REFINING MODELS OF THE
GLACIAL ISOSTATIC ADJUSTMENT PROCESS

by

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A thesis submitted in conformity with the requirements
for the degree of Doctor of Philosophy
Graduate Department of Physics
University of Toronto

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Abstract

Surface observables associated with glacial isostatic adjustment (GIA) can be employed to infer mantle viscosity structure and changes in ice mass distribution. These parameters play a key role in, respectively, our understanding of flow processes within the mantle and long-time-scale changes in the Earth's climate. The work presented in this thesis explores the importance of a number of 'second order' aspects of GIA models that are commonly neglected in the inference procedure.

Observations of sea-level change comprise the most effective data set employed in GIA modeling studies to date. For this reason, much of the work presented in this thesis focuses on a number of approximations inherent in a widely adopted GIA sea-level theory. In particular, assumptions relating to the redistribution of the water load associated with the melting of the last great ice sheets are considered. Results show that sea-level predictions are sensitive to whether or not the Earth's shorelines (which are known to have migrated by many kilometers in some areas throughout the postglacial period) are treated as 'fixed' or time dependent. The results of GIA analyses based on sea-level data obtained from regions far removed from the ancient ice masses (the 'far field') may be significantly biased if this mechanism is not incorporated into the sea-level model. Also, the sea-level theory most commonly adopted in GIA studies is shown to predict an incorrect water load redistribution in regions once covered by marine-based ice sheets (e.g., Hudson Bay). This leads to an error in the predicted relative sea-level (RSL) curves that is large enough to bias inferences of viscosity and/or ice thicknesses based on data obtained in these ('near-field') regions.

To date, most GIA sea-level analyses have assumed a non-rotating Earth model. That is, the component of sea-level change induced by GIA perturbations to the Earth's rotation vector is ignored. The first gravitationally self-consistent predictions of this sea-level component are presented in this thesis. These results show that previous estimates of the magnitude of the rotation-induced signal are too large by a factor of $3 - 5$. In contrast to conclusions based on these previous studies, the present results show that this signal is of too small a magnitude to be significant in RSL analyses based on near-field data.
However, estimates of Late Holocene melting events derived from modeling studies of far-field data can be significantly biased if this sea-level contribution is not accounted for.

Recent inferences of mantle viscosity based on convection-related data (e.g., the long-wavelength geoid) include a thin (∼100–200 km) low viscosity layer at the base of the upper mantle. Such fine-scale viscosity structure is commonly neglected in coarse (3-layer) parameterizations of mantle viscosity variation with depth. The sensitivity of a suite of GIA observables to the viscosity and thickness of such a layer is explored. In contrast to previous suggestions, the present results show that predictions of both RSL and secular change in the geoid are sensitive to this viscosity structure. Therefore, future analyses that employ these two data types to infer either mantle viscosity or changes in ice sheet mass distribution should adopt a viscosity-depth parameterization that can account for the possible existence of fine-scale, low-viscosity structure at the base of the upper mantle.
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To my parents, Len and Mary Milne
Contents

Abstract ii

Acknowledgments iv

List of Tables ix

List of Figures x

1 Introduction 1
  1.1 Glacial Isostatic Adjustment 1
  1.2 Contribution and Outline of Thesis 8
  1.3 Modeling the Glacial Isostatic Adjustment Process 10
    1.3.1 The Impulse Response Formalism 10
    1.3.2 Predicting GIA-Induced Sea-Level Change for a Non-Rotating Earth 15

2 Accurate Modeling of Water Load Redistribution in Near-Field and Far-Field Regions: Implications for Relative Sea-Level Predictions 19
  2.1 Introduction 19
  2.2 Theory: A Gravitationally Self-Consistent Sea-Level Algorithm that Incorporates a TDCM and Near-Field Water Dumping 22
2.2.1 An Efficient Method for Solving the Sea-Level Equation: The Pseudospectral Algorithm ........................................ 22
2.2.2 Extending the Pseudospectral Algorithm to Incorporate a TDCM and Near-Field Water Dumping .......................... 26
2.3 Results and Discussion ................................................. 31
  2.3.1 North Eastern Canada .............................................. 31
  2.3.2 The Australian Region ............................................. 41
2.4 Conclusions .............................................................. 47

3 Postglacial Sea-Level Change on a Rotating Earth .......... 51
  3.1 Introduction ........................................................... 51
  3.2 Theory ....................................................................... 53
    3.2.1 The Sea-Level Equation for a Rotating Earth .......... 53
    3.2.2 The Rotational Potential ......................................... 55
    3.2.3 Solving the New Sea-Level Equation ....................... 57
  3.3 Results ................................................................. 59
    3.3.1 Relative Sea-Level Predictions ................................. 59
    3.3.2 Present-Day Sea-Level Rate Predictions .................... 70
  3.4 Discussion ............................................................... 73
    3.4.1 Comparison to Previous Results ............................... 73
    3.4.2 The Significance of the Rotation-Induced Signal ........ 76
    3.4.3 The Influence of the M1 Relaxation Mode on the Rotation-Induced Component of Sea-Level Change ........................ 81
  3.5 Conclusions ............................................................. 82

4 The Sensitivity of GIA Predictions to a Low Viscosity Layer at the Base of the Upper Mantle ................................. 85
4.1 Introduction .................................................. 85
4.2 Results and Discussion ..................................... 88
  4.2.1 Viscosity Profiles ........................................ 88
  4.2.2 Relative Sea Level ....................................... 91
  4.2.3 $J_2$ and True Polar Wander Rate .................... 96
4.3 Conclusions .................................................... 103

5 Summary of Thesis ............................................ 105

A The RSL Decay Time Parameterization ......................... 109

B Predicting GIA-Induced Perturbations to the Earth's Rotation Vector 110

C Flow Charts of the Sea-Level Algorithms .................... 114
List of Tables

3.1 Predictions of Late Holocene differential high stands based on 'rotating' and 'non-rotating' Earth models .......................................................... 78

4.1 Predictions of Late Holocene high stands based on three different viscosity profiles .......................................................... 94

4.2 Predictions of Late Holocene differential high stands based on three different viscosity models .......................................................... 95
# List of Figures

1.1 Geographic extent of ice coverage at last glacial maximum and at present day based on ICE-3G model ........................................... 3

2.1 Schematic diagram illustrating the region in which the original pseudospectral algorithm predicts incorrect results when the continent margin is modeled in a time-dependent fashion ................................. 28

2.2 Schematic diagram illustrating the water load applied to sub-geoidal regions previously covered by ice ........................................... 29

2.3 ICE-3G ice thicknesses in north eastern Canada at 11 kyr BP and 10 kyr BP and the contemporaneous sea-load prediction ......................... 33

2.4 ICE-3G ice thicknesses in north eastern Canada at 9 kyr BP and 8 kyr BP and the contemporaneous sea-load prediction ............................... 34

2.5 Predictions of the sea-load increment in north eastern Canada at 11, 10, 9 and 8 kyr BP based on the standard theory ................................. 36

2.6 The difference between RSL predictions based on the standard sea-level theory to those based on the extended theory in north eastern Canada .... 38

2.7 RSL predictions at four sites in north eastern Canada ......................... 40

2.8 The predicted ocean-continent interface in the Australian region at the times 18, 14, 9 and 5 kyr BP ...................................................... 42

2.9 The difference between RSL predictions based on the standard sea-level theory to those based on the extended theory at the time of the Late Holocene high stand ...................................................... 43
2.10 Schematic diagram illustrating the error in the predicted sea load caused by assuming a fixed continent margin ................................................................. 44

2.11 RSL predictions in the Australian region .................................................. 46

2.12 The difference between RSL predictions based on the standard sea-level theory to those based on the extended theory shortly after LGM ............. 48

3.1 The postglacial RSL signal at Clinton caused by GIA-induced variations in the Earth’s rotation vector ................................................................. 59

3.2 The predicted global signal of RSL at 18 kyr BP produced by GIA-induced perturbations to the Earth’s rotation vector ........................................ 60

3.3 The predicted global signal of RSL at 18 kyr BP due to the perturbation to the rotational potential only ................................................................. 61

3.4 Postglacial sea-level curves showing predictions based on a rotating Earth compared to those based on a non-rotating Earth ................................. 64

3.5 Postglacial sea-level curves showing predictions based on a rotating Earth compared to those based on a non-rotating Earth ................................. 65

3.6 Predicted GIA-induced TPW and the corresponding ‘direct’, ‘elastic’ and ‘viscous’ sea-level responses ................................................................. 66

3.7 The rotation-induced component of RSL at Clinton for three different lower mantle viscosity values ................................................................. 68

3.8 The rotation-induced component of RSL at Clinton for three different values of lithospheric thickness ................................................................. 69

3.9 Present-day global sea-level rates caused by GIA-induced perturbations to the rotation vector ................................................................. 71

3.10 Present-day sea-level rates at Clinton due to GIA-induced change in the rotational potential as a function of lower mantle viscosity ................................. 72

3.11 Rotation-induced RSL calculated using the global ICE-3G model compared to calculations based on the Laurentide component of this model ................................. 74

3.12 The predicted degree 2 order 1 component of sea-level change induced by the rotational potential for the site Clinton ................................................................. 75
A fundamental property of Nature is not permanence, but change. This is beautifully exemplified in the processes affecting the Earth.

Adapted from Ranalli (1995), p. 5.
Chapter 1

Introduction

1.1 Glacial Isostatic Adjustment

Since the birth of the geophysical sciences, geophysicists have been concerned with determining the physical structure and chemical composition of the Earth's interior, and understanding the physical processes that have played, and continue to play, an important role in the ongoing evolution of our planet. Much of our current understanding of the Earth's composition and structure has been achieved by studying the manner in which the Earth responds to a specific type of physical forcing. For example, the response of the Earth to an applied stress field allows scientists to deduce rheological characteristics of our planet, whereas the response to an electromagnetic excitation enables the delineation of electric and magnetic properties. If the nature of the forcing is sufficiently well known and a set of data have been measured that describe the Earth's response to this forcing, then it is possible to infer Earth structure. Conversely, if certain features of Earth structure are well constrained, then it is possible to deduce the nature of the forcing from observations of the related Earth response. Regardless of the approach taken, the scientific methodology of the forward problem is well established: a physical model is created that simulates a real Earth process and is able to predict relevant observable quantities; the model parameters of interest are then varied until an acceptable fit to the observations is obtained. The best fitting model parameters are the end result of the process and represent the information attained from the modeling exercise. The work described in this thesis is concerned with the application of this methodology to the study of glacial isostatic adjustment (GIA), that is, the response of the Earth to past and present changes in surface ice mass distribution.
In the following, the nature of the Earth forcing associated with GIA is described and an overview of the current data types available for constraining models of the GIA process is presented (most of those discussed in this section are considered at various points in the following chapters). Also, a brief introduction to the current state-of-the-art in GIA modeling research is included. This is intended to provide the reader with an adequate perspective from which to view the research goals of this thesis.

Paleoclimate records show clear evidence that the volume of ice on the Earth’s surface has gone through a series of quasi-periodic variations over the past ~2 Myr (e.g., Shackleton and Opdyke 1973), with a cycle period of approximately 100 kyr evident over the last 1 Myr. Each cycle is characterized by a period of steady growth in ice volume (usually termed the glaciation phase), spanning ~90 kyr, followed by a relatively rapid reduction in ice volume (deglaciation phase), spanning ~10 kyr. At present, the global ice budget is near a minimum, since the major melting events of the last deglaciation phase ceased around 5 kyr before present (BP). The most recent glacial maximum is believed to have occurred around 20 kyr BP, at which time the volume of ice was approximately three times that of the present value. A model of the geographic extent of land-based ice sheets at last glacial maximum (LGM) is shown in Figure 1.1 along with the present-day ice coverage.

The growth and decay of the ancient ice sheets represents a dramatic global scale redistribution of ice-water mass. Throughout a typical glaciation phase, global sea level is lowered on the order of 100 m as this water volume is redistributed to the expanding ice masses. This procedure is reversed at a significantly faster rate during the deglaciation phase. The time scale of this climatically induced change in the global stress field excites a rheological response of the Earth that is characterized by both elastic and non-elastic components. Therefore, in order to realistically model the GIA process, it is necessary to adopt a rheological model that incorporates these two response types (see section 1.3.1).

The Earth’s response to the Late Pleistocene cycles of ice growth and decay is observable via a number of different phenomena. The change in sea level since LGM is clearly evident in the geological record. A global data base of past sea-level variations relative to present-day sea level has been significantly expanded over the past few decades (e.g., Pirazzoli 1991). The relative sea-level (RSL) signals evident in these data are characterized by a complex spatial and temporal dependence that is now understood to be largely a result of the GIA process. For example, in areas once loaded by the ancient ice masses, the predicted rebound of the solid surface is reflected in records showing that sea level has
Figure 1.1: Polar projections of ice extent at last glacial maximum (LGM) (based on the ICE-3G model of Tushingham and Peltier (1991)) and at present. The top projections show ice thickness in the northern hemisphere from the pole to a latitude of 30° and the lower projections show ice thickness from the south pole to a latitude of -30°.
fallen by hundreds of meters over the past ~12 kyr. In contrast, at sites far removed from these regions, the sea-level record shows a continual rise throughout the deglaciation phase (~20 ka BP to ~5 ka BP), which is primarily caused by the addition of meltwater to the oceans.

The Earth's rotational state is also affected by the Late Pleistocene ice loading cycles. This state is conventionally decomposed into two observable quantities: the change in the location of the Earth's rotation axis relative to the surface geography (true polar wander (TPW)) and the non-tidal acceleration in rotation rate. TPW has been monitored over the last ~100 yr and the observed signal shows that the secular motion of the spin pole (that is, the residual signal remaining after short term fluctuations such as the annual and Chandler wobble are removed) is characterized by a rate of ~1° per Myr with a direction southwards along the meridian defined by ~70° E (approximately in the direction of Hudson Bay). A number of early articles showed that this signal is compatible with realistic models of the Late Pleistocene GIA process (e.g., Nakiboglu and Lambeck 1980; Sabadini, Yuen and Boschi 1982; Wu and Peltier 1984), and it thus became conventional to assume that the observed signal was almost entirely due to the Earth's viscous 'memory' of the Late Pleistocene ice loading events. However, recent work suggests that the observed signal is likely also affected by processes other than the Earth's ongoing response to past ice-water loading events. For example, convective flow of mantle material may account for ~40% of the observed signal (Steinberger and O'Connell 1997). Also, the present-day mass balance of large ice sheets (such as Greenland, for example) may induce a significant TPW signal (James and Ivins 1997).

Non-tidal acceleration in the rotation rate of the Earth is a consequence of change in the moment of inertia about the Earth's spin axis. Since the moment of inertia about this axis is linearly related to the degree two zonal harmonic of the Earth's potential field ($J_2$), it follows that non-tidal secular change in rotation rate is also linearly related to $J_2$. This parameter can be measured by monitoring temporal variations in the orbit of an Earth satellite (e.g., Garland 1982). Observations of this type indicate that non-tidal processes are presently causing an increase in the spin rate of the Earth. Similar to the case of GIA-induced TPW predictions, it was initially believed that the observed signal was largely due to the isostatic disequilibrium and adjustment following the last deglaciation phase. However, it is now widely accepted that phenomena such as the present-day mass balance of small mountain glaciers and large ice sheets may contribute a significant signal (e.g., Yoder and Ivins 1985; Mitrovica and Peltier 1993b). Also, recent work indicates that tectonic processes (Vermeersen, Sabadini, Spada and Vlaar
1994) and pressure fields at the core-mantle boundary (e.g., Fang, Hager and Herring 1996) can produce a non-negligible $J_2$ signal.

Higher degree harmonic coefficients of Earth's time-varying gravity field can also be measured by monitoring the perturbations to the orbit of an Earth satellite. At present these data are well enough constrained (up to degree and order $\sim 6$) to be adopted in the GIA modeling procedure (e.g., Cheng, Shum and Tapley 1997). However, as for the case of $J_2$, present-day melting events are not well enough constrained to make this exercise unambiguous.

The Earth's gravity field displays a signature that is related to the past ice-water loading events. The largest signal is found in regions that were once loaded by the now disintegrated ice sheets. The solid surface is presently rebounding in these regions as the Earth relaxes towards a state of isostatic equilibrium. The current state of disequilibrium leads to the occurrence of a large negative free air gravity anomaly in these locations (e.g., north eastern Canada). However, attempts to model this GIA signature have not led to useful constraints due to the fact that mantle convection may contribute the dominant portion of the observed anomaly (e.g., Mitrovica and Forte 1997).

A final data set that can be used to constrain models of the GIA process consists of observations of 3D motions of the Earth's crust (James and Lambert 1993; Mitrovica, Davis and Shapiro 1993, 1994). Again, the signal is largest in 'near-field' regions (areas in the immediate vicinity of the Earth's past and present ice masses), where observations are being obtained via the use of GPS monitoring. For example, the BIFROST (Baseline Inferences for Fennoscandian Rebound Observations, Sea Level, and Tectonics) project in Fennoscandia (Davis et al. 1996) is composed of a network of 24 bedrock-mounted, continuous recording GPS receivers. The precision of the GPS observations improves with increasing monitoring time. The BIFROST project has been operational for approximately 6 years and the estimated error is now of a small enough magnitude to reveal a clear GIA component in the observed crustal deformation. Attempts to employ these data to constrain models of the GIA process in Fennoscandia are in progress (Davis, personal communication, 1997).

When adopting one or more of the above mentioned data types to constrain Earth rheology, it is conventional to assume an elastic structure that is compatible with seismic constraints (e.g., the PREM Earth model (Dziewonski and Anderson 1981)). The Earth's viscosity structure can then be constrained via the forward modeling process outlined in the first paragraph of this section. It is important to point out, however, that there ex-
ists a fundamental ambiguity inherent in modeling the GIA process that complicates this procedure: the Earth forcing (i.e., the space-time dynamics of the ice mass fluctuations) is not precisely known. All of the previously discussed observable GIA signatures are sensitive to unknowns in the GIA forcing. For example, RSL variations in the near field are especially sensitive to details of the Late Pleistocene ice loading (e.g., Tushingham and Peltier 1992). Also, anomalies associated with the rotational state of the Earth and the time variation of the low degrees of the geopotential, although less sensitive to details of the Late Pleistocene ice loading history, are sensitive to present-day ice-water mass exchange (and, possibly, a number of other non-GIA related processes).

The uncertainty in the GIA forcing is a major obstacle to researchers involved in GIA modeling. In recent years, a small number of articles have suggested methods to address this issue. As mentioned above, near-field RSL data are sensitive to details of the Late Pleistocene ice load history. This sensitivity can be reduced by considering a parameterization of recent (approximately the past 6 kyr) RSL data obtained from regions close to the center of the ancient ice masses (Mitrovica and Peltier 1993a) (see Appendix A). This parameterization acts to separate the ice dependence component of the sea-level response from the viscous Earth response component, and thus enables a more robust estimate of viscosity structure to be obtained. A second method that reduces data sensitivity to the ice load involves considering RSL observations from sites that are distant from the locations of the Late Pleistocene ice masses (the 'far field'). In such regions, the change in sea level associated with the meltwater signal is the dominant surface load. By modeling the difference in observed RSL between nearby far-field sites, the influence of the ancient ice masses is reduced (Nakada and Lambeck 1989). In analyses of this type, the magnitude of the observed signal is relatively small (on the order of a few meters) and so the sea-level model adopted to predict sea-level differentials in far-field regions is required to be accurate to less than ~1 m.

A second recent advance in GIA modeling involves the improved understanding of the viscosity depth sensitivities (or, more strictly, the resolving power) of various data sets. The resolving power of RSL data from a particular region largely depends on the scale of the ice mass that once loaded that region. Thus, the RSL data most appropriate for resolving deep viscosity structure is located in the vicinity of north eastern North America, a region once covered by the massive Laurentide ice sheet (data from this region can resolve viscosity structure to a depth of ~1800 km (e.g., Mitrovica and Peltier 1995)). The ancient ice sheet once covering Scandinavia was significantly smaller than the Laurentide ice mass (see Figure 1.1) and so the RSL data from this region display a poorer
deep mantle sensitivity. However, these data show greater sensitivity at depths around the transition zone between the upper and lower mantle, and thus supply complementary information to the North American data. In a similar manner, due to the relatively small scale of British ice sheet, RSL data from the British region can potentially resolve shallow structure within the upper mantle. In the far field, the relatively large spatial scale of the ocean load suggests that data from these regions will be sensitive to deep mantle structure.

Careful studies of selected subsets of RSL data are capable, then, of allowing relatively robust inferences of radial viscosity structure at various depths within the mantle. The resolving power of the TPW and $J_2$ observables, assuming the Late Pleistocene GIA component of the observed signal is known, would complement the RSL data well, as the rotation data are most sensitive to viscosity in the deepest mantle (e.g., Peltier 1996). However, since this component of these data is not well constrained, an alternative data type must be sought that is sensitive to deep viscosity structure. The long wavelength convection-related data sets such as the low degree coefficients of the non-hydrostatic geoid or the free air gravity anomaly fit this criterion well. A recent study has made use of these data sets together with near-field (GIA-related) RSL data in a joint inversion for the radial viscosity profile (Mitrovica and Forte 1997). The results of this study indicate that observations of the GIA process and the mantle convection process can be predicted to a good degree of accuracy using the same viscosity profile. This has important implications for previous ideas regarding the potentially transient nature of mantle rheology (e.g., Sabadini, Yuen and Gasperini 1983).

The above discussion has focused on the modeling approach that involves assuming the forcing function known in order to constrain Earth viscosity structure. A second methodology that involves adopting an a priori preferred viscosity structure in order to obtain information related to a specific Earth forcing is becoming more common given the current interest in global climate change. One example involves predicting the present-day global sea-level rate signal associated with the Earth's ongoing response to the last deglaciation in order to filter this signal from the global tide gauge data set (Peltier and Tushingham 1991). The residual signal can then be used to better estimate any globally coherent sea-level rate value, a potential indicator of climate change. Also, as stated above, the time rate of change of the long wavelength geopotential is known to be sensitive to the present-day mass balance of large high latitude ice sheets (e.g., Antarctica, Greenland). By adopting a 'best fit' mantle viscosity profile, it is possible to remove the GIA signature from the observed signal and use the residual to estimate any ongoing mass flux...
to/from these ice complexes.

1.2 Contribution and Outline of Thesis

The principal focus of the work presented in this thesis is to investigate the effect of a number of postulated 'second order' mechanisms on predictions of postglacial sea-level change (the term 'postglacial' refers to the time period between LGM and the present day). Such a study is particularly relevant given that RSL observations are currently the most effective data type for constraining models of the GIA process (see last section).

In Chapter 2, I consider the importance of accurately modeling the time-varying geometry of the continental margin in both near-field and far-field regions. In far-field regions, the change in sea level constitutes the dominant contributor to the surface load and therefore has an important affect on the predicted RSL signal. In recent years, a number of articles have appeared that discuss how to model changes in the continent margin due to GIA-induced changes in sea level (e.g., Johnston 1993). However, none of these analyses have considered the effect of a time-dependent continent margin (see section 1.3.2) on RSL predictions in the far field, where its effect is likely the most significant. In Chapter 2, I describe how to implement a time-dependent continent margin into the framework of a widely used approach for calculating GIA-induced sea-level change. This modified calculator is then applied to predict RSL in the Australian region where data have been obtained and employed to constrain Earth model and ice model parameters (Nakada and Lambeck 1989).

RSL data from near-field regions have been extensively employed to constrain models of the GIA process. In Chapter 2, I describe how the standard sea-level theory, when applied to regions once loaded by marine-based ice sheets, produces inaccurate predictions of the sea load at the times when the retreating ice mass is replaced by in-flooding meltwater.sea water. To my knowledge, this problem has not been previously addressed in the literature. A method for modeling this influx of water to previously glaciated regions while incorporating a time-dependent continent margin is introduced. This new calculator is then employed to predict RSL curves in north eastern Canada to determine the significance of this effect on predictions of RSL.

To date, mantle viscosity estimates based on RSL data have adopted a sea-level theory derived for a non-rotating Earth model; that is, only the sea-level change associated with the changing surface load has been considered in these modeling studies. However,
as mentioned in section 1.1, the rotational state of the Earth is affected by the surface mass redistribution associated with the mass balance of the Earth's past and present ice sheets, and this perturbation to the rotation vector will also influence sea-level change. In Chapter 3, I describe a new sea-level calculator that can be applied to predict GIA-induced sea-level change on a rotating Earth model. This new calculator (which also includes the time-dependent continent margin implemented in Chapter 2) is then adopted to predict RSL for a suite of different Earth models and a globally distributed range of sites. In particular, I examine the difference between these predictions and those based on a non-rotating Earth model.

I also consider the effect of the rotational excitation on predictions of global present-day sea-level rate. As an example, rotation-induced sea-level rates are predicted for sites along the US east coast where there is a high concentration of tide gauge stations, and where the predicted rotation-induced signal is relatively large.

A small number of articles have appeared in the literature that predict very different rotation-induced RSL signals (Han and Wahr 1989; Milne and Mitrovica 1996; Bills and James 1996). I compare and explain the considerable difference between my predictions of the rotation-induced component of sea-level change and those of Han and Wahr (1989) and Bills and James (1996). I then go on to address the significance of the rotation-induced sea-level signal to previous GIA modeling studies of RSL data that did not consider this sea-level excitation.

In Chapter 4, I diverge from the focus on GIA-induced sea-level change to consider the sensitivity of a suite of GIA-related observables to the existence of a thin, low viscosity layer at the base of the upper mantle. The motivation for examining the sensitivity of GIA observables to this viscosity feature arose, initially, from the modeling results of Mitrovica and Forte (1997). These results indicate that the joint data set employed (comprising of rebound and convection-related data) displays a preference for this fine scale viscosity structure. A number of recent modeling studies of the long wavelength non-hydrostatic geoid also prefer a low viscosity region at this depth (e.g., King and Masters 1992; Forte, Dziewonski and Woodward 1993). In contrast, other studies have suggested that GIA data are insensitive to such structure (Peltier 1996). I consider the sensitivity of a number of disparate GIA observables, including RSL change, the time rate of change of the low degree harmonic coefficients of the geopotential and anomalies in the Earth's present rotational state (i.e., TPW and change in the length of day) to the viscosity and thickness of this feature. The two main goals of this chapter are to
determine whether the GIA data can be used to help constrain this fine scale viscosity structure and to consider the implications of omitting such a viscosity feature in commonly adopted parameterizations of the radial viscosity profile.

Finally, in Chapter 5 the main points of Chapters 2 through 4 are summarized in the context of GIA research, and the current outlook of research in the GIA field is briefly discussed.

In the following section, two well established components of the theoretical formalism adopted to predict the GIA observables discussed in section 1.1 are outlined. The first of these is concerned with calculating the response of a specified Earth model to a general surface load. Following this, a section is included on the theory of surface load-induced sea-level change for a non-rotating Earth.

1.3 Modeling the Glacial Isostatic Adjustment Process

1.3.1 The Impulse Response Formalism

To model the Earth's response to a general surface load, the impulse response formalism of Peltier (1974) is adopted. This formalism considers the response of a spherically symmetric, Maxwell viscoelastic Earth model to a stress excitation that is instantaneous in time and point-like in space. Once the Earth model response to this idealized forcing is known, the response to a general space-time load can be obtained by convolving, in space and time, a generalized load with the impulse response. In the following, the principal components of this theory are outlined.

