The Strength of Angle-Ply Laminates and Composites with Misaligned Fibres

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
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In the first part of this work, single fibre of E-glass, carbon AS4, Kevlar 49 were embedded normally in epoxy resin and cured at room temperature. The oblique tensile strength of these fibres was measured. A 50% angle, $\phi$, was described. This was the angle of pull at which the fibre breaks at half its normal strength. These 50% angles were $20^\circ$, $30^\circ$-$40^\circ$, and $45^\circ$ for glass, carbon and Kevlar respectively. Also it was found that Kevlar fibres contributed more strength than carbon and carbon more than glass, if they were used to make random fibre composites.

In the second part, the strength of relatively wide angle-ply laminates, $[\pm \phi]_s$, $\phi = 0^\circ$-$60^\circ$, with very short gauge-lengths for glass and carbon was studied. These specimens proved to be very much stronger than the results expected based on the Tsai-Hill work. It indicated that the previous theory for tensile strength involved excessive edge effects. This was confirmed by reviewing the results from tests on pressurized tubes. These tubes strength results were an order of magnitude higher than the early laminate test results. So, it can be concluded that design with angle-ply laminates, based on the earlier theory, may be unduly conservative.

This work was extended to investigate the effect of test specimen width, 3-100 mm, on the tensile strength of $[\pm 45^\circ]_s$ laminates. The strength increased monotonically by increasing the width. It again indicated that the previous theories were relevant to edge effects. Also, the relative stiffness has been estimated from the cross head movement. The widest specimen gave the highest stiffness values. The stiffness results suggest that laminate theory for elastic constants may also be influenced by edge effects. So these results indicate that the way is open for much greater use of angle-ply laminates in real structures.
To:

my daughter *Mahshad*

&

my son *Pooyan*
I would like to thank Professor Michael R. Piggott for his supervision and guidance during this study. His encouragement and most of all his patience is appreciated.

I would like also to thank Professor D. W. Kirk, Professor T. W. Coyle and Professor C. E. Chaffey who accepted to be members in my oral committee.

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\( A_s \)  
Strength reduction factor for misaligned and short fibre composite

\( E \)  
Young’s modulus

\( E_1 \)  
Composite modulus in the fibre direction

\( E_2 \)  
Composite modulus normal to the fibre direction

\( E_f \)  
Fibre Young’s modulus

\( E_m \)  
Matrix Young’s modulus

\( E_X \)  
Modulus in the load direction (composite stiffness)

\( G_{12} \)  
Shear modulus

\( L \)  
Laminate gauge-length

\( M \)  
Moment required to bend a fibre

\( P \)  
Internal pressure of the tube

\( Pa \)  
Pascal unit

\( R \)  
Radius of curvature

\( V_f \)  
Fibre volume fraction

\( V_m \)  
Matrix volume fraction

\( W \)  
Laminate width

\( W_{\text{effective}} \)  
Laminate width which involves in fibre fracture

\( W_\phi \)  
Laminate width which involves in edge effect

\( d \)  
Fibre diameter

\( k \)  
Number of laminae

\( n \)  
Total number of pseudo laminae

\( r \)  
Tube radius

\( t \)  
Tube thickness (wall thickness)

\( t_k \)  
Thickness of each lamina
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_f$</td>
<td>Fibre tensile strain</td>
</tr>
<tr>
<td>$\varepsilon_{\text{flex}}$</td>
<td>Maximum fibre flexural strain</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>Longitudinal tensile strength</td>
</tr>
<tr>
<td>$\sigma_{1U}$</td>
<td>Tensile strength of unidirectional fibre-composite</td>
</tr>
<tr>
<td>$\sigma_{1U}\text{(short fibre)}$</td>
<td>Tensile strength of short fibre-composite</td>
</tr>
<tr>
<td>$\sigma_{2U}$</td>
<td>Transverse tensile strength of unidirectional fibre-composite</td>
</tr>
<tr>
<td>$\sigma_{cu}$</td>
<td>Tensile strength of the central part of a laminate</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Axial stress</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>Hoop stress</td>
</tr>
<tr>
<td>$\sigma_{fu}$</td>
<td>Ultimate fibre tensile strength</td>
</tr>
<tr>
<td>$\sigma_{fu\theta}$</td>
<td>Normal fibre tensile strength</td>
</tr>
<tr>
<td>$\sigma_{fu\phi}$</td>
<td>Fibre tensile strength when crossing the cracks obliquely</td>
</tr>
<tr>
<td>$\sigma_{ru}$</td>
<td>Ultimate fibre tensile strength of random composite</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Matrix tensile strength</td>
</tr>
<tr>
<td>$\sigma_{mu}$</td>
<td>Ultimate matrix tensile strength</td>
</tr>
<tr>
<td>$\sigma_{1\phi}$</td>
<td>Fibre tensile strength</td>
</tr>
<tr>
<td>$\sigma_{u\phi}$</td>
<td>Tensile strength of each lamina</td>
</tr>
<tr>
<td>$\sigma_{u\phi k}$</td>
<td>Tensile strength of $k_{\text{th}}$ lamina</td>
</tr>
<tr>
<td>$\nu_l$</td>
<td>Fibre Poisson’s ratio</td>
</tr>
<tr>
<td>$\nu_m$</td>
<td>Matrix Poisson’s ratio</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\tau_{12u}$</td>
<td>In-plane shear strength</td>
</tr>
<tr>
<td>$\tau_{cu}$</td>
<td>Composite ultimate shear strength</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Winding angle in a tube</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Angle between the fibre &amp; normal stress</td>
</tr>
</tbody>
</table>
Critical angle

$\phi_c$ Critical angle

$\phi_{cT}$ Tsai’s critical angle

$\phi_{cM}$ This experiment’s critical angle

$\phi_t$ The angle which tensile strength of the fibre at this angle is half of the normal strength

$\phi_s$ The fibre fracture mode is activated for angles of zero to $\phi_s$

$\phi_i$ The fibre fracture mode is by shear for angles of $\phi_s$ to $\phi_i$

$\phi_k$ Average misorientation

$\chi_3$ Theoretical fibre direction factor

$\chi_4$ Theoretical fibre length factor
1. INTRODUCTION

1.1. Composite Materials

Fibre composites are widely used in a variety of components because of their high strength, light weight and low cost. The main objective in a composite material is to improve matrix properties by adding a reinforcing phase, usually fibres. Fibre reinforced plastics are one of the most important types of composite materials because of their superior mechanical properties and moderate cost. This is due to the various possibility in fibre arrangements in the composite and a wide range of fibres and polymers which can be used. Laminated fibre reinforced composites enable engineers to design and make structures with high performance ability [1, Ch. 1 & 4], [2, Ch. 1].

Composites can be classified into two major categories [3, Ch. 1]:
1. Continuous fibre composites including:
   A: Uni- or bi-directionally oriented fibres.
   B: Multilayered composites with different layers of sheets stacked on to each other, also referred to as laminates. Laminates typically include 4-40 layers where the layers can have different orientations. For example a cross-ply laminate has zero degree and 90° layers and an angle-ply laminate has layers at ±Φ°, where Φ is the angle between the fibre and the direction of the normal stress.
   C: Hybrid composites, prepared based on a mixture of different fibres to produce specific properties.

All of the above composites are characterized by having high aspect ratio fibres and are usually referred to as high performance composites.

2. Discontinuous fibre or low performance composites:

This group is characterized by having short fibres with low aspect ratio and the fibres may either have random or a specified orientation.

The strength properties of composites are mainly determined by the fibre strength. The fibres are usually the main load bearing component of a composite and are
Glass, carbon and Kevlar fibres are the most common type of fibres which are used in composites.

Three types of glass fibres are commonly used in composites: E-glass (electrical resistance), S-glass (high tensile strength), and C-glass (chemical or corrosion resistance). Among the three types of glass fibres, S-glass has the highest strength, modulus and temperature resistance. Carbon or Graphite fibres are prepared with different values of strength and modulus. Kevlar which is an aromatic polyamide fibre (Aramid) is available as different types: Kevlar 29, Kevlar 49, and Kevlar 149. Kevlar 49 with a high modulus (130 GPa) has the most application in composite materials [4, Ch. 2].

In composites material the matrix can be polymer, ceramic or metal. Polymeric matrices are the most important matrices used in composites, because:

1. The mechanical properties (strength & stiffness) of different polymers are usually low and insufficient to be used for structural applications.

2. With polymers there is no need to have high pressure and high temperature to process the composite. As a result, the concern regarding the stability of the reinforcement during the processing stage is greatly reduced compared to metal and ceramic matrices. In addition, less sophisticated equipment is needed to process polymer matrix composites.

Both thermoset or thermoplastic polymers are used as a matrix material. The most popular thermosets used are epoxy, polyester, phenolic and polyimide resins and the thermoplastic polymers used include polyethylene, nylon, acrylic, PEEK, and PEKK. Table 1.1. shows tensile strength and elastic modulus of glass, carbon, Kevlar fibres and epoxy resin [1, Ch.1 & 3], [5].
### Table 1.1. Tensile Strength and Stiffness of Various Fibres and Epoxy

<table>
<thead>
<tr>
<th>Material</th>
<th>Tensile strength (GPa)</th>
<th>Tensile modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-glass</td>
<td>3.4</td>
<td>72</td>
</tr>
<tr>
<td>S-glass</td>
<td>4.8</td>
<td>85</td>
</tr>
<tr>
<td>Graphite (stiff)</td>
<td>2.3</td>
<td>377</td>
</tr>
<tr>
<td>Graphite (strong)</td>
<td>2.8</td>
<td>233</td>
</tr>
<tr>
<td>Carbon AS4 (Hercules)*</td>
<td>4.0</td>
<td>235</td>
</tr>
<tr>
<td>Kevlar 49</td>
<td>3.6</td>
<td>130</td>
</tr>
<tr>
<td>Epoxy resin</td>
<td>0.06</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**Note**

* Data obtained from Mrse and Piggott [5] and the rest of data obtained from Piggott [1, Ch. 1 & 3].
1.2. Fibre Misalignment

It is difficult to make a general statement about the effect of fibre alignment on composite strength. However, mechanical properties of composite materials strongly depend on the fibre orientation. In other words, misalignment must be taken into account in fibre reinforced materials because of its influence on the mechanical properties of composites.

It is said that there is a significant unwanted misalignment in a well aligned prepreg. Also, the alignment does change during the processing and lamination steps, e.g. by stacking different layers to prepare a continuous fibre composite. Mrse and Piggott [5] showed the presence of a significant misalignment in aligned prepreg which changed during processing. Yurgartis [6] investigated the fibre misalignment distributions for prepreg, \([0/90]_{2s}\) and \([0]_8\). His observation indicated that the misalignment increased from prepreg towards the \([0]_8\) laminate. The range of misalignment angle in high performance composites usually does not exceed \(\pm 10^\circ\) [7]. It is important to give attention to the composite processing, especially to fibre orientations which affects the final composite properties [8].

Highsmith et al [9] studied the influence of fibre waviness on the compressive behaviour of unidirectional continuous fibre composites. They showed that the compressive strength of graphite-reinforced thermoplastic was affected by fibre waviness. By visual inspection, they found numerous small “wrinkles” in that composite. These wrinkles were regions in which the fibres were quite wavy. A schematic representation of fibre waviness within a unidirectional panel is shown in Fig. 1.1. Highsmith et al, inspected the samples of prepreg tape to show that these wrinkles developed during the processing of the composite. Under compressive loading, failure of these samples originated at the wrinkled regions. This was because the straight regions were relatively stiff and strong compared to the wavy regions. Failure in the regions of high waviness resulted in overloading of fibres in the adjacent regions and eventually the composite failure.
Mrse and Piggott [5] generated fibre waviness by using weaves for the composites to increase the fibre misalignments. They investigated the effects of fibre misalignments on the compressive properties of unidirectional carbon fibre laminates and observed that the compressive strength and compressive modulus decrease by increasing the misalignments. In the presence of fibre waviness the tensile strength seemed to be reduced more than the compressive strength. Mrse and Piggott also showed a wider distribution of angles (misalignments) for the laminates made by weaving PEEK fibres and carbon fibres compared to the laminates made from the prepregs.

