Behaviour of
Offshore Reinforced Concrete Structures
under Hydrostatic Pressure

By

Amr Ibrahim I. Helmy

A thesis submitted in partial fulfillment with the requirements
for the Degree of Doctor of Philosophy
Graduate Department of Civil Engineering
University of Toronto

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At The Feet of My Parents, and
To My Little Family
ABSTRACT

Behaviour of Offshore Reinforced Concrete Structures under Hydrostatic Pressure
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A complete experimental and theoretical evaluation procedure was developed to evaluate the strength of reinforced concrete structures under external hydrostatic pressure.

The experimental approach consisted of bringing the Hydraulic Testing Facility at the University of Toronto to its full operational capacity to the study of the behaviour of reinforced concrete models under hydrostatic pressure. The most prominent component of the facility consists of a large steel pressure vessel that can withstand an internal pressure differential equivalent to 190 m of water. It was equipped with a 25 HP, variable frequency pump capable of delivering 182 litres/min. at a pressure of 207 m, flow meters to measure the inflow and outflow of water, underwater pressure and displacement transducers, and an underwater video monitoring system.

The theoretical approach was a simplified nonlinear analysis method based on the Classical Theory of Shells modified to account for the nonlinear behaviour of concrete due to concrete cracking and yielding of reinforcement. LNADS, the simplified nonlinear compatibility analysis program, and a global nonlinear finite element program RASP were used to analyze the behaviour of the model structure. LNADS can provide the design engineer with an inexpensive tool to check deformation and internal forces in storage cells under axisymmetric hydrostatic pressure.

A three-dimensional 1:13 scale model of a typical upper dome-cell wall junction of a storage cell in an early Condeep structure was constructed and tested until failure. The model resisted a maximum pressure differential equivalent to a 147 m of water. Immediately prior to failure, radial cracks could be seen at the base of the dome. Water was flowing through these cracks at a rate of 235 litres/min. The final failure was abrupt and involved a brittle shear failure of one half of the circumference of the cell wall. In these locations the dome displaced about 25mm downwards and 10mm outwards.

Before applying the results of the pressure test on the 1:13 scale model to the prototype structure the size effect in shear and the effect of in-situ strength and were examined. It was concluded that, the size effect in shear will be insignificant and it is conservative to assume a reduction of 11% of the predicted load carrying capacity of the cell wall to account for the adverse effect of the reduced strength of the cover concrete.
ACKNOWLEDGMENTS

I would like to take this opportunity to express my sincere thanks and appreciation to my supervisor Professor M.P.Collins, Department of Civil Engineering, University of Toronto, for his continuous encouragement, endless support and fruitful supervision throughout the course of this thesis.

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The author is greatly grateful to Mr. P.Leesti, Department of Civil Engineering, University of Toronto, for his precious advice, generous help and true assistance during this research. The author is grateful to Professor F.J.Vecchio for his help while carrying out the nonlinear finite element analyses using programs TRIX-97™ and RASP™. Special thanks go to Professor R.H.Mills for his generous help pertaining to the concrete mix design and assistance with the "Implosion Test Program".

The assistance of the management and personnel of the structural laboratory throughout this research is greatly appreciated. Many thanks go to Renzo Basset, Mehmet Citak, Giovanni Buzzo, Allan McLenaghen, Gayle McBurnie, John McDonald, Vasile Radulescu, Paul Amakon and Joel Babbin. Special thanks go to Peter Heliopoulos for his genius work with the control unit and the upgraded hydraulic system. The expertise of Roman Yaworsky in preparing the photographs and the illustrations is recognized.

The author wishes to expand his gratitude to Dr. D.A.Kuchma, and Mr. P.R.Gupta for their fruitful cooperation during the experimental work of the thesis. Special thanks go to Mr. E.C.Bentz for permitting the author to use his TRIXPOST™ program. The assistance and cooperation given by the author's colleagues and summer students in the Department of Civil Engineering, University of Toronto are greatly appreciated.

The author wishes to offer his wife and his little family special deep thanks for all the help and moral support they offered during the preparation of this thesis, hoping that the existence of this thesis is a remedy for all the pressure they suffered from during its preparation.
# TABLE of CONTENTS

**ABSTRACT** .................................................................................................................. iii

**ACKNOWLEDGMENTS** ............................................................................................... iv

**TABLE of CONTENTS** ................................................................................................... v

**LIST of TABLES** ........................................................................................................... x

**LIST of FIGURES** .......................................................................................................... xi

**LIST of APPENDICES** .................................................................................................. xvii

**NOTATION** .................................................................................................................... xviii

**SUMMARY** ..................................................................................................................... xxiv

**CHAPTER 1** Introduction .............................................................................................. 1

1.1 Background .................................................................................................................. 1

1.2 Review of Literature ................................................................................................... 6

1.3 Objectives of the Study ............................................................................................... 20

1.4 Thesis Layout ............................................................................................................. 21

**CHAPTER 2** Design and Construction of the Scale Model ........................................... 22

2.1 General ........................................................................................................................ 22

2.2 Dimensions of the Model ........................................................................................... 23

2.3 Reinforcement of the Model ....................................................................................... 25

2.4 Assembly of the Formwork and the Reinforcement .................................................. 29

2.5 Strain Gauging of the Reinforcement ......................................................................... 30

2.6 Concrete Used in the Construction ............................................................................ 30

2.7 Casting the Model ...................................................................................................... 37

2.8 Displacement Transducers ......................................................................................... 40

2.9 Post-Tensioning of the Model .................................................................................... 41

2.10 LNADS Linear Elastic Analysis of the Scale Model ................................................ 46
CHAPTER 3  The Hydraulic Testing Facility

3.1 General .................................................................................. 51
3.2 The Pressure Vessel ................................................................. 53
3.3 The Hydraulic Pressure System ............................................... 53
  3.3.1 The Sizing of The Pump .................................................... 53
  3.3.2 The Performance Curve ................................................... 56
  3.3.3 Behaviour During Testing ............................................... 56
  3.3.4 The Water Reservoirs ....................................................... 57
  3.3.5 The Secondary Electric Pumps ........................................... 57
  3.3.6 High Pressure Hoses ......................................................... 57
3.4 The Hydraulic Accessories ..................................................... 58
  3.4.1 The Water Inlet Port ........................................................ 58
  3.4.2 The Air Bleeding Valve .................................................... 58
  3.4.3 The Floating Valve Switch ............................................... 58
  3.4.4 The Ultrasonic Level Meter ............................................... 59
  3.4.5 The Relief Valves ............................................................ 59
  3.4.6 The Release Valve .......................................................... 59
  3.4.7 The Pressure Transducers ............................................... 60
  3.4.8 The Pressure Gauges ....................................................... 60
  3.4.9 The Water Inflow Rate .................................................... 60
  3.4.10 The Water Outflow Rate ............................................... 61
  3.4.11 The Directional Valve .................................................... 61
  3.4.12 The Check Valve .......................................................... 62
  3.4.13 The Water Outlets for the Pressure Vessel ....................... 62
3.5 The Control Panel ................................................................. 62
LIST of TABLES

CHAPTER 1 Introduction
Table 1.1 Stress Resultants and Corresponding Water Heads from Axisymmetric Finite Element Model and from Dome-Wall Slice Experiment

CHAPTER 2 Design and Construction of the Scale Model
Table 2.1 Scale Ratio for Different Geometrical Parameters of the Prototype
Table 2.2 Mechanical Properties of Reinforcing Steel Wires
Table 2.3 Concrete Mix Proportions
Table 2.4 Mechanical Properties of Prestressing Wire
Table 2.5 Shear Force at El. 60.0 Predicted by Linear Elastic Analysis by LNADS
Table 2.6 Results from the Bending Moment-Axial Force Interaction Diagram
Table 2.7 Axial Force - Shear Force Interaction Diagram at dv from El. 60.0 m

CHAPTER 5 Analysis of the Test Results
Table 5.1 Reinforcement Amounts in Different Element Types of RASP Model

CHAPTER 7 Program LNADS
Table 7.1 Reinforcement Amounts in Built-in Element Types of LNADS Model
Table 7.2 Predicted Shear Strength of a Critical Section dv from the Top of the Model Cell Wall

CHAPTER 8 Application of Test Results to Prototype
Table 8.1 Predicted Shear Strength of a Critical Section dv from the Top of the Prototype Cell Wall
Table 8.2 Summary of TRIX-97 Nonlinear Analyses Results of the Condeep Corner
LIST of FIGURES

CHAPTER 1 Introduction

Fig. 1. 1 Typical Early Condeep Platform in the North Sea - Section
Fig. 1. 2 Reinforcement Details of the Dome-Wall Connection in a Typical Early Condeep Design
Fig. 1. 3 Displacement Incompatibility at the Joint Line
Fig. 1. 4 Typical Amount of Steel Reinforcement in Offshore Structures
Fig. 1. 5 Loading for test specimens by Lenschow and Hofsoy
Fig. 1. 6 Hydraulic Testing Facility at the University of Toronto
Fig. 1. 7 Details of Pilot Test Specimen
Fig. 1. 8 View of Pilot Test Specimen prior to Testing
Fig. 1. 9 View of Pilot Test Specimen after Testing
Fig. 1. 10 Axisymmetric Finite Element Model Used in 1996 DnV Study
Fig. 1. 11 Results of Non-Linear Analysis Using Axisymmetric Finite Element Model DnV 1996
Fig. 1. 12 Appearance of 1:2.14 Scale Model of DWC1 of Dome-Wall Connection after Failure
Fig. 1. 13 Results of Non-Linear Finite Element Analysis of Specimen DWC1
Fig. 1. 14 Stress Resultants Predicted by DnV Non-Linear Finite Element Analyses Compared with Maximum Values of Stress Resultants Resisted by Laboratory Specimen DWC1

CHAPTER 2 Design and Construction of the Scale Model

Fig. 2. 1 Concrete Dimensions of Model Storage Cell
Fig. 2. 2 Reinforcement Details of Model Storage Cell
Fig. 2. 3 Stress-Strain Characteristics of Model Reinforcement
Fig. 2. 4 Construction of the Cell Wall
Fig. 2. 5 Construction of the Upper Dome
Fig. 2. 6 Preparation for Casting
Fig. 2. 7 Strain Gauge Locations and Numbering Pattern repeated at Four Locations NW, NE, SE and SW
Fig. 2. 8 Sieve Analysis of Aggregates
Fig. 2. 9 Concrete Stress-Strain Curve
Fig. 2. 10 Casting First Lift of Concrete
Fig. 2. 11 Casting Second Lift of Concrete
CHAPTER 3 The Hydraulic Testing Facility

Fig. 3.1 The Hydraulic Testing Facility
Fig. 3.2 The Pressure Vessel
Fig. 3.3 The Hydraulic Pressure System and the hydraulic Accessories
Fig. 3.4 The Performance Curve

CHAPTER 4 Testing of the Model Structure

Fig. 4.1 The Displacement Transducers Support Tree Mounted inside the Model Structure
Fig. 4.2 Interior Video Camera Mounted inside the Model Structure
Fig. 4.3 Completed Scale Model of the Typical Storage Cell
Fig. 4.4 Schematic Diagram of Model Mounted Inside Pressure Vessel
Fig. 4.5 Hydraulic Control System with Displays Giving Pressures and Flows
Fig. 4.6 Data Acquisition and Monitoring System
Fig. 4.7 Loading History for First Test
Fig. 4.8 Measured Water Flow into the Pressure Vessel during the First Test
Fig. 4.9 Vertical Displacement of the Dome during the First Test
Fig. 4.10 Model after the First Test. In this unloaded state No cracks were visible
Fig. 4.11 Loading History for Second Test
Fig. 4.12 Measured Water Flow into the Pressure Vessel during the Second Test
Fig. 4.13 Vertical Displacement of the Dome during the Second Test
Fig. 4.14 Outward Horizontal Displacement of the Joint
Fig. 4.15 Leakage of Water Through the Model at 130m of Water Head as Seen by the Interior Video Camera
Fig. 4.16 Water Leaking into Model just Prior to Shear Failure of the Wall as Seen by the Interior Video Camera
Fig. 4.17  Crushing of the Dome at Failure as Seen by Video Camera Mounted at viewport
Fig. 4.18  Air Bubbles Coming out at Model after Shear Failure of the Wall as Seen by Video Camera Mounted at Viewport
Fig. 4.19  Appearance of Model after Completion of Test. Note radial cracks and crushing of dome.
Fig. 4.20  Model Removed from the Pressure Vessel. Wires (2.64mm diameter) being inserted in shear crack. Angle formed is about 32° from vertical
Fig. 4.21  Appearance of Exterior of Model after Cracks Marked and Residual Widths Measured
Fig. 4.22  Appearance of Interior of Model after Cracks Marked and Residual Widths Measured
Fig. 4.23  Cracks on External Surface Near Top of Cell Wall: North East Quadrant
Fig. 4.24  Cracks on External Surface Near Top of Cell Wall: South West Quadrant
Fig. 4.25  Scale Model after Cutting: West Side
Fig. 4.26  Scale Model after Cutting: East Side
Fig. 4.27  West Side of Model after Cutting
Fig. 4.28  Scale Model of Storage Cell after Cutting with Diamond Saw.

CHAPTER 5  Analysis of the Test Results

Fig. 5.1  RASP Finite Element Model Used to Predict Response of 1:13 Scale Model
Fig. 5.2  Details of the Geometry of the RASP Model
Fig. 5.3  Stress-Strain Properties of Concrete and Reinforcement Used in RASP Model
Fig. 5.4  Pattern of Vertical Deformation with Increasing Applied Water Pressure Predicted by RASP Model
Fig. 5.5  Pattern of Horizontal Deformations with Increasing Applied Water Pressure Predicted by RASP Model
Fig. 5.6  Comparison of Predicted and Observed Deflections at Applied Pressure of 1.30MPa. Deflections Measured at Displacement Transducers Locations
Fig. 5.7  Measured and Predicted Vertical Displacement of the Dome
Fig. 5.8  Measured and Predicted Spreading of the Dome
Fig. 5.9  Measured and Predicted Contraction of the Cell Wall
Fig. 5.10  Patterns of Strain in Longitudinal Reinforcement with Increasing Water Pressure as Predicted by RASP
Fig. 5.11  Patterns of Strain in Circumferential Reinforcement with Increasing Water Pressure as Predicted by RASP
Fig. 5. 12 Variation of Predicted Concrete Strains near Top of Cell Wall as Applied Water Pressure Increases
Fig. 5. 13 Measured and Predicted Strains in Longitudinal Reinforcement Near Outside Face of Dome
Fig. 5. 14 Measured and Predicted Strains in Longitudinal Reinforcement Near Outside Face of Cell Wall
Fig. 5. 15 Measured and Predicted Strains in Longitudinal Reinforcement Near Inside Face of Dome
Fig. 5. 16 Measured and Predicted Strains in Longitudinal Reinforcement Near Inside Face of Cell Wall
Fig. 5. 17 Measured and Predicted Strains in Circumferential Reinforcement Near Outside Face of Dome
Fig. 5. 18 Measured and Predicted Strains in Circumferential Reinforcement Near Outside Face of Cell Wall
Fig. 5. 19 Measured and Predicted Strains in Circumferential Reinforcement Near Inside Face of Dome
Fig. 5. 20 Measured and Predicted Strains in Circumferential Reinforcement Near Inside Face of Cell Wall
Fig. 5. 21 Transition Curved Elements in RASP Model
Fig. 5. 22 Variation of Steel Strain along the Length of a Tension Specimen

CHAPTER 6 Bending of Shells and Surfaces of Revolution
Fig. 6. 1 Global Coordinate System and Internal Forces in a Surface of Revolution
Fig. 6. 2 Global Coordinate System and Internal Forces in a Cylindrical Shell
Fig. 6. 3 Compatibility in Shells and Surfaces of Revolution
Fig. 6. 4 Modelling of the Storage Cell
Fig. 6. 5 Incompatible Deformation in Cylindrical shells
Fig. 6. 6 Incompatible Deformation in Spherical Dome
Fig. 6. 7 Boundary Conditions at the Junction between the Upper Dome and the Cell Wall
Fig. 6. 8 The Global Compatibility Matrix $[A_0]$
Fig. 6. 9 Calculation of Flexural Rigidity of Reinforced Concrete Sections
CHAPTER 7  Program LNADS

Fig. 7. 1  Details of the Geometry of the LNADS Compatibility Model Used to Predict Response of 1:13 Scale Model

Fig. 7. 2  Stress-Strain Properties of Concrete and Reinforcement Used in LNADS Model

Fig. 7. 3  Pattern of Vertical Deformation with Increasing Applied Water Pressure Predicted by LNADS Model

Fig. 7. 4  Pattern of Horizontal Deformations with Increasing Applied Water Pressure Predicted by LNADS Model

Fig. 7. 5  Comparison of Observed and LNADS Predicted Deflections at Applied Pressure of 1.30MPa. Deflections Measured at Displacement Transducers Locations

Fig. 7. 6  Measured and LNADS Predicted Vertical Displacement at the Apex of the Dome

Fig. 7. 7  Measured and LNADS Predicted Vertical Displacement at One-quarter the Diameter of the Dome

Fig. 7. 8  Measured and LNADS Predicted Spreading of the Dome

Fig. 7. 9  Measured and LNADS Predicted Contraction of the Cell Wall

Fig. 7. 10 Pattern of Slope of the Deflected Line with Increasing Applied Water Pressure Predicted by LNADS Model

Fig. 7. 11 Pattern of Horizontal Shear force with Increasing Applied Water Pressure Predicted by LNADS Model

Fig. 7. 12 Pattern of Meridian/Longitudinal Bending moment with Increasing Applied Water Pressure Predicted by LNADS Model

Fig. 7. 13 Pattern of Longitudinal Membrane Force with Increasing Applied Water Pressure Predicted by LNADS Model

Fig. 7. 14 Pattern of Hoop Membrane Force with Increasing Applied Water Pressure Predicted by LNADS Model

Fig. 7. 15 Pattern of Hoop Bending Moment with Increasing Applied Water Pressure Predicted by LNADS Model

Fig. 7. 16 Predicted Ratio of Applied Forces at the Top of the Cell Wall by LNADS

Fig. 7. 17 Shear Strength-Axial Compression Interaction Diagram for Model Cell Wall

CHAPTER 8  Application of Test Results to Prototype

Fig. 8. 1  Shear Strength-Axial Compression Interaction Diagram for Cell Wall

Fig. 8. 2  TRIX-97 Finite Element Model for the Condeep Corner of Brent B
Fig. 8.3  Element Material Characteristics Used in TRIX-97 Models for the Condeep Corner of Brent B

Fig. 8.4  Nodal Loads Applied to TRIX-97 Finite Element Models for the Condeep Corner of Brent B

Fig. 8.5  Variation of Predicted Concrete Strains at the Top of the Cell Wall by TRIX-97 for BREN-75 Model

Fig. 8.6  Predicted Crack Pattern, Deflection and Stress Conditions in BREN-75 Model at 140m

Fig. 8.7  Variation of Predicted Concrete Strains at the Top of the Cell Wall by TRIX-97 for BREN-50 Model

Fig. 8.8  Predicted Crack Pattern, Deflection and Stress Conditions in BREN-50 Model at 115m

Fig. 8.9  Variation of Predicted Concrete Strains at the Top of the Cell Wall by TRIX-97 for BREN-75/50 Model

Fig. 8.10 Predicted Crack Pattern, Deflection and Stress Conditions in BREN-75/50 Model at 125m

Fig. 8.11 Influence of Concrete Strength on the Predicted Shear Strength of Cell Wall

Fig. 8.12 Net Water Flow into the Pressure Vessel during the Second Test

Fig. 8.13 Comparison of Predicted Loads and Predicted Strengths for a Brent B Storage Cell

CHAPTER 9  Summary, Conclusions and Further Work

Fig. 9.1  Typical Specimen to Study the Effect of Water Pressure in the Cracks
LIST OF APPENDICES

Appendix [A]  Program LNADS
### Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Constant used in the DE of the bending analysis of surfaces of revolution</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Cross sectional area of tensile flexural steel</td>
</tr>
<tr>
<td>$A_s^*$</td>
<td>Cross sectional area of compression flexural steel</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Arbitrary constant in the power series</td>
</tr>
<tr>
<td>$a$</td>
<td>The height of the compression block associated with the yielding of flexural steel reinforcement, The radius of the cylindrical shell, The maximum aggregate size</td>
</tr>
<tr>
<td>$B$</td>
<td>The full band width of the global compatibility matrix</td>
</tr>
<tr>
<td>$B_o$</td>
<td>Arbitrary constant in the power series</td>
</tr>
<tr>
<td>$b$</td>
<td>Constant used in the DE of the bending analysis of surfaces of revolution</td>
</tr>
<tr>
<td>$b_w, d_v$</td>
<td>The effective shear area</td>
</tr>
<tr>
<td>$C_i, C_2, C_3,$ and $C_4$</td>
<td>Four constants of integration</td>
</tr>
<tr>
<td>$C_{Dome}$</td>
<td>The constant of integration in calculating the tangential deformation of the upper dome</td>
</tr>
<tr>
<td>$C_{Shell}$</td>
<td>The constant of integration in calculating the longitudinal deformation of the shell</td>
</tr>
<tr>
<td>$D$</td>
<td>The centreline diameter of the upper dome</td>
</tr>
<tr>
<td>$D_x$</td>
<td>The flexural rigidities in the longitudinal direction</td>
</tr>
<tr>
<td>$D_{\phi}$</td>
<td>The flexural rigidities in the meridian direction</td>
</tr>
<tr>
<td>$D_{\theta}$</td>
<td>The flexural rigidities in the hoop direction</td>
</tr>
<tr>
<td>$d$</td>
<td>The distance between the most stressed fibre in the concrete compression block and the location of the tensile flexural steel reinforcement</td>
</tr>
<tr>
<td>$d_v$</td>
<td>The shear depth</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Initial Young’s modulus of concrete</td>
</tr>
<tr>
<td>$E_p$</td>
<td>Elastic Young’s modulus of prestressing wire</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Elastic Young’s modulus of steel reinforcement</td>
</tr>
<tr>
<td>$E_{sh}$</td>
<td>Strain hardening Young’s modulus of steel reinforcement in $TRIX-97$</td>
</tr>
<tr>
<td>$E'$</td>
<td>Strain hardening Young’s modulus of steel wire reinforcement</td>
</tr>
<tr>
<td>$E_x$</td>
<td>The secant modulus of elasticity in the longitudinal direction</td>
</tr>
<tr>
<td>$E_{x_i}$</td>
<td>Average secant modulus of elasticity in the longitudinal direction in the $i^{th}$ segment</td>
</tr>
<tr>
<td>$E_{\phi}$</td>
<td>The secant modulus of elasticity in the meridian direction</td>
</tr>
<tr>
<td>$E_{\theta}$</td>
<td>The secant modulus of elasticity in the hoop direction</td>
</tr>
<tr>
<td>$E'_{\phi}$</td>
<td>Average secant modulus of elasticity in the meridian direction in the $i^{th}$ segment</td>
</tr>
<tr>
<td>$E'_{\theta}$</td>
<td>Average secant modulus of elasticity in the hoop direction in the $i^{th}$ segment</td>
</tr>
</tbody>
</table>
Concrete compressive strain

Strain corresponding to $f_{c'}$

Steel Strain

Strain hardening strain

Concrete compressive stress

Cracking stress of concrete

Peak stress obtained from uniaxial compression cylinder test

$0.2\%$ Offset stress of the prestressing steel wire

Steel tensile stress

Concrete tensile strength

Ultimate Strength of reinforcement wires

Yield strength of reinforcement steel

Water head

A uniformly distributed radial load applied at the $i^{th}$ joint line between segments

Uniform radial/horizontal shear force at the $i^{th}$ joint line in the shell

Uniform horizontal shear force at the $i^{th}$ joint line in the upper dome

Constant thickness of the shell or surface of revolution

The thickness of the $i^{th}$ shell or truncated cone segment

Shell or truncated cone segments

The membrane rigidity in the longitudinal direction of the shell

The membrane rigidity in the meridian direction of the dome

The membrane rigidity in the hoop direction of the dome or the shell

The length of the cylindrical shell

The height of each shell segment

The applied bending moment at the section

The longitudinal bending moment in the dome

Uniform longitudinal bending moment at the $i^{th}$ joint line in the shell

Bending-moment-to-axial-load ratio

The elastic rate of change of the bending moment with respect to axial load

The bending-moment-to-shear-force ratio

The twisting moments in the shell

The twisting moments in the dome
\( M_\theta \) The meridian bending moment in the dome  
\( M'_\theta \) Uniform meridian bending moment at the \( i \)th joint line in the upper dome  
\( M_\theta \) The hoop bending moment in the shell or in the dome  
\( N \) The total number of segments in LNADS model  
\( N \) The axial compressive load applied on the section  
\( N_p \) The net axial load  
\( N_c \) The compression force in the web  
\( N_z \) The longitudinal membrane force in the shell  
\( N_{\theta u}, N_{\theta b} \) The membrane shear forces in the upper dome  
\( N_\theta \) The meridian membrane force in the dome  
\( N_{\theta top}, N_{\theta bottom} \) Meridian membrane force at the top and bottom of the cone segment  
\( N_{\theta u}, N_{\theta b} \) The membrane shear forces in the upper dome  
\( N_\theta \) The hoop membrane force in the dome  
\( n \) The modular ratio  
\( \Delta P \) The pressure differential across the Venturi meter  
\( p \) The intensity of the uniform external water pressure  
\( Q \) or \( q \) The water inflow rate  
\( Q_x \) The radial/horizontal shear force in the cell wall  
\( Q_y \) The radial shear force in the truncated cone  
\( Q_\theta \) The radial shear force in the surface of revolution  
\( Q_\theta \) The transverse shear force in the dome or the shell  
\( R_o \) Outer radius of upper dome  
\( R_I \) Internal radius of upper dome  
\( r_o \) Radius of the parallel circle  
\( r_{x}, r_{y} \) The principal radii of curvatures in the \( X \) and \( Y \) directions of the dome before bending  
\( r_{x}', r_{y}' \) The principal radii of curvatures in the \( X \) and \( Y \) directions of the dome after bending  
\( r_{1}, r_{2} \) The principal radii of curvatures in the meridian and hoop directions of the dome before bending  
\( r_{1}', r_{2}' \) The principal radii of curvatures in the meridian and hoop directions of the dome after bending  
\( s \) Steel wire spacing in the welded wire mesh  
\( s_x \) The crack spacing parameter or the distance between the layers of longitudinal reinforcement.  
\( u \) The vertical displacement of the wall  
\( u_{junct} \) The vertical displacement at the joint line between the dome and cell wall
V  The shear force applied at the section
V  Angle of rotation of the meridian in the shell or surface of revolution
V/N  The shear force-to-axial load
dV/dϕ  The rate of change of the angle of the deflected line
v  The tangential displacement of the upper dome
X, Y and Z  The global axis in the shell or the upper dome
x  Argument of the solution of deferential equation
x₁  Any arbitrary distance within the shell segment.
Z  Deferential equation argument
z₁ and z₂ The power series that contain the real and imaginary parts of the solution of the bending of isotropic dome

[A]_{(e×ℓ)}  The element shape matrix
[A_H]_{(e×N,2,e×N,2)}  The global compatibility matrix
[A_G]  The inverse of the global compatibility matrix
[C]_{(e×ℓ)}  The element constants vector
[C_G]_{(e×N,2,1)}  The global constants vector
[D_G]_{(e×N,2,1)}  The global discontinuity vector
[F]_{(e×ℓ)}  The element final stress-deformation vector
[I]_{(e×ℓ)}  The initial condition vector

α  The apex angle of the cone
α  A relaxation factor whose magnitude is chosen always 0 ≤ α ≤ 1
α, β, δ² Constants used in the theoretical derivations of the bending of isotropic dome
β  A factor that indicates the ability of the cracked concrete to transmit shear
β'  Constant used in the theoretical derivations of the bending of Shells
χ₁, χ₂ The curvature change of the meridian and perpendicular to the meridian in the dome
Δₜ  Predicted horizontal displacement
Δ  Measured or predicted displacement
Δₜ  Predicted Vertical Displacement
Δₜ  The horizontal displacement and extension of the radius of the parallel circle of the dome
Δₜ, Δ''ₜ The horizontal displacement at the joint line between two truncated cone segments
δₜ(joint)  The horizontal displacement at the joint line
εₜ  Strain corresponding to fc'
$\varepsilon_f$: Measured or predicted flexural steel strain

$\varepsilon_m$: Maximum elongation at rupture

$\varepsilon_x$: The maximum longitudinal strain within the shear area at the level of the flexural reinforcement steel. The strain of any lamina at distance $z$ from the middle surface due to bending of shell. Bulging strain of the cell wall

$\varepsilon_l$: The strain in the middle surface of the surface of revolution or the shell in the hoop direction

$\varepsilon_2$: The strains in the middle surface of the surface of revolution or the shell in the meridian direction. Concrete principal compressive strain

$\varepsilon_\theta$: Hoop strain in the surface of revolution

$\varepsilon_\phi$: Meridian strain in the surface of revolution

$\Phi_x$: The curvature in the longitudinal direction of the shell

$\Phi_\phi$: The curvature in the meridian direction of the dome

$\Phi_\theta$: The curvature in the hoop direction in the shell or the dome

$\phi$: Diameter of steel wire reinforcement

$\phi^o$: Central Angle in the surface of revolution made by the normal to the surface and the axis of rotation and measured from the apex

$\phi_{top}$: Central angle corresponding to the top of the cone segment

$\phi_l$: Any arbitrary central angle within the cone segment

$\phi_{bottom}$: Central angle corresponding to the bottom of the cone segment

$\phi':$ Central angle corresponding to any arbitrary cone segment

$\Phi$: Section curvature

$\gamma_{cy}$: Web Shear Strain

$\eta$: Constant used in the theoretical derivations of the bending of truncated cones

$K$: The Venturi meter constant

$\lambda$: Constant used in the theoretical derivations of the bending of truncated cones

$\mu$: Constant used in the theoretical derivations of the bending analysis of shells

$\nu$: Constant Poisson's ratio

$\theta$: The inclination of the diagonal crack to the longitudinal axis of the member. The angle measured from a datum meridian plane in the shell or surface of revolution.

$\rho$: Constant used in the theoretical derivations of the bending of surfaces of revolution

$\sigma_\phi$: The meridian stresses at any distance $z$ from the middle surface in the dome

$\sigma_\theta$: The hoop stresses at any distance $z$ from the middle surface in the dome
\( \omega \) The displacement of any point on the middle surface of the shell or surface of revolution in the Z direction.

\( \omega', \omega'' \) The horizontal displacement at the joint line between two short shell segments

\( \xi_1, \xi_2, \xi_3, \xi_4 \) The power series that contain the real and imaginary parts of the solution to the DE of bending of orthotropic truncated cone segment

\( \psi_1, \psi_2, \psi_3, \psi_4 \) The power series that contain the real and imaginary parts of the solution to the DE of bending of isotropic truncated cone segment
The aim of this thesis was to develop experimental and theoretical evaluation procedures for the strength of reinforced concrete structures under hydrostatic pressure. The experimental approach consisted of bringing the Hydraulic Testing Facility at the University of Toronto to its full operational capacity to test scale models of offshore structures under hydrostatic pressure. The theoretical approach was based on the Classical Theory of Shells modified to account for the nonlinear behaviour of concrete due to cracking and yielding.

The discoveries of large crude oil reserves in the North Sea in the early 1970's has led to the expansive use and development of reinforced concrete gravity base structures (GBS) commonly known as Condeep structures to harvest oil. The dominant load applied on the storage cells that make up the foundation of a typical GBS, is the net external hydrostatic pressure. Under current operation of early platforms, the pressure differentials are kept low by a header tank usually located in the main utility shaft. As the platforms are aging with time, deterioration and the possibility of failure of the old hydraulic systems may raise a serious consideration about the safety of the platforms. If certain components of the hydraulic system fail, the pressure differential across the cell walls would increase substantially. The questions then become how to evaluate the strength of the early concrete offshore platforms and whether this increase in pressure differential will jeopardize the safety of these platforms triggering a failure of the whole structure.

The Hydraulic Testing Facility at the University of Toronto is a new facility dedicated to the study of the behaviour of reinforced concrete structures under external water pressure. The most prominent component of the facility consists of a large steel pressure vessel certified to safely withstand an internal pressure differential equivalent to 190 m head of water. It has been brought to its full operational capacity by installing a 25 HP, variable frequency pump capable of delivering 182 litres/min. at a pressure of 207 m. By adjusting the frequency of the pump, the pressure inside the pressure vessel could be controlled to within 2 kN/m². The facility was equipped with flow meters to measure the inflow and outflow of water, underwater pressure and displacement transducers, and an underwater video monitoring system along with an underwater light system.

A theoretical approach based on the Classical Theory of Shells, modified to account for the nonlinear behaviour of reinforced concrete due to cracking of concrete and yielding of...
reinforcement, is introduced. The method is based on the compatibility approach and the bending analysis of shells. It performs a simplified nonlinear analysis providing the design engineer with an inexpensive tool to check deformation and internal forces in reinforced concrete storage cells under axisymmetric hydrostatic pressure.

A three-dimensional 1:13 scale model of a typical upper dome-cell wall junction of a storage cell in an early Condeep structure was constructed and tested until failure in the Hydraulic Testing Facility. The model resisted a maximum pressure differential equivalent to a 147 m head of sea water. Immediately prior to failure, radial cracks could be seen near the base of the dome. Water was flowing through these cracks at a rate of 235 litres/min. which would have filled the storage cell in about 7 min. As the failure was imminent, internal and external longitudinal reinforcement at the top of the cell wall crossing the yet-to-form shear crack and the hoop reinforcement at the junction yielded, accompanied with concrete crushing due to high compressive strain on the inside face of the cell wall at the junction. The final failure was abrupt and involved a brittle shear failure of one half of the circumference of the cell wall. In these locations the dome displaced about 25mm downwards and 10mm outwards. Several longitudinal bars in the wall buckled while several hoop bars ruptured at the failure zone.

LNADS, the simplified nonlinear compatibility analysis program based on the Classical Theory of Shells and a global nonlinear finite element program RASP based on the Modified Compression Field theory were used to analyze the behaviour of the model structure. Both analyses predicted a flexural shear failure at the top of the cell wall. However, there was some deviation between the theoretical predictions and the observed response of the model structure since both analyses ignored the detrimental effect of water pressure in the cracks.

Before applying the results of the pressure test on the 1:13 scale model to the prototype structure the size effect in shear was examined using the Modified Compression Field theory. For the load combinations experienced by the cell wall the prototype wall is predicted to be able to resist a differential water pressure of 137 m head of sea water corresponding to a shear stress in the prototype cell wall that is 6.2% higher than the model. It is concluded that for the load combinations experienced by the cell wall, the size effect in shear will be insignificant.

Based on the work carried out in this thesis, it is concluded that a complete process was achieved that could be applied to evaluate the strength of reinforced concrete structures subjected to axisymmetric hydrostatic pressure both theoretically and experimentally.
CHAPTER 1

Introduction

1.1 Background

The discoveries of large crude oil reserves in the North Sea in the early 1970's has led to the extensive use and development of Gravity Base reinforced concrete platforms (GBS) commonly known as Condeep Structures used to harvest oil. In a typical GBS, there might be 19 or 24 storage cells to provide stability and buoyancy requirements and 3 or 4 shafts to carry the large deck payloads. An early typical concrete deep water platform operating at a North Sea site under a water depth of about 145 m is shown in Fig. 1.1. The platform is one of the first Condeep designs, being completed in the early 70's, and as such, contains much less reinforcement than comparable platforms built more recently. The upper dome-cell wall connection of the storage cells was identified as a critical region, which may govern the capacity of the platform. See Fig. 1.1. The lack of shear reinforcement in this region, see Fig. 1.2, resulted in the belief that a shear failure of the cell wall would restrict the pressure differential that the structure could resist. As a result of these concerns, an internal air pressure of several atmospheres was usually induced during deck mating of such structures. This internal pressure reduced the maximum pressure differential during deck mating to roughly one half the water head at the junction between the upper dome and the cell wall. Under current operation of early platforms, the
Fig. 1.1 Typical Early Condeep Platform in the North Sea - Section
Fig. 1.2 Reinforcement Details of the Upper Dome-Cell Wall Connection in a Typical Early Condeep Design

- 16 tendons each with 12 1/2” dia. strands
- Force in each 1114 kN
- 20 @ 210 U-bars both faces
- 16 @ 150
- 2 x 20 @ 210 + 20 @ 630
- 20 @ 210 U-bars + 20 @ 210 both faces
pressure differentials are restricted to that value by a header tank located in the main utility shaft at mid-height between the junction and the sea level. See Fig. 1.1. In extending the expected life span of the early existing rigs, a serious consideration should be given to the possibility of failure of the old internal hydraulic system. As the structure is aging certain components of the system that were originally embedded in the concrete may fail, and a storage cell may drain. Thus the pressure differential across the dome would approximately double. The question then becomes whether this increase in pressure differential will jeopardize the safety of the platform and will result in the failure of the structure.

The upper dome-cell wall connection in a storage cell, commonly known as the Condeep Corner, is a joint between two different types of thin shells, a cylindrical shell and a spherical dome. See Fig. 1.3 (a). Under the applied axisymmetrical water pressure the spherical dome with its flat central angle chosen to suit the construction requirements, tends to flatten out horizontally at the joint line, while the cylindrical shell tends to deform inwards. See Fig. 1.3 (b). This creates displacement incompatibility between the upper dome and the cell wall along the joint line consisting of an angular and a radial discontinuity. To restore the compatibility at the joint line, a uniformly distributed bending moment and radial shear force are applied along it, known as the Discontinuity Stresses. See Fig. 1.3 (b). These undesired stresses have a local nature and damp out quickly.

The Condeep Corner is subjected to compressive axial force, bending moment, radial shear force, and tensile hoop force. See Fig. 1.3 (c). Although reinforcement detailing in the corner and the immediate vicinity is extremely critical, early concrete platforms were lightly reinforced and the arrangement of reinforcement was simple with no additional shear reinforcement. Consequently, any shear failure would possibly be brittle with very few warning signs of imminent failure.

The early Condeep GBS structures typically contained about 140 kg of reinforcing bars for every cubic metre of concrete, (i.e. 140 kg/m³). By comparison, the reinforcement amount used in the more recent offshore structures is about 400 kg/m³. See Fig. 1.4. For the more recent platforms the upper dome-cell wall junction has become heavily reinforced and the arrangement of steel reinforcement has become progressively more complex. This of course raises serious questions as to the safety of the early platforms.
Introduction

(a) Storage cell of an offshore platform
(b) Discontinuity stresses required to restore displacement compatibility at joint line
(c) Stress resultants acting on corner

Fig. 1.3 Displacement Incompatibility at the Joint Line

Fig. 1.4 Typical Amount of Steel Reinforcement in Offshore Structures
Most Data Courtesy of Dr. Techn. Olav Olsen a. s., Lysaker, Norway
1.2 Review of Literature

Research work carried out in the past 20 years related to this thesis was driven primarily by the demands of the expanding oil industry which pushed the state-of-the-art concerning our understanding of the behaviour of reinforced concrete under hydrostatic pressure.

Two areas were investigated namely; the behaviour of the dome-wall connection of the storage cells, and the implosion capacity of reinforced concrete cylinders. The latter area had the lion’s share of the experimental work\textsuperscript{1-3}. However, it is far cry from the scope of this research work. Besides, they recommended that experimental work be carried out on a closely associated topic which was “the shear capacity of the dome-cylinder junction of concrete cell structures”.

In 1976 a 1:5 scale model of a storage cell of the Statfjord A Condeep platform was loaded to failure by lowering it into the sea. The test was part of the Marbet project\textsuperscript{4} conducted by the Norwegian concrete research institute, FCB. The model had a cylindrical wall with an outside diameter of 4.0 m and a thickness of 160 mm, giving a diameter-to-thickness ratio of 25. One end of the Statford model was capped with a dome 550 mm high and 108 mm thick, giving a cell diameter-to-dome rise ratio of 7.3 and a dome thickness-to-cell wall-thickness ratio of 0.68. The bottom of the model, which was about 12 m high, was closed with a 1 m thick circular slab. The concrete strength was about 60 MPa. The model was lowered into the North Sea in 10 m load increments with a 30 minute load stage being taken at each increment, for data observations. After a test lasting for 13 hours the model failed when the net water pressure applied on the upper dome-cell wall connection was equivalent to a water head of about 164 m. The failure was preceded by a substantial leakage of water into the model, which increased the air pressure inside the unvented model. The final failure was explosive and occurred near the top of the cell wall resulting in the disappearance of the top 150 mm to 200 mm of the cell wall. The failure was identified as a shear failure of the top of the cell wall. A two dimensional finite element mesh was used for the capacity check of the model on the assumption that shear failure occurred first in the cell wall and the later breakage of the dome was a secondary failure. The report concluded that the failure load for the model “occurs between 150 m and 165 m of water” for the case with no water pressure on the
cracks while it “occurs between 136 m and 150 m” for the case with water pressure on the cracks. Accounting for the detrimental effects of water pressure in the cracks acted like an internal load in the model accelerating the cracking phenomenon.

Lenschow and Hofsoy\textsuperscript{5} in 1976, conducted a series of experiments to investigate the shear strength of dome-cell wall connections in Condeep structures. The objective was to increase the knowledge of the load carrying capacity of construction elements that are connected to each other at angles other than 90\textdegree. Twenty three “one-dimensional” beam-bends were tested looking for the effect of high compressive stresses on the shear capacity of the connection. The specimens were loaded with a force \( P \), to simulate the effect of the hydrostatic load on the cell wall. See Fig. 1. 5. As the magnitude of the applied load \( P \) increased, it resulted in a proportional increase in the axial force, shear force, and bending moment at the critical cross section with their maximum magnitudes at the intersection. By changing the angle \( \alpha \) of the test specimen, the loading ratio was changed. An interesting key finding in the investigation was that for a range of wall thicknesses varying from 180 mm to 510 mm, the specimens tested under a combination of high compression and shear showed negligible size effect. The investigation concluded that for their specimens the then current design code grossly underestimated the shear capacity of concrete without stirrups under compressive stresses and overestimated the beneficial effect of stirrups. Although, the failure for all specimens was shear dominated, specimens with stirrup reinforcement experienced a modest increase in ultimate strength.

Fig. 1. 5 Loading for test specimens by Lenschow and Hofsoy\textsuperscript{5}
In 1990, Koltuniuk started an investigation into the effect of high water pressure on cracked surfaces of reinforced concrete structures. His objective was to design and implement laboratory equipment necessary to study the behaviour of offshore reinforced concrete models. A large steel pressure vessel was designed and manufactured. See Fig. 1.6. A 1:7 scale model of a buoyancy compartment of a small offshore oil platform along with the collapsible steel formwork required to construct it, was designed and built. The cylindrical model, shown in Fig. 1.7 and Fig. 1.8, had an external diameter of 1524 mm and a height of 2210 mm. The cylinder, which had a wall thickness of 51 mm and was topped with a flat slab 86 mm thick was tested later. The study provided valuable data about the challenges facing the construction of scale models in a laboratory environment.

Fig. 1.6 Hydraulic Testing Facility at the University of Toronto

In 1992, The Hydraulic Testing Facility, was brought into operation at the University of Toronto to study the behaviour of reinforced concrete structures subjected to external water pressure. As shown in Fig. 1.6, the most prominent component of the facility consists of a large steel pressure vessel 8 ft - 6 in. (2.59 m) in diameter and
14 ft - 2 in (4.32 m) high with hemispherical ends. This vessel is certified to safely withstand an internal pressure of 275 psi gauge, that is a pressure differential of 1.90 MPa, which is equivalent to a water head of about 190 m. A full line of peripheral systems were designed and implemented such as an air driven pressurizing system, underwater displacement and pressure transducers and an underwater video monitoring system.

In 1992 Helmy© carried out the first major test using the facility on a 1 : 7 scale model of a buoyancy cell for a proposed platform in relatively shallow water off the coast of Australia. The cylindrical model, shown in Fig. 1. 7 and Fig. 1. 8, had an external diameter of 1524 mm and a height of 2210 mm. The cylinder, which had a wall thickness of 51 mm, was topped with a flat slab 86 mm thick. During the test, two large circumferential cracks developed on the outside face of the cylindrical wall near the junction with the bottom of the slab. See Fig. 1. 9 (a). At a pressure differential of 0.44 MPa, the vertical reinforcement crossing these cracks yielded and water began to leak into the specimen at a rate of about 16 gallons per minute (73 litres per minute). While the concrete compression zone was capable of preventing water flow through concrete cracks prior to yielding of the reinforcement, after yielding significant water flow occurred. The hydraulic pressure system used at that time, which was driven by a 1.5 horsepower (1120 Watt) pump, was not capable of increasing the pressure differential for this rate of water flow and hence the test had to be stopped prior to structural failure of the model. The maximum pressure differential reached was about three times the design pressure, i.e. $P_{(Design)} = 0.15$ MPa (22 psi) and $P_{(Maximum)} = 0.440$ MPa (64 psi). This was due to the fact that the reinforcing steel spacing was governed by the construction requirements of the model rather than by the applied water pressure.

