is fig.4.3.8, in which the profiles are symmetric due to the bubble distortion in the plane perpendicular to the plane in which bubble has been moved, as shown in fig.4.3.1 - b. As a result, in fig. 4.3.9, there is basically no difference in the profiles of the measured drag force between the normal and distorted nose bubbles and strangely enough, neither in their values. Disregarding inherent off-centering of the bubble, most of the data are characterized by asymmetry in the drag force profiles with respect to the pipe axis, and furthermore, the skewness of asymmetry seems to reverse at higher Re numbers.

The convergent flow areas located at the deformed bubble nose are similar to a convergent nozzle, with curvilinear walls. For convergent nozzles with rectilinear boundaries for example, it is known that the resistance coefficient depends on the convergence angle and the ratio of the extreme cross-sectional areas of flow. The larger the angle and the smaller the ratio, the greater the resistance of the convergent nozzle. When the convergent nozzle has also curvilinear walls, the pressure losses become smaller.

At higher Reynolds numbers, according to figs.4.3.12 and 4.3.14 the drag force decreases less sharply when the bubble is moved from the tube axis toward the left than the right (bubble positioned with the deformed nose toward left, as shown in fig.4.3.1 - a). This is due to the fact that when the bubble is moved toward left, the ratio between the cross-sectional areas $3-3'$ and $1-1'$, is increasing. Thus, the pressure losses decrease in this highly convergent area, while the reverse is true when the bubble is pushed in the opposite direction. In fig.4.3.13, the difference in symmetry between the profiles is obvious: symmetric for the normal nose and asymmetric for the deformed one. Still, the difference in drag forces between the two different nose shapes - about 8% for fig 4.3.13 - is not large, since deviations of about 10% were obtained sometimes for the same normal nose bubble, under the same flow conditions. As an example compare fig.4.3.13 with fig.4.3.16.

Fig.4.3.8 Drag Force and Coefficient Variations with Radial Displacement for a Deformed Nose, 15 cm Length Bubble (Liquid Reynolds No.=2203)

Fig.4.3.9 Drag Force Variation with Bubble Radial Displacement for a Normal and Deformed Nose, 15 cm Length Bubble

Fig.4.3.10 Drag Force and Coefficient Variations with Radial Displacement for a Deformed Nose, 15 cm Length Bubble (Liquid Reynolds No.=2937)

Fig.4.3.11. Drag Force Variation with Bubble Radial Displacement for a Normal and Deformed Nose, 15 cm Length Bubble

Fig.4.3.12 Drag Force Variation with Radial Displacement for a Deformed Nose, 15 cm Length Bubble (Liquid Reynolds No.=3389)

Fig.4.3.13 Drag Force Variation with Radial Displacement for Normal and Deformed Nose, 15 cm Length Bubble

Fig.4.3.14 Drag Force and Coefficient Variations with Radial Displacement for a Deformed Nose, 15 cm Length Bubble (Liquid Reynolds No.=4349)

Fig.4.3.15 Drag Force Variation with Radial Displacement for Normal and Deformed Nose, 15 cm Length Bubbles

Fig.4.3.16 Drag Force Variation with Radial Displacement for a Normal and Deformed Nose, 15 cm Length Bubble

At smaller Re numbers, the drag force variation is opposite to that previously described. An explanation for this is difficult to find. But, it is possible that overlooked mechanisms become important at smaller Reynolds numbers. Characteristics for this situation are shown in fig.4.3.10 in which the bubble was positioned as in fig.4.3.17. Although most of the graphs for the 15 cm bubble do not present a huge difference, this is not the case with fig.4.3.15 at liquid Re number of 4349.

On the other hand, if one allows just for the effect of bubble misalignment for the previous graphs, the flattening at high Re numbers of the drag force profile in the proximity of $e=0$ might be simply attributed to the transition phenomenon from laminar to turbulent flow during which the velocity profile changes from a parabolic to a more flat pattern.

Fig.4.3.17 Streamlined Flow over a Plastic Taylor Bubble
for two Complementary Eccentricities

However, it is clear that the deformed nose bubbles experience a lower drag force than the normal nose bubbles, especially in the case of 7.5 cm length bubbles. For the 15 cm bubbles, the reduction is less significant. When comparing the short and the long bubbles, the differences in the velocity profiles and pressure fields at the nose are better “preserved” for the shorter bubble, while for the longer one, they are “lost”. For the deformed nose bubbles, changes in velocity profiles in the annular region between the nose and tail are more significant for the 15 cm length bubble than the 7.5 cm length bubble. This may be the reason why the drag force differences between the deformed and normal nose bubbles are less for 15 cm compared to 7.5 cm length bubbles. Thus, the longer the bubble, the less important the shape of the nose becomes at least at lower values of the Re number, which is not true for the gas-liquid slug flow. This might be one of the shortcomings of the solid model constructed. Clearly enough, more experiments with longer bubbles and some flow visualization in the liquid film along the side of the bubble would bring more information related to the flow structure to verify the previous speculations.

4.4 Influence of Leading Bubble on Trailing Bubble

The measurements were made with a 15 cm leading bubble placed upstream, for five flow rates, at nine different separation distances for both the leading and trailing bubbles located at the pipe axis. The drag force on the 15 cm long trailing bubble is plotted in figure 4.4.1. It can be seen that at small separation distances, i.e. 0 to approximately $2D$, the drag force increases continuously till a peak value is reached, and then gradually decreases or remains relatively constant with the increasing separation distance. The lower drag on the trailing bubble is due, of course, to the low pressure field in the wake of the leading bubble, as it has been reported for flow over different shaped bodies placed in an array. As the pressure recovers partially, the drag force increases until the wake influence is not 'sensed' any more by the trailing bubble.

The same type of experiment has been repeated for three of the five flow rates. The drag force data for the trailing bubble, presented in fig.4.4.1.b, show fairly good agreement in the wake region, i.e. $0 < x < 2D$ for all flow rates. In the far wake region, for 4.593 l/min, the results are almost identical, while they are off by about 14% for 6.124 l/min and about 8% for 8.244 l/min. Nonetheless, they clearly follow the same trend: a sharp increase with the separation distance, followed by a nearly constant plateau.

Further investigation of the drag force variation with the bubble eccentricity, at different locations of the trailing bubble with respect to the leading one, has been performed. Results are shown in figs. 4.4.2 to 4.4.10. It is clear that these data display the same characteristics as the single bubble data shown earlier: a significant decrease when the bubble is laterally displaced. The drag force value for the central bubble position in these figures and the maximum value obtained in the single bubble experiment, at the corresponding flow rates (see figs. 4.1.1, 4.1.3 and 4.1.6) give a fairly good agreement, as one can observe from fig. 4.4.11. The fact that all the trailing bubble drag force values are higher than those for the single bubble, no matter the separation distance, proves that the wake influence disappears when the separation distance is greater than $3D$. At the wall, a comparison of the minimum drag force values leads to fig.4.4.12.