Ifju et al [10] used textile composites composed of AS4 graphite fibres and a polymeric matrix. They made panels with significantly different mechanical properties by varying the yarn size and the braider yarn direction. They found out that textile composites (woven fibre composites) have more strict misalignments in comparison with laminated composites.
Also in short fibre composites, processing affects the resulting composite fibre orientations [2, Ch. 4], [8]. For example, during the injection moulding processing, it is important to control the flow of fibre and polymer in order to reduce the fibres alignment gradient over the thickness of the final product. In an injection moulded composite fibres on the surface are preferentially oriented parallel to the injection direction while those in the centre are almost oriented perpendicular to the direction of injection [8]. It is found that, if prepared under identical conditions, discontinuous fibre cannot be aligned as well as continuous ones [11]. The effective tensile modulus of short fibre composites also will improve by alignment of the fibres [12].

Conclusion may be drawn that, the presence of misalignment is distinct in laminated and textile as well as in short fibre composites. Also, the influence of small misalignments on the compressive strength is particularly noticeable.
1.5. Fibre Strength

The variability of fibre strength is well known. In a batch of carbon fibres the strength may vary between 0.5-4.3 GPa (see e.g. Proctor [13].) A typical histogram of the strength distribution for glass fibres is shown in Fig. 1.2. This demonstrates the flaw controlled nature of the strength. The difference between the results from tensile and flexure tests is due to a shorter fibre length used in the flexure. The probability of a critical flaw will increase with increasing gauge length of the test section. Thus the longer the fibre, the greater the probability for more flaws to exist which reduces the strength of the fibre [1, Ch. 6].

Surface damages can also be introduced to the fibre during handling and processing which may further increase the scatter in the strength of the fibres. This can be reduced by sizing and application of better protective coatings to the fibres. However, some damage cannot be avoided and will be randomly distributed along the fibre [13].

![Histogram of fibre strength distribution](image)

**Fig. 1.2.** Tensile and flexural strengths of fibres at 20° C. (After Piggott, M. R., and Yokom, J. (1968) Glass Technol 9,172.) [1, Ch.6].
1.4. Laminates

Laminates composed of unidirectional fibre layers are high performance composites. Different mechanical properties can be achieved by using different orientations [14]. These materials are of especial interest where high ratios of strength/weight are required.

Laminates are a form of fibre reinforced composites which are commonly used in such structural application areas as aircraft, space, automotive, chemical plants, sporting goods and marine engineering.

Prepregs, which can be resin-impregnated tape and are partially cured, can be used to make laminates. The laminates are produced by stacking a number of layers and further hot pressing. The fibre orientation for each ply and the stacking sequence of different plies are chosen to achieve different mechanical properties [15, Ch.1].

Using the material properties of the components and the construction of a laminate, the stress that each layer can carry can be calculated. Classical plate theory is used to predict the mechanical properties of the laminates. In this theory, interlaminar shear stresses caused by bending or edge effects, are neglected [14].

It is said that fibre composite laminates can fail according to three different failure modes. These are:

1. Fibre fracture.
2. In-plane matrix cracking, or in other words, the propagation of the crack parallel to the direction of the fibre in a lamina.
3. Delamination or the propagation of the crack between the laminae.

Specimens can be designed in order to show different modes of failure. For the fibre fracture mode, axial load is applied to unidirectional fibre reinforced composite specimens. Angle ply laminates are used to enforce the other two failure modes. For in-plane matrix cracking, specimens with $[\pm 45^\circ]$ are used. A constant load is applied to the samples until cracks start to appear at many points along the fibres. Delamination is usually observed using specimens with $[\pm 30^\circ]$. At this angle, high interlaminar shear stresses at the free boundary of the specimen cause the delamination. Usually the
The fibre fracture mode is very different from matrix failure modes and this can be seen in carbon fibre composites.

The lay-up sequence of the composite affects matrix cracking under mechanical loads. The highest number of cracks is usually observed in cross-ply laminates. In angle ply laminates, the number of cracks will increase by increasing the angle [16].

1.5. Filament Winding & Pressure Vessel

Using a filament winding process is the best way to produce cylindrical structures such as pressure vessels, pipes, storage tanks and aerospace sections like blades. Filament winding is one of the most common forms in which continuous fibre-reinforced polymer composites are produced for fundamental and constructional usage. In this process, continuous fibres in the form of tow (carbon) or rovings (fibreglass) are used. First, the fibres are impregnated with a resin, usually epoxy or polyester, under a required tension. Then using a computer to control the orientation of the fibres, they will be led on to a mandrel (former) for winding. Different types of mandrels, including fixed and rotating, can be used. Fibres are then wound on the mandrel in a helical shape with a specific angle based on the required application. Then the resin has to be cured at room temperature. The mandrel has to be strong enough under the pressure and the tube has to be removed easily after completing the process [1, Ch. 9], [2, Ch.1], [3, Ch. 5]. This process is usually called “wet winding” which is not a clean process but is cheaper and more flexible than dry winding.

Applying a uniform internal or external load to a vessel will lead to pressure build-up in the walls. The state of stress can be described as follows:

1. An axial stress, $\sigma_A$.
2. A hoop or circumferential stress, $\sigma_H$.

In addition, both bending and shear stress can also occur. $\sigma_A$ and $\sigma_H$ can be described using following equations:
where \( P \) is the internal pressure, \( t \) is the wall thickness and \( r \) is the tube radius [2, Ch.8].

The angle-ply laminate structure \([ \pm \phi ]_S\) (i.e. \( +\phi, -\phi, -\phi, +\phi \)) is usually used in filament wound tubes and pressure vessels and has excellent mechanical characteristics. Hull et al [17] performed experimental work on tubes and showed that the deformation and the failure mechanisms were strongly dependent on the winding angle. Some references specify \( 55^\circ \) as an optimum angle for a filament wound tube which is the same as \( \phi = 35^\circ \), however selection of the winding angle for pressure vessels depends on the final working condition.

It is advantageous to test tubes instead of angle-ply laminates to eliminate the edge effects. In particular the properties of the tubes are similar to the properties of angle-ply laminates tested in uniaxial tension. Hull et al used glass-polyester filament wound pipes to investigate the deformation and the failure modes. These pipes had four layers, i.e., two layers of roving in \(+\theta\) (winding angle to the axis of the mandrel. see Fig. 1.3) direction and two layers in the \(-\theta\) direction \( (+\theta, -\theta, +\theta, -\theta) \). During the test and at the onset of a failure the pipes showed white streaks parallel to the fibres. These streaks expanded in length and increased in numbers as the pressure increased. Finally, fibre breakage occurred in the pipes.

Experimental data showed that the winding angle strongly affected the deformation and the failure modes. For example, the strength properties of the pipes tested under Mode 3 increased sequentially by increasing the angle, \( \theta \) [17, 18]. Rosenow [19] showed that the optimum winding angle for tubes tested for biaxial pressure loading is \( 54.75^\circ \), while for hoop pressure loading, is \( 75^\circ \) and for tensile loading the lowest possible angle is the best one. He also used glass-polyester tubes and his results showed that in some cases the tubes weep before bursting. Some results from Hull et al. [17, 18] and Rosenow [19] are shown in Fig. 1.4.
Highton et al [20] and Soden et al [21, 22] used E-glass epoxy filament wound tubes to determine the weeping and fracture strengths at different angles of $\theta$. They used netting analysis to predict the axial and circumferential failure stresses for their filament wound tubes. In this analysis, resin is ignored and it is assumed that all the loads are carried by the fibres. All the axial tests were carried out on an Instron testing machine. For burst tests, the internal pressure was applied with a hand pump. Fibre fracture was observed in final failure for all tubes. Also in the fracture region, whitening was observed in all specimens. Although the origin of the failure was not very obvious, most of the specimens failed close to the centre of the gauge-length. All the experimental results were scattered but they were usually about $\pm 12\%$ of the mean values. These results which were obtained for a fibre volume fraction of 0.6 were higher than some previous results. Some result values of Highton et al and Soden et al are shown in Fig. 1.5.
Fig. 1.4. Burst tests [18] and weeping tests [19] on filament wound glass fibre reinforced tubes.

Fig. 1.5. Comparison of the results of Soden et al. [20-22] for filament-wound tubes.
1.6. Tensile Behaviour of Composites

A considerable amount of experimental work has been done on the tensile properties, especially the ultimate tensile strength of composites. It is known that the tensile strength of reinforced aligned fibre materials is very high in the direction of the fibres. But in many applications the stresses will be applied in different directions [23].

One of the major weaknesses of a composite material is the off-axis strength. This is true for both unidirectional and angle-ply laminates.

The maximum stress theory has been used to estimate the oblique strength properties of composites and angle-ply laminates. In this theory three modes of failure can be described according to the three following equations:

1. Fibre fracture, \[\sigma_{u\phi} = \frac{\sigma_{1u}}{\cos^2 \phi} \] (1.3)

2. Matrix and/or interface shear, \[\sigma_{u\phi} = \frac{\tau_{12u}}{\cos \phi \sin \phi} \] (1.4)

3. Matrix and/or interface cleavage, \[\sigma_{u\phi} = \frac{\sigma_{2u}}{\sin^2 \phi} \] (1.5)

where \(\phi\) is the angle between fibre and the normal stress applied to the composite body, \(\sigma_{1u}, \sigma_{2u}, \) and \(\tau_{12u}\) are the axial, transverse and the shear strength of the unidirectional composite, respectively. The failure mode for any given angle, is the mode which gives the lowest strength for that angle [24]. A plot of tensile strength \(\sigma_{u\phi}\) vs \(\phi\) has three curves meeting at cusps. These plots show that the strength falls off rapidly with increasing the angle [1, Ch. 4].

The fibre fracture mode occurs at small angles from zero to \(\phi_s\) where

\[\tan \phi_s = \frac{\tau_{12u}}{\sigma_{1u}} \] (1.6)

i.e. when \(\sigma_{u\phi}\) given by equation (1.3) is equal to \(\sigma_{u\phi}\) from equation (1.4). For \(\phi_s \leq \phi \leq \phi_t\), failure is by shear where
(obtained by equating $\sigma_{u\phi}$ in equations (1.4) and (1.5)). For $\phi_1 \leq \phi \leq 90^\circ$ failure is by cleavage.

Fig. 1.6. shows the maximum stress criterion and the experimental results for oblique tensile strength of aligned silica-aluminium composites. Even though it gave good agreement for theoretical and experimental results of reinforced metals, it did not accurately predict the oblique strength of the fibre reinforced polymers [1, Ch.4].

