Sequential Scouring: alternating patterns of erosion and deposition
– laboratory experiments and mathematical modelling

by

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A thesis submitted in conformity with the requirements
for the degree of Doctor of Philosophy
Graduate Department of Geography
University of Toronto

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Sequential Scouring: Alternating Patterns of Scour and Deposition - Laboratory Experiments and Mathematical Modelling
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Abstract:

Erosion and deposition by flowing water commonly produce patterns of soil type, topography and vegetation. Sometimes they produce distinctive, spatial patterns whereby erosion and deposition alternate downslope. These alternating patterns are common in arid and semi-arid regions on gentle slopes with low rainfall amounts and are typically associated with banded vegetation. Scour and fill processes in natural alluvial rivers commonly produce riffle-pool sequences. A similar, but smaller-scale, pattern of erosion and deposition was produced by this study – sequential scouring.

Sequential scouring is best described as series of scour chutes separated by deposition zones. These patterns of scour and deposition exhibit quasi-periodic behaviour and have 2 to 6 m distances between successive deposition zones. Scour chutes are small erosional channels which are not deeply incised, are not associated with headcuts and have non-vertical sidewalls. Deposition zones are lobate deposits where the thickest portion of the deposit occurs near the head of the deposit. Scour chutes are generally 5 - 10 cm wide and less than 1 cm deep, while deposition zones are between 20 -50 cm wide.

The purpose of this research was to: 1) produce sequential scouring in the laboratory: 2) determine the relationship between initial microtopography and the initiation of scour and deposition; 3) investigate the downslope sediment sorting associated with
sequential scouring; 4) determine the optimal conditions for the occurrence of sequential scouring; 5) determine the interactions between scour and deposition; and 6) produce a one-dimensional mathematical model for simulating sequential scouring.

The initiation of sequential scouring was strongly influenced by local convexities and concavities in the initial surface. Through time, deposition zones became dominated by coarser soil material, while scour chutes were typified by finer sediments. The variations in surface material altered downslope surface roughness and flow conditions. Strong positive feedback mechanisms developed which caused downslope variations in water and sediment fluxes. The interactions between scour and deposition produced a system which varied spatially and temporally. The mathematical model adequately simulated the initiation and development of sequential scouring as it reproduced the quasi-periodic behaviour, positive feedbacks and interactions between scour and deposition associated with sequential scouring.
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Glossary:

A  cross-sectional area
Css  sediment concentration
d  flow depth
Di  particle size of ith percentile
eb  coefficient for bedload efficiency
ffe  Darcy-Weisbach friction factor
Fr  Froude number
g  gravitational constant
I  infiltration rate
Ii  initial infiltration rate
K  relative roughness factor
MWD  mean weight diameter
n  Manning's roughness coefficient
Pr  rainfall rate
P  wetted perimeter
Q, Qw  water discharge
Qs  sediment discharge
Qh  header-tank discharge
q, qw  unit water discharge
q  unit sediment discharge
Re  Reynolds number
R  hydraulic radius
Rr  rainfall rate
S  bed slope, local gradient
Ss  surface energy gradient
t  time
tanα  coefficient for sliding friction
V  effective fall velocity
v  flow velocity
u  mean velocity
u*  shear velocity
w  flow width
wb  width of channel bed
x  slope position
z  bed elevation
Δz  change in bed height

α  slope angle
β  thalweg side-slope angle
βc  channel side-wall angle
θ  volumetric soil moisture
μE  hydraulic variable mean for scour chutes
μD  hydraulic variable mean for deposition zones
ξ  counter variable
ρb  soil bulk density
ρw  density of water
ρ  population correlation coefficient
τ  shear stress (Pa)
v  kinematic viscosity
ϕ1, ϕ2  spatial autoregression coefficients
Ω  stream power (W m⁻²)
ω  stream power per unit bed area
1.0 Sequential Scouring: Alternating Patterns of Erosion and Deposition

- Laboratory Experiments and Mathematical Modelling

1.1 Introduction:

Soil erosion research encompasses many topics ranging from broad-based environmental issues, such as the viability of arable land, to more specific studies concerning the physical processes acting on hillslopes. Many studies have examined the processes which lead to significant erosion from hillslopes (cf. Bryan, 1987); while others have focussed on major factors which influence these processes (cf. Selby, 1982; Lal, 1994; Morgan, 1995). However, most soil erosion research has concentrated on erosional processes with less attention paid to depositional processes. Comparatively few studies have dealt with the role of deposition in hillslope development (e.g. Moss and Walker, 1978; Pennock and de Jong, 1987; Hairsine and Rose, 1991). Furthermore, most studies simply infer the dominance of processes by measuring total runoff and sediment output (Selby, 1994). As these studies tend to spatially aggregate all active processes, no information can be gained about the importance of individual processes, the factors controlling them, or the interactions between them. The interactions between scour and deposition, in particular, have been largely unexplored. In order to gain a more complete understanding of erosional and depositional processes, it is necessary to determine how they interact to redistribute sediment and their significance in landscape evolution.

The hydraulic characteristics of flowing water are often highly variable due to, for
example, differences in bed roughness, flow depth and local slope. Localised sites of erosion can result and can sometimes produce distinctive, spatial patterns whereby erosion and deposition alternate downslope (Moss and Walker, 1978). Govers (1986) discussed alternating patterns of rilling and alluviation within agricultural furrows. Bryan and Poesen (1989) and Bryan and Oostwoud-Wijdenes (1992) have produced analogous forms (i.e. cyclic rilling\(^1\) and scour steps\(^2\), respectively) in laboratory experiments. Distances between deposition zones range from 3 – 5 m for cyclic rills (Bryan and Poesen, 1989) and 3 – 20 m for scour steps (Bryan and Oostwoud-Wijdenes, 1992). Larger-scale alternating patterns of erosion and deposition are common in arid and semi-arid regions on gentle slope gradients with low rainfall amounts and are typically associated with banded vegetation, *brousse tigrée* (Tiger Brush), desert ripples, or vegetation arcs (White, 1969, 1970, 1971; Cornet et al., 1988; Cornet et al., 1992; Thiery et al., 1995; Wallace and Holwill, 1997). Distances between deposition zones are on the order of 100 m for *brousse tigrée*. Scour and fill processes in natural alluvial rivers commonly produce riffle-pool sequences (cf. Sear, 1996; Clifford, 1993a and b). Spacings between successive ripples are roughly 5 to 7 times channel width (Keller and Melhorn, 1978).

However, most hypotheses concerning the initiation and development of

\(^{1}\) Cyclic rilling refers to alternating patterns of rilling and alluviation (Bryan and Poesen, 1989). They differ from sequential scouring in that rills are more deeply incised and, thus, comparatively more sediment is redistributed along the hillslope and alluviation zones tend to be larger.

\(^{2}\) Scour steps, or microsteps, are step-like feature which are characterized by broad scour surfaces (i.e. scouring is not confined to channels) separated by broad zones of deposition. Crenulate micro-steps occur at the head of the scour surfaces (Bryan and Oostwoud-Wijdenes, 1992).
alternating patterns of erosion and deposition, whether on hillslopes or in river channels, remain untested. Thus, the current understanding of processes leading to the development of erosion and deposition sequences remains incomplete. An understanding of the conditions under which alternating patterns of erosion and deposition occur, and the processes which act to redistribute sediments between erosion and deposition zones, should promote the development and use of more realistic models for sediment routing.

Alternating patterns of erosion and deposition in this laboratory study, here termed *sequential scouring*, are best described as a series of scour chutes\(^3\) separated by deposition zones\(^4\), oriented along a downslope transect. They exhibit an undulating longitudinal profile coupled with oscillations in bed width (Figure 1.1). Distances between successive deposition zones range between 1 – 5 m.

Scour chutes are zones of flow concentration where flows are relatively narrow (5 – 10 cm) and deep (0.5 – 1.0 cm). Incision increases local slope gradients upstream and enhances flow convergence subsequently increasing local velocities, shear stresses and stream powers. Increased sediment transport capacities cause net removal of sediments, greater incision and enhanced flow convergence. Sediment is transported only relatively

\(^3\) Scour chutes are defined as small erosional channels, differing from rill channels in that they are not as deeply incised, are not associated with headcuts and have non-vertical sidewalls. In contrast, rill channels tend to be deeply incised, are usually associated with headcutting and have near-vertical sidewalls and rectangular cross-sectional areas (Torri et al., 1987; Bryan, 1987; Moss and Walker, 1978).

\(^4\) Deposition zones were lobate deposits where the thickest part of the mass occurred near the head of the deposit (i.e. where runoff exited upslope scour chutes). The deposits thinned outward and downward from the head as runoff broke down into distributaries. The deposition zones were similar to large scale alluvial fans, except they were more ellipsoidal rather than fan-shaped with their long axes extending downstream.
Figure 1.1: Photo and outline of sequential scouring pattern taken from a preliminary rainfall experiment, July 13, 1995.
short distances (~0.25 – 5 m) downslope before deposition. The deposition zones obstruct runoff causing it to widen (20 – 50 cm) and thin (0.1 – 0.5 cm). Deposition decreases local slope gradients and enhances flow divergence. As a result, local flow velocities, shear stresses and stream powers decrease, diminishing the capacity to transport sediment. Through time, clear zonation develops with marked variations in capacity to transport sediment which leads to the development of sequential scouring.

1.2 Research Objectives:

There are two major goals for this research: 1) to explore the initiation and development of sequential scouring; and 2) to determine the interactions between scour and deposition. Neither has been adequately described or explained. The specific objectives of this research are to use laboratory simulations to:

1) produce alternating patterns of erosion and deposition;
2) determine the active processes and their influence on water and sediment output;
3) determine the optimal conditions under which sequential scouring develops;
4) document the quasi-periodicity and degree of spatial and temporal variability associated with sequential scouring;
5) investigate how microtopography, antecedent soil moisture and bulk density influence scour chute and deposition zone initiation;
6) isolate the hydraulic conditions associated with scour chutes and deposition zones and determine the interactions between scour and deposition; and
7) produce a simple, one-dimensional, mathematical model to simulate sequential scouring.

1.3 Significance:

Three contributions to soil erosion research and hillslope geomorphology will result from this study. First, it will provide insight to the initiation and development of sequential scouring. Second, it will address the dependencies between scour and deposition. Third, rather than simply inferring the dominance of onslope processes from water and sediment output, this study will directly relate the two.
2.0 Literature Review: Soil Erosion Processes and Modelling

2.1 Introduction:

Soil erosion by wind or water poses significant problems worldwide. It is responsible for causing a wide range of deleterious effects including decreases in agricultural productivity of the world’s arable land, an increased reliance on fertilizer usage, sediment clogging of natural and artificial waterways and decreases in water quality. Although wind erosion is a significant problem, especially in arid and semi-arid regions, erosion by water has been far more destructive (Lal, 1994). It has been estimated that, since the beginning of settled agriculture, roughly 430 million hectares of once productive land has been, and continues to be, made non-productive through erosion losses (Morgan, 1995; Lal, 1994). In order to create and select appropriate mitigation measures and conservation strategies for the prevention of soil erosion, it is necessary to have a detailed understanding of the processes and factors which influence the movement and redistribution of soil (Morgan, 1995).

Soil detachment occurs when the tractive forces exerted by moving water overcome the forces acting to hold particles, or aggregates, in place (Rose, 1994; Morgan, 1995). The relative magnitude of the resisting forces is related to the combined effects of grain, fluid and flow properties (Allen, 1994; Julien, 1995). These factors combined determine entrainment threshold conditions. However, as there are many degrees of freedom which must be considered, predicting entrainment thresholds is a daunting task (Richards, 1982).
There are two dominant groups of detachment and transport agents (Morgan, 1995). The first group includes the processes of rainsplash and unconcentrated overland flow. These processes act areally to remove a relatively uniform layer of hillslope material. The second group includes concentrated overland flow where detachment and transport are confined to channels.

Rainsplash and running water are the two primary tractive forces involved in sediment entrainment. While running water exerts a shear stress on the soil surface, it is not the most efficient detachment agent as a large proportion of the stress is used to overcome friction (Morgan, 1995). The most effective detachment agent is rainsplash (Morgan, 1978; Abrahams et al., 1991; Morgan, 1995); however, it is not an efficient transport agent. The processes of rainsplash and running water work in tandem to mobilise and redistribute material on hillslopes, and are primary agents in hillslope evolution.

Once the stress applied to the bed materials exceeds a critical shear stress and entrainment occurs, soil particles may be easily transported downslope by running water. The mode of transport also depends on the combined effects of the grain, fluid and flow properties (Allen, 1994). There are three general modes of transport on lower gradient slopes: bedload, which includes sliding and rolling, intermittent suspension and suspension (Moss and Walker, 1978). However, there is no clear division between the three (Allen, 1994). Clay and silt particles are usually transported as suspended load, while sands and larger particles are moved downslope as bedload (Julien, 1995). When the available kinetic energy is no longer sufficient to maintain sediment transport, deposition occurs (Morgan, 1995). As the processes of sediment entrainment, transport and deposition are all
dependent upon the grain, fluid and flow properties, all of these processes can occur simultaneously resulting in the generation of a variety of bedforms (e.g. ripples, dunes, antidunes, chutes and pools; Julien, 1995). These bedforms, in turn, affect flow properties and may enhance erosion and deposition. For instance, Moss and Walker (1978) noted that at net deposition sites, flow widths increased and flow depths decreased. This reduced velocities and available flow energy enhancing further deposition. At erosion sites, flow widths decreased and flow depths increased, enhancing erosion.

2.2 Rainsplash and Rainflow Erosion:

The significance of entrainment by rainsplash and rainflow transportation (i.e. raindrops impacting thinflow) has been documented by numerous authors (e.g. Wischmeier and Smith, 1958; Yariv, 1976; Morgan, 1978; Poesen, 1981; Moss, 1988; Guy et al., 1990; Proffit et al., 1991; Parsons et al., 1994). Raindrop impact has been shown to have an important effect upon both rillflow and sheetflow. Although raindrops have little effect on flow velocities and net drag force in deep flows, raindrops can provide a sediment-free water source to rillwash increasing its transport capacity (Savat, 1979).

The momentum transfer associated with raindrops striking thinflow can increase the shear stresses imparted to the soil surface (Palmer, 1963). These stresses increase with increasing water depth up to a critical depth of approximately three times the drop diameter. The additional stress imposed by the falling drops become negligible with greater depths. Rainflow transportation has been shown to act on slope gradients less than
0.005 (Savat, 1977); it can operate in both supercritical and subcritical flows and transport particles up to 3 mm in diameter (Moss et al., 1979). Rainsplash entrainment and rainflow transportation from interrill areas has been shown to transport sufficient quantities of sediment to rill channels, reducing the transport capacity of rill flow and resulting in channel infilling and elimination (Dunne and Aubry, 1986).

Contrary to Savat and Poesen (1981), Luk et al. (1993) demonstrated that raindrop impact on interrill areas was relatively aselective up to size ranges of several millimetres. If larger particles exist than can be moved by rainsplash on the interrill slopes, then armouring can occur. If a lag deposit develops on interrill regions, a relatively sediment-free water source will be supplied to rill channels increasing its transport capacity. Other studies have looked at the effect of surface slope and raindrop detachment (Torri and Poesen, 1992), seal development and slope on rainfall erosion (Mah et al., 1992), rainfall energy and surface seal (Römkens et al., 1990; Betzazel et al., 1995), raindrop impact and the breakdown of aggregates (Ghadiri and Payne, 1977), and splash detachment and kinetic energy of raindrop impact (Wischmeier and Smith, 1958; Sharma and Gupta, 1989; Sharma et al., 1993).

2.3 Rill and Interrill Erosion:

A great deal of attention has been focused on determining the potential geomorphic and hydrologic significance of the rilling process. Horton (1945), Schumm (1956) and Schumm et al. (1987) discussed the development of rills as agents of landform
development in terms of drainage basin evolution. Rills have been defined as small, intermittent, steep-sided erosional channels, usually only a few centimetres deep, which present no obstacle to normal tillage practices (Torri et al., 1987; Bryan, 1987). Moss and Walker (1978) observed that rill channels are also associated with active deposition within the channel. Dunne and Aubry (1986) explained rill system dynamics as a balance between rill and inter-rill processes. Inter-rill erosion is largely due to the combined processes of raindrop impact and sheetflow (Meyer, 1985; Luk et al., 1993; Lal and Elliot, 1994).

2.3.1 Threshold Conditions for Rill Initiation:

Of primary importance to an understanding of rill development is the need to determine the threshold condition for rill initiation (Govers, 1985). Most research has been centred on determining the critical tractive force for rill inception, while little attention has been focussed on soil resistance (Bryan, 1987). Rill incision may commence soon or immediately after the hydraulic tractive forces overcome the resistance thresholds of the soil material (Horton, 1945; Schumm, 1956; Rauws and Govers, 1988; Bryan, 1987; Selby, 1994). However, even though critical threshold conditions for rill incision may be surpassed, the development of rills may not occur (Dunne and Dietrich, 1980a,b; Dunne, 1980).

Many studies have determined a variety of hydraulic threshold conditions associated with incipient rilling. Among these are critical values of the Froude (Fr) and Reynolds (Re) numbers, shear velocity ($u_\tau$), shear stress ($\tau$), unit discharge ($q$) and stream
power (Ω) (Table 2.1). Gilley et al. (1993) isolated critical shear stresses and critical discharge rates for the initiation of rilling.

Slattery and Bryan (1992a) found that no single hydraulic threshold value could be isolated and used as a sole predictor for rill initiation. Controlled laboratory experiments showed that the Froude number and critical shear velocity are critically dependent on flow depth which changes dramatically due to rainfall intensity, surface roughness, slope, crusting, and variable grain size. Rather, ranges of hydraulic threshold values (i.e. zones of transition), in which rilled and unrilled flow occurred, was better suited to distinguished between rilled and unrilled surfaces.