In its equilibrium state, before the application of the impulse load, the Earth model is in a state of gravitationally induced hydrostatic pre-stress. If small displacements are assumed when the body is perturbed, its response will satisfy the following linearized version of the momentum equation:

\[ \nabla . \hat{T}_1 + \nabla . (\hat{u} . \nabla ) \hat{T}_0 - \rho_1 \nabla \phi_0 - \rho_0 \nabla \phi_1 = 0, \quad (1.1) \]

where \( \hat{T}_0, \rho_0 \) and \( \phi_0 \) are, respectively, the stress tensor, the density and the gravitational field in the equilibrium state (these parameters are a function of radius, \( r \), only). \( \hat{T}_1, \rho_1 \)
and $\phi_1$ are the perturbations to these respective fields and $\bar{u}$ is the load-induced vector displacement field. Note that the $\phi_1$ term is the sum of two effects: the perturbation to the background potential due to the direct effect of the applied load (which is conventionally denoted by $\phi_2$) and the perturbation due to internal mass redistribution ($\phi_3$) (i.e., $\phi_1 = \phi_2 + \phi_3$). Also note that the particle acceleration (i.e., inertia) term is neglected in this application of the momentum equation since the GIA process operates on a time scale of $\sim 10$ kyr.

The first term in equation (1.1) denotes the stress field gradient perturbing the body from its equilibrium state. The second term describes the property known as the advection of pre-stress, which occurs when a pre-stressed elemental volume is displaced through the background hydrostatic stress field. The third term is a consequence of the equilibrium gravity field acting on the perturbation to the background density field, while the fourth term is, in contrast, the perturbation to the background gravity field acting on the equilibrium density structure.

In addition to equation (1.1), two other relations must be simultaneously satisfied. To ensure that matter is conserved, the continuity equation is applied. This equation relates perturbations in the background density field to the divergence of the displacement field, and is written (to first order) as,

$$\rho_1 = -(\nabla \cdot \bar{u})\rho_0.$$  \hspace{1cm} (1.2)

Second, to ensure that any changes in the mass distribution influence the potential field, one can apply Poisson's equation (again to first order),

$$\nabla^2 \phi_1 = -4\pi G(\nabla \cdot \bar{u})\rho_0.$$ \hspace{1cm} (1.3)

In order to predict the GIA observables discussed in the previous section, solutions to equation (1.1) are sought for $\bar{u}$ and $\phi_1$. Therefore, the unknown fields $\rho_1$ and $\bar{T}_1$ must be expressed in terms of the displacement field, $\bar{u}$, and the perturbation to the gravitational potential, $\phi_1$. Equation (1.2) allows $\rho_1$ to be substituted for an expression involving the displacement field. To replace the $\bar{T}_1$ term, a constitutive equation is chosen that relates the perturbing, deviatoric stress field ($\bar{T}_1$) to the displacement field. As previously mentioned, a Maxwell viscoelastic rheology is adopted. The constitutive equation for this
Chapter 1: Introduction

Rheology takes the mathematical form,

\[ \tau_{ij} + \frac{\mu}{\nu} (\tau_{ij} - \frac{1}{3} \tau_{rr} \delta_{ij}) = 2\mu \dot{\varepsilon}_{ij} + \lambda \dot{\varepsilon}_{rr} \delta_{ij}, \]  \hspace{1cm} (1.4)

where \( \tau_{ij} \) is the \( ij \)th component of \( \vec{T}_1 \), \( \lambda \) and \( \mu \) are the elastic Lamé parameters, \( \nu \) is viscosity and \( \delta_{ij} \) is the Kroneker delta symbol (which equals unity when \( i = j \), and zero otherwise). The dot above certain terms denotes time differentiation and repeated indices follow the summation convention (i.e., \( \tau_{rr} = \tau_{11} + \tau_{22} + \tau_{33} \)).

At this point, the usual procedure (Peltier 1974) is to invoke the so-called correspondence principle (e.g., Biot 1954). A first step in this procedure involves determining the Laplace transform of equation (1.4). Doing this gives,

\[ \hat{\tau}_{ij} = \lambda(s) \dot{\varepsilon}_{rr} \delta_{ij} + 2\mu(s) \dot{\varepsilon}_{ij}, \]  \hspace{1cm} (1.5)

where the inverted 'hat' denotes implicit dependence on the Laplace transform variable \( s \), and \( \lambda(s) \) and \( \mu(s) \) are given by,

\[ \lambda(s) = \lambda + \frac{2\mu(s)}{s + \frac{s}{\nu}}, \quad \mu(s) = \frac{\mu s}{s + \frac{s}{\nu}}. \]  \hspace{1cm} (1.6)

Equation (1.5) is identical to the constitutive equation for a Hookean elastic solid, with the exception that the Lamé parameters are functions of the transform variable \( s \). This is the basis of the correspondence principle which states that the Laplace transform domain solution of the viscoelastic problem can be obtained by solving the elastic problem and by substituting \( \lambda(s) \) for \( \lambda \) and \( \mu(s) \) for \( \mu \). The time-domain solution of the visco-elastic problem is obtained by solving this system for a wide range of \( s \) values and then inverting the transform.

To apply the correspondence principle, a Laplace transformed version of equation (1.1) is adopted in which the \( \rho_1 \) term is replaced by \(-\nabla \cdot \vec{u}\rho_0\) and the deviatoric stress components are replaced by strains via the constitutive equation (1.5). This equation is then solved to obtain \( s \) dependent perturbations to the gravitational potential and the displacement field. To ensure that the solutions are gravitationally self consistent, the Laplace transform of the Poisson equation (1.3) must be solved simultaneously. Details of the above described procedure are not necessary here. The reader is referred to Peltier (1974) and Wu and Peltier (1981) for a detailed discussion of this mathematical/numerical exercise.
Chapter 1: Introduction

The time domain form of the solutions to the viscoelastic problem may be expressed in terms of an infinite series of Legendre polynomials,

\[ u(a, \gamma, t) = \sum_{\ell=0}^{\infty} u_{\ell}(a, t) P_{\ell}(\cos(\gamma)) \]  
\[ v(a, \gamma, t) = \sum_{\ell=0}^{\infty} v_{\ell}(a, t) P_{\ell}(\cos(\gamma)) \]  
\[ \phi_{1}(a, \gamma, t) = \sum_{\ell=0}^{\infty} \phi_{1,\ell}(a, t) P_{\ell}(\cos(\gamma)) , \]

where \( u(a, \gamma, t) \) and \( v(a, \gamma, t) \) are the radial and tangential components of the induced displacement field at the Earth's undeformed surface (defined at \( r = a \)), and \( \phi_{1}(a, \gamma, t) \) is the perturbation to the geopotential on this surface. (Note that the tangential component of the impulse response deformation field on a spherically symmetric Earth model must lie in a direction away from or towards the point of application of the impulse load.) The \( P_{\ell} \) are Legendre polynomials and \( \gamma \) is the great circle angle between the impulse load point and the observation point. The Legendre coefficients in equations (1.7) to (1.9) are,

\[ u_{\ell}(a, t) = u_{E}^{\ell} \delta(t) + \sum_{k=1}^{K} \delta u_{k,\ell}^{NE} \exp(-s_{k}^{t} t) \]  
\[ v_{\ell}(a, t) = v_{E}^{\ell} \delta(t) + \sum_{k=1}^{K} \delta v_{k,\ell}^{NE} \exp(-s_{k}^{t} t) \]  
\[ \phi_{1,\ell}(a, t) = \phi_{1,E}^{\ell} \delta(t) + \sum_{k=1}^{K} \delta \phi_{1,k,\ell}^{NE} \exp(-s_{k}^{t} t) , \]

in which the \( \delta(t) \) term denotes the Dirac delta function. From (1.10) to (1.12), it is evident that the temporal aspect of the response is composed of an immediate elastic component, defined by \( u_{E}^{\ell} \), \( v_{E}^{\ell} \) and \( \phi_{1,E}^{\ell} \), and a non-elastic component (which, notably, has a non-zero magnitude at the instant the load is applied). The non-elastic component consists of a series of exponential functions, each of which are weighted by an amplitude \( \delta u_{k,\ell}^{NE} \), \( \delta v_{k,\ell}^{NE} \) and \( \delta \phi_{1,k,\ell}^{NE} \) and are characterized by an inverse decay time \( s_{k}^{t} \).

Instead of directly using \( u(a, \gamma, t) \), \( v(a, \gamma, t) \) and \( \phi_{1}(a, \gamma, t) \) to predict observables associated with GIA, it has become conventional to use a triplet of non-dimensional numbers known as viscoelastic load Love numbers. These are related to the displacement field
and the perturbation to the geopotential in the following manner,

\[
\begin{pmatrix}
  h^L(t) \\
  l^L(t) \\
  k^L(t)
\end{pmatrix}
= \frac{g}{\phi_{2,\ell}(a)}
\begin{pmatrix}
  u_\ell(a, t) \\
  v_\ell(a, t) \\
  -\frac{\phi_{2,\ell}(a, t)}{g}
\end{pmatrix},
\]  

(1.13)

where \(g\) is the Earth's surface gravitational acceleration and \(\phi_{2,\ell}(a)\) represents the degree \(\ell\) component in the Legendre expansion of the perturbation to the surface gravity field caused by the direct attraction of the surface load. The parameters \(h^L, l^L\) and \(k^L\) are the surface load Love numbers associated with the radial displacement field, the tangential displacement field and the perturbation to the gravitational field, respectively. The subscript \(L\) denotes that these Love numbers are associated with the surface mass loading problem. (There also exists a set of tidal Love numbers that describe the response of an Earth model to a general potential forcing that does not load the surface of the Earth (e.g., body tides due to the sun and moon; the rotational potential associated with the spin of the Earth).)

The explicit time dependence of the load Love numbers follows from equation (1.10) through to equation (1.13),

\[
h^L(t) = h^{L,E}\delta(t) + \sum_{k=1}^{K} r^{L,L}_{k}\exp(-s^L_k t) \\
\]  

(1.14)

\[
l^L(t) = l^{L,E}\delta(t) + \sum_{k=1}^{K} r^{L,L}_{k}\exp(-s^L_k t) \\
\]  

(1.15)

\[
k^L(t) = k^{L,E}\delta(t) + \sum_{k=1}^{K} r^{L,L}_{k}\exp(-s^L_k t). \\
\]  

(1.16)

In analogy to the above expressions for the displacement field components \(u\) and \(v\) and the perturbation to the geopotential \(\phi_1\), the time dependence of the Love numbers is characterized by an immediate elastic component (the first term in the above three equations) and a series of weighted exponential functions that decay exponentially with time subsequent to the impulse loading.

The surface observables discussed in the previous section (with the exception of TPW) can be calculated using the load Love numbers introduced above. As an example, this thesis is largely concerned with the prediction of sea-level change; this requires the calculation of the radial displacement of the solid surface and the perturbation to the Earth's
gravitational potential at the undeformed surface. The load-induced impulse responses for these quantities are, respectively,

\[ \Gamma^L(\gamma, t) = \frac{a}{M_e} \sum_{t=0}^{\infty} h^L_t P_z(\cos \gamma), \] (1.17)

and

\[ \Upsilon^L(\gamma, t) = \frac{aq}{M_e} \sum_{t=0}^{\infty} (\delta(t) + k^L_t) P_z(\cos \gamma). \] (1.18)

In equations (1.17) and (1.18), \( M_e \) represents the mass of the Earth and the \( \delta(t) \) term accounts for the perturbation to the Earth’s gravitational field due to the presence of the surface load. The result \( \phi_{2,\epsilon}(a) = \frac{aq}{M_e} \) is used in equations (1.17) and (1.18). Equations (1.17) and (1.18) are known as, respectively, the radial displacement and gravitational potential Green’s functions for the surface loading problem.

The Earth’s response to a more general surface load can be determined by calculating the space-time convolution of the general load function with the relevant Green’s function. For example, the deformation of the solid surface induced by a general load, \( L(\theta, \psi, t) \), where \( \theta \) is co-latitude and \( \psi \) is east longitude, is given by,

\[ R^L(\theta, \psi, t) = \int_{-\infty}^{t} \int_{\Omega} a^2 L(\theta', \psi', t') \Gamma^L(\gamma, t - t') d\Omega' dt', \] (1.19)

where \( R \) is the surface radial displacement and \( \Omega \) signifies spatial integration over the unit sphere. Similarly, the perturbation to the geopotential on the undeformed surface \( (r = a) \) is given by,

\[ \Phi^L(\theta, \psi, t) = \int_{-\infty}^{t} \int_{\Omega} a^2 L(\theta', \psi', t') \Upsilon^L(\gamma, t - t') d\Omega' dt'. \] (1.20)

The following section describes how equations (1.19) and (1.20) can be applied to predict GIA-induced sea-level change for a non-rotating Earth.

### 1.3.2 Predicting GIA-Induced Sea-Level Change for a Non-Rotating Earth

In GIA studies of sea-level change, it is conventional to define sea level as the vertical height difference between the ocean surface, or geoid, and the Earth’s solid surface. The change in sea level due to an applied surface load is, therefore, simply the height change of the geoid minus the height change of the solid surface. This can be expressed as,

\[ S(\theta, \psi, t) = C(\theta, \psi)[G^L(\theta, \psi, t) - R^L(\theta, \psi, t)], \] (1.21)
where the function $S(\theta, \psi, t)$ defines sea-level change at some time $t$ after the onset of the loading period; $C(\theta, \psi)$ is the ocean function, defined to be unity over ocean regions and zero over land regions (Munk and MacDonald 1960), and $G^L(\theta, \psi, t)$ and $R^L(\theta, \psi, t)$ denote the load-induced perturbations to the geoid and solid surface, respectively.

The load-induced deformation of these two surfaces can be calculated by adopting the impulse response formalism outlined above. The deformation of the Earth's solid surface is calculated directly from (1.19). The deformation of the geoid surface is obtained by considering the perturbation to the geopotential at the Earth's undeformed surface. Since the ocean is a fluid, its equilibrium surface (i.e., the surface the geoid would define if the ocean were unperturbed by the winds and tides) must lie on an equipotential of the Earth's gravitational field. Thus, the ocean surface, or geoid, must deform to reflect any perturbations in the gravity field. To first order, the magnitude of geoid warping is given by,

$$G^L(\theta, \psi, t) = \frac{\Phi^L(\theta, \psi, t)}{g}, \quad (1.22)$$

where $G^L(\theta, \phi)$ represents the incremental height shift of the geoid and $\Phi^L(\theta, \psi, t)$ is the load-induced perturbation to the gravity field (given by equation (1.20)). Equation (1.22) is simply a statement of Bruns' formula (e.g., Heiskanen and Moritz 1967). To accurately determine the height shift of the geoid due to GIA, processes that affect the mean height of this surface also require consideration. Throughout a glacial cycle, the water volume contained within the ocean basins varies due to mass transfer between the ice sheets and the oceans. This change in ocean water volume produces a globally uniform shift in geoid height. Also, the load-induced deformation causes a net volume shift between the geoid and the solid surface (over ocean regions) which acts to produce a uniform height shift of the geoid. Consideration of these two processes ensures that ice-water mass is conserved. The total shift of the geoid is then,

$$G^L(\theta, \psi, t) = \frac{1}{g}\Phi^L(\theta, \psi, t) + G(t), \quad (1.23)$$

where,

$$G(t) = \frac{M_I(t)}{\rho_w A_o} - \frac{1}{A_o} \langle G^L(\theta, \psi, t) - R^L(\theta, \psi, t) \rangle, \quad (1.24)$$

in which $M_I(t)$ is the total mass of the ice sheets, $A_o$ is the area of the oceans and the angle brackets denote a spatial integration over the oceans.

Substituting equations (1.19), (1.20) and (1.23) into equation (1.21) yields the following expression for GIA-induced sea-level change on a Maxwell viscoelastic planet,
Chapter 1: Introduction

\begin{equation}
S(\theta, \psi, t) = C(\theta, \phi) \left[ \int_{-\infty}^{t} \int_{\Omega} a^2 L(\theta', \psi', t') \left\{ \frac{\gamma^L(\gamma, t-t')}{g} - \gamma(\gamma, t-t') \right\} d\Omega' dt' + \mathcal{G}^L(t) \right]. \tag{1.25}
\end{equation}

Equation (1.25) is termed the ‘sea-level equation’ and was first introduced by Farrell and Clark (1976).

It is important to note that the function describing the surface load redistribution has both an ice component and an ocean component, and is conventionally written in the form,

\begin{equation}
L(\theta, \psi, t) = \rho_I I(\theta, \psi, t) + \rho_W S(\theta, \psi, t), \tag{1.26}
\end{equation}

where \( \rho_I \) and \( \rho_W \) are the densities of ice and water respectively and \( I(\theta, \psi, t) \) is a model of the space-time geometry of the ancient ice sheets. Equation (1.25) is an integral equation since the unknown functional \( S(\theta, \psi, t) \) also appears on the right-hand-side of the formula as an implicit component of the load, \( L(\theta, \psi, t) \). Accurate solutions of equation (1.25) are generally obtained via a process of iteration (e.g., Clark, Farrell and Peltier 1978; Mitrovica and Peltier 1991; Johnston 1993).

The integral nature of the sea-level equation reflects the interdependence between the ocean surface, or geoid, and the internal and external mass redistribution. A change in the surface mass redistribution perturbs the geopotential by its direct gravitational effect and by inducing a mass redistribution within the Earth. This perturbation to the geopotential deflects the geoid which, in turn, causes a redistribution of the ocean load. It is necessary to incorporate the loading effect of this secondary mass redistribution in order to obtain an accurate solution of the sea-level equation. That is, one must ensure that the predicted sea-level change is consistent, in a gravitational sense, with the total mass redistribution. Solutions of the sea-level equation which account for this interdependence are, accordingly, termed gravitationally self consistent.

Solutions of the sea-level equation (Farrell and Clark 1976; Clark et al. 1978) have indicated that many features of the observed global sea-level record are a result of the redistribution of ice-water mass occurring during the most recent deglaciation phase of the current ice age. The largest signal, not surprisingly, is found in near-field regions. The rebound of the solid surface is the dominant process resulting in a sea-level fall of several hundreds of meters from LGM to the present day. Observed sea-level curves from regions immediately outside the the central location of the large ice masses are characterized by an initial sea-level fall followed by a sea-level rise. This is the result of an initial crustal uplift evolving into a crustal subsidence as the solid surface bulge peripheral to the ice
masses migrates towards the center of the region of uplift. At a further distance from the large ice masses, the solid surface deformation takes the form of monotonic subsidence associated with the collapsing peripheral bulge. This subsidence causes a sea-level rise that adds to the signal associated with the addition of meltwater to the oceans.

Far removed from the locations of the Late Pleistocene ice sheets, the meltwater signal is dominant and the influence of the ocean load becomes more evident. For example, RSL data from far-field sites exhibit a continual sea-level rise throughout the postglacial period until approximately 5 kyr BP when all major melting events are believed to have essentially ceased. After this time most far-field data indicate a subtle sea-level fall of a few meters, resulting in a RSL 'high stand' at the time when major melting events ended. Two mechanisms are postulated to account for this sea-level fall. One, termed 'continental levering', is a result of the load differential across the ocean-continent interface producing a tilting of the crust in these regions (e.g., Clark et al. 1978). A second mechanism involves the ice load-induced deformation of the solid surface. The Late Pleistocene ice sheets were generally located in continental regions and so solid surface deformation in ocean areas consists of large regions of peripheral bulge subsidence during the deglaciation period. This subsidence results in a continuous, spatially uniform drop in the height of the geoid as water fills the regions vacated by the subsiding bulges. This mechanism is termed 'equatorial ocean syphoning' (Mitrovica and Peltier 1991). Neither continental levering nor ocean syphoning are large effects relative to the change in sea-level due to mass loss from the Late Pleistocene ice sheets. As a consequence, these effects are only evident in the RSL records after major melting events have ceased (i.e., after ~5 kyr BP).
Chapter 2

Accurate Modeling of Water Load Redistribution in Near-Field and Far-Field Regions: Implications for Relative Sea-Level Predictions

2.1 Introduction

The sea-level equation introduced in Chapter 1 governs load-induced sea-level change on a non-rotating viscoelastic Earth model. The work presented in this chapter considers two approximations relating to predictions of the water load redistribution that are based on the sea-level equation. The significance of these with regard to predicting RSL in both near-field and far-field regions is addressed.

In their derivation of the sea-level equation, Farrell and Clark (1976) assumed that the continent function defining the perimeter of the ocean load was time independent. The majority of subsequent articles have adopted the same approximation (e.g., Wu and Peltier 1983; Nakada and Lambeck 1989; Tushingham and Peltier 1993). This approximation will be most inaccurate in locations where the coastal bathymetry exhibits a gentle gradient, and where the existence of the ancient ice masses affected the geometry of the ocean periphery (e.g., Hudson Bay in north eastern Canada).

Johnston (1993) was the first to consider the effect of a time-dependent continent margin (TDCM) on RSL predictions. Johnston (1993) employed simple, axially symmetric continent functions throughout much of his analysis due to the computational intensity of the problem. He also carried out a more realistic calculation of GIA-induced sea-level
change in the British Isles region. This calculation shows that the coastline of the British Isles has migrated dramatically over the past ~14 kyr, and neglect of this change in the ocean load geometry can induce an error of ~20% in RSL predictions at 10 kyr BP.

Peltier (1994) has also calculated the change in the Earth's coastlines induced by sea-level variations associated with GIA. His results confirmed those of Johnston (1993) for the British Isles region and indicated other locations where the shorelines have migrated a considerable distance over the past ~18 kyr, such as the Bering Strait and Indonesia. Peltier (1994) was specifically concerned with applying his results to determine the influence of a changing global topography on paleoclimate predictions. He did not consider the effect of a TDCM on predictions of sea-level change.

In more recent work, Lambeck (1996a, 1996b) calculated the paleoshorelines in specific regions of anthropological interest such as the Persian Gulf (Lambeck 1996a). He considered the implications of shoreline migration in these regions with regard to early human settlement and migration routes.

In this chapter the effect of a TDCM on RSL predictions in two specific regions is considered. The Australian region is of special interest for a number of reasons. The results of Peltier (1994) show that the northern shoreline of Australia may have migrated a considerable distance over the postglacial period. This is a consequence of the relatively shallow continental shelf adjacent to this area. Also, data from this region have been employed to constrain Earth and ice model parameters (Nakada and Lambeck 1989). The Australian RSL data are especially significant since they are in the far field of the Late Pleistocene ice sheets and therefore provide constraints on Earth viscosity structure that are relatively insensitive to variations in the adopted ice history (see section 1.1). The differential RSL signal in the Australian region is highly sensitive to the ocean load (Nakada and Lambeck 1989) and, since the ocean periphery is an integral aspect of this load, it is important to determine the significance of a TDCM in this region.

Calculations that consider the effect of a TDCM on RSL predictions in north eastern Canada are also of interest. This region was once covered by the massive Laurentide ice mass and so the location of the ocean periphery will depend on the space-time variations of the ice sheet geometry as well as the local bathymetry. It is well known that the ancient Laurentide ice sheet was a marine-based ice sheet. That is, a considerable portion of the area it covered was below contemporaneous sea level (e.g., Denton and Hughes 1981). This property has important implications for the manner in which the sea load is calculated when this ice sheet (or any of the ancient marine-based ice sheets, e.g., the
Chapter 2: Accurate Modeling of Water Load Redistribution

Fennoscandian ice mass) disintegrated.

There are a number of ice model reconstructions of the Laurentide ice mass (e.g., Peltier and Andrews 1976; Tushingham and Peltier 1991). A common element in such models is the catastrophic down wasting of the component of the ice sheet over the Hudson Bay region around 9 kyr BP. At this time the solid surface in Hudson Bay was considerably lower than contemporaneous sea level, largely due to the local crustal depression induced by the overlying ice mass. It follows that the removal of ice from this region will be accompanied by a massive influx of meltwater/sea water to fill the void existing between the geoid and the solid surface at this time.

The sea-level equation introduced by Farrell and Clark (1976) does not account for the accumulation of water in sub-geoidal regions once covered by ice sheets. In fact, previous solutions of the sea-level equation predict a negative sea load in near-field regions throughout the postglacial period (e.g., Wu and Peltier 1983; Johnston 1993). This is because there is a net decrease in height between the geoid and the solid surface that is largely due to the rebound of the solid surface induced by ice unloading. This prediction is accurate at times subsequent to the above described water influx, but it is inaccurate at times when marine-based ice sheets retreat, exposing sub-geoidal solid surface regions.

Another component of this chapter is the consideration of the influence of this 'water dumping' effect, as well as the effect of a TDCM, on RSL predictions in north eastern Canada. The combination of these two mechanisms will be potentially more significant than that of considering a TDCM alone, given the unusually large magnitude of the above described water dumping phenomenon. Also, this region is particularly interesting since RSL data from north eastern Canada have been extensively employed to constrain models of the Late Pleistocene ice sheets (e.g., Wu and Peltier 1983; Tushingham and Peltier 1991) and models of Earth rheology (e.g., Han and Wahr 1995; Mitrovica and Forte 1997).

The algorithm employed to solve the sea-level equation while incorporating a TDCM and water dumping is described in the next section. This sea-level calculator is then applied, in the following section, to determine the significance of these effects for predicting RSL.
2.2 Theory: A Gravitationally Self-Consistent Sea-Level Algorithm that Incorporates a TDCM and Near-Field Water Dumping

2.2.1 An Efficient Method for Solving the Sea-Level Equation: The Pseudospectral Algorithm

As mentioned in Chapter 1, solving the sea-level equation (1.25) generally requires an iterative procedure. At each iteration step, the temporal and spatial convolutions in (1.25) are evaluated. The temporal convolution is straightforward to carry out if a simple model of the time history of the loading is assumed. For example, it is common to assume that the loading history is comprised of a series of discrete Heaviside load increments (e.g., Farrell and Clark 1976). This model is adopted for all the calculations presented in this thesis. The temporal convolution for such a Heaviside loading history is evaluated below. The spatial convolution is less straightforward and more computationally intensive. Early solutions of the sea-level equation calculated the spatial convolution by adopting a 'finite element' approach in which the surface load (ice and ocean components) is discretized into a large number of circular disks (e.g., Clark et al. 1978; Wu and Peltier 1983). The spatial convolution in (1.25) could then be replaced by a sum of a priori determined disk convolutions. However, constructing the grid of disk loads was very time consuming, especially in areas where the coastline (and therefore ocean load periphery) exhibited complex, short wavelength structure. Also, it was inevitable that disks would overlap in some areas and there would be gaps between disks in others, resulting in some degree of error.

The spatial convolution in (1.25) can also be evaluated in the spherical harmonic domain by representing the load as a weighted series of spherical harmonic functions. Nakiboglu, Lambeck and Aharon (1983) were the first to employ a spectral technique to solve the sea-level equation. However, their solutions were approximate since the ocean load was assumed to be geographically uniform (i.e., the change in ocean load was simply the volume of meltwater divided by the surface area of the oceans). The first accurate, gravitationally self-consistent solutions of the sea-level equation obtained by adopting a spectral approach were calculated by Mitrovica and Peltier (1991). They introduced two spectral approaches, one of which involves no iterative process. However, due to computing constraints, this non-iterative spectral technique was limited to a harmonic truncation of degree and order ~30. Their iterative approach enables computationally efficient so-
olutions of the sea-level equation up to at least degree and order $\sim256$. This efficiency is achieved by performing the projection of globally defined sea level onto the oceans (see equation (1.25)) within the space domain, hence the pseudospectral terminology.