Fig.4.4.1.a Variation of the Drag Force with the Separation Distance
Trailing Bubble Length = 15 cm, Leading Bubble Length = 15 cm)

Fig.4.4.1.b Variation of the Drag Force with the Separation Distance

Fig.4.4.2 Drag Force Variation with Lateral Displacement for a Normal Nose, 15 cm Long Trailing Bubble at $x=3D$ from the 15 cm Leading Bubble

Fig.4.4.3 Drag Force Variation with Lateral Displacement for a Normal Nose, 15 cm Long Trailing Bubble at $x=5D$ from the 15 cm Leading Bubble

Fig.4.4.4 Drag Force Variation with Lateral Displacement for a Normal Nose, 15 cm Long Trailing Bubble at $x=7D$ from the 15 cm Leading Bubble

Fig.4.4.5 Drag Force Variation with Lateral Displacement for a Normal Nose, 15 cm Long Trailing Bubble at $x=3D$ from the 15 cm Leading Bubble

Fig.4.4.6 Drag Force Variation with Lateral Displacement for a Normal Nose, 15 cm Long Trailing Bubble at $x=5D$ from the 15 cm Leading Bubble

Fig.4.4.7 Drag Force Variation with Lateral Displacement for a Normal Nose, 15 cm Long Trailing Bubble at $x=7D$ from the 15 cm Leading Bubble

Fig.4.4.8 Drag Force Variation with Lateral Displacement for a Normal Nose, 15 cm Long Trailing Bubble at $x=3D$ from the 15 cm Leading Bubble

Fig.4.4.9 Drag Force Variation with Lateral Displacement for a Normal Nose, 15 cm Long Trailing Bubble at $x=5D$ from the 15 cm Leading Bubble

Fig.4.4.10 Drag Force Variation with Lateral Displacement for a Normal Nose, 15 cm Long Trailing Bubble at $x=7D$ from the 15 cm Leading Bubble

Fig.4.4.11 Maximum Drag for Single and Trailing Bubbles at Different Separation Distances at Different Flow Rates

Fig.4.4.12 Minimum Drag for Single and Trailing Bubbles at Different Separation Distances at Different Flow Rates

Here, one could expect higher values for the two bubble system, when the trailing bubble is placed at the maximum eccentricity because the wire suspending the trailing bubble is passing axially through the leading bubble to the load-cell. Thus, at small separation distances, the leading bubble may lean upon the wire, exerting an extra force. Still the minimum drag force on the trailing bubble is close to the minimum drag force obtained in the single bubble configuration.

There are a few other observations to be made regarding the very first graph, Fig.4.4.1-a: the increase in the drag force over the wake region is higher for the higher Reynolds number, which can be related to the increased intensity of mixing in the wake. Due to the increased pressure field in the wake region, a boundary layer separates at the tail of the solid bubble and the liquid film penetrates at the rear. With an increase in Reynolds number, the side boundaries of the wake become more unstable such that the adjacent flow layers undergo more intensive turbulent agitation. Also, the higher the flow rate, the bigger the wake volume. For example, at the lowest flow rate, 4.593 l/min, the maximum drag force value is approximately at $x=D$, while for the highest, 8.244 l/min,

the maximum drag is around $x=3D$. This is probably related to the changes in the wake size and flow pattern. Unfortunately there are no data regarding the flow over oblong bodies in pipes (with a ratio of bubble diameter to pipe diameter of the current case), such that values for the critical Reynolds number and wake characteristics can be known. However, for the dual gas bubble system, experimental data obtained from flow visualization work using a PDA technique show a drastic change in the magnitude of the velocity field two pipe diameters below the leading bubble tail, while nine diameters below the tail, the effect becomes totally negligible [DeJesus, 1997]. A certain similarity between the actual and real data was almost unexpected, considering that even if the geometrical similitude is satisfied and Reynolds numbers are the same for the two systems, the boundary conditions are different, so the liquid film hydrodynamics should not be the same.

A comparison between the observed drag force invariance outside the wake region and the fact that for the gas bubble system, the acceleration of the trailing bubble starts far downstream of the leading one, might lead to the supposition that this is possible due to the off-centering, thus decreasing drag of the trailing bubble. Further experiments should be designed for the gas-liquid system to certify more thoroughly this hypothesis.

5 ANALYSIS OF DRAG FORCE

5.1 Total Drag Coefficient

From drag force measurements, a total drag coefficient, c_D , has been calculated, using equation 5.1.1:

$$c_D = 2 \cdot \frac{f_D}{\rho \cdot u^2 \cdot A_{\text{cross}}} \quad (5.1.1)$$

where f_D is the total drag force (N), ρ is the liquid density (kg/m^3), u is the mean liquid velocity and A_{cross} is the bubble cross-sectional area.

For the 7.5 cm length bubble the drag coefficient variation with liquid Reynolds number, for different eccentricities is plotted in fig. 5.1.1, and for the 15 cm length bubble, in fig.5.1.2. In both cases, there is an expected decrease in this parameter with the increasing Reynolds number, as the previous literature for flow over immersed bodies indicates [Schlichting, 1979]. Regarding the eccentricity influence, it is obvious that the drag coefficient is at a maximum when the bubble is placed at the center, i.e. $e=0$. For non-zero eccentricities, of equal absolute value, the data should lie on the same side and equally distanced from the c_D curve for $e=0$. The slight deviation is due to the small misalignment or the non-uniform flow field as it has been already discussed in Section 4.1.

It can be observed that at a certain value of Re number, there is an increase in the drag coefficient, particularly for the 15 cm length bubble, which can be related to the flow transition phenomenon. It is known that transition from laminar to turbulent starts with sinusoidal oscillations which become stronger with an increase in the Re number. This is true for flow in pipes but also in annuli. Prengle and Rothfus [1954] were among the first to study the flow in different sized annuli by injecting dye in the stream. They found that at a film Reynolds number, $Re_f=1200$, the first disturbance eddy is cast off and the progression to full turbulence is characterized by more frequent formation of disturbance eddies. Bird et al.[1960] stated also a critical $Re_f \leq 2000$ for an annulus. Usually, for flow over immersed bodies, transition is associated with a shift in the

separation zone along the surface of the body. Thus, a strong decrease in the drag coefficient takes place. For the actual experimental data, there seems to be a critical Reynolds number, $Re=3400$ ($Re_f=1800$). However, the separation point of the boundary layer is located at the same point, at the tail of the plastic bubble. Still, the data obtained for the 15 cm length bubble in run 1 and presented in Appendix 1, indicate the same critical Re number value.

A new correlation for the drag coefficient as a function of Re number for flow over bullet shaped, 7.5 cm solid bubbles in pipes can be proposed, by using a curve fitting function in SigmaPlot software. For all calculated values, the normalized standard error was less than 4%. This equation has the form of the Colebrook correlation, namely:

$$\frac{1}{\sqrt{c_D}} = a + \ln(Re_f \cdot \sqrt{c_D}) \quad (5.1.2)$$

where Re_f is liquid film Reynolds number, a is a constant weakly dependent on the bubble eccentricity, as one can observe from Table 5.1.1 below. The average value for coefficient a is 10.386, and the corresponding c_D versus Re curve is represented with a thick dashed line in fig.5.1.1. On the other hand, a similar correlation for the 15 cm bubble could not be established due to greater oscillations of the c_D value.

