Fast Radio-Frequency Current Density Imaging With Spiral Acquisition

by

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A thesis submitted in conformity with the requirements for the degree of

Master of Applied Science

Graduate Department of Electrical and Computer Engineering

University of Toronto

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Fast Radio-Frequency Current Density Imaging With Spiral Acquisition

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University of Toronto, 1997

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Abstract

Radio-Frequency Current Density Imaging (RF-CDI) is a unique magnetic resonance technique that measures electrical current density. A potential application for RF-CDI is brain functional imaging. However, the existing RF-CDI imaging sequence is too slow to capture the rapidly occurring cortical conductivity changes associated with brain functions.

This thesis describes the design and the implementation of a RF-CDI system that achieves the required temporal resolution while fulfilling other imaging requirements. The system is optimized to ensure the highest sensitivity within the shortest possible imaging time. Power deposition is also considered to ensure safety. It is demonstrated that the final system achieves a noise performance close to theoretical limits and a very flexible tradeoff between temporal resolution and sensitivity. Compared to the previously optimized technique, the new system reduces the imaging time by 13 times at a 2.5 times loss in sensitivity within specific absorption rate. With this promising tradeoff, the new system is fast enough for functional imaging of the brain.
To my family and friends,
Acknowledgments

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Raymond T. H. Yan

University of Toronto
March 1997
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ABBREVIATIONS

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<th>Description</th>
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<tr>
<td>BEPI</td>
<td>Blipped Echo-Planar Imaging</td>
</tr>
<tr>
<td>BW</td>
<td>Bandwidth</td>
</tr>
<tr>
<td>CD</td>
<td>Current Density</td>
</tr>
<tr>
<td>CPCB</td>
<td>Current Pulse Control Box</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>EPI</td>
<td>Echo-Planar Imaging</td>
</tr>
<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>FID</td>
<td>Free Induction Decay</td>
</tr>
<tr>
<td>IEPI</td>
<td>Interleaved Echo-Planar Imaging</td>
</tr>
<tr>
<td>ISPI</td>
<td>Interleaved Spiral Imaging</td>
</tr>
<tr>
<td>FOV</td>
<td>Field of View</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
</tr>
<tr>
<td>MRI</td>
<td>Magnetic Resonance Imaging</td>
</tr>
<tr>
<td>NEX</td>
<td>Number of Excitations</td>
</tr>
<tr>
<td>PC</td>
<td>Phase Encoding</td>
</tr>
<tr>
<td>PSF</td>
<td>Point Spread Function</td>
</tr>
<tr>
<td>RF-CDI</td>
<td>Radio-Frequency Current Density Imaging</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>SAR</td>
<td>Specific Absorption Rate</td>
</tr>
<tr>
<td>SPI</td>
<td>Spiral Imaging</td>
</tr>
<tr>
<td>SLR</td>
<td>Shinnar Le-Roux</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>TX/RX</td>
<td>Transmit and Receive</td>
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SYMBOLS AND VARIABLES

\( B_o \)  
static magnetic field of the imager

\( B_1 \)  
an radio frequency pulse, normally refer to the rotary echo

\( \tilde{B}_x, \tilde{B}_y \)  
rotating frame fields due to RF current

\( \tilde{B}_J \)  
total field due to RF current, \( \tilde{B}_J \equiv \tilde{B}_x + j \tilde{B}_y \)

\( \tilde{B}_t \)  
all rotating frame fields in the transverse plane

\( \Delta B_o \)  
static field inhomogeneity

\( F_x, F_y \)  
derivative template filter weight noise weights

\( f(x, y) \)  
off resonance frequency as a function of position

\( \Delta H \)  
static field inhomogeneity gradient

\( \tilde{J}_z \)  
rotating frame current density

\( k \)  
RF-CDI sequence dependent noise weight factor

\( M_{xy} \)  
component of the net magnetization in the transverse plane

\( n I \)  
number of spiral excitation

\( N_{pixels} \)  
number of pixels in a region of interest

\( N_{images} \)  
number of repeated excitation in noise measurement

\( \phi_1(x, y), \phi_2(x, y) \)  
phase map angle

\( S_{SE} \)  
signal in a standard spin-echo pulse sequence

\( S_{SE/RE} \)  
signal in an RF-CDI pulse sequence

\( T_1, T_2 \)  
longitudinal and transverse relaxation time

\( T_{2p} \)  
relaxation time for magnetization precession orthogonal to \( B_1 \)

\( T_{Acq} \)  
total image acquisition time

\( T_c \)  
duration of RF current pulse

\( T_E \)  
echo refocus time

\( T_R \)  
pulse sequence repetition time

\( \tilde{x}, \tilde{y}, \tilde{z} \)  
rotating frame co-ordinate axes

\( \Delta x, \Delta y \)  
pixel dimensions

\( \gamma \)  
gyromagnetic constant for hydrogen

\( \Gamma^+, \Gamma^- \)  
precession angle about +\( \tilde{z} \) axis and -\( \tilde{z} \)

\( \mu_o \)  
free space magnetic permeability

\( \rho \)  
density [kg/m\(^3\)]

\( \sigma_J \)  
current density error standard deviation

\( s(t) \)  
FID signal measured by receive coil

\( \Delta z \)  
imaging slice thickness
Chapter 1

Introduction

This thesis describes the design and the optimization of a fast implementation for Radio Frequency Current Density Imaging (RF-CDI). RF-CDI is a technique for imaging electrical current using a magnetic resonance imager. Previous work in RF-CDI focused on improving its robustness, portability and sensitivity [1]. Recent work adapted it to a large bore clinical MRI system that allows imaging of large animals and human [2]. The technique was also optimized to measure conductivity in biological tissues with enough sensitivity and image quality [3]. These endeavours have paved the way for future medical applications of RF-CDI. However, its imaging time was too long to capture the fast cortical conductivity changes associated with brain functions. This thesis concerns the research and development of a fast realization of RF-CDI having sufficient temporal resolution to monitor such dynamically changing conductivity.
1.1 Thesis Outline

This chapter outlines the goals and the requirements of this project. Chapter 2 provides background on RF-CDI and existing fast MRI acquisition schemes. It also outlines the performance criteria that will be used throughout the thesis for system testing and enhancement. Included with these criteria is the theoretical performance bound that will be used to evaluate the final system.

The body of this thesis lies in chapter 3 and 4. In chapter 3, the most appropriate fast imaging technique for RF-CDI is identified based on the discussions in chapter 2. The design and implementation of this technique are described. The performance of this system is then measured using the criteria defined in chapter 2. Unsatisfactory performance, potential problems and deviations from theoretical behavior are identified.

Chapter 4 describes the improvements to the system developed in chapter 3. Theories are proposed to explore the problems identified in chapter 3. Based on these hypotheses, appropriate solutions are implemented. The system performance is then re-evaluated and it satisfies all the initial system requirements.

Chapter 5 summarizes the design components and performance of the final optimized system. It also gives recommendations for the future RF-CDI research direction in light of the success of this project.

1.2 System Requirements

The requirements of the system are dictated by the system applications. These requirements define the design objectives and outline all the functions
and characteristics that the final system must encompass in order to be fully operational.

- **Sensitivity Requirement:** The system must be capable of achieving the best possible measurement precision within acceptable power deposition. Studies dealing with cortical conductivity changes [5] during spreading depression and other sensorimotor stimuli must detect 10% changes in current density or less. To achieve this requirement, artifacts and noise in the final system must be minimized.

- **Temporal Resolution Requirement:** To be able to monitor the ongoing conductivity changes during in-vivo studies, the imaging time must be significantly shorter than the time of event by a factor of 10 or more. Stimulation of a rat neocortex results in spreading depression episodes which last for 7-8 minutes [5] and thus, an imaging time of shorter than \( \sim 50 \) seconds is required. This number was chosen as the required temporal resolution.

- **Safety Requirement:** As the intent is to bring this technique to in-vivo imaging, its safety issues have to be carefully addressed. The requirement is that the power deposited by the final system has to be within generally accepted safety limits, which is less than \( 8W/kg \) for the body and less than \( 25W/kg \) for the body surface.
Chapter 2

Background

2.1 RF Current Density Imaging

Radio frequency current density imaging (RF-CDI) [8] images the amplitude and phase of an externally applied RF current at the Larmor frequency. The applied current produces a magnetic field which distorts the MR imager homogeneous RF magnetic field. These distortions of the magnetic field are then imaged using a commercial wide-bore MR imager. Image post-processing allows the external current to be calculated using Maxwell’s equations.

Central to any MR technique are two essential components, magnetization preparation and data acquisition. In RF-CDI, there is an additional image post-processing component as current density images are derived from the MR raw data. This chapter briefly describes the magnetization preparation and the post processing required to obtain current density images. The chapter also defines the assumptions and performance criteria pertaining to the operation and the analysis of the temporally enhanced RF-CDI system.
2.1.1 Theory of Rotating Frame RF-CDI

Two implementations of RF-CDI have been developed, the polar decomposition method [9] and the rotating frame approach [8]. Because of the inferior sensitivity and robustness of the polar decomposition implementation, recent research has been dedicated to enhance the rotating frame approach. In the rotating frame approach, current is assumed to flow predominantly in the \( \hat{z} \) direction parallel to the static field \( B_0 \) of the imager to allow measurement of current density with one sample orientation. Based on this assumption, for an arbitrary heterogeneous medium, the current density in the laboratory frame of reference can be denoted by:

\[
J(t) = j_z \cos(\omega t + \phi_z) \hat{z}
\]  
\( (2.1) \)

where \( j_z \) and \( \phi_z \) are the magnitude and the spatially dependent phase angle of the current respectively. This RF current will produce a magnetic field \( \tilde{B}_J \) transverse to \( B_0 \) with \( x \) and \( y \) components given by:

\[
B_x(t) = b_x \cos(\omega t + \theta_x)
\]  
\( (2.2) \)

\[
B_y(t) = b_y \cos(\omega t + \theta_y)
\]  
\( (2.3) \)

Since the MR imager can measure only the rotating field with respect to its synchronized rotating frame whose \( \hat{x} \) axis is defined as the left rotating magnetic field component produced by its transmit coil, the magnetic fields measured by the MR imager in the rotating frame are:

\[
\tilde{B}_x = \frac{[b_x \cos(\theta_x - \Psi) + b_y \sin(\theta_y - \Psi)]}{2}
\]  
\( (2.4) \)

\[
\tilde{B}_y = \frac{[b_y \cos(\theta_y - \Psi) - b_x \sin(\theta_x - \Psi)]}{2}
\]  
\( (2.5) \)
where $\Psi$ is the difference in phase between the rotating and the laboratory frame of reference due to spatial variations in the transmit and receive coils, which is assumed ideally constant throughout the region of interest. Since the current magnitude and phase are encoded in $\vec{B}_x$ and $\vec{B}_y$, also noting that $\vec{B} = \mu_0 \vec{H}$, the complex current phasor flowing in the region of interest can be estimated using the curl operation [8]:

$$J_z = \frac{2}{\mu_0} \left\{ \left[ \frac{\partial \vec{B}_y}{\partial x} - \frac{\partial \vec{B}_x}{\partial y} \right] + 2j \left[ \frac{\partial \vec{B}_x}{\partial x} + \frac{\partial \vec{B}_y}{\partial y} \right] \right\} \quad (2.6)$$

The above estimation is valid based on the assumption that $|\partial H_z/\partial z| \ll J_z$ [8]. Practically, this condition holds if the dominant current flow is in the direction of $B_z$. Violation of this condition necessitates imaging multiple orientations of the sample as there exist other components of the magnetic field that cannot be encoded by rotation on a single plane.

### 2.1.2 Magnetization Preparation & Post Processing

The magnetization preparation in RF-CDI for encoding the RF current is shown pictorially in Fig. 2.1 with a step by step explanation. The improved sensitivity and robustness to noise, relaxation and field inhomogeneity of rotating frame RF-CDI are attributed to the constrained direction of rotation by the transmission of a spatially uniform RF pulse along the transverse rotating frame axes. This rotary echo pulse $B_1$ and the magnetic fields of the injected current will cause the magnetization to precess in a plane orthogonal to the $B_1$ field. Midway through the RF pulse, the direction of $B_1$ is reversed, causing the magnetization to precess in the opposite direction. This field reversal thus undoes the precession due to $B_1$, leaving the net magnetization rotated solely
by the magnetic field of the RF current $J_z$. Since only the transverse components are mapped by the MR imager, the net magnetization has to be flipped onto the transverse plane by a hard 90° rotation pulse. The current encoded information - the original rotation angle $\Gamma_z$ now manifests as the phase angle $\Psi$ in the MR signals in the transverse plane. Mathematically, with $B_1$ aligned to $\tilde{x}$, the rotation angles before and after field reversal are [8]:

$$\Gamma_+ = -\gamma|\tilde{B}_+|T_c/2 \quad \Gamma_- = -\gamma|\tilde{B}_-|T_c/2$$

with the net magnitude fields rotated along the axes of rotation:

$$\tilde{B}_+ = (B_1 + \tilde{B}_x, \tilde{B}_y, \Delta B_o) \quad \tilde{B}_- = (-B_1 + \tilde{B}_x, \tilde{B}_y, \Delta B_o)$$

$$N_+ = \tilde{B}_+/|\tilde{B}_+| \quad N_- = \tilde{B}_-/|\tilde{B}_-|$$

From the rotation axes $N_{+/-}$, it can be seen that for $B_1$ to constrain the axes of rotation along $\tilde{x}$ and $\tilde{y}$ for undistorted measurement of $\tilde{B}_x$ and $\tilde{B}_y$, it is essential to satisfy the rotary echo criterion defined by:

$$|B_1| \gg |\tilde{B}_x|, |\tilde{B}_y|, |\Delta B_o|$$

(2.7)

where $\Delta B_o$ is the field inhomogeneity arising from off-resonance components and imperfect field shimming [10]. To subtract off systematic phase variations arising from the receiver and the gradient coils, two excitations ($NEX_J = 2$) with $B_1$ phase cycling$^1$ are acquired for signal averaging before the reference phase image with zero current is subtracted to calculate the net phase wrap angle due to the current. The wrap angles are related to the $B$ fields in Eq. 2.6 by:

$$\frac{\partial \tilde{H}_x}{\partial x} = -\frac{1}{\gamma \mu_o T_c} \left[ \frac{\partial \Psi_{x+}}{\partial x} - \frac{\partial \Psi_R}{\partial x} \right]$$

(2.8)

$^1$Phase Cycling is an averaging technique in MRI. Its details can be found in [8].
Figure 2.1: An illustration of how current is encoded and mapped by the imager using a spin-echo sequence for rotating frame RF-CDI. It highlights the steps and the intermediate magnetic field for the entire magnetic field preparation process to be followed by data acquisition.
The other components in Eq. 2.6 can be derived similarly. Eq. 2.6 and Eq. 2.8 define the post processing from the MR phase images to obtain current density images.

2.1.3 Tradeoffs in RF-CDI Imaging Parameters

As with other MR techniques, to optimize the performance of CDI, it is desirable to achieve the highest spatial resolution with minimal artifact and noise within the shortest imaging time. It is essential to compromise between these performance criteria because of their conflicting requirements.

Imaging Time

The total imaging time for a single MR image is:

\[ T_{\text{Acq}} = T_R \times N_{\text{excitation}} \] (2.9)

where \( T_R \) is the repetition time between successive non-modulated excitation and \( N_{\text{excitation}} \) is the number of excitations in the non-frequency encoding direction. To reduce the imaging time, \( T_R \) or \( N_{\text{excitation}} \) can be reduced. However, reducing \( T_R \) will cause MR signal loss due to insufficient longitudinal magnetization recovery from \( T_1 \) relaxation\(^2\) [10]. Trimming down \( N_{\text{excitation}} \) will introduce aliasing in the non-frequency encoding direction. The shortest scan time is therefore bounded by the minimum possible values of these two parameters.