The pseudospectral algorithm is currently the most efficient method available for solving the sea-level equation, and has consequently been adopted by a number of research groups involved in predicting GIA-induced sea-level change. For these reasons, this approach is adopted here as the basic framework within which to implement an algorithm that incorporates both a TDCM and the above described near-field water dumping effect. (The pseudospectral methodology is also applied to solve the sea-level equation derived for a rotating Earth model in the following chapter.) In the remainder of this section, the key aspects of this sea-level algorithm will be briefly discussed.

### A Spectral Form of the Sea-Level Equation

To solve the temporal convolution in (1.25) the time history of the load is modeled as a series of Heaviside loading increments (e.g., Farrell and Clark 1976; Peltier and Andrews 1976). It follows from (1.26) that,

$$L(\theta, \psi, t) = \sum_{n=1}^{N} [\rho_\delta I^n(\theta, \psi) + \rho_W \delta S^n(\theta, \psi)] H(t - t_n).$$  

Using (2.1) in equation (1.25), and applying equation (1.24), gives the result,

$$S(\theta, \psi, t) = C(\theta, \psi, t) \left[ \int_{\Omega} a^2 \left( \rho_I I(\theta', \psi', t) + \rho_W S(\theta', \psi', t) \right) Z^{L,E}(\gamma) d\Omega' + \sum_{n=1}^{N} H(t - t_n) \int_{\Omega} a^2 \left( \rho_\delta I^n(\theta', \psi') + \rho_W \delta S^n(\theta', \psi') \right) Z^{L,NE}(\gamma, t - t_n) d\Omega' + C^L(t) \right],$$  

where,

$$Z^{L,E}(\gamma) = \frac{a}{M_e} \sum_{\ell=0}^{\infty} E^L_{\ell} P_{\ell}(\cos\gamma),$$  

$$Z^{L,NE}(\gamma, t - t_n) = \frac{a}{M_e} \sum_{\ell=0}^{\infty} \beta^L_{\ell}(t - t_n) P_{\ell}(\cos\gamma),$$

with,

$$E^L_{\ell} = 1 + h^L_{\ell} - h^L_{\ell}.$$
and,
\[ \beta^L_t(t - t_n) = \sum_{k=1}^{K} \frac{(r^t_{k,L} - r^t_{k,E})}{s^t_k} \left[ 1 - \exp\left( -s^t_k(t - t_n) \right) \right]. \] (2.6)

The superscripts 'E' and 'NE' represent parameters associated with the elastic and non-elastic response, respectively.

The next step is to evaluate the spatial convolution in (2.2). This can be done by transforming the problem into the domain of spherical harmonics. Using the general spectral decomposition,
\[ X(\theta, \psi) = \sum_{l,m} X_{l,m} Y_{l,m}(\theta, \psi), \] (2.7)
where,
\[ \sum_{l,m} \equiv \sum_{l=0}^{\infty} \sum_{m=-l}^{l}, \] (2.8)

one can define the harmonic coefficients of the ocean function, the ice and sea load functions, and the Heaviside increments of the ice and sea load functions as \( C_{l,m}, I_{l,m}, S_{l,m}, \delta I_{l,m} \) and \( \delta S_{l,m} \), respectively. The following normalization of the spherical harmonic basis functions is adopted throughout this thesis,
\[ \int_{\Omega} Y^\dagger_{l',m'}(\theta, \psi) Y_{l,m}(\theta, \psi) \sin \theta \, d\theta \, d\psi = 4\pi \delta_{l,l'} \delta_{m,m'}, \] (2.9)

where \( \dagger \) denotes the complex conjugate. The spatial convolution in (2.2) can now be performed analytically by applying the result (e.g., Mitrovica and Peltier 1991),
\[ \int_{\Omega} X(\theta, \psi) P_{\ell}(\cos \gamma) \, d\Omega' = \frac{4\pi a^2}{(2\ell + 1)} \sum_{l=0}^{\ell} X_{l,m} Y_{l,m}(\theta, \psi). \] (2.10)

This leads to the following spectral-domain form of the sea-level equation for a Maxwell viscoelastic, non-rotating Earth model excited by a Heaviside loading history (Mitrovica and Peltier 1991),
\[ \sum_{l,m} S_{l,m}(t) Y_{l,m}(\theta, \psi) = \sum_{r,s} C_{r,s}(t) Y_{r,s}(\theta, \psi) \sum_{l,m} \left\{ E^L_{l} T_{l} \left( \rho_l I_{l,m}(t) + \rho W S_{l,m}(t) \right) \right. \]
\[ + T_{l} \sum_{n=1}^{N} \left( \rho_l \delta I_{l,m}^{n} + \rho W \delta S_{l,m}^{n} \right) \beta^L_t(t - t_n) H(t - t_n) \]
\[ + G^L(t) \delta_{l,0} \delta_{m,0} \left\} Y_{l,m}(\theta, \psi), \] (2.11)

where
\[ T_{l} = \frac{4\pi a^3}{M_s(2l + 1)}. \] (2.12)
Solving the Sea-Level Equation using the Pseudospectral Algorithm

The pseudospectral method introduced above is adopted to solve the equation (2.11) in a gravitationally self-consistent manner. The key aspects of this procedure are outlined below. For more information the reader is referred to the original article by Mitrovica and Peltier (1991).

By defining the time dependence of the load as a series of Heaviside increments, solving (2.11) reduces to determining the increments in sea-level, $\delta S^i_{\ell,m} \ (n = 1, 2, 3, \ldots)$ for the time period of interest. Consider the $j^{th}$ Heaviside ocean increment, which can be defined as,

$$\delta S^j_{\ell,m} = S_{\ell,m}(t_j) - S_{\ell,m}(t_{j-1}). \quad (2.13)$$

At any time $t = t_j$, it is assumed that the preceding sea-level increments have been calculated and thus $S_{\ell,m}(t_{j-1})$ is known. The task at hand, therefore, is to determine the coefficients $S_{\ell,m}(t_j)$ and then to apply equation (2.13) in order to determine the $\delta S^j_{\ell,m}$.

The $S_{\ell,m}(t_j)$ can be determined from equation (2.11), which can be written in the form,

$$\sum_{\ell,m} S_{\ell,m}(t_j) Y_{\ell,m}(\theta, \psi) = \sum_{r,s} C_{r,s}(t)Y_{r,s}(\theta, \psi) \times \sum_{\ell,m} \left\{ E^L_{\ell} T_{\ell} \left( \rho I_{\ell,m}(t) + \rho w S_{\ell,m}(t_{j-1}) \right) \right\} + \left\{ \sum_{n=1}^{j-1} \left[ T_{\ell} (\rho I_{\ell} \delta I^n_{\ell,m}) + \rho w \delta S^n_{\ell,m} \right] \delta_{\ell,0}\delta_{m,0} \right\} Y_{\ell,m}(\theta, \psi). \quad (2.14)$$

The right-hand side of (2.14) describes the operation of projecting the predicted net displacement between the global geoid and solid surface from the onset of loading to the time $t_j$ onto the ocean function. This aspect of the calculation is carried out in the space domain (hence the term ‘pseudospectral’) and the resulting field, $S(\theta, \psi, t_j)$, is then decomposed into its spectral components $S_{\ell,m}(t_j)$. On inspection of (2.14), it is evident that there is a single unknown term, the $\delta S^j_{\ell,m}$, on the right-hand side of the equation. This term can be evaluated by employing the following procedure. First, the $\delta S^j_{\ell,m}$ are approximated by calculating the incremental change in sea-level due to meltwater addition/subtraction to the ocean,

$$[\delta S^j_{\ell,m}]^{i=1} = \left[ -\frac{\rho I}{\rho w C_{00}(t_j)} \right] C_{\ell,m}(t_j), \quad (2.15)$$

where the superscript $i = 1$ denotes the first iterate of $\delta S^j_{\ell,m}$. Substituting the $[\delta S^j_{\ell,m}]^{i=1}$ into the right-hand-side of equation (2.14) yields a first iterate solution for $S_{\ell,m}(t_j)$ which
is applied in equation (2.13) to calculate the second iterate harmonic coefficients of the sea-level increment at \( t = t_j \) (denoted by \([\delta S_{t,m}^j]^i=2\)). This procedure is repeated (i.e., the new iterate of \( \delta S_{t,m}^j \) is inserted into the right-hand-side of equation (2.14)) until,

\[
\sum_{t,m} \frac{||[\delta S_{t,m}^j]^{i+1}| - |[\delta S_{t,m}^j]^i||}{||[\delta S_{t,m}^j]^i||} < \varepsilon ,
\]

(2.16)

where \( \varepsilon \) is a predetermined convergence parameter and the vertical bars represent the modulus of the complex variable. Three to five iterations are required for \( \varepsilon = 10^{-4} \). Trial calculations have shown that this choice of tolerance level ensures convergence to within a few centimeters for a spherical harmonic truncation of degree and order 256. The accuracy of the pseudospectral algorithm is attested to by the fact that calculations adopting spherical harmonic expansions truncated at degree and order 32, 64, 128 and 256 converge to a consistent solution (within \( \sim 0.1 \) m) by degree and order 128 at most sites. Sites that are located in geographic regions characterized by a complex, short wavelength coastline geometry exhibit convergence by degree and order 256. A flow chart describing the pseudospectral algorithm is given in Appendix C.

### 2.2.2 Extending the Pseudospectral Algorithm to Incorporate a TDCM and Near-Field Water Dumping

The Earth's coastlines mark the location of the intersect between the geoid and the solid surface. Therefore, to be able to calculate the geometry of the coastlines at some time in the past, one must know the height difference between these two surfaces at previous times. By solving the sea-level equation using an assumed ice load and Earth model, the relative shift between the geoid and the solid surface can be calculated at any time subsequent to the onset of loading. Therefore, by calculating sea-level change to the present day from some time in the past, the absolute height difference between these two surfaces at a previous time can be determined if the present-day topography of the solid surface relative to the geoid is known.

The change in sea level can be calculated from the onset of loading to the present day by first assuming a fixed continent margin equal to the present-day coastline and by computing,

\[
RSL(t_j) = S(\theta, \psi, t_j) - S(\theta, \psi, t_p),
\]

(2.17)

where \( t_p \) represents the present time. The absolute height difference between the geoid
and the solid surface at any $t_j$ is then given by,
\[ T(\theta, \psi, t_j) = T(\theta, \psi, t_p) - RSL(\theta, \psi, t_j), \] (2.18)

where $T(\theta, \psi, t_p)$ is the present-day topography of the Earth (relative to the height of the geoid) and $T(\theta, \psi, t_j)$ is the topography at some previous time $t_j$. The predicted coastline at $t_j$ will be located wherever the field $T(\theta, \psi, t_j)$ is zero.

The $T(\theta, \psi, t_j)$ allow a set of ocean functions $C(\theta, \psi, t_j)$ to be determined. However, this set of ocean functions is not yet derived in a gravitationally self-consistent manner since the ocean-land interface defines an integral aspect of the ocean load geometry and so any change in the $C(\theta, \psi, t_j)$ will affect the predicted sea level. Therefore, in order to calculate a series of gravitationally self-consistent ocean functions, a second iterative procedure is necessary (e.g., Johnston 1993). In this regard, a series of ocean functions $C(\theta, \psi, t_j)$ are defined, where as a first guess, the $C(\theta, \psi, t_j)$ are assumed to only depend on the migration of the ice-ocean boundary specified by the adopted ice model. That is, a present-day coastline geometry is adopted except in near-field regions where the ice migrates into present-day ocean areas. Using this first series of ocean functions, one can employ the methodology outlined above to calculate a second iterate set of $C(\theta, \psi, t_j)$, $j = 1, n$. This procedure is repeated until the desired convergence defined by equation (2.16) is attained.

As mentioned previously, the pseudospectral algorithm (Mitrovica and Peltier 1991) is adopted in the above described method for calculating a TDCM. Before doing so however, a few minor changes to the original pseudospectral algorithm are necessary. During a period of sea-level rise or fall, say between the times $t_{j-1}$ and $t_j$, the movement of the coastline at a specific location is determined by the slope of the topography at the ocean-land interface and the incremental height change in sea level. When calculating the change in sea-level using the original pseudospectral algorithm, an error is incurred when the ocean function changes at each Heaviside time step. For the purposes of the following discussion, it is useful to re-write (2.13) in the following space domain form,
\[ \delta S(\theta, \psi, t_j) = C(\theta, \psi, t_j)SG(\theta, \psi, t_j) - C(\theta, \psi, t_{j-1})SG(\theta, \psi, t_{j-1}), \] (2.19)

where $SG(\theta, \psi, t_j)$ and $SG(\theta, \psi, t_{j-1})$ denote the change in sea-level defined over the entire globe (that is, not projected onto the ocean function) from the beginning of the loading period until the times $t_{j-1}$ and $t_j$, respectively. The function $SG(\theta, \psi, t_j)$ is, from
Chapter 2: Accurate Modeling of Water Load Redistribution

Figure 2.1: Schematic diagram illustrating the region in which the original pseudospectral algorithm predicts incorrect results when the continent margin is modeled in a time-dependent fashion. The symbols $C_{j-1}$ and $C_j$ are shorthand notation for the fields $C(\theta, \psi, t_{j-1})$ and $C(\theta, \psi, t_j)$ respectively.

\begin{equation}
SG(\theta, \psi, t_j) = \sum_{\ell, m} \left[ E^\ell_\ell T_\ell \left( \rho_1 I_{\ell,m}(t) + \rho_w S_{\ell,m}(t_{j-1}) \right) + \left[ E^\ell_\ell T_\ell \delta S^j_{\ell,m} \right] 
+ \sum_{n=1}^{j-1} \left[ T_0 (\rho_1 \delta I^0_{\ell,m} + \rho_w \delta S^0_{\ell,m}) \beta^0_\ell(t_j - t_n) \right] + C^L(t) \delta_{\ell,0} \delta_{m,0} \right] Y_{\ell,m}(\theta, \psi). \tag{2.20}
\end{equation}

A scenario of sea-level rise is schematically illustrated in Figure 2.1(A). Sea-level has risen by some amount, and the coastline has migrated landwards. Applying (2.19) to calculate the sea-level increment at this time step would produce an incorrect result within the area of shoreline migration (shown as area II in figure). Within this region, $C(\theta, \psi, t)$ is zero at $t = t_{j-1}$ (i.e., this area is not ocean at this time). Thus equation (2.19) will give the result $\delta S(\theta, \psi, t_j) = S(\theta, \psi, t_j)$ (since $C(\theta, \psi, t_j) = 1$). That is, the calculated incremental sea-level rise at $t = t_j$ is equal to the total displacement between the geoid and the solid surface from the onset of loading to the time $t_j$.

Figure 2.1(B) schematically portrays the scenario of sea-level fall at some geographic location causing the coastline to migrate seawards. In a similar manner to above, applying equation (2.19) would produce the incorrect result $\delta S(\theta, \psi, t_j) = -S(\theta, \psi, t_{j-1})$ within the region migrated by the shoreline. This prediction is equal in magnitude but opposite in sign to the net sea-level change since the beginning of the glacial loading cycle to the time $t_{j-1}$.

Clearly, equation (2.19) is not valid when $C(\theta, \psi, t_{j-1}) \neq C(\theta, \psi, t_j)$. This problem can be avoided in a straightforward manner by subtracting the functional $SG(\theta, \psi, t_{j-1})$ from
Chapter 2: Accurate Modeling of Water Load Redistribution

Figure 2.2: Schematic diagram illustrating the water load applied (shaded region) to sub-geoidal regions once covered by ice. In this special case, the water load is defined by the total height displacement between the geoid and the solid surface at the time of ice retreat.

$SG(\theta, \psi, t_j)$ first and then multiplying the result of this operation by the ocean function valid for the time $t = t_j$. That is,

$$\delta S(\theta, \psi, t_j) = C(\theta, \psi, t_j) \left[ SG(\theta, \psi, t_j) - SG(\theta, \psi, t_{j-1}) \right].$$  \hspace{1cm} (2.21)

Application of equation (2.21) in either of the above scenarios gives the proper result at all sites when the pseudospectral algorithm is extended to consider a TDCM.

As described in the first section of this chapter, water will flood into surface regions exposed by a retreating ice sheet wherever the geoid lies above the solid surface. This idea is schematically illustrated in Figure (2.2). At the time $t_j$, the ice sheet retreats and water floods into the region vacated by the ice. The sea load in this region is the total height difference between the geoid and solid surface at the time of ice retreat. Thus, in analogy to the method adopted to determine the geometry of paleoshorelines, in order to accurately model the water load applied to previously glaciated regions, it is necessary to calculate the paleotopography.

A standard, gravitationally self-consistent, solution of the sea-level equation involves the determination of the relative height shift between the geoid and the solid surface within some specified time increment. During a period of deglaciation in near-field regions, the sea-level equation will predict a negative sea-load increment due, largely, to the isostatic rebound of the solid surface. Thus, the sea-level equation does not accurately describe the water load shown in Figure 2.2 at these specific times during deglaciation (since this
Chapter 2: Accurate Modeling of Water Load Redistribution

water load is governed by the total, rather than the incremental, displacement between the geoid and the solid surface).

A method is sought which predicts the desired, relatively large, positive sea-load increment when ice immediately vacates a region and yet produces the correct, standard result obtained via a conventional application of the sea-level equation at times subsequent to the influx of water. This can be done by introducing a functional, \( \zeta(\theta, \psi, t_j) \), which takes the value zero in areas where the ice load has retreated and the value unity elsewhere. Equation (2.21) is then replaced by,

\[
\delta S(\theta, \psi, t_j) = C(\theta, \psi, t_j) \left[ \zeta(\theta, \phi, t_j) \left[ SG(\theta, \psi, t_j) - SG(\theta, \psi, t_{j-1}) \right] \right.
+ \left[ \zeta(\theta, \psi, t_j) - 1 \right] T(\theta, \psi, t_j). \tag{2.22}
\]

The first term in the square brackets describes the same field as equation (2.21) except in regions characterized by ice retreat, in which case a value of zero is given. The second term, in contrast, describes a field that is zero everywhere except in regions where ice has retreated. In these regions this term gives the negative of the paleotopography field at the time \( t_j \).

The reader should note that application of equation (2.22) assumes that the ice mass was grounded on the solid surface, which is assumed to exhibit the same topography (not accounting for load-induced deformation) as that existing today (i.e., changes in topography due to such processes as sediment transport are ignored).

In the same manner as that adopted to model a TDCM, a second iterative procedure is employed to obtain a gravitationally self-consistent solution of the sea-level equation which incorporates both a TDCM and water dumping. This entails solving the sea-level equation over an entire glacial cycle while adopting the present-day ocean function (except modified in near-field regions to account for ice sheet growth and retreat) and the present-day topography as approximations for \( C(\theta, \psi, t_j) \) and \( T(\theta, \psi, t_j) \) respectively.

The calculated sea-level change can they be applied to predict a second iterate set of functionals \( T(\theta, \psi, t_j) \) and \( C(\theta, \psi, t_j) \). These are then used in a second 'run' through a full glacial cycle to calculate sea-level change. This procedure is repeated until the solution converges to within a predetermined tolerance. Three such iterations are normally adequate to satisfy the tolerance level defined in equation (2.16). A flow chart describing this extended pseudospectral algorithm is given in Appendix C.
2.3 Results and Discussion

The calculations presented in this thesis are based on the impulse response of a spherically symmetric, self-gravitating, compressible, Maxwell viscoelastic Earth model. The elastic and density structure of the Earth model are derived from seismic constraints (Dziewonski and Anderson 1981). For the purposes of this chapter, two viscosity profiles are adopted that are broadly compatible with those obtained from a recent joint inversion study of GIA and convection-related observables (Mitrovica and Forte 1997). The chosen viscosity profiles are characterized by an upper mantle with a uniform viscosity of $5 \times 10^{20}$ Pa s and an isoviscous lower mantle of viscosity $5 \times 10^{21}$ Pa s. To predict RSL change in north eastern Canada a lithosphere of thickness 100 km is adopted, whereas, results for the Australian region are based on a lithosphere of thickness 70 km. These values were chosen to be compatible with constraints imposed by data obtained within these regions.

The ice model adopted throughout this thesis is the ICE-3G deglaciation model (Tushingham and Peltier 1991). The ICE-3G model is temporally discretized into 14 Heaviside loading increments over the period 18 kyr BP to 5 kyr BP in 1 kyr time intervals. A glaciation phase is incorporated by reversing the deglaciation history in time and extending the Heaviside time increment to 7 kyr.

For ease of discussion in this section, the term 'standard theory' shall be used to denote sea-level predictions based on a standard gravitationally self-consistent solution of the sea-level equation that incorporates a continent margin which varies only in near-field regions due to change in ice coverage. In a similar manner, the term 'extended theory' will denote results based on the theory that incorporates a fully TDCM and the water dumping effect.

2.3.1 North Eastern Canada

Calculations based on the extended theory shall be considered first. Figures 2.3 and 2.4 illustrate the Heaviside ice and sea load increments in NE Canada at 11, 10, 9 and 8 kyr BP. This particular time window was chosen because it is during this time that much of the land in this region becomes ice free. The left-hand frames show spherical harmonic representations of the ICE-3G model, truncated at degree and order 128, while the right-hand frames show the predicted sea-load increments at the corresponding times. The shaded maps in Figures 2.3 and 2.4 are determined by employing the harmonic
coefficients of the ice load and the predicted, global sea-level change (also up to degree and order 128) to calculate these respective fields on a 10' by 10' grid. Equation (2.22) is then applied by adopting these grid files and a 10' by 10' present-day topography data set. Note that the sea-level increments are confined to sub-geoidal solid surface regions not covered by ice. Hence, any regions showing ice cover on the left-hand-side of Figures 2.3 and 2.4 are devoid of a sea-load increment on the associated right-hand panel.

Consider the change in the ice coverage over the period 11 kyr BP to 10 kyr BP in Figure 2.3. Note that the shallow ice coverage over the Boothia Penninsula and the Gulf of Boothia (the light shaded region centered on approx. 71°N and 265°E) at 11 kyr BP has disintegrated by 10 kyr BP, as has ice in the mouth of Hudson Strait (approx. 62°N and 290°E). The bottom right-hand frame in Figure 2.3 shows the predicted sea-load increment associated with this (instantaneous) change in the ice load. There is a substantial influx of water into the region of Hudson Strait that has become ice free. A similar sea-load increment is applied to the present-day ocean area surrounding the Boothia Penninsula. Note that, at these locations, the predicted ocean load does not coincide with the black lines indicating the present-day coastline. An extreme example is King William Island (69°N, 262°E), which is calculated to be below the geoid at this time and thus entirely submerged. Note also the thin strip of darker shading adjacent to the retreating ice mass near the west shore of James Bay and within Lake Superior indicating that these areas are also predicted to be below the geoid at this time.

Now consider the change in the ice and sea loads occurring over the time interval 10 - 9 kyr BP. In the ICE-3G reconstruction, the Laurentide ice sheet melts catastrophically during this period. Hudson Strait opens up and Hudson Bay becomes largely ice free. In fact, there is only a small volume of ice remaining westwards of ~280°E at 9 kyr BP in the geographic region considered in Figures 2.3 and 2.4. As one would expect, such a major change in ice coverage is accompanied by a dramatic influx of water into Hudson Bay and the ocean region surrounding Prince of Wales Island and Somerset Island (dark shaded area in the north west section of the top right-hand frame). This water load is of an unusually large magnitude, reaching values of several hundreds of meters. Note that locations in which a large water load was added at 9 and 10 kyr BP are characterized, at 8 kyr BP, by a relatively small negative sea load. The sign and magnitude of this load are typical of predictions based on the standard theory.

The sea-load increment at 9 kyr BP (Figure 2.4, top right-hand frame) illustrates that the geometry of the Hudson Bay coastline has changed considerably from this time to the
Figure 2.3: The left-hand frames display ICE-3G thicknesses in north eastern Canada at 11 kyr BP and 10 kyr BP. The darkest shade (not included on the 'ice thickness' scale) denotes ice free regions. The right-hand frames display the predicted sea-load increments corresponding to the change in the ice load (e.g., the sea-load increment shown at 10 kyr BP corresponds to the instantaneous change in the ice load at 10 kyr BP). The lightest shade (not shown on the 'sea load' scale) denotes solid surface regions that are either ice covered or above the geoid at the time indicated.
Figure 2.4: Analogous to Figure 2.3 with the exception that a different time period is considered.
present day. Not surprisingly, the predicted change is most significant along the south west shore where the present-day topography is relatively low and flat. Also note that, at this time, a small section of Prince of Wales Island is now above the geoid, as the solid surface has risen relative to the geoid during the previous thousand years.

The following Heaviside ice load increment, at 8 kyr BP, results in Hudson Strait and Hudson Bay becoming totally ice free. At this time the predicted water load, except in a few locations, is due to the incremental relative displacement between the geoid and the solid surface. This signal is negative in near-field regions largely due to the rebound of the solid surface in response to the ice unloading. The perturbation to the geoid is a second order signal which is essentially the sum of two competing effects: the globally uniform height increase due to meltwater addition to the oceans and the local height decrease caused by a diminishing gravitational attraction between the disintegrating ice mass and the surrounding ocean water.

Figure 2.5 illustrates the predicted sea-load increments for the same region considered in Figures 2.3 and 2.4 except that, in this case, the results are based on the standard theory. The same Earth and ice model are adopted. The continent functions at each of the four times shown are taken from the first iterate set described in the previous section (i.e., the present-day coastline is assumed except where ice exists in ocean areas). Therefore, the maps showing ice thicknesses in Figures 2.3 and 2.4 also apply to the sea-load increments in Figure 2.5.

There are a number of points of comparison with the results based on the extended theory. First, note that the sea load is always negative in regions once loaded by ice with a magnitude not exceeding 80 m at any time step. (The positive signal evident in the Labrador sea is due to a combination of effects such as the geoid height increase due to meltwater addition to the oceans plus subsidence of the solid surface in this region caused by a collapsing peripheral bulge.) Second, the water load predicted in regions that become ice free are confined to the geometry of the present-day coastline. For example, the loading increment at 10 kyr BP in the vicinity of the Boothia Peninsula is not applied over Prince William Island. Also, at 9 kyr BP, since the predicted water loading of Hudson Bay is, in this case, confined to the present-day geometry, it extends over a considerably smaller area than that predicted in the extended calculation.

(There are a number of small light colored land regions that exist where the black coastline indicates there should be ocean. This is noticeable in coastline areas that exhibit complex, short wavelength structure (e.g., Bathurst Inlet, approx. 67°N, 253°E – see 9 kyr BP
Figure 2.5: Predictions of the sea-load increment at the same time steps considered in Figures 2.3 and 2.4. These results are based on the standard sea-level theory and the same Earth and ice models as those adopted in constructing Figures 2.3 and 2.4.
frame) and at seemingly random locations such as in the south east section of Hudson Bay. This is simply due to the fact that the plotting software employed to draw the coastlines is not related to the algorithm adopted to calculate the ocean-continent interface. From the results shown in Figure 2.4, it is evident that the method applied in the sea-level calculation is less efficient at resolving fine-scale coastline structure, but is able to 'pick out' small islands that are not shown by the plotting software. For example, the light shaded land region in south east Hudson Bay is Belcher Island (56°N, 281°E).