Table 5.1.1. Coefficient a Dependence on Bubble Eccentricity
(7.5 cm Length)

Bubble Eccentricity	Coefficient a
0.006	10.392
0.294	10.402
0.582	10.400
0.870	10.386
0.942	10.389
-0.714	10.368
-0.930	10.367

For a gas Taylor bubble, the drag coefficient, c_D , should be independent of the bubble length, unlike in the case of a solid Taylor bubble. If the total drag force is due mainly to the form drag, then it may be possible to estimate the c_D value for gas Taylor bubbles from the present data. To do this, the contribution of the friction drag must be evaluated for the present experiments, as discussed in the next section.

Fig 5.1.1 Drag Coefficient Variation with Liquid Reynolds No.
for Different Bubble Eccentricities
(Normal Nose, 7.5 cm Length Bubble)

Fig.5.1.2 Drag Coefficient Variation with Liquid Reynolds No.
for Different Bubbles Eccentricities
(Normal Nose, 15 cm Length Bubble)

5.2 Skin Friction Estimation

The liquid flow in the annular space between the solid Taylor bubble and pipe wall is similar to the flow through eccentric annular ducts. For the latter, with $0 \leq e \leq 1$ and $r^* \rightarrow 1$ and fully developed flow, Tiedt propose the following equation to calculate the Fanning friction factor [Kakac, 1989],

$$f \cdot Re_f = \frac{24}{1 + 1.5 \cdot e^2} \quad (5.2.1)$$

where Re_f is the hydraulic diameter Reynolds number: $Re_f = \frac{\rho v_f D_h}{\mu}$, and v_f is the liquid film velocity. On the solid bubble wall, when the liquid enters the annular region, a boundary layer starts to form, such that along the bubble one has a hydrodynamically developing region, followed by a fully developed one. For the first zone, there are some data available for the friction factor from numerical analysis [Feldman, 1982], but not for the geometrical characteristics of the present case. So, the assumption of fully-developed flow was made. Results are presented in Appendix 3. All the calculated skin friction forces are much smaller than the measured experimental data for the total drag force, which means that the form drag is the most important variable in this system.

Due to the viscous flow in the annular gap, one can also relate the present flow to the lubrication mechanics, characterized by a high ratio of load to friction. For the situation in discussion, skin friction contribution to the force balance has been estimated to be about 5%, thus almost negligible. The procedure involved the calculation of a velocity profile in the gap region, assuming flow in this region as a flow between flat plates and a force balance for the solid bubble. This is explicitly shown in Appendix 4.

Graphically, the negligible effect of skin friction compared to the form drag is presented in figs.5.2.1, 5.2.2 and figs.5.2.3 , 5.2.4, for the 7.5 cm and the 15 cm long bubbles, respectively. The total drag and form drag have been plotted, while the skin friction, calculated with equation 5.2.1, is the difference between the two curves.

Fig.5.2.1 Total drag and Form Drag Force vs. Bubble Eccentricity (4.593 l/min, Normal Nose, 7.5 cm Length Bubble)

Fig.5.2.2 Total Drag and Form Drag vs. Bubble Eccentricity (9.068 l/min, Normal Nose, 7.5 cm Length Bubble)

Fig.5.2.3 Total Drag and Form Drag vs. Bubble Eccentricity
(4.593 l/min, Normal Nose, 15 cm Bubble)

Fig.5.2.4 Total Drag and Form Drag Variation vs. Bubble Eccentricity
(9.068 l/min, Normal Nose, 6 cm Length Bubble)

In the developing flow region along the bubble, the skin friction should be higher than the values calculated, especially as the Re number is increasing. It is estimated that an increase of almost 3-4% will occur in the calculated skin friction component for the highest Re number used, as shown in Tables A5.1 and A5.2, in Appendix 5. The aforementioned percentage might be higher than the actual value due to the estimation made for a ratio of bubble and pipe diameters, $r^*=0.5$, instead of the actual case $r^*=0.91$. However, the dependence of f_{app} on r^* is less strong compared to the dependence of f_{app} on bubble eccentricity, e , as discussed in Appendix 5. Even the latter does not cause significant variations in the skin friction when the entrance region is also considered. Thus, it can be stated that the form drag has the predominant effect. To explain better the results for form drag, more information should be obtained related to the bubble wake characteristics.

Finally, a graph representing the contribution of skin friction to the measured drag force for the 15 cm length bubble as a function of liquid Re number and bubble eccentricity has been plotted in fig.5.2.5. The fact that the contribution of the skin friction to the total drag is decreasing with Re number even though the friction force is actually increasing with Re, is a proof that the rate of increase for the form drag is higher than that for the skin friction. The maximum value in this graph is 10%. Even if the effect of the entrance region is added, the skin friction contribution remains less significant than that of the form drag.

Fig.5.2.5 Skin Friction Variation with Bubble Eccentricity and Liquid Reynolds Number (15 cm Length)

5.3 Error Analysis

One of the problems that often occurred during the experimental work was the measurement of different values of the drag force for the same bubble position, at the same flow rate, but at different time intervals. As it has already been mentioned in Section 4.1, one possible cause could have been the changes in the flow structure. Another possibility is the drifting of the load-cell readings. With respect to the drift, at the beginning of the experimental work, the pump was stopped every five minutes and the load-cell reading for buoyancy force was recorded. Every time, the reading was the same, which suggests that the drift cannot be a serious source of error. One remote possibility, which is hard to prove though, is the influence of the electromagnetic field from other labs. This might explain the fact that good reproducibility of a certain run was sometimes obtained, while at other times, the whole range of values was shifted, although the relative variation in the drag force measured remained the same. Still, no

relative variation in the drag force measured remained the same. Still, no more than 10% variation in drag force has occurred for the same experimental arrangement.

Concerning the vertical alignment of the tube and the bubble, the extreme values of the drag force obtained are presented in fig.5.3.1. For run2, better alignment of the column and the bubble was achieved, but contrary to expectations the results were lower than for the previous setting.

Another possible source of error in the drag force measurement was the liquid level fluctuations in the top tank during a given run. To check this, separate experiments were performed at a flow rate of 5.064 l/min, for the central position of the bubble, at different liquid levels. The variation in readings was no more than 2.7% for a total level change of 5 cm. To further diminish the error, all the experiments were done with the liquid level varying no more than 2 cm, such that the error was limited to less than 1%. Regarding the pressure exerted by the liquid column on the bubble, its influence on the readings has been eliminated by subtracting the buoyancy force measured in the tube filled with stagnant kerosene.

Another important source of error was the variation in the kerosene temperature after a period of time, due to the energy dissipation as heat. The cooling system used assured a relatively constant temperature during a given run, when temperature variation of 0.2°C occurred, which was considered to have little effect on the recorded drag force as long as the symmetry of the drag force profiles was preserved. As some experiments were done in spring or summer and during day or night, the absolute temperature of kerosene among different runs has varied over the range of 20°C-25°C, due to limitations in the control of the cooling water temperature.

The load-cell sensor used was calibrated a few times with standard weights for both tension-compression load. In the linear regression analysis, basically the same value of the correlation coefficient, $r^2=0.9997$ was found. The batteries used for a power supply for the load cell were frequently changed, such that the sensor would have the same sensitivity.