\(^2\)When the static field is flipped onto the transverse plane, its longitudinal component is momentarily zero. This longitudinal component will recover to its full magnitude with a time constant of \( T_1 \).
Signal to Noise in MR Images

The signal to noise ratio of an MR magnitude image is a function of many hardware and imaging parameters which can be expressed as:

$$SNR = K_{SNR} \frac{FOV_x FOV_y \delta Z \sqrt{NEX}}{\sqrt{N_x N_y BW_{f_{req}}}}$$ (2.10)

where $K_{SNR}$ is a proportionality constant incorporating all hardware and non-adjustable imaging parameters, $FOV_{x,y}$ are the field of views in the imaging plane, $\delta Z$ is the imaging slice thickness, $NEX$ is the number of excitation for signal averaging, $N_{x,y}$ is the number of pixels in the two orthogonal directions and $BW_{f_{req}}$ is the data acquisition bandwidth of the receiver. In systems which use high encoding gradients, it is essential to widen the receiver bandwidth [10]. Therefore, from the SNR point of view, it is desirable to have weaker encoding gradients. In addition, $BW_{f_{req}}$ has to satisfy the Nyquist sampling criterion which is determined by the number of required samples and the available readout time in each excitation. To minimize the readout time while obtaining maximum samples in each excitation, it is necessary to reduce the sampling time at the expense of a lower SNR. Lengthening the readout time for more samples preserves the SNR. However, it exacerbates the non-linear weighting due to $T_2$ relaxation which distorts the data as explained in the following discussion.

Signal Loss in Rotating Frame RF-CDI

In any 90° dip angle spin-echo imaging sequence, the signal strength at the echo refocus time ($T_E$) is given by:

$$M_{SE} = k_{SE} N_{PD} e^{-\frac{T_E}{T_2}} \left[ 1 - 2e^{-\frac{(T_R - T_E)}{T_1}} + e^{-\frac{T_R}{T_1}} \right]$$ (2.11)
During the rotary echo $B_1$, the signal strength will decay to a value approximated by:

$$M_{SE|B_1} \approx M_{SE} e^{\frac{T_c}{T_2p}}$$  \hspace{1cm} (2.12)

where $T_c$ is the duration of the $B_1$ pulse, $N_{PD}$ is the proton density and $T_{2p}$ is the relaxation time constant for the precessing magnetization given by $\frac{1}{2}(T_1^{-1} + T_2^{-1})^{-1}$. In the absence of static field inhomogeneity, Eq. 2.11 is valid when $T_c \ll T_R$ and $\gamma B_1 T_c \gg 1$. This signal loss resulting from $B_1$ is an unavoidable problem and no design strategy can recover the signal. The resulting drop in SNR causes noise in the CDI images, as will be discussed in the noise model described in the next section.

**Tradeoff in $B_1$ strength - Image Quality and SAR**

An important operating condition for rotating frame RF-CDI is the rotary echo criterion. In general, the stronger $B_1$, the more constrained are the axes of rotation and thus the more accurate is the estimation of $\vec{H}_x$ and $\vec{H}_y$. To satisfy the rotary echo criterion, a stronger $B_1$ is often required for higher current density images. However, there exists an upper limit of $B_1$ strength due to RF amplifier limitations and the maximum specific absorption rate (SAR) for safety consideration. The SAR for $B_1$ is [8]:

$$SAR = \frac{T_c \sigma}{T_R 2\rho} \frac{16\pi^4}{\gamma^2} R^2 f_o^2 f_1^2$$  \hspace{1cm} (2.13)

where $f_1 (f_1 = \gamma B_1/2\pi)$ is the rotary echo strength, $f_o$ is the Larmor frequency (63.6 MHz for GE 1.5 T Signa), $\sigma$ and $\rho$ are the conductivity and density of the subject respectively. Since the fast imaging technique will ultimately be used for in-vivo imaging, the total SAR from $B_1$ and $J_z$ has to satisfy the Canadian EM field exposure limits for occupational exposure, which will be verified in Chapter 4.
2.2 Performance Analysis in RF-CDI

Unlike other MR techniques, the image quality of CD images cannot be directly quantified by SNR measurements as current density is calculated from a differentiation operation on the phase images. To gauge the performance of any RF-CDI system, it is essential to develop a model to mathematically quantify its noise and artifacts. Various performance criteria are defined in this section and will be used in chapters 3 and 4 to compare the performance of the original RF-CDI system and its fast implementation.

2.2.1 Performance Measures for RF-CDI

Throughout this thesis, the performance of the RF-CDI system will be measured with respect to [11]:

1. Stochastic Noise

2. Systematic Artifacts

Both are measured with respect to CD images. To measure these variables, we made multiple acquisitions of a region in which current density is expected to be uniform. Consider a region of interest (ROI) consisting of $N_{ROI}$ pixels and of which we have $N_{images}$ images with the same parameters, the model of the current density value for each pixel in each image is:

$$p^i_k = P_k + n^i_k$$

(2.14)

where $p^i_k$ is the measured magnitude of the RF current density $|\vec{J}_z|$ for pixel $k$ in image $i$, $P_k$ is the mean value of pixel $k$ for all $N_{images}$ and $n^i_k$ is a zero mean random variable with a variance of $\sigma^2_{n_k}$ representing the stochastic noise. The
mean value for each pixel is defined as:

\[ P_k = J_k + A_k \]  

(2.15)

where \( J_k \) is the true current density of pixel \( k \) and \( A_k \) is a systematic artifact due to the imaging technique. These two quantities are assumed to be constant across multiple acquisitions of the same region of interest.

### 2.2.2 Stochastic Noise Analysis

In this work, the stochastic random noise \( n_k \) defined in Eq. 2.14 is measured by its estimated variance:

\[ \hat{\sigma}_n^2 = \frac{1}{N_{ROI}} \sum_{k=1}^{N_{ROI}} \left[ \frac{1}{N_{Images} - 1} \sum_{i=1}^{N_{Images}} (P_k^i - \hat{P}_k)^2 \right] \]  

(2.16)

where

\[ \hat{P}_k = \frac{1}{N_{Images}} \sum_{i=1}^{N_{Images}} P_k^i \]  

(2.17)

In regions where SNR is high (i.e. \( \text{SNR} \gg 5 \)), this stochastic noise is theoretically [8]:

\[ \sigma_J = \frac{1}{\gamma \mu_0 T_c \text{SNR}_{MR}} \sqrt{\left( \frac{F_x}{\delta x} \right)^2 + \left( \frac{F_y}{\Delta y} \right)^2} \approx \sigma_n \]  

(2.18)

where \( T_c \) is the duration of the current pulse, \( \delta x \) and \( \delta y \) are the pixel sizes, \( F_x \) and \( F_y \) are the derivative weighting\(^3\) of the Sobel filter [8] used in the derivative operation in Eq. 2.6, \( k \) is the weighting factor\(^4\) for the phase-cycling method and SNR is the signal to noise ratio of the MR magnitude image:

\[ \text{SNR}_{MR} = \frac{\frac{1}{N_{Signal}} \sum_{i=1}^{N_{Signal}} x_i}{\sqrt{\frac{1}{N_{Noise}} \sum_{j=1}^{N_{Noise}} x_j^2}} \]  

(2.19)

---

\(^3\)These numbers are related to the dimensions of the convolution template used for the derivative operations in calculating current density. In this thesis, \( F_x = F_y = \sqrt{3}/4 \). See [8] for details.

\(^4\)For NEX=2 phase cycling.
where \( x_i \) is the MR intensity over a signal region and \( x_j \) is the MR signal in a signal-free noise region (i.e. no ghosting or artifacts). If a signal free region cannot be found, the noise term in Eq. 2.19 can be estimated using the approach of Eq. 2.16 with \( x_j \) in place of \( p \).

Eq. 2.18 is valid only for the signal region of CD images with an average current level \( \bar{J} \gg \sigma_J \). In regions where there is no RF current, the calculated current will be dominated by noise with a root mean squared value of \( \sqrt{2} \sigma_J \) [8]. The \( \sigma_J \) calculated in Eq. 2.18 is the theoretical lower bound of the random noise in CD images post processed from the MR images with SNR defined in Eq. 2.19. Thus under ideal conditions, the random noise measured in Eq. 2.16 should be equal to the value calculated in Eq. 2.18 if the noise model is valid and the dominant noise is Gaussian.

### 2.2.3 Systematic Artifacts Analysis

The value of \( \hat{P}_k \) in Eq. 2.17 is an estimate of the noise free or mean current value at pixel \( k \). Within a ROI where current density is expected to be uniform, the values of \( \hat{P}_k \) should be independent of \( k \), the specific pixel in the image. Practically, the value of \( \hat{P}_k \) is \( k \) dependent due to the systematic artifacts of the imaging process. This RMS systematic artifact \( \sigma_s \) is:

\[
\sigma_s = \sqrt{\frac{1}{N_{ROI}} \sum_{k=1}^{N_{ROI}} (\hat{P}_k - \hat{J})^2}
\]  
(2.20)

with

\[
\hat{J} = \frac{1}{N_{ROI}} \sum_{k=1}^{N_{ROI}} \hat{P}_k
\]  
(2.21)

where \( \hat{J} \) is the average current in a region of uniform current density.
The sensitivity measurement is the most representative performance criterion of the final RF-CDI system. It is defined as the ability of the RF-CDI technique to detect differences in current density. A lower value implies a higher sensitivity. Mathematically, it is defined as:

\[ \text{Sensitivity} = \frac{\sigma_s}{j} \]  

(2.22)

2.3 Fast Imaging Techniques & Principles

By shortening the imaging time, fast imaging techniques open a broad spectrum of new MRI applications. Recently established techniques such as Echo Planar Imaging (EPI) [12] and Spiral Imaging (SPI) [13] have been employed in a wide range of applications. This section briefly describes their principles, advantages and disadvantages in the context of RF-CDI. It is based on this discussion that the best implementation is chosen for current density imaging.

2.3.1 Principles of Fast Imaging

The long imaging time associated with the conventional Fourier approach in MR imaging is the consequence of the requirement to acquire all the k-space data on equally spaced rectangular grids for 2-dimensional DFT reconstruction. The phase encoding scheme used in the Cartesian approach\(^5\) requires \(N_{PE}\) excitations successively separated by a relative long \(T_R\) time, each for encoding one line of k-space. The imaging time can be reduced with the dimension of the k-space array at the expense of a lower spatial resolution along the phase encoding direction [14]. However, there exists a smallest

\(^5\)In the Cartesian approach, the magnetization are phase and frequency encoded along the two orthogonal directions for the 2-dimensional k-space data array.
array size, typically 128 by 128 or 256 by 256 to prevent aliasing. The rationale of most fast imaging techniques is that certain pattern of k-space data are skipped and later interpolated from a set of sufficiently sampled k-space points. This can be achieved by traversing different k-space acquisition trajectories and applying interpolation algorithms prior to 2-D DFT reconstruction (see appendix A.2.2).

2.3.2 Echo Planar Imaging

An Overview of Echo Planar Imaging

Echo Planar Imaging (EPI) [12] permits the measurement of an entire MR image in less than 100ms. In this technique, k-space data are obtained from more than one value of the phase encoding gradient within one $T_R$ period. EPI is normally designed to acquire all k-space data within a single excitation. It uses a very rapid series of spin echoes generated by rapidly switching a strong phase encoding gradient in the presence of a weaker readout gradient. This can be considered conceptually as an additional frequency modulation in which both spatial directions are simultaneously frequency modulated.

Several versions of EPI are available. Typical ones are zig-zag and blipped EPI. Their corresponding k-space sampling pattern and encoding gradients are shown in Fig. A.2 and Fig. A.3.

Problems with Echo-Planar Imaging

Since the entire image is acquired in a single (EPI) or at most a few interleaved excitations (IEPI), each readout is limited in duration to avoid $T_2$ relaxation. This presents extremely demanding requirements on the synchronization and gradient systems of the imager. IEPI mitigates these stringent
requirements by completing the data acquisition with multiple excitations. However, the high switching frequencies and gradient strength achieved within short rise-time as shown in Fig. A.3 are required. These spike-like gradients are presently beyond the capability of most clinical gradient systems. Hardware limitations have always been the major deterrent to EPI practicality.

In theory, the SNR associated with EPI is low because it uses a high sampling rate and requires a larger receiver bandwidth (Eq. 2.10) up to 500 kHz compared to 32 kHz generally used in the conventional Cartesian technique. Operating at a high static field strength alleviates the SNR loss\(^6\) it but necessitates very precise field shimming to avoid intolerable geometrical distortion arising from magnetic susceptibility which is proportional to the field strength. However, this precise shimming is practically difficult. Spatial resolution is also low as EPI typically yields 64 by 128 image matrix. Due to the high switching rates in EPI, peripheral nerve and muscle stimulations are possible. When \(\frac{\partial B}{\partial t}\) is greater than 60 T/s, physiological stimulations were reported from different groups [15]. The potential hazards associated with these stimulations are not yet established. However, they have definite implications on the practicality of EPI when physiological issues are concerned.

**Strength of Echo-Planar Imaging**

The most important advantage of EPI is its ultra fast imaging capability. It is one of the fastest imaging techniques in MR imaging as the entire image can be acquired in one single continuous excitation. The short imaging time reduces the complications due to chemical shift and motion artifacts. Inevitable ghosting artifacts[10] arising from echo shift typical in EPI can be

\(^6\)Increasing field strength improves the SNR by increasing the proportional constant \(K_{SNR}\) in Eq. 2.10.
corrected for to a satisfactory degree by many pulse sequence design and post-correction algorithms [16]. Interpolation reconstruction is relatively simple as the k-space sampling is non-uniform in only one direction.

### 2.3.3 Spiral Imaging

**Overview of Spiral Imaging**

Spiral Imaging (SPI) [13] was first proposed as an alternative to EPI for fast imaging. Conceptually, it is an MR acquisition technique with time varying readout gradients being modulated to achieve a spiral sampling pattern in k-space. The k-space trajectory and the corresponding readout gradients for SPI are shown in Fig. A.4. Like other fast imaging technique, SPI can significantly reduce the imaging time to a few (interleaved spiral ISPI) or even one excitation time.

**Problems with Spiral Imaging**

Important considerations for SPI are its ease of implementation on clinical imagers and the complexity of the required image reconstruction algorithm. With the two orthogonal readout gradients being modulated simultaneously, the resulting k-space sampling pattern is non-uniform in both encoding directions. This non-standard acquisition scheme requires substantial post-imaging interpolation [17] and density compensation compared to EPI whose k-space trajectory is non-uniform only in the phase encoding (PC) direction. Since ISPI is a non-standard acquisition scheme, it is not supported in most clinical imager proprietary pulse sequence design and image reconstruction routines [18]. Therefore, the final ISPI system will inevitably be limited in functionality and flexibility. Another major disadvantage of ISPI in common with most non
Cartesian acquisition technique is its susceptibility to the detrimental effects of inhomogeneity and the exacerbation of the resulting image quality degradation by the ISPI interpolation algorithm [19].

**Advantages of Spiral Imaging**

Spiral imaging has many advantages for ultra fast data acquisition. The most important strength of ISPI is its smooth encoding gradients which make this technique realizable in most clinical gradient systems. The spiral trajectory also has the highest scanning efficiency as it makes optimal use of the gradient systems with minimal slew rate and abrupt gradient reversal. This helps to alleviate possible eddy current artifact due to gradient transients [16]. Since spiral scan covers the Fourier plane with uniform $T_2$ weighting in all directions, it eliminates artifacts that may appear when $T_2$ weighting is different in the x and y directions. Motion artifacts are therefore minimal and isotropic compared to other conventional spin-wrap techniques [13].

Another important benefit of ISPI is a very flexible trade-off between image quality and imaging time by changing the number of spiral interleaves. The use of multiple interleaved spirals also shortens the readout time for each excitation without widening the receiver bandwidth for optimal SNR (Eq. 2.10). Despite the susceptibility of ISPI to inhomogeneity and off-resonance complications, effective post processing techniques such as field map correction [20] is available to improve the final image quality, making ISPI a promising candidate for ultra fast imaging.