By comparing the sea-load increments shown in Figures 2.3 and 2.4, it is evident that application of the standard theory produces an unrealistic result at times when solid surface regions below the geoid become ice free. The error incurred is on the order of hundreds of meters. Calculations based on the standard theory predict a considerable negative load (ice plus water) from regions such as Hudson Bay at 9 kyr BP. In contrast, the results based on the extended theory indicate that the net load over Hudson Bay at this time is quite small, as the water load is similar in magnitude but opposite in sign to the ice load. Thus one would expect that the RSL predictions based on the two calculations will be considerably different.

This issue is addressed in Figure 2.6, which shows the predicted RSL signal (see equation (2.19)) derived from the extended theory minus the same predictions based on the standard theory. (The signal shown in Figure 2.6 is noticeably smoother than that shown in Figures 2.3 and 2.4 because the results shown in Figure 2.6 consider only the deformation of the geoid and the solid surface due to the predicted sea loads shown in Figure 2.3, 2.4 and 2.5. The short wavelength structure of the ocean load shown in Figures 2.3 and 2.4 is not evident in the predicted RSL response due to the filtering effect of the elastic lithosphere.) Not surprisingly, the largest error is incurred in the vicinity of Hudson Bay, and reaches a magnitude of ~75 m in this region around the time 10 kyr BP. (The error becomes larger at older times because the predicted RSL time series are defined to converge at present day when RSL is zero; see equation (2.17).) The sign of the error reflects the fact that a larger load is removed in the standard theory, thus producing a greater amount of uplift and so a larger sea-level fall.

The labeled dots in the 8 kyr BP frame in Figure 2.6 indicate a selection of sites taken from the RSL data base of Tushingham (1989). These particular sites were chosen to indicate the geographical distribution of data available for modeling purposes. Data from these sites have been used in a number of GIA modeling studies (e.g., Tushingham and Peltier 1991; Han and Wahr 1995; Mitrovica and Forte 1997).
Figure 2.6: The difference between RSL predictions based on the standard sea-level theory to those based on the extended theory. The largest discrepancy occurs in the immediate vicinity of Hudson Bay where the water load predicted using the extended theory is most significant.
The significance of the results in Figure 2.6 depends on the magnitude of the RSL signal and the observational uncertainty for the data at each site. Figure 2.7 shows predicted RSL curves for four sites chosen such that the difference between the standard and extended sea-level predictions is a maximum. Sites H3, H6 and H7 lie within the area of Hudson Bay while site H18 is on the northern shore of North Prince of Wales Island (northwestern most site shown in Figure 2.6). Results from the three sites within the Hudson Bay area exhibit a ~20% difference in predictions at 10 kyr BP. The oldest data from these sites date back to approximately 7 kyr BP. The estimated 1-σ error on these data is 15 m to 17 m (Tushingham 1989). The predicted difference for the sites H3 and H6 exceeds this error, while that for site H7 falls slightly within this value. Data from site H18 extends back to ~10 kyr BP and has an estimated 1-σ error of about 15 m, which is slightly larger (by a few meters) than the predicted difference at this time. Thus, at certain sites, the difference in the RSL predictions based on the standard and extended theories is larger than the estimated observational error.

Note, from Figure 2.4, that the difference in RSL is negative at most sites, indicating that application of the standard theory will introduce a systematic error to the predictions. Modeling analyses that adopt the standard theory are therefore likely infer ice sheet thicknesses that are too low, since the predicted water component of the unloading is too large in this case. Furthermore, the common method of assuming the ice model known in order to constrain mantle viscosity structure will produce biased viscosity estimates if the standard theory is employed.

As discussed in the previous chapter, the decay time parameterization of RSL data (see Appendix A) from near-field regions, such as north eastern Canada, enables Earth and ice parameters to be separated (to some degree), and so relatively robust inferences of mantle viscosity can be obtained. In a recent application of this method, data from a number of sites from north eastern Canada and Scandinavia were employed to infer viscosity structure (Mitrovica and Forte 1997). Among the sites considered in north eastern Canada were H6 and H7 shown in Figure 2.6. The decay time corresponding to the solid curve for site H6 (i.e., corresponding to calculations based on the standard sea-level theory) is 10.0 kyr, whereas that for the dashed curve is 10.5 kyr. This difference is not significant given that the 1-σ error in the decay time obtained from observations at this site is 1.2 kyr (Mitrovica and Forte 1997). At site H7, where the 1-σ error in the observed decay time is also 1.2 kyr, there is no difference in the predicted decay times (to one decimal place) obtained from the two theories. These results confirm the robustness of the decay time parameterization.
Figure 2.7: RSL predictions at four sites in the region under consideration (see Figure 2.6, bottom right-hand frame, for specific location of sites). The solid line represents predictions based on the standard sea-level theory, while the dashed line is based on the extended sea-level theory.
On comparing the predicted water load redistributions shown in Figures 2.3, 2.4 and 2.5, it is evident that the extended sea-level theory predicts a more realistic result than the standard theory. However, comparing the results shown in Figure 2.3 and 2.4 to geological reconstructions of the deglaciation of the Laurentide ice sheet (e.g., Dawson 1995), one specific weakness of the model becomes apparent. The modeling results shown in Figures 2.3 and 2.4 illustrate that melt water is confined to areas below the height of the geoid at the time considered. This assumption prohibits the model from predicting such features as ice dammed lakes or the pooling of meltwater in topographic lows that may lie above the geoid. Furthermore, the model assumes that all regions below the geoid are interconnected and therefore an outward flux of water from areas characterized by a decrease in the volume bounded by the geoid and the solid surface is always possible. Of course, uplift of the solid surface may simply raise the water level of a lake that is isolated from the oceans. In this case there would be no change in the local water load.

It may be important to consider large-scale redistributions of meltwater that are not predicted by the extended sea-level theory for the reasons discussed above. For example, geological evidence suggests that a massive ice dammed lake (Lake Agassiz) existed to the south west of Hudson Bay prior to the catastrophic down-wasting of ice from this region. A volume of water on the order of $10^5$ km$^3$ (this is on the same order as the water volume presently occupying Hudson Bay) is estimated to have accumulated in Lake Agassiz, which drained into the Hudson Basin at the time of the catastrophic ice collapse. This redistribution of water mass may have a significant effect on RSL predictions around the western shore of Hudson Bay.

### 2.3.2 The Australian Region

The far-field region considered here includes Australia, Tasmania, New Zealand, the southern coast of New Guinea and numerous small Pacific islands. This region was chosen specifically because the northern Australian coastline is predicted to have changed dramatically (Peltier 1994) and RSL data from Australia and New Zealand have been employed to constrain Earth and ice model parameters (Nakada and Lambeck 1989).

In their analysis, Nakada and Lambeck (1989) concluded that in order to accurately predict RSL in the Australian region, the ocean function should be resolved to a spherical harmonic truncation of approximately degree and order 180. Therefore, the calculations presented in this subsection are based on spherical harmonic expansions of the surface load truncated at degree and order 256. Also, in a similar manner to above, the shaded
plots presented below are based on the harmonic output of the sea-level calculations superimposed onto a 10' by 10' topography grid.

Figure 2.8: The predicted ocean-continent interface in the Australian region at the times 18, 14, 9 and 5 kyr BP. The small, dark shaded region in southern mid-Australia, evident at 9 kyr BP and 8 kyr BP, is Lake Eyre (much of the lake basin lies below the present-day geoid).

Figure 2.8 shows the predicted land area exposed at the times 18, 14, 9 and 5 kyr BP. Shortly after the last glacial maximum, significant exposure of the Great Australian Bite (southern shore of Australia) is predicted. Also, the Bass Strait is exposed linking mainland Australia to Tasmania. The northern shoreline of Australia from the northern section of the Great Barrier Reef in the east to the North West Cape in the west shows the most dramatic change. The Arafura Shelf immediately south of New Guinea, and the Gulf of Carpentaria are completely exposed, connecting New Guinea to Australia at this time. The two main islands of New Zealand are also connected during the early stages of the deglaciation period. Note also that a significant body of land lying between
the north eastern shore of Australia and the island of New Caledonia (-20°N, 159°E) is exposed until sometime between 9 kyr BP and 5 kyr BP.

The focus of this section is to determine the effect of the migrating shorelines illustrated in Figure 2.8 on predictions of RSL in this region. Most sea-level data from sites in the far field of the Late Pleistocene ice sheets date back to around 5 kyr BP. This is because sea-level has risen on the order of 100 m in this region throughout the deglaciation period, except over the past ~5 kyr, during which time the far-field data generally indicate a small sea-level fall on the order of a few meters (the mechanisms for this change are described in section 1.3). Since sea-level markers older than the time of this sea-level ‘high stand’ are submerged, the clearest and most accessible geological marker of ancient sea level in this region is the high stand itself.

![Figure 2.9: The difference in predicted RSL (results based on extended theory minus those based on standard theory) at the time of the sea-level high stand. Sites from which observed RSL high stands have been employed to constrain GIA model parameters (Nakada and Lambeck 1989) are indicated.](image)

Figure 2.9 shows the difference between RSL predictions based on the standard sea-level theory to predictions based on the extended theory at the time of the predicted Late Holocene high stand. It is clear that the largest difference occurs in regions where the ocean periphery has moved the greatest distance. Notice that the six local maxima located around the Australian coastline occur where the most significant land exposure is predicted. Also note that each of these maxima are more or less located at the center of the continental shelf area exposed at last glacial maximum. This spatial correlation
can be explained in a straightforward manner.

Consider Figure 2.10, which is a schematic representation of the difference in the predicted ocean load when applying the two sea-level algorithms employed to generate Figure 2.9. The change in sea level between some past time \( t_- \), say at LGM, and the present day \( t_p \), is shown in Figures 2(a)-2(b). A sea-level calculation that adopts a fixed continent margin places the ocean perimeter at the present-day intersect of the geoid (sea surface) and the solid surface. Thus, the total change in the sea load, assuming a fixed margin, is shown in Figure 2(c). The actual change in the sea load in this region (which is approximated in a step-like fashion by the TDCM theory) is shown in Figure 2(d). The difference between the ‘fixed’ case and the ‘actual’ case is shown in Figure 2(e). If one considers that the total height of the sea load shown in Figure 2(c) is on the order of 100 m, it is clear that the theory adopting a fixed ocean margin is significantly overestimating the load in regions characterized by a broad, shallow shelf area. This explains why the maximum discrepancy apparent in Figure 2.9 is located approximately in the middle of the continental shelf area exposed around the time of the LGM.

![Figure 2.10: Schematic diagram illustrating the error in the predicted sea load caused by assuming a fixed continent margin. See text for a discussion of this figure.](image)

The increased load predicted by the standard theory will induce an additional subsidence of the solid surface compared to predictions based on the extended theory. This will
contribute a sea-level rise from the time of the LGM to the present day, thus reconciling the sign of the signal in Figure 2.9.

Nakada and Lambeck (1989) used sea-level high stands obtained from 5 sites in Australia (sites A1 to A5 in Figure 2.9) and one site in New Zealand (A6 in Figure 2.9) to constrain both mantle viscosity and the amount of melt water added to the oceans since the time of the high stands to the present day. In particular, they employed the difference between high stands (‘differential high stands’) to constrain viscosity structure. They then adopted the resulting, ‘preferred’, viscosity model to predict absolute high stand amplitudes which they used to constrain the melt signal.

Figure 2.11 shows predictions of RSL at the six sites shown in Figure 2.9. The two curves in each plot correspond to calculations based on the standard theory (solid line) and the extended theory (dashed line). Nakada and Lambeck (1989) considered RSL differentials between the following site pairs: Karumba and Halifax Bay, Halifax Bay and Moruya, Port Pirie and Cape Spencer, Moruya and Christchurch. The results shown in Figure 2.11 indicate a discrepancy of 0.6 m, -1.1 m, 1.0 m and 0.2 m for these differential high stands, respectively, when the two sea-level calculators are adopted. The predicted discrepancies for the site pairs Halifax Bay-Moruya and Port Pirie-Cape Spencer may be large enough to significantly bias the viscosity inference of Nakada and Lambeck (1989). To determine if this is the case, a careful analysis of the type carried out by Nakada and Lambeck (1989) is required. It is worth noting that Nakada and Lambeck (1989) adopted a lithosphere of thickness 50 km in their calculations. This reduction in lithospheric thickness will act to magnify the predicted discrepancies illustrated in Figures 2.9 and 2.11.

The height of the Late Holocene high stands observed at far-field sites is a function of Earth rheology and the amount of global melting since the timing of the high stands. Adopting an Earth model based on their differential high stand analysis, Nakada and Lambeck (1989) noted that their predicted high stands were consistently too high. They postulated that this discrepancy was due to a melt event from the Antarctic ice sheet of magnitude 2–3 m over the past ~5 kyr. By adopting a TDCM, the predicted high stands are consistently higher than those calculated using a fixed continent margin. Of the six sites considered by Nakada and Lambeck (1989), the largest predicted difference occurs at the site Cape Spencer with magnitude of 1.3 m (see Figure 2.9) and the smallest discrepancy, at Moruya, is less than 0.1 m. The average discrepancy for the six sites considered in Figure 2.8 is around 0.6 m. This suggests that calculations based on a TDCM could increase the Nakada and Lambeck (1989) estimate of Antarctica melting
Figure 2.11: RSL predictions at the six sites labeled in Figure 2.9. The solid lines represent predictions based on the standard theory and the dashed lines represent predictions based on the extended theory.
by up to 30%.

A considerable amount of sea-level high stand data has been obtained from the Australian region following the analysis of Nakada and Lambeck (1989). It is clear from Figure 2.9 that an accurate analysis of the extended data set will require a sea-level calculator that adopts a TDCM. An error of several meters in the predicted high stands and their associated differentials can be easily introduced (see Figure 2.9). The magnitude of this error will depend on the geographic distribution of data sites (as well as the choice of Earth and, to a lesser extent, ice parameters).

Figure 2.12 is an analogous plot to that shown in Figure 2.9 with the exception that in this case the time considered is 18 kyr BP. This figure is included because it illustrates the maximum RSL error introduced when applying the standard sea-level theory (for the Earth and ice model assumed here). Although most data from far-field regions post-date the timing of the high stands, there are some notable exceptions. For example, a continuous RSL record dating from shortly after the timing of the late glacial maximum to the present has been obtained by analyzing drilled sections of coral reefs from offshore Papua New Guinea (-3°N, 147°E). Records such as this are ‘corrected’ for any deformation induced sea-level change and are then employed to provide constraints on the global eustatic melt curve throughout the postglacial period. The results shown in Figure 2.9 indicate that application of this procedure by employing the standard theory could result in a significant error (on the order of tens of meters) if the region under consideration is characterized by a shallow and broad continental shelf.

2.4 Conclusions

The above results show that the magnitude of the water load predicted in north eastern Canada can be on the order of hundreds of meters at times when the Laurentide ice sheet retreats exposing sub-geoidal solid surface regions. Calculations based on a standard solution of the sea-level equation predict, in contrast, a negative load with an amplitude significantly less than 100 m.

The ICE-3G reconstruction of the Laurentide ice sheet deglaciation includes a catastrophic down-wasting of the ice sheet in the Hudson Bay region at 9 kyr BP. By applying a sea-level theory that incorporates water influx to sub-geoidal regions exposed by the retreating ice, a positive water load of a similar magnitude to the negative ice load at this time is predicted. The net load over Hudson Bay at this time is, consequently, relatively
Figure 2.12: The difference between predicted RSL (results based on extended theory minus those based on standard theory) shortly after LGM.

small compared to the net negative load predicted by a standard solution of the sea-level equation. This overestimate of load removal causes RSL predictions from the Hudson Bay region that are based on the original theory to exhibit a larger sea-level fall (up to \(~75\) meters at 10 kyr BP).

The overestimate of load removal from previously glaciated regions can potentially lead to biased estimates of ice thickness (to lower values). In a similar manner, viscosity constraints obtained from data within such regions will be biased if predictions are based on the standard theory. In contrast, decay time estimates are relatively insensitive to the difference in the water load predicted by the standard and extended theories. Therefore, previous inferences based on this data parameterization will not be affected.

The new theory does not predict the occurrence of water accumulation in regions above the geoid, and so features characteristic of ice sheet deglaciation such as ice dammed lakes are not predicted. The influence of the build up and catastrophic drainage of large-scale ice dammed lakes (e.g., Lake Agassiz) on RSL predictions remains an important issue for future study.

The assumption of a fixed continent margin in far-field sea-level calculations causes an overestimate of the ocean load in regions that are characterized by a shallow and broad
continental shelf. This assumption results in RSL predictions exhibiting a larger sea-level rise compared to those based on a calculator that incorporates a TDCM. Also, the maximum error is evident in the center of shelf areas exposed at LGM.

The present results indicate that the error incurred by assuming a fixed continent margin can be as large as 20 - 25 m at LGM and 4 - 5 m at the time of the Late Holocene high stands. Therefore, it is clearly important to adopt a sea-level calculator that includes a TDCM when constraining the eustatic melt signal using data obtained near broad and shallow continental shelf regions.

Predictions of differential sea-level high stands can be affected by as much as 4 - 5 m when a TDCM is adopted (depending on site locations; see Figure 2.9). The predicted differential high stands for the locations considered by Nakada and Lambeck (1999) are shown to differ, at most, by a little over 1 m due to the influence of a TDCM for the model parameters chosen in this study. This is a systematic error which may significantly bias their estimate of mantle viscosity. It is clear that future analyses of this type must adopt a sea-level theory that incorporates a TDCM.
Chapter 3

Postglacial Sea-Level Change on a Rotating Earth

3.1 Introduction

The sea-level equation introduced in Chapter 1 governs sea-level change caused by a surface mass redistribution on a non-rotating Earth. However, as briefly discussed in Chapter 1, the surface mass redistribution and consequent Earth deformation associated with the growth and decay of the Late Pleistocene ice sheets also induces change in the Earth's rotation vector (e.g., Sabadini et al. 1982; Wu and Peltier 1984). The corresponding change in the rotational potential deforms both the solid surface and the geoid, contributing a sea-level signal that is not incorporated into the sea-level theory of Farrell and Clark (1976).

On time scales of 1-10 kyr and longer, perturbations to the rotation vector are associated with both GIA and mantle convection (e.g., Spada, Ricard and Sabadini 1992; Steinberger and O'Connell 1997). In considering the influence of mantle-convection-induced true polar wander (TPW) on sea-level change, Sabadini, Doglioni and Yuen (1990) found that a uniform TPW of $1^\circ$/Myr can produce a sea-level signal of up to 50 m when only the effect of rotational potential forcing is considered (i.e., the loading effect produced by this sea-level change was not included). Perturbations to the rotation vector occurring over shorter time scales, for example, the Annual and Chandler wobbles, produce the well studied sea-level phenomenon of pole tides (e.g., Miller and Wunsch 1973; Wahr 1985). The sole purpose of this chapter is to consider the rotational signature induced by GIA and its corresponding influence on postglacial sea-level change. A number of studies have
considered this topic (Han and Wahr 1989; Milne and Mitrovica 1996; Bills and James 1996); however, the results of these studies appear to be in significant disagreement.

Han and Wahr (1989) were the first to consider the effect of GIA-induced change in the rotation vector on sea level. Using a viscoelastic Earth model with an isoviscous mantle of 10^{21} Pa s and a lithospheric thickness of 120 km, their results show that the predicted change in sea level may be described by a degree 2 order 1 spherical harmonic function with a maximum (peak to peak) signal of ~30 m. Their predicted RSL curves are characterized by a monotonic form with an amplitude variation of up to ~15 m from 18 kyr BP to present. In their analysis, Han and Wahr (1989) assumed a eustatic ocean load and modeled the Laurentide ice sheet using a single ice disk. Also, they did not consider the direct effect of the changing rotational potential on sea level. (The term 'direct' is used to denote the component of the sea-level response that is independent of Earth deformation for a specified surface load or potential forcing.)

The most recent paper on the subject, by Bills and James (1996), is similar to the analysis of Han and Wahr (1989) in that only the Laurentide ice mass and a eustatic ocean load are considered. Bills and James (1996) conclude that calculations based on a rigid Earth model give a "reasonable first order estimate of the polar motion contribution to relative sea level". If this suggestion is correct, then the problem is greatly simplified since only the direct effect of the rotational potential requires consideration (i.e., the sea-level response associated with the deformation induced by the changing rotational potential can be neglected). Using the rigid Earth approximation, Bills and James (1996) predicted a rotation-induced postglacial sea-level signal that is of similar magnitude but of opposite sign to the prediction of Han and Wahr (1989).

There are four principal components to this chapter. First, an extended sea-level equation is derived that includes the effect of a changing rotational potential on sea-level. A brief description is then given of the spectral algorithm employed to solve this equation. Second, the new theory is applied to calculate sea-level change for a rotating Earth model and to consider the sensitivity of the predictions to variations in model parameters. Also, the unique spatial and temporal dependence of the rotation-induced sea-level signal is explained in terms of the relative magnitudes of the Earth's different sea-level response types (i.e., direct, elastic and viscous) to the changing rotational potential. Third, the significant discrepancy evident in the literature regarding predictions of postglacial rotation-induced sea-level change is addressed. Lastly, in order to determine the significance of the rotation-induced signal, the magnitude of this signal relative
to the load-induced signal is considered for regions where data have been obtained and consequently employed to constrain model parameters.

3.2 Theory

This section follows on from the discussion in sections 1.3 and 2.2.1 by considering the effect of a time-varying rotational potential on sea-level change. The spectral form of the new sea-level equation is first derived and then applied to calculate sea-level change induced by a surface load redistribution and a general potential forcing. The theory adopted to calculate GIA-induced changes to the Earth's rotational potential is then introduced, followed by a discussion on how this theory is incorporated into an algorithm that can be employed to solve the new sea-level equation in a gravitationally self-consistent manner.

3.2.1 The Sea-Level Equation for a Rotating Earth

On a rotating Earth, the geoid and solid surface are perturbed by a changing rotational potential as well as by a changing surface load. The sea-level response of the adopted Earth model to a time-varying potential excitation can be calculated using a method which is analogous to that used in the loading problem described above. To calculate the sea-level response to a varying potential, as opposed to a surface load, tidal Love numbers are applied that describe the response of the Earth model to a general potential forcing that does not involve a loading of the Earth's surface.

When subject to a changing rotational potential, the sea-level response of the adopted Earth model depends on the viscoelastic tidal Love numbers,

$$ h^T_t(t) = h^T_{t,E}(t) + \sum_{k=1}^{K} r_k^T \exp(-s_k^t t) , \quad (3.1) $$

and,

$$ k^T_t(t) = k^T_{t,E}(t) + \sum_{k=1}^{K} r_k^T \exp(-s_k^t t) , \quad (3.2) $$

where the superscript 'T' indicates parameters associated with a general potential (tidal) forcing. Equations (3.1) and (3.2) are the tidal analogues to the load Love numbers in equations (1.14) and (1.16), and they can be used to estimate the Earth's response to a general potential forcing. (Note that the inverse decay times, $s_k^t$, in equations (3.1) and
The tidal Green’s functions for the radial displacement of the solid surface and the gravitational potential perturbation at the undeformed surface are, respectively,

$$\Gamma^T(t) = \frac{1}{g}[h^T_{t,E}(t)\delta(t) + \sum_{k=1}^{K} r^T_{k,E} \exp(-s_k^T t)], \quad (3.3)$$

and,

$$\Upsilon^T(t) = \delta(t) + k^T_{t,E}(t)\delta(t) + \sum_{k=1}^{K} r^T_{k,E} \exp(-s_k^T t). \quad (3.4)$$

In contrast to the loading Green’s functions in equations (1.17) and (1.18), (3.3) and (3.4) are Green’s functions in time only. That is, the perturbations to the solid surface radial displacement and the geopotential are calculated by convolving the time variation of the rotational potential with (3.3) and (3.4). In analogy with equation (1.18), the first Dirac delta function on the right-hand-side of (3.4) signifies the direct effect of the rotational potential on the Earth’s gravity field.

By denoting the harmonic coefficients of the perturbing rotational potential as $\Lambda_{t,m}(t)$, the deflections of the radial position of the solid surface and the geoid are given by,

$$G^T_{t,m}(t) = \frac{1}{g} \int_{-\infty}^{t} \Lambda_{t,m}(t') \Upsilon^T_{t}(t - t') \, dt' + G^T(t)\delta_{t,0}\delta_{m,0}, \quad (3.5)$$

and,

$$R^T_{t,m}(t) = \int_{-\infty}^{t} \Lambda_{t,m}(t') \Gamma^T_{t}(t - t') \, dt'. \quad (3.6)$$

To be consistent with equation (2.1), the temporal form of the rotational potential is modeled as a series of Heaviside increments:

$$\Lambda_{t,m}(t) = \sum_{n=1}^{N} \delta\Lambda_{t,m}^n H(t - t_n). \quad (3.7)$$

Using equation (3.7) in (3.5) and (3.6), and applying (2.7) and (2.8), the following expressions are obtained:

$$R^T(\theta, \psi, t) = \sum_{t,m} \left\{ h^T_{t,E} \Lambda_{t,m}(t) \right\} \frac{\Lambda_{t,m}(t)}{g}$$

$$+ \sum_{n=1}^{N} \frac{\delta\Lambda_{t,m}^n}{g} \sum_{k=1}^{K} \frac{r^T_{k,E}}{s_k} \left[ 1 - \exp(-s_k^T(t - t_n)) \right] \} Y_{t,m}(\theta, \psi), \quad (3.8)$$

and,
\[ G^T(\theta, \psi, t) = \sum_{\ell, m} \left\{ (1 + k_{\ell}^{T,E}) \frac{\Lambda_{\ell, m}(t)}{g} \right\} + \sum_{n=1}^{N} \frac{\delta \Lambda_{\ell, m}^n}{g} \sum_{k=1}^{K} \frac{r_k^{T,E}}{s_k} \left[ 1 - \exp(-s_k(t - t_n)) \right] Y_{\ell, m}(\theta, \psi) + G^T(t). \] (3.9)

The difference between (3.9) and (3.8) multiplied by the ocean function gives the contribution of the varying rotational potential forcing to sea level change. Adding this contribution to equation (2.11) yields,

\[ \sum_{\ell, m} S_{\ell, m}(t) Y_{\ell, m}(\theta, \psi) = \sum_{r, s} C_{r, s}(t) Y_{r, s}(\theta, \psi) \times \]
\[ \sum_{\ell, m} \left\{ E_{\ell}^T T_{\ell} \left( \rho_I I_{\ell, m}(t) + \rho W S_{\ell, m}(t) \right) + E_{\ell}^T \frac{\Lambda_{\ell, m}(t)}{g} \right\} \]
\[ \sum_{n=1}^{N} \left[ T_\ell (\rho_I \delta I_{\ell, m}^n + \rho W \delta S_{\ell, m}^n) \beta_{\ell}^n(t - t_n) + \frac{\delta \Lambda_{\ell, m}^n}{g} \beta_{\ell}^n(t - t_n) \right] H(t - t_n) \]
\[ + G^{L,T}(t) \delta_{\ell, 0} \delta_{m, 0} Y_{\ell, m}(\theta, \psi), \] (3.10)

with,
\[ E_{\ell}^T = 1 + k_{\ell}^{T,E} - h_{\ell}^{T,E}, \] (3.11)

and,
\[ \beta_{\ell}^n(t - t_n) = \sum_{k=1}^{K} \frac{(r_k^{T,E} - r_k^{T,E})}{s_k} \left[ 1 - \exp(-s_k(t - t_n)) \right]. \] (3.12)

Equation (3.10) is the spectral domain form of the sea-level equation for a Maxwell viscoelastic Earth model excited by both a surface mass redistribution and a changing rotational potential (both varying with a Heaviside time dependence). Next, the geographic form of the rotational potential shall be described, followed by a brief discussion on the method presently adopted to calculate this sea-level forcing.