The fluctuations in the pump outlet have been reduced due to flow rate control at the exit of the column with a globe valve. Still, extremely small fluctuations of the rotameter float could not be totally eliminated.

Fig.5.3.1 Extreme Values for the Total Drag Force for Two Different Runs
15 cm Length, Normal Nose Bubble

5.4 Comparison between Solid and Gas Bubbles in Slug Flow

As previously mentioned, due to the difficulties in measuring the velocity profiles in the wake of a gas bubble in order to estimate the drag force acting on it and to prove the assumption of drag reduction with bubble radial displacement from the tube axis, a solid Taylor bubble was placed inside a 2.54 cm I.D. tube in which kerosene flowed downwards. Obviously, a boundary layer developed along the solid bubble wall, unlike in the gas bubble-liquid interface case characterized by the absence of shear stress.

In gas-liquid slug flow, the characteristics of the flow field (velocity profile and turbulence eddies) in the nose region are important because the gas bubbles having a

free interface “respond” to any pressure fluctuations. In the solid bubble case, the velocity profiles just ahead of the plastic bubble nose have no effect on the shape of the boundary and are thought to be less important given the solid wall of the bubble. Actually, for the real slug flow, the bubble nose shape is one of the factors influencing the rise velocity.

There is also a slight difference in the shape of the bubbles: the solid bubble had a cylindrical shape and the annulus has a constant cross section while the real Taylor bubble has a gradually divergent shape and convergent annulus cross-sectional area. Since the average radial velocity component for the gas bubble is extremely small compared to the axial one and zero for the solid bubble, it is clear that the difference in the bubble shape will not lead to different flow characteristics.

The hydrodynamics of the liquid film are, however, different: for the gas bubble, one has a free falling liquid film, because of the constant pressure inside the bubble, while for the solid bubble, the liquid film flow is controlled by the solid boundaries, thus by the length of the bubble at least for the entrance region.

Despite the differences noted, Moissis and Griffith [1960] performed some preliminary experiments in order to prove that having a solid bubble wall instead of a free surface is an insignificant fact. Allowing an air bubble to be attached to the flat surface of the plastic bubble, they found that the velocity profile behind the air bubble remained identical to the profile behind the plastic one. Even though this might not seem true, it is possible that the differences between the boundary conditions at the gas-liquid and solid-liquid interfaces for the velocity profile in the liquid film entering the bubble wake do not cause major modifications of the wake. Comparing the wake size in the present experiments with that of a Taylor gas bubble, the results were basically the same, two pipe diameters.

Flow visualization in the wake of different length gas bubbles has indicated existence of a counterclockwise vortex immediately below the tail of the bubble and also the presence of small scale eddies further below [Kawaji et al., 1996]. In the present work simple visual observations of the solid bubble wake where small diameter spherical bubbles were used as tracers, indicated similar flow patterns as for a real Taylor bubble. This is not surprising considering that in both cases a large adverse pressure gradient

causes the separation of the liquid film at the tail of the bubble. Also, for both the gas and solid Taylor bubbles, the downward flow of the falling film entering the liquid slug whose core is moving upwards, leads to the appearance of a strong shear layer at the two stream interface. On the other hand, a clear statement regarding the magnitude of the flow in the wake region for the two situations is difficult to make.

Further experiments providing information about the microstructure of the flow field in both near-wake and far-wake regions of the plastic bubble should be done so that a better comparison with the gas bubble wake becomes possible

5.5 Why do the Taylor Bubbles Coalesce?

For many years the process of Taylor bubble coalescence occurring in slug flow has been studied and recently more information related to the hydrodynamics of Taylor bubbles in single and both leading and trailing configurations has been obtained. Nonetheless, the question of what causes the Taylor bubbles to accelerate and coalesce when the separation distance is smaller than a critical value remains still to be answered.

Previous studies have shown that the initial acceleration of the Taylor bubble is not due to the liquid film penetrating into the liquid core as far as the trailing bubble, if the liquid slug is longer than about two pipe diameters. Another explanation supported by the present study relies on a decrease in the drag force experienced by the Taylor bubble when its axis is displaced from the tube axis. This is based on previous experimental observations of the pseudo-random lateral motion of the Taylor bubble as it sways over to one side or the other inside the tube. Also, even though different from the actual case, the wall effects on the hydrodynamics of the dispersed phase in semiconfined media are known, such as the changes in external flow patterns, in drag force and as a consequence in the terminal velocity, and bubble elongation [Cheremisinoff and Gupta, 1983].

The present study was aimed at making a step forward in answering the aforementioned question. Undoubtedly, further experimental work has to be performed to understand the mechanisms regarding the phenomena leading to bubble coalescence. The following sequence is based partially on the experimental observations of Taylor bubbles

in slug flow, the results of this study and some presumptions that future experiments have to prove further.

Many authors have related the bubble nose deformation to the increase in the bubble rise velocity. Additionally, the liquid flow dynamics in the nose region is extremely important in the increase of speed of the trailing Taylor bubble relative to the lead bubble [Laird and Chisholm, 1956]. Shemer and Barnea [1987] indicated that the trailing Taylor bubble's nose shape resembled the distorted velocity profiles in the liquid obtained at a similar distance in the wake of a single Taylor bubble. There is no doubt [DeJesus,1997] that the wake of the leading bubble contains many large and small eddy structures. Their presence probably determines modifications of the shape of the bubble nose. Also, they might be considered the source of energy necessary to cause bubble displacement from the tube axis. When the bubble is displaced from the vertical axis of the tube, velocity gradients increase in the annular space and, according to the continuity equation the rise velocity of the Taylor bubble increases.

Consequent to bubble off-centering, the wake becomes asymmetric and the strong vortices present in the wake region below the wider area of the annular gap will induce a lower pressure field in this region. The tail of the bubble which is a free surface will "respond" by modifying its slope from configuration *a* to *b*, as shown in fig.5.5.1. The inclined surface of the Taylor bubble tail is clearly seen as well from experimental observations in figs.5.5.2 a and b photographed by Kagawa et al. [1983] for upward slug flow. The superficial gas and liquid velocities, j_g and respectively, j_l , are indicated below the figures. At the same time, the bubble starts to move toward the opposite side of the tube wall such that a new eccentric position is reached. As the bubble shifts from one side to another, the same sequence described above is repeated: the nose deformation, local increase in the velocity gradient, and tail inclination. The rise velocity increases in response to a decrease in the drag force as the bubble sways and approaches one side of the tube wall.

As the separation distance between the Taylor bubbles decreases, the eddies encountered by the trailing bubble become more energetic. Actually, Shemer and Barnea [1987] found that at distances of seven pipe diameters from the tail of the trailing Taylor

bubble only eddies with scales comparable to the pipe radius did survive, while at a distance of twelve pipe diameters a fully developed velocity profile was attained (where liquid slug Reynolds number indicated laminar flow). Thus, the nose of the trailing Taylor bubble deforms the closer it rises to the leading Taylor bubble. Also, there should be an increase in the frequency with which the bubble sways from one side to the other, contributing to the acceleration of the bubble as it moves upwards. Although further experiments have to be performed, coalescence of Taylor bubbles is postulated to be due to the changes in the nose shape and bubble position with respect to the vertical tube axis caused by the small and large scale eddy motion remaining in the liquid core.