Given all the strengths and weaknesses of these two fast imaging schemes, the candidate for the ultra-fast RF-CDI system will be selected for fast RF-CDI, as described in Chapter 3.
Chapter 3

Spiral Current Density Imaging

Present RF-CDI uses the conventional spin-echo Cartesian approach and acquires all k-space data on Cartesian grids for 2-D DFT reconstruction. With phase cycling for signal averaging, it takes six MR images to construct one CD image. Therefore, the long imaging time inherent in the conventional Cartesian approach significantly lengthens the time for CD imaging. With a $T_R$ of 300ms, it takes 4 minutes to obtain one 128x128 CD image using the existing RF-CDI sequence. This temporal resolution is far from adequate for monitoring the dynamic electrical conductivity changes which occurs in the order of seconds inside the brain and other biological tissues. To make RF-CDI feasible for functional imaging, its temporal resolution has to be enhanced. This chapter describes the choice of the fast imaging implementation for CDI and its detailed design. Phantom studies will be used at the end to quantify the performance of the fast RF-CDI system.
Table 3.1: EPI and ISPI Comparison

<table>
<thead>
<tr>
<th>Method</th>
<th>ISPI</th>
<th>EPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardware Requirement</td>
<td>Moderate</td>
<td>Demanding</td>
</tr>
<tr>
<td></td>
<td>Realizable on Signa</td>
<td>Impractical</td>
</tr>
<tr>
<td>Severity of Artifacts</td>
<td>Blurring</td>
<td>Echo Shift</td>
</tr>
<tr>
<td></td>
<td>Correctable</td>
<td>Hardly Correctable</td>
</tr>
<tr>
<td>Signal to Noise Ratio</td>
<td>≈ 1/2 of Cartesian</td>
<td>≈ 1/4 of Cartesian</td>
</tr>
<tr>
<td>Temporal Resolution/Image Quality Tradeoff</td>
<td>Flexible</td>
<td>Inflexible</td>
</tr>
<tr>
<td>Imaging Time¹</td>
<td>Longer ≈ 3-12 seconds</td>
<td>Shorter ≈ 1-4 second</td>
</tr>
<tr>
<td>Reconstruction Complexity</td>
<td>Relatively</td>
<td>Relatively</td>
</tr>
<tr>
<td></td>
<td>Complex</td>
<td>Simple</td>
</tr>
<tr>
<td>Physiological Stimulations</td>
<td>Not</td>
<td>Identified</td>
</tr>
<tr>
<td></td>
<td>Identified</td>
<td></td>
</tr>
</tbody>
</table>

3.1 Choice of Implementation for Fast RF-CDI

The design and initial testing of the fast RF-CDI system was performed on a GE 1.5T Signa clinical imager running in research mode. At the time of this project, it was equipped with a gradient system capable of delivering an orthogonal set of gradients at 1 G/cm in each direction with a slew rate of 1.67 G/cm/ms. The contemporary imager firmware was EPIC 5.4 [18] under which ISPI and IEPI were not standard. To choose the fast imaging scheme for RF-CDI, considerations for hardware limitations, the strength and weaknesses of the two candidates ISPI and IEPI as outlined in subsection 2.3.1 were given decreasing order of importance as tabulated in table 3.1.

With reference to the relative importance of the factors listed in table 3.1, it was concluded that interleaved Spiral Imaging is the more appropriate implementation for fast RF-CDI largely attributed to its realizable hardware requirements and superior image quality after image correction.
3.2 Detailed Implementation of Spiral RF-CDI

There are four main components in the Spiral RF-CDI system:

1. Pulse Sequence for Field Encoding
2. Readout Gradients Generation
3. Gridding Algorithm for k-space Interpolation
4. Post Image Processing for Current Density Images

3.2.1 Pulse Sequence for Field Encoding

The pulse sequence for Spiral RF-CDI was modified from an existing RF-CDI spin-echo sequence [3] that uses a hard 90° and a selective 180° pulse pair for refocusing. This existing sequence has been proved to have good performance with minimal random noise and artifacts [3]. However, certain design aspects of this sequence were re-examined to ensure the best performance with respect to the criteria described in section 2.2. The following describes the design of each component in the new Spiral RF-CDI sequence shown in Fig. 3.1.

Rotary Echo $B_1$ Pulse

The rotary echo pulse reverses its direction with the same amplitude at its mid point theoretically causes the magnetization to unwind back to its longitudinal direction with a signal loss given by Eq. 2.12 and a zero transverse component. This pulse is synchronized with the external RF current via the current pulse control box (CPCB) [2] to achieve the rotary echo criterion in Eq. 2.7. In Spiral RF-CDI, this pulse is implemented as an external pulse [18] whose duration, pulse shape and amplitude are user-specified with a resolution
Figure 3.1: Fast Spiral RF-CDI pulse sequence with: (1) RF pulses (rho1 components in figure, from left to right) - Rotary Echo $B_1$, Hard 90° excitation pulse and Selective 180° refocusing pulse. (2) Z-directional encoding gradients (zgrad) - Crushers Gradients, Z-profile Selective Gradient (work in conjunction with the selective 180° refocusing pulse), Crusher Gradients and a Spoiler Gradient. (3) Y-Directional encoding gradients (ygrad) : Spiral Readout Gradients with trapezoid Rewinder. (4) X-Directional encoding gradients (xgrad) - Spiral Readout Gradients with trapezoid Rewinder and a Spoiler Gradient to dephase magnetization on X-Y plane. The details of these component design are described in section 3.2.1.
of 1000 sample points over its entire duration. Therefore, other than RF amplifier imperfection, this pulse should be perfectly anti-symmetric and in the absence of other excitation pulse, it should give zero transverse magnetization and a signal free image upon transverse magnetic fields.

To verify that this pulse is not giving any unwanted magnetization and artifact, a phantom (to be described in section 3.3.1) was imaged only with this $B_1$ pulse turned on for 20ms at 800 Hz\textsuperscript{2} followed by data acquisition at $T_E = 25\text{ms}$. The final magnitude image was analyzed in two regions, one within the phantom and one in the free space of the FOV. These regions were found signal free and dominated by system noise. No signal pattern was identified in both the spatial and k-space domains. The MR signal magnitude was found indifferent within a measurement accuracy of 18% from the same image taken with this rotary pulse turned off. It was thus verified that this pulse was functioning as expected without introducing any detectable artifact due to unwanted transverse magnetization other than the signal loss predicted by Eq. 2.12.

**Hard 90° Excitation Pulse**

This non-selective RF pulse flips all the current encoded magnetic fields onto the transverse plane for mapping. It is delayed 350us from the end of the $B_1$ pulse to the RF amplifier the time to switch between waveform instructions. Since the duration of the pulse and its strength completely define the flip angle $\theta$ as given by:

$$\theta = \gamma B \tau$$  \hspace{1cm} (3.1)

\textsuperscript{2}In MRI, magnetic field strengths are expressed in Hz to facilitate calculation of rotation angle by RF pulses and comparison with the static field strength, which is also expressed in Hz. This representation is related to the standard field strength in Tesla by $\gamma B/2\pi$ where $\gamma$ is the gyromagnetic constant for hydrogen.
specifying the duration of the pulse $\tau$ at 250us with the required $\theta$ at 90° fixes the strength of the hard 90° pulse at 1000Hz. The advantage of fixing the strength of the hard 90° at this relatively high strength is to allow a stronger rotary echo $B_1$ whose strength is upper limited to and scaled down from that of this hard 90° during prescan3 [18]. It was experimentally determined that with $T_c = 20ms$, the maximum $B_1$ field with the extremity coil loaded by the tri-concentric phantom (to be described in 3.3.1) is 900 Hz. This hard 90° configuration should allow a $B_1$ strong enough to satisfy the rotary echo criterion in most phantom and animal experiments under the existing RF amplifier capabilities.

**Selective 180° Refocusing Pulse**

The selective 180° pulse is redesigned and enhanced from the one used in the pulse sequence for the Cartesian approach [3]. The accuracy of this selective 180° pulse is especially important when a hard 90° pulse is used because slice selectivity depends solely on its ability to refocus in-slice magnetization and suppress unwanted off-slice signal4.

Further enhancement of the 180° pulse is possible because of the elimination of the frequency rewinder when spiral readout gradients starting at $T_E$ are used, thus permitting a longer 180° pulse without having it overlapping with other gradients or RF pulses. Governed by the time-bandwidth tradeoff, lengthening the duration of the selective 180° pulse gives it a sharper frequency spectrum and thus a better slice selectivity [21]. This is particularly useful for imaging inhomogeneous media in which partially refocused magnetization in

---

3During prescan, the imager calibrates the required RF power necessary for the RF pulses to achieve their desired flip angles depending on the loading by the object.

4See appendix A.1 for explanation.
Figure 3.2: Crushed Signal Profile $M_{xy}$ for (a) a 3.2ms Spin-Echo 180° pulse previously optimized for the Cartesian Approach. (b) a 6.4ms Spin-Echo pulse used for Spiral RF-CDI.

The slice transition regions often contributes additional noise to the current density images.

The Shinnar-Le Roux (SLR) [21] technique which transforms RF pulse design problems to filter design problems available from the GE RF Profile Tool [18] was used to design a 180° spin-echo refocusing pulse with the following design parameters: pulse duration 6.4ms, 1.0% ripple in the passband and stopband. The optimization algorithm gives a RF pulse with band transition characteristics\(^5\) given by $F_p = 0.3652$, $F_s = 0.6348$, $W = 0.2696$, $D_{in_f} = 1.725$. The $M_{xy}$ crushed signal profile for the previously optimized and the new 180° refocusing pulse are shown in Fig. 3.2. It can be seen that the new pulse offers a sharper transition and thus a better slice selectivity for inhomogeneous media.

---

\(^5\)Assuming the design of a filter with a passband at $F_p$, a stopband at $F_s$, a transition band of $W$ and a time-bandwidth product of $D_{in_f}$. Refer to [21] for details regarding the meaning of these FIR filter parameters and how they are mapped into the characteristics of the RF pulse.
Crusher Gradient Design

It is important to eliminate any unwanted magnetization at the boundaries of the selected slice that got partially refocused by the transition band frequencies of the imperfect selective 180° pulse. This residual magnetization is "crushed" by crusher gradients placed before and after the selective 180° pulse. Instead of the iterative approach used in [3], a more accurate analytical approach is used to determine the required crusher size.

Consider a slice of thickness $\delta z$ at $z = z'$, the signal contributed by a thin strip along $z$ within this slab is given by:

$$M_{x,y} = \int_{z'-\frac{\delta z}{2}}^{z'+\frac{\delta z}{2}} e^{j\gamma G_z T_c dZ}$$  \hspace{1cm} (3.2)

After integrating Eq. 3.2, minimal (zero) signal occurs when:

$$G_z T_c = \frac{2n\pi}{\gamma \delta z}$$  \hspace{1cm} (3.3)

The value of $G_z T_c$ with $n = 1$ in Eq. 3.3 represents the minimum crusher size for complete dephasing. Since the exact transition slice thickness $\delta z$ cannot be determined practically, it is reasonable to use the transition width of the crushed signal profile for calculation. With $F_s - F_p = 0.2696$, the corresponding $\delta z$ is 2.696 mm for a 10 mm slice. With $n = 1$ in Eq. 3.3, the minimum crusher size $G_z T_c$ is 871 Gcm$^{-1}$μs. Any larger crusher implies $n > 1$ and a dephasing of more than one cycle (i.e. $2\pi$) across the unwanted slab. Considering the required minimum crusher size and the possible duration of the crusher gradients without gradients or RF pulses overlapping, a size of 2000 Gcm$^{-1}$μs corresponding to $n=2.29$ was chosen. It was iteratively verified that further increasing the crusher gradients show no measurable sensitivity improvement in CD images.
3.2.2 Generation of Spiral Readout Gradients

Choice of $k$-space Trajectory

Interleaved spiral trajectories are used because they can be readily implemented on standard clinical imagers. The principal advantage of interleaving is a higher SNR [13]. With $N$ spirals, the required gradient power drops by about $1/N^2$ with the data collection bandwidth reduced by $1/N$ and the SNR is thus improved by $\sqrt{N}$ compared to single-shot spiral. Another advantage of interleaved spiral is its narrower point spread function\(^6\) (PSF) with more power concentrated inside a narrower main-lobe[22].

Interleaved Spiral Trajectories

The radius at any point on the Archimedean spiral is proportional to the cumulative angle traced out to that point. The $k$-space trajectory is:

$$k(t) = r_1(t)e^{i\omega \tau_2(t)}$$

(3.4)

where $r_1(t)$ and $r_2(t)$ are functions of time that define the spiral trajectory. The complex encoding gradient $g(t) = g_x(t) + ig_y(t)$ is related to $k(t)$ by:

$$g(t) = dk(t)/\gamma dt :$$

(3.5)

where

$$g(t) \approx i\omega r_1(t) \frac{\partial r_2(t)}{\partial t} e^{i\omega \tau_2(t)}$$

(3.6)

From Eq. 3.6, it can be seen that setting both $r_1(t)$ and $r_2(t)$ equal to $t$ and $\sqrt{t}$ respectively gives a constant angular-velocity spiral and a constant linear-velocity spiral. Constant linear velocity was chosen because it is more desirable\(^6\) The ideal PSF is a delta function $\delta(\vec{r})$.\n
---

\(^6\)The ideal PSF is a delta function $\delta(\vec{r})$.\n
28
from the SNR point of view as k-space data are more equally weighted with more points taken further away from the origin [13]. However, choosing \( r_1(t) \) and \( r_2(t) \) to be \( \sqrt{t} \) overdrives the gradients and violates the rise-time constraints because the gradients initially start at full amplitude. The algorithm used to generate risetime-limited spiral gradients described in [23, 24] was adopted to design realizable gradients that best approximate the ideal condition \( \tau(t) = \sqrt{t} \). Given the maximum gradient amplitudes, slew-rates and a given \( \tau_1(t) \), the algorithm iteratively modifies the given function to a new \( \tau'_1(t) \) while ensuring the resulting gradients are not over-driving the gradient system. The final gradients can be expressed as[24]:

\[
g_x(t) = \mu_1(t)\sin(\omega t_2(t)) + \mu_2(t)\cos(\omega t_2(t)) \tag{3.7}
\]

\[
g_y(t) = \mu_1(t)\cos(\omega t_2(t)) - \mu_2(t)\sin(\omega t_2(t)) \tag{3.8}
\]

where \( \mu_1(t) \) and \( \mu_2(t) \) are functions of time derived from the given \( \tau_1(t) \) and \( \tau_2(t) \) using Eq. 3.5. These gradients are initially rise-time limited and then constant amplitude limited for the rest of the scan. An example of a 20-interleaved spiral with FOV=12cm is shown in Fig. 3.3. The high gradient amplitude maintained for most of the scan time offers a higher SNR given that a wider bandwidth is required for spiral acquisition (section 2.3.3).

To traverse all the \( n_l \) interleaves in k-space, a complete set of \( n_l \) readout gradients for each excitation can be individually calculated. Another way is to take one set of \( x \) and \( y \) gradients and shift them with respect to each other for interleaving. However, the simplest way is to make use of the imager hardware waveform multiplier [18] which rotates the logical axes of the imager between successive excitations. This has the same effect as rotating the k-space using the same gradients. This approach is used here because it greatly simplifies
Figure 3.3: Spiral Readout Gradients for Spiral RF-CDI system designed for nI=10 and FOV=12cm. They were designed using the method described in [23, 24] and are mathematically expressed by Eq. 3.7 and Eq. 3.8. The gradients are risetime limited initially and then constantly amplitude limited to the capability of the Signa gradient system at 1.0 G/cm. The total readout time synchronized for data acquisition is 17.49 ms. Following the readout portions are the gradient rewinders (trapezoids) with an area that bring the k-space trajectory back to the origin prior to successive excitations. They ensure that every excitation starts from the k-space origin, a crucial requirement for the prevention of phase accrual.
Figure 3.4: Spiral k-space trajectories for Spiral RF-CDI system designed for \( n_l=10 \) and FOV=12cm. The trajectory magnitude is normalized and is derived from the gradients in Fig. 3.3 using the inverse relation depicted in Eq. 3.5.
the design process as one set of pre-calculated $x$ and $y$ gradients suffices $nl$
excitations.

In this thesis, spiral gradients for FOV=8cm, 12cm, 20cm and 36cm
with $nl = 6, 10, 16$ and 20 were developed for the Spiral RF-CD1 system. The
technical details for generating these gradients for other required FOV and
$nl$ are given in Appendix B.4. The detailed calculations for generating spiral
gradients can be found in [13, 23, 24].