Sheetflow characteristics, leading to the development of proto-channels and rills, have been observed in a number of studies. The flow of thin water films has been shown to be highly complex due to the relatively thick laminar sublayer layer in relation to the flow depth (Torri et al., 1987). In thin sheetflows, laminar flow was found to be the dominant flow pattern. However, as unit discharge increased, laminar flow became transitional to turbulent. The initial transition was characterized by the appearance of small vortices created by surface roughness. With further increases in discharge, these vortices transformed into isolated turbulent spots. These spots formed on 2° slopes with shear velocities between 3.0 and 3.5 cm s⁻¹. The vortices locally increased shear velocity, subsequently increasing transport capacity and rills were initiated (Torri et al., 1987). Conversely, Savat (1979) concluded that rill initiation had little to do with vortex erosion.

Moss et al. (1982) observed that proto-channels were associated with dominant secondary flow cells. Near the threshold conditions for bed load transport, these cells
Table 2.1: Hydraulic threshold conditions for rill and headcut incision.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Fr</th>
<th>Re</th>
<th>$u_o$ (cm s$^{-1}$)</th>
<th>$\Omega$ (W m$^{-2}$)</th>
<th>$q_o$ (cm s$^{-1}$)</th>
<th>$\tau$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merz and Bryan (1993)</td>
<td>0.1-0.3 RI</td>
<td>&gt;400 RI</td>
<td>5.0-7.0 RI</td>
<td>4.0-5.5 K</td>
<td>0.1-0.3 RI</td>
<td>4.0-8.0 RI</td>
</tr>
<tr>
<td>Slattery and Bryan (1992a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concentrated Flow Rill Flow</td>
<td>0.6</td>
<td>275</td>
<td>4.59</td>
<td>0.234</td>
<td>2.75</td>
<td>2.17</td>
</tr>
<tr>
<td></td>
<td>1.12</td>
<td>1314</td>
<td>6.62</td>
<td>1.119</td>
<td>13.32</td>
<td>4.89</td>
</tr>
<tr>
<td>Bryan (1990)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without Knickpoint</td>
<td>0.541</td>
<td>3.06</td>
<td>0.05</td>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With Knickpoint</td>
<td>1.02</td>
<td>4.67</td>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bryan and Poesen (1989)</td>
<td>&lt;1.0 RI</td>
<td>&gt;400 RI</td>
<td>3.1 RI</td>
<td>0.6-1.2 RI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Govers (1985)</td>
<td>1.5 RI</td>
<td>3.0 RI</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rauws and Govers (1988)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Rauws (1987)</td>
<td>&gt;0.5 RI</td>
<td>&gt;400 RI</td>
<td>3.2-3.4 RI</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torri et al. (1987)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{50}$ (µm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>&gt;3.0 RI</td>
<td></td>
<td>&gt;2.60 RI</td>
<td>0.95-1.1 RI</td>
<td>1.70-2.0 RI</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1.0-1.4 RI</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>4</td>
<td>0.8-0.95 RI</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Merritt (1984)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Overland Flow</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rill with Knickpoint</td>
<td>10.4</td>
<td>6.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rill w/o Knickpoint</td>
<td></td>
<td>4.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hodges (1982)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Savat and De Ploey (1982)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without Rainfall</td>
<td>2.8 RI</td>
<td>2.5 RI</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With Rainfall</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Savat (1980)</td>
<td>1.2 RI</td>
<td>3.0-3.5 RI</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Karez and Kersey (1980)</td>
<td>0.5 RI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$RI =$ rill inception; $K =$ knickpoint development; $D_{50}$ = median particle size
could spontaneously form on smooth beds initiating channel development. It was found that raindrop impact inhibit secondary flow cell development and channel initiation.

In order for rills to form, the hydraulic conditions must allow coarser fragments to be transported as easily as finer particles (i.e. aselective transport). Selective transport can produce surface lag deposits which can inhibit channel incision (Moss and Walker, 1978; Loch and Donnollan, 1983; Govers, 1985; Rose et al., 1990). It has been shown that rill channel flows are quite capable of transporting larger rock fragments as bedload with diameters of up to 6 cm (Moss and Walker, 1978) and 9 cm (Poesen, 1987) depending on hydraulic conditions and material characteristics. Torri et al. (1990) determined that surface flow became aselective at shear velocities greater than 3.2 cm s\(^{-1}\).

2.3.2 Headcut Development and Migration:

It has been determined that rill incision always begins with knickpoint development. However, not all knickpoints lead to subsequent rill formation (Slattery and Bryan, 1992a; Bryan, 1990; Merz and Bryan, 1993). Once formed, the knickpoints followed various evolutionary sequences including recession, bifurcation, and the development of serial knickpoints. All were related to localised deposition within the channel. Knickpoint decay was related to hydraulic changes and bank collapse. DePloey (1989) theoretically showed that smaller headcuts should retreat headward at higher rates than larger ones. Smaller headcuts should coalesce, if they are not destroyed, to form larger headcuts.

Schumm et al. (1987) discussed knickpoint migration in terms of drainage basin
dynamics. As headcuts migrate upslope, they supply sediment to downslope portions of a hillslope. This can potentially inundate the downstream flow with sediment, reducing transport capacity and channel incision downslope. Therefore, upslope degradation can cause downslope aggradation as channel erosion progresses upstream. This produces zones of deposition and erosion and may produce episodic sediment yields represented by pulses of sediment moving downstream.

2.4 Rills and Gully Development:

More permanent rill systems which exist over long periods of time may eventually deepen and widen downslope forming gullies (Bryan, 1987). Gullies can be classified as either continuous or discontinuous channel systems (Heede, 1974). Discontinuous gullies usually form in a series along the length of the hillside. Once initiated, their depth decreases rapidly downslope such that their bottom gradient is much gentler than the hillslope gradient. Where both gradients intersect, a sediment fan is built. According to Heede (1974), the largest rates of sediment production were associated with the discontinuous stages of gully development. The loose alluvial sediments deposited downslope of the plunge pool (at the foot of the headcut) could be mobilized periodically. This in combination with lateral channel migration and sidewall collapse produced appreciable amounts of sediment.
2.5 Factors Influencing Surface Erosion:

Factors affecting the rates of erosion in general are reviewed by Evans (1980) and Morgan (1995). Some of the most important factors determining the amount of soil erosion are soil texture, soil aggregation, surface roughness, antecedent moisture conditions, slope angle, vegetation and state of the surface (e.g. compacted, sealed). Soil erodibility is a complex property and is influenced by vegetation, topography, soil conditions, temperature and biological and chemical factors (Bryan et al., 1989; Young et al., 1990). The resistance to concentrated flow erosion has been shown to be strongly affected by initial soil moisture content and bulk density (Govers et al., 1990).

2.5.1 Influence of Sub-Surface Moisture:

The position of the water table in relation to the soil surface has been shown to have profound impacts on the processes of rilling on low to moderate slope angles (Bryan and Rockwell, 1997). Torri et al. (1994) found that runoff detachment was limited by the rate at which the water front penetrated the regolith. Michiels et al. (1989) suggested that the development of perched water tables should have major implications for surface erosion.

Bryan and Rockwell (1997) simulated hillslopes composed of freely-draining soils underlain by a layer of lower permeability at a relatively shallow depth (e.g. similar to a plough-pan). It was shown that almost no knickpoint development or sediment discharge occurred until a water table formed at the base of the soil. The rise in the water table
coincided with an immediate decrease in soil strength, the development of knickpoints and a substantial increase in sediment discharge. This study showed that surface hydraulic conditions (See § 2.3.1) may not be useful as a sole predictors for rill initiation. In contrast, Shainberg et al. (1994) suggested that detachment by rill erosion depended on surface properties and not on the bulk properties of the subsoil.

2.5.2 Surface Sealing:

The processes and micromorphology associated with surface sealing have been described by numerous authors (e.g. McIntyre, 1958; Tackett and Pearson, 1965; Norton; 1987; Mualem et al., 1990; Le Bissonais, 1990; Luk et al., 1990; Slattery and Bryan, 1994). Briefly, there are three types of seals: 1) disruptional, or structural, seals formed by the physical breakdown (e.g. slaking and microcracking), rearrangement and compaction of aggregates at the soil surface; 2) sedimentational, or depositional, seals formed by particle deposition on interrill areas or within rill channels during storm events; and 3) afterflow, or lamellar, seals formed by the parallel packing of clays during flow cessation. Sealing, in effect, can produce a two-layered soil from previously homogeneous conditions. The top layer, usually only a few mm thick, has greater strength and is less erodible, while the subsoil remains relatively weaker and more erodible.

Seals have a profound effect upon infiltration, runoff and surface erosion. Seals are known to dramatically decrease infiltration, increase runoff, and increase soil strength (Tackett and Pearson, 1965; Mualem et al., 1990; Moore and Singer, 1990; Le Bissonais
and Singer, 1993). During the period of increasing runoff, the sediment detachment rate reaches a maximum. Under steady-state runoff conditions, sediment detachment decreases and then tends towards an equilibrium suggesting that surface seals decrease the erodibility of the soil surface (Moore and Singer, 1990; Slattery and Bryan, 1992b). If the seal is breached by incision, erosion can be substantially increased as the subsoils are generally weaker (Bryan and Poesen, 1989). With regard to knickpoint formation, the surface seal can provide the necessary resistant layer protecting weaker soils in the headcut face (Bryan, 1990). Once rills develop, because the surface seal is breached, differences in infiltration rates can occur between rills and interrills (Poesen, 1984).

2.6 Spatial and Temporal Variability of Soil Erosion:

Evans (1980) and Morgan (1995) provide comprehensive reviews of the spatial and temporal controls on erosion rates. Many studies neglect within-plot heterogeneity and assume uniform conditions such as constant rainfall rates. Lane et al. (1995) have demonstrated that significant deviations between measured and modelled erosion rates can result when uniformity is assumed.

The classic Hortonian model (Horton, 1945) assumes runoff to be a simple function of rainfall and infiltration at a distance from the divide. Although it is well-known that this concept is not suited for humid regions (e.g. Dunne and Black, 1970a,b), recent studies have cast doubt on its applicability to arid and semi-arid regions also. For example, Bryan et al. (1978), working in badlands, observed spatially non-uniform runoff generation
after rainfall under conditions of spatially uniform antecedent soil moisture and infiltration. The variations in runoff were attributed to differences in the composition and structure of surface material. Scoging (1982) suggested that in arid and semi-arid regions, the spatial and temporal variability in infiltration and runoff generation is most often a response to differences in surface properties such as soil texture and structure, susceptibility to surface sealing, topography and vegetation cover.

In arid and semi-arid regions, observed spatial variability in soil loss has been attributed to across-slope differences in flow depth and velocity (Abrahams et al., 1989; 1991), vegetation and ground cover (Lane et al., 1995), microtopography (Dunne and Dietrich, 1980a,b; Dunne et al., 1991; Dunne et al., 1995), and discontinuities in overland flow due to differences in soil moisture, rainfall and infiltration rates (Lavee and Yair, 1990). Temporal variability in soil erodibility has been attributed to variations in soil moisture, temperature, tillage practice, bioturbation and chemical factors (Young et al., 1990). Govers (1987, 1991) showed that the spatial and temporal variability, associated with the rill erosion processes, are strongly controlled by topography and the mechanical and hydrological properties of the soil profile.

2.7 Alternating Patterns of Erosion and Deposition:

Many studies have discussed the initiation and development of alternating patterns of erosion and deposition on hillslopes and in natural alluvial channels. Bryan and Poesen (1989) and Kirkby (1990a) attributed the formation of cyclic rills to downslope alterations
in infiltration rates, soil erodibility and transport capacity. As surface seals are absent in rill channels, infiltration rates tend to be higher and soils are generally more erodible in rill beds. Material is eroded from rill beds and headcuts until transport capacity is reached in the rill channel. Due to infiltration losses, transport capacity is reduced a finite distance downslope and deposition occurs causing the flow to spread laterally. Beyond the deposit, the flow regains competence and erosion begins renewing the cycle.

The first stage in scour step, or microstep, development was observed to be bed moulding due to standing waves in shallow flows on very smooth surfaces (Bryan and Oostwoud-Wijdenes; 1992). The local increases in runoff energy created by the standing waves caused the formation of numerous, small crenulate scoursteps. These scoursteps eventually receded headward and coalesced to form wider, deeper microsteps.

Although much research has focussed on the morphology and maintenance of riffle-pool sequences (Tinkler, 1970; Richards, 1976a and b, 1978a and b; Keller and Melhorn, 1978; Thompson, 1986; Carling, 1991, Keller and Florsheim, 1993), comparatively few studies have focussed on their origin (Clifford, 1993a). A comprehensive review of riffle-pool sequence characteristics is given by Sear (1996). Clifford (1993a) described the initiation of riffle-pool sequences as an autogenetic process resulting from major flow obstructions. Roller eddies upstream and downstream of major flow obstructions create scour pools. The obstacle is eventually undermined and removed leaving two depositional areas (i.e. riffles) separated by a scour pool. The downstream riffle acts as a flow obstruction itself and promotes further riffle-pool sequences downstream. Yang (1971) and Cherkauer (1973) attributed the formation of riffle-pool
sequences as a means of stream self-adjustment to minimize the rate of energy expenditure.

2.8 Longitudinal Sediment Sorting:

The selectivity and distribution of entrainment, transport and deposition processes can produce systematic distributions of bed material on hillslopes from otherwise homogeneous initial conditions (Farenhorst and Bryan, 1995). These variations can then have cascading affects on the magnitude and frequency of subsequent processes. Grain size catenae can result from the translocation of solids by surface wash and their subsequent differential redeposition downslope (Milne, 1936; Robinson, 1936). Abrahams et al. (1985), Scheidegger (1986), Armstrong (1987) and Band (1985a and b; 1990) discussed relationships between slope gradient and sediment size in the creation of grain-size catenae. In general, these authors report that, on relatively unvegetated slopes where surface wash dominates, selective removal of fines leaves behind coarse lag deposits on steeper slopes. Finer sediment is transported downslope and redeposited differentially so that the finest fractions settle out on the most gentle gradients.

Sediment sorting in natural alluvial channels has been attributed to mean travel distance of bed particles and has shown to be dependent on discharge and grain-size (Kirkby, 1991). In general, smaller particles tend to move more frequently and travel further than larger ones with mean travel distance directly proportional to discharge.

Sediment sorting processes in riffle-pool sequences have been discussed in a
number of studies (cf. Sear 1996). The most widely accepted hypothesis for sediment transport in riffle-pool sequences is that finer sediments are transported from riffles to pools during low flows since velocities are maximized over the steeper riffle sections. However, at large discharges, it is hypothesized that a velocity reversal occurs whereby near-bed velocities in the pools will exceed riffle velocities. Coarser sediments are evacuated from the pools and transported to riffles. Through time, coarser-grained sediments come to dominate riffles and finer sediments in pools (Keller, 1971; Lisle, 1979; Hirsch and Abrahams, 1981; Keller and Florsheim, 1993).

2.9 Modelling Soil Erosion:

There are two general reasons for modelling soil erosion. First, predictive models are useful to policy-makers (e.g. planners and conservationists) as an aid to the decision-making process (Morgan, 1995), providing simple methods for estimating soil loss. The second reason for modelling, which is the focus of this research, is to describe how specific systems function. These explanatory models are predominantly used by researchers in the field of soil erosion to gain a better understanding of complex systems and how these systems respond to changes in environmental conditions (Freeze, 1978; Kirkby, 1980; Nearing et al., 1994; Morgan, 1995).

The three primary types of models are physical, analogue and mathematical (Morgan, 1995). Physical models, such as soil erosion flumes, are simply controlled, scaled-down versions of reality. Under the assumption that these systems adequately
simulate the larger-scale environmental scenarios under investigation, these small-scale physical models allow researchers to gain insight to large-scale systems by holding many conditions constant and allowing only a few to vary. In this way, the most important factors which influence a system’s development can be ascertained. Analogue models use one type of system to gain an understanding of another. An example is to use the flow of electricity to simulate water routing. These models assume that the two systems are analogous and that results from one can be directly applied to the other. Mathematical models use sets of mathematical expressions to represent reality. However, since mathematical models are simple abstractions of nature’s complexities, all of them are to some degree incorrect (Lane et al. 1988).

Mathematical models can be categorized as either empirical, conceptual or physically-based (Nearing et al., 1994). With empirical models, the physical processes, which act on the input variables, are represented by statistical relationships. These models use a black-box approach and are primarily used for predicting soil loss (Kirkby, 1980). With increases in the level of understanding of the processes involved in soil erosion, empirical modelling gave way to more physically-based techniques. Physically-based models rely more on the principles of mass and energy conservation. By applying these principles to individual slope segments, physically-based approaches are used to produce models for water and sediment routing. Physically-based models attempt to use a white-box approach with the intention of being used as diagnostic tools (Nearing et al., 1994). These models represent the essential mechanical processes which govern the dominant processes as a set of individual, physically-based mathematical expressions. Each
expression can be assessed individually to determine how it affects the overall system. In this way, each expression can be used as a diagnostic tool to ascertain the most important parameters which govern the whole system. However, although physically-based models are more attractive for reasons cited above, the majority of soil erosion models produced thus far remain largely empirical (Freeze, 1978; Kirkby, 1980; Addiscott, 1993; Morgan, 1995). Physically-based models which incorporate some degree of empiricism have been termed process-based models (Schmidt, 1991). Conceptual models usually incorporate spatially aggregated forms of the continuity equation (i.e. in contrast to physically-based models where the continuity equations are applied to discrete slope segments; Lane et al., 1988).

Gilbert (1877, 1909) was the first to consider changes in hillslope form as a result of the active transport processes (i.e. soil creep, rainsplash, and overland flow). Models for predicting soil erosion by water, particularly at the plot scale, first appeared in the 1940's. For example, Zingg (1940) produced an empirical formula which related soil loss to slope length and slope steepness. Though these initial attempts were purely empirical, they played a crucial role in leading the way to improved erosion prediction methods. Numerous process-based models have been produced since in an attempt to model the development of hillslopes as a result of various combinations of dominant transport processes (e.g. Scheidegger, 1970; Kirkby, 1971; Ahnert, 1977; Band, 1985a; Rose and Freebairn, 1985; Kirkby, 1990a,b; Schmidt, 1991; Wright and Webster, 1991). Modifications of Zingg's (1940) approach by subsequent researchers eventually lead to the development of the Universal Soil Loss Equation (USLE). The USLE (Wischmeier and
related soil loss to rainfall erosivity, soil erodibility, slope length, slope steepness, crop management and conservation practices.