Fig. 5.5.1. Different Tail Shapes of Taylor Bubbles

a

b

Fig.5.5.2 Shape of Taylor Bubbles in Upward Slug Flow

a. $j_l = 0.2 \text{ m/s}$

$j_g = 0.0728 \text{ m/s}$

b. $j_l = 0 \text{ m/s}$

$j_g = 0.0875 \text{ m/s}$

6 CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

An experimental test-section was designed and constructed to conduct investigations into the drag force variation for a single plastic Taylor bubble with the radial position and length of the bubble, the shape of the nose, and also, the drag force variation for a trailing bubble placed in the wake of a leading bubble with different bubble separation distances and liquid flow rates.

The radial displacement of the bubble axis from the tube axis caused a significant decrease in the measured drag force, for the entire range of flow rates. The shapes of the drag force profiles obtained for the 15 cm long bubble were symmetrical, with a peak at or near the pipe axis (zero eccentricity). Similar observations related to the drag force variation as a function of the bubble's lateral displacement were also applicable to the 7.5 cm length bubble. The only difference was the lower drag force values for the shorter length bubble, measured under the same flow conditions.

The drag forces measured for plastic Taylor bubbles having a deformed nose shape were compared with the values obtained for bubbles with normal nose, and shown to be significantly smaller, especially for the shorter bubbles, by 20-30%. For the 15 cm long bubbles, the drag forces for the deformed bubble were never greater than those for the normal bubbles, but the difference was also not clear. In addition, the shape of the drag force variation with bubble lateral displacement with respect to the orientation of the deformed nose was different at high Reynolds numbers compared to lower values of Re .

Regarding the influence of a leading bubble on a trailing bubble, both 15 cm long, it was clearly seen that at small separation distances, up to $2-3D$, the drag force increases continuously till a peak value is reached, and then remains basically the same with the increasing separation distance. Further experiments to measure the variation of drag force on a trailing bubble with lateral displacement, at different separation distances, $3D$, $5D$ and $7D$, showed almost the same values as in single bubble measurements. This

proved that the wake influence is limited and the bubble radial displacement is responsible for bubble acceleration.

From the drag force measurements at different flow rates, total drag coefficient variation with liquid Reynolds number was plotted for 7.5 cm and 15 cm long bubbles. For the shorter bubbles, a correlation for the drag coefficient-liquid Re number relationship was established. This is applicable for the liquid Re range from 2200 up to 4400.

An estimation of the skin friction contribution to the total drag of the plastic bubble in fully developed flow conditions, indicated approximately 10%, while the rest is due to the form drag. Even higher values for the skin friction when allowance is made for the entry region, still leave the form drag contribution significantly higher. This means that the liquid flow around the solid bubble is closer to that for the gas Taylor bubble. Thus, it is postulated that the gas Taylor bubble accelerates due to the decrease in bubble form drag when the bubble moves off-center from the tube axis and bubble nose deformation occurs. This is possible due to the small and large scale eddy motion remaining in the liquid core in the wake of the leading bubble.

6.2 Recommendations

The results obtained in this experiments can be used for further calculations in order to quantify more precisely the similarity between the gas-liquid slug flow and the present solid-liquid flow. An extensive study and additional experiments should be designed based on the conclusions drawn in this work to explain the underlying phenomena regarding Taylor bubble acceleration and coalescence in slug flow and to develop better predictive models. Some of these suggestions are presented below:

1. One of the limitations in the test-section design was related to the bubble alignment with respect to the tube wall due to a certain degree of flexibility in the joint created when the cylindrical piece containing the transverse mechanism for the bubble was inserted into the glass tube. The problem can be solved using a one piece tube of a certain length with two diametrically opposite holes bored to allow installation of the

transverse mechanism. The tube can be attached with flanges to the original glass column.

2. To be able to obtain a more uniform flow field, changes are recommended in the liquid feeding system by placing the inlets symmetrically.
3. The actual experiments on the drag force variation with bubble displacement were conducted at a constant liquid flow rate to keep the axial pressure gradient constant. A variation in the axial pressure gradient will affect the falling film velocity, thus, the mixing intensity in the wake region. An experiment which allows for a variation in the flow rate while the bubble is off-centered, at a constant axial pressure gradient, might bring a different absolute value for the drag force, but for sure the same meaningful variation with the bubble eccentricity.
4. One of the uncertainties in these experiments was related to the length of the hydrodynamically developing region. Flow visualization using a Photochromic Dye Activation technique in the falling film region will provide accurate information regarding this matter. Also, the flow patterns in the wake region may be obtained by applying the same PDA technique and a comparison with the flow characteristics in the Taylor bubble wake is recommended to enhance the similarities between the gas-liquid and solid-liquid systems.
5. Another possibility to further support the hypothesis is to obtain a correlation for the trailing bubble drag force as a function of bubble eccentricity and obtain the frequency with which a bubble gets off-centered when traveling in the wake of a leading Taylor bubble. The latter parameter has to be experimentally determined for the real slug flow (see recommendation 6) and probably will depend on the bubble separation distance. Afterwards, using a force balance which includes added mass effects and considering the aforementioned correlation, a differential equation can be obtained for the rise velocity of the trailing Taylor bubble. Integrating over the entire separation distance and considering the initial bubble velocity equal to the rise velocity of a single bubble, one can get the Taylor bubble rise velocity right before the coalescence process takes place and compare it with the real values.

6. The main result of this study has established a decrease in the drag force when the Taylor bubble is radially displaced. To prove how often a bubble traveling in the wake of another bubble is radially displaced as it approaches the leading bubble, a video camera moving at the same speed as the trailing bubble and a time code generator will provide information regarding the falling film thickness variation in time on both sides of the bubble. Thus, one can obtain the frequency the Taylor bubble gets off-centered. Considering that the bubble trajectory might be helical, a second camera moving at the same time with the bubble should be used. The two cameras must be placed in perpendicular planes. The images obtained from different cameras can be correlated with the time, such that a three dimensional path of the trailing bubble as it approaches the leading one may be obtained.
7. Using one of the commercial codes available to calculate the velocity and pressure distributions around the solid Taylor bubble, calculation of the skin friction, form drag and thus the total drag of the bubble should be conducted. In this way, a comparison and verification of the experimental data are possible.

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APPENDICES

Appendix 1

More results from run1 for a 15 cm long bubble related to the drag force and drag coefficient variation with the lateral displacement and film liquid Reynolds number are presented below in figs.A1.1 to A1.5.

Also, this appendix contains the results obtained when the symmetry of the flow field was checked, i.e. the drag force variation versus bubble eccentricity at different orientations of the micrometer head: figs.A1.7, A1.8, A1.9 and A1.10. Another auxiliary information is related to the symmetry of the data (F_D vs. e) when non-symmetric feeding of the liquid into the tube was used - figs. A1.6 and A1.6*.

