A Study of the Bubble Removal Mechanism from a Flat Plate
Under Both Normal-Gravity and Reduced-Gravity Conditions

by

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A thesis submitted in partial fulfillment of the requirements for the Degree of Master of Applied Science

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ABSTRACT

A rectangular flow channel was built and used in conjunction with a liquid flow loop to visualize water and kerosene flows near departing air bubbles that were injected into the channel and attached to the surface under both normal-gravity and reduced-gravity conditions. The reduced gravity condition was obtained by performing the experiments in a DC-9 microgravity aircraft undergoing parabolic flight.

The photochromic dye activation technique has been successfully used to obtain velocity profiles of both fluids near departing bubbles, ranging in size from 1.32mm to 1.96mm in bubble departure diameter for kerosene flow with velocities from 0 to 0.22m/s under normal-gravity conditions; 2.10mm to 3.10mm in bubble departure diameter for water velocities ranging from 0.026 to 0.20m/s under normal-gravity conditions; and 3.25mm to 6.55mm in bubble departure diameter for water velocities ranging from 0.017 to 0.173m/s under reduced-gravity conditions.

The results revealed several important hydrodynamic aspects that have not been clearly seen previously. Departing bubble shapes were all ellipsoidal and became more elongated with increasing liquid velocity. Gas bubble shapes changed their form from ellipsoidal to spherical immediately after departure. The departing bubble diameters also decreased exponentially with increasing liquid velocity. Bubbles were inclined towards the liquid flow direction as the liquid velocity increased. In addition, a theoretical model has been developed to predict the departure bubble diameter and its trajectory path along the test channel. Predictions of the departing bubble size did not agree well with the available experimental results in the case of normal gravity and microgravity conditions.
However, the model showed satisfactory agreement with the bubble trajectory data in all experimental runs.

Comparisons of the departing bubble shapes have been made with those assumed by Al-Hayes and Winterton (1981) and Kandlikar and Stumm (1995). The truncated spherical shape assumed by previous researchers failed to match with the ellipsoidal bubble shapes obtained in the present study.
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**NOMENCLATURE**

\[ A_p = \text{Projected Area of the Bubble in the Flow Direction, } m^2 \]

\[ C_D = \text{Drag Coefficient} \]

\[ D = \text{Deformation Factor} \]

\[ D_b = \text{Bubble Departure Diameter, } m \]

\[ D_H = \text{Hydraulic Diameter of the Test Channel, } m \]

\[ D_N = \text{Orifice Nozzle diameter, } m \]

\[ D_v = \text{Equivalent Bubble Diameter, } m \]

\[ F_B = \text{Buoyancy Force, } N \]

\[ F_D = \text{Drag Force, } N \]

\[ F_g = \text{Gravitational Force, } N \]

\[ F_{GAS} = \text{Gas Momentum Force, } N \]

\[ F_{LM} = \text{Force resulting from Added-Mass, } N \]

\[ F_p = \text{Pressure Force, } N \]

\[ F_s = \text{Surface Tension Force, } N \]

\[ g = \text{Gravitational Acceleration, } m/s^2 \]

\[ h = \text{Half Height of the Flow Channel, } m \]

\[ l = \text{Half Width of the Flow Channel, } m \]

\[ M_{\text{add}} = \text{Added-Mass of the Gas Bubble, } kg \]

\[ M_{\text{bubble}} = \text{Mass of Gas Bubble, } kg \]

\[ M_{\text{total}} = \text{Sum the of Added-Mass and the Mass of Gas Bubble, } kg \]

\[ P_B = \text{Pressure of the Gas Bubble, } N/m^2 \]
$P_{\omega} =$ Pressure of the Surrounding Liquid, N/m$^2$

$Q_G =$ Volumetric Gas Injection Rate, m$^3$/s

$R =$ Bubble Radius, m

$Re =$ Liquid Reynolds Number

$Re_b =$ Bubble Reynolds Number

$R^* =$ Time Derivative of the Bubble, m/s

$R^{**} =$ Second Derivative of the Bubble Radius with respect to Time, m/s$^2$

$r_b =$ Bubble Departure Radius, m

$r_s =$ Bubble Base Radius, m

$t =$ Time, s

$U =$ Mean Liquid Velocity, m/s

$U_B =$ Bubble Rise Velocity, m/s

$U_f =$ One Dimensional Flow Velocity in the Flow Direction, m/s

$U_{upstream} =$ Mean Upstream Liquid Velocity, m/s

$V(z,y) =$ Two Dimensional Liquid Velocity, m/s

$V_b =$ Departure Bubble Volume, m$^3$

$V_x =$ Bubble Velocity in the x-direction, m/s

$V_z =$ Bubble Velocity in the z-direction, m/s

$x =$ Coordinate Axis, in the Flow Direction

$y =$ Coordinate Axis, Normal to the Flow Direction along the Width of the Test Channel

$z =$ Coordinate Axis, Normal to the Flow Direction along the Height of the Test Channel

$\beta_m =$ Mean Contact Angle, degree
\[ \beta_u \] = Upstream Contact Angle, degree
\[ \beta_d \] = Downstream Contact Angle, degree
\[ \mu_L \] = Viscosity of Flowing Liquid, kg/m \cdot s
\[ \mu_b \] = Viscosity of Gas Bubble, kg/m \cdot s
\[ \rho_l \] = Density of Flowing Liquid, kg/m\(^3\)
\[ \rho_G \] = Density of Gas Bubble, kg/m\(^3\)
\[ \sigma \] = Surface Tension, N/m
\[ \kappa \] = Viscosity Ratio of Gas Bubble to Flowing Liquid
\[ \gamma \] = Angle Measured from the Front Edge of a Bubble, degree
\[ \nu_l \] = Kinematic Viscosity of Liquid, m\(^2\)/s
\[ \phi \] = Inclination Angle of a Bubble, degree
1.0 INTRODUCTION

Space utilization is becoming more important in today's technological world, as evidenced by technologies such as communication, navigation, military and weather forecasting. Often, space utilization work involves complicated automation equipment and devices which operate in the space environment, and have, in most cases, extensive thermal requirements in the form of heating, cooling and refrigeration.

In the case of heating, boiling often occurs when a heated surface is exposed to a liquid. Bubbles begin to form at the beginning of nucleate boiling, grow to certain size and either collapse on the surface or depart from the surface because of buoyancy action.

In space, gravity level is very low with the result that the buoyancy force has only a small effect on the two-phase liquid and gas bubble system. The removal of these bubbles from the heating surface, usually a solid wall, is difficult under this microgravity condition, and bubbles can accumulate on the heating surface. Because a gas bubble has a lower thermal conductivity than the liquid, a reduction in heat flux can occur from the heating surface to the bulk liquid. In such cases, heating surfaces can burn out and be physically damaged. Controlling the bubble sizes and their departure from the heating surface is necessary for efficient thermal transport in space. An additional force is required for detachment of the bubble under microgravity conditions.

One practical way to obtain this additional force is to induce liquid flow near the bubble. This could probably create enough drag force that pushes the bubble away from the solid surface. Because the liquid is travelling in a confined environment within a particular system, the collection of the bubble is also important after its departure to prevent bubble build up in the system.
In order to have a better understanding of the bubble removal mechanism under microgravity conditions, a detailed hydrodynamics study of fluid and bubble behavior is required.

The objective of this study is, therefore, to investigate the hydrodynamics aspects of bubble departure with several test fluids, water and kerosene, flowing past an air bubble located on a flat plate.

To achieve a microgravity environment, the experiments were aboard a DC-9 microgravity aircraft at the NASA (National Aeronautics and Space Administration) Lewis Research Center, located in Cleveland, Ohio. This aircraft was capable of achieving a 20 s microgravity period with a gravity level of 0.02 ± 0.02g during its parabolic flight.

This thesis contains seven main parts: introductory; literature review; theoretical principles; experimental apparatus; experimental procedure; results and discussions; conclusions; and recommendations.

The beginning of the thesis provides a review of the previous works on the investigations of bubble removal and related topics. This gives a better understanding of the difficulties experienced by earlier workers and the amount of research that has been done up to date in this field. The review is presented in detail in Chapter 2. Theoretical modeling and the photochromic dye method are discussed in Chapter 3.

Detailed equipment specifications are listed in Appendix A. A computer model written in Fortran is presented in Appendix B.
2.0 LITERATURE REVIEW

Very few studies have been reported on bubble removal under both microgravity and normal gravity conditions. In general, the mechanism of bubble departure is not well understood. This chapter provides a survey of both the experimental and theoretical investigations of bubble removal from a solid surface.

Experimental studies on bubble removal from a solid surface under normal gravity have been extensively conducted in the past. Fritz (1935) investigated the manner in which a growing bubble detaches from an orifice under a normal gravity condition. He found, for low gas injection rates, the detaching bubble radius could be determined from a static force balance between the surface tension and buoyancy. From his results, Fritz suggested a correlation between the size of the departing bubble from an orifice and the bubble-growing rate at low gas injection rates.

Winterton (1972) proposed an overall force balance for liquid flowing past a departing gas bubble on a solid plate. Three different forces were mentioned in the force balance, the surface tension force $F_S$, the buoyancy force $F_B$, and the drag force $F_D$. The summation of all forces was equal to zero as shown in equation 2.0.

$$F_S - F_B - F_D = 0$$  \hspace{1cm} (2.0)

A correlation, equation 2.1, for the net surface tension force acting on a departing bubble was also introduced by Winterton (1972) by assuming a reasonable variation in the contact angle between the departing bubble and the solid surface.

$$F_S = \frac{\pi}{2} \tau_0 \sigma \sin \beta_m (\cos \beta_d - \cos \beta_u)$$  \hspace{1cm} (2.1)
\( r_b \) is the bubble departure radius, \( \sigma \) is the interfacial tension between the bubble and wall, \( \beta_u \) and \( \beta_d \) are the upstream and downstream contact angles as shown in Figure 2.1, and \( \beta_m \) is the mean contact angle defined as

\[
\beta_m = (\beta_u + \beta_d)/2 \tag{2.2}
\]

![Diagram of bubble contact angles](image)

Figure 2.1 Illustration of upstream and downstream contact angles

Al-Hayes and Winterton (1981) obtained results on the departure diameter of bubbles, which were adhered to the wall of various cylindrical tubes of different surface roughness. Their data showed that there was at most a 10% difference in bubble departure diameters with different surfaces. Their experimental range for bubble departure diameter was between 1 mm to 3 mm, and their results showed that the bubble departure diameter was a function of liquid flow velocity and bubble contact angle with the solid surface. This work led to an introduction of a correction factor to Winterton's correlation, which improved the model for the net surface tension force acting on a departing bubble. The expression for the net surface tension force was
For the drag force, the following expression was given for a truncated sphere,

\[
F_d = \frac{1}{2} C_d \rho_f U^2 r_b \left[ \frac{1}{4} \pi \beta_m + \cos \beta_m \sin \beta_m \right]
\]  

(2.4)

where \( \rho_f \) is the density of the fluid, \( C_d \) is the drag coefficient for a sphere (\( C_d = 1.22 \) for \( 20 < \text{Re}_b < 400 \), and \( C_d = 24 / \text{Re}_b \) for \( 4 < \text{Re}_b < 20 \)). The Reynolds number, \( \text{Re}_b \), for the bubble is defined as

\[
\text{Re}_b = \frac{\rho_f U D_b}{\mu_L}
\]  

(2.5)

where \( D_b \) is the bubble departure diameter, and \( \mu_L \) is the fluid viscosity, and \( U \) is the mean velocity of the fluid.

