EXPERIMENTAL STUDY OF LAMINAR LIQUID FILMS FALLING ON AN INCLINED PLATE

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
Department of Chemical Engineering and Applied Chemistry
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Experiments have been conducted to investigate instantaneous hydrodynamic characteristics of laminar falling films of 20 cS silicon oil on an inclined plate using a photochromic dye activation (PDA) technique over Reynolds numbers of 11-220.

The film thickness was measured directly from the video pictures and the data showed good agreement with those of previous investigations. Time-averaged mean velocity data were over-predicted by classical theory, while time-averaged wall shear stress data were in reasonable agreement. Instantaneous velocity distributions indicated that the over-prediction of the time-averaged mean velocity was a result of severe over-estimations of velocity in waves. Good agreement with classical theory was obtained with instantaneous velocity profiles in non-wavy regions, while wave-front and wave-back regions exhibited limited agreement. Non-dimensionalized velocity distributions did not follow classical laminar film theory, due to the existence of interfacial fluctuations causing unsteadiness in the local velocities, or turbulent models. Waves were determined to carry between 74%-85% of the liquid mass. Comparisons of instantaneous wall shear stress to classical theory showed under-prediction in wavy regions and good agreement in non-wavy regions. A steady-state force balance suggests that wavy regions experience acceleration, non-wavy regions experience deceleration, and wave-front and wave-back regions exhibit both acceleration and deceleration.
I would like to take this opportunity to express my gratitude to the people who assisted me in this project. Without their help it would not have been the enjoyable experience that it was.

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Abstract

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**Dimensional Parameters**

The following nomenclature is presented in terms of a dimensional analysis, as opposed to a unitary evaluation, so as to allow for flexibility in the discussion of, and references to, the quantities, which they represent. The dimensional parameters utilized to describe the physical quantities in the present experiment are given here.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>L</td>
<td>length</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>mass</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>time</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>constant in equation 4.2</td>
<td>(-)</td>
</tr>
<tr>
<td>b</td>
<td>constant in equation 4.2</td>
<td>(-)</td>
</tr>
<tr>
<td>c</td>
<td>constant in equation 4.6</td>
<td>(-)</td>
</tr>
<tr>
<td>d</td>
<td>constant in equation 4.6</td>
<td>(-)</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity</td>
<td>(L/T²)</td>
</tr>
<tr>
<td>h_max</td>
<td>maximum film thickness</td>
<td>(L)</td>
</tr>
<tr>
<td>h_mean</td>
<td>time-averaged mean film thickness</td>
<td>(L)</td>
</tr>
<tr>
<td>h_min</td>
<td>minimum film thickness</td>
<td>(L)</td>
</tr>
<tr>
<td>m</td>
<td>constant in equation 4.9</td>
<td>(-)</td>
</tr>
<tr>
<td>n</td>
<td>number of data points</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>volumetric flow rate</td>
<td>(L³/T)</td>
</tr>
<tr>
<td>r²</td>
<td>coefficient of determination</td>
<td>(-)</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds Number</td>
<td>(4Q/(Wv))</td>
</tr>
<tr>
<td>s</td>
<td>standard deviation</td>
<td>(L)</td>
</tr>
<tr>
<td>u</td>
<td>stream-wise velocity</td>
<td>(L/T)</td>
</tr>
<tr>
<td>u⁺</td>
<td>dimensionless velocity</td>
<td>(-)</td>
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\( W \)  span-wise channel width (L)
\( x \)  Cartesian coordinate in direction of flow (-)
\( y \)  Cartesian coordinate perpendicular to bounding wall (-)
\( y^+ \)  dimensionless location in Cartesian y direction (-)

**Greek Symbols**

\( \delta \)  film thickness (L)
\( \mu \)  dynamic viscosity (M\*L/T)
\( \nu \)  kinematic viscosity (L^2/T)
\( \rho \)  density (M/L^3)
\( \theta \)  inclination angle to the horizontal (-)
\( \tau \)  shear stress, momentum flux (M/L^3)
\( \Gamma \)  mass flow per unit width (M/(L*T))

**Subscripts**

ave  average
cal  calibrated
max  maximum
sub  substrate
s/a  substrate or average (dependent upon argument, equations 4.10 and 4.11)
w  wall
Liquid films on a vertical or inclined surface falling under the influence of gravity are important in numerous industrial applications, including wetted-wall absorbers, condensers, vertical tube evaporators, and falling film chemical reactors. In order to design these industrial equipment reliably the transport rates of heat and mass must be accurately predicted. However, before the heat and mass transport properties can be evaluated, the momentum transfer and the hydrodynamic characteristics of the falling liquid film must be fully understood so that the film thickness, velocity distribution, and wall shear stress that govern this viscous flow system can be modeled and predicted.

Falling liquid films may be described through the use of three progressive regimes, based on the Reynolds number, Re, of the flow. In the present work, the Reynolds number is defined as four times the volumetric flow rate per unit of channel width divided by the kinematic viscosity (equation 3.1). The smooth laminar regime, usually considered to exist in the Reynolds number range up to 25, exhibits a thin film with a smooth, flat interface. The intermediate regime, indicated by laminar flow with a rippling surface, is observed in flows with Re ranging from 25 to 1000. This regime is characterized by a wavy interface with a fluctuating film thickness. It has been noted in the past that this intermediate regime can exist at Reynolds numbers as low as four and as high as 2000. The turbulent regime is characterized by flows above a Reynolds number of 1000 (Bird et al., 1960). The wide degree of variation of the rippling laminar regime gives rise to the question of a transition zone between laminar and turbulent flows. Several studies have been conducted to examine this aspect of falling liquid films and the transition zone has been observed at Reynolds numbers as low as 370 and as high as 1700.

Flows of falling films on inclined surfaces have been noted to exhibit spatially distinct regions as functions of distance from the leading edge of the flow plane. Immediately following the leading edge an acceleration region is observed in which the flow is not fully developed. Following the acceleration zone is a region of smooth, apparently fully developed, flow in which little or no rippling can be seen regardless of the Reynolds number. The smooth interface region quickly develops ripples, defining a region characterized by a wavy interface. Initially, the wavy interface appears to be two-dimensional in nature, with uniform waves extending span-wise across the channel width or
down and a random, three-dimensional wavy interface predominates the falling film structure (figure 1.1). This final region is observed to extend along the remaining length of the test section (Bird et al., 1960; Cook and Clark, 1971; Dukler, 1977).

![Wavy interface](image)

**Figure 1.1. Typical Wavy Interface on an Inclined Falling Film**

Historically, the study of the hydrodynamic characteristics of falling liquid films has been impeded due to the thin nature of these flows, often observed to be in the order of millimeters. Many of the measurement methods that have typically been applied to study falling film flow fields, such as hot wire anemometry (HWA), cinematic techniques following particles or bubbles in the fluid, electrical conductivity, and electrochemical probe techniques, to determine such characteristics as the film thickness and velocity profiles can often interfere with the flow structure or suffer from limited spatial resolution, yielding somewhat inaccurate data.

The majority of studies on falling liquid films to date, employing both intrusive and non-intrusive techniques, such as laser Doppler anemometry (LDA) for the determination of velocity, have the ability to provide local, instantaneous velocity data and, in some cases, shear stresses. However, these methods are able to provide only time-averaged data for
instantaneous flow field data has prevented us from understanding the important effect of local film thickness variations on the local velocity profiles and shear stress at the wall.

Some work has been conducted to examine the instantaneous velocity profiles in falling films, providing some useful insights for the present experiment. However, the measurements were conducted over a very limited flow range, and involved an intrusive technique to obtain the data.

Much numerical work, especially on low Reynolds number flows, has been conducted on the hydrodynamic aspects of falling films, with the aim of solving momentum equations and predicting the fluctuations of the free interface. In the absence of accurate, instantaneous velocity and film thickness data the validity of these models may be in question.

Decades of research into falling liquid films have produced no conclusive model to describe and predict the hydrodynamics characteristics of such films. It is the purpose of the present study to attempt to elucidate some of the hydrodynamic characteristics of a viscous fluid falling on an inclined plate in a rectangular channel. This will be accomplished through the use of technique known as photochromic dye activation (PDA), a non-intrusive flow visualization method. This technique will yield instantaneous film thickness, velocity profile, and wall shear stress data, over a wider range of flow rates than previously reported. Once the data are obtained, an attempt will be made to discern the relationships between instantaneous film thickness, velocity and other instantaneous hydrodynamic characteristics.

This information, concerning the hydrodynamics of falling liquid films, will hopefully allow for the better understanding of the momentum transfer characteristics which will, in turn, lead to improved heat and mass transfer analogies and predictions. Improvements in the design and efficiency of industrial units designed to effect these physical transfer operations represent one of the long-term goals of this work.

A background highlighting previous work in the field of falling film hydrodynamics, as it applies to the present work, is given in Chapter 2. This section gives a historical survey of the experimental techniques used for measuring the pertinent hydrodynamic properties and briefly discusses some results. In addition, a somewhat chronological account of some of the numerical work involving falling films at low Reynolds numbers is presented. A detailed
methodology, and sources of error incurred in this work may be reviewed in Chapter 3. The experimental results and a detailed discussion are presented in Chapter 4. This section discusses the experimental data obtained including instantaneous film thickness, velocity profile, and wall shear stress measurements. The present data are discussed with reference to results given in previous studies. A comprehensive analysis of the mass transported by waves in falling films is presented, indicating the importance of the development of standard definitions of film thickness characteristics. New and interesting insights into the dependence of the velocity profiles and wall shear stress on the local film thickness are then discussed. Finally, Chapter 5 discusses the major conclusions of the present work and makes reference to the possible direction of future studies.
2.1. Classical Models

2.1.1. Constant Film Thickness

The first study of falling liquid films was that conducted by Nusselt (1916). He made several assumptions including a steady state system, a flat interfacial region, and rectilinear flow (i.e., the streamlines of the system are straight lines), and set up a shell momentum, or force, balance over a thin layer of the fluid. Then, the thickness of this layer was allowed to approach zero, as the fundamental definition of a derivative is used to generate a differential equation, expressed in terms of a momentum flux gradient, which could be integrated to give the momentum flux. The resultant differential equation, relating the momentum flux to the velocity gradient, was integrated to give the velocity profile of the fluid. The boundary conditions used in order to solve these equations for falling liquid films were: (a) the velocity of the fluid at the wall must be zero relative to the velocity of the wall and, (b) the momentum flux, or shear stress, and velocity gradient must be equal to zero at the liquid-gas interface. A detailed analysis of these aspects is given by Bird et al. (1960).

By following the steps indicated above, for a film falling on an inclined rectangular channel, equations describing the velocity profile \( u(y) \), wall shear stress \( \tau_w \), and film thickness \( \delta \), can be derived as follows.

\[
u(y)=\frac{\rho g \delta^2 \cos \theta}{2 \mu} \left[ 1 - \left( \frac{y}{\delta} \right)^2 \right] \quad (2.1)
\]

\[
\tau_w = \rho g \delta \cos \theta \quad (2.2)
\]

\[
\delta = \left[ \frac{3 \mu u_{ave}}{\rho g \cos \theta} \right]^{1/2} \quad (2.3)
\]

Here, \( g \) is the gravitational acceleration downward, \( \rho \) is the liquid density, \( \mu \) is the liquid viscosity, and \( \theta \) is the angle of inclination of the plate from the horizontal. In equation (2.1)
wall. The average velocity, $u_{ave}$, in equation (2.3) may be replaced with the volumetric flow rate per unit channel width, if the exponent is replaced by a value of $1/3$, in order to calculate Nusselt's (1916) prediction of film thickness in terms of experimentally determined variables. However, it is necessary to define the average velocity to provide some meaningful analysis, along with the maximum velocity, in the present work. The average velocity may be obtained by integrating equation (2.1) with respect to the channel width and film thickness, and dividing this value by the cross-sectional area of the film. The maximum velocity may be obtained by evaluating equation (2.1) at a value of $y = 0$ (i.e., the interface location).

$$u_{ave} = \frac{\rho g \delta^2 \cos \theta}{3\mu}$$  \hspace{1cm} (2.4)

$$u_{max} = \frac{\rho g \delta^2 \cos \theta}{2\mu}$$  \hspace{1cm} (2.5)

It is clear, from equation (2.1) that the velocity profile predicted by a straightforward force balance analysis is parabolic in shape.

### 2.1.2. Variable Film Thickness

Experimental observations of laminar falling films indicated that the gas-liquid interface was not a constant, flat surface, but exhibited a wavy structure at Reynolds numbers as low as 20. By 1948, to address shortcomings in previously developed theories, Kapitsa (1965) analyzed falling films, accounting for this undulatory flow pattern observed, by considering the effects of surface tension. The argument of Kapista (1965) stated that the surface tension should play a significant role in the hydrodynamics of free falling films since the relative magnitude of the surface tension force is comparable to the viscous forces acting in thin liquid films. This work detailed the development of first order and second order approximations for the undulatory flow, and the predictions of wave amplitude and phase velocity. The prediction of film thickness (equation 2.6) agreed well with the experimental data obtained, and was of a form similar to, but consistently less than that of Nusselt (1916).
Here, $Q$ is the volumetric flow rate, $\nu$ is the kinematic viscosity, and $W$ is the channel width. Kapitsa’s (1965) theory appeared to account for the wavy nature of falling films only up to the condition that the wavelength was less than 13.7 times as great as the film thickness, which corresponded to a Reynolds number of 50 for a vertical falling film of water.

2.1.3. Universal Velocity Distribution

Nikuradse (1933) developed the universal velocity distribution, which provided a means by which flows could be examined in terms of a boundary layer analysis based on dimensionless parameters (Bird et al., 1960). This theory was derived for fully developed, single phase, turbulent pipe flow. However, in many cases, as will be presented in following sections, applications to falling film hydrodynamic systems, in both the turbulent and laminar regimes, prove useful. In addition, the dimensionless approach provides a convenient method by which velocity profiles close to the wall, in the laminar sublayer, may be examined. The universal velocity distribution may be derived from equations proposed by Deissler (1955) for regions near a wall, where equations developed by von Karman (1939) and Prandtl (1925) are inadequate. By integrating Deissler’s (1955) equation, Nikuradse (1933) developed a set of equations that represent the laminar sublayer and buffer regions of the flow field. These equations are given by:

$$ u^+ = y^+ \quad \text{for } 0 \leq y^+ \leq 5 \quad (2.7) $$
$$ u^+ = -3.05 + 5\ln y^+ \quad \text{for } 5 \leq y^+ \leq 30 \quad (2.8) $$

where equation (2.7) represents the laminar sublayer and equation (2.8) represents the buffer region between the laminar sublayer and the turbulent zone. The non-dimensionless velocity profile parameters, $u^+$ and $y^+$, are defined as,

$$ u^+ = \frac{u}{u_*} \quad (2.9) $$
$$ y^+ = \frac{yu^+}{\nu} \quad (2.10). $$
The falling film may become turbulent if the dimensionless film thickness, obtained by substituting the local film thickness for \( y \) in equation (2.10), reaches a value of 30. This corresponds to a Reynolds number of 270, which appears to significantly under-estimate the transition to turbulent film flow reported in previous reports. Although the turbulent regime has been cited to cover a wide range, a value of \( Re = 1000 \) appears to be the most common (Dukler and Bergelin, 1952; Tailby and Portalski, 1962; Salazar and Marschall, 1978), whereas Karapantsios et al. (1989) reported a value of 1700. Salazar and Marschall (1978) cite a transition \( Re \) range of 362 to 1700, while Aragaki et al. (1990) reported a \( Re \) range from 150 to 1000, suggesting that the transition zone may appear at relatively low dimensionless film thickness values. Therefore, a falling film non-dimensional velocity distribution would be expected to deviate from the equations developed by Nikuradse (1933), possibly in the buffer region or the laminar sublayer.

### 2.2. Experimental Work

Liquid falling films of all types have been extensively studied in the past. Dukler (1977) estimated that between 1942 and 1977 over 7,000 papers had been published in gas-liquid flows, many of which dealt with falling film hydrodynamics. An excellent review of the research up to 1964, both experimental and theoretical, is presented by Fulford (1964). Some of the previous investigations will be briefly mentioned in this section, as they pertain to the current work.

#### 2.2.1. Film Thickness

Apart from the classical theorists, one of the first to measure the falling film thickness was Kirkbride, who in 1933, using a micrometer, found that Nusselt (1916) under-estimated the mean film thickness (Kapitsa, 1965). Since then several different techniques have been utilized to measure this characteristic of the film including physical contact probes,
Dukler and Bergelin (1952), utilizing a falling film of water on a vertical plate, developed a new theory based on the universal velocity profile. They measured the film thickness and wave profile using capacitance probes, consisting of two parallel conducting plates mounted vertically. One of these plates, the flow plate, was fixed in position while the second plate was arranged such that it could be moved to vary the normal distance between the plates. The capacitance of this sensor was directly proportional to the dielectric constant of the fluid between the plates and the area of the plates. Their results, at low Reynolds numbers agreed well with the new theory as well as those of Nusselt (1916). Portalski (1963) tested this new development at Reynolds numbers as low as 400, also using a glycerol-water falling film on a vertical plate and measuring the film thickness by means of hold-up data. In this method the fluid flow was halted and the fluid drained from the wetted wall into a collection vessel. From the volume collected and the known surface area of the vertical plate the average film thickness may be calculated. At these low Reynolds numbers the universal velocity distribution prediction, to which their data fit well, was significantly below those of Nusselt (1916) and Kapitsa (1965). However, it should be noted that a theoretical analysis, extending the work of Kapitsa (1965) to account for eddy formations in falling films on vertical plates, was conducted by Portalski (1964). This work gave good agreement with qualitative results, obtained by photographic methods, of other investigators.