There are six general steps which need to be followed when constructing any mathematical model (compiled from Carson and Kirkby, 1972; Freeze, 1978; Addiscott, 1993; Nearing et al., 1994; Morgan, 1995). The first step is to define the modelling objectives, intended uses and the environments in which the model should be used. This explains to the user, not only what kind of modelling approach is used, but also how and where the model should be applied. The second phase is model construction. This usually involves examining the physical system, determining the most important factors and processes to be included in the model framework, replacing these by equivalent mathematical expressions, and solving the system of equations by acceptable mathematical techniques (Freeze, 1978). This stage is analogous to hypothesis building (Addiscott, 1993) in that one is inherently, by virtue of the equations, hypothesizing that these are the only factors and processes which need be included to simulate the system under investigation. All assumptions and limitations should also be clearly outlined in this stage. The third step is model calibration. This is especially necessary if the model relies on any experimental data. In this step, the model parameters are adjusted for specific environmental conditions. The fourth phase is model validation. This should include a rigorous statistical analysis whereby the model output is compared to measured data. Since no model of complex systems can accurately and precisely reproduce reality, this stage should also explicitly state a range of acceptable results. Fifth, a sensitivity analysis should be included in all modelling exercises. This determines how sensitive the model
outputs are to changes in input parameters. The final stage of the modelling process is to interpret the model results in terms of the physical environment.

To date, there are literally hundreds of soil erosion models. Morgan (1995), Nearing et al. (1994) and Kirkby (1980) provide summaries of a few. As each model incorporates different sub-components, for example, for routing water and sediment, and numerical techniques, it would be difficult to discuss all of them on an individual basis. Furthermore, since the model proposed in this study is designed to simulate sequential scouring primarily as a function of sediment redistribution, the discussion will be restricted to the various approaches used to model overland flow and sediment transport.

2.9.1 Modelling Overland Flow:

The objective of some hydrologic models is to predict the spatial and temporal variations in overland flow (Rose, 1985). Most models for soil erosion simply assume an areally uniform rate of runoff. Although this may be appropriate for unvegetated hillslopes, this assumption is limited in practice in more humid, vegetated regions (Kirkby, 1980). In these regions, the generation of overland flow is much more complex. For example, given that soil moisture levels tend to increase progressively downslope and that near saturated areas may generate a disproportionate amount of runoff (Dunne and Black, 1970a,b), modelling surface water in humid regions can be a difficult task. As this study is concerned with the development of sequential scouring on unvegetated slopes, this discussion will be restricted accordingly.
A basic approach for approximating overland flow is to model overland flow as a simple function of areally uniform rainfall rate, \( P_r \), and infiltration rate, \( I \):

\[
R_r = P_r - I \tag{2.1}
\]

where \( R_r \) is excess rainfall rate and \( I \) and \( P_r \) are considered variable in time, but not over space (Kirkby, 1980; Rose, 1994). Thus, runoff generation per unit area, \( q \), is defined as:

\[
q = R_r x \tag{2.2}
\]

where \( x \) is the distance from the divide. This is known as the approximate analytic solution for overland flow (Rose, 1985) and has been used by a number of authors (e.g. Ahnert, 1977; Rose et al., 1983a,b; Khanbilvardi and Rogowski, 1984; Band, 1985a,b; Rose and Freebairn, 1985; Moore and Burch, 1986a; Kirkby, 1990a; Schmidt, 1991; Hairsine and Rose, 1992b; Rose, 1994). An exact analytical solution for overland flow uses the principles of mass conservation coupled with a kinematic flow approximation:

\[
\frac{\partial q}{\partial x} + \frac{\partial d}{\partial t} = R \tag{2.3}
\]

where \( d \) is flow depth. The kinematic flow approximation, \( q \), is expressed as:

\[
q = K_1 d^m \text{ and } K_1 = \frac{\frac{1}{2}}{n} \tag{2.4}
\]

where \( n \) is Manning's roughness coefficient, \( S \) is the sine of the local slope and \( m \) is an empirical exponent (Freeze, 1978; Parlange et al., 1981; Rose, 1985; Lane et al., 1988; de
Lima, 1989). The exact solution has been used in a number of models (e.g. Smith and Woolhiser, 1971; Kinsel, 1980; Lane and Nearing, 1989; Kirkby, 1990b; Hairsine and Rose, 1992b; Calver and Cammeraat, 1993).

Although these models use similar approaches for modelling runoff, they differ in their treatment of rainfall and infiltration. For instance, Band (1985b) incorporated variable rainfall rates, while Schmidt (1991) assumed rainfall to be spatially and temporally uniform. Infiltration is treated using a wide variety of infiltration equations from the more simple as an exponential decay function (Horton, 1945) to more complex using the Richard’s equation (Smith and Woolhiser, 1971).

2.9.2 Modelling Sediment Transport:

There are a wide variety of methods for mathematically describing sediment transport in both transport- and detachment-limited cases and under the influence of various combinations of active processes and environmental conditions (cf. Simons and Şentürk, 1977; Julien, 1995). A basic approach to modelling sediment transport ($q_s$) is to treat it as a simple power function of water discharge ($q_w$):

$$q_s = a q_w^b$$

where $a$ and $b$ are empirically derived (Morgan, 1995; Julien, 1995). More complex methods determine the rates of detachment for each of the active processes (e.g. splash, rill and interrill detachment) and combine them to produce the total amount of sediment
emanating from a slope segment (e.g. Anhert, 1977; Kirkby, 1980; Kinsel, 1980; Rose, 1985; Lane and Nearing, 1989; Kirkby, 1990a, Hairline and Rose, 1992a,b; Rose et al., 1983a,b; Rose, 1994). Other methods employ the various sediment transport equations developed for bedload, suspended or total load (e.g. Einstein, Yalin or Bagnold equations; cf. Julien, 1995; Rose and Freebairn, 1985; Moore and Burch, 1986b; Wright and Webster, 1991).

Numerous authors have discussed the effects of slope gradient on the rates of soil loss (e.g. Dunne et al., 1978; Bryan, 1979) and on sediment transport (e.g. Kirkby, 1971; Smith and Bretherton, 1972; Carson and Kirkby, 1972; Band, 1985a,b; Julien and Simons, 1985; Dunne and Aubry, 1986; Mathier et al., 1989; Julien, 1995; Mathier and Roy, 1996). A common power function used to express sediment transport as a function of unit discharge and slope gradient (S) is

\[ q_s = k q_w^m S^n \]  

where \( q_s \) is sediment discharge per unit width, \( q_w \) is water discharge per unit width, S is the tangent of the slope angle, k is an empirically-derived rate constant, and m and n are empirically derived exponents.

Coupling the various sediment transport equations with the principles of mass conservation, the rate of soil loss or gain can be determined for each hillslope segment such that:

\[ \rho_b \frac{\partial z}{\partial t} = -\frac{\partial q_s}{\partial x} \]
where \( z \) is surface elevation and \( \rho_b \) is soil bulk density. Equation 2.7 forms the basis for the process-based modelling approach used in this thesis.

2.9.3 Problems with Soil Erosion Modelling:

Most models are plagued by a number of general problems: 1) sediment storage is poorly understood and inadequately quantified; 2) the quantification of episodic events, which can account for a large proportion of sediment movement and redistribution, is not well understood; and 3) the role of biotic processes are usually not incorporated into modelling schemes (Rawat, 1987). Further, in terms of quantifying the various parameters involved, many process-based models employ some degree of empiricism; this limits their universal applicability. In the case of complex models (e.g. CREAMS (Kinsel, 1980) and WEPP (Lane and Nearing, 1989)), they can be of limited use solely because they require large numbers of input parameters, some of which may be unknown or unquantifiable (Schmidt, 1991). Many models are limited in use solely due to their inability to characterise flow width (Darby and Thornes, 1996). Most models simply neglect variable flow width and assume it to be constant. This, of course, can have extreme consequences for the proper quantification of the water and sediment fluxes. Scale issues continue to plague most modelling endeavours. Models constructed for large-scale drainage basins may not be useful for smaller-scale systems (e.g. hillslopes) simply because the dominance of a specific set of processes is scale-dependent (Rawat, 1987). Few models incorporate the effects of soil and pore fluid chemistry. These can have profound influence on soil
aggregation and crusting (Bryan, 1987) and, thus, affect the resistance to entrainment. Lastly, spatial heterogeneity in environmental conditions (e.g. soil type, soil moisture) are largely neglected.

2.10 Summary:

The state of knowledge of soil erosion processes has significantly improved since the time of Horton (1945). However, many important gaps still remain. First, as most soil erosion studies treat the processes separately, little is known about the complex interactions between the processes. Second, few studies consider the spatial and temporal variability of hillslope processes (e.g. Scoging, 1982; Lavee and Yair, 1990; Young et al., 1990; Abrahams et al., 1991; Lane et al., 1995). As this significantly affects sediment yield and rates of hillslope denudation, this area needs much more attention. Third, most research has been preoccupied with defining hydraulic threshold conditions for entrainment using indices of surface characteristics. Although the influence of subsurface flow as a contributor to rill initiation, for instance, has been recognized (Bryan, 1987; Michiels et al., 1989), few studies have attempted to quantify these relationships (e.g. Bryan and Rockwell, 1997). Lastly, few studies deal directly with biotic influences (e.g. vegetation, bioturbation). Although this simplifies reality, the effects of vegetation, for instance, on the processes of soil erosion are not well-understood.
3.0 Materials and Methods for Laboratory Simulations:

3.1 Introduction:

In order to isolate the variables which contribute to the initiation and development of sequential scouring, controlled laboratory experiments were performed on unvegetated artificial slopes. These experiments were carried out in the Soil Erosion Laboratory, University of Toronto at Scarborough, to produce sequential scouring and generate data for the following purposes:

1) to determine factors contributing to sequential scouring;
2) to test a series of hypotheses for soil erosion and deposition; and
3) for validating the mathematical model.

The simulations were designed specifically to permit measurement of hydraulic variables, the longitudinal bed profiles and planimetric morphologies associated with erosion and deposition under various slope and discharge conditions.

3.2 Slope and Soil:

An artificial soil was prepared by combining different proportions of two natural Canadian soils: Pontypool fine sand and Peel clay. These soils originated from an area north of Metropolitan Toronto near the Oak Ridges Moraine. The soils were originally air-dried, mixed and sieved through a 4 mm screen producing a mixture of approximately 80%
Pontypool fine sand and 20% Peel clay. Particle size analyses (pipette method; Gee and Bauder, 1986) showed that the soil mixture was a fine sandy loam (Table 3.1; Figure 3.1). However, since the mixture consisted predominantly of small, relatively stable clay aggregates (~2 to 4 mm) surrounded by a matrix of fine sand, it was considered that the soil was better described by non-dispersive particle size analysis (dry sieving; Kemper and Rosenau, 1986). The soil was dry sieved through 7 screens (i.e. 0.1, 0.25, 0.5, 1.0, 2.0, 2.8, 4.75 mm). Results from the dry sieve analysis were more representative of the particle size distribution of the soil mixture (Figure 3.1). The soil mixture is bimodal with two dominant grain size fractions: 60% of the total material by weight is accounted for by the 50 to 200 μm and 2.0 to 4.75 mm size fractions.

Table 3.1: Particle size distribution of soil mixture from pipette method.

<table>
<thead>
<tr>
<th>Particle Size</th>
<th>%weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 63 μm (sand)</td>
<td>73.2</td>
</tr>
<tr>
<td>2 - 63 μm (silt)</td>
<td>14.3</td>
</tr>
<tr>
<td>&lt; 2 μm (clay)</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Prior to an aggregate stability analysis, roughly 34% of the material by weight had diameters greater than 2.0 mm (Figure 3.1). After testing for aggregate stability (water stable aggregate test; Kemper and Rosenau, 1986), only 9.6% of the total by weight had diameters greater than 2.0 mm (Table 3.2) showing that larger aggregates were only moderately stable.
Figure 3.1: Sediment size distribution of soil mixture, both aggregated and dispersed.

Table 3.2 Aggregate stability for Peel clay and soil mixture (water stable aggregate test).

<table>
<thead>
<tr>
<th>Water Stable Aggregates (mm)</th>
<th>Peel Clay (% weight)</th>
<th>Soil Mixture (% weight)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;2.8</td>
<td>52.7</td>
<td>5.2</td>
</tr>
<tr>
<td>2.8 - 2.0</td>
<td>7.8</td>
<td>4.4</td>
</tr>
<tr>
<td>2.0 - 1.0</td>
<td>10.8</td>
<td>5.7</td>
</tr>
<tr>
<td>1.0 - 0.5</td>
<td>7.2</td>
<td>3.4</td>
</tr>
<tr>
<td>0.5 - 0.25</td>
<td>4.5</td>
<td>15.0</td>
</tr>
<tr>
<td>&lt;0.25</td>
<td>16.9</td>
<td>66.3</td>
</tr>
<tr>
<td>Pre-Test Moisture Content (% weight)</td>
<td>12.4</td>
<td>4.1</td>
</tr>
</tbody>
</table>
The soil mixture was removed, re-sieved, blended with unused soil and placed back into the flume for each experiment (a total of 12 experiments; see below). Prior to each experiment, a 1 kg sample was taken from the bed slope for dry sieve analysis. The resulting frequency distribution (by percent weight in each sieve) of the soil mixtures used in each experiment are shown in Table 3.3. The frequency distributions were tested to determine if similar soils were used in each experiment or if the original soil mixture changed significantly through time as a result of experimental procedures. A chi-square goodness-of-fit test, at the 0.05 significance level, was used to compare the frequency distribution of the original soil mixture (expected distribution) to those used in each experiment (observed distributions; See Freund and Simon, 1992). The chi-square statistic for each comparison shows that the soils used in each experiment were similar, except for experiments 9 and 11 (Table 3.3). The soil mixture used in experiment 9 had a finer overall particle size distribution, while that used in experiment 11 was coarser.

After each simulation, samples were taken from the near surface layer of deposition and scour zones. The mean weight diameter (MWD) was determined for each sample by dry sieve analyses. The MWD equals the sum of the products of:

1) the mid-point between two sieves, determined from the 7 sieve sizes; and

2) the percentage of the total weight occurring in each sieve (Van Bavel, 1949; Youker and McGuinness; 1956).

The summation is carried out over all sieve sizes including the material which passes through the finest sieve.
Table 3.3: Chi-squared Goodness-of-fit test comparing the frequency distribution of the original soil mixture (expected) to the soil mixture used in each experiment (observed).

<table>
<thead>
<tr>
<th>Sieve Size (mm)</th>
<th>Expected frequency distribution of original soil (%)</th>
<th>Observed frequency distributions for the soil mixture used in experiments 2 – 12 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment 1</td>
<td>2</td>
</tr>
<tr>
<td>4.75</td>
<td>4.6</td>
<td>5.5</td>
</tr>
<tr>
<td>2.8</td>
<td>17.4</td>
<td>19.7</td>
</tr>
<tr>
<td>2.0</td>
<td>11.3</td>
<td>9.6</td>
</tr>
<tr>
<td>1.0</td>
<td>12.0</td>
<td>11.4</td>
</tr>
<tr>
<td>0.50</td>
<td>9.3</td>
<td>7.6</td>
</tr>
<tr>
<td>0.25</td>
<td>20.8</td>
<td>22.4</td>
</tr>
<tr>
<td>0.106</td>
<td>21.1</td>
<td>21.5</td>
</tr>
<tr>
<td>0</td>
<td>3.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Chi-square Goodness-of-fit test comparing observed distributions (exps. 2 – 12) to expected frequency distribution (exp 1). At 0.05 significance level, reject null hypothesis (observed and expected are not significantly different) if $\chi^2 > \chi^2_{0.05} (= 15.507)$.

<table>
<thead>
<tr>
<th>df = 7</th>
<th>Experiment 1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment 1</td>
<td>2.623</td>
<td>1.346</td>
<td>2.999</td>
<td>1.513</td>
<td>1.838</td>
<td>6.195</td>
<td>3.144</td>
<td>16.58</td>
<td>1.171</td>
<td>22.34</td>
<td>11.68</td>
</tr>
</tbody>
</table>
The soil was placed in a 8.5 m × 0.8 m perspex flume and smoothed to reduce significant undulations. The soil was lightly compacted to a depth of ~5 cm with average initial bulk densities ranging between 1.4 and 1.6 kg m⁻³. The flume was tilted to simulate gradients of 0.026, 0.061 and 0.087. Runoff and sediment load were measured from a terminal weir. A thalweg with mean side-slopes of 5° was sculpted longitudinally down the soil surface (Figure 3.2). The soil was placed directly on top of the impermeable flume base. Although this inhibited drainage, it was done to decrease the time to saturation, reduce infiltration losses and hasten the experimental process. The impermeable boundary

Figure 3.2: Photo of a typical cross-sectional profile. Bedstead rods show the thalweg with mean side slopes of 5°.
is not dissimilar to plough pan layers found in agricultural fields. A similar methodology was used by Bryan and Rockwell (1997). In their simulations, the perspex flume base caused the rapid creation of a perched water table. Bryan and Poesen (1989) and Bryan and Oostwoud-Wijdenes (1992) used a similar soil mixture for their cyclic rilling and scour step simulations, respectively. It is considered that the results from their studies can be applicable to the present simulations.

3.3 Runoff:

From twelve preliminary rainfall analyses, the temporal and spatial distribution of the simulated rainfall intensities produced in the Soil Erosion Laboratory were found to vary significantly, ranging from ~40 to more than 120 mm hr\(^{-1}\). As a consequence, it was difficult: 1) to produce rainfall events of sufficiently low intensity comparable to conditions under which patterns of erosion and deposition were known to occur; and 2) to replicate similar rainfall events due to the temporal and spatial variability. However, three of the preliminary analyses did demonstrate that sequential scouring only occurred with weir discharges less than 3.0 \(l\) \(min^{-1}\). In place of rainfall simulations, a constant-head tank was mounted at the upper end of the flume to produce replicable, low magnitude surface water discharges (Figure 3.4). This apparatus simulated discharges up to 5.0 \(l\) \(min^{-1}\). Four discharges were used for the simulations: 1.0, 1.6, 2.2 and 3.0 \(l\) \(min^{-1}\).