The buoyancy force, \( F_B \), in their model was expressed as:

\[
F_B = \frac{1}{3} \rho_f g \pi r_b^3 \left[ 2 + 3 \cos \beta_m - \cos^3 \beta_m \right]
\]  

(2.6)

where \( g \) is the gravitational acceleration.

In the above equations, the parameters, \( \beta_m, \beta_a, \beta_d, \) and \( \sigma \) are unknown and must be determined experimentally for different bubbles and liquids by measuring the bubble departure radius, \( r_b \), at different liquid velocities.

The bubble, which Al-Hayes and Winterton analyzed, were generated by heating a saturated liquid on a heating surface. Bubbles started to grow slowly on the heating surface until departure.

Volkov (1992) performed a numerical study of the liquid flow regime near the rising bubbles under low gravity conditions. The liquid flow regimes were obtained numerically by a mathematical simulation based on the Navier-Stokes equations.
Kirk et al. (1992) have investigated the bubble departure phenomena at low velocities of liquid flowing normal to the bubble formed under nucleate boiling conditions at various orientations of a flat heating surface. They found that the bubble departure diameter, at any orientation, decreased with an increase in liquid velocity.

Mironov and Starikov (1992) investigated gas bubble break-up in liquids by impulsive accelerations under microgravity conditions. Their experimental results showed that the break-up of a gas bubble attached to a flat surface was proportional to the accelerations applied to the system.

Ervin and Merte (1993) studied nucleate boiling and propagation of vapor bubbles on a flat plate under both normal and microgravity conditions. They observed that the heating surface had a higher temperature under microgravity conditions than in a normal gravity situation for the same size of vapor bubble at departure.

Oguz (1993) investigated the dynamics of bubble growth and the detachment from a needle tip. They found that the diameter of the bubble leaving the needle tip was inversely proportional to the growth rate of the bubble.

Oguz & Prosperetti (1993) obtained results on the size of bubbles departing from an orifice under conditions of high gas flow rates and with liquid flowing normal to the bubble. Based on their experimental results, they developed a theoretical model.

Kim et al. (1994) developed a theoretical model of the bubble departure mechanism. This model was based on the application of a force balance on a gas bubble with uniform liquid velocity flowing upstream to the bubble. It can be used to predict the progress of the bubble formation in a flowing liquid. Kim et al.’s model can be applied to both normal gravity and microgravity conditions. The model showed satisfactory
agreement with available experimental results in the case of normal gravity. However, in the case of microgravity, no experimental results are available for comparison with the theoretical predictions.

Kandlikar and Stumm (1995) conducted similar experiments as Al-Hayes and Winterton (1981) but the diameters of the departing bubbles were less than 500μm. They also found that the departing bubble diameter was a function of the liquid flow velocity and the bubble contact angle with the solid surface. However, the theoretical model developed by Al-Hayes and Winterton (1981) was not applicable within Kandlikar and Stumm's experimental range. Kandlikar and Stumm introduced a numerical simulation model that could match their results based on the momentum and force balances for a liquid flowing around a bubble.

They suggested that the net surface tension force acting on a bubble should be,

$$
F_S = 2 \sigma r_S \int_0^\gamma \cos(\beta(\gamma)) \cos(\gamma) \, d\gamma + 2 \sigma \pi r_b (1 - \beta_m/\pi)
$$

where $\sigma$ is the interfacial tension between the bubble and wall; $r_S$ is the departure bubble radius; and $\gamma$ is the angle, in radian, between the radical lines, one joining the front edge of the bubble to the center of the bubble base, and the other line joining a given location on the base periphery to the center. The illustration of $\gamma$ is shown in Figure 2.2

For $\gamma<\pi/2$;

$$
\beta(\gamma) = \beta_u - (\beta_u - \beta_m) \gamma/(\pi/2)
$$

For $\pi/2<\gamma<\pi$;

$$
\beta(\gamma) = \beta_m - [(\beta_m - \beta_d) (\gamma - \pi/2)/(\pi/2)]
$$
The drag force acting on the bubble, according to Kandlikar and Stumm(1995), is given by

$$F_D = \frac{8}{Re_b} \left[ \left(2 + 3\kappa\right)/(1 + \kappa) \right] \left(\rho_l U^2/2\right) A_P$$  \hspace{1cm} \text{(2.9)}

where $\rho_l$ is the density of the fluid, and $A_P$ is the projected area of the bubble in the flow direction, and $\kappa$ is the viscosity ratio between the fluid($\mu_L$) and the bubble($\mu_b$), and is defined as

$$\kappa = \frac{\mu_b}{\mu_L}$$  \hspace{1cm} \text{(2.10)}

For the force of buoyancy, their expression was:

$$F_B = \frac{1}{2} V_b (\rho_l - \rho_g) g$$  \hspace{1cm} \text{(2.11)}

where $V_b$ is the departure bubble volume, and $\rho_b$ is the density of the bubble.

Bubble departure condition is reached when the surface tension force given by equation 2.7 equals the sum of drag and buoyancy forces given by equations 2.9 and 2.11, respectively. The parameters, $\beta_u$, $\beta_d$, $\sigma$, and $r_s$ are unknowns that must be determined experimentally for different bubbles and liquids. Other parameters, $\rho_h$, $\rho_G$,
and $\kappa$, are the physical properties of the liquid and gas. The only unknowns that are left are the bubble Reynolds number, $Re_b$, and the mean liquid velocity, $U$. Once the Reynolds number for a departing bubble is determined for a particular liquid velocity, $U$, the bubble departure radius, $r_b$, at this liquid velocity can be determined. The departure bubble radius, $r_b$, is embedded inside the bubble Reynolds number expression.

Preliminary results reported by Leung (1996) on the bubble removal mechanism from a flat plate with liquid flowing normal to a growing bubble, showed that the size of the departure bubble decreased exponentially with increasing liquid mean velocity. The bubble departing shapes obtained in his experiments were not similar to those of previous researchers; Al-Hayes and Winterton (1981) and Kandlikar and Stumm (1995).

Peng (1996) investigated the interfacial shape and Marangoni effect around a bubble attached to a flat surface within a thermal boundary layer. Numerical simulations based on mass, momentum, and energy balances for a liquid were conducted to obtain Marangoni flow and temperature profiles around the gas bubble. However, no observations were reported on the departure diameter of gas bubbles from a solid surface.

Misawa et al. (1996) observed the bubble detachment phenomenon and measured its trajectory path before and after detachment from a solid plate with liquid flow patterns in shear flow under microgravity conditions. Their experimental results showed that a gas bubble attached to a flat surface became elongated as the shear stress of the liquid flowing past the bubble increased. Also, the departure diameter of the bubbles decreased as the shear stress of the liquid increased.

Misawa et al. (1997) proposed a correlation between the departing bubble size with the liquid flowing in a shear flow under a microgravity environment. Their model
predicted the size of the departing bubble for a given mean velocity of the flowing liquid and the trajectory of bubble’s center of gravity before departure. Misawa et al.’s model has been used to compare the results obtained from microgravity experiments. However, the comparison showed a large deviation between the experimental data and their predictions.

Although a number of results have been reported on the removal of gas bubbles from a solid surface using a flowing liquid, the experimental data are not consistent numerically. More reliable data are required in order to have a clearer understanding of the bubble removal mechanism under both normal gravity and microgravity conditions.
3.0 Theoretical Background

3.1 Photochromic dye activation method

Photochromism is the reversible transition of a chemical species between two structural states with distinguishable absorption spectra, where the reaction in at least one direction is induced by the absorption of electromagnetic radiation. The application of this technique in measuring velocity profiles utilizes the photochromic properties of a colorless indicator, which is dissolved in a test fluid.

A focused and pulsed ultra-violet(UV) laser beam incident on the test section changes the color of the indicator, thereby producing sharp traces in the fluid. By filming the trace as it is displaced by the flow, velocity profiles can be obtained. The advantages of this technique are that it is non-invasive to the liquid flow and can be used to measure instantaneous velocity profiles in liquid that are very near the walls of the test section and gas-liquid interfaces.

In the past, a photochromic dye (6-nitro-1',3',3' tri-methylspiro- 2H-1 benzopyran-2-2'-indoline) or simply called TNSB was utilized in all studies. This dye has a short UV absorption spectrum and is soluble only in organic liquids. Its performance is excellent in liquid kerosene. The photochromic reaction is shown in Figure 3.1.

![Photochromic reaction](image)

**Figure 3.1** Photochromic reaction

Recently, a photochromic dye, which is soluble in water, was developed at the University of Toronto. This dye, when dissolved in water, has the same performance as
TNSB in kerosene. The aqueous dye solution will change its color from pale yellow to dark blue once activated by ultra-violet (UV) laser.

### 3.2 Velocity profile determination

The experimental determination of the velocity profile, which utilized the photochromic dye activation technique, is a simple matter.

Figure 3.2 shows the dye trace displacement along the liquid flow direction.

![Figure 3.2 Illustration of the dye trace displacement](image)

If the liquid flow is induced only in the x-direction, the expression for the velocity is,

\[ U_r = \frac{\Delta x}{\Delta t} \]  \hspace{1cm} (3.1)
where $\Delta x$ is the displacement of the dye trace in the $x$-direction and $\Delta t$ is the time elapsed between the two frames. The value of $\Delta x$ can be obtained as $(x_1 - x_2)$. If the framing rate of the dye traces is known, $\Delta t$ can also be determined.

It follows that the knowledge of successive trace displacement profiles and the framing rate of the dye traces should be sufficient to determine the velocity profile of liquid within a flow channel.

3.3 Theoretical velocity profile

Langlois (1964) outlined an expression for the velocity profile of an incompressible, fully-developed Newtonian fluid flowing within a rectangular flow channel with a constant cross-section. The orientation of the co-ordinate system for a rectangular flow channel is illustrated in Figure 3.3.

![Co-ordinate system for a rectangular flow channel](image)

The channel is bounded by the width, $Y = \pm l$, and the height, $Z = \pm h$. The velocity of a flowing liquid within the co-ordinate system is then given by,
\[
V(z,y) = \frac{13U}{D_H^2} \left[ h^2 - z^2 + \frac{32h^2}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n \cosh(2n+1) \pi y/2h \cos(2n+1) \pi z/2h}{(2n+1)^3 \cosh(2n+1) \pi l/2h} \right]
\]

(3.2)

At the centerline of the channel, \( y = 0 \) the summation term of \textit{equation 3.2} becomes zero. \textit{Equation 3.2} is then simplified to

\[
V(0,z) = \frac{13U}{D_H^2} (h^2 - z^2)
\]

(3.3)

The theoretical velocity profile of the liquid flowing in the central plane of the rectangular flow channel can be obtained once \( U \), the mean liquid velocity, and \( D_H \), the hydraulic diameter of the flow channel, are specified.