Several investigations have examined the statistical properties of falling liquid films. Telles and Dukler (1970), studying water falling down parallel vertical rectangular channels at Reynolds number of 900-6000, measured the film thickness through the use of parallel electrodes mounted flush with the wall. These probes measured the conductance of the liquid, proportional to the film thickness, by the voltage drop detected across a resistor. This work, which appears to be the pioneering research in this area, details the methods used for calculating statistically meaningful wave celerity, amplitude, separation distance, frequency and wave shape. Of particular interest to the present work are the data relating the ratio of film thickness rms-to-mean film thickness to Reynolds number. In addition, their results indicated that, at low Reynolds numbers, approximately 40% of the liquid mass was carried by the waves. Chu and Dukler (1974, 1975) continued this work, studying an annular falling
investigations of the wave frequency, amplitude, and wave profile, the first statistical
definition of the substrate film thickness was presented. Chu and Dukler (1974) defined the
substrate thickness in three ways, based on: (a) expected values of substrate, (b) fractional
time exposure of substrate, and (c) the probability density function (PDF) of film thickness.
In Chu and Dukler (1974, 1975), the increase in right-directed skewness of the film thickness
PDF as a function of Reynolds number is clearly observed. Wasden and Dukler (1992),
studying an annular falling film of an aqueous solution, between the Reynolds numbers of
174 and 805 reported good agreement with the film thickness PDF trends. The film thickness
was measured using probes consisting of two parallel platinum-rubidium wires whose
resistance measured was directly proportional to this quantity.

The oscillogram and shadowgraph methods were used to measure the artificially
oscillated film thickness of water on the exterior of a vertical tube, at Reynolds numbers
ranging from 20-50 by Nakoryakov et al. (1976) and Nakoryakov et al. (1977). Film
thickness measurements agreed with those of Nusselt (1916), based on the velocity profiles
calculated.

Salazar and Marschall (1978) measured the film thickness of a water film falling over
a vertical plate, between the Reynolds numbers of 145-4030, by using a light scattering
technique. The local film thickness was measured by detecting changes in the length of a
column of light scattered by latex particles in the fluid, which was proportional to the liquid
film thickness. Important comparisons to previous measurements, such as those of Portalski
(1963) which gave good agreement, were presented.

Intensive investigations of the mean, minimum and maximum film thickness were
conducted by Takahama and Kato (1980). They measured the thickness of a water film
falling on the exterior side of a brass tube, by means of a needle contact and electrical
capacity, similar to those described above. A formula for mean film thickness was
presented, based on Nusselt’s (1916) equation (equation 2.3). Their results indicate that the
minimum film thickness is independent of Reynolds number. However, the maximum film
thickness is highly dependent on Reynolds number, as is the mean film thickness, though to a
lesser degree. Similar trends were observed by Karapantsios et al. (1989), who studied
annular falling films of water, and measured the film thickness by means of the parallel wire
addition, statistical studies concerning the standard deviation of the film thickness and film thickness ratios agreed with results disclosed by Telles and Dukler (1970) and some new data were provided concerning the nature of film thickness fluctuations down to a Reynolds number of approximately 500. More recent work by Karapantsios and Karabellas (1990, 1995), utilizing the same apparatus, working fluid, and measurement technique as Karapantsios et al. (1989), provides useful comparisons of film thickness measurements to those predicted by Nusselt (1916) at a Reynolds number as low as 370, as well as new fits to the equation proposed by Takahama and Kato (1980).

More recently, Karimi and Kawaji (1996) measured the film thickness of an annular falling film of kerosene by two methods. A laser displacement sensor, in which a beam was emitted, reflected off the film surface, and detected with a voltage corresponding to the film thickness, gave similar results as those of high-speed video recordings of the film thickness as a function of Reynolds number. A new correlation similar to that proposed by Takahama and Kato (1980) was presented.

2.2.2. Velocity Measurements and Velocity Profiles

Velocity profiles are very difficult to measure experimentally in falling liquid films, mainly due to the thin nature, often less than one millimeter, of these films. The small thickness makes it nearly impossible to employ any standard, intrusive fluid velocity measuring equipment without disturbing the flow patterns.

Grimley (1945) was one of the first to report velocity profile measurements for a falling film of water. The measurements were conducted by following colloidal particles suspended in the film using an ultramicroscope technique. Results indicated significant deviations from a parabolic profile with wavy films, due to fluctuating wavy velocities near the free surface (Fulford, 1964). Clayton (1958), using a chronophotographic technique, measured the velocities of falling films in the smooth laminar regime and found that the profiles agreed well with those of Nusselt (Fulford, 1964).

Wilkes and Nedderman (1962), to obtain velocity profiles, employed a stereoscopic photography method, similar to that used by Clayton (1958), in an annular falling films at very low Reynolds numbers. This technique involved the cinematic observation of small air
observed with both wavy and non-wavy flows (achieved by the addition of surface-active agents), though the wavy films showed a large degree of fluctuation around the parabolic fit. The rate of acceleration was accounted for by the surface tension of the liquid, as proposed by Kapitsa (1965).

A fundamental technique to measure the average velocity of a water film falling on a vertical plate was first reported by Portalski (1964). This method involved measuring the time required for the liquid front to travel a determined distance at known volumetric flow rates. From the experimental (average) velocity, the maximum (or free surface) velocity was established and the ratio of maximum-to-average velocity examined over the Reynolds number range of 4-5000. This ratio was found to be constant at a value of 1.5, indicative of parabolic velocity profiles, as deduced by Nusselt (1916), in the laminar regime from Re = 4-1000. Above this range this velocity ratio decreased steadily to a value of 1.15 at Re = 4000.

An electrochemical technique, somewhat similar to those used for film thickness measurements, was applied to falling liquid films to obtain velocity profiles. This method was first reported by Reiss and Hanratty (1962, 1963) with the aim of studying the flow field close to the bounding surface, and was used to examine the velocity gradients around a sphere at low Reynolds numbers by Dimopoulos and Hanratty (1968). This technique involved an electrolytic reaction between a small electrode, mounted flush with the wall, and a much larger electrode located in the fluid downstream. The concentration of the electrolyte at the surface electrode was assumed constant and equal to zero, resulting in a diffusion controlled reaction. The mass transfer coefficient could then be calculated from the measured current, through Faraday's Law, between which a proportional relationship existed. The mass transfer coefficient fluctuations were then related to variations in the velocity gradient. This technique is important to falling liquid films in its applications to the measurement of wall shear stress, as discussed in the following section.

Cook and Clark (1971) developed a stroboscopic, stereoscopic technique, based on the work of Wilkes and Nedderman (1962), utilizing a telemicroscope and a high speed camera. The method involved the still photography of micron sized aluminum oxide particles dispersed in a water film falling over a vertical plate with Re ranging from 300 to 1000. Parabolic velocity profiles, in agreement with several previous studies, were reported.
an inclined (at 20° to the horizontal) plate in a rectangular channel, with spindle oil as the working fluid, was reported by Ueda and Tanaka (1975). The measurement technique used consisted of a hot wire probe. In constant current hot wire anemometry, the wire current is held constant while variations in the rate of heat transfer (or wire temperature), caused by local flow variations, are measured as variations in the wire resistance or voltage drop. Parabolic velocity profiles were obtained, in this turbulent regime, which conform to laminar prediction. Excellent agreement with the universal velocity distribution was shown, up to a value of $y^+ = 10$. Above this value the experimental values fell above the law of the wall.

Koziol et al. (1981) also studied falling films in inclined rectangular channels and calculated the average velocity from hold-up data. In their investigation, water was used as the working fluid, over a flow range of $Re = 500-3000$, with the channel inclined at an angle of 20° to the horizontal. At known volumetric flow rates the amount of liquid on the solid surface was determined, from which the film thickness could be calculated. Subsequently, the mean volumetric velocity was found by dividing the mass flow rate per unit width by the film thickness and density. The maximum velocity was found by measuring the time of flow over a known distance by following small (1 mm diameter) pieces of paper carried by the surface of the film. The maximum-to-average velocity ratio was calculated was calculated and they report that the laminar film data of Friedman and Miller (1941) deviated significantly from the parabolic prediction, showing a peak discrepancy at a Reynolds number of 90. However, in the smooth laminar regime parabolic profiles were confirmed. To account for the parabolic nature of velocity profiles in the laminar sublayer a modification to the universal velocity distribution was presented, and good agreement was obtained with their data.

All investigations of velocity and velocity profiles mentioned thus far were limited to time-averaged studies, due to limitations in the experimental apparatus available at the time. This did not allow for a study of the transient effects on these velocity profiles, imparted by the fluctuating film thickness.

In 1977, a new technique, involving two synchronized methods, developed by Nakoryakov et al. (1977), allowed for the instantaneous measurement of velocity profiles in falling liquid films. The shadowgraph method, to determine the instantaneous film thickness
velocity profiles. Spherical aluminum particles were photographed, in a similar manner as described by Cook and Clark (1971), and the time trace recorded on a photographic film allowed for the calculation of the velocity profiles. This work was further extended by Alekseenko et al. (1985), who studied a vertical falling film of water flowing down the exterior surface of a highly polished stainless steel tube with Re ranging from 20 to 50. Their results indicate that the velocity profiles under interfacial waves are well behaved and conform to a self-similar parabolic velocity profile very well, with little fluctuations. In regions of extended substrate the velocity profiles were reported to agree well with the parabolic velocity profile predicted by Nusselt (1916). The most interesting result, however, was that in regions immediately surrounding waves the fluctuations in velocity profile were very large, and no parabolic fit was presented for data in this region.

Falling film flow has also been studied, using laser-Doppler velocimetry (LDV), by Mudawar and Houpt (1993). Films of highly viscous propylene glycol-water mixtures, falling on the exterior of a vertical tube in the flow range of Re = 209-5000, were seeded with small air bubbles to facilitate the operation of the LDV apparatus. Parabolic velocity profiles were obtained, though a large degree of fluctuation was observed. Dimensionless velocity profiles gave good agreement to data reported by Cook and Clark (1971). The time-averaged velocity profiles were integrated, in order to assess the amount of liquid passing through large waves, and it was determined that this value was 40% at a flow corresponding to a Reynolds number of 209.

**2.2.3. Wall Shear Stress**

Historically, the wall shear stress in falling liquid films has been difficult to measure, as the velocity profile very close to the wall must be accurately established. Prior to 1965 only two reports detailed the measurement of wall shear stress. Brauer (1956), studying film flow on the exterior of a vertical tube, and Fulford (1964), examining falling films on an inclined channel, reported time-averaged wall shear stress values in the smooth laminar regime which conform to the classical prediction of Nusselt (1916). However, in the wavy laminar region, Fulford (1964) indicated the wall shear stress was greater than the classical prediction.
previous section, to measure mass transfer rates in falling films, with applications leading to the calculation of velocity profiles. However, years later, Miya et al. (1971) utilized this methodology to measure shear stress profiles in falling liquid films. In addition, Miya et al. (1971) obtained wall shear stress data by measuring the rate of heat transfer from a water film to a thin platinum film fixed to a vertical flat plate. Data collected from an inclined rectangular channel, at Reynolds numbers ranging from 800 to 2000, indicate a sudden relaxation of the shear stress upstream of a wave, with a sharp increase in the magnitude of this value in the wave front. Mao and Hanratty (1986) used this electrochemical probe to study the wall shear stress in developed, pulsatile pipe flow. Their results indicate that the wall shear stress increases with Reynolds number, due to stronger velocity gradients observed. Considering a boundary layer analysis, these results may be applied to falling film flow. The regions close to the wall, where the shear stress measurement is important in viscous flows, may be considered to be in the laminar sublayer of the boundary layer flow. Therefore, in the sense of the universal velocity distribution, it may be stated in general, that the wall shear stress will always increase with Reynolds number, and in the case of falling films, this corresponds to an increase in film thickness and waviness. An excellent review of the electrochemical, or polarographic technique to measure wall shear stress is given by Hanratty (1991).

Alfredsson et al. (1988) studied wall shear stress in boundary layer flows using the hot wire probe method. They noted significant fluctuations in the standard deviation of wall shear stress values, indicating large fluctuations around the mean value, while the largest degree of fluctuations occurred in the viscous sublayer. These data indicated that, even in apparently steady flow, the region close to the wall experiences significant effects imparted by the potential flow region. Applied to falling film flow, this argument suggests that the wall region should experience even greater effects due to the transient film thickness variations.

The hot wire probe technique, to measure wall shear stress, was applied by Govan et al. (1989) on a vertical annular falling film of water. In the presence of counter current air flow, their data suggested that experimental wall shear stress values were slightly higher than those predicted by theoretical considerations. In addition, they found that the fluctuations of
Differential pressure transducers and an electrolytic system was used by Wasden and Dukler (1989) to evaluate the wall shear stress in a vertical annular falling film at a Reynolds number of 880. As with Miya et al. (1971), they reported that the wall shear stress achieved peak values in the wave front region, as opposed to the wave peak. Similar trends, in other transport phenomena, were reported by Lyu and Mudawar (1991) and Brauner (1982).

The electrochemical technique was employed by Aragaki et al. (1990) to measure the wall shear stress in vertical annular falling film with a Re ranging from 100 to 8000. Their results indicated excellent agreement with the shear stress values calculated from the mean film thickness (equation 2.3) except in the range of Re = 600-4000, in which the experimental values were significantly overestimated. They reasoned that this discrepancy was due to a transition region between the laminar and turbulent regimes.

2.3. The Photochromic Dye Activation Technique

The photochromic dye activation (PDA) technique for flow visualization was first developed at the University of Toronto by Popovich and Hummel (1967), and was originally known as the flash photolysis method. This method involved the formation of a visual record in the fluid, in the form of a well-defined trace, perpendicular to the bounding wall. The dye material undergoing the photochromic reaction (an almost instantaneous, reversible reaction involving a colour change due to the absorption of electromagnetic radiation) in these experiments was 2-(2,4-dinitro-benzyl)-pyridine (DNBP). The reaction was initiated through the use of an ultraviolet light that imparted the required energy such that the DNBP, dissolved in the working fluid at low concentrations, formed a thin, coloured, parallelepiped trace which was subsequently captured by a series of photographs taken at a predetermined time interval.

This PDA technique was first applied to study turbulent flow in a square pipe (Popovich and Hummel, 1967). The universal velocity distribution obtained from these results indicated that the law of the wall holds only in the range of \(y^*\) of zero to a value of 1.6, indicating that a linear velocity gradient exists close to the wall.
the energy source responsible for causing the photochromic reaction, as they studied the turbulent nature of liquid streams.

Ho and Hummel (1970) utilized an ultraviolet light source and DNBP to study falling liquid films. Time-averaged velocity distributions were obtained, for vertical, annular falling films of aqueous alcohol solutions and alcohol-glycerol mixtures, over a Reynolds number range of 124-2800. Analysis of these data indicated that the velocity structure followed that of the universal velocity distribution quite well close to the wall, from \( y^+ = 1 \) to 10. Above this range, the experimental non-dimensional velocity distribution fell above the law of the wall. Based on their data, a new empirical fit to the velocity distribution was proposed, based on a correlation proposed by Spalding (1961).

Several other investigators have used this PDA technique in order to visualize the flow field including such phenomena as flow in mixing vessels (Sovona et al., 1973), liquid drop formation (Humphrey et al., 1974), and flow around a sphere (Seeley et al., 1975). This technique appeared to be so robust and useful that it was published in the Journal of the Society of Motion Picture and Television Engineers (Smith and Hummel, 1973).

The modern era of the PDA technique appeared to begin in the late 1980's, a result of advances in scientific technology, in particular the advent of the high-speed video camera, powerful lasers with better resolution, and digital computer equipment. These new developments allowed for the accurate recording of vast amounts of data at sufficiently high resolution, allowing for more detailed studies and analysis than was possible in the past. In addition, a new dye material was developed, to be used for the reversible photochromic reaction. The first report documenting these developments involved the study of pulsatile flow in a constricting passage (Ojha et al., 1988). The new chemical species utilized in this study, and the others that follow in this discussion, was 1',3',3'-trimethylindoline-2-spiro-2-benzapyran (TNSB).