### 3.2.2 The Rotational Potential

A right-handed Cartesian co-ordinate system is adopted in the following analysis (denoted by \((x_i)\)) with its origin located at the center of mass of the Earth model in its unperturbed state. The \(x_1\) axis is aligned along Greenwich longitude and the \(x_2\) axis is 90 degrees east of \(x_1\). In the equilibrium state (i.e. before surface loading), the rotation vector, \(\overline{\omega}(t)\), is assumed to be \((0,0,\Omega)\), where \(\Omega\) is the present-day rotation rate of the Earth. Subsequent
to the onset of surface mass redistribution, the inertia tensor of the system is perturbed and the components \( \omega_i \) \((i = 1, 2)\) generally become non-zero. It is conventional to write the \( \omega_i \) in the form (e.g., Munk and MacDonald 1960),

\[
\omega_i(t) = \Omega(\delta_{i3} + m_i(t)), \tag{3.13}
\]

where the \( m_i(t) \) are small changes from the equilibrium state.

The rotational potential at the surface of a spherical Earth can be written in the form (e.g., Lambeck 1980),

\[
U_R(\gamma) = \frac{1}{3} \omega^2 a^2 - \frac{1}{3} \omega^2 a^2 P_{2,0}(\cos \gamma), \tag{3.14}
\]

where \( \gamma \) is now the angular distance between \( \vec{\omega}(t) \) and an arbitrary field point \((\theta, \psi)\). Using the result

\[
P_{2,0}(\cos \gamma) = \frac{1}{5} \sum_{m=-2}^{2} Y_{2,m}^1(\theta', \psi')Y_{2,m}(\theta, \psi), \tag{3.15}
\]

in equation (3.14), where \((\theta', \psi')\) are the co-ordinates of \( \vec{\omega}(t) \), the following expressions are obtained for the perturbation to the rotational potential from the equilibrium value,

\[
\Lambda(\theta, \psi, t) = \Lambda_{0,0}(t)Y_{0,0}(\theta, \psi) + \sum_{m=-2}^{2} \Lambda_{2,m}(t)Y_{2,m}(\theta, \psi), \tag{3.16}
\]

where

\[
\begin{align*}
\Lambda_{0,0}(t) &= \frac{a^2 \Omega^2}{3}[m^2(t) + 2m_3(t)] \\
\Lambda_{2,0}(t) &= \frac{a^2 \Omega^2}{6\sqrt{5}}[m_1^2(t) + m_2^2(t) - 2m_3^2(t) - 4m_3(t)] \\
\Lambda_{2,1}(t) &= \frac{a^2 \Omega^2}{\sqrt{30}}[m_1(t)(1 + m_3(t)) - im_2(t)(1 + m_3(t))] \\
\Lambda_{2,2}(t) &= \frac{a^2 \Omega^2}{\sqrt{5}\sqrt{24}}[(m_2^2(t) - m_1^2(t)) + 2m_1(t)m_2(t)], \tag{3.17}
\end{align*}
\]

with,

\[
\Lambda_{2,-m} = (-1)^m \Lambda_{2,m}^\dagger. \tag{3.18}
\]

The symbol \( i \) in equation (3.17) represents the complex number \( \sqrt{-1} \). Note that the rotational potential is completely described by degree 0 and degree 2 harmonics. Calculations show that the \( m_3 \) perturbations due to GIA are \( \sim 2 \) orders of magnitude smaller than either \( m_1 \) or \( m_2 \). As a consequence, the \( \Lambda_{2,1}(t) \) term in equation (3.17) is dominant since it is the only coefficient of the perturbed potential which contains first order terms in
Chapter 3: Postglacial Sea-Level Change on a Rotating Earth

$m_1$ or $m_2$ (neglecting the other harmonic components will introduce an error of no more than a few percent). Therefore, the sea-level response due to GIA will be dominated by the degree 2 order 1 harmonic signature (as discussed by Han and Wahr 1989). TPW, rather than change in rotation rate, is thus by far the dominant mechanism influencing sea-level change.

A number of publications describe the theory used to determine the $m_i$ associated with the surface loading of a spherically symmetric, Maxwell viscoelastic Earth model. The two most commonly adopted methodologies are described, for example, in Sabadini et al. (1982) and Wu and Peltier (1984). Although these two approaches are similar, the final equations used to calculate the $m_i$ are different. Recent work by Vermeersen and Sabadini (1996) and Mitrovica and Milne (1997) has shown, however, that these two methodologies yield essentially the same GIA-induced $m_i$ for a specific load and Earth model. The approach followed here is reviewed in Appendix B. Note that, from equations (2.1), (B.8), (B.14) and (B.15), the $m_i$ are dependent on the surface load $L(\theta, \psi, t)$; therefore, sea-level change appears explicitly on the right-hand-side of (3.10) as a contributor to the surface mass load and implicitly as a functional argument of the perturbations to the rotational potential.

### 3.2.3 Solving the New Sea-Level Equation

A modified version of the pseudospectral algorithm is adopted to solve the sea-level equation (3.10) in a gravitationally self-consistent manner while incorporating a TDCM and the water dumping phenomenon discussed in the previous chapter. By defining the rotational potential as a series of Heaviside increments in time, solving (3.10) can be achieved in a similar manner to that described in section 2.2.1.

Equation (3.10) can be written in the form,

$$\sum_{\ell, m} S_{\ell, m}(t_j) Y_{\ell, m}(\theta, \psi) = \sum_{r, s} C_{r, s}(t) Y_{r, s}(\theta, \psi) \sum_{\ell, m} \left\{ E_{\ell}^T T_{\ell} \left( \rho I_{\ell, m}(t) + \rho W S_{\ell, m}(t_{j-1}) \right) 
+ E_{\ell}^T \frac{\Lambda_{\ell, m}(t_{j-1})}{g} \right. 
+ \left. \left[ E_{\ell}^T T_{\ell} \delta S_{\ell, m}^T + E_{\ell}^T \frac{\delta \Lambda_{\ell, m}^T}{g} \right] \right. 
+ \left. \sum_{n=1}^{j-1} \left[ T_{\ell} \left( \rho I_{\ell, m}^n + \rho W S_{\ell, m}^n \right) \beta_{\ell}^T(t_j - t_n) + \frac{\delta \Lambda_{\ell, m}^n}{g} \beta_{\ell}^T(t_j - t_n) \right] \right. 
+ G_{\ell}^T(t) \delta_{\ell, o}^T \delta_{m, o} \} Y_{\ell, m}(\theta, \psi).$$

Equation (3.19) is analogous to equation (2.14). In this case, however, there are two
unknowns in equation (3.19), the $j$th sea-load increment, $\delta S^j_{t,m}$; and the $j$th increment of the rotational potential, $\delta \Lambda^j_{t,m}$. The $\delta S^j_{t,m}$ are determined in a similar manner to before. A first iterate of the $\delta S^j_{t,m}$ is determined by assuming that the sea-level change is geographically uniform and defined by the amount of meltwater added or subtracted to/from the oceans (see equation (2.15)). This first iterate is then applied to determine the $\delta \Lambda^j_{t,m}$ via equations (B.8), (B.14), (B.15) and (3.17). The first iterates of $\delta S^j_{t,m}$ and $\delta \Lambda^j_{t,m}$ can be employed to calculate the harmonic coefficients describing global sea-level change from the onset of the loading period to the time $t_j$, via the equation,

$$SG_{t,m}(t_j) = E^T_{t} T_{t} \left( \rho I_{t,m}(t) + \rho S_{t,m}(t_{j-1}) \right) + E^T_{t} \frac{\Lambda_{t,m}(t_{j-1})}{g} + \left[ E^T_{t} T_{t} \delta S^j_{t,m} + E^T_{t} \frac{\delta \Lambda^j_{t,m}}{g} \right] + \sum_{n=1}^{j-1} \left[ T_{t} (\rho I_{t,m} + \rho \delta S_{t,m}^n) \beta^T_{l}(t_j - t_n) + \frac{\delta \Lambda_{t,m}^n}{g} \beta^T_{l}(t_j - t_n) \right] + G^{T,T}(t) \delta_{t,0} \delta_{t,0}^m.$$  \hspace{1cm} (3.20)

The $SG_{t,m}(t_j)$ are then applied to calculate a second iterate set of the $\delta S^j_{t,m}$. This aspect of the calculation, which is more efficiently carried out in the space domain, is based on equation (2.22). The above described procedure is repeated until the desired tolerance is achieved (see equation (2.16)). Note that the $\delta \Lambda^j_{t,m}$ are necessarily computed at each iteration step since the $m_i$ are dependent on the surface load redistribution and thus sea-level change. Also, to incorporate a TDCM and near-field water dumping in a gravitationally self-consistent manner, the second iterative procedure described in section 2.2. is employed. A flow chart of the algorithm adopted to solve equation (3.19) is given in Appendix C.

As mentioned in section 3.1, perturbations to the Earth's rotation vector are caused by a number of processes acting over different time scales. For example, recent calculations have shown that convective flow within the mantle may significantly contribute to the present-day secular motion of the rotation pole (Steinberger and O'Connell 1997). It is of interest, therefore, to consider how the theory described above can be modified to calculate sea-level change resulting from an a priori defined TPW path. For this case, the governing equation is a modified form of equation (3.10) in which the $\delta I^p_{t,m}$ and $I_{t,m}(t)$ are set to zero. This equation can be solved using the same iterative approach discussed above. However, the water dumping effect is not relevant in this case and so equation (2.21) should be employed rather than equation (2.22).
3.3 Results

3.3.1 Relative Sea-Level Predictions

Since TPW is sensitive to the number of glacial cycles included in the modeling calculation (e.g., Wu and Peltier 1984), it is necessary to determine whether postglacial RSL predictions are also sensitive to this parameter. Figure 3.1 shows the predicted rotational contribution to RSL at Clinton, Massachusetts (northeastern United States) after one glacial cycle and seven glacial cycles for an isoviscous mantle of $10^{21}$ Pa s and a lithospheric thickness of 120 km. This particular viscosity structure yields the maximum difference between the one and seven cycle predictions for the suite of Earth models considered in this analysis. Nevertheless, the discrepancy is (in general) less than a tenth of a meter. Accordingly, one glacial cycle is adopted in the subsequent analysis.

![Graph showing relative sea-level predictions](image)

Figure 3.1: The postglacial RSL signal at Clinton (41.2N, -72.5E) caused by GIA-induced variations in the Earth's rotation vector calculated using an Earth model with an isoviscous mantle of $10^{21}$ Pa s and a lithospheric thickness ($LT$) of 120 km. The 'squares' indicate predictions calculated using a single glacial cycle, while the 'triangles' show predictions calculated using 7 glacial cycles.

An example of the spatial form of the sea-level signal associated with GIA-perturbations to the rotation vector is shown in Figure 3.2. (Note, the reader can compare this rotation-induced RSL signal to the 'non-rotating' case by referring to Figures 3.4 and 3.5. This figure shows RSL curves at a number of globally distributed sites calculated by solving...
Figure 3.2: The predicted global signal of RSL at 18 kyr BP (i.e., the change in sea level from 18 kyr BP to the present) produced by GIA-induced perturbations to the Earth's rotation vector. The Earth model adopted in the calculation is characterized by: $LT = 120$ km, $\nu_{um} = 10^{21}$ Pa s and $\nu_{im} = 10^{22}$ Pa s. Note the dominant degree 2 order 1 spherical harmonic component of the signal. The shorter wavelength harmonics are caused by the effect of the water load induced by the perturbation to the rotation vector.
Figure 3.3: The predicted global signal of RSL at 18 kyr BP due to the perturbation to the rotational potential only (that is, the water loading effect associated with this potential is not considered). This prediction is based on the Earth model described in Figure 3.2.
Chapter 3: Postglacial Sea-Level Change on a Rotating Earth

the new and the conventional sea-level equation.) This figure is obtained by differencing a prediction calculated using the new sea-level theory (equation (3.10)) with one based on the theory appropriate to a non-rotating Earth (equation (2.11)). For comparison, Figure 3.3 shows the analogous sea-level change associated with the effect of the perturbing rotational potential alone (i.e., the sea load induced by the rotational forcing is not considered). Figures 3.2 and 3.3 were computed using the same Earth model and both represent global plots of RSL at 18 kyr BP. As discussed in the context of equation (3.18), Figures 3.2 and 3.3 are dominated by a degree 2 order 1 harmonic signal (the combined contribution of orders 0 and 2 is less than a few percent). A comparison of Figures 3.2 and 3.3 indicates that the ocean load deformation associated with the rotational signal excites higher degree harmonics (note the short wavelength structure in Figure 3.2) and acts, in general, to increase the amplitude of the RSL signal. For example, the total RSL signal range in Figure 3.3 is \( \sim 9 \) m, which compares with \( \sim 12 \) m in Figure 3.2. This increase in amplitude can be explained in a straightforward manner. Consider, for example, the east coast of North America. The direct effect of the rotational potential induces a sea-level fall of approximately 4 - 5 m over the last 18 kyr in this region (see Figure 3.3). This sea-level fall represents a 'negative' load which will be accompanied, in the full calculation, by an incremental uplift of the solid Earth. This uplift acts to increase the amplitude of the predicted sea-level fall and hence the results in Figure 3.2 show an enhanced sea-level fall in this region of about 5 - 6 m. This enhancement is evident in both the western Atlantic and the southern Indian Ocean. A related effect occurs over southern South America and Japan. In this case the sea-level rise associated with the direct effect of the perturbing rotational potential (from 18 kyr BP to present) acts to induce an incremental solid surface subsidence and hence an enhanced sea-level rise. Clearly, an accurate prediction of rotation-induced postglacial sea level requires the application of the gravitationally self-consistent theory of equation (3.10)).

Since the dominant degree 2 order 1 sea-level signal is due to TPW, as opposed to change in the length of day, the spatial orientation of the signal is, to a large extent, defined by the line of longitude along which the rotation pole moves, on average, during the postglacial period (Han and Wahr 1989). The present predictions show that the rotation pole traces an average direction along the great circle defined by longitudes (\( \sim 106^\circ \)E, \( \sim 74^\circ \)W), with the pole moving towards north eastern Canada during the deglaciation phase. Therefore, the rotational potential will show maximum change along this meridian at mid-latitudes and a minimum change at 90 longitudinal degrees from this meridian and along the equator. The sea-level signals shown in Figures 3.2 and 3.3 clearly reflect
The temporal form of the rotationally induced sea-level signal is evident in Figures 3.4 and 3.5, which show predicted RSL curves for rotating and non-rotating Earth models (left side) and the difference between these ('rotating' minus 'non-rotating'; right side) for sites distributed around the globe. As in Figures 3.2 and 3.3, these predictions were calculated using an Earth model with $LT = 120$ km, $\nu_{um} = 10^{21}$ Pa s and $\nu_{vm} = 10^{22}$ Pa s. The specific sites were chosen to illustrate the spatial symmetry and maximum and minimum amplitudes of the rotation-induced sea-level signal. (Clinton and Recife were also chosen to facilitate comparison with the results of Han and Wahr (1989) - see below.) Most of the sites included in Figures 3.4 and 3.5 are from far-field regions where the predicted RSL curves are not significantly affected by ice-induced deformation (Tientsin, Perth, Recife and Bremerhaven fall into this category). The site ‘Bahia Gente Grande’ is located near the southern section of the Andes mountain range where a small ice mass existed at LGM producing a significant amount of deformation. Clinton is located near the edge of the massive Laurentide ice mass and so exhibits an RSL curve that is characteristic of such regions; that is, the initial sea-level fall due to solid surface rebound is followed by sea-level rise as the peripheral bulge migrates towards the center of the ice mass (e.g., Clark et al. 1978).

The symmetry of the rotation-induced sea-level signal is evident when comparing the 'rotating minus non-rotating' predictions for Clinton, Bahia Gente Grande, Tientsin and Perth. All of these sites are at mid-latitudes near the great circle meridian described above. Sites that are 180 latitudinal degrees apart on this meridian exhibit the same time-dependent rotation-induced RSL curve (e.g., Clinton and Perth, Bahia Gente Grande and Tientsin). Sites near the equator, such as Recife, display a considerably smaller amplitude, as do sites that are located ~90 longitudinal degrees from the polar wander meridian (e.g., Bremerhaven).

The postglacial rotation-induced sea-level signal is related to the rate of TPW during the deglaciation period. To illustrate this relationship, consider Figure 3.6, which shows TPW (i.e., rotation pole displacement) (Figure 3.6(A)), and the direct, elastic, viscous and total sea-level response to the changing rotational potential at the site Clinton (Figure 3.6(B)). Note, first, that the direct and elastic effects are proportional to the time series of TPW, whereas the viscous effect is a time-lagged response to the displacement of the rotation pole. The direct effect of the rotational potential acts to produce a sea-level fall at this site during deglaciation, whereas the viscoelastic deformation associated with the
Figure 3.4: Postglacial RSL curves calculated using an Earth model with $LT = 120$ km, $\nu_{um} = 10^{21}$ Pa s and $\nu_{lm} = 10^{22}$ Pa s. The left hand frames show solutions of the sea-level equation for a rotating (squares) and non-rotating (triangles) Earth. The right hand frame shows the difference ('squares' minus 'triangles') between these predictions.
Figure 3.5: See caption for Figure 3.4.
Figure 3.6: Frame (A) shows the predicted GIA-induced TPW for an Earth model with \( LT = 120 \) km, \( \nu_m = 10^{21} \) Pa s and \( \nu_{im} = 10^{22} \) Pa s. Frame (B) shows the corresponding rotation-induced component of RSL at Clinton, U.S. (41.2N, -72.5E). The ‘squares’ show the direct effect of the rotational potential on sea level, while the ‘circles’ and ‘triangles’ show, respectively, the contributions from elastic and viscous deformations to the sea-level response. The black circles indicate the sum of these three sea-level contributions.
rotational potential is dominated by solid surface subsidence and sea-level rise. Notice that the direct effect dominates the rotation-induced sea-level signal until about half way through the deglaciation phase (~12 kyr BP). Subsequent to this time, the viscoelastic deformation, particularly the non-elastic component of this deformation, dominates. This evolution in the strength of the direct and viscous contributions leads to the marked non-monotonicity of the rotation-induced sea-level signals shown in all frames of Figures 3.4 and 3.5.

Previous studies have shown that predictions of TPW are sensitive to variations in \( \nu_m \) and lithospheric thickness (e.g., Yuen, Sabadini and Boschi 1982; Wu and Peltier 1984) and are relatively insensitive to variations in \( \nu_m \) (Mitrovica and Milne 1997). Accordingly, the rotational component of sea-level change at Clinton is calculated for a suite of Earth models in which \( \nu_m \) (Figure 3.7(A)) and LT (Figure 3.8) are varied significantly. A comparison of Figures 3.7(A) and 3.8 indicates that the RSL signal induced by perturbations in the rotation vector is more sensitive to variations in \( \nu_m \) than variations in LT.

Figure 3.7 shows the total RSL signal associated with perturbations in the rotational potential (frame A) together with the three contributions (viscous, elastic, direct) to this signal. The RSL signal at 18 kyr BP increases by a factor of ~2 as the lower mantle viscosity is increased from \( 10^{21} \) Pa s to \( 10^{23} \) Pa s. This trend arises because the sea-level signal associated with viscoelastic deformation (frames B and C) less effectively compensates for the direct RSL contribution from the rotational potential (frame D) as \( \nu_m \) is increased.

Each prediction in Figure 3.7(A) is characterized by a transition from a period of sea-level fall (prior to about 9 kyr BP) to an interval (extending to the present) of sea-level rise. Although the amplitude of the latter signal is a function of the viscoelastic structure of the adopted Earth model, the timing of the transition is relatively insensitive to this structure. Once again, this timing marks the transition to a period in which the viscous response to the rotational potential dominates the direct effect of this potential. A discussion of the timing of this transition can be found in section 3.4.1.

Figure 3.8 shows the predicted rotation-induced RSL variation at Clinton for LT values of 70 km, 120 km and 170 km with adopted \( \nu_m \) values of \( 10^{21} \) Pa s, \( 10^{22} \) Pa s, and \( 10^{23} \) Pa s. The results show that the rotation-induced component of RSL is most sensitive to this range of LT for the isoviscous mantle model (frame A in Figure 3.8), in which an increase in LT of 100 km leads to an increase in the predicted RSL at 18 kyr BP of ~1.2
Figure 3.7: Frame (A) displays the rotation-induced component of RSL at Clinton for three different Earth models; each with $LT = 120 \text{ km}$ and $\nu_{um} = 10^{21} \text{ Pa s}$ and contrasting $\nu_{im}$ values of $10^{21} \text{ Pa s}$ (squares), $10^{22} \text{ Pa s}$ (triangles) and $10^{23} \text{ Pa s}$ (circles). Frames (B), (C) and (D) show, respectively, the viscous, elastic and direct sea-level responses contributing to the RSL signals shown in plate (A).
Figure 3.8: Predicted RSL change at Clinton (41.2N, -72.5E) due to variations in the rotation vector. Calculations are based on Earth models with $\nu_{um} = 10^{21}$ Pa s and $\nu_{lm} = 10^{21}$ Pa s (frame A), $10^{22}$ Pa s (frame B) or $10^{23}$ Pa s (frame C). Within each frame, the curves correspond to calculations based on a lithospheric thickness of 70 km (squares), 120 km (circles) or 170 km (triangles).
A similar trend is evident for models with a more viscous lower mantle (frames B and C in Figure 3.8), although the sensitivity of predictions to the range of $LT$ explored here becomes smaller as $\nu_{im}$ is increased.

### 3.3.2 Present-Day Sea-Level Rate Predictions

Figure 3.9 shows calculated present day sea-level rates due to GIA-induced perturbations in the rotation vector. As before (i.e., see Figure 3.2), the signal is predominantly that of a degree 2 order 1 harmonic with some minor perturbations caused by the rotation-induced component of the water load. As for the case of RSL predictions, the rotation-induced water load also acts to increase the magnitude of the predicted sea-level rate signal. As an example, the rotation-induced sea load has a maximum effect in the region of southern South America where it increases the signal magnitude by $\sim 25\%$. This compares to an approximately $10\%$ predicted increase along the north east coast of the US, resulting in a total peak to peak increase of the sea-level rate signal in the western hemisphere of $\sim 35\%$. The prediction shown in Figure 3.9 exhibits a maximum amplitude of $\sim 0.15$ mm/yr. This signal amplitude is approximately $10\%$ of that recently estimated for present-day eustatic sea-level change (e.g., Peltier and Tushingham 1991; Davis and Mitrovica 1996).

The sensitivity of the predicted, rotation-induced, sea-level rate to variations in lower mantle viscosity shall now be investigated. To illustrate this sensitivity Figure 3.10 considers the sea-level rate shown in Figure 3.9 at Clinton as a function of $\nu_{im}$. The predictions are remarkably insensitive to variations of $\nu_{im}$ in the range $3 - 50 \times 10^{21}$ Pa s. The small amplitude of the prediction for $\nu_{im} \sim 10^{21}$ Pa s may be surprising given that the predicted present-day TPW rate for this model is large (e.g., Milne and Mitrovica 1996). However, this small amplitude arises because the the direct RSL effect due to the rotational potential is almost the same magnitude, but of opposite sign, to the RSL signal associated with viscoelastic deformation. (This near cancellation is evident by examining the slopes of the curves in Figure 3.7 (squares) at present day.) In contrast, the predicted amplitude for the case $\nu_{im} = 10^{23}$ Pa s is large, despite the small present-day TPW rate prediction for this class of model (e.g., Milne and Mitrovica 1996). In this case, the direct component of the present-day RSL response is negligible compared to the viscous component (see Figure 3.7 (circles)). The present-day sea-level rate predictions based on models with $\nu_{im}$ between these two end-members can, analogously, be explained in the context of competing direct and viscoelastic RSL signals. Note that the predictions shown in Figure 3.10 are sensitive to the amount of time elapsed since the end of the
Figure 3.9: Present-day sea-level rates caused by GIA-induced perturbations to the rotation vector. Calculations are based on an Earth model with $LT = 120$ km, $\nu_{um} = 10^{21}$ Pa s and $\nu_{lm} = 10^{22}$ Pa s. The signal is dominated by a degree 2 order 1 harmonic geometry with shorter wavelength perturbations to this signal induced by the water load component associated with polar motion.
Chapter 3: Postglacial Sea-Level Change on a Rotating Earth

Figure 3.10: Predicted present-day sea-level rates at the site Clinton (41.2N, -72.5E) due to GIA-induced change in the rotational potential as a function of $\nu_{lm}$, for Earth models characterized by $LT = 120$ km and $\nu_{um} = 10^{21}$ Pa s.

final melting event (this occurs at 5 kyr BP in the ICE 3G model). For example, if the final melting event is assumed to have occurred 1 kyr later (at 4 kyr BP), the predicted sea-level rate at Clinton has a value of $\sim 0.14$ mm/yr for $\nu_{lm} = 1 \times 10^{22}$ Pa s. The general trend of the curve shown in Figure 3.10 is not altered by this modification to the ice model.

Predictions of present-day sea-level rates for lithospheric thicknesses ranging from 70 km to 170 km were also considered while adopting the values of lower mantle viscosity: $10^{21}$ Pa s, $10^{22}$ Pa s and $10^{23}$ Pa s. The predictions based on Earth models characterized by an isoviscous mantle were most sensitive to the chosen range of $LT$. In this case, adopting thin (70 km) lithosphere increased predicted rates by $\sim 40\%$ at selected sites, compared to the model with $LT = 120$ km, while the model with a 170 km lithosphere decreased the predicted present-day sea-level rate by $\sim 15\%$. The sensitivity of predictions to variations in $LT$ for Earth models with a $\nu_{lm}$ of $10^{22}$ Pa s or $10^{23}$ Pa s is approximately half of that for the isoviscous mantle models, with the same trend evident (thinner lithosphere, larger sea-level rate). These results, together with the predictions shown in Figure 3.10, indicate that models characterized by a thin lithosphere and a factor of 5 – 20 increase in viscosity from upper to lower mantle will produce the largest sea-level rate signal. The calculations presented here show that such models produce a global peak to peak sea-level rate signal of $\sim 0.3$ mm/yr.
3.4 Discussion

3.4.1 Comparison to Previous Results

In their study of sea-level change arising from GIA-induced perturbations in the Earth's rotation vector, both Han and Wahr (1989) and Bills and James (1996) (hereafter referred to as HW and BJ, respectively) included only the Laurentide ice mass component of the Late Pleistocene ice cover. It is therefore of interest to examine how predictions of the rotation-induced component of sea-level change are affected by adopting this simpler ice model. Figure 3.11 shows predictions of the rotation-induced component of the sea-level signal calculated using the complete global ice cover (circles) and using only the Laurentide component of the global ice model (squares). The discrepancy is small, indicating that GIA-induced TPW is driven almost entirely by the growth and decay of the Laurentide ice sheet. This is due to the fact that the cryospheric mass flux in the eastern hemisphere (i.e., between 0 and 180° east longitude) during the last deglaciation contributed a $J_{23}^L$ inertia component (see Appendix B) which was only ~20% of the magnitude (and of opposite sign) to the contribution associated with the Laurentide ice complex. Thus, motion of the rotation pole along the $x_2$ axis (which dominates the calculated GIA-induced TPW) is largely controlled by the growth and decay of the Laurentide ice sheet. Similar predictions for $\nu_{lm}$ ranging from $10^{21}$ Pa s to $10^{23}$ Pa s show that, in general, a maximum error of ~10% is introduced to predictions of rotationally induced RSL when only the Laurentide ice mass is considered.