3.4 Description of the hydrodynamic forces acting on a non-growing bubble.

Figure 3.4 shows the hydrodynamic forces acting on a bubble which is attached to a solid surface with liquid flowing in the \( x \)-direction. The three forces as shown in Figure 3.4 are: 1) the surface tension force \( F_S \); 2) the buoyancy force, \( F_B \); and 3) the drag force acting on the bubble in the \( x \)-direction, \( F_{D(x)} \).

The bubble departure condition is reached when the resultant of the bubble removal forces, i.e., drag and buoyancy forces acting on the entire bubble, exceed the net surface tension force pulling the bubble towards the plate.
In the case of microgravity, the buoyancy force becomes zero or negligibly small. Bubble departure occurs when the drag force acting on the entire bubble in the liquid flow direction exceeds the net surface tension force acting on the bubble in the opposite direction.

3.5 Description of the hydrodynamic forces acting on a growing bubble.

Figure 3.5 shows the hydrodynamic forces acting on a growing bubble which is attached to a solid surface with liquid flowing in the x-direction. The four forces, which keep the bubble attached to the wall, are 1) the surface tension force acting between the bubble and the edge of the orifice, $F_S$; 2) the gravitational force acting on the bubble in the negative direction, $F_g$; 3) the drag force acting on the bubble while it is being pumped out from the orifice in the z-direction, $F_{D(z)}$;
and, 4) the force resulting from the added-mass of the bubble during its growth in the z-direction, $F_{LM}$.

![Diagram of hydrodynamic forces acting on a growing bubble]

Figure 3.5 Hydrodynamic forces acting on a growing bubble

The forces which push the bubble away from the wall are as follow: 1) the pressure force, $F_p$, obtained from the pressure difference between the bubble and its surrounding; 2) the gas momentum force, $F_{gas}$, of the bubble while it is being injected out from the orifice; 3) the buoyancy force acting on the bubble, $F_B$; 4) the liquid drag acting on the bubble in the x-direction, $F_{D(x)}$.

3.6 Bubble growth and departure model.

Performing an overall force balance on the gas bubble as shown in Figure 3.5 gives
for the Z-direction, \( F_{D(Z)} + F_{LM} + F_g + F_{S(Z)} = F_B + F_P + F_{Gas} \) \( (3.4a) \)

and for the X-direction, \( F_{D(X)} = F_{S(X)} \) \( (3.4b) \)

The surface tension force, \( F_S \), between the edge of the orifice and the bubble is defined as

\[ F_S = (F_{S(X)}^2 - F_{S(Z)}^2)^{1/2} = \pi d_N \sigma \] \( (3.5) \)

where \( d_N \) is the orifice nozzle diameter and \( \sigma \) is the surface tension of the liquid.

The gravitational force, \( F_g \), is given by

\[ F_g = \rho_G (4/3) \pi R^3 g \] \( (3.6) \)

where \( \rho_G \) is the density of the gas inside the bubble, \( R \) is the radius of the bubble at a particular instant in time with \( R(t)=0 \) at \( t=0 \), and \( g \) is the gravitational acceleration.

The drag force, \( F_{D(Z)} \), acting on the gas bubble in the z-direction during its growth is

\[ F_{D(Z)} = 0.5 C_D U_B \pi R^2 \] \( (3.7) \)

where \( C_D \) is the drag coefficient of the bubble which depends on the bubble Reynolds number, and \( U_B \) is the bubble rise velocity in the z-direction measured at the centre of gravity of the bubble. The bubble Reynolds number, \( Re_B \), is defined as

\[ Re_B = (\rho_l U_{upstream} D_B) / \mu_L \] \( (3.7a) \)

where \( \rho_l \) is the density of the flowing liquid and \( U_{upstream} \) is the mean upstream liquid velocity facing the bubble; \( D_B \) is the bubble diameter and \( \mu_L \) is the dynamic viscosity of the liquid.

For \( Re_B < 1 \), the drag coefficient, \( C_D \), is defined as \( (Happel,1965) \)

\[ C_D = (24 / Re_B) \] \( (3.7b) \)
For \( \text{Re}_B > 1 \), Peebles & Garber (1953) suggested the drag coefficient of a deformed gas bubble can be approximated by

\[
C_D = (18.7 / \text{Re}_B^{0.64})
\]  

(3.7c)

If the bubble is growing in a spherical manner, the added-mass, \( M_{\text{add}} \), of the bubble is equal to the mass of half of the bubble volume,

\[
M_{\text{add}} = 0.5 \rho_l (4/3) \pi R^3
\]

(3.8)

A momentum balance on the bubble momentum due to added-mass and bubble rise velocity yields,

\[
d \frac{(M_{\text{add}} \cdot U_B)}{dt} = \sum F_i
\]

(3.9)

Thus, for the \( Z \)-direction,

\[
\sum F_z = U_{B(z)} \cdot \partial \frac{M_{\text{add}}}{\partial t} + M_{\text{add}} \cdot \partial \frac{U_{B(z)}}{\partial t}
\]

(3.9a)

Since the bubble is growing in a spherical manner, the force resulting from the added-mass only applies to the \( Z \)-direction. Simplifying equation 3.9a, the equation becomes

\[
F_{LM} = (2/3) \pi \rho_l R^2 (3 R^* + R R^{**})
\]

(3.10)

where \( F_{LM} \) is the force resulting from the added-mass acting on the bubble during its growth, and \( R^* \) and \( R^{**} \) are the first and second time derivatives of the bubble radius.

Because of the pressure difference between the gas bubble and its surrounding liquid, there exits a pressure force, \( F_P \), which pushes the bubble away from the wall in the \( z \)-direction

\[
F_P = (\pi / 4) d_n^2 (P_B - P_\infty)
\]

(3.11)
where $P_B$ is the pressure inside the bubble and $P_\infty$ is the pressure of the surrounding liquid.

The gas momentum force, $F_{Gas}$, is given by:

$$F_{Gas} = \rho_G Q_G \left[ \frac{1}{2} \pi \frac{dN}{2} \right]$$ (3.12)

where $Q_G$ is the volumetric injection rate of the gas.

The buoyancy force, $F_B$, acting on the bubble is:

$$F_B = \rho_L \left( \frac{4}{3} \pi \right) R^3 g$$ (3.13)

The drag force in the $x$-direction, $F_{D(x)}$, acting on the gas bubble is given by

$$F_{D(x)} = 0.5 C_D U_{\text{upstream}} \pi R^2$$ (3.14)

The bubble departure condition is reached when the resultant of the bubble removal forces, i.e., $F_B + F_P + F_{Gas} + F_{D(x)}$, acting on the entire bubble exceeds the forces, i.e., $F_{D(Z)} + F_{LM} + F_g + F_{S(Z)} + F_{S(\infty)}$ pulling the bubble towards the solid plate. The parameters, $\sigma$, $\rho_G$, $\rho_L$ and $\mu_L$ are the physical properties of the gas bubble and flowing liquid and the values of $dN$, $P_B$, $P_\infty$, and $Q_G$ were determined experimentally. The value of $C_D$ was calculated once $U_{\text{upstream}}$ was determined using equation 3.3. The bubble radius, $R$, and the first and second time derivatives of the bubble radius, $R^*$ and $R^{**}$, were calculated using the Euler method. The above equations were solved simultaneously to obtain the size of the departing bubble with liquid flowing by. A time interval, $\Delta t$, of $1 \times 10^{-5}$ s was used in the calculations to determine the values of $R$, $R^*$ and $R^{**}$. This $\Delta t$ value was sufficient in predicting an accurate value of the departing bubble size.

### 3.7 Bubble trajectory path model

After the bubble departs from the wall and travels along the test channel, Kariyasaki (1987) proposed that a lift force is exerted on the bubble. This is due to the
shear created by the liquid velocity gradient upstream of the bubble. The expression for
the lift force is given by

$$F_L = 6.84 \pi \rho_l U^2 V^2 D^2 R_K^{-0.2} R_e^{-1}$$

(3.15)

where $U$ is the bubble travelling velocity defined as

$$U = (V_X^2 + V_Z^2)^{0.5}$$

(3.16)

and $V_X$ and $V_Z$ are the velocities of the gas bubble in $X$ and $Z$ directions; $D$ is a
dimensionless deformation factor given as

$$D = 0.43 \left[ (K \rho_l \nu_l D_V) / \sigma \right]^{0.6}$$

(3.17)

(where $K$ is the velocity gradient of the liquid upstream of the bubble and $\nu_l$ is the
kinematics viscosity of the liquid); $D_V$ is the bubble equivalent diameter as compared to a
sphere and is obtained as $D_V = (ab^2)^{1/3}$ where $a$ and $b$ are the major and minor axes of the
departing bubble as illustrated in Figure 3.6.

![Figure 3.6 Illustration of the major and minor axes of a departing gas bubble](image)

The parameters, $R_K$, is defined as
and, the Reynolds number, is defined as

\[ \text{Re} = \left( \frac{U \cdot D_v}{v_l} \right) \]  

(3.19)

Next, a momentum balance is performed on the gas bubble after it has departed from the wall;

for the X-direction,

\[ \frac{d}{dt} (M_{\text{bubble}} \, U)_x = (\text{Drag force})_x \]  

(3.20)

for the Z-direction,

\[ \frac{d}{dt} (M_{\text{bubble}} \, U)_z = (\text{Buoyancy force})_z - (\text{Drag force})_z + (\text{lift force on bubble})_z \]  

(3.21)

These equations can be written as,

for the X-direction,

\[ \frac{dV_x}{dt} = \frac{3 \cdot C_p}{2 \cdot D_v} \, U \, (U_{\text{upstream}} - V_x) \]  

(3.22)

and for the Z-direction,

\[ \frac{dV_z}{dt} = [2g - \frac{3 \cdot C_p}{2 \cdot D_v} \, U \, V_y] + \left[ 6.84 \times 12 \, U^2 \, D_v^2 \, R_k^{-0.2} \, \text{Re}^{-1} \, (U_{\text{upstream}} - V_z) / U \right] \]  

(3.23)

The co-ordinate of the bubble trajectory along the test channel can be obtained by using the following relationship

\[ dz = dV_z \cdot dt \]  

(3.24)

\[ dx = dV_x \cdot dt \]  

(3.25)

The parameters, \( \sigma \), \( \rho_h \), and \( v_l \), are the physical properties of the flowing liquid and the value of \( D_v \) was determined experimentally. The values of \( D \) and \( R_k \) were calculated once the value of \( K \) was determined using equation 3.3. The Reynolds number, \( \text{Re} \), can
be obtained when the value of \( U \) is known. \( \text{Equations 3.24 and 3.25} \) were solved concurrently by Euler method to obtain the co-ordinate of the bubble along the test channel with respect to time. The above equations can be integrated simultaneously with the following initial conditions:

\[
\begin{align*}
Dv &= 2R \text{ at departure} \\
V_x(0) &= 0 \text{ at } t=0 \\
V_z(0) &= 0 \text{ at } t=0 \\
Z(0) &= R \text{ at departure} \\
X(0) &= 0
\end{align*}
\]

Results showed that a time interval, \( \Delta t \), of \( 1 \times 10^{-9} \) s used in the Euler method calculations was sufficient to predict an accurate value for the co-ordinates of the bubble along the test channel with respect to time.