Fig.A1.1 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 15 cm Length Bubble (Liquid Reynolds No.=2429)

Fig.A1.2 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 15 cm Length Bubble (Liquid Reynolds No.=2937)

Fig.A1.3 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 15 cm Length Bubble (Liquid Reynolds No.=3389)

Fig.A1.4 Drag Force and Coefficient Variations with Radial Displacement for a Normal Nose, 15 cm Length Bubble (Liquid Reynolds No.=4349)

Fig.A1.5 Drag Coefficient Variation with Liquid Film Reynolds No. for different Bubble Eccentricities (Normal Nose, 15 cm Length Bubble)

Fig.A1.6 Drag Force Variation with Radial Displacement for a Normal Nose, 15 cm Length Bubble (nonsymmetrical feeding)

Fig.A1.6* Drag Force Variation with Radial Displacement for a Normal Nose, 15 cm Length Bubble (nonsymmetrical feeding)

Fig.A 1.7 Drag Force Variation with Bubble Eccentricity
Azimuthal Angle=0°
Liquid Flow Rate=4.59l/min

Fig.A 1.8 Drag Force Variation with Bubble Eccentricity
Azimuthal Angle=60°
Liquid Flow Rate=4.59l/min

Fig.A 1.9 Drag Force Variation with Bubble Eccentricity
Azimuthal Angle=90⁰
Liquid Flow Rate=4.59l/min

Fig.A 1.10 Drag Force Variation with Bubble Eccentricity
Azimuthal Angle=120⁰
Liquid Flow Rate=4.59l/min

Appendix 2

The flow in an annular space can be reduced to the flow between parallel plates when the cylinder diameter ratio, D / D_B , is close to unity, as shown in Fig.A.2.1.

The Navier-Stokes equation for flow between two parallel plates in y-direction is:

$$\rho \cdot \left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) = \mu \cdot \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right) - \frac{\partial p}{\partial y} + \rho \cdot g_y \quad (1)$$

Fig.A2.1- Flow between Parallel Plates

Fig.A2.2- Eccentric Annulus Duct

$$x = e(D - D_B) / 2$$

The following assumptions are made:

- steady, fully-developed laminar flow;
- no flow in x and z directions, thus $v_x, v_z = 0$;
- y-velocity component does not vary along x and y directions, so $\frac{\partial v_y}{\partial x}, \frac{\partial v_y}{\partial y} = 0$ and

$$\frac{\partial^2 v_y}{\partial x^2}, \frac{\partial^2 v_y}{\partial y^2} = 0.$$

Then, a simpler form of equation (1) can be used.

$$\frac{\partial p}{\partial y} - \rho \cdot g_y = \mu \cdot \frac{\partial^2 v_y}{\partial z^2} \quad (2)$$

The following boundary conditions are used to calculate the integration constants,

$$\begin{aligned} z = 0 & \quad v_y = 0 \\ z = h & \quad v_y = 0 \end{aligned} \quad (3)$$

where h is the gap between the plates.

The velocity distribution for a liquid flow between two parallel plates is:

$$v_y = \frac{1}{2\mu} \left(\frac{\partial p}{\partial y} - \rho \cdot g_y \right) z^2 - h \cdot z \quad (4)$$

For an eccentric annulus, the gap h can be expressed as a function of the gap, h_0 , between the concentric inner and outer cylinders, and the coordinate ζ (see fig.A2.2):

$$h = h_0 \left[1 + e \cos \left(\frac{\zeta}{\frac{D_B}{2}} \right) \right] \quad (5)$$

According to this equation,

$$\text{for } e = 0, \quad h = h_0$$

$$\text{and for } e = 1, \quad h = 0 \quad \text{with } \zeta = \frac{\pi D_B}{2}$$

Integration over z and ζ will allow determination of the total flow rate as a function of bubble eccentricity:

$$Q = 2 \int_0^{\frac{\pi D_B}{2}} \int_0^h v_y \cdot dz \cdot d\zeta \quad (6)$$

Substituting equations (4) and (5) into (6), one obtains:

$$Q = \frac{\pi D_B}{12\mu} h_0^3 \left(\frac{\partial p}{\partial y} - \rho g_y \right) \left(1 + \frac{3}{2} e^2 \right) \quad (7)$$

At the same axial pressure gradient, the ratio between the flow rates at eccentricity $e \neq 0$ and $e = 0$ is given by:

$$\frac{Q_{e \neq 0}}{Q_{e=0}} = 1 + \frac{3}{2} e^2 \quad (8)$$

which clearly shows an increase in the flow rate with the bubble eccentricity. Or for constant flow rate imposed, the ratio of the axial pressure gradients at $e \neq 0$ and $e = 0$ is:

$$\frac{\left(\frac{\partial P}{\partial y}\right)_{e \neq 0}}{\left(\frac{\partial P}{\partial y}\right)_{e=0}} = \frac{1}{1 + \frac{3}{2} e^2} \quad (9)$$

where $P = p - \rho g_y$ is the total pressure.

It is interesting to compare this result with an equation proposed by Tiedt [1971] for the friction factor for a fully-developed flow through an eccentric annulus.

$$f \cdot Re = \frac{24}{1 + 1.5e^2}$$

This equation has been used to estimate the skin friction for the plastic bubbles in this work.

Appendix 3

The skin friction factor, f , for an annular space between the bubble and pipe wall has been estimated using the equation 5.2.1, in Section 5.2 assuming a fully-developed, laminar flow. The following tables contain the values of: $f Re_f$, f , bubble eccentricity- e , friction force for 7.5 cm and 15 cm length bubbles at different liquid film Reynolds numbers, Re_f .

Table A3.1. $Re_f = 1153.3$

$f Re_f$	Friction Factor f	Bubble Eccentricity e	Friction Force(N) $L=7.5$ cm	Friction Force (N) $L=15$ cm
9.600	0.008	1.000	0.013	0.026
10.835	0.009	0.900	0.015	0.030
12.245	0.011	0.800	0.017	0.033
13.833	0.012	0.700	0.019	0.038
15.584	0.014	0.600	0.021	0.043
17.455	0.015	0.500	0.024	0.048
19.355	0.017	0.400	0.026	0.053
21.145	0.018	0.300	0.029	0.058
22.642	0.020	0.200	0.031	0.062
23.645	0.021	0.100	0.032	0.065
24.000	0.021	0.000	0.033	0.066
23.645	0.021	-0.100	0.032	0.065
22.642	0.020	-0.200	0.031	0.062
21.145	0.018	-0.300	0.029	0.058
19.355	0.000	-0.400	0.026	0.053
17.455	0.015	-0.500	0.024	0.048
15.584	0.014	-0.600	0.021	0.043
13.833	0.012	-0.700	0.019	0.038

12.245	0.011	-0.800	0.017	0.033
10.835	0.009	-0.900	0.015	0.030
9.600	0.008	-1.000	0.013	0.026

Table A3.2. $Re_f = 1271.6$

fRe_f	Friction Factor f	Bubble Eccentricity e	Friction Force (N) L=7.5 cm	Friction Force (N) L=15 cm
9.600	0.008	1.000	0.014	0.029
10.835	0.009	0.900	0.016	0.033
12.245	0.010	0.800	0.018	0.037
13.833	0.011	0.700	0.021	0.042
15.584	0.012	0.600	0.023	0.047
17.455	0.014	0.500	0.026	0.053
19.355	0.015	0.400	0.029	0.058
21.145	0.017	0.300	0.032	0.064
22.642	0.018	0.200	0.034	0.068
23.645	0.019	0.100	0.036	0.071
24.000	0.019	0.000	0.036	0.072
23.645	0.019	-0.100	0.036	0.071
22.642	0.018	-0.200	0.034	0.068
21.145	0.017	-0.300	0.032	0.064
19.355	0.015	-0.400	0.029	0.058
17.455	0.014	-0.500	0.026	0.053
15.584	0.012	-0.600	0.023	0.047
13.833	0.011	-0.700	0.021	0.042
12.245	0.010	-0.800	0.018	0.037
10.835	0.009	-0.900	0.016	0.033
9.600	0.008	-1.000	0.014	0.029