The dominance of the Laurentide ice sheet contribution to the rotationally induced RSL change raises an interesting issue. As discussed in the last section, the predicted RSL perturbation due to rotation is characterized by a transition from a phase of sea-level fall (rise) to a period of sea-level rise (fall) in quadrants associated with Australia and North America (South America and eastern Asia). The timing of this transition is linked to the end of the deglaciation event (and hence the end of the interval in which the direct effect of the rotational potential on sea level is significant). Although the adopted ice model (ICE-3G) is defined by a global deglaciation which ends at 5 kyr BP, the melting of the Laurentide ice sheet component of the model is essentially complete by 9 kyr BP (Tushingham and Peltier 1991). It is this specific aspect of the ice model which governs the timing of the sea-level transitions evident in Figures 3.4 and 3.5.

HW presented predictions for a Maxwell viscoelastic Earth model with $LT = 120$ km, and an isoviscous mantle of $10^{21}$ Pa s (see their Figure 3). The temporal form of the
Figure 3.11: Rotation-induced RSL calculated using the global ICE-3G based model (see text) (circles) and the Laurentide component of this model (squares). The Earth model adopted in this calculation is characterized by a lithospheric thickness of 120 km and an isoviscous mantle of $10^{21}$ Pa s.
Figure 3.12: The predicted degree 2 order 1 component of sea-level change induced by the rotational potential for the site Clinton (41.2N, -72.5E) (the associated water loading effect has not been included). The ‘squares’ denote the direct effect of the rotational potential on sea level while the ‘triangles’ denote the viscoelastic sea-level response. The sum of these curves gives the total sea-level signal shown by the black circles. These calculations are based on an Earth model with $LT = 120$ km and an isoviscous mantle of $10^{21}$ Pa s.

rotationally induced RSL curves are significantly different from those shown in HW’s Figure 3. This issue is explored in Figure 3.12 which shows the predicted rotation-induced RSL change (and its various contributions) at Clinton for the same Earth model used by HW. (To be consistent with the HW predictions, the calculations in Figure 3.12 do not include the influence of the rotation-induced water load and consider only the degree 2 order 1 component of the rotational potential.) Clearly, the direct effect of the rotational potential on sea level (squares) cannot be ignored since it is of an equal and opposite magnitude to the deformational effect (triangles); indeed, its omission leads to a significantly larger signal which lacks the important transition at ~9 kyr BP.

As stated in section 3.1, BJ concluded that calculations based on a rigid Earth model produce a reasonable first order prediction of rotation-induced sea-level change. The sea-level response, for the case of a rigid Earth model, is due solely to the direct effect of the rotational potential. It is clear from Figure 3.12 that the direct effect cannot provide an accurate description of the total rotation-induced RSL response since LGM. As an example, a prediction based on the direct effect alone will not be characterized by the non-monotonicity evident in the total rotation-induced RSL response. Furthermore, the
former calculation will yield an RSL amplitude that is significantly too large. BJ claim that the peak-to-peak amplitude of the total rotation-induced RSL signal reaches 40 m (± 20 m). This is consistent with the 'direct effect' in Figure 3.12, which is a factor of ~5 larger than the actual rotation-induced signal.

The results in Figure 3.12 also demonstrate why the BJ analysis led to predictions that are of opposite sign and comparable magnitude to those presented in HW (simply compare the squares and triangles in Figure 3.12). As a specific example, Figure 1 of BJ implies a fall in sea level over the deglaciation period at Clinton, while Figure 3 of HW indicates a sea-level rise at this site.

### 3.4.2 The Significance of the Rotation-Induced Signal

In this section, the magnitude of the rotation-induced component of sea-level change is compared to the load-induced component for geographic regions from which data have been obtained and consequently employed to constrain models of the GIA process.

As previously discussed, inferences of mantle viscosity based on RSL predictions have commonly utilized data from north eastern Canada (e.g., Mitrovica and Peltier 1993; Han and Wahr 1995) and Australia (e.g., Nakada and Lambeck 1989). These are regions in which the TPW-induced sea-level change is near a maximum (see Figure 3.2), leading BJ to state that "...failure to include it (the rotation-induced RSL contribution) in previous analyses of the sea-level problem largely invalidates many ... quantitative inferences (of mantle viscosity)" (p. 3023). Figure 3.13 shows predictions of RSL variations at the Richmond Gulf site in Hudson Bay for the cases of a sea-level theory valid for a rotating and non-rotating Earth model and for two different viscosity profiles ($\nu_m = 10^{21}$ Pa s or $10^{22}$ Pa s). It is clear, in contrast to the assertion by BJ, that the rotational RSL signal is too small in magnitude to affect previous analyses using data from north eastern Canada. Note, in this regard, that the uncertainties in the observed RSL curves from this region (see, for example, Walcott 1972) are also significantly larger than the maximum predicted RSL contribution due to GIA-induced TPW.

RSL predictions in the Australian region and its vicinity will now be considered. As discussed previously, Nakada and Lambeck (1989) used the difference in the measured Holocene high stands from five Australian sites and one site in New Zealand to infer upper and lower mantle viscosity. Table 3.1 shows predictions of differential sea-level high stands between sites considered by Nakada and Lambeck (1989) for the case of a sea-level theory
Figure 3.13: Predicted postglacial RSL curves at the site Richmond Gulf (57.0N, -77.0E) in north eastern Canada. The 'squares' denote predictions based on the sea-level equation valid for a rotating Earth while the 'triangles' denote predictions based on the sea-level equation for a non-rotating Earth. The solid lines represent calculations that adopted an Earth model with LT = 120 km, and an isoviscous mantle of $10^{21}$ Pa s, whereas the dotted lines represent calculations that adopted the same Earth model with the exception that $\nu_{tm}$ is increased $10^{22}$ Pa s.

appropriate to a rotating and non-rotating Earth. As in Figure 3.13, the calculations are based on Earth models with $LT = 120$ km, $\nu_{um} = 10^{21}$ Pa s and $\nu_{tm} = 10^{21}$ Pa s ('Viscosity Model 1') or $10^{22}$ Pa s ('Viscosity Model 2'). Since the rotation-induced sea-level geometry is dominated by a long wavelength degree two spherical harmonic function, errors in differential sea-level high stands will be exceedingly small for sites in close proximity. From Table 3.1, the largest error incurred by neglecting the rotational contribution to the differential sea-level high stands is 0.2 m for the case of the Halifax Bay-Moruya and Moruya-Christchurch pairs. These errors will have no bearing on the viscosity inference based on these observables since the observational uncertainties are significantly larger than 0.2 m. This result also holds for the other differential high stands considered in Table 3.1, therefore, the Nakada and Lambeck (1989) viscosity inference (despite the assertions of BJ) would not be affected by the inclusion of a rotation signal in the GIA sea-level theory.

BJ also argue that the contribution of GIA-induced TPW to sea-level change will likely bias previously constructed models of glacial loading and unloading based on RSL data.
Table 3.1: Predictions of Late Holocene differential high stands for sites in the Australian region. See text for a definition of 'Viscosity Model 1' and 'Viscosity Model 2'.

<table>
<thead>
<tr>
<th>Site Pairs</th>
<th>Viscosity Model 1</th>
<th>Viscosity Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rotating</td>
<td>Non-Rotating</td>
</tr>
<tr>
<td>Karumba-Halifax Bay</td>
<td>0.6 m</td>
<td>0.6 m</td>
</tr>
<tr>
<td>Port Pirie-Cape Spencer</td>
<td>1.2 m</td>
<td>1.1 m</td>
</tr>
<tr>
<td>Halifax Bay-Moruya</td>
<td>-0.1 m</td>
<td>-0.2 m</td>
</tr>
<tr>
<td>Moruya-Christchurch</td>
<td>-1.1 m</td>
<td>-1.1 m</td>
</tr>
</tbody>
</table>

(e.g., Tushingham and Peltier 1991; Peltier 1994). The RSL constraints adopted in Tushingham and Peltier (1991), for example, are generally younger than 10 kyr BP and were obtained from previously glaciated regions. Neglecting the rotational signal of RSL change would introduce a maximum error of ~2 m over this time range (see Figure 3.4). The RSL signals from previously glaciated regions and their associated error bars (e.g., Tushingham and Peltier 1991) are significantly larger than this signal, therefore, inferences of the space-time geometry of the late Pleistocene ice loads will, in fact, be unaffected by the inclusion of a rotation-induced RSL signature.

Global models of late Pleistocene ice mass history have been tuned to match the eustatic sea-level history determined by Fairbanks (1989) on the basis of coral records from Barbados (e.g., Peltier 1994). Since the Barbados site is at low latitudes, the rotationally induced RSL signal will be small. In fact, the signal is roughly ~3 m at 18 kyr BP in Figure 3.2, which is consistently smaller (by a factor of 2 - 3) than the observational uncertainty in RSL data of this age. Therefore, the previously applied 'tuning' procedure remains valid.

Although differential sea-level high stands are not significantly affected by the rotationally induced RSL signal (Table 3.1), this signal is large enough to bias estimates of melting events since the end of the deglaciation period (~5 kyr BP to present) which are based on high stands for specific sites (i.e., not differential high stands). As previously discussed, Nakada and Lambeck (1989) used their preferred viscosity model to predict the absolute value of the observed Late Holocene high stands. The present results indicate that the inclusion of a rotationally induced sea-level signal will act to lower the predicted high stands in the Australian region relative to those based on a theory valid for a static Earth by, on average, 0.5 m (see the Perth site in Figure 3.5). Therefore, the new sea-level theory described in this chapter will, when applied to the Nakada and Lambeck
Chapter 3: Postglacial Sea-Level Change on a Rotating Earth

(1989) analysis, lead to lower estimates of late Holocene melting from Antarctica.

Davis and Mitrovica (1996) analyzed U.S. east coast tide-gauge data using a suite of Earth models identical to those considered here and a sea-level algorithm based on a non-rotating Earth. They found that a model characterized by a LT of 120 km, an upper mantle viscosity of $10^{21}$ Pa s and a lower mantle viscosity of $5 \times 10^{21}$ Pa s removed the geographic trend in the tide-gauge rates. They then adopted this model to estimate a uniform sea-level rise of 1.5 (± 0.3) mm/yr from the tide-gauge data. The results in Figure 3.9 indicate that the rotation-induced component of the predicted present-day sea-level rate signal is relatively large in this region. Therefore, the significance of this signal shall briefly be considered for the type of analysis carried out by Davis and Mitrovica (1996).

The 1-σ error associated with the 33 sea-level records analyzed by Davis and Mitrovica (1996) ranges from ~0.3 mm/yr to ~0.8 mm/yr. Since the rotation-induced signal does not exceed ~0.15 mm/yr for the range of Earth models considered in this chapter, it is evident that the signal is of too small a magnitude to alter the viscosity inference of Davis and Mitrovica (1996). To determine the significance of the rotation-induced signal for their estimate of a geographically uniform sea-level rise, Figure 3.14 considers a prediction of the sea-level perturbation due to the rotational excitation for the 33 sites considered in their analysis. The predictions are based on their 'preferred' Earth model described above. The locations of the tide gauge sites are also shown in Figure 3.14.

All of the sites considered in Figure 3.14 exhibit a sea-level rise. This is because the viscous component of the sea-level response, which causes postglacial solid surface subsidence in this region, dominates at present. The general trend of the data illustrates the degree 2 order 1 geometry of the sea-level signal. As described earlier, the signal magnitude occurs at mid-latitudes along the great circle defined by the TPW. The northernmost sites shown in Figure 3.14 are the nearest to the signal maximum in this region, and so as one moves progressively southwards, the signal magnitude generally decreases. The short wavelength structure evident in the signal is a result of the higher harmonics excited by the water load component of the rotation-induced signal (see Figure 3.9), as well as the random scatter evident in the location of the tide gauge sites.

The net effect of the rotation-induced contribution to sea-level change is to reduce the regional sea-level trend estimated by Davis and Mitrovica (1996) by just under 10%. The magnitude of this effect should not be compared to the statistically determined 1-σ uncertainty (0.3 mm/yr) for the regional trend, since the rotation-induced signal is
Figure 3.14: The top frame is a map of the US east coast showing the location of the 33 tide gauge sites considered by Davis and Mitrovica (1996). Sites are numbered beginning from the northeastern most site and progressing (generally) southwards along the coast. The lower frame indicates the rotation-induced component of the present-day sea-level rate signal at each of the sites illustrated. The Earth model adopted in the calculation is described in the text.
a systematic effect. The magnitude of the rotation-induced signal is notable given that other sources contaminating estimates of the present-day global melt signal with a similar magnitude have been discussed in the literature (see Davis and Mitrovica 1996).

### 3.4.3 The Influence of the M1 Relaxation Mode on the Rotation-Induced Component of Sea-Level Change

To this point the normal mode nature of the load and rotational responses associated with the GIA process has been neglected (see equations (3.1) and (3.2) for example). A recent study by Mitrovica and Milne (1997) has shown that the M1 mode of relaxation plays an important role in predictions of present day TPW rate due to GIA for values of lower mantle viscosity below $\sim 10^{22}$ Pa s. The M1 mode is associated with the buoyancy-induced stress field resulting from the deflection of the density (seismic) discontinuity at 670 km depth. However, the mode will only contribute to the GIA process if the boundary behaves non-adiabatically on GIA time scales. (Non-adiabaticity can arise as a consequence of either a chemical discontinuity at this depth or a phase change that is characterized by a time scale considerably longer than the GIA process.) Since this behavior is a point of some contention (e.g., Fjeldskaar and Cathles, 1984; Mitrovica and Peltier, 1989) it is worthwhile determining how sensitive the predictions of rotation-induced sea-level change are to the existence of this relaxation mode. Figure 3.15 shows the rotational component of sea-level change calculated for two Earth models. These are characterized by: $LT = 120$ km, $\nu_{um} = 10^{21}$ Pa s and either $\nu_{lm} = 10^{21}$ Pa s or $\nu_{lm} = 10^{22}$ Pa s. For each model the signal is shown for the cases in which the M1 mode of relaxation is included or removed. (Note that, in the present analysis, the effect of a perfectly adiabatic boundary at 670 km depth is modeled by simply removing the M1 mode calculated for an Earth model which includes the seismically inferred density jump at this depth. This procedure, which has been adopted in previous GIA analyses (e.g., Yuen, Sabadini, Gasperini and Boschi 1986; Mitrovica and Milne 1997), is, however, only an approximate treatment of this problem. The reader is referred to recent work by Johnston, Lambeck and Wolf (1997) for a more rigorous approach.) The M1 mode has a relatively large effect in the case of the isoviscous model, producing a $\sim 50\%$ reduction in the RSL signal over the last 18 kyr. In accord with the TPW predictions of Mitrovica and Milne (1997), the effect of the M1 mode decreases monotonically as the value of $\nu_{lm}$ is increased, and is only significant (discrepancy greater than $\sim 20\%$) for models with $\nu_{lm}$ in the range $1 - 5 \times 10^{21}$ Pa s.
3.5 Conclusions

In this chapter a new sea-level equation was derived that incorporates the perturbations to the geoid and the solid surface caused by change in the Earth's rotational potential as well as surface mass redistribution. Also, a technique for solving this equation in a gravitationally self-consistent manner while adopting a TDCM was outlined.

The rotation-induced component of the postglacial sea-level signal exhibits a spatial form that is described almost completely by a degree 2 order 1 spherical harmonic function (Han and Wahr 1989). The rotation-induced water load acts to increase the signal due to the potential forcing and excites lower amplitude, higher degree harmonics in the sea-level response. The orientation of the dominant degree 2 order 1 signal relative to the surface geography is dependent on the direction of TPW during the postglacial period. This direction is largely determined by the degree 2 signature of the adopted ice model. The calculations presented herein, based on the ICE-3G model (Tushingham and Peltier 1991), indicate that the pole traces a path defining the meridian $\sim$106°E during the deglaciation period. This path direction is largely controlled by the Laurentide
component of the global ice mass distribution.

The magnitude of the rotation-induced RSL signal is dependent on the adopted Earth model. The results in this chapter indicate that the predicted magnitude is most sensitive to variations in lower mantle viscosity. Increasing this parameter from $10^{21} \text{ Pa s}$ to $10^{23} \text{ Pa s}$ changes the magnitude of the RSL signal by a factor of $\sim 2$ shortly after LGM. This dependence is a result of the fact that the Earth deformation component of the RSL response to the rotational potential is significantly reduced for larger values of $\nu_m$ and so the opposing direct effect becomes more dominant at this time. The relatively small signal predicted for models characterized by a weaker lower mantle viscosity may be surprising given that the magnitude of polar motion is largest for this class of models. However, in this case the Earth deformation induced by the rotational potential is large and so more completely compensates the direct effect of the rotational potential.

The temporal form of the rotation-induced RSL signal exhibits a transition from a period of sea-level fall (rise) to a period of sea-level rise (fall) (the specific trend is site dependent). This change reflects the shift in the relative magnitudes of the deformational and direct sea-level response components. Early on in the postglacial period, the direct contribution is dominant whereas, approximately half way though this period, the viscous component becomes dominant. The specific timing of this transition depends on the time at which the surface mass redistribution ends. The above results indicate that the massive Laurentide component of the global ice coverage largely controls the GIA-induced TPW signal. Therefore, since this ice mass catastrophically melts over the period 11 kyr BP to 9 kyr BP within the ICE-3G reconstruction, the timing of the above mentioned transition occurs at $\sim 9$ ky BP in the above predictions.

The magnitude and temporal form of the rotation-induced RSL signal obtained by solving the new sea-level equation are not consistent with previous theoretical results (Han and Wahr 1989; Bills and James 1996). The signal predicted in these studies is $\sim 3 - 4$ times larger at LGM than the results given here, and displays a temporal form that is monotonic. The reason for these contrasting results can be explained in a straightforward manner. From the previous discussion, it is evident that the magnitude and temporal form of the rotation-induced sea-level signal is dependent on the the relative magnitudes of the competing viscoelastic and direct components of the response. Han and Wahr (1989) considered only the viscoelastic sea-level response, while Bills and James (1996) concluded that results based on the direct effect alone give a good first approximation to the total rotation-induced signal. Thus, Bills and James' (1996) and Han and Wahr's
predictions are of a comparable magnitude and of opposite sign.

The rotation-induced RSL signal is of too small a magnitude to be significant in near-field regions. Previous modeling constraints from these regions that have been based on a sea-level theory appropriate to a non-rotating Earth are not biased in a significant manner. The assertion of Bills and James to the contrary is likely a result of their incorrect conclusion that the direct sea-level response gives a good first approximation to the total signal.

The rotation-induced RSL signal is most significant in far-field regions where the observed, recent (~ 5 kyr BP onwards) RSL signal is of a small amplitude. For example, the rotation-induced signal is relatively large in the Australian region and can bias sea-level high stand predictions to be as much as ~1 m lower than those based on the original sea-level equation. Therefore, neglect of the rotation-induced signal can result in Late Holocene melt estimates that are too high by ~0.5 m. Predictions of differential high stands in the Australian region are, however, not significantly affected by the rotation-induced signal. Therefore, inferences of mantle viscosity based on these data need not consider the rotation-induced signal.

Predictions of the global present-day sea-level rate due to change in the rotational potential reach a magnitude of ~0.15 mm/yr for the class of models considered in this chapter. Again, the signal takes the geographic form of a degree 2 order 1 spherical harmonic function with minor shorter wavelength structure superimposed on this signal due to the rotation-induced water load. Predictions of this observable are surprisingly insensitive to variations of \( \nu_{1m} \) in the range \( 3 - 50 \times 10^{21} \) Pa s. As discussed above, the nature of the sensitivity of this signal to \( \nu_{1m} \) is a result of the competing direct and deformational sea-level response components.

Estimates of the degree zero present-day sea-level rate signal may be biased if a considerable proportion of the data employed in the analysis are obtained from a region in which the rotation-induced signal is large. For example, the present-day sea-level rate signal estimated by Davis and Mitrovica (1996) (1.5 mm/yr), on the basis of tide gauge data from the US east coast, is reduced by just under 10% when the rotation-induced sea-level signal is considered.
Chapter 4

The Sensitivity of GIA Predictions
to a Low Viscosity Layer at the Base
of the Upper Mantle

4.1 Introduction

The previous two chapters focussed on the influence of second order mechanisms contributing to the GIA forcing on predictions of RSL change. In this chapter, the sensitivity of a suite of predicted GIA signatures to second order structure within the Earth’s viscosity profile will be considered.

Recent viscous flow modeling of the low degree nonhydrostatic geoid indicates that these convection-related data are highly sensitive to radial viscosity variations at depths in the vicinity of the transition zone (e.g., King and Masters 1992; Forte, Dziewonski and Woodward 1993; Panasyuk, Hager and Forte 1996). This sensitivity is a consequence of the high correlation between the seismically inferred 3-D density structure in this region and the low degree nonhydrostatic geoid (King and Masters 1992). A number of authors have used this sensitivity to argue for a thin low viscosity layer at the base of the upper mantle (e.g., King and Masters 1992; Forte et al. 1993; Pari and Peltier 1995). These studies have generally assumed a whole mantle convection scenario and, in this context, there are several physical mechanisms that may explain the existence of a thin low viscosity layer. These involve, for example, transformational superplasticity (Sammis and Dein 1974; Panasyuk and Hager 1996) or the latent-heat release associated with upwelling mantle material passing through the 670 km phase boundary (e.g., Forte et al. 1993). If mantle material convects as a two-layer system with separate reservoirs
in the upper and lower mantle, then a thermal boundary layer will develop at the 670 km discontinuity producing high temperature gradients and thus large variations in viscosity over short radial length scales (e.g., Peltier 1985; Forte et al. 1993). It is notable, however, that the preference for a low viscosity layer at 670 km depth is weaker in the case of geoid modeling which assumes a layered mantle system (e.g., Corrieu, Ricard and Froidevaux 1994).

As discussed in Chapter 1, observables associated with the Earth's response to the redistribution of ice-water mass occurring throughout the Late Pleistocene and up to the present day can also be employed to infer viscosity structure. Attempts to infer viscosity on the basis of these observables have invariably been based on forward analyses characterized by a restricted class of viscosity models. Indeed, by far the most common choice is a three layer model which includes a high viscosity (elastic) lithosphere, and isoviscous upper and lower mantle regions (with the boundary between the two regions located at a depth of 670 km). Almost no effort has been made to consider the sensitivity of the GIA observables to fine scale viscosity structure at depths in the vicinity of the transition zone.

One of the few exceptions is the study of Peltier (1985), who used an observational constraint on the non-tidal acceleration of the Earth's rotation rate (or, effectively, the present-day secular variation in the degree two zonal harmonic of the Earth's geopotential, \( J_2 \)) to infer the average viscosity within the lower mantle. In particular, Peltier (1985) investigated the influence of a thin (50 km) high viscosity layer located immediately beneath the 670 km boundary on \( J_2 \) predictions; he argued that this "internal lithosphere" would be the net result of a thermal boundary layer at the 670 km discontinuity. (Forte et al. (1993) and Pari and Peltier (1995) have, in contrast to Peltier (1985), argued that a thermal boundary layer at 670 km depth would also give rise to a pronounced low viscosity region at this depth.) The results of Peltier's (1985) sensitivity analysis show that \( J_2 \) predictions are relatively insensitive to this type of fine scale viscosity structure, with a maximum effect of \( \sim 15\% \) for Earth models in which the lower mantle viscosity is \( \sim 2 - 3 \) times greater than an assumed upper mantle viscosity of \( 10^{21} \) Pa s. In general, the influence of the high viscosity layer does not exceed a few percent. Ivins, Sammis and Yoder (1993) considered a similar class of models and obtained a consistent result.

A sensitivity analysis similar to that of Peltier (1985) has not been carried out for a low viscosity layer at the base of the upper mantle. Recent results from long-wavelength
geoid modeling described above suggest that such an analysis is warranted. There have been suggestions that GIA observables are relatively insensitive to such a layer (e.g., Peltier, Forte and Mitrovica 1992; Peltier 1996) although no quantitative support for this assertion has appeared. Indeed, this assumption of insensitivity plays an important role in a recent analysis by Peltier (1996) which considered an inversion of GIA data (including RSL variations, $\dot{J}_2$ and TPW speed). Peltier (1996) claimed that his inverted viscosity profile was compatible with an independent profile derived from geoid modeling despite the absence of a low viscosity 'notch' in the former. He argued that the addition of a low viscosity 'notch', which would be necessary to fit the geoid data, would have no effect on the GIA observables used in the inversion. Recent simultaneous joint inversions of GIA and convection-related observables (Forte and Mitrovica 1996; Mitrovica and Forte 1997) are characterized by a low viscosity layer immediately above the 670 km boundary; however, it is not clear whether the GIA data played a role in constraining this feature.

The goal of this chapter is to explicitly examine the sensitivity of a suite of GIA observables to the viscosity and thickness of a low viscosity layer in the transition zone region. Three GIA observables: RSL variations, $\dot{J}_2$ and TPW speed, are chosen for this purpose. The reader should note from the outset that these observables shall not be employed in this study to set constraints on the viscosity and/or thickness of such a low viscosity layer.

Predictions of the GIA component of $\dot{J}_2$ and TPW speed have been shown to be sensitive to lower mantle viscosity and relatively insensitive to upper mantle viscosity (e.g., Mitrovica and Peltier, 1993b). These data have, consequently, been used to infer deep mantle viscosity (e.g., Sabadini and Peltier, 1981; Yuen et al. 1982; Wu and Peltier, 1984; Peltier 1985) with some models even including $D''$ structure in the analysis (Ivins et al. 1993). The inference of viscosity is, however, complicated by the fact that these observables are also sensitive to, for example, present-day mass flux associated with ice sheets and mountain glaciers (see section 1.1) (e.g., Yoder and Ivins 1985; Trupin, Meier and Wahr 1992; Mitrovica and Peltier 1993b). Clearly, the $\dot{J}_2$ and TPW observables cannot be used to constrain mantle viscosity until these contributions are independently estimated. Alternatively, these observables might be applied to constrain this present-day mass balance once the Earth's viscosity structure is rigorously inferred from independent data sets (see, for example, Mitrovica and Forte (1997)). Regardless of the application, it is important to determine whether predictions of $\dot{J}_2$ and TPW are sensitive to the presence of a thin low viscosity region at the base of the upper mantle. Another goal
of this chapter is to quantify this sensitivity and compare it with the predicted range of signal expected from processes other than the surface loading associated with the Late Pleistocene glaciation cycles. The latter is now understood to include not only present-day cryospheric mass changes, but also tectonic processes (e.g., Vermeersen et al. 1994), the advection of mantle density heterogeneities (Steinberger and O'Connell 1997) and pressure changes in the fluid core (e.g., Fang et al. 1996).