The PDA technique has been applied to several two-phase flow studies, and Kawaji et al. (1993) have provided an excellent review. Some recent works utilizing this PDA technique include turbulent momentum transfer (Lorencez et al., 1991), liquid turbulence
Annular falling films have also been examined using the PDA technique to determine
the instantaneous velocity profiles (Pun et al., 1994). A large degree of scatter was observed
in these profiles, confirming the fluctuations observed in previous studies of velocity profiles
in falling films. The wall shear stress was found, on average, to be significantly larger than
that predicted by the appropriate two-phase flow model, proposed by Bharathan and Wallis
(1986). However, the time-averaged velocity distribution agreed well with the law of the
wall in the range of $y^+ = 0.5-8$. Above this range, the experimental profile fell below the law
of the wall, contrary to time-averaged data reported by Ho and Hummel (1970) for a smooth,
falling film.

2.4. Numerical Studies

Several investigators have attempted to model falling liquid films through various
means. Most models make assumptions in order to simplify the equations necessary to solve,
the Navier-Stokes equations and a wave profile equation. A brief review of some of these
models will be presented here.

Shkadov (1968), using a periodic wave assumption, developed a model based on that
of Kapitsa's (1964) original solution. This simulation gave a successful solution for wavy
film flow for a very limited flow rate of less than $Re = 100$. This work was furthered by
Hirshburg and Florschuetz (1982), who included more expansion terms in their solution,
based on a Fourier series truncated after six harmonics. Their equation of free surface
deflection was developed from the equation of motion, making use of a permanent wave
translation. Yang et al. (1991) developed a numerical spectral method to predict the heat and
mass transfer coefficients, based on a periodic wave state assumption. This method, building
on the work of the above two investigations, gave good agreement with experimental data up
to a Reynolds number of 150.

An interesting approach was adopted by Brauner and Moalem-Maron (1983), Brauner
et al. (1985), and Brauner (1989), in which the wave structure was divided into sections and
each section was solved using a different solution scheme with different assumptions as
observed variations in velocity profiles in different regions, appeared to give good agreement with wavy falling films in many different regions.

A finite element method was employed by Bach and Villadsen (1984), to solve low Reynolds number flows, for wavy films developing from smooth interfaces. This model, based on a two-dimensional, laminar, vertical falling liquid film, predicted the wave celerity and wave form in addition to solving the flow field equations.

An integral relation method was developed by Alekseenko et al. (1985) to derive a non-linear two-wave equation for long waves on the surface of vertical falling films. This equation was found to be very robust, as it was applicable to both high and low Reynolds number flows. This two-wave equation, derived on the basis of the integral approach beginning with the Navier-Stokes equation assuming boundary layer conditions, was further developed for the modeling of weakly non-linear long waves on gravity driven liquid films moving on an inclined plane (Alekseenko et al., 1995).

Wasden and Dukler (1989) examined wave structures of interacting waves in regions where the Navier-Stokes equations could be solved. Their model, based on flows with a Reynolds number of 880, predicted the existence of recirculation cells within large disturbance waves, velocity profiles of a cubic nature (as opposed to parabolic) and shear stress peaks in the wave front. In addition, this computational simulation depicted regions of acceleration in the wave fronts, and deceleration zones in the wave backs. This work was continued by Yu et al. (1995), examining the stability of the two-dimensional Navier-Stokes equations using non-linear wave evolution assumptions. They reported good agreement with experimental data, however, the wall shear stress simulation seriously under predicted the values experimentally found under large waves.
3.1. Flow Loop and Test Section

The flow loop used to conduct this experiment consisted of several units arranged such that the delivery of fully developed flows could be obtained in the test section in a consistent manner (figure 3.1). The working fluid, described in detail below, stored in a covered, opaque, plastic storage tank, was maintained at a volume of 25 L. The fluid was kept dark while in storage to prevent degradation of the photochromic dye dissolved in it.

The fluid was pumped from the storage tank to the constant head tank, constructed of Lucite, by means of a Nemo progressive pump (Netzsch Inc., model number NE15A), operating at a maximum speed of 415 rpm. The motor driving the pump was a single phase DC drive motor operating on a 115 VAC input supply, and had a maximum output of 0.25 HP. The motor was controlled by means of a Bronco II Seco DC drive (model 160), which also ran on 115 VAC, 60 Hz input. As the fluid was pumped to the constant head tank it was passed through a particle filter (Cole Parmer, casing model no. E-01508-30, filter model no. E-01508-50) to remove fine particles and debris introduced into the fluid stream from external sources.

From the constant head tank, the fluid flowed down into the upper plenum, a short term holding reservoir before the fluid entered the test section. The flow loop between the constant head tank and upper plenum was always full of fluid to ensure that a constant pressure gradient was maintained prior to the test section.

The flow rate was monitored and controlled by means of two rotameters, located in the flow loop upstream of the upper plenum. One rotameter was used to control the flow at very low rates (Brooks Instrument Division Emerson Electric Canada, model no. 1307 DO7F1B1A, tube size R-7M-25-1F), while the other one controlled flows at higher rates (Brooks Instrument Division Emerson Electric Canada, model no. 1307 EJ21CK1AA, tube size R-8M-25-4F). The maximum flow rate obtainable, based on calibrations conducted with these rotameters was approximately 6.2 LPM, while the pump was operated at 100%.

The upper plenum, also constructed of Lucite, was designed to reduce the velocity of the fluid and allow a smooth entry into the test section. The fluid was collected from the end of the test section, which was inclined at an angle of 45° to the horizontal, in a Lucite
1. Constant Head Tank
2. Overflow Line
3. Upper Plenum
4. Test Section
5. Rotameter One
6. Rotameter Two
7. Storage Tank
8. Liquid Pump
9. Collection Vessel
10. Drainage Line
11. Filter
12. Thermometer
Gate, Ball Valves
Needle Valve

Figure 3.1. Experimental Flow Loop
Clear, reinforced plastic tubing (Kuriyama Kuritec K3150 RF) of ½ “and ¼” nominal inside diameters (ID) connected all sections. All joints in the flow system, elbows, tee’s, and other connectors of various configurations, were made of PVC (Spears, APF). The ball valves used were ½” 125 WOG from B&K, the needle valve was supplied by Grinnell-Saunders, and the gate valve was a Chemkor 1” PVC model. The super structure of the experimental apparatus was constructed from 1” nominal ID steel tubing, Dexion 225, and Kee-clamp fittings.

The test section consisted of a smooth, polished copper plate with translucent polycarbonate sidewalls. It was necessary for the sidewalls to be translucent because the traces formed had to be video recorded from the side to obtain the desired data. The total length of the test section, including the upper plenum was 2.27m, while the copper plate was 1.92m long. The open flow channel was 8.0 cm wide. At the entrance to the copper plate, to ensure evenly distributed flow across the entire channel, the plate was smoothly rounded. This test section also contained an enclosed section under the copper plate to allow for heat transfer experiments in the future. A detailed drawing of the test section is shown in figure 3.2.

3.2. Working Fluid and Photochromic Dye

The working fluid used in this study was a silicon oil manufactured by Dow Corning, known as Silicon Fluid 200. This fluid was chosen for use in the present experiment for several reasons. One of the most important aspects of this fluid was its high viscosity. The relatively high viscosity was desired in order to study the hydrodynamics fluids widely used in industry, such as lubricants, electrical insulating fluids, and many consumer products, among others. In addition, it is very safe to work with as it is non-toxic, is considered a low fire hazard, exhibits low reactivity and vapour pressure, and has high temperature serviceability.

Silicon Fluid 200 is a transparent, colourless, organic fluid, and is compatible with the PDA technique presently used. The transparent aspect allows for easy visualization of the traces formed. The organic nature of the fluid is a requirement in order to dissolve the
Figure 3.2. Test Section Design (a) plan, (b) section
Fluid 200, at a temperature of 22°C, are as follows: kinematic viscosity 20 cS, density 960 kg/m³.

The photochromic dye used in this experiment was 1',3',3'-trimethylindoline-2-spiro-2-benzapyran (TNSB). This dye was dissolved in the silicon oil to a maximum concentration of 0.015%, by weight. Throughout the course of experiments, some additional dye was added to the original 0.01%, by weight, to make up for spillage and degradation due to exposure to light sources. This dye provided advantages over previously used dyes (Ho and Hummel, 1967), due to the almost instantaneous, reversible, photochromic reaction upon exposure to UV light in the short wavelength absorption spectra, allowing for the acquisition of data at high frequencies.

It is commonly known that the surface tension of a fluid is extremely sensitive to additives that may be dissolved in it. However, in the present experiment the TNSB exerted a negligible effect on surface tension due to the low concentration used and the fact that the dye molecules remain evenly dispersed throughout the entire fluid layer.

3.3. Data Acquisition System

The PDA technique was utilized to collect all data in the present work. A laser source was used to create a photochromic trace in the working fluid, which was subsequently captured by a video camera and transferred to a videocassette.

A nitrogen gas (N₂) laser (Photonics PRA UV-24) was used as the energy source to excite the TNSB in the silicon oil, powered by standard 115 VAC power input. To operate optimally, a low pressure environment was required in the laser. This was achieved through the use of a vacuum pump (Busch Inc., model no. RA 0068-A005-1101), giving a displacement of 41 CFM. This pump was powered by a 3 HP AC motor (Lincoln AC motors, model no. V40), running at a maximum rpm of 1760. This UV laser was placed in front of the test section and, operating at 335 nm, gave an average maximum power of 330 mW, creating dark purple traces in the oil. However, the beam emitted by the laser exhibited a great divergence. To focus the beam and enhance the formation of a well-defined, thin trace, an optical lens with a focal length of 25 cm was placed at the aperture of the laser. In order
**Figure 3.3.** Nitrogen Laser, Mirror, and Lens Configuration (side view)

**Figure 3.4.** High Speed Video Equipment Configuration (plan view)
above the test section. A pictorial description of the laser and lens system is given in figure 3.3.

The traces formed by the laser pulses were captured through the use of a high speed CCD video camera unit (Photron HVC 11B). A 35-mm camera lens (Minolta MD) was used in conjunction with a 20-mm extension ring (Vivitar Automatic Extension Tube, model AT-5) and a Minolta C-mount adapter. The high-speed video camera was operated under the following conditions: shutter speed 1/500 s, frame rate 186 s⁻¹, and f-stop at 2.8 s.

Due to the nature of high-speed cameras a large amount of back-lighting was required, in order to obtain good images. In the present experiment, this was achieved through the use of a high intensity sodium lamp (General Electric Lucalox, model LU1000) operating at 1000W. To supply enough power to this light system, a high voltage transformer was required (Hammond Manufacturing Company (HMC), model no. K15 9M) in conjunction with a variable high frequency inverter (HMC, model no. VAR-5).

Small time segments of data were recorded by the high-speed video camera. These sets of data were transferred to an SVHS videocassette by means of a VCR (Panasonic, model no. AG-7355). The data were visually observed at the time of the experiments through the use of a video monitor (Panasonic, model no. CT-1400 MGC). The optical setup is given by figure 3.4.

### 3.4. Experimental Procedure and Conditions

Experiments were conducted to determine the hydrodynamic nature of liquid films falling on an inclined plate in a rectangular channel under the effect of gravity, using the photochromic dye activation method.

The flow system was first calibrated by measuring the volume of fluid collected, in the collection vessel, over predetermined time intervals and rotameter settings. The rotameter calibration curves may be seen in Appendix A.

Before data were collected the flow system was run for at least 30 minutes to allow for conditions to become stabilized at the operating temperature. This was determined through the use of a mercury thermometer (Canlab, model no. T2025) located in the upper plenum (figure 3.1).
rotameters, over a four-day period. Data were collected at five flow rates in the low flow rate range, and an additional six flow rates were examined in the higher flow rate range. Data were obtained in segments of 2.8 seconds, as determined by the operation of the high-speed video camera, and immediately transferred to a videocassette. The temperature in the upper plenum was also recorded for each run. A summary of experimental conditions is given in Table 3.1.

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<tr>
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<td>135.4</td>
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<td>50</td>
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<td>167.5</td>
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<td>50</td>
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<tr>
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<td>199.2</td>
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<td>50</td>
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<tr>
<td>11</td>
<td>5.45</td>
<td>227.1</td>
<td>3</td>
<td>50</td>
</tr>
</tbody>
</table>

The sampling frequency in Table 3.1 refers to the frequency at which the laser was pulsed. Low sampling frequencies at low flow rates were required such that each trace could be identified, as the separation distance between traces in this range was very small.

To account for minor changes in the camera and laser positions, caused by air movement and vibrations, a section of a ruler was recorded after each run. This would ensure that the proper scale be applied to the videocassette recordings when analyzed.

### 3.5. Data Reduction

A total of twenty video segments, two from most runs listed in Table 3.1, were randomly chosen for further data analysis. This was a process of extracting some physical, quantitative values from the traces recorded on the SVHS videocassettes. The traces were
Mocha 2.1 image analysis software. The frame grabber card enabled video image frames to be individually captured and digitized in the computer.

The Mocha 2.1 image analysis software allowed laser-generated traces to be recorded as vectors of data points, in Cartesian coordinates. Thus, a two-column matrix generated for each trace captured, detailing its shape, was saved in a coded file for future reference.

A computer program based on Matlab 4.2 was developed to calculate many of the hydrodynamic properties of the falling film, using the coded trace files as input data (Appendix B). Since the trace vectors were recorded as pixel values (corresponding to pixel location within the Mocha 2.1 software) it was necessary to convert these values to length dimensions. This was accomplished through a scale factor, determined from the video image of a ruler recorded on the tape. In addition, an aspect ratio was required to accurately define the length scale of the traces. This ratio was provided through the Mocha software, and verified by experimental measurements in both the horizontal and vertical directions.

The instantaneous film thickness was determined directly from the difference in Cartesian coordinates perpendicular to the flow, corrected by the scale factor, at the wall and the maximum height attained by the trace.

All traces were then fit with polynomials varying from second to fifth order, with the best fit being chosen to represent the trace. The instantaneous stream-wise velocity was obtained through the comparison of the positions of the same trace from two successive frames. The stream-wise distance traveled by the fluid defined a pair of trace coordinates, calculated at 0.005 mm intervals perpendicular to the wall. This stream-wise displacement of the trace, corrected by the scale factor, was then divided by the time interval between the frames, in this case 1/186 seconds, to obtain the instantaneous velocity profiles. In addition, the mean and maximum velocities were determined.

The instantaneous flow rate was calculated by integrating each velocity profile, from the wall to the interface, and multiplying this by the channel width. In addition, the Reynolds number, defined as

\[ Re = \frac{4 * Q}{\nu * W} \]  
(4.1)
The instantaneous wall shear stress was calculated by fitting the velocity profile data close to the wall (estimated to be within 0.2 mm from the wall) to a polynomial equation to obtain the velocity gradient in this region, and multiplying by the dynamic viscosity.

To perform comparisons with measured instantaneous wall shear stress and velocities the Nusselt (1916) equations were also evaluated based on the average instantaneous film thickness obtained between two frames.

The time-averaged results, obtained from the instantaneous data, were collected for easy reference through the use of a Fortran 77 program (Appendix B).

### 3.6. Sources of Error

Errors involved in the application of the PDA technique, pertaining to the present work, may be attributed to several causes. Random vibrations from the laboratory floor caused the slight movement of both the laser and the high speed camera. This error was minimized through the recording of a ruler segment following each run, such that it was deemed negligible.

The inclination of the channel, at 45° to the horizontal, was measured to an accuracy of 0.5°. This deviation of one-half of a degree could introduce small errors when comparing to Nusselt's (1916) predictions of film thickness, velocity and wall shear stress. The maximum error, occurring in the velocity calculations, due to inclination angle errors, was determined to be 0.5%.

Since the traces were not defined lines of a single pixel width, errors were introduced upon quantification of the trace. An approximation was made, using the Mocha software, by following the center-line of the trace from the point of contact with the wall to the interface. With the resolution used, 256 by 256 pixels, it was observed that the average width of a trace was about 10 pixels. Using the maximum scaling factor, this introduced an error of approximately 0.071 mm. Based on the time-averaged velocities calculated this resulted in a maximum error of 22.1%, at low flow rates, and a maximum error of 5.4% at higher flow rates. However, most instantaneous errors were estimated to be far less than these values, which represent maximum deviations.
approximately three pixels wide and the location of the wall could accurately be determined to within four pixels. These deviations exhibit a maximum error of 0.05 mm, or an error range of 5.4% to 2.2%, based on time-averaged measurements, from low to high Re flows.

Errors contributing to the wall shear stress included those incorporated in the velocity measurements and those concerning the measurement of location from the wall. Propagation of errors analysis indicates an error range of 11.8% to 5.85%, from low to high Re flows.

Additional errors were introduced through the reading of the rotameters, the thermometer, and the rulers used for scaling. The low flow rate rotameter was accurate to within 0.02 LPM, representing a maximum error of 5.83%, while the higher flow rate rotameter, accurate to within 0.2 LPM, represented a maximum error of 11.0%. The thermometer, calibrated in degrees Celsius, was accurate to one degree and errors incurred may be considered to have negligible effect on the density and viscosity of the silicon oil used for calculations. When digitized, the ruler markings were about three pixels thick. The scaled-width of the ruler markings represented a maximum error of 0.02 mm. This error was also considered negligible, relative to the errors encountered with the trace thickness.

