JUDGING DIFFERENCES BETWEEN VISUALLY TEXTURED MATERIALS

by

Raymond Young-Jin Cho

A thesis submitted in conformity with the requirements for the degree of Master of Science Graduate Department of Physiology University of Toronto

© Copyright by Raymond Young-Jin Cho 1997
The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author’s permission.

L’auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L’auteur conserve la propriété du droit d’auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-28808-0
Abstract

Observers were asked to compare pairs of natural materials in two ways: ratings of innate overall dissimilarity ($\Delta$) and ratings of specific dissimilarity according to each of six candidate attributes ($\Delta^4$) (Lightness, Contrast, Edginess, Coarseness, Directionality and Regularity). Unlike some previous studies, we present the overall dissimilarity task first to avoid possible bias from knowledge of attributes. Multidimensional scalings of $\Delta$ ratings suggest a dimensionality of around four. Dimensionality is also low when the task is identifying materials. It is particularly interesting that cross-validation and redundancy analyses show good agreement for $\Delta$ ratings across observers and texture sets. All six $\Delta^4$ attributes were significant in accounting for the $\Delta$ ratings, including texture Lightness, a factor not previously examined in the literature. The consistently best predictors were Coarseness and Regularity. Although there was a range where judgements were fairly constant, sufficiently distant viewing or dim lighting reduced dimensionality and Lightness became the important attribute.
Acknowledgements

First, great thanks goes to my parents for a thorough education and their continued support.

Special thanks goes to Peter Hallett, my supervisor, who has been an exemplary academic, and instrumental in the confirming of all my suspicions about how gratifying research could be. For fruitful discussions and many helpful suggestions, I would also like to thank Bruce Schneider and Robert Tibshirani, the two other two members of my thesis committee. Pat Bennett has also been generous, lending his computing facilities for my analysis and providing numerous occasions to present this work.

There are many fellow lab members who have helped make my research experience both stimulating and enjoyable. Dariush Ebrahimi, Stan Hamstra, and Tim Sinha always made themselves available for questions, discussion, and technical assistance. Vicky Yang has been very helpful with the experimental portion of the thesis and with some of the attendant number-crunching. Thanks also to Li-Wing Lau for many stimulating discussions and for helping me out of my computer fixes.

Finally, I am much indebted to Young-Ha Cho, Jennifer Huynh, Elizabeth Small, Jason Gold, and my brother Ronald Cho, for their readings of the paper, helpful discussions (not always about texture), and moral support.
## Contents

1 Introduction ................................................................. 1

2 Methods ........................................................................ 8
   2.1 Stimulus Characteristics ................................................. 8
   2.2 Procedure ................................................................... 9
      2.2.1 Ratings of Overall Dissimilarity ($\Delta$)......................... 11
      2.2.2 Ratings of Attribute Dissimilarity ($\Delta^i$)..................... 11
   2.3 Stimulus Presentation and Data Collection ....................... 13
   2.4 Impoverished Viewing Conditions and Identification Task .... 13
   2.5 Analysis .................................................................... 15

3 Results ........................................................................... 19
   3.1 Dimensionality: Overall dissimilarity ($\Delta$)...................... 20
   3.2 Attribute Dissimilarities $\Delta^i$ as Predictors of Overall Dissimilarity $\Delta$ .................................................. 23
      3.2.1 Regression Analyses Using $\Delta$ vs. $\Delta^i$ Data .................. 23
      3.2.2 Regression Analysis Using MDS Transformation of the $\Delta$ and $\Delta^i$ Data ............................................. 26
   3.3 Variability Across Texture Sets and Observers ($\Delta$, $\Delta^i$) ...... 29
      3.3.1 Across Texture Sets: Cross-validation ($\Delta$, $\Delta^i$) ............ 30
      3.3.2.1 Across Observers: Cross-validation ......................... 30
      3.3.2.2 Across Observers: Redundancy Analysis ($\Delta$) .......... 32
   3.4 Nature of the Attributes .................................................. 37
      3.4.1 Correlations Between Attributes ($\Delta^i$) .......................... 37
3.4.2 Dimensionality of the Individual Attributes ($\Delta^i$)................................. 37
3.5 Judgements Under Impoverished Viewing Conditions........................................ 41
  3.5.1 Dimensionality: Overall Dissimilarity ($\Delta$)................................................. 41
  3.5.2 Relevant Attributes: $\Delta$ vs. $\Delta^i$ Relationship........................................... 41
  3.5.3 Direct Comparisons of $\Delta$ Ratings............................................................... 47
3.6 Identification Task............................................................................................... 49

4 Discussion ............................................................................................................. 56
  4.1 Dimensionality..................................................................................................... 56
    4.1.1 High Dimensionality of Early Neural Representations.................................... 56
    4.1.2 Behavioral Studies of Low-level Representations........................................... 57
    4.1.3 Studies of Higher-Order Attributes............................................................... 57
    4.1.4 At least four dimensions............................................................................... 60
    4.1.5 Low dimensionality: Real or Artefact?......................................................... 62
  4.2 Attributes........................................................................................................... 65
  4.3 Variability of Texture Space............................................................................... 70
    4.3.1 Across Texture Sets and Observers............................................................... 70
    4.3.2 Task Dependence............................................................................................ 71
    4.3.3 Impoverished Viewing Conditions.................................................................. 72
  4.4 Summary and Conclusions.................................................................................. 73
  4.5 Future Directions................................................................................................ 74

References .................................................................................................................. 76

Appendix 1: Statistical Analyses .................................................................................. 80

Appendix 2: Stimulus-Response Matrices for Identification Task .................................. 81
List of Figures

3.1 $\Delta$ Data -- Dimensionality of the Implied Space ................................................................. 21
3.2 $\Delta$ vs. $\Delta^4$ relationship -- Exhaustive Survey of the 63 Possible Attribute Combinations 25
3.3 $\Delta$ vs. $\Delta^4$ Relationship -- Cross Validation Across Texture Sets ..................................... 31
3.4 $\Delta$ vs. $\Delta^4$ Relationship -- Cross Validation Across Observers ...................................... 33
3.5 $\Delta^4$ Ratings -- Dimensionality of the Individuals Attributes .............................................. 40
3.6 $\Delta$ Ratings -- MDS Analyses of the Varying Luminance and Distance Conditions .............. 43
3.7 $\Delta$ vs. $\Delta^4$ Relationship -- Distance and Luminance Data .................................................. 45
3.8 $\Delta$ Ratings -- Comparisons Between Impoverished and Optimal Conditions .................. 48
3.9 Information Transmitted Over Each of the Five Distances .................................................... 51
3.10 Identification-Dissimilarity Relationship for the Four Longer Distances ............................ 53
3.11 Stresses for MDS Configurations Derived from Identifications ........................................... 55
List of Tables

2.1 Stimulus Set..........................................................................................................................10
3.1 $\Delta$ vs. $\Delta'$ relationship -- Regressions of Attribute Scales on Coordinates......................28
3.2 $\Delta$ Ratings -- Redundancy Coefficients for Individual Observers' Spaces..........................36
3.3 $\Delta'$ Inter-Relations -- Correlations Between Attributes.....................................................38
3.4 Sample Stimulus-Response Matrix for the Observers' Pooled Responses............................50
4.1 Summary of Experiments Involving Comparative Judgements of Textures..........................58
A2.1 Stimulus-Response Matrices for Identification Task...........................................................81
Chapter 1

Introduction

Visual texture is the spatial distribution of reflectance that characterizes a material. Visual texture is an important cue as to the physical properties of the material. It has been suggested that inherent in the perception of texture is a process of comparison, whether between two or more simultaneously presented stimuli or, between stimulus and memory representations (Harvey & Gervais, 1981). What can be said about how the visual system that performs these comparative judgements?

A preponderance of studies to date have employed simple artificial textures as stimuli. One line of research has focused primarily on the analysis of simple texture patterns consisting of composites of sine-wave gratings. For example, Richards & Polit (1974) proposed that a small number of physiological spatial filters (types of receptive field) were sufficient in explaining the perception of noisy one-dimensional grating textures. They had assumed the existence of independent spatial filters (Campbell & Robson, 1968) and aimed to determine their number and shapes. Borrowing the methods of colour matching (Maxwell, 1855) for discovering the number of chromatic filters, Richards’ laboratory demonstrated that a four-filter model sufficed to explain observers’ “texture matches”. Following their lead.
Harvey & Gervais (1978, 1981) employed textures of the type used by Richards & Polit (1974) and explored whether similarities in the appearance of the samples were matched by concomitant similarities in the activities of the four spatial filters. Tasks included groupings, and paired or triadic comparisons of textures. Using linear discriminant and multidimensional scaling (MDS) analyses, it was found that the comparative judgements were largely predictable by either the spatial frequency components of the textures or the responses of the four spatial filters. Others have examined the effects of differing spatial phase on the perceived similarity of simple one-dimensional compound gratings. Kahana & Bennett (1994) found that a two-channel model of phase discrimination was useful in explaining their triadic comparisons of f+2f (fundamental and second harmonic) gratings.

It may not be unexpected that the outputs of spatial frequency and phase discrimination filters are successful at explaining the perceived similarities of simple synthetic textures that differ only in the characteristics of a relatively small number of component gratings. What of more complicated textures? Many studies have examined the perception of two-dimensional patterns consisting of simple, repeated elements. Julesz (1981) focussed on preattentive perception, using discrimination tasks with very short viewing times, and attempted to explain texture discrimination in terms of the statistics of local spatial features or "textons" such as terminators, corners and intersections. Texton-based explanations have been criticised for lacking general applicability, and models invoking lower-level mechanisms have been proposed. For example, simple size-tuned mechanisms were suggested as underlying the discriminations of synthetic textures consisting of "X" and "L" elements.
CHAPTER 1. INTRODUCTION

(Bergen & Adelson, 1988). Malik & Perona (1990) provide an instance of a more comprehensive model that employed spatial filters, motivated by physiologic findings which included outputs of simple cells, local inhibitory mechanisms, and boundary detection using odd-symmetric mechanisms. Hallett & Hofmann (1991) studied the discrimination of mesh-derived textures, and found that either Fourier (low-frequency harmonic amplitude) or specific spatial (hole size) characteristics of the stimuli contributed towards discriminability of figure from ground. Hofmann & Hallett (1993a, 1993b) explained rating comparisons of three component sine-wave textures with a particularly economical cortical model.

It is clear that invoking reasonably simple descriptions of the stimuli and physiologic spatial filters is useful in explaining the perception of simple patterns. The interest of this study, however, is in elucidating what governs rapid innate judgements of the much more complex, natural textures that are encountered in ecological conditions. It has yet to be firmly established what attributes are used in judgements of natural textures. Are texture judgements an essentially objective process insensitive to observer idiosyncrasies, choice of texture, task or viewing condition? Colour science has faced similar challenges and has managed to capture the regularities of colour processing in a three-dimensional colour space which is valid for all colours and all (i.e. trichromatic) observers (e.g. Gouras, 1991; Boynton & Olsen, 1987; Uchikawa & Boynton, 1987). Although there have been several claims of similarity between texture perception and colour vision (Richards, 1979; Rao & Lohse, 1996), it is far from clear that this is more than a hoped for analogy. As of yet, no standard texture analogue to colour space has been developed, and issues concerning the dimensionality and
axes of such a "texture space" have yet to be resolved.

Texture perception could, in principle, involve a very high dimensionality in the light of some neurophysiological and psychophysical evidence. At each point of the striate cortex, there exists many different flavours of simple cells (e.g. 5 spatial frequency bands × 6 orientations × 4 phases = order 100-200 cells; e.g., Sakitt & Barlow, 1982; Daugman, 1987; De Valois, Albrecht & Thorell, 1982; Field, 1987; Fleet & Jepson, 1989). A perceptual study by Richards (1979) posited the existence of six spatial frequency channels (for one-dimensional textures) and six orientation channels, and proposed that perceptually similar textures elicited similar responses in these channels. It would seem that very many channels are necessary to accurately characterize a small patch of texture.

It is quite possible, however, that not all the information carried by the initial high dimensional representation is preserved in rapid innate judgements of natural textures. These judgements may, instead, be based on transformed and reduced versions of the lower-level representations. A number of studies have attempted using higher-order features to explain relatively coarse judgements of the similarities in the appearances of natural textures (ratings, groupings, etc.). In attempts to simulate ecological viewing conditions, these studies have employed a set of textures taken from the Brodatz (1966) photographic album of man-made and naturally occurring textures. What is interesting about these studies is that only a handful of features were proposed as candidate attributes, as opposed to the high number that obtains when enumerating the different types of channels or cells. These views imply not only that
higher-order features are useful in the judgements of complex textures, but that the number of useful features is small.

Tamura et al. (1978) studied paired comparisons of textures according to six textural attributes (i.e. coarseness, contrast, directionality, line-likeness, regularity, roughness). The emphasis was placed, however, on capitalizing on observers' judgements of textures according to these features in the interest of developing computational measures of the same features, and not on whether they were those actually used in everyday untutored judgements of textures. Amadasun & King (1989) performed another study of the Brodatz textures with the same objective, using a similar set of features (coarseness, contrast, busyness, complexity and texture strength). Both studies were only moderately successful in their predictions of similarities between of textures using their computational approximations of textural attributes.