### 4.2 Results and Discussion

#### 4.2.1 Viscosity Profiles

As described in the previous section, a number of recent modeling studies of nonhydrostatic geoid data indicate the presence of a low viscosity zone above the 670 km discontinuity. The inferred viscosity profile from one such study (Forte et al. 1993), in which both geoid and free air gravity data were considered, is shown as profile 'thn' in Figure 4.1 (solid line). Note that these data are sensitive only to relative variations of viscosity. Hence, the right-hand axis of Figure 4.1 (which represents the logarithm of the viscosity variation) can be arbitrarily normalized. In this respect, the logarithm of the viscosity of the layer above the low viscosity notch is set to zero.

The sensitivity of both the nonhydrostatic geoid data and the free air gravity data to perturbations in the viscosity of the notch in profile 'thn' is explored in Figure 4.2. In particular, the figure shows the variance reduction associated with profile 'thn' and with a suite of models in which the viscosity of the notch is increased over an order of magnitude from the value which characterizes this profile. Note that the optimum viscosity for this thickness of notch is (not surprisingly) the value obtained by Forte et al. (1993), which is \( \sim 100 \) times lower than the viscosity of the adjacent upper mantle material (see Figure 4.1). In this case, the variance reduction for both the geoid and free-air gravity harmonics is roughly 80%. It is clear that the predictions are sensitive to the viscosity of this thin layer, with the goodness of fit deteriorating rapidly as the notch viscosity is increased. Thus, the existence of a significant low amplitude notch is a robust requirement of the convection modeling when the Forte et al. (1993) viscosity structure is adopted.

In calculations described below the profile 'thn' (and variations on it) is adopted to investigate the sensitivity of a subset of GIA observables to the viscosity of the notch. Since GIA predictions are sensitive to the absolute value of viscosity within the Earth
Figure 4.1: A subset of the viscosity profiles considered in this study. Profile 'thn' (solid line) corresponds to the relative viscosity model proposed by Forte et al. (1993) on the basis of nonhydrostatic geoid data, and it is thus arbitrarily scaled (see right hand axis). The left-hand axis shows the absolute scaling associated with the Forte et al. (1993) profile which Mitrovica and Forte (1997) found provides a best fit to a suite of RSL decay times. The thickness of the low viscosity notch is 70 km. Profiles 'thk' and 'nlvl' are derived from profile 'thn' by either increasing the thickness of the low viscosity region (to 220 km) or increasing the viscosity of the region to the value which characterizes the average upper mantle viscosity. The model 'thk\textsuperscript{o}' is derived by varying the viscosity at the base of the upper mantle in model 'thk' until a best fit to convection-related observables is obtained (see Figure 4.2).

(As well as depth variations of this parameter), the scaling determined by Mitrovica and Forte (1997) is selected (see left hand axis in Figure 4.1). This scaling, which provides an optimal fit to a large set of decay times determined from RSL variations in Canada and Fennoscandia, yields a mean lower mantle viscosity value of \( \sim 5 \times 10^{21} \) Pa s.

Although the geoid data seem to prefer a thin (\( \sim 100 \) km) low viscosity layer (e.g., Pari and Peltier 1995), these data do not uniquely constrain the thickness or viscosity of this structure (e.g., King 1995). Accordingly, the present analysis will also consider GIA predictions based on models in which the thickness of the low viscosity layer is increased to \( \sim 220 \) km (or approximately the depth extent of the transition zone - see the 'thk' models in Figure 4.1). A low viscosity layer of this approximate thickness has been
Figure 4.2: The variance reduction associated with model fits to the nonhydrostatic geoid (solid lines) and free air gravity (dashed lines) harmonics for degrees 2 to 8 as a function of the perturbation to the logarithm of the viscosity of the notch in profile 'thn' (circles) and profile 'thk' (triangles). The left-most abscissa value, '0.0', refers to either model 'thn' (circles) or 'thk' (triangles) in Figure 4.1. The right-most value, '2.0', represents model 'nlvl' in Figure 4.1. Finally, the abscissa value 0.6, for the case of the curves denoted by triangles, is model 'thk°'.

proposed in other inversion studies (e.g., King and Masters 1992; Forte and Mitrovica 1996). The sensitivity of nonhydrostatic geoid data and free air gravity data to the viscosity of this thicker notch is shown in Figure 4.2. Note that the variance reduction is a maximum when the change in the logarithm of the viscosity of this notch is \( \delta \log \nu_{\text{notch}} \sim 0.60 \) (approximately corresponding to a four-fold increase in viscosity) relative to its value in profile 'thn' or 'thk' (this optimal model is denoted as 'thk°' - see Figure 4.1). This result indicates that there is a trade-off between notch thickness and viscosity (compare models 'thn' and 'thk°').

For purposes of comparison, a profile in which the notch viscosity is increased to the value associated with the main upper mantle layer is also considered (see Figure 4.1). This profile, denoted as model 'nlvl' (an abbreviation for 'no low viscosity layer'), is thus characterized by an isoviscous upper mantle (i.e., the low viscosity feature has been removed) below the high viscosity lithosphere. It is clear from Figure 4.2 that model 'nlvl' (corresponding to \( \delta \log \nu_{\text{notch}} = 2.0 \)) does an extremely poor job in fitting the convection related observables. Variance reductions for this model are roughly -2.44 for the nonhydrostatic geoid and -1.44 for the free-air gravity harmonics.
Since TPW speed predictions are sensitive to the number of glacial cycles before present (e.g., Wu and Peltier 1984), the following predictions of GIA induced anomalies in the Earth's rotation vector were calculated using seven full glacial cycles. The complementary ocean load is modeled as having a eustatic (i.e., geography independent) variation. The viscosity profiles shown in Figure 4.1 do not include an elastic lithosphere (although the viscosity of the top 80 km layer is more than an order of magnitude greater than the viscosity of the underlying upper mantle). A thin elastic outer layer is strongly preferred on the basis of independent GIA constraints. Accordingly, the present GIA predictions will adopt an elastic lithosphere of 80 km depth (in practice this rheology is obtained by increasing the viscosity of the top layer in Figure 4.1 to very high values).

4.2.2 Relative Sea Level

Previously Glaciated Regions

Figure 4.3 investigates the sensitivity of predicted postglacial RSL variations at four sites located in previously glaciated regions to variations in viscosity within the transition zone. Oslo and Angerman River are situated in the region once covered by the Fennoscandian ice sheet, while Richmond Gulf and Ipik Bay were covered, respectively, by the Laurentide and the Arctic ice complexes. Predictions are shown for profiles 'thn', 'thko' and 'nlvl'. As discussed in the Introduction, previous studies have suggested that the decay times associated with the most recent portion of these RSL variations provide a robust constraint on viscosity (Mitrovica and Peltier 1993a, 1995) (see Appendix A).

The decay times determined from models 'thn', 'thko' and 'nlvl' are listed on each frame of Figure 4.3. The two-sigma observational uncertainty ($\Delta \tau_{ob}$) is also listed on each frame (these uncertainties are taken from Mitrovica and Forte 1997). The two sites from each geographic region (Hudson Bay and Fennoscandia) were chosen because they exhibit roughly the minimum and maximum effect of the low viscosity notches on RSL predictions in these areas.

The results shown in Figure 4.3 indicate that the predicted RSL curves for three of the four chosen sites are significantly influenced by the presence of the low viscosity layer in profiles 'thn' and 'thko'. The maximum effect on RSL ranges from $\sim12$ m at the Ipik Bay site to $\sim50$ m at the Angerman River site over the time window considered in Figure 4.3. This sensitivity is, in general, larger than the observational uncertainties at these sites (e.g., the error in RSL height for data obtained at Richmond Gulf is $\sim\pm2.5$ m for index
Chapter 4: Sensitivity of GIA Predictions to a Low Viscosity Layer

Figure 4.3: Predicted RSL curves at four different sites (as labeled). The dotted, solid and dashed lines refer to predictions calculated using the viscosity profiles 'nlvl', 'thn' and 'thk0', respectively, in Figure 4.1 (these line types correspond to those employed in Figure 4.1 to specify various viscosity models). The parameters $\tau_{nlvl}$, $\tau_{thn}$ and $\tau_{thk0}$ are the decay times corresponding to each of these RSL curves. $\Delta \tau_{ob}$ is the two-$\sigma$ observational uncertainty in the decay time for each site.
Chapter 4: Sensitivity of GIA Predictions to a Low Viscosity Layer

points dating back approximately 6 kyr (Hillaire-Marcel 1980)). Therefore, inferences of viscosity based on raw RSL records from these regions may be biased if the class of viscosity models considered does not include fine scale structure in the transition zone. Data from previously glaciated regions are commonly used to constrain Late Pleistocene ice thicknesses (e.g., Tushingham and Peltier 1991) and these analyses generally assume a simple three-layer viscosity profile (lithosphere-upper mantle-lower mantle). For the case of centrally located sites (e.g., Angerman River, Richmond Gulf), Figure 4.3 illustrates that the presence of a thin low viscosity notch directly above 670 km depth could lead to significantly higher estimates of ice sheet thicknesses in these regions since the introduction of a low viscosity notch would have to be compensated by a large increase in local ice thickness.

Inferences of viscosity based on the RSL decay times \( \tau_i \) (equation (A.1)) are characterized by less sensitivity to the ice load history than the raw RSL time series. Mitrovica and Forte (1997; Figure 5) have shown, via calculations of Frechet kernels, that decay time data become progressively more sensitive to viscosity variations at greater depth as one considers sites closer to the edge of the ancient ice complex at its glacial maximum. This is consistent with the results in Figure 4.3, which indicate that the decay times for central sites (Angerman River, Richmond Gulf) show more sensitivity to the existence of the low viscosity notch than the decay times from Oslo and Ipik Bay, which are closer to the edge of their respective ice complexes.

As one would expect, the predicted decay times are reduced by the presence of a low viscosity layer relative to predictions based on profile 'nlvl'. As shown by a number of authors (e.g., Peltier 1976), GIA scales naturally with the logarithm of the viscosity. Thus, a measure of the 'strength' of the notch is the logarithm of the viscosity drop times the thickness of the region. Using this measure, the notch in profile 'thkO' has approximately twice the 'strength' of the notch which characterizes profile 'thn'. This is roughly in accord with the predictions in Figure 4.3, where the difference \( \log \tau_{thn} - \log \tau_{nlvl} \) is on average \( \sim 50\% \) of the difference \( \log \tau_{thko} - \log \tau_{nlvl} \). These relative effects vary from site to site since the detailed depth dependent sensitivity of decay time predictions to variations in viscosity is a function of the geographic location.

To determine whether the variation of the predicted decay times in Figure 4.3 is significant, it is necessary to compare the range in predicted values to the observational uncertainty at the four sites. (The 'edge' sites Oslo and Ipik Bay have relatively higher observational uncertainties for two reasons. First, the amplitude of the raw RSL varia-
tions at these sites is smaller than that of the central sites and the decay time estimates are thus less precise. Second, the observational uncertainties quoted in Figure 4.3 are augmented to include a contribution associated with ice load uncertainties. This contribution generally increases as one consider sites closer to the edge of the ancient ice complexes.) The difference \( \tau_{\text{nlvl}} - \tau_{\text{thk}} \) for the sites Angerman River and Richmond Gulf is larger than the two-sigma error. The predicted difference \( \tau_{\text{nlvl}} - \tau_{\text{thn}} \) exceeds the two-sigma error only in the case of the Richmond Gulf site. Thus, the decay time predictions at central sites are significantly sensitive to the presence of the thick and, to a lesser extent, the thin notch.

The Far Field

Numerical predictions of GIA-induced RSL change in the far field of the Late Pleistocene ice sheets are characterized by a moderate (sub-mm/yr) sea level fall subsequent to the end of the final deglaciation event. This fall leads to the occurrence of a sea level high stand with an amplitude of a few meters around 5 kyr BP.

Table 4.1 shows the predicted high stands at a number of sites in the Australian region and its vicinity. The predictions indicate that the calculated high stand is lowered at all sites by the inclusion of a low viscosity notch. (Once again, the effect of the thicker low viscosity notch, relative to predictions using profile 'nlvl', is roughly twice, on average, the effect associated with the thin notch). The sensitivity varies from site to site, with the largest predicted variation of \( \sim 1.2 \) m at Karumba, and the smallest of \( \sim 0.1 \) m at Cape Spencer.

<table>
<thead>
<tr>
<th>Site</th>
<th>thn</th>
<th>thk</th>
<th>nlvl</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port Pirie, Australia</td>
<td>4.9 m</td>
<td>4.5 m</td>
<td>5.4 m</td>
</tr>
<tr>
<td>Cape Spencer, Australia</td>
<td>2.8 m</td>
<td>2.9 m</td>
<td>3.0 m</td>
</tr>
<tr>
<td>Halifax Bay, Australia</td>
<td>3.3 m</td>
<td>3.1 m</td>
<td>3.9 m</td>
</tr>
<tr>
<td>Karumba, Australia</td>
<td>4.3 m</td>
<td>3.9 m</td>
<td>5.1 m</td>
</tr>
<tr>
<td>Moruya, Australia</td>
<td>3.3 m</td>
<td>2.9 m</td>
<td>3.6 m</td>
</tr>
<tr>
<td>Christchurch, New Zealand</td>
<td>2.9 m</td>
<td>2.1 m</td>
<td>3.0 m</td>
</tr>
</tbody>
</table>

There are three mechanisms that affect the value of the predicted high stands (see section
1.3.2): ocean syphoning caused by the collapse of peripheral bulges (Mitrovica and Peltier 1991), continental levering associated with local ocean loading (e.g., Clark et al. 1978; Nakada and Lambeck 1989) and ongoing melting of global ice from the time of the high stand to present (Nakada and Lambeck 1989). The first two of these mechanisms are dependent on viscosity structure and thus contribute to the sensitivity observed in table 4.1. Since the peripheral bulge collapse produces a more geographically uniform sea-level fall than the ocean loading effect within far-field regions, the latter likely accounts for most of the site dependent variation evident in table 4.1.

Nakada and Lambeck (1989) investigated the sensitivity of high stand amplitudes to variations in three Earth model parameters (upper and lower mantle viscosity, lithospheric thickness). The significance of the results in table 4.1 is best illustrated by comparison to these earlier predictions. As an example, Nakada and Lambeck (1989; Figure 7) found that a two order of magnitude increase in lower mantle viscosity produced less than a 2 m change in the predicted high stand at Halifax Bay. The introduction of a low viscosity notch has an effect which reaches ~30-40% of this value. Comparable (significant) percentages are obtained for the other sites listed in table 4.1.

Predicted differential sea level high stands for various pairs of sites are listed in table 4.2. These predictions are less sensitive to the presence of a low viscosity notch than the predictions of the individual high stands. The effect of a low viscosity zone is no more than 0.4 m except for predictions based on the ‘thk’ model for the Port Pirie-Cape Spencer site pair. This is the only result which may be of consequence to the Nakada and Lambeck (1989) analysis. In general, the results in Table 4.2 indicate that the viscosity inference of Nakada and Lambeck (1989) would not be significantly affected by the existence of the fine-scale viscosity structure in profiles ‘thn’ and ‘thk’.

Table 4.2: Predictions of Late Holocene differential high stands for sites in the Australian region based on the three viscosity profiles ‘thn’, ‘thk’ and ‘thk0’ shown in Figure 4.1.

<table>
<thead>
<tr>
<th>Site Pairs</th>
<th>thn</th>
<th>thk0</th>
<th>nvl</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karumba-Halifax Bay</td>
<td>1.0 m</td>
<td>0.8 m</td>
<td>1.2 m</td>
</tr>
<tr>
<td>Port Pirie-Cape Spencer</td>
<td>2.1 m</td>
<td>1.6 m</td>
<td>2.4 m</td>
</tr>
<tr>
<td>Halifax Bay-Moruya</td>
<td>0.0 m</td>
<td>0.1 m</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Moruya-Christchurch</td>
<td>0.4 m</td>
<td>0.8 m</td>
<td>0.6 m</td>
</tr>
</tbody>
</table>

As previously discussed, Nakada and Lambeck (1989) adopted their optimum viscosity
model based on differential high stand data to predict the high stands at a number of geographic sites. The discrepancy between the predicted high stand amplitudes and the observed values was then used to estimate ongoing melting of the Antarctic ice sheet from the timing of the high stands to present. Since the high stand predictions are sensitive to a low viscosity layer (table 4.1), estimates of this melt signal will vary depending on whether or not such a layer is included in the viscosity model parameterization. For example, on the basis of the results in table 4.1, failure to account for the existence of a low viscosity region would result in an overestimate of the Late Holocene melting from Antarctica by as much as 0.2 mm/yr of equivalent eustatic sea level rise (or \( \sim 1 \text{ m in 5 kyr} \)).

### 4.2.3 \( \hat{J}_2 \) and True Polar Wander Rate

The sensitivity of predictions of present day \( \hat{J}_2 \) and TPW speed to the existence of a low viscosity notch at the base of the upper mantle will now be considered. Consider Figure 4.4(A) which shows predictions of \( \hat{J}_2 \) as the viscosity of the notch is increased two orders of magnitude from the values which characterize profiles 'thn' and 'thk' to the case of the isoviscous upper mantle profile 'nlvl' (i.e., no notch). The sensitivity of the predictions to the viscosity within either the thin or thick notch is dramatic. Relative to the results for profile 'nlvl', the presence of a thin low viscosity feature favored by modeling of convection related observables (profile 'thn' - see Figure 4.2) reduces the \( \hat{J}_2 \) prediction by 40%. The analogous results for the thick notch show a maximum reduction of approximately 30%. It is interesting, though likely coincidental, that the minimum \( \hat{J}_2 \) amplitude predicted for the thick notch calculations (\( \delta \log \nu_{\text{notch}} = 0.6 \)) is obtained for the model which simultaneously maximizes the fit to the convection related observables for this class of viscosity models (that is, model thk\(^{0} \); compare Figure 4.2 - dashed lines with Figure 4.4(A) - solid triangles). Note, that the two curves in Figure 4.4(A) converge, as they must, when the viscosity of both notches reach that of the overlying upper mantle (the two viscosity profiles 'thn' and 'thk' (or 'thk\(^{0} \)) are equivalent to profile 'nlvl' at this value of notch viscosity).

Figure 4.4(B) shows analogous predictions of TPW speed as a function of ('thn' and 'thk') notch viscosity. In contrast to the \( \hat{J}_2 \) results, predictions of TPW speed are relatively insensitive to the existence of a low viscosity layer at the bottom of the upper mantle. Indeed, the prediction for model 'thn' differs by only \( \sim 9\% \) from the result obtained using the no-notch profile 'nlvl'. The discrepancy between predictions based on the 'thk\(^{0} \) and
Figure 4.4: (A) Predictions of the GIA-induced $\dot{J}_2$ signal as a function of the perturbation to the logarithm of the viscosity of the notch in profile ‘thn’ (circles) and profile ‘thk’ (triangles). (B) As in frame (A), except for predictions of TPW speed.
Chapter 4: Sensitivity of GIA Predictions to a Low Viscosity Layer

The sensitivity of $J_2$ predictions to the existence of a low viscosity notch at the base of the upper mantle has important implications for a variety of potential geophysical applications based upon this observable. One such application is the estimate of bounds on the present-day global cryospheric mass flux. This mass flux can be separated into two sources; small mountain glaciers and ice sheets, and the large polar ice caps (Greenland, Antarctica). Meier (1984) has tabulated the retreat of 31 small mountain glaciers and ice sheets and the $J_2$ signal associated with this forcing has been predicted to be $\sim 10^{-11}$ yr$^{-1}$ (e.g., Sabadini, Yuen and Gasperini 1988; Peltier 1988; Mitrovica and Peltier 1993b). The difference between the $J_2$ predictions for models 'nlvl' (no notch) and 'thn' (thin low viscosity notch) is $2 \times 10^{-11}$ yr$^{-1}$ (Figure 4.4(A)), or twice the signal from Meier's sources. Mitrovica and Peltier (1993b) have found that the $J_2$ signal associated with mass flux in the Antarctic and Greenland ice sheets is approximately $4 \times 10^{-11}$ yr$^{-1}$ per mm/yr of net eustatic sea level rise. (Variations in the mass of the Antarctic and Greenland ice sheets are equally efficient at exciting a $J_2$ response, and one need not consider their independent contributions.) Therefore, the thin low viscosity notch which characterizes model 'thn' has an effect on $J_2$ which is comparable to the signal associated with a polar melting equivalent which yields a eustatic sea level rise of 0.5 mm/yr. Thus, failure to account for the possible presence of such a low viscosity layer could lead to a $\sim 0.5$ mm/yr overestimate of the ongoing melt signal associated with the large polar ice sheets.

The observed $J_2$ likely has contributions from a number of geophysical phenomena unrelated to cryospheric forcings. For example, Fang et al. (1996) have argued that pressure changes in the fluid outer core of the Earth may contribute as much as approximately $-1.3 \times 10^{-11}$ yr$^{-1}$. (Other effects associated with the core-mantle-boundary have also been described (see Lefftz and Legros 1992) which produce comparable signals.) This contribution is approximately 60% of the signal associated with the presence of a thin low viscosity zone immediately above the 670 km boundary.

A second common application of the $J_2$ observable involves the inference of lower mantle viscosity (e.g., Wu and Peltier 1984; Peltier and Jiang 1994). This application has generally involved a standard three-layer parameterization of the radial viscosity structure which includes an elastic lithosphere and isoviscous upper and lower mantle regions. Figure 4.5 (dotted line) indicates predictions of $J_2$ for the case of an 80 km lithosphere, an upper mantle viscosity of $10^{21}$ Pa s, and a lower mantle viscosity which ranges from $10^{21}$ Pa s to $10^{23}$ Pa s (as shown on the abscissa). The 'classic' inverted parabolic form of
the predicted curve is now well understood; in particular, small amplitudes are associated both with low viscosity models, which have relaxed close to equilibrium in the 5 kyr since the end of major melting events, and high viscosity models, which are characterized by slow rates of adjustment at all times. The model ‘nlvl’ (see Figure 4.1) has an average lower mantle viscosity of $\sim 5 \times 10^{21}$ Pa s. The $J_2$ prediction for this profile, $\sim -5.3 \times 10^{-11}$ yr$^{-1}$ (Figure 4.4(A)), is essentially the same as the prediction for the simple three layer model in which the isoviscous lower mantle viscosity is set to $5 \times 10^{21}$ Pa s (see Figure 4.5, dotted line). This suggests that when a pronounced low viscosity layer is not present, the $J_2$ predictions are largely determined by the mean viscosity within the lower mantle, and so a simple three layer parameterization is adequate.

![Graph](image)

Figure 4.5: Predictions of the GIA-induced $J_2$ signal for a suite of viscosity models. In all cases a lithospheric thickness of 80 km is used. Dotted line - the upper mantle viscosity is fixed to $10^{21}$ Pa s and the lower mantle viscosity is varied according to the abscissa. Solid line - same as the dotted line, with the exception that a 70 km thick zone of low ($10^{19}$ Pa s) viscosity is introduced immediately above the 670 km discontinuity. Long-dashed-dotted line - same as the dotted line, with the exception that a 70 km thick zone of high ($10^{23}$ Pa s) viscosity is introduced immediately below the 670 km discontinuity. Short-dashed-dotted line - same as the dotted line with the exception that both the high and low viscosity 70 km zones are introduced to generate a ‘dipole’ viscosity structure at 670 km depth (see text).
Chapter 4: Sensitivity of GIA Predictions to a Low Viscosity Layer

The solid line in Figure 4.5 is analogous to the dotted line with the exception that a thin (70 km) low viscosity notch has been added to the base of the upper mantle. To be consistent with the profile 'thn', the notch viscosity is taken to be two orders of magnitude lower than the average upper mantle value (in this case, the notch viscosity is $10^{19}$ Pa s). For the case of a lower mantle viscosity of $5 \times 10^{21}$ Pa s, the introduction of the low viscosity notch increases the value of the $\dot{J}_2$ prediction from $\sim -5.3 \times 10^{-11}$ yr$^{-1}$ to $\sim -3.7 \times 10^{-11}$ yr$^{-1}$, which is consistent with the results in Figure 4.4(A). (That is, it is consistent with the difference in $\dot{J}_2$ predictions based on models 'thn' and 'nlvl'.) A comparison of the dotted and solid lines in Figure 4.5 leads to two important conclusions. First, the existence of a thin low viscosity layer may significantly alter inferences of lower mantle viscosity. In particular, the $\dot{J}_2$ signal associated with a lower mantle viscosity of $5 \times 10^{21}$ Pa s and the presence of a thin low viscosity notch (solid line) is the same as the signal derived for an average lower mantle viscosity of $\sim 2 \times 10^{21}$ Pa s in the absence of the notch (as indicated Figure 4.5). Therefore, three layer modeling of the type associated with the solid line in Figure 4.5 (lithosphere, upper mantle, lower mantle) will have significantly underestimated the mean value of lower mantle viscosity if fine scale viscosity structure exists at the base of the upper mantle. Second, examination of Figure 4.5 indicates that $\dot{J}_2$ predictions are significantly influenced by the presence of a low viscosity notch at the base of the upper mantle for a wide range of lower mantle viscosities.

Mitrovica and Peltier (1993b; Figure 3) predicted the $\dot{J}_l$ ($2 \leq l \leq 10$) signal for a suite of three layer viscosity profiles similar to those used to calculate the solid line in Figure 4.5. They found that a reduction in the average viscosity of the upper mantle from $10^{21}$ Pa s to $10^{20}$ Pa s reduced the maximum amplitude of the $\dot{J}_2$ prediction by $\sim 0.8 \times 10^{-11}$ yr$^{-1}$. This is approximately half the effect of introducing a thin (70 km) structure of viscosity $10^{19}$ Pa s within an upper mantle of viscosity $10^{21}$ Pa s.

As previously discussed, Peltier (1985) and Ivins et al. (1993) have considered the influence on $\dot{J}_2$ predictions of a thin, highly viscous, layer located at 670 km depth. The dashed-dotted lines in Figure 4.5 revisit this issue. The long-dash-dotted line is analogous to the dotted line with the exception that a thin (70 km) high viscosity ($10^{23}$ Pa s) layer is introduced just below 670 km depth (in this case the abscissa refers to the viscosity of the remaining portion of the lower mantle). The results are consistent with those appearing in Peltier (1985) and Ivins et al. (1993) and they suggest that the influence of a high viscosity layer is significantly less than the effect of a low viscosity layer at the upper mantle-lower mantle boundary. Indeed, in the case of a lower mantle viscosity of...
5 × 10^{21} \text{ Pa s}, the influence of the latter is roughly four times the former; this factor rises to an order of magnitude for a lower mantle viscosity of 10^{22} \text{ Pa s}.

Pari and Peltier (1995) have recently explored the influence of a 'dipole' viscosity structure at 670 km depth on convection-related observables. That is, a viscosity structure characterized by a thin low viscosity layer above the 670 km discontinuity and a high-viscosity zone of equal thickness below the discontinuity. The short-dashed-dotted line in Figure 4.5 represents such a model. It is the same as the model used to generate the solid line with the exception that a 70 km thick layer of viscosity 10^{23} \text{ Pa s} is added below the 670 km boundary. Comparison of the solid and short-dashed-dotted lines in the figure indicates that the high viscosity layer within the dipole structure has a relatively minor effect on the $J_2$ predictions.