Another source of error dealt with the energy imparted to the fluid system as the laser activated the TNSB dye. This energy would not all go towards the photochromic reaction, as some of it would be used locally to heat the working fluid, causing some convective dispersion of the trace. This convective motion of the fluid was noticed even in stagnant solutions of silicon oil and TNSB. However, it was noticed that this convective action in the fluid was quite small relative to the thickness of the trace. Therefore, when one considers the width of the trace formed, this error may also be considered negligible.

As with all scientific studies, there is an error involved in the computation and processing of data. In the present study, these errors may become evident in the goodness of fit of the regression lines modeling the traces. In all cases, the regression lines were fit such that a coefficient of determination of 0.95 (95%) was obtained and may be considered scientifically accurate. Due to some sampling inconsistencies, caused by the quality of the trace in certain locations, some error may have been incurred in the time averaging of the data. However, the frequency of these inconsistencies was so low any resulting errors may be considered negligible.
4.1. Qualitative Observations

The development of the wave structure could be observed by following the liquid as it flowed through the test section. The falling liquid film exhibited a smooth, flat gas-liquid interface upon immediate entrance to the test section. Approximately ten centimetres from the test section entrance small ripples appeared at the interface. The ripples were two dimensional in nature at this location. Tailby and Portalski (1962) observed a similar ripple structure in a vertical falling film on a plate. At approximately 40 cm from the inlet, the ripples had proliferated and developed a wavy structure, while maintaining their two-dimensional, roughly sinusoidal, character.

By the time the waves reached a location of 50-60 cm from the test section inlet the two-dimensional structure of the waves began to break down, resulting in a three-dimensional gas-liquid interface characterized by large waves. In between these larger wave structures, smaller waves of saw-toothed shape could be observed. This type of developed wave structure has previously been reported in numerous publications, including Dukler (1977) and Takahama and Kato (1980). The falling film appeared to become fully developed at about 1.0 m from the test section entrance, as seen by the complex, but consistent, wave development observed at this location, as well as the relatively little change to the transient wave structures exhibited at a location 1.3 m downstream of the entrance (where all measurements were recorded). In general the degree of waviness increased with an increase in flow rate. Similarly, Dukler and Bergelin (1952) reported an increase in the waviness of a gas-liquid interface as the Reynolds number was increased from $Re = 482-2770$. Typical video recordings of substrate and wave regions are presented in figure 4.0.

4.2. Film Thickness Measurements

The time variation of the falling film thickness, at 20 different flow rates, was recorded and digitally analyzed. Figure 4.1 indicates the film thickness variations for the freely falling films at different Reynolds numbers, as well as the time averaged mean value of the film thickness for each run. Clearly, this figure depicts several general trends in the time-dependent liquid film thickness variations: (a) the increase in the wavy structure, or
Figure 4.0a. Typical Sequential Recordings of Wave Structures, Re=24 (f5r3)

Figure 4.0b. Typical Sequential Recordings of Substrate Regions, Re=122 (f72r1)
to the mean film thickness, increased with the Reynolds number, and (c) the increase in flow rate also appeared to increase the mean film thickness. Several other researchers have published similar results concerning the time-dependent thickness variation of falling liquid films, including Chu and Dukler (1975), Dukler (1976), Brauner and Maron (1981, 1983) Wasden and Dukler (1989, 1992), Karapantsios et al. (1989), and Karapantsios and Karabelas (1990).

It is interesting to note that in the present study, the local film thickness to average film thickness ratio never appeared to exceed a value of 2.5, as evident in figure 4.1. This result compares well with the values reported by Nakoryakov et al. (1976), ranging from 1.1 to 3.0, and a maximum film thickness to average film thickness ratio of about 3.0 reported by Karapantsios et al. (1989). The present experiment, and those of Nakoryakov (1976), and Karapantisos et al. (1989), are in general agreement with the results of other researchers, such as Chu and Dukler (1974) who reported a maximum film thickness to average film thickness ratio of approximately 3.6.

The random, stochastic nature of the film thickness variations, clearly evident in figure 4.1, lends itself well to statistical time series analysis. The following sections will consider the mean, the probability density function, and the standard deviation of the film thickness.

4.2.1. Average Film Thickness

The average, or mean, film thickness for each of the runs was calculated by sampling 60 evenly spaced data points for each of the 20 runs analyzed. The time series average film thickness is defined as

$$\tilde{\delta} = \frac{\sum_{i=1}^{N} \delta_i}{N}$$

(4.1)

The mean film thickness as a function of Reynolds number has been presented (Figure 4.2). A non-linear increase in mean film thickness is noted which follows the general trend first proposed by Nusselt's force balance model (1916) based on a smooth laminar film, whose
Figure 4.1. Typical Local Film Thickness Fluctuations.
Nusselt (1916) found the values of the constants to be \(a=1.102\) and \(b=0.333\) for a film flowing over a flat plate with a 45° inclination. The best fit to equation 4.2 using the current experimental data reveals constants of \(a=0.991\) and \(b=0.328\), resulting in a correlation coefficient \((r^2)\) of 96.77% (follows approximately the same line as that of Salazar and Marchall, 1978). The maximum deviation of the present experimental data from the Nusselt

\[
\delta = a^* \left( \frac{\nu^2}{g} \right)^{1/(1/3)} \left( \frac{\Gamma}{\mu} \right)^b
\]  

(4.2)

![Figure 4.2. Time Averaged Film Thickness as a function of Reynolds Number](image)
the Nusselt (1916) theoretical prediction.

Similar trends have been noted by other investigators. Dukler and Bergelin (1952), Ho and Hummel (1970), Cook and Clark (1971) all reported experimental film thickness measurements that agree well with those of Nusselt’s (1916) theory in the laminar regime. However, Karapantsios et al. (1989) found that Nusselt’s (1916) theory underpredicts the mean film thickness. Karapantsios and Karabelas (1995) found also that the mean film thickness increases non-linearly in both the laminar (as seen in the present work) and turbulent regimes.

Comparisons of the mean film thickness data with other theoretical predictions and empirical correlations are also presented in figure 4.2. The Kapitsa (1965) theoretical model, attempting to account for regular periodic waves in a falling laminar film, underpredicts both the present experimental film thickness data and Nusselt’s (1916) model by 22-25%. Another theoretical prediction, derived from the universal velocity distribution (Dukler and Bergelin, 1952), seriously underpredicts all models presented at low Reynolds numbers (Portalski, 1963). However, the data obtained by Portalski (1963) appear to support the reduced mean film thickness predicted by the Dukler and Bergelin (1952) model. Salazar and Marschall (1978) presented an empirical model for estimating the average film thickness by correlating their data in terms of the Reynolds number, disregarding any other physical parameters previously used. A similar fit is presented in figure 4.2. Experimental film thickness measurements were conducted by Karimi and Kawaji (1996) and the data fitted to equation 4.2, resulting in the constants a=0.2325 and b=0.5. Although they report good agreement to Nusselt’s (1916) values in the laminar film regime for Re > 1000, at very low Reynolds numbers the fit appears to deviate.

### 4.2.2 Maximum and Minimum Film Thickness

The maximum, minimum and mean film thickness variations with Reynolds number are presented in figure 4.3. The values for the maximum and minimum film thickness represent the absolute maximum and minimum for each run examined. Since the maximum and minimum film thickness data are randomly fluctuating data as a function of time, the
probability of occurrence of 5% and 95% display the same general trends as in figure 4.3.

Figure 4.3. Maximum, Minimum and Mean Film Thickness Variations with Re.

As with the mean film thickness, the maximum film thickness tends to increase with an increase in the Reynolds number. However, the rate of increase of the maximum film thickness with the Reynolds number is greater than that of the mean data. Similar results were reported by Karapantsios et al. (1989). In addition, the minimum film thickness remains constant up to a Reynolds number of about 25. Above this value the minimum film thickness also increased as a function of Reynolds number, only to a much lesser extent compared to the mean and the maximum film thickness values. Karapantsios et al. (1989) found that the minimum film thickness remained constant until the flow conditions reached a Reynolds number of 5000.
the ratio of mean film thickness to maximum film thickness as a function of Reynolds number (figure 4.4).

![Graph showing the ratio of mean to maximum film thickness as a function of Reynolds number.]

**Figure 4.4. Ratio of Mean to Maximum Film Thickness.**

At low Reynolds numbers figure 4.4 exhibits large values and a strong negative slope, indicative of the lack of large wave structures in this region. Above a Reynolds number of 50 this ratio appears to level off and reach a minimum value of about 0.35, noting that some scatter exists. This may be accounted for when one considers the strong effect that large waves may have on the mean film thickness (Karapantsios et al., 1989).

### 4.2.3. Probability Density Function (PDF)

The probability density function was introduced in this study with the aim of defining the substrate film thickness in a consistent, statistical manner. The interval chosen for the PDF analysis was $\Delta \delta = 0.1$ mm. However, due to the inherent errors that may exist when an interval is chosen, several intervals were initially considered ranging from 0.4 to 0.15 mm.
The definition of the PDF is as follows:

\[ F_\delta = \text{Prob}[\delta(t) < \delta] \quad (4.3a) \]

\[ \text{PDF} = \frac{dF_\delta(\delta)}{d\delta} \quad (4.3b) \]

From equations 4.3a and 4.3b it is evident that the probability density function of the film thickness describes the likelihood of the film assuming a thickness within the interval stated.

Figure 4.5 depicts PDF’s of the instantaneous film thickness for four runs. Similar results were obtained for all runs. Some general trends may be noted from this figure: (a) as the Reynolds number increased, the substrate film thickness (the most probable film thickness, indicated by a peak in the PDF plot) tends to increase, (b) the degree of right-directed skewness increases as the flow is increased, and (c) the peak value of the probability density function decreases with increasing Reynolds number. The increase in skewness and the decrease in PDF peak value with Reynolds number may be attributed to greater waviness of the film. As the waviness is enhanced, the probability that the film thickness will fall within a particular interval is decreased and the skewness is increased to balance out the PDF summation. Similar results were found by Chu and Dukler (1974), Wasden and Dukler (1992), Mudawar and Houpt (1993), and Karapantsios et al. (1989).

The substrate (PDF) film thickness remained at a relatively constant value of 0.85-0.95 mm in the range of Re=11-26. Above this Reynolds number the substrate thickness increased slightly (similar to the trend noted for the minimum film thickness in figure 4.3), at a reduced rate, however, compared to the increase in mean film thickness (as evident in figure 4.3). It is interesting to note that the deviation between the mean film thickness and the substrate film thickness increases as the Reynolds number is increased. Mudawar and Houpt (1993) reported similar findings. This may suggest that the film substrate assumes a defined thickness and the waves and wavy structure act on top of the substrate. Plots of PDF analysis for all runs are presented in figure 4.6. It is clearly evident that the right directed
number flows indicates the lack of wavy structure in this range, as seen in figure 4.1.

Figure 4.5. Typical Local Film Thickness Probability Density Function Plots.
Figure 4.6. PDF Plots of Local Film Thickness a) Low Re  b) high Re.
The standard deviation of the instantaneous film thickness represents a means to evaluate the fluctuations around the mean film thickness. In this regard it can also represent the waviness of the gas-liquid interface. This quantity is the positive square root of the second central moment and may be given as

\[ s = \left( \lim_{T \to \infty} \frac{1}{T} \int_{T} \left[ (\delta(t) - \bar{\delta})^2 \right] dt \right)^{1/2} \]  

(4.4)

The standard deviation of the instantaneous film thickness as a function of Reynolds number is presented in figure 4.7.

![Figure 4.7. Variation of Standard Deviation with Reynolds Number.](image)

As the Reynolds number was increased, the standard deviation of the local film thickness also increased in an asymptotic manner. This indicates that the waviness is
suggesting that the wavy film structure is becoming fully developed. Significant amounts of scatter are present at high Reynolds numbers. Karapantsios et al. (1989) found similar trends except that the actual values of standard deviation appear to be lower than those found in the present study, although the degree of scatter appears to be consistent with this experiment.

To evaluate the relative degree of variation in the instantaneous film thickness data the ratio of standard deviation to average film thickness, also known as the ‘coefficient of variation’, was plotted as a function of Reynolds number (figure 4.8).

![Figure 4.8. Effect of Reynolds Number on the Coefficient of Variation.](image)

It is clear from figure 4.8 that at low Reynolds numbers the flow takes on a more uniform, less wavy profile. As the flow rate is increased the wavy structure increases, evident from the increase in the coefficient of variation, reaching a maximum around a Reynolds number of 50, after which point the coefficient of variation appears to decrease slightly in a linear manner. This may be an indication that the flow structure has already reached its maximum limit for growth or amplification of waves (Karapantsios et al., 1989).
It has previously been reported that the wavy structure of falling films enhances heat and mass transfer across the gas-liquid interface (Dukler, 1977). Therefore, it may be of some interest to investigate the frequency with which these waves occur. In particular, since large waves may carry a large portion of the liquid mass (Mudawar and Houpt, 1993), the frequency of large waves only will be considered here. Due to a lack of data points required for a power spectral density function, the wave frequency was determined by visual observation of the instantaneous local film thickness traces, such as those of figure 4.1. A wave was considered large if the local film thickness to substrate film thickness ratio was 1.2 or greater.

Figure 4.9 indicates that the wave frequency increases as a function of Reynolds number. The frequency of large waves was very low at low liquid flow rates, as should be expected. Dukler and Bergelin (1952) reported that waves of any form did not appear until a Reynolds number of 25 had been reached. However, Wilkes and Nederman (1962) indicate that waves were present at a Reynolds number as low as one. It is apparent in the present investigation that large waves occurred at Reynolds numbers as low as 11, which falls in the middle of the range defined by the two experiments mentioned above. The maximum frequency of large waves of about 7 Hz occurred at the peak Reynolds number of the present study (Re=220).
Figure 4.9. Frequency of Large Waves.
Experiments were run at 20 different flow rates in order to study the film thickness, flow rate, velocity profiles, and wall shear stress characteristics of falling liquid films. The time-averaged results of this investigation, at 22°C ± 0.5°C, are summarized in Table 4.1.

### Table 4.1. Summary of Time-Averaged Flow Characteristics

<table>
<thead>
<tr>
<th>Run No.</th>
<th>$Q_{cal}$ (LPM)</th>
<th>$Re_{cal}$</th>
<th>$\delta$ (mm)</th>
<th>$Q_{ave}$ (LPM)</th>
<th>$U_{ave}$ (m/s)</th>
<th>$U_{max}$ (m/s)</th>
<th>$Re_{ave}$</th>
<th>$\tau_W$ (Pa)</th>
<th>$\rho\delta\cos(\theta)$ (Pa)</th>
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</tbody>
</table>

The second and third columns of Table 4.1 are the flow rate and Reynolds number obtained from the calibrated flow meter used in the flow loop (Appendix A). The data in column five, $Q_{ave}$, represents the time-averaged flow rate, with the corresponding average Reynolds number ($Re_{ave}$) in column eight, obtained by integrating the velocity profiles calculated using a Matlab 4.2 program (Appendix B). It is important to note that the volumetric flow rate
rotameters (Figure 4.10). This was also evident through a reduction in the time-averaged Re calculated from the instantaneous data compared to the Re obtained from the rotameters.

![Figure 4.10. Deviation of Calculated Average Flow Rate from Calibrated Flow Rate.](image)

It is interesting to note that an increase in flow rate at the center of the channel, relative to that obtained from the rotameters, should be observed considering the parabolic velocity profile that would be expected in the transverse direction. However, as mentioned above, the present data indicate a flow reduction in the span-wise center of the plate. This minor flow reduction may be attributed to the side wall effects (due to the limited width of the channel) causing minor increases in film thickness at the walls and small reductions in
wide due to the PDA technique employed, which requires extensive and powerful back lighting to enhance the quality of the trace images recorded by the high speed video camera. Since the film thickness data compared well with that of Nusselt (1916), it is apparent that the width of the channel had little or no effect on the falling film structure or hydrodynamic characteristics of the system, in spite of this very minor reduction in flow.

The experimental time-averaged, mean and maximum velocities, shown in Table 4.1, exhibited minor deviations from those as predicted by Nusselt (1916), to the order of 11% to

Figure 4.11. Time-Averaged Mean and Maximum Velocities.
least squares regression analysis, with resulting correlation coefficients of 97.7% and 98.3% respectively. In Nusselt’s theory the mean and maximum velocities are dependent on film thickness to the second power. An analysis of substrate data suggests that Nusselt’s (1916) equations fit quite well in this region, whereas the larger deviations occur in the wavy films. These time-averaged quantities are over estimated by Nusselt (1916). It may be assumed that the dependence of velocity and flow rate on film thickness alone is not as strong as Nusselt (1916) suggests and that the transient nature of the wavy interface in falling films accounts for much of this discrepancy. A more detailed analysis of the velocity profiles will be discussed in a following section, particularly the structure of these profiles and their relation to film thickness.