### 3.8 Selection of computational device

The equations and correlations as outlined in the above sections were solved by a computer program written in \textit{Fortran 77}. For precise computation in the models, fine grid size was required in the simulation. An Alpha\textsuperscript{\textregistered} chip workstation, model 500, (100 times faster than a Pentium\textsuperscript{\textregistered} 166 desktop computer in computation speed) was used to perform all calculations in the present project.
4.0 EXPERIMENTAL

The apparatus which was designed and constructed to perform experiments in the present project is shown in Figure 4.1a. It consisted of (1) a fluid pumping and recirculating system (a liquid flow loop), (2) a flow channel with a device for injecting gas bubbles (test section), and (3) a video system for recording the fluid velocity profile data. It was essential that this apparatus be usable under both a normal and microgravity environment.

4.1 Apparatus

4.1.1 Fluid pumping and re-circulation system (Flow Loop System)

A schematic diagram of the liquid flow loop is shown in Figure 4.1a. The required test fluids were supplied from a 10 L storage drum, filtered and circulated using a centrifugal pump through a turbine flowmeter, and then to the test section and back to the storage tank. A 5 μm filter was used to remove particulates. An infrared turbine flowmeter was used to obtain accurate measurements of the liquid flow rate. This flowmeter was connected to an electronic rate-meter, which provided a digital output of the liquid flow rate.
Figure 4.1a  Schematic diagram of the experimental apparatus
4.1.2 Test fluids

Two liquids were used in the experiments, deionized water and deodorized kerosene (Shell-Sol 715). Because kerosene is highly flammable, this test fluid was prohibited in the cabin of the DC-9 plane. Therefore, kerosene was used only for studies performed under normal gravity conditions. The advantage of using Shell-Sol 715 was that the photochromic dye, TNSB, was soluble in this fluid at room temperature. A concentration of 0.01 weight % TNSB was used to provide sharp velocity profile traces. The viscosity and density of the liquid at 20°C are 1.43mPa-s and 755kg/m³ respectively. It was found that the surface tension of the solution increased slightly with increasing dye concentration. For the 0.01 weight % solution, the surface tension was 2.14x10⁻² N/m.

Water was used as the test fluid for both normal gravity and microgravity experiments. An inorganic photochromic dye was developed specifically for this fluid. Furthermore, there were no hazardous issues associated with using this fluid inside the cabin of a DC-9 plane.

The viscosity and density of the liquid at 20°C were 1.12mPa-s and 999kg/m³, respectively. It was found that the surface tension of the solution increased slightly with the dissolved dye. For the aqueous dye solution, the surface tension was 7.23x10⁻² N/m.

4.1.3 Test section

The test section, which enclosed the flow channel, was 120cm long, 15cm wide, and 5cm high. It was constructed with clear acrylic plastic. The details of the fabrication are given in Appendix A. The flow channel was rectangular in shape and of constant cross section. The dimensions were 110cm in length, 6.0cm in width and 2.5cm in height. Pyrex® glass of 0.3 cm thickness was used for the bottom surface of the flow channel. At
the center of the bottom plate of the channel, a circular hole, 1.58 mm in diameter, was drilled 8 cm from the exit end of the channel for connection of the bubble injector. Both sides of the test section were polished for optimum viewing of the experimental progress. The viewing area of the test section is illustrated in Figure 4.1b.

Clear acrylic plastic was chosen as the building material for the test section because of its transparent properties, which, allows observation into the rectangular channel at any position. Optical grade Pyrex® glass was used as the bottom plate of the channel because of its low UV absorption and its durability. The length to hydraulic diameter ratio of the flow channel was less than one hundred. Nevertheless, it provided a steady, nearly fully developed flow of liquid near the bubble.

4.1.4 Bubble injector

To generate bubbles of various sizes in the flow channel, a bubble injector was attached to the base of the flow channel at the bottom of the test section. This injector consisted of three main parts; 1) a syringe; 2) a 3-way release valve; and 3) a repeating dispenser.

Two syringes were used in this study to obtain different bubble sizes. These Pyrex® glass syringes had a capacity of 5 ml and 100 μL. The plunger tips of the syringes were made with Teflon®, which were free from gas leakage. The syringes were attached to a 3-way release valve, which was connected to the 1.58 mm diameter hole on the bottom plate of the flow channel with a 0.8 mm I.D Teflon® tubing. The function of the release valve was to allow the air to pass through without liquid back flow.

The final part of the bubble injector unit consisted of a repeating dispenser. This
Figure 4.1b  Schematic diagram of the test section showing viewing area
dispenser was attached to the syringe and was able to discharge air with 1/50 th of the syringe's capacity at each push of the button on the dispenser.

4.1.5 Laser and optical equipment

The optical arrangement is shown in Figures 4.2a and 4.2b. An ultra-violet continuous laser beam from a commercially available He-Cd laser was fired at between 1 to 5 Hz to activate the photochromic dye dissolved in the test solution. The laser was used to produce a 325 nm wavelength pulse. The laser beam was reflected through a fused silica front surface mirror to eliminate the weaker parts of the beam. A lens was used to focus the laser beam to produce a sharp intense point at the target.

The trace sequences were recorded using a Hi-8 video camera capable of capturing 30 composite frames per second. A 100 watt high-intensity halogen lamp was used as background lighting for the video camera. To diffuse the light evenly, an opal plate glass was mounted between the lamp and the test section. A cooling fan was used to minimize the heating effect on the glass and the test section.

4.1.6 Image analysis equipment

The trace displacement images were recorded from the video camera and transferred to a Super-VHS video tape. The image analysis devices consisted of a S-VHS VCR, a data monitor, and an image analysis software named Mocha 1.2. The information was processed and analyzed in a Pentium® 166MHz personal computer with 4MB ATI 3D Expression video card.
Figure 4.2a  Top view of the optical arrangement

Figure 4.2b  Side view of the optical arrangement
4.2 Experimental Procedure

4.2.1 Preliminary experiments

To insure experimental success during microgravity conditions, it was essential to develop and test the experimental procedures under normal gravity conditions. These tests included selection of the working fluid, optimum concentration of dye, and the determination of the optimum lighting and filming conditions. These are listed in Appendix B.

4.2.2 Experimental start-up procedure

The experimental procedure for aligning the laser beam to produce the optimum fluid velocity trace is outlined below.

1. For normal gravity experiments, the test section was pre-filled with either dyed water solution or Shell-Sol 715 containing the TNSB dye. However, for microgravity experiments, only water was used as the test fluid.

2. Both exit and entrance valves of the test channel were closed. Care was taken to ensure that no liquid flowed through the test channel.

3. The He-Cd laser was started up. The warm-up time for the laser was 30 minutes before full power of the laser beam was attained.

4. The laser beam was then directed normal to the test section using the front surface mirrors.

5. The final focusing was carried out by adjusting the distance of the beam-focussing lens to the liquid in the flow channel.
4.2.3 Description of an experimental run

Once the laser beam was properly adjusted, the experiment was started.

1. The 100-watt high intensity lamp was turned on and the cooling fan was started.

2. The Hi-8 video camera was adjusted to the proper height and the desired viewing position, first from the side then at the top of the test section.

3. The liquid flow was restarted at the desired flow rate.

4. Bubbles were discharged into the flow channel by manually opening and closing the release valve while pushing the dispensing button on the repeating dispenser.

5. The laser was fired at between 1 to 5 Hz depending on the liquid flow rate.

6. The liquid motion, laser dye traces and bubble images were recorded using the Hi-8 video camera for data analysis.

Eight experiments using kerosene as the test fluid were performed under normal gravity conditions as indicated in Table 4.1a; where Q is the liquid flow rate, \( V_{\text{Mean}} \) is the mean liquid velocity within the flow channel, and \( D_B \) is the departure bubble diameter.

Five experiments using water under normal gravity conditions and five under microgravity environment were performed at conditions indicated in Table 4.1b and Table 4.1c.

4.2.4 Analysis of video tape

Qualitative analyses were performed on all runs. The recorded trace motion was played frame by frame on the video monitor manually. The desired frames were then captured and transferred into the computer. Traces were digitized using the image analysis software, Mocha 1.2, which was able to calculate the velocity profile after
digitizing the co-ordinates of each trace and inputting the framing rate of the video camera.

Figure 4.3 Reduced gravity conditions; water mean flow velocity is 0.0167m/s, departure bubble diameter is 6.55mm.

4.2.5 Experimental uncertainties

The errors in the experiments were mainly due to the following factors. The unstable liquid flow within the liquid flow loop which contributed an estimated 10% error in the liquid flow measurements. The vibrations induced during manual injection of air bubbles into the flow channel, which caused the bubbles to vibrate and introduced an estimated uncertainty of 5% in the image analyses. Glass was chipped near the edge of the orifice hole that was on the bottom plate of the test channel for gas bubble injection. This imperfect fabrication caused a 100% variance in the orifice diameter used in the theoretical model. Small gas bubbles trapped in the liquid stream as shown in Figures 4.3
under microgravity conditions, which, caused inaccuracy in liquid flow analysis where flowing liquid was supposed to be single phase. For void fraction of 0.1, the error in liquid flow measurements was estimated to be 10%. Due to fluctuations in the gravity level of ± 0.02g inside the aircraft cabin, the bubble was subjected varying in magnitudes and directions of buoyancy force.

Table 4.1a Summary of experimental conditions for kerosene under normal gravity conditions

<table>
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<th>Run</th>
<th>Q (m³/s) x 10⁻⁵</th>
<th>Vₘₑₐₙ (m/s)</th>
<th>Dₜ (m) x 10⁻³</th>
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<td>0</td>
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<td>8</td>
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### Table 4.1b Summary of experimental conditions for water under normal gravity conditions

<table>
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<th>Q (m³/s) x 10⁻⁵</th>
<th>V&lt;sub&gt;Mean&lt;/sub&gt; (m/s) x 10⁻³</th>
<th>D&lt;sub&gt;B&lt;/sub&gt; (m) x 10⁻³</th>
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<td>2</td>
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<td>8.80</td>
<td>2.80</td>
</tr>
<tr>
<td>3</td>
<td>18.0</td>
<td>12.0</td>
<td>2.60</td>
</tr>
<tr>
<td>4</td>
<td>26.0</td>
<td>17.3</td>
<td>2.40</td>
</tr>
<tr>
<td>5</td>
<td>30.0</td>
<td>20.0</td>
<td>2.10</td>
</tr>
</tbody>
</table>

### Table 4.1c Summary of experimental conditions for water under microgravity conditions

<table>
<thead>
<tr>
<th>Run</th>
<th>Q (m³/s) x 10⁻⁵</th>
<th>V&lt;sub&gt;Mean&lt;/sub&gt; (m/s) x 10⁻³</th>
<th>D&lt;sub&gt;B&lt;/sub&gt; (m) x 10⁻³</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.50</td>
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<td>6.55</td>
</tr>
<tr>
<td>2</td>
<td>3.80</td>
<td>2.56</td>
<td>5.65</td>
</tr>
<tr>
<td>3</td>
<td>6.30</td>
<td>4.20</td>
<td>5.05</td>
</tr>
<tr>
<td>4</td>
<td>13.3</td>
<td>8.80</td>
<td>3.95</td>
</tr>
<tr>
<td>5</td>
<td>26.0</td>
<td>17.3</td>
<td>3.25</td>
</tr>
</tbody>
</table>
5.0 Results and Discussion

This study was the first that implemented the photochromic dye activation technique to obtain velocity profiles in liquid moving past gas bubbles under both normal and microgravity conditions. This is a significant improvement over previous investigations, which studied only the bubble departure behavior. The advantages of the photochromic dye method are that: 1) immediate visualization of liquid streamline is possible, and; 2) qualitative and quantitative analyze of the liquid motion near bubbles are possible.