Fig. 1.6. Effect of fibre orientation on strength for silica-aluminum. Lines are drawn for maximum stress criterion for failure. (After Cooper, G. A. (1966) J. Mech. Phys. Solids, 14, 103.) [1, Ch.4].
composite materials, to determine failure stresses. This is referred to as the Tsai-Hill criterion which is based on anisotropic yield criterion. Hill adapted this theory to anisotropic structures and Tsai applied it to glass-epoxy composites. The advantage of this theory compared to maximum stress theory is having a relation between the $\sigma_{u\phi}$ (the oblique strength of the composite at any angle) and the $\sigma_{1u}$, $\sigma_{2u}$ and $\tau_{12u}$, which is written as:

$$
1/\sigma^2_{u\phi} = \cos^4 \phi/\sigma^2_{1u} + (1/\tau^2_{12u} - 1/\sigma^2_{1u} \) \cos^2 \phi \sin^2 \phi + \sin^4 \phi/\sigma^2_{2u} \quad (1.8)
$$

To determine the oblique tensile strength of the laminate based on equation (1.8), $\sigma_{1u}$, $\sigma_{2u}$ and $\tau_{12u}$ have to be calculated. The rule of mixtures is one of the most popular methods of estimating the tensile strength of unidirectional fibre composites, $\sigma_{1u}$ and may be written as:

$$
\sigma_{1u} = V_f \sigma_{fu} + V_m \sigma_{mu} \quad (1.9)
$$

where $\sigma_{fu}$ and $\sigma_{mu}$ are fibre and matrix stresses at failure and $V_f$ and $V_m$ are their volume fractions. $\sigma_{2u}$ and $\tau_{12u}$ can be approximately evaluated by the following equations [26]:

$$
\sigma_{2u} = \sigma_{mu} \quad (1.10)
$$

$$
\tau_{12u} = \sigma_{mu}/2 \quad (1.11)
$$

According to Tsai [25] and using equation (1.8), glass-epoxy unidirectional laminates show a rapid fall-off in strength with increasing angle between the fibres and the direction of the applied force, see Fig. 1.7. For example at $5^\circ$, the strength has fallen to about 50% of the $0^\circ$ value. For angle ply laminates the effect is not quite as high, see Fig. 1.8. [25].
Fig. 1.7. Strength of unidirectional composites [25].
Fig. 1.8. Strength of angle-ply composites [25].

The maximum stress theory was used for experiments on reinforced metals [23, 27], and Tsai's work was on glass-epoxy [25]. All the specimens [25, 28] were
failure process. In other words, there was no fibre failure mode. Tsai had a length/width ratio of 14, Rotem and Hashin [29] used 11.6, and Yeow and Brinson [28] had a ratio of 12.0. Due to the narrow specimens used, some edge effects were to be expected.

The Tsai-Hill theory was also used to estimate the tensile strength of short fibre composites. Sanadi and Piggott showed that [26], for aligned short fibre composites the strength can be calculated from:

\[
\sigma_{tu(\text{short-fibre})} = \chi_3 \chi_4 V_f \sigma_{fu} + V_m \sigma_{mu}
\]  
(1.12)

Also a modified rule of mixtures expression can be used to estimate the tensile strength of aligned short fibre composites as follows:

\[
\sigma_{tu(\text{short-fibre})} = A_s V_f \sigma_{fu} + V_m \sigma_{mu}
\]  
(1.13)

To have experimental work and theoretical estimation in agreement, we compare equations (1.12) and (1.13). So, the theoretical value (\(\chi_3 \chi_4\)) and the empirical parameter may be related in \(A_s\) [strength reduction factor for misaligned and short fibre composite]:

\[
A_s = \chi_3 \chi_4
\]  
(1.14)

Hence, a plot of \(A_s\) vs \(\chi_3 \chi_4\) should give a straight line for an acceptable estimation of tensile strength of moderately aligned short fibre composites.

\(\sigma_{tu(\text{short-fibre})}\) is the strength of the aligned short fibre composite in the fibre direction, \(\chi_3\) is the fibre direction factor and \(\chi_4\) is the fibre length factor (\(\chi_3\) and \(\chi_4\) are the
follows. First of all, the strength of each pseudo lamina, $\sigma_{u\phi k}$, should be estimated from equation (1.8). Then it is necessary to know the relative thickness of each lamina, $t_k$. To have $t_k$, a plot of the number of fibres in a batch of aligned short fibre specimen vs fibre angle is needed. This plot gives the relative number of fibres or $t_k$ in the range of angles matching each $\phi$ value [26].

So, according to the thickness of each lamina, $t_k$, and with the average misorientation $\phi_k$, $\chi_3$ can be calculated using equation (1.15).

\[ \chi_3 = \frac{\sum \sigma_{u\phi k} t_k}{\sigma_{1u} \sum t_k} \]  \hspace{1cm} (1.15)

where $n$ is the total number of pseudo laminae and $\sigma_{1u}$ is a scaling factor which could be the rule of mixtures strength, i.e., equation (1.9).

When equation (1.15) was used to calculate the $\chi_3$ for aligned short fibre composites by Chuang [30], it was found that experimental results, $A_S$, were not in good agreement with the theoretical ones, $\chi_3\chi_4$, as shown in Fig. 1.9.
As shown in Fig. 1.9, the Chuang experimental results were low. These may have been caused as a result of a milling process done on the fibers prior to composite production. This milling process could have damaged the fibres and, therefore, affected the tensile strength of the composite. Hence, it can be concluded that this approach led to considerable error when put to use for moderately aligned short fibre composites.

In this approach, $\chi_4$ was estimated from Kelly and Tyson work, who used reinforced metals for their experimental works, and was applied to the reinforced polymers. So, it seems that part of the reason for the disagreement between experimental results and theoretical estimations is an incorrect value for the parameter $\chi_4$ [31]. Later Piggott and Dai used glass and Kevlar fibres embedded in ductile polymers. They found that even with ductile polymers interfaces fail in a brittle manner [32]. In addition, it was found that fully embedded fibres debond at the ends when the strain is low [33]. Moreover, it is probable that composite failure occurs by a developing crack which initiates at the debonded fibre ends [34].

So, instead of a yielding process during failure of a composite, a crack will develop. Then this crack will obliquely cross the misaligned fibres in the composite.
It is also useful to have a model to predict the effective fibre strength as a function of the obliqueness angle. It seems that there are not too many experimental data corresponding to oblique tensile strength of single fibres. Hence, it is important to understand and quantify the effect of fibre alignment on composite strength in order to develop a reliable basis for the design of composites. The perfect alignment of fibres in practical applications is impossible.

As also previously mentioned Yurgartis showed [6] that laminates have a significant degree of misalignment. This is also valid for woven fibre composites [10] and short fibre composites [8, 27]. It can, therefore, be concluded that the Tsai-Hill equation (1.8) cannot be used to estimate the effect of the fibre alignment on the strength of composites, as it does not make an accurate prediction for the strength in the presence of a small degree of fiber misalignment in composite with short fibres.

Therefore, the problem with interpretation of strength results for short fibre composite led to measure the oblique strength of a single fibre. Also, attempts to determine the value of elastic constants and strengths for unidirectional laminae and angle-ply laminates will be useful, as also pointed out in [35], and some attention should be paid to this area, as well. It will be also useful to predict the failure of a fibre composite laminate [29].

Noting that models like the one given by Tsai are applied to composite design [36, 37], the importance of oblique strength assessment will become more obvious. Such an assessment could be of strong potential importance in design methodologies where strength and not stiffness is the design criterion.
1.7. Objective of This Research

According to the above introduction, tensile strengths of angle-ply laminates were not investigated properly. Using narrow specimens, while applying the Tsai-Hill theory, cannot properly predict the tensile strength, since the results can be affected dramatically by edge effects. Also, it was shown by Hull et al that when using a tube as a specimen to measure tensile strength, higher strengths were obtained compared to Tsai’s work. This suggests that the real strength properties of a composite laminate material are different from the data obtained in earlier works which have been used in the design field for the past almost 35 years. Looking at these problems and hoping to have new criteria for design of composite materials, following objectives where set for the current research:

1. To measure the oblique tensile strength of a single fibre to obtain data for composites with misaligned fibres. In other words, the misorientation effect can be investigated. These data can be used when the crack crosses the fibres in an angle in a composite. In fact, oblique tensile strength depends on both fibre and resin behaviour.

2. To measure the tensile strength of shorter and wider samples of angle-ply laminates with $[\pm \phi]_s$ and $\phi$ varying between $0^\circ$-$60^\circ$ to check the Tsai’s and other previous works.

3. To investigate the edge effect on specimens with the width varying from 3-100 mm and a constant gauge length (20 mm) for $[\pm 45^\circ]$ and $[\pm 30^\circ]$ balanced angle ply laminates.

4. To develop a more realistic design criteria based on the above approaches.

5. To have a model to relate the single fibre results to angle ply laminates and to predict the properties of composites using this model when the design criteria are based on tensile strength.
2. EXPERIMENTAL METHOD

2.1. Oblique Strength of Single Fiber

2.1.1. Materials

To measure oblique tensile strength of single fibres, E-glass fibres (diameter, \( d = 22 \, \mu m \)) from Fiberglas Canada, Kevlar 49 fibres (\( d = 12 \, \mu m \)) from Du Pont, and carbon AS4 fibres (\( d = 8 \, \mu m \)) from Hercules were used. An epoxy resin (EPON 815) from Shell with 10-12\% hardener (triethylenetetramine) was used as the matrix.

2.1.2. Specimen Manufacture and Testing

A carousel arrangement [32] (seen in Fig. 2.1) was used for the fibre embedment process. Every time, 39 samples were prepared using this arrangement. First, plastic capsules (5 mm diameter and 11 mm long) were inserted in the carousel. The liquid resin and the hardener was then poured in the capsules using a pipette. The fibre, held vertically in a capillary tube, was advanced into the resin using a micrometer pressing against the end of capillary while being observed through a binocular microscope. The embedment depth was 3 mm so that the fibre could not be pulled out of the resin after being cured. Finally, the samples (for all three different kinds of the fibres) were allowed to cure at room temperature at least for 22 hours. For glass fibres, some other specimens were cured at \( 80^\circ \pm 3^\circ \)C in an oven for three hours.

The fibre tensile strength tests were done using a special set up that is shown in Fig. 2.2. This set up was placed in the table Instron machine, to apply the load. The plastic capsule including the cured resin and the fibre was set to the desired angle of test, so that the fibre would be at the required angle from the vertical. To measure the oblique strength of the specimens, the free end of the fibre was glued to the copper plate using a Krazy (cyanoacrylate) glue. Its position was carefully adjusted, so that the exposed fibre lays along the surface of the copper plate while leaving a free length of about 1 mm. Tensile strength tests for samples were done at a cross head speed of 0.5 mm/min.
Fig. 2.1. General view of carousel used for fibre embedment [32].

Fig. 2.2. Schematic drawing of set-up used for oblique fibre fracture tests.
2.2.1. Materials

S-glass/epoxy prepregs (Toughened Epoxy Glass Prepreg: S2-R7376) and carbon/epoxy prepregs (High Impact Resin System: T650-R6376) from Ciba-Geigy were used to make laminates. Prepreg properties are shown in table 2.1.

TFE release agent dry lubricant from Miller-Stephenson Chemical Co., Inc. was used to coat the mould before use.

Aluminium sheet (thickness = 3 mm) was used as end tabs for the tensile tests.

Table 2.1. Prepreg Properties Manufacture’s Data

<table>
<thead>
<tr>
<th></th>
<th>Glass Prepreg</th>
<th>Carbon Prepreg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resin Content</td>
<td>35 ± 4%</td>
<td>35% nominal</td>
</tr>
<tr>
<td>Volatile Content</td>
<td>2% max. @ -177°C</td>
<td>1.5% max.</td>
</tr>
<tr>
<td>Gel Time</td>
<td>18 ± 7 min. (~177°C)</td>
<td>18 ± 5 min. (~ ~177°C C)</td>
</tr>
<tr>
<td>Shelf Life</td>
<td>6 months @ ~ -18°C</td>
<td>6 months @ ~ -18°C or below</td>
</tr>
<tr>
<td>Out Time</td>
<td>21 days @ ambient temp.</td>
<td>21 days at ambient temp.</td>
</tr>
</tbody>
</table>

2.2.2. Specimen Manufacture

The prepregs were kept in a cold room at -20°C until required. A rectangular steel mould (Fig. 2.3) with dimensions 89 mm x 127 mm (inside) was used to prepare the laminate.

Layers were cut from the prepreg at room temperature with the desired angle to fit simply into the mould. Prepregs came with a backing paper which was removed before laminating.