The time-averaged wall shear stress values calculated from the experimental velocity gradients agree well with the Nusselt (1916) predictions for a smooth, laminar film, which exhibits a linear dependence on average film thickness (Table 4.1). In the majority of experimental runs, the calculated wall shear stress was slightly greater, by a maximum of 4.3%, than what Nusselt (1916) expected, whereas in a few runs Nusselt’s (1916) predictions over estimated the time-averaged wall shear stress, by a maximum of 6.03%. The largest positive deviation from the Nusselt (1916) theory appear to occur at lower Reynolds numbers, while the largest negative deviation occurred at higher Reynolds numbers. These minor variations from Nusselt’s (1916) theory may be explained by the fact that Nusselt’s (1916) work was based on a steady state force balance where the gas-liquid interface was assumed to be a smooth, flat surface lacking wave structures. Therefore, these deviations may be due to the transient effects imparted by the wavy interface.

Next, the instantaneous fluctuations of the parameters mentioned above are presented in figures 4.12-4.15.

The time-averaged values of the parameters examined in figures 4.12-4.15, which are typical representative plots of all data collected, are indicated by the dashed lines. All experimental runs were carefully examined in the above manner in order to compare
Figure 4.12 Instantaneous Hydrodynamic Properties at Re=15 (f2r4).
Figure 4.13 Instantaneous Hydrodynamic Properties at Re=71 (f52r1).
Figure 4.14. Instantaneous Hydrodynamic Properties at Re=122 (f72r1).
Figure 4.15. Instantaneous Hydrodynamic Properties at Re=202 (f10r1).
Some interesting trends were noted:

(a) The film thickness was always found to decrease dramatically before a large wave. This decrease was less for smaller waves.

(b) At low Reynolds numbers all hydrodynamic parameters were found to be less chaotic than those at high Reynolds numbers, with a majority of values closer to the time-averaged mean.

(c) The instantaneous volumetric flow rate, average velocity and maximum velocity appeared to follow the local fluctuations in film thickness quite closely. It did not, however, appear that the variations followed any proportional relationship with the film thickness.

(d) The instantaneous wall shear stress exhibited a lesser tendency (relative to other parameters) to follow the variations in the local film thickness variations, although in many cases it seemed to follow these fluctuations rather closely. In addition, in lower Reynolds number flows, the wall shear stress appears to oscillate around the time-averaged value in extended regions of substrate, or non-wavy, regions.

A more detailed analysis of many of the instantaneous hydrodynamic parameters mentioned above will be presented in the following sections.

4.3.1. The Nature of Laminar Velocity Profiles

According to Nusselt (1916) the laminar velocity profiles for a freely falling film can be described by parabolic equations. Through an analysis of these equations, based on a flat interface and steady state system, it may be shown that the ratio of maximum-to-average velocity in the laminar regime is given by the following:

\[
\frac{U_{\text{max}}}{U_{\text{ave}}} = \frac{3}{2}
\]  

(4.5)
examine this velocity ratio under transient, wavy conditions to evaluate the nature of the velocity profiles in order to determine whether this parabolic structure predominates. Some typical instantaneous results are presented in figure 4.16 over the range of flow conditions used in the current study.

It is apparent from this figure that the local velocity ratio oscillates around the steady state, parabolic velocity ratio value of 1.5, represented by the dashed line in each plot (note: the solid straight line represents the time-averaged mean of the experimental data). The degree of oscillation does not change appreciably with increases in Reynolds number, and the common upper and lower limits, with a few exceptions, are approximately 1.6 and 1.35, respectively, for most cases examined. It is important to note that no significant trend with regards to film thickness (velocity ratio peaks in wave, or non-wave, regions etc.) was observed with this velocity ratio.

This fluctuation around the parabolic velocity ratio may be accounted for by considering the transient effects of the wavy interface. Although the falling film studied in this experiment was laminar, the waves caused considerable variations in the local velocity profiles. In the wave front, it is reasonable to assume that higher velocity liquid travelling in the wave will have some effect on the preceding substrate, possibly causing acceleration of the fluid. Similarly, in the wave back region the presence of a wave, decelerating by this point, may also play a role in accelerating the liquid in the substrate following the wave. In regions of prolonged substrate, as noted in section 4.3, Nusselt's (1916) estimates of average and maximum velocity were most accurate, however, large deviations from the parabolic velocity profiles were still observed.

The average velocity ratio for all data analyzed indicates that the Nusselt (1916) parabolic velocity profile holds true when considering the time-averaged velocities (figure 4.17). The range of time-averaged velocity ratio in the present experiment into which most data fell was 1.42 to 1.56, representing a maximum deviation from Nusselt’s (1916) prediction of 5.6%. It may be of some interest to note that in the present study, instantaneous values for velocity ratio as high as 2.0 were obtained although the average
Figure 4.16. Instantaneous Fluctuations of Velocity Ratio.
it may be said that the velocity profiles calculated in this study are generally parabolic in nature.

Other researchers have studied this velocity ratio to evaluate the parabolic nature of the velocity profile. Portalski (1964), who studied water-glycerine solutions falling down a vertical flat plate, found the velocity ratio, based on time-averaged velocity profile measurements, to be very nearly equal to 1.5 throughout the laminar range. Significant deviations from the parabolic velocity ratio value, in the laminar flow regime of current interest, were reported by Koziol et al. (1981). They presented time-averaged velocity ratio values as high as 2.2, with a curve fitted value of approximately 1.93 at a Reynolds number of 80.

Figure 4.17. Time-Averaged Velocity Ratio
attempt to determine if a parabolic curve provides the best fit. Wilkes and Nederman (1962) found that the velocity profiles were of a parabolic nature in the laminar range. However, a significant amount of scatter around the fitted curve was evident. This is in agreement with the present research, and is most likely due to the transient nature of the wave structure. Parabolic velocity profiles, in the laminar regime, were confirmed by several other studies, including Ho and Hummel (1970), Cook and Clark (1971), Ueda and Tanaka (1974), Nakoryakov et al. (1978), and Mudawar and Houpt (1993). All the above papers mention a maximum deviation from the parabolic profile of about 15%, occurring at the interface region. As well, in several of these studies, the parabolic fit was improved upon the addition of a surface-active agent, which suppressed the wave formation. These results lend support to the idea that the wavy profile alters the parabolic nature of the velocity profiles, as predicted by Nusselt (1916). Wasden and Dukler (1989) found that velocity profiles obtained at a Reynolds number of 880 were fit best to a cubic curve. It must be noted here, however, that all previous research pertaining to this topic, with the exception of Alekseenko et al. (1985), was based on time-averaged velocity profiles. Alekseenko et al. (1985) obtained instantaneous velocity profile and film thickness data. They reported a maximum velocity profile deviation of 15% from a parabolic curve, over a Reynolds number range of 20-120. In addition, much of the scatter in the velocity profiles was found in substrate region surrounding the waves.

It appears that significant fluctuations in the velocity profile may have also occurred in the substrate regions, presumably just preceding and immediately following a wave, in the present work. To examine this idea a detailed analysis of the velocity ratio was conducted in terms of a dimensionless film thickness to evaluate the effect of the film thickness on the parabolic nature of the velocity profile. The dimensionless film thickness parameter utilized was the local film thickness divided by the average film thickness ($\delta/\delta_{ave}$). This parameter was chosen to reflect the waviness of the gas-liquid interface, while disregarding small changes in the interface that would become evident if the local film thickness-to-most probable substrate film thickness ratio, $\delta/\delta_{sub}$, were used. In effect, this parameter is believed
surrounding the waves.

Figure 4.18. Effect of Waves on Velocity Profiles.

An initial investigation of figure 4.18 appears to indicate little as to the effect of large waves on the velocity profiles of near wave substrate regions, since the degree of scatter is roughly consistent along the entire film ratio range. This indicates that instantaneous velocities deviate from the parabolic form in all regions of a wavy laminar film regardless of local position. However, a trend may be elucidated through an examination of a subset of the data (figure 4.19). It is much more evident from this figure (figure 4.19) that the peak scatter occurs at the dimensionless film thickness ratio of approximately one, or slightly less than one, with decreasing amounts of scatter in either direction from this point. This is an indication that the waves have a definite effect on the velocity profiles of substrate regions immediately surrounding the wave structures, causing them to deviate from the parabolic form. At low values on the abscissa, which may be considered to contain large amounts of data for the substrate regions, the scatter is much reduced and the data falls around a value of 1.5, as was previously indicated. However, this is only a preliminary analysis and this result
collected the degree of scatter may become similar to that of figure 4.18, and no trend could be observed.

Figure 4.19. Effect of Waves on Velocity Ratio at Low Reynolds Number.

The scatter at a dimensionless film thickness greater than 1.5 (wavy regions) in figure 4.18 may indicate some degree of turbulence, or minor mixing motions in wave peaks at the higher Reynolds numbers studied. Similar results have been reported by Ueda and Tanaka (1975) in flows with a Reynolds number as low as 180, noting the presence of velocity fluctuations at the gas-liquid interface. Previous evidence of this observation may also be obtained from Wilkes and Nedderman (1962), Ho and Hummel (1970), and Mudawar and Houpt (1993) whose time-averaged velocity profiles show larger amounts of variation at the interface as the film thickness increases. However, the instantaneous data of Alekseenko et al. (1985) appear to contradict this view, as they indicate the velocity profiles within regions of maximum film thickness do not fluctuate significantly.
In the majority of previous work the experimental techniques only allowed for the calculation of time-averaged velocity profiles. These studies were thereby limited in that the time dependent hydrodynamic effects were averaged out, providing little or no information on the local effects of waves in wavy falling films. In addition, the previous work concerned with instantaneous velocity profiles only discussed a very limited range of data. The interesting results obtained in the previous sections demand that the instantaneous velocity profiles calculated in the present study be examined more closely. By doing so, the transient effects of the film thickness fluctuations on local velocity profiles may be better understood, providing further insights into the hydrodynamics of falling films.

The local velocity profiles were calculated for all experimental runs and approximately 60 to 80 profiles were obtained per run. The number of velocity profiles obtained was a result of different laser pulsing frequencies used. At low flow rates lower frequencies were utilized to enhance the resolution of the traces formed, while at higher flow rates a higher sampling frequency was required to record the increased transient nature of the film. Typical plots are presented in figure 4.20. Each plot in figure 4.20 contains all velocity profiles calculated for the designated run. The scales for these plots are different in order to effectively examine each set of profiles. Some interesting observations can be made based on careful examination of all experimental velocity profiles, such as those given in figure 4.20:

(a) The substrate film regions, evident by the relatively small maximum distance from the wall, were associated with velocity profiles that achieved relatively low maximum and average velocities (i.e., on the left side of each plot). The wavy regions of the falling film, presumably at relatively large distances from the wall, gave resultant velocity profiles characterized by relatively large maximum and average velocities (i.e., on the right side of each plot).

(b) At low Reynolds numbers, relatively few waves were present, as seen by the small number of velocity profiles reaching large distances from the wall with high velocities,
n - number of instantaneous velocity profiles obtained in each run

Figure 4.20. Typical Instantaneous Velocity Profiles.
measurements indicated in a previous section. In addition, at low flow rates, the film can be separated into two distinct regions, the substrate and wave regions, with more scatter present in the substrate region. This observation agrees with that made by Alekseenko et al. (1985) who found, at a Reynolds number of 50, that the velocity fluctuations in thin film regions were greater than those in the thick film regions (waves). In other words, the substrate film surrounding a wave structure was influenced by the presence of the wave, causing fluctuations of the velocity profiles. This may be attributed to the transient effects imparted by the waves on the flow structure. In the present data, the fluctuations in the velocity profile in the wave region appear to be more significant than those reported by Alekseenko et al. (1985), possibly indicating local transient effects occurring within the waves.

(c) As the Reynolds number was increased, an increase in the number of thick film regions (presumably waves) was noticeable, due to the extra data evident at large distances from the wall at high velocities. Consequently, the degree of velocity profile fluctuations, due to reasons mentioned above, increased in both the substrate and wavy regions. Above a Reynolds number of 69 the boundary between the substrate and wavy regions became less clear, and the two regions were no longer distinct.

(d) At the higher Reynolds numbers, the maximum velocity was not always present at the gas-liquid interface, primarily in the wavy regions. This indicates that some interfacial shear stress was present, violating Nusselt’s (1916) assumption of zero shear at this point. Another explanation for this phenomenon is that some mixing within the wavy regions was beginning to occur as the flow became increasingly transient and unsteady.

It is evident that interfacial waves appear to play a significant role in the hydrodynamics of falling films on an inclined plate, including their obvious effect on the velocity profiles studied in the present work. In this regard, it is of great interest to study individual velocity profiles directly with the fluctuating gas-liquid interface to further evaluate the effect of waves on the hydrodynamics, particularly the velocity profiles, of laminar falling films. To this end, a series of figures, representative of typical results, are
Figure 4.21. Instantaneous Velocity Profile Variation with Film Thickness, Re=18 (f3r2)
Figure 4.22. Instantaneous Velocity Profile Variation with Film Thickness, Re=26 (f5r4)
Figure 4.23. Instantaneous Velocity Profile Variation with Film Thickness, Re=95 (f62r2)
Figure 4.24. Instantaneous Velocity Profile Variation with Film Thickness, Re=156 (f82r3)
Figure 4.25. Instantaneous Velocity Profile Variation with Film Thickness, Re=202 (f10r1)
Figure 4.26. Instantaneous Velocity Profile Variation with Film Thickness, Re=220 (f10r3)
presented depicting small time-wise portions (in the order of 0.43s) of the film thickness fluctuations and the associated velocity profiles (figures 4.21-4.26). It is important to note that, due to the differences in average velocities obtained over the range of the present experiments, the scales of the plots are different to allow for effective visualization.

Careful examination of figures 4.21-4.26, and several others of similar nature, have revealed some interesting observations:

(a) In all cases, the velocity profiles exhibited larger gradients, with greater velocities reached under waves, relative to the substrate regions, confirming observations from figure 4.20.

(b) Although the time-averaged mean and maximum velocities were over predicted by Nusselt’s (1916) theory (figure 4.11), the velocity profiles are seen to fluctuate around Nusselt’s (1916) prediction. However, the majority of velocity profiles, in both substrate and wave regions, were over predicted by this theory.

(c) The velocity profiles in extended non-wavy, or substrate, regions followed Nusselt’s (1916) predictions quite closely, with typical deviations of less than 15%. The velocity profiles obtained by Alekseenko et al. (1985) were slightly under predicted by Nusselt (1916), which appears to conflict with the present results. The relatively small degree of fluctuation reported by Alekseenko et al. (1985) agrees well with results of the present experiment.

(d) Significant over predictions of velocity profiles, by Nusselt (1916), were observed in the large wave peaks, with deviations of the order of 100% in many instances. These severe deviations may account for some of the over prediction of time-averaged mean and maximum velocities (figure 4.11). This observation appears to conflict with data reported by Alekseenko et al. (1985), who cite that velocity profiles in wave regions compare very well with a self-similar parabolic profile, with little deviation. They did not, however, compare their velocity profiles in large waves to Nusselt’s (1916) theory.
In regions immediately preceding and following waves (wave-front and wave-back regions, respectively), the velocity profiles were observed to give better agreement with Nusselt's (1916) theory than in waves. In addition, these profiles appeared to deviate from the theoretical prediction to a greater extent than those obtained in the extended substrate regions far away from the waves. In most cases, the velocity profiles in the wave-front and wave-back regions were over predicted by Nusselt (1916), to a maximum value of 25%. The degree of fluctuations in these velocity profiles agrees well with data given by Alekseenko et al. (1985). The velocity profiles they calculated, however, were slightly under predicted by Nusselt (1916), which appears to conflict with the present work.

The observations from figures 4.21-4.26 are consistent with the arguments already presented in the present work concerning the transient effect of waves on velocity profiles.

### 4.3.3. Non-Dimensional Velocity Distribution

The previous section concerning velocity profiles dealt primarily with transient effects closer to the interface than the wall. It would be interesting, therefore, to examine the present data with the aim of investigating the transient effects of waves on velocity profiles close to the wall, through non-dimensional parameters. Also, some models will be examined for goodness of fit to the present data.

Although the universal velocity distribution was derived from, and developed for, fully developed turbulent pipe flow, several falling film investigations have compared their data to this so-called law of the wall, and data has previously been reported concerning laminar falling films. The universal velocity distribution, in a convenient dimensionless manner, gives indications as to the nature of the velocity profiles in the laminar sublayer and transition zone of film flow.

Non-dimensional velocity distributions were calculated for all velocity profiles obtained over the full range of the present experiment. In these calculations, the time-averaged wall shear stress was utilized in the calculation of the non-dimensional velocity
Figure 4.27. Typical Non-dimensional Velocity Distributions at Low Re.
Figure 4.28. Typical Non-dimensional Velocity Distributions at High Re.
was to provide a means for accurate comparison to the work of previous investigations, which could only use a time-averaged value for this parameter, and for ease in the calculations of averages for analysis in the present work. However, in order to specifically address this issue, the non-dimensional velocity distributions were calculated using the instantaneous wall shear stress values, and were found to agree extremely favourably with the distributions calculated with the time-averaged wall shear stress value. Therefore, it was determined that the use of the time-averaged wall shear stress value was acceptable. Typical non-dimensional velocity distributions (individual curves) and the mean non-dimensional velocity profiles (solid circles) are presented in figures 4.27 and 4.28, along with the universal velocity distribution.