Periodically during each experiment, runoff was halted in order to measure the longitudinal profiles and planimetric morphologies. Therefore, each experiment consisted
Figure 3.4: Diagram and photo of constant-head tank used to generate runoff.
of two or three sub-experiments, each lasting between 25 and 65 minutes. The gap in time between sub-experiments ranged between 0.5 and 2 hrs. The duration of each sub-experiment was varied as necessary to: 1) obtain a sufficient number of onslope water and sediment samples (~30 per sub-experiment); and 2) reach a runoff equilibrium (i.e. when weir and header-tank discharges were equal). After a runoff equilibrium was established in each experiment, surface morphologies appeared to stabilise (e.g. erosion and deposition zones did not change in location or areal extent) and the experiments were terminated.

3.4 Soil Moisture:

Volumetric soil moisture was monitored using eight time domain reflectometers (TDR’s) designed by Hawke (1997). These were placed at one metre distances between 2.5 and 5.5 m from the weir on either side of the flume at a depth of approximately 3.5 cm (Figure 3.5). These instruments measured the antecedent, volumetric soil moisture, the temporal changes in soil moisture and the time to soil saturation. Five core samples were taken at one metre distances between 3.5 and 5.5 m from the weir and used to determine (gravimetrically) antecedent wet and dry bulk densities and volumetric soil moisture for each experiment. Antecedent soil moisture measured from the TDR’s and gravimetric measurements were compared and found to agree within ±5%. Infiltration rates, corresponding to a variety of soil moisture and bulk density conditions, were determined independently using constant-head infiltration columns (See Bouwer, 1986).
3.5 Longitudinal Profile and Planimetric Morphology:

Longitudinal bed profiles were measured at 0.25 m distances down the centre of the thalweg before and after each sub-experiment using a bedstead and ruler (See Figure 3.2). These measurements are considered accurate to 0.5 mm. The longitudinal profiles were used to determine local slope gradients and changes in bed elevation. The planimetric morphologies were measured before and after each sub-experiment at 0.10 m distances.
using a ruler. Thus, a detailed survey was made of the surface morphology, bed height, local gradients and areal extent of erosion and deposition zones.

3.6 Sediment and Water Measurements:

Sediment and water characteristics were monitored at the terminal weir and on-slope at 0.5 m distances between 3.0 and 5.5 m from the weir (Figure 3.5). The weir was mounted horizontally across the slope base. The height of the weir was set equal to the elevation of the vertex of the V-shaped thalweg moulded in the initial surface. Water and sediment discharge (i.e. total load) were sampled from the weir at 1 minute intervals. This consisted of taking two types of samples from the weir. First, water discharge was determined every 2 minutes by measuring the fill time of approximately 500 ml of flow in graduated cylinders. Second, at alternate 2 minute intervals, sediment discharge and sediment concentration were determined by measuring the fill time of 500 ml beakers and the volume of water contained in the beakers. The sediment collected in the beakers was later oven dried, weighed and coupled with the measured volumes to produce both sediment concentration and sediment discharge. The flow width immediately above the weir and water temperature were monitored at 10 minute intervals. Flow width was used to determine the water and sediment fluxes at the weir.

Onslope measurements of flow velocity, width and depth, and sediment and water discharge were taken simultaneously at successive downslope locations with a sampling frequency of approximately once per minute. Thus, a set of sediment and water
characteristics for all locations between 3.0 and 5.5 m was obtained approximately every 6 minutes. No samples were taken between 0 to 3.0 m and 5.5 to 8.5 m as these regions were regarded as potentially influenced by the weir and header tank, respectively.

3.6.1 Sampling Procedures and Data Reliability:

Flow velocity along the downslope transect was determined by manually timing (with a stopwatch) the downslope movement of a potassium permanganate (KMnO₄) dye tracer over 0.5 m distances. Because of dye tracer diffusion and operator error, it is difficult to obtain accurate measurements of velocity. To obtain better estimates of velocity, Slattery and Bryan (1992a) timed the movement of dye tracers over 0.5 m intervals using both stopwatches and a video playback monitor accurate to 0.03 s (i.e. one frame is 1/29th of a second). In general, it was found that: 1) increasing the flow velocity and decreasing the length over which it was measured, decreased the reliability of manual velocity measurements; and 2) manual velocity measurements tend to overestimate velocity (determined by video monitor) above 20 cm s⁻¹. As velocities measured in this study were mainly above 20 cm s⁻¹ and the experimental design and procedures were similar to those of Slattery and Bryan (1992a), their correction algorithm \( y = 0.855x + 0.938, \ R^2 = 0.894, \) where the dependent variable is video velocity and the independent is measured velocity) was used to adjust the manual velocity measurements and obtain better velocity estimates.

Flow width was determined by directly measuring the width of the potassium permanganate dye tracer as it passed the downslope end of each measurement section.
using thin rulers. Errors associated with this technique are: 1) it is difficult to determine the actual boundaries of flow even with the aid of dye tracers; and 2) because of more sluggish water near flow boundaries, due to frictional drag, dye tracers do not penetrate these regions instantaneously. Therefore, although the dye tracer width most likely accounts for total width, it cannot account for it entirely. Further, it is nearly impossible to accurately measure flow width of unconcentrated runoff. This is mainly because of the irregular nature of the flow boundaries as runoff flows around and through surface deposits such as aggregates. However, through observations it was noted that the unmeasured portion of flow width, due to “edge-effects”, was on the order of less than 0.5 cm and depended on the local water flux and flow depth. Since most measured widths were greater than 5 cm, these errors should not amount to more than 10%.

Flow depth was measured to the nearest 0.5 mm at the downslope end of each measurement section using thin rulers. Accurate depth measurements in thin flow are difficult to obtain for a variety of reasons: 1) under highly mobile bed conditions or in sediment laden flows, it is difficult to determine the bed location; 2) manual depth readings can only be obtained to the nearest half millimetre in flows which are on average less than a centimetre deep; 3) the thin rulers used for measurement can pierce the channel bed; 4) the water surface height is artificially raised at the leading edge of the ruler due to hydraulic jumps and surface tension; and 5) in turbulent flows, the water surface is not always easily discernible. Given that all flow depths measured in these experiments were less than 1.0 cm, the manual depth measurements most likely represented significant overestimations of the true water depths. Errors with manual flow depth measurements
have been considered to range from 30% (Dunne and Dietrich, 1980a,b) to 50% (Merz and Bryan, 1993). Furthermore, since there is no way of determining the actual depths associated with these samples, it is impossible to objectively correct for the measurement errors. The manual depth measurements were used for the analyses with knowledge of their inherent inaccuracies.

Sediment concentration and water and sediment discharge (i.e. total load) were measured using miniature samplers. These samplers are thin-walled (~100μm) cylinders (diameter: 4.2 cm; length: 15 cm) with a single open end. Due to the small thickness of the container walls, these devices could sample surface flows with very small depths. The samplers were lowered into the flow at angles similar to local bed slopes until slight contact with the bed was made. The samplers were lowered into the flow so that bed disturbance was negligible. The samplers were allowed to fill with sediment and water to approximately 5 cm from their opening. This avoided overfilling and flow back-up. Inserting them into the flow, the time to fill was recorded. The samplers are sufficiently large to allow for 1 to 3.5 s of flow sampling, depending on the water discharge rate. The water volume and dry weight of sediment captured by the sampler was later determined. Since the samplers only account for a small portion of the total flow cross-section, and, thus, only a small portion of total water and sediment discharge, these samplers were calibrated against the more reliable weir data. An experiment was run independently in which samples were obtained simultaneously at the weir and immediately above the weir. Comparing these sediment and water discharge values, it was found that the samplers on average overestimated water discharge per unit width by a factor of two and sediment
discharge per unit width by a factor of three (Figure 3.6a and b). A correction algorithm was produced using linear regression and applied to the experimental data set.

The onslope flow velocities, widths, depths and local slope gradients were used to calculate the various hydraulic parameters (Table 3.4). However, given the errors associated with the flow measurements, in particular depth, the computed hydraulic characteristics should be considered only as indices for the flow conditions.

3.7 Experimental Design:

The various slope and water discharge conditions under which alternating patterns of erosion and deposition (i.e. cyclic rilling and scour steps) have been shown to occur are listed in Table 3.5. Bryan and Poesen (1989) and Bryan and Oostwoud-Wijdenes (1992) both observed these features: 1) using runon experiments, stream power ranged from 0.015 to 0.019; and 2) under simulated rainfall, stream power was larger than 0.049.

Using this range of conditions, a total of fifteen experiments (i.e. three rainfall, P1 – P3, and 12 runon experiments) were undertaken for this study in which water discharge and slope gradient were varied with the goal of producing sequential scouring. Table 3.6 summarizes these fifteen simulations in terms of their gradients, discharges and stream powers. Table 3.7 summarizes the initial conditions and duration of each sub-experiment used for experiments 1 – 12 and P1, P2 and P3. The variation in initial conditions (e.g. soil moisture) reflected the inability to control the environmental conditions in the laboratory.
Figure 3.6a: Calibration of sampler for unit water discharge.

\[
\text{Sampler } q_w = 0.38 \text{ (weir } q_w), \quad r^2 = 0.7, \quad n = 30
\]

Figure 3.6b: Calibration of sampler for unit sediment discharge.

\[
\text{Sampler } q_s = 0.25 \text{ (weir } q_s), \quad r^2 = 0.7, \quad n = 30
\]
Table 3.4: Hydraulic characteristics of open channel flow.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Definition</th>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydraulic Radius</td>
<td>R</td>
<td>( R = \frac{A}{P} )</td>
<td>( A = ) cross-sectional area, ( g = ) gravitational constant, ( P = ) wetted perimeter</td>
</tr>
<tr>
<td>Stream power per unit bed area</td>
<td>( \omega )</td>
<td>( \omega = \frac{\rho_w g OS_e}{w} = \tau v )</td>
<td>( S = ) bed slope, ( S_e = ) surface energy gradient, ( w = ) flow width, ( v = ) velocity, ( \nu = ) kinematic viscosity</td>
</tr>
<tr>
<td>Froude No.</td>
<td>Fr</td>
<td>( Fr = \frac{v}{\sqrt{gR}} )</td>
<td>( \rho_w = ) density of fluid, ( \tau = ) shear stress</td>
</tr>
<tr>
<td>Reynold’s No.</td>
<td>Re</td>
<td>( Re = \frac{vR}{\nu} )</td>
<td></td>
</tr>
<tr>
<td>Shear Velocity</td>
<td>( \nu_s )</td>
<td>( \nu_s = \sqrt{\frac{\tau}{\rho_w}} = \sqrt{gRS} )</td>
<td></td>
</tr>
<tr>
<td>Shear Stress</td>
<td>( \tau )</td>
<td>( \tau = \rho_w gRS )</td>
<td></td>
</tr>
<tr>
<td>Darcy-Weisbach friction factor</td>
<td>( ff )</td>
<td>( ff = \frac{8gRS}{\nu^2} )</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.5: Slope and water discharge under which alternating patterns of erosion and deposition occur.

<table>
<thead>
<tr>
<th>Experiment #</th>
<th>Gradient</th>
<th>Water Discharge (l min⁻¹)</th>
<th>Stream Power (W m⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bryan and Poesen, 1989</td>
<td>~0.07</td>
<td>~3.6</td>
<td>0.040 *</td>
</tr>
<tr>
<td>Bryan and Oostwoud-Wijdenes, 1992</td>
<td>0.026</td>
<td>~11.5</td>
<td>0.049 *</td>
</tr>
<tr>
<td></td>
<td>0.026</td>
<td>~4.8</td>
<td>0.020 *</td>
</tr>
<tr>
<td></td>
<td>0.014</td>
<td>~7.8</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>0.014</td>
<td>~7.8</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>0.014</td>
<td>~8.9</td>
<td>0.020</td>
</tr>
</tbody>
</table>

* denotes rainfall experiments.

Table 3.6: Summary of experimental design.

<table>
<thead>
<tr>
<th>Experiment #</th>
<th>Gradient</th>
<th>Water Discharge (l min⁻¹)</th>
<th>Stream Power (W m⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.087</td>
<td>2.9</td>
<td>0.042 *</td>
</tr>
<tr>
<td>P2</td>
<td>0.087</td>
<td>3.3</td>
<td>0.055 *</td>
</tr>
<tr>
<td>P3</td>
<td>0.087</td>
<td>4.1</td>
<td>0.059 *</td>
</tr>
<tr>
<td>1</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
</tr>
<tr>
<td>2</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
</tr>
<tr>
<td>3</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
</tr>
<tr>
<td>5</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
</tr>
<tr>
<td>6</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
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<tr>
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<td>0.087</td>
<td>1.0</td>
<td>0.014</td>
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<tr>
<td>8</td>
<td>0.061</td>
<td>2.2</td>
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<tr>
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<td>10</td>
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<td>11</td>
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<td>3.0</td>
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</tr>
<tr>
<td>12</td>
<td>0.061</td>
<td>2.2</td>
<td>0.022</td>
</tr>
</tbody>
</table>

* denotes rainfall experiments.
Table 3.7: Summary of initial conditions and duration of each sub-experiment for experiments 1 – 12 and P1, P2 and P3.

<table>
<thead>
<tr>
<th>Exp #</th>
<th>Initial Dry Bulk Density (kg m(^{-3}))</th>
<th>Slope Angle (°)</th>
<th>Sub-Experiment &amp; Duration (min)</th>
<th>Soil Moisture (m(^3) m(^{-3}))</th>
<th>Header Tank Discharge (m m(^{-1}))</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1.44</td>
<td>5</td>
<td>A 32</td>
<td>0.20</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B 33</td>
<td>0.28</td>
<td>1.6</td>
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<td></td>
<td></td>
<td></td>
<td>C 44</td>
<td>0.42</td>
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</tr>
<tr>
<td>2</td>
<td>1.44</td>
<td>5</td>
<td>A 23</td>
<td>0.23</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B 25</td>
<td>0.33</td>
<td>1.6</td>
</tr>
<tr>
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<td></td>
<td>C 31</td>
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<td></td>
<td></td>
<td>B 25</td>
<td>0.36</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>C 32</td>
<td>0.36</td>
<td>1.6</td>
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<td></td>
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<td>B 28</td>
<td>0.27</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>C 22</td>
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<tr>
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<td>0.22</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B 21</td>
<td>0.30</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>C 22</td>
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</tr>
<tr>
<td>6</td>
<td>1.22</td>
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</tr>
<tr>
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<td></td>
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<td>1.6</td>
</tr>
<tr>
<td>7</td>
<td>1.27</td>
<td>5</td>
<td>A 53</td>
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<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B 28</td>
<td>0.30</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>C 19</td>
<td>0.30</td>
<td>1.0</td>
</tr>
</tbody>
</table>

A, B and C denote the sub-experiments for each experiment.
<table>
<thead>
<tr>
<th>Exp #</th>
<th>Initial Dry Bulk Density (kg m$^{-3}$)</th>
<th>Slope Angle (°)</th>
<th>Sub-Experiment &amp; Duration (min)</th>
<th>Initial Soil Moisture (m$^3$ m$^{-3}$)</th>
<th>Header Tank Discharge (l min$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1.31</td>
<td>3.5</td>
<td>A 43</td>
<td>0.11</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B 26</td>
<td>0.22</td>
<td>2.2</td>
</tr>
<tr>
<td>9</td>
<td>1.32</td>
<td>3.5</td>
<td>A 46</td>
<td>0.13</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>B 23</td>
<td>0.35</td>
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<td></td>
<td>B 26</td>
<td>0.26</td>
<td>1.0</td>
</tr>
<tr>
<td>11</td>
<td>1.20</td>
<td>1.5</td>
<td>A 69</td>
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<td>1.13</td>
<td>3.5</td>
<td>A 72</td>
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<td></td>
<td></td>
<td></td>
<td>B 38</td>
<td>—</td>
<td>2.2</td>
</tr>
<tr>
<td>P1</td>
<td>1.44</td>
<td>5</td>
<td>—</td>
<td>120</td>
<td>0.12</td>
</tr>
<tr>
<td>P2</td>
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<td>5</td>
<td>—</td>
<td>120</td>
<td>0.10</td>
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<td>P3</td>
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<td>5</td>
<td>—</td>
<td>80</td>
<td>0.09</td>
</tr>
</tbody>
</table>

The three preliminary analyses (P1, P2, and P3) had slope gradients of 0.087 and final equilibrium discharges of 2.9, 3.3 and 4.1 l min$^{-1}$, respectively. Experiments 1 – 6 were designed as replicates having 0.087 slope gradients and 1.6 l min$^{-1}$ header-tank discharges. Experiments 7 – 12 used various combinations of slope gradient and header-tank discharge. It was believed that sequential scouring would develop in experiments 1 – 12, given the conditions under which cyclic rilling and scour steps had been shown to occur (Table 3.5).
4.0 General Experimental Results and Discussion:

4.1 Introduction:

Generally, it was expected that sequential scouring would develop in all experiments, given the results of Bryan and Poesen (1989) and Bryan and Oostwoud-Wijdenes (1992). However, sequential scouring was exclusive to experiments 1 – 6 and 12. Experiment P1 produced well-defined cyclic rilling. The remaining simulations (P2, P3 and 7 – 11) produced a variety of erosional and depositional forms such as braided washes, deposition zones and scour chutes, but no patterns of alternating erosion and deposition. All experiments (except experiment 11) were run until a runoff equilibrium was established.

4.2 Characteristics of Scour Chutes and Deposition Zones:

Scour chutes were on average 5 to 10 cm wide and incised to a depth of roughly 1 cm (Figure 4.1a). They were characterized by trapezoidal-shaped channel cross-sections with steep, non-vertical side-slopes (Figure 4.1b). Scour chutes were relatively straight with lengths ranging between 0.25 m and 4 m. Scour chute bed materials were typified by a relative lack of aggregates. Flow depths ranged between 0.1 and 1.0 cm with an average of 0.3 cm. Average flow velocities for scour chutes were approximately 27 cm s⁻¹. Few knickpoints and no well-defined rill channels formed in experiments 1 – 12. Furthermore, knickpoints which did form were shallow, highly unstable and retreated headward only a
Figure 4.1a: Photo of a typical scour chute. Note the fine scour chute bed materials in comparison to coarser materials in the deposition zones.