In the following sections, both qualitative and quantitative results are presented and discussed. Also, a theoretical model developed to predict the bubble motion is compared with the experimental results obtained under normal and microgravity conditions.

5.1 Qualitative results

Prior to the discussion of the experimental results, it is pertinent to first present a visual perspective of the actual bubble shape at departure under different liquid flow rates, and the bubble deformation along its trajectory path.

Two different liquids, water and kerosene, were examined. Each liquid was directed over the air bubble at various flowrates to determine the corresponding gas bubble detachment diameter. Figures 5.1a to 5.1f show the side view of the departing bubble shape and the surrounding liquid both upstream and downstream of the bubble. The liquid flow is from the right to left-hand side of the pictures in Figures 5.1a to 5.1f. Figures 5.1a and 5.1b show departing bubbles with diameters of 3.00 mm and 2.10 mm for water velocities of 0.026m/s and 0.20m/s respectively under normal gravity
conditions. Figures 5.1c and 5.1d show departing bubbles with diameters of 6.55 mm and 3.25 mm for water velocities of 0.017 m/s and 0.17 m/s, respectively under microgravity environment. Figures 5.1e and 5.1f show departure bubble diameters of 1.93 mm and 1.32 mm for kerosene velocities of 0.05 m/s and 0.22 m/s, respectively. Six different drawings of the bubble trajectories and their deformations after departure under different experimental conditions as described above are shown in Figures 5.2a to 5.2f.

The shapes of departing bubbles were different from the observations reported by Al-Hayes and Winterton (1981) and Kandlikar and Stumm (1995). They suggested that the shape of the departing bubbles should be truncated spheres. However, the shapes of the departing bubbles, obtained in the present investigation, as shown in Figures 5.1a to 5.1f, were all ellipsoidal under both normal and microgravity conditions. Furthermore, as seen in Figures 5.2a to 5.2f, the bubbles changed their shape rapidly from ellipsoidal to spherical after departure. This effect was mainly due to the surface energy minimization of the gas bubbles after their detachment from the solid plate.
Figure 5.1a  Normal gravity conditions; water mean flow velocity is 0.0256m/s, departure bubble diameter is 3.0mm.

Figure 5.1b  Normal gravity conditions; water mean flow velocity is 0.20m/s, departure bubble diameter is 2.10mm.
Figure 5.1c  Reduced gravity conditions; water mean flow velocity is 0.0167 m/s, departure bubble diameter is 6.55 mm.

Figure 5.1d  Reduced gravity conditions; water mean flow velocity is 0.173 m/s, departure bubble diameter is 3.25 mm.
Figure 5.1e  Normal gravity conditions; kerosene mean flow velocity is 0.05 m/s, departure bubble diameter is 1.93 mm.

Figure 5.1f  Normal gravity conditions; kerosene mean flow velocity is 0.22 m/s, departure bubble diameter is 1.32 mm.
Figure 5.2a  Bubble shape along test channel, normal gravity conditions; water mean flow velocity is 0.0256 m/s, departure bubble diameter is 3.0 mm.

Figure 5.2b  Bubble shape along test channel, normal gravity conditions; water mean flow velocity is 0.20 m/s, departure bubble diameter is 2.10 mm.
Figure 5.2c  Bubble shape along test channel, reduced gravity conditions; water mean flow velocity is 0.0167 m/s, departure bubble diameter is 6.55 mm.

Figure 5.2d  Bubble shape along test channel, reduced gravity conditions; water mean flow velocity is 0.173 m/s, departure bubble diameter is 3.25 mm.
Figure 5.2e  Bubble shape along test channel, normal gravity conditions; water mean flow velocity is 0.05 m/s, departure bubble diameter is 1.93 mm.

Figure 5.2f  Bubble shape along test channel, normal gravity conditions; water mean flow velocity is 0.22 m/s, departure bubble diameter is 1.32 mm.
5.2 Quantitative results

Velocity profiles in the liquids were captured 3mm upstream of the gas bubbles under different experimental conditions and analyzed. The results were compared with the theoretical predictions based on equation 3.3. In addition to the liquid velocity measurements, other experimental parameters were analyzed in various ways under both normal gravity and microgravity conditions. The analyses include: 1) representation of the typical gravity level within the microgravity plane cabin; 2) correlation of mean liquid velocities; 3) the bubble departure diameters with comparison between mean liquid velocities and the ratio of major and minor axes of bubbles at their departure; 4) bubble inclination as a function of the mean liquid velocity; 5) comparison between experimental values and theoretical predictions of the relationship between the mean liquid velocity and bubble departure diameter; and 6) a comparison of measured and predicted bubble trajectories after departure. Each data point shown in the following sections represents an average of five experimental runs for normal gravity experiments, however, for microgravity experiment, each data point represents the average of three experimental runs due to time limitation for experiments conducted aboard the aircraft.

5.3 Velocity profiles

Figures 5.3a, 5.3b, and 5.3c are plots of experimental and theoretical values of liquid velocity profiles near departing gas bubbles. Velocity profile measurements for all experimental runs were taken 3mm upstream of the departing gas bubbles. Figure 5.3a shows the results for water as the test fluid with a mean liquid velocity of 0.12m/s under normal gravity conditions. Figure 5.3b shows the results for water with a mean liquid velocity of 0.042m/s under microgravity conditions. On the other hand, figure 5.3c shows
the results for kerosene with a mean liquid velocity of 0.15m/s under normal gravity conditions.

The differences in all data are quite small between theoretical predictions and experimental data. In addition to the data shown in figures 5.3a to 5.3c, the data from other runs were also compared and were observed to agree with the predictions.

In the present project, the investigation of velocity profiles for different liquids was limited to within 3mm height due to the limited penetration of the laser beam into the liquids containing photochromic dyes. However, for the mathematical models described in sections 3.6 and 3.7, the values of upstream velocity and velocity gradient must be provided for different liquids in order to predict the bubble departure diameters and trajectory paths. This required liquid velocity profiles up to half of the channel height, 12.7mm. Because of this, the liquid velocity profiles were extrapolated using equation 3.3 based on the assumption that the discrepancy was small between the experimental values and theoretical prediction between 3mm height and the half height of the test channel.
Figure 5.3a  Velocity profile plot for water with mean liquid flow of 0.12 m/s under normal gravity

Figure 5.3b  Velocity profile plot for water with mean liquid flow of 0.042 m/s under microgravity
5.4 Gravity level

All experiments were conducted in either normal gravity or microgravity conditions. Normal gravity experiments were all conducted in the Thermal Hydraulics laboratory of the University of Toronto. The gravitational acceleration under normal gravity environment is 1.0g. The microgravity experiments were conducted aboard the DC-9 parabolic aircraft at the NASA (National Aeronautics and Space Administration) Lewis Research Center, in Cleveland, Ohio. The aircraft was capable of achieving a minimum gravity level of 0 ± 0.02g during each 20-second microgravity period. A plot representing the typical acceleration levels in all three directions aboard the aircraft during the reduced gravity period is shown in Figure 5.4.
The vertical axis represents the acceleration levels in all directions, X, Y, and Z. The coordinate of the axis is described in Figure 3.2 in the theoretical section. The horizontal axis indicates the time starting from the beginning of the weightless period. As one can observe, there exists a considerable amount of gravity level fluctuations during the weightless period. These fluctuations are called g-jitter and have serious effects on certain types of experiments. One major contribution to this distortion was the wind turbulence around the aircraft. This caused the aircraft to tremble during its microgravity period.

Figure 5.4  Plot of a typical g-level for all co-ordinates during parabolic flight
5.5 Correlation of departure bubble diameter with mean liquid velocity

Figure 5.5 shows a plot of departure bubble diameter varying with liquid mean velocity for all runs. The numerical values are given in Tables 5.1a to 5.1c. The bubble departure diameters for all runs were computed as \( D_V = (ab^2)^{1/3} \), where \( D_V \) is the equivalent bubble diameter as compared to a sphere, and \( a \) and \( b \) are the major and minor axes of the departing bubbles as illustrated in Figure 3.6.

From the plot, the equivalent bubble departure diameter decreased with increased liquid velocity for both liquids. An interesting phenomenon is observed for the results of water under microgravity conditions. The equivalent bubble departure diameter decreased in an exponential fashion with an increase in liquid velocity. However, for water and kerosene under normal gravity conditions, the equivalent bubble departure diameters decreased initially in a fairly linear fashion with an increase in liquid velocity. At zero liquid velocity for kerosene, air bubbles departed from the solid plate of the test channel when the diameter reached a value of 1.96mm. This phenomenon was due to the buoyancy force acting on the bubble exceeding the net surface tension force acting on the bubble attached to the solid plate.

It would be interesting to see how small bubbles behave with liquid flow for different liquids. However, due to the technical limitations, for example, difficulties in fabricating an extremely small gas injection hole on the glass plate of the test channel with a suitable tubing arrangement, the size of the bubble that could be injected was limited during the experiment.
Figure 5.5 Variation of departing bubble diameters with mean liquid velocity

Figure 5.6 Variation of aspect ratio (a/b) of departing bubbles with mean liquid velocity
5.6 Variation of aspect ratio of departing bubbles with mean liquid velocity

Figure 5.6 shows a plot of the departing bubble aspect ratio (a/b), which is a ratio of major and minor axes, versus mean liquid velocity for different liquids.

As observed in Figure 5.6, the ratio of a/b for different bubbles increases slowly as the mean liquid velocity increases. The aspect ratio indicates the shape of air bubbles at their departure. In all cases, the a/b ratios have values greater than unity. The a/b ratio equal to unity represents a spherical bubble shape at departure. The data shown in Figure 5.6 clearly indicate that an air bubble attached to a solid plate subjected to liquid flow undergoes an ellipsoidal deformation at its departure.

5.7 Variation of bubble inclination angle with mean liquid velocity

Figure 5.7 illustrates the inclination angle, $\Theta$, of a departing gas bubble. In Figure 5.8, the bubble inclination angle is plotted against the mean liquid velocity for different liquids.