The demonstration that a low viscosity zone at the base of the upper mantle has a significant effect on (GIA) predictions of $J_2$ suggests that it would be of interest to consider the effect on higher degree zonal harmonics. Figure 4.6 examines this issue for the case of $J_l$ ($3 \leq l \leq 6$). The dotted line in each frame corresponds to the three layer viscosity parameterization adopted to calculate the dotted curve in Figure 4.5 (80 km lithosphere, 10^{21} \text{ Pa s} upper mantle viscosity and a lower mantle mantle viscosity given by the abscissa). The solid line uses the same viscosity structure with the exception that a 70 km thick low viscosity zone is introduced immediately above 670 km (this suite of models is identical to the one adopted to generate the solid line in Figure 4.5). Clearly, the inclusion of a low viscosity zone has a significant effect on the secular variations of the higher degree zonal harmonics. As an example, the maximum amplitude of the $J_4$ prediction drops by \sim 30\% when the thin low viscosity structure is introduced. Also note that the low viscosity zone acts to reduce the slope of the high (lower mantle) viscosity limb on each of the predicted curves (as in Figure 4.5 for the case of $J_2$). The $J_l$ ($3 \leq l \leq 6$) signal is also calculated for a sequence of models in which a thin high viscosity layer was introduced immediately below 670 km depth (as in the dashed and dashed-dotted lines in Figure 4.5). Results (not shown in Figure 4.6) indicate that the inclusion of the high viscosity feature has a maximum effect of \sim 10\% for models with a moderate increase in viscosity (2 – 3 times) across the 670 km boundary. There is only a few percent effect, at most, for models with higher lower mantle viscosities.
Figure 4.6: Predictions of the GIA-induced $\tilde{J}_l$ ($3 \leq l \leq 6$) coefficients for a suite of viscosity models. All predictions adopt an 80 km thick lithosphere. The dotted line is computed using an upper mantle viscosity fixed to $10^{21}$ Pa s and a lower mantle viscosity varied according to the abscissa. The solid line uses the same viscosity structure with the exception that a 70 km thick zone of low viscosity is introduced immediately above the 670 km boundary.
Chapter 4: Sensitivity of GIA Predictions to a Low Viscosity Layer

4.3 Conclusions

The above results show that a thin low viscosity notch at the base of the upper mantle, recently inferred on the basis of convection-related long-wavelength nonhydrostatic geoid and free-air gravity harmonics, can have a significant influence on a suite of observables associated with GIA. Perhaps the most important effect isolated in this chapter involves the sensitivity of predicted $\dot{J}_2$ (or rotation rate) variations to the viscosity of the notch region. This sensitivity can bias constraints on either the mean value of lower mantle viscosity or the present-day mass balance of large polar ice sheets which have been derived from simple models that do not include the presence of fine scale low viscosity structure at 670 km depth. These results counter recent suggestions that GIA observables are insensitive to such structure (e.g., Peltier et al. 1992; Peltier 1996).

The above predictions of RSL variations for previously glaciated regions indicate that decay time estimates can be significantly reduced by the presence of a low viscosity zone, with sites nearer the center of the rebound area showing greater sensitivity. Also, the presence of a thin low viscosity layer above the 670 km discontinuity reduces the predicted amplitude of Late Holocene high stands in the far field of the Late Pleistocene ice sheets. While this sensitivity will have bearing on estimates of Late Holocene melting events, inferences of lower mantle viscosity based on differential high stands from these far-field regions (Nakada and Lambeck 1989) will not be significantly altered by the presence or absence of the low viscosity notch.

Forte and Mitrovica (1996) and Mitrovica and Forte (1997) jointly inverted RSL decay time data and long wavelength free-air gravity harmonics for radial mantle viscosity structure. The solution profile they obtained is characterized by a low viscosity region immediately above the 670 km boundary which extends through the transition zone. The convection-related harmonics played an important role in constraining this low viscosity region (Figure 4.2). The present results indicate that the site-dependent RSL decay times also provided constraints on this feature of the inversion.

Peltier (1996) used $\dot{J}_2$, TPW speed and RSL decay time data associated with GIA to invert for the radial profile of mantle viscosity. The inverted profile was compared to an independent model inferred by Pari and Peltier (1995) on the basis of low degree nonhydrostatic geoid data alone. The latter profile was scaled to provide a qualitative fit between the two predictions and the general fit led Peltier (1996) to conclude that the rebound-inferred viscosity profile simultaneously reconciled both GIA and convection-
related observables. The absence of a thin low viscosity zone at the base of the upper mantle in the GIA inversion (but not in the Pari and Peltier 1995 model) was not considered to be of significance. Indeed, Peltier (1996) assumed that the fine scale low viscosity feature "does not represent a significant departure from the GIA profile, because the GIA data are not particularly sensitive to its presence" (p.1364). The above analysis indicates that this assumption is invalid, since it demonstrates that a subset of RSL decay times, and in particular the $J_2$ datum, are significantly sensitive to the presence of the low viscosity region. The determination of a viscosity model which reconciles both GIA and convection observables requires a simultaneous consideration of both data sets (Forte and Mitrovica 1996, Mitrovica and Forte 1997).
Chapter 5

Summary of Thesis

The three principal research contributions presented in this thesis are treated as independent analyses in Chapters 2, 3 and 4. For this reason, the main results of each of these analyses are summarized at the end of their respective chapters. The purpose of the present chapter is to summarize the results of this thesis within the more general context of current GIA research.

Two primary goals of GIA research are to provide quantitative information on both the Earth's viscosity structure and the surface mass exchange between the Earth's oceans and ice sheets (both in the past and at present). The former will help to constrain the dynamics of the Earth's interior and can be applied, for example, to consider the relationship between internal mantle processes and the Earth's surface geology (e.g., Gurnis 1992; Pysklywec and Mitrovica 1997). The latter will provide insights into the Earth's climate change, from time periods ranging from ~100 kyr to ~10 yr. To achieve these goals, the basic methodology of GIA research involves comparing quantitative predictions based on numerical models to observations that are sensitive to the GIA process.

As described in section 1.1, a number of data types exist that are sensitive to both viscosity structure and surface ice-water mass redistribution. This dual sensitivity gives rise to a fundamental problem in GIA modeling research since neither the GIA forcing nor the Earth's viscosity structure are, in most cases, sufficiently well resolved to permit independent inferences of viscosity and/or surface mass exchange. Therefore, an important aspect of current research involves seeking data that exhibit a greater sensitivity to either the GIA forcing or the Earth's response to this forcing. As described previously, RSL data from two specific geographic locations can be parameterized in such a way as to reduce their sensitivity to the GIA forcing.
By considering RSL data obtained from regions once loaded by the Late Pleistocene ice sheets, and applying a straightforward parameterization of the data, it is possible to separate the sensitivity to load and Earth model parameters (Mitrovica and Peltier 1993a; see Appendix A). Adopting this procedure permits a relatively robust inference of viscosity structure to be obtained. The resulting preferred viscosity profile can then be applied to constrain ice thicknesses in the region under consideration.

A second procedure involves considering RSL data from regions far removed from the locations of the ancient ice masses. In such regions, a sea-level high stand is commonly observed around the time at which the major melting events of the previous deglaciation phase ended (~5 kyr BP). The difference between observed high stands at two nearby sites is sensitive to the local, water induced deformation while being relatively insensitive to ice sheet parameters (Nakada and Lambeck 1989). Therefore, this data parameterization also permits relatively robust inferences of mantle viscosity structure to be obtained. In this case, the resulting, preferred viscosity profile can be employed to predict the amplitude of the far-field RSL high stands. The difference between the predicted and the observed amplitude can be used to estimate the amount of global ice melting that has occurred from the time of the high stands to the present.

These parameterizations of data from near-field and far-field regions provide, arguably, the most rigorous GIA constraints on Earth viscosity and ice-water mass redistribution. For this reason, the work presented in Chapters 2 and 3 investigates whether or not these types of RSL analyses are influenced by approximations inherent in the most commonly adopted sea-level theory.

In Chapter 2, the pseudospectral sea-level algorithm (Mitrovica and Peltier 1991) is extended to more accurately model the water load redistribution in near-field and far-field regions. In particular, a TDCM and near-field water dumping are incorporated into an extended pseudospectral sea-level calculator. The application of this extended calculator to predict RSL in the Australian region indicates that both differential high stands and absolute high stands are significantly sensitive to the TDCM aspect of the sea-level model.

Predictions of RSL in north eastern Canada based on the extended sea-level calculator indicate that the neglect of the water dumping mechanism in previous studies leads to a significant overestimate of the unloading (ice plus water) from solid surface regions lying below the geoid at the time of ice retreat. This neglect can bias estimates of viscosity structure and/or ice thickness based on 'raw' observed RSL time series. However, the
present results indicate that viscosity inferences based on the decay time data parameterization (Appendix A) are not affected by this aspect of the water load.

In Chapter 3, a new sea-level equation appropriate to a rotating Earth model was derived and solved in a gravitationally self-consistent manner for a large range of viscosity profiles. The results indicate that previous analyses have significantly overestimated the influence of a time-varying rotational potential on sea-level change. The predicted, rotation-induced component of the RSL signal is of too small a magnitude to be significant in modeling analyses based on near-field RSL time series. In the far field, the rotation-induced signal does not affect the prediction of differential high stands in a significant manner due to the long wavelength of the predicted signal (essentially a degree two, order 1 spherical harmonic function). However, the signal does affect predictions of the absolute value of the high stands such that Late Holocene melting estimates may be biased if the rotation-induced sea-level component is ignored. Future analyses of the type employed by Nakada and Lambeck (1989) should adopt the extended sea-level methodology outlined in Chapters 2 and 3.

A second important aspect of GIA research involves considering the detailed depth-dependent sensitivity of different data types to variations in viscosity. For example, as described in section 1.1, RSL data have a relatively poor sensitivity to viscosity below \( \sim 1800 \) km depth. Other GIA-related data that are more sensitive to deep mantle viscosity, such as TPW and \( \dot{J}_2 \), are, unfortunately, also sensitive to present-day ice-water mass redistribution and a potential range of non-GIA related processes (see section 1.1), and so cannot be applied to complement any RSL-derived viscosity constraint. For this reason, long wavelength convection-related data have been incorporated in order to provide the required deep mantle viscosity sensitivity (e.g., Mitrovica and Forte 1997).

In addition to being sensitive to viscosity variations in the deep mantle, the convection-related data are particularly sensitive to viscosity structure at depths in the vicinity of the 670 km seismic discontinuity. This sensitivity has led a number of recent studies to postulate the existence of a relatively thin, low viscosity layer in this region (e.g., Forte et al. 1993). A number of recent articles have concluded that GIA observables are relatively insensitive to such fine scale, low viscosity structure (e.g., Peltier 1996). The results in Chapter 4 clearly indicate that this is not the case for predictions of RSL variations and present-day \( \dot{J}_k \) harmonics. This demonstration has important implications for studies that adopt a three-layer viscosity parameterization to constrain either mantle viscosity or surface ice-water mass redistribution on the basis of RSL and \( \dot{J}_k \) observations.
The work presented in this thesis indicates that certain aspects of some commonly applied GIA models require improvement. These include the theoretical treatment of sea-level change and the depth discretization of the viscosity profile. These refinements will likely become increasingly important as the observational constraints on the GIA process continue to improve. There are a number of exciting prospects in this regard that are of particular relevance to the theoretical advances described in this thesis.

High-precision, space geodetic observations of 3D crustal deformations in Scandinavia are now being employed to constrain GIA models (Davis, personal communication, 1997). Predictions of GIA-induced 3D deformations of the solid surface are sensitive to the load history (e.g., Mitrovica et al. 1994). Therefore, it will be of interest to explore the influence of the water dumping effect introduced in Chapter 2 on predictions of present-day crustal rates in the vicinity of the Gulf of Bothnia in Scandinavia (the region where this effect will be most significant).

Also of interest is a future planned satellite gravity mapping mission (Wahr 1996). Existing satellite gravity data lack the spatial resolution required to resolve the signal associated with current ice-water mass redistribution from the signal caused by the most recent major Late Pleistocene deglaciation. The GRACE (Gravity Recovery and Climate Experiment) mission planned for early next century is expected to provide observations of the secular change in the geoid to a resolution of spherical harmonic degree and order $\sim$32. This increase in spatial resolution will have to be met by concomitant improvements in the GIA modeling methodology. For example, in Chapter 3, the influence of perturbations to the rotation vector on predictions of sea-level change was explored. This theory can be readily applied to examine the influence of the changing rotational potential on predictions of present-day geoid deformation rates (calculating perturbations to the geoid is an integral aspect of predicting sea-level change). Previous models have considered only the load-induced component of the signal.
Appendix A

The RSL Decay Time Parameterization

Observed RSL curves from geographic locations near the center of once ice covered regions exhibit a monotonic, exponential-like sea-level fall throughout the postglacial period. Previous studies have suggested that the decay times associated with the most recent portion of these RSL variations provide a robust constraint on viscosity (Mitrovica and Peltier 1993a, 1995). The choice of RSL time window adopted to estimate the decay time for a specific site is based on two criteria: (1) that the solid surface be in a state of free rebound (i.e., that the region be essentially ice free); and (2) that global eustatic sea-level changes do not contaminate the local decay time estimates. (Mitrovica and Forte (1997) argued that these requirements are met by considering data over the past 6.5 kyr in north eastern Canada and the past 5 kyr in Fennoscandia.) Over this time window, the site-dependent decay times, $\tau_i$, and amplitudes, $A_i$, are estimated via a Monte-Carlo determination of the best fitting exponential form defined by,

$$RSL_i(t) = A_i[\exp(t/\tau_i) - 1],$$

(A.1)

where $i$ specifies the chosen site. The time series $RSL_i(t)$ may represent either sea-level observations or predictions based on a specific GIA sea-level model. By adopting a two parameter model of the form (A.1) it is possible to separate sensitivity to load parameters (embedded in the $A_i$) from the sensitivity to Earth parameters (embedded in the $\tau_i$).
Appendix B

Predicting GIA-Induced Perturbations to the Earth’s Rotation Vector

The theory outlined below reviews and extends the work of Wu and Peltier (1984) to consider perturbations to the rotation vector associated with an arbitrary surface mass redistribution. In the equilibrium state (before surface mass redistribution) it is assumed that the inertia tensor is diagonal with components $I_{11}^0 = A, I_{22}^0 = A$ and $I_{33}^0 = C$ corresponding to the principal moments of inertia of the Earth, and the rotation vector is $(0,0,\Omega)$. At any subsequent time during the loading episode, it is conventional to define (e.g., Munk and MacDonald 1960),

\begin{align*}
\omega_i(t) &= \Omega(\delta_{i3} + m_i(t)) \quad \text{(B.1)} \\
I_{ij}(t) &= \delta_{ij}I_{ij}^0 + J_{ij}(t), \quad \text{(B.2)}
\end{align*}

where the $m_i$ and $J_{ij}$ represent small changes from the equilibrium state. Substituting equations (B.1) and (B.2) into the standard Euler equations governing the coupling between the inertia tensor and angular velocity components of a rotating body (with a set of axes fixed to the body), and dropping terms second order or higher in small quantities, the result is,

\begin{align*}
\frac{m_1(t)}{\sigma_r} - \frac{\dot{m}_2(t)}{\sigma_r} &= \frac{J_{13}(t)}{[C - A]} + \frac{J_{23}(t)}{\Omega[C - A]} \\
\frac{m_2(t)}{\sigma_r} + \frac{\dot{m}_1(t)}{\sigma_r} &= \frac{J_{23}(t)}{[C - A]} - \frac{\dot{J}_{13}(t)}{\Omega[C - A]} \\
\dot{m}_3(t) &= -\frac{J_{33}(t)}{C}, \quad \text{(B.3)}
\end{align*}
where $\sigma_r$ is the Chandler wobble frequency of a rigid Earth given by,

$$\sigma_r = \Omega \frac{[C - A]}{A}. \tag{B.4}$$

From equations (B.3), the $m_i$ (and thus the $\omega_i$) can be calculated if the products of inertia, $J_{13}$ and $J_{23}$, and the moment of inertia $J_{33}$, are known as functions of time. There are three physical mechanisms influencing these inertia terms in the context of GIA. These are the inertia perturbations due to: the direct effect of the surface load, the surface load-induced deformation and the rotation-induced deformation. Thus, one can write,

$$J_{33}(t) = J_{33}^L(t) + J_{33}^{LD}(t) + J_{33}^{RD}(t), \quad (i = 1, 2, 3). \tag{B.5}$$

The superscripts $L$, $LD$, and $RD$ refer to load, load deformation and rotational deformation effects respectively.

The change in inertia due to the surface load is given by,

$$J_{33}^L(t) = \int_{\Omega} (\rho_l I(\theta, \psi, t) + \rho_w S(\theta, \psi, t))(a^2 \delta_{33} - x_3 x_3) d\Omega, \tag{B.6}$$

under the assumption that the height of the load is negligible relative to the radius of the Earth. Following equations (2.1) and (3.7), the $J_{33}^L(t)$ can be expanded as a series of Heaviside step increments,

$$J_{33}^L(t) = \sum_{n=1}^{N} [\delta J_{33}^L]^n H(t - t_n). \tag{B.7}$$

From (B.6) and (2.10) one can show that the $[\delta J_{33}^L]^n$ are related to the surface load increments in the following manner,

$$[\delta J_{13}^L]^n = \frac{4}{3} \sqrt{\frac{6}{5}} a^4 \pi \text{Re}[\rho_l \delta I_{21}^n + \rho_w \delta S_{21}^n],$$

$$[\delta J_{23}^L]^n = -\frac{4}{3} \sqrt{\frac{6}{5}} a^4 \pi \text{Im}[\rho_l \delta I_{21}^n + \rho_w \delta S_{21}^n],$$

$$[\delta J_{33}^L]^n = \frac{8}{3} a^4 \pi [\text{Re}(\rho_l \delta I_{00}^n + \rho_w \delta S_{00}^n) - \frac{1}{\sqrt{5}} \text{Re}(\rho_l \delta I_{20}^n + \rho_w \delta S_{20}^n)] = -\frac{8\pi}{3\sqrt{5}} a^4 \pi \text{Re}(\rho_l \delta I_{20}^n + \rho_w \delta S_{20}^n), \tag{B.8}$$

where the $\delta I_{lm}^n$ and $\delta S_{lm}^n$ are the degree $l$ order $m$ spherical harmonic coefficients of the $n^{th}$ Heaviside increment of the ice and water loads, respectively. The final expression in
Appendix B: GIA-Induced Perturbations to the Earth’s Rotation Vector

(B.8) has been simplified by invoking mass conservation (i.e., \( \rho_1 \delta I_{90} + \rho_0 \delta S_{90} = 0 \)). ‘Re’ and ‘Im’ refer to the real and imaginary parts of the complex numbers, respectively.

Wu and Peltier (1984) derived expressions for the \( J_{i3}^{L}(t) \) for the case of highly simplified disk load geometries. The above expressions (B.8) are general in the sense that they relate the \( J_{i3}^{L}(t) \) to the spherical harmonic coefficients of an arbitrary surface load.

The change in inertia due to the surface load deformation, \( J_{i3}^{LD}(t) \), is determined via a temporal convolution of the degree two load Love number \( k_2^L(t) \) with \( J_{i3}^{L}(t) \) (Sabadini and Peltier 1981; Wu and Peltier 1984). This procedure yields:

\[
J_{i3}^{LD}(t) = J_{i3}^{L}(t)k_2^L \sum_{n=1}^{N} [\delta J_{i3}^{L}]^n H(t - t_n) \sum_{k=1}^{K} \frac{r_{k}^{t=2,L}}{s_{k}^{t=2}} \left[ 1 - \exp(-s_{k}^{t=2}(t - t_n)) \right].
\]  

(B.9)

Finally, the Earth deformation induced change in the \( J_{i3}(t) \) caused by the time varying rotational potential is derived by utilizing MacCullagh’s formula to give (to first order in small quantities)(e.g., Munk and MacDonald 1960; Lambeck 1980),

\[
J_{i3}^{RD}(t) = \frac{k_2^{T}(t)}{k_f} * m_1(t)[C - A]
\]

\[
J_{23}^{RD}(t) = \frac{k_2^{T}(t)}{k_f} * m_2(t)[C - A]
\]

\[
J_{33}^{RD}(t) = \frac{4k_2^{T}(t)}{3k_f} * m_3(t)[C - A],
\]

in which * denotes a temporal convolution and \( k_f \) is the ‘fluid’ Love number defined as,

\[
k_f = k_2^{T,E} + \sum_{p=1}^{K} \frac{r_{p}^{t=2,T}}{s_{p}^{t=2}}.
\]

(B.11)

If the time dependence of the \( \omega_i \) (and thus the \( m_i \)) is defined as a series of Heaviside step increments, then the equations (B.10) can be re-written as:

\[
J_{i3}^{RD}(t) = \frac{[C - A]}{k_f} \left[ m_i(t)k_2^{T,E} + \sum_{n=1}^{N} [\delta m_i]^n H(t - t_n) \times \right.
\]

\[
\sum_{k=1}^{K} \frac{r_{k}^{t=2,T}}{s_{k}^{t=2}} \left[ 1 - \exp(-s_{k}^{t=2}(t - t_n)) \right] \right] \quad (i = 1, 2),
\]

(B.12)

and,

\[
J_{33}^{RD}(t) = \frac{4[C - A]}{3k_f} \left[ m_3(t)k_2^{T,E} + \sum_{n=1}^{N} [\delta m_3]^n H(t - t_n) \times \right.
\]

\[
\sum_{k=1}^{K} \frac{r_{k}^{t=2,T}}{s_{k}^{t=2}} \left[ 1 - \exp(-s_{k}^{t=2}(t - t_n)) \right],
\]

(B.13)
Appendix B: GIA-Induced Perturbations to the Earth's Rotation Vector

in which the \([\delta m_i]^n\) represent the Heaviside increments used to construct the \(m_i(t)\), \((i = 1, 2, 3)\).

Transforming equations (B.3) to the Laplace transform domain and employing equations (B.8), (B.12) and (B.13), solutions can ultimately be obtained for the \(m_i\) which have the time-domain form,

\[
<m_i(t)> = \frac{\Omega}{A\sigma_0} \left[ D_1 J_{13}^L(t) + D_2 \int_0^t J_{13}^L(t')dt' + \sum_{k=1}^{K-1} E_k (J_{13}^L(t) * \exp(-\lambda_k t)) \right],
\]

\((i = 1, 2) \quad (B.14)\)

and,

\[m_3(t) = -\frac{1}{C} \left[ D_1 J_{33}^L(t) + \sum_{k=1}^K r_k \tau^{L*} \{ J_{33}^L(t) * \exp(-s_{k}^{L*} t) \} \right]. \quad (B.15)\]

The symbol <> signifies that the Chandler wobble has been removed from the response.

Furthermore, the \(\lambda_k\) are roots of the polynomial,

\[
Q(s) = \frac{\sum_{k=1}^K \left( \frac{\pi^{L*}}{s_{k}^{L*}} \prod_{p \neq q} (s + s_{p}^{L*}) \right)}{\sum_{j=1}^K \frac{\pi^{L*}}{s_{j}^{L*}}}. \quad (B.16)\]

Also,

\[D_1 = 1 + k_2^{L*}, \quad D_2 = \frac{l_s \prod_{p=1}^K s_{p}^{L*}}{\prod_{p=1}^{K-1} \lambda_p}, \quad (B.17)\]

and,

\[
E_k = \frac{-\left[ l_s \prod_{p=1}^K (s_{p}^{L*} - \lambda_k) \right]}{\prod_{p=1}^K \lambda_p} + \sum_{q=1}^K \frac{r_q^{L*}}{s_{q}^{L*}} \prod_{p \neq q} (s_{p}^{L*} - \lambda_k) \quad (B.18)\]

The parameter \(\sigma_0\) in (B.14) is the Chandler wobble frequency of a deformable Maxwell viscoelastic Earth model and is given by,

\[
\sigma_0 = \frac{\sigma_r}{k_f} \sum_{p=1}^K \frac{r_p^{L*}}{s_{p}^{L*}}, \quad (B.19)\]

and,

\[l_s = 1 + k_2^{L*} + \sum_{p=1}^K \frac{r_p^{L*}}{s_{p}^{L*}}. \quad (B.20)\]
Appendix C

Flow Charts of the Sea-Level Algorithms

Before showing the flow charts that describe the sea-level algorithms discussed in Chapters 2 and 3, it is useful to introduce the following two expressions. The harmonic coefficients of globally defined sea-level change from the onset of the loading period to the time $t_j$ are given by,

$$SG_{\ell,m}(t_j) = E_{\ell}^{T}T_{\ell}(\rho_{I}I_{\ell,m}(t) + \rho_{W}S_{\ell,m}(t_{j-1})) + \left[ E_{\ell}^{T}T_{\ell}\delta S_{\ell,m}^{I} \right]$$

$$+ \sum_{n=1}^{j-1} \left[ T_{\ell}(\rho_{I}\delta I_{\ell,m}^{n} + \rho_{W}\delta S_{\ell,m}^{n})\beta_{\ell}^{T}(t_j - t_n) \right] + C_{\ell}^{I}(t)\delta_{\ell,0}\delta_{m,0}. \quad (C.1)$$

The parameters in equation (C.1) are defined in sections 1.3.2 and 2.2.1. The convergence attained in a given iteration loop of the sea-level algorithms is given by,

$$\xi_{p}^{j} = \sum_{\ell,m} \frac{||\delta S_{\ell,m}^{I}||^{p} - ||\delta S_{\ell,m}^{I}||^{p-1}}{||\delta S_{\ell,m}^{I}||^{p}}. \quad (C.2)$$

The coefficients $[\delta S_{\ell,m}^{I}]$ are the sea-level increment harmonics at the $j^{th}$ time step. The superscript $p$ denotes the iterate number of these coefficients.

A flow chart describing the original pseudospectral algorithm is shown in Figure C.1. A flow chart describing the extended pseudospectral algorithm incorporating a time dependent continent margin (TDCM) and the water dumping effect is given in Figure C.2. This figure also indicates the modification required to include the effect of GIA-induced perturbations to the Earth’s rotation vector in a gravitationally self-consistent manner.
Figure C.1: Flow chart of the original pseudospectral algorithm. The indices $i$ and $j$ correspond to the iterate number and the time increment, respectively (this is the same convention as that adopted in Chapters 2 and 3). The parameter $\varepsilon$ is a predetermined quantity that specifies the tolerance level of the algorithm.
Figure C.2: Flow chart of the extended pseudospectral algorithm that incorporates a TDCM and the water dumping effect. The modification required to incorporate the effect of Earth rotation is also shown. The indice $k$ denotes the second iteration loop required to incorporate a TDCM and the water dumping effect in a gravitationally self-consistent manner.
References


References


James, T. S., and A. Lambert, 1993. A comparison of VLBI data with the ICE-3G glacial


Nakiboglu, S. M., K. Lambeck, and P. Aharon, 1983. Postglacial sea levels in the Pacific:
Implications with respect to deglaciation regime and local tectonics, *Tectonophysics*, 91, 335–358.