As can be seen from Fig. 1.4, Brent B is one of the first Condeep platforms, being completed in 1974, and as such, contains much less reinforcement than comparable platforms built recently. Thus Brent B provides an excellent example to review the theoretical and the experimental work carried out to evaluate the strength of reinforced concrete offshore structures subjected to hydrostatic pressure.
$f_{c'} = 54$ MPa

No. 6 bars area = 25.7 mm$^2$

$f_y = 420$ MPa
(a) Cracks on Exterior Face near Slab-Wall Junction

(b) Cracks on Interior Face of Slab. Residual Crack Widths Given in Millimetres.

Fig. 1. 9 View of Pilot Test Specimen after Testing
Early analytical studies of Brent B had identified the upper dome-cell wall connection of its storage cells as a critical region, due to the lack of shear reinforcement in this zone, and resulted in the belief that a shear failure of the cell wall would restrict the pressure differential that the structure could resist. A number of recent studies have investigated the load bearing capacity of the Brent B upper dome-wall connection. These include "global" non-linear finite element analyses of axisymmetric models representing the upper portion of a typical storage cell; more detailed "local" two-dimensional, non-linear finite element analyses of a slice of the structure near the dome-wall connection; and laboratory experiments on 1:2.14 scale models of a slice of the structure near the dome-wall connection.

The axisymmetric finite element model used in the 1996 Det Norske Veritas (DnV) Research Division study of the Brent B dome-wall connection is shown in Fig. 1. This study concluded that failure of the connection would typically be caused by crushing of the concrete near the inside face at the dome-wall junction. Because of this, the predicted failure load was quite sensitive to the assumptions made about the in-situ strength of the concrete in this region. See Fig. 1. For a concrete strength of about 50 MPa, the structure was predicted to resist a pressure differential of about 127 m of water. The analytical model accounted for the detrimental effects of water in the cracks. However, its ability to accurately predict a shear failure in the cell wall is open to question.

The influence of reinforcement detailing on the behaviour of the upper dome-cell wall intersection was investigated through an experimental program. Three reinforced concrete specimens, representing models of the dome-wall connection were tested in the Shell Element Testing facility under simulated water pressure. The observed crack pattern and failure mode of one of the dome-wall connection slice models is shown in Fig. 1. This specimen, DWC1, had a concrete cylinder strength of 50.2 MPa, and failed when the concrete stress resultants reached the values shown in Fig. 1. Based on the value of axial compression in the cell wall, and neglecting the influence of water pressure in the cracks, the failure was reported to occur at a pressure differential of 118 m of water. The cracking pattern was initiated with flexural cracking at the top of the cell wall followed by flexural cracking at the base of the dome. Then the flexural cracks developed into flexural shear cracks. Yielding of steel reinforcement at both locations was followed shortly after
by the crushing of concrete at the inside corner of the specimens. Failure occurred after crushing of the concrete. Measured deformation during testing indicated that the dome response was more flexible than the cell wall region. The ultimate load carrying capacity was well predicted with program TRIX, and it accurately predicted the failure mode for DWC1 as flexural shear failure in the cell wall. The two-dimensional, non-linear finite element model analysis results for this specimen are shown in Fig. 1. 13.

The experiments on the 1:2.14 scale models of the dome-wall connections used a number of different loading ratios to simulate the patterns of axial loads, moments, and shears that are predicted to occur as the structure is subjected to an increasing pressure differential. The 1996 DnV predictions for the manner in which these stress resultants increase with the pressure differential for two locations in the cell wall is shown in Fig. 1. 14. Also shown in these plots are the maximum values of the stress resultants at these locations, reached during the experiment on specimen DWC1, scaled up to prototype values (i.e., shears and axial forces are increased by a factor of 2.14, while moments are increased by a factor of 2.14²). Accounting for the influence of the water pressure in the cracks, which reduces the effective compression in the wall, means that the axial force applied to specimen DWC1 was equivalent to a head of 134 m. The moments corresponded to heads between 133 m and 145 m, while the shears corresponded to pressure differentials between 158 m and 165 m. See Fig. 1. 14 and Table I. 1. These results suggest that it would take a pressure differential greater than about 150 m to cause a shear failure of the dome-wall connection.

Table I. 1 Stress Resultants and Corresponding Water Heads from Axisymmetric Finite Element Model and from Dome-Wall Slice Experiment

<table>
<thead>
<tr>
<th>Method of Determination</th>
<th>Location Elev. m</th>
<th>Axial Load Value kN/m</th>
<th>Water head m</th>
<th>Moment Value kN·m</th>
<th>Water head m</th>
<th>Shear Value kN/m</th>
<th>Water head m</th>
</tr>
</thead>
<tbody>
<tr>
<td>DnV 1996</td>
<td>60.0</td>
<td>-5990</td>
<td>127</td>
<td>2520</td>
<td>127</td>
<td>2600</td>
<td>127</td>
</tr>
<tr>
<td>Non-Linear Finite Element Analysis</td>
<td>59.7</td>
<td>-5990</td>
<td>127</td>
<td>1800</td>
<td>127</td>
<td>2180</td>
<td>127</td>
</tr>
<tr>
<td>Toronto 1994 DWC1</td>
<td>60.0</td>
<td>-6353</td>
<td>134</td>
<td>2788</td>
<td>145</td>
<td>3242</td>
<td>158</td>
</tr>
<tr>
<td>Experimental Values</td>
<td>59.7</td>
<td>-6353</td>
<td>134</td>
<td>1870</td>
<td>133</td>
<td>2894</td>
<td>165</td>
</tr>
</tbody>
</table>
Fig. 1.10 Axisymmetric Finite Element Model Used in 1996 DnV Study
Introduction

Ultimate Load Bearing Capacity vs. Concrete Strength

<table>
<thead>
<tr>
<th>Case</th>
<th>$f_c$ (MPa)</th>
<th>$f_r$ (MPa)</th>
<th>$\kappa$</th>
<th>$f_t$ (MPa)</th>
<th>$T_{fail}$ (MPa)</th>
<th>ULT - head (m)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Br</td>
<td>26.9</td>
<td>1.80</td>
<td>5</td>
<td>348</td>
<td>138</td>
<td>88</td>
<td>Compression failure</td>
</tr>
<tr>
<td>Ar</td>
<td>33.6</td>
<td>2.25</td>
<td>5</td>
<td>397</td>
<td>164</td>
<td>105</td>
<td>Compression failure</td>
</tr>
<tr>
<td>Cr</td>
<td>39.1</td>
<td>2.25</td>
<td>5</td>
<td>397</td>
<td>207</td>
<td>109</td>
<td>Compression failure</td>
</tr>
<tr>
<td>Dr</td>
<td>48.7</td>
<td>2.55</td>
<td>5</td>
<td>397</td>
<td>225</td>
<td>127</td>
<td>Steel yield at $T_{fail} = 227$</td>
</tr>
<tr>
<td>Er</td>
<td>59.4</td>
<td>2.55</td>
<td>5</td>
<td>397</td>
<td>227</td>
<td>129</td>
<td>Steel yield es &gt; 2.0 o/oo</td>
</tr>
</tbody>
</table>

Crack pressure included, ie -R cases
Self weight is covered as separate load

NC-86 NC -ABAQUS 1986 $f_c = 26.1$ ULT -load = 88 m head

Fig. 1. 11 Results of Non-Linear Analysis Using Axisymmetric Finite Element Model (DnV 1996)
Fig. 13 Results of Non-Linear Finite Element Analysis of Specimen DWCl
Fig. 1. 14 Stress Resultants Predicted by DnV Non-Linear Finite Element Analyses Compared with Maximum Values of Stress Resultants Resisted by Laboratory Specimen DWC1
The recent studies summarized above provided encouraging evidence that the shear capacity of the cell wall is greatly in excess of what would be calculated using the simple, conservative expressions given in building codes such as ACI 318-95\textsuperscript{11}. See the line labelled “ACI Shear Capacity” in Fig. 1. 14 (c). However, some questions still remained about the structural performance of the dome-cell wall region of platforms such as Brent B.

1. The two-dimensional scale models of a slice of the dome-wall connection ignored the three-dimensional effect of a complete model and could not account for the influence of the hoop tensions that are predicted to develop as the pressure differential increases. Since they were based on a statically determinate setup, by definition, the bending moment was not a function of the flexural stiffness as it is in the actual structure.

2. The mode of failure of the dome under water pressure was not clear. The investigations did not provide any information on the vital signs of when failure was imminent. Would water leak into the cell fast enough to relieve the pressure differential and prevent a brittle shear failure?

3. Water under pressure seeping through the short concrete cracks would increase the tensile stresses in the steel reinforcement and reduce the compressive stresses on the concrete. With the reduced compressive stresses, the flexural capacity of the critical sections would be lower than theoretically expected. Would water pressure in the cracks reduce the aggregate interlock mechanisms required for shear transfer and hence, reduce the capacity even more than the 5 to 6 m reduction in differential head capacity suggested by the finite element analyses?

4. Besides, any theoretical investigation was, by definition, based on formulae derived from tests carried out in the air and at the ambient temperature. Indeed, this was not the environment that surrounded the actual structure.

It was decided that the most direct way of answering these questions was to investigate the possibility of developing an evaluation process both theoretically and experimentally dedicated to developing a better understanding of the behaviour of reinforced concrete structures under hydrostatic pressure.
1.3 Objectives of the Study

The main objective in this research project is aimed towards developing an evaluation process that could be applied to evaluate the strength of existing reinforced concrete offshore structures subjected to hydrostatic pressure both experimentally and theoretically. The objectives of the research work are as follows;

1. To bring the Hydraulic Testing Facility at the University of Toronto, into its full operational capacity. This was achieved by implementing various components such as a powerful pressurization system with a custom design control unit, an underwater pressure and displacement transducers, an underwater video monitoring system along with underwater light system, and inflow and outflow measuring devices.

2. To evaluate the strength of such structures is by no means a straightforward task. The closed form solutions developed by the Classical Theory of Shells were not ideally suited for the highly non-linear behaviour of reinforced concrete. A theoretical approach based on the Classical Theory of Shells, modified to account for the nonlinear behaviour of concrete due to cracking and yielding of reinforcement was developed, referred to as the compatibility approach.

3. To construct a 1:13 prestressed concrete scale model of a typical storage cell of a GBS in the North Sea, then to test this model until failure in the University of Toronto's Hydraulic Testing Facility. The prime objectives of this phase were to gain insight into the behaviour of the model under loading, the failure mechanisms, the failure load and vital signs that failure is imminent under monotonically increased water pressure.

4. To investigate the theoretical behaviour of the model structure using a nonlinear analysis based on the Finite Element Method and the Modified Compression Field theory and a simplified nonlinear analysis carried out using the compatibility approach based on the Classical Theory of Shells. A comparison between the analytical predictions and experimental observations was carried out for the model structure.

5. To apply the experimental test results to the prototype structure including, consideration of the size effect in shear, the mode of failure, the effect of concrete strength and a comparison between several analytical and experimental works carried out for the same structure and similar structures.
1.4 Thesis Layout

The research work carried out during this project is presented in this thesis in 9 chapters and one appendix. Chapter 1 presents the background, literature review, objectives of the work, and the thesis layout. Chapter 2 presents the design and construction of the scale model, details of reinforcement in the model, the various laboratory steps during construction of the model, the instrumentation, the concrete mix design, the casting and curing of the model, the design and application of the external prestressing, linear elastic analysis results for the behaviour of the model, and a simplified check on the load carrying capacity of the critical sections. Chapter 3 presents the various components of the experimental test setup and the upgrading of the Hydraulic Testing Facility. Chapter 4 presents the testing of the scale model, the preparation of the model for testing, observations during testing, load history and water inflow measurements during both hydrostatic tests, the failure observations and failure sequence. Chapter 5 presents the predicted behaviour of the model structure using a three-dimensional mesh analyzed by program RASP and a comparison between the theoretical predictions and the experimental results from the pressure test namely; the deformation of the model and strain gauge readings. Chapter 6 presents a theoretical investigation into the bending of thin shells and surfaces of revolution based on the Classical Theory of Shells. Chapter 7 presents a simplified method of analysis that was capable of tracking the nonlinear behaviour of reinforced concrete based on a compatibility approach, the outlines of the computer program LNADS that applies the method to the storage cell model structure and the results of program LNADS compared to the experimental results. Chapter 8 presents an application of the test results to the prototype platform including an investigation of the size effect in shear, a parametric study using a two-dimensional model analyzed using program TRIX addressing the effect of in-situ strength of the concrete cover and a comparison between the test results and several relevant earlier experimental and theoretical investigations. Chapter 9 presents the conclusions of the research work and areas of potential research work. Appendix A presents the manual for program LNADS, a description of the input and output data files, and a hard copy of the program.
2.1 General

A three-dimensional, 1:13 scale model of a storage cell of a typical early Condeep platform was designed and constructed. The choice of the scale ratio was governed by the available space in the test facility and the available steel formwork. The reinforcement ratio in the actual structure and the model were kept constant. The concrete dimensions, the detailed drawings for the scale model and the construction sequence are presented. An elevated casting platform was built. Collapsible steel forms were used as the external and the internal forms of the cylindrical wall, while a wood form was used as the internal form for the upper dome. The steel cage was built out of steel wires and welded wire fabric. The mechanical properties of the steel reinforcing wires were checked. A specially-designed high strength concrete mix was used for the model. The model was cast in two days with a construction joint away from the test zone. It was cured for 28 days to ensure high quality concrete. An external prestressing system was designed. It was applied right before testing to minimize the effect of creep. A linear elastic analysis to predict the stress resultants at critical sections of the model was carried out under an effective load of 49.2m
Design and Construction of the Scale Model

of sea water using the compatibility approach based on the Classical Theory of Shells. A check on the critical sections suggested a flexural failure at the top of the cell wall at 103.5 m head of sea water and a shear failure in the cell wall at 101.0 m head of sea water.

2.2 Dimensions of the Model

The choice of model size was governed by the available space within the hydraulic test facility and the size of the existing steel formwork. This formwork can be used to produce a cylinder with an internal diameter of 56 inches (1422 mm). See Fig. 1.7. As the storage cells of the typical platform have an internal diameter of 18.54 m, see Fig. 1.1, this would result in a scale factor of about 1:13. The chosen concrete dimensions for the model are shown in Fig. 2.1. Table 2.1 represents a comparison between the main geometrical parameters in the prototype and the scale model. Note that in both the prototype and the model, the thickness of the dome increases near the edge. In those regions of the dome where the interior radius makes an angle of more than 30° to the vertical, the upper surface of the dome is conical and has the shape formed by projecting the tangent of the dome at the location where the radius is 30° to the vertical. Also, the centre line of the cell wall and the dome in the central region were assumed to intersect at Elevation 60.0 m.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Prototype (mm)</th>
<th>Scale Model (mm)</th>
<th>Scale Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell wall internal radius</td>
<td>10,000</td>
<td>767.2</td>
<td>1 : 13.03</td>
</tr>
<tr>
<td>Cell wall external radius</td>
<td>9,230</td>
<td>711.2</td>
<td>1 : 12.98</td>
</tr>
<tr>
<td>Cell wall thickness</td>
<td>730</td>
<td>56</td>
<td>1 : 13.04</td>
</tr>
<tr>
<td>Dome internal radius</td>
<td>14,000</td>
<td>1082</td>
<td>1 : 12.94</td>
</tr>
<tr>
<td>Dome external radius</td>
<td>14,520</td>
<td>1120</td>
<td>1 : 12.96</td>
</tr>
<tr>
<td>Dome thickness in central region</td>
<td>520</td>
<td>39</td>
<td>1 : 13.33</td>
</tr>
<tr>
<td>Dome height</td>
<td>3,510</td>
<td>267</td>
<td>1 : 13.15</td>
</tr>
<tr>
<td>Cell wall internal height</td>
<td>__</td>
<td>2134</td>
<td>__</td>
</tr>
</tbody>
</table>
Design and Construction of the Scale Model

<table>
<thead>
<tr>
<th>Description</th>
<th>Prototype</th>
<th>Model</th>
<th>Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal diameter of cell</td>
<td>18.54m</td>
<td>1.42m</td>
<td>13.05</td>
</tr>
<tr>
<td>Thickness of cell wall</td>
<td>730mm</td>
<td>56mm</td>
<td>13.04</td>
</tr>
<tr>
<td>Internal radius of dome</td>
<td>14.0m</td>
<td>1.08m</td>
<td>12.96</td>
</tr>
<tr>
<td>Dome height</td>
<td>3.51m</td>
<td>267mm</td>
<td>13.15</td>
</tr>
<tr>
<td>Dome thickness in central region</td>
<td>520mm</td>
<td>39mm</td>
<td>13.33</td>
</tr>
</tbody>
</table>

*Fig. 2.1 Concrete Dimensions of Model Storage Cell*
2.3 Reinforcement of the Model

The reinforcement used in constructing the model was gauge 12 galvanized steel wire. It was used in the form of welded wire fabric for reinforcing the cylindrical cell walls and as individual wires for reinforcing the dome and to provide the additional vertical cell wall reinforcement near the junction with the dome. See Fig. 2.2. The galvanized wire was used because it had stress-strain characteristics that were rather similar to those of the hot-rolled reinforcing bars used in the prototype structure. See Fig. 2.3. In the critical region near the outer face at elevation 60 m, 50% of the vertical reinforcement is individual wires, while 50% is part of welded wire fabric. As a result, the average yield stress of the reinforcement in the model is 417 MPa. It is believed that the average yield stress of the reinforcement in the actual structure is 427 MPa. The mechanical properties of the reinforcing steel wires are summarized in Table 2.2.

Table 2.2 Mechanical Properties of Reinforcing Steel Wires

a) Galvanized Welded wire fabric 51x51 MW 5.6 x MW 5.6

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel wire, Gauge 12, diameter</td>
<td>2.64 mm (0.104in)</td>
</tr>
<tr>
<td>Cross sectional area As</td>
<td>5.47 mm²</td>
</tr>
<tr>
<td>Steel wire spacing s</td>
<td>51.0 mm (2.0 in)</td>
</tr>
<tr>
<td>Yield strength fy</td>
<td>368 MPa</td>
</tr>
<tr>
<td>Ultimate Strength fu</td>
<td>465 MPa</td>
</tr>
<tr>
<td>Young's Modulus E</td>
<td>185,800 MPa</td>
</tr>
<tr>
<td>Percentage of Elongation εₘₐₓ</td>
<td>13.03 %</td>
</tr>
</tbody>
</table>

b) Galvanized steel wire gauge 12

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel wire, Gauge 12, φ</td>
<td>2.64mm (0.104in)</td>
</tr>
<tr>
<td>Cross Sectional Area As</td>
<td>5.47mm²</td>
</tr>
<tr>
<td>Yield Strength fy</td>
<td>466MPa</td>
</tr>
<tr>
<td>Ultimate Strength fu</td>
<td>512MPa</td>
</tr>
<tr>
<td>Young's Modulus E</td>
<td>180,500MPa</td>
</tr>
<tr>
<td>Percentage of Elongation εₘₐₓ</td>
<td>12.86%</td>
</tr>
</tbody>
</table>
(a) Reinforcement layering details near dome-wall connection.

*Fig. 2.2 Reinforcement Details of Model Storage Cell.*
Design and Construction of the Scale Model

27

(b) Reinforcement details near dome-wall connection

(c) Layout of basic reinforcement grid in dome. Extra wires near edge not shown

Fig. 2.2 Reinforcement Details of Model Storage Cell [Cont.]
**Galvanized steel wire**

Gauge 12  
Diameter = 2.64 mm  
As = 5.46 sq.mm

**Mechanical properties**

\( E_s = 180,500 \text{ MPa} \)  
\( E_{th} = 235 \text{ MPa} \)  
\( f_y = 466 \text{ MPa} \)  
\( f_c = 512 \text{ MPa} \)  
\( \varepsilon_0 = 12.86 \% \)

---

**Galvanized Welded wire fabric designation**

51 x 51 MM 5.6 x MW 5.6

**Mechanical properties**

\( E_s = 185,800 \text{ MPa} \)  
\( E_{th} = 385 \text{ MPa} \)  
\( f_y = 368 \text{ MPa} \)  
\( f_c = 465 \text{ MPa} \)  
\( \varepsilon_0 = 13.03 \% \)

---

**Fig. 2.3 Stress-Strain Characteristics of Model Reinforcement**
Almost all of the reinforcement in the prototype structure consisted of 20 mm diameter bars. In the cell walls the basic vertical bars were spaced at 210 mm centres. These basic 20 mm diameter bars were supplemented by additional 20 mm U bars, also spaced at 210 mm, in the critical region near the top of the wall. The result is that at elevation 60 m there is a 20 mm bar every 105 mm near the exterior face, giving an area of flexural tension reinforcement of 2990 mm²/m. At this location in the model, there is a gauge 12 wire with a diameter of 2.64 mm every 25.4 mm, giving an area of flexural tension reinforcement of 215 mm²/m. This amount of reinforcement in the model is about 7% less than the 230 mm²/m obtained by dividing 2990 mm²/m by the scaling factor of 13.

In the joint region of the prototype structure, there is one 16 mm diameter bar spaced at 150 mm giving an area of steel reinforcement of 1340 mm²/m. At this location in the model, there is a gauge 12 wire every 51 mm, giving an area of steel reinforcement of 107.5 mm²/m. This amount of reinforcement in the model is 4.3% higher than the 103.1 mm²/m obtained by dividing 1340 mm²/m by the scaling factor of 13.

In the ring beam, due to the construction requirements of the scale model, an additional gauge 12 wire closed stirrup every 51 mm was added. The meridian reinforcement in the upper dome was reduced as the reinforcement wires go towards the apex. Using individual wires allowed the elimination of every other wire at one half the diameter, with only 6 continuous wires near the apex. See Fig. 2.2(c).

2.4 Assembly of the Formwork and the Reinforcement

To gain access to the inside form, an elevated casting platform was assembled by placing a special base plate on the top of two sturdy beams. The base plate was a perfect copy of the pressure vessel closure plate with the same number and location of holes to ensure a trouble-free operation while fastening the model inside the pressure vessel.

The inside form for the cylindrical wall consisted of four steel quarter cylinders, bolted together at internal vertical flanges. The dimensions of the form were controlled by three sets of adjustable diagonal spacers placed at three different heights within the form. See Fig. 2.4(a). After the internal form was covered with a plastic film the welded wire fabric reinforcement was then assembled. See Fig. 2.4(b).
The internal formwork for the dome was made up of four quarter domes, each of which was made from wooden ribs covered with a plywood skin. See Fig. 2.5(a). The dome reinforcement was assembled on the dome form, see Fig. 2.5(b), and then the dome and the cylinder were joined together. See Fig. 2.6(a). Note that to improve the bond characteristics of the plain individual wires used in fabricating the dome reinforcement, each wire ended with a 180° hook. See Fig. 2.5(b).

The external formwork for the cylindrical wall consisted of 12 steel quarter cylinders bolted together at external steel flanges, with each set of 4 occupying about one-third of the specimen height. See Fig. 2.6(b) and Fig. 1.8. The dimensions of the external forms were maintained by 56 mm long, 6 mm diameter steel threaded spacer rods, which were attached to the reinforcing cage.

2.5 Strain Gauging of the Reinforcement

A total of 72 strain gauges were installed on the reinforcement of the model. Because the reinforcing wires had a diameter of only 2.6 mm very small strain gauges with a gauge length of 2 mm were used. The gauges were attached after the wires had been bent and placed in the reinforcing cage. Eighteen strain gauges were placed at four locations separated by 90° around the perimeter of the model. The locations and identification numbers used for the strain gauges are illustrated in Fig. 2.7. At each quarter point of the model the lead wires for the strain gauges were taken along the vertical and radial wires to a location one-third of the way up the dome towards the apex. Here the wires were collected into a 25 mm diameter conduit that was passed through the inner dome form.

2.6 Concrete Used in the Construction

The concrete mix used in the construction of the model was designed to avoid some of the early leakage problems that had been exhibited by the pilot test specimen. See Fig. 1.8. The objective was to achieve a dense, impervious concrete that was easy to place and had an appropriate strength. The mix proportions chosen are summarized in Table 2.3. The coarse aggregate used was crushed limestone with a maximum aggregate size of 4.5
Design and Construction of the Scale Model

Fig. 2.4 Construction of Cell Wall

(a) Internal Formwork of the cell wall

(b) Cell Wall Reinforcement
Design and Construction of the Scale Model

(a) The Internal Formwork of the Upper Dome

(b) The Upper Dome Reinforcement Assembly

Fig. 2.5 Construction of the Upper Dome
Design and Construction of the Scale Model

Fig. 2. 7 Strain Gage Location and Numbering Pattern Repeated at Four Locations NW, NE, SE and SW
mm, which was called manufactured sand. The sieve analyses for the aggregate are shown in Fig. 2.8. The sand-to-coarse aggregate ratio that produced the maximum dry rode density was 3:2, while the maximum dry rode density was 1681 kg/m³.

At an age of 28 days this concrete had an average cylinder strength of 62 MPa and a density of 2380 kg/m³. The concrete showed no segregation, was easy to place and resulted in a very watertight specimen. The model was loaded to failure 21 months after casting. By this time the average cylinder strength of the concrete had increased to 75 MPa. The stress-strain curve for the concrete at this time is shown in Fig. 2.9. The average tensile strength, as determined from a split cylinder test was 4.16 MPa.

The construction records of the prototype platform indicate that the average 28 day cube strength of the cell walls was 53 MPa. Based on experimental results for the effect of sea water submersion on concrete it was observed that similar concrete mixes gained about 8 MPa over a period of 8 years without any signs of slowing down or adverse effect. Extending the finding to the prototype over twenty three years and adjusting for the cement content in the prototype (480 Kg OPC per m³ of concrete) relative to the cement content in the experimental program (420 Kg OPC per m³ of concrete), it is expected that this strength will have increased by 26 MPa resulting in an average cube strength of 79 MPa. This would correspond to an average cylinder strength of about 68 MPa. From this it is concluded that the average concrete strength of the model is about 9.1% higher than the average concrete strength of the prototype.

Table 2.3 Concrete Mix Proportions

<table>
<thead>
<tr>
<th>Component</th>
<th>Dry Weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>400</td>
</tr>
<tr>
<td>Fly ash</td>
<td>80</td>
</tr>
<tr>
<td>Silica fume</td>
<td>30</td>
</tr>
<tr>
<td>Manufactured sand (coarse aggregate)</td>
<td>663(1)</td>
</tr>
<tr>
<td>Sand</td>
<td>994(1)</td>
</tr>
<tr>
<td>Water</td>
<td>193.8(2)</td>
</tr>
<tr>
<td>Superplasticizer</td>
<td>2.55</td>
</tr>
<tr>
<td>Voids</td>
<td>1% (by volume)</td>
</tr>
</tbody>
</table>

(1) The exact weight should account for the natural moisture content in the aggregate.
(2) The actual mixing water should account for the difference between the natural moisture content in the aggregate and the saturated-surface-dry condition.
Design and Construction of the Scale Model

Fig. 2.8 Sieve Analysis of Aggregates

Fig. 2.9 Concrete Stress-Strain Curve

**Concrete compressive strength**
- Standard cylinder
- \( \phi = 300 \text{mm} \)

**Mechanical properties**
- \( f'_c = 75.0 \text{ MPa (21 months)} \)
- \( f_{cr} = 4.17 \text{ MPa (Split cylinder test)} \)
- \( e'_c = -2.53 \text{ mm/m} \)
- \( \gamma = 2380 \text{ Kg/m}^3 \)

<table>
<thead>
<tr>
<th>Strain ( e_c ) [mm/mm]</th>
<th>0</th>
<th>-0.0005</th>
<th>-0.001</th>
<th>-0.0015</th>
<th>-0.002</th>
<th>-0.0025</th>
<th>-0.003</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive Strength ( f_c ) [MPa]</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>

\( f'_c = 75 \text{ MPa} \)
2.7 Casting the Model

The casting of the model was conducted in two stages, with the first cast involving the bottom flange of the specimen and about the bottom third of the cylinder wall. See Fig. 2.10. After curing the concrete for two weeks, the top surface of the first cast was roughened with an air-driven chisel, thoroughly water cleaned, and then given a bonding coat of a proprietary product called XYPEX. On January 17, 1995 the remaining portion of the model, including the critical upper wall-dome region was cast. See Fig. 2.11.

All of the concrete for the model was mixed in 100 litre batches in a 150 litre pan mixer. Allowing for the 5 test cylinders made from each batch, each mix would cast about a 250 mm high segment of the model and took about an hour to mix and place. The concrete was compacted with air-driven external form vibrators and 12 mm diameter needle nose vibrators. Considerable care was taken to duplicate the concrete batching, mixing, placing and compacting for each of the eleven batches used in casting the model.

The concrete batch used for casting the concrete dome did not contain superplasticizer. This resulted in a concrete with low slump that could be formed to the shape of the dome. A wooden profile board was rotated around the top edge of the steel forms to form the exterior surface of the dome. See Fig. 2.11(b). Two hours after the casting of the model, the surface was finished and then covered with wet burlap and plastic sheets.

Three days after casting, the external form was removed, the internal form was unbolted and the internal spacer bars were removed. The entire model was then wrapped in wet burlap and covered with plastic sheets. A small water pump ensured that the burlap remained wet for the next four weeks. The model was then lifted by the overhead crane and the inside forms were removed. Both the outside and inside faces of the model had a smooth marble finish that showed no evidence of honeycombing or segregation. The formwork seam lines were clearly visible, but only the construction joint at the bottom third of the cylinder caused any concerns about being possible leakage locations. To alleviate these concerns this area was patched using an epoxy material.
Fig. 2.11 Casting Second Lift of Concrete
2.8 Displacement Transducers

An aluminum frame was mounted inside the model specimen to hold nine waterproof displacement transducers. Three of these transducers measured the vertical displacements of the dome, while the other six measured the horizontal displacements of the cell wall. The locations and names of these nine displacement transducers are shown in Fig. 2.12.

![Diagram showing LVDT numbering and locations](image-url)
2.9 Post-Tensioning of the Model

To simulate the post-tensioned tendons in the region of the dome-wall connection the model was post-tensioned with one external 7 mm diameter high strength wire. This wire was located at the level of the intersection of the upper dome and cell wall (i.e., elevation 60.0 m in the prototype). The stress-strain characteristics of the prestressing wire are shown in Fig. 2.13. The mechanical properties of the prestressing wire, given in Table 2.4, were consistent with the standard mechanical properties of low relaxation wires.

![Stress-Strain Characteristics of Model Prestressing Wire](image)

**Fig. 2.13 Stress-Strain Characteristics of Model Prestressing Wire**

**Table 2.4 Mechanical Properties of Prestressing Wire**

<table>
<thead>
<tr>
<th>Mechanical and Physical Properties</th>
<th>Prestressing Wire</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of Prestressing wire #4</td>
<td>7.0 mm</td>
</tr>
<tr>
<td>Cross Sectional Area As</td>
<td>38.50 mm²</td>
</tr>
<tr>
<td>Yield Strength</td>
<td>1479 MPa</td>
</tr>
<tr>
<td>Ultimate Strength</td>
<td>1688 MPa</td>
</tr>
<tr>
<td>Young's Modulus</td>
<td>217,000 MPa</td>
</tr>
<tr>
<td>Minimum Elongation at Rupture</td>
<td>4.22%</td>
</tr>
</tbody>
</table>
The system used for post-tensioning the prestressing wire is described in Fig. 2.14. It consisted of 24 aluminum blocks, 51 mm wide and 100 mm long, mounted around the circumference of the cell wall. Two of these blocks, numbered 1 and 24, anchored the ends of the wire and were joined together by threaded rods. See Fig. 2.15. These rods were tightened to remove the slack from the wire. The other 22 blocks supported Teflon plates with the prestressing wire sitting in a groove in the Teflon plate. The wire was tensioned by using small jacks to push the wire away from the specimen. See Fig. 2.16. After the jacks had been extended at a particular location, a packing plate was placed behind the Teflon plate. This procedure was repeated for all 22 spacing blocks around the specimen. The prestressing force was applied as the last step prior to testing to minimize the losses due to creep. The prestressing force in the wire was monitored by 23 strain gauges, one between each pair of blocks from blocks 1 to 2 until blocks 23 to 24. After the jacking was completed the actual strain readings in the strain gauges prior to testing are shown in Fig. 2.17. The average hoop strain in the prestressing wire was 2.5 mm/m with a standard deviation of 0.265 mm/m and a coefficient of variation of 10.91%. The average stress in the wire was 543 MPa.

The diameter of the center line of the prestressing wire was \( D_{\text{wire}} = 1619 \text{mm} \). If the hoop strain in the prestressing wire was \( \varepsilon_{\text{hoop}} = 0.25\% \), then the wire resulted in radial confinement of 27.2 kN/m, which was equivalent to an additional 27 m of sea water acting at the top 100 mm of the cell wall.

The single wire tendon used for post-tensioning the model had a cross-sectional area of 38.5 mm\(^2\). This would correspond to a tendon in the prototype with a cross-sectional area of 6507 mm\(^2\). The tendons used in the prototype contained twelve 13 mm diameter 7-wire strands giving an area, for each tendon, of 1188 mm\(^2\). Thus, the model tendon corresponded to 5.5 prototype tendons. Different storage cells in the prototype platform contain different numbers of tendons in the dome-cell wall region. The cell modelled by "DnV" contained 5 tendons in the "ring beam" region and 11 more close by in the dome. See Fig. 1.2. Using the compatibility program LNADS, that was based on the Classical Theory of Shells, a set of linear elastic analyses were carried out on the actual structure with different prestressing cable arrangements. Three different cases were considered namely; 11 tendons in the dome, 5 tendons in the joint and 1 tendon in the
Design and Construction of the Scale Model

Presstressinpre$
\begin{align*}
\text{Prestressing wire} & \quad \sigma=7\text{mm} \\
D=1570\text{mm} \\
D=1244\text{mm} \\
D=1534\text{mm}
\end{align*}$

Supporting block

Upper dome

Aluminum plate
$3 \times 2 \times 3/16 \text{in}$

Prestressing wire

Teflon plate
$7 \times 2 \times 3/16 \text{in}$

Supporting block
$7 \times 2 \times 1.5 \text{in}$

Cell wall

a) Plan view.

b) Section AA

(c) Plan view of the model during prestressing

*Fig. 2.14 External Prestressing System*
Fig. 2. 15 Details of Anchorage System of Prestressing Wire

(a) Support block.  (b) Loading block  (c) Loading block  (d) Support block
No packing plates    (fully closed)   (fully opened)   with packing plate

Fig. 2. 16 Application of Prestressing Force

joint. The magnitude of the radial shear force at Elevation 60.0 for each case is given in Table 2. 5. Analysis indicated that, in terms of their ability to reduce the shear in the cell wall at elevation 60 m, the 11 tendons in the dome are equivalent to about 4.3 tendons in the ring beam. That is, to match the prototype tendons the model should have had the
equivalent of 9.3 prototype tendons rather than the 5.5 provided. In addition to having only about 60% \( \frac{5.5}{9.3} \times 100 \) of the "required" area for the tendons, the prestress in the model tendons was only about 54% of the assumed stress in the prototype tendons. Hence, the total prestress in the model was only about 32% of that required to simulate the prestress in the prototype.

From Fig. 1.14 (c) it can be seen that, for the prototype structure, it takes a pressure differential of about 15 m to overcome the beneficial effects of the prestress. This would imply that the reduced prestress in the model would reduce the capacity of the model by a pressure differential of about 10 m.

![Fig. 2.17 Hoop Strain in the Prestressing Wire Prior to Testing](image)

**Table 2.5 Shear Force at El. 60.0 Predicted by Linear Elastic Analysis by LNADS**

<table>
<thead>
<tr>
<th>Analysis Case</th>
<th>Radial Shear Force (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case I</strong>: 11 tendons in the dome</td>
<td>282</td>
</tr>
<tr>
<td><strong>Case II</strong>: 5 tendons in the joint</td>
<td>331</td>
</tr>
<tr>
<td><strong>Case III</strong>: 1 tendon in the joint</td>
<td>66.2</td>
</tr>
</tbody>
</table>
2.10 Linear Elastic Analysis of the Scale Model

To predict the behaviour of the model, a linear elastic analysis using program *LNADS*, was carried out. *LNADS* is a compatibility program for the analysis of reinforced concrete thin shells under axisymmetric loads based on the *Classical Theory of Shells*. The initial uncracked stiffness of the model was used. Program *LNADS* will be discussed in details in *Chapter 6* and *Chapter 7*.

The first load applied to the model was intended to simulate the prestressing of the ring beam in the form of a line load of 27.2 kN/m applied at the joint line. Under the current operating conditions the external pressure acting on the prototype storage cell is 94.2 m head, while the internal pressure is 45 m head of sea water. Additional water pressure load equivalent to 49.2 m head of sea water was then applied to all the model.

Using Program *RESPONSE*\textsuperscript{12,13}, two critical sections were checked in the cell wall namely at Elev. 60 m and at 225 mm below Elev. 60 m corresponding to the maximum negative and positive longitudinal bending moment. In addition, two critical sections were checked in the cell wall namely at the ring beam and at 225 mm below Elev. 60 m corresponding to the maximum negative and positive hoop bending moment in the cell wall. See Fig. 2. 18 (a). Similarly, two critical sections were checked in the upper dome namely; at the base of the dome where $\phi = 38.1^\circ$ from the apex and at $\phi = 24.3^\circ$ from the apex corresponding to the maximum negative and positive meridian bending moment in the upper dome. Also, two critical sections were checked in the hoop direction, namely at the maximum positive bending moment which occurred at $\phi = 26.8^\circ$ from the apex and at the maximum negative bending moment in the ring beam at $\phi = 41.1^\circ$ from the apex. See Fig. 2. 18 (b). The applied forces, namely the axial load and bending moment at the critical sections were checked versus the bending moment-axial force interaction diagram.

Table 2. 6 summarizes the stress resultants at the critical sections during the prestressing of the ring beam, under the application of 49.2 m head of water and under the maximum predicted failure water pressure. The analysis results suggested a flexural failure at El. 60.0 m at 103.5 m head of sea water.

A check on the shear strength at the top of the cell wall was carried out using the *Modified Compression Field* theory\textsuperscript{12,13} and taking into account the influence of axial load
### Table 2.6 Results from the Bending Moment-Axial Force Interaction Diagram

<table>
<thead>
<tr>
<th>Upper Dome</th>
<th>Angle from Apex deg.</th>
<th>Water Head m</th>
<th>Axial Load kN/m</th>
<th>Bending Moment kN.m/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meridian Direction</td>
<td>38.1°</td>
<td>Prestressing</td>
<td>-7.86</td>
<td>0.437</td>
</tr>
<tr>
<td></td>
<td>49.2</td>
<td>-218</td>
<td>-3.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>306</td>
<td>-1316</td>
<td>-22.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Prestressing</td>
<td>0.424</td>
<td>-0.1042</td>
<td></td>
</tr>
<tr>
<td></td>
<td>49.2</td>
<td>-274</td>
<td>0.690</td>
<td></td>
</tr>
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and bending moment on the shear capacity. The nominal shear strength of members without stirrups is expressed as

\[ V = \beta \sqrt{f_c b_w d_v} \]  

(2-1)

where \( b_w d_v \) is the effective shear area and \( \beta \) is a factor that indicates the ability of the cracked concrete to transmit shear. The factor \( \beta \) is a function of the applied bending moment at the section \( M \), the applied axial load at the section \( N \), the crack spacing parameter, \( s_\alpha \), the maximum longitudinal strain, \( \varepsilon_\alpha \), that occurs at within the shear depth,
Design and Construction of the Scale Model

Fig. 2.18 The Bending Moment-Axial Load Interaction Diagram
and the area of reinforcement on the flexural tension side $A_t$. The crack spacing parameter, $s_n$, was taken as equal to the distance between the layers of longitudinal reinforcement. Thus, for the cell wall of the model, $s_n$ would be 37.4 mm (1.47 inches). Since, the shear strength is highly sensitive to the highest longitudinal tensile strain within the shear depth, it would be conservative to take the strain at the flexural tensile steel reinforcement. Using a plane-section analysis with the strain at the flexural tensile reinforcement set to $\varepsilon_n$, the strain profile that is corresponding to the applied bending moment was found. The associated normal force $N_p$ has to balance the applied axial load $N$ and the additional tensile force $N_v$ carried by the top and bottom chords of the section due to the compression stresses associated with field of compression in the cracked concrete.

A diagonal shear failure spreads over a length of the member about $d, \cot \theta$ in extent, where $\theta$ is the inclination of the diagonal crack to the longitudinal axis of the member. The calculations were performed for a particular section representing a length of the member $d, \cot \theta$ long, with the calculated section being in the middle of this length. Thus, near the joint between the upper dome and the cell wall, the first section to be checked is the section $0.5 d, \cot \theta$ from the face of the joint. As a simplification, the length $0.5 d, \cot \theta$ was taken as $d_v$ when determining the location of this critical section. The shear force-to-axial compressive force interaction diagram, for the model cell wall, is shown in Fig. 2.19, while the calculations are summarized in Table 2.7. The relationship has been calculated for bending moment-to-axial load ratio $dM/dN \approx 44.7 \text{mm}$ according to the linear elastic analysis from $LNADS$. As the axial compression is applied, the maximum shear force carried by the cell wall section increases. For the axial compression-to-shear force ratio calculated by $LNADS$, the model wall is predicted to resist a shear force that is corresponding to a differential pressure of 101.0 m head of water.

Both predictions described above should be used cautiously since they were based on the linear elastic results. The results were sensitive to the ratios between the applied forces at the critical sections. Due to the non-linear behaviour of reinforced concrete members, the bending moment-to-axial load and bending moment-to-shear force ratios will change from the predicted linear results as the applied water pressure increases. Hence, the failure water pressure was expected to increase.
Predicted failure shear force by the Modified Compression Field Theory "expect at the flexural reinforcement":

Predicted failure of the model: 
= 101 m head of sea water

Applied loads at section $d_0=46\text{mm}$ away from El. 60.0 m "linear elastic analysis results by LNADS"

$p=49.2 \text{ m head of sea water}$

$fc' = 75 \text{ MPa}$

$f_{cr} = 4.17 \text{ MPa}$

**Fig. 2.19 The Shear Force - Axial Load Interaction Diagram**

**Table 2.7 Axial Force - Shear Force Interaction Diagram at $d_0$ from El. 60.0 m**

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<th>Water Head</th>
<th>$N$ [kN/m]</th>
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<th>$V$ [kN/m]</th>
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<th>$\phi$ [rad/km]</th>
<th>$N_1$ [kN/m]</th>
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CHAPTER 3

The Hydraulic Testing Facility

3.1 General

The Hydraulic Testing Facility, HTF, is a new facility at the University of Toronto dedicated to studying the behaviour of reinforced concrete structures subjected to external water pressure. The facility, shown in Fig. 3.1, was substantially upgraded bringing it into its full operational capacity. The most prominent component of the facility consists of a large steel pressure vessel certified to safely withstand an internal pressure of 275 psi gauge, that is, a pressure differential of 1.90 MPa, which is equivalent to a water head of about 190 m. The hydraulic pressure system of the facility consisted of a 25 horsepower (19 kW), variable frequency pump capable of delivering 40 gallons per minute (182 litres per minute) at a pressure of 275 psi (1.90 MPa). By adjusting the frequency of the pump through a custom made control unit, the pressure inside the pressure vessel could be controlled to within 2 kN/m². Flow meters were installed in the pipe by which water entered the pressure vessel and in the pipe by which water could drain from the inside of the test specimen. Two pressure transducers measured the applied water pressure at two different levels inside the vessel. The facility was equipped with internal and external video systems to monitor the model from inside and outside during testing.
Fig. 3.1 The Hydraulic Testing Facility
3.2 The Pressure Vessel

As shown in Fig. 3.2, the pressure vessel is a large cylindrical steel vessel 8 ft - 6 in. (2.59 m) in diameter, 14 ft - 2 in. (4.32 m) high and 7/8 in (22.1 mm) thick, with hemispheres at both ends. It is used to house the model during testing. It is certified to safely withstand an internal pressure of 275 psi gauge, that is, a pressure differential of 1.90 MPa, which is equivalent to a water head of about 190 m. It is located in a special pit on the south east corner of the Mark Huggins Structural Laboratory, at The Department of Civil Engineering, in University of Toronto. Its design followed the ASME code for pressure vessels.