3.2.3 Spiral Reconstruction

The gridding algorithm[17] is used to interpolate k-space data onto
Cartesian grids for 2D-DFT before the required CDI post processing. It can
be summarized as a convolution of the sampled k-space data with a kernel
followed by sampling onto rectangular grids. Mathematically, with $M_{sp}(u, v)$
denoting the spiral samples falling on the grids $S(u, v)$ traced out by the spiral
in Eq. 3.4 and $C(u, v)$ denoting the chosen 2-D kernel, the interpolated k-space
data $M_{grid}(u, v)$ are:

$$M_{grid}(u, v) = [M_{sp}(u, v) * C(u, v)] \cdot \Pi(u, v)$$

(3.9)

with the sampling function $\Pi(u, v)$ given by the 2-D comb function

$$\Pi(u, v) = \sum_{i} \sum_{j} \delta(u - i, v - j)$$

(3.10)

To compensate for the non-uniform sampling density, the spiral samples are
weighted by an area density function $\rho(u, v)$ defined by:

$$\rho(u, v) = S(u, v) * C(u, v)$$

(3.11)

The density-compensated k-space data $M_{sp}(u, v)/\rho(u, v)$ is then substituted
into Eq. 3.9 to calculate $M_{grid}(u, v)$.
Theoretically, the ideal convolution kernel \( C(u, v) \) is the sinc function of infinite extent [17]. For practical calculations, it has to be truncated into a finite kernel. The zero-order Kaiser-Bessel function of the first kind:

\[
\frac{1}{W} I_o \left[ \beta \sqrt{1 - (2u/W)^2} \right]
\]  

(3.12)

with width \( W \) and \( u \) defined for \( u \leq W/2 \) is chosen for \( C(u, v) \) because of its good performance [17]. This finite convolving function however contributes side-lobes that will be aliased back into the imaged FOV. The roll-off in the central lobe of \( c(x, y) \) will also attenuate the edge of the spatial image. To compensate for this attenuation, the final spatial image is first divided pixel-wise by \( c(x, y) \) which has the effect of flattening the effective principal transfer function (PTF). To avoid aliasing, the spatial image is masked by the 2-D rect or boxcar function \( \Pi^2(x, y) \). Incorporating these two operations into Eq. 3.9 and noting the convolution multiplication duality in the frequency and spatial domains, the final spatial image obtained from the gridding algorithm is:

\[
m_{grid}(x, y) = \frac{\Pi^2(x, y)}{c(x, y)} \text{IDFT}_{(x,y)} \left( \frac{M_{sp}(u, v) \ast \ast C(u, v)}{S(u, v) \ast \ast C(u, v)} \Pi(u, v) \right)
\]  

(3.13)

Equation 3.13 is the complete gridding algorithm used to obtain the final 2-D spatial MR images from the spiral k-space data. The \( m_{grid}(x, y) \) interpolated in Eq. 3.13 is post-processed according to Eq. 2.6 to obtain the final current density images.

**Practical Implementation**

Technically, there are two modes of image reconstruction - online and off-line. Online reconstruction means producing an MR image on the imager console for on-site viewing and debugging. This is normally done by the imager
itself using its own proprietary reconstruction firmware. Off-line reconstruction refers to any other means of obtaining images directly from the imager raw-data file using user-written routines. Both off-line and online spiral reconstruction are realization of the algorithm described in Eq. 3.13. To make online spiral reconstruction possible, an experimental reconstruction package developed in [13] exclusively for Sigma was adopted. It produces MR magnitude and phase images which reveal many useful debugging information for the new RF-CDI system (Appendix B.5). The operations and limitations of this unsupported software were explored experimentally and given in appendix B.2. To obtain final CD images, off-line gridding was performed on the spiral data followed by post processing done in the CDI graphical user interface (GUI)[3].

### 3.3 Performance of the Spiral RF-CDI System

The purpose of this section is to describe the methodology used to measure and analyze the performance of the above-mentioned spiral RF-CDI system. The experimental setting is first described followed by some experimental current density images and a detailed performance analysis and comparison with theory with respect to the performance criteria described in section 2.2. It is based on this analysis that the spiral RF-CDI system is further enhanced, as will be described in Chapter 4.

#### 3.3.1 Methodology for Phantom Studies

To measure the performance of the spiral RF-CDI system, imaging experiments were performed on a tri-concentric tube phantom constructed from plexiglass as shown in Fig. 3.5. The phantom was filled with $CuSO_4$ doped
saline solution in the innermost tube and aqueous $CuSO_4$ in the outer two annular compartments. The resulting electrolyte in the outer tubes was a 4 mM $CuSO_4$ (4.0 g per 1 L) solution with relaxation times $T_1 \approx 340 ms$, $T_2 \approx 270 ms$ and a complex conductivity $\sigma \approx 0.38 \angle 84.6^\circ S/m$ at the 63.6 MHz Larmor frequency of the Signa 1.5 T imager. The extra doping in the innermost tube by the addition of 0.154 mM NaCl solution (9.0 g per 1 L) increased the complex conductivity approximately four times to $1.53 \angle 14.5^\circ S/m$ at the Larmor frequency. The conductive pathway within the phantom was established by copper electrodes covering only the inner and the middle annular regions. The return wires were connected to a BNC connector which provides the front end connection for the phantom.
**Phantom Wiring and Setup for External Current Source**

To avoid possible RF coupling between the phantom and the transmit/receive coil, the return wires were re-soldered and carefully re-routed to ensure that the entire current return path lies on a single plane so that the phantom as a whole can be rotated to align the wire loop for minimal RF coupling. It has been found experimentally that careful routing of return wires with straight wire segments and sharp 90° bending at corners along the loop significantly reduces random noise and artifact in the final CD images [25].

To inject current into the phantom, a RG58/U 50 Ω coaxial cable of half wavelength at the Larmor frequency was used to connect the phantom to a PI-matching network which was in turn connected to the AMT 3206 RF amplifier via a patch panel that was grounded to the shielding of the imaging room. The purpose of the matching network was to match the impedance of the phantom seen by the amplifier to 50∠0°Ω for maximum power transfer and minimum signal reflection. The AMT 3206 RF amplifier [26] with blanking capability was used to deliver synchronized RF current to the phantom. The synchronization was brought about by the current pulse control box (CPCB) which interfaces the RF amplifier with the internal clock and the Integrated Pulse Generator (IPG) of the imager. The detailed connections between the imager, the CPCB and the RF amplifier is illustrated in appendix B.1.

**Phantom Placement and Choice of RF Coil**

The imaging sequence described earlier was used to scan the tri-concentric phantom using the standard extremity transmit and receive coil. The extremity coil was chosen because compared to the head or body coil, it gives higher

---

7 Coils are antenna that transmit RF pulses and/or receive FID signals from the object.
SNR and pixel intensity in the signal regions of the MR images [3]. Its physical specifications (a 16 cm diameter vertically linear polarized saddle coil) also justify this choice because it housed the 12 cm diameter phantom in closer proximity for better signal sensitivity and MR noise suppression. To avoid any possible RF coupling, the phantom was rotated with the plane containing the three return wire segments aligned with the polarization of the extremity coil (vertical). This leaves two possible choices of running the axial return wire segment either above or below the phantom. It was experimentally determined that images with better visual quality and signal uniformity were obtained under the later configuration with the axial wire running closer to the center of the RF coil. This observation reinforced the fact that non-uniformity inside most RF coils is spatial dependent and increases away from their centers. The phantom was thus oriented in this way inside the extremity coil for minimal non-uniform RF complications. This represents the standard phantom setup for all subsequent experiments.

**Imaging Parameters & Spiral Sequence Fine Tuning**

To compare the performance of the new spiral CDI system with its conventional Fourier spin echo counterpart [3], typical imaging parameters for previous phantom studies were selected: $T_E = 25 \text{ ms}$, $T_R = 300 \text{ ms}$, $NEX_F = 2$, $FOV = 12\text{cm}$, $\delta Z = 10 \text{ mm}$. The strength of the rotary echo pulse $B_1$ was 800 Hz and its duration was 20 ms.

In each of the $n_l$ excitation, 2187 k-space samples were acquired at a sampling rate of 125 kHz. Depending on the exact FOV (8cm, 12cm, 16cm and 20cm), rewinders of different sizes were appended to the readout portion of the encoding gradients, making the final gradients waveforms with varying total durations. In this thesis, $n_l = 6, 10, 16, 20$ representing different
imaging time were studied. The spiral readout gradients were advanced in
time by $\delta t_{sp}$ and were played out at $T_E - \delta t_{sp}$ because experimental obser-
vations showed that this shifting alone significantly enhanced the sharpness
and the high frequency details of the MR images. Advancing these compo-
nents in time probably achieved desirable k-space weighting since k-space data
further away from the origin (i.e. high frequency components) were taken at
higher signal strength when transverse magnetization exactly refocused at $T_E$.
This may account for the observed improvement in image detail. Using the
visual quality of the standard GE phantom image (Fig. 4.1) for fine-tuning,
the optimal $\delta t_{sp}$ was iteratively determined to be $448 \mu s$. The data acquisi-
tion, however was delayed by $\delta t_{DAB}$ with respect to the readout gradients and
started at $T_E - \delta t_{sp} + \delta t_{DAB}$ to account for the hardware instruction delay
between playing out the readout gradients and starting the data acquisition.
The optimal $\delta t_{DAB}$ was iteratively found to be $48 \mu s$. This slight shift in the
relative placement of the readout gradients and data acquisition compensated
for the hardware limitations of the imager and were incorporated in all imag-
ing experiments.

**Image Reconstruction**

The spiral data were gridded onto 256x256 Cartesian grids in accordance
with Eq. 3.13. The phase images were then post-processed UN-truncated and
unpadded according to Eq. 2.6 to obtain 256x256 CD images.

**3.3.2 Results of Phantom Studies**

The new spiral RF-CDI system operated under the above-mentioned
experimental setup was used to obtain current density images with different
number of interleaved spirals and different current levels. To illustrate the visual image quality, two examples of current density images taken with different $nl = 10$ and 16 are shown in Fig. 3.6 and 3.7 together with their complete MR data set.

These images are illustrative as they demonstrate the common features in spiral imaging. The magnitude images exhibit significant ringing due to the convolution by a 2-D symmetric kernel when k-space data are interpolated onto Cartesian grids. This has the visual effect of smoothing out the spatial image, giving poorly defined edges and blurring fine image details. From the MR magnitude images, it can be seen that the inner compartment wall which presents a higher curvature (ie. higher spatial frequency) got widened and more blurred than the exterior wall. The sinc like point spread function (PSF) inherent in gridding exemplifies the ringing in these phantom images which display radial symmetry. The phantom proximity contains residual signals coming from the PSF side-lobes, causing the SNR of the final gridded MR image to be significantly lower than that by the 2-D Fourier technique. This implies that spiral RF-CDI has a higher theoretical random noise, as predicted by Eq. 2.18.

Another newly observed phenomenon with spiral RF-CDI is the non-uniform signal attenuation by the rotary echo $B_1$, as seen in the gridded MR magnitude images in Fig. 3.6 and Fig. 3.7. The MR signal loss described by Eq. 2.12 was experimentally observed to be uniform across the entire image in the conventional Cartesian approach [3]. However, the non-uniform MR signal loss even in the absence of $B_1$ and $J_z$ (ie. MR magnitude image with reference phase in Fig. 3.6 and Fig. 3.7) suggests that signal non-uniformity is not RF-CDI specific. Instead, it may be attributed to the non-linear interpolation algorithm or other spiral imaging complications.
Figure 3.6: Current Density Images taken with 10 interleaved spiral excitations ($nl = 10$), $B_1 = 800Hz$ turned on for 20ms. Significant ringing is visible in the MR magnitude images even in the absence of $B_1$. MR signal attenuation by $B_1$ is easily observed when the two MR magnitude images are compared. The signal loss in the central annular region is non-uniform, which is a new phenomenon with Spiral CDI. The middle column shows the unmasked current-encoded phase images. The right column shows the masked magnitude and phase of the RF current. Averaged $|J_z|$ in the central annular region is $510A/m^2$. Artifacts are prominent in the final current density image. Current ramps in the central annular region exists where current density is expected to be constant. This current density image is taken in 18 seconds at a $T_R$ of 300ms.
Figure 3.7: Current Density Images taken with 16 interleaved spiral excitations ($nI = 16$) and $B_1 = 800Hz$ turned on for 20ms. Compared to Fig. 3.6, ringing in the magnitude image is alleviated with increasing number of spirals. Uneven MR signal loss still persists. However, current density is more uniform with less artifacts in the inner tube of the phantom. Averaged $|J_z|$ in the central annular region is 504$A/m^2$. Imaging time for this CD image is 28 seconds at a $T_R$ of 300ms.
An observable artifact can be seen in the inner tube of the phantom in the current density image of Fig. 3.6 with \( nl = 10 \). However, increasing the number of interleaved spiral at the expense of a longer imaging time significantly reduced the artifact, as shown in Fig. 3.7. This suggests that the artifact may arise from undersampling, even though an analytical approach to confirm this underlying cause is not readily available. This observation highlights the possible tradeoff between visual image quality and temporal resolution, an issue that will be investigated further in section 3.3.4 and Chapter 4.

### 3.3.3 Analytical Performance in Phantom Studies

Visual quality is an important aspect for the creditability of spiral RF-CDI. This section gives an analytic assessment of the performance of the spiral system using the criteria defined in section 2.2.

**Sensitivity Analysis**

The region of interest used for analysis was a 8x8 square patch in the central annular region where conduction current was expected to be maximum and uniform. Current density images were taken for various current densities and different number of spirals under identical experimental conditions. The performance curve for the spiral RF-CDI system depicting the standard deviation of the current (Eq. 2.20) and sensitivity (Eq. 2.22) is given in Fig. 3.8.

All of the data points in Fig. 3.8 came from current density images with observable artifacts. An example was already shown in Fig. 3.6. The artifact, though small and confined within a relatively uniform region, gives a significant \( \sigma_J \) upon calculation and renders a relatively poor sensitivity figure. The four interpolated performance curves demonstrate the tradeoff between CD
Figure 3.8: Sensitivity Analysis for Spiral RF-CDI. The region of interest is a 8x8 square patch in the central annular compartment. Operating conditions stated in section 3.3.2 were repeated for various number of spirals (nI) and different operating current densities. The standard deviation was calculated based on Eq. 2.20. The number associated with each data point is the sensitivity defined in Eq. 2.22.
sensitivity and imaging time. It is observed that increasing $nl$ from 6 to 10 offers the greatest reduction in $\sigma_J$ over the lower operating current range. With higher operating current, increasing $nl$ from 10 to 16 shows more effective cuts in $\sigma_J$. This observation reveals that the optimal number of interleaves depends on the current magnitude used in a particular scan. However, the sensitivity was found decreasing with increasing current for any choice of $nl$, as seen in Fig. 3.8.

**Current Independent Artifacts**

The current variance in the central region of interest in any single CD image is contributed by two components: a systematic current independent artifact and a current dependent artifact. The former is present in all MR technique and it comes from the imperfection of the imager and the signal acquisition scheme. These errors exist in the MR component images and propagate through the k-space interpolation and CDI post-processing algorithms. The current dependent component, however, arises from the magnetic field distortion due to the RF current. The most prevalent example is the phase distortion artifact which occurs when the rotary echo criterion is violated.

The investigation of the current independent artifact provides insights into the additional artifact introduced by the extra k-space gridding in spiral imaging. To measure the systematic current independent artifact, the same experimental procedures were repeated except with the RF-amplifier input terminated on a 50 $\Omega$ load. The resulting images were similarly analyzed for sensitivity. The current independent artifact, which is the measured current standard deviation under zero applied RF current, is tabulated in table 3.2 for different number of excitations.

The results in table 3.2 show that the current independent artifacts
diminish with increasing number of spirals. In comparison with the same artifact arising from the conventional Cartesian approach, these values are 2 to 3 times higher. These artifact figures also closely coincide with the extrapolated values of $\sigma_J$ in Fig. 3.8 when $J_z$ is zero.