Several interesting features are obvious upon careful examination of these figures. The point of deviation of the data from the universal distribution appears to increase as the Reynolds number is increased, ranging from $y^+ = 0.35$ (Re=11) to approximately $y^+ = 1.0$ (Re=220). This is an indication that the waves have a decreased effect on the laminar sublayer (typically below $y^+ = 5$) as the Reynolds number increases. This is in contradiction to what may be initially expected. However, if one considers the general increase in the average (figure 4.2), the minimum (figure 4.3), and substrate (most probable) film thickness as the flow rate is increased a reasonable explanation may be given. The increase in these film thickness parameters, particularly the substrate film thickness, may effectively dampen the effects of the waves closer to the wall at higher flow rates, causing the average velocity distribution for each run to follow the universal law of the wall further from the wall. In all the distributions examined, the average experimental distribution deviated significantly below the law of the wall for $y^+ < 1$, indicating a reduction in the dimensionless velocity (and actual velocities) in the laminar sublayer and buffer regions. This indicates that the waves have significant effects on the velocity profiles in this region, as suggested in previous sections. The degree of scatter of these distributions for $y^+ > 1$ is indicative of the velocity fluctuations imparted on the falling film due to its transient nature. It should also be noted that some degree of fluctuation was observed at $y^+ < 1$ as well, suggesting that velocity profile fluctuations are significant in this region, similar to observations of velocity fluctuations in turbulent flows.
Figure 4.29. Time-Averaged Non-Dimensional Velocity Distributions.
from the average of all experimental runs (figure 4.29). The average value was calculated by fitting each experimental run to a fourth order polynomial equation, with a correlation coefficient range from 95.12% to 99.80%, and then plotting these equations as functions of dimensionless distance from the wall (figure 4.29). Some average values were not reported in figure 4.29 at approximately $y^+ = 15$, since the polynomials did not give a good fit in this region. The average experimental distribution curve began to deviate from the law of the wall at a dimensionless velocity of approximately 0.7, indicating that the waves had a significant effect close to the wall, well within the laminar sublayer region, and that the velocity distribution was not linear in this region. In addition, at $y^+ > 1.0$, it was evident that large fluctuations of the distribution were present, presumably due to the transient nature of the waves. Significantly reduced fluctuations were observed at locations close to the wall, indicating that the average velocity profiles attain some degree of conformity in this region.

Ho and Hummel (1970), who used a similar PDA technique with a vertical annular flow apparatus, obtained time-averaged non-dimensional velocity distributions that fit quite closely with the law of the wall in the laminar sublayer and buffer regions, to a minimum value of $y^+ = 1$. Their non-dimensional velocity distributions were reported for low Reynolds number flows down to $Re = 124$. They fitted their data to a single curve of the form

$$y^+ = u^+ + c(e^{d^+} - 1)$$

where $c=0.111$ and $d=0.281$. Equation 4.6 was fit to the present data with constants of $c=0.976$ and $d=0.415$, suggesting that the general form of this equation is quite reasonable. Possible discrepancies between the present results and Ho and Hummel (1970) may be attributed to a difference in the film geometry. The present film, in a rectangular test section of narrow width, may be subjected to wall effects from the side walls, which are absent in an annular film. In addition, Ho and Hummel (1970) utilized a surfactant to suppress the waves in the falling film. Therefore, the lack of a transient wavy interface is sufficient to cause the discrepancy with the present work.

Ueda and Tanaka (1975), studying falling films on an inclined rectangular channel, found that the law of the wall predicted their non-dimensional velocity distributions well in
little effect on velocity distributions close to the wall. Discrepancies with the present work may be attributed to the range of Reynolds numbers examined. It appears that in the turbulent regime the law of the wall describes the velocity profiles for inclined falling films in a rectangular geometry quite well.

However, Koziol et al. (1981) argued that the velocity distribution within the laminar sublayer should be parabolic in nature. To account for the parabolic nature of velocity profiles in the laminar sublayer and buffer regions of non-dimensional velocity distributions, Koziol et al. (1981) proposed modifications to the law of the wall. As indicated in figure 4.29, these corrections do not account for the deviations observed above \( y^+ = 0.8 \) in the present work. Therefore, some of the discrepancy from law of the wall must be attributed to the transient nature of the gas-liquid interface in wavy falling films, as has been suggested previously in this work.

The velocity distribution data reported by Pun et al. (1994), obtained using PDA in an annular flow test section at Reynolds numbers as low as 100, agreed with the law of the wall from values of \( y^+ = 1 \) to 8, in conflict with the present results. This discrepancy may be explained due to the differences in test section geometry, as discussed above.

Lorencez et al. (1991), using PDA on a horizontal rectangular duct, found that the law of the wall significantly over predicted the experimental universal velocity profiles in the laminar sublayer region and the buffer region. These results agree well with the present work, especially considering the experimental technique used and the geometry of the test section.

Since the falling films in the present experiment were in the laminar film regime, it is prudent to compare the dimensionless velocity profiles with laminar film theory. The velocity distribution for a laminar film proposed by Nusselt (1916) was converted into dimensionless form, and the resulting relations are given by

\[
\begin{align*}
  u^* &= \frac{\sqrt{\frac{\mu}{\rho g}}}{\sqrt{\delta}} (\delta y - 0.5y^2) \sqrt{\cos \theta} \\
  u^* &= \sqrt{\rho g \delta \cos \theta}
\end{align*}
\]  

The non-dimensional distance from the wall is given by equation (2.10), evaluated with the friction velocity of equation (4.8). The film thickness (\( \delta \)) used in the above
4.30). It is apparent that this analysis gives a good fit for the present data up to $y^+ = 0.5$, presumably due to the laminar nature of the profiles close to the wall. However, above this $y^+$ value significant deviations occurred, and the dimensionless Nusselt profile over predicts the present results. This finding is consistent with instantaneous velocity profile and average and maximum velocity data discussed earlier (figure 4.11), indicating that the Nusselt (1916) theory is inadequate to properly describe transient wavy film flow.

It is clear that significant velocity fluctuations are present close to the wall in laminar falling films (figure 4.20). These fluctuations may be considered as oscillations commonly observed in turbulent flows. To further investigate the nature of the instantaneous velocity profiles close to the wall in inclined channel flow, it may be of some interest to examine a turbulence model with the aim of modeling the dimensionless velocity distribution in the laminar sublayer and transition regions. Deissler (1955) proposed, in his momentum transfer model, that both molecular and turbulent processes be accounted for even close to the wall ($y^+ = 0$). Upon integration, his model yielded the following equation for the dimensionless velocity profile.

$$y^+ = \sqrt{\frac{\pi}{2m}} \exp(0.5mu^+) \text{erf}\left(\frac{m}{2}u^+\right)$$

Equation 4.9 was intended for pipe flow, and the value of $m$ was empirically determined to be 0.0119.

Deissler's (1955) model, however, did not fit the present data, possibly due to the free boundary flow structure of falling films (figure 4.30). To account for the free boundary in falling films, different values of $m$ were examined to determine if Deissler's (1955) equation could be used to model laminar falling films. Figure 4.30 clearly indicates that equation 4.9 does not fit the present data well, regardless of the values of the empirical constant. This indicates that the flow in laminar falling films is definitely not turbulent in nature, in spite of fluctuations in the velocity close to the wall which are a result of the local variations in film thickness, not localized turbulence effects.
Figure 4.30. Laminar and Turbulent Non-dimensional Velocity Distributions
Hummel (1970) with modified constants, can describe the non-dimensional velocity distribution adequately for wavy, laminar falling films. Clearly, further work must be conducted to better understand and model this type of flow.

4.3.4. Mass Transport in Waves

The increase in heat and mass transfer across the interface in wavy falling films has been largely attributed to the wavy, transient nature of these films. One mechanism, among many others, for the increase in these transport phenomena has been attributed to the large amounts of mass transported in the waves (Dukler, 1977). Therefore, it is of interest in the current work to examine the amount of liquid that is actually carried by the waves, with the aim of better understanding the mechanisms by which transport phenomena are enhanced.

In the present study, regions of thick film (waves) were found to transport more liquid than regions of thin films (figure 4.31). However, waves appear to carry less liquid than

![Figure 4.31. Relationship of Mass Transport to Film Thickness.](image)
profiles and average and maximum velocities.

In figure 4.31, the same dimensionless film thickness parameter, $\delta/\delta_{ave}$, was used along with the dimensionless volumetric flow rate, $Q/Q_{ave}$, in order to evaluate the relative amount of mass transported in waves, directing the analysis to waves that are larger in amplitude than small variations of substrate film thickness. A plot using the substrate flow rate and film thickness, in place of the time-averaged values, as determined from a PDF analysis gave similar results. Clearly, from this figure, it is obvious that regions of large film thickness, assumed to be waves, carry a significantly greater amount of mass than the regions of thin films, presumably the substrate regions. It is interesting to note that a large degree of fluctuation is present in all regions, due to the transient hydrodynamic nature of the film. With the above result, it may prove beneficial to examine the transient nature of the flow rate with film thickness fluctuations, with the aim of studying the amount of liquid flowing through individual waves. For this analysis, another dimensionless parameter was used, obtained by dividing the local flow rate by the local substrate flow rate ($Q/Q_{sub}$). The main impetus for the use of this new parameter is for direct comparison to the data of other investigators. The substrate flow rate, in the above ratio, was determined by plotting the local flow rate against the film thickness for each run and interpolating through the data to obtain the most probable flow rate at the most probable film thickness, representing the flow rate in the substrate film. Similar results were obtained from a direct PDF analysis of the flow rate data. It was apparent that individual large waves indeed carry a large fraction of the liquid at any given instance (figure 4.32). The largest waves, throughout all runs, appear to carry approximately up to five times the amount of liquid as carried in the average substrate film, with other waves carrying anywhere between one and five times as much liquid as the substrate. The number of large waves increased as the flow increased. As a result, one may expect that the fraction of total mass transported by the waves should increase as a function of the Reynolds number.

To further examine the effect of Reynolds number on mass transport in waves, the following analysis was conducted. The data from all runs was separated into two general categories, waves and substrate. The definition of a wave was considered in two different ways. The first was based on the dimensionless film thickness parameter mentioned
Figure 4.32. Instantaneous Mass Transport Under Waves
substrate film thickness, $\delta/\delta_{\text{sub}}$, (where the substrate film thickness was determined by PDF analysis). These two different parameters were used to address apparent shortcomings, or lack of adequate description, in previous investigations. In addition, the percentage of mass carried by the waves was calculated by two different methods, based on differing definitions of a wave. The first method considered a wave as an entire entity from the wall to the interface. The second method considered a wave to be only that part of the wave region that existed between the edge of the substrate (defined by either $\delta/\delta_{\text{ave}}=1$ or by $\delta/\delta_{\text{sub}}=1$) and the interface. The first method was defined by

$$\%Q_w = \frac{Q}{Q_t} \times 100$$  \hspace{1cm} (4.10)$$

and the second method given by

$$\%Q_w = \frac{(Q - Q_{s/a})}{Q_t} \times 100$$  \hspace{1cm} (4.11)$$

where the subscript s/a refers to the appropriate volumetric flow rate to be used when considering mean or substrate (most probable) film thickness in these equations.

Thus, by combining the two film thickness ratios with the two different calculations, four methods to determine the amount of liquid transported by the waves were developed in the present study (figure 4.33). All data in figure 4.33 were fitted with fourth order polynomials, with the resultant correlation coefficients ranging from 95.71%-97.69% for all runs.

Figure 4.33(a) indicates the percentage of mass transported by the waves, considering a wave to extend from the wall to the interface. When the substrate is defined as the most probable film thickness the percentage of mass carried by the waves ranges from 74% at low Reynolds numbers to 85% at higher Reynolds numbers. When the average film thickness is considered to be the substrate, the percentage of mass carried by the waves ranges from 40%-65%, with the peak occurring at Re=75. Figure 4.33(b) depicts the mass carried by waves, when waves are considered to extend from the interface to the substrate surface (either the PDF film thickness, or the average film thickness) only. This method was used to address the idea of a wave acting as a lump flowing over the substrate (Chu and Dukler, 1975). Both curves in this plot follow nearly identical sine curves, with minimum values at low Reynolds
Figure 4.33. Time-Averaged Mass Transport Under Waves.
20%), and local maximums at the maximum flow rates (43%, 22%). These local reductions in mass transported by waves are most likely due to increases in the average thickness and most probable film thickness as the Reynolds number increases. Clearly, this figure indicates that different definitions of substrates and waves results in significantly different outcomes.

The percentage of mass transported by waves at high Reynolds numbers (Re=900) reported by Telles and Dukler (1970) was in the range of 20%-30%. Their calculation appears to be based on the integration of the wave profile, multiplied by the frequency of the waves. This method, apparently considering a wave from the wall to the interface, seriously underestimates the data of the present experiment.

Dukler (1977) found, by comparing the total liquid Reynolds number to the average substrate Reynolds number, that approximately 87% of the mass was transported by waves in vertical, annular falling films at a Reynolds numbers of 300. In his study, the substrate film thickness was considered to be the most probable one derived through PDF analysis. This method appears to consider the wave as extending from the wall to the interface. An extrapolation of Dukler's (1977) line of best fit through his data shows that, at a Reynolds number of 200, approximately 85% of the total liquid was carried by the waves. This result is in agreement with the present study based on the substrate film thickness (when considering the upper curve of figure 4.33(a)).

Mudawar and Houpt (1993), studying vertical, annular falling films at low Reynolds numbers, found that, at Re=209, 40% of the mass was transported by large waves. Their results appear to be derived by integrating time-averaged velocity profiles, presumably from the wall to the interface. The discrepancy in their work lies in the lack of definition of a large wave. By comparison to the present data, using a film thickness ratio of $\delta/\delta_{ave} > 1.2$ to define a large wave, and considering the wave to extend from the wall to the interface, a value of 40% for the mass transported in the waves could be obtained.

The dimensionless velocity distributions ($u^+ = f(y^+)$) calculated in the present study indicate that waves affect the velocity profiles close to the wall, well into the laminar sublayer region (figures 4.29 and 4.30). Based on this observation, a wave may be considered to extend from the wall to the interface. Therefore, it may be concluded that the percentage of mass transported by waves in this study was in the range of 74-85%.
4.3.5. Wall Shear Stress

The wall shear stress has historically been a very difficult value to directly measure experimentally, mainly due to the requirement of the instantaneous or time-averaged velocity gradient close to the wall. However, this quantity is one of the more important aspects when considering the hydrodynamics of viscous fluid flow, in terms of both industrial applications and theoretical modeling. Therefore, it is of interest to study the wall shear stress in falling liquid films in order to examine the transient hydrodynamic behavior of this characteristic.

Classical theory predicts that wall shear stress is a linear function of film thickness (Nusselt, 1916). Results from this study indicate a similar dependence of wall shear stress on film thickness (figure 4.34), however, the constant of proportionality is less than unity. The dimensionless wall shear stress and film thickness parameters for this figure were chosen to

Figure 4.34. Relationship of Wall Shear Stress to Film Thickness.
cut as that observed with flow rate (Figure 4.31), or velocity (Figures 4.12-4.15), as denoted by the relatively large scatter at a film thickness ratio of approximately unity and the number of outlying data points at all film thickness ratios. This variation may be attributed to the transient nature of the film. It is expected that a large degree of fluctuation be present at a film thickness approximately equal to the average film thickness, that often represents transitions between wavy and substrate regions, as discussed in previous sections. In addition, large fluctuations were observed for the wave peaks, evident through the scatter at large values of the abscissa.

Based on the above results, it may be of interest to conduct a more detailed examination of the fluctuations in wall shear stress as a function of film thickness variations. Two dimensionless parameters, $\tau_w/\tau_{w,ave}$ (shear ratio) and $\delta/\delta_{ave}$ (film ratio), were studied as functions of time, with the aim of exploring the dependence of local wall shear stress to local film thickness (Figure 4.35). Again, these parameters were chosen to reflect effects of significant interface fluctuations, while minimizing the effect of small variations in film thickness about the substrate film. These parameters may be considered indicative of actual wall shear stress and film thickness values and their fluctuations. Through this interpretation, in some cases the wall shear stress either preceded or lagged behind the film thickness peak, but in many cases the two peaks coincided with each other. In the regions of extended substrate film, the wall shear stress often appeared to follow the film thickness in a consistent, sometimes oscillatory, manner with quite good agreement. The wall shear stress as predicted by Nusselt (1916), therefore, holds reasonably well in non-wavy regions of falling films. As the Reynolds number was increased, the deviations between film thickness and wall shear stress ratios were more common. The largest deviations, consistent throughout all runs, occurred in the wave regions, where it was observed that the wall shear stress ratio, in most instances, did not achieve the same maximum values as the corresponding wave-to-substrate thickness ratio. It is clearly evident that, in transient, wavy films, the wall shear stress may not be expressed explicitly as a proportional function of film thickness.