3.3 The Hydraulic Pressure System

The hydraulic pressure system along with the hydraulic accessories is shown in Fig. 3.3. The most prominent component of the system is the 25 horsepower electric pump.

3.3.1 The Sizing of The Pump

The choice of the proper pump size depended on an estimate for the amount of water flow through cracked concrete and the pressure at which these cracks would occur. During the Pilot Test Program, at a pressure differential of 64 psi (440 kN/m²), water began to leak into the specimen at a rate of about 16 gallons per minute (73 litres per minute). The hydraulic pressure system used at that time, which was driven by a 1.5 horsepower (1120 Watt) pump, was not capable of increasing the pressure differential for this rate of water flow and hence, the test had to be stopped. Thus, the water flow rate near failure of the model, estimated at 40 gpm, and the maximum differential pressure that the pressure vessel could withstand of 275 psi were used as the design parameters. The pump horsepower is estimated as:

\[
Pump\ Horsepower = Factor\ of\ Safety \times \frac{Flow\ rate\ (imp.\ gpm) \times\ pressure\ (psi)}{Efficiency \times\ Conversion\ factor}
\]

\[
= 2 \times \frac{40 \times 275}{0.65 \times 1566} = 21.6\ Horsepower.
\]

A 25 horsepower pump was required to satisfy the design parameters.
Fig. 3.2 The Pressure Vessel
Fig. 3.3 The Hydraulic Pressure System and the Hydraulic Accessories
3.3.2 The Performance Curve

Figure 3.4 illustrates the performance curve for the 3SV series from GOULDS. By specifying the output pressure in meters of water \( \equiv 190 \) m, and the water output flow in US gpm \( \equiv 48.0 \) US gpm, the nearest curve above the identified point on the graph is selected to allow for restrictions normally found with standard fluid control units. The 3SVC 13-stage electric pump with standard 25 HP, 3-phase (575 Volts) electric motor and a maximum working pressure of 360 psi would satisfy the design requirements with a 65% efficiency. The 2 in discharge and suction openings dictated the use of the spare port G4 on the pressure vessel for water inlet.

![Performance Curve](image)

**Fig. 3.4 The Performance Curve**

3.3.3 Behaviour During Testing

Using the 25 horsepower electric pump may create a potential problem in controlling the rate of building up the pressure inside the pressure vessel. By adjusting the frequency of the pump using an electrical inverter, the pressure inside the pressure vessel could be controlled. Also, a directional valve was mounted downstream the pump to divert a
portion of the water back to the reservoir. The pump would keep on pumping water with a small amount of it delivered to the pressure vessel only at the beginning of the test. If the pump failed to maintain or increase the differential pressure under the actual water flow rate at failure, water may be prevented from leaking through concrete cracks by placing a non-permeable membrane around the model. Another way is to reduce the prestressing force on the model forcing it to fail at a lower differential pressure.

3.3.4 The Water Reservoirs

Since the water flow rate in the nearest water supply to the facility was 8 gpm and was less than the pump demand, a reservoir(s) was needed to store water during testing. If the test were to run for 30 min under a high flow rate of water \( q = 40 \text{ gpm} \), then the maximum volume of water that should be stored was 1200 gal. Two 500-gallon reservoirs were used while, the difference would be delivered by the city water over the same period of time (30 min). The upper reservoir, located on the laboratory floor, directly fed the main pump. Since its water level was higher than the main pump, there was no losses in the power of the pump due to suction of water. The lower reservoir, located in the pit, stored water that may be transferred to the main reservoir through the secondary pumps.

3.3.5 The Secondary Electric Pumps

Two 1.5HP electric turbine pumps located in the HTF pit were used to transfer the water from the lower reservoir to the upper reservoir upon demand. They could work simultaneously or alternatively. They were controlled through the control panel based on electrical signals from the ultrasonic level meter mounted on the upper reservoir.

3.3.6 High Pressure Hoses

Two 2 in diameter high pressure hoses, rated at 800 psi, were used to connect the main GOULDS pump to the pressure vessel and the main reservoir. Full-flow stainless steel fittings were used on both ends of the hoses allowing no obstruction to the water flow. Four 2 in stainless steel control valves were mounted at the hoses’ ends. All 4 valves should be fully opened before, and while, running the pump. If not, the pump would run
dry and could get severely damaged or an over-pressurizing and potential explosion of the hoses or the fittings may take place.

3.4 The Hydraulic Accessories

The hydraulic accessories, shown in Fig. 3.3, were used to control and measure the applied water pressure and water flow. They consisted of different types of hydraulic valves, transducers and gauges. Some were electrically driven, others were mechanically set and the rest were manually operated giving a wide range of safety factor against over-pressurizing, as it was highly unlikely that three different systems could fail simultaneously.

3.4.1 The Water Inlet Port

Filling the vessel directly with city water was a highly time consuming procedure. The pump could transfer the water stored in the reservoirs to the vessel in a substantially short time. Under these conditions, the pump could deliver more than 65 gpm through the 2 in diameter port (G4). The Venturi meter and the differential pressure transducer were sharing the same port. Port (J) is used as an air bleeding port at that time.

3.4.2 The Air Bleeding Valve

It was a ¼ in needle valve mounted on port (A) on the vessel apex. At the beginning of the test, it was important to keep it fully opened until all air was bled from the vessel and water starts coming out through it. Failure to do this would interfere with the rate of building up the pressure inside the vessel since air is highly compressible. However, it should be closed gently at the beginning of the test to prevent any pressure surge inside the vessel. During testing, it should be kept firmly closed. After testing, it was opened to allow air to replace the drained water.

3.4.3 The Floating Valve Switch

It was a ½ in x 2 in float valve mounted inside the upper reservoir 8 in from the bottom. If the water level inside the reservoir was higher than that level, the switch was normally
opened. If the water level inside the reservoir dropped down below that level, the switch was immediately closed preventing the main pump from running dry. The switch could not be overridden. The float switch was connected to the control panel with its status displayed to alert the user. If the water level started rising up again after the switch was tripped, the pump would not start unless the user resets the start button for the pump.

3.4.4 The Ultrasonic Level Meter

The ultrasonic level meter was mounted on the upper reservoir, detecting its water level up to 2.5 mm (0.1 in) accuracy. Then the control unit uses its signal to activate or deactivate the secondary transfer pumps in the pit. When the water level in the upper reservoir was receding below a certain limit, the meter signals the transfer pumps to start pumping up water from the lower reservoir to the upper one. When the water level reached its highest level in the upper reservoir, the ultrasonic meter would shut off the transfer pump. The meter was connected to the control panel to display its status and to allow the user to pick up which pump should do the transfer. The meter may be overridden by the user either to shut off or to run the transfer pumps.

3.4.5 The Relief Valves

Two relief valves were installed in the system. The first valve was installed on port (B) on the vessel head while the second valve was installed on the manifold downstream the electric pump. They prevented over-pressurising of the hydraulic system acting like pressure regulators. Both were mechanically set to the maximum working pressure of the vessel and could be manually overridden. When water pressure either inside the vessel or downstream the electric pump exceeded that pressure level, the valves started diverting part of the water back to the main reservoir keeping water pressure at that level.

3.4.6 The Release Valve

It permits the hydraulic circuit to de-pressurize safely. It was an air piloted, normally closed valve mounted on port (C). It was connected to the shop-air supply through an operating switch. If the switch was activated "manually", the air pressure would open the
The valve, allowing the water to safely drain to the lower reservoir until there was no pressure inside the system or until the switch was turned off. Since it was an air-actuated valve, it was ideal for the set-up since it used a different mechanism other than the relief valves. Accompanied with its two special switches, the valve could be used as a pressure control to simulate wave actions on offshore structures, fatigue test or even cyclic loading.

3.4.7 The Pressure Transducers

Two "Priabran" diaphragm-type pressure transducers, with maximum working pressure of 300 psi, were used to measure water pressure inside the pressure vessel. Pressure inside the model was kept at atmospheric pressure. Thus only water pressure inside the vessel was measured. Pressure readings represented actual differential pressure acting on specimen since the pressure transducer cables had their own breather opened to the atmosphere. Also, since these transducers were submerged in water they used DC power, thus they do not interfere with the LVDTs and strain gauges readings. Both transducers were connected to the control panel to display the pressure readings to the user in psi.

3.4.8 The Pressure Gauges

Two dial gauges were used to monitor the outlet water pressure of the pump. A 6 in diameter gauge was mounted on the manifold immediately downstream the main electric pump sharing it with the directional and check valves. The active pressure rang was from 10 psi to 400 psi. A small 2 in diameter gauge was mounted on port (B) sharing it with the vessel relief valve with an active pressure rang from 30 psi to 600 psi.

3.4.9 The Water Inflow Rate

The flow rate of water through the concrete cracks was of great importance to understand when water leakage started and how it increased. The main obstacle in measuring the water inflow rate was that the measurements should not interfere or restrict the water flow specially when high demand of water was needed near failure. A Venturi meter was mounted on the pipe by which the water entered the vessel. The pre-defined restriction in the Venturi meter would result in increasing the fluid velocity and reducing the fluid
The flow rate in the Venturi test in Facility 61 was measured by applying Bernoulli's equation assuming zero losses, the pressure differential was directly related to the water flow;

$$Q = K \sqrt{\Delta P},$$

where; $Q$ is the flow rate in imp. gpm, $\Delta P$ is the pressure differential between the initial and the restricted cross sections of the Venturi meter in "inches of water", and $K$ is a constant that depends on the properties of the Venturi meter and the fluid used where $K = 6.725$. The pressure differential between the two sections in the Venturi meter was measured using a "Semmens" pressure differential transducer with a maximum range of 100.55 inches of water giving a maximum water flow rate of 67.4 gpm. The transducer was connected to the Venturi meter through pressure rated 1/8 in Teflon hoses. Readings from the transducer were connected to an integrator to calculate the total amount of water pumped into the system at any time. Both readings for the water flow rate in gpm and the total amount of water in imperial gallons were displayed on the control panel, and were directly incorporated with the data acquisition system.

3.4.10 The Water Outflow Rate

A paddle wheel flow meter was installed in the pipe by which water could drain from the inside of the test specimen, i.e. the secondary drainage port (I4). As the water passed through the drainage pipe, it turned a paddle wheel. Each turn of the wheel would produce an electric pulse. As the outflow rate increased, the velocity of water increased and the frequency of the electric pulses increased. The pulse frequency was calibrated versus a known flow rate to establish a conversion factor. The electrical pulses were connected to an integrator to display the total amount of outflow water. Readings were displayed on the control panel to the user and incorporated with the data acquisition system.

3.4.11 The Directional Valve

To help building up the pressure inside the pressure vessel at a slow and controlled rate, a directional valve was mounted on the pump manifold. It is a standard 1 in ball valve with an electric actuator mounted on the top. When the valve is opened (or partially opened), it diverts a portion of the water from the downstream of the pump back to the main
reservoir. Thus, the valve should be fully closed when the specimen is near failure since there would be a high demand for water flow. On the other hand, it could be fully (or partially) opened during testing depending on the required rate of building up the pressure inside the vessel. The valve is not equipped with a manual override. The electric actuator was connected to the control panel. Through it, the user could set the actuator to any position (closed, partially opened, or fully opened) during testing.

3.4.12 The Check Valve

The hydraulic system was equipped with a 2 in brass check valve downstream the electric pump, rated at 300psi to allow the water to flow from the main pump to the pressure vessel, however, it would minimize water flow in the reverse direction. The valve should not interfere or restrict the water flow specially when high demand of water was needed near failure. The flap type can satisfy this condition with its free gate inside the valve body that opens in the direction of flow with no required pressure and closes under the hydrostatic pressure difference of the reversed flow.

3.4.13 The Water Outlets for the Pressure Vessel

The pressure vessel had two water outlets. The main outlet, for water surrounding the specimen, was port (F) on the lowest point of the vessel. The 2 in stainless steel pressure rated ball valve should be kept fully closed during filling and testing. The secondary outlet, for water inside the specimen, was sharing port (I4) with the outflow peddle wheel meter.

3.5 The Control Panel

The control panel was a movable custom-made unit used to control and display the feedback of all the test set-up components. It consisted of several units as follows;

1. The Hitachi Electric Invertor was the main component of the control unit that allows the user to adjust the frequency of the pump from 0 Hz to 120Hz with an accuracy of 0.1Hz using an accurate potentiometer. The target and actual frequencies of the pump were displayed to the user. Also, it was displayed as a percentage of the maximum frequency, 120 Hz. The invertor had different built-in options to allow the
user to change the maximum frequency (but not beyond 120Hz), the frequency-to-voltage conversion, the acceleration/deceleration time, ..etc. The inverter was connected to a special 575Volts power supply and the pump electric motor through shielded heavy duty cables. To prevent any arcing in the electrical motor due to the frequency control method, two line reactors were connected before and after the electric motor. These line reactors helped minimizing any undesired harmonics and spikes in the input power and reduced the motor arcing.

2. **The Starting unit** was used to start/stop/reset the control unit. It was connected to the float switch and two remote “kill” switches that were mounted in the pit and on the platform. The unit and its accessories allowed the user to precede with or stop the test at any time or reset the test when it was abruptly stopped. The unit would stop the test automatically if the water level in the upper reservoir was below the float switch. The remote “kill” switches added an extra safety feature to the setup.

3. **The Pressure Transducers unit** was connected to the underwater pressure transducers to display the applied water pressure inside the vessel in psi.

4. **The Flow Meters unit** was connected to both of the differential pressure transducer and the peddle wheel flow meter. The unit was used to display the inflow and outflow rate in imp. gpm, and the total inflow and outflow water in imperial gallons.

5. **The Transfer Pump unit** was connected to the ultrasonic level meter and the secondary transfer pumps and was used to display the status of the water level in the upper reservoir (high or low), and the status of the transfer pumps. It allowed the user to control/override the transfer pumps and to choose which pump would work.

3.6 **The Video Monitoring System**

It consisted of an internal unit mounted inside the specimen and an external unit mounted on the vessel top viewport. The former system was accompanied with water-tight light units, while the latter was accompanied with pressure-proof underwater light units.
3.6.1 The Internal Video Monitoring System

Mounting the system inside the model would provide a better understanding of behaviour of the model and would satisfy the minimum focal distance usually found in most cameras.

1. **The Underwater Video Camera** A HITACHI colour camera CCTV VKC150 was used accompanied with COSMICAR C31210-6 CCTV lens. The lens was equipped with a remote control for focus and zoom. Due to the small angle of vision $\leq 20^\circ$, the camera was accompanied with a pan and tilt mechanism providing a good control on the line of sight by rotating the camera around two perpendicular axes with a maximum angle of $\pm 35^\circ$ in each direction. A control circuitry was provided for the whole system to control the 12 volts DC power, the pan and tilt mechanism, and the focus and zoom drive. An underwater enclosure was essential to protect the camera assembly from water pressure. It consisted of a cylindrical aluminum housing 200 mm in diameter $\times$ 200 mm long capped with a hemispherical optical dome.

2. **The Water-tight Light System** consists of eight 5½ in-diameter halogen sealed beam headlights with light diffusers. Four units were mounted on each side of the camera just behind the optical dome. A 12 Volts DC power was supplied through a separate cables for each light.

3.6.2 The External Video Monitoring System

The vessel top viewport provided an excellent view of the specimen. The system consisted of an ordinary camcorder mounted on the top viewport of the vessel and four pressure rated underwater light units. The light could be manually-adjusted to eliminate any hot spots or shadows using a special light-control unit. Each unit used 12 Volts DC and provided 100 Watts which was adequate for any video camera with high shutter speed.

3.7 Conclusion

All parts of the test set-up were addressed. The set-up had brought the HTF to its full operational capacity with scope for improvements. It would serve any test that may use the HTF easily and safely.
CHAPTER 4

Testing of the Model Structure

4.1 General

The model structure was tested in the *Hydraulic Testing Facility*. Two pressure tests were carried out on the model. During both tests, the pressure was increased gradually to a predetermined level and then it was held constant while the interior of the model was scanned for leaks, the exterior of the model was examined, and the displacement and strain data were monitored. During the first pressure test, the differential water pressure reached as high as 125 psi (85.6 m of water head). In the second pressure test, as the water pressure reached 200 psi (137 m of water head), radial cracks were visible at the base of the dome. At an applied pressure of 215 psi (147 m of water head) the amount of water flowing into the pressure vessel increased greatly and the applied pressure dropped to 186 psi (127 m of water head). As the pressure reached 205 psi (140 m of water head) a loud "thump" was heard from the model followed by a gentle rocking of the whole pressure vessel. The northwest quadrant of the cell wall below the dome showed clear signs of a shear failure occupying about 165°, while the upper surface of the dome in the same quadrant showed evidence of concrete crushing. The concrete in compression on the interior surface of the model at the joint between the cell wall and the upper dome
showed clear signs of crushing. Several hoop wires were ruptured while several longitudinal wires buckled at that location. When the model structure was cut along an east-west diameter, it was evident that the west side of the cell wall failed in shear, while the east side of the wall did not experience any cracks at that location.

### 4.2 Preparation of the Model for Testing

The strain gauge cables were extended to 15 m each and were protected against water. The model was checked for water tightness by filling the interior space of the model with water while allowing the air to escape through a stand pipe and subjecting the model to a small hydrostatic pressure. A few small leaks were detected in the lower third of the model. A minor leak was also detected at the construction joint. All these leaks were patched using an epoxy.

The displacement transducer support tree, see Fig. 4.1, was mounted inside the specimen and the displacement transducers were positioned. The remotely controlled video camera in its waterproof housing and the video lights were also installed inside the specimen. See Fig. 4.2. The model was then lifted into the pressure tank. See Fig. 4.3.

As the model was lowered into the pressure tank, the strain gauge leads, the displacement transducer leads and the cables for the video camera and lights were all fed into pressure sealed conduits, which passed through the steel closure plate at the bottom of the specimen and through the bottom of the pressure vessel. See Fig. 4.4. The specimen was then bolted down to the steel closure plate. At this stage, the external post-tensioning system was put in place and the prestressing wire stressed as described in Section 2.9. After the prestressing was completed the lid of the pressure vessel was bolted into place and the vessel filled with water from the city supply.

The control panel for the custom designed hydraulic system used for controlling the water pressure inside the pressure vessel is shown in Fig. 4.5. The panel displayed the water pressure in pounds per square inch (psi), the flow into the pressure vessel in imperial gallons per minute (gpm) and the pump frequency in Hertz (Hz). A separate data acquisition and monitoring system, see Fig. 4.6, was used to record the output of the sensors and to monitor these values during the test.
Fig. 4.1 The Displacement Transducers Support Tree Mounted inside the Model Structure.

Fig. 4.2 Interior Video Camera Mounted inside the Model Structure.
Fig. 4.3 Completed Scale Model of the Typical Storage Cell
Fig. 4.4 Schematic Diagram of Model Mounted Inside Pressure Vessel.
Fig. 4. 5 Hydraulic Control System with Displays Giving Pressures and Flows

Fig. 4. 6 Data Acquisition and Monitoring System
4.3 First Test of the Model - October 22, 1996

The time-pressure relationship for the first test of the model is shown in Fig. 4.7. The pressure differential applied to the dome and cell wall of the model is given both in terms of pounds per square inch (psi) and in terms of an equivalent head of sea water. It is assumed that a 100 m head of sea water applies a pressure of 1007 kN/m² (146 psi). The intended loading was to increase the water pressure in 10 psi increments, stopping after each increment to scan the interior of the model for evidence of leaks and to monitor the displacement and strain gauge readings.

At relatively low pressure, 25 psi, a substantial quantity of water, 4 gpm, was leaking out of the pressure vessel, see Fig. 4.8, even though the video camera showed only a few drops of water leaking through the concrete specimen. A large portion of the water flow was leaking around the five “sealed” openings in the base of the pressure vessel. See Fig. 4.4. The remaining flow was coming out of the pipe that drained the inside of the model. It is believed that almost all of this flow was coming into the model by flowing around the five “sealed” openings in the steel base plate of the specimen.

As the water pressure was being increased to 40 psi, a pin-hole water jet was detected on the north-east side of the cell wall about 50 mm below the joint line with the dome. At 50 psi a new leak was detected at the joint line with a small water jet. From 50 psi to 130 psi there was no evidence of any major new leakage through the walls of the concrete specimen. The plot of applied pressure versus resulting vertical deformation of the dome, Fig. 4.9, began to deviate significantly from a linear response at a pressure of about 95 psi (65 m of water head). When the pressure reached 125 psi the water draining out from the inside of the model, most of which was coming through the openings in the steel base plate, suddenly stopped. See Fig. 4.8. The water level inside the model began to rise quickly, submerging the video camera. A decision was made to release the pressure and investigate the cause of the problem. As the pressure was reduced, the drainage port came back into operation and in a few minutes the water inside the model drained out. It was concluded that the drainage hose had elastically buckled under the high pressure, sealing off the inside of the model.
After the completion of the first test, the water was drained from the pressure vessel and the lid of the vessel was removed. A close examination of the exterior of the model structure failed to find any cracks. Presumably, when the water pressure was removed, the circumferential cracks in the cell wall near the dome junction almost completely closed, rendering them invisible. See Fig. 4.10.

To correct the drainage problems experienced in the first test, the drainage hose connecting the interior of the model structure with the exterior of the pressure vessel was replaced. The new hose had a diameter of 25 mm and was reinforced with an interior steel coil so that it could withstand an external water pressure of 800 psi prior to collapse. An attempt was also made to tighten the water seals around all of the conduits. In addition, adjustments were made to reduce the electronic noise that had been exhibited by the middle vertical displacement transducer. See Fig. 4.9.

4.4 Second Test of the Model - October 24, 1996

The second test followed the same loading procedure as that used in the first test, namely the pressure was increased to a predetermined level and then it was held constant while the interior of the model was scanned for new leaks using the video camera, the exterior of the model was examined though the viewports of the pressure vessel, and the displacement and strain data were monitored. The manner in which the applied pressure varied with time during the second test is shown in Fig. 4.11.

During the second test, there was still substantial leakage around the sealed openings in the walls of the steel pressure vessel and around the sealed openings in the steel base plate of the specimen. However, at a given pressure level, the flow of water was only about half of what it had been in the first test. Compare Fig. 4.12 and Fig. 4.8. As the pressure reached about 90 psi an overflow of water from the main reservoir feeding the pump required the pressure to be reduced to close to zero. After the problem was corrected, the pressure was again increased. See Fig. 4.11.
Testing of the Model Structure

Fig. 4.7 Loading History for First Test

Fig. 4.8 Measured Water Flow into the Pressure Vessel during the First Test
Testing of the Model Structure

Fig. 4. 9 Vertical Displacement of the Dome during the First Test

Fig. 4. 10 Model after the First Test. In this unloaded state No cracks were visible
As the water pressure was increased it was noted that the rate of water flow into the pressure vessel increased linearly with increasing pressure until, at a pressure of 190 psi (130 m of water head), the flow reached 10 gallons per minute (45.4 litres per minute). See Fig. 4.12. For pressures above the previously reached maximum of 130 psi, the relationship between the applied pressure and the vertical displacement of the dome deviated more and more from a straight line, indicating that significant cracking was occurring. See Fig. 4.13. Even at a pressure of 190 psi (130 m of water head) relatively little water was leaking through the concrete model. A number of small water jets could be seen coming from the cell wall. See Fig. 4.15. The largest of these leaks occurred on the south side of the model at a location about 50 mm below the junction of the dome and the cell wall.

At an applied water pressure of 200 psi (137 m of water head) radial cracks on the upper surface of the dome were observed through the viewport of the pressure vessel. Two cracks on the northwest side of the model, extending from the edge of the dome for about 150 mm towards the apex, could be seen. As had happened during previous load stages, the pump frequency was kept constant while the specimen was being examined. During the 7 minutes that elapsed there was no significant change in pressure, indicating that the radial cracks had not yet greatly influenced the leakage rate of the specimen.

It was intended to take the next load stage at an applied pressure of 215 psi (147 m of water head). As this pressure was reached and the frequency of the pump was held constant, the amount of water flowing into the pressure vessel began to increase greatly and the applied pressure dropped. Within about 5 minutes the flow rate increased from 15 gallons per minute (68 litres per minute) to 25.6 gallons per minute (116 litres per minute) and the pressure dropped from 215 psi (147 m of water) to 186 psi (127 m of water). See Figs. 4.11 and 4.12. As the pressure dropped the vertical deflections of the dome stayed nearly constant, at about 2.5 mm, but the outward deflection near the base of the dome increased from about 0.54 mm to about 0.69 mm. See Figs. 4.13 and 4.14. As an outward displacement of the cell wall of 0.69 mm corresponds to an average tensile strain in the hoop direction of 0.97 mm/m, it was not surprising that many radial cracks near the
edge of the dome could be seen at this load stage. Because the large quantity of water leaking through the model had almost emptied the reservoir it was decided to reduce the pressure from the 186 psi value by decreasing the frequency of the pump. The decrease in frequency reduced the pressure to 25 psi and the water flow to about 6 gallons per minute. After about one hour at a low pressure the 500 gallon reservoir had been refilled and the test could resume.

During the last loading of the model the pressure was increased relatively quickly in order to reduce the total quantity of water flowing into the pressure vessel. See Fig. 4.11. As the pressure was increased beyond the previous 186 psi (127 m of water head) the rate at which water was leaking through the model dramatically increased. See Fig. 4.12. At a pressure of 190 psi (130 m of water) water was flowing into the pressure vessel at 30.7 gallons per minute (139 litres per minute), which, based on the assumption that leaks through the pressure seals would account for about 10 gallons per minute at this pressure, would imply that about 20.7 gallons per minute (94 litres per minute) was leaking through cracks in the concrete model. See Fig. 4.16. As the pressure reached 205 psi (140 m of water head) a loud "thump" was heard from the model followed by a gentle rocking of the whole pressure vessel. In the next few seconds the water pressure dropped dramatically as water poured into the model structure. See Fig. 4.17. Through the viewports, air could be seen bubbling out of the cell wall of the model. See Fig. 4.18. The final values for the downward displacements of the dome, for essentially zero pressure, were 12.8 mm, 14.0 mm and 8.0 mm for the apex, north and south transducers, while the maximum outward displacement near the base of the dome was 1.63 mm for the transducer on the inside of the cell wall. Because of the choice of scale, these final values are not shown in Fig. 4.13 and Fig. 4.14.
Max applied pressure $p = 215\text{psi}(147\text{m})$

Flow increased $p = 185\text{psi}(127\text{m})$

Visible cracks $p = 200\text{psi}(137\text{m})$

Violent failure $p = 205\text{psi}(140\text{m})$

Water overflow

Storing Water

Fig. 4.11 Loading History for Second Test

Fig. 4.12 Measured Water Flow into the Pressure Vessel during the Second Test
Testing of the Model Structure

"Fig. 4.13 Vertical Displacement of the Dome during the Second Test"

"Fig. 4.14 Outward Horizontal Displacement of the Joint"
Fig. 4. 15 Leakage of Water Through the Model at 130m of Water Head as Seen by the Interior Video Camera

Fig. 4. 16 Water Leaking into Model just Prior to Shear Failure of the Wall as Seen by the Interior Video Camera
Fig. 4.17 Crushing of the Dome at Failure as Seen by Video Camera Mounted at Viewport

Fig. 4.18 Air Bubbles Coming out at Model after Shear Failure of the Wall as Seen by Video Camera Mounted at Viewport
4.5 Crack Patterns and Failure Mode

After the failure of the model the water was drained out and the lid of the pressure vessel was removed. The northwest quadrant of the cell wall below the dome showed clear signs of a shear failure (there was a shear displacement of about 10 mm across a circumferential crack), while the upper surface of the dome in the same quadrant showed evidence of concrete crushing. See Fig. 4.19. When the model was lifted out of the pressure vessel it was possible to push a 2.6 mm diameter wire through the cell wall of the model at the location of the shear crack. As can be seen in Fig. 4.20, this wire was inclined at about 32° to the vertical.

To make it possible to record the crack patterns more clearly the cracks were marked. The residual widths of the cracks were measured to a precision of 0.05 mm with a crack comparator and the residual crack widths labelled. As reference points, the locations of the 24 post-tensioning support blocks were labelled. In addition, compass directions were marked on the specimen. See Figs. 4.21, 4.22, 4.23, and 4.24. The shear crack near the top of the cell wall was found to extend from about block location 10 to block location 21, thus occupying about 165° of the 360° circle. The failure was accompanied with the crushing of the concrete in compression on the interior surface at the joint line in the top of the cell wall. The crushing of the concrete on the inside surface at the joint line extended from about blocks location 22 and 23 to blocks location 9 and 10, thus occupying about 195° of the 360° circle.

There was a circumferential crack in the cell wall at Elev. 60 m, extending completely around the specimen. This crack varied in width from about 0.4 mm to 1.0 mm. In the sector where the shear failure had occurred, a regular pattern of radial cracks in the dome and vertical cracks in the cell wall was evident. See Figs. 4.21 and 4.22. The spacing of these cracks matched the spacing of the underlying reinforcement (i.e., 51 mm) and the width varied from about 0.1 mm to 2.0 mm. Several hoop reinforcement steel wires were ruptured at the failure zone while the upper dome was clearly pushed outside and down the cell wall. Measurements of the damaged model indicated that at locations of shear failure the cell wall shortened about 25 mm more than at locations that did not fail in shear. Several longitudinal wires crossing the shear crack buckled at that location.
A wide hoop crack was formed in the middle of the dome due to concrete crushing on the exterior surface of the dome. See Fig. 4.21 and Fig. 4.22. It was found to extend from block location 13 to block location 19 thus covering 90° of the 365° circle, remarkably where the additional meridian reinforcement at the joint ended and the constant thickness portion of the dome started. It was in the same failure zone where the shear crack was formed in the cell wall. The concrete crushing was due to the fact that this portion of the dome is unstable under the applied loads due to the shear crack at the one end and two radial cracks on the side. The small two-dimensional strip would rotate in the counter clockwise direction resulting in the formation of that crack.

It is believed that the resulting distortion of the dome is responsible for the pattern of cracks seen in Figs. 4.21 and 4.22. Comparing both patterns of cracks on the exterior and the interior surfaces of the upper dome revealed that the cracks on the exterior surface were perpendicular to those on the interior surface. On the interior surface, the cracks were pointing towards where the shear crack was initiated i.e., North West, while, on the exterior surface, the cracks were going form the North East towards the South West in a hoop-like cracks.

In order to obtain photographs of the internal crack pattern, the model structure was cut along an east-west diameter. See Figs. 4.25, 4.26, 4.27 and 4.28. In these photographs, the difference in crack patterns between the west side, where the wall failed in shear, and the east side, where the wall did not fail, is evident. The shear crack was formed in an uncracked zone at the top of the cell wall. It was inclined by 32° to the vertical pointed towards the joint line between the upper dome and the cell wall and started about 75 mm (3 inches) below the joint line. The circumferential crack on the unfailed portion of the cell wall went as deep as 8 mm from the interior surface of the cell wall while the circumferential crack on the base of the upper dome went as deep as 12 mm from the interior surface of the dome.

It is of interest to note that the failure crack pattern of the 1:13 storage cell model was similar to the failure crack pattern of the 1:2.14 dome-wall slice model\textsuperscript{10}. Compare Fig. 4.28 with Fig. 1.12.
Fig. 4. 19 Appearance of Model after Completion of Test. Note radial cracks and crushing of dome.

Fig. 4. 20 Model Removed from the Pressure Vessel. Wires (2.64mm diameter) being inserted in shear crack. Angle formed is about 32° from vertical.
Fig. 4.21 Appearance of Exterior of Model after Cracks Marked and Residual Widths Measured

Fig. 4.22 Appearance of Interior of Model after Cracks Marked and Residual Widths Measured
Testing of the Model Structure

Fig. 4. 23 Cracks on External Surface Near Top of Cell Wall: North East Quadrant

Fig. 4. 24 Cracks on External Surface Near Top of Cell Wall: South West Quadrant
Fig. 4.25 Scale Model of Storage Cell after Cutting with Diamond Saw.
Fig. 4. 26 Scale Model after Cutting: West Side

Fig. 4. 27 Scale Model after Cutting: East Side
Fig. 4.28 West Side of Model after Cutting
CHAPTER 5

Analysis of the Test Results

5.1 General

Predicting the response of such model is by no means a straight forward task. The ability to use the *Finite Element* method has to be assessed in predicting the behaviour of the model. The theoretical behaviour of the model was investigated using a nonlinear analysis based on the *Finite Element* method and the *Modified Compression Field* theory using program *RASP*. A comparison between the analytical results and experimental behaviour of the model was carried out namely, the measured and predicted vertical and horizontal deflections of the structure, the predicted pattern of strains and the measured and predicted reinforcement strains.

5.2 *RASP* Finite Element Model of Tested Structure

In order to have a prediction of the behaviour of the model, a non-linear finite element analysis, using program *RASP*, was conducted just prior to testing the model. The results of this analysis were useful during the loading of the test structure, but after the test it was found that some errors had been made in inputting reinforcement areas into the analytical model. These errors were corrected and a new analysis was conducted. In this chapter
the results of this corrected analysis will be discussed and compared with the experimental observations.

*RASP* is a non-linear finite element program for the analysis of reinforced concrete shell structures. The program uses degenerate isoparametric shell elements and can account for both geometric and material non-linearities. It uses three-dimensional stress-strain characteristics of cracked concrete based on the modified compression field theory\(^\text{14}\). As the 8-layered elements employed allow for “out-of-plane” shear deformations, it can predict “through wall” shear failures. An important feature of the program is that it takes into account the influence of shear force and hence, is capable of predicting shear failures\(^\text{15}\) of the type which may occur at the top of the cell wall. The program was developed by Vecchio, Seracino\(^\text{16}\) and Polak\(^\text{17}\), all of the *Department of Civil Engineering, University of Toronto*. Full details of program *RASP* are given in Ref. [16] and [17].

The geometry of the finite element model is described in Fig. 5.1 and Fig. 5.2, while the material properties used are summarized in Fig. 5.3. A total of 168 shell elements were used to model one-quarter of the model structure, down to 437 mm from the base. Note that the *RASP* model had a 94 mm radius enoculus near the apex. In program *RASP* the amounts of reinforcement are described in terms of different element types. A total of six different element types were used in the model, see Fig. 5.2. The quantities of reinforcement in each of these element types are given in Table 5.1.

The first load applied to the finite element model was intended to simulate the prestressing of the ring beam. The load consisted of a water pressure of 0.57 MPa applied to the vertical outside face of the ring beam elements number 97 through 102. See Fig. 5.2. Additional water pressure loads were then applied to all exterior surfaces of the model. This water pressure load was increased until the maximum load capacity of the finite element model was reached. Convergence could be achieved for water pressures from 0.05 MPa up to 1.35 MPa with 0.05MPa increment but could not be achieved for 1.40MPa. This water pressure of 1.35 MPa, corresponds to a differential head of 134 m of sea water.
Analysis of the Test Results

Fig. 5.1 *RASP Finite Element Model Used to Predict Response of 1:13 Scale Model*
Analysis of the Test Results

Fig. 5.2 Details of the Geometry of the RASP Model
Analysis of the Test Results

(a) Base Curve Compressive Stress-Strain for Concrete

(b) Stress-Strain Curve for Reinforcement

Fig. 5.3 Stress-Strain Properties of Concrete and Reinforcement Used in RASP Model
Analysis of the Test Results

Table 5.1 Reinforcement Amounts in Different Element Types of RASP Model

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Average Thickness (mm)</th>
<th>Longitudinal</th>
<th>Circumferential</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Outside (mm$^2$/m)</td>
<td>Inside (mm$^2$/m)</td>
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</tr>
<tr>
<td>6</td>
<td>39</td>
<td>107.5</td>
<td>107.5</td>
</tr>
</tbody>
</table>

5.3 Measured and Predicted Deflections of the Structure

The predicted patterns of vertical deflections as the applied water pressure is increased are illustrated in Fig. 5.4. Program RASP outputs the deflections of the centreline of the model. To simplify the presentation of the results, locations in the structure are defined in terms of the distance along this centreline from the apex of the structure. In this coordinate system the junction of the dome and the cell wall occurs at 806 mm from the apex. From Fig. 5.4 it can be seen that, due to the axial compression in the cell wall, this junction deflects downwards by about 0.25 mm at an applied water pressure of 1.35 MPa. The downward deflections of the dome are predicted to approximately double as the pressure is increased from 1.30 MPa to 1.35 MPa. It can be seen that this substantial increase in deflection is associated with significant kinking of the structure at the location of the dome-wall junction.

The predicted patterns of horizontal deflections as the applied water pressure is increased are illustrated in Fig. 5.5. It can be seen that, due to the circumferential compression, the circumference, and hence, the radius of the lower cell wall becomes smaller. At a water pressure of 1.35 MPa, the wall is predicted to move inwards by about 0.35 mm. Dividing this movement by the original radius of the middle line of the cell (738 mm) we see that this deflection corresponds to a circumferential compressive strain of 0.47 mm/m. For lower pressures, a zone of the structure about 270 mm long, centred on the junction between the dome and the wall, moves outwards rather than inwards. As the applied pressure reached 1.35 MPa, this zone increases in extent to about 400 mm long.
and the outward deflections increase greatly. Thus, the outward deflection of the junction line increases from 0.32 mm to 1.15 mm as the pressure increases from 1.30 MPa to 1.35 MPa. An outward deflection of 1.15 mm corresponds to a circumferential tensile strain of 1.56 mm/m.

During the loading of the 1:13 scale model structure a total of 9 displacement transducers recorded the deflections of the model, see Fig. 2.12. In Fig. 5.6 the pattern of deflections measured by these transducers is compared with the pattern of deflections predicted by the RASP finite element analysis for an applied water pressure of 1.30 MPa. It can be seen that while the measured and predicted cell wall deflections are in good agreement, the measured deflections of the dome are somewhat greater than the predicted deflections. Note that in all three locations where the vertical deflection of the dome was measured, the dome is predicted to come down by essentially the same amount. As can be seen from Fig. 4.13, the three measured vertical deflections were also essentially equal.

Figure 5.7 is a more complete comparison of the measured and predicted vertical displacements of the dome. Note that there was a 94 mm radius enoculus in the RASP model near the apex. Thus the theoretical prediction for the deflection at the apex was extrapolated from the last elements results (i.e. elem. # 173 through elem. # 176). It can be seen that the measured deflections are somewhat greater than the predicted deflections. Part of this increased deflection was due to the unloading and reloading of the model. It will be recalled that during the test the structure was unloaded twice. The first unloading, labelled 1 in Fig. 5.7, occurred at the end of the first day of testing when the drainage hose buckled. The second major unloading, labelled 2 in Fig. 5.7, occurred near the end of the second day of testing as the pressure was reduced so that the water reservoir of the pump could be refilled, just prior to final failure of the structure. The analytical model assumes the pressure is increased monotonically until failure is reached. Note that the experimental results shown for the first pressure test carried out on the model structure started at the origin, while the deformation recorded at the application of the prestressing force were omitted. These recorded data were so small to be recorded accurately using the ±25 mm LVDTs used in most locations.
To facilitate the comparisons between the predicted and measured values, in the remaining plots of this chapter only the “envelope” of experimental values will be given. That is, data associated with the two unloading and reloading “loops” shown in Fig. 5.7 will be omitted. The transitions from one loading line to the next will be shown by dotted lines.

The measured and predicted horizontal spreading at the base of the dome are compared in Fig. 5.8. It can be seen that for most of the test, the measured horizontal deflections near the dome-wall junction were significantly greater than the predicted values. Only as the failure pressure was reached did the two sets of values converge. The initial slopes of the two lines in Fig. 5.8 are in reasonable agreement but at a pressure of about 0.30 MPa the two lines begin to diverge significantly. The influence of water pressure in radial cracks at the base of the dome and at the top of the cell wall, which was neglected in the analytical model, may be responsible for part of this discrepancy. As the model structure cracked near the junction water pressure inside the cracks preys the cracks open, resulting in increasing the outwards spreading at the base of the dome.

It can be seen from Fig. 5.6 that the predicted inward deflection of the cell wall is essentially constant for distances greater than about 300 mm from the dome. The measured deflections increased somewhat from locations 5 to 6 and from location 6 to 7 and then reduced from location 7 to 8. See Fig. 5.9. Presumably the steel base plate restrained somewhat the inward deflection of lower portions of the cell wall.
Analysis of the Test Results

Fig. 5.4 Pattern of Vertical Deformation with Increasing Applied Water Pressure Predicted by RASP Model.
Fig. 5. 5 Pattern of Horizontal Deformations with Increasing Applied Water Pressure Predicted by RASP Model
Analysis of the Test Results

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Fig. 5.6 Comparison of Predicted and Observed Deflections at Applied Pressure of 1.30 MPa. Deflections Measured at Displacement Transducers Locations

Magnification Factor of Displacement = 75
Analysis of the Test Results

Fig. 5.7 Measured and Predicted Vertical Displacement of the Dome

Displacement Plotted = (Δ1 + Δ2 + Δ4)/3

Experimental Results

RASP Predictions

Applied Water Pressure p [MPa]

Vertical Displacement Δv [mm]
Fig. 5.8 Measured and Predicted Spreading of the Dome
Fig. 5.9 Measured and Predicted Contraction of the Cell Wall
5.4 Predicted Pattern of Strains

The predicted strains for the vertical reinforcement in the cell wall and for the radial reinforcement in the dome are shown in Fig. 5.10. Both the reinforcement near the inside face of the storage cell and the reinforcement near the outside face are predicted to remain in compression for most locations over the height of the structure and for most levels of load. Near the junction between the dome and the cell wall the reinforcement strains increase dramatically as the water pressure is increased from 1.30 MPa to 1.35 MPa. Also note that at locations about 100 mm below and 150 mm above the junction, the strain in the reinforcement near the inside face changes from compressive to tensile as the water pressure is increased from 1.30 MPa to 1.35 MPa. Such a change is indicative of significant diagonal cracking and the influence of shear on the strain in the longitudinal steel.

The predicted strains for the circumferential reinforcement are shown in Fig. 5.11. As would be expected the strains in the reinforcement near the inside face are essentially identical to those in the reinforcement near the outside face. Note that while the circumferential strains are predicted to increase substantially as the water pressure is increased from 1.30 MPa to 1.35 MPa the strains are predicted to remain below the yield strain of the reinforcement.

The predicted increases in longitudinal strains, circumferential strains and shear strains as the applied water pressure increases, for a section of the cell wall just below the dome, are shown in Fig. 5.12. Note that for water pressures higher than 0.8 MPa both the vertical tensile strain on the outside face and the shear strain across the thickness of the wall increase substantially. This would indicate that both flexural cracking and diagonal cracking are predicted to occur at this location. The pattern of increasing strains is indicative of a flexure-shear failure in which yielding of the flexural tension reinforcement triggers a shear failure.
Fig. 5. 10 Patterns of Strain in Longitudinal Reinforcement with Increasing Water Pressure as Predicted by RASP
Fig. 5.12 Variation of Predicted Concrete Strains near Top of Cell Wall as Applied Water Pressure Increases
5.5 Measured and Predicted Reinforcement Strains

Because of the difficulty of placing and waterproofing the small strain gauges and their associated wires it was expected that many of the gauges would not be operating at the time of the test. Unfortunately, during the period prior to testing, the labels of the eighteen sets of wires in one of the four strain gauge conduits (the south-east conduit) were destroyed rendering the gauges unusable. At the time of testing 11 of the 18 gauges in both the north-west and the south-west conduits were working, while 8 of the 18 gauges in the north-east conduit were operating. In this section, the readings from these 30 working gauges will be compared with the predictions from the *RASP* model.

Again, to facilitate the comparisons between the predicted and measured values of strains only the “envelope” of experimental values will be given. That is, data associated with the two major unloading and reloading “loops” will be omitted. The transitions from one loading line to the next will be shown by dotted lines. Note that the experimental results shown for the first pressure test started at the origin, while the strain gauges’ readings recorded at the application of the prestressing force were omitted. These recorded data were so small to be recorded accurately.

Figure 5.13 compares the measured and predicted strains in the radial reinforcement near the outside face of the dome. At gauge location 1, some 267 mm from the junction with the cell wall, the recorded strains were compressive and agreed closely with the *RASP* predictions. At location 5, just 25 mm from the junction, the measured strains were initially compressive but became tensile for water pressures higher than 0.75 MPa. The measured strains were substantially smaller than the predicted values.