### 3.3.4 Discussion and Conclusions

The relationship between $\sigma_J$ and $J_z$ for $200 A/m^2 < J_z < 1000 A/m^2$ is different from that of the Cartesian system. It was found experimentally in [3] that a linear and more gentle increase in $\sigma_J$ with $J_z$ is possible using identical experimental parameters with Cartesian acquisition. Magnetic field mapping had been used in [3] to analytically verify that phantom studies under these chosen experimental conditions satisfy the rotary echo criterion with $B_J/B_1 \approx 0.19$. The absence of a threshold current in Fig. 3.8 after which $\sigma_J$ starts to increase sharply further indicates that the error in $J_z$ is not dominated by the primordial phase distortion artifacts. This artifact is new and specific to the spiral scheme. It has yet to be identified and reduced.

The sensitivity as a function of imaging time is plotted in Fig. 3.9. Since the sensitivity decreases with current amplitude, the figures shown for each $nl$ are averaged over the entire tested current range. The plot also includes the point representing the performance of the existing Cartesian approach. With

<table>
<thead>
<tr>
<th>Operating Parameters</th>
<th>Standard Deviation [$A/m^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral, $nl=6$</td>
<td>28.6</td>
</tr>
<tr>
<td>Spiral, $nl=10$</td>
<td>25.9</td>
</tr>
<tr>
<td>Spiral, $nl=16$</td>
<td>15.7</td>
</tr>
<tr>
<td>Spiral, $nl=20$</td>
<td>16.7</td>
</tr>
<tr>
<td>Cartesian 256x128</td>
<td>8.8</td>
</tr>
</tbody>
</table>
increasing $nl$, the spiral RF-CDI sensitivity is asymptotically approaching the 2% sensitivity promised by the Cartesian approach. However, excessive spiral excitations entail long imaging time and deprive the creditability of this fast imaging technique.

Further design strategy should therefore aim at improving the existing spiral system without relying on more spiral excitations. Previous observations suggest that the artifacts are new and they arise from the new spiral imaging scheme which is highly susceptible to field inhomogeneity and off-resonance complication. Since the quality of the final CD images ultimately depends on the quality of the current-encoded MR images, solutions that restore the quality of the MR images are anticipated to improve CD image quality upon post-processing.

### 3.4 Summary

The following summarizes the design and the performance of the basic spiral RF-CDI system.

**Spiral RF-CDI Pulse Sequence**

The sequence was adapted from the existing RF-CDI system with the following enhancement and new design components:

- The prescan tuning procedure for RF pulse scaling was modified to accommodate higher $B_1$ up to a maximum of 1000 Hz. This extends the range of feasible $|J_z|$ without phase distortion artifacts.

- The selective 180° refocusing pulse was lengthened in duration to obtain sharper slice profile beneficial for imaging inhomogeneous media.
Figure 3.9: The sensitivity is averaged over the images taken with $J_z$ ranging from $200 A/m^2$ to $1000 A/m^2$ for different $n_l$. Operating conditions stated in section 3.3.2 were used. The point representing the existing Cartesian RF-CDI system adapted from [3] is shown to illustrate the substantial gain in imaging speed with the spiral system. The interpolation curve terminates shortly outside the tested imaging time as the relationship between imaging time and sensitivity cannot be accurately predicted by extrapolation.
• The spiral readout gradients were specifically designed for the given rise-time and amplitude constraints. Their placement in time relative to the other excitation pulses were iteratively optimized for the best visual quality of the MR magnitude images.

• The size of the crusher gradients for adequate off-slice dephasing was analytically determined to reduce systematic artifacts.

The above design gives a rudimentary RF-CDI system with the following qualitative and quantitative performance:

• The visual quality of CD images are acceptable and free from major artifact only if 16 or more spirals are used, corresponding to an imaging time of 28 seconds or more with $T_R$ at 300 ms.

• The systematic current independent artifact is 2 to 3 times more compared to the conventional Cartesian approach under identical operating parameters.

• The sensitivity can be arbitrarily traded off with imaging time. For CD images with acceptable visual quality, 16 or more spirals are necessary. This represents an eight times gain in imaging speed at the expense of a six times decrease in sensitivity compared with the original Cartesian approach.

As this technique will be used ultimately in monitoring dynamic conductivity changes in the brain cortex, the required sensitivity depends on the specific investigation in question. An example in monitoring cortical conductivity changes during spreading depression requires a minimum sensitivity of 10%. The basic spiral RF-CDI system at this point is functioning but it fails
to give the required sensitivity and visual image quality. The next chapter is devoted to improve the performance of the existing system so that it meets the required sensitivity with the highest possible temporal resolution.
Chapter 4

Enhancement for Spiral RF-CDI

This chapter describes the design strategy that improves the performance of the basic spiral RF-CDI system. It begins with the rational for the approach used to hypothesize the causes of the new artifacts in spiral CD images. The hypotheses are then verified by theories. An appropriate solution is proposed and integrated into the existing system. The performance of this enhanced system is re-evaluated by phantom studies. It will be shown that the improved system achieves much higher sensitivity for any imaging time. Compared to its conventional Cartesian counterpart, the improved system achieves a promising temporal resolution and sensitivity tradeoff that meet all the required specifications outlined in chapter 2. With this enhanced system, the goal of this project is accomplished.

4.1 Approach Rational

As CD images are obtained from derivative operations on the MR phase images, it follows that improving the MR raw images as the starting point for
the differentiation process yields better CD images upon post-processing. The enhancement strategy therefore starts with visually identifying the new artifacts in the raw images produced by the spiral RF-CDI sequence. Theories are then sought to explain these artifacts. Once their nature is identified, an appropriate modification, known to alleviate them in other spiral imaging applications, is incorporated into the spiral RF-CDI system. The performance of the enhanced system is then re-evaluated analytically. Any observed improvement confirms the hypotheses as to the causes of the artifacts and produces a better spiral RF-CDI system.

4.2 Artifacts in Spiral RF-CDI

4.2.1 Visual Perception of Spiral RF-CDI Artifacts

With reference to the current-encoded and reference MR magnitude images in Fig. 3.7, there are several observations specific to the new spiral RF-CDI system. They are listed below followed by an explanation for each.

- Significant ringing in the proximity of the phantom.
- Non-uniform MR signal across the theoretically homogeneous phantom.
- Blurred and widened walls between the three concentric compartments, with more blurring around the higher curvature inner wall.
- In the current-encoded magnitude image, the MR signal is least uniform in the central annular compartment where \( J_z \) is highest and in the proximity of the return wire (underneath the phantom) which has the largest residual \( \partial H_z/\partial z \) component.
The first observation may arise from the finite sinc like point spread function (PSF) of the gridding algorithm which spreads out unwanted residual signal to the proximity of the phantom. Due to the large bandwidth used in the receiver, the acquired data are vulnerable to system noise coming from the imager hardware. Spikes, which are erroneous k-space data of unreasonable magnitude, are found when a line of spiral data are analyzed. The second observation may be attributed to the smoothing of these spikes by the PSF, causing confined patches of signal non-uniformity within the phantom. Any means that broadens the PSF main lobe or increases its side lobes will exacerbate these artifacts and degrade visual image quality as seen in Fig. 3.7. To obtain good visual CD images, it is essential to eliminate complications that broadens the effective PSF of the system.

The fact that higher curvature was blurred to a larger degree suggests that fine image details consisting of higher spatial frequency were selectively degraded. This may be due to undersampling high frequency regions of k-space that are far away from the origin. The sampling pattern described in section 3.2.2 theoretically covers the entire k-space without undersampling (see appendix A.2.2). Any imperfection that deviates the actual k-space trajectories from the theoretical trajectories as shown in Fig. 3.4 may elicit localized undersampling that violates the Nyquist sampling criterion[27]. On top of the detrimental effects of undersampling, this trajectory distortion also impairs the accuracy of the gridded images when actual data acquired along the deviated trajectories are mistakenly processed as if they lie along the theoretical interleaves. Therefore, mechanisms that distort the k-space trajectories should be eradicated.

The field produced by the RF conduction current and the return wire perturbs the local field homogeneity when superimposed onto the imager static
field $B_0$. The observed correlation between the extent of signal non-uniformity and current magnitude in different regions of the images supports the hypothesis that MR signal attenuation in spiral RF-CID images comes from static field inhomogeneity. In addition, a direct comparison of the two MR magnitude images in Fig. 3.7 reveals an exacerbation of their common artifacts when current is flowing. These two observations reinforce field inhomogeneity as the main cause of MR signal attenuation under spiral imaging. It is hypothesized that eliminating the effects of field inhomogeneity may restore the MR signal. In light of Eq. 2.18, this will improve the random noise and artifact performance of the final RF-CID system.

In summary, observations from the artifacts in the spiral MR magnitude images establish the need for the system to:

1. Avoid excessive spreading of its effective PSF.
2. Minimize deviation between the actual and theoretically required k-space trajectories to avoid undersampling and error in gridding.
3. Minimize and/or correct for $B_0$ field inhomogeneity in the imaged FOV.

### 4.2.2 Underlying Causes of the Observed Artifacts

This section briefly discusses the theories behind factors that may affect the PSF and the k-space trajectories in spiral imaging. These theories verify the hypothesis made in the last section and propose a solution for these observed artifacts.

**Broadening of Effective PSF**

Off-resonance effects are the principal challenges in spiral imaging. In
conventional Cartesian approach, off-resonance causes a local shift in the direction of the readout gradient without geometric distortion [29]. For any pulse sequence based on non-2DFT acquisition, the same off-resonance can cause significant local blurring [28] similar to that observed in Fig. 3.7.

There are two main sources of off-resonance. The largest source is due to the 3.4 ppm chemical shift between fat and water, corresponding to a 216 Hz offset under Signa at a Larmor frequency of 63.6 MHz. The second source comes from static field inhomogeneity. Typically, it is in the order of 1 ppm after automatic shimming[20]. For all phantom experiments in this thesis, any possible off-resonance must arise from static field inhomogeneity as the phantom used is aqueous and homogeneous.

An analytical study conducted in [28] has simulated the PSF of a spiral scan in the presence of a first order linear static field inhomogeneity $\Delta H(\tilde{r})$. The phase rotation $\Delta \phi$ caused by $\Delta H$ during a single FID of duration $T$ is:

$$\Delta \phi = \gamma \Delta H(\tilde{r}) T$$  (4.1)

It is necessary to satisfy $\Delta \phi << 2\pi$ to avoid any zero and first order phase accrual that broadens the PSF. For the gradients in Fig. 3.3, this requires $\Delta H(\tilde{z}) \ll 1.5 \times 10^{-2} G$, corresponding to a stringent $\ll 1$ ppm requirement on a 1.5 T system. This is beyond the shimming capability of existing imagers. Based on the first order simulation in [28], violation of this condition results in a broadened PSF shown in Fig. A.5. This simulation is an under-estimate for RF-CDI because any axial magnetic field produced by the RF current and the return wire ($\partial h_z/\partial z$) may induce higher order inhomogeneity, resulting in complex distortions which cannot be easily predicted by simulation. It is hypothesized that the problem of a broadened PSF reduces to problem of field

\footnote{Inhomogeneity is expressed as part per million of the static field strength.}
ihomogeneity.

**Skewed k-space Trajectory**

Undersampling is hypothesized in the last section as one of the reasons for the poor CD images shown in Fig. 3.7. An analytical approach done in [27] confirmed that time-varying spiral gradients in the presence of field inhomogeneity skews the $k$-space trajectory, resulting in undersampling and images with typical artifacts as those seen in Fig. 3.7. Under a spatial and time varying field inhomogeneity $\Delta H(\vec{r},t)$, the effective complex gradient $\vec{G}_r(t)^2$ realized is:

$$\vec{G}_r(t) = G_r(t) + \text{grad} \Delta H(\vec{r},t)$$

Upon the inverse relation in Eq. 3.5, the $j$-th actual sample $k(t_j)$ along the $k$-space trajectories are shifted:

$$k(t_j) \rightarrow k(t_j) + \gamma \int_0^{t_j} \text{grad} \Delta H(\vec{r},t) \, dt$$

Even though the measurement of the imager intrinsic $\Delta H(\vec{r})^3$ is possible for super-conducting magnets, exact $\Delta H(\vec{r},t)$ under RF-CDI settings cannot be determined practically due to the external current, making $\Delta H(\vec{r},t)$ time and current pattern dependent. Neglecting the complications due to the RF current, a first order analysis in [27] reveals a skewed $k$-space trajectory under a moderate (2 ppm) inhomogeneity as shown in Fig. A.6. There are two types of deformation: radial and angular deformation. The angular deformation is not a uniform rotation which would only produce a corresponding rotation in the reconstructed image. The nature of the distortion is such that the angular

\[^2\vec{G}_r(t) = G_x(t) + jG_y(t).\text{ It is the complex representation of the two orthogonal gradients.}\]

\[^3\text{The imager static field inhomogeneity is assumed time independent.}\]
sampling rate has increased on one half-plane of $k$-space. Compounded with the radial distortion, the distances between certain samples exceed the upper limit imposed by Nyquist criterion. This confirms the nature of the visual distortions in the MR images of Fig. 3.7.

4.2.3 Artifacts Elimination Strategy

The above discussions all converge to the need for minimizing the effects of field inhomogeneity, which is supported by simulations as the principal cause of the visual artifacts identified in section 4.2. Since the static field cannot be shimmed to a required homogeneity in the prescan due to hardware limitations, the only solution is to correct the acquired $k$-space data prior to interpolation, as described in the next section.

4.3 Inhomogeneity Correction for RF-CDI

4.3.1 Choice of Correction Algorithm

There are various inhomogeneity correction algorithms with different performance and computational complexity. These algorithms all include an extra acquisition and gridding reconstruction for a field map. Examples are frequency-segmented reconstruction, time-segmented reconstruction [30] and linear field map correction [20]. Best deblurring is possible with the two segmentation schemes but they are too computationally heavy and their run times are impractically long in particular for RF-CDI where six MR images are required to reconstruct one CD image.

The linear field map method is generally less robust but it yields visually comparable images in less than one-tenth the run time of the segmenta-
tion methods. It also outperforms them in places where field inhomogeneity changes abruptly. It was shown in [20] that this new algorithm has a high efficiency and it provides the most important image quality improvement within the least incremental reconstruction time. This method was therefore chosen and incorporated in the RF-CDI system.

4.3.2 Principles of Linear Field Map Correction

This section briefly summarizes the operating principles of the linear field map correction algorithm. Extensive references can be found in [20].

Inhomogeneity Modeling and Actual Correction

In the presence of field inhomogeneity which causes a local frequency deviation $f(x, y)$ from the demodulation frequency (Larmor frequency), the FID from an object $m(x, y)$ ignoring $T_2$ relaxation and the effects of the rotary echo $B_1$ is:

$$ s(t) = \int \int m(x, y) e^{-2i\pi(k_x(t)x+k_y(t)y+tf(x,y))} dx \ dy $$

where $k_x(t)$ and $k_y(t)$ are the desired $k$-space trajectories encoded by the spiral gradients in Eq. 3.5. Field inhomogeneity $f(x, y)^4$ under linear field map correction is modeled as:

$$ \hat{f}(x, y) = f_0 + \alpha x + \beta y $$

Combining Eq. 4.4 and Eq. 4.5, the distorted FID $s'(t)$ is:

$$ s'(t) = e^{-2i\pi f_0} \int \int m(x, y) e^{-2i\pi(k'_x x+k'_y y)} dx \ dy $$

Field inhomogeneity $f(x, y)$ expressed in Hz is given by $\gamma \Delta B_0(x, y)/2\pi$. 

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The last two equations completely define the operations of the linear field map correction. The first step is to demodulate the FID by a mean off-set frequency $f_o$ as depicted in Eq. 4.6. The second step is to correctly skew the spiral trajectories according to Eq. 4.7 to avoid improper assignment of data in $k$-space. The $k$-space data from the demodulated $s(t)$ is then gridded according to Eq. 3.13 assuming the corrected trajectories. The next section describes how to obtain the required parameters to do the correction.