Rao & Lohse (1993, 1996) recognized the need to investigate features that are relevant to actual judgements of natural textures. Comparing ratings of the Brodatz textures (1966) along feature scales with the results of a sorting task in which no explicit criteria were specified, they found that these high-order judgements could be embodied by relatively low-dimensional representations. For example, they proposed that three dimensions were important to texture perception, namely, "repetitive", "contrast/directional", and "granular/coarse/low-complexity". Many of the findings are consistent with our independent study; however, there are two remaining issues to be resolved: the objectivity of texture
perception (intersubject variability) and the homogeneity of the texture space.

In the present study, we examine six attributes as candidate descriptors of texture space: Lightness, Contrast, Edginess, Coarseness, Directionality and Regularity. Texture "Lightness", a feature not previously used in the texture literature, seemed a fairly obvious choice as it has a physical correlate in mean surface reflectance and should be of importance in comparisons of most monochrome texture pairs; most of our other attributes have found prior use in the literature (e.g. Tamura et al., 1978). Some might argue that Lightness is not a textural attribute per se, but past perceptual studies of the Brodatz textures (1966) have neglected in excluding Lightness as a "confounding" factor, either by manipulation of the stimuli or instruction to the observers. We have found that some of these candidate attributes, including Lightness and, in particular, Coarseness and Regularity, account for the greater part of the variance in rapid untutored or "gestalt"-type judgements of the overall dissimilarity of texture pairs. Both our regression and multidimensional scaling (MDS) analyses suggest that despite the rich early neural representation, the practical differentiation of textured materials is based on a relatively low number of dimensions, somewhat larger than three and that our six attributes are useful directions in that space despite nonorthogonality and other complexities. Further, using the techniques of cross-validation and redundancy analysis, we find that judgement of natural textures is an objective experience across observers, relatively insensitive to observer idiosyncracy, and that texture space is relatively homogeneous. We also asked to what extent the objectivity of the judgements would generalize. Accessory experiments suggest that texture judgements access a common
representation, whether viewed under impoverished viewing conditions, or when different types of judgements are asked of the observers.

There are a number of features unique to our study: a) to avoid possible bias from knowledge of attributes, we took the simple measure of first requesting ratings of innate overall dissimilarity, which would avoid bias by the later task involving ratings of specific dissimilarity according to each of the six candidate attributes; b) we examined *Lightness* as a candidate attribute and found it to be a significant predictor of overall dissimilarity; c) we suggest the dimensionality of around four, slightly higher than previous estimates, which may be due to our inclusion of *Lightness* as an additional candidate attribute; d) our explicit treatment of intersubject variability has demonstrated good agreement between observers; e) we examined judgements of textures under a wide range of viewing conditions and with an identification task.
Chapter 2

Methods

2.1 Stimulus Characteristics

In selecting the stimuli, we endeavoured to satisfy a number of criteria. Most importantly, to examine judgements of complex textures that are ecologically valid, stimuli were limited to prints of real textures, photographed at the correct magnification for viewing at arm’s length (Brodatz, 1966). Attempts were also made to maximize the number of samples, limited mostly by concerns of the duration of an experiment with too large a set of textures. The selection also attempted to capture some of the diversity of textured materials -- the salience of attributes might easily vary with unbalanced selections of samples. It was apparent that the set of textures from which our sample was selected (Brodatz, 1966) was only a small subset of all natural and man-made textures, but they were considered to be sufficiently representative to merit study, and also allowed comparison to past studies that used samples from the same source.

The stimulus set consisted of 60 texture samples cut from half-tone prints of man-
made and naturally occurring materials (Brodatz, 1966: Table 2.1). For convenience, these will be referred to collectively as natural textures. For ease of presentation and comparisons of textures to self, nonidentical samples of the textures with same Brodatz numbers were cut from another copy of the Brodatz book, for a total of 120 samples. The textures were divided into five sets of twelve patterns to avoid the prohibitive numbers of pairings that would result with such a large sample. Also, this division into subsets permitted a cross-validation between groups of textures (see Results 3.3). Viewing distance was 90 cm. Each texture sample was 10 x 10 cm, subtended 6.4 deg of visual angle, and was mounted on an 18 x 18 cm piece of grey matte board with a reflectance of 0.30. Pairs of textures were presented with the side borders adjacent to each other so that the angular subtense between the centres of the texture pairs was 11.4 deg.

Lighting was northern daylight supplemented by fluorescent light. Retinal illuminance by the grey borders of the textures (reflectance 0.30) ranged from 37.2 to 158.2 Td (or luminance of 19 to 50 cd/m$^2$; mean 94.8 Td or 37.5 cd/m$^2$; SD 38.7 Td or 8.9 cd/m$^2$; the average pupil diameter was 2.5 mm) due to variations of daylight (Trolands will be used henceforth).

2.2 Procedure

Twelve observers (6 male, 6 female) participated in the experiment. All observers had normal or corrected-to-normal vision. Six observers had previous experience with texture
### Stimulus Set

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D9 grass lawn</td>
<td>D2 fieldstone</td>
<td>D4 pressed cork</td>
</tr>
<tr>
<td>D20 French canvas</td>
<td>D5 expanded mica</td>
<td>D12 bark of tree</td>
</tr>
<tr>
<td>D22 reptile skin</td>
<td>D11 woolen cloth</td>
<td>D14 woven wire</td>
</tr>
<tr>
<td>D29 beach sand</td>
<td>D35 lizard skin</td>
<td>D17 herringbone weave</td>
</tr>
<tr>
<td>D34 netting</td>
<td>D49 straw screening</td>
<td>D18 raffia weave</td>
</tr>
<tr>
<td>D52 oriental straw cloth</td>
<td>D55 straw matting</td>
<td>D21 French canvas</td>
</tr>
<tr>
<td>D66 plastic pellets</td>
<td>D64 oriental rattan</td>
<td>D31 beach pebbles</td>
</tr>
<tr>
<td>D68 wood grain</td>
<td>D70 wood grain</td>
<td>D50 raffia</td>
</tr>
<tr>
<td>D87 sea fan</td>
<td>D73 soap bubbles</td>
<td>D60 European marble</td>
</tr>
<tr>
<td>D93 fur</td>
<td>D92 pigskin</td>
<td>D82 oriental straw cloth</td>
</tr>
<tr>
<td>D98 crushed rose quartz</td>
<td>D101 cane</td>
<td>D107 Japanese rice paper</td>
</tr>
<tr>
<td>D112 plastic bubbles</td>
<td>D109 handmade paper</td>
<td>D111 plastic bubbles</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Set 4</th>
<th>Set 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1 woven wire</td>
<td>D6 woven wire</td>
</tr>
<tr>
<td>D23 beach pebbles</td>
<td>D7 fieldstone</td>
</tr>
<tr>
<td>D24 calf leather</td>
<td>D15 straw</td>
</tr>
<tr>
<td>D32 pressed cork</td>
<td>D19 woolen cloth</td>
</tr>
<tr>
<td>D62 European marble</td>
<td>D27 beach sand and pebbles</td>
</tr>
<tr>
<td>D65 oriental rattan</td>
<td>D37 water</td>
</tr>
<tr>
<td>D72 tree stump</td>
<td>D47 woven brass mesh</td>
</tr>
<tr>
<td>D77 cotton canvas</td>
<td>D74 coffee beans</td>
</tr>
<tr>
<td>D80 oriental straw cloth</td>
<td>D79 oriental grass fibre cloth</td>
</tr>
<tr>
<td>D83 woven matting</td>
<td>D84 raffia</td>
</tr>
<tr>
<td>D100 ice crystals</td>
<td>D86 ceiling tile</td>
</tr>
<tr>
<td>D104 loose burlap</td>
<td>D105 cheesecloth</td>
</tr>
</tbody>
</table>

Table 2.1: This table is a listing of the five sets of textures (from Brodatz, 1966) used as stimuli. The letter/number combinations identifying the textures follow the nomenclature used by Brodatz.
judgements. The other six observers were naive to the specific aims of the experiment.

2.2.1 Ratings of Overall Dissimilarity ($\Delta$)

Observers first familiarized themselves with the textures by viewing five 8.5" x 11" panels held in hand, each panel providing samples of the 12 textures comprising one of the five sets. The time for familiarization was not restricted, generally lasting a few minutes. The observers were then asked to make two types of judgements. The first half-session required judging the innate or overall dissimilarity, with the criteria for judgement left to the observer’s discretion (the symbol $\Delta$ will frequently be used to denote overall dissimilarity judgements in the following text). A 7-point scale ranging from 1 (“very similar” pairs) to 7 (“very dissimilar” pairs) was used, with the appropriate gradations in between. The $\Delta$ ratings were completed before the instructions for the second half-session were explained in attempts to avoid observer bias.

2.2.2 Ratings of Attribute Dissimilarity ($\Delta^4$)

The second type of judgement involved ratings of dissimilarity with respect to a set of candidate attributes ($\Delta^4$ ratings). Our analyses examine whether the candidates were well chosen. The attributes were defined for the observers as:
CHAPTER 2. METHODS

i) "Lightness" (l) was used as an assessment of the average perceived brightness from all parts of the texture plate.

ii) "Contrast" (c) was used as a measure of the degree of variation of light levels within each texture. For example, a plate without much contrast has a relatively uniform reflectance.

iii) "Edginess" (e) was used as a measure of edge definition within the texture. Edginess may be thought of as the opposite of "fuzziness".

iv) "Coarseness" (f) of textures can vary in two different ways in keeping with the definition of Tamura et al. (1978). Assuming a texture can be represented as a repeated placement of fundamental elements, one texture will be coarser than another if either a) its elements or b) their spacings are larger than those of the other. A texture that is not coarse may be thought of as "fine".

v) "Directionality" was defined by both its element shape and placement rule (again, in agreement with Tamura et al., 1978). Directionality could exist for more than one orientational axis, but a texture that was uniformly directional for only one axis was assessed as having the highest "Directionality".

vi) "Regularity" (r) was used as an assessment of order within a texture. Regularity may be thought of as the opposite of "randomness".

The observer's task was to rate each texture pair according to each of the above attributes on an integer scale. A coarser rating scale was employed with the aim of speeding

For brevity the attributes are sometimes referred to by their first letters (l, c, e, f, d, r). To distinguish from Contrast, Coarseness will be denoted by "f" for "Fineness".
the task and eliciting rapid, robust judgements. Observers were instructed to assign a rating of "-2" if the texture on the left was deemed to have much more of the assessed attribute than the one on the right, and a rating of "+2" for the converse case. If there was little or no difference between the textures, observers were to assign a rating of "0", and "+1" or "-1" to intermediate judgements.

2.3 Stimulus Presentation and Data Collection

A "Ross ordering" (Ross, 1934) balanced the number of presentations on each side and ensured that specific textures were repeated at relatively well-spaced intervals. Viewing times were not limited, but most comparisons took approximately 5 seconds to complete. For the Δ ratings, all 78 possible pairs (including self-comparisons) of a set of 12 textures were presented to each of the 12 observers. When a pattern was compared with itself, different samples were used (nonidentical samples taken from the same Brodatz (1966) texture print). Either of the samples was used in the rest of the comparisons. The total number of Δ ratings was 4680 (12 observers/pairing x 78 texture pairings x 5 texture sets). For the Δ⁴ ratings, the number of attributes made it impractical for each observer to exhaustively complete all pairings. Instead, pooling across observers provided four replications for each texture pair. The total number of Δ⁴ ratings amounted to 9360 (6 attributes x 4 observers/pairing x 78 texture pairings x 5 sets). A full session for one observer lasted about 2.5 hours. The analyses pool ratings across observers except where specified.
2.4 Impoverished Viewing Conditions and Identification Task

How do the judgements of textures change with impoverished viewing conditions? And do different tasks differ markedly in their processing of textural information? Since we suspected that there existed robust regularities in the judgements of textures, it seemed reasonable to ask these questions in attempting to delimit the extent of these regularities.

Past studies in this lab have explored the effects of systematic decreases in luminance (99.3, 17.7, 3.14, 0.56, 0.10 Td: Yang, 1995) or increases in viewing distance (0.9, 8.2, 15.5, 22.9, 30.2 m: Cho & Hallett, 1995) on the judgements of texture pairs (\(\Delta_1\) and \(\Delta_4\) ratings). Cho & Hallett (1995) also examined observers' performances on a task involving the identification of 20 different texture samples with varying distance. Ten sets of twenty different 12 x 12 cm Brodatz texture samples were mounted in randomly ordered 4 x 5 arrays on ten grey panels of reflectance 0.30. In addition to all 12 textures of Set A (Table 2.1), the following were included: D4 (Pressed cork); D12 (Bark of tree); D16 (Herringbone weave); D37 (Water); D55 (Straw matting); D76 (Oriental grass fibre cloth); D86 (Ceiling tile); D109 (Handmade paper). The subjects provided their best guesses at identifying the distant samples, referring to a small panel of all 20 textures (each 3 x 3 cm of a Brodatz plate) held in the hand to avoid any confusion over names. With some additional analysis, selected portions of the results of these studies will be presented with a view to addressing the above questions. Further details of methods will be presented with their respective findings in Results 3.5 and 3.6. Complete descriptions can be found in Yang (1995) and Cho & Hallett (1995).
2.5 Analysis

A brief description of the methods of analysis is provided here. More detailed accounts of the application of each of the methods accompany their respective findings in the Results section. For algebraic descriptions, see Appendix 1. The software used for the analyses is as follows: for Multidimensional scalings, Systat 5.2.1 (1992); for Multiple linear regressions and the Cross validations, Fastat 2.0 (1991) and Excel 5.0 (1993); for Redundancy analyses, Excel 5.0 and Matlab 3.5 (1991).