The response of the cell wall to flexure can be seen in the recorded strains in the vertical reinforcement near the outer face, which are shown in Fig. 5.14. Gauge location 11 is just 13 mm below the El. 60.0 m junction and at this location high tensile strains were both predicted and recorded. Location 15, on the other hand, which is 165 mm below the junction, experienced compressive strains, which were predicted to increase markedly just prior to failure.

The strains in the radial reinforcement near the inside face of the dome are shown in Fig. 5.15. At gauge location 2, 279 mm from the cell wall junction, the predicted and measured pattern of strains are in very good agreement. Note that just prior to failure the
strains begin to decrease in magnitude as the water pressure increases. A similar pattern is evident at location 6, but at this location the measured strains are substantially smaller than the predicted strains.

Figure 5.16 compares the measured and predicted strains in the vertical reinforcement near the inside face of the cell wall. Note that at location 12, which is just 13 mm from the cell wall junction line, high compressive strains were recorded prior to failure, while at location 16, 165 mm below the junction, significant tensile strains were being recorded prior to failure.

The strains in the circumferential reinforcement near the outer face of the dome and the cell wall are shown in Fig. 5.17 and Fig. 5.18, while those near the inner face are shown in Figs. 5.19 and 5.20. Because the predicted circumferential strains are very similar for the outer face and the inner face, it is convenient to discuss both sets of strains at the same time.

Locations 3 and 4 are in the dome about 254 mm and 266 mm away from the cell wall junction. At these locations the predicted and measured circumferential strains remained compressive as the water pressure was increased, with the measured compressive strains being higher than those predicted. The somewhat comparable locations in the cell wall, 17 and 18, showed closer agreement between measured strains and predicted strains. At these locations the strains were initially compressive but became tensile prior to failure.

The four circumferential locations close to the junction, 7, 13, 8 and 14, all showed a similar pattern. The measured hoop strains at these locations were all substantially greater than the predicted hoop strains until just prior to failure, when the predicted strains increased greatly. The gauges 7 and 8 were at the base of the upper dome while gauges 13 and 14 were at the top of the cell wall. The measured strains at these locations, suggested that the radial and vertical cracks were almost extended all way through the concrete section. Water pressure in the crack would be responsible for increasing the tensile strains at these locations significantly.
Fig. 5.13 Measured and Predicted Strains in Longitudinal Reinforcement Near Outside Face of Dome
Analysis of the Test Results

1.50
1.25
1.00
0.75
0.50
0.25
0.00
-0.20
-0.00
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
1.60
1.80
Strain $\epsilon$ [mm/m]

(a) Strain Gauge Location #11

(b) Strain Gauge Location #15

Fig. 5.14 Measured and Predicted Strains in Longitudinal Reinforcement
Near Outside Face of Cell Wall
Fig. 5.15 Measured and Predicted Strains in Longitudinal Reinforcement Near Inside Face of Dome
Fig. 5.16 Measured and Predicted Strains in Longitudinal Reinforcement Near Inside Face of Cell Wall
Fig. 5.17 Measured and Predicted Strains in Circumferential Reinforcement Near Outside Face of Dome
Fig. 5.18 Measured and Predicted Strains in Circumferential Reinforcement Near Outside Face of Cell Wall
Fig. 5.19 Measured and Predicted Strains in Circumferential Reinforcement Near Inside Face of Dome
**Fig. 5.20 Measured and Predicted Strains in Circumferential Reinforcement Near Inside Face of Cell Wall**

(a) Strain Gauge Location #14

(b) Strain Gauge Location #18
5.6 Closure

Program RASP predicted that the maximum load carrying capacity of the model structure would be reached between a differential head of 129 m and 134 m of sea water. The model structure was predicted to fail through a flexure-shear failure in which yielding of the flexural tension reinforcement triggers a shear failure. While the ultimate load carrying capacity of the model was reasonably predicted by program RASP, it was less successful in predicting the abrupt failure of the model structure. Instead, it provided a long post peak ductility plateau due to the fact that it is a load control program.

It should be appreciated that in program RASP, like in all other global finite element programs, a finer mesh would have enhanced the quality of the results. Due to the RASP formulations, the centre line of the structure near the junction was chosen to pass through the mid points between the inside and outside nodes with a tangent normal to the line joining both nodes resulting in a significant disturbance to the centre line near the junction in the mesh used. See Fig. 5.21 (a). For future analytical studies it is recommended to use a mesh with curved transition elements at the junction to yield better results. See Fig. 5.21 (b).

![Current Mesh](a) Current Mesh ![New Mesh](b) New Mesh

Fig. 5.21 Transition Curved Elements in RASP Model.

Water pressure on the cracks reduces the axial compressive load acting on the critical sections and accelerates the yielding of reinforcement. Water under pressure seeping through the short concrete cracks would increase the tensile stresses in the steel reinforcement and reduce the compressive stresses on the concrete. With the reduced compressive stresses, the flexural capacity of the critical sections would be lower than
Analysis of the Test Results

what was theoretically expected resulting in a softer response. Furthermore, water pressure in the concrete pores "pore water pressure" can apply effective tensile stresses to the solid phase\textsuperscript{32-33,34,35}. It was reported\textsuperscript{32,33} that axisymmetrical water pressure of 6 MPa applied to the curved surface of a concrete standard cylinder without intervening membrane can apply an axial tensile strain of 80 µm/m. Although such an effect is small, under a condition of fully saturated pores it will slightly accelerate cracking of concrete.

As can be seen from Figs 5.13 (a), 5.14 (b), 5.15 (a), and 5.16 (b), for the strains in the longitudinal reinforcement far from the junction, that the effect of water pressure on the cracks is negligible and good agreement was observed between the measured strains and the predicted values by \textit{RASP} at all four locations. On the other hand, it can be seen from Figs 5.14 (a) for the strain in the outside longitudinal reinforcement at the top of the cell wall, the effect of water pressure on the circumferential cracks resulted in considerable increase in the measured strain over the predicted values by \textit{RASP}. As can be seen from Fig. 5.15 (b) the measured compressive strain on the inside face at the base of the dome is considerably less than the predicted values by \textit{RASP}.

As can be seen from Figs 5.17 (b), 5.18 (a), 5.19 (b), and 5.20 (a), for the strains in the circumferential reinforcement near the junction, the effect of water pressure on the vertical and radial cracks resulted in substantial increase in the measured strain over the predicted values by \textit{RASP}. On the other hand, it can be seen from Figs 5.17 (a), 5.18 (b), 5.19 (a) and 5.20 (b) for the strains in the circumferential reinforcement away from the junction that the effect of water pressure on the cracks is negligible and good agreement was obtained between the measured strains and the predicted values by \textit{RASP} at all four locations. As can be seen from Figs. 5.13 (b) and 5.16 (a), the strain in the outside longitudinal reinforcement at the base of the dome and at the inside longitudinal reinforcement at the top of the cell wall do not follow the same trend.

The measured horizontal spreading at the base of the dome indicated a softer response than what was predicted by program \textit{RASP}. Again, part of the explanation may be due to the effect of pore water pressure and the effect of water pressure on the cracks. The tensile stresses developed due to pore water pressure and water under pressure acting on the vertical and radial cracks near the junction would pry the cracks open resulting in increasing the outwards spreading of the dome.
The vertical response of the dome was softer than what was predicted by program RASP. Part of the difference is due to RASP own formulation that accounts for the beneficial effect of the biaxial compression in most of the upper dome that may have led to a stiffer response. Another source for the difference might be due to the effect of water pressure on the radial and vertical cracks at the base of the dome. Increased horizontal spreading of the junction will result in increased vertical deformation of the upper dome.

It should be appreciated that program RASP, like all programs that use the Modified Compression Field theory, works in terms of average tensile strains. That is, the tensile strains are measured over base lengths long enough to average out the peaks in strain that occur near crack locations. Figure 5.22 gives an example of the type of strain variation that can be experienced by a reinforcing bar in concrete under tension. Note that the strains measured by gauges at crack locations can be three times as high as the strains measured by gauges midway between the cracks. This expected variation in local strains is part of the explanation for why some of the measured strains greatly exceed or were less than the average strain values predicted by the RASP model.

![Diagram showing strain variation](image)

**Fig. 5.22** Variation of Steel Strain along the Length of a Tension Specimen.
6.1 General

A theoretical approach, based on the *Classical Theory of Shells*\textsuperscript{18,19}, is presented to calculate the deformation and internal forces in a storage cell under axisymmetrical loads. A clear distinction between the material properties in the hoop and longitudinal directions was maintained during theoretical derivation. The general formulae for the bending of thin orthotropic truncated cone and short shell with constant thickness under axisymmetric loads were reached. An analysis method based on the compatibility conditions within the structure is presented. It is *not* limited to linear elastic behaviour and is extended to account for the material nonlinear behaviour, such that of reinforced concrete, by tracking the bending moment-curvature diagram. The method requires several iterations for each load level and is geared towards a flexural failure in the structure. The method provides the design engineer with a quick alternative for the analysis of shells subjected to axisymmetric loading.
6.2 Definition of the Problem

Storage cells in offshore oil platforms are regarded as thin cylindrical shells capped with spherical domes subjected to an axisymmetrical hydrostatic pressure. The dome is usually less than a hemisphere with a flat apex angle to suit the construction requirement, a constant diameter, and variable thickness. Under Loading, the dome flattens out horizontally at the joint line, while the cylindrical shell deforms inwards. To restore compatibility at the joint line, longitudinal bending moment and horizontal shear force, called the discontinuity stresses are needed. A mechanism should be reached with which those local stresses and the distribution of other internal forces and deformation along the dome and the cylindrical shell are calculated taking into account the material nonlinear behaviour accurately.

Any analysis method that is based solely on the Membrane Theory would fail to account for bending stresses specially near the Condeep corner. However, it could easily be modified to account for the material nonlinear behaviour based on the level of internal membrane forces. Bending stresses were of local nature only in domes with no holes, large domes with small holes on the top or, long shells. To account for the effect of bending stresses, at both ends of the dome or shell segment, on deformation, the dome-with-a-hole and the short shell solutions should be used.

Furthermore, the storage cell carries the axisymmetrical water pressure by two mechanisms namely; the longitudinal direction and the hoop action. As the internal longitudinal bending moment increases, it results in cracking of concrete in the hoop direction or yielding of the reinforcement in the longitudinal direction. Thus the shell loses significant amount of its stiffness in the longitudinal direction resulting in a redistribution of the load sharing between both mechanisms. As the load sharing of the hoop action is increased, the structure develops additional cracks in the radial and longitudinal direction and hoop reinforcement may also yield resulting in reducing the stiffness of the structure in the hoop direction as well. Thus the analysis method should differentiate between the material behaviour and properties in each direction; i.e. meridian and hoop, separately. With different material properties in each direction, the material is no longer an isotropic material and should be treated as an orthotropic material. In the upper dome, bending of thin orthotropic dome-with-a-hole and the membrane behaviour are included in the
analysis to reach reasonably accurate results. Since such solution was complicated to reach, the solution of a truncated orthotropic cone is used instead. Similarly, in the cylindrical shell, bending of thin orthotropic short shell and the membrane behaviour are included in the analysis.

It is worth to mention that the same technique of reducing the stiffness of the shell to account for the redistribution of the applied loads was used extensively in the design practice of the early Condeep platforms. Using the Classical Theory of Shells and reducing the stiffness of these sections based on the designer engineer own practical experience, more reasonable values for the internal forces at critical sections were reached other than the highly overestimated linear elastic analysis results.

6.3 Notation

A surface of revolution is formed by rotating a plane curve, called the meridian, about an axis laying in its plane, called the meridian plane. For a spherical dome, the meridian is a circular arc and the vertical axis is passing through the center of the circle. For a cylindrical shell, the meridian is a straight line parallel to the vertical axis. Any point on the meridian rotating around the axis will intersect the dome or the shell in a parallel circle. The shell was assumed to have a constant thickness, denoted as $h$. The surface that bisects the thickness was called the middle surface. The basic assumptions in the analysis were; 1) plane sections perpendicular to the middle surface remained plane and perpendicular to the middle surface after deformation, 2) the shell was thin and the thickness was assumed small compared to all other dimensions of the shell including its radii of curvature, 3) the stresses in the direction normal to the middle surface were negligible to all other stresses, and 4) the deformations were small and negligible compared to the radii of curvature of the middle surface.

6.3.1 Global Coordinate System

Figure 6. shows an element $ABCD$ on the surface of revolution defined by two adjacent meridian planes and two planes perpendicular to the meridians and contain the principal curvature of the dome. Its position in the dome is defined by angle $\phi$ made by the
Fig. 6.1 Global Coordinate System and Internal Forces in a Surface of Revolution
normal to the surface and the axis of rotation, and measured from the apex, and angle $\theta$
measured from a datum meridian plane. The length of the element sides at point $A$ were $r_1 d\phi$
and $r_\circ d\theta = r_2 \sin \phi d\theta$, giving an area of $r_1 r_2 \sin \phi d\phi d\theta$. The global axis of $X$, $Y$ and $Z$ are
taken at point $D$ where $X$ axis is in the hoop direction tangent to the parallel circle, $Y$ axis is in
the meridian direction tangent to the meridian, and $Z$ axis is in the radial direction normal to the
middle surface pointing inwards. The displacement of any point on the middle surface of the
element was resolved into three components namely; $u$, $v$ and $\omega$ in the $X$, $Y$ and $Z$ directions.

Figure 6.2 shows an element $ABCD$ cut from the shell by two adjacent vertical
planes and two horizontal planes that contain the principal curvature of the shell. Its position in
the shell is defined by; distance $x$ measured from the top of the shell and, angle $\theta$ measured
from a datum meridian plane. The length of the element sides at point $A$ were $r d\theta$ and $dx$
giving an area of $r d\theta dx$. The global axis $X$, $Y$ and $Z$ axes are taken at point $D$ where $X$ axis is
in direction of the longitudinal axis of the cylinder, $Y$ axis is the horizontal tangent to the
parallel circle, and $Z$ axis is in the radial direction normal to the middle surface pointing
inwards. The displacement of any point on the middle surface of the element was resolved into
three components namely; $u$, $v$, and $\omega$ in the $X$, $Y$ and $Z$ directions respectively.

6.3.2 Compatibility

During bending of the dome, the trapezoidal and rectangular sides of the element $ABCD$ shown
in Fig. 6.1 (a), that were normal to the middle surface of the dome, remained straight and normal
to the deformed middle surface of the dome. The middle surface may stretch with $\varepsilon_U$ and $\varepsilon_V$
in the $X$ and $Y$ directions of the dome. Thus, these sides were displaced parallel to themselves.
Before bending, the principal radii of curvatures in the $X$ and $Y$ directions of the dome were $r_x$
and $r_y$, while, $r'_x$ and $r'_y$ were the new values of the radii of curvatures after deformation. See
Fig. 6.3 (a). The strain $\varepsilon$ of any lamina at distance $z$ from the middle surface was given by;

$$
\varepsilon_x = \left( \frac{l_2 - l_1}{l_1} \right),
$$

where;

$$
l_1 = ds \left( 1 - \frac{z}{r_x} \right) \quad \text{and} \quad l_2 = ds \left( 1 + \varepsilon_1 \right) \left( 1 - \frac{z}{r'_x} \right).
$$

By substituting,

$$
\varepsilon_x = \frac{\varepsilon_1}{1 - z/r_x} - \frac{z}{(1 - z/r_x)} \left[ \frac{(1 + \varepsilon_1)}{r'_x} - \frac{1}{r_x} \right].
$$
Bending of Shells and Surfaces of Revolution

Since the thickness of the dome was assumed small w.r.t. the radii of curvature of the dome, the term \( z/r \) is neglected compared to unity. The strain components in the middle surface is neglected compared to unity, i.e. \((I + \epsilon) \cong I\). Thus the final expressions for the strains in any lamina at distance \( z \) from the middle surface in the \( x \) and \( y \) directions are given as follows:

\[
\epsilon_x = \epsilon_1 - z \chi_x \quad \text{and} \quad \epsilon_y = \epsilon_2 - z \chi_y,
\]

where \( \chi_x \) and \( \chi_y \) were the curvature change of the meridian and perpendicular to the meridian. Consider the side \( AB \) of the meridian, due to axisymmetrical condition, the hoop displacement vanishes, \( u=0 \). Due to the tangential displacement \( v \) and \( v + (dv/d\phi) \, d\phi \) at both ends, the
element elongates by \((\frac{dv}{d\phi})\) \(d\phi\). While, due to the inward radial displacement of \(\omega\) and \(\omega + (\frac{d\omega}{d\phi})\) \(d\phi\) of both ends, the element decreases in length by \(\omega\) \(d\phi\). See Fig. 6.3 (b). The total change in length of the element was \((\frac{dv}{d\phi})\) \(d\phi\) + \(\omega\) \(d\phi\). Dividing the change in length by the original length of the element; \(r d\phi\), an expression for the strain component of the dome middle surface in the meridian direction, \(\phi\) is given by;

\[
\varepsilon_2 = \frac{1}{r_1} \frac{dv}{d\phi} = \frac{\omega}{r_1}.
\]
Again, consider the side $AB$ of the meridian, see Fig. 6.3 (b), due to the axisymmetrical displacements $\omega$ and $v$ at end $A$ (or $D$), the side $AD$ elongates by, $(v \cos \phi - \omega \sin \phi)$. The circumference of the parallel circle increases by the same amount. An expression for the strain component of the dome middle surface in the hoop direction, $\theta$ is given by;

$$
\varepsilon_\theta = \frac{1}{r_o} (v \cos \phi - \omega \sin \phi) = \frac{v}{r_2} \cot \phi - \frac{\omega}{r_2}.
$$

Consider the same side $AB$ during bending of the surface of revolution. The element sides, normal to the middle surface of the shell, remain straight and normal to the deformed middle surface of the shell and stretch with $\varepsilon_1$ and $\varepsilon_2$ in the $\theta$ and $\phi$ directions. The effect of the strain in the middle surface of the shell on the change in curvature is neglected. The angle of rotation of the tangent to the meridian is defined as $V$. The change in curvature was defined as the angular change of $V$ divided by the arc length $r_1 d\phi$. The initial angle between the upper and lower sides of the element is $d\phi$. See Figs. 6.3 (b, c and d). Due to the tangential displacement $v$ and the inward radial displacement of $\omega$ at the upper side, the element rotates by;

$$
V = \frac{v}{r_1} + \frac{d\omega}{ds} = \frac{v}{r_1} + \frac{d\omega}{r_1 d\phi}.
$$

Similarly, due to the tangential displacement $v + (d\omega d\phi) d\phi$ and the inward radial displacement of $\omega + (d\omega d\phi) d\phi$, the lower side of the element rotates by;

$$
\frac{v}{r_1} + \frac{d\omega}{ds} + \frac{d}{ds} \left( \frac{v}{r_1} + \frac{d\omega}{ds} \right) ds = \frac{v}{r_1} + \frac{d\omega}{r_1 d\phi} + \frac{d}{d\phi} \left( \frac{v}{r_1} + \frac{d\omega}{r_1 d\phi} \right) d\phi = V + \frac{dV}{d\phi} d\phi.
$$

The expression for the curvature change of the meridian is given by;

$$
\chi_\phi = \frac{1}{r_1} \frac{d}{d\phi} \left( \frac{v}{r_1} + \frac{d\omega}{r_1 d\phi} \right) = \frac{1}{r_1} \frac{dV}{d\phi}.
$$

As can be seen from Fig. 6.3 (c) and (d), as the lateral sides of the element $ABCD$, i.e. $AB$ and $DC$, rotate in their meridian plane with the angle $V$, the middle surface radius changes by;

$$
\begin{align*}
    r_2 - r_2' &= \frac{v}{r_1} r_2' \cot \phi + \frac{d\omega}{r_1 d\phi} r_2' \cot \phi.
\end{align*}
$$

The expression for the curvature change of the perpendicular to the meridian is given by;

$$
\chi_\theta = \left( \frac{v}{r_1} + \frac{d\omega}{r_1 d\phi} \right) \frac{\cot \phi}{r_2} = \frac{V}{r_2} \cot \phi.
$$
By substituting for the expressions for \( \varepsilon_1, \varepsilon_2, \chi_\phi \) and \( \chi_\theta \) the expressions for the strains in any lamina at distance \( z \) from the middle surface in the \( \phi \) and \( \theta \) directions were given by;

\[
\varepsilon_\phi = \left( \frac{1}{r_1} \frac{dV}{d\phi} - \frac{\omega}{r_1} \right) z \left( \frac{1}{r_1} \frac{dV}{d\phi} \right),
\]

\[
\varepsilon_\theta = \left( \frac{v}{r_2} \cot \phi - \frac{\omega}{r_2} \right) z \left( \frac{v}{r_2} \cot \phi \right) \quad \text{and} \quad V = \frac{v}{r_1} + \frac{\omega}{r_1} d\phi.
\]

The compatibility equations between strains and deformations in a cylindrical shell are driven in a similar way and recognizing the facts that subscript \( \phi \) is substituted with \( x, r_1 = \infty, r_1 d\phi = dx, r_2 = a \) (radius of the cylinder), \( \phi = 90^\circ \) and displacement \( v \) is substituted with \( u \). The strains in the middle surface were \( \varepsilon_1 \) and \( \varepsilon_2 \) in the hoop and longitudinal direction respectively. Thus;

\[
\varepsilon_1 = -\frac{\omega}{a}, \quad \varepsilon_2 = \frac{du}{dx}, \quad \gamma_{\theta x} = \gamma_{x \theta} = 0,
\]

\[
\chi_x = -\frac{d^2 \omega}{dx^2}, \quad \chi_\theta = 0, \quad \varepsilon_x = \frac{du}{dx} z \frac{dV}{dx}, \quad \varepsilon_\theta = -\frac{\omega}{a} \quad \text{and} \quad V = \frac{d\omega}{dx}.
\]

The axisymmetrical condition resulted in no change in the curvature in the hoop direction.

### 6.3.3 Constitutive Relation

By applying generalized Hook's law for an orthotropic material as the constitutive relation between strains and stresses, and assume a plain stress condition, we obtain;

\[
\varepsilon_\phi = \frac{\sigma_\phi}{E_\phi} - \nu \frac{\sigma_\theta}{E_\theta} \quad \text{and} \quad \varepsilon_\theta = \frac{\sigma_\theta}{E_\theta} - \nu \frac{\sigma_\phi}{E_\phi}.
\]

Where, \( E_\phi \) and \( E_\theta \) are the secant moduli of elasticity in the meridian and hoop directions of the dome respectively. Since concrete is not a perfectly orthotropic material, Poisson's ratio is assumed constant. The corresponding stresses in any lamina, at any distance \( z \) from the middle surface in the meridian and hoop directions of the dome, are given as follows;

\[
\sigma_\phi = \frac{E_\phi}{(1 - \nu^2)} (\varepsilon_\phi + \nu \varepsilon_\theta) \quad \text{and} \quad \sigma_\theta = \frac{E_\theta}{(1 - \nu^2)} (\varepsilon_\theta + \nu \varepsilon_\phi).
\]

The internal longitudinal and hoop membrane forces in the dome are given by;

\[
N_\phi = \int_{-h/2}^{h/2} \sigma_\phi \, dz = K_\phi (\varepsilon_2 + \nu \varepsilon_1) \quad \text{and} \quad N_\theta = \int_{-h/2}^{h/2} \sigma_\theta \, dz = K_\theta (\varepsilon_1 + \nu \varepsilon_2),
\]
where, \[ K_\phi = \frac{E_\phi \cdot h}{(1 - \nu^2)}, \quad K_\theta = \frac{E_\theta \cdot h}{(1 - \nu^2)} \],
and \( K_\phi \) and \( K_\theta \) are the membrane rigidities in the meridian and hoop directions respectively.

The meridian and hoop bending moments in the dome are given as follows;

\[
M_\phi = -\int_{-h/2}^{h/2} \sigma_\phi z \, dz = -\frac{E_\phi \cdot h^3}{12 \cdot (1 - \nu^2)} (\chi_\phi + \nu \chi_\theta) = -D_\phi \left[ \frac{1}{r_1} \frac{dV}{d\phi} + \nu \frac{V}{r_2} \cot \phi \right],
\]
\[
M_\theta = -\int_{-h/2}^{h/2} \sigma_\theta z \, dz = -\frac{E_\theta \cdot h^3}{12 \cdot (1 - \nu^2)} (\chi_\theta + \nu \chi_\phi) = -D_\theta \left[ \frac{V}{r_2} \cot \phi + \nu \frac{dV}{d\phi} \right],
\]
where;
\[
D_\phi = \frac{E_\phi \cdot h^3}{12 \cdot (1 - \nu^2)}, \quad D_\theta = \frac{E_\theta \cdot h^3}{12 \cdot (1 - \nu^2)},
\]
and \( D_\phi \) and \( D_\theta \) are the flexural rigidities in the meridian and hoop directions respectively.

Using the previously mentioned substitutions, these equations may be adopted to the case of cylindrical shell as follows;

\[
\sigma_z = \frac{E_z}{(1 - \nu^2)} (\varepsilon_z + \nu \varepsilon_\theta), \quad \sigma_\theta = \frac{E_\theta}{(1 - \nu^2)} (\varepsilon_\theta + \nu \varepsilon_z),
\]
\[
N_z = K_z (\varepsilon_z + \nu \varepsilon_1), \quad N_\theta = K_\theta (\varepsilon_\theta + \nu \varepsilon_z),
\]
\[
M_z = -D_z \frac{dV}{d\phi}, \quad M_\theta = -\nu \cdot D_\theta \frac{dV}{d\phi},
\]
where;
\[
K_z = \frac{E_z \cdot h}{(1 - \nu^2)}, \quad D_z = \frac{E_z \cdot h^3}{12 \cdot (1 - \nu^2)},
\]
\[
K_\theta = \frac{E_\theta \cdot h}{(1 - \nu^2)}, \quad D_\theta = \frac{E_\theta \cdot h^3}{12 \cdot (1 - \nu^2)},
\]
and \( K_z \) and \( K_\theta \) are the membrane rigidities while \( D_z \) and \( D_\theta \) are the flexural rigidities in the longitudinal and hoop directions of the shell respectively.

### 6.3.4 Displacement

The strains of the middle surface of the dome are written as follows;

\[
\varepsilon_1 = \frac{1}{r_2} (\nu \cot \phi - \omega) = \frac{N_\theta}{E_\theta h} - \nu \frac{N_\phi}{E_\phi h}, \tag{a}
\]
\[
\varepsilon_2 = \frac{1}{r_1} \left( \frac{d\nu}{d\phi} - \omega \right) = \frac{N_\phi}{E_\phi h} - \nu \frac{N_\theta}{E_\theta h}.
\]
By eliminating \( \omega \) from both equations, we get;

\[
\left( \frac{d\nu}{d\phi} - \nu \cot \phi \right) = \frac{(r_1 + \nu r_2)}{E_\phi h} N_\phi - \frac{(r_2 + \nu r_1)}{E_\theta h} N_\theta. \tag{b}
\]
The general solution for this differential equation is written as follows;

\[ \nu = \sin \phi \left[ \int \frac{r_1 + \nu r_2}{E_\phi h \sin \phi} N_\theta \, d\phi - \int \frac{r_2 + \nu r_1}{E_\phi h \sin \phi} N_\theta \, d\phi \right] + C_{Dome}. \]

The constant \( C_{Dome} \) is calculated based on the boundary condition at the base of the dome. The radial displacement \( \omega \) is found by substituting in the first equation of (a). Similarly, the strain of the middle surface of the shell is given as follows;

\[ \varepsilon_z = \frac{du}{dx} = \frac{N_x}{E_z h} - \nu \frac{N_\theta}{E_\theta h}. \]

By integrating, the vertical deformation of the shell is written as follows;

\[ u = \left[ \int N_x \, dx \right] \left[ \int \frac{N_\theta}{E_\theta h} \, dx \right] + C_{Shell}. \]

The constant \( C_{Shell} \) is calculated based on the boundary condition at the base of the shell.

### 6.4 Membrane Theory

The shell is subjected to a uniform external hydrostatic pressure of intensity \( p \). Due to symmetry in both geometry and loading, the membrane shear forces vanish i.e. \( N_{\theta \phi} = N_{\phi \theta} = 0 \) and \( N_{\theta x} = N_{x \theta} = 0 \), the membrane hoop force \( N_\theta \) is constant along the circumference, and the transverse shear force vanishes i.e. \( Q_\theta = 0 \).

#### 6.4.1 Circular Cylindrical Shells

In reviewing the equilibrium of the element, shown in Fig. 6.2 (c) cut from the shell by two adjacent generators and two perpendicular cross sections, and summing up the forces in the \( X \), \( Y \) and \( Z \) directions respectively the equilibrium equations are given by;

\[ \frac{\partial N_x}{\partial x} = 0 \quad \text{and} \quad N_\theta = -p \, a. \]

The hoop stress, strain and the inward radial displacement are given by;

\[ \sigma_\theta = -\frac{p \cdot a}{h}, \quad \varepsilon_\theta = \frac{p \cdot a}{E_\theta \cdot h} \quad \text{and} \quad \omega = \frac{p \cdot a^2}{E_\theta \cdot h} \]

where; \( h \) is the thickness of the shell, \( E_\theta \) is the secant modulus of elasticity in the hoop direction and positive sign indicates compression in the radius of the cylinder.
6.4.2 Spherical Domes

In reviewing the equilibrium of the element, as shown in Fig. 6.1 (b), cut from the dome by two adjacent meridians and two parallel circles, and summing up the forces in the Z direction, we get \( N_z/r_1 + N_\theta/r_2 = -p \). Taking the equilibrium of a portion of the dome above a certain parallel circle, under the applied uniform hydrostatic loading, \( p \), the meridian membrane force is; \( N_\phi = -p.D/4 \). See Fig. 6.1 (c). By substituting, the hoop membrane force is; \( N_\theta = -p.D/4 \). The strains in the middle surface, \( \varepsilon_1 \) and \( \varepsilon_2 \) are given as follows;

\[
\varepsilon_1 = -\frac{pD}{4h} \left( \frac{1}{E_\theta} - \frac{\nu}{E_\phi} \right) \quad \text{and} \quad \varepsilon_2 = -\frac{pD}{4h} \left( \frac{1}{E_\theta} - \frac{\nu}{E_\phi} \right).
\]

The horizontal displacement; i.e. extension of the radius of the parallel circle, is given by,

\[
\delta_H = -\frac{pD^2}{2g} \left( \frac{1}{E_\theta} - \frac{\nu}{E_\phi} \right) \sin \phi.
\]

6.5 Bending Analysis of Cylindrical Shells

In addition to the previously outlined conditions due to symmetry in both geometry and loading, the twisting moments vanish; \( M_{x\theta} = M_{x\phi} = 0 \), and the hoop bending moment \( M_\theta \) is constant along the circumference. In reviewing the equilibrium of the element shown in Fig. 6.2 (c) and (d), the three equilibrium equations are rewritten as follows;

\[
\frac{dN_z}{dx} = 0, \quad \frac{dQ_z}{dx} + \frac{l}{a} N_\phi = -p \quad \text{and} \quad \frac{dM_z}{dx} + Q_x = 0.
\]

The strain of the middle surface in the hoop direction is given by;

\[
\varepsilon_1 = \frac{-\omega}{a} = \frac{N_\theta}{E_\theta h} - \nu \frac{N_z}{E_x h}.
\]

By eliminating \( Q_x \) and substituting in the equilibrium equation with the expressions for the hoop membrane forces and the longitudinal bending moment the general DE is given;

\[
-\frac{d^2}{dx^2} \left( D_x \frac{d^2 \omega}{dx^2} \right) + \frac{l}{a} \left( -\frac{E_\theta}{a} \frac{h}{a} \omega + \nu \frac{E_\theta}{E_x} N_z \right) = -p.
\]

All problems of axisymmetrical deformation of orthotropic (and isotropic) circular cylindrical shells subjected to uniform radial shear force and meridian bending moment at both ends
thus reduce to the integration of the DE. Assuming that the flexural rigidity was constant within the shell segment, the DE\textsuperscript{18,19} was rewritten as follows:

\[
\frac{d^4 \omega}{dx^4} + 4 \beta^4 \omega = \frac{1}{D_x} \left( p + \nu \frac{E_\theta}{E_x} \frac{N_z}{a} \right).
\]

where:

\[
\beta^4 = \frac{E_\theta h}{4 a^2 D_x} = \frac{3n(1-\nu^2)}{a^2 h^2} \quad \text{and} \quad n = \frac{E_\theta}{E_x}.
\]

The general solution of the governing differential equation is written as follows:

\[
\omega = e^{\beta x} \left( C_1 \cos \beta x + C_2 \sin \beta x \right) + e^{\beta x} \left( C_3 \cos \beta x + C_4 \sin \beta x \right) + f(x),
\]

in which \(C_1, C_2, C_3, \text{ and } C_4\) are the four constants of integration determined from four different end conditions for the shell segments and \(f(x)\) is the particular solution expressed as follows;

\[
f(x) = \frac{p a^2}{E_\theta h} + \nu \frac{N_z a}{E_x h}.
\]

The particular solution represents the membrane deformation of the free-edge orthotropic cylindrical shell. Positive sign indicates compression in the radial direction. Positive longitudinal membrane forces indicate tensile forces.

### 6.6 Bending Analysis of Surfaces of Revolution

In addition to the previously outlined conditions due to symmetry in both geometry and loading, the twisting moments vanish; \(M_{\theta\phi} = M_{\phi\theta} = 0\), and the hoop bending moment \(M_\theta\) is constant along the circumference. To establish the set of the governing differential equations for bending of spherical domes, it is necessary to assume there is no applied load on the dome except the horizontal shear force and the meridian bending moment at the base of the dome. The equations are written in term of two variables namely; \(V\) the angle of rotation of the meridian, and \(U = r_2 Q_\phi\). Summing up the moment around the \(Z\) axis, the equilibrium equation of the element shown in Fig. 6.1 (b) and (d) is;

\[
\frac{d}{d\phi} \left( M_\phi r_\phi \right) - M_\theta r_1 \cos \phi - Q_\phi r_1 r_\theta = 0.
\]

By substituting for the bending moments with the expression given in the constitutive relation, the first equation between the chosen variables is written as follows;
\[ \frac{-U}{D_\phi} = \frac{r_2}{r_1^2} \frac{d^2V}{d\phi^2} + \frac{1}{r_1} \left[ \frac{d}{d\phi} \left( \frac{r_2}{r_1} \right) \right] \cot \phi + \nu (1-n) \cot \phi \] 
\[ \frac{dV}{d\phi} - \frac{1}{r_1} \left[ \nu + n \cot^2 \phi \left( \frac{r_1}{r_2} \right) \right] V \]

where, \( n = E_\phi / E_\theta \). By summing up the forces in the Z direction, the second equilibrium equation for the element shown in Fig. 6.1(c) and (e) is written as follows;

\[ N_\theta r_o + N_\phi r_1 \sin \phi + \frac{d(Q_\phi r_o)}{d\phi} = 0. \]

Taking the equilibrium of a portion of the dome above the parallel circle, the meridian force is \( N_\phi = -Q_\phi \cot \phi = -U \cot \phi / r_2 \). See Fig. 6.1(e). By substituting, the hoop membrane force with \( N_\theta = -(dU/d\phi)/r_1 \), differentiating the first equation from (a), then eliminating the derivative \( dv/d\phi \) using the second equation of (a), we get;

\[ r_1 V = \cot \phi \left[ \left( r_1 + \nu r_2 \right) \frac{N_\phi}{E_\phi h} - \left( r_2 + \nu r_1 \right) \frac{N_\theta}{E_\theta h} \right] - \frac{d}{d\phi} \left[ \frac{r_1}{r_2} \frac{N_\theta}{E_\phi h} - r_1 \nu \frac{N_\phi}{E_\phi h} \right]. \]

Substituting for the membrane forces, the second equation between \( V \) and \( U \) is given by;

\[ E_\phi h V = \frac{r_2}{r_1^2} \frac{d^2U}{d\phi^2} + \frac{d}{d\phi} \left( \frac{r_2}{r_1} \right) \cot \phi + \nu (1-n) \cot \phi \left( \frac{d}{d\phi} \right) - \frac{1}{r_1} \left[ \nu + n \cot^2 \phi \left( \frac{r_1}{r_2} \right) \right] U. \]

The linear differential operator \( L \), known as Messines' operator, is defined by;

\[ L(\cdot) = \frac{r_2}{r_1^2} \frac{d^2(\cdot)}{d\phi^2} + \left[ \frac{d}{d\phi} \left( \frac{r_2}{r_1} \right) + \frac{r_2}{r_1} \cot \phi + \nu (1-n) \cot \phi \right] \frac{d(\cdot)}{d\phi} - \frac{1}{r_1} \left[ \nu + n \cot^2 \phi \left( \frac{r_1}{r_2} \right) \right]. \]

Thus, the system of the simultaneous differential equations is written as follows;

\[ - \frac{U}{D_\phi} = L(V) - \frac{\nu}{r_1} V \quad \text{and} \quad E_\phi h V = L(U) + \frac{n\nu}{r_1} U. \]

By substitution, the governing differential equation\(^{18,19}\) is written as follows;

\[ L(L(U)) + \frac{\nu}{r_1} (n-1) L(U) + \left( \frac{E_\phi h}{D_\phi} - \frac{\nu^2 n}{r_1^2} \right) U = 0. \]

All problems of bending of orthotropic (and isotropic) surfaces of revolution subjected to uniform radial shear force and meridian bending moment at its base thus reduce to the integration of the simultaneous system of second order homogeneous differential equation.

In the case of isotropic materials, \( n=1 \), the equations agree fully with the standard equations for the bending of isotropic surfaces of revolution\(^{18,19}\). However, it is fundamentally different for orthotropic material by the presence of the middle term on the left...
hand side making the solution overwhelmingly complicated. To eliminate this term, there are two cases either the Poisson’s ratio $\nu=0$ or the radius $r_1=\infty$. The second case means that the dome’s behaviour was approximated by the behaviour of a series of truncated cones stacked on the top of each other. The assumption will facilitate the solution of the governing differential equation. It is acceptable providing that the number of elements used to model the actual structure is reasonably high. However, the top of the dome is modelled as an isotropic spherical dome without a hole. Three General solutions are presented namely; a) An orthotropic truncated cone used in modelling parts of the spherical dome with non-linear behaviour, b) an isotropic truncated cone used in modelling parts of the spherical dome with the same stiffness in both directions, and c) an isotropic dome with no hole used in modelling the top part of the dome.

6.6.1 Bending of Isotropic Spherical Dome

The linear differential operator and the governing differential equation are given as follows;

$$L(U) = \frac{1}{a^2} \frac{d^2 (\theta)}{d\phi^2} + \frac{1}{a} \left[ \cot \phi \frac{d(\theta)}{d\phi} - \frac{1}{a} \left[ \cot^2 \phi \right] \right] \quad and \quad L(L(U)) + \mu^4 \cdot U = 0,$$

where; $\mu^4 = E_\phi h^2 / D_\phi - \nu^2/a^2$. A solution for the governing DE is given in the form;

$$L(U) \pm i \mu^2 \cdot U = 0,$$

while the differential equation is rewritten as follows;

$$\frac{d^2 Q_\phi}{d\phi^2} + \cot \phi \frac{dQ_\phi}{d\phi} + (-\cot^2 \phi + 2i\rho^2) Q_\phi = 0,$$

where; $\rho^2 = a \mu^2 / 2$. Since the solution is sought near the apex, the substitutions, $x = \sin^2 \phi$ and $Q_\phi = Z \sin \phi$, will ensure convergence of the hypergeometric series\(^{18,19}\) given as follows;

$$Z = \sum_{n=1}^{\infty} A_n x^n = z_1 + iz_2 \quad and \quad A_n = A_n^{-2} \left( \alpha + 1 \right) \left( \beta + 1 \right) \frac{n(n+1)}{n},$$

where; $\alpha = (3 + \delta^2)/4$, $\beta = (3 + \delta^2)/4$, $|x| \leq 0$ and $A_\alpha \neq 0$ is any arbitrary constant. The power series $z_1$ and $z_2$ contain the real and imaginary parts of the solution and represent two independent solutions to the DE. An expression for the radial shear force $Q_\phi$ in the dome and the slope angle of the meridian $V$ are given as follows;

$$Q_\phi = C_1 z_1 \sin \phi + C_2 z_2 \sin \phi,$$

$$V = C_1 (\nu - a\mu^2) z_1 \sin \phi + C_2 (a\mu^2 - \nu) z_2 \sin \phi.$$
where \( C_1 \) and \( C_2 \) are the constants of integration, determined from two different end conditions at the dome base. The bending moments, membrane forces, displacements and strains of the middle surface are calculated directly from these equations.

### 6.6.2 Bending of Truncated Cone

The linear differential operator and the governing differential equation are given as follows;

\[
L() = \tan \alpha \left[ y \frac{d^2()}{dy^2} + A \frac{d()}{dy} - \frac{1}{y} () \right] \quad \text{and} \quad L(L(U)) + \mu^2 \cdot U = 0,
\]

where, \( \alpha \) is the apex angle of the cone and \( \mu^2 = E_h h / D_\phi \). A solution for the governing DE is given in the form; \( L(U) \pm i \mu^2 \cdot U = 0 \), while the differential equation\(^{18,19} \) is given by;

\[
y \frac{d^2(yQ_y)}{dy^2} + A \frac{d(yQ_y)}{dy} + (-1 \pm i\lambda^2 y) Q_y = 0,
\]

where; \( \lambda^2 = \mu^2 / \tan \alpha \) and \( A = 1 + (1 - n) v \). Taking the first of the set of equations, and using the substitution, \( x = 2\lambda \sqrt{i} \sqrt{y} \) and \( Z = y Q_y \), the DE is rewritten as follows;

\[
\frac{d^2Z}{dx^2} + b \frac{dZ}{dx} + (-\frac{4n}{x^2} \pm 1) Z = 0,
\]

where; \( b = 2A - 1 \) and \( n = E_\theta / E_\phi \). There are two forms of solution depending primarily on whether the truncated cone is isotropic, \( n = 1 \) and \( b = 0 \), or orthotropic, \( n \neq 1 \) and \( b \neq 0 \).

### 6.6.3 Bending of Isotropic Truncated Cone

For isotropic truncated cones, the general solution for \( Q_y \) is presented as follows;

\[
Q_y = \frac{1}{y} \left[ \psi_1(\eta) + \frac{2}{\eta} \psi_2(\eta) \right] C_1 + \frac{1}{y} \left[ \psi_3(\eta) - \frac{2}{\eta} \psi_4(\eta) \right] C_2
+ \frac{1}{y} \left[ \psi_5(\eta) + \frac{2}{\eta} \psi_6(\eta) \right] C_3 + \frac{1}{y} \left[ \psi_7(\eta) - \frac{2}{\eta} \psi_8(\eta) \right] C_4,
\]

where; \( \eta = 2\lambda \sqrt{i} \sqrt{y} \), and \( C_1, C_2, C_3, \) and \( C_4 \) are the four constants of integration determined from four different end conditions for the truncated cone segment. The power series \( \psi_1, \psi_2, \psi_3, \) and \( \psi_4 \) contain the real and imaginary parts of the solution and each represents an independent solution to the DE\(^{18,19} \) given as follows;
Bending of Shells and Surfaces of Revolution

\[ \psi_1(\eta) = A_0 + \sum_{n=4,6,12}^{\infty} A_{n-2} \frac{\eta^n}{n^2}, \quad \psi_2(\eta) = \sum_{n=2,6,10}^{\infty} -A_{n-2} \frac{\eta^n}{n^2}, \quad A_n = \frac{-1}{(n^2 - 1)} A_{n-2} \]

\[ \psi_3(\eta) = \frac{1}{2} \psi_1(\eta) - \frac{2}{\pi} \left[ R_1 + \ln \frac{B_0}{2} \psi_2(\eta) \right], \quad \psi_4(\eta) = \frac{1}{2} \psi_2(\eta) - \frac{2}{\pi} \left[ R_2 + \ln \frac{B_0}{2} \psi_1(\eta) \right], \]

\[ R_1 = \sum_{n=1,3,5}^{\infty} (-1)^{(n-1)/2} R_n(\eta/2)^2, \quad R_2 = \sum_{n=2,4,6}^{\infty} (-1)^{(n-2)/2} R_n(\eta/2)^2, \]

\[ R_n = R_{n-1} \frac{S(n)}{n^2}, \quad S(n) = \sum_{k=1}^{n} 1/k \quad \text{and} \quad \ln \beta = 0.57722. \]

In which \( A_0 \neq 0 \) is any arbitrary constant. An expression for the slope angle of the meridian \( V \) is given as follows,

\[ \frac{E_{\phi h}}{\mu^2 \tan \alpha} V = \left[ \psi_2(\eta) - \frac{2}{\eta} \psi_1(\eta) \right] C_1 - \left[ \psi_1(\eta) + \frac{2}{\eta} \psi_2(\eta) \right] C_2 \]

\[ + \left[ \psi_3(\eta) - \frac{2}{\eta} \psi_1(\eta) \right] C_3 - \left[ \psi_2(\eta) + \frac{2}{\eta} \psi_3(\eta) \right] C_4, \]

Using the expressions for \( Q_y \) and \( V \) the bending moments, membrane forces, displacements, and strains of the middle surface are calculated directly.