Wasden and Dukler (1989) reported a general increase in wall shear stress with an increase in film thickness. It is interesting to note that the experimental wall shear stress
Figure 4.35. Instantaneous Wall Shear Stress Dependence on Film Thickness.
In addition, they cite that the wall shear stress peaks slightly ahead of the wave peak for large waves. Similar results were reported by Miya et al. (1971), who measured the variation of wall shear stress as a function of film thickness in horizontal co-current gas-liquid flow. Evidences in the present study were found to corroborate the results of both studies.

Fluctuating wall shear stress was measured by Govan et al. (1989), in vertical annular flow. They reported that the fluctuating values for wall shear stress occurred at characteristic peak frequencies corresponding to the disturbance wave peaks. It is evident, from the present data that local shear stress peaks do occur at approximately the same time as the peak of large waves.

Most recently, Yu et al. (1995) discussed the development of a new model for prediction of hydrodynamic properties of falling liquid films. Their model indicates that the wall shear stress under waves remains at approximately a constant value equal to that under the substrate. Clearly, their model conflicts with the present data as well as previous results in that a local peak in wall shear stress is observed in wavy regions in nearly all experimental runs.

The classical analysis of falling liquid films, conducted by Nusselt (1916), was based on a force balance on a control volume of liquid. In this model, the interface was assumed to be flat and the system steady state. Since it is obvious that, in wavy falling liquid films, the interface fluctuates and the system appears to be in a transient state, it would be of interest to compare the experimental shear stress to the steady state model.

Figure 4.36 depicts the ratio of local wall shear stress to the gravitational force per unit area, plotted against the film thickness ratio. The ordinate in this figure represents a quantity that is predicted to be unity by Nusselt (1916), $\tau_w/\rho g\delta \cos \theta$, for a smooth laminar film. It is evident from figure 4.36 that the wall shear stress in waves (i.e., large values of $\delta/\delta_{ave}$) was found to be less than what Nusselt (1916) predicted (this was also evident from the velocity profiles in figures 4.21-4.26). In the region of average film thickness and below ($\delta/\delta_{ave}=1$ or less) the data appear to straddle Nusselt's (1916) theory, with a large number of data points lying above this line. Thus, a trend of negative slope may be visualized, from above the theoretical prediction (in the general region of the substrate film thickness) to

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below this line (for large wavy structures). In all experimental runs, the local value of $\tau_w/\rho g \delta \cos \theta$ fluctuated around the equilibrium value of 1.0.

From the point of view of a force balance, the data obtained at a large film thickness (i.e., the waves) appears to indicate that the fluid is accelerating ($\tau_w < \rho g \delta \cos \theta$). This may be deduced by considering that, if the wall shear stress is below the steady state equilibrium value, the velocity of the fluid would increase over the distance, or time. In the region of a substrate film, a similar argument could be made for the data falling below the theoretical line. For the data lying above this line, however, the opposite argument may be developed. In order to satisfy the force balance for these data points, in which the wall shear stress is greater than that predicted by Nusselt (1916), one must consider the fluid to be decelerating ($\tau_w > \rho g \delta \cos \theta$). The present work, therefore, suggests a transient state of the falling liquid film in which the fluid is primarily accelerating in the large waves, decelerating as it passes through the wave back regions (with intermediate values of $\delta/\delta_{ave}$) and into the substrate region (with the lowest values of $\delta/\delta_{ave}$). The fluid begins to accelerate again as it enters the wave front region, represented by intermediate values of $\delta/\delta_{ave}$.

Individual waves studied by Wasden and Dukler (1989) resulted in a computational model that predicted similar results to those discussed in the present experiment, concerning the transient developments of wall shear stress.
5.1. Conclusions

Experiments were conducted to study the instantaneous hydrodynamic characteristics of a laminar falling liquid film on a flat plate in an inclined rectangular channel using a viscous fluid in the Reynolds number range of 11-220. Photochromic dye activation (PDA) was utilized to obtain data. The advantage of the PDA technique is that instantaneous velocity profiles may be obtained in a visual record. This reduces errors incurred through the calibration of equipment previously used, allows for data to be collected close to the wall and, as a result, allows for the accurate calculation of the wall shear stress.

The film thickness was examined carefully to assess the precision and applicability of the PDA technique to the study of falling films. Several parameters of the film thickness were examined including the mean, probability distribution, standard deviation, wave frequency, and minimum and maximum values. Results of the present experiment gave good agreement to that of previous results, with mean film thickness measurements giving a maximum of 6% error from that predicted by Nusselt (1916).

The mean and maximum velocity data and wall shear stress values were time-averaged and compared to Nusselt’s (1916) theory. Nusselt (1916) over predicted both the maximum and mean velocities by a maximum of 18% and 11% respectively, due to severe over predictions under waves. The wall shear stress obtained in the present study agreed well with that predicted by Nusselt (1916), although most values were slightly under-estimated.

The instantaneous film fluctuations and qualitative results concerning the development of the falling film agreed well with previous reports. The following summarizes the major findings involving the instantaneous data.

The ratio of maximum-to-average velocity was examined to determine if a parabolic distribution, indicative of laminar flow, existed. Results showed that the ratio fluctuated around, and agreed well with, the laminar value of 1.5, with a mean value of 1.47. However, a large degree of scatter was present indicating that, particularly around the average film thickness, significant local deviations from the parabolic nature existed.

Instantaneous velocity profiles were examined with respect to local film thickness and compared to the theory of Nusselt (1916). The maximum deviations occurred in the
regions exhibited the best agreement with Nusselt’s (1916) predictions, with a maximum of 15% deviation (positive and negative). The wave-front and wave-back regions exhibited intermediate deviations, typically to a maximum of 25%.

Non-dimensional velocity profiles were calculated to assess the effect of waves in the laminar sublayer region close to the wall. A fit proposed by Ho and Hummel (1970) gave the best agreement with present data through the use of empirically determined constants. A comparison to the universal velocity distribution indicated that the velocity profiles in the laminar sublayer were not linear in nature. In addition, it appeared that these distributions were not parabolic, except at locations very close to the wall ($y^+ << 1$), as seen by a dimensionless version of Nusselt’s (1916) velocity profile, and work done by Koziol et al. (1981). Large amounts of scatter were observed close to the wall, in a manner commonly associated with turbulence. To assess this, the data was fit to Deissler’s (1955) turbulence model, with a number of values for the empirical constant ($m$). It was determined that Deissler’s (1955) turbulence model was not applicable to laminar falling films, and that the velocity fluctuations could not be modeled as turbulent oscillations.

A comprehensive analysis was conducted to assess the mass transported in the waves of falling films. Clearly, wavy regions were observed to carry more liquid than non-wavy regions. However, due to complications arising from definitions of the substrate film thickness and waves, considerable discrepancy exists in the literature. Therefore, different methods to evaluate this parameter were examined, explaining some of these previous discrepancies. It was determined in the present work that waves, extending from the wall to the interface, constitute any film region whose thickness was greater than the substrate film thickness (PDF definition) and that 74%-85% of the mass is transported by waves in the flow range examined.

Wall shear stress data indicated a general linear dependence with film thickness, although the constant of proportionality was less than unity. Instantaneous wall shear stress peaks were observed to precede, follow and coincide with the film thickness peaks. To assess the falling film in terms of a steady state force balance, a dimensionless shear ratio ($\tau_w/\rho g \delta \cos \theta$) was compared to a dimensionless film ratio ($\delta/\delta_{ave}$). This analysis showed that wavy regions experienced acceleration. In addition, regions of thin films were indicative of
accelerating zones in the waves (and presumably wave-fronts), and decelerating zones in the non-wavy regions (and presumably wave-backs). However, a transient force balance may indicate both acceleration and deceleration in all regions due to the presence of velocity and film thickness quantities in the time-dependent and convective terms.

5.2. Recommendations

The present experiment, involving the study of hydrodynamic characteristics of falling films in inclined rectangular channels, represent some initial investigations into instantaneous velocity profiles and wall shear stress, made possible through the PDA technique. This section briefly describes new directions in which analysis may be conducted. In addition, improvements to the experiments, which may improve the accuracy of the PDA technique, can help reduce the errors incurred in future work.

Data should be analyzed in a more rigorous manner so as to clearly elucidate the characteristics in the wave-front and wave-back regions. This may help to precisely determine the velocity profile fluctuations and regions of acceleration and deceleration caused by deviations from a steady state force balance. Perhaps an investigation considering the convective and time-dependent terms of the Navier-Stokes equations is appropriate. In addition, it may be of some interest to examine the profiles to generate streamlines to describe the flow field in falling liquid films.

The transient nature of other transport phenomena, such as heat transfer, may be studied and related to the momentum transfer results of the present work. The test section is currently fitted with a core to enable heat exchange fluid to control the temperature, allowing for experimentation of this nature.

In the present experiment, no adequate model was suggested to describe the non-dimensional velocity profile. It is prudent that a single equation model be developed to describe falling film flow, in a non-dimensional manner, taking into account the wavy laminar interfacial effects found in the present work.

Some improvements in the experimental apparatus will invariably lead to more precise results. The N₂ laser used in the present experiments, although quite powerful, had a badly diverging rectangular beam. This made it very difficult to focus the energy to a fine
the maximum pulsing frequency was 50 Hz, limiting the collection of data. Therefore, a new laser with a refined, highly focused beam and the ability to operate at higher frequencies (100 to 200 Hz) would be an asset. A laser of this nature would reduce the error when analyzing data, as the traces formed would be very fine. In addition, the increased frequency would allow for more data to be collected and for closer analysis of the hydrodynamic characteristics of the wave structures. The resolution of the high-speed camera somewhat limited the quality of the video recordings obtained in the present experiment. Thus, a new camera with improved resolution would allow for improved picture quality, making it easier to analyze the traces formed by the laser. The sodium lamp back-lighting, required with the use of the high-speed camera, was very tedious to use although it provided excellent contrast for the visualization of the traces. Experimentation would be enhanced if a better back-lighting system was developed.


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APPENDICES

Appendix A: Rotameter Calibrations

Figure A.1. Calibration Curve for Rotameter 1 (low Re flows)
Figure A.2. Calibration Curve for Rotameter 2 (high Re flows)

Appendix B: Computer Programs

B.1. Data Reduction Program
B.2. Data Collection Program
$y = 1.4x + 0.4332$

$R^2 = 0.9937$

Figure A.1. Calibration Curve for Rotameter 1 (low flows)
Figure A.2. Calibration Curve for Rotameter 2 (high Re flows)
function vel

% This program calculates the film thickness, velocity profile, and
% wall shear stress from Cartesian representation of two simultaneous
% traces by fitting them to a polynomial (order 2-5). Comparisons to
% Nusselt (1916) predictions of velocity profile and wall shear stress,
% based on the experimental film thickness are also computed.
% VARIABLES:
% ar - aspect ratio
% ave - average film thickness (mm)
% cxxx - polynomial fit velocity gradient (mm/s)
% datax - noramlized position vector (mm)
% delta - distance between velocity profile calculations (mm)
% dev - difference in height of two consecutive traces (%)
% dispx - displacement of trace (mm)
% fitxxx - polynomial fit to position vector
% fr - camera frame rate
% g - acceleration due to gravity (m/s^2)
% i, nn, m - counter variables
% minx - height of position vector (mm)
% miu - dynamic viscosity (kg/(m*s))
% qx - volumetric flow rate (LPM)
% ro - density (kg/m^3)
% Re - Reynolds number (4Q/nu)
% sf - scale factor (1/mm)
% twcalx - calculated wall shear stress (kg/(m*s^2))
% twnus - Nusselt (1916) wall shear stress
% velcx - velocity (m/s)
% velcn - Nusselt (1916) velocity (m/s)
% vmaxx - maximum velocity (m/s)
% vmeanx - mean velocity (m/s)
% w - channel width (m)
% xx - polynomial order
% yyx - output variable
% This program written and modified by Reza Karimi and Kevin Moran at the
% University of Toronto, 1996-97 in Matlab 4.2.
clc

x1=2;
x2=3;
x3=4;
x4=5;
sf=121;
ar=0.857;
fr=186;
delta=0.005;

w=0.08;
ro=960;
g=9.81;
miu=0.0192;
```matlab
file = strrep(file,'.TXT','');
eval(['datas1 =', file, ';']);

[file, path] = uigetfile('*.txt', 'Load the Second File');
if file == 0,
    eval(['load ', path, file, '-ascii'])
    file = strrep(file,'.TXT','');
    eval(['data2 =', file, ';']);
end

% Normalize data vectors

data1=(1/sf)*(data1-ones(length(data1(:,1)),1)*data1(1,:));
data2=(1/sf)*(data2-ones(length(data2(:,1)),1)*data2(1,:));

% Fit displacement vectors to polynomial of order 2 to 5

tl=data1(:,2);
disp1=-ar*data1(:,1);
t2=data2(:,2);
disp2=-ar*data2(:,1);
c12=polyfit(t1,disp1,x1);
c22=polyfit(t2,disp2,x1);
c2=c22-c12;
fitl2=polyval(c12,t1);
fit22=polyval(c22,t2);
c13=polyfit(t1,disp1,x2);
c23=polyfit(t2,disp2,x2);
c3=c23-c13;
fitl3=polyval(c13,t1);
fit23=polyval(c23,t2);
c14=polyfit(t1,disp1,x3);
c24=polyfit(t2,disp2,x3);
c4=c24-c14;
fitl4=polyval(c14,t1);
fit24=polyval(c24,t2);
c15=polyfit(t1,disp1,x4);
c25=polyfit(t2,disp2,x4);
c5=c25-c15;
fitl5=polyval(c15,t1);
fit25=polyval(c25,t2);

% Plot normalized displacement vectors, by polynomial order

subplot(3,4,1),plot(-fitl2,t1,-disp1,t1,'o',-fit22,t2,-disp2,t2,'x');grid;
title('2');
axis([-max([max([disp1]) max([disp2])]) 0 max([max([t1]) max([t2])])]);
ylabel('[mm]');

subplot(3,4,2),plot(-fitl3,t1,-disp1,t1,'o',-fit23,t2,-disp2,t2,'x');grid;
title('3');
axis([-max([max([disp1]) max([disp2])]) 0 max([max([t1]) max([t2])])]);

subplot(3,4,3),plot(-fitl4,t1,-disp1,t1,'o',-fit24,t2,-disp2,t2,'x');grid;
title('4');
axis([-max([max([disp1]) max([disp2])]) 0 max([max([t1]) max([t2])])]);
```

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Calculate mean, local film thickness from displacement vectors

\[
\begin{align*}
min1 &= \max(data1(:,2)); \\
min2 &= \max(data2(:,2)); \\
ave &= \text{mean}([\text{min1} \ \text{min2}]); \\
dev &= ((\text{min1} - \text{min2})/\text{ave})^*100;
\end{align*}
\]

Calculate velocity at \(nn*\delta) to generate velocity profile for experimental data and Nusselt (1916) prediction

\[
\begin{align*}
nn &= \text{abs} \left( \text{round} \left( \text{ave}/\delta \right) \right); \\
\text{for } i &= 1:nn+1, \\
& \quad t(i) = (i-1)*\delta; \\
velc2(i) &= \left( \text{polyval}(c2,t(i))/1000/\text{fr} \right); \\
velc3(i) &= \left( \text{polyval}(c3,t(i))/1000/\text{fr} \right); \\
velc4(i) &= \left( \text{polyval}(c4,t(i))/1000/\text{fr} \right); \\
velc5(i) &= \left( \text{polyval}(c5,t(i))/1000/\text{fr} \right); \\
velcn(i) &= 0.7*\rho*g*(-\text{ave}^*0.001*t(i)^*0.001-\text{t}(1)^*0.001)^*2)/(2*\mu); \\
\end{align*}
\]

Calculate flow rate and Reynolds number for each polynomial fit

\[
\begin{align*}
q2 &= \delta^*\text{sum}(\text{velc2})^*60^*0.08; \\
re2 &= 4*q2/w*\rho/\mu/1000/60; \\
q3 &= \delta^*\text{sum}(\text{velc3})^*60^*0.08; \\
re3 &= 4*q3/w*\rho/\mu/1000/60; \\
q4 &= \delta^*\text{sum}(\text{velc4})^*60^*0.08; \\
re4 &= 4*q4/w*\rho/\mu/1000/60; \\
q5 &= \delta^*\text{sum}(\text{velc5})^*60^*0.08; \\
re5 &= 4*q5/w*\rho/\mu/1000/60;
\end{align*}
\]

Calculate maximum and mean velocity for each polynomial fit

\[
\begin{align*}
\text{vmax2} &= \max(\text{velc2}); \\
\text{vmax3} &= \max(\text{velc3}); \\
\text{vmax4} &= \max(\text{velc4}); \\
\text{vmax5} &= \max(\text{velc5}); \\
\text{vmean2} &= -q2/(w*\rho*60); \\
\text{vmean3} &= -q3/(w*\rho*60); \\
\text{vmean4} &= -q4/(w*\rho*60); \\
\text{vmean5} &= -q5/(w*\rho*60);
\end{align*}
\]

Fit displacement vectors to polynomials close to wall and calculate velocity gradient to obtain wall shear stress.