Figure 4.1b: Typical cross-sectional profile of a scour chute.
few centimetres before decaying (i.e. Type B knickpoints; Bryan, 1990).

Most deposition zones were 20 to 50 cm wide, consisted of an accretion of approximately 1 to 1.5 cm (measured in the centre thalweg) and were composed mainly of coarse soil aggregates (Figure 4.2). Average flow velocities were roughly 24 cm s\(^{-1}\), or slightly less than scour chute velocities. Initially, flow width was continuous across the deposit, but as discharge and alluviation increased, surface flow became discontinuous and separated into several, small channels which oscillated back and forth over the deposition zone. The channels in the deposition zones were shallow (~0.1 to 0.2 cm deep), confined by their own deposits (as opposed to scour chutes which were incised into the bed material) and can be best characterised as depositional channels (Moss and Walker, 1978).

4.3 Experiments 1 – 6:

Experiments 1–6 used a header tank discharge of 1.6 \( t \) min\(^{-1}\) and an overall slope gradient of 0.087. Classic sequential scouring developed relatively quickly. Well-defined scour chutes and relatively large deposition zones developed within forty minutes after the onset of runoff. This was most notable in experiment 1 (Figure 4.3); in general, the narrowest flow regions represent scour chutes, while wider areas depict deposition zones. The planimetric morphologies for all remaining experiments are given in Appendix A.

When runoff reached each hillslope segment, it appeared that the critical thresholds for entrainment were achieved almost immediately. Initially, relatively thin (i.e. < 0.2 – 0.5 cm deep) concentrated surface wash dominated, but was capable of mobilising the largest
Figure 4.2a: Photo of a typical deposition zone. Note the coarser surface materials in comparison to the scour chutes.

Figure 4.2b: Typical transverse profiles of deposition zones showing relatively uniform deposition across the width of the bed.
Figure 4.3: Planimetric morphologies for sub-experiments 1A, 1B and 1C. Hashed areas represent regions of no flow. (Note: planimetric morphologies for all remaining experiments are in Appendix A).
surface aggregates (~0.4 cm) as bedload. As infiltration losses declined and the volume and depth of runoff increased, the relatively aselective\(^5\) surface wash progressively detached greater quantities of surface aggregates transporting them downslope. The aggregates tended to congregate in microtopographic depressions resulting in the formation of numerous, randomly located aggregate masses. The lower gradient on the downslope portion of the depression caused the flow to diverge, widen and thin. Aggregates were preferentially deposited at these sites most likely due to both reductions in transport capacity and because the largest aggregates sizes exceeded flow depth. The largest aggregates (~0.4 cm) were roughly 2 to 4 times larger than the depth of flow (0.1 to 0.2 cm). Since the aggregates protruded through the flow in the microtopographic depressions, less stress was imparted on the aggregate surfaces and they were no longer maintained in transport. In contrast, flow depths in scour zones were similar to aggregate sizes. Runoff in scour zones could easily mobilise the submerged aggregates. Bryan and Oostwoud-Wijdenes (1992), using a similar soil mixture, also noted that the earliest stages in scour step evolution was marked by the movement of loose surface sediments into topographic depressions.

Through continued localised scouring and incision, surface water discharge slowly concentrated and formed well-defined scour chutes. This increased flow depths and reduced flow widths, resulting in larger unit discharges and shear stresses on the bed. The

\[^5\] Aselectivity refers to entrainment processes which can easily mobilise all bed particle sizes. That is, it does not select only a particular range of sizes. For example, Poesen (1981) found that particles ranging between 63 to 250 µm are more vulnerable to rainsplash detachment (i.e. a selective process). Selective and aselective terms are also applied to transport and depositional processes.
associated local increases in runoff energy incised the bed material further, mobilised more aggregates and transported them downslope. As a result, the aggregate masses in the microtopographic depressions increased in height and areal extent and formed well-defined deposition zones. Upon reaching a deposition zone, runoff was significantly impeded and forced to expand laterally around and through the aggregate masses. Deposition zone flow became increasingly more divergent. As flow depth dropped below the surface of protruding aggregates, the aggregates were significant obstacles to the flow and increased surface roughness, thus capturing more incoming sediments. Through time, some aggregate masses coalesced to form larger deposition zones, while others were eradicated by surface wash forming distinct sites of scour and deposition (e.g. Figure 4.3).

4.4 Experiment 7:

A slope gradient of 0.087 and a larger header tank discharge of 1.0 l min\(^{-1}\) was used for experiment 7. As in the previous experiments, loose surface aggregates were initially entrained and transported downslope into topographic depressions. However, shortly thereafter, no further scouring occurred and the surface appeared to stabilise. That is, during the last two stages of this experiment, there was virtually no change in surface morphology (See Appendix A). The only notable deposition zone formed between 6 and 7 m from the weir, but this was likely due to the influence of the header-tank outlet.

The absence of sequential scouring and, indeed any appreciable scour or deposition, was attributed to the smaller stream power (0.014 W m\(^{-2}\); See Table 3.6). In
comparison, stream powers for experiments 1 – 6 were roughly 0.023 W m\(^{-2}\), or ~40% greater. This suggested that the slope and discharge conditions chosen for experiment 7 were not conducive to the development of sequential scouring. Insufficient sediment was mobilised and as a result only minor deposition zones were formed. Without further significant sediment entrainment, the deposition zones could not enlarge sufficiently to significantly impede surface wash. Localised regions of strong flow convergence and divergence were not established and the development of sequential scouring was inhibited.

4.5 Experiment 8:

In experiment 8, the slope gradient was reduced to 0.061 and the runon discharge was increased to 2.2 \(\ell\) min\(^{-1}\). Artificial incision upslope, caused by the header-tank, created a large deposition zone between 4.5 and 7.5 m from the weir. Downslope of 4.5 m, no bed scouring or deposition occurred (See Appendix A). The bed downslope of 4 m appeared to be laminated with a thin veneer of fine sediments (i.e. a depositional crust). Beds with small slope gradients and small discharges have been shown to armour rapidly, significantly altering sediment entrainment and transport (Moss et al., 1982). The initial particle movement falls to zero quickly as sediments are either: 1) removed entirely from the region; or 2) settle out of the flow in protected bed positions (i.e. depositional crust). The depositional crust inhibited scouring downslope of 4 m and, in turn, this limited deposition zone development. Indeed, there was virtually no change in the form downslope of 4 m throughout the duration of experiment 8.
4.6 Experiment 9:

A gradient of 0.061 and a header-tank discharge of 1.6 l min⁻¹ were used for experiment 9. During sub-experiment A, surface runoff was sluggish, few bed materials were entrained and transported downslope and, as a result, few form changes occurred (See Appendix A). During sub-experiments B and C (See Appendix A), major incision below the header-tank outlet caused sediments to build-up beginning near 7.5 m from the weir. Through time, this deposition zone extended progressively downslope. Sediments, which were not trapped upstream of the alluvial deposit, were transported to the downstream end of the sediment lobe and deposited. As this process continued, a braided wash formed which extended over roughly two-thirds the length of the flume from 2.5 to 8 m above the weir.

4.7 Experiment 10:

Experiment 10 used a 0.061 gradient and a small header-tank discharge of 1.0 l min⁻¹. Only minor deposition occurred upslope between 5.0 and 6.5 m above the weir (See Appendix A) due to minimal incision at the header-tank. The enlarged area of flow in the mid-section of the flume (See Appendix A) was caused by ponding in a topographic depression and was not a site of alluviation. A depositional crust formed downslope of 3.0 m and caused similar effects as those caused by the depositional crust in experiment 8. A large deposition zone formed between 5 and 7 m above the weir, but this was most likely due to the influence of the header-tank.
4.8 Experiment 11:

There were virtually no changes in surface morphology during experiment 11 (See Appendix A). Even though a large header tank discharge was used (3.0 l min⁻¹), the small gradient (0.026) created sluggish runoff. The changes in form width shown in Appendix A are the result of ponded water and do not represent deposition zones. Sediment samples taken from the onslope measurement locations indicated only clear water conditions. Few sediments and almost no surface aggregates were mobilised and transported. As a result, no scour chutes or deposition zones developed. This experiment was halted after one hour prior to reaching a runoff equilibrium and was excluded from subsequent analyses. Clearly, the slope and discharge conditions of experiment 11 were not conducive to the development of sequential scouring.

4.9 Experiment 12:

Experiment 12 was designed as a replacement for experiment 8 and had a 0.061 slope gradient and a header-tank discharge of 2.2 l min⁻¹. However, in sub-experiment 12A, problems with the header-tank produced an unexpected equilibrium discharge of only 1.6 l min⁻¹. As such, sub-experiment 12A had slope and discharge conditions similar to experiment 9. During sub-experiment 12A, only two minor deposition zones formed in the mid-section of the flume (See Appendix A). The header-tank discharge was increased for sub-experiment 12B until an equilibrium runoff of 2.2 l min⁻¹ was established. As a result,
more sediments were entrained and the deposition zones enlarged. Near the end of experiment 12, three distinct deposition zones formed between 3.0 and 8.5 m above the weir. Below 3.0 m, relatively few changes in surface morphology occurred.

4.10 Preliminary Rainfall Experiments:

Three preliminary rainfall experiments were run prior to experiments 1 - 12. The goal of these preliminary experiments was to determine conditions under which sequential scouring occurred. Each had gradients of 0.087, but different final equilibrium weir discharges (i.e. 3.0, 3.3 and 4.1 l min⁻¹). The lowest magnitude runoff simulation (P1) produced well-defined cyclic rilling. The two remaining simulations (P2 and P3) each produced single, deeply incised rill channels extending almost the entire length of the flume. No deposition zones formed during the larger magnitude runoff simulations. From these preliminary experiments, it was considered that sequential scouring could not occur with weir discharges exceeding 3.0 l min⁻¹ on 0.087 gradients (i.e. stream power greater than 0.051 W m⁻²).

4.11 Influence of the Header-Tank:

Deeply incised scour chutes formed between 8.0 and 8.5 m above the weir in all runoff experiments (See Appendix A). These scour chutes were believed to be caused by the discontinuity between the soil and the perspex header-tank outlet. Attempts were made to
reduce the amount of incision by overcompacting the surface or placing large obstacles in the path of flow. Although this reduced flow velocity and increased soil strength, it did little to counteract the amount of incision. Therefore, the uppermost scour chute of all experiments was considered to be an artifact of the experimental design. Consequently, although it is shown on all relevant plots and diagrams, the region between 7.5 and 8.5 m above the weir was discarded from most analyses.

4.12 Weir Hydrographs:

The weir hydrographs are shown in Figure 4.4. The amount of elapsed time for runoff to reach the weir, the time to reach a runoff equilibrium and the temporal variations in water discharge varied substantially between and within experiments. The objective of this section is to provide physical explanations for these variations.

4.12.1 Timing of Weir Runoff:

As runoff was produced by applying water at the head of the slope, infiltration losses along the slope produced significant variations in the elapsed time for runoff to reach the weir, ranging from between 11 minutes to more than 1 hour. To determine possible causes for these differences, the time to weir runoff for each experiment was regressed against its associated: 1) antecedent soil properties of wet and dry bulk densities and volumetric moisture content; 2) flume slopes; and 3) header-tank discharges. Due to large inter-
Figure 4.4: Weir water and sediment discharge for experiments 1 - 12. Marked on each figure are: 1) the durations of each sub-experiment (A - C); 2) the time to runoff at the weir; and 3) the approximate time when sequential scouring was initiated.
Figure 4.4 continued.
correlations between wet and dry bulk density and between wet bulk density and volumetric moisture content, the wet bulk density was removed from the analysis.

Backwards stepwise multiple linear regression was used to determine the variables which best described the differences in the time to runoff (Table 4.1). All variables, except dry bulk density, were included in the regression equation (Table 4.1). Initial slope explains the largest amount of variance and header-tank discharge the second largest. As the t-values for each coefficient are greater than \( t_{0.05} = 1.812 \), the null hypothesis that the intercept and coefficients of each variable are zero is rejected. Furthermore, since \( F \) is greater than \( F_{0.05} = 2.98 \), the model sufficiently explains much of the statistical variation in time to reach saturation and shows the influence of experimental conditions on runoff production.

Table 4.1: Regression results: Time to reach runoff (dependent) regressed against antecedent soil moisture, dry bulk density, header-tank discharge and slope gradient.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>Std. Coeff</th>
<th>t</th>
<th>P(2 tail)</th>
<th>% variance explained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>167.4</td>
<td>13.3</td>
<td>0.0</td>
<td>12.560</td>
<td>0.00000</td>
<td>—</td>
</tr>
<tr>
<td>Header-tank dis.</td>
<td>-45.2</td>
<td>5.7</td>
<td>-0.81</td>
<td>-7.893</td>
<td>0.00010</td>
<td>0.50</td>
</tr>
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<td>Slope</td>
<td>-806.9</td>
<td>108.3</td>
<td>-0.90</td>
<td>-7.448</td>
<td>0.00014</td>
<td>0.42</td>
</tr>
<tr>
<td>Soil Moisture</td>
<td>-55.8</td>
<td>29.9</td>
<td>-0.21</td>
<td>-1.865</td>
<td>0.10442</td>
<td>0.02</td>
</tr>
</tbody>
</table>

\( F = 39.33, r^2 = 0.94 \)

\( n = 11; \) df = 10; \( t_{0.05} = 1.812; F_{0.05} = 2.98 \)
4.12.2 Establishment of a Runoff Equilibrium:

Differences in the time for runoff to reach an equilibrium were not easily explained. The differences between hydrographs were due mostly to the experimental design such as shutting off the header-tank to take measurements. These artificial delays produced significant variations in the time to reach runoff equilibrium (e.g. Figure 4.4f – experiment 7C). Furthermore, this methodology seemed to have major effects in some experiments (e.g. experiments 1, 2, 4 and 7), but little in others. However, runoff equilibrium was established by the middle of sub-experiment C in most experiments with the exception of experiments 4, 6, 7C and 11.

4.12.3 Temporal Variations in Water Discharge:

Since the flow and bed conditions were continuously changing throughout experiments and because of the experimental design, it was difficult to determine exact physical causes for the observed temporal variations in the magnitude of weir water discharge. However, neglecting the influence of the experimental design by analysing the within sub-experiment hydrographs, pulsatory flow was exhibited in many of the hydrographs. For example, prior to soil saturation, two peaks in weir water discharge occur in sub-experiment 1A within 20 minutes (Figure 4.4a). Post soil saturation, there are four peaks in water discharge occurring within 31 minutes in sub-experiment 2C (Figure 4.4b). The header-tank could not be responsible for these variations as it is designed to maintain a constant flow.
True pulsatory flows have been attributed to the formation of roll waves, also termed slug flows, kinematic waves and free-surface instabilities (Karcz and Kersey, 1980; Schumm et al., 1982). In general, there are two types of pulsating flow: 1) those dependent on bedform growth and decay; and 2) those dependent only upon hydraulic processes (i.e. independent of bedform development). Non-bedform dependent pulsating flows, or roll waves, are most common in thin-sheet flows subject to high intensity rainfall. They appear as a series of uniformly spaced waves in which runoff and runoff energy are concentrated (Horton, 1945; Karcz and Kersey, 1980). Roll waves occur due to free-surface instabilities where the inertial forces exceed the resistance forces and the flow breaks down into propagating waves. Schumm et al. (1982) discussed bedform-dependent pulsating flow as a result of anti-dune growth and decay. Unstable anti-dunes were observed to store water temporarily and release it upon anti-dune wash-out. Pulsating flows, associated with scour steps, have been shown to have periods ranging between 0.25 and 3 minutes (Bryan and Oostwoud-Wijdenes, 1992).

The pulses shown in the weir hydrographs have durations ranging from 5 to 10 minutes (e.g. Figure 4.4a between 10 and 20 min). Since, theoretically, the largest possible non-bedform dependent flow pulses (i.e. 8.5 m wavelength) with the smallest wave propagation speeds (~20 cm s⁻¹ with a flow depth of 0.4 cm) would reach the weir approximately every 40 s (i.e. much more frequent than the recorded pulses), it is believed that these pulses were not due to free-surface instabilities. Rather, they were likely due to changes in surface morphology. Prior to flume saturation, progressive widening of the deposition zones would create greater surface areas over which infiltration losses are
sustained. As deposition zones form and decay, the local differences in infiltration losses could alter the magnitude of weir discharge and cause temporal variations in the weir hydrographs (Figure 4.4a; sub-experiment 1A). By the end of sub-experiment 1A, three large deposition zones were in the process of developing near 0.5, 3.5 and 6.5 m above the weir, while smaller alluvial deposits were disappearing near 2.5 and 5.5 m (Figure 4.3). As the soil in experiment 1 was not saturated by this time (See Table 3.7), the growth and decay of deposition zones may have caused the temporal variations in weir discharge.

4.13 Sediment Transport:

Sediment discharges ranged from 2 to 230 g min\(^{-1}\) with sediment concentrations between 2 and 100 g l\(^{-1}\). Average and peak sediment concentrations and discharges for experiments 1 – 12 are given in Table 4.2. Marked variations in sediment discharge were exhibited in all experiments (Figure 4.4).