6.6.4 Bending of Orthotropic Truncated Cone

For orthotropic truncated cones, the general solution for \( Q_y \) is presented as follows;

\[ Q_y = \frac{1}{y} [\xi_1(\eta)] C_1 + \frac{1}{y} [\xi_2(\eta)] C_2 + \frac{1}{y} [\xi_3(\eta)] C_3 + \frac{1}{y} [\xi_4(\eta)] C_4, \]

where; \( \eta = 2\lambda \sqrt{y} \), and \( C_1, C_2, C_3, \) and \( C_4 \) are the four constants of integration determined from four different end conditions for the truncated cone segment. The power series \( \xi_1, \xi_2, \xi_3, \) and \( \xi_4 \) contain the real and imaginary parts of the solution and each represents an independent solution to the differential equation\(^{20,21}\) given as follows;

\[ R_{1,2} = \left( -b - 1 \pm \sqrt{(b - 1)^2 + 16n} \right) / 2, \]

\[ A_{2m} = \frac{-A_{2m-2}}{(2m)^2 + 2m(2R_1 - 1 + b)}, \quad B_{2m} = \frac{-B_{2m-2}}{(2m)^2 + 2m(2R_2 - 1 + b)}. \]
\[ \xi_1(\eta) = \sum_{m=0,2,4,...}^{\infty} (-1)^m A_{2m} \eta^{2m}, \quad \xi_2(\eta) = \sum_{m=1,3,5,...}^{\infty} (-1)^2 A_{2m} \eta^{1+2m}, \]
\[ \xi_3(\eta) = \sum_{m=0,2,4,...}^{\infty} (-1)^m B_{2m} \eta^{R2-2m}, \quad \xi_4(\eta) = \sum_{m=1,3,5,...}^{\infty} (-1)^2 B_{2m} \eta^{R2-2m}. \]
in which \( A_0 \neq 0 \) and \( B_0 \neq 0 \) are any arbitrary constants. An expression for the slope angle of the meridian \( V \) is given as follows;
\[ \frac{E\phi h}{\mu^2 \tan \alpha} V = \xi_2(\eta) C_1 - \xi_1(\eta) C_2 + \xi_4(\eta) C_3 - \xi_3(\eta) C_4. \]
The expressions for \( Q_y \) and \( V \) are used in calculating the bending moments, membrane forces, displacements, and strains of the middle surface directly.

6.7 Deformation along the Joint Line between Segments

Figure 6.4 (a) illustrates the front profile of a storage cell where the cylindrical shell has radius \( a \) and length \( L \), and the upper dome has a diameter \( D \). It is subjected to a uniform external water pressure of intensity \( p \). The end conditions at the base of the cell wall may be either free, hinged, fixed or symmetric type.

6.7.1 Modelling

The spherical dome is modelled with a fair number of truncated cones stacked on the top of each other. See Fig. 6.4 (b). Each segment is defined with two meridian angles \( \phi = \phi_{top} \) and \( \phi = \phi_{bottom} \), and a thickness \( h' \). See Fig. 6.4 (c). The segments may have different central angle \( \phi' = \phi_{bottom} - \phi_{top} \), and different thicknesses. Each segment is subjected to a uniform external water pressure of intensity \( p \) and a uniform meridian membrane forces \( N_{\phi_{top}} \) and \( N_{\phi_{bottom}} \). Each segment may have different secant moduli of elasticity in both directions: \( E_{\phi} \) and \( E_{\theta} \).

Similarly, the cylindrical shell is discretized into short shell segments. See Fig. 6.4 (d). Each segment has a length \( L' \) and a thickness \( h' \). The segments may be of different lengths and thicknesses. Each segment is subjected to a uniform external water pressure of intensity \( p \) and a uniform longitudinal membrane forces \( N_x \) at both ends. Each segment may have different secant moduli of elasticity in both directions: \( E_x \) and \( E_{\theta} \).
**Fig. 6.4** Modelling of the Storage Cell
In addition to the uniformly distributed hydrostatic pressure, there may be a uniformly distributed radial load applied at any (or all) joint lines between segments $H$.

### 6.7.2 Compatibility

Consider, two successive shell segments $i$ and $i+1$. See Fig. 6.5 (a). Under the action of the applied loads, the radial displacement at the bottom end of shell segment $i$ and at the top end of shell segment $i+1$ are given by;

$$\omega^i = \frac{p}{E_y H'} \cdot \frac{a^2}{a'} + \nu \frac{N_x}{E_x H}$$

and

$$\omega^{i+1} = \frac{p}{E_{y}^{i+1} H'} \cdot \frac{a^2}{a'} + \nu \frac{N_x}{E_{x}^{i+1} H^{i+1}}$$

Different thicknesses and material properties in each segment, will give rise to displacement discontinuity between segments along joint line. Since membrane deformation produces no change in the slope angle, under the same applied load, there is no rotational gapping in the slope angle of the deflected line. The displacement gapping is given as follows;

$$\omega^i - \omega^{i+1} = \frac{p}{E_y H'} \cdot \left( \frac{1}{E_y H'} - \frac{1}{E_{y}^{i+1} H^{i+1}} \right) + \nu N_x \frac{a}{E_x H'} \left( \frac{1}{E_x H'} - \frac{1}{E_{x}^{i+1} H^{i+1}} \right)$$

The first result is that there must be horizontal shear force $Q'_x$ and longitudinal bending moment $M'_x$ uniformly distributed along the circumference at the joint line $i$ to eliminate that discontinuity. In the case of long segments, the bending will be of local nature. The deformation dies quickly as the distance increases further away from the edges. The physical meaning for this is that the bending at one end of the segment does not affect the bending of the other end and vice versa. Since there is no restrictions on the minimum length of the shell segment; the load does not have a local nature. The shear force and bending moment along both ends of each shell segment will affect the displacement at the joint line. Thus, there are $Q'_x$ and $M'_x$ along the top edge and $Q'_x$ and $M'_x$ along the bottom edge of the top segment. Similarly, there are $Q'_x$ and $M'_x$ applied at the top edge and $Q'_{i+1}$ and $M'_{i+1}$ applied at the bottom edge of the bottom segment. See Fig. 6.5 (b).

Consider, two successive truncated cone segments $i$ and $i+1$. See Fig. 6.6 (a). Under the action of the applied loads, the horizontal displacement at the bottom end of cone segment $i$ and at the top end of cone segment $i+1$ are given by;
Bending of Shells and Surfaces of Revolution

\[ \delta_h = \left( \frac{N_\theta}{E_\theta h'} + \nu \frac{N_\theta}{E_\theta h'} \right) \frac{D \sin \phi}{2} \quad \text{and} \quad \delta_{h+1} = \left( \frac{N_\theta}{E_\theta^{+1} h^{+1}} + \nu \frac{N_\theta}{E_\theta^{+1} h^{+1}} \right) \frac{D \sin \phi}{2}. \]

Different thicknesses and material properties in each segment, will give rise to displacement discontinuity between segments along the joint line given as follows;

\[ \delta_h - \delta_{h+1} = N_\theta \left( \frac{1}{E_\theta h'} - \frac{1}{E_\theta^{+1} h^{+1}} \right) \frac{D \sin \phi}{2} + \nu N_x \left( \frac{1}{E_\theta h'} - \frac{1}{E_\theta^{+1} h^{+1}} \right) \frac{D \sin \phi}{2}. \]

Since membrane deformation produces no change in the slope angle, under the same applied load, there is no discontinuity in the slope angle of the deflected line.

Applying the same result discussed for the shell segments to the successive truncated cone segments, there are uniform horizontal shear force \( H_{h'} \) and meridian bending moment \( M_{h'} \) at the top edge and \( H_{h'} \) and \( M_{h'} \) along the bottom edge of the upper segment. Similarly, there are uniform horizontal shear force \( H_{h'} \) and bending moment \( M_{h'} \) along the top edge and \( H_{h+1} \) and \( M_{h+1} \) along the bottom edge of the lower segment. See Fig. 6.6 (b). These internal forces are needed to eliminate deformation incompatibility along the joint line.

### 6.7.3 Elimination of Incompatible Deformation

The conditions at the joint line between any two successive segments are: a) there is no displacement discontinuity, b) there is one common slope to the deflected line, c) the horizontal shear forces are continuous, and d) there is one value for the longitudinal or meridian bending moment. Using the general form of the solution for the shell segment, the final deformation and final stress resultants at either end of the segment is given in a matrix form as;

\[ \begin{bmatrix} \omega \\ V \\ Q_x \\ M_x \end{bmatrix} = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 & \omega_4 \\ \nu_1 & \nu_2 & \nu_3 & \nu_4 \\ q_{x1} & q_{x2} & q_{x3} & q_{x4} \\ m_{x1} & m_{x2} & m_{x3} & m_{x4} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} + \begin{bmatrix} \omega' \\ 0 \\ H' \\ 0 \end{bmatrix}. \]

For the truncated cone segment, the final deformation and final stress resultants at either end of the segment is written in a matrix form as follows;
Fig. 6.5 Incompatible Deformation in Cylindrical shells

Fig. 6.6 Incompatible Deformation in Spherical Dome
In a matrix notation, it is written as follows;

\[
[F]_{(x_1)} = [A]_{(x_1)} \cdot [C]_{(x_1)} + [f_0]_{(x_1)} \cdot [H^r].
\]

where; \([F]_{(x_1)}\) is called the element final stress-deformation vector, matrix \([A]_{(x_1)}\) is called the element shape matrix and contains the multipliers coefficients based on the bending of shell (dome, or truncated cone) approach, \([C]_{(x_1)}\) is called the element constants vector that contains the four constants of integration and \([f_0]_{(x_1)}\) is the initial condition vector that contains the membrane deformation and the applied loads. To eliminate any displacement discontinuity and to maintain a common slope to the deflected line at the joint line between any successive shell segments, the compatibility equations at the joint line are written as follows;

\[
\omega^r_{lower} = \omega^r_{upper}, \quad V^r_{lower} = V^r_{upper}, \quad Q_{X}^r_{lower} + H^r = Q_{X}^r_{upper}, \quad \text{and} \quad M_{X}^r_{lower} = M_{X}^r_{upper}.
\]

while for the cone segments they are written as follows;

\[
\delta^r_{lower} = \delta^r_{upper}, \quad V^r_{lower} = V^r_{upper}, \quad H^r_{\phi} + H^r = H^r_{\phi upper}, \quad \text{and} \quad M^r_{\phi lower} = M^r_{\phi upper}.
\]

In a matrix form, they are written as follows;

\[
[F]_{lower} = [F]_{upper} \cdot [A]_{(x_1)} \cdot [C]_{(x_1)} = [f_0]_{(x_1)} \cdot [H^r].
\]
A similar matrix form may be written for the cone segments. The right hand side of the equation represents the displacement and force discontinuity along joint line. It follows from the previous compatibility that there will be four equations in eight unknowns. These unknowns are the constants of integration in both segments. Matrices $[A]_{4x4}$ are not symmetric and depends only on the material the geometric parameters of each segment.

6.7.4 The Effect of Boundary Conditions

The effect of "boundary conditions" in the storage cell is broadened to account for the end condition at the apex, the compatibility at the joint line between the upper dome and the cell wall and the end condition at the base of the cell wall. The first element in the model is the isotropic dome. See Fig. 6.4 (b). The bending of a complete dome required only two constants of integration. The final deformation and stress resultants at the base of the dome is given by;

$$[F]_{4x1} = [A]_{4x4} \cdot [C]_{4x4} + [f_0]_{4x1}.$$ 

Thus there are four equations of compatibility at the base of the dome in two unknowns.

At the joint line between the upper dome and the cell wall, the membrane meridian force $N_\phi$ is resolved into two component. See Fig. 6.7. The longitudinal component is the constant $N_x$ force acting on the cell wall, while the horizontal component is added to the applied ring force at the joint line, usually due to the prestressing force. Also, due to the different set of global axes used for the shell and the dome, the positive directions of the horizontal displacement and the angle of rotation of the deflected line are different. Thus the four compatibility equations at the joint line are modified as follows;

$$\delta_h^{\text{NC}} = -\omega^{n+1}U, \quad \nu^{\text{NC}} = V^{n+1}U, \quad Q_x^n = H_x^n + H^n + N_\phi \sin \phi_x, \quad \text{and} \quad M_x^n = M_x^n.$$ 

The boundary conditions at the base of the cylinder will result in only two equations of compatibility at that location. The last two equations are written differently, depending on the type of boundary conditions at the bottom of the cylinder namely;

a) Fixed end condition; $V=0$ and $\omega_{\text{final}} = 0$, i.e. $\omega_{\text{bending}} = -\omega_{\text{membrane deformation}}.$

b) Hinged end condition; $M_x = 0$, and $\omega_{\text{final}} = 0$ i.e. $-\omega_{\text{bending}} = \omega_{\text{membrane deformation}}.$

c) Symmetrical end condition; $Q_x = 0$ and $V = 0$ and

d) Free end condition; $Q_x = 0$ and $M_x = 0.$
Fig. 6. Boundary Conditions at the Junction between the Upper Dome and the Cell Wall
6.7.5 Global Compatibility Matrix

The problem of eliminating the displacement discontinuity at the joint lines in a storage cell is reduced to the solution of a system of simultaneous equations where the unknowns are the constants of integration within each segment. The structure is modelled by an N number of segments with four unknowns at each segment, except for the top isotropic dome, there is only two unknowns. The total number of unknowns is 4N-2. There are N-1 joint lines and a bottom boundary with four equations of compatibility written at each joint line and two equations at the bottom boundary due to the end conditions for the cell wall resulting in 4N-2 equations. The total number of equations is exactly the same as the number of unknowns. The global compatibility matrix is written as follows;

\[
[A_G]_{(4N-2,4N-2)} \cdot [C_G]_{(4N-2,1)} = [D_G]_{(4N-2,1)}
\]

where; matrix \([A_G]_{(4N-2,4N-2)}\) is called the global compatibility matrix, vector \([C_G]_{(4N-2,1)}\) is called the global constants vector and vector \([D_G]_{(4N-2,1)}\) is called the global discontinuity vector.

Figure 6.8 illustrates the global compatibility matrix \([A_G]\), for a structure made up of three elements in the upper dome and two elements in the cell wall and free end condition at the base of the cell wall. It can be seen that matrix \([A_G]\), is a non-symmetric banded "stepped" matrix. It requires the storage of the full band width to solve this system of equations. The finer the modelling, the higher the number of equations and the better the results. However, the full band width is constant and is independent on the number of segments; i.e. the bandwidth \(B=8\).

The only unknown in the above equation was the global constants vector \([C_G]_{(4N-2,1)}\). Solving for the constants of integration is carried out using any standard full band-width matrix inversion / solver routine. By pre-multiplying both sides of the equation by \([A_G]^T\), the global constants vector \([C_G]\) is given by;

\[
[C_G]_{(4N-2,1)} = [A_G]^T \cdot [D_G]_{(4N-2,1)}
\]

As can be seen from Fig. 6.8 the matrix format was not changed to account for the fact that there is only two significant unknowns at the first joint line and two significant equations at the last joint line rather than four. The maximum benefit would be the reduction of the matrix size by a mere 4 rows. Besides, the current matrix structure will accommodate any future upgrade to account for holes at the top of the upper dome.
Fig. 6.8 The Global Compatibility Matrix \([A_0]\)
6.7.6 Final Deformation and Stress Resultants

Having the constants of integration handy, the final deformation and internal forces at any point within any segment are evaluated. In a shell segment at any arbitrary coordinate \( x = x_1 \) within the shell segment they are expressed in terms of the constants of integration as follows;

\[
\begin{bmatrix}
\omega \\
V \\
Q_x \\
M_x
\end{bmatrix}
= \begin{bmatrix}
\omega_1 & \omega_2 & \omega_3 & \omega_4 \\
\nu_1 & \nu_2 & \nu_3 & \nu_4 \\
q_{x1} & q_{x2} & q_{x3} & q_{x4} \\
m_{x1} & m_{x2} & m_{x3} & m_{x4}
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4
\end{bmatrix}
+ \begin{bmatrix}
\omega' \\
0 \\
0 \\
0
\end{bmatrix}
, \quad x = x_1
\]

where all the multipliers are evaluated at the distance \( x = x_1 \). The rate of change of the slope angle in the meridian direction \( (dV/dx) \) is calculated based on the meridian bending moment \( M_x \).

The hoop bending moment is then evaluated. The longitudinal force \( N_x \), calculated based on equilibrium of vertical forces for the whole structure, is constant and is not affected by bending. Using the horizontal displacement \( \omega \) and the longitudinal membrane force \( N_x \), the middle surface strain in the longitudinal direction \( \varepsilon_x \) is calculated. The strain of the middle surface in the hoop direction \( \varepsilon_\theta \) is calculated directly from the horizontal displacement \( \omega \). Finally the hoop force \( N_\theta \) is calculated. It is summarized as follows;

\[
\frac{dV}{dx} = -\frac{M_x}{D_z}, \quad M_\theta = -\nu D_\theta \frac{dV}{dx}, \quad N_x = N_x(\text{membrane condition}),
\]

\[
\varepsilon_\theta = -\frac{\omega}{a}, \quad \varepsilon_x = \frac{N_x}{K_x} - \nu \varepsilon_\theta, \quad \text{and} \quad N_\theta = K_\theta (\varepsilon_\theta + \nu \varepsilon_x).
\]

The vertical displacement of the wall; \( u \), is calculated using the numerical integration method and the trapezoidal rule for the results of the longitudinal strain of the middle surface by;

\[
u_{@x=x_1} = \int_{x=0}^{x=x_1} \varepsilon_x dx = \int \frac{N_x}{E_x h} dx - \int \nu \frac{N_\theta}{E_\theta h} dx + C_{\text{Shell}}.
\]

The constant of integration \( C_{\text{Shell}} \) is evaluated using the condition that the base of the cell wall is restricted from movement in the vertical direction; i.e., \( u = 0 \) at \( x = L \). The vertical displacement at the joint line between the dome and cell wall is calculated as \( u_{\text{joint}} \).

For the truncated cone segment \( i \), the final deformation and internal forces at any arbitrary angle \( \phi = \phi_i \) within the segment is written in a matrix form as follows;
where all the multipliers are evaluated at angle $\phi=\phi_l$. The rate of change of the angle of the deflected line $dV/d\phi$ is calculated based on the meridian bending moment $M_{\phi}$ and the angle of rotation $V$ at that location. The hoop bending moment $M_{\theta}$ is then evaluated. The radial shear force is calculated based on vertical equilibrium of forces at that location, while the meridian force $N_{\phi}$ is calculated based on vertical equilibrium of forces at that location in addition to the membrane case. The strain of the middle surface in the hoop direction $\varepsilon_1$ is calculated directly from the horizontal displacement $\delta_{h}$. Using the hoop strain $\varepsilon_1$ and the meridian membrane force $N_{\phi}$, the middle surface strain in the meridian direction $\varepsilon_2$ is calculated. Finally the hoop force $N_{\theta}$ is calculated. It is summarized as follows;

$$
\frac{dV}{d\phi} = -\frac{a M_{\phi}}{D_{\phi}} - V V \cot \phi, \quad M_{\theta} = \frac{-D_{\theta}}{a} \left( V \cot \phi + \nu \frac{dV}{d\phi} \right),
$$

$$
Q_{\phi} = H_{\phi} \sin \phi, \quad N_{\phi} = H_{\phi} \cos \phi + N_{\phi}^{(membrane\ condition)},
$$

$$
\varepsilon_1 = \frac{\delta_{h}}{a \sin \phi}, \quad \varepsilon_2 = \frac{N_{\phi}}{K_{\phi}} - \nu \varepsilon_1, \quad \text{and} \quad N_{\theta} = K_{\theta} (\varepsilon_1 + \nu \varepsilon_2).
$$

The tangential displacement of the upper dome; $v$, is calculated as;

$$
\frac{v_{\phi=\phi_l}}{\sin \phi} = \frac{D}{2} \left( \int (1 + \nu)N_{\phi} d\phi - \int \frac{(1 + \nu)N_{\theta}}{E_{\phi} h \sin \phi} d\phi \right) + C_{\text{Dome}} = \Sigma + C_{\text{Dome}}.
$$

Using the numerical integration method with the trapezoidal rule, the constant of integration $C_{\text{Dome}}$ is evaluated from the horizontal displacement, $\delta_{h(joint)}$, at the joint line between the upper dome and cell wall where;

$$
v_{\phi=\phi_{\text{edge}}} = \delta_{h(joint)} \cos \phi_{\text{edge}} \quad \text{and} \quad C_{\text{Dome}} = \delta_{h(joint)} \cos \phi_{\text{edge}} - \Sigma/\sin \phi.
$$

The radial and vertical displacement of the dome are given as follows;

$$
\omega = \nu \cot \phi_{\text{edge}} - \delta_{h}/\sin \phi_{\text{edge}} \quad \text{and} \quad \delta_{v} = \nu \sin \phi_{\text{edge}} + \omega \cos \phi_{\text{edge}} - u_{\text{joint}}.
$$

Vertical displacement at the joint line would affect the dome by a rigid body motion that creates no additional stresses. The negative sign of $u_{\text{joint}}$ will account for the fact the downward vertical deformation in the dome is positive while it is negative for the shell. See Fig. 6.7 (b).
6.7.7 Checking The Modulus of Elasticity

To assess the change in material properties, the moduli of elasticity in both directions are checked and updated. While solving for the constants of integration, each segment is assumed to have constant Young's moduli in each direction and a constant thickness. The Young's modulus in each direction is estimated as the average Young's modulus between both ends and the thickness is estimated as the average thickness between both ends. Each segment is assumed to have different cross section details at each end in each direction separately. At the end of each iteration, the longitudinal and hoop bending moments at both ends of each segment are calculated. For each section detail, using a layered approach the bending moment-curvature response is calculated and the associated curvature, in each direction, is calculated. See Fig. 6.9. The flexural rigidities, are calculated as the secant slope given as follows;

\[
D_x = \frac{M_x}{\Phi_x}, \quad D_\phi = \frac{M_\phi}{\Phi_\phi} \quad \text{and} \quad D_\theta = \frac{M_\theta}{\Phi_\theta},
\]

where; \(D_x, D_\phi\) and \(D_\theta\) are the flexural rigidities in the longitudinal, meridian and hoop directions respectively, and \(\Phi_x, \Phi_\phi\) and \(\Phi_\theta\) are the associated curvature in the longitudinal, meridian and hoop directions respectively. It is worth to mention that if the applied bending moment at any section is zero, the above equation will yield an infinite value for the flexural rigidity. Instead the uncracked stiffness of the concrete section is calculated and used for this particular case.

Since the secant moduli of elasticity \(E_x, E_\phi\) and \(E_\theta\) were used in the constitutive relation, they are estimated from the bending moment-curvature response as follows;

\[
E_x = \frac{12(1-\nu^2)}{h^3} \cdot \frac{M_x}{\Phi_x}, \quad E_\phi = \frac{12(1-\nu^2)}{h^3} \cdot \frac{M_\phi}{\Phi_\phi} \quad \text{and} \quad E_\theta = \frac{12(1-\nu^2)}{h^3} \cdot \frac{M_\theta}{\Phi_\theta}.
\]

Repeating the previous steps at both ends, two values for the secant moduli of elasticity, in each direction, are calculated. An average values for the moduli of elasticity, in each direction, is estimated as follows;

\[
E_x = \frac{E_{x\text{top}} + E_{x\text{bottom}}}{2}, \quad E_\phi = \frac{E_{\phi\text{top}} + E_{\phi\text{bottom}}}{2} \quad \text{and} \quad E_\theta = \frac{E_{\theta\text{top}} + E_{\theta\text{bottom}}}{2}.
\]

The approximation is acceptable providing that the segments are small enough. If the calculated moduli of elasticity for ALL segments in both directions are within practical
acceptable error; 0.5%, from the initial moduli of elasticity, then the iteration is called successful. If not, new values for the moduli of elasticity are estimated and another iteration is carried out where the new moduli of elasticity are chosen as follows;

\[ E_{\text{new}} = E_{\text{old}} + \alpha \cdot \left( E_{\text{calculated}} - E_{\text{old}} \right), \]

where, \( \alpha \) is a relaxation factor whose magnitude is chosen always between 0 and 1. For \( \alpha = 0 \), it represents a linear elastic analysis with constant moduli of elasticity. For \( 0 < \alpha \leq 1 \), it represents a nonlinear analysis with variable moduli of elasticity under each load increment. It is recommended to use a small value for \( \alpha; 0.05 \leq \alpha \leq 0.10 \). As the analysis approaches the ultimate load, it is recommended to reduce the relaxation factor to achieve a monotonic convergence.
6.8 Conclusions and Further Work

The governing differential equations for the bending of different shells and surfaces of revolutions are presented along with their solutions. The membrane behaviour of the both types of shells is also reviewed. An analysis method using the compatibility approach based on the Classical Theory of Shells modified to account for the nonlinear behaviour of reinforced concrete is presented. The method is capable of tracking the loss of stiffness in the reinforced concrete due to cracking of concrete and yielding of steel reinforcement. The method is limited to the analysis of storage cells subjected to axisymmetric loads only.

It provides the design engineer with quick non-linear analysis of storage cells under axisymmetric loading and an inexpensive alternative to global non linear finite element analysis. The method calculates the deformation and internal forces at critical sections in a storage cell. With many storage cells in a typical GBS platform to check, each may have different reinforcement details, dimensions or material properties, the simplified non linear approach is relatively quicker than the traditional nonlinear global finite element programs while providing accurate and reliable results.

Further work may be carried out to avoid the limitations inherent in this analysis method. The axisymmetrical bending of three different types of shells was covered. However, all shells are assumed to have constant thickness and stiffness. The effect of variable thickness and variable stiffness within the shell segment need to be investigated. The problem is important since it will allow fewer elements to model variable thickness shells. The method is geared towards a flexural failure in the structure. Check against imminent shear failure should be implemented and used as a cut off for the analysis results. Several types of loading may be introduced in the analysis such as own weight, horizontal and vertical uniform hydrostatic pressure, and uniformly distributed vertical and tangential loads.
CHAPTER 7

Program *LNADS*

7.1 General

The closed form solutions from the *Classical Theory of Shells*\textsuperscript{18,19} could not be readily applied to predict the response of the model since they are based on linear elastic behaviour only. Instead program *LNADS*, (Linear and Non-linear Analysis of Domes and Shells) which uses a linear and non-linear compatibility analysis method based on the *Classical Theory of Shells*, is used to investigate the theoretical behaviour of the model. The results of this analysis are discussed and compared with the experimental observations of the model structure namely; the measured and predicted vertical and horizontal deflections. The comparison between the measured and predicted deformation was in good agreement. *LNADS* is geared towards a flexural failure in the model structure while it is not capable of predicting shear failures of the type which may occur at the top of the cell wall. To check against imminent shear failure, the internal forces for the critical section in the cell wall are used to manually check the shear capacity of this section using the *Modified Compression Field* theory for members without web reinforcement. The results are used as a cut off for the analysis method. Based on *LNADS* results a flexural shear failure was predicted at the top of the cell wall in the model structure at 129.0 m head of sea water.
7.2 Analysis using Program LNADS

LNADS, "Linear and Non-linear Analysis of Domes and Shells", is a compatibility analysis program for the bending analysis of reinforced concrete shells and domes under axisymmetric hydrostatic pressure based on the Classical Theory of Shells. The program can account for both geometric and material non-linearities. Appendix [A] represents a user manual, a hard copy of the program LNADS and a sample data file. Detailed checks were carried out on shells with known closed form solution from Ref. [18]. The linear elastic results for internal forces and deformation were in good agreement.

LNADS analyzes storage cells under axisymmetric hydrostatic pressure. The storage cell is made out of one cylindrical shell capped with a spherical dome with or without a hole at the top. The dome is modelled with a fair number of small truncated cones topped with a solid isotropic dome, while the cylindrical shell is modelled with a fair number of short shell segments. Since, the truncated cone and short shell solutions were adopted in the first place; there is no limitation on the minimum height of each segment. Each segment has a constant or variable thickness defined with the thicknesses at both ends, while the analysis is carried out for the average thickness. The material properties in each segment are defined by the secant Young's moduli at each end in the meridian and hoop directions separately. However, during the analysis, the meridian Young's modulus is assumed constant taken as the average of the top and bottom values. Similarly, the hoop Young's modulus is assumed constant taken as the average of the top and bottom values. In addition to the radial water pressure, the prestressing force is modelled as axisymmetric ring force at any joint line between segments.

The deformation along the joint line between any two successive segments are calculated based on the actual thickness in the structure at that joint line. Thus, the membrane case will not give rise to any displacement discontinuity except at the joint line between the upper dome and the cell wall. Any incompatibility in the deformation is eliminated by applying a uniformly distributed horizontal shear force and meridian (or longitudinal) bending moment known as discontinuity stresses. Four compatibility equations are written at each joint line to satisfy the four compatibility conditions. At any joint line, there is one value for the horizontal displacement, and a common tangent to the deflected line. The discontinuity stresses, acting along the joint line are constant (i.e.
horizontal shear force and meridian/longitudinal bending moment). This has resulted in a system of banded simultaneous equations in the coefficients of integration within each element. The higher the number of segments is, the higher the number of equations. However, the band width of the compatibility matrix is constant. Having the coefficients of integration within each segment handy, the distribution of the deformation and stress resultants within the segment are directly calculated.

The program could carry out linear and non-linear analysis depending on the chosen value for the relaxation factor M-Φ. It possesses its own bending moment-curvature routine that calculate the response for various cross sections. The response accounts for the enhancement due to the axial load only in the longitudinal/meridian direction. It does not account for any enhancement or reduction in the moment capacity due to the axial load in the hoop direction. The effect of the material non-linear behaviour is introduced by tracking the M-Φ diagram for the cross sections at the top and the bottom of each segment in each direction. The secant moduli of elasticity in each direction are checked at both ends of the segment. If the estimated average moduli of elasticity are within reasonable practical error from the initial moduli, i.e. 0.5%, for all segments, the iteration is called successful. Otherwise, new moduli of elasticity are estimated for the next iteration based on the initial and the estimated moduli of elasticity and the relaxation factor. The non-linear analysis requires several iterations at each load level to reach an acceptable error in all segments.

The analysis was geared towards a flexural failure. In order to introduce a different mode of failure, the ratios between the applied forces at the critical section at the top of the cell wall near failure are assumed constant. The bending moment-to-shear force ratio and the shear force-to-axial load ratio are determined. These ratios are used to "manually" check against imminent shear failure using the Modified Compression Field theory for members without web "shear" reinforcement.

7.3 **LNADS Model of Tested Structure**

The geometry of the model is shown in Fig. 7.1. A total of 200 elements are used to define the model described as 1 isotropic dome, 105 truncated cone segments and 94 short shell segments used to model the full model structure, down to 133 mm from the base. In
Program LNADS the built-in amounts of reinforcement are described in terms of different element types. A total of 8 different element types were used in the model. See Fig. 7.1. The quantities of reinforcement in each of these element types are given in Table 7.1. Program LNADS extracts the actual thickness of the section from the mesh data. The effect of the ring beam at the joint between the upper dome and the cell wall is introduced as an additional reinforcement in the hoop direction in the 6 elements at the base of the dome and 5 elements at the top of the cell wall from element number 101 to element number 111. The material properties, predefined in program LNADS, for the concrete and the reinforcement steel wires are shown in Fig. 7.2.

The first load applied to the model was intended to simulate the prestressing of the ring beam. The load consisted of axisymmetric radial pressure of 25.0 kN/m applied at the joint line between the upper dome and the cell wall. This was the intersection of the central lines of both portions of the model at elevation 60.0 m between element number 106 and 107. See Fig. 7.1. Additional water pressure loads were then applied to all segments of the model. This water pressure load was increased until the maximum load capacity of the model was reached. Convergence could easily be achieved for water pressures form 0.05 MPa up to 1.45 MPa with 0.05MPa increment but could not be achieved for 1.50MPa. This water pressure of 1.45 MPa, corresponds to a differential pressure of 144.0 m head of sea water.

**Table 7.1 Reinforcement Amounts in Built-in Element Types of LNADS Model**

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Element Location</th>
<th>Longitudinal</th>
<th></th>
<th>Hoop</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Outside (mm²/m)</td>
<td>Inside (mm²/m)</td>
<td>Outside (mm²/m)</td>
</tr>
<tr>
<td>1</td>
<td>Dome</td>
<td>107.5</td>
<td>107.5</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Dome</td>
<td>-</td>
<td>-</td>
<td>107.5</td>
</tr>
<tr>
<td>3</td>
<td>Dome</td>
<td>322</td>
<td>358</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>Cell Wall</td>
<td>215</td>
<td>215</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>Cell Wall</td>
<td>107.5</td>
<td>107.5</td>
<td>107.5</td>
</tr>
<tr>
<td>6</td>
<td>Cell Wall</td>
<td>-</td>
<td>-</td>
<td>322</td>
</tr>
<tr>
<td>7</td>
<td>Cell Wall</td>
<td>322</td>
<td>322</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>Dome</td>
<td>-</td>
<td>-</td>
<td>215</td>
</tr>
</tbody>
</table>
Fig. 7.1 Details of the Geometry of the LNADS Compatibility Model

Used to Predict Response of 1:13 Scale Model
(a) Base curve compressive stress-strain for concrete.

(b) Stress-strain curve for reinforcement.

Fig. 7.2 Stress-Strain Properties of Concrete and Reinforcement Used in LNADS Model
7.4 Measured and Predicted Deflections of the Model Structure

Figure 7.3 presents the predicted patterns of vertical deflections as the applied water pressure is increased. Program LNADS outputs the deflections of the centreline of the model. Using the same simplified presentation of the results, previously used for RASP analysis results in Chapter 5, locations in the structure are defined in terms of the distance along this centreline from the apex of the structure. In this co-ordinate system the joint between the upper dome and the cell wall occurs at 792 mm from the apex. It can be seen from Fig. 7.3 that, the junction deflects downwards by about 0.47 mm at an applied water pressure of 1.45 MPa due to the axial compression in the cell wall. There is no significant change in the deflection response predicted by LNADS as the maximum load was reached at the location of the dome-wall junction.

Figure 7.4 illustrates the predicted patterns of horizontal deflections as the applied water pressure is increased. It can be seen that, due to the hoop compression, the radius of the lower cell wall gets smaller. At a water pressure of 1.45 MPa, the wall is predicted to move inwards by about 0.34 mm which corresponds to a hoop compressive strain of 0.461 mm/m. For lower pressures, a zone of the structure about 300 mm long, (170 mm above the joint + 130 mm below the joint between the dome and the wall) moves outwards rather than inwards. As the maximum pressure is reached, this zone increases in extent to about 435 mm long. The outward deflection of the junction line reaches 0.36 mm at water pressure of 1.45 MPa which corresponds to a circumferential tensile strain of 0.49 mm/m in the ring beam.

As early mentioned, 9 displacement transducers recorded the deflections of the 1:13 scale model during loading, see Fig. 2.12. In Fig. 7.5 the pattern of deflections predicted by the LNADS compatibility analysis method is compared with the pattern of deflections measured by these transducers for an applied water pressure of 1.30 MPa. The measured deflection of the dome at the apex is in excellent agreement with the predicted deflections while they are greater at one-quarter the diameter. Note that in the three locations where the vertical deflection of the dome was measured, the dome is predicted to come down more at the apex than at one quarter the inside diameter. The three measured deflections at these locations were essentially the same. See Fig. 7.5.
Figure 7.6 presents a more comprehensive comparison of the measured and predicted vertical displacements of the dome at the apex. As can be seen from Fig. 7.6 the measured and the predicted vertical deflection by program LNADS at the apex are in excellent agreement up to 0.40 MPa, when the predicted deflections started to deviate significantly resulting in somewhat greater predicted deflection than the measured deflection. This increased deflection may be in part due to the fact that using the secant moduli of elasticity to represent the section stiffness resulted in a less stiffer response than the actual behaviour near the apex. The measured response of the model structure becomes softer as the applied load is increased. Consequently, the difference between the predicted and the measured deflections diminishes. Yet, the predicted deflection remained higher until 1.25 MPa when both of the measured and the predicted deflections are in excellent agreement. Just prior to final failure of the model structure the measured deflection of the model increased higher than the predicted deflection by program LNADS starting from 1.30 MPa. As mentioned earlier, program LNADS assumes the pressure is increased monotonically until flexural failure is reached and does not account for any shear failure in the model. Thus it could not predict the post peak behaviour of the model.

Similarly, Fig. 7.7 presents another comparison of the measured and predicted vertical displacements of the dome at one-quarter of the inside diameter. As can be seen from Fig. 7.7 the slope and the values of the measured and the predicted vertical deflection are in excellent agreement up to 0.80 MPa, during the first pressure test. However, it can be seen that the measured deflections started to increase at the first unloading, labelled 1 in Fig. 7.7, occurred at the end of the first pressure test when the drainage hose buckled. Part of this increased deflection was due to the unloading and reloading of the model. Since that point on, the measured deflection are somewhat greater than the predicted deflection. This increased deflection may also be in part due to the fact that the model structure cracked near the junction between the upper dome and the cell wall. Water pressure inside the cracks preys the cracks open, resulting in increasing the outwards spreading at the base of the dome and the vertical deflection of the dome. The latter is shown as a softer response of the dome in Fig. 7.6. The analytical model assumes the pressure is increased monotonically until failure is reached and does not account for water pressure in the cracks.
To facilitate the next two comparisons between the predicted and measured values, only the “envelope” of experimental values will be given. During testing the model structure was unloaded twice. The first unloading, labelled 1 in Fig. 7.6 and Fig. 7.7, occurred at the end of the first pressure test when the drainage hose buckled. The second major unloading, labelled 2 in Fig. 7.6 and Fig. 7.7, occurred near the end of the second pressure test as the pressure was reduced so that the water reservoir of the pump could be refilled, just prior to the final failure of the structure. The data associated with the two unloading and reloading “loops” shown in Fig. 7.6 and Fig. 7.7 will be omitted. The transitions from one loading line to the next will be shown by dotted lines.

Figure 7.8 presents a comparison between the measured and predicted horizontal spreading at the base of the dome. The measured horizontal deflections near the dome-wall junction were significantly greater than the predicted values during most of the test. Although the initial slopes of the both lines in Fig. 7.8 are in reasonable agreement, the two lines begin to diverge significantly at a pressure of about 0.30 MPa. The influence of water pressure in vertical and radial cracks, which was neglected in the analytical model, may be responsible for part of this discrepancy. Again, water under pressure would prey the cracks open, increasing the spreading of the junction. Also, this increased deflection was partially due to the unloading and reloading of the model. The analytical model does not account for both factors.

In Fig. 7.9, it can be seen that the measured and predicted cell wall deflections are in reasonably good agreement. LNADS predicted that beyond 600 mm from the junction, the cell wall will behave like a long pipe where the inward deflection of the cell wall is essentially constant. The measured deflections increased somewhat from locations 5 to 6 and from location 6 to 7 and then reduced from location 7 to 8 while the predicted deflection for all 4 locations were essentially the same. See Fig. 7.9. Presumably the steel base plate restrained somewhat the inward deflection of lower portions of the cell wall. Program LNADS assumed that the base of the cell wall is restricted from movement in the longitudinal direction while it has a free end condition in the radial direction. Thus it predicts that the four locations will essentially have the same deflection. See Fig. 7.9.
Fig. 7.3 Pattern of Vertical Deformation with Increasing Applied Water Pressure Predicted by LNADS Model.
Fig. 7.4 Pattern of Horizontal Deformations with Increasing Applied Water Pressure Predicted by LNADS Model
Deflected shape predicted by LNADS analysis

Measured deflection at transducer locations

Magnification Factor of Displacement = 75

Fig. 7.5 Comparison of Observed and LNADS Predicted Deflections at Applied Pressure of 1.30MPa. Deflections Measured at Displacement Transducers Locations
Fig. 7.6 Measured and LNADS Predicted Vertical Displacement at the Apex of the Dome.
Fig. 7.7 Measured and LNADS Predicted Vertical Displacement at One-quarter the Diameter of the Dome.
Experimental Result

Displacement Plotted = \frac{(\Delta o + \Delta 3)}{2}

Fig. 7.8 Measured and LNADS Predicted Spreading of the Dome.
Fig. 7.9 Measured and LNADS Predicted Contraction of the Cell Wall.
7.5 Internal Forces Predicted by LNADS

Program LNADS calculates the value of the deformation and internal forces at the boundaries between segments, one-quarter the height from each end of the segment and the mid-height of each segment. Figures 7.4, 7.10, 7.11 and 7.12 present the predicted patterns of the horizontal deflections, the slope of the deflected line, the horizontal shear force and the meridian bending moment respectively as the applied water pressure is increased. It can be seen that, all four variables are continuous within each segment and across the inter-segments boundaries. These are the four prime variables that dictated the compatibility conditions.

The hoop strain in the middle surface is calculated directly from the horizontal displacement resulting in continuous function within each segment and across the inter-segments boundaries. The meridian membrane force and radial shearing force in the upper dome are continuous within each segment and across the inter-segments boundaries because of the equilibrium of forces based on the horizontal shear force transmitted between segments. The longitudinal membrane force in the cell wall is constant along the cell wall due to the equilibrium of forces in the longitudinal direction. Figure 7.13 presents the predicted patterns of the longitudinal / meridian membrane force as the applied water pressure is increased.

The longitudinal strain then the hoop membrane force are calculated based on the longitudinal membrane force and the hoop strain resulting again in continuous values within each segment and across the inter-segments boundaries. Figure 7.14 presents the predicted patterns of the hoop membrane force as the applied water pressure is increased.

Similarly, the hoop bending moment is calculated based on the continuous results of the meridian bending moment and the slope angle of the deflected line. Thus the hoop bending moment is continuous within each segment and across the inter-segments boundaries. Figure 7.15 present the predicted patterns of the hoop bending moment as the applied water pressure is increased.
Fig. 7.10 Pattern of Slope of the Deflected Line with Increasing Applied Water Pressure Predicted by LNADS Model
Fig. 7.11 Pattern of Horizontal Shear force with Increasing Applied Water Pressure Predicted by LNADS Model
Fig. 7.12 Pattern of Meridian/Longitudinal Bending moment with Increasing Applied Water Pressure Predicted by LNADS Model
Fig. 7.13 Pattern of Longitudinal Membrane Force with Increasing Applied Water Pressure Predicted by LNADS Model
Fig. 7.14 Pattern of Hoop Membrane Force with Increasing Applied Water Pressure Predicted by LNADS Model
Fig. 7.15 Pattern of Hoop Bending Moment with Increasing Applied Water Pressure Predicted by LNADS Model
7.6 Check on Shear Strength

Program LNADS is geared towards a flexural failure in the model structure loaded until failure under monotonically increased water pressure. It is not capable of predicting shear failures of the type which may occur at the top of the cell wall. Thus, before the results of the LNADS analysis of the model structure can be used, a careful examination is made of the shear capacity of the critical section at the top of the cell wall.

While the calculations for the shear capacity described later will be carried out for a specific section it is recalled that a shear failure does not occur at just one section but rather, the diagonal failure spreads over a length of the member about $d, \cot \theta$ in extent, where $\theta$ is the inclination of the diagonal crack to the longitudinal axis of the member and $dv$ is the shear depth. Thus, the calculations for a specific cross section may be taken as representing a length of the member $d, \cot \theta$ long, with the calculated section being in the middle of this length. Thus, near the junction with the dome the first section that could be checked is the section $0.5 d, \cot \theta$ from the face of the junction. As a simplification, the length $0.5 d, \cot \theta$ is taken as $dv$ when determining the location of this critical section.

The shear depth $dv$ is given as $dv = d - a/2$, where $d$ is the distance between the most stressed fibre in the concrete compression block and the location of the tensile flexural steel reinforcement and $a$ is the height of the compression block associated with the yielding of flexural steel reinforcement. For the cell wall of the model structure; $d = 46.7$ mm, $a = 1.4$ mm and $dv = 46.0$ mm.