Each angle ply laminate had four alternate layers (+φ, −φ, −φ, +φ). A hot press machine was used to press and cure the laminate under the applied pressure of 85 psi.
2.2.3. Curing

For glass samples, the cure began at room temperature and involved heating at a constant rate (2°C/min.), followed by 150 minutes at 180°C, and cooling by natural convection, as shown in Fig. 2.4. The carbon samples were cured in a similar fashion, except for the hold time at the maximum temperature which was set to 120 minutes.
Length

In this part, the tensile strength of glass and carbon laminates at different angles was studied. Each moulding had four layers, i.e. $[\pm \phi]_5$. Five angles were chosen for the laminates ( $\phi = 0^\circ, 15^\circ, 30^\circ, 45^\circ$ and $60^\circ$). After cure, the laminates were cut lengthwise into two equal halves to have 44 mm width and 127 mm length.

Two gauge lengths, 2 and 20 mm, were used. They were notched 5 mm deep at the centre on each edge. The notches were cut using a hacksaw for the shorter gauge length and using a diamond impregnated saw, 1 mm thick, for the longer gauge length.

For the 2 mm gauge length, aluminium end tabs were used in the mould. But for the longer gauge length, specimens were tabbed after curing.

For the $0^\circ$ laminates, narrow specimens were prepared. They had 20 mm gauge lengths with 10 mm widths. These specimens were not notched.

MTS (Material Testing System) machine at a cross head speed of 0.8 mm/min. was used to measure the tensile strength of laminates.

2.2.5. Laminates with Constant Gauge-Length and Different Width

In this part, the width effect on the tensile strength of the laminates was investigated. For this purpose, experimental analysis were carried out on glass and carbon angle ply laminates with four layers at $[\pm 45^\circ]_5$ and $[\pm 30^\circ]_5$. They had constant gauge length (20 mm) and different width (3, 10, 30, 100 mm). All the specimens had the same total lengths of 127 mm except for the wide samples which were 89 mm long. The specimens under consideration were prepared using the mould shown in Fig. 2.3.

For all specimens, the cut edges were ground using a # 1200 SiC-paper to the required width. The surfaces were prepared for end tabbing by lightly ground using # 1200 SiC paper, leaving the centre test section untouched. The aluminum tabs were 53.5 mm long with a width equal to the specimens width. They were ground using # 120 SiC paper and then were coated with Lepage 12 epoxy glue and held on using paper clamps.
designed in order to accommodate fitting the specimens into the MTS machine grips. Samples were protected from slipping in the grips using three 6.3 mm bolts in each set of the end tabs. A torque wrench set at 350 kg-cm was used to tighten the end tabs. Fig. 2.5 shows the special end tabs.

The MTS servohydraulic machine at a cross head speed of 0.4 mm/min. was used to test the tensile strength of the angle-ply laminates.

Fig. 2.5. End tabs used for 100 mm wide specimens.
3.1. Oblique Strength of Single Fibre

Typical force-displacement plots for glass fibre samples cured at room temperature are shown in Fig. 3.1. For 0° (normal specimen), there is a linear region near the origin, then the slope changed when the force reached about half of the breaking force value. For 30° and 60° specimens, the plot was totally different from the normal specimen’s result. At increasing angles of pull, the initial build-up of the stress was more gradual, afterwards being linear until failure occurred.

Oblique strength results at different angles for glass (room temperature and 80°C), carbon, and Kevlar fibres are recorded in tables A-1, A-2, A-3 and A-4 in the appendix I. For each angle a minimum of ten replicate strength measurements were performed. Only for glass fibre cured at 80°C, at angles greater than 10°, five replicate tests were made.

Figs. 3.2 and 3.3 show the tensile strength of glass vs angle at room temperature and 80°C, respectively. The fibre breaking stress decreased quite steeply with increasing angle, φ. It dropped to 50% of its normal value at about 20° in both cases. This angle, φh, is defined as the angle for which the tensile strength is half of the normal strength of the fibre. Samples which were cured at higher temperature had about the same tolerance to oblique pull as the room temperature cured samples. Tests at oblique angles for carbon fibre revealed higher strength compared to glass fibre. The φh for the carbon fibre was approximately 40°, as shown in Fig. 3.4. Kevlar, see Fig. 3.5, was more tolerant than the other fibres as it retained the strength until 40°. The φh for Kevlar was 45° and it still showed some strength at 90°.

The scattering of the tensile strength at each angle, φ, for the specimens which were cured at room temperature narrowed for almost every fibre at φ≥φh.

The SEM (Scanning Electron Microscope) method was used to examine the broken ends of the fibre emerging from the polymer. Among the three different fibres, Kevlar showed completely different features, as shown in Fig. 3.6. The Kevlar fibres broke far from the surface of the polymer. Fracture was involved with fibrillation of the
Glass and carbon fibre. In all cases, were broken close to the polymer surface (Figs. 3.7-3.8), where the fibre flexural strain is expected to be the greatest.

**Fig. 3.1.** Typical force-displacement plots obtained with glass fibres, pulled obliquely at the angles shown (i.e. $0^\circ$, $30^\circ$, $60^\circ$).
Fig. 3.2. Fibre strength as a function of angle of pull relative to the direction of the fibre as it emerges from the polymer. Glass fibres, 20°C cure.
Fig. 3.3. Fibre strength as a function of angle of pull relative to the direction of the fibre as it emerges from the polymer. Glass fibres, 80°C cure.
Fig. 3.4. Fibre strength as a function of angle of pull relative to the direction of the fibre as it emerges from the polymer. Carbon fibres, 20°C cure.
Fig. 3.5. Fibre strength as a function of angle of pull relative to the direction of the fibre as it emerges from the polymer. Kevlar fibres, 20°C cure.
Fig. 3.6. Stubs of broken Kevlar fibres in the polymer. Top 0° pull; middle 30° pull; bottom 60° pull.
Fig. 3.7. Stubs of broken glass fibres in the polymer. Top 0° pull; middle 30° pull; bottom 60° pull.
Fig. 3.8. Stubs of broken carbon fibres in the polymer. Top 30° pull; bottom 60° pull.
3.2. Tensile Strength of Angle Ply Laminates

3.2.1. Laminates with Constant Width and Different Gauge-Length

Figs. 3.9-3.12 show the typical force-displacement plots for glass and carbon angle ply laminates, with 20 mm and 2 mm gauge length, respectively. For the longer gauge length laminates, Figs. 3.9 and 3.11, a sharp drop from the maximum load at lower angles can be observed, while at higher angles some load recovery is evident. For the shorter gauge length specimens, Figs. 3.10 and 3.12, the same features with a more pronounced recovery after the load drop can be seen. For glass specimens, except at 60°, a stepwise failure can be observed in the force-displacement. For carbon specimens, Figs 3.11 and 3.12, catastrophic failure occurred at the maximum load.

The strength values for all specimens are given in tables A-5 and A-6 in the appendix I. By increasing the angle φ, for [±φ]s laminates, strength decreased for both of the fibre types. The decrease in tensile strength between 15°-45° was linear for both glass and carbon fibre laminates with 20 mm gauge length.

The tensile strength of glass and carbon angle-ply laminates are plotted in Figs. 3.13-3.16 as a function of angle φ for specimens with a gauge length of 20 and 2 mm, respectively. For glass fibre laminate specimens with 2 mm gauge length, the specimen strength decreased faster than for the 20 mm gauge length, as shown in Figs. 3.13 and 3.14. For carbon fiber laminate specimens with 2 mm gauge length, the strength at 15°, 30° and 45° angle was almost half of the value for the 20 mm gauge length specimens, as shown in Figs. 3.15 and 3.16. At 60°, the strengths for both gauge lengths overlapped.

Nominal stiffnesses are given in table A-7 in the appendix I. For both of these materials, nominal stiffnesses decreased with increasing lamination angle except at 60°. For the 20 mm gauge length specimens, the average nominal stiffnesses were greater for the carbon than for the glass fiber laminates. With the shorter specimens, 2 mm gauge length, the stiffnesses were slightly more than those of the other gauge lengths.
gauge length specimens in MTS machine. For the glass laminates, the fibres were partially separated from the matrix during the failure and the fibre fracture was not so obvious. In fact the main failure occurred in the resin. This caused the specimen to have white appearance along the sample width between the notches. This whitening spread out along the fibres towards the end tabs. For carbon laminates, a smaller number of fibers remained intact after the complete failure of the specimens. Fig. 3.17 shows a 20 mm gauge length, 15° broken carbon laminate specimen. The centre region between the notches involved a very high proportion of fibre fracture. The percentages of broken fibres decreased with increasing lamination angle, see table A-8 in the appendix I.
Fig. 3.9. Force-displacement plots for 20 mm gauge-length for glass specimens. The numbers on the curves are the $\phi$ values.

Fig. 3.10. Force-displacement plots for 2 mm gauge-length for glass specimens. The numbers on the curves are the $\phi$ values.
Fig. 3.11. Force-displacement plots for 20 mm gauge-length for carbon specimens. The numbers on the curves are the $\phi$ values.

Fig. 3.12. Force-displacement plots for 2 mm gauge-length for carbon specimens. The numbers on the curves are the $\phi$ values.
Fig. 3.13. Strength of glass angle ply laminates vs angle (20 mm gauge-length).

Fig. 3.14. Strength of glass angle ply laminates vs angle (2 mm gauge-length).
Fig. 3.15. Strength of carbon angle ply laminates vs angle (20 mm gauge-length).

Fig. 3.16. Strength of carbon angle ply laminates vs angle (2 mm gauge-length).
Fig. 3.17. Failed 15° specimen. The gauge length was 20 mm and the sample was notched.
For this section, [±45°]_S and [±30°]_S specimens were prepared. The typical force-displacement curves are shown in Figs. 3.18 and 3.19. Inelastic behaviours or yielding were observed in all cases. All the plots were curved and in most cases were followed by a sudden failure. The wider samples behaved somewhat different compared with the others. Nominal stiffnesses were calculated based on cross head movement, see table A-9 in the appendix. The values were almost the same for 3, 10 and 30 mm wide samples, but for the widest specimens (100 mm width), the nominal stiffness attained the highest value, see Fig. 3.20.

Tensile strength values are given in tables A-10 and A-11 in the appendix, for [±45°]_S and [±30°]_S specimens respectively. For both of the fibres, [±45°]_S laminates, the tensile strength increased monotonically with increasing sample width, as seen in Fig. 3.21. Fig. 3.22 shows the results for [±30°]_S, carbon laminates.

The micrographs for [±45°]_S, Figs. 3.23-3.26, showed that the glass and carbon angle-ply laminates responded entirely differently during failure. In carbon specimens, effective damage including fibre breakage and matrix splitting was quite evident. But glass specimens involved much less fibre breakage at least for 3 mm and 10 mm wide laminates. Instead, delamination of the layers was observed.

For 3 mm specimens with an aspect ratio (gauge length/width) of 6.7 many carbon fibres were broken, as shown in Fig. 3.23. For glass samples, however, only delamination was obvious. The tensile strength of carbon specimens was about twice as high as that of glass laminates. In both fibres, and 10 mm wide specimens, the layers were extensively split, as shown in Fig. 3.24. Again, the strength of carbon specimens was about twice as high as that of the glass ones.

Fig. 3.25 shows a failed 30 mm specimens for glass and carbon. Carbon fibres fracture occurred obviously for the entire gauge length and it also involved splitting. In the case of glass, fibre fracture and splitting were less than that of the carbon fibre specimens and the tensile strength was almost half of the carbon fibre laminates.