4.13.1 Sediment Discharge and Sediment Concentration:

Although significant temporal variations in sediment discharge occurred in all experiments (Figure 4.4), conspicuous peaks in sediment discharge occurred in all sub-experiments in which sequential scouring formed. These peaks were especially pronounced in sub-experiments 2B and 3A (Figure 4.4b and c). In experiments 1, 4, 5 and 6, although peaks in sediment discharge occurred near the time of the development of sequential scouring,
Table 4.2: Maximum and average sediment concentrations and discharges for experiments 1 – 12.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Sediment Discharge (g min⁻¹)</th>
<th>Sediment Conc. (g l⁻¹)</th>
<th>Stream Power (W m⁻²)</th>
<th>Sequential Scouring</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>Average</td>
<td>Maximum</td>
<td>Average</td>
</tr>
<tr>
<td>1</td>
<td>173.9</td>
<td>53.4</td>
<td>106.1</td>
<td>49.8</td>
</tr>
<tr>
<td>2</td>
<td>93.9</td>
<td>44.3</td>
<td>67.1</td>
<td>37.5</td>
</tr>
<tr>
<td>3</td>
<td>119.1</td>
<td>57.9</td>
<td>82.2</td>
<td>44.2</td>
</tr>
<tr>
<td>4</td>
<td>56.5</td>
<td>30.9</td>
<td>54.6</td>
<td>31.7</td>
</tr>
<tr>
<td>5</td>
<td>89.2</td>
<td>44.8</td>
<td>62.1</td>
<td>35.8</td>
</tr>
<tr>
<td>6</td>
<td>41.4</td>
<td>24.9</td>
<td>33.7</td>
<td>22.5</td>
</tr>
<tr>
<td>7</td>
<td>28.1</td>
<td>9.9</td>
<td>30.6</td>
<td>13.9</td>
</tr>
<tr>
<td>8</td>
<td>53.3</td>
<td>19.3</td>
<td>25.7</td>
<td>10.6</td>
</tr>
<tr>
<td>9</td>
<td>18.3</td>
<td>7.1</td>
<td>11.8</td>
<td>5.5</td>
</tr>
<tr>
<td>10</td>
<td>6.9</td>
<td>3.8</td>
<td>21.3</td>
<td>5.78</td>
</tr>
<tr>
<td>11</td>
<td>6.4</td>
<td>2.6</td>
<td>15.9</td>
<td>4.6</td>
</tr>
<tr>
<td>12</td>
<td>75.2</td>
<td>19.6</td>
<td>34.3</td>
<td>14.6</td>
</tr>
</tbody>
</table>

These peaks were not as pronounced. Well-defined peaks in sediment discharge did not occur in experiments 7 – 12, although they may have been masked by the relatively small overall sediment discharge. All peaks and variations in the sediment discharge graphs do not reflect runoff cessation at the end of each sub-experiment as these periods are not included on the hydrographs.

These peaks suggest that the development of sequential scouring produces a form of clockwise hysteresis in sediment concentration and has the ability to alter sediment
output. Clockwise hysteresis reflects changes in sediment supply due to, for example, sediment supply exhaustion (Richards, 1982). Prior to the formation of the deposition zones, sediment concentrations should increase with corresponding increases in water discharge. However, as sediment storage increases due to deposition zone formation, sediment concentrations should decrease even under steady or steadily increasing water discharge. Temporal variations in bedload transport (the dominant mode of transport observed in this study) have been attributed to: 1) changes of channel sediment storage resulting from aggradation and degradation processes (Jackson and Beschta, 1982; Ashmore, 1988); 2) changes in channel morphology of braided rivers (Hoey and Sutherland, 1991); 3) the migration of bedforms (Gomez et al., 1989); and 4) changes in the availability of superficial bed materials through longitudinal sediment sorting (Iseya and Ikeda, 1987).

4.13.2 Influence of Slope Angle:

Figure 4.5 relates weir sediment and water discharge and shows the effects of slope angle on soil loss. All sediment discharges from the 0.087 slopes, regardless of the magnitude of water discharge, plot higher than those of the 0.061 slopes. The single 0.026 slope, or experiment 11, was not used in this analysis. In general, holding water discharge and other factors constant (e.g. soil conditions and the active processes), increases in slope angle are shown to substantially increase the rates of soil erosion.
Figure 4.5: Relationship between water and sediment discharge for experiments with differing initial slopes

A common power function used to express sediment transport as a function of unit discharge and slope gradient is

\[ q_s = k q_w^m S^n \]  \hspace{1cm} 4.1

where \( q_s \) is sediment discharge per unit width, \( q_w \) is water discharge per unit width, \( S \) is slope gradient, \( k \) is an empirically-derived rate constant, and \( m \) and \( n \) are empirically derived exponents (Kirkby, 1971; Carson and Kirkby, 1972; Smith and Bretherton, 1972; Band, 1985a,b; Julien and Simons, 1985; Dunne and Aubry, 1986; Mathier et al., 1989;
Julien, 1995; Mathier and Roy, 1996). Using non-linear multiple regression, the following sediment transport equation was derived from the weir data set:

\[ q_s = 0.001 \, q_w^{2.45} \, S^{1.73}, \text{ adjusted } r^2 = 0.76, \, n = 404 \]

where \( q_s \) is in g s\(^{-1}\) cm\(^{-1}\), \( q_w \) is in cm\(^3\) s\(^{-1}\) cm\(^{-1}\) and \( S \) was taken to be the local slope gradient (\( \sin \theta \)) immediately above the weir measured prior to the beginning of each sub-experiment. A plot of measured versus predicted values for unit sediment discharge (Figure 4.6) shows the adequacy of this power function for describing sediment transport.

Figure 4.6: Measured versus predicted unit sediment discharge.
The values of $m = 2.45$ and $n = 1.73$ agree well with those found by other authors for predicting unit sediment discharge from sheet and rill erosion (Julien and Simons, 1985; Julien, 1995). The $m$ and $n$ exponents have been found experimentally to range from 1.4 to 2.4 and from 1.2 to 1.9, respectively (Julien, 1995; pg. 223). Smith and Bretherton (1972), Carson and Kirkby (1972), Band (1985) and Mathier et al., (1989) discuss the dependence of sediment transport, derived from the equation, on the values of $m$ and $n$. For all $m > 0$ and $0 < n < 1$, soil erosion rates increase more slowly with increasing slope, and prevailing conditions are considered to be detachment-limited. When $n > 1$ (i.e. with these data), soil loss increases at a faster rate with increases in slope and prevailing conditions are considered to be transport-limited. With $n = 1.73$, the prevailing flow conditions throughout these experiments were transport-limited.

4.14 Summary:

Experiments 1 – 6 and 12 produced varying degrees of sequential scouring. Comparing experiments (1 – 12), sequential scouring was shown to occur under conditions associated with the greatest stream powers and sediment transport rates (Tables 4.2). Sequential scouring did not occur in experiments associated with low stream powers or low sediment discharges (i.e. experiments 7 – 11). This topic is addressed further in Chapter 5.

The weir hydrographs showed differences between experiments in the time for runoff to reach the weir and the time to reach runoff equilibrium, and within experiments
in variations in the magnitude of water discharge. The time for runoff to reach the weir was explained by the differences in antecedent properties, slope gradients and header-tank discharges. The time to reach runoff equilibrium could not be related to physical causes due to the experimental design. Lastly, changes in surface morphology were shown to have the potential ability to alter weir runoff by locally increasing or decreasing infiltration losses.

Variations in weir sediment discharge were partially explained by changes in surface morphology. Peaks in the sediment output and hysteresis in the relationships between sediment concentration and water discharge were related to sequential scouring. A general sediment transport formula was determined and found to: 1) adequately describe sediment transport; and 2) agree well with the literature. Lastly, the slope exponent in the sediment transport equation showed that the prevailing sediment transport conditions throughout most experiments were transport-limited.
5.0 Optimal Conditions for Sequential Scouring

5.1 Introduction:

Results from experiments 1 – 12, P1, P2 and P3 seem to suggest that sequential scouring is associated with specific slope, discharge and sediment transport characteristics. The objective of this chapter is to determine the optimal conditions for the occurrence of sequential scouring by comparing the results of experiments 1 – 12, P1, P2 and P3 with the scour step and cyclic rilling experiments of Bryan and Oostwoud-Wijdenes (1992) and Bryan and Poesen (1989).

5.2 Stream Power and Sediment Output:

Table 5.1 is a summary of the slope, water and sediment discharge, and stream power conditions in experiments 1 – 12, P1, P2 and P3 and the experiments of Bryan and Oostwoud-Wijdenes (1992) and Bryan and Poesen (1989). To determine if there are preferred stream powers under which sequential scouring occurs, experiments 1 – 12, P1, P2 and P3, BO1 – 5 and BP were plotted according to slope and water discharge (Figure 5.1). Superimposed on Figure 5.1 are lines of equal stream power ($\bar{Q} = \rho_w g S Q_w$). Figure 5.2 is a plot of average sediment discharge against stream power for all experiments.

Figures 5.1 and 5.2 show four general categories of experiments based on morphologic features and mode of formation.
Table 5.1: Slopes, water and sediment discharges, and stream powers of experiments 1 – 12, P1, P2, P3, Bryan and Oostwoud-Wijdenes (1992) and Bryan and Poesen (1989).

<table>
<thead>
<tr>
<th>Experiment #</th>
<th>Slope (θ)</th>
<th>Water Discharge (l min⁻¹)</th>
<th>Stream Power (W m⁻²)</th>
<th>Average Sediment Discharge (g min⁻¹)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.087</td>
<td>2.9 *</td>
<td>0.051</td>
<td>52.7</td>
<td>Cyclic Rilling</td>
</tr>
<tr>
<td>P2</td>
<td>0.087</td>
<td>3.3 *</td>
<td>0.049</td>
<td>78</td>
<td>Rilling</td>
</tr>
<tr>
<td>P3</td>
<td>0.087</td>
<td>4.1 *</td>
<td>0.058</td>
<td>75</td>
<td>Rilling</td>
</tr>
<tr>
<td>1</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
<td>53.9</td>
<td>Sequential Scouring</td>
</tr>
<tr>
<td>2</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
<td>44.3</td>
<td>Sequential Scouring</td>
</tr>
<tr>
<td>3</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
<td>57.9</td>
<td>Sequential Scouring</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
<td>30.9</td>
<td>Sequential Scouring</td>
</tr>
<tr>
<td>5</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
<td>44.8</td>
<td>Sequential Scouring</td>
</tr>
<tr>
<td>6</td>
<td>0.087</td>
<td>1.6</td>
<td>0.023</td>
<td>24.9</td>
<td>Sequential Scouring</td>
</tr>
<tr>
<td>7</td>
<td>0.087</td>
<td>1.0</td>
<td>0.014</td>
<td>9.9</td>
<td>No Sequential Scouring</td>
</tr>
<tr>
<td>8</td>
<td>0.061</td>
<td>2.2</td>
<td>0.022</td>
<td>19.3</td>
<td>No Sequential Scouring</td>
</tr>
<tr>
<td>9</td>
<td>0.061</td>
<td>1.6</td>
<td>0.016</td>
<td>7.1</td>
<td>No Sequential Scouring</td>
</tr>
<tr>
<td>10</td>
<td>0.061</td>
<td>1.0</td>
<td>0.010</td>
<td>3.8</td>
<td>No Sequential Scouring</td>
</tr>
<tr>
<td>11</td>
<td>0.026</td>
<td>3.0</td>
<td>0.013</td>
<td>2.6</td>
<td>No Sequential Scouring</td>
</tr>
<tr>
<td>12</td>
<td>0.061</td>
<td>2.2</td>
<td>0.022</td>
<td>19.6</td>
<td>Sequential Scouring</td>
</tr>
<tr>
<td>BP</td>
<td>~0.07</td>
<td>~4.5</td>
<td>0.051</td>
<td>66.6</td>
<td>Cyclic Rilling</td>
</tr>
<tr>
<td>BO1</td>
<td>0.026</td>
<td>~11.5</td>
<td>0.049</td>
<td>—</td>
<td>Scour Steps</td>
</tr>
<tr>
<td>BO2</td>
<td>0.026</td>
<td>~4.81</td>
<td>0.020</td>
<td>—</td>
<td>Scour Steps</td>
</tr>
<tr>
<td>BO3</td>
<td>0.014</td>
<td>~7.8</td>
<td>0.017</td>
<td>44</td>
<td>Scour Steps</td>
</tr>
<tr>
<td>BO4</td>
<td>0.014</td>
<td>~7.8</td>
<td>0.017</td>
<td>30</td>
<td>Scour Steps</td>
</tr>
<tr>
<td>BO5</td>
<td>0.014</td>
<td>~8.9</td>
<td>0.020</td>
<td>49</td>
<td>Scour Steps</td>
</tr>
</tbody>
</table>

BO = Bryan and Oostwoud-Wijdenes (1992); BP = Bryan and Poesen (1989); * denotes rainfall experiment. Note: water discharge is the final equilibrium discharge for rainfall experiments or header-tank discharge for runon experiments.
Figure 5.1: Plot of water discharge versus slope showing stream power conditions for all experiments.

Figure 5.2: Plot of average end-of-slope sediment discharge versus the index of stream power showing the optimal conditions for alternating patterns of erosion and deposition.
These categories are:

1) rainfall experiments P2 and P3, with relatively large stream powers (i.e. > 0.05 W m$^{-2}$) and sediment discharges near 80 g min$^{-1}$, produced actively eroding surfaces each with a single, well-defined rill channel – no alternating patterns of erosion and deposition developed;

2) rainfall experiments P1, BP and BO1, with stream powers between 0.04 and 0.05 W m$^{-2}$ and sediment discharges between 50 and 70 g min$^{-1}$, produced cyclic rilling$^6$;

3) runon experiments 1–6, 12, BO3, BO4 and BO5, with stream powers between 0.015 and 0.025 W m$^{-2}$ and sediment discharges between 20 and 60 g min$^{-1}$, produced sequential scouring or scour steps; and

4) runon experiments 7–11, with low stream powers (i.e. < 0.015 W m$^{-2}$) and low sediment discharges (i.e. < 20 g min$^{-1}$), produced no alternating patterns of erosion and deposition.

Figures 5.1 and 5.2 suggest that alternating patterns of erosion and deposition occur under specific stream power and sediment discharge conditions. All experiments which produced sequential scouring had stream powers ranging between 0.015 and 0.025 W m$^{-2}$ and sediment discharges between 20 and 60 g min$^{-1}$.

Experiments 7, 8 and 10 had low average sediment discharges (< 20 g min$^{-1}$) and did not produce sequential scouring. Although experiment 8 fell within the stream power

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$^6$ Cyclic rilling differs from sequential scouring in that rill channel incision is greater than that of scour chutes. As such, more sediment is mobilised and transported to the deposition zones with cyclic rilling.
category for sequential scouring, its average sediment discharge was slightly lower.

Results and observations suggest that the low sediment loads of experiments 7, 8 and 10 were most likely due to sediment supply exhaustion and depositional crust development. As less sediment was redistributed in experiments 7, 8 and 10, the deposition zones could not increase sufficiently in height and areal extent to substantially increase roughness, impede runoff and alter downslope water and sediment fluxes. These processes are believed to be important factors in the development of sequential scouring.

5.3 Transport Capacity:

Figure 5.3 relates transport capacity to measured transport rate and illustrates a general relationship between sequential and non-sequential scouring experiments. The measured transport rate is a conversion of sediment discharge per unit width (kg s\(^{-1}\) m\(^{-1}\)) into W m\(^{-2}\).

Unit sediment discharge is multiplied by the acceleration due to gravity (m s\(^{-2}\))

\[
\text{Unit sediment discharge} \times \frac{m}{s^2} \times \frac{1}{m} = \frac{kg}{s^2 m} \times \frac{m}{s^3 m} = \frac{N}{m s m^2} = \frac{J}{m^2} = \frac{W}{m^2}
\]

(See Bagnold, 1966). The measured transport rate was calculated from ~850 unit sediment discharges measured mid-flume between 3.5 and 5.0 m in experiments 1 – 12. Transport capacity was determined using a reduced form of Bagnold’s (1966) total load sediment transport equation:

\[
i = \omega \left( \frac{e_b}{\tan \alpha} + 0.01 \frac{u}{\nu} \right)
\]

where \(i\) is the transport rate in W m\(^{-2}\), \(\omega\) is stream power per unit bed area, \(e_b\) is an
empirical coefficient for bedload load efficiency, \( \tan \alpha \) is the coefficient for sliding friction, \( \bar{u} \) is mean flow velocity and \( V \) is effective fall velocity. The coefficient for bedload efficiency should range from 0.11 for large grain sizes and large flow velocities to 0.15 for fine grains and small flow velocities (Bagnold, 1966). The coefficient for bedload efficiency was assumed to be 0.15 given the small flow velocities and grain sizes used in this study. The effective fall velocity, calculated to be 11.9 cm s\(^{-1}\), was determined using the particle size distribution shown in Figure 3.1 and Bagnold (1966). Stream power per unit bed area
and mean flow velocity were derived from measurements between 3.5 and 5.0 m above the weir. Local bed slope gradient was used for calculating stream power rather than local water surface gradient. Errors due to this substitution should be minimal since bed slope and water surface slope are similar in thinflows. With an average Reynolds number of ~3000 (See Table 7.4), tanα was assumed to be 0.4 from Bagnold (1966). Since equation 5.1 is applicable to relatively deep, turbulent flows and because of the assumptions for ea and tanα, it is applied here only as an index for transport capacity and serves to illustrate differences between sequential scouring and non-sequential scouring experiments.

Data from sequential scouring experiments indicate 96.6% of the measurement locations were at or below transport capacity, while 93.1% of the measured transport rates for the non-sequential scouring experiments were below 50% of transport capacity. The points shown above transport capacity on Figure 5.3 illustrate the errors associated with measuring the on-slope unit water and sediment discharges. Figure 5.3 shows that non-sequential scouring experiments were likely detachment-limited which corresponds with observations from experiments 7, 8 and 10.

The comparison of measured transport rates and transport capacity derived from the on-slope data contradicts the sediment transport equation (q_s = k q_w^n S^m) derived from weir data (See §4.13.2). With n = 1.73, the sediment transport equation indicated that experiments 1 − 12 combined were transport-limited, while the results shown in Figure 5.3 suggest that experiments 7 − 10 were most likely detachment-limited. As such, the sediment transport formula was re-applied only to the weir data from experiments 7 − 10. Again, the corresponding sediment transport formula (q_s = 0.47 q_w^{3.3} S^{2.6}, r^2 = 0.7)
indicated that experiments 7 – 10 were transport-limited when the weir data are used. The
contradiction may be the result of using two different data sources and different
techniques for generating data.