*Multidimensional Scaling* (MDS): Individual texture samples are viewed as points populating a multidimensional space where the distances between the points match, as closely as possible, the experimentally derived Δ ratings. If the space is given the correct number of dimensions, pairs of textures that elicit larger Δ ratings (*i.e.* look more dissimilar) will tend to be situated farther apart in the space, and those with lesser ratings, situated closer together. We employed a non-metric MDS algorithm (Kruskall, 1964) to discover the correct dimensionality. This algorithm attempts to match the ordinal values of the Δ ratings for texture pairs with the respective distances between the texture pairs in the MDS space. A perfect match, then, would mean that the MDS distances between pairs of textures would be a monotonically increasing function of the Δ ratings. To assess the goodness-of-fit, we used Kruskall's "Stress" (Kruskall & Wish, 1978), which ranges from 0 to 1 for good to bad fits respectively. We also provide the accompanying $R^2$'s, which give the proportion of variance in the Δ ratings accounted for by the corresponding MDS distances. The dimensionality of
the MDS solutions can be increased until the Stress is satisfyingly low. Recommendations for assessing the reliability of a solution include finding an "elbow" in a plot of Stress vs. Dimensions plot. As our plots showed fairly gradual decrease, we adopted the usual practice of considering Stresses lower than 0.10 as sufficiently low. After determining dimensionality, the experimenter can then search for meaningful axes in the MDS representation of perceptual space ("MDS-Δ space"). This can be done by mere visual examination of the solutions. However, there is great difficulty in inspecting solutions that have dimensions higher than two. A more objective method uses independently derived MDS scales (based on Δ^4 ratings) which place the textures along a 1-dimensional MDS-Δ^4 scale defined by an attribute \( i \) (\( i = 1-6 \)) that has possible relevance to the judgements of overall dissimilarity (Δ). After regressing the MDS-Δ^4 values of the texture set against the MDS-Δ coordinates, the relevance and orientation of scale \( i \) in the space can be ascertained from the resulting \( R^2 \) and Beta (standardized) coefficients, respectively.

*Multiple Linear Regression* (MLR): MLR aims to predict the behaviour of a dependent variable using a weighted sum of independent predictor variables. In our case, we attempt to predict the Δ ratings using the Δ^4 ratings as predictor variables. That is, we are proposing that a linear combination of the attribute ratings may explain some of the variance in the Δ ratings. In assessing the goodness-of-fit of the regression model, the most commonly employed is \( R^2 \) which ranges from 0 to 1 for bad to good fits respectively: an \( R^2 \) of 1 would mean there was perfect prediction of the Δ ratings by the Δ^4 ratings.
CHAPTER 2. METHODS

Cross Validation: Cross validation provides another measure of the reliability of the regression model. The disadvantage of the usual $R^2$ is that it assesses only the fit of the model to the training data—-but this does not necessarily lead to accurate predictions of the model's performance when confronted with new data. Also, intermediate values of $R^2$ are often difficult to interpret. Cross validation attempts to address both these concerns by making predictions of new data in the actual units of the predicted variable. Another concern is that correlated predictor variables can cause instability in the coefficients of the regression equation, which can cause adverse effects when predicting new data. While cross validation does not remedy this, it does provide a measure of the severity of the adverse effects. In our case, the technique assesses a regression model's predictive power using the difference between predicted $\Delta$ ratings and actual $\Delta$ ratings as an estimate of the prediction error. First the data is partitioned into a training set and a test set. The regression coefficients are obtained using the $\Delta$ and $\Delta^i$ values from the training set. Then, the $\Delta^i$ values of the test, together with the regression coefficients, are used to predict $\Delta$ values. The average difference between these predicted $\Delta$ values and the actual test-set $\Delta$ values, is then used as the estimate of prediction error.

Redundancy Analysis: This provides a measure of the amount of variance in one set of variables explained by another set (Stewart & Love, 1968). One of the main interests of this study was variability between observers' ratings, and specifically, the $\Delta$ ratings since these represented everyday, untutored judgements of natural textures. To use redundancy analysis as a measure of intersubject variability, separate MDS solutions based on each individual's
Δ ratings were derived. A given texture had one set of coordinates in one observer’s MDS-Δ space, and another set for another observer’s. The coordinates of an individual’s MDS-Δ space were then employed as predictor variables for those of another individual’s MDS-Δ space. A perfect prediction would mean that the two observers had identical MDS spaces. The technique is equivalent to regressing, in turn, each of the variables (MDS coordinates) of the first set over all the variables of the second set and averaging the resulting multiple correlation coefficients. More explicitly, the technique produces a redundancy coefficient $R^2$, which is an average of the proportion of explained variance ($R^2$) that is accounted for by a particular canonical variate, each $R^2$ weighted by its associated squared canonical correlation (Stewart & Love, 1965). The redundancy coefficient $R^2$ can be regarded as an average multiple-$R^2$ (of linear regression analysis). Of course, this measure of intersubject variability rests on the assumption that the MDS approach is appropriate for our data. The high $R^2$s of the redundancy analyses and the meaningful interpretations gained from the MDS analyses provide evidence that the assumption is at least a useful one.
Chapter 3

Results

The question we have attempted to address is whether regularities in observers' \( \Delta \) judgements of textures can be usefully represented by a "texture space" analogous to the "colour space" that describes various aspects of colour perception. In attempting to characterize a hypothetical texture space, we observe the following organization in the Results section. First, the relatively simple models of linear regression analysis and multidimensional scaling (MDS) are used to ascertain the dimensionality of the \( \Delta \) judgements and to evaluate the relevance of the candidate \( \Delta^4 \) attributes to these judgements. It should be noted that a city-block metric is implied whenever raw \( \Delta \) and \( \Delta^4 \) rating values are used, and a Euclidean metric, when dealing with MDS representations of the \( \Delta \) and \( \Delta^4 \) data (however, the differences between the metrics are probably obscured by the noise in our data). We then use the techniques of cross-validation and redundancy analysis to check that such spatial representations have general utility across observers and texture sets. In attempts to better characterize the attributes themselves, correlations and MDS analyses were performed on the \( \Delta^4 \) judgement data. Finally, we examine how resistant the implied representation of textures
CHAPTER 2. METHODS

is to changing task and viewing conditions.

3.1 Dimensionality: Overall dissimilarity (Δ)

Multidimensional scaling (MDS) analysis provides evidence for a low dimensionality in the innate overall dissimilarity judgements (Δ). MDS aims to discover the underlying structure in proximity (e.g. dissimilarity) data, by converting the dissimilarities between stimuli into distances between points in a geometric space, analogous to converting inter-city distances into city positions in a 2-D geographic map. In our case, the texture samples can be viewed as points that inhabit a multidimensional texture space where the distances between the points are matched, as closely as possible, to the experimentally derived dissimilarity ratings (Δ). The most appropriate metric can vary according to the particularities of how the stimuli are distinguished from each other. When there are no a priori grounds for assuming a particular relationship between the stimuli, a Euclidean metric is commonly employed, and was used in all the following MDS analyses. Various goodness-of-fit measures are used to assess the match. We used Kruskall’s “Stress” (Kruskall & Wish, 1978) which ranges continuously from 0 to 1, with lower values indicating better fits and values less than 0.10 being conventionally regarded as acceptable.

Separate Stress values for dimensionalities varying from 1 to 4 were determined for the mean observer Δ ratings of each of the five texture sets. The results are summarized in Figure 3.1 which provides average values for each dimension. Stress values start at 0.31
Δ Data -- Dimensionality of the Implied Space

Figure 3.1: Stress values, averaged over the five texture sets, are shown for analyses of increasing dimensionality. A dimensionality of 3 or 4 is suggested with Stress values of 0.07 ($R^2=0.96$) and 0.03 ($R^2=0.99$) respectively.
(average for five sets) at one dimension, and decline to 0.07 by three dimensions: associated \( R^2 \) values are 0.68 and 0.96, respectively. The values for the four-dimensional solutions show some improvement (Stress=0.03, \( R^2=0.99 \)), but these might be considered with caution because of the danger of overfitting (n=12 for each texture set). There is already a good fit, however, with a three dimensional solution. We may conclude that the implied texture space derived from MDS analyses of \( \Delta \) ratings is at least 3 dimensional (we will henceforth refer to estimates of texture space based on overall innate dissimilarity ratings (\( \Delta \)) and MDS procedures as "MDS-\( \Delta \) space").
3.2 Attribute Dissimilarities $\Delta^A$ as Predictors of Overall Dissimilarity $\Delta$

3.2.1 Regression Analyses Using $\Delta$ vs. $\Delta^A$ Data

When observers make judgements of overall dissimilarity ($\Delta$) between textures, it is reasonable to ask whether that there are specific attributes of the textures which are consistently used to make these judgements. For example, a texture pair may appear very dissimilar when differing greatly in Coarseness, while another pair would appear very similar due to similarity in Coarseness. We attempted to explore the dependency of innate judgements $\Delta$ on our candidate attribute judgements $\Delta^A$ by using multiple linear regression (MLR). MLR aims to predict the behaviour of one dependent variable, using a weighted sum of multiple independent (predictor) variables. In our case, we are attempting to predict overall dissimilarity ($\Delta$) judgements using the attribute judgements ($\Delta^A$) as predictor variables.

To obtain the optimal model for any given number of predictors, an exhaustive survey of all possible regression equations was undertaken (Figure 3.2). Each point in Figure 3.2 represents the proportion of variance (multiple $R^2$) in the $\Delta$ judgements accounted for by a regression model that includes the attribute judgements as labelled in order of decreasing importance. By four or five parameters, the best regression models (the top points in each column of points) attain multiple $R^2$ values that plateau with values exceeding 0.70, with no gains offered by the six predictor model. The two best overall predictors are Coarseness and Regularity (models with filled circles), though all attributes have some use. Significant but
CHAPTER 3. RESULTS

Figure 3.2: $\Delta$ vs. $\Delta^4$ relationship -- Exhaustive survey of the 63 possible attribute combinations. Each point represents the proportion of variance (multiple $R^2$) in the overall dissimilarity ($\Delta$) judgements accounted for by a regression model that includes the attributes as labelled (abbreviated) in order of decreasing importance. The x-axis indicates the number of attributes entered in the respective equations. The best of the six possible one-predictor models (not visible) just fail to exceed $R^2=0.5$. Of the 15 two-predictor models, the one using Coarseness ($f$) and Regularity ($r$) accounts for 0.64 of the variance in the $\Delta$ judgements. By four or five parameters, the best regression models attain $R^2$'s exceeding 0.70. The six-predictor model offers no real improvement. The two best predictors overall are Coarseness ($f$) and Regularity ($r$) (filled circles denote models including $f$ and $r$). Lesser predictors are Lightness ($l$), Directionality ($d$), Contrast ($c$) and Edginess ($e$).
Δ vs. Δ^A Relationship -- Exhaustive Survey of the 63 Possible Attribute Combinations
lesser predictors for this viewing condition (94.8 Td, 90 cm distance) are Lightness, Directionality, Contrast and Edginess.

3.2.2 Regression Analysis Using MDS Transformation of the $\Delta$ and $\Delta^4$ Data

The above regression analyses assumed that overall dissimilarity ($\Delta$) of pairs of textures can be predicted by linear combinations of attribute differences ($\Delta^4$) for the same pairs. Here, we evaluate the importance of each $\Delta^4$ attribute in overall dissimilarity judgements ($\Delta$) by assessing the proportion of variance in individual MDS-$\Delta^4$ scales accounted for by the MDS-\Delta space coordinates (Shiffman et al., 1981). First, six 1-dimensional MDS scales were constructed from the ratings for the six individual $\Delta^4$ attributes; each texture is a point on such a scale. Each texture also has 3-D or 4-D coordinates already obtained with the MDS-\Delta estimate of texture space in Results 1 above. So we can regress the six 1-D $\Delta^4$ derived coordinates (one attribute at a time) against the 3- or 4-D MDS-$\Delta^4$ derived coordinates as predictors. This optimally orients each $\Delta^4$ attribute axis in the MDS-$\Delta$ space, and goodness-of-fit of the MDS-$\Delta^4$ axis to the MDS-$\Delta$ space is shown by $R^2$ (Shiffman et al., 1981). For example, if Coarseness was an attribute that was relevant to observers' $\Delta$ judgements of textures, the textures should vary according to Coarseness along a particular direction in the MDS-$\Delta$ space. $R^2$ should be high if the 1-D MDS-$\Delta^4$ Coarseness scale faithfully captured the variance along this direction in MDS-$\Delta$ space. It might be noted that these $R^2$s can be elevated in comparison to the $\Delta$ vs. $\Delta^4$ regressions (Results 3.2.1) since in the present regressions, the $\Delta$-derived 3- or 4-D MDS coordinates are used to explain the
variance of each of the individual attribute scales in turn; in the previous section (3.2.1) the
Δ^4 ratings, in combinations, were used to account for the variance in the Δ ratings.