There were some exceptions for the widest specimens (100 mm wide) with 0.2 aspect ratio, see Fig. 3.26. The grips were not strong enough to hold onto the
specimen, and as a result, some slip and shear occurred in the composite and around the bolts. The type of damage was similar for both fibres, with more fibre fracture in carbon specimens, even though they had almost the same tensile strength (0.3 GPa). In the case of carbon specimens, it was noticed that failure started from the centre of the samples and then expanded toward the edges. For the glass, the starting point of the damage was not as obvious as for the carbon.
Fig. 3.18. Force-displacement plots for the 20 mm gauge length glass samples at $[\pm 45^\circ]$s; the numbers on the curves are the width values.
Fig. 3.19. Force-displacement plots for the 20 mm gauge length carbon samples at $[\pm 45^\circ]_S$; the numbers on the curves are the width values.
Fig. 3.20. Nominal stiffness vs specimen width for glass and carbon [±45°]s laminates at fixed length (20 mm).
Fig. 3.21. Tensile strength vs specimen width for [$\pm 45^\circ$]s glass and carbon angle ply laminates at fixed length (20 mm).
Fig. 3.22. Tensile strength vs specimen width for $[\pm 30^\circ]_s$ carbon angle ply laminates at fixed length (20 mm).
Fig. 3.23. Failed 3 mm wide specimens; glass at left and carbon at right.

Fig. 3.24. Failed 10 mm wide specimens; glass at left and carbon at right.
Fig. 3.25. Failed 30 mm wide specimens; glass at left and carbon at right.

Fig. 3.26. Failed 100 mm wide specimens; glass at left and carbon at right.
This experimental work done on wide laminates was designed to check the laminate theory which is commonly used for the Young's modulus of angle-ply laminates. Based on this theory the Young’s modulus of the specimen ($E_x$), is given approximately by

$$E_x = \left[ \cos^4\phi/E_1 + \left\{ 1/G_{12} - 2v_{12}/E_1 \right\} \sin^2\phi \cos^2\phi + \sin^4\phi/E_2 \right]^{-1} \tag{3.1}$$

where $E_1$ and $E_2$ are the Young’s modulus of the composite in the fibre direction and normal to the fibre direction respectively. $G_{12}$ is the shear modulus and the $v_{12}$ is the Poisson’s ratio. (This equation works very well for narrow and long angle-ply laminates based on Tsai [25]). Furthermore, if the plot for the balanced angle-ply laminates in Tsai’s original paper (Fig. 8 [25]) is compared with the experimental results for tensile strength of unidirectional laminates (Fig. 4 [25]) they are seen to be very similar.

In this experimental work on angle-ply laminates with different width, the gauge-length was too short (20 mm). Hence, it was not possible to use the standard extensometer, which is 25 mm long, to measure the modulus. Thus, cross head movement was used to estimate the stiffness of the specimens. The results are shown in Fig. 3.20. These are taken from the slope of the earlier part of the force-displacement plot. This stiffness was further divided by $W$. In both cases, glass and carbon, the widest specimens (100 mm) have higher nominal stiffnesses than the others (3, 10, 30 mm wide specimens). This is a noticeable effect which is supported by the results from the angle-ply laminates with the constant width, section 3.2.1. The normalized nominal stiffnesses for glass-epoxy angle-ply laminates are also plotted vs angle, Fig. 3.27. (These results are from table A-7 in the appendix I, such that normalized the unidirectional laminates agree with the value 48 GPa at $\phi = 0^\circ$). In this figure, the dashed line is given by using equation (3.1), while $G_{12} = 10$ GPa. The other elastic constants are calculated using the Rule of Mixtures:

$$E_i = V_iE_i + V_mE_m \tag{3.2}$$
where $E_1 = 48$ GPa, $E_2 = 9.4$ GPa, $v_{12} = 0.27$. The solid line is given using equation (3.1) for $G_{12} = 3.51$ GPa. The dashed line with 10 GPa shear modulus fits this experimental results much better than the others.

Also, a comparison between theoretical and experimental values of stiffness for carbon angle-ply laminates is given in Fig. 3.28. For this, Rule of Mixtures was used to obtain the elastic constants such as $E_1 = 190$ GPa, $E_2 = 10.0$ GPa and $v_{12} = 0.35$. The dashed line, which goes through the experimental values, is plotted for $G_{12} = 30$ GPa. It fits the results much better than the solid line which corresponds to the $G_{12} = 3.72$ GPa (inverse Rule of Mixtures).

There is probably a large interaction effect between the layers in a laminate. Having -$\phi$ layer beside the +$\phi$ layer should prevent the shearing action, which is the main component of the compliance, once $\phi$ is larger than a few degrees. The effect depends on the Young's modulus of the fibres (and probably their shear modulus to an even greater extent.)

Designs for structural application usually imply stiffness rather than strength. Hence, the modulus results from this work even though was based on using short and wide specimens to obtain the elastic constants, is more crucial than the failure of the explained tensile strength theories. As mentioned here, the experimental result for the stiffness is at least three times (glass specimens) as high as the theoretical estimation. So, if this is true, then the angle-ply laminates could be considered to be three times more efficient than it was thought. This might be the case except that some other properties, like lack of adequate delamination resistance, could also affect the modulus.

Modulus results from Soden's work on glass-epoxy filament wound tubes are also shown in Fig. 3.27 [22]. Our 15$^\circ$ result is slightly more than Soden's values, but the rest of those are in a very good agreement. Also at 45$^\circ$, the hoop and axial values are slightly less than estimated values on dashed line for $G_{12} = 10$ GPa.
Fig. 3.27. Nominal stiffness vs $\phi$ for glass-epoxy balanced angle-ply laminates with constant gauge-length (34 mm) (Solid circles). Also Soden's results obtained with glass-epoxy filament wound tubes are shown with diamond marks [22]. Curves based on equation (3.1) with $G_{12} = 3.51$ GPa (Solid curve) and $G_{12} = 10$ GPa (Dashed curve).
Fig. 3.28. Nominal stiffness vs $\phi$ for carbon-epoxy balanced angle-ply laminates with constant gauge-length (34 mm) (Solid rectangles). Curves based on equation (3.1) with $G_{12} = 3.72$ GPa (Solid curve) and $G_{12} = 30$ GPa (Dashed curve).
4.1. Oblique Strength of Single Fibre

Variability is an important aspect of the mechanical properties of fibres. Tensile tests on single fibre of different materials used in this experimental work showed a wide range of strength. The large standard deviations in Figs. 3.2-3.5 originates from the effect of the flaws in single fibre tests. Fig. 4.1 compares the variation of the tensile strength for the three different fibres at zero degree obtained in this work. In all of the cases, the free length of the single fibre was one mm. In glass batch, for example, the strength varied between 1.3-5.5 GPa while for carbon and Kevlar it varied from 1.6-4.8 GPa and 2.6-4.9 GPa, respectively. For glass, most of the specimens (60%) were characterized by strengths between 2-3 GPa while 45% of the carbon fibres had a tensile strength around 3 GPa. Kevlar was the strongest one and most of the specimens (70%) had about 3-4 GPa strength.

The other factor which affects the mechanical properties is the fibre diameter. Insufficient flexibility leads to difficulties in handling fibres and finally, to fibre breakage. The diameter of fibres is very important for the easiness of the bending e.g. in filament winding. Bending of fibres causes surface tensile stresses which lead to fibre fracture. Flexibility of a fibre is characterized by the moment “M” which is the moment required to bend a fibre with a circular cross-section to a given radius of curvature R. This is equal to:

\[
M = \pi Ed^4 / 64 R
\]

Therefore, M depends on Young’s modulus “E” as well as fibre’s diameter “d”. For a fibre with the same E, the moment will increase by increasing the diameter. Also the longer the length or the gauge-length of the specimen, the more the flaw.
Fig. 4.1. Comparison of tensile strength distribution for glass, carbon and Kevlar single fibres specimens at zero degree.

The plots of tensile strength of different fibres vs oblique angles (for single fibres) were not straight lines. Plots given in Figs. 3.2, 3.4, 3.5 are presented together without error bars in Fig. 4.2. Comparison between these results and those from equation (1.8), representing Tsai’s work [25], show quite different drop for the strength of the fibres. For example, for glass-epoxies the 50% angle, $\phi_h$, for unidirectional laminates was about 3° [25] and for angle-ply laminates about 8° [25]. This is due to the fact that a different mechanism was considered to occur when stress is applied to the fibre in Tsai’s work compared to this study for both unidirectional and angle-ply laminates [25]. In Tsai’s work, it was assumed that fracture involved matrix shearing, and not fibre fracture.

So, it can be concluded that for imperfectly aligned short fibre composites, such as those described by Sanadi and others [26, 30], $\chi_3$ should not be calculated using equation (1.8). Also $\chi_3$ should not be estimated on the basis of the resolved fibre stress, as was the case in works by Fukuda and Chou [38] or Kallmes et al [39].
Fig. 4.2. Fibre strength as a function of angle of pull relative to the direction of the fibre as it emerges from the polymer. Glass, carbon and Kevlar fibres.

To investigate the properties of the obliquely-stressed fibrous composites, a pair of steel wires were embedded in blocks of polycarbonate and E-glass fibre bundles were embedded in polyester [40]. The results for the required force to break the fibres obliquely showed that the strength fell off more or less linearly as a function of $\tan \phi$. This approach seems not to be applicable for these experimental results. Figs. 4.3, 4.4 and 4.5, show the experimental results for glass, carbon and Kevlar fibres, respectively, as a function of $\tan \phi$. For carbon, Fig. 4.7, the results might obey the $\tan \phi$ relationship [40] if we ignore the $60^\circ$ result. But this approach fits neither the glass nor the Kevlar results.
Fig. 4.3. Tensile strength of the glass fibres plotted as a function of the tan $\phi$.

Fig. 4.4. Tensile strength of the carbon fibres plotted as a function of the tan $\phi$. 
If one assumes that these experiments are similar to the elastica test (a loop was made and this was pulled and made tighter, i.e. smaller, until fibre broke because the flexural stress became greater than the material strength) which was performed by Sinclair [41], it is possible to reach an agreement about the intrinsic strength of the fibres. First, an approximate analysis which has been confirmed by numerical analysis in the appendix II, gives a very simple equation for the maximum flexural strain, $\varepsilon_{\text{flex}}$ [42], for $\phi$ from $0^\circ$ up to $90^\circ$.

$$
\varepsilon_{\text{flex}} = 2\phi \sqrt{\varepsilon_f}
$$

For the angles of $\phi < 60^\circ$, the estimated error is small and negligible but it becomes quite significant at $90^\circ$ (this is true for a fibre tensile strain of $\varepsilon_f \leq 0.01$).

Fig. 4.6 shows $(\varepsilon_f + \varepsilon_{\text{flex}})$ vs angle $\phi$ for glass and carbon fibres. Kevlar fibre results have been omitted since the fibre kinked during testing, and therefore, was not elastic. Kinking of the Kevlar fibres can be seen clearly in Fig. 3.6. Hence, this
cannot be applied to the Kevlar. The total $\varepsilon$ values for glass fibres which is about 12% up to $\phi = 30^\circ$, are higher than the results for silica which were about 8%, as given by Proctor et al [43]. The values obtained were also a little higher than values which were reported by Piggott and Yokom in flexure tests at room temperature [44]. The latter used fibres with 50$\mu$m diameter while the glass fibre's diameter in the current research was 22$\mu$m. Their modal for strength was 6 GPa, i.e., a strain of about 11% (considering $E_f = 69$ GPa for silica). So, their value of strain is very close to the experimental results for glass fibres in this research.

Finally, it seems that the test gives the intrinsic failure strain for brittle fibres. Therefore, it can be concluded that the intrinsic failure strain of AS4 carbon fibre is between 9%-14% which is in agreement with a stress level of about 30 GPa.

For composites with misaligned fibres or random fibres, the data in Figs. 3.2-3.6 can be used to predict the fracture tensile strength assuming the fibres break during the testing. It is noticeable that even though Kevlar fibre is weaker than glass fibre, it is expected to give stronger random fibre composites.