5.4 Summary:

Results suggest that sequential scouring occurs in a limited range of slope and discharge
conditions and that the occurrence of sequential scouring can be related to transport
capacity. If sediment transport capacities are relatively large, then although relatively large
amounts of sediment may be mobilised, a large proportion of the sediment will be
transported through the system and minimal deposition will occur. Conversely, if the
capacity to transport sediment is small, then few sediments will be mobilised and, if
deposition occurs, sufficiently large deposition zones will not develop. It appears that the
combination of slope and water discharge must produce moderate transport capacities
which are capable of mobilising modest amounts of sediment, but are incapable of
transporting sediment large distances. Future research should attempt to refine these
relationships using a wider range of slope, discharge and soil conditions.
6.0 Spatial and Temporal Variability and Quasi-Periodicity of Sequential Scouring:

6.1 Introduction:

Spatial variability is assessed in this study by analysing the differences between the measured final longitudinal profiles and planimetric morphologies of experiments 1 – 6. Temporal variability is assessed within each experiment by analysing changes in hydraulic properties. Spatial and temporal variability were evaluated using experiments 1 – 6 as they were designed as exact replicates and, thus, expected to yield similar results. Experiments 7 – 12 could not be used in these analyses since they had different initial conditions and, thus, different outcomes were expected.

6.2 Spatial Variability:

One of the most striking features observed with alternating patterns of erosion and deposition is that they sometimes exhibit semi-periodic longitudinal wavelengths. Figures 6.1 and 6.2 are the initial and final longitudinal profiles and planimetric morphologies for experiments 1 – 6, respectively, and serve to illustrate the development of each morphology. Since these experiments used similar soil mixtures, slope angles and headertank discharges (See Tables 3.3 and 3.6), it was expected that their final longitudinal profiles and planimetric morphologies would be similar. Furthermore, since each
Figure 6.1: The initial and final longitudinal profiles of experiments 1 – 6.

6.1a: Experiment 1

6.1b: Experiment 2

6.1c: Experiment 3
Figure 6.1 continued.

6.1d: Experiment 4

6.1e: Experiment 5

6.1f: Experiment 6
Figure 6.2: The final planimetric morphologies for experiments 1 – 6. Hashed areas represent zones of no flow.
Figure 6.2 continued.
experiment was run until it appeared to stabilise with no further change in form within each experiment, the shapes of longitudinal profiles and planimetric morphologies were expected to reflect the "equilibrium" form associated with sequential scouring. The equilibrium form should show sequential scouring to be associated with similar distances between successive deposition zones (i.e. longitudinal wavelengths) as defined by both the final longitudinal profiles and planimetric morphologies of experiments 1 – 6.

Figures 6.1 and 6.2 seem to suggest that no two experiments produced similar longitudinal wavelengths and, thus, no equilibrium form was established. Spatial autoregression was used on the final longitudinal profiles and planimetric morphologies of experiments 1 – 6 to determine if the longitudinal wavelengths were different.

Second-order autoregressive models, AR(2), can be used to predict pseudo-periodic behaviour (See Box et al., 1994). Autoregression (AR) is simply a series of observations regressed onto itself. A lag of 1 (i.e. AR(1)) compares observations in a series with their preceding observations (i.e. n with n-1, n-1 with n-2,...,and 2 with 1). A lag of 2 (i.e. AR(2)) compares n with n-2, n-1 with n-3,..., or every other observation. Spatial autoregression has been shown to adequately predict the pseudo-periodic behaviour of stream bed profiles and bed widths associated with riffle-pool sequences (Richards, 1976a). Sequential scouring is considered to be morphologically similar to riffle-pool sequences. Deposition zones are topographic high points in a longitudinal profile, analogous to riffles; while scour chutes are topographic low points, analogous to pools.

The general solution of a second-order autoregressive process applied here to the
longitudinal profiles and planimetric morphologies is

\begin{align*}
    z_i &= \phi_1 z_{i-1} + \phi_2 z_{i-2} + \epsilon_i \\
    w_i &= \phi_1 w_{i-1} + \phi_2 w_{i-2} + \epsilon_i
\end{align*}

where \( z_i \) is bed elevation and \( w_i \) is form width of the \( i \)th segment, \( \phi_1 \) and \( \phi_2 \) are coefficients determined from an ARIMA (Autoregressive Integrated Moving Average) analysis, and \( \epsilon_i \) is an error term. Equation 6.1 explains the variance in bed elevation or form width at location \( i \) as a function of bed elevations or form widths upslope. There are two assumptions with this technique: 1) equal measurement intervals and 2) stationarity. Assumption one is satisfied (See §3.5). Stationarity implies the process varies about some constant mean level (See Box et al., 1994). If so, then the coefficients from the ARIMA analysis will satisfy the following inequalities:

\begin{align*}
    \phi_1 + \phi_2 &< 1 \\
    \phi_2 - \phi_1 &< 1 \\
    -1 &< \phi_2 < 1.
\end{align*}

Furthermore, if the following inequality is satisfied: \( \phi_1^2 + 4\phi_2 < 0 \), then the autoregressive process will exhibit pseudo-periodic behaviour. In which case, the frequency and period can be determined by the following equation:

\begin{equation}
    \cos (2\pi f) = \frac{|\phi_1|}{2\sqrt{-\phi_2}}
\end{equation}

where \( f \) is the frequency and \( \frac{1}{f} \) is the spatial periodicity.
The coefficients determined for the final longitudinal bed profiles did not satisfy the above inequalities. If non-stationarity exists, a filtering technique can be used to de-trend the data (See Box et al., 1994). Similar to Crickmore (1970) and Nordin and Algert (1966), a linear filter was used to de-trend the final longitudinal profiles, after which $\phi_1$ and $\phi_2$ satisfied the inequalities.

The final, linearly de-trended longitudinal profiles for experiments 1 – 6 were analysed to determine if they exhibited quasi-periodic behaviour. Only measurements below 7.5 m were used in order to limit the header-tank influence. Table 6.1 is a summary of the results from the AR(2) analysis. Although the final longitudinal profiles (Figures 6.1) seem to suggest that experiments 1 – 6 behaved periodically, the values of $\phi_1$ and $\phi_2$ indicate that only four of the six experiments exhibited pseudo-periodic behaviour.

For experiments 1, 3 and 4, the computed wavelengths are significant at the 0.05 significance level (Table 6.1) and similar to wavelengths observed in Figures 6.1a, c and d. Spatial autoregression on experiment 6 generated a wavelength of 1.8 m, which is strikingly different from observations (Figure 6.1f). However, the corresponding $r^2$ is not significant at the 0.05 significance level. Spatial autoregression on the longitudinal profiles showed that the longitudinal wavelengths associated with sequential scouring were spatially variable and an equilibrium profile did not develop.

The planimetric morphologies of experiments 1 – 6 were also linearly detrended to satisfy the assumptions of stationarity. The results from spatial autoregression on the planimetric morphologies showed that only experiments 1, 3 and 4 exhibited quasi-periodic behaviour which agrees with the longitudinal profile analysis (Table 6.2).
Table 6.1: Spatial autoregression on the longitudinal profiles of experiments 1 – 6.

<table>
<thead>
<tr>
<th>Experiment #</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$r^2$</th>
<th>quasi-periodic</th>
<th>$f$</th>
<th>wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.02</td>
<td>-0.48</td>
<td>0.48</td>
<td>Yes</td>
<td>0.117</td>
<td>2.1 m</td>
</tr>
<tr>
<td>2</td>
<td>1.09</td>
<td>-0.28</td>
<td>0.73</td>
<td>No</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>0.71</td>
<td>-0.14</td>
<td>0.40</td>
<td>Yes</td>
<td>0.045</td>
<td>5.6 m</td>
</tr>
<tr>
<td>4</td>
<td>0.83</td>
<td>-0.20</td>
<td>0.47</td>
<td>Yes</td>
<td>0.063</td>
<td>4.0 m</td>
</tr>
<tr>
<td>5</td>
<td>1.19</td>
<td>-0.19</td>
<td>0.70</td>
<td>No</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>6</td>
<td>0.70</td>
<td>-0.30</td>
<td>0.29</td>
<td>Yes</td>
<td>0.141</td>
<td>1.8 m</td>
</tr>
</tbody>
</table>

However, the computed wavelengths are different from observed wavelengths (Figure 6.2) and from those produced by spatial autoregression on the longitudinal profiles (Table 6.1). This suggests that the planimetric morphologies of experiment 1 – 6 also exhibited spatial variability and no equilibrium form was evident in the planimetric morphologies.

Table 6.2: Spatial autoregression on planimetric morphologies of experiments 1 – 6.

<table>
<thead>
<tr>
<th>Experiment #</th>
<th>$\phi_1$</th>
<th>$\phi_2$</th>
<th>$r^2$</th>
<th>quasi-periodic</th>
<th>$f$</th>
<th>wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.32</td>
<td>-0.50</td>
<td>0.87</td>
<td>Yes</td>
<td>0.058</td>
<td>4.3 m</td>
</tr>
<tr>
<td>2</td>
<td>1.22</td>
<td>-0.37</td>
<td>0.76</td>
<td>No</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>1.40</td>
<td>-0.52</td>
<td>0.77</td>
<td>Yes</td>
<td>0.038</td>
<td>6.5 m</td>
</tr>
<tr>
<td>4</td>
<td>1.31</td>
<td>-0.44</td>
<td>0.83</td>
<td>Yes</td>
<td>0.025</td>
<td>9.9 m</td>
</tr>
<tr>
<td>5</td>
<td>1.26</td>
<td>-0.37</td>
<td>0.81</td>
<td>No</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>6</td>
<td>1.31</td>
<td>-0.37</td>
<td>0.90</td>
<td>No</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
6.3 Temporal Variability:

Figure 6.1 shows that, although some locations exhibited only scour or deposition throughout each experiment, numerous locations experienced periods of both degradation and aggradation. Figure 6.3 illustrates changes in velocity recorded at the on-slope measurement locations for experiments 1 – 6. It is clear that most locations experience accelerating and decelerating flow conditions during each experiment. This illustrates the temporal variability of the flow properties.

6.4 Possible Causes of Spatial and Temporal Variability

It was expected that experiments 1 – 6 would have produced similar final longitudinal profiles and planimetric morphologies since their initial conditions were assumed to be similar. However, the longitudinal wavelengths and the hydraulic properties were found to exhibit spatial and temporal variability. This suggested that not all initial conditions were similar between experiments and that “between-experiment” differences in initial conditions may have caused the variation. The particle size distributions of the soil mixtures, slope angles and header-tank discharges were similar for experiments 1 – 6 and, thus, these initial conditions most likely had no influence on the observed variability. Three remaining factors, antecedent soil moisture and bulk density and the initial shapes of the longitudinal profiles, will be tested for significant differences to determine if they potentially influenced the observed spatial and temporal variability.
Figure 6.3: On-slope velocity measurements for experiments 1 – 6.

6.3a: Experiment 1

6.3b: Experiment 2

6.3c: Experiment 3
Figure 6.3 continued.

6.3d: Experiment 4

6.3e: Experiment 5

6.3f: Experiment 6
6.4.1 Antecedent Volumetric Soil Moisture and Bulk Density:

Differences in antecedent soil moisture and bulk density between experiments can strongly influence the uniformity of runoff generation and local rates of soil loss or gain through their influence on infiltration and soil erodibility. Any differences in these antecedent soil properties may have caused the observed variability in the longitudinal profiles and planimetric morphologies.

Wet and dry bulk densities and volumetric soil moisture contents ranged from 1.30 to 1.64 kg m\(^{-3}\), 1.27 to 1.44 kg m\(^{-3}\) and 0.08 to 0.27 m\(^{3}\) m\(^{-1}\), respectively (Table 6.3). A Kruskal-Wallis Analysis of Variance (Freund and Simon, 1992) at the 0.05 significance level was performed to determine if significant differences existed in antecedent soil moisture contents and wet and dry bulk densities between experiments 1 – 6 (Table 6.3). This test does not require the assumption that data are normally distributed and is well-suited for small sample sizes. The \(\chi^2\) approximations for all antecedent soil conditions exceeded the critical \(\chi^2_{0.05}\) and, therefore, the null hypothesis was rejected demonstrating that the antecedent soil properties were significantly different between experiments.

6.4.2 Initial Slope Profiles:

Microtopography plays an important role in determining the amount and location of runoff generation through its influence on infiltration, flow depth and velocity and slope gradient, (Dunne et al., 1995) and ultimately on the amount and location of erosion or
Table 6.3: Kruskal-Wallis ANOVA test results for differences between antecedent conditions of experiments 1 – 6.

<table>
<thead>
<tr>
<th>Exp #</th>
<th>Antecedent soil properties</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Wet Bulk Density g cm$^{-3}$</td>
<td>Mean Dry Bulk Density g cm$^{-3}$</td>
<td>Mean Moisture Content ml ml$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.64</td>
<td>1.28</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.64</td>
<td>1.44</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.63</td>
<td>1.43</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.56</td>
<td>1.39</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.53</td>
<td>1.37</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.24</td>
<td>1.20</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\chi^2 = 18.765$</td>
<td>$\chi^2 = 17.170$</td>
<td>$\chi^2 = 19.386$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 df</td>
<td>5 df</td>
<td>5 df</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\chi^2_{0.05} = 11.070$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

deposition. It is hypothesized that initial microtopography, more specifically local convexities and concavities in the surface, influence the initial locations and magnitudes of scour and deposition. Any differences in the shapes of the initial longitudinal profiles between experiments 1 – 6 could create differences in the location and magnitude of scour and deposition and produce the variability observed in the final longitudinal profiles.

The initial longitudinal profiles for experiments 1 – 6 are shown in Figure 6.1 and represent initial soil thickness measured at 0.25 m distances along the flume. Measures of skewness and kurtosis indicated that the frequency distributions of elevation for each measured profile were not normally distributed. Spearman rank-order correlation
coefficients (Freund and Simon, 1992) were used to determine whether any two initial longitudinal profiles of experiments 1 – 6 were significantly different. Table 6.4 shows the Spearman rank-order correlation coefficients and their corresponding z-values for comparisons between the longitudinal profiles of experiments 1 – 6. At the 0.05 significance level, 12 of the 15 correlation coefficients are not significant indicating that most initial longitudinal profiles were significantly different. This suggests that any high- or low-points in the initial longitudinal profiles, or any local convexities or concavities, do not occur in the same locations in each experiment. If local convexities and concavities influence the location and magnitude of scour and deposition, then the differences between the initial longitudinal profiles of experiments 1 – 6 may have caused the observed spatial variability.

Table 6.4: Spearman rank-order correlation coefficients and corresponding z-values for comparisons between the initial longitudinal profiles of experiments 1 – 6 (n = 35).

<table>
<thead>
<tr>
<th>Exp. #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_s$</td>
<td>z</td>
<td>$r_s$</td>
<td>z</td>
<td>$r_s$</td>
</tr>
<tr>
<td>2</td>
<td>-0.02</td>
<td>-0.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.42</td>
<td>2.45*</td>
<td>-0.18</td>
<td>-1.05</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.24</td>
<td>1.40</td>
<td>0.18</td>
<td>1.05</td>
<td>0.30</td>
</tr>
<tr>
<td>5</td>
<td>0.45</td>
<td>2.62*</td>
<td>0.14</td>
<td>0.82</td>
<td>0.31</td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
<td>0.29</td>
<td>0.20</td>
<td>1.17</td>
<td>0.03</td>
</tr>
</tbody>
</table>

* denotes significance at 0.05 significance level.
6.5 Summary:

Under assumptions of similar initial conditions, it was expected that experiments 1 – 6 would produce similar final longitudinal profiles and planimetric morphologies, and an equilibrium form. On the contrary, the final longitudinal profiles and planimetric morphologies of experiments 1 – 6 were found to be significantly different and no such equilibrium form developed. It was found that the variability in the final forms was probably due to between-experiment differences in antecedent soil moisture and bulk density, random variations in initial microtopography, and the transient nature of the flow properties.

The spatial variability analysis also shows that sequential scouring has an inherent stochastic component introduced by random variations in the initial slope profile. Kirkby (1990a) produced a process-based model for predicting cyclic rill wavelengths and explaining the processes associated with cyclic rilling on gentle slope gradients (i.e. 0.035). The model predicted wavelengths of 5 m on shorter hillslopes (20 m) and 20 m on longer hillslopes (100 m). Although Kirkby’s model may be an adequate tool for explaining the processes of cyclic rilling, it may not be able to adequately describe the ranges in wavelengths observed between successive alluvial zones, as it did not account for random variations in the initial slope profile. The spatial and temporal variability analyses in this study suggest that, in order to obtain adequate model results, any model of sequential scouring must account for non-uniform initial conditions and downslope changes in the water and sediment fluxes.
7.0 Initiation and Development of Sequential Scouring and
the Interactions Between Scour and Deposition

7.1 Introduction:

Based on theory and a review of the literature, the following sequence of events can be hypothesized concerning the initiation and development of sequential scouring. On relatively smooth slopes, once runoff is initiated and thresholds for entrainment are surpassed, sheetwash erosion is initiated. This produces relatively uniform degradation across and down the slope. If, at any point, the transport capacity of the surface wash is sufficiently reduced, deposition occurs. For example, microtopographic depressions can cause an increase in the erosive power on the steeper upslope, portion of the depression and a reduction in power on the more gentle downslope side (Smith and Bretherton, 1972; Loewenherz, 1991; Smith and Merchant, 1995; Dunne et al., 1995). Generalizing this statement, Smith and Bretherton (1972) postulated that erosion is enhanced in local concavities (i.e. $\frac{\partial^2 z}{\partial x^2} < 0$, where $z$ is bed elevation and $x$ is slope position) and deposition in local convexities (i.e. $\frac{\partial^2 z}{\partial x^2} > 0$) (Figure 7.1). A major obstacle with this postulation is that those concavities with the greatest curvature create maximum disturbance and promote the largest amount of incision. Smith and Bretherton (1972)

7 Microtopography refers to slight undulations in the surface which create local convexities and concavities. Microtopography in this study does not refer micro-relief between, for instance, soil particles. Rather, in this study microtopography refers to surface undulations on the order of a few centimetres of vertical change over 50 cm horizontal distances.
countered this obstacle by stating that at some scale this theory must break down. Dunne and Aubry (1986) suggested that the dispersive process of rainsplash acted to suppress the smallest perturbations.