For the 3-D configurations, Coarseness, Regularity and Directionality demonstrate
impressive R^2's (around 0.87; see Table 3.1). There is only modest improvement in these
particular R^2's for the 4-D configurations, suggesting that each of these three attributes could
individually serve as dominant axes or useful directions in a low-dimensional texture space.
In proceeding to the 4-D solutions, Contrast, Lightness and Edginess attain R^2's of around
0.70, meaning that all six candidates are satisfactory axes in a 4-D MDS-Δ estimate of texture
space.
\( \Delta \text{ vs. } \Delta^A \text{ relationship -- Regressions of Attribute Scales on Coordinates} \)

<table>
<thead>
<tr>
<th>Attributes</th>
<th>f</th>
<th>r</th>
<th>d</th>
<th>c</th>
<th>l</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D</td>
<td>0.85</td>
<td>0.87</td>
<td>0.87</td>
<td>0.54</td>
<td>0.48</td>
<td>0.64</td>
</tr>
<tr>
<td>4-D</td>
<td>0.91</td>
<td>0.90</td>
<td>0.90</td>
<td>0.70</td>
<td>0.66</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Table 3.1: One-dimensional MDS scales of the textures were constructed from the ratings for each individual \( \Delta^A \) attribute. The scales were then regressed over the corresponding 3-D and 4-D MDS spaces derived from \( \Delta \) ratings of the same textures. In the 3-D solutions, attributes f, r, and d show impressive \( R^2 \)s, suggesting that any of these attributes could serve as axes in a low dimensional texture space. The \( R^2 \)s for these attributes show only modest improvement for 4-D solutions. In proceeding to the 4-D solutions, c, l, and e demonstrate notable increases in \( R^2 \)s. All attributes are satisfactory candidate axes when 1-D attribute scales are regressed against 4-D MDS spaces.
3.3 Variability Across Texture Sets and Observers ($\Delta$, $\Delta^4$)

One possibly serious concern about the literature, and the above analyses, is the use of seemingly very subjective judgements. To explore this issue, we use the method of cross-validation (Efron & Tibshirani, 1993) to assess the reliability of a model by confronting the model with new data. Another problem is that a fixed body of data is generally better fitted by more parameters. This is not the case in cross-validation where introducing irrelevant parameters (dimensions) will eventually worsen the fit of new data (for an analytic estimate of the same, see Schwartz, 1978). We partitioned the data set and developed the model (the regression equation) on one portion (the training set), estimating the prediction error on the remaining data (the test set). Variance in the prediction error estimate can be reduced by alternating the portion of the data set that is utilised as the test set. In other words, the data is split into $K$ equal parts, the $k$th part taken as the test set and the prediction error determined using the remaining $K-1$ parts as training data. Performing this for each of $k = 1, 2, \ldots, K$ parts, one can combine the $K$ estimates of prediction error (Efron & Tibshirani, 1993). One can repeat this procedure for models with varying numbers of dimensions.

For our data, the regression coefficients (the weightings for the $\Delta^4$ judgements) were obtained using training sets, and were then used to predict new $\Delta$ ratings using the $\Delta^4$ ratings of the remaining (test) data. The average absolute value of the difference between predicted $\Delta$ ratings and actual $\Delta$ ratings of the test set was then used as an estimate of prediction error. Prediction errors are in $\Delta$ rating units on a scale of 1 to 7. Typically, a plot of the prediction
error versus the number of parameters should demonstrate a minimum prediction error for an optimal number of parameters, beyond which any further elaboration of the model overfits the data and increases the prediction error. In our case all six attributes improved the fit, confirming the above finding (3.2.2) that none are irrelevant.

3.3.1 Across Texture Sets: Cross-validation ($\Delta$, $\Delta^4$)

A real concern is the possibility that texture discrimination is considerably texture-set dependent. To examine this, the cross-validation exploited the fact that the 60 textures were divided into five sets. We calculated the average prediction error using cross-validation, rotating the set that served as the test set. The results, shown in Figure 3.3, confirm that the regression models are applicable across texture sets. With only one attribute the predictive power of the models is poor with errors of around 2.0 rating units. With four attribute models, however, the error decreases to 0.9 rating units on a scale of 1-7. Considering that the dissimilarity judgements ($\Delta$) of wholly new sets of textures are being predicted, this result gives good support to the reliability of the simple, low-dimensional, regression models.

3.3.2.1 Across Observers: Cross-validation

Intersubject variability is another important consideration when positing a stable, unified texture space common to all observers. So far (1-3.1), our analyses have data pooled across all observers, and have suggested that for the observers as a group, there are robust
\( \Delta \text{ vs. } \Delta^A \text{ Relationship -- Cross Validation} \\
\text{Across Texture Sets} \\

Figure 3.3: Prediction error was used as a measure of across-set variability. Shown are the average prediction errors for models with increasing numbers of attributes. This confirms that the regression models are applicable across different sets of textures. With one attribute the predictive power of the models is poor with errors of around 2.0 rating units. With four predictor models, the error decreases to 0.9, suggesting that at least four attributes are required to account for the variance in the \( \Delta \) judgements.
regularities in the judgements of texture pairs. One may still worry, however, whether rapid untutored judgements (ratings) by one observer have much relevance to another’s. Figure 3.4 shows intersubject variability as measured by prediction error derived from cross-validation. There were four repeats of each $\Delta^4$ rating, so each point is an average of four calculations of prediction errors. Shown are the prediction errors for models with increasing numbers of attributes. Since, in this case, we predict one observer’s $\Delta$ ratings using regression models trained on three other observers’ ratings, we obtain a useful measure of intersubject variability; these errors can be directly compared against the $\Delta$ rating scale which ranges from 1 to 7. With one predictor, there is a reasonable prediction of the remaining observers’ ratings. There is a steady decrease in prediction error until we have models with 5 attributes, with average errors of around 1.2 rating units. It appears that observers’ innate $\Delta$ judgements have a surprising common ground.

3.3.2.2 Across Observers: Redundancy Analysis ($\Delta$)

The cross-validation of the observer data provides one measure of intersubject variability. The interpretation of the prediction errors, however, is compromised by the errors generated by the imperfect $\Delta$-$\Delta^4$ linear regression models used in making the predictions. To assess the similarity of observers’ perceptual spaces more directly, redundancy analysis (Stewart & Love, 1968) was employed. We gain the advantage of excluding the $\Delta^4$ attributes from the analysis--they may be imperfect descriptors for texture judgements--and examining only the innate $\Delta$ judgements. A given texture has one set of coordinates in the MDS-$\Delta$


Δ vs. Δ\textsuperscript{A} relationship -- Cross Validation Across Observers

Figure 3.4: Prediction error is used as a measure of the intersubject variability in texture spaces. Shown are the average prediction errors for models with increasing numbers of attributes. Cross-validation is explained in the Methods section. Since, in this case, we predict one observer's Δ ratings using regression models trained on other subjects Δ and Δ\textsuperscript{A} values, we measure intersubject variability in rating units. Notice that with one predictor, there is already reasonable prediction of another subject's Δ ratings. There is a steady decrease in prediction error with average errors of around 1.2 for models with 5 attributes.
space of one observer, and another set for another observer. The technique is equivalent to regressing, in turn, each of the variables (MDS coordinates) of the first set over all the variables of the second set and averaging the resulting squared multiple correlation coefficients. The redundancy coefficient $R^2$ can be regarded as an average multiple-$R^2$ (of linear regression analysis).

For each of the five sets of textures, the $R^2$'s were determined for all observer combinations. Those for set A are summarized in Table 3.2. These were computed for the 3-D and 4-D configurations which provided the most reasonable fits to the $\Delta$ data (3-D: stress=0.08, $R^2=0.90$; 4-D: stress=0.04, $R^2=0.96$). The average over all texture sets of the 4-D $R^2$'s is 0.63 (SD=0.11, range: 0.28-0.88). For 3-D MDS configurations, the $R^2$'s are slightly lower (0.54, SD=0.11, range: 0.21-0.88). Rarely, the MDS-$\Delta$ estimate of one observer's texture space may deviate somewhat from the rest; in texture set A, one observer (DE, no. 7), had consistently lower $R^2$ values with a 4-D $R^2$ average of 0.50 and a 3-D $R^2$ average of 0.32. It is concluded that matches between observers' perceptual spaces are reasonable to very good with the 4-D configurations being better than 3-D. This approach provides a separate measure of intersubject variability to the $\Delta-\Delta^4$ model cross-validation in 3.2.1 above and yields the same conclusion. As a simple check, direct correlations between different observers' $\Delta$ ratings also average to $R^2 = 0.61$. 

Table 3.2: Δ ratings -- Redundancy Coefficients for Individual Observers' Spaces. As another measure of intersubject variability, the variances shared between individual observers' MDS configurations were determined using redundancy analysis. A sample (texture set A) of redundancy coefficients (R²) for all observer combinations is given in the first two tables; the larger the value, the more variance two observers have in common. The third table summarizes the redundancy coefficients for all of the sets, with average R² values of 0.54 and 0.63 for the 3-D and 4-D MDS solutions respectively. Matches between observers' perceptual spaces are reasonable to very good, the 4-D configurations being the better.
### RESULTS

**Redundancy Coefficients for Set A**

#### 3-D MDS Solutions

<table>
<thead>
<tr>
<th>Subject</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.556</td>
<td>0.592</td>
<td>0.351</td>
<td>0.407</td>
<td>0.465</td>
<td>0.335</td>
<td>0.539</td>
<td>0.371</td>
<td>0.460</td>
<td>0.495</td>
<td>0.507</td>
</tr>
<tr>
<td>2</td>
<td>0.628</td>
<td>0.560</td>
<td>0.732</td>
<td>0.805</td>
<td>0.301</td>
<td>0.614</td>
<td>0.434</td>
<td>0.450</td>
<td>0.623</td>
<td>0.691</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.609</td>
<td>0.582</td>
<td>0.522</td>
<td>0.314</td>
<td>0.541</td>
<td>0.434</td>
<td>0.608</td>
<td>0.597</td>
<td>0.534</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.531</td>
<td>0.565</td>
<td>0.536</td>
<td>0.421</td>
<td>0.507</td>
<td>0.376</td>
<td>0.459</td>
<td>0.489</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.783</td>
<td>0.225</td>
<td>0.670</td>
<td>0.559</td>
<td>0.530</td>
<td>0.635</td>
<td>0.529</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.328</td>
<td>0.531</td>
<td>0.369</td>
<td>0.445</td>
<td>0.509</td>
<td>0.545</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.254</td>
<td>0.305</td>
<td>0.230</td>
<td>0.367</td>
<td>0.309</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.526</td>
<td>0.607</td>
<td>0.507</td>
<td>0.492</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.310</td>
<td>0.305</td>
<td>0.339</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.534</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.629</td>
</tr>
</tbody>
</table>

#### 4-D MDS Solutions

<table>
<thead>
<tr>
<th>Subject</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.578</td>
<td>0.651</td>
<td>0.379</td>
<td>0.570</td>
<td>0.386</td>
<td>0.336</td>
<td>0.575</td>
<td>0.639</td>
<td>0.405</td>
<td>0.588</td>
<td>0.653</td>
</tr>
<tr>
<td>2</td>
<td>0.531</td>
<td>0.505</td>
<td>0.797</td>
<td>0.563</td>
<td>0.470</td>
<td>0.516</td>
<td>0.470</td>
<td>0.514</td>
<td>0.540</td>
<td>0.655</td>
<td>0.623</td>
</tr>
<tr>
<td>3</td>
<td>0.546</td>
<td>0.608</td>
<td>0.516</td>
<td>0.391</td>
<td>0.608</td>
<td>0.712</td>
<td>0.593</td>
<td>0.610</td>
<td>0.559</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.652</td>
<td>0.701</td>
<td>0.735</td>
<td>0.483</td>
<td>0.593</td>
<td>0.438</td>
<td>0.599</td>
<td>0.478</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.695</td>
<td>0.497</td>
<td>0.696</td>
<td>0.601</td>
<td>0.583</td>
<td>0.677</td>
<td>0.580</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.616</td>
<td>0.548</td>
<td>0.472</td>
<td>0.481</td>
<td>0.526</td>
<td>0.584</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.489</td>
<td>0.445</td>
<td>0.388</td>
<td>0.602</td>
<td>0.487</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.624</td>
<td>0.565</td>
<td>0.638</td>
<td>0.612</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.636</td>
<td>0.654</td>
<td>0.576</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.484</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.665</td>
</tr>
</tbody>
</table>

**Summary of Redundancy Coefficients for All Texture Sets**

<table>
<thead>
<tr>
<th>MDS Dimensionality</th>
<th>Average</th>
<th>SD</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-D</td>
<td>0.54</td>
<td>0.11</td>
<td>0.21-0.88</td>
</tr>
<tr>
<td>4-D</td>
<td>0.63</td>
<td>0.11</td>
<td>0.28-0.88</td>
</tr>
</tbody>
</table>
3.4 Nature of the Attributes

3.4.1 Correlations Between Attributes ($\Delta^4$)

Although all the $\Delta^4$ attributes may serve as useful axes (direction) in a multidimensional texture space, they need not be mutually orthogonal or cardinal. And there are a priori grounds for suspecting that a number of the attributes are inter-related, e.g., Tamura et al. (1976) noted the possible contributions of sharpness of edges (Edginess) and period of repeating patterns (Coarseness) to judgements of contrast (Contrast).