To check against imminent shear failure, the ratios between the applied forces at the critical section at the top of the cell wall are assumed constant near failure. The ratios between the applied forces namely; the bending moment-to-shear force "$M/(V, dv)$" ratio and the shear force-to-axial load "$V/N$" ratio are shown in Fig. 7.16 for three sections in the model cell wall; namely at the El. 60.0 m, at $dv/2$ and $dv$ from the junction. It can be seen from Fig. 7.16 a that the bending moment-to-shear force ratio is fairly constant at $0.75 dv$ near failure and from Fig. 7.16 b, it can be seen that the shear force-to-axial load ratio $= 0.34$ and it is essentially constant near failure.
Fig. 7.16 Predicted Ratio of Applied Forces at the Top of the Cell Wall by LNADS
If the tensile stresses in the reinforced concrete are neglected, then members with no web “shear” reinforcement, like the scale model, are predicted to have no shear resistance. The Modified Compression Field theory\textsuperscript{12,22,23,24} accounts for the tensile stresses in the concrete between cracks. Thus members with no shear reinforcement will have a considerable post cracking shear strength depending on the deformation of the member. The shear stress to cause failure of the section will depend on the applied axial load and associated bending moment. The axial load and bending moment are carried by the concrete compressive stresses and the longitudinal reinforcement stresses near the top and the bottom chords of the section, while the shear force is carried by a field of compression in the concrete web. The diagonal compressive struts in the cracked concrete pushes the top and bottom chords while the tensile stresses in the cracked concrete pulls them together. The compression force in the web $N_t$ must be balanced by tensile stresses in the longitudinal reinforcement thus increasing the applied axial load acting on the section. The shear stresses are assumed constant over the shear depth, $d_s$, and that the nominal shear strength of members without stirrups is expressed as

$$ V_n = \beta \sqrt{f'_c} \ b_w \ d_s $$  \hspace{1cm} (7-1)

where $b_w d_s$ is the effective shear area and $\beta$ is a factor that indicates the ability of the cracked concrete to transmit shear. The factor $\beta$ is a function of the longitudinal strain, $\varepsilon_x$, within the shear area, the crack spacing parameter, $s_x$, the bending moment at the section $M$, taken as positive, the axial load $N$, taken as positive for tension and negative for compression, the area of reinforcement on the flexural tension side $A_t$.

Before the analysis is carried out, a decision should be made where the location of the longitudinal strain $\varepsilon_x$ is taken within the shear depth. Increased longitudinal strain reduces the shear capacity of the section. Sections with web reinforcement possess a considerable shear capacity and an ability to redistribute the stresses from the highly stressed areas to the low stressed areas in the section. Thus the longitudinal strain $\varepsilon_x$ may be chosen at the mid depth of the section. However, since sections with no web reinforcement do not possess such ability, it will be conservative to take the longitudinal strain $\varepsilon_x$ at the most highly stressed location in the section, i.e. the location of the flexural tensile steel reinforcement.
The crack spacing parameter, \( s_x \), can be taken as equal to the distance between the layers of longitudinal reinforcement. Thus, for the cell wall of the model structure, \( s_x \) would be 37.4 mm (1.47 inches). The compressive axial load \( N \) applied on the critical section will affect the angle of the inclined cracks \( \theta \). The higher the compressive stresses, the lower "flatter" the angle \( \theta \). Using a plane section analysis, with the strain at the flexural tensile steel reinforcement set to \( \varepsilon_x \), the strain distribution corresponding to the applied bending moment \( M \) and the total axial load \( N_p \) are found. The total axial load has to balance the applied axial load \( N \) and an additional tensile force \( N_v \) required to balance the horizontal compressive stresses in the cracked concrete web; i.e. \( N_p = N + N_v \).

The failure shear stress-axial compressive stress interaction diagram, for the model cell wall, predicted by the Modified Compression Field theory, is shown in Fig. 7.17. Table 7.2 presents a summary of the calculations results. This relationship has been calculated for the section \( d_v = 46 \text{ mm} \) below the junction. The bending moment-to-shear force ratio according to the LNADS analysis is estimated at \( 0.75 d_v \) near failure. Thus, this ratio is used in the calculations. It can be seen from Fig. 7.17 that as the axial compressive stress is applied, there is a substantial increase in the predicted shear strength of the cell wall. For the shear stress-to-axial compression ratio calculated by program LNADS, the cell wall of the model structure is predicted to be able to resist a differential water pressure of 129.0 m head of sea water.
Fig. 7.17 Shear Strength-Axial Compression Interaction Diagram for Model Cell Wall

$fc' = 75\, MPa$

$ft = 4.17\, MPa$

$M/(V.dv) = 0.75$
Table 7.2  Predicted Shear Strength of a Critical Section $dv$ from the Top of the Model Cell Wall

dv = 46.0 mm,
s_r = 37.4 mm,
$\sigma_y' = 75$ MPa, $f_y = 4.17$ MPa,
$As = As' = 215.5 \text{ mm}^2$/m.

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CHAPTER 8

Application of Test Results to Prototype

8.1 General

The results of the water pressure test on the 1:13 scale model of the storage cell loaded to failure could not be applied directly to the prototype unless the issue of the size effect in shear is addressed. There is clear evidence from cores drilled in the prototype structure that the concrete cover zone may only be about two-thirds as strong as the concrete core inside the reinforcing cage. There are some concerns about how sensitive the strength of the dome-wall connection is to the strength of the cover concrete. The failure mode of the model structure was clear, with adequate signs of visible radial cracks near the dome-wall junction. The question now is whether or not the prototype structure would undergo a similar behaviour while the peak pressure was maintained. This chapter is aimed at shedding some light on the three questions posed above.
8.2 Size Effect in Shear

Before the results of the experiment on the 1:13 scale model of the storage cell can be applied to the full sized structure a careful examination must be made of the so-called size effect in shear. This “effect” refers to the observation that in many situations the shear stress to cause failure of large reinforced concrete members is lower than the shear stress to cause failure of geometrically similar but smaller members.

While the calculations, described later, are performed for a particular section it should be appreciated that a shear failure does not occur at just one section but rather, the diagonal failure spreads over a length of the member about \( d \cdot \cot \theta \) in extent, where \( \theta \) is the inclination of the diagonal crack to the longitudinal axis of the member and \( dv \) is the shear depth. Thus, the calculations for a particular section can be taken as representing a length of the member \( d \cdot \cot \theta \) long, with the calculated section being in the middle of this length. Thus, near the junction with the dome the first section to be checked is the section \( 0.5d \cdot \cot \theta \) from the face of the junction. As a simplification, the Modified Compression Field theory\(^{12,13,23} \) suggests the length \( 0.5d \cdot \cot \theta \) be taken as \( d \), when determining the location of this critical section. The shear depth “\( dv \)” is given as \( dv = d - a/2 \), where \( d \) is the distance between the top fibre in the concrete compression block and the location of the tensile flexural steel reinforcement and \( a \) is the height of the concrete compression block associated with the yielding of flexural steel reinforcement. For the prototype cell wall; \( d = 625 \text{ mm}, \ a = 19.0 \text{ mm} \) and \( dv = 615 \text{ mm} \). It will be recalled that the shear depth for the model cell wall is \( dv = 46 \text{ mm} \).

Using the results obtained from the compatibility analysis program LNADS to predict the behaviour of the model structure, described in Chapter 7, the comparable ratios between the applied forces at a distance \( dv \) from elevation 60.0 m in the prototype structure are calculated. The axial force applied at the section was calculated from vertical equilibrium while the ratios between the applied loads namely; the bending moment-to-shear force “\( M/(V \cdot dv) \)” ratio and the shear force-to-axial load “\( V/N \)” ratio remained the same for the prototype and the model structures. It will be recalled that the bending moment-to-shear force ratio is fairly constant at \( 0.75 \, dv \) near failure and that the shear force-to-axial load ratio \( \approx 0.34 \) and is essentially constant near failure. Hence, these
ratios were used to calculate the comparable applied loads on the critical section in the prototype structure. See Fig. 7.16.

In the modified compression field theory\textsuperscript{12,13} the size effect in shear is accounted for by the crack spacing parameter, $s_x$. The nominal shear strength of members without stirrups is expressed as

$$V_n = \beta \sqrt{f_c} b_w d_v$$  \hspace{1cm} (8-1)

where $b_w d_v$ is the effective shear area and $\beta$ is a factor that indicates the ability of the cracked concrete to transmit shear. The factor $\beta$ is a function of the highest longitudinal strain, $\varepsilon_x$, that occurs within the shear area; i.e. at the location of the flexural tensile steel reinforcement, and the crack spacing parameter, $s_x$. As $s_x$ becomes larger, $\beta$ value becomes smaller. The crack spacing parameter, $s_x$, can be taken as equal to the distance between the layers of longitudinal reinforcement. Thus, for the cell wall of prototype structure, $s_x$ would be 520 mm (20.5 inches), while for the model structure, $s_x$ would be 37.4 mm (1.47 inches).

The failure shear stress - axial compressive stress interaction diagrams, for both the prototype cell wall and the model cell wall, calculated from the Modified Compression Field theory shear strength expressions, are shown in Fig. 8.1 and are summarized in Table 8.1 for the prototype structure and Table 7.2 for the model cell wall. These relationships have been calculated for the section $d$, below the junction. At this location the moment-to-shear-force ratio according to program LNADS is about $0.75d$, and hence, this ratio was used for the predictions. From Fig. 8.1 it can be seen that the predicted shear strengths when the axial compressive stress is zero indicate somewhat a small size effect. For this case, the prototype wall is predicted to fail at a shear stress 90% as large as the failure shear stress for the model wall. However, as axial compression is applied, the size effect diminishes. For the shear-force-to-axial-compression ratio calculated by program LNADS, the prototype wall is predicted to be able to resist a differential water pressure of 137.0 $m$ head of sea water, while the model cell wall is predicted to be able to resist a differential water pressure of 129.0 $m$ head of sea water. These differential water pressures correspond to a shear stress in the prototype cell wall that is 6.2% higher than the model cell wall.
Predicted Shear Strength of the Cell Wall by the Modified Compression Field theory:

- For Prototype Structure
- For Model Structure

Shear Stress at Failure [MPa]

Compressive Stresses $f_x$ [MPa]

Experiment: 147.0 m
Predicted Failure: Prototype 137.0 m
Predicted Failure: Model 129.0 m

$fc' = 75$ MPa
$ft = 4.17$ MPa
$M/(V.dv) = 0.75$

Fig. 8.1 Shear Strength-Axial Compression Interaction Diagram for Cell Wall
Application of Test Results to Prototype

Table 8.1  Predicted Shear Strength of a Critical Section dv from the Top of the Prototype Cell Wall

<table>
<thead>
<tr>
<th>dv</th>
<th>dN/</th>
<th>V</th>
<th>M</th>
<th>m</th>
<th>V</th>
<th>N</th>
<th>F</th>
<th>M</th>
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<td>119.2</td>
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<td>0.480</td>
<td>5.33</td>
<td>-0.1459</td>
</tr>
</tbody>
</table>
It is predicted by the above calculations that for the load combinations experienced by the cell wall, the “size effect” in shear will not be significant. It is of interest to note that a similar conclusion was reached by Lenschow and Hofsøy in 1976. They conducted a series of experiments to investigate the shear strength of dome-cell wall connections in Condeep structures. For a range of wall thicknesses varying from 145 mm to 470 mm they concluded that, for specimens tested under a combination of high compression and shear, the size effect was negligible.

8.3 Concrete Strength

At the time of testing the model structure the average cylinder crushing strength of the concrete from which it was made was 75 MPa. It is estimated that the average cylinder crushing strength of the concrete in the prototype structure is 69 MPa. However, there is evidence from cores drilled in the shaft splash zone in 1994 that the concrete cover zone may only be about two-thirds as strong as the concrete in zones inside the reinforcing cage. It has been speculated that this difference in strength may be due to the construction practices used in early Condeep structures, which may have resulted in the cover concrete receiving inadequate vibration. The question then becomes how sensitive the strength of the dome-wall connection is to the strength of the concrete cover.

8.3.1 TRIX Finite Element Models

The irregular shape and geometry of the Condeep corner and the rapid change of strains in the vicinity of the upper dome-cell wall junction make the use of the classical engineering beam theory inadequate as a design tool. Instead, the nonlinear finite element analysis provides a valuable tool in the design of such disturbed regions. To gain insight into the question posed above, a series of analyses were conducted on a TRIX model of the dome-wall connection. TRIX is a nonlinear finite element program developed at the University of Toronto. The formulation of program TRIX is found in the Ref. 25 and 26 by Vecchio. Program TRIX is based on an iterative, secant stiffness formulation and uses the biaxial stress-strain formulations of the Modified Compression Field theory. Program TRIX utilizes a 4-node with 8-degree-of-freedom plane stress rectangle (PSR) and a 3-node with 6-degree-of-freedom plane stress triangle (PST). The reinforcement may be modelled
as discrete truss elements extending between nodes or as distributed “smeared” ratio of reinforcement within a group of elements. Program TRI/X accounts for the beneficial effect of biaxial compression and allows for a variable Poisson’s ratio.

Figure 8.2 presents the concrete model of the junction between the upper dome and the cell wall of a storage cell of Brent B platform. The model consists of 308 plane stress rectangles (PSR) and 304 plane stress triangles (PST). The reinforcement near both faces of the cell wall and the upper dome were represented using the smeared approach, while the reinforcement crossing the junction was modelled using 22 discrete truss elements. The bottom face of the mesh was restrained against any displacement. This two-dimensional, non-linear finite element model had been formulated as part of an earlier study on 1:2.14 two dimensional strips to study the effect of reinforcement detailing and loading paths on the strength of the dome-cell wall connection of Brent B platform.

In this new series of analyses, which used TRI/X-97, the concrete strength was varied. The first analysis, BRNET-75, was conducted with a uniform concrete strength of 75 MPa, while the second, BRNET-50, used a uniform concrete strength of 50 MPa. In the third analysis, BRNET-75/50, the elements that represented the concrete cover were assigned a strength of 50 MPa while the rest of the elements were assigned a strength of 75 MPa. The associated concrete tensile strength for the 75 MPa concrete was taken as 2.86 MPa, while it was taken as 2.33 MPa for the 50 MPa concrete. Both types of concrete were assumed high strength concrete with a zero maximum aggregate size.

Figure 8.3 illustrates the material characteristics used in all three models. The crack spacing factor for all models was taken as the distance between the reinforcement layers. Thus for the cell wall using the TRI/X Cartesian coordinate system, it is $s_y = 520$ mm.

The applied water pressure was modelled by a series of nodal loads on selected nodes on the outside face of the cell wall and the top face of the upper dome. The loading ratio suggested by program LNADS at El. 60.0 m was used as a reference location for section shear force, bending moment and axial load. The applied forces corresponding to 100 m head of sea water are given in Fig. 8.4. The applied loads in the analyses were incremented in steps representing 5 m head of sea water. It is worth to mention that the applied forces on the model at different critical section did not deviate by more than 2.8% from the loading ratios suggested by program LNADS at these locations.
Application of Test Results to Prototype

Fig. 8.2 TRIX-97 Finite Element Model for the Condeep Corner of Brent B
Fig. 8.3 Element Material Characteristics Used in TRIX-97 Models for the Condeep Corner of Brent B
Equivalent sectional forces at El. 60.0 m

\[ V = 2681 \text{ kN} \]
\[ M_x = 2302 \text{ kN.m} \]
\[ N_x = 5226 \text{ kN} \]

All loads are in kN applied to 1000 mm wide strip of the Condeep corner in Brent B storage cell correspond to a depth of 100 m head of sea water.

Fig. 8.4 Nodal Loads Applied to TRIX-97 Finite Element Models for the Condeep Corner of Brent B
8.3.2  *TRIX-97 Analyses Results*

Figure 8.5 illustrates the pattern of strains in the *BRENT-75* model as the applied water pressure increases. The *BRENT-75* model was predicted to fail between an equivalent differential water pressure of 145 m and 150 m head of sea water through a flexural shear failure at the top of the cell wall. The cell wall was predicted to crack at a water depth of 25 m head of sea water. The concrete compressive stresses on the inside face at the top of the cell wall (i.e., elm. 220) reached its highest value of 86 MPa and the corresponding principal compressive strain of -2.79 mm/m at a water depth of 150 m head of sea water. With the next load increment to 155 m, the principal compressive stresses decreased to 84.7 MPa while the principal compressive strain continued to increase to -3.33 mm/m indicating a crushing of the concrete at that location. Since *TRIX-97* accounts for the beneficial effect of biaxial compression, it allows the strength to exceed the strength of fc' = 75 MPa assumed for the concrete. Yielding of flexural tensile reinforcement (i.e., elm. 222) was predicted to commence between water depth of 140 m and 145 m. Between 145 m and 150 m, the strain in several elements representing the flexural tensile reinforcement on the outside face of the cell wall (i.e. elm. 212 and below) was beyond yielding. The flexural reinforcement on the inside face of the cell wall (i.e., elm. 219) was predicted to yield at 180 m. In the load increment from 145 m and 150 m a large increase occurred in the shear strain in the web elements below the top of the cell wall (i.e. elm. 185). By 155 m the shear strain in several elements in the cell wall web experienced the same large increase (i.e. elm. 196). The yielding of the reinforcement on the flexural tension side accompanied with the increase of shear strain suggested a flexural shear failure in the cell wall. Figure 8.6 illustrates the predicted cracking pattern, the deflected shape and the principal compressive stress distribution of *BRENT-75* model at 145 m head of sea water.

Figure 8.7 illustrates the pattern of strains in the *BRENT-50* model as the applied water pressure increases. The *BRENT-50* model was predicted to fail between an equivalent differential water pressure of 120 m and 125 m head of sea water through a flexural shear failure at the top of the cell wall. As with the analysis of *BRENT-75*, the cell wall was predicted to crack between a water depth of 20 m and 25 m head of sea water.
The concrete compressive stresses on the inside face at the top of the cell wall (i.e. elm. 220) reached its highest value of 57.4 MPa and the corresponding principal compressive strain of -2.44 mm/m at a water depth of 110 m head of sea water. With the next load increment to 115 m, the principal compressive stresses decreased to 56.7 MPa while the principal compressive strain continued to increase to -2.83 mm/m indicating a crushing of the concrete at that location. Yielding of flexural tensile reinforcement (i.e. elm. 222 and elm 212) was predicted to commence between 125 m and 130 m of water. By 130 m the strain in several elements representing the flexural tensile reinforcement on the outside face of the cell wall (i.e. elm. 192 and below) was well beyond yielding. The reinforcement on the inside face of the cell wall (i.e. elm. 219) was predicted to yield between 140 m and 145 m. In the load increment from 115 m to 120 m a large increase occurred in the shear strain in the web elements below the top of the cell wall (i.e. elm. 185). By 125 m the shear strain in several elements in the cell wall web experienced the same large increase (i.e. elm. 196). The yielding of the flexural reinforcement on the tension side accompanied with the increase of shear strain suggested a flexural shear failure in the cell wall. Figure 8.8 illustrates the predicted cracking pattern, the deflected shape and the compressive stress distribution of BREN'T-50 model at 115 m.

Figure 8.9 illustrates the pattern of strains in the BREN'T-75/50 model as the applied water pressure increases. The BREN'T-75/50 model was predicted to fail between a differential water pressure of 130 m and 135 m head of sea water through a flexural shear failure at the top of the cell wall. Similar to the former analyses, the cell wall was predicted to crack at a water depth of 25 m head of sea water. The concrete compressive stresses on the inside face at the top of the cell wall (i.e., elm. 220) reached its highest value of 57.1 MPa at a water depth of 115 m head of sea water and a corresponding principal compressive strain of -2.45 mm/m. With the next load increment to 120 m, the principal compressive stresses decreased to 55.8 MPa while the principal compressive strain continued to increase to -2.90 mm/m indicating a crushing of the concrete at that location. It will be recalled that this is the element with the reduced concrete compressive stresses. However, the concrete principal stresses on the second highly stresses element (i.e., elm. 219) reached 83.9 MPa at 150 m head of sea water and an associated principal compressive strain of -2.74 mm/m. With the next load increment to 155 m, the principal
compressive strains continued to increase and the principal compressive stresses decreased to -79.6 MPa indicating crushing of the concrete core. It will be recalled that this is the first element within the high strength concrete core near the inside face of the cell wall. Thus, the concrete in the highly stressed element in the high strength concrete core crushed after the maximum load carrying capacity was reached. Yielding of flexural tensile reinforcement (i.e. elm. 222 and elm. 212) was predicted to commence between 125 m and 130 m of water. By 135 m the strain in several elements representing the flexural tensile reinforcement on the outside face of the cell wall (i.e. elm. 192 and below) was beyond yielding. The flexural reinforcement on the inside face of the cell wall (i.e. elm. 219) was predicted to yield between 150 m and 155 m. In the load increment from 125 m to 130 m a large increase occurred in the shear strain in the web elements below the top of the cell wall (i.e. elm. 185). With the next load increment to 135 m, several web elements exhibit the same increase in the shear strain (i.e. elm. 196). Again, the yielding of the reinforcement on the flexural tension side accompanied with the increase of shear strain suggested a flexural shear failure in the cell wall. Figure 8.10 illustrates the predicted cracking pattern, the deflected shape and the compressive stress distribution of BREN'T-75/50 model at 125 m head of sea water.

In viewing Figs. 8.6, 8.8 and 8.10, it is worth to note that the plots do not predict the actual spacing or number of cracks in the actual structure since the cracks are dictated mainly by the mesh size. It can be seen that the cracks form a fan-like pattern near the corner with the cracks pointing towards the inner corner. The cracks that run parallel to the concrete surface indicate splitting in the concrete due to high compressive straining. The thick line cracks indicate where the principal tensile strains exceed 2.0 mm/m. The deflection shape clearly indicates that most of the distortions are close to the base of the junction where the bulging indicates a flexural shear failure. The zone of high compressive stresses is shown in the stress contour plot indicating the line of thrust forces in the models.

The results of the three analyses are summarized in Table 8.2. When diagonal cracks form in the cell wall the thickness of the wall increases. The resulting "bulging strain" is thus a good indicator of an impending shear failure. Fig. 8.11 shows the predicted relationship between the average shear stress in the cell wall located about $dv$ from the junction with the dome and the bulging strain (i.e., the expansion) of elm. 153.
Fig. 8.5 Variation of Predicted Concrete Strains at the Top of the Cell Wall by TRIX-97 for BREN-75 Model.
Fig. 8.6 Predicted Crack Pattern, Deflection and Stress Conditions in BRENT-75 Model at 140m.
Fig. 8.7 Variation of Predicted Concrete Strains at the Top of the Cell Wall by TRIX-97 for BRENT-50 Model.
Fig. 8.8 Predicted Crack Pattern, Deflection and Stress Conditions in BRENT-50 Model at 115m.
Fig. 8.9 Variation of Predicted Concrete Strains at the Top of the Cell Wall by TRIX-97 for BREAT-75/50 Model.
Application of Test Results to Prototype

Fig. 8.10 Predicted Crack Pattern, Deflection and Stress Conditions in BREN'T-75/50 Model at 125 m.

Deflected Shape

Displacement Magnification: X-20

Crack Pattern

Principal Strain
thin < 1.0 mm/m
tight > 2.0 mm/m

Principal Compressive Stresses

< -52.25
< -44.50
< -36.76
< -29.00
< -21.25
< -13.50
< -5.76
< 2.00

199
Fig. 8.11 Influence of Concrete Strength on the Predicted Shear Strength of Cell Wall.
Application of Test Results to Prototype

Table 8.2 Summary of TRIX-97 Nonlinear Analyses Results of the Condeep Corner

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<tr>
<th>Analyses Case</th>
<th>Water Head</th>
<th>Concrete Compressive strain $\varepsilon_c$ [mm/m]</th>
<th>Flexural Reinforcement Strain $\varepsilon_f$ [mm/m]</th>
<th>Web Shear Strain $\gamma_{sv}$ [mm/m]</th>
<th>Bulging Strain $\varepsilon_b$ [mm/m]</th>
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<td>2.27 2.22</td>
<td>1.74 0.35</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td>125 m</td>
<td>-3.85 -1.59</td>
<td>2.47 2.36</td>
<td>3.19 1.93</td>
<td>3.28</td>
</tr>
</tbody>
</table>

8.3.3 Conclusion

Based on the results of the three analyses, it can be seen that reducing the concrete compressive strength from 75 MPa to 50 MPa (and the associated concrete tensile strength from 2.86 MPa to 2.33 MPa) has reduced the load carrying capacity of the cell wall from 147 m to 122 m or by about 20%. If only the cover concrete is reduced in strength the predicted load carrying capacity is reduced to 132 m which corresponds to a mere 11% reduction. The final failure for the high strength concrete model does not involve crushing of the cover concrete near the inner face of the cell at the dome-wall junction but this crushing occurs after large deformations have occurred at load levels dictated by yielding of the reinforcement and cracking of the core concrete. In contrast, the final failure was preceded by the weakening of the compression zone at the inner corner of the junction as the concrete crushed at this location for the 50 MPa concrete model and the model with the reduced cover strength prior to yielding of the reinforcement and diagonal cracking of the core concrete. For the latter model with the high strength concrete core and weaker concrete cover, the crushing of the core concrete occurs after large deformations have occurred due to yielding of the flexural tensile reinforcement and diagonal cracking of the core concrete. For this lightly reinforced section the shear failure is triggered by yielding of the vertical reinforcement near the outer face of the cell wall and by the spreading of diagonal cracks into the core of the cell wall. Neither of these factors is critically influenced by the strength of the concrete cover. It is conservative to assume a reduction of 11% of the predicted load carrying capacity of the cell wall to account for the adverse effect of the reduced strength of the cover concrete.
8.4 Failure Mode of the Storage Cell

In the pressure test of the model structure the first noticeable sign of failure was the occurrence of visible radial cracks near the dome-wall junction. As these cracks opened, a large amount of water began to flow through the concrete dome and cell wall into the storage cell. Just prior to the violent shear failure of the cell wall, which terminated the test, the water was flowing through the cracks at a rate of about 235 litres per minute. See Fig. 8.12. This rate would have been enough to fill the empty model storage cell in about 7 minutes. However, even with the reduction in pressure differential from the peak of 147 m of water head to 127 m of water head, it only took about 2 minutes to fail the specimen as the pressure was increased again from 127 m to 140 m.

If the prototype structure undergoes a comparable amount of cracking as the model did prior to the final shear failure, the leakage rate for the prototype would be about 530 cubic metres per minute, which would fill the storage cell at a rate of about 2 metres every minute. However, in the actual structure the applied external pressure will not reduce as significant leakage starts. If the peak pressure is maintained then, rather than failing in two minutes as the model did, the prototype may fail just a few seconds after significant leakage commences.

![Water Inflow Rate Graph](image)

*Fig. 8.12 Net Water Flow into the Pressure Vessel during the Second Test*
8.5 Comparison of Loading Estimates and Strength Estimates for Brent B Storage Cells

Several analytical studies and several experimental studies have now been completed, which address the question "Will the storage cells of the Brent B platform fail if the pressure differential across these cells is substantially increased because of a failure in the system that maintains the internal pressure?" In addition, some 20 years ago a number of significant studies were made to address a very similar concern for the Statfjord A platform. Figure 8.13 is an attempt to highlight the significant findings of a number of these studies.

Shown in Fig. 8.13 is an estimate of the maximum pressure differential that the storage cells of the Brent B platform are likely to experience at the dome-cell wall junction under current operating conditions. This 49.2 m head of water is arrived at by subtracting the internal pressure of a 45 m head of water from the external pressure of a 94.2 m head. The external pressure is composed of an 84.2 m static head, a 0.8 m allowance for subsidence, a 5.5 m allowance for a 100-year wave, a 2.7 m allowance for surge and a 1.0 m allowance for the caisson effect.

In the event that a failure in the internal piping system causes the internal pressure in a storage cell to drop to atmospheric pressure, the pressure differential across the cell wall would increase to 94.2 m. If the draw down of the storage cell occurred with no leakage then it is theoretically possible that a full vacuum could develop inside the dome, which would increase the pressure differential by about a 10 m head of water. Thus, after a failure of the internal piping it is possible that the pressure differential across the wall could reach a maximum value of 104.2 m of water.

An analytical study by Norwegian Contractors (NC) in 1986\textsuperscript{27}, using the non-linear finite element program ABAQUS, concluded that the maximum pressure differential that the storage cells of Brent B could safely resist was 88 m. The 1996 analytical study by Det Norske Vertias (DnV)\textsuperscript{8}, using the non-linear finite element program SOLVIA, concluded that if the concrete strength was about 50 MPa, the storage cells would fail when the pressure differential reached 127 m.
Fig. 8.13 Comparison of Predicted Loads and Predicted Strengths of Brent B Storage Cell
In 1994, experimental studies were conducted at the University of Toronto\textsuperscript{10} on 1:2.14 scale models of a slice of the structure near the dome-wall connection. With a concrete strength of 50.2 MPa, specimen \textit{DWC1}, which was a very close model of the \textit{Brent B} dome-wall connection, resisted a shear force in the cell wall comparable to that caused by a 158 m head of water.

The 1:13 scale model of a \textit{Brent B} storage cell that was loaded under water pressure at the \textit{University of Toronto} in October 1996 resisted a pressure differential equivalent to a water head of 147 m prior to failure. Immediately prior to failure, radial cracks could be seen near the base of the dome. Water was flowing through these cracks at a rate of 235 liters/min. The final failure was abrupt and involved the shear failure of about one half of the circumference of the cell wall, while the upper surface of the dome in the same location showed evidence of concrete crushing. In these locations the dome displaced about 25 mm downwards and about 10 mm outwards.

The global finite element program \textit{RASP} predicted that the maximum load carrying capacity of the model structure would be reached between 1.30 MPa and 1.35 MPa, corresponding to a differential head between 129 m and 134 m head of sea water through a flexure-shear failure in which yielding of the flexural tension reinforcement triggers a shear failure.

Program \textit{LNADS} that applies the simplified nonlinear analysis method based on the \textit{Classical Theory of Shells} modified to account for the nonlinear behaviour of reinforced concrete suggested a flexural shear failure at the top of the cell wall at 129 m head of sea water for the model structure.

Using the non-linear finite element program \textit{TRIX-97} to analyze a two-dimensional strip of the dome-wall connection, concluded that if the concrete strength was about 75 MPa, the storage cells of the prototype structure would fail when the pressure differential reached 147 m through a flexural shear failure.

In 1976 a 1:5 scale model of a storage cell of the \textit{Statfjord A Condeep} platform was loaded to failure by lowering it into the sea. The test was part of the \textit{Marbet} project\textsuperscript{4} conducted by the Norwegian Concrete Research Institute, FCB. The model had a cylindrical wall with an outside diameter of 4.0 m and a thickness of 160 mm, giving a
diameter-to-thickness ratio of 25. For Brent B, this ratio was 27.4. One end of the Statford model was capped with a dome 550 mm high and 108 mm thick, giving a cell-diameter-to-dome-rise ratio of 7.3 and a dome-thickness-to-cell-wall-thickness ratio of 0.68. For Brent B these ratios were 5.7 and 0.71. The bottom of the model, which was about 12 m high, was closed with a 1 m thick slab. The concrete strength was about 60 MPa. The model was lowered into the North Sea in 10 m load increments with a 30 minute load stage being taken at each increment, for data observations. After testing the model failed when the net water pressure applied on the upper dome-cell wall connection was equivalent to a water head of 164 m. The failure was preceded by a substantial leakage of water into the model, which increased the air pressure inside the unvented model. The final failure was explosive and occurred near the top of the cell wall resulting in the disappearance of the top 150 mm to 200 mm of the cell wall. The failure was identified as a shear failure of the top of the cell wall.

The very similar failure behaviour of the 1:13 Brent B model and the 1:5 Statford A model and the rather similar failure loads for all three of the experimental models discussed above, mean that we can predict with reasonable confidence that the Brent B storage cells will fail at a pressure differential equivalent to a water head of about 150 m. With the maximum possible pressure differential being calculated as 104.2 m, the above estimate of strength implies a factor of safety against this type of failure of 1.4. In view of the very low probability of occurrence of the 10 m head associated with a full vacuum, particularly in view of the large water leakage, prior to failure, experienced by both the Statford A model and the Brent B model, it is believed that this factor of safety is acceptable.
CHAPTER 9

Summary, Conclusions and Further Work

9.1 Summary and Conclusions

The research work carried out under the scope of this thesis was aimed towards developing a better understanding of the behaviour of reinforced concrete offshore structures under axisymmetric hydrostatic pressure. An evaluation process that can be applied both theoretically and experimentally to evaluate the strength of existing offshore structures was developed. The theoretical approach was a simplified nonlinear analysis based on the Classical Theory of Shells modified to account for the nonlinear behaviour of concrete, while the experimental approach was achieved by bringing the Hydraulic Testing Facility at the University of Toronto into its full operational capacity. The evaluation process was applied to an early Condeep structure in the North Sea by constructing a 1:13 prestressed concrete scale model of a storage cell in the prototype then testing it until failure under monotonically increasing hydrostatic pressure in the upgraded facility. The theoretical behaviour of the model was investigated using the theoretical approach correlated with a nonlinear global finite element program. Before the results from the evaluation process could be applied to the prototype several factors such as the size effect, the effect of concrete strength and the mode of failure were addressed. Based on the experimental and analytical work carried out the following conclusions can be drawn:
1. The Hydraulic Testing Facility, HTF, at the University of Toronto was substantially upgraded bringing it into its full operational capacity. The most prominent component of the facility consists of a steel pressure vessel that can safely withstand an internal pressure of 275 psi gauge, that is, a pressure differential equivalent to a water head of about 190 m. The hydraulic pressure system of the facility consisted of a 25 HP, variable frequency pump capable of delivering 40 gallons per minute (182 litres per minute) at a pressure of 275 psi (1.90 MPa). Flow meters were installed in the pipe by which water entered the pressure vessel and in the pipe by which water could drain from the inside of the test specimen. Based on the experimental investigation, it is concluded that the facility is capable of carrying out tests on models of reinforced concrete offshore structures under hydrostatic pressure. It is believed that through the application of various load combination, achievable only in a pressurized environment, the design relationships for reinforced concrete subjected to hydrostatic pressure can be enhanced, resulting in well designed, safe and economical offshore structures.

2. The closed form solutions developed by the Classical Theory of Shells were not ideally suited for predicting the highly non-linear behaviour of reinforced concrete. A compatibility approach based on the bending analysis of thin shells and the membrane theory modified to account for the nonlinear behaviour of cracked concrete was used to investigate the theoretical behaviour of offshore structures under axisymmetric water pressure. The simplified nonlinear analysis method is based on an iterative procedure and uses the secant modulus of elasticity to account for the loss of stiffness in the reinforced concrete and tracks the actual bending moment-curvature response of different sections. Based on the theoretical investigation, it is concluded that the relationships for membrane behaviour and bending of shells derived from the Classical Theory of Shells was successfully modified to account for the nonlinear behaviour of reinforced concrete due to cracking and yielding of reinforcement.

3. The design and construction of a 1:13 prestressed concrete scale model of a typical storage cell of a gravity base offshore structure in the North Sea was carried out successfully. Gauge 12 galvanized steel wires were used in the form of welded wire fabric and as individual wires for reinforcing steel. The amount of reinforcement in the
Conclusions and Further Work

The model in the critical section at the top of the cell wall was about 7% less than the corresponding amount in the prototype structure. It is believed that the average cylinder strength of the prototype is about 69 MPa which is about 9% lower than the 75 MPa average concrete strength in the model. One external 7 mm diameter high strength wire located at the top of the cell wall was used to simulate the post-tensioned tendons. The total prestress force in the model was only about 32% of that required to simulate the prestress in the prototype reducing the capacity of the model by a pressure differential of about 10 m. It is concluded from the case study that it is possible to design and construct a scale model of a reinforced concrete offshore structure in a laboratory environment even if the arrangements of the reinforcement in the prototype structure are very complex. The choice of a 1:13 scale ratio was governed by the available space in the test facility. This ratio preserved all the structural aspects of the prototype but reduced the model to a manageable size for the laboratory.

4. The model structure was loaded to failure in the Hydraulic Testing Facility. As the water pressure reached 137 m head of sea water (200 psi), radial cracks were visible at the base of the dome. At an applied pressure of 147 m (215 psi) the amount of water flowing into the pressure vessel increased greatly and the applied pressure dropped to 127 m (186 psi). As the pressure increased again and reached 140 m (205 psi) a loud "thump" was heard from the model followed by a gentle rocking of the whole pressure vessel. The final failure was abrupt and involved the shear failure of the northwest half of the circumference of the cell wall, while the concrete in compression on the inside surface of the model at the junction showed clear signs of crushing. In these locations the dome displaced about 25 mm downwards and about 10 mm outwards. When the model structure was cut along an east-west diameter to expose the internal cracks, it was evident that the west side of the cell wall failed in shear, while the east side of the wall did not experience any cracks at that location. The shear failure in the cell wall had started in an uncracked zone of the cell wall, and was inclined by 32° to the longitudinal axis of the cell wall pointed towards the junction between the upper dome and the cell wall. It is concluded from the pressure test carried out on the model...
structure that the mode of failure of the storage cell is an abrupt and violent flexural shear failure at the top of the cell wall. Although immediately prior to failure, water was flowing into the model at a rate of 235 liters/min. that would have filled the storage cell in about 7 minutes, it took less than 2 minutes to fail the model. It is concluded that it would not be prudent to rely on water leaking fast enough into the actual structure prior to failure to reduce the differential pressure on the dome.

5. Based on program RASP results, the model structure was predicted to fail between a differential head of 129 m and 134 m head of sea water through a flexure-shear failure in which yielding of the flexural tension reinforcement triggers a shear failure. Both the vertical tensile strain on the outside face and the shear strain across the thickness at the top of the wall were predicted to increase substantially between 129 m and 134 m suggesting that both flexural cracking and diagonal cracking will occur at that location. A substantial increase in the deflections of the dome is predicted as the pressure is increased from 129 m and 134 m associated with significant kinking of the model at the dome-wall joint indicating crushing of the concrete at that location. It is concluded that the ultimate load carrying capacity of the model was predicted reasonably well by program RASP. Yet program RASP was less successful in predicting the abrupt failure of the scale model and instead it provided a long post peak ductility plateau due to the fact that it is a load control program. It should be appreciated that in program RASP, like in all global finite element programs, a finer mesh was needed to enhance the quality of the results. Due to the RASP formulations, it was also concluded that a mesh with a transition curved elements at the junction would have yielded better results.

6. Good agreement was obtained between the measured values from the pressure test for the vertical deformation in the dome and the radial inwards deformation in the cell wall and the predicted results from program LNADS. The ratios between the applied loads at the top of the cell wall were used to check against imminent shear failure using the Modified Compression Field theory suggesting a flexural shear failure at the top of the cell wall at 129 m head of sea water for the model structure. The compatibility method based on the Classical Theory of Shells, modified to account for the nonlinear behaviour of
Conclusions and Further Work

Reinforced concrete can provide the design engineer with a quick and an inexpensive alternative for the analysis of storage cells under axisymmetric loading. It is concluded that program LNADS, that applies the simplified nonlinear analysis method, can provide accurate predictions for the deformation and internal forces at critical sections in a storage cell. With many storage cells in a typical GBS platform to check, each one having different reinforcement details, dimensions or material properties, program LNADS has the capability to perform simplified nonlinear analysis relatively more quickly than the available nonlinear global finite element programs while providing accurate and reliable results.

7. The measured horizontal spreading at the base of the dome indicated a softer response than what was predicted by both analytical models namely; RASP and LNADS. Based on the current state of the knowledge, it is believed that part of this increased deflection was due to the adverse effect of water pressure on the cracks and in the concrete pores. Neither analytical model accounts for the detrimental effect of water pressure on the cracks or in the concrete pores.

8. The possibility of testing a full-scale model of a Condeep structure is unrealistic. Before applying the results of the pressure test on the 1:13 scale model to the prototype structure the size effect in shear was examined using the Modified Compression Field theory. Under zero axial compressive stresses it was predicted that the prototype cell wall would fail at a shear stress 90% as large as the failure shear stress for the model wall. However, as axial compression is applied, the size effect diminishes. For the load combination calculated by program LNADS, the prototype wall is predicted to be able to resist a differential water pressure of 137.0 m head of sea water. It is concluded that for the load combinations experienced by the cell wall, the size effect in shear will be insignificant.

9. Evidence from cores drilled in the shaft splash zone of the prototype in 1994 suggest that the concrete cover zone may only be about two-thirds as strong as the concrete core inside the reinforcing cage. To gain insight about how sensitive the strength of the dome-wall connection is to the strength of the concrete cover, a parametric analysis, using program TRIX-97, was carried out using a two-dimensional strip
Conclusions and Further Work

loaded with the same load combination predicted by program LNADS at El. 60.0 m as a reference point. Based on the results of the analyses, it is concluded that reducing all of the concrete's compressive strength from 75 MPa to 50 MPa will reduce the load carrying capacity of the cell wall from 147 m to 122 m or about 20%. If only the cover concrete is reduced in strength the predicted load carrying capacity is reduced to 132 m which corresponds to a mere 11% reduction in capacity. For this lightly reinforced section the shear failure is triggered by yielding of the vertical reinforcement near the outer face of the cell wall and by the spreading of diagonal cracks into the core of the cell wall. Although neither of these factors is critically influenced by the strength of the concrete cover, it is conservative to assume a reduction of 11% of the predicted load carrying capacity of the cell wall to account for the adverse effect of the reduced strength of the cover concrete.

10. The maximum pressure differential that the storage cells of the prototype platform at the dome-cell wall junction under current operating condition is 49.2 m head of sea water. In the event that a failure in the internal piping system causes a full vacuum in a storage cell the pressure differential across the wall could reach a maximum value of 104.2 m. The strength evaluation carried out within the scope of this thesis concludes that, based on the results of the pressure test on the model structure and of other experimental and analytical studies, the factor of safety against a failure of the storage cells, in the event of a loss of the internal pressurization system, will be about 1.4.

9.2 Further Work

The Effect of Water Pressure in the Cracks

Design formulae used in the design of offshore concrete structures are based on relations derived from specimens tested in air and at ambient temperature although this is not the environment under which the concrete is serving. It is believed that the introduction of water under pressure into the cracked concrete may significantly reduce the strength of a structure. The wedging action of water under pressure would reduce the axial load applied at the critical sections and would accelerate yielding of reinforcement. Reducing
the axial load means that the predicted response of the cracked section will not reflect the actual behaviour of that section. Also, water under pressure may lubricate the crack surfaces and widening them altering the ability of a cracked concrete surface to transmit shear across the crack significantly and may degrade the reinforcement bond at the crack location.

In order to investigate the effect of water pressure on the cracked concrete surfaces, three identical reinforced concrete specimens should be tested in the Hydraulic Testing Facility at the University of Toronto. Each cube-shape specimen, see Fig. 9.1, would be 1220 mm wide capped with a square flat slab of the same dimension. The bottom of the cube, which is 1370 mm long, would match the circular base plate inside the pressure vessel with an outer diameter of 1778 mm and 150 mm thick. The hollow cube-like specimen would have a constant thickness of 100 mm and would be designed to avoid flexural failure at all corners and in the center of all the slabs. The concrete used may be a high strength concrete since most of the concrete used in offshore structure belongs to that category.

The first specimen would be tested in the usual manner that was illustrated in the report for testing the scale model structure, referred to later as the “bare model”. In the second specimen, water could be prevented completely from leaking through the cracked concrete surface by placing an impermeable membrane on the concrete surface referred to latter as the “membrane model”. The third specimen will be tested similar to the first specimen except that air pressure would be used rather than water pressure referred to latter as the “air model”.

The difference between the “bare model” and the “membrane model” will illustrate the total effect of water under pressure in the cracks on the actual response i.e., the lubricating and wedging effects combined. The difference between the “bare model” and the “air model” will illustrate only the lubricating effect of water in the cracks. The difference between the “air model” and the “membrane model” will illustrate only the wedging effect of water under pressure in the cracks. If this research program were to be carried out successfully the total effect of water pressure on the cracks could be more fully understood.
Fig. 9. 1 Typical Specimen to Study the Effect of Water Pressure in the Cracks
REFERENCES


11. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-95) and Commentary ACI 318R-95," American Concrete Institute, Detroit, 1995, 369 pp.


APPENDIX A

Program LNADS

A.1 General

Program LNADS "Linear and Nonlinear Analysis of Domes and Shells" analyzes storage cells under axisymmetric loads. It is a FOR-77 program that runs on IBM and compatible’s personal computers. Program LNADS consists of 18 subroutines. The theoretical derivations could be found in Chapter 6 while this appendix includes details of the input data file, a list of the output data files and a hard copy of the program.