Field Map Computation and Linear Fit

The off-resonance frequency map $f(x, y)$ in the presence of field inhomogeneity is computed from two images acquired at $TE$ and $TE + \Delta T$. If the two spatial images are $M_1(x, y) = m_1(x, y)e^{i\phi_1(x,y)}$ and $M_2(x, y) = m_2(x, y)e^{i\phi_2(x,y)}$, the field map $f(x, y)$ is:

$$f(x,y) = \frac{\phi_2(x,y) - \phi_1(x,y)}{2\pi \Delta t}$$  (4.8)

The actual field map in Eq. 4.8 is fit into a linear model in Eq. 4.5 using the parameter vector $\theta = [f_o \alpha \beta]$ obtained from the maximum likelihood $\chi^2$ fit:

$$\min_\theta \chi^2(\theta) = \sum_{i=1}^{N} \left( \frac{f(x_i, y_i) - \hat{f}(x_i, y_i)}{\sigma_i} \right)^2$$  (4.9)

where $\sigma_i = 1/m(x,y)$ are the weights for different pixels, accounting for the fact that map data are less accurate in regions of low MR intensity. The detailed mathematical solution for $\theta$ can be found in [20]. The parameters obtained from Eq. 4.9 are substituted into Eq. 4.6 and Eq. 4.7 which completely define the operation of the correction algorithm.
4.3.3 Practical Implementation in Spiral RF-CDI

To accommodate the linear map correction algorithm, the spiral RF-CDI system developed in chapter 3 were modified in two main aspects:

1. Pulse sequence: Acquisition of a frequency map under steady state.
2. Post Processing: Demodulation and k-space skewing prior to gridding.

Pulse Sequence Modification

The pulse sequence first acquired the frequency map which was used to correct for all the six data images obtained thereafter. All the existing elements of the pulse sequence as described in section 3.2.1 were unchanged for the map acquisition except the spiral readout gradients. To minimize the time penalty, these gradients were redesigned for a lower resolution to allow each of the two MR images\(^5\) used in calculating the entire frequency map to be scanned with one single spiral excitation. A low resolution\(^6\) frequency map is justified because field inhomogeneity is assumed to be a slowly varying function.

The delay between the two acquisitions \(\Delta T\) was set at 5 ms. From Eq. 4.8, the resulting field map can correct for inhomogeneity up to \(\pm 100\text{Hz}\) without phase wraps. In the absence of the 3.4 ppm shift between fat and water as in all the homogeneous phantom experiments, this correctional bandwidth is adequate. A slightly longer \(\Delta T\) for a larger correctional bandwidth is appropriate when imaging inhomogeneous media. However, unnecessarily long \(\Delta T\) impairs the accuracy of the frequency map due to \(T_2\) relaxation.

\(^5\)As indicated in Eq. 4.8, two MR images \(M_1(x, y)\) and \(M_2(x, y)\) are necessary for calculating the frequency map.
\(^6\)In this thesis, frequency map resolution is 32 by 32 for all FOV.
In the spiral RF-CDI sequence, the frequency map was taken under steady state with $B_1$ turned off. The sample was excited twice for steady state before the map was calculated from the third and forth MR images. The six data images were then acquired thereafter.

**Post-Processing with Frequency Map Correction**

The post processing is similar to that described in chapter 3 but with the following extra steps. The frequency map is first calculated from the two low resolution MR images. The $\chi^2$ fit is performed to obtain the parameter vector $\theta$ of the linear map. Each of the $nl$ FID for the six MR data images is demodulated by the center frequency $f_0$. The k-space trajectories are corrected using the parameters $\alpha$ and $\beta$ as in Eq. 4.7. Gridding then proceeded as in chapter 3 to obtain the MR images which derive the CD images.

4.4 **Performance Analysis of The Enhanced Spiral RF-CDI System**

4.4.1 **Methodology**

The experimental setup and imaging parameters were exactly the same as in chapter 3. This allows a standard comparison between this improved spiral RF-CDI system with its previous version and its conventional Cartesian counterpart. More extensive analytical studies are used here to assess the image quality on top of visual perception. The results obtained are compared to the performance bound predicted by theory.
4.4.2 Visual Enhancement by Field Map Correction

To illustrate the effectiveness of the correction algorithm, a standard GE testing phantom\(^7\) containing sharp edges and fine details was imaged using the spiral RF-CDI system with and without frequency map correction. Different number of excitations were tested and the resulting images are shown in Fig. 4.1.

Frequency map correction was found to greatly improve the visual quality. The corrected image taken with \( nl = 10 \) is visually better than the uncorrected image with \( nl = 20 \). Fine details near the 'comb' and the GE symbol are enhanced and become more conspicuous. Signal intensity is more uniform across the entire image. Many of the artifacts mentioned in section 4.2 are alleviated. The next section demonstrates that the quality of the CD images post-processed from MR images also benefited from the correction algorithm.

4.4.3 Visual Enhancement in Current Density Images

The tri-concentric phantom was imaged using the same experimental settings and imaging parameters as described in section 3.3.2. Two examples of complete data sets for \( nl = 10 \) and \( nl = 16 \) are shown in Fig. 4.2 and Fig. 4.3. Their corresponding field maps are shown in Fig. 4.4.

Visually, these images are much better than the two uncorrected images in Fig. 3.6 and Fig. 3.7 taken with the same or more excitations. The MR images obtained from the enhanced system are less blurry and contain sharper boundaries. Ringing in the signal free regions is greatly suppressed. The MR signal is more uniform in the signal regions as seen in the reference-phase (no

\(^7\)As this phantom contains fine details and other demanding patterns, it was used for fine tuning the placement for spiral gradients as described in section 3.3.1.
Figure 4.1: Comparison between MR magnitude images of a standard GE phantom with (right column) and without (left column) frequency map correction using the Spiral RF-CDI sequence. The FOV is 20 cm, TR is 300 ms, $B_1$ is 800 Hz and $T_c$ is 20 ms but with no applied current. The three sets of images (from top to bottom) were taken with $nl = 10, 16$ and 20 respectively, corresponding to imaging times of 18 seconds, 28 seconds and 36 seconds for the uncorrected images. The imaging time is 1 second longer for any of the corrected images because of the extra excitations for the frequency map.
Figure 4.2: Complete data set for CD images taken from the enhanced Spiral RF-CDI system with n'l = 10 and frequency map correction. The imaging parameters were the same as those used to obtain Fig. 3.6. A direct comparison between these images and Fig. 3.6 reveals significant visual quality improvement in both the MR and CD images. Edges are sharper and more defined. Ringing around the phantom is effectively suppressed. MR signal is more uniform even when current is flowing. The CD image is almost artifact free and the current in the central compartment is uniform. This image was taken in 19 seconds (1 second longer because of the extra excitations for the frequency map) at a TR of 300 ms with a sensitivity of 4.1%.
Figure 4.3: Complete data set for CD images taken from the enhanced Spiral RF-CDI system with $nl = 16$ and frequency map correction. Compared to Fig. 3.7 taken within the same imaging time, these images display similar perceptual improvement as discussed in Fig. 4.2. This CD image was taken in 29 seconds at a TR of 300 ms. The sensitivity is 3.3 %.
current) magnitude images. When current is flowing, the same pattern of signal loss correlating to localized current density persists but the attenuation is less severe. The most important improvement lies in the CD images. There are fewer observable artifacts and current density is more uniform in the central compartment where it is expected to be constant.

With reference to all the four data sets shown, it can be concluded that frequency map correction improves CD images to an extent that is inachievable by merely increasing $n_l$. Yet, the correction algorithm necessitates only four\(^8\) extra excitations for the frequency map. This entails an increase in imaging time that is relatively insignificant in particular for large $n_l$ where many excitations are required\(^9\). Given the significant visual improvement, this benign

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\(^8\)Two excitations are used to promote steady state, followed by another two excitations, one for each of the two low resolution MR images used in calculating frequency map.

\(^9\)Consider the worst case when $n_l = 10$, a total of 60 excitations are required for the six data images. The additional 4 excitations for the frequency map represents only a 6.7% increase in imaging time.
increase in imaging time essential for frequency map correction is justified.

The enhanced Spiral RF-CDI system achieves a promising tradeoff between temporal-resolution and visual quality. To appreciate its performance, a complete CD data set obtained from the conventional Cartesian approach is shown in Fig. 4.5 for comparison. Visually, the CD images from the Cartesian approach and the enhanced spiral system are of comparable visual quality. However, the new system reduces the imaging time by a factor of 13. From the perceptual quality point of view, the creditability of the spiral system is well established.

4.4.4 Analytical Performance in Phantom Studies

The section quantifies the performance of the enhanced system. The focus is to compare the effectiveness of this final ultra fast implementation and the conventional Cartesian approach. It will also be shown that the new system provides an excellent tradeoff between temporal-resolution and sensitivity. It also closely achieves the theoretical noise performance limit.

Sensitivity Analysis

With frequency map correction, the sensitivity calculated from Eq. 2.22 for various \( nl \) at different current levels is shown in Fig. 4.6. Similar to the observations made under no frequency map correction, the sensitivity improves the most when \( nl \) is increased from 6 to 10. The differential performance gain tapers when \( nl \) is increased further. For all \( nl \) tested, the sensitivity is found current dependent in which it is lower at the two extremes of the tested current range and is highest when \( |J_z| \) is between 400\( A/m^2 \) and 800\( A/m^2 \).
Figure 4.5: Complete data set for CD images obtained from the conventional Cartesian system with 128 phase-encoding steps (i.e. excitations) under the same experimental conditions as in Fig. 4.2 and Fig. 4.3. The imaging time was 3 minutes 57 seconds at a TR of 300 ms. Compared to the spiral system with \( n_l = 10 \), this CD image is of similar visual quality with a sensitivity of 1.9% but its imaging time is 13 times longer.
Table 4.1: Current Independent Artifact with Frequency Map Correction

<table>
<thead>
<tr>
<th>Operating Parameters</th>
<th>Standard Deviation [A/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral, nl=6</td>
<td>21.6</td>
</tr>
<tr>
<td>Spiral, nl=10</td>
<td>16.6</td>
</tr>
<tr>
<td>Spiral, nl=16</td>
<td>12.7</td>
</tr>
<tr>
<td>Spiral, nl=20</td>
<td>11.3</td>
</tr>
<tr>
<td>Cartesian 256x128</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Under identical operating conditions\textsuperscript{10}, frequency map correction alone enhances the sensitivity 2.4 to 3.8 times for all \( nl \) across the entire current range. Analytically, this improvement is significant and it further ameliorate the tradeoff between sensitivity and temporal-resolution.

The averaged sensitivity of the improved spiral RF-CDI system as a function of imaging time is shown in Fig. 4.7. The absolute imaging time is based on a TR of 300 ms. The extra point denoting the performance of the Cartesian system is used to illustrate the relative imaging time of the two implementations. The enhanced spiral system meets the minimum sensitivity requirement and it provides a minimum sensitivity of 5\% with 10 or more excitations. It is therefore concluded that the spiral system reduces the imaging time 13 times while fulfilling all performance specifications.

Current Independent Artifacts

The insights provided by this investigation were already discussed in section 3.3.3. The same experimental approach was used and the results after frequency map correction are shown in table 4.1. Depending on \( nl \), the current independent artifact is approximately 1.3 to 2.4 times that of the Cartesian approach. Compared to the previous spiral system, the frequency map correction

\textsuperscript{10}The extra 1 second required for frequency map acquisition is assumed negligible.
Figure 4.6: Sensitivity Analysis for Spiral RF-CDI with frequency map correction. The region of interest is the same 8x8 square used in Fig. 3.8. Different n1 and |Jz| were analyzed. The standard deviation was calculated based on Eq. 2.20. The percentage associated with each data point is the sensitivity defined in Eq. 2.22.
Figure 4.7: The sensitivity as a function of imaging time for the RF-CDI system with frequency map correction. The sensitivity is averaged over all the images taken with the same \( n_I \) for different \( J_z \). Comparison with Fig. 3.9 shows a 2.4 to 3.8 times gain in sensitivity for different imaging time. The point representing the existing Cartesian RF-CDI system is also shown to illustrate the relative speed of the two implementations. The enhanced system achieves the minimum required sensitivity of 10% (see chapter 2) for \( n_I \geq 10 \), corresponding to a minimum imaging time of 19 seconds at a TR of 300 ms.
only reduces this current-independent artifact by 28%\textsuperscript{11}, which is relatively ineffective compared to the 240% ~ 380% gain in overall sensitivity. Frequency map correction is therefore more effective in reducing the current dependent component of the observed artifacts, eliminating which significantly improves the overall system performance.

**Random Noise Performance**

As explained in section 2.2.3, the $\sigma_J$ used in calculating sensitivity comprises of both artifacts and stochastic noise. It is difficult to explicitly quantify the artifact $A_K$ (Eq. 2.15) because the actual current density $J_K$ is not known at every pixel. However, the random noise $n_{i,k}$ can be determined by multiple acquisitions of the same object using Eq. 2.16.

The random noise in CD images ultimately depends on the SNR of the MR images from which they are derived. Given a set of MR images of a known SNR, there exists a theoretical random noise as described in Eq. 2.18 for the resulting CD images. This theoretical value is the lower bound of the stochastic noise that the system can achieve at best. By measuring the SNR of the MR magnitude images defined in Eq. 2.19, the theoretical random noise in CD images can be calculated and compared to the values obtained by actual measurement (Eq. 2.16). This comparison indicates how far the system performance is from perfection.

The measured random noise in CD images taken with different $nl$ at various $J_z$ after frequency map correction are plotted in Fig. 4.8. The theoretical $\sigma_J$ calculated from the SNR measured in their respective MR images are tabulated in table 4.2. In calculating the SNR, the signal region is a 8x8 patch in the central annular tube of the phantom whereas the noise region is another

\textsuperscript{11}An average value for $nl \geq 10$ using the figures from table 3.2 and 4.1.
Table 4.2: Theoretical and Measured Stochastic Noise

<table>
<thead>
<tr>
<th>Operating Parameters</th>
<th>SNR (Linear)</th>
<th>Ave Measured Random Noise ([A/m^2])</th>
<th>Theoretical Random Noise ([A/m^2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral, (n_l=6)</td>
<td>61</td>
<td>5.30</td>
<td>3.15</td>
</tr>
<tr>
<td>Spiral, (n_l=10)</td>
<td>71</td>
<td>4.54</td>
<td>2.71</td>
</tr>
<tr>
<td>Spiral, (n_l=16)</td>
<td>85</td>
<td>3.63</td>
<td>2.26</td>
</tr>
<tr>
<td>Spiral, (n_l=20)</td>
<td>90</td>
<td>3.01</td>
<td>2.14</td>
</tr>
<tr>
<td>2-DFT 256x128</td>
<td>232</td>
<td>1.19</td>
<td>0.83</td>
</tr>
</tbody>
</table>

8x8 patch immediately to the left of the phantom where noise is unaffected by the post-convolution masking\(^{12}\).

From Fig. 4.8, the measured random noise for any \(n_l\) is almost constant\(^{13}\) within measurement accuracy. This constancy agrees with the noise model as this stochastic noise is systematic and current independent. For \(n_l = 10\), the measured random noise is approximately 3 times that of the Cartesian method. This is expected because the SNR is intrinsically lower in the spiral system by approximately the same factor (see table 4.2).

In theory, the SNR of the MR images should vary linearly with \(\sqrt{n_l}\) (section 3.2.2). According to Eq. 2.18, the stochastic noise in CD images should be inversely proportional to \(\sqrt{n_l}\). However, the SNR in table 4.2 is not increasing linearly with \(\sqrt{n_l}\). The lower SNR may be due to the RF current which causes an excessive signal attenuation greater than the theoretical value predicted in Eq. 2.12. However, the measured random noise more closely resembles a linear relationship with \(1/\sqrt{n_l}\) for \(n_l \geq 10\). Despite the discrepancy between the measured and the theoretical relationship of SNR and \(n_l\), the

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\(^{12}\)The masking is due to the boxcar function in the gridding algorithm described in Eq. 3.13.