![Graph](image)

**Fig. 4.6.** Estimated maximum fibre strain at the point where the fibre emerges from the polymer surface. ($\varepsilon_{\text{flex}} + \varepsilon_f$) vs $\phi$. 
4.2.1. Laminates with Constant Width and Different Gauge-Length

In previous works \([25,28,36]\), the specimens were too narrow and long. Different \(L/W\) (\(L=\) gauge-length and \(W=\) width) such as \(\approx 14\) \([25]\), \(\approx 12\) \([36]\) and \(\approx 11.6\) \([28]\) were used. In order to be closer to a practical value and to eliminate the edge effects, the \(L/W\) ratio was set at about 0.5 for this experimental work.

Tensile strength measurements of short and wide specimens of angle-ply laminates involved fibre breakage, Fig. 3.17. This fibre fracture was much more clear for carbon balanced angle-ply laminates compared to the same angle in glass specimens. Since fibres were broken in the case of carbon specimens and they were close to breaking for the glass laminates, the final tensile strengths were much higher than the values obtained in the previous works \([23, 25, 27]\). Fig. 4.7, shows this experimental results obtained for glass and carbon specimens with 20 mm gauge-length. These samples were notched towards final width of 34 mm. The other data in Fig. 4.7 are taken from Tsai’s work \([25]\) (specimen size 63 mm long x 4.6 mm wide) for glass fibre-epoxy laminates and Snell’s work \([45]\) (specimen size 20 mm x 20 mm) for carbon fibre reinforced plastics laminates. Comparison of these results in both cases (glass and carbon) are as follows:

1. At \(15^\circ\), for this study the results are much higher than the others, i.e., they are five times as high as Tsai’s results and about three times as high as values reported by Snell.

2. For \(30^\circ\), the above ratios changed to six and two, respectively.

3. At \(45^\circ\), the results from this work are four times higher than Tsai’s and two times higher than Snell’s work.

4. For \(60^\circ\), the difference is negligible.

Hence, we can conclude that at low angles, e.g. from \(15^\circ\)-\(45^\circ\), this work gave results which were much higher than the earlier results. For the first three angles, the samples contained fibres that extended from one end to the other end of the specimen.
specimens the fibre arrangement in the sample was such that individual fibres could not extend over the entire length of the sample. This is similar to the case in Tsai’s work [25], where selection of a fairly small W/L ratio could not accommodate the extension of individual fibers over the entire length of the specimen beyond a certain fibre angle. This could mean that fibre breakage during testing and measuring the tensile strength of the fibres has not been possible. That is why the tensile strength results reported in [25,28,36] are low and the 60° laminate specimens in this research replicated such condition.

Fig. 4.7. Results for glass and carbon angle-ply laminates (20 mm x 34 mm) compared with Snell’s work [45] and Tsai’s experiments [25].
In Fig. 4.7, referring to Tsai’s work (1965), there is a rapid fall off of tensile strength with increasing the angle. This would indicate that angle-ply laminates cannot be very useful for practical applications. The other earlier theory belonging to Stowell and Liu [24] (1961, the maximum stress theory) gave results similar to these, with some difference for the cusps. It seems that these theories neglected the edge effects. Snell used instead, shorter and wider specimens with a L/W = 1 (20/20). He obtained much higher values for tensile strength at lower angles (from zero at least up to 35°). So, in this case he reduced the edge effects.

Table 4.1 gives the effective width, defined as $W - 2L \cdot \tan(\phi)$, for the angle-ply laminate specimens used in this work.

**Table 4.1.** The effective width for fibre breakage regarding to the different angles

<table>
<thead>
<tr>
<th>$\phi$ (°)</th>
<th>$\tan \phi$</th>
<th>$W_\phi$ (mm)</th>
<th>$W_{\text{effective}}$ (mm) = $W - 2W_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.27</td>
<td>5.4</td>
<td>33.2</td>
</tr>
<tr>
<td>30</td>
<td>0.58</td>
<td>11.6</td>
<td>20.8</td>
</tr>
<tr>
<td>45</td>
<td>1.00</td>
<td>20.0</td>
<td>4</td>
</tr>
<tr>
<td>60</td>
<td>1.73</td>
<td>34.6</td>
<td>0</td>
</tr>
</tbody>
</table>

Fibre breakage can only be expected if $W_{\text{effective}} > 0$ and the larger $W_{\text{effective}}$, the more fibre breakage. As can be seen from the data in table 4.1 there should exist no fibre breakage in 60° angle-ply laminates, and as a result, the data referring to this specimen are in good agreement with Tsai’s work.

For an unnotched specimen, we can define a critical angle, $\phi_c$, at which 100% shear failure is to be expected:

$$\phi_c = \arctan \left( \frac{W}{L} \right)$$ (4.3)
4.6/63, marked as $\phi_{CT}$ on Fig. 4.7) and Snell had $\phi_c = 45^\circ$ (W/L = 20/20), while for this experimental work $\phi_c = 60^\circ$ (notched specimens, W/L = 34/20). In this study for unnotched specimens $\phi_c = 66^\circ$ since W/L = 44/20, and the presence of the double notch reduced the local width of the specimen such that $\phi_c$ decreased from 66° to 60°.

The 2 mm specimens gave lower results for tensile strength than 20 mm specimens for both materials, glass and carbon. This was due to the different methods used to prepare the specimens. For this case (shorter gauge-length), the tabs were used inside the mould, and as a result, the region of the specimen between the two end tabs could not get exposed to the pressure applied during the curing stage. The nominal stiffnesses for shorter gauge-length specimens were usually higher than those of longer specimens, except for the value from carbon 45° angle-ply specimen (refer to table A-7 in the appendix I). The increase in stiffness values was not as great for the 2 mm specimens since the fibre kinked in the short gauge-length due to the non-uniform pressure field during the cure processing. This kinking occurred because this part of the specimen was not entirely restrained. So finally these kinked fibres were able to straighten out more than the others under pressure [46]. Moreover, the fibres which were straighten in the 2 mm gauge-length should have been much stiffer than the 20 mm gauge-length. However, these results showed, see table A-7 in the appendix I, that the 2 mm specimens were not very much stiffer.

Looking at the actual 20 mm gauge-length specimen tensile strength results, it can be concluded that in spite of having much greater specimen width relative to the gauge-length, there is still a large edge effect. Now, if one assumes that the fibre which breaks during loading is characterized by a strength of $\sigma_{1\phi}$, then there are two regions in the composite corresponding to the tensile strength. For now we assume that the fibre strength is not affected by the angle of orientation. So:

1. Near to the edge of the specimen, the fibres slide off and they obey equation (1.4) or ($\sigma_{w\phi} = \tau_{12\phi}/\cos\phi \sin\phi$). For an unnotched specimen, the fraction of the fibre in this section is $\tan\phi/\tan\phi_0$. When $\phi_0$, the minimum angle for cleavage at a stress of
2. Near to the centre of the specimen the fibres break. So the fraction of the fibre in this section is $1 - \tan \phi / \tan \phi_0$.

The expression for composite strength is:

$$\sigma_{u\phi} = \sigma_{1\phi} (1 - \tan \phi / \tan \phi_0) + \tau_{12\phi} / \cos^2 \phi \tan \phi_0$$

(4.3)

and Fig. 4.8 shows the plot of equation (4.3) for $\phi_0 = 48^\circ$, $\tau_{cu} = 60$ MPa and $\sigma_{1\phi} = 2.0$ GPa along with the experimental results. The region $\phi > \phi_0$ in Fig. 4.8 was plotted using equation (1.5) and $\sigma_{2u} \equiv 60$ MPa for $\phi$ greater than $\phi_0$.

The experimental results and the theoretical curves are in good agreement. However, the fraction of the fibre fracture in carbon specimens used in this study was somewhat greater than the predicted value. For example, at $15^\circ$ this model gives 76% for the fiber fracture but the experimental observation indicates $(86 \pm 19)\%$. At higher angles, the predicted values for fraction of fibre breakage are 48%, 10% and 0% (no fibre should break) corresponding to the $30^\circ$, $45^\circ$, $60^\circ$ laminates specimens. However, experimental observation showed the following fraction values of fibre fracture for the angles mentioned in above: $(81 \pm 35)\%$, $(72 \pm 13)\%$, $(50 \pm 27)\%$ (table. 4.2). For glass specimens, whitening was observed in most of the cases and only a few percent of fibre fracture was noticed at any angle.
In addition, regarding the data in table A-6 in the appendix I, \( \sigma_{1\phi} = 2.0 \) GPa is not a very good estimation for \( \sigma_{1\phi} \). In fact, \( \sigma_{1\phi} \) at 0° angle should be calculated by the Rule of Mixtures. For the carbon fibre specimens, with \( V_f = 0.65\% \) and the fibre tensile strength of 4.5 GPa, the \( \sigma_{1\phi} \) at 0° angle should be about 3.0 GPa. The tensile strength of the narrow specimens from this work, with 10 mm width and at 0°, was 2.6±0.1 GPa. Considering that some excessive stress exists in the grip (almost 11% of the measured stress\[47]\), this experimental tensile strength is close to the one predicted by the Rule of Mixtures. For the glass fibre laminates, the fibre tensile strength was not known, but the manufacturer (Ciba-Geigy) gave a value of 2.02±0.09 GPa for the prepreg. Referring to this experimental work, table A-5 in the appendix I, the tensile strength of the narrow specimen of glass-epoxy is 1.7±0.2 GPa which is slightly lower than what was expected.

Also it is important to remember that the \( \sigma_{1\phi} \) is affected by the angle \( \phi \), as was mentioned in single fibre results in section 4.1. This will be discussed more extensively in section 4.2.1.2.
Fig. 4.8. Equation (4.3) fitted from 0° up to 45°, and equation (1.5) fitted for the 45° up to 90° for the glass and carbon balanced angle-ply laminates.

4.2.1.1 Comparison with Recent Results from Burst Tests on Tubes

It was mentioned that earlier theories did not consider the edge effects. By using tubes instead, the edge effects are diminished. So, it is important to look at the recent works on filament wound tubes.

Soden et al used filament wound glass-epoxy tubes in their experimental work [20]. They obtained higher values for tensile strengths compared to the previous results while burst strengths could be very high. Fig. 4.9 gives a comparison of the results from this work for glass-epoxy laminates with 20 mm x 34 mm dimensions and Soden’s work. The material used was similar in both cases (glass-epoxy specimens). However, the detailed specimen geometry was different.
In tubes, for angles of $\phi \leq 45^\circ$, the pure hoop stress tests with the open ends were used. As was already mentioned in the introduction, a tube with the $\theta = \pm 75^\circ$ winding angle will give the same results as $\phi = 90^\circ - \theta$ for the laminates. Hence, Soden’s results at 15° are much less than the values for glass-epoxy laminate in this study (about 20%). The $\pm 55^\circ$ winding angle gives the $\pm 35^\circ$ result which is quite close to our values for the laminate at 30°. The $\pm 45^\circ$ hoop stress result is somewhat higher than our experimental results at $\pm 45^\circ$. In general, a good agreement between the data from the tube and flat specimens at higher angle can be observed.

Concluding these comparisons, it is noticeable that the pressurized filament wound tubes described by Soden et al. behaved similar to balanced angle-ply laminates in this study. These tube results were definitely much higher than those predicted by the maximum stress theory (Stowell & Liu) or Tsai-Hill’s approach (an order of magnitude greater at $\pm 35^\circ$ [21]). Also, both hoop stresses and axial stresses at $\pm 45^\circ$ are analogous to $\pm 45^\circ$ angle-ply laminates.
Angle-ply laminates are used widely in the design of real structures. For example, in aerospace engineering an airplane wing can be made from a simple angle-ply laminate. In this case, the aspect ratio of the wing might be of ten to one (W/L), wide and short. So, to design this with a required strength, a value for $\phi_0$ can be estimated and the strength based on equation (4.3) is calculated. Additionally, it should be noticed that microcracking usually occurs before final burst in the pressurized filament wound tubes and as a result strength is expected to be lower [22]. In addition $\sigma_{1\phi}$ depends on the angle of $\phi$, which was already explained in section 4.1.