Table A3.3. $Re_f = 1537.8$

fRe_f	Friction Factor f	Bubble Eccentricity e	Friction Force (N) $L=7.5$ cm	Friction Force (N) $L=15$ cm
9.600	0.006	1.000	0.017	0.035
10.835	0.007	0.900	0.020	0.039
12.245	0.008	0.800	0.022	0.045
13.833	0.009	0.700	0.025	0.050
15.584	0.010	0.600	0.028	0.057
17.455	0.011	0.500	0.032	0.064
19.355	0.013	0.400	0.035	0.071
21.145	0.014	0.300	0.039	0.077
22.642	0.015	0.200	0.041	0.082
23.645	0.015	0.100	0.043	0.086
24.000	0.016	0.000	0.044	0.087
23.645	0.015	-0.100	0.043	0.086
22.642	0.015	-0.200	0.041	0.082
21.145	0.014	-0.300	0.039	0.077
19.355	0.013	-0.400	0.035	0.071
17.455	0.011	-0.500	0.032	0.064
15.584	0.010	-0.600	0.028	0.057
13.833	0.009	-0.700	0.025	0.050
12.245	0.008	-0.800	0.022	0.045
10.835	0.007	-0.900	0.020	0.039
9.600	0.006	-1.000	0.017	0.035

Table A3.4. $Re_f = 1774.3$

fRe_f	Friction Factor f	Bubble Eccentricity e	Friction Force (N) L=7.5 cm	Friction Force (N) L=15 cm
9.600	0.005	1.000	0.020	0.040
10.835	0.006	0.900	0.023	0.046
12.245	0.007	0.800	0.026	0.051
13.833	0.008	0.700	0.029	0.058
15.584	0.009	0.600	0.033	0.066
17.455	0.010	0.500	0.037	0.073
19.355	0.011	0.400	0.041	0.081
21.145	0.012	0.300	0.044	0.089
22.642	0.013	0.200	0.048	0.095
23.645	0.013	0.100	0.050	0.099
24.000	0.014	0.000	0.050	0.101
23.645	0.013	-0.100	0.050	0.099
22.642	0.013	-0.200	0.048	0.095
21.145	0.012	-0.300	0.044	0.089
19.355	0.011	-0.400	0.041	0.081
17.455	0.010	-0.500	0.037	0.073
15.584	0.009	-0.600	0.033	0.066
13.833	0.008	-0.700	0.029	0.058
12.245	0.007	-0.800	0.026	0.051
10.835	0.006	-0.900	0.023	0.046
9.600	0.005	-1.000	0.020	0.040

Table A3.5. $Re_f = 2070$

fRe_f	Friction Factor f	Bubble Eccentricity e	Friction Force (N) L=7.5 cm	Friction Force (N) L=15 cm
9.600	0.005	1.000	0.024	0.047
10.835	0.005	0.900	0.027	0.053
12.245	0.006	0.800	0.030	0.060
13.833	0.007	0.700	0.034	0.068
15.584	0.008	0.600	0.038	0.076
17.455	0.008	0.500	0.043	0.086
19.355	0.009	0.400	0.047	0.095
21.145	0.010	0.300	0.052	0.104
22.642	0.011	0.200	0.056	0.111
23.645	0.011	0.100	0.058	0.116
24.000	0.012	0.000	0.059	0.118
23.645	0.011	-0.100	0.058	0.116
22.642	0.011	-0.200	0.056	0.111
21.145	0.010	-0.300	0.052	0.104
19.355	0.009	-0.400	0.047	0.095
17.455	0.008	-0.500	0.043	0.086
15.584	0.008	-0.600	0.038	0.076
13.833	0.007	-0.700	0.034	0.068
12.245	0.006	-0.800	0.030	0.060
10.835	0.005	-0.900	0.027	0.053
9.600	0.005	-1.000	0.024	0.047

Table A3.6. $Re_f = 2277$

fRe_f	Friction Factor f	Bubble Eccentricity e	Friction Force (N) L=7.5 cm	Friction Force (N) L=15 cm
9.600	0.004	1.000	0.026	0.052
10.835	0.005	0.900	0.029	0.058
12.245	0.005	0.800	0.033	0.066
13.833	0.006	0.700	0.037	0.075
15.584	0.007	0.600	0.042	0.084
17.455	0.008	0.500	0.047	0.094
19.355	0.009	0.400	0.052	0.104
21.145	0.009	0.300	0.057	0.114
22.642	0.010	0.200	0.061	0.122
23.645	0.010	0.100	0.064	0.128
24.000	0.011	0.000	0.065	0.129
23.645	0.010	-0.100	0.064	0.128
22.642	0.010	-0.200	0.061	0.122
21.145	0.009	-0.300	0.057	0.114
19.355	0.009	-0.400	0.052	0.104
17.455	0.008	-0.500	0.047	0.094
15.584	0.007	-0.600	0.042	0.084
13.833	0.006	-0.700	0.037	0.075
12.245	0.005	-0.800	0.033	0.066
10.835	0.005	-0.900	0.029	0.058
9.600	0.004	-1.000	0.026	0.052

Appendix 4

The velocity profile characteristic of a flow between two parallel plates is given by equation (4) from Appendix 2:

$$v_y = \frac{1}{2\mu} \left(\frac{\partial P}{\partial y} \right) (z^2 - h \cdot z)$$

The shear stress τ_{zy} varies across the gap according to:

$$\tau_{zy} = \left(-\frac{\partial P}{\partial y} \right) \left(z - \frac{h}{2} \right) \quad (1)$$

Considering the plastic Taylor bubble to be positioned at the center of the pipe, in a flowing liquid, one can make a force balance over the entire bubble,

$$\text{Weight - Buoyancy} = \text{Shear Stress Force} + \text{Pressure Force} \\ (\text{Skin Friction}) \quad (\text{Form Drag})$$

or, more explicitly,

$$\pi D_B^2 L (\rho_B - \rho) g = \tau_{zy} \pi D_B L + (p_0 - p_L) \pi D_B^2 \quad (2)$$

where L is the Taylor bubble length, ρ_B and ρ are the plastic bubble and liquid densities, respectively, and p_0 and p_L are the pressures at the nose and tail of the bubble, respectively.

Assuming a linear distribution of the pressure along the length of the bubble and using the shear stress along the bubble, evaluated from equation (1) with $z = h$, equation (2) becomes:

$$\frac{\Delta P}{L} = \frac{D_B (\rho_B - \rho) g}{\frac{h}{2} + D_B} \quad (3)$$

so that the form drag is given by,

$$F_{fD} = \Delta P \pi D_B^2 = \frac{\pi D_B^3 (\rho_B - \rho) g L}{\frac{h}{2} + D_B} \quad (4)$$

Equations (1), (2) and (4) were used to estimate the relative contributions of skin friction to the total drag force. The result is 4.65% skin friction, while the rest, 95.35% is due to the form drag.