![Diagram](image)

**Figure 5.7** Illustration of the concept of inclination angle of a departing gas bubble

50
The bubble inclination angle decreased in a fairly linear fashion with increasing mean liquid velocity. When the liquid flow was stagnant, the inclination angle of the bubble reached 90 degrees, which corresponds to a vertical upward orientation.

![Bubble inclination angle graph](image)

**Figure 5.8** Plot of bubble inclination at departure versus mean liquid flow

### 5.8 Qualitative comparison between present study with previous investigations

From Figures 5.1a to 5.1f, one can clearly observe that the departing bubble shapes are ellipsoidal. These shapes are inconsistent with previous investigations of departure bubble behavior reported by Al-Hayes and Winterton (1981) and Kandlikar and Stumm (1995). These researchers observed that the shape of bubble at departure was a
truncated sphere. They generated gas bubbles in their analysis by either a pressure method or heating method. However, because surface tension is a function of temperature and pressure, these methods might have created an unstable surface tension at the liquid and gas interface, which is a possible reason for the variations in their experimental results.

In the present work, the tube injection method was used as described in the experimental section, to discharge air bubbles into the liquid flow channel. This method eliminated the effect of pressure and temperature gradients on the interface between the departing gas bubble and surrounding liquid.

Because the shapes of the departing bubbles in the present study were not the same as in previous investigations; models presented by the previous researchers can not be applied in the present work. For this reason, a new model was developed to predict the bubble departure diameters under forced liquid flow conditions as discussed in section 3.6. In addition, the new model, after few adjustments have been made as described in section 3.7, could predict the trajectory path of the center of gravity for a gas bubble after its departure under any gravity conditions. In the following two sections, the predictions of the present model are compared with the experimental results obtained under both normal gravity and microgravity conditions.

5.9 Comparison of predicted and measured departure bubble diameters

Figures 5.9 through 5.11 show comparisons of the bubble departure diameter between experimental results and theoretical predictions with variance in orifice diameter, \( D_n \).
Figures 5.9 and 5.10 show the results for water as the test fluid under normal gravity and reduced gravity conditions, respectively. Figure 5.11 illustrates the results for kerosene under normal gravity conditions. For all plots, the experimental results and theoretical predictions show the same tendency. The equivalent bubble departure diameter decreased as the liquid velocity increased. Better agreement between the experimental results and theoretical predictions is obtained as the orifice diameter is increased in the theoretical predictions in all plots. The major factor contributing to the poor predictions for higher liquid velocities was the deformation of the gas bubbles during their growth. At low liquid velocities, the deformation of the bubbles was small, but it became more significant at higher liquid velocities. However, under a microgravity environment, the major factor contributing to the poor prediction was the fluctuations in magnitude and direction of the gravitational field during the experiments.

To account for the deformation of gas bubbles due to liquid shear, the drag coefficient (equation 3.7c) was introduced into the present model.

An interesting phenomenon is observed in the above figures. Curves representing theoretical predictions in Figures 5.9 through 5.11 show for liquid velocity greater than 0.15 m/s, there would be no further changing in bubble departure diameter with increasing in liquid flow. In addition, both figures show bubble diameters decreasing in a fairly linear fashion with an increase in liquid velocity.
A hypothesis can be made based on the above results. There exists a critical point for all bubble-liquid flow system under different gravity conditions. This critical point changes the sensitivity of the bubble departure size with the velocity of liquid flowing around it. Further investigations must be carried to verify the existence and the validity of this critical point.

The major factor, which contributed to the deviations between the experimental results and the theoretical predictions in the comparisons presented, was the use of an incompatible drag coefficient in the present model. The mathematical model as proposed in section 3.6 has low sensitivity to the bubble shape deformation at higher liquid
velocity. A better drag coefficient has to be developed for bubbles with different shapes at their departure.

5.10 Comparison of measured bubble trajectories with model predictions

The coordinate system for the trajectory path of departed bubble is shown in Figure 5.12. Figures 5.13 through 5.18 show comparisons of measured bubble trajectories with theoretical predictions under different conditions. The theoretical results showed negligible differences (less than 5%) for bubble trajectory path with and without the lift force applied to the bubble. Thus, for simplicity, the figures show theoretical results without the lift force applied to the departed bubble. For the experimental results, the trajectory paths of departed gas bubbles were obtained by determining the center of gravity for each bubble image and obtaining the coordinates of the bubble center of gravity within the test channel. For the theoretical predictions, the trajectory paths of the center of gravity for departed bubbles of different sizes were obtained after the bubble departure diameters and their corresponding mean liquid velocities were defined with experimental values. The detailed description of the theoretical model was presented in section 3.7.
Figure 5.10  Comparison of measured departure bubble diameters with theoretical predictions under reduced gravity with water as the test fluid

Figure 5.11  Comparison of measured departure bubble diameters with theoretical predictions under normal gravity with kerosene as the test fluid
Figures 5.13 and 5.14 show the results for the trajectories of the center of gravity for bubbles with the highest and lowest liquid velocities of water under normal gravity conditions; Figures 5.15 and 5.16 show the results for the highest and lowest liquid velocities of water under microgravity environment. Figures 5.17 and 5.18 show the results for the trajectories of the center of gravity for bubbles with the highest and lowest liquid velocities of kerosene, under the normal gravity environment. In all plots, the experimental results and theoretical predictions show the same tendency. The coordinate of the departed bubble trajectory in the vertical direction, Z, increased in an exponential fashion with an increase in bubble traveling distance in the horizontal direction, X. As shown in all plots, theoretical predictions of the bubble trajectory paths matched closely with experimental values at low mean liquid velocities. At higher liquid velocities, larger than 0.15m/s, theoretical predictions started to deviate more from experimental values. Liquid shear at higher liquid velocities increases the deformation of departed gas bubbles. As mentioned previously, the drag coefficient employed in the present model lacks sensitivity to the deformation of bubble shape at higher liquid flow rates. The deviation
of the theoretical predictions from experimental data could be attributed to the drag coefficient used in the present model. In general, the mathematical model proposed in section 3.8 showed roughly 20% deviation from the experimental results obtained in this work.
Figure 5.13  Trajectory path of departed bubble with mean water velocity of 0.026 m/s under normal gravity

Figure 5.14  Trajectory path of departed bubble with mean water velocity of 0.20 m/s under normal gravity
Figure 5.15  Trajectory path of departed bubble with mean water velocity of 0.017 m/s under reduced gravity

Figure 5.16  Trajectory path of departed bubble with mean water velocity of 0.173 m/s under reduced gravity
Figure 5.17  Trajectory path of departed bubble with mean kerosene velocity of 0.05m/s under normal gravity

Figure 5.18  Trajectory path of departed bubble with mean kerosene velocity of 0.22m/s under normal gravity
6.0 Conclusions

An experimental test section with a rectangular cross section was designed and constructed. The test section was connected to a liquid flow loop in order to conduct experiments on the removal of bubbles attached to a surface by a flowing liquid. The photochromic dye activation technique was successfully used in conjunction with a CCD micro-video camera to visualize water and kerosene flow near departing air bubbles. Qualitative and quantitative results from the experiments were obtained and analyzed. Departure bubble diameters ranging from 1.32mm to 1.96mm were obtained for kerosene flow velocities which ranged from 0 to 0.22m/s under normal-gravity conditions; 2.10mm to 3.10mm in bubble departure diameter for water flow velocities ranging from 0.0256 to 0.20m/s under normal-gravity conditions; and 3.25mm to 6.55mm in bubble departure diameter for water flow velocities ranging from 0.0167 to 0.173m/s under reduced-gravity conditions.

Detailed velocity profiles of liquid were obtained and other hydrodynamic analyses, both qualitative and quantitative, of the liquid flowing past departing bubbles were also performed. Moreover, a theoretical model has been developed to predict the bubble departure diameter and its trajectory of the center of gravity along the test channel after departure.

As observed in the present work, liquid velocity profiles for all fluids were linear near the departing bubble. Departing gas bubble shapes were all ellipsoidal and they changed their shapes immediately from ellipsoidal to spherical after departure. Departing bubble diameters decreased exponentially with liquid velocity and gas bubbles elongated as the liquid velocity increased. The bubble inclination angle decreased as the bubble
diameter decreased. Furthermore, predictions of the bubble size using the theoretical model did not agree with the available experimental results in both normal gravity and microgravity conditions. However, the model showed satisfactory agreement (<20% deviation) with the experimental data obtained for the bubble trajectory of the center of gravity along the test channel, in all runs. Lastly, none of the departing bubble shapes obtained in the present study agreed with the bubble shapes assumed by previous investigators.
7.0 Recommendations

The results obtained in this study provide valuable information for future development of the hydrodynamics of bubble departure and its trajectory. Given below are the comments for further research that may be useful for improving the predictive model and developing new correlations for bubble departure from a surface and its trajectory.

1. Two dimensional velocity profiles of the liquid flowing past departing bubbles of various sizes could be obtained if the laser beam is directed simultaneously in both normal and tangential directions.

2. Smaller bubble sizes could be analyzed if the injection hole on the Pyrex glass plate was made with a finer diameter.

3. Using a high-speed video camera, faster liquid flow can be recorded and analyzed.

4. Another laser with higher power and proper optical arrangement could enhance the visibility of the dye traces for engineering analysis.

5. More experiments have to be done to verify the validity of the model developed in the present project.
References


Appendix A: Equipment specifications

Test section

The detailed sketch of the test section is shown in Figures A.1 to A.4. The test section was made from a clear acrylic plastic and assembled from three main parts; the main body section and the bottom and top plates. Thirteen screws on each side of the test section were used to bolt the three pieces together. Two large o-rings were used to seal the leak between the main body section and the bottom and top plates. Five 0.1 cm thick copper plates, 0.3 cm apart from each other, were inserted at the inlet of the test section for evenly distributing liquid flow within the test section.

Bubble injector unit

Bubble injector unit consisted of three main parts: 1) a release valve; 2) a syringe; and 3) a repeating dispenser.

The release valve was a two-way miniature valve made by Hamilton Scientific. The syringes, (models 1701 and 1705), were 100μL and 5mL gas tight Pyrex® syringes made by Hamilton Scientific. The repeating dispenser, model PB-600-1, manufactured by Hamilton Scientific, was a steel syringe dispenser that could discharge 1/50th of the syringe’s capacity at each push of the release button on the dispenser.

Pump and tank

A 1/8 h.p. horizontal centrifugal pump, capable of delivering 75L/min of fluid, model number TE-5c-MD, manufactured by MARCH Electric was used to deliver fluids into and recycle within the flow loop. The pump was thermally protected to prevent sparks. A 10-L acrylic storage tank was used to store liquids for different experiments.
Diagram A.1  Top view of the test section
Diagram A.2  Cross sectional view of the test section
Diagram A.3  Test section's top plate

Diagram A.4  Test section's bottom plate
Flowmeters

A Cole-Parmer [model 94788] infrared turbine flow meter with digital display was used to monitor the liquid flow rate. The flowmeter was pre-calibrated and produced signals between 4-20mA. In addition, K-factors for different fluids were pre-adjusted for different experiments to ensure correct signal output.