The single fibre results suggest a method to predict the strength of composites (angle-ply laminates) as follows:

$$\sigma_{f_{u\phi}} = \sigma_{f_{u0}} f(\phi)$$

(4.4)

where $\sigma_{f_{u\phi}}$ is the fibre strength when crossing cracks obliquely at angle $\phi$ to the normal and $\sigma_{f_{u0}}$ is the normal fibre strength. Using our experimental data, following equation was obtained for a $\phi$ by curve fitting procedure.

$$f(\phi) = \left[ 1 + \left( \frac{1-\phi}{\phi_h} \right)^{1/3} \right] / 2$$

(4.5)

for $\phi \leq 2\phi_h$ and assume $\sigma_{f_{u\phi}} = 0$ for $\phi > \phi_h$

This is shown in Fig. 4.10. Fig. 4.10 presents a comparison of theoretical (equation 4.5) and experimental results for $\sigma_{f_{u\phi}}$ vs $\phi$ for glass, carbon and Kevlar fibres. The $\phi_h$ was described in results section and depends on properties of both the fibre and the matrix. In general, the definition given in equation (4.5) gives a moderate fit to all of the fibre used. The best fit corresponds to the carbon fibre results. Equation (4.5) gives a good estimation for lower angles while it tend to underestimate the results at higher angles. We, therefore propose the use of equations (4.4) and (4.5) to predict the tensile strength of the composite for design purposes. However, more work has to be done to extend the validity range of this estimation.
where $\sigma_{1\phi}$ is the composite strength. For the angle-ply laminates by substituting $\sigma_{1\phi}$ in equation (4.3) with $\cot\phi_0$ equal to the aspect ratio of the structure, the composite strength can be calculated.
FIG. 4.10  Single fibre results fitted using equation (4.5). (Data from single fibre results.)
employing the normal summation methods. Hence, prediction of the tensile strength will include the strength of the central part of the specimen plus the edge effects. The tensile strength of the central part of the laminate, \( \sigma_{cu} \), can be estimated from

\[
\sigma_{cu} = \sum t_n \sigma_{1\phi n} / \sum t_n
\]  

(4.7)

Here \( t_n \) is the thickness of the \( nth \) ply. One has to still take into account the edge effect for this case. This also can be applied to short fibre composite. For instance, in a short fibre composite which is aligned randomly, the tensile strength, \( \sigma_{ru} \), can be estimated using equations (4.4) and (4.5). We have to integrate these equations with respect to \( \phi \) from \( 0^\circ \) to \( 90^\circ \), then divide by \( 90^\circ \). The result should be added to the matrix effect. Thus

\[
\sigma_{ru} = V_f \sigma_{fu0} \phi_h / 90 + V_m \sigma_{mu}
\]  

(4.8)

for \( \phi \) in degrees. (We have also to take into account the “fibre efficiency” factor for short fibres which will reduce the \( \sigma_{fu0} \) as explained by Sanadi and Piggott [26]). In this kind of composites, i.e. randomly oriented short fibre composites, there would be no significant edge effect. Table 4.3. shows a comparison of mean tensile strength for single fibre results of this study and what was obtained from equation (4.8). The experimental values and the estimated one are quite close. The best fit correspond to the carbon single fibre which were cured at \( 20^\circ \) C. The glass fibre results do not fit the equations (4.4) and (4.5), as well as the others which was already shown in Fig. 4.10.
### Table 4.3: The 90° angles tensile strength, normalized mean experimental strengths, and normalized mean strength estimated from equation (4.8).

<table>
<thead>
<tr>
<th>Fibre</th>
<th>Resin</th>
<th>Cure</th>
<th>$\phi_h$</th>
<th>$\sigma_{fu}/\sigma_{fu0}$ Experimenta l</th>
<th>$\sigma_{fu}/\sigma_{fu0}$ Eq. (4.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>glass</td>
<td>Epoxy</td>
<td>20°C, 24 h</td>
<td>20°</td>
<td>0.31</td>
<td>0.22</td>
</tr>
<tr>
<td>Kevlar</td>
<td>Epoxy</td>
<td>20°C, 24 h</td>
<td>45°</td>
<td>0.53</td>
<td>0.50</td>
</tr>
<tr>
<td>Carbon</td>
<td>Epoxy</td>
<td>20°C, 24 h</td>
<td>40°</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>Carbon</td>
<td>Epoxy</td>
<td>80°C, 3 h</td>
<td>30°</td>
<td>0.35</td>
<td>0.33</td>
</tr>
</tbody>
</table>

4.2.2. **Laminates with Constant Gauge-Length and Different Width**

In aerospace material design, [0°, 90°] laminates are usually used. Balanced [±45°] angle ply laminate is the closest arrangement to the laminate mentioned. Therefore, in this study tensile test on [±45°] balanced angle-ply laminate was performed.

Tensile strength results of ±45° balanced angle-ply specimens increased by increasing the width. In fact the strength of these specimens clearly depends on the laminate width as long as they have the same constant short gauge-length. In these cases (provided high width value and short gauge-length for the specimen), the applied stresses were able to break the fibres instead of just sliding out. Referring to most of the real structures produced from composite materials, we will notice that they have large $W/L$ ratios. So, it seems highly likely that the earlier theories [24,25] did not pay enough attention to the edge effects. This tells us that in reality angle-ply laminates are much stronger than what is expected (at least according to their tensile strength). Moreover the angle-ply laminates are widely used in filament winding and pressure vessels. This experimental work on glass fibre laminates showed a good agreement with Soden's work on glass epoxy filament wound tubes. As mentioned earlier, this was because the edge
since the pure hoop strength was about twice as great as the axial strength. This irregularity corresponds to the fact that the longitudinal stress can cause an inner collapse in a filament wound tube. In fact this kind of collapse occurs at low winding angles, i.e. \(<45^{\circ}\) (similar to the angle-ply laminates with angles \(>45^{\circ}\)) which cannot be applied in tubes under just pure hoop stress.

The micrographs Figs. 3.23-3.26 show the specimens after the tensile strength test. Some interesting features which occurred during loading the specimens, can be observed. The glass-epoxy laminates involved slide off for the narrow specimen up to the 30 mm width. This was also observed by Tsai. But the carbon-epoxy laminates behaved quite differently, and for these samples, there was a significant fibre breakage. Because of this, they gave greater tensile strength values (Fig. 3.21). In the case of carbon specimens, it seems that by increasing the width more than 100 mm, the tensile strength would not increase too much. This can be concluded from the small increase of the strength values between 30 mm and 100 mm, Fig. 3.21. However, this does not happen for the glass laminates since the 100 mm specimen is more than two times as strong as the 30 mm wide one, table A-10 in the appendix.

The fibre fracture occurs even for the narrowest specimens (3 mm wide), Fig. 3.23, and perhaps this corresponds to the higher stiffness of the carbon fibres. Presumably, the slide off process requires a certain degree of compliance which the glass has, but the carbon does not.

In glass as well as in the carbon specimens, the greater the width the more the fibre breakage. So the widest specimens with 100 mm width have the most percentage of fibre breakage even though this is not too easy to observe in the micrographs (Fig. 3.26).

The matrix also suffers much damage, with severe inter-fibre splitting.
Experiments were performed to measure the oblique tensile strength of single fibre for glass, carbon and Kevlar embedded in epoxy resin, in 0°-90° range. The same property was measured for balanced angle-ply laminates of glass and carbon fibres, at angles of 0° to 60°, for a constant gauge-length and width (20 mm & 44 mm). Additionally, the aspect ratio was changed (the width was increased from 3 mm to 100 mm) for glass and carbon specimens at [±30°]_s and [±45°]_s to diminish the edge effects.

1. The 50% angle definition, \( \phi_h \), corresponds to the oblique tensile strength of the single fibres. At this angle, the tensile strength of the single fibre is half of the normal strength. \( \phi_h \) is 20°, 30°-40° and 45° for glass, carbon and Kevlar, respectively. The oblique single fibre tests showed that these fibres retain their tensile strength to angles beyond than 10°.

2. The Tsai-Hill criterion is inappropriate to predict the strength of composites with misaligned fibres or imperfectly aligned fibres relative to each other.

3. The strength of single fibres obliquely stressed is not in agreement with \( \tan \phi \) relationship which was used for glass fibre bundles and steel wires. Instead, the results can be fitted using a one-third-power relationship.

4. Measurements of the oblique tensile strengths of single fibres at 15°-30° gives information about the intrinsic strength of the material. For carbon fibre the intrinsic strength may be about 30 GPa, assuming that the Young's modulus is constant.

5. Single fibre results show that there is a fibre effect. Although, Kevlar fibres are weaker than glass, they should give stronger random fibre composites.

6. The maximum stress theory and Tsai-Hill criterion should not be used to design composite, since testing wider and shorter angle-ply laminates gives higher results than these theories.

7. A possible design criterion is suggested when only tensile strength of the composites is considered. This involves determination of the effective aspect ratio of the specimens (W/L) so that edge effects can be quantified. The fibre fractures occur at the
8. Also, using this approach the tensile strength of randomly aligned short fibre composites can be better estimated.

9. The tensile strength of \([\pm 45^\circ]_s\) and \([\pm 30^\circ]_s\) angle-ply laminates increases monotonically by increasing specimen width.

10. The nominal stiffness is highest for the widest specimens (100 mm). A careful look at the stiffness results obtained from angle-ply laminates with constant gauge-length and width indicates that the laminate theory may not be suitable to estimate the Young’s modulus as well as the tensile strength. A reasonable fit corresponds to higher values for \(G_{12}\). Moreover, tests on filament wound glass-epoxy tubes are in a good agreement with the tensile strength as well as the stiffness results in this study.
REFERENCES

Table A-1. Strength of Glass Fibers vs Different Angle Cured at Room Temperature.

<table>
<thead>
<tr>
<th>$\phi$ (degrees)</th>
<th>Strength (GPa)</th>
<th>CV%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.0 ± 1.0</td>
<td>33</td>
</tr>
<tr>
<td>5</td>
<td>3.0 ± 1.0</td>
<td>33</td>
</tr>
<tr>
<td>10</td>
<td>3.3 ± 0.7</td>
<td>21</td>
</tr>
<tr>
<td>15</td>
<td>2.9 ± 0.7</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>1.4 ± 0.6</td>
<td>43</td>
</tr>
<tr>
<td>25</td>
<td>1.0 ± 0.3</td>
<td>30</td>
</tr>
<tr>
<td>30</td>
<td>0.7 ± 0.2</td>
<td>29</td>
</tr>
<tr>
<td>40</td>
<td>0.3 ± 0.1</td>
<td>33</td>
</tr>
<tr>
<td>50</td>
<td>0.2 ± 0.1</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>0.2 ± 0.1</td>
<td>50</td>
</tr>
<tr>
<td>70</td>
<td>0.1 ± 0.01</td>
<td>10</td>
</tr>
<tr>
<td>80</td>
<td>0.04 ± 0.01</td>
<td>25</td>
</tr>
<tr>
<td>90</td>
<td>0.04 ± 0.02</td>
<td>50</td>
</tr>
</tbody>
</table>
Table A-2. Strength of Glass Fibers vs Different Angle Cured at 80°±3°C for Three Hours.