Correlations were determined for all possible pairs of attributes (Table 3.3). Highest correlations include the pairs Regularity and Directionality ($r = 0.72$), and Contrast and Edginess ($r = 0.64$). Lesser correlations include Contrast and Coarseness ($r = 0.39$), Edginess and Regularity ($r = 0.41$), and Coarseness and Edginess ($r = 0.45$). These pairs, then, cannot be strictly orthogonal dimensions in a texture space. This also explains how each of the six 1-D attribute scales could attain such high $R^2$'s when regressed with the corresponding 4-D MDS configurations (Results 3.2.2) - six scales could only serve as meaningful axes in a 4-D MDS space if they were not all mutually orthogonal.

3.4.2 Dimensionality of the Individual Attributes ($\Delta^4$)

The $\Delta^4$ judgements have thus far been assumed to have implied truly unidimensional
$\Delta A^1$ Inter-Relations -- Correlations Between Attributes

<table>
<thead>
<tr>
<th></th>
<th>l</th>
<th>r</th>
<th>d</th>
<th>c</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>-0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>-0.06</td>
<td>0.72</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>0.20</td>
<td>-0.06</td>
<td>-0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>-0.03</td>
<td>-0.09</td>
<td>-0.18</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>0.16</td>
<td>0.41</td>
<td>0.27</td>
<td>0.64</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Table 3.3: Shown are the correlations between all the possible pairings of $\Delta A^1$ ratings for each set, and pooling across sets. Shaded boxes mark notable correlations. The highest correlations include the pairs r:d and c:e (bold, italic); lesser correlations include c:f, e:r and f:e (italic). These pairs, then, cannot be strictly orthogonal dimensions in a texture space.
attribute scales. There are good reasons to suspect, however, that the $\Delta^4$ judgements themselves may be multidimensional. For instance, Tamura et. al (1976) suggest that four factors contribute towards contrast judgements, namely, the dynamic range and polarity of the grey level distribution the sharpness of edges and the period of repeating patterns. It remains a possibility, then, that the variance in $\Delta^4$ judgements for Contrast might be captured by a four-dimensional "Contrast space". Coarseness may deserve a two-dimensional space since both the size and space of repeating elements figure into judgements. Similar arguments could be made for the other attributes, except perhaps Lightness. In the latter case, one might hope for a unidimensional representation because spatially averaged reflectance is a single physical variable; however, it is not entirely clear how receptive field signals are combined to estimate this physical quantity.

To explore the possibility of multidimensionality of attributes. MDS analyses were performed on $\Delta^4$ ratings of each separate attribute (Figure 3.5). None of the attributes attain stress levels below 0.10 with only one-dimensional solutions. Lightness, Regularity and Directionality all attain stresses below 0.10 by two dimensions. Edginess, Coarseness and Contrast require two or three dimensions. These results suggest that the individual attributes are not infinitesimally narrow 1-D axes in texture space. The result is, however, consistent with the above discussion. It should be noted that these configurations were derived from fairly coarse $\Delta^4$ ratings (before averaging), taking on only one of three values (0, 1 or 2). This would tend to contribute noise to the process of configuring an MDS space and thus, on the whole, raise Stress values and so, bias towards a dimensionality higher than one.
Δ^4 Ratings -- Dimensionality of the Individuals Attributes

Figure 3.5: Shown are Stress values for MDS analyses of the Δ^4 ratings of each separate attribute. None of the attributes attain stresses below 0.10 with only one MDS dimension. Lightness, Regularity and Directionality all attain stresses below 0.10 by two dimensions. Edginess, Coarseness and Contrast require two or three dimensions. This suggests that the individual attributes are not infinitesimally narrow 1-D axes in texture space.
3.5 Judgements Under Impoverished Viewing Conditions

Since the innate dissimilarity judgements seem relatively stable across different observers and sets of textures, it can be asked how unvarying they remain when the stimuli are presented under impoverished viewing conditions. Observers were asked to make the same types of judgment of texture pairs ($\Delta$ and $\Delta^4$ ratings), with systematic decreases in luminance (Yang, 1995) or increases in viewing distance (Cho & Hallett, 1995).

3.5.1 Dimensionality: Overall Dissimilarity ($\Delta$)

At the most general level, the dimensionality of the implied spaces is preserved in the MDS-$\Delta$ analyses (Figure 3.6). For the varying luminance condition, all luminance conditions show satisfactory Stress values of below 0.10 for 4-D configurations. A similar pattern held when distance was varied. These results are not significantly different from those of the main experiment (Figure 1, 94.8 Td, 0.9 m). However, the Stresses of Figure 3.6 (unfilled circles, top and bottom panels) are a little lower for the extreme viewing conditions (0.1 Td or 30 m), suggesting a lower dimensionality, as might be expected.

3.5.2 Relevant Attributes: $\Delta$ vs. $\Delta^4$ Relationship

To examine which attributes were significant, regressions were performed with $\Delta$ ratings being predicted by the $\Delta^4$ ratings (Figure 3.7), in the manner of Figure 3.2. If we
CHAPTER 3. RESULTS

Figure 3.6: Δ Ratings -- MDS Analyses of the Varying Luminance and Distance Conditions. 

a) **Luminance** The first four luminances show similar declines in Stress ending below 0.10 by the 4-D solutions. 

b) **Distance** All the distances achieve stresses below 0.10 by the 4-D solutions. In general the Stresses are lowest for the most impoverished viewing conditions, suggesting a lower dimensionality in such cases as might be expected.
CHAPTER 3. RESULTS

Stresses For Varying Luminance Condition

Dimensions

Stresses For Varying Distance Condition

Dimensions
Figure 3.7: Δ vs. Δ³ Relationship -- Distance and Luminance Data. Multiple regressions for varying viewing conditions. Regressions were performed for ratings gathered for systematic decreases in luminance or increases in distance. 

a) Luminance All luminance levels demonstrate high maximum $R^2$'s which increase with added attributes and plateau at around 0.80 (arrows denote plateaus). The $R^2$'s for the four highest luminances follow a similar distribution. There is a sharp reduction in the number of useful attributes at the lowest luminance. In that case, closer scrutiny would show that Lightness accounts for most of the explained variance.

b) Distance There is a more gradual change in the distributions of $R^2$'s over distance. Again, at each distance, maximum $R^2$'s increase with added attributes and in this case plateau at around 0.70 (arrows). At the furthest distance, there are few viable attributes, Lightness explaining most of the variance. Lightness is the major variable when viewing is impoverished.
CHAPTER 3. RESULTS

Luminance 1 (99.3 Td)

Luminance 2 (17.7 Td)

Luminance 3 (3.14 Td)

Luminance 4 (0.56 Td)

Luminance 5 (0.10 Td)

Distance 1 (0.9 m)

Distance 2 (8.2 m)

Distance 3 (15.5 m)

Distance 4 (22.9 m)

Distance 5 (30.2 m)

Number of Attributes

Multiple R²
focus on the top-most points for each attribute number (x-axis), we see that all luminance levels demonstrate high maximum $R^2$'s which increase with added attributes and plateau at around 0.80 (arrows). While the $R^2$'s for the four highest luminances follow a similar pattern, the early plateau of the maximum $R^2$'s at the lowest luminance denotes a sharp reduction in the number of useful attributes at that luminance. With increases in viewing distance, there is a more gradual change in the distributions of maximum $R^2$'s (top-most points). Again, at each distance, maximum $R^2$'s increase with added attributes and in this case, plateau at around 0.70. At the furthest distance, however, $R^2$ is reduced and there is effectively only one viable attribute. Closer scrutiny of the regression equations, in either of the conditions, would show that Lightness explains most of the variance. Even under the best viewing conditions (highest luminance, closest distance), Lightness had more significance as a predictor than in the main experiment where the textures were viewed under comparable conditions. The different viewing conditions may have increased the salience of Lightness as an attribute. Most (80%) observers began the distance experiment at a distance considerably greater than the 0.9 m of the main experiment; also, in comparison to brightest condition. Lightness gained in salience in the four dimmest conditions of the luminance experiment, where retinal illuminance was at least 1 log lower than in the main experiment (17.7, 3.14, 0.56, 0.10 Td vs. 94.8 Td). Still, Coarseness and Regularity, which were the major attributes in the main experiment, figured as important predictors of overall dissimilarity judgements ($\Delta$) for the first four luminances (99.6-0.56 Td) and the first three distances (0.9-15.5 m).
3.5.3 Direct Comparisons of $\Delta$ Ratings

As a more stringent measure of the invariance of judgements, one could make direct comparisons of the $\Delta$ ratings under impoverished and optimal (highest luminance and closest distance) conditions. Figure 3.8 (squares) shows Pearson correlations between $\Delta$ ratings at the closest distance (0.9 m) with comparable ratings made at the further distances (8.2 to 30.2 m). High correlations would mean that the relative values of the judged differences are well preserved. This is the case at distance 2 (8.2 m: $r = 0.91, r^2 = 0.81$), with rapid decrease in $r$ values for further distances. Figure 3.8 (diamonds) also shows a similar plot for the varying luminance condition. The correlations remain consistently high at around $r = 0.95$ with a sharp drop at the lowest (0.1 Td) illuminance level ($r = 0.83$). The shapes of these plots are in good agreement with the changes in the distributions of $R'$s of Figure 3.7, that is, for varying luminances, the sharp change at luminance 4 (Figure 3.8, L4 or 0.56 Td); and for the distance case, more gradual changes over the five distances (Figure 3.8, squares). If we take a cutoff $r = 0.90$ to mark highly significant correlations, these results delimit the range over which innate judgements of overall texture dissimilarity are effectively constant: for distance, 1-8 m; for luminance, 100-1 Td.
Δ Ratings -- Comparisons Between Impoverished and Optimal Conditions

Figure 3.8: a) Distance The squares denote Pearson correlations between Δ ratings at the closest distance (0.9 m) with comparable ratings made at the further distances (8.2 to 30.2 m or D2 to D5, respectively). High correlations would mean that the relative values of the judged differences are well preserved. This is the case at D2 (8.2 m: \( r = 0.91, r^2 = 0.81 \)), with rapid decrease in \( r \) values for further distances. b) Luminance The diamonds denote correlations between Δ ratings at the brightest illuminance (99.3 Td) with comparable ratings made at dimmer illuminances (17.7 to 0.10 Td or L2 to L5, respectively). The correlations remain consistently high at around \( r = 0.95 \) with a sharp drop at the lowest (L5 or 0.1 Td) illuminance level (\( r = 0.83 \)).
3.6 Identification Task

To compare the $\Delta$ and $\Delta^4$ dissimilarity ratings to a more objective and natural measure of observers' perceptions, an ancillary experiment was performed which tested their ability to identify texture samples. From varying distances (as in Results 3.5), observers viewed panels of 20 texture samples and provided their best guesses at identifying the distant samples, referring to a panel of thumbnail samples of the textures to avoid any confusion over names. All other aspects of the viewing conditions were comparable to the other experiments of the study; with 10 observers, pooled data consisted of 2000 identifications for each of the five distances.