Liquid line

The liquid line from the pump was a ½” I.D. vinyl pipe. A ½” I.D. PVC needle valve was used to accurately control the liquid flow rate. Two ½” I.D. solenoid valves were placed at the inlet and outlet of the flow channel. Clear flexible Vinyl tubing was used for connection between the test section, liquid storage drum and the pipeline from the centrifugal pump.

Laser and optics

A Kimmon Electric’s, Japan, continuous He-Cd laser was used with an AC 110V operating voltage. The power consumption of its transformer is 1000W. The laser was able to produce a beam of 325 nm wavelength with a 4 sq. mm cross section. The beam energy was 105mW.

A Melles Griot LQP-025 lens with a 25cm focal length was used to focus the laser beam. Three Melles Griot P9996 ultra-violet light reflection mirrors with an energy loss of less than 0.5% were used to direct the laser beam into the targeted location.

Video camera

A Hitachi Hi-8 CCD video camera with a framing rate of 30 pictures per second was used to visualise the dye traces.
Light

A 100 Watt high intensity light, LPL model L-2641 was used to create adequate background lighting for the CCD micro-video camera.

VCR

A Panasonic super-VHS videocassette recorder with a digital frame memory function, model ERG-7355-P, was used to store the trace sequences from the CCD micro-video camera.

Monitor

A Panasonic 14" colour video data monitor, model CT-1400 MGC, was used in conjunction with the CCD micro-video camera and the VCR.

Digitizer

A Pentium® 166 personal computer with video interface hardware was combined with software called MOCHA 1.2 supplied by Jandel Scientific to digitize the dye traces.
Appendix B: Fortran codes for the mathematical model

C
C THIS PROGRAM IS TO MODEL THE DEPARTING BUBBLE MECHANISM
C AND ITS TRAJECTORY PATH ALONG A
C RECTANGULAR TEST CHANNEL
C
C ~~~~~ IMPORTANT ~~~~ ALL DATA IN SI UNITS
C
C
C$NOEXT
C$NOWARNING

PROGRAM BUBBLE
IMPLICIT DOUBLE PRECISION(A-Y)
CHARACTER*5 ZTrajectory
CHARACTER*5 ZDeparture
PRINT*, ' CHOOSE (1) FOR STAGNANT FLOW :'
PRINT*, ' CHOOSE (2) FOR LIQUID FLOWING :
PRINT*, ' CHOOSE (3) FOR TRAJECTORY PATH OF BUBBLE :
PRINT*, '
PRINT*, 'PLEASE ENTER :
READ(S,*), CHOICE
IF(CHOICE.EQ.1) THEN
  GOTO 1
  END IF
IF(CHOICE.EQ.2) THEN
  GOTO 2
  END IF
IF(CHOICE.EQ.3) THEN
  GOTO 3
  END IF
STOP

C
C MODELS-> STAGNANT FLOW
C
1 PRINT*, 'PLEASE ENTER THE FOLLOWING:'
PRINT*, '
PRINT*, 'G - THE GRAVITY LEVEL'
READ(5,*) G
PRINT*, 'DN - NOZZLE INNER DIAMETER'
READ(5,*) Dn
PRINT*, 'O - SURFACE TENSION OF THE LIQUID'
READ(5,*) O
PRINT*, 'V - DYNAMIC VISCOSITY OF LIQUID'
READ(5,*) V
PRINT*, 'OG - GAS INJECTION RATE'
READ(5,*) OG
PRINT*, 'Pg - DENSITY OF THE GAS'
READ(5,*) Pg
PRINT*, 'Pl - DENSITY OF THE LIQUID'
READ(5,*) Pl
PRINT*, 'Pb - PRESSURE OF THE BUBBLE'
READ(5,*) Pb
PRINT*, 'Pp - PRESSURE OF THE SURROUNDING'
READ(5,*) Pp
PRINT*, 'dt - TIME INTERVAL BETWEEN EACH CALCULATION'
READ(5,*) dt
PRINT*, '

C CALCULATION

C FRITZ MODEL
RF=((3.0D0*OG*0.5D0*Dn)/(2.0D0*Pl*G*9.81D0))**(1.0D0/3.0D0)

C OGUZ MODEL
PO=(1.0D0/5.0D0)
PR1=(22.0D0/7.0D0)
Ro=((9.0D0*OG**2.0D0)/(8.0D0*PR1**2.0D0*G*9.81D0))**PO

C MY MODEL

C ** SIMPLIFIED MODEL **
PP1=(22.0D0/7.0D0)
Rmt=((PP1*Dn*OG*(4.0D0*Pg*OG**2.0D0))/(PP1*Dn**2.0D0))
Rmb=(4.0D0/3.0D0)*PP1*(Pl-Pg)*G*9.81D0
Rs=(Rmt/Rmb)**(1.0D0/3.0D0)

C ** COMPLICATED MODEL **
Rc=0.0D0
Rc1o=0.0D0
Rc2=0.0D0
CD=0.0D0
DO 100 ZCOUNT=1,100000
TIME=ZCOUNT*dt/2.0D0
AA=(1.0D0/3.0D0)-Rc
Rc1n=(OG*dt*3.0D0/4.0D0/PP1+Rc**3.0D0)**AA/dt
Rc2=(Rc1n-Rc1o)/dt
C CD CALCULATION

\[ Reb = \pi R_c L_n \times 2.0 \times D_0 \times R_c / V \]
IF(Reb.NE.0.0D0) THEN
IF(Reb.LT.1.0D0) THEN
CD=24.0D0/Reb
ELSE
CD=18.7D0/Reb**0.68D0
END IF
END IF

C FORCE BALANCE

\[ F_{push} = (22.0D0/7.0D0)/4.0D0 \times D_n \times 2.0D0 \times (P_b - P_i) \]
\[ F_{b} = \pi (4.0D0/3.0D0) \times (22.0D0/7.0D0) \times R_c \times 3.0D0 \times G \times 9.81D0 \]
\[ F_{GAS} = P_g \times \pi \times 2.0D0 / ((22.0D0/7.0D0)/4.0D0 \times D_n \times 2.0D0) \]
\[ F_{l} = 2.0D0 \times P_l / 3.0D0 \times \pi \times R_c \times 2.0D0 \times (3.0D0 \times R_c L_n \times 2.0D0 + R_c \times R_c 2) \]
\[ F_s = (22.0D0/7.0D0) \times D_n \times 0 \]
\[ F_{g} = P_g \times (4.0D0/3.0D0) \times (22.0D0/7.0D0) \times R_c \times 3.0D0 \times G \times 9.81D0 \]
\[ F_{d} = (22.0D0/7.0D0) \times P_l / 2.0D0 \times \pi \times R_c L_n \times 2.0D0 \times R_c \times 2.0D0 \]

C CRITERIA

\[ F_{push} = F_p + F_b + F_g \]
\[ F_{pull} = F_l + F_s + F_{GAS} + F_d \]
\[ R_c = R_c + R_c l \times dt / 2.0D0 \]
\[ R_c l_{0} = R_c l_{n} \]
IF(Fpush.GT.Fpull) THEN
GOTO 200
END IF

100 CONTINUE

STOP

C RESULTS OF MODELS

200 PRINT*, 'FRITZ OGUS MODEL(S) MODEL(C)'
PRINT*, Rf,Ro,Rs,Rc
PRINT*, 'TIME'
PRINT*, TIME

C END OF MODELS->STAGNANT FLOW
STOP
C END END END END STAGNANT FLOW MODEL

C ** MODEL-> LIQUID FLOWING **

2 OPEN (UNIT=20,FILE = 'ZDeparture.dat',STATUS='NEW')
ZHZZN=0
PRINT*, 'PLEASE ENTER dT'
READ(5,*) dT
PRINT*, 'PLEASE ENTER MICROGRAVITY LEVEL'
READ(5,*) G
ZeN=1
ZL=1

3000 IF(ZL.EQ.1) THEN
   Dn=1.58D-3
   O=7.34D-2
   V=1.12D-3
   PI=999.0D0
   END IF
IF(ZL.EQ.2) THEN
   Dn=1.58D-3
   O=2.14D-2
   V=1.43D-3
   PI=755.0D0
   END IF
OG=4.0D-10
Pg=1.23D0
Pb=101325.0d0
Pl=101325.0d0

IF (ZeN.EQ.1) THEN
   UA=0.0167D0
END IF

IF(ZeN.EQ.2) THEN
   UA=0.0420D0
END IF

IF (ZeN.EQ.3) THEN
UA=0.0880D0
END IF

IF (ZeN.EQ.4) THEN
UA=0.1730D0
END IF

IF(ZeN.EQ.5) THEN
UA=0.0256D0
G=1.0D0
END IF

IF(ZeN.EQ.6) THEN
UA=0.0880D0
G=1.0D0
END IF

IF(ZeN.EQ.7) THEN
UA=0.173D0
G=1.0D0
END IF

IF(ZeN.EQ.8) THEN
UA=0.20D0
G=1.0D0
ZL=2
END IF

IF(ZeN.EQ.9) THEN
UA=0.22D0
G=1.0D0
END IF

IF(ZeN.EQ.10) THEN
UA=0.20D0
G=1.0D0
END IF

IF(ZeN.EQ.11) THEN
UA=0.18D0
G=1.0D0

END IF

IF(ZeN.EQ.12) THEN
UA=0.13D0
G=1.0D0
END IF

IF(ZeN.EQ.13) THEN
UA=0.1D0
G=1.0D0
END IF

IF(ZeN.EQ.14) THEN
UA=0.05D0
G=1.0D0
ZHZN=1
END IF

TIME=0.0D0
CALCULATION OF MODEL
PP2=22.0D0/7.0D0
Rc=0.0D0
Rc2=0.0D0
AA=(1.0D0/3.0D0)-Rc
Rcl0=(OG*(dt/1000)*3.0D0/4.0D0/PP2+Rc**3)**AA/dt
CDv=0.0D0
CDh=0.0D0
PR=(22.0D0/7.0D0)
DO 101 ZTCOUNT=1,((1.0D-3/dT)*100000000)
TIME=TIME+(dt/2)
AA=(1.0D0/3.0D0)-Rc
Rcln=(OG*dt**3.0D0/4.0D0/(22.0D0/7.0D0)+Rc**3)**AA/dt
AAR=(1.0D0/3.0D0)-Rc
RclR=(OG*dt/2.0D0*3.0D0/4.0D0/PR+Rc**3)**AA/dt/2.0D0
Rc2=(Rcln-Rcl0)/(dt/2.0D0)
VSUM=0.0D0
Rc=Rc+RclR*(dt/2)
RcD=Rc/500.0D0
dRcD=RcD
DO 102 ZVCOUNT=1,1000
V=10432.63D0*UA*(0.0127D0**2-(0.0127D0-RcD)**2)
VSUM=VSUM+V
RcD=RcD+dRcD
102 CONTINUE
   Vupstream=VSUM/999.0D0

C   CD CALCULATION

C   VERTICAL DIRECTION
Rebv=Pl*Rc1n**2.0D0*Rc/V
   IF(Rebv.NE.0.0D0) THEN
   IF(Rebv.LT.1.0D0) THEN
      CDv=24.0D0/Rebv
   ELSE
      CDv=18.7D0/Rebv**0.68D0
   END IF
END IF