\(^{13}\)Note that the scale on the figure is very fine. The \(\sigma_I\) variation along any \(n_l\) is upper bounded by \(1.5A/m^2\), which is insignificant relative to \(J_z\) that is two orders of magnitude larger.
Figure 4.8: Stochastic Noise Analysis for Spiral RF-CDI with frequency map correction. The region of interest is a 8 x 8 patch in the central annular region of the tri-concentric phantom where current is highest. Five images (i.e. $N_{images} = 5$) were used to obtain the noise figures according to Eq 2.16. The dotted line shows the performance of the conventional Cartesian approach for comparison.
random noise in CD images still agrees with the trend predicted jointly by the spiral SNR theory and the current density noise model.

The measured random noise is larger than the theoretical value by an average of 56% for $nl \geq 10$, indicating that the system is not functioning ideally. However, the same imperfection exists in the Cartesian system with its measured noise average 44% higher than its theoretical floor. Noting that the random noise of the two systems are higher than their respective theoretical limits by a comparable amount, it is concluded that their stochastic noise performance are similar.

4.5 Safety Considerations in Spiral RF-CDI

Previous discussions show that the enhanced spiral RF-CDI system achieves the temporal-resolution and sensitivity to capture conductivity changes for functional imaging. As this technique will ultimately be extended to in-vivo imaging, its safety\textsuperscript{14} has to be carefully considered. This section shows that the power deposited by the spiral RF-CDI system is within safety limits, which warrants its feasibility for future medical imaging. The tradeoff between power deposition and sensitivity is also studied to ensure highest image quality under permissible SAR.

\textsuperscript{14}A broad range of physiological and neurological effects can result from magnetic field and RF current stimulations. It is assumed in this thesis that the main effect of RF current on biological tissues is heating. Although this section only addresses safety on power deposition, it is evident that further work is required to investigate other safety considerations.
4.5.1 Power Deposition in Biological Media

The SAR in spiral RF-CDI imaging mainly comes from the rotary echo and the RF current. Assuming $T_c = 20$ ms, $TR = 500$ ms\textsuperscript{15} and the power from the current and the magnetic field are additive, the total power deposited onto a typical biological medium\textsuperscript{16} as a function of $J_z$ is shown in Fig. 4.9 together with the permissible SAR for various regions of the body based on the Canadian regulations. The SAR for $J_z$ is calculated by $T_c|J_z|^2/Tr^2\rho\sigma$ and the contribution from $B_1$ is obtained from the GE RF Heat Tool. The figure shows that there exists a range of $J_z$ for every $B_1$ such that the safety limits for human body and its surface are satisfied. Therefore, the spiral RF-CDI system operating under these parameters is safe for in-vivo imaging with respect to power deposition.

4.5.2 Sensitivity and Power Deposition Tradeoff

As it is essential to satisfy the rotary echo criterion, the value of $B_1$ has to be carefully chosen for any operating $J_z$ in order to maximize the sensitivity while avoiding excessive SAR. The sensitivity with $nl = 10$ at a 400 Hz and a 800 Hz $B_1$ (i.e. different SAR) as a function of $J_z$ are shown in Fig. 4.10.

The difference in sensitivity between $B_1$ at 400Hz and 800Hz is relatively small for $J_z \leq 500A/m^2$. As expected, the difference emerges after this breakpoint, revealing a possible violation of the rotary echo criterion and error from phase distortion artifacts. When $J_z$ is small, SAR is mainly contributed by $B_1$. Lowering $B_1$ reduces a substantial proportion of the SAR without incurring any significant sensitivity penalty, as seen in Fig. 4.10. At higher $J_z$,

\textsuperscript{15}This is a more reasonable value for typical biological media with long $T_2$.
\textsuperscript{16}Assuming typical values for a human head, $\rho = 1000kg/m^3$, $\sigma = 0.75S/m$ and a weight of 16 lb.
Figure 4.9: The specific absorption rate on a typical biological medium ($\rho = 1000 kg/m^3$, $\sigma = 0.7 S/m$) imaged by the spiral RF-CDI system using a $T_R$ of 500 ms. The main power from $J_z$ and $B_1$ are assumed additive. The dotted lines shows the permissible safety limits for EM field exposure.
Figure 4.10: The sensitivity of spiral RF-CDI at two different $B_1$ strengths as a function of $J_z$. The same imaging parameters for previous phantom studies are used and $n/\ell$ is 10. The values in brackets are total power depositions by $J_z$ and $B_1$ as calculated for Fig. 4.9.
SAR is dominated by $J_z$. Large $B_1$ in this case is desirable because it maintains sensitivity at a small proportional increase in SAR. In light of these findings and the fact that sensitivity is current dependent (Fig. 4.6), appropriate values of $B_1$ for different $J_z$ ensure an optimal sensitivity at minimal SAR. This also allows the system to operate within permissible SAR for a wider range of $J_z$.

4.6 Discussions and Conclusions

The approach taken to improve the performance of the basic spiral RF-CDI system was proved successful. The hypothesis that field inhomogeneity and off resonance were causing the poor performance of the bare spiral system was verified as the inhomogeneity correction algorithm effectively mitigated the artifacts in CD images. The following summarizes the performance of the enhanced system and suggests room for improvement.

Performance Summary

The qualitative and quantitative performance of the enhanced spiral system are summarized below.

- Utilizing frequency map correction reduces complications from static field inhomogeneity and off-resonance. Blurring and signal non-uniformity in the magnitude images are significantly reduced, giving final current density images of improved visual quality with fewer excitations.

- The average sensitivity attains 5\% with 10 spiral excitations. Compared to the conventional approach, this represents a 13 times gain in imaging speed with the sensitivity slightly reduced by a factor of 2.5.

- The current independent artifacts in the presence of random noise ranges
from $11A/m^2 \sim 16A/m^2$ for 10 or more excitations. On average, this is approximately 1.5 times worse than that of the Cartesian system.

- The stochastic noise characteristics of the spiral system closely agrees with theory and is similar to that of the Cartesian approach. On average, its noise is 56% higher than its theoretical limit while the Cartesian system has a noise 44% higher than predicted.

**Discussions and Implications**

The images produced by the two techniques share different features under scrutiny. Edges in spiral CD images are smoothed because of the interpolation whereas in Cartesian images, they display Gibbs ringing due to finite $k$-space sampling. However, all these images are of acceptable visual quality for practical applications. Which image characteristics are preferable depends on the features on the actual object and the subjective assessment criteria.

Despite the satisfactory sensitivity, the $\sigma_I$ used in its calculation is significantly larger than the current-independent variance after inhomogeneity correction. This observation suggests that current dependent error is much larger than the current independent component. Hence, the error contributed by the RF current dominates over the systematic current independent artifacts which in turn exceeds the stochastic noise. This implies that further endeavors to improve system performance should focus on current dependent errors, as opposed to minimizing the current independent artifacts and random noise.

The measured stochastic noise being higher than the theoretical limit may be related to the signal loss in the $J_z$ encoded magnitude image (Fig. 4.2 and Fig. 4.3) where current is flowing. As this residual unevenness remains after inhomogeneity correction and its severity depends on current magnitude,
it may be caused by the RF current. This phenomenon is specific to the new spiral system and its causes are not yet identified. Future work should explore any potential interference between the RF current and the spiral acquisition scheme. Under this unknown complication, the stochastic noise is only 56% higher than the theoretical value, which is still acceptable seeing that the same problem exists in the conventional approach with its actual noise 44% higher than its theoretical limit.

On top of a better temporal-resolution, another beauty of the spiral system is its lower energy deposition. Given that the RF-CDI SAR is inherently larger than other clinical MR applications, the fewer excitations required under spiral implementation can significantly reduce the total energy deposition and the overall bioheating, making the technique more suitable for in-vivo imaging and other biomedical applications.
Chapter 5

Conclusions and Discussions

The ultra-fast RF-CDI system developed in this thesis is capable of producing current density images that meet all the visual and analytical specifications from a commercial imager within safe power deposition. The technique allows very flexible tradeoff between sensitivity, visual quality and imaging time. By choosing appropriate imaging parameters, it was shown that the new system has a sensitivity and temporal resolution needed to image the fast occurring cortical conductivity changes in the brain for studying spreading depression, sensorimotor stimulations and other functional imaging applications.

The following list contains a detail summary of conclusions and results in the context of the initial system requirements:

- **Sensitivity Requirement:** The enhanced spiral RF-CDI system developed in this thesis allows measurement of current densities with variable precisions controlled by the imaging time. To satisfy the initial system requirement of a minimum of 10% sensitivity, 10 or more excitations are needed. With 10, 16 and 20 excitations, the averaged sensitivity for $J_2$ over the range of $200A/m^2$ and $1000A/m^2$ are 5.1%, 4.1% and 3.8% re-
respectively. This represents a 2.1 to 2.7 times decrease compared to the previously optimized Cartesian approach whose averaged sensitivity is 1.8%.

- **Temporal Resolution Requirement:** The imaging time of the spiral system is variable by adjusting the number of excitations. Assuming $T_R = 300ms$ and with frequency map correction, the imaging time for 10, 16 and 20 excitations are 19 seconds, 30 seconds and 37 seconds. When compared to the previous technique, this represents a 13, 8 and 6 times reduction in imaging time respectively. These imaging times make real-time snapshot possible for fast conductivity changes during typical cortical stimulation scenario (eg. spreading depression) which last for minutes.

- **Power Deposition Requirement:** Based on simulations and calculations, the SAR from the spiral system meets the power deposition safety limits $8W/kg$ with properly chosen imaging parameters. The spiral system therefore satisfies the initial safety requirement in the scope of power deposition.

As summarized above, all the initial requirements for this project are completely and simultaneously satisfied. The full creditability of the new spiral system is fully warranted. Although this work focus exclusively on phantom studies for the purpose of system testing and enhancement, it essentially paves the temporally enhanced system for imaging biological media and ultimately in-vivo studies.
5.1 Recommendations for Future Work

Previous research in current density imaging has focused mainly on developing a better RF-CDI system that is robust, sensitive and capable of contrasting conductivity in biological media. This thesis is the first RF-CDI work that addresses the issue of temporal resolution, which is an essential performance aspect that has to be optimized before the technique can image the fast conductivity changes for its ultimate medical applications.

The improved temporal resolution opens a new realm of in-vivo and biomedical applications for RF-CDI. To fully explore its potential, it is logical that future work should follow two parallel paths:

1. Further development of the imaging technique with particular focus on its performance and safety issues.

2. Identify clinical and medical applications of the RF-CDI technique.

To improve the performance of the technique, the following projects and investigation are worthwhile:

- Further enhance the existing spiral RF-CDI system:
  - Replace the existing hard 90°-selective 180° encoding scheme by spectral-spatial excitation pulse. This helps to achieve better slice selectivity and fat suppression which are often beneficial when imaging animal and heterogeneous media.
  - Investigate the possible interference between the current in RF-CDI and the spiral acquisition scheme. This may help to identify the causes of the signal attenuation mentioned earlier and to improve the overall system performance.
• Develop a multi-slice imaging sequence to improve functionality.

• Investigate how impedance contrast varies with current frequency. This identifies the required current that gives optimal contrasts for different tissues and biological media.

To ensure RF-CDI safety, the following approaches may be appropriate:

• Assess power deposition and temperature rise from in-vivo experiments.

• Perform finite element simulations for bioheat dissipation.

• Investigate the possibility and severity of physiological and neurological effects elicited by the RF current, such as increased blood-brain barrier permeability, cell depolarization and neuron synchronization.

5.2 The future of the RF-CDI Technique

Much of the past research in current density imaging focused on porting its implementation to better imaging systems and improving the overall performance with respect to robustness, sensitivity, noise and artifact minimization. The resulting system was able to image the static conductivity in biological media satisfactorily given adequate imaging time. The work from this thesis eliminates the need for long imaging time and thus opens the technique to a broader range of new applications that require contrasting dynamic conductivity. Diagnostic imaging for stroke and migraine which mimic spreading depolarization are just a few of the many examples. With the improved temporal resolution, the future RF-CDI will be a more useful research tool and a new medical imaging modality with unique contrast mechanism and values.
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Appendix A

MRI and Fast Imaging

This appendix provides the fundamental theory and rationale for magnetic resonance imaging (MRI) and its fast implementations. Its objectives are to supplement readers with the necessary background to understand the information presented in the thesis. Excellent references can be found in [10, 14].

A.1 Principles of MRI

A.1.1 Magnetization Preparation

What the MR technique is measuring are the signals coming out from the resonating proton of the object at the Larmor frequency - one dictated by the imager static field $B_0$ along the direction normally denoted by $z$.

Without excitation, the magnetization of all these protons aligns with this external static field along $z$. Their signal cannot be measured because they average out to zero due to their random phase. When these protons are subjected to a magnetic field $B_1$ rotating at the Larmor frequency transverse to their static $B_0$ magnetization, their net magnetization originally along $z$
will precess at the Larmor frequency defined by:

\[ f_0 = \frac{\gamma \times |B_0|}{2\pi} \]  (A.1)

with \( \gamma = 2.6751 \times 10^8 \text{ } \text{rad s}^{-1} \). While precessing, the magnetization will be 'tipped' towards the transverse plane. The tip angle is related to the duration \( (T_{B_1}) \) and the strength of the external \( B_1 \) field and is given by \( \theta_{\text{tip}} = \gamma B_1 T_{B_1} \). With \( T_{B_1} \) and \( B_1 \) appropriately chosen for a 90° pulse, the final magnetization can be tipped 90° and thus lands on the transverse plane to be mapped by the imager. Upon the cessation of \( B_1 \), the net magnetization will return to its original z direction with magnitude \( B_0 \) through decaying its transverse component and recovering its longitudinal component at half times of \( T_2 \) and \( T_1 \) respectively, a phenomenon known as relaxation.

### A.1.2 Spin-Echo Sequence and Slice Selectiveness

Immediately after the 90° tipping, all the transverse magnetization are in phase as they are instantaneously orthogonal to \( B_1 \) and \( B_0 \). Due to hardware limitations, the imager cannot measure these required magnetization at the instance when they are first tipped. As the medium is never perfectly homogeneous, the regional magnetization precess with their own Larmor frequencies and eventually get out of phase. To measure the original tipped magnetization without being distorted by the dephasing effect, a spin-echo is formed by refocusing the signal at an echo-refocused time \( T_E \) using a 180° 'reversing pulse' at \( T_E/2 \). This pulse reverses the precession direction of all precessing magnetization, thus restoring at \( T_E \) their initially tipped orientations except with \( T_2 \) relaxation.

To refocus the signal for detection, the refocusing pulse must rotate the magnetization by exactly 180°. Refocusing by any other angle will yield a null
signal at $T_E$ due to dephasing. Therefore, a way to achieve slice selectiveness (along $z$ for an axial image on $x$-$y$ plane) is to superimpose a $z$ gradient\(^1\) to make the Larmor frequency $z$ dependent (Eq. A.1) and to apply a refocusing pulse composing of a band of Larmor frequencies necessary to refocus the slice of interest. This method signal refocus and slice selection is known as the hard (non-selective) 90°-selective 180° excitation and is used in RF-CDI.

**A.1.3 Encoding by Readout Gradients**

The refocused signal on the transverse plane are measurable but they bare no information about the imaged object. Readout gradients, which are small magnetic field along $z$ with field strength varying spatially along a transverse plane, are used to encode the object. Their principle is to distort the homogeneous $B_0$ field when superimposed onto the imager static field, producing tipped transverse precessing magnetization having resonance frequencies which are spatially dependent. The spatial proton density variation is thus encoded in amplitude of the frequency spectrum of the collected signal $s(t)$ and can be revealed after the two-dimensional Fourier transform (2D-DFT).

**A.2 Principles in Fast Imaging**

**A.2.1 Long Imaging Time in MR Imaging**

Unlike other imaging techniques which capture the entire projection of an object in one snap-shot, MR acquires the image data in the spatial frequency domain and then reconstructs the visual image using the Fourier

\(^1\)A gradient field is a small magnetic field along the static field direction with field strength varying spatially along the axis of interest.
An example of the acquired $k$-space data and its corresponding transformed image are shown in Fig. A.1.