Calculate Nusselt (1916) wall shear stress.

\[
\begin{align*}
mm &= 40; \\
cll2 &= \text{polyfit}(t1(1:mm),\text{displ}(1:mm),1); \\
c2l2 &= \text{polyfit}(t2(1:mm),\text{displ}(1:mm),1); \\
twcal2 &= \left( \text{polyval}(cl22,mm*\delta) - \text{polyval}(cl12,mm*\delta) \right) * \text{fr}/(mm*\delta) * \mu;
\end{align*}
\]

\[
\begin{align*}
cll3 &= \text{polyfit}(t1(1:mm),\text{displ}(1:mm),2); \\
c2l3 &= \text{polyfit}(t2(1:mm),\text{displ}(1:mm),2); \\
twcal3 &= \left( \text{polyval}(cl23,mm*\delta) - \text{polyval}(cl13,mm*\delta) \right) * \text{fr}/(mm*\delta) * \mu;
\end{align*}
\]

\[
\begin{align*}
cll4 &= \text{polyfit}(t1(1:mm),\text{displ}(1:mm),3); \\
c2l4 &= \text{polyfit}(t2(1:mm),\text{displ}(1:mm),3); \\
twcal4 &= \left( \text{polyval}(cl24,mm*\delta) - \text{polyval}(cl14,mm*\delta) \right) * \text{fr}/(mm*\delta) * \mu;
\end{align*}
\]

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twcal5 = polyval(cl25, mm * delta) - polyval(cl15, mm * delta); 

twcal_ave = (twcal5 + twcal2 + twcal3 + twcal4) / 4;
twnus = -ro * g * ave * 0.001 * 707;

% Plot velocity gradients close to wall, and the wall shear stress - both with % comparisons to Nusselt (1916) predictions

subplot(3, 4, 5), plot(-velc2, t, '-', velcn, t, 'r:'); grid;
max([velcn]) 0 0 -ave));
ylabel('[m/s]');

subplot(3, 4, 6), plot(-velc3, t, '-', velcn, t, 'r:'); grid;
max([velcn]) 0 0 -ave));

subplot(3, 4, 7), plot(-velc4, t, '-', velcn, t, 'r:'); grid;
max([velcn]) 0 0 -ave));

subplot(3, 4, 8), plot(-velc5, t, '-', velcn, t, 'r:'); grid;
max([velcn]) 0 0 -ave));

subplot(3, 4, 9), plot(-velc2, t, '-', velcn, t, 'r:'); grid;
axis([-0.05 0 0.1]);
xlabel('[mm]');
ylabel('[m/s]');

subplot(3, 4, 10), plot(-velc3, t, '-', velcn, t, 'r:'); grid;
axis([-0.05 0 0.1]);
xlabel('[mm]');

subplot(3, 4, 11), plot(-velc4, t, '-', velcn, t, 'r:'); grid;
axis([-0.05 0 0.1]);
xlabel('[mm]');

subplot(3, 4, 12), plot(-velc5, t, '-', velcn, t, 'r:'); grid;
axis([-0.05 0 0.1]);
xlabel('[mm]');

tab = [x1, x2, x3, x4; -ave; -ave; -ave; -ave; dev, dev, dev, dev; q2, q3, q4, q5; vmean2, vmean3, vmean4, vmean5; vmax2, vmax3, vmax4, vmax5; ... 

twcal2, twcal3, twcal4, twcal5, twcal_ave; twcal_ave, twcal_ave, twcal_ave, twcal_ave; twnus, twnus, twnus, twnus]

pause(2)

% Choose best polynomial fit for both the velocity profile and wall shear stress. % Display output at terminal and prompt for data file name

m = menu('Choose Polynomial Order', '2', '3', '4', '5');
k = menu('Choose Shear Stress', 'twcal2', 'twcal3', 'twcal4', 'twcal5', 'twcal_ave', 'other');

if k == 1,
twm = twcal2;
elseif k == 2,
twm = twcal3;
elseif k == 3,
twm = twcal4;
elseif k == 4,
twm = twcal5;
elseif k == 5
    tmw = twcal_ave;
end;
yy3_out = [nn, x2, 0; ave, 0, dev, 0, q3, 0, vmean3, 0, vmax3, 0; twm, 0, twnus, 0];
yy4_out = [nn, x3, 0; ave, 0, dev, 0, q4, 0, vmean4, 0, vmax4, 0; twm, 0, twnus, 0];
yy5_out = [nn, x4, 0; ave, 0, dev, 0, q5, 0, vmean5, 0, vmax5, 0; twm, 0, twnus, 0];

[file, path] = uiputfile('*.out', 'Save the Output File');
if file ~= 0,
  if m==1,
    eval(['save ', path, file, ' yy2_out -ascii']);
    tab=[x1; ave; dev; q2; vmean2; vmax2; twm; twnus]
  elseif m==2
    eval(['save ', path, file, ' yy3_out -ascii']);
    tab=[x2; ave; dev; q3; vmean3; vmax3; twm; twnus]
  elseif m==3
    eval(['save ', path, file, ' yy4_out -ascii']);
    tab=[x3; ave; dev; q4; vmean4; vmax4; twm; twnus]
  elseif m==4
    eval(['save ', path, file, ' yy5_out -ascii']);
    tab=[x4; ave; dev; q5; vmean5; vmax5; twm; twnus]
  else
    eval(['save ', path, file, ' yy5_out -ascii']);
    tab=[x4; ave; dev; q5; vmean5; vmax5; twm; twnus]
  end
end
end
end;
PROGRAM DATCOL

C

This program collects the instantaneous data from vel.m and calculates the time-averaged mean for a number of hydrodynamic characteristics. In addition, it stores the experimental velocity profile in a separate file. As well, it calculates, and stores in separate files, Nusselt (1916) velocity profiles.

C

VARIABLES

C ro - density (kg/m^3)
C miu - dynamic viscosity (kg/(m*s))
C fname - file name
C delta - film thickness (mm)
C dev - difference between two consecutive trace heights
C flow - volumetric flow rate (LPM)
C ua - mean velocity (m/s)
C um - maximum velocity (m/s)
C twa - experimental wall shear stress (Pa)
C twn - Nusselt (1916) wall shear stress (Pa)
C ynuus - incremental location for velocity profile caculations
C extension "P" - summations
C extension "A" - time-averaged values
C y,yy - location within film for velocity profile measurement (mm)
C u,uu - velocity at y or yy. (m/s)
C

This program written in Fortran 77 compiler by Kevin Moran, 1997.

INTEGER NUM, J, K, DH, KK, NPP
REAL NP, MIU
CHARACTER*20 FNAME(200), FN(200), X, Q, Z, W, S, P
DIMENSION Y(500), U(500), VELNUS(500), VELN(500), YY(500), UU(500)
DIMENSION YYNUS(500), UUNUS(500)
WRITE(6,*)('ENTER MATLAB OUTPUT FILE (XXX.IN): ', )
READ(6,10) X
WRITE(6,*)('ENTER FILENAME FOR GENERAL DATA (FXRY.OUT): ', )
READ(6,10) Q
WRITE(6,*)('ENTER NAME FOR VELOCITY DISTR (FXRY.VEL): ', )
READ(6,10) Z
WRITE(6,*)('ENTER NAME FOR NUSSELT VEL. DIST. (FXRY.NUS): ', )
READ(6,10) W
WRITE(6,*)('ENTER NAME FOR EXPLICIT VEL PROF. (FXRYOUT.DEL): ', )
READ(6,10) S
WRITE(6,*)('ENTER NAME FOR EXPLICIT NUSSELT VP (FXRYNUS.DEL): ', )
READ(6,10) P

109
OPEN (UNIT=1, FILE=X, STATUS='OLD')
OPEN (UNIT=6, FILE='TERMINAL')
OPEN (UNIT=7, FILE=Q, STATUS='UNKNOWN')
OPEN (UNIT=8, FILE=Z, STATUS='UNKNOWN')
OPEN (UNIT=9, FILE=W, STATUS='UNKNOWN')
OPEN (UNIT=10, FILE=S, STATUS='UNKNOWN')
OPEN (UNIT=20, FILE=P, STATUS='UNKNOWN')

WRITE(6,120)
WRITE(7,120)

RO=960.00
MIU=0.0192

DEL=0
DEVP=0
FLOWP=0
UAVEP=0
UMAXP=0
TWAP=0
TWNP=0
YNUS=0

C Open and read Matlab data files.
READ(1,*) NUM
WRITE(6,*) NUM
DO 11 J=1,NUM
READ(1,100) FNAME(J)
11 CONTINUE
CLOSE (UNIT=1)
C WRITE(6,*) NUM
DO 22 K=1,NUM
OPEN(UNIT=2,FILE=FNAME(K), STATUS='OLD')
READ(2,*), NP
READ(2,*), PORD
READ(2,*), DELTA
READ(2,*), DEV
READ(2,*), FLOW
READ(2,*), UA
READ(2,*), UM
READ(2,*), TW
READ(2,*), TWA
READ(2,*), TWN
WRITE(6,110) FNAME(K), PORD, DELTA, DEV, FLOW, UA, UM, TWA, TWN
WRITE(7,110) FNAME(K), PORD, DELTA, DEV, FLOW, UA, UM, TWA, TWN

CLOSE(2)
DO 33 I=1, NUM
  DEL=DEL+DELTA
  DEVP=DEVP+DEV
  FLOWP=FLOWP+FLOW
  UAVEP=UAVEP+UA
  UMAXP=UMAXP+UM
  TWAP=TWAP+TWA
  TWP=TWAP+TWN
CONTINUE

DO 22 CONTINUE
  DELA=DEL/NUM
  DEVA=DEVP/NUM
  FLOWA=FLOWP/NUM
  UAVEA=UAVEP/NUM
  UMAXA=UMAXP/NUM
  TWAA=TWAP/NUM
  TWA=TWNP/NUM

WRITE(6,130) NUM, DELA, DEVA, FLOWA, UAVEA, UMAXA, TWAA, TWA
WRITE(7,130) NUM, DELA, DEVA, FLOWA, UAVEA, UMAXA, TWAA, TWA
WRITE(6,150)

C Read Matlab files and collect instantaneous velocity profile data

WRITE(10,14) NUM
DO 44 L=1, NUM
  OPEN(UNIT=2, FILE=FNAME(L), STATUS='OLD')
  READ(2,*) NP
  READ(2,*) PORD
  READ(2,*) DELTA
  READ(2,*) DEV
  READ(2,*) FLOW
  READ(2,*) UA
  READ(2,*) UM
  READ(2,*) TW
  READ(2,*) TWA
  READ(2,*) TWN
  NPP=NINT(NP)
  WRITE(10,12) FNAME(L), NPP
  NPPC=NPP+1
  DO 55 M=1, NPPC
    READ(2,*) Y(M), U(M)
    WRITE(10,190) Y(M), U(M)
CONTINUE

DH=NINT(NP/9)
INP=NINT(NP)
44 CONTINUE
CLOSE(2)
CLOSE(8)
WRITE(6,160)

C Calculate Nusselt (1916) velocity profiles, based on exptl film thickness

WRITE(20,14) NUM
DO 88, LLL=1, NUM
OPEN(UNIT=2,FILE=FNAME(LLL),STATUS='OLD')
READ(2,*) NP
READ(2,*) PORD
READ(2,*) DELTA
READ(2,*) DEV
READ(2,*) FLOW
READ(2,*) UA
READ(2,*) UM
READ(2,*) TW
READ(2,*) TWA
READ(2,*) TWN

DH=NINT(NP/9)
INP=NINT(NP)
YNUS=0.0

DELTAN=DH*0.005*0.001
DELTAP=DELTA*0.001

DO 77 KK=1,8
VELNUS(KK)=RO*9.81*0.707*KK*DELTAN*(DELTA*0.001-KK*DELTAN/2)/MIU
IF (KK.GE.8) THEN
VELNUS(9)=RO*9.81*0.707*DELTAP*(DELTAP-DELTAP/2)/MIU
ENDIF
77 CONTINUE

NPP=NINT(NP)
WRITE(20,12) FNAME(LLL),NPP
NPPC=NPP+1
DO 99 JJJ=1,NPPC
IF (YNUS.LE.DELTA) THEN
VELN(JJJ)=RO*9.81*0.707*YNUS*0.001*(DELTA-YNUS*0.001/2)/MIU
ELSE IF (YNUS.GT.DELTAP) THEN
YNUS=DELTAP
VELN(JJJ)=RO*9.81*0.707*DELTAP*(DELTAP/2)/MIU
ENDIF
99 CONTINUE

WRITE(20,190) YNUS,VELN(JJJ)
YNUS=YNUS+0.005

99 CONTINUE
C Write experimental velocity profile data to individual files

OPEN (UNIT=65, FILE=S, STATUS='OLD')
READ (65, *) NUMB
WRITE (6, *) NUMB
DO 119 IJ = 1, NUMB
   READ (65, 15) FNAME (IJ), NNP
   WRITE (6, *) FNAME (IJ), NNP
   FN (IJ) = FNAME (IJ) (:3) // '.DAT'
   WRITE (6, *) IJ, (' '), FN (IJ), NNP
   OPEN (UNIT=66, FILE=FN (IJ), STATUS='UNKNOWN')
   WRITE (66, 220) FN (IJ), NNP
   NNPC = NNP + 1
   DO 109 IK = 1, NNPC
      READ (65, 240) YY (IK), UU (IK)
      WRITE (6, *) IK, YY (IK), UU (IK)
C      WRITE (66, 230) YY (IK), UU (IK)
   109 CONTINUE
   CLOSE (66)
   119 CONTINUE
   CLOSE (65)

C Write Nusselt (1916) velocity profile data to individual files

C PROGRAM VPROFNUS.FOR

OPEN (UNIT=75, FILE=P, STATUS='OLD')
READ (75, *) NUMB
WRITE (6, *) NUMB
DO 139 JI = 1, NUMB
   READ (75, 15) FNAME (JI), NNP
   WRITE (6, *) FNAME (JI), NNP
   FN (JI) = FNAME (JI) (:3) // '.NUS'
   WRITE (6, *) JI, (' '), FN (JI), NNP
   OPEN (UNIT=76, FILE=FN (JI), STATUS='UNKNOWN')
   WRITE (76, 220) FN (JI), NNP
   NNPC = NNP + 1
   DO 129 KI = 1, NNPC
      READ (75, 240) YYNUS (KI), UUNUS (KI)
      WRITE (6, *) KI, YYNUS (KI), UUNUS (KI)
C      WRITE (76, 230) YYNUS (KI), UUNUS (KI)
   129 CONTINUE
   CLOSE (76)
   139 CONTINUE
10 FORMAT(A15)
12 FORMAT(A12,2I4)
14 FORMAT(I5)
15 FORMAT(A13, I3)
100 FORMAT(A10)
110 FORMAT(A10, 8F8.3)
120 FORMAT('FILE1, 9X, 'POLYN', 3X, 'DELTA', 2X, 'DEV', 4X, 'FLOW', 5X,
   + 'VEL(A)', 2X, 'VEL(M)', 2X, 'TW(A)', 3X, 'TW(N)'/
   + '-----', 9X, '-----', 3X, '-----', 2X, '-----', 3X, '-----', 5X, '-----',
   + 2X, '-----', 2X, '-----', 3X, '-----')
130 FORMAT('---------------', 4X, '-----', 3X, '-----', 2X, '-----', 4X, '-----', 5X,
   + '---------------', 2X, '---------------', 2X, '---------------', 3X, '---------------'/
   'AVERAGE', I11, 7F8.3///)
140 FORMAT(A4, F7.4, I3, 9F7.4)
150 FORMAT(15X, 'VELOCITY DISTRIBUTION'/15X, '--------------------------'/
   + '5', 5X, '6', 6X, '7', 7X, '8', 3X, 'U(IN)'/ '------', 2X, '------', 1X, '--', 2X,
   + '------', 3X, '------', 3X, '------', 3X, '------', 3X, '------', 3X, '------', 3X,
   + '------', 3X, '------', 3X, '------')
160 FORMAT(///5X, 'NUSSELT VELOCITY DISTRIBUTION'/15X, '--------------------------'/
   + 'FILE', 1X, 'YPOSITION', 3X, '1', 6X, '2', 6X, '3', 6X, '4', 7X,
   + '5', 5X, '6', 6X, '7', 7X, '8', 3X, 'U(IN)'/ '------', 1X, '------', 2X,
   + '------', 3X, '------', 3X, '------', 3X, '------', 3X, '------', 3X,
   + '------', 3X, '------', 3X, '------')
170 FORMAT(A4, E10.3, 9F7.4)
190 FORMAT(2F10.7)
220 FORMAT(A12, 15)
230 FORMAT(2F10.7)
240 FORMAT(2F10.7)

END