Table 3.4 illustrates an example of the raw data in the form of a "stimulus/response" matrix (viewing distance 8.2 m; others in Appendix 2). The labels identify the Brodatz textures, rows the stimuli and columns the observers' pooled identifications. When all stimuli are equally likely, as here, it is convenient to reduce such confusion matrices to the information received in bits. Figure 3.9 shows the information received at each distance, calculated in the standard manner (Garner & Hake, 1951), and averaged in two different ways. As expected, performance is nearly perfect at 0.9 m (ideally 4.3 bits for 20 equally likely textures), and declines at longer distances. Performance at the longer distances is higher if one averages each individual's received information, rather than first pooling the responses across observers. This is because individuals differ in their patterns of error, and so pooling increases the number of categories (cells) in which there are errors.
Sample Stimulus-Response Matrix for the Observers’ Pooled Responses

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>9</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>22</th>
<th>29</th>
<th>34</th>
<th>37</th>
<th>52</th>
<th>55</th>
<th>68</th>
<th>86</th>
<th>87</th>
<th>93</th>
<th>98</th>
<th>109</th>
<th>112</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>33</td>
<td>49</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>t</td>
<td>58</td>
<td>35</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>i</td>
<td>12</td>
<td>1</td>
<td>77</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>m</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>65</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>17</td>
<td>11</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>u</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>99</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>l</td>
<td>20</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>62</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>16</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>u</td>
<td>22</td>
<td>0</td>
<td>0</td>
<td>97</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>s</td>
<td>34</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>73</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>i</td>
<td>37</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>64</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>21</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>m</td>
<td>52</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>u</td>
<td>55</td>
<td>0</td>
<td>0</td>
<td>23</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>72</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>l</td>
<td>66</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>u</td>
<td>68</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>s</td>
<td>76</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>11</td>
<td>78</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>i</td>
<td>86</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>m</td>
<td>87</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>u</td>
<td>93</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>l</td>
<td>98</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>u</td>
<td>109</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>s</td>
<td>112</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>57</td>
</tr>
</tbody>
</table>

Table 3.4: Rows represent the texture shown and columns the identification. Correct responses lie on the diagonal. The row and column labels are the Brodatz numbers of the textures. The boxes mark an example of mirror-position cells, the sum totals of which are a measure of confusion of one texture for another (in this case, D37 sent/D68 received and D68 sent/D37 received). The stimulus-response for the other distances can be found in Appendix 2.
Figure 3.9: As expected, there is a general decrease in the information received with increasing distance. Notice that as the distance increases, the discrepancies between the pooled and unpooled data information measures increase. This means that even though individual subjects are receiving similar amounts of information (unpooled data), the distributions of responses are different. This information about the differences between subjects is lost when the data is pooled; the scatter for an individual subject’s matrix is less than that for the pooled matrix.
CHAPTER 3. RESULTS

Accurate identifications lie on the diagonal and errors off it. If there is a single perceptual space that is accessed by both identification and comparison tasks, a reasonable question might be whether the error rate in a pair of mirror-position cells (e.g., the cells "D37 sent/D68 received" and "D68 sent/D37 received") is correlated with the "overall" dissimilarity judgement Δ for the same texture pair rating in varying distance condition (Results 3.5). Figure 3.10 shows scatter plots pooling all pairs of cells and all observers at the four further distances, against Δ. The general inverse relation means that the more dissimilar a two textures are rated, the less likely they will be confused in the identification task. This provides assurance that the present subjective ratings are related to natural objective measures of visual performance, such as identifications.

A MDS analysis was performed using transformed paired-error rates as the dissimilarity input to the MDS algorithm. The transformation into dissimilarities rested on the assumption that the confusion between a pair of textures would be inversely proportional to the dissimilarity between them. For each distance, the dissimilarity matrix was derived as follows: the stimulus/response matrix (M) was added to its own transpose, resulting in a symmetric matrix, say, A. The dissimilarity matrix (D) was then calculated by subtracting A from a matrix composed solely of elements whose values equal that of the highest element of A (k, a scalar). So, in algebraic notation, \[ D = k \mathbb{1} \mathbb{1}^T - A \] where \( \mathbb{1} \mathbb{1}^T \) is matrix with "1"s as elements and \( A = M + M^T \). It should be noted that this transformation was an ad hoc method of attaining the inverse relation between confusion and dissimilarity; for an alternate method, see Getty et al. (1979). Figure 3.11 shows that all solutions at each distance achieve Stresses...
CHAPTER 3. RESULTS

Identification-Dissimilarity Relationship for the Four Longer Distances

Figure 3.10: The ordinate shows errors of confusion in the identification task for the 12 textures that were studied in both the identification and overall dissimilarity (Δ) tasks. The abscissa is the mean overall dissimilarity Δ ratings of Results 3.5.
below 0.10 by four dimensions. Consistent with the hypothesis that dimensionality should decrease with distance, distances 2 to 4 (8.2-22.9 m) maintain a reasonable order, with distance 4 consistently maintaining lower Stress values. The fifth distance misbehaves somewhat according to this reasoning, which may be due to difficulties inherent in any task at this distance--the regression models for the $\Delta$ and $\Delta^4$ dissimilarity ratings at same distance also suffered lesser $R^2$'s than those for the four closer viewing distances.
Stresses for MDS Configurations Derived from Identifications

Figure 3.11: All the distances achieve Stresses below 0.10 by four dimensions. Distances 2 to 4 maintain a reasonable order, with distance 4 consistently maintaining the lowest Stress until the 4-D solutions. This is consistent with the hypothesis that dimensionality should decrease with distance. However, distance 5 misbehaves according to this reasoning.
Chapter 4

Discussion

4.1 Dimensionality

4.1.1 High Dimensionality of Early Neural Representations

"Bottom-up" information processing models assume a hierarchical structure for the processing of sensory information, the outputs of each level serving as inputs for the next. If strict serial processing is assumed, the dimensionality of such a system is constrained by that of the most limiting stage preceding the final output. In the study of colour vision, discovery of the three kinds of cone photopigments (Brown & Wald, 1963) confirmed that the dimensionality was confined to a maximum of three, a number suggested by early experiments which found that a three channel model could account for the observed colour-matching behaviour (Maxwell, 1855). In the texture case, if we consider both the large number of types of receptive field at each point in the primary visual cortex (e.g., Sakitt & Barlow, 1982; Daugman, 1987; De Valois, Albrecht & Thorell, 1982; Field, 1987; Fleet & Jepson, 1989), and the fact that these must be replicated to represent a patch of texture, we are not limited (by this measure) to as low a dimensionality as 3. It is possible that at the
behavioral level, the high dimensionality of early neural representations is preserved.

4.1.2 Behavioral Studies of Low-level Representations

Harvery & Gervais (1978, 1981) have demonstrated that similarity judgements can indeed be based on relatively low-level representations of textures (see Table 4.1 for summary of studies of comparative judgements of textures). Information concerning the spatial frequency components of their textures, or the activities of spatial frequency channels, largely accounted for the results of their grouping and comparison tasks. Similar studies have shown that texture judgements could be based on relative phase (Kahana & Bennett, 1994) and orientation (Richards, 1979) information. It is therefore clear that behavioral responses can rely on low-level representations, if only for very simply textured patterns. However, due to attentional limitations (e.g., Broadbent, 1958; Attneave, 1974; Tsotsos, 1990), basing our judgements of much more complex textures solely on lower-level representations may be impracticable.

4.1.3 Studies of Higher-order Attributes

If textural information processing involves only linear transformations, the characterization of responses at an early level would preclude the uniqueness of response functions at any other level (Richards, 1979). In other words, the response functions at this earlier level would suffice as a basis set for the processing of all kinds of textures. However,
<table>
<thead>
<tr>
<th>Study</th>
<th>Task and Sample characteristics</th>
<th>Stimulus characteristics</th>
<th>Prediction characteristics</th>
<th>Analysis</th>
<th>Dimensionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harvey and Cervais, 1978</td>
<td>One-dimensional compound gratings</td>
<td>Triadic (2/30) and paired comparisons</td>
<td>Discriminant analysis, MDS, canonical correlation</td>
<td>4 channels</td>
<td>4 channels</td>
</tr>
<tr>
<td>Harvey and Cervais, 1981</td>
<td>One-dimensional compound gratings</td>
<td>Triadic comparisons (16/20)</td>
<td>MDS, eigenvalues</td>
<td>4 channels</td>
<td>3 MDS axes</td>
</tr>
<tr>
<td>Kahana and Bennett, 1994</td>
<td>Compounds of fundamental (2/30) and second harmonic (2f)</td>
<td>Triadic comparisons (2f)</td>
<td>MDS, regression</td>
<td>4 channels</td>
<td>2 channels</td>
</tr>
<tr>
<td>Tamura et al., 1989</td>
<td>Staircase</td>
<td>Contrast, order, directionality, roughness</td>
<td>Thurstonian scaling, correlations, least squares, prediction</td>
<td>MDS, regression</td>
<td>56 (not specified)</td>
</tr>
<tr>
<td>Nakayama and Ungerleider, 1989</td>
<td>Textures</td>
<td>Coarseness, complexity, periodicity, texture strength, prediction</td>
<td>MDS, cluster analysis</td>
<td>55 (not specified)</td>
<td></td>
</tr>
<tr>
<td>Rao and Lohse, 1997</td>
<td>Groupings (20/30)</td>
<td></td>
<td>MDS, regression, discriminant analysis, partial and composite analysis</td>
<td>3 MDS axes</td>
<td>3 MDS axes</td>
</tr>
<tr>
<td>Rao and Lohse, 1996</td>
<td>Groupings (20/30)</td>
<td></td>
<td>MDS, regression, discriminant analysis, partial and composite analysis</td>
<td>3 MDS axes</td>
<td>3 MDS axes</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of Experiments Involving Comparative Judgements of Textures
basing all judgements of textures on such representations might be computationally unwieldy. The complex, natural textures such as those used in this study would require the full complement of receptive field types at this level; the small subset of receptive fields types that would suffice when viewing simple textures such as one-dimensional compound gratings would not be adequate in this instance. Getty et al. (1979) note that information-processing models propose that a selective reduction in information is desired for the extraction of perceptually more important attributes. Without speculating about what determines the importance of attributes, a number of studies have shown that a handful of higher-order attributes, such as "coarseness" and "contrast", are, in fact, useful in explaining the perceived similarities of natural textures (Tamura et al., 1978; Amadasun & King, 1989; Rao & Lohse, 1996; see Table 4.1). In addition to its lower dimensionality, the implied representation is characterized by attributes that are not simple linear sums of lower-level channel activities (e.g.: Tamura et al., 1978; Amadasun & King, 1989; Rao, 1990). This means that new and partially independent sensory dimensions are produced (Richards, 1979). To summarize, studies that have examined judgements of natural textures have posited that a small number higher-order attributes underlie the processing of texture vision. These attributes are not related to lower-level Fourier representations in a simple linear fashion.

Our own study has confirmed that a low-dimensional representation can explain the judgements of dissimilarity of texture pairs. The MDS analyses suggest that 3 or 4 dimensions can account for the Δ ratings. Regression and cross-validation analyses also suggest that 4 to 5 of the 6 studied higher-order (not necessarily orthogonal) attributes were
good predictors of overall dissimilarity ($\Delta$). These findings are consistent across all five sets of textures.

The studies by Tamura et al. (1976) and Amadasun & King (1989) made no explicit claims about the dimensionality of texture space. Tamura et al. (1976) note the perceptual importance of their computationally defined "coarseness" and "directionality" but these gave poor predictions of texture similarity. Amadasun & King (1989) had reasonable success in the prediction of similarity rankings and classifications using their computationally defined features. But their study would only permit a maximal claim for a dimensionality of three since three of their five attributes (coarseness, texture strength, busyness) were highly correlated. Rao & Lohse (1993, 1996) presented texture samples taken from the same set (Brodatz, 1966) and asked observers to perform rating and grouping tasks. While noting that relevant dimensions may have been neglected in their model due to limited texture sample size, they concluded that for their 56 texture sample set, judgements could be based on a three dimensional MDS representation.

4.1.4 At least four dimensions

Our findings suggest that the dimensionality is at least four. There is only marginal improvement in Stress when proceeding from three to four dimensional solutions in our MDS analyses (Figure 3.1). But various results suggest a dimensionality greater than three. First, there is great improvement in the fits of the 1-D attribute scales to the MDS-$\Delta$ space when
proceeding from 3 to 4 dimensional solutions (Table 3.1) with all six attribute scales attaining $R^2$ values above 0.65. Stress cannot, by itself, distinguish a lower dimensionality with noisy data, from a higher dimensionality with little noise in the data; MDS overestimates dimensionality given noisy data. In our case, it seems that an increase in dimensionality from 3 to 4 uncovers regularities previously obscured in the three dimensional configuration. If one takes into account the correlations between the attributes (Table 3.3), it is possible that four of the six attributes, say, Coarseness, Regularity, Contrast and Lightness, could serve as candidate approximations to orthogonal axes in the 4-D MDS-$\Delta$ space.

Further evidence for a dimensionality greater than three is provided by our exhaustive search for regression equations that employ the six $\Delta^4$ attribute ratings as candidate predictors of $\Delta$ ratings. Figure 3.2 shows steady improvements in $R^2$'s until 5 attribute models, with the "frdlc" model being the best with an $R^2$ of 0.75. These regression equations, however, are fitted to the whole body $\Delta$ and $\Delta^4$ data. It is difficult to predict the exact performance of the models on new data, especially since some of the attributes are correlated (e.g. see Table 3, Directionality and Regularity) producing unstable regression coefficients. The cross validation analyses provide a measure of the actual performance of the regression models and show that increasing the number of predictors up to 4 attributes results in continued decrease in the prediction error of the models (Figure 3.3). It is, of course, possible that further decreases in prediction error would obtain with the addition of other viable attributes not examined in this study.
4.1.5 *Low dimensionality: Real or Artefact?*

There are a number of reasons to suspect that the observed low dimensionality of texture judgements merely provides a lower bound. First, the experimental set of textures may be too small to stress other dimensions. Second, the coarse nature of the judgements (i.e. our ratings and the grouping tasks of Rao & Lohse, 1996) may reduce the observed dimensionality by excluding a number of otherwise useful perceptual axes. Even with a greatly refined resolution, it is unlikely that, with a low-dimensional representation, coordinate values for, say, *Coarseness, Regularity,* and *Contrast* axes would be able to recover the appearance of a texture in the manner that, say, hue, saturation and brightness values would for colour perception. It may be, then, that texture perception *in toto* is of high dimensionality but that a much lower-dimensional subspace suffices for practical judgements of a small subset of possible textures.