The long imaging time arises out of the need to reconstruct images from a two dimensional $k$-space array whose data cannot be collected within a single excitation. The standard Cartesian approach encodes the object using frequency and phase modulation along the two orthogonal directions so that the encoded data correspond to different lines in $k$-space to be acquired sequentially. This necessitates a long imaging time as the dimension of the data array is usually large (eg. 128 or 256) and successive acquisitions have to be separated by a long sequence repetition time $T_R$ to allow the magnetization to recover from relaxation.

Existing fast imaging technique deviates from the conventional approach by skipping certain $k$-space data and traversing different trajectories while ensuring that no data points are too sparse that may result in irrecoverable data loss and unacceptable image quality.
A.2.2 Encoding Gradients and $k$-Space Trajectories For Fast Imaging

This section supplements the discussion in section 2.3.1 by showing the gradients and $k$-space trajectories for the two proposed candidates. In any encoding scheme, the $k$-space trajectories $k(t)$ are related to the encoding gradients $G(t)$ by:

$$k(t) = \gamma \int_{0}^{t} G(t') dt'$$

(A.2)

With the above relation, the resulting gradients for different fast imaging methods characterised by their $k$-space trajectories are shown in Fig. A.2 to A.4. They demonstrate the features that are described in Chapter 2. The spike-like EPI gradients fail many commercial imager gradient systems and are causing phase accrual during abrupt directional changes in $k$-space. The spiral gradients are smooth and more readily realizable using moderate hardware available in existing clinical MR systems.

A.3 Complications in Spiral Imaging

A.3.1 Point Spread Function Broadening

The point spread function (PSF) is the transfer function of the entire imaging system and can be visualized as the output image of a point. The ideal PSF is the delta function $\delta(\tilde{r})$ so that the image is reproduced undistorted.

Deviation of the PSF from the ideal delta function results in image degradation. This normally occurs in the presence of inhomogeneity when two orthogonal time varying encoding gradients are used. If the inhomogeneity is significant, the PSF will become shift variant. Based on the simulation results
Figure A.2: Trajectories and Readout Gradients of Zig-Zag EPI.

Figure A.3: Trajectories and Readout Gradients for Blipped EPI (BEPI).

Figure A.4: Trajectories and Readout Gradients for Spiral Imaging (SPI).
in [28], the 2-D PSF for spiral imaging under no and a moderate $\delta H(x) = 5 \times 10^{-3} G/cm$ inhomogeneity are shown in Fig. A.5. It can be seen that the PSF is heavily broadened under static field inhomogeneity, which deteriorate high frequency components and causes unwanted smoothing in the final image.

A.3.2 Skewed $k$-space Trajectories

The effective encoding gradients are distorted when they are superimposed by spatially varying inhomogeneity. The resulting $k$-space trajectories will be deformed due to the integral relationship between $k(t)$ and $G(t)$. This deformation is complicated and cannot be analytically determined for any arbitrary inhomogeneity. However, to illustrate the severity of $k$-space skewing, a simulation for the $k$-space trajectory done in [27] under a linear 50 ppm/m inhomogeneity is shown in Fig. A.6 together with its expected trajectory. The skewing elicits angular and radial deformation, resulting in under-sampling and misinterpretation of acquired data.
Figure A.5: Spiral PSF under no (left) and a $\delta H(x) = 5 \times 10^{-3} G/cm$ (right) static field inhomogeneity. The PSF is greatly broadened in the presence of inhomogeneity.

Figure A.6: Spiral trajectory deformation under a homogeneous (left) and inhomogeneous field at 50 ppm/m (right) along two orthogonal axes.
Appendix B

System Operating Instructions

The operating instructions for Spiral RF-CDI are more complicated than the scan operations for most clinical sequences because it involves additional hardware setup and the spiral scan mode it uses is not supported directly under the the imager firmware at the time of this research. This appendix gives detailed instructions on how to set up the hardware, perform the scanning, do the post-processing from spiral data to CD images and how to further expand the capability of the system for different operating parameters. The following discussions are current at the time of this publication and are based on EPIC 5.4 and the newest spiral sequence cdisp3v04.

B.1 Hardware Setup

Compared the Spiral RF-CDI system to its conventional Cartesian approach, the new system requires no major modification to the existing hardware setup. Extensive hardware instructions documented in [3] can be followed directly with the following additional step:
Before powering down the TPS and doing the hardware setup, convert the imager mode to an unsupported spiral operation mode. This mode of operation supports spiral imaging, the original Cartesian RF-CDI sequence \texttt{rfcdi03h90} and most standard stock sequences. To do the switching on the console, open a C-Shell (select UTILITIES \rightarrow C-SHELL) and issue the command 'dospiral'. A printed feedback will confirm a successful switching. After this switching, reset the TPS (select UTILITIES \rightarrow OPERATOR MODE \rightarrow RESET TPS) and then power down the TPS to do the hardware setup. After completion of the setup, power up the TPS. If an error message states "TPS download unsuccessful", reset the TPS again. For further details, refer to [31].

For a quick reference, the additional hardware connection adopted from [3] is summarized in Fig. B.1. The connections shows the case where a patch panel is used. In most applications, the patch panel is unnecessary as the running cables are generally short enough to avoid intolerable signal loss.

**B.2 Spiral RF-CDI Operation on Console**

**Land-Marking**

The imaging slice and its orientation have to be selected before the scanning parameters can be prescribed. This operation is known as land-marking and has to operated directly in front of the imager.

1. Use the \texttt{in} and \texttt{out} buttons to adjust the axial position of the object so that the imaging slice lands on the landmark indicator.

2. Press \textit{advance scan}.
Figure B.1: The hardware setup required to deliver synchronous RF current using the CPCB and the AMT RF amplifier. The patch panel shown is optional and in most applications where the signal cables are short, it is not required and direct connections can be made. The ST, S1 and S2 are scope inputs for triggering, viewing the RF pulses and the blanking signals respectively. Recommended values for S1 and S2 in most CD experiments are 50mV/div and 5ms/div. For proper triggering, connect S3 to channel 3 and use normal trigger on it. Power down the TPS before performance any of the above connections.
Prescription of Imaging Parameters

The control variables for the sequence can be completely specified on the Signa touch screen console. Table B.1 summarized the required parameters and how they are entered sequentially to prescribe a scan. A complete reference for the meaning of these operation can be found in [18].

Static Field Shimming

As the pulse sequence was developed under EPIC 5.4, field shimming was not performed during prescan. While field homogeneity is particularly crucial for spiral imaging, shimming must be performed before any experiment. To do the static field shimming, use a 'stock' sequence that has shimming built in. The most suitable one is the standard clinical sequence, which can be specified by giving the PSD filename in table B.1 a null input. Prescriptions for this shimming process are the same as in table B.1 except with: (1) PSD filename: a null entry, (2) Prescan Option: AutoShim On, and (3) skip all Modify CV's operations in table B.2. Then run Auto-Prescan with the RF amplifier connected but turned OFF. The shimming values are then stored in the imager and will be used for successive scanning until another prescan with shimming enabled is prescribed.

Prescan Instructions for Spiral Scan

The prescan instructions are described in table B.1. If field shimming were performed right before prescan as described above, the prescription can be mere modifications to the appropriate parameters. (i.e. (1) AutoShim Off, (2) re-enter the pulse sequence name instead of a null entry.) After completion
of auto-prescan\textsuperscript{1}, turn on the RF amplifier and reset the CPCB to initialize the
timing and phase cycling counters and to synchronize the box with the imager.

**Scanning Instructions**

Once pre-scan is completed without error, the control variables of the
spiral sequence have to be set before scanning. The instructions are detailed
in table B.2. Pressing *scan* will then initiate the spiral RF-CDI sequence.
The remaining scan time will be displayed during the scan. Upon completion,
online reconstruction will be performed and the MR magnitude\textsuperscript{2} images will
be shown on the console.

**Raw Data File for Post-Processing**

The raw data files for the spiral scan are stored in `/usr/g/mrraw` under
signa2 or equivalently, `/net/signa2/usr/g/mrraw` under the Sunnybrook UNIX
NFS network where post-processing is done. These files have to be moved to
private directory as the raw file directory is purged periodically. These files all
have names PXXXXX where XXXXX are numbers starting from 00512 with
512 increment for each scan. To identify the file for a particular scan, note the
time of the scan and the time stamp on the raw data files.

### B.3 Post-Processing

There are two major post-reconstruction steps. First is the gridding
operation to obtain the the Cartesian MR images from the spiral data. Second
is the derivative operation to obtain the CD images from MR phase images.

\textsuperscript{1}Manual prescan is optional. Normally, auto prescan is used except under situation
where fine tuning and investigations are necessary.

\textsuperscript{2}To show the phase images, set rhrcctrl=2 in the modify CV section of table B.2.
Table B.1: Scan Prescription for *cdisp3v04.e*

<table>
<thead>
<tr>
<th>Field</th>
<th>Type</th>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Exam</td>
<td>Keyed</td>
<td>Id</td>
<td>cdispiral ←</td>
</tr>
<tr>
<td></td>
<td>Keyed</td>
<td>Name</td>
<td>Op. Initials ←</td>
</tr>
<tr>
<td></td>
<td>Keyed</td>
<td>Weight</td>
<td>200 ←</td>
</tr>
<tr>
<td>Patient Pos.</td>
<td>Touch</td>
<td>Patient Entry</td>
<td>Head First</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Patient Pos.</td>
<td>Supine</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Axial/Sag. Lndmrk</td>
<td>Nasion</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Coil Type</td>
<td>Coil Type</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Scan Plane</td>
<td>Axial</td>
</tr>
<tr>
<td>Imag. Params.</td>
<td>Touch</td>
<td>Image Mode</td>
<td>2D</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Pulse Seq.</td>
<td>Spin Echo</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Imaging Options</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>Keyed</td>
<td>PSD Filename</td>
<td>/yanr/cdisp3v04</td>
</tr>
<tr>
<td>Scan Timing</td>
<td>Touch</td>
<td>Num Echos.</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>TE</td>
<td>25 ms</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>TR</td>
<td>300 ms</td>
</tr>
<tr>
<td>Scan Setup</td>
<td>Touch</td>
<td>PrescanOpts.</td>
<td>Autoshim OFF (unhighlighted)</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Auto CF</td>
<td>Water</td>
</tr>
<tr>
<td>Scan Range</td>
<td>Touch</td>
<td>FOV</td>
<td>12 cm</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Scan Thickness</td>
<td>10 mm</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Interscan</td>
<td>5.0 mm</td>
</tr>
<tr>
<td></td>
<td>Keyed</td>
<td>Start (I/S)</td>
<td>0 ←</td>
</tr>
<tr>
<td></td>
<td>Keyed</td>
<td>End (I/S)</td>
<td>0 ←</td>
</tr>
<tr>
<td>Acq. Timing</td>
<td>Touch</td>
<td>Acq. Freq</td>
<td>256</td>
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<td></td>
<td>Touch</td>
<td>Acq. Phase</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Frequency Direction</td>
<td>R/L</td>
</tr>
<tr>
<td></td>
<td>Touch</td>
<td>Imaging Time</td>
<td>NEX 1</td>
</tr>
</tbody>
</table>
### Table B.2: Control Variables Entry for cdisp3v04.e

<table>
<thead>
<tr>
<th>PAGE</th>
<th>SUB-PAGE</th>
<th>CV</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scan Ops</td>
<td>Modify CVs</td>
<td>nl</td>
<td>$10^a$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>rhtype</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mapon</td>
<td>$1^b$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>dotardis</td>
<td>$1^c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>opfov</td>
<td>$120^d$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>rhrecon</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>pw_curmag</td>
<td>$20000^e$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>curmag_amp</td>
<td>$0.8^f$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>rhrcctrl</td>
<td>$1^g$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>tlead1</td>
<td>500</td>
</tr>
</tbody>
</table>

**Switch on RF Amplifier, Reset CPCB**

**SCAN**

- **a** Number of spiral excitations for each image. $nl=6,10,16$ and 20 are available.
- **b** Setting 1 for frequency map acquisition, 0 skipping Map acquisition.
- **c** Do online reconstruction. Setting 0 will not produce any image on console.
- **d** The field of view in mm. Available values are 80, 120 and 200.
- **e** Duration of $B_1$ rotary echo field in $\mu$s.
- **f** Strength of Rotary Echo $B_1$ as a fraction of the strength of hard $90^\circ$ (1000Hz).
- **g** Setting rhrcctrl=1 for magnitude and 2 for phase image online reconstruction.
The first operation is specific to the new spiral system and is described below. It produces the necessary MR images in an intermediate file to be processed by the GUI which is modified to interface\(^3\) with the new file format. The other operations of the GUI can be found in [3].

**Gridding Operation**

The raw image file PXXXXX is first copied into a directory\(^4\) which contains the programs for gridding reconstruction. The script `cdirecon` is then invoked by 'cdirecon PXXXXX' and it will produce a file readable by the GUI for further processing. In performing the gridding, the `cdirecon` routines reads the configuration file `sequence-name_nL_FOV_0.in` which describes the format of the raw data file and the scan parameters (eg. for map correction, the value of $\Delta T$, etc.). Modification to this file is unnecessary in most applications and its content are self-explanatory.

**B.4 Generation of Readout Gradients & $k$-space Trajectories**

In this thesis, gradients for $nL=6, 10, 16, 20$ and for FOV $8\text{cm}$, $12\text{cm}$, $20\text{cm}$ had been generated. To run the RF-CDI spiral sequence for other FOV or $nL$, new gradients and $k$-space files have to be generated.

**Readout Gradients**

Routines in Matlab 4.2c in conjunction with some pre-compiled pro-

---

\(^{3}\)Select from the GUI: *Current Density Imaging $\rightarrow$ Rotating Frame RF-CDI $\rightarrow$ Process Spiral Pre-Processed Data File*

\(^{4}\)At the time of this project, it is sunnybrook:/home/yanr/Thesis/spiral_recon.
grams were used to generate the readout gradients and append the corresponding rewinders to bring the $k$-space trajectories back to their original. The front end matlab scripts used are listed below. Their detail operations can be found in directory and program documentations.

- `makereadout.m` generates readout gradients for a given FOV and nl. It is essential for the pulse sequence and off-line reconstruction.

- `find_nptout.m` calculates the length of the gradient with rewinder. This length information is essential for sequence coding.

$k$-space Trajectory Files

The following routines\textsuperscript{5} are used to obtain $k$-space trajectories:

- Online recon: Use calge5 to get the .kk files from the gradient files.

- Offline recon: Use cdigenoffline.m to get the .k files.

All the generated files are then properly placed in the pulse sequence and reconstruction directories to enable new imaging parameters for the existing RF-CDI system.

B.5 Debugging Information

Some common problems are outlined below as a trouble-shooting guide:

- *RF amplifier failure* normally occurs when the magnitude/duration of $B_1$ are improperly prescribed during prescan/scan. Recheck prescription or use a weaker/shorter $B_1$.

\textsuperscript{5}Complete references can be found in sunnybrook:/home/yanr/Thesis/generate_grads/README.
- **AP failure.** *Reset TPS.* happens when data acquisition fails. This may be due to incorrect number of echo (opnecho) or spiral (nl). Ensure opnecho is 6 and *Redownload* the variables every time after changing nl or *mapon* (manually hit the download plasma key) before prescan/scan.

- **IPG Failure.** This may be due to invalid or non-existent spiral readout gradients being called from the pulse sequence. Ensure that any (new) gradients are of correct length and in proper directory as hard-coded in the pulse sequence.

- **Symptoms:** The image is totally distorted and is unrecognisable. The problem is often field inhomogeneity and can be alleviated by shimming the field by re-running the prescan of the clinical sequence. Ensure that the RF amplifier is off but connected to the object when shimming.

- **Symptoms:** A bright dot appears in the center of the tri-concentric phantom with significant blurring. This is again due to field inhomogeneity and the solution is to re-run the automatic shimming of the clinical sequence or manually edit the shim values. Typical values below 100 for X, Y and Z shimming are normally appropriate. Experience shows that repeating a prescan with the power amplifier connected and off may mitigate the observed phenomenon.

- **Symptoms:** Hollow $J_z$ derived from correct phase images. This happens when the CPCB are out of phase with the imager. Reseting the CPCB will re-synchronize the reference signal of CPCB with the imager IPG.