Appendix 5

The boundary layer formed on the solid bubble surface, in the annular space between the bubble and pipe walls, has an entrance region characterized by a continuously developing velocity profile along the axial coordinate, y , and a fully developed region, with an invariant velocity profile across any section. Consequently, for the latter, the wall shear stress does not change axially, and the average friction factor, f , is the same as the local friction factor, f_y , while for the former region, the pressure drop results from the wall shear and the change in momentum flow rate across the two relevant annular sections. Due to difficulties in calculation, an apparent Fanning friction factor is used, f_{app} .

As the skin friction on the bubble has been calculated for fully developed conditions, an attempt to estimate the pressure drop in the hydrodynamic entrance length is presented below. Also, a comparison between the two situations will reveal how much the neglecting of the entrance effect influences the final results.

For eccentric annular ducts, Feldman et al. [1982] analyzed the hydrodynamically developing flow by using a model including continuity and momentum equations and assuming a maximum velocity locus on a circle, between the outer and inner pipes with a relation between the two transverse components of velocity. The processed data for the extreme cases of concentric annuli and circular pipes were in excellent agreement with the previous studies. From the cases presented in their paper, the closest of the actual situation ($r^* = 0.91$) was characterized by : $r^* = 0.5, e = 0.9$. First, the apparent friction coefficient for the entrance region as a function of the axial coordinate has been plotted, for the six values of liquid film Reynolds number from the experimental test-matrix, as shown in fig.A5.1. Using Sigma Plot's hyperbolic curve fitting in the form:

$$f_{app} Re_r = \frac{ay}{b + y} \quad (1)$$

the two coefficients **a** and **b** have been determined for every Re_r , with a normalized standard error of 20% for **a** and 8% for **b**. The corresponding values are presented in

Table A5.1. An average value of the apparent Fanning friction factor, f_{app_avg} was obtained by integrating equation (1) along the axial distance considered.

Fig.A5.1 Axial Variation of the Apparent Friction Factor in an Annulus in Entrance Region ($e=0.9$, $r^*=0.5$) for Different Liquid Film Reynolds Number, Re_f

To calculate the force exerted on the bubble in the developing region due to skin friction and axial momentum changes, F_{entr} , one has to estimate L_{hy} , the length of this region. As the annular space can be approximated by two parallel plates, - see Appendix 2 -, an equation proposed by Chen [Ward and Smith, 1980] for this specific flow, in the laminar Re_f range up to 4000, will be used:

$$\frac{L_{hy}}{D_h} = \frac{0.315}{0.0175 Re_f + 1} + 0.011 Re_f \quad (2)$$

where: $D_h = D - D_B$. Table A5.1 contains this parameter. The next step is to estimate over the entire length of the entrance region the aforementioned force, F_{entr} , from equation:

$$F_{entr} = f_{app_avg} \frac{\rho v_y^2}{2} \pi D_B L_{hy} \quad (3)$$

and the force due to skin friction over that part of the bubble beyond the hydrodynamic entry length is given by,

$$F_{fd} = f \frac{\rho v_y^2}{2} \pi D_B (L_B - L_{hy}) \quad (4)$$

with friction factor values for the fully developed region, f from Appendix 2.

Finally, the friction force when allowance is made for the entrance region and the friction force for fully developed region over the entire length of the bubble were calculated using respectively, the equations:

$$(F_t)_{(entr+fd)} = F_{entr} + F_{fd}$$

$$(F_t)_{(fd)} \equiv F_{fd} \quad (5)$$

Table A5.1. Friction Factors and Forces for 7.5 cm and 15 cm Length Bubbles for Two Cases: Entrance Region Considered and Fully Developed Flow along the Bubble

Crt. No.	Re_f	L_{hy} (m)	Coeffic. a	Coeffic. b	f_{app_avg} (entrance region)	f (fully developed region)
1.	1153	0.029	28.8	-0.002	0.025	0.009
2.	1272	0.032	28.8	-0.002	0.023	0.009
3.	1538	0.039	28.8	-0.003	0.019	0.007
4.	1774	0.045	28.8	-0.003	0.016	0.006
5.	2070	0.052	28.8	-0.004	0.014	0.005
6.	2277	0.058	28.8	-0.004	0.013	0.005

Table A5.1. continued

 $L_B = 7.5 \text{ cm}$

Crt. No.	Friction Force (N) $(F_t)_{(entr+fd)}$	Friction Force (N) $(F_t)_{fd}$	$\frac{(F_t)_{(entr+fd)}}{(F_t)_{fd}}$	$(\% \Delta F_D)_{e=0.9}$
1.	0.024	0.015	1.636	2.5
2.	0.028	0.016	1.701	2.8
3.	0.036	0.020	1.849	3.3
4.	0.045	0.023	1.980	3.5
5.	0.057	0.027	2.144	3.9
6.	0.066	0.029	2.260	4.0

Table A5.1. continued

 $L_B = 15 \text{ cm}$

Crt. No.	Friction Force (N) $(F_t)_{(entr+fd)}$	Friction Force (N) $(F_t)_{fd}$	$\frac{(F_t)_{(entr+fd)}}{(F_t)_{fd}}$	$(\% \Delta F_D)_{e=0.9}$
1.	0.039	0.030	1.318	2.1
2.	0.044	0.033	1.351	2.4
3.	0.056	0.039	1.424	2.6
4.	0.068	0.046	1.490	2.4
5.	0.084	0.053	1.572	2.7
6.	0.095	0.058	1.630	3.0

The very last column of the table, $\% \Delta F_D$ indicates the percentage the experimental force for bubble eccentricity, $e=0.9$, would vary if the influence of hydrodynamic entrance region were considered. As one can observe, the highest values are 4% for the 7.5 cm length bubble and 3% for the 15 cm length bubble. The same algorithm has been applied to the data for the entrance region in an annulus characterized by eccentricity $e=0.5$. The latter $\% \Delta F_D$, presented in Table A5.2, is, as one would have

expected, smaller than that for the previous eccentricity, $e=0.9$, but still not much different. We can then consider that the variation of the drag force with the radial bubble displacement from the central position is almost entirely due to the reduced form drag. The contribution of the skin friction to the total drag force when one considers the hydrodynamic entry length is approximately 15%, the rest of 85% being due to the form drag. This means that skin friction is not important compared to the form drag and that the assumption of fully developed flow through the annular space between the bubble and pipe walls is good enough for the present analysis.

Table A5.2. The Contribution of the Hydrodynamic Entry Length
to the Measured Drag Force (%)

Crt. No.	$(\% \Delta F_D)_{e=0.5}$ $L_B=7.5$ cm	$(\% \Delta F_D)_{e=0.5}$ $L_B=15$ cm
1.	2.2	1.6
2.	2.7	1.9
3.	3.0	2.3
4.	3.2	1.9
5.	3.8	2.1
6.	3.6	2.4

IMAGE EVALUATION TEST TARGET (QA-3)

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