C   HORIZONTAL DIRECTION
Rebh=Pl*Vupstream*2.0D0*Rc/V
   IF(Rebh.NE.0.0D0) THEN
      CDh=18.7D0/Rebh**0.68D0
   END IF

C   FORCE BALANCE
Fp=(22.0D0/7.0D0)/4.0D0*Dn**2.0D0*(Pb-Pi)
Fb=Pl*(4.0D0/3.0D0)*(22.0D0/7.0D0)*Rc**3.0D0*G*9.81D0
FGAS=Pg*OG**2.0D0/((22.0D0/7.0D0)/4.0D0*Dn**2.0D0)
Fl=2.0D0*PP2/3.0D0*Pl*Rc**2.0D0*(3.0D0*Rc1n**2.0D0+Rc*Rc2)
Fs=(22.0D0/7.0D0)*Dn*O
Fg=Pg*(4.0D0/3.0D0)*(22.0D0/7.0D0)*Rc**3.0D0*G*9.81D0
Fdv=(22.0D0/7.0D0)*Pl/2.0D0*CDv*Rc1n**2.0D0*Rc**2.0D0
Fdh=(22.0D0/7.0D0)*Pl/2.0D0*CDh*Vupstream*2.0D0*Rc**2.0D0

C   CRITERIA
Fpushy=Fp+Fb+Fg
Fpully=Fdv+Fl+FGAS
Rc1o=Rc1n
   IF(Fpully.GT.Fpushy) THEN
      Fpushy=Fpully
   END IF
FYNET=Fpushy-Fpully
FTOTAL=(Fdh**2+FYNET**2)**0.5D0
   IF(FTOTAL.GT.Fs) THEN
      GOTO 202
   END IF

101 CONTINUE
STOP

202 WRITE(20,*) 'TIME TO DEPARTED'
WRITE(20,*) TIME
PRINT*, 'TIME TO DEPARTED'
PRINT*, TIME
WRITE(20,*) 'DEPARTURE BUBBLE DIAMETER'
PRINT*, 'DEPARTURE BUBBLE DIAMETER'
Dv=Rc*2
WRITE(20,*) Dv
PRINT*, Dv
WRITE(20,*) ''

IF(ZHZZN.EQ.1) THEN
STOP
END IF
ZeN=ZeN+1
GOTO 3000

STOP
C END OF MODELS->LIQUID FLOWING

C ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
C END END END END LIQUID FLOWING MODEL
C ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

C ** MODELS-> TRAJECTORY PATH OF BUBBLE **

3 OPEN (UNIT=21,FILE = 'ZTRAJECTORY.dat',STATUS='NEW')
PRINT*, 'PLEASE ENTER THE FOLLOWING:'
PRINT*, ''
PRINT*, 'DN - NOZZLE INNER DIAMETER'
READ(5,*) Dn
PRINT*, 'G - THE GRAVITY LEVEL'
READ(5,*) G
ZHZZN=0

PRINT*, 'PLEASE ENTER dT'
READ(5,*) dT
PRINT*, ''
PRINT*, 'CHOICE 1-6'

80
READ(5,*) ZeN
PRINT*, ''

PRINT*, 'LIFT (1) OR NOT (2)'
READ(5,*) ARZ
PRINT*, ''

PRINT*, 'COMPARSION BETWEEN LIFT AND NOT IN %'
PRINT*, 'YES(1) OR NO(2)'

READ(5,*) ARZZ
PRINT*, ''

IF(ZeN.GT.2) THEN
  G=1.0D0
  IF(ZeN.GT.4) THEN
    ZL=2
    END IF
  END IF
IF(ZeN.LE.2) THEN
  PRINT*, 'PLEASE ENTER g-LEVEL'
  READ(5,*) G
  END IF
PRINT*, ''

ZCZC=0
ZL=1
4000  IF(ZL.EQ.1) THEN
  O=7.34D-2
  V=1.12D-3
  VK=1.12D-6
  Pl=999.0D0
  END IF
IF(ZL.EQ.2) THEN
  O=2.14D-2
  V=1.43D-3
  VK=1.89D-6
  Pl=755.0D0
  END IF
OG=4.0D-10
Pg=1.23D0
Pb=101325.0d0
Pi=101325.0d0
  CTIME=0.0D0
  TIMEH=0.0D0

81
IF (ZeN.EQ.1) THEN
UA=0.0167D0
Dv=6.55D-3
    AA=1
END IF

IF(ZeN.EQ.2) THEN
UA=0.173D0
Dv=3.2D-3
AA=2
END IF

IF(ZeN.EQ.3) THEN
UA=0.0256D0
Dv=3.0D-3
G=1.0D0
    AA=3
END IF

IF(ZeN.EQ.4) THEN
UA=0.20D0
Dv=2.10D-3
G=1.0D0
    AA=4
ZL=2
END IF

IF(ZeN.EQ.5) THEN
UA=0.22D0
Dv=1.32D-3
G=1.0D0
AA=5
END IF

IF(ZeN.EQ.6) THEN
UA=0.05D0
Dv=1.93D-3
G=1.0D0
AA=6
END IF

CALCULATION OF MODEL
U=0.0D0
VHO=0.0D0
VZO=0.0D0
VZONL=0.0D0
POINTZ=Dv/2.0D0
POINTX=0.0D0
NN=-1.0D0

TTIME=0.0D0
ZZN=1
DO 103 ZTIMEC=1,((1.0D-5/dT)*1000000)

C VELOCITY OF LIQUID

dD=Dv/1000.0D0
dDD=dD
VSUM=0.0D0
DO 106 ZVCOUNT=1,1000
GRID=(POINTZ-Dv/2.0D0+dD)
V=10432.63D0*UA*(0.0127D0**2-(0.0127D0-GRID)**2)
VSUM=VSUM+V
dD=dD+dDD
106 CONTINUE
ZZN=ZZN+1
VL=VSUM/999.0D0

C
C VELOCITY GRADIENT OF LIQUID

dD=Dv/1000.0D0
dDD=dD
VO=0.0D0
VGS=0.0D0

DO 107 ZVGCOUNT=1,999
GRID=(POINTZ-Dv/2.0D0+dD)
V=10432.63D0*UA*(0.0127D0**2-(0.0127-GRID)**2)
IF(ZVGCOUNT.GT.1) THEN
VG=(V-VO)/dD
dD=dD+dDD
VGS=VGS+VG
END IF
VO=V
107 CONTINUE

KV=VGS/998.0D0
IF(KV.LT.0.0D0) THEN
KV=0.0D0
END IF
\[ \text{RKV} = KV \times Dv \times 2.0D0/Vk \]

\[ U = (VL - VHO)^{2} + VZO^{2} \times (0.5D0) \]

\[ \text{CD CALCUATION} \]
\[ \text{Reb} = PI \times U \times Dv/V \]
\[ \text{IF(Reb.NE.0.0D0) THEN} \]
\[ \text{IF(Reb.LT.1.0D0) THEN} \]
\[ \text{CD} = 24.0D0/Reb \]
\[ \text{ELSE} \]
\[ \text{CD} = 18.7D0/Reb \times 0.68D0 \]
\[ \text{END IF} \]
\[ \text{END IF} \]

\[ \text{DVERTIC} = 0.43D0 \times (KV \times PI \times V_k \times Dv/O) \times 0.6D0 \]
\[ \text{LIFTz} = 82.08D0/Dv \times U \times DVERTIC \times 2 \times RKV \times (-0.2D0)/Reb \times (VL - VHO) \]

\[ \text{IF(LIFTz.LT.0.0D0) THEN} \]
\[ \text{LIFTz} = 0.0D0 \]
\[ \text{END IF} \]

\[ \text{IF(ARZ.EQ.2) THEN} \]
\[ \text{LIFTz} = 0.0D0 \]
\[ \text{END IF} \]

\[ dVZ = 2.0D0 \times G \times 9.81D0 \times (1.5D0) \times CD/Dv \times U \times VZO + LIFTz \]
\[ dVZNL = 2.0D0 \times G \times 9.81D0 \times (1.5D0) \times CD/Dv \times U \times VZO \]

\[ \text{IF(VHO.GE.VL) THEN} \]
\[ \text{VHO} = VL \]
\[ \text{END IF} \]
\[ dVH = (1.5D0) \times CD/Dv \times U \times (VL - VHO) \]
\[ VZ = VZO + dVZ \times dT \]
\[ VZL = VZOL + dVZL \times dT \]
\[ VH = VHO + dVH \times dT \]
\[ \text{IF(VH.LT.0.0D0) THEN} \]
\[ \text{GOTO 2000} \]
\[ \text{END IF} \]

\[ \text{IF(VH.GE.VL) THEN} \]
\[ \text{VH} = VL \]
\[ \text{END IF} \]

\[ \text{IF(POINTZNL.LT.0.0127D0) THEN} \]
\[ \text{POINTZNL} = \text{POINTZNL} + (VZNL + VZONL) / 2 \times dT \]
\[ \text{END IF} \]
IF(POINTZ.LT.0.0127D0) THEN
  POINTZ=POINTZ+(VZ+VZO)/2*dT
END IF

IF(POINTZ.GT.0.0127D0) THEN
  POINTZ=0.0127D0
END IF

POINTX=POINTX+(VH+VHO)/2.0D0*dT

VZONL=VZNL
VZO=VZ
VHO=VH
TTIME=TTIME+dT

DIFF=POINTZ-POINTZNL
IF(POINTZNL.GT.0.0D0) THEN
  PER=(DIFF)/POINTZNL*100
  IF (PER.LT.PERO) THEN
    PER=PERO
  END IF
  PERO=PER
END IF

IF(NN.LE.0.0D0) THEN
  WRITE(21,*) POINTX,POINTZ
  PRINT*, POINTX,POINTZ
  NN=(1.0D-5/DT)*100
END IF

IF(POINTZ.LE.0.0D0) THEN
  TIMEH='mME
  WRITE(21,*) POINTZ
  GOTO 1000
END IF

C TIME TO REACH THE CENTER LINE
   
   CTIME=TTIME

WRITE(21,*) POINTX,POINTZ
GOTO 1000
END IF

IF(POINTX.GT.15.0D-3) THEN
  TIMEH=TTIME
  WRITE(21,*) POINTX,POINTZ
  GOTO 1000
END IF
103 CONTINUE
   STOP

1000 WRITE(21,*) 'TIME TO REACH CENTER LINE'
      WRITE(21,*) CTIME

WRITE(21,*) 'TIME TO REACH 10MM HORIZONTAL'
WRITE(21,*) TIMEH
WRITE(21,*) 'LARGEST DIFFERENCE BETWEEN LIFT AND NOT %'
WRITE(21,*) PER
STOP

2000 WRITE(21,*) 'ERROR IN CALCULATION, FINER dT REQUIRES'
      ZCZC=ZCZC+1
      IF(ZCZC.EQ.6) THEN
      STOP
      END IF
      ZeN=ZeN+1
      GOTO 4000
      STOP

END

C  ~~~~~~~~~~ END OF PROGRAM ~~~~~~~~~~