A.2 Input Data File

Program LNADS accepts one data file for the whole structure called "DATA.DAT". The maximum number of elements in both of the upper dome and the cell wall may not exceed 200 elements. As mentioned before, there are 8 types of built in reinforcement areas in the program. The data file consists of 5 major input data blocks. Data in any block may be separated with spaces or tabs only. Do not enclose any commas or slashes at the end of any data line. The data should be entered in the following order, Job Title, Geometrical Data Block, Mesh Data Block, Dome Elements Data and Shell Elements Data. All variables should use "mm" for lengths, radius and thicknesses, "degrees" for angles, "MPa" for stresses, pressure and Young's moduli, and "N/mm" for radial forces.
Job Title Block:

Line 1  C1
Variable Description  Job title
Format  \{A72\}
Type  \{Alphanumeric string\}

Geometrical Data Block:

Line 2  C2
Variable Description  Geometrical data title
Format  \{A72\}
Type  \{Alphanumeric string\}

Line 3  RAD, FIC, FIO, XLNG, EI, XNU, Q, JTOP, JBOT

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<th>Description</th>
<th>Type</th>
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<td>RAD</td>
<td>Upper dome radius</td>
<td>Real</td>
</tr>
<tr>
<td>FIC</td>
<td>Central angle at the junction</td>
<td>Real</td>
</tr>
<tr>
<td>FIO</td>
<td>Central angle of the top opening</td>
<td>Real</td>
</tr>
<tr>
<td>XLNG</td>
<td>Length of the cell wall</td>
<td>Real</td>
</tr>
<tr>
<td>EI</td>
<td>Initial Young's modulus of concrete</td>
<td>Real</td>
</tr>
<tr>
<td>XNU</td>
<td>Poisson's ratio</td>
<td>Real</td>
</tr>
<tr>
<td>Q</td>
<td>Unit weight of concrete</td>
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</tr>
<tr>
<td>JTOP</td>
<td>End condition at the top of the dome</td>
<td>Integer</td>
</tr>
<tr>
<td>JBOT</td>
<td>End condition at the base of the cell wall</td>
<td>Integer</td>
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</table>

In the case of solid dome, $FIO = 0$ and $JTOP = 0$, while for a dome with a whole at the top, $JTOP = 1$ for free end condition. The end condition at the base of the cell wall $JBOT = 1$ for hinged, $JBOT = 2$ for fixed, $JBOT = 3$ for free or $JBOT = 4$ for symmetric.

Mesh Data Block:

Line 4  C3
Variable Description  Mesh data title
Format  \{A72\}
Type  \{Alphanumeric string\}

Line 5  NELED, NELES, NELTP, ERR, ALFA, IURLS, INX, FACINI, FACFIN, FACSTP

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<td>NELED</td>
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<td>Integer</td>
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<tr>
<td>NELES</td>
<td>Number of elements in the cell wall</td>
<td>Integer</td>
</tr>
<tr>
<td>NELTP</td>
<td>Number of elements type ($\leq 8$)</td>
<td>Integer</td>
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<tr>
<td>ERR</td>
<td>Percentage of Maximum allowable error (0.5)</td>
<td>Real</td>
</tr>
<tr>
<td>ALFA</td>
<td>Relaxation factor ($0 \leq \alpha \leq 1$)</td>
<td>Real</td>
</tr>
<tr>
<td>IURLS</td>
<td>Relaxation identifier (1, 2, 3 or 4)</td>
<td>Integer</td>
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Again, the total number of elements in the upper dome and the cell wall do not exceed 200 elements. The relaxation identifier allows the user to specify the convergence to be carried in the longitudinal / meridian direction alone with IURLS=2, or in the hoop direction alone with IURLS=3, or for both directions simultaneously with IURLS=4 (recommended). Note that when IURLS=1, it is a linear elastic analysis with no check on the stiffness degradation while the relaxation factor ($\alpha$) should be set to $\alpha = 0$. The Axial load identifier allows the user to account for the effect of axial load on the initial membrane deformation in the cell wall INX=1 (recommended) or to neglect it with INX=0.

### Dome Elements Data Block:

**Line 5**

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<td>FACINI</td>
<td>Initial load factor</td>
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<tr>
<td>FACFIN</td>
<td>Final load factor</td>
<td>Integer</td>
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<tr>
<td>FACSTP</td>
<td>Load factor step</td>
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<td>Maximum number of iteration</td>
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**Line 6**

<table>
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<td>IELE</td>
<td>Element number</td>
</tr>
<tr>
<td>XLNG1</td>
<td>Element central angle</td>
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<td>THK1</td>
<td>Top and bottom thicknesses of the dome element</td>
</tr>
<tr>
<td>THK2</td>
<td>of the dome element</td>
</tr>
<tr>
<td>WP</td>
<td>Applied water pressure</td>
</tr>
<tr>
<td>H</td>
<td>Hoop force at the bottom of the element</td>
</tr>
<tr>
<td>EFI1</td>
<td>Meridian Young's moduli at the top</td>
</tr>
<tr>
<td>EFI2</td>
<td>and the bottom of the element</td>
</tr>
<tr>
<td>ETH1</td>
<td>Hoop Young's moduli at the top</td>
</tr>
<tr>
<td>ETH2</td>
<td>and the bottom of the element</td>
</tr>
<tr>
<td>IETFI1</td>
<td>Element type in the meridian direction</td>
</tr>
<tr>
<td>IETFI2</td>
<td>at the top and bottom of the element</td>
</tr>
<tr>
<td>IETTH1</td>
<td>Element type in the hoop direction</td>
</tr>
<tr>
<td>IETTH2</td>
<td>at the top and bottom of the element</td>
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</tbody>
</table>
There is one line 6 for each element in the upper dome and the elements should be in ascending order starting from 1 to NELED. Each element may be subjected to a different water pressure and a different hoop force at the bottom edge. No element generation is permitted.

**Cell Wall Elements Data Block:**

**Line 7**  
**Variable Description**  
Cell wall elements title  
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<td>Top and bottom thicknesses</td>
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<td>THK2</td>
<td>of the short shell element</td>
<td>Real</td>
</tr>
<tr>
<td>WP</td>
<td>Water pressure</td>
<td>Real</td>
</tr>
<tr>
<td>H</td>
<td>Hoop force at the bottom of the element</td>
<td>Real</td>
</tr>
<tr>
<td>EX1</td>
<td>Longitudinal Young's moduli at the top and the bottom of the element</td>
<td>Real</td>
</tr>
<tr>
<td>EX2</td>
<td>Longitudinal Young's moduli at the top and the bottom of the element</td>
<td>Real</td>
</tr>
<tr>
<td>ETH1</td>
<td>Hoop Young's moduli at the top and the bottom of the element</td>
<td>Real</td>
</tr>
<tr>
<td>ETH2</td>
<td>Hoop Young's moduli at the top and the bottom of the element</td>
<td>Real</td>
</tr>
<tr>
<td>IETX1</td>
<td>Element type in the longitudinal direction at the top and bottom of the element</td>
<td>Real</td>
</tr>
<tr>
<td>IETX2</td>
<td>Element type in the hoop direction at the top and bottom of the element</td>
<td>Real</td>
</tr>
<tr>
<td>IETFH1</td>
<td>Element type in the hoop direction at the top and bottom of the element</td>
<td>Real</td>
</tr>
<tr>
<td>IETFH2</td>
<td>Element type in the hoop direction at the top and bottom of the element</td>
<td>Real</td>
</tr>
</tbody>
</table>

There is one line 8 for each element in the cell wall and the elements should be in an ascending order starting from 1 + NELED to NELED + NELES. Each element may be subjected to a different water pressure and a different hoop force. Again, no element generation is permitted.

### A.3 Output Data File

Program **LNADS** pipes out two different results files for each load increment, both are in the ASCII format. The data used in the analysis is printed out in “data0000.dat”, while the analysis results at the end of the last iteration is printed out in “rslt0000.res”, where
“0000” identifies the load factor used in the analysis. The data file has the same format as the original input data file “DATA.DAT”, thus any new analysis at a higher load can be carried out by simply changing the initial, final and increment load factor. The data file provides the user with the maximum number of iterations used to reach these results, the error encountered in each element, whether it is a converged or unconverged load stage and the most probable reason for the unconverged results. The result file contains the displacement and internal forces in the upper dome and the cell wall calculated at the top, bottom, mid-height and quarter points of each element. All output results are in “mm” for deformation, lengths and thicknesses, “degrees” for angles, “radians” for the slope angles, “mm/mm” for longitudinal and hoop strains of the middle surface, “MPa” for stresses, water pressure and Young’s Moduli, “N/mm” for internal forces and “N.mm/mm” for internal bending moments.

A.4 Hard Copy of Program LNADS

A hard copy of Program LNADS is printed. The program was written in FOR77 and consists of a main routine and 18 subroutines, totaling 1800 lines of instructions.
**Program LNA**

```fortran
C ####################################################################################################################
C # PROGRAMME NAME LNA.DS FOR V5.00 #
C # Linear & Nonlinear Analysis of Domes & Shells #
C # BENDING ANALYSIS OF CYLINDERS AND #
C # SPHERICAL DOMES UNDER EXTERNAL WATER PRESSURE. #
C # BASED ON THE CLASSICAL THEORY OF SHELLS #
C ####################################################################################################################
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION FG*(800,11), RS*(804,2),
   XONGLI*(200), THK*(200,2), WP*(200), HH(200),
   EX*(200,2), EFI*(200,2), IETX*(200,2), IETFH*(200,2),
   FA*(8,9), R(14), C(4), S(8)
CHARACTER B2*18, B3*80, B4*18, B5*80
CHARACTER C1*80, C2*20, C3*20, C4*20, C5*20
CALL GDATINC1,
   C2, RAD, FIC, FIO, XLONG, EI, XNU, Q, JTOP, JBOT,
   C3,NELED,NELES,NELEP,ERR,ALFA,IURLS,INX,FACINI,FACFIN,
   FACSTP,ITEM, C4, XONGLI, THK, WP, IETX, IETFH, EX, EFI, H,
   C5, INERFL)
IF (INERFL.NE.0) THEN
WRITE (*,9010) 'ERROR ENCONTERED IN DATA FILE', INERFL.
GOTO 30
END IF
WRITE (*,9010) 'NO errors encountered in DATA file'
WRITE (*,9020) C1
ITE = 0
NELE = NELED + NELES
B2 = 'Load Factor #'
B4 = 'Iteration #'
DO 20 FAC = FACINI, FACFIN, FACSTP
WRITE (*,9030) B2,FAC
20 ITE = ITE + 1
DO 12 J = 1, 4 * NELE
   FG(I,J) = 0D0
12 CONTINUE
CALL FLEXD (RAD, FIC, FIO, Q, EI, XNU, NELED, NELES,
   ALFA, IURLS, FAC, JTOP, JBOT,
   XONGLI, THK, WP, EX, EFI, HI, REC, FG, RS, FA, S)
CALL FLEXS (RAD, FIC, XLONG, Q, EI, XNU, NELED, NELES,
   ALFA, IURLS, INX, FAC, JTOP, JBOT,
   XONGLI, THK, WP, EX, EFI, HI, REC, FG, FA, S)
CALL BINDINV (NELE, FG, RS, FA)
DO 14 J = 1, 4 * NELE
   FG(I,J) = 0D0
14 CONTINUE
IF(ALFA.EQ.0.0 OR. IURLS.EQ.1) THEN
   IYN = 2
```
C  #希望能看见我输入的代码
C  #### General Data Input Routine ####
C  #希望能看见我输入的代码
  SUBROUTINE GDATIN(C1,
  . C2, RAD, FIC, FIO, XLNG, EI, XNU, Q, JT, JBOT,
  . C3, NELED, NELES, NELTP, ERR, ALFA, IURLS, INX, FACINI, FACFIN, FACSTP, ITEM,
  . C4, XLNG1, THK, WP, IETX, IETF, EX, EFI, H,
  . C5, INERFL)
  IMPLICIT DOUBLE PRECISION (A-H, O-Z)
  DIMENSION XLNG1(200), THK(200), WP(200),
  IETX(200,2), IETF(200,2), EX(200,2), EFI(200,2), H(200)
  CHARACTER C1*80, C2*20, C3*20, C4*20, C5*20
  INERFL = 0
  OPEN (UNIT=31, FILE="DATA.DAT")
  READ (31, 90000) C1
  READ (31, 90100) C2
  READ (31,*) RAD, FIC, FIO, XLNG, EI, XNU, Q, JT, JBOT
  READ (31,90100) C3
  READ (31,*) NELED, NELES, NELTP, ERR, ALFA, IURLS, INX,
  . FACINI, FACFIN, FACSTP, ITEM
  IF (RAD .LE. 0.0) INERFL = 1
  IF (FIC .LE. 0.0) INERFL = 2
  IF (FIO .GT. 0.0 .OR. FIO .GE. FIC) INERFL = 3
  IF (FIO .LT. 0.0 .OR. FIO .GE. FIC) INERFL = 3
  IF (XLNG .LE. 0.0) INERFL = 4
  IF (EI .LE. 0.0 .AND. ALFA .GE. 0.0) INERFL = 5
  IF (XNU .GT. 0.0 .OR. XNU .GT. 0.5) INERFL = 6
  IF (Q .LT. 0.0) INERFL = 7
  C  End Conditions
  C  JT, JBOT: HINGED=1, FIXED=2, FREE=3, SYMMETRIC=4
  IF (JT .NE. 0 .AND. JT .NE. 1) INERFL = 8
  IF (JT .NE. 0 .AND. FIO .EQ. 0.0) INERFL = 8
  IF (JBOT .NE. 1 .AND. JBOT .NE. 2 .AND. JBOT .NE. 3 .AND. JBOT
  . .NE. 4) INERFL = 9
  IF (NELED .LT. 1) INERFL = 11
  IF (NELES .LT. 1) INERFL = 12
  IF (NELED+NELES .GT. 200) INERFL = 10
  IF (NELTP .EQ. 0 .AND. ALFA .NE. 0.0) INERFL = 13
  IF (NELTP .GT. 200) INERFL = 13
  IF (ERR .LT. 0.0 .OR. ERR .GT. 10000) INERFL = 14
  IF (ERR .EQ. 0.0 .AND. ERR .GT. 0.0) INERFL = 13
  IF (ALFA .LT. 0.0 .OR. ALFA .GT. 1.0) INERFL = 15
  IF (IURLS .NE. 1 .AND. IURLS .NE. 2 .AND. IURLS .NE. 3 .AND. IURLS .NE. 4)
  . . INERFL = 16
  IF (ALFA .NE. 0.0 .AND. IURLS .EQ. 1) INERFL = 16
  IF (INX .NE. 0 .AND. INX .NE. 1 .AND. INX .NE. 2) INERFL = 17
  IF (FACINI .LT. 0.0 .OR. FACINI .GT. 99) INERFL = 18
  IF (FACFIN .LT. FACINI .OR. FACFIN .GT. 99) INERFL = 19
  IF (FACSTP .GT. (FACFIN-FACINI)) INERFL = 19
  IF (ITEM .GT. 0 .OR. ITEM .GT. 400) INERFL = 20
  C  DOME Elements Data
  READ (31, 90100) C4
  X = FIO
  IF (FIO .EQ. 0.0) X = 0.0
  DO 1010 I = 1, NELED
  CALL ELM(I, NELTP,
  . XNLI(I), THKL(I,1), THKL(I,2), WP(I), H(I),
  . EXL(I), EXL(I,2), EFI(I,1), EFI(I,2),
  . IETX(I,1), IETX(I,2), IETF(I,1), IETF(I,2),
  . . X, INERFL, 30)
  1010 CONTINUE
  IF (X .NE. FIC) INERFL = 30
  C  SHELL Elements Data
  READ (31, 90100) C5
  X = 0.0
  DO 1020 I = NELED+1, NELES+NELED
  CALL ELM(I, NELTP,
  . XNLI(I), THKL(I,1), THKL(I,2), WP(I), H(I),
  . EXL(I), EXL(I,2), EFI(I,1), EFI(I,2),
  . IETX(I,1), IETX(I,2), IETF(I,1), IETF(I,2),
  . . X, INERFL, 40)
  1020 CONTINUE
  IF (X .NE. XLNG) INERFL = 40
  IF (ALFA .EQ. 0.0 .OR. IURLS .EQ. 1) GOTO 1050
  1050 CLOSE (UNIT=31, STATUS=\KEEP)
  9000 FORMAT (A80)
  9010 FORMAT (A20)
  RETURN
  END
Program LMADS

SUBROUTINE ELM(N, NELTP, XLNG1, THK1, THK2, WP, HI, EX1, EX2, EFI1, EFI2, IETX1, IETX2, IETFI1, IETFI2, X, INERFL, I)  
IMPLICIT DOUBLE PRECISION (A-H, O-Z)  

C Data for Non-Linear Analysis  
READ(31,*) IELE, XLNG1, THK1, THK2, WP, HI,  
& EX1, EX2, EFI1, EFI2,  
& IETX1, IETX2, IETFI1, IETFI2  
IF (IELE .NE. N) INERFL = 1  
IF (XLNG1 .LE. 0.0) INERFL = 1  
IF (THK1 .LE. 0.0) INERFL = 1  
IF (THK2 .LE. 0.0) INERFL = 1  
IF (EX1 .LE. 0.0) INERFL = 1  
IF (EX2 .LE. 0.0) INERFL = 1  
IF (EFI1 .LE. 0.0) INERFL = 1  
IF (EFI2 .LE. 0.0) INERFL = 1  
IF (IETX1 .LE. 0.0 OR. IETX1 .GT. NELTP) INERFL = 1  
IF (IETX2 .LE. 0.0 OR. IETX2 .GT. NELTP) INERFL = 1  
IF (IETFI1 .LE. 0.0 OR. IETFI1 .GT. NELTP) INERFL = 1  
IF (IETFI2 .LE. 0.0 OR. IETFI2 .GT. NELTP) INERFL = 1  
X = X + XLNG1  
RETURN  
END

C Null model data routine C
C
C Subroutine for spherical Domes routine C
C
SUBROUTINE FLEXD(RAD, FIC, Fig, Q, E, XNU, NELED, NELES,  
& ALFA, IURLS, FAC, JTOP, JBOT,  
& FI, THK, WP, EFI, ETII, HI, REC, FG, RS, FA, S)  
IMPLICIT DOUBLE PRECISION (A-H, O-Z)  
DIMENSION FG(800, 11), HI(200), RS(804, 2),  
& FI(200), THK(200, 2), WP(200),  
& EFI(200, 2), ETFI(200, 2), FA(8, 8), CS(4)  
IF (ALFA .NE. 0.0 OR. IURLS .NE. 1)  
& OPEN (UNIT=32, FILE=COEFF.DAT)  
2001 PI = 3.141592653589793238462644D0  
DR = PI / 180D0  
IELE = 0  
REC = 0D0  
FI2 = FI  
IF (Q EQ. 0D0) THEN  
IELE = 1  
DO 2007 J = 1, 8  
S(I) = 0D0  
DO 2007 J = 1, 4  
FA(I, J) = 0D0  
2007 CONTINUE  
THKV = THK(1,2)  
EFIV = EFI(1,2)  
ETHV = ETII(1,2)  
X = 0D0  
CALL DOME (1, RAD, XNU, X, FA, THKV, EFIV, ETHV)  
X = FI(1) * DR  
W = WP(1) * FAC  
FNFI0 = -W * RAD / 2D0  
FNT0 = -(W * Q * THKV) * RAD / FNFI0  
CALL DOME (2, RAD, XNU, X, FA, THKV, EFIV, ETHV)  
REC = -W * PI * (RAD * DSIN(X))**2D0  
- 2D0 * Q * THKV * PI * RAD**2D0 * (1D0 - DCOS(X))  
FNFI1 = REC / (2D0 + PI * RAD * (DSIN(X))**2D0)  
FNT1 = -(W * Q * THKV * DCOS(X)) * RAD - FNFI1  
S(5) = FNT1 / (EI * THKV) - XNU * FNFI1 / (EI * THKV)  
S(8) = -H(I) / RAD  
IF (FI2 EQ. FIC) S(8) = -(H(I) + FNFI * DCOS(FIC * DR)) / RAD  
IF (ALFA .NE. 0.0 OR. IURLS .NE. 1) THEN  
WRITE (32, 9050) (FA(I, J), J=1,4), I=1,8  
WRITE (32, 9050) S(1), FNFI, S(5), FNFI  
END IF  
CALL GBLMT(1, FG, FA, S, JTOP, 3, NELED, 0)  
FI2 = FI(IELE)  
IF (NELED .EQ. 1) GOTO 2050  
END IF
IN1 = IELE + 1
DO 2020 IN1 = IN1, NELED
DO 2021 I=1,8
S(I) = 0D0
DO 2021 J=1,4
FAJ(J) = 0D0
2021 CONTINUE
W = WP(IELE) * FAC
FI = F12
F12 = F11 + FI(IELE)
APEX = 90D0 - (FI + F12) / 2D0
THKV = (THK(IELE,1) + THK(IELE,2)) / 2D0
EFIV = (EFI(IELE,1) + EFl(IELE,2)) / 2D0
ETHV = (ETHI(IELE,1) + ETHI(IELE,2)) / 2D0
CALL CONE(1, RAD, XNU, FI, F12, 0, APEX, FA,
THKV, EFIV, ETHV, X)
X = FI1 * DR
FNFI = REC / (2D0 * PI * RAD * (DSIN(X))**2D0)
FNTHI = -(W + Q * THKV * DCOS(X)) / RAD - FNFI
FNFI0 = FNFI
S(I) = (FNTHI + XNU*FNFI/EI) / THK(IELE,1)
XI = X
CALL CONE(2, RAD, XNU, FI, F12, 4, APEX, FA, THKV, EFIV, ETHV, X)
X = FI2 * DR
REC = REC
- W * PI * RAD**2D0 * (DSIN(X)**2D0 * (DSIN(XO)**2D0)
- 2D0 * Q * THKV * PI * RAD**2D0 * (DCOS(XO) - DCOS(X))
FNFI = REC / (2D0 * PI * RAD * (DSIN(X))**2D0)
FNTHI = -(W + Q * THKV * DCOS(X)) / RAD - FNFI
S(5) = (FNTHI + XNU*FNFI/EI) / THK(IELE,2)
S(6) = X(IELE) / RAD
IF (F12.EQ.FIC) S(6) = -(THI(IELE) + FNFI * DCOS(FIC*DR)) / RAD
IF (ALFA.NE.0.0 OR JUR.LS.NE.1) THEN
WRITE (32, 9050) (FA(I, J), J=1,4, I=1,8)
WRITE (32, 9050) S(5), FNFI0, S(5), FNFI
END IF
CALL GLBLMT(IELE, FG, FA, S, JTOP, 3, NELED, 0)
2020 CONTINUE
EPSO = -S(5)
IQ = S(5)
DO 2022 I = 1, 8
S(I) = 0D0
2022 CONTINUE
OPEN (UNIT=31, FILE="FG.DAT")
WRITE (31, 9060) (FG(I, J), J=1,8, I=1, 4, NELED)
CLOSE (UNIT=31, STATUS=\"KEEP\")
CALL BNDINV (NELED, FG, RS, FA)
DO 2025 I = 1, 4, NELED
RS(I, 2) = RS(I, 1)
8X(I, 1) = 0D0
2025 CONTINUE
DO 2030 I = 1, 4, NELED
DO 2030 J = 1, 11
FG(L, J) = 0D0
2030 CONTINUE
OPEN (UNIT=31, FILE="FG.DAT")
READ (31, *) (FG(L, J), J=1,8, I=1, 4, NELED)
CLOSE (UNIT=31, STATUS=\"DELETE\")
IELE = NELED
K = 4D0 * (IELE - 1D0)
DO 2035 I = 1, 4
CS(I) = RS(1 + K, 2)
2035 CONTINUE
DO 2040 I = 1, 8
S(I) = 0D0
DO 2040 J = 1, 9
FAJ(J) = 0D0
2040 CONTINUE
FI = FIC - FIC(IELE)
F12 = FIC
APEX = 90D0 - (F11 + F12) / 2D0
THKV = (THK(IELE,1) + THK(IELE,2)) / 2D0
EFIV = (EFI(IELE,1) + EFl(IELE,2)) / 2D0
ETHV = (ETHI(IELE,1) + ETHI(IELE,2)) / 2D0
CALL CONE(1, RAD, XNU, FI, F12, 0, APEX, FA, THKV, EFIV, ETHV, X)
CALL CONE(2, RAD, XNU, FI, F12, 4, APEX, FA, THKV, EFIV, ETHV, X)
CALL MATMLT (S, 8, FA, CS, S)
S(5) = EPSO + S(5)
S(6) = S(6)
S(7) = 0D0
S(8) = S(8) + IQ
CALL GLBLMT(IELE, FG, FA, S, JTOP, JBOT, NELED, NELES)
905O FORMAT (4(2X, E24.18))
9OS6 FORMAT (4(2X, E24.18))
2050 RETURN
END
SUBROUTINE FLENS(RAD1, FIC, XLNG, Q, EI, XNU, NELED, NELES,
    ALFA, IURLS, INX, FAC, JTOP, JBOT,
    XLNGI, THK, WP, EX, EFI, H, REC1, FG, FA, S)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION FG(800,11), H(200), XLNGI(200), THK(200,2).
    WP(200), EX(200,2), EFI(200,2), FA(8,9), S(8)
PI = 3.141592653589793238462644)

END

DO 2000 IELE = NELED + 1, NELE
DO 2003 J = 1, 4
FA(IJ) = 0D0

2005 CONTINUE

THKV = (THK(IELE, 1) + THK(IELE, 2)) / 2D0
EXV = (EX(I,IELE, 1) + EX(I,IELE, 2)) / 2D0
EFIV = (EFI(I,IELE, 1) + EFI(I,IELE, 2)) / 2D0

IF (INX.EQ.1 .OR. INX.EQ.0) THEN
    GAM = BTA
    XIE = BTA
    ALF = 0D0
END IF

IF (REC.GT.0D0 .AND. INX.EQ.2) THEN
    GAM = DSQRT((BTA**2D0 + ALF**2D0)
    IF (BTA.GE.ALF) XIE = DSQRT((BTA**2D0 - ALF**2D0)
    IF (BTA.LT.ALF) XIE = DSQRT((ALF**2D0 - BTA**2D0)
END IF

IF (REC.LT.0D0 .AND. BTA.GE.ALF .AND. INX.EQ.2) THEN
    XIE = DSQRT((BTA**2D0 + ALF**2D0)
    GAM = DSQRT((BTA**2D0 - ALF**2D0)
END IF

IF (REC.LT.0D0 .AND. BTA.LT.ALF .AND. INX.EQ.2) THEN
    RO = (ALF**2D0 - BTA**2D0)**.5D0
    GAM = DSQRT((2D0 * ALF**2D0 + 2D0 * RO**2D0)
    XIE = DSQRT((2D0 * ALF**2D0 - 2D0 * RO**2D0)
Program LV4

SUBROUTINE DOME (ID, RAD, XNU, X, FA, THK, EFI, ETI)

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION FA(K, 9), RI(2), XI(2), DI(2), RIO(2), DV(2)

DFI = EFI * THK**3D0/(12 * (1 - XNU**2D0))
DTH = ETH * THK**3D0/(12 * (1 - XNU**2D0))
K = 4D0 * (ID - 1D0)
DO 7010 I = 1, K + 4
DO 7010 J = 1, 4
FA(I,J) = 0D0
7010 CONTINUE

RO = (3D0*RA*D**2D0 - (1 - XNU**2D0)*THK**2D0)*2D0*XNU**2D0**2D0
RIO(1) = 1D0
RIO(2) = 0D0
XI(1) = RAD * DSIN(X)
XI(2) = 0D0
DI(1) = RAD * DCOS(X)
DI(2) = 0D0

DO 7030 I = 1, 50
DENO = DENO * 1D0 * (I + 1D0) * 16D0
C = (4D0 - 1D0)**2D0 - 5D0
R(1) = C * RIO(1) - (-8D0 + RO**2D0)*RIO(2)
R(2) = C * RIO(2) + (-8D0 + RO**2D0)*RIO(1)
DO 7020 J = 1, 2
RIO(J) = R(I)
C = RAD * RIO(J) / DENO
XI(J) = XI(J) + C * DSIN(X) ** (2D0 * I + 1D0)
DI(J) = DI(J) + (2D0 - I + 1D0) * C * DCOS(X) * DSIN(X)**(2D0*1)
7020 CONTINUE

7030 CONTINUE

C V (MERIDINAL ROTATIONAL ANGLE)
FA(K, 2, 3) = (XNU * XI(I) + 2D0 * RO**2D0 * XI(2)) / (ETH*THK*RAD)
FA(K, 2, 4) = (XNU * XI(2) - 2D0 * RO**2D0 * XI(2)) / (ETH*THK*RAD)

C DV (RATE OF CHANGE OF MERIDINAL ROTATIONAL ANGLE)
DV(1) = (XNU * DI(1) + 2D0 * RO**2D0 * DI(2)) / (ETH*THK*RAD)
DV(2) = (XNU * DI(2) - 2D0 * RO**2D0 * DI(1)) / (ETH*THK*RAD)
IF (X, NE. 0.0) THEN
DO 7040 J = 1, 2
QFI = XI(J) / RAD
FNTI = -DI(J) / RAD
FNFI = -XI(J) / (RAD * DTAN(X))

C eth
FA(K, 1, 2, J) = FNTI / (ETH*THK) - XNU * FNFI / (EFI*THK)
C Mi
FA(K, 3, 2, J) = (DFI/RAD*(DV(J)+XNU*FA(K, 2, J)*DTAN(X)))/RAD**2
C H
FA(K, 4, 2, J) = (QFI / DSIN(X)) / RAD

7040 CONTINUE
END IF
C This is for the apex (i = 0.0)
IF (X, EQ 0.0) THEN
DO 7050 J = 1, 2
QFI = XI(J) / RAD
FNTI = -DI(J) / RAD
FNFI = -XI(J) / (RAD * DTAN(X))
C eth
FA(K, 1, 2, J) = FNTI / (ETH*THK) - XNU * FNFI / (EFI*THK)
C Mi
FA(K, 3, 2, J) = (DFI/RAD*(DV(J)+XNU*FA(K, 2, J)*DTAN(X)))/RAD**2
C H
FA(K, 4, 2, J) = (QFI / DSIN(X)) / RAD

7050 CONTINUE
END IF
RETURN
END
SUBROUTINE CONE(I,D,RAD,XNU,F1I,F12,K,APEX,FA,THK,EF1,ETH,EXH,X)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION FA(8,9),W(4),D(4),DV(4)
PI = 3.14159265358979323846264D0
DR = PI / 180D0
DF1 = EF1 + THK**2D0 / 12D0 + (1-XNU**2D0))
ETH = ETH + THK**2D0 / 12D0 + (1-XNU**2D0))
DN = ETH + EF1
Y1 = RAD + DSIN(F11 * DR) / DSIN(APEX * DR)
Y2 = RAD + DSIN(F12 * DR) / DSIN(APEX * DR)
Y = Y1 + 1 / 2 * (Y1 - Y2) / 4D0
X = (F11 * K * (F12 - F11 / 4D0) * DR)
DDN = DMAX1(ETH,EF1) / DMN1(EF1,ETH)
C (ETH,NE,E,F1)
IF (DDN .GT. 1.05D0) THEN
AA = 1D0 + XNU - XNU * DN
BB = 2D0 * AA - 1D0
XLAMDA = (ETH * THK / DF1)**0.25D0 / DSQRT(THN (APEX * DR))
ETA = 2D0 * XLAMDA / DSQRT(Y)
R1 = ((BB - 1D0) + DSQRT((BB - 1D0)**2D0 + 16D0*DDN)) / 2D0
R2 = ((BB - 1D0) - DSQRT((BB - 1D0)**2D0 + 16D0*DDN)) / 2D0
IF (R2 .GT. R1) THEN
RR = R1
R1 = R2
R2 = R1
RR = 0D0
END IF
A = 1D0
B = 1D0
W(1) = ETA**(R1)
W(2) = ETA**(R1 - 1D0) * XLAMDA / DSQRT(Y)
W(3) = ETA**(R1 - 2D0)
W(4) = ETA**(R1 - 3D0) * XLAMDA / DSQRT(Y)
IS - 1D0
DO 5020 M = 1, 50, 2
IS = -IS
A = -A(W(2D0*M+R1) * (2D0*M+R1-1D0) + BB(2D0*M+R1-4D0))
W(2) = W(2) + IS*A*ETA**(R1 + 2D0*M)
W(2) = W(2) + IS*A*ETA**(R1 + 2D0*M)
W(3) = W(3) + IS*A*ETA**(R1 + 2D0*M)
W(4) = W(4) + IS*A*ETA**(R1 + 2D0*M)
W(1) = W(1) + IS*A*ETA**(R1 - 1D0)*BB(2D0*M+R1+2D0-4D0)
W(2) = W(2) + IS*A*ETA**(R1 - 2D0*M)
W(3) = W(3) + IS*A*ETA**(R1 - 3D0) + BB(2D0*M+R1+2D0-4D0)
W(4) = W(4) + IS*A*ETA**(R1 - 4D0)
5020 CONTINUE
END IF
C (ETH,EQ,E1)
IF (DDN .LE. 1.05D0) THEN
XLAMDA = (ETH * THK / DF1)**0.25D0 / DSQRT(THN (APEX * DR))
ETA = 2D0 * XLAMDA / DSQRT(Y)
PSI11 = 1D0
PSI11 = 1D0
PSI120 = 1D0
PSI121 = 1D0
PSI121 = 1D0
R10 = 1D0
R11 = 1D0
R20 = 1D0
R21 = 1D0
S = 1D0
DEN01 - 1D0
DEN02 - 1D0
DO 5030 M = 1, 50, 2
IS = -IS
DEN01 - DEN01 - (2D0 * M)**2D0
PSI210 = PSI210 + IS*ETA**(2D0 * M) / DEN01
PSI211 = PSI211 + IS*ETA**(2D0 * M - 1D0) / DEN01
DEN01 = DEN01 - (2D0 * M + 1D0)**2D0
PSI110 = PSI110 + IS*ETA**(2D0 * M + 2D0) / DEN01
PSI111 = PSI111 + IS*ETA**(2D0 * M + 2D0) / DEN01
DEN02 - DEN02 - M**2D0
S = S + 2D0 / M
R10 = R10 + IS*S + ETA/2D0)**2D0 / DEN02
R11 = R11 - IS* ETA/2D0)**(2D0 * M - 1D0)/(2D0*DEN02)
DEN02 = DEN02 - (M + 1D0)**2D0
S = S + 2D0 / M
R20 = R20 - IS*S + ETA/2D0)**(2D0 * M + 2D0) / DEN02
R21 = R21 - IS* ETA/2D0)**(2D0 * M + 2D0) / DEN02
5030 CONTINUE
BETA = DEXP(0.57722D0)
PSI30 = (PSI10/2D0 - PSI10/2D0) / PSI2O*DLOG(BETA*ETA/2D0)*1R10)
.RAD**2D0
PSI31 = (PSI10/2D0 - PSI10/2D0) / PSI2O*DLOG(BETA*ETA/2D0)*1R11/PSI2O*ETA)
.RAD**2D0
PSI40 = (PSI20/2D0 - PSI20/2D0) / PSI10*DLOG(BETA*ETA/2D0)*R20)
.RAD**2D0
PSI41 = (PSI21/2D0 - 2D0/PI*(PSI111*DLOG(BETA*ETA/2D0) + R21 + PSI10/ETA))
* RAD**2D0
PSI12 = PSI10 - PSI11 / ETA
PSI22 = PSI10 - PSI21 / ETA
PSI32 = PSI40 - PSI31 / ETA
PSI42 = PSI30 - PSI41 / ETA
W(1) = PSI10 + 2D0 * PSI21 / ETA
W(2) = PSI10 - 2D0 * PSI11 / ETA
W(3) = PSI30 + 2D0 * PSI41 / ETA
W(4) = PSI30 - 2D0 * PSI31 / ETA
C = XLAMDA / DSQRT(Y)
D(1) = C * (PSI21 + 2D0 * PSI12 / ETA - 2D0 * PSI21 / ETA**2D0)
D(2) = C * (PSI21 - 2D0 * PSI12 / ETA + 2D0 * PSI11 / ETA**2D0)
D(3) = C * (PSI31 + 2D0 * PSI42 / ETA - 2D0 * PSI41 / ETA**2D0)
D(4) = C * (PSI31 - 2D0 * PSI32 / ETA + 2D0 * PSI31 / ETA**2D0)
END IF
K = 4D0 * (ID - 1D0)
A = 1D0 + XNU - XNU * DN
IF (ETH.EQ.EFI) A = 1D0
C V (MERIDINAL ROTATIONAL ANGLE)
C = (XLAMDA * DTAN(DR * APEX))**2D0 / (ETH * THK)
FA(2 + K, 1) = C * W(2)
FA(2 + K, 2) = -C * W(1)
FA(2 + K, 3) = C * W(4)
FA(2 + K, 4) = -C * W(3)
C dv (RATE OF CHANGE OF MERIDINAL ROTATIONAL ANGLE)
DV(1) = C * D(2)
DV(2) = -C * D(1)
DV(3) = C * D(4)
DV(4) = -C * D(3)
DO 5040 J = 1, 4
QFI = W(J) / Y
FNTN = DTAN(DR * APEX) * D(J)
FNFN = DTAN(DR * APEX) * QFI
C eth
FA(1 + K, J) = FNTN / (ETH * THK) - XNU * FNFN / (EFI * THK)
C Mi
FA(3 + K, J) = -DFI * (DV(J) + XNU * FA(2 + K, J) / Y) / RAD**2D0
C H
FA(4 + K, J) = W(J) / (Y * DCOS(DR*APEX)) / RAD
5040 CONTINUE
RETURN
END

SUBROUTINE SHELL (K, RAD, DX, X, GAM, XIE, ID, FA, RAD1)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION FA(8,9)

**** Solution ID = 1 ****
IF (ID.EQ.1) THEN
C Radial Displacement Coefficients (e11, e12, e13 and e14)
FA(1 + K, 1) = (DENP(GAM * X) * DCONS(XIE * X)) / RAD
FA(1 + K, 2) = (DENP(GAM * X) * DSIN(XIE * X)) / RAD
FA(1 + K, 3) = (DENP(GAM * X) * DCONS(XIE * X)) / RAD
FA(1 + K, 4) = (DENP(GAM * X) * DSIN(XIE * X)) / RAD
C Rotation Coefficients (w1, w2, w3 and w4)
FA(2 + K, 1) = (GAM * FA(1 + K, 1) - XIE * FA(1 + K, 2)) / RAD
FA(2 + K, 2) = (GAM * FA(1 + K, 2) + XIE * FA(1 + K, 1)) / RAD
FA(2 + K, 3) = (-GAM * FA(1 + K, 3) - XIE * FA(1 + K, 4)) / RAD
FA(2 + K, 4) = (-GAM * FA(1 + K, 4) + XIE * FA(1 + K, 3)) / RAD
C Longitudinal bending moment Coefficients (m1, m2, m3 and m4)
A = (GAM ** 2D0 - XIE ** 2D0) * RAD / RAD1**2D0
B = (2D0 * GAM * XIE) * RAD / RAD1**2D0
FA(3 + K, 1) = -DX * (A * FA(1 + K, 1) + B * FA(1 + K, 2)) / RAD
FA(3 + K, 2) = -DX * (A * FA(1 + K, 2) + B * FA(1 + K, 1)) / RAD
FA(3 + K, 3) = -DX * (A * FA(1 + K, 3) + B * FA(1 + K, 4)) / RAD
FA(3 + K, 4) = -DX * (A * FA(1 + K, 4) + B * FA(1 + K, 3)) / RAD
C Radial shearing force Coefficients (q1, q2, q3 and q4)
A = (GAM ** 3D0 - 3D0 * GAM * XIE ** 2D0) * RAD / RAD1
B = (XIE ** 3D0 - 3D0 * XIE * GAM ** 2D0) * RAD / RAD1
FA(4 + K, 1) = -DX * (A * FA(1 + K, 1) + B * FA(1 + K, 2)) / RAD
FA(4 + K, 2) = -DX * (A * FA(1 + K, 2) + B * FA(1 + K, 1)) / RAD
FA(4 + K, 3) = -DX * (-A * FA(1 + K, 3) - B * FA(1 + K, 4)) / RAD
FA(4 + K, 4) = -DX * (-A * FA(1 + K, 4) - B * FA(1 + K, 3)) / RAD
END IF
RETURN
END
SUBROUTINE GBLMAT(IELE, FG, FA, S, JTOP, JBOT, NELE, NELES)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION FG(800, 11), S(8), FA(8, 9)
C JTOP: CLOSED=0, FREE=1
C JBOT: HINGED=1, FIXED=2, FREE/ROLLER=3, GUIDE=4
NELE = NELE + NELED
DO 6710 1 = 1, 4
  DO 6705 J = 1, 4
    FA(I, J) = FA(I, J)
  END DO
6705 CONTINUE
  SI(I + 4) = -SI(I + 4)
6710 CONTINUE
C CLOSED DOME JTOP = 0
IF (JTOP .EQ. 0) THEN
  FG(1, 5) = 1D0
  FG(2, 6) = 1D0
END IF
C FREE DOME JTOP = 1
IF (JTOP .EQ. 1) THEN
  DO 6720 1 = 1, 2
    DO 6715 J = 1, 4
      FG(I, J+4) = FA(I+2, J)
    END DO
6715 CONTINUE
  FG(I, 9) = SI(I)
6720 CONTINUE
END IF
C ROLLER JBOT = 3
IF (JBOT .EQ. 3) THEN
  DO 6735 J = 1, 4
    FG*(4NELE - 1, J) = FA(7, J)
  END DO
6735 CONTINUE
C GUIDE JBOT = 4
IF (JBOT .EQ. 4) THEN
  DO 6740 J = 1, 4
    FG*(4NELE - 1, J) = FA(8, J)
  END DO
6740 CONTINUE
END IF
IF (IELE .EQ. NELE) THEN
  C HINGED JBOT = 1
  IF (JBOT .EQ. 1) THEN
    DO 6755 J = 1, 4
      FG*(4NELE - 1, J) = FA(5, J)
    END DO
  6755 CONTINUE
  FG*(4NELE, 9) = FG*(4NELE, 9) + S(7)
END IF
C FIXED JBOT = 2
IF (JBOT .EQ. 2) THEN
  DO 6760 J = 1, 4
    FG*(4NELE - 1, J) = FA(6, J)
  END DO
  FG*(4NELE, 9) = FG*(4NELE, 9) + S(6)
END IF
SUBROUTINE CHECK (C1, C2, C3, C4, C5, 
    . RAD, FIC, FIO, XDN1, E1, XNU, Q, JT0P, JTB0T, 
    . NELED, NELES, NELTP, ERR, ALFA, IURLS, INX, 
    . FAC, FACFIN, FACSTP, ITEM, 
    . FI, THK, WP, JETX, IETTH, EX, EFL, H, 
    . ITE, B2, B3, B4, B5, 
    . FG, RS, RA, R, C, S, 
    . ERX1M, ERFM, NLX, NLF, NFALX, NFALF, IYN) 
IMPLICIT DOUBLE PRECISION (A-H, O-Z) 
DIMENSION RS(804,2), FG(800,11), 
    . FA(8,9), C(4), S(8), R(14), 
    . FI(200), THK(200,2), WP(200), H(200), 
    . JETX(200,2), IETTH(200,2), EX(200,2), EFL(200,2) 
CHARACTER B2*18, B3*60, B4*18, B5*60 
CHARACTER C1*80, C2*20, C3*20, C4*20, C5*20 
NELE = NELED + NELES 
PI = 3.14159265358979323846264D0 
DR = PI / 180D0 
ERXM = 0D0 
ERFM = 0D0 
NFALX = 0D0 
NFALF = 0D0 
NLX = 0D0 
NLF = 0D0 
X = FIO * DR 
OPEN (UNIT=32, FILE="COEFF.DAT") 
DO 4015 IELE = 1, NELE 
    K = 4D0 * (IELE - 1D0) 
    DO 4005 J = 1, 4 
        C(J) = RS(K + J, 1) + RS(K + J, 2) 
        DO 4010 J = 1, 4 
            READ (32,*),(FA(J, J), J=1, 4), I=1, 8) READ (32,*),(R(J), J=1, 4) DO 4010 J=1, 4, 
            FA(3,J) = FA(3,J) * RAD**2D0 
            FA(4,J) = FA(4,J) * RAD 
            FA(7,J) = FA(7,J) * RAD**2D0 
            FA(J) = FA(J) * RAD 
      4010 CONTINUE 
      CALL MATMLT (1, 4, FA, C, S) 
      THKV = THK(IELE, 1) 
      DFI = EX(IELE,1) * THKV**3D0 / (12D0 * (1D0 * XNU**2D0)) 
      DTH = EFL(IELE,1) * THKV**3D0 / (12D0 * (1D0 * XNU**2D0)) 
      IF (X, EQ. 0D0, S(2) = DTH / DFI * (1D0 + XNU)* S(3) IF (X, NE. 0D0, 
            S(2) = DTH / DFI * (S(2)**(1-XNU**2D0)*DTAN(X)-XNU*S(3)*RAD/DFI)
            S(1) = S(1) * R(1) 
            S(4) = S(4) * DCOS(X) * R(2) 
            FG(K + 1, 1) = S(3) 
            FG(K + 1, 2) = S(4) 
            FG(K + 3, 1) = S(2) 
            FG(K + 3, 2) = 0D0 
        X = FA * F(IELE) * DR 
        IF (IELE. EQ. NELED) THEN 
            CALL MATMLT (5, 8, FA, C, S) 
            THKV = THK(IELE, 2) 
            DFI = EX(IELE,2) * THKV**3D0 / (12D0 * (1D0 * XNU**2D0)) 
            DTH = EFL(IELE,2) * THKV**3D0 / (12D0 * (1D0 * XNU**2D0)) 
            S(6) = S(6) * DTH / DFI * (1D0 + XNU**2D0)/S(7)*S(7)**RAD/DFI 
            S(5) + S(5) = R(3) 
            S(8) = S(8) * DCOS(X) + R(4) 
            FG(K + 2, 1) = S(7) 
            FG(K + 2, 2) = S(8) 
            FG(K + 4, 1) = S(6) 
            FG(K + 4, 2) = 0D0 
        END IF 
    4015 CONTINUE 
    DO 4030 IELE = 1, NELE 
        K = 4D0 * (IELE - 1D0) 
        DO 4020 J = 1, 4 
            C(J) = RS(K + J, 1) 
            DO 4010 J = 1, 4 
    4020 CONTINUE 
    READ (32,*),(FA(J, J), J=1, 4), I=1, 8) READ (32,*),(R(J), J=1, 4) DO 4025 J=1, 4, 
        FA(3,J) = FA(3,J) * RAD**2D0 
        FA(4,J) = FA(4,J) * RAD 
        FA(7,J) = FA(7,J) * RAD**2D0 
        FA(J) = FA(J) * RAD 
    4025 CONTINUE 
    CALL MATMLT (1, 4, FA, C, S) 
    THKV = THK(IELE, 1) 
    DX = EX(IELE,1) * THKV**3D0 / (12D0 * (1D0 * XNU**2D0)) 
    DFI = EFL(IELE,1) * THKV**3D0 / (12D0 * (1D0 * XNU**2D0)) 
    S(1) = S(1) * R(1) 
    S(2) = DFI * XNU * S(3) / DX 
    X = X + F(IELE) 
    FG(K + 1, 1) = S(3) 
    FG(K + 1, 2) = R(2) 
    FG(K + 3, 1) = S(2) 
    FG(K + 3, 2) = 0D0 
    IF (IELE. EQ. NELE) THEN 
        CALL MATMLT (5, 8, FA, C, S) 
        THKV = THK(IELE, 2) 
        DX = EX(IELE,2) * THKV**3D0 / (12D0 * (1D0 * XNU**2D0)) 
    4030 CONTINUE
DFI = EFL(ELE,3) * THKV**3D0 / (12D0 * (1D0 - XNU**2D0))
S(5) = S(5) + R(3)
S(6) = DFI * XNU * S(7) / DX
FG(K+2, 1) = S(7)
FG(K+2, 2) = R(4)
FG(K+4, 1) = S(6)
FG(K+4, 2) = 0D0
END IF