A comparison of the MDS studies helps in elucidating the issue of dimensionality. The Rao & Lohse (1993, 1996) studies employed single sets of 30 and 56 textures respectively. Our study used 60 textures in total, consisting of 5 separate sets of 12 textures. The results of our MDS analyses (Figure 3.1) were averages of solutions for each of the sets of twelve separately. Although Rao & Lohse (1996) conjectured that they may have overlooked dimensions due to limitations of texture set size, no such increase is observed when comparing our MDS dimensionality of 3 or 4 to their dimensionality of three, with increases of sample size from 12 (our study) to 30 or 56 (Rao & Lohse, 1993, 1996) textures.
Of course, the exact relationship between texture set size and dimensionality is unknown. But we suspect that the dimensionality for judgements of natural textures is low (order 10). Harvey & Gervais (1978) note that in addition to their own three dimensional MDS solutions, a number of general perceptual studies (e.g., Aiken & Brown, 1969) demonstrate dimensionalities of three. They suggest that this commonly found dimensionality is consistent with Attneave’s proposal that all intellectual judgements are performed in a "working space" that is spanned by three dimensions (Attneave, 1974). We agree in principle with Attneave’s suggestion that there is a low limit to the dimensionality of perceptual judgements but do not feel that the number 3 should be dogmatic. The number of dimensions might be increased by extensive practice or for expert observers but we suspect that the total would still be low.

In keeping with Attneave’s suggestion that a "working space" might be common to all kinds of intellectual judgements, we predict that judgements on any texture set would yield low-dimensional representations, whether the set includes simple textures, complex ones or combinations thereof. For judgements on any particular set of textures, nonrelevant attributes/dimensions may lie "dormant", but are presumably available for new tasks or sets of textures, e.g. a discrimination task with a set of "wood grain" textures. In fact, the studies listed in Table 4.1 may demonstrate that this is the case: dimensionalities remain low while the specific attributes that define the axes are highly contingent on the types of stimuli that are used. As a thought experiment, one could envision experiments where dissimilarity judgements are made on a set of one-dimensional compound grating textures (e.g. Harvey
& Gervais, 1981). The members of the set are then progressively replaced by Brodatz (1996) textures. One would expect that the MDS representations might initially be definable by the activities of low-level spatial channels. But the higher-order attributes would gain importance as the more complex stimuli replaced the simpler ones.

Therefore, the dimensionality of everyday texture judgements may be low, though not as low as 3. If the variation in the Brodatz textures resembles that found in textures normally encountered in our environments, it may be that in ecological conditions, the "working" dimensionality of our judgements is low. Moreover, it is not so much the case that the visual system is incapable of distinguishing textures through use of a large number attributes, but rather that which specific attributes gain salience depends heavily on the nature of the texture set. Even with a particular set of textures, observers may have to adjust in order to optimize their performance on different tasks. Getty et al. (1979) describe the results of an identification task which are consistent with such a hypothesis. Observers were asked to identify different subsets of a stimulus set. They seemed to tune the weightings of the dimensions in order to conform to the attributes of the particular subset and thereby maximize the probability of correct identifications.
4.2 Attributes

We have shown that all six Δ⁴ attributes are suitable predictors of the overall dissimilarity ratings (Δ), either directly, as in the linear regressions using the Δ⁴ attributes as predictors (Results 3.2.1), or as 1-D MDS scales which were regressed over the 3- and 4-D MDS-Δ spaces (Results 3.2.2). As regression equation predictors, five Δ⁴ attributes account for about 0.70 of the variance in the Δ ratings; other equations with a different set of five attributes as predictors were also comparable (Figure 3.2, e.g. "fdrle"). The cross-validation analyses further demonstrated the utility of the four- or five-attribute regression models which show good predictive power, with errors of around 1 rating unit, when predicting Δ ratings for either new texture sets or new observers.

A portion of the unaccounted variance may be due to the coarseness of the Δ⁴ attribute judgements. It was thought that the Δ⁴ ratings were a less natural task than the judgements of overall dissimilarity (Δ) and that a less refined scale might elicit more robust results. This was, of course, at the risk of forcing a resolution too crude to capture all the potential predictive power of the Δ⁴ ratings. Another experiment using finer gradations might be fruitful. In the event, the regressions still showed quite reasonable fits. If observers would find some advantage in a finer Δ⁴ rating scale, the predictability of Δ ratings could only be improved.

_Coarseness_ and _Regularity_ figured as the two most important predictors, but the
remaining four were also significant (Figure 3.2). Coarseness has long been touted as the most important textural feature and its primacy in our observers' judgements is not surprising. All perceptual studies of the Brodatz (1966) textures have noted its import (Table 4.1). The importance of Regularity as an attribute may reflect an implementation by the visual system of a taxonomy commonly cited in the computer vision literature, namely, the differentiation of statistical vs. structural analysis of texture (Haralick, 1979). Regularity may be a summary measure of the relative weightings of these functionally distinct approaches.

The importance of Directionality is probably comparable to that of Regularity. In Figure 3.2. "frdlc" and "fdrle" are the first and second best regression models; Directionality and Regularity switch in their order of importance between the two models (the abbreviations are placed in order of importance). Also, in Table 3.1, both the Directionality and Regularity 1-D MDS scales have identically high significances when regressed over \( \Delta \)-MDS spaces \((R^2 = 0.90\) for 4-D \( \Delta \)-MDS spaces\). The relevance of both Regularity and Directionality have been noted previously. Rao & Lohse (1996) had "repetitiveness" as an important dimension, which was highly correlated with "regularity". "Directionality" was one of the strongest cues for similarity judgements in the study by Tamura et al. (1976).

The utility of Lightness was examined since, although some argue that Lightness is not a textural attribute \textit{per se}, it has a physical correlate in mean surface reflectance and is likely needed in comparisons of monochrome texture pairs under ecological conditions. In the event, Lightness achieved significance as a predictor in the \( \Delta \)-\( \Delta^4 \) regression equations and
as a candidate axis \( (R^2 = 0.66) \) in the 4-D \( \Delta \)-MDS space. None of the previous perceptual studies have explored Lightness, even as a "confounding" factor. Since none of the natural-texture studies of Table 4.1 controlled for Lightness, neither by stimulus manipulation nor by instruction to the observers, it would seem reasonable in exploring Lightness, if only in attempts to account for some of the variance left unexplained by their models. Even if Lightness were examined, however, determining its relevance would have been difficult since the previous studies requested judgements according to their candidate attributes before judgements of overall similarity, which may have biased the latter task (Amadasun & King, 1989; Rao & Lohse, 1996; Tamura et al. (1976) also performed the experiments in this order but it is not clear whether they used the same observers for both tasks).

*Contrast* and Edginess were two of the least significant attributes. It is not surprising that they are comparably poor since they are significantly correlated \( (r = 0.64) \), an association noted by Tamura et al. (1976). Measures of contrast, by themselves, were never the most important predictors of similarity in previous perceptual studies. The relevance of Edginess may have been obscured due to poor intersubject consistency in the Edginess \( \Delta \) ratings.

There were no *a priori* grounds for assuming that the cardinal axes of a texture space would lend themselves to labelling; the english language may lack the proper adjectives to name orthogonal directions in the space. Since all six of our 1-D MDS attribute scales gained significance as axes in the 4-D MDS-\( \Delta \) space, it was apparent that these could not all be orthogonal. Four of the attributes do attain near-orthogonality (Table 3.3), namely.
Coarseness, Regularity, Lightness and Contrast may be considered as candidate axes of the 4-D MDS representation of our Δ data. It may be that wholly new attributes, or better synonyms of the six attributes of this study, could offer improvement, though it seems that we are limited in choices of words that are sufficiently general in application. Words that may suffice for a subset of textures may be too specific for general surveys of textures. The optimal coordinate system, then, may require that axes receive a few arbitrary labels, or be named by exemplars or by compound names that are combinations of attributes (Rao & Lohse, 1996).

Related to the problem of labelling is the possibility that a portion of the unexplained variance may be due to the attributes not being truly unidimensional. Multidimensionality of the attributes has been already implied in various computational definitions of the attributes (Tamura et al., 1976; Amadasun & King, 1989). For example, Coarseness might be derived from a two-dimensional space since both the size and spacing of repeating elements figure into judgements. It may be that a particular factor that contributes to the attribute, is most relevant to similarity judgements, or that a different relation between the contributing factors would optimize their relevance. Our MDS scalings of the attributes (Figure 3.5) are based on coarse ΔΔ ratings but they do raise the possibility that the information supplied in summary form in our attributes, might, in different form, lead to better prediction of overall dissimilarity. Essentially, this is just one way of possibly discovering better axes.

It may also be the case that our results are based on faulty assumptions about the
processes that underlie representations and comparisons of textures. Both Tamura et al. (1976) and Amadasun & King (1989), experienced poor to moderate success in predicting texture similarities by calculating two measures of similarity, namely, the minimum Mahalanobis distance and the Euclidian distance. Rao & Lohse (1996) achieved reasonable predictions with their Classification and Regression Tree (Brieman et al., 1984) analysis and discriminant analysis with estimated "true" classification rates of 70% and 49%, respectively. Our regression and MDS analyses were employed in the interest of ease of comprehension and implementation. Dr. R.J. Tibshirani has kindly examined our varying-distance data from the viewpoint of general additive models (Hastie & Tibshirani, 1993). This alternative view of the \( \Delta \) vs. \( \Delta^4 \) relationships is very similar in detail to ours, with cubic salines offering improved fits in a few instances. The probable reason why these nonlinear fits are not much more successful than our linear regressions is that the our mean ratings do not change in fine steps, so linear fits are about as good as nonlinear ones, given measurement noise. Other possible elaborations of the models include: an exhaustive search for the optimal metric in our MDS analyses; "trajectory mapping", a technique that traces paths through a feature space that is not necessarily metric or homogeneous (Richard & Koenderink, 1993). But, since we account for a large portion of the variance in, and have good predictability for, our \( \Delta \) ratings, and can produce satisfactory MDS representations of the same data accompanied by good candidate axes, it would seem that the process of comparing textures can be largely explained by simple models, and that any further elaborations would elicit only marginal gains.
4.3 Variability of Texture Space

4.3.1 Across Texture Sets and Observers

The issue of variability across texture sets has already indirectly addressed in the previous section. It is difficult to speculate how much of the unaccounted-for variance (or prediction error) is due to inhomogeneity of the texture space and not due to inadequacy of the model, intersubject variability, etc. But the average prediction error from the across-set cross validations (1 Δ rating unit) indicates that there is relative homogeneity across texture space.

Another important issue is variability across observers. In colour research, studies have shown that colour-normal observers are consistent individually, and achieve consensus as a group, in colour naming tasks, even displaying cross-cultural agreement (Boynton & Olson, 1987; Uchikawa & Boynton, 1987). None of the studies of natural textures to date (Table 4.1), have explicitly examined intersubject variability. Rao & Lohse (1996), by visual inspection, noted resemblances between their MDS plot and that of their previous study (1993). We have shown that there is relatively good agreement across observers, with prediction errors of around 1 rating unit using regression models, an average shared variance of 0.63 using redundancy analysis and an $r^2 = 0.61$ for direct comparisons of observers’ Δ ratings. It is difficult to assess how much these measures reflect intersubject vs. intrasubject variability, since we had insufficient data to reliably ascertain the latter. But the general
agreement, regardless, was almost a surprise given the seeming subjectivity of the $\Delta$ and $\Delta^4$ ratings.

4.3.2 Task Dependence

One might question the exclusivity of a texture space that is directly accessed by different tasks: each task may define its own space. An alternate expression is that a single space is accessed differently by different tasks. Is the implied configuration of textures resistant to changes in task? In a sense, this question has been answered operationally in the affirmative. The two tasks of this study, the $\Delta$ and $\Delta^4$ judgements, have found close association, whether through the predictions of new $\Delta$ ratings using regression models trained on coupled $\Delta$-$\Delta^4$ data (figures 2, 3 and 4), or through orienting the 1-D MDS attribute scales in the 4-D MDS configurations (Table 3.1). The strength of these associations suggests a lower bound to the confidence with which we can posit a single space that yields to types of texture judgements in this study.

The identifications of textures (Results 3.6) lends further evidence to support the notion of a representation common to different tasks. Qualitative support is given by the general inverse relation between confusions of pairs of textures and their overall dissimilarity (Figure 3.10). It is difficult, however, to predict the exact theoretical expectations for Figure 3.10. Getty et al. (1979) suggest a relation between confusion and interstimulus distance in their study of the prediction of confusion matrices from similarity judgements. The
dimensionalities of the MDS configurations based on transformed stimulus-response matrices are generally low (3–4), with suggestion that viewing from further distances reduces dimensionality (Figure 3.11). However, even these estimates are elevated, if only for the further distances. For the furthest distance, both our regression analyses, which indicate that the only significant attribute is Lightness, and our information calculation (1.6–2.4 bits; Figure 3.9), which is consistent with judgements at a distance being largely based on a few shades of Lightness (e.g. 3–5 shades), indicate a dimensionality of one. It may be that a more suitable transformation of the stimulus-response matrices would improve the dimensionality estimates.