<table>
<thead>
<tr>
<th>$\phi$ (degrees)</th>
<th>Strength (GPa)</th>
<th>CV%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.9 ± 0.5</td>
<td>17</td>
</tr>
<tr>
<td>10</td>
<td>2.7 ± 0.6</td>
<td>22</td>
</tr>
<tr>
<td>20</td>
<td>1.3 ± 0.3</td>
<td>23</td>
</tr>
<tr>
<td>30</td>
<td>0.8 ± 0.1</td>
<td>13</td>
</tr>
<tr>
<td>40</td>
<td>0.7 ± 0.3</td>
<td>43</td>
</tr>
<tr>
<td>50</td>
<td>0.3 ± 0.1</td>
<td>33</td>
</tr>
<tr>
<td>60</td>
<td>0.20 ± 0.05</td>
<td>25</td>
</tr>
<tr>
<td>70</td>
<td>0.10 ± 0.03</td>
<td>30</td>
</tr>
<tr>
<td>80</td>
<td>0.10 ± 0.05</td>
<td>50</td>
</tr>
<tr>
<td>90</td>
<td>0.04 ± 0.02</td>
<td>50</td>
</tr>
</tbody>
</table>

Table A-3. Strength of Carbon Fibres vs Different Angle.

<table>
<thead>
<tr>
<th>$\phi$ (degrees)</th>
<th>Strength (GPa)</th>
<th>CV%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.7 ± 1.7</td>
<td>46</td>
</tr>
<tr>
<td>10</td>
<td>3.8 ± 1.1</td>
<td>29</td>
</tr>
<tr>
<td>20</td>
<td>3.2 ± 1.2</td>
<td>38</td>
</tr>
<tr>
<td>30</td>
<td>3.0 ± 1.4</td>
<td>47</td>
</tr>
<tr>
<td>40</td>
<td>1.6 ± 1.0</td>
<td>63</td>
</tr>
<tr>
<td>50</td>
<td>0.4 ± 0.2</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>0.2 ± 0.1</td>
<td>50</td>
</tr>
</tbody>
</table>
Table A-4. Strength of Kevlar Fibers vs Different Angle

<table>
<thead>
<tr>
<th>φ (degrees)</th>
<th>Strength (GPa)</th>
<th>CV%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.7 ± 0.7</td>
<td>19</td>
</tr>
<tr>
<td>10</td>
<td>3.2 ± 0.9</td>
<td>28</td>
</tr>
<tr>
<td>20</td>
<td>3.1 ± 0.8</td>
<td>26</td>
</tr>
<tr>
<td>30</td>
<td>3.1 ± 0.7</td>
<td>23</td>
</tr>
<tr>
<td>40</td>
<td>2.7 ± 0.6</td>
<td>22</td>
</tr>
<tr>
<td>45</td>
<td>1.7 ± 0.5</td>
<td>29</td>
</tr>
<tr>
<td>50</td>
<td>1.2 ± 0.5</td>
<td>42</td>
</tr>
<tr>
<td>60</td>
<td>1.0 ± 0.3</td>
<td>30</td>
</tr>
<tr>
<td>70</td>
<td>1.0 ± 0.2</td>
<td>20</td>
</tr>
<tr>
<td>80</td>
<td>0.4 ± 0.2</td>
<td>50</td>
</tr>
<tr>
<td>90</td>
<td>0.4 ± 0.2</td>
<td>50</td>
</tr>
</tbody>
</table>

Table A-5. Strength of Glass Angle-Ply Laminates

<table>
<thead>
<tr>
<th>φ (degrees)</th>
<th>20 mm gauge length</th>
<th>2 mm gauge length</th>
</tr>
</thead>
<tbody>
<tr>
<td>0(^a)</td>
<td>1.7 ± 0.2</td>
<td>--</td>
</tr>
<tr>
<td>0</td>
<td>1.6 ± 0.4</td>
<td>1.2 ± 0.1</td>
</tr>
<tr>
<td>15</td>
<td>1.4 ± 0.1</td>
<td>0.7 ± 0.1</td>
</tr>
<tr>
<td>30</td>
<td>0.7 ± 0.1</td>
<td>0.26 ± 0.02</td>
</tr>
<tr>
<td>45</td>
<td>0.17 ± 0.03</td>
<td>0.08 ± 0.01</td>
</tr>
<tr>
<td>60</td>
<td>0.05 ± 0.01</td>
<td>0.05 ± 0.03</td>
</tr>
</tbody>
</table>

\(^a\) 10 mm wide specimens.
Table A-6. Strength of Carbon Angle-Ply Laminates

<table>
<thead>
<tr>
<th>φ (degrees)</th>
<th>20 mm gauge length</th>
<th>2 mm gauge length</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>2.6 ± 0.1</td>
<td>--</td>
</tr>
<tr>
<td>0</td>
<td>1.1 ± 0.2</td>
<td>1.6 ± 0.2</td>
</tr>
<tr>
<td>15</td>
<td>1.3 ± 0.3</td>
<td>0.8 ± 0.1</td>
</tr>
<tr>
<td>30</td>
<td>0.8 ± 0.1</td>
<td>0.5 ± 0.1</td>
</tr>
<tr>
<td>45</td>
<td>0.3 ± 0.1</td>
<td>0.14 ± 0.01</td>
</tr>
<tr>
<td>60</td>
<td>0.10 ± 0.02</td>
<td>0.06 ± 0.01</td>
</tr>
</tbody>
</table>

* 10 mm wide specimens.

Table A-7. Nominal Stiffnesses (kN/mm) for Glass and Carbon Laminates

<table>
<thead>
<tr>
<th>Angle (degrees)</th>
<th>glass</th>
<th>carbon</th>
<th>glass</th>
<th>carbon</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>3.9 ± 0.2</td>
<td>12 ± 2</td>
<td>6 ± 1</td>
<td>12 ± 1</td>
</tr>
<tr>
<td>30</td>
<td>2.3 ± 0.2</td>
<td>5 ± 1</td>
<td>4 ± 1</td>
<td>8 ± 1</td>
</tr>
<tr>
<td>45</td>
<td>1.5 ± 0.3</td>
<td>1.5 ± 0.4</td>
<td>0.3 ± 0.1</td>
<td>4 ± 1</td>
</tr>
<tr>
<td>60</td>
<td>1.0 ± 0.1</td>
<td>1.8 ± 0.4</td>
<td>9 ± 3</td>
<td>4 ± 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle (degrees)</th>
<th>Carbon</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>86 ± 19</td>
</tr>
<tr>
<td>30</td>
<td>81 ± 35</td>
</tr>
<tr>
<td>45</td>
<td>72 ± 13</td>
</tr>
<tr>
<td>60</td>
<td>50 ± 27</td>
</tr>
</tbody>
</table>

Table A-10. Tensile Strength of Glass and Carbon Balanced Angle Ply Laminates, $\phi=45^\circ$

<table>
<thead>
<tr>
<th>Width (mm)</th>
<th>Glass $\sigma$ (GPa)</th>
<th>Carbon $\sigma$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.06 ± 0.02</td>
<td>0.12 ± 0.01</td>
</tr>
<tr>
<td>10</td>
<td>0.11 ± 0.01</td>
<td>0.22 ± 0.04</td>
</tr>
<tr>
<td>30</td>
<td>0.14 ± 0.02</td>
<td>0.27 ± 0.06</td>
</tr>
<tr>
<td>100</td>
<td>0.32 ± 0.04</td>
<td>0.28 ± 0.03</td>
</tr>
</tbody>
</table>
Table A-11. Tensile Strength of Carbon Balanced Angle Ply Laminates, $\phi=30^\circ$

<table>
<thead>
<tr>
<th>Width (mm)</th>
<th>Aspect Ratio Length/width</th>
<th>Carbon $\sigma$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6.67</td>
<td>0.46 $\pm$ 0.05</td>
</tr>
<tr>
<td>10</td>
<td>2.00</td>
<td>0.56 $\pm$ 0.05</td>
</tr>
<tr>
<td>30</td>
<td>0.67</td>
<td>0.73 $\pm$ 0.07</td>
</tr>
<tr>
<td>100</td>
<td>0.20</td>
<td>NA</td>
</tr>
</tbody>
</table>
**APPENDIX H**

**Governing equation for the fibre axis**

We choose our x axis along the direction of the pull on the fibre; see Fig. A1. The force F is extended at some indefinitely large distance from the origin, as will be shown. With the origin as shown, the fibre intersects the y axis at \( y_0 > 0 \).

It will be seen that the moment of the force, M, is always \( Fy \). Since the radius of curvature, R, is given by \( MR = E_f I \) where \( E_f \) is the Young's modulus of the fibre and \( I \) is the moment of area, equal to \( \pi r^4/4 \) where \( 2r \) is the fibre diameter, we can write

\[
1/R = 4Fy/E_f \pi r^4 \tag{A1}
\]

and writing \( \varepsilon_f = F/\pi r^2 E_f \), we have

\[
1/R = 4\varepsilon_f y/r^2 \tag{A2}
\]

Thus

\[
(d^2 y / dx^2) / [1 + (dy/dx)^2]^{2/3} = 4\varepsilon_f y/r^2 \tag{A3}
\]

Writing \( \tan \phi = dy/dx \), it can be readily shown that \( \phi = 2\sqrt{\varepsilon_f} y/r \) is a solution of this equation for small \( \phi \). (The limit is when \( \sin \phi \) is significantly less than \( \phi \); \( \phi \) is in radians.) This gives an equation most easily presented in the form

\[
x = (r / 2\sqrt{\varepsilon_f}) \ln \left\{ \sin(2\sqrt{\varepsilon_f} y/r) / \sin(2\sqrt{\varepsilon_{f0}}/r) \right\} \tag{A4}
\]

eqn (A4) gives \( x = -\infty \) for \( y = 0 \).

Numerical analysis appear to give the result

\[
\phi = 2\sqrt{\varepsilon_f} y/r \tag{A5}
\]
to values of $\phi$ up to very close to $\pi/2$, at least for $\varepsilon_f = 0.01$ or less. For this we note that, for $\Delta$ indicating very small but finite values

$$\Delta\phi = 1/R (\Delta s) \quad \text{(A6)}$$

where $\Delta s$ is the length of arc. Since $\Delta s = \Delta x / \cos \phi$ we can write

$$\Delta\phi = 4 (\Delta x / r) \varepsilon_f (y / r) \sec \phi \quad \text{(A7)}$$

We can solve for $\phi$ as a function of $y / r$ using step sizes $\Delta x = cr$. This was done, starting at $\phi = \phi_0 = 0.1$ radian ($5.73^\circ$), and with $y_0 / r = \phi_0 / 2\sqrt{\varepsilon_f}$ and $y_{n+1} / r = y_n / r + c \tan \phi_n$. The step size factor, $c$, was 1 for $\phi$ to 12°, then 0.5 to 29°, 0.2 to 48°, 0.1 to 66°, 0.05 to 78°, 0.02 to 83°, 0.01 to 86.4°, 0.001 to 88.9°, then 0.0001 to 89.7°.

The result was a straight line, see Fig. A2, very close to eqn (A5) with $\varepsilon = 0.01$. Thus writing $\varepsilon_{\text{flex}} = r / R$, we can conclude that, using eqns (A2) and (A5)

$$\varepsilon_{\text{flex}} = 2\phi \sqrt{\varepsilon_f} \quad \text{(A8)}$$

with $\phi$ in radians, with an error of less than 36% at $\phi = 90^\circ$ for $\varepsilon_f \leq 0.01$, and much smaller errors at angles. (The error is $1 - \sin \phi / \phi$.)

Finally Fig. A3 shows the $x / y$ plot obtained for $\varepsilon_f = 0.01$, compared with eqn (A4). It can be seen that the equation predicts the $x / y$ plot very well.
Fig. A1. Schematic drawing showing fibre emerging from polymer at right, and axes used for the development of the equation for the curve taken by the fibre.

Fig. A2. The points are the numerical solution to eqn (A7) and the line is given by eqn (A5).
Fig. A3. Curve assumed by the fibre. The outer curve is the approximate result and the inner curve is the numerical solution.