4030 CONTINUE
IF (IURSL.EQ.3 .or. iurls .eq. 4)
  CALL ESCNTZ (2, NELED,NELES,XNU,ALFA,ERR,EFI,JETFI,
  FG, THK, ERFM, NLF, NFAFL, ITE, ITEM)
IF (IURLS .EQ. 2 .or. iurls .eq. 4)
  CALL ESCNT2 (0, NELED, NELES, XYU, IDO, ERR,
  EX, IETX, FG, THK, ERXM, NIX, NFAIX, ITE, ITEM)
4031 OPEN (UNIT=36, FILE=STATUS.DAT)
WRITE (36, 9010)((FG(I,J), J=1,11),I=1,1,4*NELE)
CLOSE (UNIT=36, STATUS=’KEEP’)
CLOSE (UNIT=32, STATUS=’DELETE’)
IF (NLX .EQ. 0 .AND. NLF .EQ. 0 .AND.
(NFALX .NE. 0 .OR. NFAFL .NE. 0)) THEN
B3 = "Un-Successful Analysis For Load Factor #"
B5 = 'Needs Finer Mesh OR Reduce Applied Load - Failure Data'
IYN = 0
GOTO 4040
END IF
IF((NLX .NE. 0 .OR. NLF .NE. 0) .AND. ITE .EQ. ITEM) THEN
B3 = "Un-Successful Analysis for Load Factor #"
B5 = 'Max. number of iterations was exceeded - Unconverged Data'
IYN = 0
GOTO 4040
END IF
IF(NLX .NE. 0 .OR. NLF .NE. 0) THEN
IYN = 1
GOTO 4090
END IF
IF(NLX .EQ. 0 .AND. NLF .EQ. 0) THEN
B3 = 'Successful Analysis for Load Factor #'
B5 = 'Converged Data'
IYN = 2
END IF
4040 DO 4050 I = 1, 4 * NELE
DO 4050 J = 1, 11
FG(I, J) = 0D0
4050 CONTINUE
CALL FLESD (RAD, FIC, FIO, Q, EI, XNU, NELED, NELES,
  ALFA, IURLS, FAC, JTOP, JBOT,
  FI, THK, WP, EX, EFI, II, REC, FG, RS, FA, S)
CALL BNDINV (NELE, FG, RS, FA)
4080 CALL FORCES (C1, C2, C3, C4, C5,
  RAD, FIC, FIO, XLNG, EI, XNU, Q, JTOP, JBOT,
  NELED, NELES, NELTP, ERR, ALFA, IURLS, INX,
  FAC, FACFIN, FACSTP, ITEM,
  FI, THK, WP, IETX, IETF, EX, EFI, II,
  ITE, B2, B3, B4, B5,
  RS, FG, FA, R, C, S,
  ERXM, ERFM, NIX, NLF, NFALX, NFAFL)
9010 FORMAT((2X,E12.6))
4090 RETURN
END
SUBROUTINE ESCN2 (ID, NELE0, NELES, XNU, ALFA, ERR, E, IET, FG, THK, ERM, NFAL, ITE, ITEM)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION FG(800,11), THK(200,2), IET(200,2), E(200,2), SS(2), EPS(2), EPC(2)
NELE = NELE0 + NELES
ERM = 0
IEN = 0
NL = 0
NFAL = 0
DO 4120 I = 1, NELE
  J = 1
4102 EI = 0
  ER1 = 0
  THK0 = THK(I, J)
  L = 4 * (IEN + 1)
  BM = FG(L+1, ID, 1)
  FN = FG(L+1, ID, 2)
  K = IET(I, J)
  EE = E(I, J)
  ifl0 = 0
  ifs0 = 0
CALL MFIN(BM, FN, THK0, EE, XNU, K, EI, SS, EPS, EPC, IFLS, IFLC)
  EC = 12 * EI ** 2 * THK**2
  ER1 = E(I, J) / EC
  EI = E(I, J) + (EC - E(I, J)) * ALFA
  IF (DFABS(ERM).LE.DABS(ER1)) ERM = ER1
  IF (IFLS.EQ.1.OR.IFLC.EQ.1) NFAL = NFAL + 1
  IF (DABS(ER1).GT.ERR/100) THEN
    NL = NL + 1
  ELSE IF (ITE.LT.ITEM) THEN
    E(I, J) = EI
  ELSE IF (IELE.EQ.1.OR.IELE.EQ.NELE) GOTO 4103
    IF (IELE.EQ.NELE.OR.IELE.EQ.NELE).AND.J.EQ.2) GOTO 4103
    E(I, J, 2) = EI
  END IF
END IF
4103 FG(L+1+ID, 2) = FI
  FG(L+1+ID, 9) = IFLC
  FG(L+1+ID, 10) = IFLS
  FG(L+1+ID, 11) = ER1
  DO 4105 K = 1, 2
  FG(L+1+ID, K) = SS(K)
  FG(L+1+ID, K+4) = EPS(K)
  FG(L+1+ID, K+6) = EPC(K)
4105 CONTINUE
  IF (I.EQ.2) GOTO 4108
  IF (IELE.EQ.1.AND.IELE.EQ.NELE+1) THEN
  END IF
4108 IF (IELE.EQ.NELE.OR.IELE.EQ.NELE) THEN
  IF (I.EQ.1) THEN
    J = 2
    GOTO 4102
  END IF
END IF
4120 CONTINUE
4125 FORMAT (1X, I3, 3, 3(2X, F12.4))
RETURN
END
SUBROUTINE FORCES (C1, C2, C3, C4, C5,)

RAD, FIC, FIO, XNGL, EI, XNU, Q, JTOP, JHOT,
NELED, NELES, NELTP, ERR, ALFA, JURLS, INX,
FAC, FACFIN, FACSTP, ITEM,
XNGL1, THK, WP, IETX, IETF, EX, EFI, II,
ITE, B2, B3, B4, B5,
RS, FG, FA, R, C, S,
ERXM, ERFM, NFLX, NFLF, NFALX, NFALF)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION RS804,2), FG80011),
XNGL1(200), THK(200,2), WP(200),
IETF(200,2), IETX(200,2), EX(200,2), EFI(200,2), HI(200),
R(14), FA(9,4), C(4), S(8)
CHARACTER B2*18, B3*80, B4*80, B5*210
CHARACTER C1*80, C2*20, C3*20, C4*20, C5*20
CHARACTER OUTDAT*12, OUTRES*12
I = DINT10000 * FAC + 0.940)
WRITE(OUTDAT, 9000) 'DATA', 1, 'DAT
WRITE(OUTRES, 9000) 'RSLT', I, 'RES
NELE = NELED + NELES
CD = 0D0
CN = 0D0
OPEN (UNIT=41, FILE='TEMP.RES')
CALL DEFORMD(RAD, FIC, FIO, Q, NELED, EI, XNU, XNGL1, THK, WP,
FAC, EX, EFI, RS, CD, REC, FA, R, C, S)
CALL DEFORMS(RAD, FIC, FIO, XNGL1, EI, XNU, Q, NELED, NELES, INX, FAC,
XNGL1, THK, WP, EX, EFI, RS, REC, FA, R, C, S)
CLOSE (UNIT=41, STATUS='KEEP')
OPEN (UNIT=38, FILE='OUTDAT')
WRITE (38, 9001) C1
WRITE (38, 9004) B2, FAC
WRITE (38, 9006) C2
WRITE (38, 9010) RAD, FIC, FIO, XNGL1, EI, XNU, Q, JTOP, JHOT
WRITE (38, 9006) C3
WRITE (38, 9015) NELED, NELES, NELTP, ERR, ALFA, JURLS, INX,
FAC, FACFIN, FACSTP, ITEM
WRITE (38, 9006) C4
DO 5005 IELE = 1, NELED
WRITE (38, 9020) IELE, XNGL1(IELE), THK(IELE,1), THK(IELE,2),
WP(IELE), EX(IELE,1), EX(IELE,2), EFI(IELE,1), EFI(IELE,2),
IETX(IELE,1), IETX(IELE,2), IETF(IELE,1), IETF(IELE,2)
5005 CONTINUE
WRITE (38, 9006) C5
DO 5010 IELE = NELED+1, NELED+NELES
WRITE (38, 9020) IELE, XNGL1(IELE), THK(IELE,1), THK(IELE,2),
WP(IELE), H(IELE),
EX(IELE,1), EX(IELE,2), EFI(IELE,1), EFI(IELE,2),
IETX(IELE,1), IETX(IELE,2), IETF(IELE,1), IETF(IELE,2)
5010 CONTINUE
IF (ALFA.EQ.0.0) OR (JURLS.EQ.1) GOTO 5035
WRITE (38, 9003) B3, FAC
WRITE (38, 9001) B3
WRITE (38, 9003) B3, B5, B6, B7, B8
WRITE (38, 9010) FAC, TER, ERXM, NFLX, NFLF, NFALF
OPEN (UNIT=36, FILE='DATA')
READ (38, *) (FG(I,J), I = 1, 14, NELE)
DO 5025 IELE = 1, 4 * NELE
Z = 1.0 D00 + 0.9990
L = Z
WRITE (38, 9035) L, (FG(I,J), J = 1, 11)
5025 CONTINUE
CLOSE (UNIT=38, STATUS='KEEP')
CLOSE (UNIT=36, STATUS='DELETE')
5035 OPEN (UNIT=40, FILE='TEMP.RES')
OPEN (UNIT=39, FILE='OUTRES')
B0 = Dione i Dv V Nih Npl Mih Mpi H Qf1 dV v w Dv eth efi
WRITE (39, 9050) J6
Pi = 3.141592653589793238462640
J = PI / 180.0DO
IELE = 0
IF (FIO.EQ.0.0) THEN
IELE = 1
DO 5040 K = 0, 10
IF (K.EQ.0) THEN
READ (40, *) IELE, X(I), R(I,1-1,9), SUM, DUMMY1, DUMMY2
XO = X
READ (40, *) IELE, X(I), R(I,1-1,9), SUM, DUMMY1, DUMMY2, R(I,13), R(I,14)
R(I,10) = (SUM + CD) * DSIN(DR * X)
Y1 = R(I,10) / DSIN(DR * X) - R(I,10) / DSIN(X * DR)
X1 = X
READ (40, *) IELE, X(I), R(I,1-1,9), SUM, DUMMY1, DUMMY2, R(I,13), R(I,14)
R(I,10) = (SUM + CD) * DSIN(DR * X)
Y2 = R(I,10) / DSIN(DR * X) - R(I,10) / DSIN(X * DR)
X2 = X
CLOSE (UNIT=40, STATUS='KEEP')
OPEN (UNIT=40, FILE='TEMP.RES')
READ (40, *) IELE, X(I), R(I,1-1,9), SUM, DUMMY1, DUMMY2, R(I,13), R(I,14)
R(I,10) = (SUM + CD) * DSIN(DR * X)
R(I,11) = Y1 - (Y2 - Y1) * (X1 - X0) / (X2 - X1)
R(I,12) = R(I,10) * DSIN(DR * X) + R(I,11) * DCOS(X * DR) * CN
WRITE (39, 9055) IELE, X(I), R(I,1-1,14)
GOTO 5040
END IF
READ (40, *) IELE, X(I), R(I,1-1,9), SUM, DUMMY1, DUMMY2, R(I,13), R(I,14)
R(10) = (SUM + CD) * DSIN(DR * X)
R(11) = R(10) / DTAN(DR * X) - R(1) / DSIN(X * DR)
R(12) = R(10) * DSIN(DR * X) + R(11) * DCOS(X * DR) - CN
WRITE (39, 9055) IELE, X, (R(I), I = 1, 14)

5040 CONTINUE
END IF
INI = IELE + 1
IF (FIO .EQ. 0.0 .AND. NELED .EQ. 1) GOTO 5055
DO 5050 J = INI, NELED
DO 5045 K = 0, 4
READ (40,*) IELE, X, (R(I), I = 1, 19)
SUM = DUMMY1 + DUMMY2 + R(13) + R(14)
R(10) = (SUM + CD) * DSIN(DR * X)
R(11) = R(10) / DTAN(DR * X) - R(1) / DSIN(X * DR)
R(12) = R(10) * DSIN(DR * X) + R(11) * DCOS(X * DR) - CN
WRITE (39, 9055) IELE, X, (R(I), I = 1, 14)
5045 CONTINUE
5050 CONTINUE

5055 WRITE (39, 9051) B6
DO 5065 IELE = NELED + 1, NELE
DO 5060 K = 0, 4, 1
READ (40,*) IELE, X, (R(J), J = 1, 10)
R(1) = R(1)
R(8) = R(8) - CN
WRITE (39, 9060) IELE, X, (R(J), J = 1, 10)
5060 CONTINUE
5065 CONTINUE
5090 CLOSE (UNIT=39, STATUS='KEEP')
CLOSE (UNIT=40, STATUS='DELETE')

9000 FORMAT (A4, I4, 4, A4)
9001 FORMAT (A80)
9004 FORMAT (A18, 2X, F8.2)
9005 FORMAT (A18, 2X, I3.3)
9006 FORMAT (A80)
9007 FORMAT (A80, 2X, F8.3)
9015 FORMAT (3(2X,F10.3),2(2X,F6.3),2(2X,13),3(2X, F8.3), 2X, 13)
9020 FORMAT (2X,I3,9(2X,F10.3),4(2X,13))
9035 FORMAT (1X,I3,3,11(1X,E12.6))
9040 FORMAT (R(2X,F12.6))
9050 FORMAT (2X, A210)
9051 FORMAT (2X, A180)
9055 FORMAT (2X,I3,3,2X,F10.3,14(2X, E12.6))
9060 FORMAT (2X,I3,3,2X,F10.3,10(2X,E12.6))
RETURN
END
C #*************************************************************************
C # DEF0R Mation in spherical Dome #*
C #*************************************************************************

SUBROUTINE DEFORMD (RAD, FIC, FIO, Q, NELED, EI, XNU, FI, THKV, WP, 
   FAC, ETH, RS, CD, REC, FA, R, CS, S)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION RS(804,2,2), FI(200), THKV(200,2,2), WP(200), ETH(200,2), 
   FA(8,9), CS(4), S(8), R(14)
PI = 3.14159265358979323846264D0
DR = PI / 180D0
DO 8002 I = 1, 3
DO 8001 J = 1, 4
FA(I,J) = 0D0
8001 CONTINUE
8002 CONTINUE
IELE = 0
REC = 0D0
FNFI0 = 0D0
FNTHIO = 0D0
SUM = 0D0
FI2 = FIO
XO = FIO * DR
IF (FIO .EQ. 0D0) THEN
   IELE = 1
   W = WP(1) * FAC
   DO 8010 I = 1, 4
      CS(I) = RS(I,1) + RS(I,2)
8010 CONTINUE
   DO 8005 K = 0, 10
      DO 8003 I = 1, 14
         R(K) = 0D0
8005 CONTINUE
   X = FI(1) * DR * K / 10D0
   THKV = THK(1,2)
   ETHV = ETH(1,2)
   EFIV = EFIV(1,2)
   DFIV = EFIV * THKV**3 / (12 * (1D0 - XNU**2D0))
   DTHKV = ETHV * THKV**3 / (12 * (1D0 - XNU**2D0))
   CALL DOME (1, RAD, XNU, X, FA, THKV, EFIV, ETHV)
   X = FI(1) * DR * K / 10D0
   DO 8020 J = 1, 4
      FA(3,J) = FA(3,J) * RAD**2D0
      FA(4,J) = FA(4,J) * RAD
8020 CONTINUE
   CALL MATMLT(1, 4, FA, CS, S)
   REC = W * PS + (RAD * DSIN(X))**2D0
   R(K) = W / RAD / 2D0
   IF (X .GE. 0D0) R(4) = REC / (2D0 * PS + RAD**2D0) *(1D0 - COCOS(X))
   R(3) = (W * Q * THKV * DCOS(X) + RAD * R(4)
   R(1) = (R(3) / (E1 * THKV) - XNU * R(4) / (E1 * THKV)
   R(13) = S(1) + S(1)
   R(1) = R(13) * RAD + DSIN(X)
   R(7) = S(4)
   R(8) = R(7) * DSIN(X)
   R(9) = R(8) * DCS(C) * R(4)
   R(1) = R(4) / (EI * THKV) *(1 - XNU**2D0) - XNU * R(13)
   R(3) = (ETHV*THKV)(1-XNU**2D0) *(R(13) - XNU**R(14))
   R(2) = S(2)
   R(6) = S(3)
   IF (X .GE. 0D0) THEN
      R(9) = RAD * S(3) / DFIV - XNU * S(2) / DTAN(X)
      R(5) = DTHKV / RAD * (S(2) + DTAN(X) + XNU) * R(9)
   END IF
   IF (X .EQ. 0D0) THEN
      R(9) = RAD * S(3) / DFIV / (1D0 + XNU)
      R(5) = DTHKV / RAD * (1D0 + XNU) * R(9)
   END IF
   IF (X .GE. 0D0) THEN
      D = (N - XO)
   C = RAD * (1D0 + XNU) / (THKV * ETHV)
   H1 = R(3) / DSIN(X)
   (XI, XI) = 0D0
   IF (X .LE. 0D0) H1 = 0D0
   IF (X .GE. 0D0) H1 = H1 / DSIN(X)
   SUM = SUM + C * (H1 + H2) * D / 2D0
   C = RAD * (1D0 + XNU) / (ETHV * EFIV)
   H1 = R(4) / DSIN(X)
   IF (X .EQ. 0D0) H1 = 0D0
   IF (X .GE. 0D0) H1 = H1 / DSIN(X)
   SUM = SUM + C * (H1 + H2) * D / 2D0
   IF (X .LE. 0D0) H1 = 0D0
   IF (X .GE. 0D0) H1 = H1 / DSIN(X)
   SUM = SUM + C * (H1 + H2) * D / 2D0
   FORTRAN = R(3)
   (XI, XI) = R(4)
   XO = X
8035 WRITE(41,9100) IELE, X/DR,(R(I),I=1,19),SUM,0,0,0,0,0,R(13),R(14)
8040 CONTINUE
F1Z = F1(1)
XO = F1(1) * DR
IF (NELEDEQ) GOTO 8090
END IF
INI = IELE
DO 8080 IELE = INI, NELED
F1 = F1Z
F1Z = F1(1) + F1(4)
APX = 9D0 - (F1 + F1Z) / 2D0
K = 4D0 * (EI - E1)
W = WP (IELE) * FAC
DO 8050 I = 1, 4
CS(I) = RS(K+1,1) + RS(K+1,2)
8050 CONTINUE
DO 8070 K = 0, 4, 1
DO 8045 J = 1, 14
R(J) = 0.D0
8045 CONTINUE
XI = K
THKV = (THK(IELE, 1) + THK(IELE, 2)) / 2.D0
EFIV = (EFI(IELE, 1) + EFI(IELE, 2)) / 2.D0
ETIV = (ETH(IELE, 1) + ETH(IELE, 2)) / 2.D0
CALL CONE (1, RAD, XNU, FI1, FI2, APEX, FA, THKV, EFIV, ETIV, X)
DO 8060 J = 1, 4
FA(3, J) = FA(3, J) * RAD**2.D0
FA(4, J) = FA(4, J) * RAD
8060 CONTINUE
XI = (FI1 + (FI2 - FI1) * K / 4.D0) * DR
CALI. MATMLT (1, 4, FA, CS, 8)
THKV = THK(IELE, 1) + K * (THK(IELE, 2) - THK(IELE, 1)) / 4.D0
EFIV = EFi(IELE, 1) + K * (EFI(IELE, 2) - EFI(IELE, 1)) / 4.D0
ETIV = ETH(IELE, 1) + K * (ETH(IELE, 2) - ETH(IELE, 1)) / 4.D0
DFI = EFIV * THKV**3 / (12 * (1.D0 - XNU**2.D0))
DTIV = ETHV * THKV**3 / (12 * (1.D0 - XNU**2.D0))
REC = REC
-W * PI * RAD**2.D0 * ((DSIN(XO))*2.D0 - (DSIN(XO))*2.D0)
-2.D0 * Q * THKV * PI * RAD**2.D0 * (DCOS(XO) - DCOS(X))
R(4) = REC / (2.D0 * PI * RAD * (DSIN(XO))*2.D0)
R(3) = -(W + Q * THKV * DCOS(X0)) * RAD - R(4)
R(1) = R(3) / (EI * THKV) - XNU * R(4) / (EI * THKV)
R(13) = S(1) + R(1)
R(1) = R(13) * RAD * DSIN(X)
R(7) = S(4)
R(8) = R(7) * DSIN(X)
R(4) = -R(7) * DCOS(X) + R(4)
R(14) = R(4) / (EFIV * THKV) * (1 - XNU**2.D0) - XNU * R(13)
R(3) = (ETHIV*THKV)/(1-XNU**2.D0)*R(13)/XNU*R(14))
R(2) = S(2)
R(6) = S(3)
R(9) = RAD * S(3) / DFI - XNU * S(2) / DTAN(X)
R(5) = DTIV / RAD * (R(2) / DTAN(X) - XNU * R(9))
D = (X - XO)
C = RAD * (1.D0 + XNU) / (THKV * ETHV)
HI1 = R(3) / DSIN(X)
HI2 = FNTHO / DSIN(XO)
SUM = SUM - C * (HI1 + HI2) * D / 2.D0
C = RAD * (1.D0 + XNU) / (THKV * EFIV)
HI1 = R(4) / DSIN(X)
HI2 = FNTIO / DSIN(XO)
SUM = SUM + C * (HI1 + HI2) * D / 2.D0
FNTHO = R(3)
FNTIO = R(4)

WRITE(41, 9100) IELE, XI, DR, (R(I), I = 1, 14), SUM, 0, 0, 0, 0, R(13), R(14)
8070 CONTINUE
8080 CONTINUE
8090 CD = R(I) / DTAN(FIC * DR) - SUM
9100 FORMAT (13, 1X, 15(1X, E18.12))
9110 FORMAT (5(2X, E18.12))
RETURN
END
C

C -------------------------------------------------------------
C    #DEFORMation and stress resultant in Shells #
C -------------------------------------------------------------
C
SUBROUTINE DEFOMS (RAD, FIC, EI, XNU, Q, NELED, NELES, INX, FAC,
   .  XNLGI, THK, WP, EX, EFI, RS, REC1, FA, R, CS, S, CN)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION RS(804,2),XNLGI(200),THK(200,2),WP(200),EX(200,2),
   .  EFI(200,2),FA(8,9),CS(4),S(8),R(14)
NELE = NELED + NELES
PI = 3.14159265358979323846264D0
DR = PI / 180D0
RAD = RAD1 * DSIN(DR * FIC)
REC = REC1
XLO = 0D0
CN = 0D0
XO = 0D0
EXO = 0D0
IF (INX.EQ.0D0) REC = 0D0
DO 8080 IELE = NELED + 1, NELE
   K = 4D0 *(IELE - 1D0)
   W = WP(IELE)*FAC
   DO 8050 I = 1, 4
   CS(I) = RS(I + K, 1)
8050 CONTINUE
DO 8075 K = 0D0, 4D0, 1D0
   DO 8057 I=1, 9
      R(I) = 0D0
8057 CONTINUE
EXV = (EX(IELE,1) + EX(IELE,2))/2D0
EFIV = (EFI(IELE,1) + EFI(IELE,2))/2D0
THKV = (THK(IELE,1) + THK(IELE,2))/2D0
DEN0 = EXV * RAD**2D0 * THKV**2D0
BTA = (3D0 * (1D0 - XNU**2D0) * EFIV / DEN0) ** 0.25D0
DX = EXV * THKV**3D0 / (12D0 * (1D0 - XNU**2D0))
DFI = EFIV * THKV**3D0 / (12D0 * (1D0 - XNU**2D0))
ALF = DSQRT(DABS(REC)/(4D0*DX))
IF (INX.EQ.1.0 OR. INX.EQ.0) THEN
   BTA = GM
   XIE = BTA
   ALF = 0D0
END IF
IF (REC .GT. 0D0 AND. INX. EQ. 2) THEN
   GAM = DSQRT (BTA**2D0 + ALF**2D0)
   IF (BTA .GE. ALF) XIE = DSQRT (BTA**2D0 - ALF**2D0)
   IF (BTA .LT. ALF) XIE = DSQRT (ALF**2D0 - BTA**2D0)
END IF
IF (REC .LT. 0D0 AND. BTA .GE. ALF. AND. INX. EQ. 2) THEN
   XIE = DSQRT (BTA**2D0 + ALF**2D0)
   GAM = DSQRT (BTA**2D0 - ALF**2D0)
END IF
IF (REC .LT. 0D0 .AND. BTA .LT. ALF. AND. INX. EQ. 2) THEN
   RO = -(ALF**4D0 - BTA**4D0)**0.25D0
   GAM = DSQRT (2D0 * ALF**2D0 + 2D0 * RO**2D0)
   XIE = DSQRT (2D0 * ALF**2D0 - 2D0 * RO**2D0)
END IF
ID = 1
IF (BTA .LT. ALF .AND. REC .GT. 0D0) ID = 2
IF (BTA .LT. ALF .AND. REC .LT. 0D0) ID = 3
X = XNLGI(IELE) * K / 4D0
C = PI / 180D0
DO 8060 J=1, 4
   FA(1, J) = FA(1, J) * RAD1**2D0
   FA(4, J) = FA(4, J) * RAD1
8060 CONTINUE
CALL MATMUL(1, 4, FA, CS, S)
THKV = THK(IELE,1) + K * (THK(IELE,2) - THK(IELE,1))/4D0
EXV = EX(IELE,1) + K * (EX(IELE,2) - EX(IELE,1))/4D0
EFIV = EFI(IELE,1) + K * (EFI(IELE,2) - EFI(IELE,1))/4D0
DX = EXV * THKV**3D0 / (12D0 * (1D0 - XNU**2D0))
DFI = EFIV * THKV**3D0 / (12D0 * (1D0 - XNU**2D0))
THKV = DSQRT(DABS(REC)/(4D0*DX))
R(4) = REC / (2D0 * PI * RAD)
R(3) = W * RAD
R(1) = R(3) / (EI * THKV) * XNU * R(4) / (EI * THKV)
IF (INX.EQ.0) R(1) = R(3) / (EFIV * THKV)
R(8) = S(1) + R(1)
R(1) = -R(8) * RAD
R(2) = S(2)
R(4) = R(4)
R(7) = S(4)
R(9) = R(4)/(EXV * THKV) * (1 - XNU**2D0) - XNU * R(8)
R(3) = (EFIV * THKV*(1 - XNU**2D0) - R(8) + XNU * R(9))
R(6) = S(3)
R(5) = XNU * R(6) * DFI / DX
X = XNU + X
CN = CN + (XL - XO) * (R(9) + EXO) / 2D0
WRITE (41, 9100) IELE, X, (R(I),I=1,7), CN, R(8), R(9)
XO = XL
EXO = R(9)
8075 CONTINUE
XLO = XL
8080 CONTINUE
9100 FORMAT (I3,1X,11(1X,E18.12))
RETURN
END
SUBROUTINE MFIN(BM,FNC,THKV,EE,XNU,K,FI,FI,S,ES,EC,IFS,IFS)
IMPLICIT DOUBLE PRECISION (A-I, O-Z)
DIMENSION FS(2), ES(2), SS(2), FI, FT(2), FSCR(2), ESCR(2), SSC(2), ECCR(2)
EI = 0
BMX = 0.0
BMO = 0.0
BMC = 0.0
CO = 0.0
IF (K.EQ.1 .OR. K.EQ.2 .OR. K.EQ.3 .OR. K.EQ.8) THEN
EPSY = 2.693D-03
EPSU = 0.1
END IF
IF (K.EQ.4 .OR. K.EQ.5 .OR. K.EQ.6 .OR. K.EQ.7) THEN
EPSY = 2.26D-03
EPSU = 0.13
END IF
FC = -75.0
FCR = -3.00D-03
ECR = 92D-06
FCR = 3.10D0
FCR = 1.8550
IF (KEY .LE. 0) THEN
EPSY = 1.80D-03
FCR = 4.65D0
FTCR = 2.78D0
END IF
IF (XN .LE. 0.81ID0) THEN
ECR = FC / EPS * XN / (XN - 1D0)
END IF
IF (END = 0)
IFL = 0
IFLS = 0
IFC = 0
IFC = 0
IF = (EPSCR - (-4D-03)) / THKV
IF (BM.LT.0.0D0) FI = -FI
EPSM = EPSCR + (-4D-03) / 2D0
EC(1) = EPSM - THKV / 2D0 * FI
EC(2) = EPSM - THKV / 2D0 * FI
C = -EC(1) / FI
IF (FI.LT.0.0D0) C = EC(2) / FI
CALL FTS(K, EPSM, THKV, FI, FS, ES, SS, BM, FNL, FNU)
CALL CONC(FNC, FNT, BMCC, BMCT, THKV, C, FI, FCR, FTCR, XN, EPSC,
EPSM, EC, ESCR, ECR, ES, EC)
FNN = FNT + FNC + FS(1) + FS(2)
CN = C
FI = (EPSCR - 0.99D0*EPSU) / (THKV - 8D0)
IF (BM.LT.0.0D0) FI = -FI
EPSM = EPSCR + (0.99D0*EPSU- EPSCR)*THKV / (2D0*(THKV - 8D0))
EC(1) = EPSM - THKV / 2D0 * FI
EC(2) = EPSM - THKV / 2D0 * FI
C = -EC(1) / FI
IF (FI.LT.0.0D0) C = EC(2) / FI
CALL FTS(K, EPSM, THKV, FI, FS, ES, SS, BMS, FNL, FNU)
CALL CONC(FNC, FNT, BMCC, BMCT, THKV, C, FI, FCR, FTCR, XN, EPSC,
EPSM, EC, ESCR, ECR, ES, EC)
FNN = FNT + FNC + FS(1) + FS(2)
CN = C
SSCR(I) = SS(I)

4204 CONTINUE
IF (DABS(BM) .LE. DABS(BMCR)) THEN
    B = BM / BMCR
    DO 4205 I = 1, 2
      FS(I) = B * FS(I)
      EC(I) = B * EC(I)
      ES(I) = B * ES(I)
      SS(I) = B * SS(I)
 4205 CONTINUE
    FI = B * FICR
    BMCC = BM
  GOTO 4260
ENDIF

4207 EPSM = 0.99DO*EPSU - (THKV / 2DO - 8DO) * DABS(FI)
    IF (FI.I .LT. OW) THEN
      EPSM = EPSM + FI
      EPSM = EPSM / FI
      C = EPSM
      IF (FI.I .LT. 0DO) C = EPSM
      CALL FTS(K, EPSM, THKV, FI, ES, SS, BMCC, FICR, FICR, XN, EPSC, EPSM, FC, EPSC, ECR, EC, IFLC)
      FNP = FNT + FNC + FS(I) + FS(2)
      CP = C
      EPSM = (-4D03) + THKV / 2DO * DABS(FI)
      EPSM = EPSM + FI
      IF (FI.I .LT. 0DO) C = EPSM
      CALL FTS(K, EPSM, THKV, FI, ES, SS, BMCC, FICR, FICR, XN, EPSC, EPSM, FC, EPSC, ECR, EC, IFLC)
      FNP = FNT + FNC + FS(I) + FS(2)
      CP = C
      IF (FIN#GT.FNP. OR. FIN#GT.FNN) STOP
      ITE = 0
 4210 C = CP + (FNP-FNG)*(CN-CP)/(FNP-FNN)
    IF (FI .LT. C) THEN
      DO 4210 FI = FI + C
    END IF
    EPSM = (EC(I) + EC(2)) / 2DO
    CALL FTS(K, EPSM, THKV, FI, ES, SS, BMCC, FICR, FICR, XN, EPSC, EPSM, FC, EPSC, ECR, EC, IFLC)
    CALL CONC(FNC, NTC, BMCC, BMCT, THKV, C, FI, FICR, FICR, XN, EPSC, EPSM, FC, EPSC, ECR, EC, IFLC)
    FNO = FNC + FNT + FS(I) + FS(2)
    BMC = BMCT + BMCC + BMS
    ITE = ITE + 1
    IF (DABS(FNO-FNG) .LE. 1DO) GOTO 4212
    IF (FNO.GT.FNG) CN = C
    IF (FNO.LT.FNG) CN = C
    GO TO 4210
 4212 ER = (1.05DO * BMCR / BMC - 1DO) * 100DO
    IF (DABS(ER) .GT. 0DO) THEN
      FI = FI * (1.05DO * BMCR) / BMC
  GOTO 4207
    END IF
    B = (BM - BMCR) / (BMCC - BMCR)
    FI = FICR + B * (FI - FICR)
    DO 4213 I = 1, 2
      EC(I) = ECCR(I) + B * (EC(I) - ECCR(I))
      ES(I) = ESCR(I) + B * (ES(I) - ESCR(I))
      SS(I) = SSCR(I) + B * (SS(I) - SSCR(I))
      FS(I) = FSCR(I) + B * (FS(I) - FSCR(I))
 4213 CONTINUE
    BMCC = BM
    GOTO 4260
    END IF
    FI = 1DO * (1DO-XNU)**2DO * BM / (EE*THKV**3DO)
    AA = 0.999D0*EPSU
 4230 EPSM = AA - (THKV / 2DO - 8DO) * DABS(FI)
    EPSM = EPSM - FI
    EPSM = EPSM / FI
    C = EPSM
    IF (FI.I .LT. 0DO) C = EPSM
    CALL FTS(K, EPSM, THKV, FI, ES, SS, BMCC, FICR, FICR, XN, EPSC, EPSM, FC, EPSC, ECR, EC, IFLC)
    FNP = FNT + FNC + FS(I) + FS(2)
    CP = C
    EPSM = (-4D03) + THKV / 2DO * DABS(FI)
    EPSM = EPSM + FI
    IF (FI.I .LT. 0DO) C = EPSM
    CALL FTS(K, EPSM, THKV, FI, ES, SS, BMCC, FICR, FICR, XN, EPSC, EPSM, FC, EPSC, ECR, EC, IFLC)
    FNP = FNT + FNC + FS(I) + FS(2)
    CN = C
    ITE = 0DO
 4235 C = CP + (FNP-FNG)*(CN-CP)/(FNP-FNN)
 4237 EC(I) = FI * C
IF (FILT.0D0) THEN
  EC(1) = F1 * (THKV - C)
  EC(2) = F1 * C
END IF

EPSM = (EC(1) + EC(2)) / 2D0
ITE = ITE + 1
CALL. FT1K, EPSM, THKV, F1, F2, ES, SS, BMS, IFLT, FNU)
CALL. CONC(FNC, FNT, BMCC, BMCT, THKV, C, F1, FCR, FTCR, XN, EPSM,
  FPSM, FC, EPSCR, ECR, EC, IFLC)
FNO = FNC + FNT + FS(1) + FS(2)
BMC = BMCT + BMCC + BMS
if (ite.gt.200) then
  AA = 5*EPSY
  ite = 0
  F1 = FICR
  F1L = 0
  F1L = 0
  F1END = 0
GOTO 4230
end if
IF (AFLT(1) + IFLT(2))
IF (IFLEND.EQ.1) goto 4250
IF (IFL.GE.10 .AND. (FNO+FNU).GE.1FNG) THEN
  CP = C
  FNP = FNO + FNU
GOTO 4235
END IF
IF (IFL.GE.10 .AND. (FNO+FNU).LT.1FNG) THEN
  F1L = 1
  F1L = IFLC
  CN = C
  F1 = F1CR
  F1L = 0
  F1END = 0
GOTO 4230
END IF
END IF
END IF
IF (IFL.FEQ.1) THEN
  IF (FNO.LT.1FNG) THEN
    F1 = 0
  ELSE
    F1 = 0
  END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
IF (FNO .LT. FNG) THEN
  iFLend = 1
  IF (CO.NE.0D0) THEN
    C = CO
    F1 = F10
    GO TO 4237
  END IF
END IF
IF (DABS(FNO-FNG) .GE. 1D0) GOTO 4240
IF (FNO.GT.1FNG) THEN
  CP = C
  FNP = FNO
END IF
END IF
IF (FNO.LT.1FNG) THEN
  CN = C
  FNN = FNO
END IF
GOTO 4230
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
END IF
C               CONConcrete forces routine C
C
SUBROUTINE CONC(FNC,FNT,BMCC,BMCT,THKV,C,F,FC,FTCR,YN,EPSC,
               EPSM,EC,EPSCR,ECR,EC,FLC)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION EC(2)
FNC = 0D0
FNT = 0D0
BMCC = 0D0
BMCT = 0D0
IF (C.GT.0D0) THEN
HI = DMIN1(C,THKV) / 2D0
DO 4210 I=1, 20
   Y1 = -THKV / 2D0 + (I - 0.5D0) * HI
   IF (F(I).LT.0D0) Y1 = -Y1
   EPSI = EPSM + F1 * Y1
   S = EPSI / EPSC
   IF (EPSI .LT. -4.0D-03) IFLC = 1
   XK = 1D0
   IF (S.GT.1D0) XK = 0.67D0 - FC / 62D0
   SCI = FC*XXN* S (XXI+SP*XXK)
   FC1 = SCI * HI
   FNC = FNC + FC1
   BMCC = BMCC + Y1 * FC1
4210 CONTINUE
END IF
IF (C.LT.0D0) THEN
HCRY = EPSCR / DABS(F1)
   HCR = DMIN1 (HCRY,(THKV-C))
   YCR = -THKV / 2D0 + C * 2D0 / 3D0 * HCR
   FCNCR = + FCR * (HCRY/HCRY)/2D0 * HCR
   IF (HCRY.LT.0D0) YCR = -YCR
   DL1 = THKV - C - HCR
   IF (DL1.LT.0D0) DL1 = 16DO
   Y1 = THKV/2D0 - DL1/2D0
   IF (F(I).LT.0D0) Y1 = -Y1
   DL2 = 16DO - C - HCR
   IF (DL2.LT.0D0) DL2 = 0D0
   Y2 = -THKV/2D0 + 16DO - DL2/2D0
   FNT2 = DL2 * FTCR / 2D0
   IF (F(I).LT.0D0) Y2 = -Y2
   FNT = FNT1 + FNT2 + FNC
   BMCT = BNCR * YCR + FNT1 * Y1 + FNT2 * Y2
END IF
C ###############################################
C ####### STeel Forces routine #######
C ###############################################

SUBROUTINE FTS(K, EPSM, THKV, FI, FS, ES, SS, BMS, IFLT, FNUL)
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION FS(2), ES(2), SS(2), Y(2), AS(2), U(2), IFLT(2)
FNUL = 0D0
DO 4310 I = 1, 2
  IFLT(I) = 0D0
  FS(I) = 0D0
  SS(I) = 0D0
  ES(I) = 0D0
4310 CONTINUE

  IF (K.EQ.1 .OR. K.EQ.2 .OR. K.EQ.3 .OR. K.EQ.8) THEN
    EPSY = 2.6931D-03
    FY = 486D0
    E = 180500D0
    EP = 235D0
    FU = 512D0
    EPSU = 0.1286D0
  END IF
  IF (K.EQ.4 .OR. K.EQ.5 .OR. K.EQ.6 .OR. K.EQ.7) THEN
    EPSY = 2.260D-03
    FY = 420D0
    E = 185800D0
    EP = 351.45D0
    FU = 465D0
    EPSU = 0.1303D0
  END IF

  IF (K.EQ.1 .OR. K.EQ.2 .OR. K.EQ.5) THEN
    AS(1) = 0.1073D0
    AS(2) = 0.1073D0
  END IF

  IF (K.EQ.3) THEN
    AS(1) = 0.3218D0
    AS(2) = 0.3577D0
  END IF

  IF (K.EQ.4 .OR. K.EQ.8) THEN
    AS(1) = 0.2145D0
    AS(2) = 0.2145D0
  END IF

  IF (K.EQ.6 .OR. K.EQ.7) THEN
    AS(1) = 0.3218D0
    AS(2) = 0.3218D0
  END IF

  U(1) = 8D0
  U(2) = 8D0
  Y(1) = THKV / 2D0 + U(1)
  Y(2) = THKV / 2D0 - U(2)

  ES(1) = EPSM + FI * Y(1)
  ES(2) = EPSM + FI * Y(2)
  BMS = 0D0
  DO 4320 I = 1, 2
    IFLT(I) = 0
    IF (DAHS(ES(I)).LE.EPSY) THEN
      SS(I) = ES(I) * E
    ELSEIF (DAHS(ES(I)).GT.EPSY .AND. DAHS(ES(I)).LE.EPSU) THEN
      SS(I) = ES(I)/DAHS(ES(I)) * (FY + (DAHS(ES(I)) - EPSY) * EP)
    ELSE
      SS(I) = FU
    END IF
    IFLT(I) = 10 * I
    FNUL = FNUL + SS(I) * AS(I)
  END IF

  IF (FS(I) = SS(I) * AS(I)) THEN
    BMS = BMS + FS(I) * Y(I)
  END IF

4320 CONTINUE
RETURN
END
IMAGE EVALUATION
TEST TARGET (QA-3)

1.0
1.1
1.25
1.4
1.6

1.0
1.1
1.25
1.4
1.6

150mm
6"

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