4.3.3 Impoverished Viewing Conditions

Since there is reasonable agreement for judgements of natural textures across texture sets, observers, and tasks, we can ask how resistant the implied representation is to changing viewing conditions. We examined two types of impoverished viewing conditions, namely, varying distance and varying luminance. Extremes of both conditions had the effect of low-pass filtering the texture samples, borne out in the increased significance of Lightness with increased distance and decreased illuminance. Direct comparisons of the Δ ratings delimited the range over which innate judgements of overall texture dissimilarity showed constancy (1–8 m; 100–1 Td). Judgemental constancy does not have a simple perceptual explanation as texture samples look obviously different under the varying conditions; there must be a cognitive component.
4.4 Summary and Conclusions

The present study has attempted to characterize regularities in observers’ comparisons of complex, natural textures. We assumed that the regularities implied a "texture space" representation, and asked questions concerning the dimensionality and axes of such a space. It seems that at least four dimensions are required to explain the judgements. Comparisons of this dimensionality, to that of other studies suggests that increases in texture set size does not necessarily increase dimensionality; the dimensionality may not be much greater than four. All of the studied attributes proved useful, with Coarseness, Regularity, Contrast and Lightness possibly serving as candidate orthogonal axes for a four-dimensional representation. Coarseness, Regularity, Contrast have found prior use in the literature. Texture Lightness has not been examined before and its relevance to texture comparisons may explain why our dimensionality estimate is slightly higher than that of previous work. There is also suggestion that some of the attributes themselves are not strictly unidimensional. We also depart from previous studies in our task ordering: our observers were asked to make the judgements of overall dissimilarity before the attribute judgements, so that the latter would not bias the former. Intersubject variability is another issue which has not received explicit treatment to date. Our findings show that despite using a seemingly very subjective rating task, there is good agreement between observers’ texture comparisons. Finally, we examined the judgements of textures under a wide range of viewing conditions and with an identification task. The results suggest that there is a representation that is common to different tasks, and is insensitive to changes in illumination and viewing distances over certain ranges (1-8 m:...
4.5 Future Directions

Topics for further study of the vision of complex textures include many different possibilities. First, there are issues that stem directly from this study. Our models were useful but imperfect. Predictions of new judgements might be improved by additional or better attributes, models that make better use of the attributes, or by use of finer rating scales. In the interest of ecological validity, we employed prints of natural stimuli: obviously, use of real materials would be an improvement. We provided evidence for good agreement between different observers' judgements. It would of interest to examine whether this holds universally, as there may be variations across culture, profession, habitat, etc. There was also a suggestion that there may be a relatively low limit to the number of dimensions that could be employed in the judgements of textures. One could test this hypothesis by use of large texture sets and different tasks though current indications (Discussion 4.1.5) are that present estimates of dimensionality approach an upper limit. The question of limitations to dimensionality could extend to sensory perception in general, with exploration of cross-modal interactions. Of particular interest would be tasks combining visual and tactile aspects of texture judgements. There have already been studies of the dimensions of tactile surface texture (Hollins et al., 1993) and attempts at examining visual-tactual interactions (Yoshida, 1968). Finally, our understanding of the vision of complex textures at the cognitive level, may help us in forming hypotheses about possible neurophysiologic correlates, which have been investigated for viewings of simple patterns (Optican & Richmond, 1987; Victor &
Purpura, 1996).
References


REFERENCES


Appendix 1

Multidimensional Scaling

The best MDS configuration minimizes the following goodness-of-fit measure. "Stress" (Kruskall & Wish, 1978):

\[
\sqrt{\frac{\sum_{i} \sum_{j} \left[ f(\Delta_{ij}) - d_{ij} \right]^2}{\sum_{i} \sum_{j} d_{ij}^2}}
\]

where \( \Delta_{ij} \) is the overall dissimilarity between the \( i^{th} \) and \( j^{th} \) textures, \( d_{ij} \) is the distance between the points in the MDS configuration representing the \( i^{th} \) and \( j^{th} \) textures, \( f(\Delta_{ij}) \) is some monotonically increasing function of \( \Delta_{ij} \). The best nonmetric MDS solution is one that, assuming an optimal function \( f \), configures the textures stimuli so that the ordinal values of the \( \Delta_{ij} \) are as closely matched to those of the \( d_{ij} \) as possible.

Multiple Linear Regression

Given \( k \) predictor variables, \( x_i \), and one dependent variable, \( y \), multiple linear regression determines a set of coefficients (\( B_i, i = 0 \) to \( k \)), that will optimally (in a least-squares sense) weight a linear combination of independent variables (\( x_i, i = 1 \) to \( k \)) for the best prediction of the dependent variable, \( y \) (e.g. the prediction of \( \Delta \) ratings by the \( \Delta^A \) ratings in Results 3.2.1):

\[
y = B_0 + B_1 x_1 + \cdots + B_k x_k
\]

Cross Validation

The prediction error is estimated by the following formula:

\[
\frac{\sum_{i=1}^{n} | y_{ai} - y_{pi} |}{n}
\]

where \( y_{ai} \) is the \( i^{th} \) actual \( \Delta \) rating of the test set, and \( y_{pi} \) is the \( i^{th} \) \( \Delta \) predicted using the \( \Delta^A \) ratings of the test set and the regression model trained on the \( \Delta \) and \( \Delta^A \) ratings of the training set.

Redundancy Analysis

Redundancy analysis provides a measure of the shared variance between two sets of variables. The development is somewhat involved, and the reader is referred to Stewart & Love (1968) for details.
### Table A2.1: Stimulus-Response Matrices for Identification Task

This appendix includes the remaining (distance 1, 3, 4, 5) stimulus response matrices for the identification task (Results 3.6). The matrix for distance 2 can be found in Table 3.4 (page 50).

| Response (Distance 1, 0.9 m) | 4  | 9  | 12 | 16 | 20 | 22 | 29 | 34 | 37 | 52 | 55 | 66 | 68 | 76 | 86 | 87 | 93 | 98 | 109 | 112 |
|-----------------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 4                           | 99 | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 9                           | 1  | 97 | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  |
| 12                          | 0  | 0  | 100 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 16                          | 0  | 0  | 0  | 99 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  |
| 20                          | 0  | 0  | 0  | 0  | 100 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 22                          | 0  | 0  | 0  | 0  | 0  | 10 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| S                           | 29 | 3  | 1  | 0  | 0  | 0  | 0  | 96 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| t                           | 34 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 99 | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| i                           | 37 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 100 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| m                           | 52 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 100 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| u                           | 55 | 0  | 0  | 0  | 2  | 0  | 0  | 0  | 0  | 0  | 0  | 98 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| l                           | 66 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 100 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| u                           | 68 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 100 | 0  | 0  | 0  | 0  | 0  | 0  |
| s                           | 76 | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 98 | 0  | 0  | 0  | 0  |
| 86                          | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 1  | 0  |
| 87                          | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 100 | 0  | 0  |
| 93                          | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 100 | 0  |
| 98                          | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 100 |
| 109                         | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 100 |
| 112                         | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 100 |
### Response (Distance 3, 15.5 m)

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>9</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>22</th>
<th>29</th>
<th>34</th>
<th>37</th>
<th>52</th>
<th>55</th>
<th>65</th>
<th>76</th>
<th>86</th>
<th>93</th>
<th>98</th>
<th>109</th>
<th>112</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>29</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>70</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>t</td>
<td>34</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>95</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>i</td>
<td>37</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>17</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>m</td>
<td>52</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>28</td>
<td>2</td>
<td>45</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>u</td>
<td>55</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>67</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>l</td>
<td>66</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>42</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>u</td>
<td>68</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>53</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>23</td>
<td>13</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>s</td>
<td>76</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>7</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>44</td>
<td>1</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>86</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>89</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>87</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>19</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>18</td>
<td>26</td>
<td>3</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td>93</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>98</td>
<td>10</td>
<td>1</td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>0</td>
<td>57</td>
</tr>
<tr>
<td>109</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>112</td>
<td>1</td>
<td>0</td>
<td>17</td>
<td>0</td>
<td>1</td>
<td>19</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Response (Distance 4, 22.9 m)

<table>
<thead>
<tr>
<th>S</th>
<th>4</th>
<th>9</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>22</th>
<th>29</th>
<th>34</th>
<th>52</th>
<th>55</th>
<th>66</th>
<th>68</th>
<th>76</th>
<th>86</th>
<th>87</th>
<th>93</th>
<th>98</th>
<th>109</th>
<th>112</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>38</td>
<td>0</td>
<td>6</td>
<td>15</td>
<td>0</td>
<td>8</td>
<td>12</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>34</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>81</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>37</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>13</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>8</td>
<td>33</td>
<td>14</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>52</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>13</td>
<td>18</td>
<td>0</td>
<td>31</td>
<td>0</td>
<td>9</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>55</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>16</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>44</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>66</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>17</td>
<td>4</td>
<td>0</td>
<td>14</td>
<td>0</td>
<td>43</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>68</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>41</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>18</td>
<td>15</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>75</td>
<td>10</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>17</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>13</td>
<td>16</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>19</td>
<td>0</td>
</tr>
<tr>
<td>86</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>10</td>
<td>72</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>87</td>
<td>2</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>22</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>14</td>
<td>25</td>
<td>6</td>
<td>14</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>93</td>
<td>1</td>
<td>0</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>48</td>
<td>0</td>
<td>0</td>
<td>34</td>
</tr>
<tr>
<td>98</td>
<td>6</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>20</td>
<td>21</td>
<td>0</td>
<td>34</td>
<td>0</td>
</tr>
<tr>
<td>109</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>58</td>
</tr>
<tr>
<td>112</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>63</td>
<td></td>
</tr>
</tbody>
</table>
## Response (Distance 5, 30.2 m)

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>9</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>22</th>
<th>29</th>
<th>34</th>
<th>37</th>
<th>52</th>
<th>55</th>
<th>66</th>
<th>68</th>
<th>76</th>
<th>86</th>
<th>87</th>
<th>93</th>
<th>98</th>
<th>109</th>
<th>112</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>18</td>
<td>12</td>
<td>2</td>
<td>15</td>
<td>2</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
<td>18</td>
<td>2</td>
<td>8</td>
<td>3</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>18</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>14</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>2</td>
<td>36</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>8</td>
<td>13</td>
<td>17</td>
<td>5</td>
<td>10</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>19</td>
<td>0</td>
<td>35</td>
<td>3</td>
<td>2</td>
<td>10</td>
<td>0</td>
<td>8</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>32</td>
<td>2</td>
<td>0</td>
<td>12</td>
<td>3</td>
<td>6</td>
<td>18</td>
<td>0</td>
<td>4</td>
<td>8</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>14</td>
<td>3</td>
<td>9</td>
<td>4</td>
<td>5</td>
<td>20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>23</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>10</td>
<td>1</td>
<td>13</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>S</td>
<td>29</td>
<td>2</td>
<td>10</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>25</td>
<td>1</td>
<td>6</td>
<td>16</td>
<td>1</td>
<td>9</td>
<td>13</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>t</td>
<td>34</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>75</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>i</td>
<td>37</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>12</td>
<td>5</td>
<td>1</td>
<td>17</td>
<td>22</td>
<td>15</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>m</td>
<td>52</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>8</td>
<td>9</td>
<td>0</td>
<td>40</td>
<td>0</td>
<td>12</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td>u</td>
<td>55</td>
<td>18</td>
<td>3</td>
<td>8</td>
<td>2</td>
<td>3</td>
<td>22</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>29</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>l</td>
<td>66</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>14</td>
<td>5</td>
<td>3</td>
<td>20</td>
<td>0</td>
<td>23</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td>u</td>
<td>68</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>32</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>16</td>
<td>16</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>9</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>s</td>
<td>76</td>
<td>6</td>
<td>11</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>0</td>
<td>30</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>2</td>
<td>2</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>86</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>14</td>
<td>57</td>
<td>7</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>87</td>
<td>4</td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>12</td>
<td>0</td>
<td>24</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>20</td>
<td>12</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>93</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>1</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>52</td>
<td>3</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>98</td>
<td>1</td>
<td>2</td>
<td>20</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>24</td>
<td>6</td>
<td>1</td>
<td>33</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>109</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>17</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>17</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>49</td>
<td>0</td>
</tr>
<tr>
<td>112</td>
<td>2</td>
<td>0</td>
<td>13</td>
<td>0</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>65</td>
<td>0</td>
</tr>
</tbody>
</table>
IMAGE EVALUATION
TEST TARGET (QA-3)

1.0
1.1
1.25
1.4
1.6

2.0
1.25
1.4
1.6

2.2
2.5
2.2
1.8

150mm
6"

APPLIED IMAGE, Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989

© 1993, Applied Image, Inc. All Rights Reserved