Once the power of surface wash is reduced and deposition occurs, coarser sediments are deposited first; while finer sediments are maintained in transport. Bryan (1990), Bryan and Oostwoud-Wijdenes (1992) and Slattery and Bryan (1992a), using a similar soil type to that used in this study, noted that the earliest stage of sediment transport was marked by the redistribution of surface aggregates into topographic depressions. For instance, since the downstream half, or convex portion, of a depression promotes flow divergence and thinning, surface aggregates which are transported to this site may exceed local flow depth. Aggregates would be preferentially deposited, while finer material would be maintained in transport. Furthermore, because aggregates tend to have lower densities than solid soil particles, in low runoff conditions, they may be the
only sediments initially mobilised and transported.

Larger quantities of aggregates are transported to the deposition sites through continued upslope scouring. These aggregates increase surface roughness in deposition zones and obstruct surface wash. In response, flow velocities are reduced and the flow becomes wider, shallower and divergent. Since scouring incises into the original soil material, surface roughness in scour chutes remains largely unchanged. Thus, through longitudinal sediment sorting, marked differences in surface roughness can develop along the slope producing downslope alternations in flow convergence and divergence.

Through continued scour and deposition, the initial bed profile is progressively transformed so that topographic high- and low-points are created in deposition and scour sites, respectively. Simultaneous deposition in the deposition zones and erosion in the scour chutes decreases local gradients in the transition from scour to deposition and increases the local gradients in the transition from deposition to scour (Figure 7.2). Positive feedback mechanisms occur such that increases in local gradients upslope of scour chutes enhance flow convergence and further scouring; while decreases in local gradients upslope of deposition zones enhance flow divergence and further deposition. Continued scour upslope enhances downslope deposition, while upslope deposition enhances downslope scour.

Depending on the size of the deposition zones, runoff may be forced to flow around deposition zones. Sediment deposition will occur on the sides and upslope portion of the deposition zones resulting in their upslope and lateral expansion. Lateral expansion further enhances the positive feedback mechanisms as progressively more sediments are
Figure 7.2: Idealized local slope gradients in deposition zones and scour chutes.

Caught by the deposition zones. Headward scouring, from downslope scour chutes, may progressively excavate materials from downslope portion of the deposition zones. These feedback mechanisms may cause the upslope migration of deposition zones.

7.2 The Initiation of Sequential Scouring:

It is hypothesized that the local slope gradient plays a crucial role in determining the initial location and magnitude of erosion and deposition and, thus, the subsequent development of sequential scouring. As Dunne et al. (1995) noted, the relationship between microtopography and sediment redistribution is not well understood and needs further
attention. Comparing the initial and sub-experiment A longitudinal profiles, it appears that the locations of large positive changes in bed elevation, or deposition zones, in sub-experiment A tend to coincide with the locations of the most gentle local slope angles in the initial profile (Figure 7.3). For example, deposition zones, near 4 m in experiment 1, 3 m in experiment 2 and 1 m in experiment 3, occur at locations where pronounced decreases in local gradient occur (Figures 7.3a, b, c). This suggests a relationship exists between the location and magnitude of initial scour and deposition and local concavities and convexities in the surface, respectively. The research hypothesis to be tested is

\[
H_0 : \rho \left( \frac{\partial^2 z_t - \text{initial}}{\partial x^2}, \frac{\partial z_t > \text{initial}}{\partial t} \right) = 0 \tag{7.1}
\]

\[
H_A : \rho \left( \frac{\partial^2 z_t - \text{initial}}{\partial x^2}, \frac{\partial z_t > \text{initial}}{\partial t} \right) \neq 0
\]

where \( z \) is surface elevation, \( x \) is slope position, \( t \) is experimental duration, and \( \rho \) is the population correlation coefficient. If strong positive correlations exist between initial microtopographic variations and the location and magnitude of initial scour and deposition, the null hypothesis is rejected. Microtopography is here defined as vertical changes on the order of a few centimetres over 50 cm horizontal distances.

Only locations between the weir and 7.0 m were used for the analysis (n = 29) to offset the influence of artificial incision caused by the header-tank. Histograms, probability plots and skewness and kurtosis coefficients indicated that, for each experiment, the rate of change in bed height (i.e. \( \frac{\partial z}{\partial t} \)) and the rate of change of slope (i.e. \( \frac{\partial^2 z}{\partial x^2} \)) exhibited normal distributions. Thus, Pearson correlation coefficients were used to determine the
Figure 7.3: Initial and sub-experiment A profiles for experiments 1 – 12.

7.1a: Experiment 1

7.1b: Experiment 2

7.1c: Experiment 3
Figure 7.3 continued.

7.1d: Experiment 4

7.1e: Experiment 5

7.1f: Experiment 6
Figure 7.3 continued.

7.1g Experiment 7

7.1h: Experiment 8

7.1i: Experiment 9
Figure 7.3 continued.

7.1j: Experiment 10

7.1k: Experiment 11

7.1 L: Experiment 12
relationship between initial scour and deposition and variations in microtopography. Table 7.1 documents the Pearson correlation coefficients for the comparisons between
\[
\frac{\partial^2 z_{t = initial}}{\partial x^2} \quad \text{and} \quad \frac{\partial z_{(x \cdot t = A)}}{\partial t}
\]
for all experiments. From the t statistics, the null hypothesis was rejected for experiments 1 – 7 and 12. These experiments exhibited positive correlations between initial local concavities and convexities in the surface and subsequent scour and deposition, respectively. The null hypothesis was not rejected for experiments 8, 9 and 10. Observations and results showed that experiments 8, 9 and 10 were most likely operating under detachment-limited conditions. This suggests that initial microtopography does not play a strong role in the initiation scour chutes and deposition zones under detachment-limited conditions. This was expected since erosion, and thus the amount deposited, is constrained by soil erodibility, for example, and not by initial surface geometry. However, experiment 7 was also found to be associated with sediment-supply exhaustion, or detachment-limited conditions, but the null hypothesis was rejected for this experiment (Table 7.1). This apparent contradiction can be explained by the fact that detachment-limited conditions did not prevail until sub-experiment B. The Pearson correlation coefficients were for comparisons between initial and sub-experiment A profiles.

The correlation coefficients show that microtopography played a key role in the initiation of sequential scouring. As microtopographic variations in any experiment were not defined by experimental design, the locations of local convexities and concavities in the surface can be considered random. Since the locations of deposition zones and scour chutes in the sequential scouring experiments were partially controlled by local convexities
Table 7.1: Pearson correlation coefficients and corresponding t statistics for the relationship between initial local convexities and concavities, \( \frac{\partial^2 z_t}{\partial x^2} =_{\text{initial}} \), and the change in bed elevation measured at the end of sub-experiment A, \( \frac{\partial z(x, t + \Delta t)}{\partial t} \) (* denotes significant at 0.05 significance level).

<table>
<thead>
<tr>
<th>Experiment #</th>
<th>Sequential Scouring</th>
<th>No alternating patterns of scour and deposition.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r</td>
<td>t</td>
</tr>
<tr>
<td>1</td>
<td>0.56</td>
<td>3.512*</td>
</tr>
<tr>
<td>2</td>
<td>0.34</td>
<td>1.878*</td>
</tr>
<tr>
<td>3</td>
<td>0.32</td>
<td>1.755*</td>
</tr>
<tr>
<td>4</td>
<td>0.38</td>
<td>2.134*</td>
</tr>
<tr>
<td>5</td>
<td>0.36</td>
<td>2.005*</td>
</tr>
<tr>
<td>6</td>
<td>0.46</td>
<td>2.692*</td>
</tr>
<tr>
<td>12</td>
<td>0.33</td>
<td>1.817*</td>
</tr>
</tbody>
</table>

and concavities, respectively, this suggests that the wavelengths exhibited by sequential scouring partially reflect random variations in the initial slope profile. Thus, initial microtopography may have been a primary cause for variability in the wavelengths.

7.3 Longitudinal Sediment Sorting:

The downstream sediment sorting observed with sequential scouring is believed to reflect the selective nature of entrainment, transport and deposition processes. Initially, aggregates tended to move first and were transported downslope as bedload. This was probably due to their lower densities and greater sizes relative to flow depth. Since
aggregates tend to have lower densities than solid particles of comparable size, they are more easily mobilised. At the onset of runoff, concentrated wash dominated where steeper local gradients existed (Figure 7.4). Flow depth ranged from 0.2 to 0.5 cm and was equal to or slightly less than the aggregate sizes. Aggregates tended to protrude into higher velocity flow layers above the bed and their size restricted entrapment. The probability of aggregate entrainment increases in concentrated runoff zones due to greater shear stresses, the lower densities of aggregates and their size relative to flow depth. Aggregates rolled downslope and appeared to remain in motion until they reached the leeside of surface depressions (Figure 7.4).

The leeside of depressions, with their lower gradients, cause flow divergence, widening and thinning, and decreased unit water discharges, transport capacities and shear stresses accordingly. As slopes and flow depths decreased at the leeside of depressions, the local shear stresses must drop below a critical value for aggregate entrainment and transport. In these zones of unconcentrated runoff, aggregates were the first particles deposited. Presumably, this occurred because aggregate diameters exceeded flow depth and local shear stresses were reduced. Flow depths ranged from 0.1 to 0.2 cm in zones of unconcentrated wash. Aggregate diameters tended to be 2 to 4 times flow depth on the leeside of the depressions.

Figure 7.5 illustrates a relationship between shear stress and the rate of change in surface declivity for various scour and deposition zones, measured within the first 15 minutes of each experiment. There is a clear distinction between initial scour and deposition zones with regard to shear stress. Figure 7.5 also suggests that a critical rate of
Figure 7.4: Influence of initial surface configuration on flow properties and aggregate movement.

Figure 7.5: Relationship between the rate of change in gradient and shear stress for scour and deposition zones measured within the first 15 minutes of runoff.
change in slope may exist, which alters flow depth and shear stress, above which aggregates cannot be maintained in transport.

Observations suggest that there are clear differences in the particle size distributions between scour chutes and deposition zones (See Figure 4.1 and 4.2). As scour chutes are erosional forms, they reflect the original soil material. Deposition zones are depositional forms which reflect re-worked sediment. In order to determine whether significant differences existed between scour chute and deposition zone particle size distributions, the samples taken from the near surface layer of various scour chutes and deposition zones were dry sieved to determine particle size distributions (Figure 7.6). Table 7.2 gives the percent by weight held within each sieve. Deposition zones tend to have coarser particle size distributions than scour chutes. The null hypothesis that the particle size distributions of deposition zones and scour chutes are not significantly different and the alternative hypothesis is that deposition zones have a coarser particle size distribution.

A chi-square goodness-of-fit test at the 0.05 significance level was used to determine whether the post-experiment particle size distributions of scour chutes and deposition zones differed significantly (Table 7.2). With df = 7, the $\chi^2$ statistic ($= 28.513$) is greater than the critical $\chi_{0.05}^2$ and shows that deposition zone particle size frequency distributions differed significantly from those of scour chutes. However, Figure 7.6 illustrates that scour chutes and deposition zones were composed of similar proportions of the finer fractions, but deposition zones had a larger proportion of coarser material than scour chutes.
Figure 7.6: Particles size distributions of scour chutes and deposition zones.

Table 7.2: Chi-square goodness-of-fit test at the 0.05 significance level for the differences between scour chute and deposition zone particle size frequency distributions.

<table>
<thead>
<tr>
<th>Sieve Size</th>
<th>Scour chute (expected)</th>
<th>Deposition zone (observed)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(%) by weight</td>
<td>(%) by weight</td>
</tr>
<tr>
<td>4.75</td>
<td>4.68</td>
<td>14.59</td>
</tr>
<tr>
<td>2.8</td>
<td>17.62</td>
<td>16.50</td>
</tr>
<tr>
<td>2.0</td>
<td>11.50</td>
<td>10.72</td>
</tr>
<tr>
<td>1.0</td>
<td>12.18</td>
<td>8.20</td>
</tr>
<tr>
<td>0.50</td>
<td>9.54</td>
<td>5.61</td>
</tr>
<tr>
<td>0.25</td>
<td>20.75</td>
<td>14.88</td>
</tr>
<tr>
<td>0.106</td>
<td>20.50</td>
<td>23.40</td>
</tr>
<tr>
<td>0</td>
<td>3.27</td>
<td>6.11</td>
</tr>
</tbody>
</table>

\( df = 7; \chi^2 = 28.513; \chi^2_{0.05} = 12.592 \)
The mean weight diameter (MWD) was used to determine the dominant grain size for each sample excavated from scour chutes and deposition zones. Each MWD was paired with the post-experiment local gradient measured at the sampling location. Since the experiments used different overall flume gradients, the post-experiment local gradients were divided by their respective initial slopes (i.e. slope index). Figure 7.7 is a plot of MWD versus the slope index. A slope index greater than unity implies scouring and below unity implies deposition. Using non-linear regression, an inverse power function was shown to adequately describe the relationship between MWD and the slope index, S:

\[ MWD = 1.56 S^{-0.89} \]  

7.2

This implies that the deposition zones with coarser particle size distributions were also associated with lower gradients. This is opposite to that found in riffle-pool sequences. Riffles, analogous to the deposition zones, tend to have steeper gradients and coarser particle sizes. The inverse relationship between dominant grain-size and slope is also contrary to the direct relationships between grain-size and slope found on hillslopes (Band, 1985a and b; Abrahams et al., 1985; Scheidegger, 1986; Armstrong, 1987). However, in this study, the inverse relationship was not unexpected. It likely differs from other grain-size catena results due to the fact that the coarsest particle sizes are lower density aggregates. They are preferentially entrained from the steepest gradients and preferentially deposited on the upslope side of local convexities. If the aggregates were replaced by comparable-sized solid particles, a surface lag deposit would probably have developed and no sequential scouring would have occurred. If the aggregates were removed from the
soil, although variations in grain-size may develop, the differences in bed roughness would probably be insufficient to alter local flow conditions and generate sequential scouring.

7.4 Interactions Between Scour and Deposition:

For this analysis, each of the on-slope hydraulic and sediment measurements obtained during each sub-experiment between 3.0 and 5.5 m above the weir, was associated with a categorical variable (E or D) denoting whether the site was characterised by erosion or deposition. This classification was based on the cumulative amount of soil loss or gain
measured at the end of each sub-experiment. Each hydraulic and sediment characteristic was tested at the 0.05 significance level to determine if significant differences exist between scour chutes and deposition zones. These differences were used to investigate the interactions between scour and deposition.

7.4.1 Reynolds and Froude Numbers:

The Froude number (Fr) is an indication of the stability of the free water surface. Flow is critical when Fr is equal to unity, subcritical or tranquil when Fr < 1, and supercritical or rapid when Fr > 1. Reynolds numbers give an indication of flow stability and describes whether a particular flow is laminar or turbulent. For deep open channel flow, when Re < 500, the flow is laminar and viscous forces are significant; while Re > 2000, the flow is turbulent and the inertial forces dominate. When 500 < Re < 2000, the flow is said to be transitional. With Re > 2000, turbulent mixing transfers high energy surface water to the bed and increases the potential for particle entrainment (Allen, 1994). The Froude and Reynolds numbers for overland flow typically range from 0.1 to 15 and 1200 to 3000, respectively (Selby, 1994). Overland flow is mainly supercritical and turbulent (Moss and Walker, 1978) due to, for example, raindrops impacting thinflow or flow obstructions (e.g. aggregates) forcing thinflow to diverge and converge locally.

The measured Froude numbers in deposition zones ranged from 0.11 to 3.54; while Reynolds numbers ranged from 397 to 5925. Scour chutes were associated with 0.15 < Fr < 3.87 and 427 < Re < 6920.
7.4.2 Shear Velocity:

Most studies have reported substantial increases in sediment concentration once shear velocities exceed 3.0 to 3.5 cm s\(^{-1}\) and have used this parameter for defining threshold conditions for incipient rilling (e.g. Savat, 1980; Govers, 1985; Rauws, 1987; Rauws and Govers, 1988; Merz and Bryan, 1993). In this study, increases in sediment concentration coincided with shear velocities above 3.5 cm s\(^{-1}\) (Figure 7.8). Below 3.5 cm s\(^{-1}\), sediment concentrations range from \(\sim 1\) to 40 g l\(^{-1}\) (\(\mu = 15.5\) g l\(^{-1}\) \(\pm 14.4\)), and above, sediment concentrations increase to between \(\sim 1\) and 100 g l\(^{-1}\) (\(\mu = 32.0\) g l\(^{-1}\) \(\pm 22.3\)).

Since the hydraulic forces necessary for rilling appeared to be present in all experiments, the lack of rilling was most likely due to the low soil strength conditions in the absence of rainfall. The occurrence of rill channels or scour chutes, may be related to rainfall and surface sealing. Surface seals can provide a necessary resistant layer protecting weaker soils below (Bryan, 1990). This allows development of steep side-walls and headcut faces and the formation of deeply incised rill channels. The strength of the soil mixture, without rainfall, may have been too low to support steep near-vertical side-walls necessary for headcut and rill develop. As a result, only scour chutes, rather than well-defined rill channels formed in this study.
Figure 7.8: Plot of sediment concentration versus shear velocity. Shaded region denotes the shear velocity threshold condition for incipient rilling.

7.4.3 Velocity, Width and Depth:

Scour chutes had smaller flow widths and larger velocities, on average, than deposition zones (Table 7.3) in agreement with the findings of Moss and Walker (1978). Flow conditions were convergent and divergent in scour chutes and deposition zones, respectively. Pennock and deJong (1987) found net erosion and deposition zones to be associated with more convergent and more divergent flow conditions, respectively.
Table 7.3: Averages and ranges of flow velocities, widths and depths for scour chutes and deposition zones.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Average</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Scour chute</td>
<td>Deposition zone</td>
</tr>
<tr>
<td>Flow velocity ((\text{cm s}^{-1}))</td>
<td>24.6</td>
<td>26.6</td>
</tr>
<tr>
<td>Flow Depth (cm)</td>
<td>0.31</td>
<td>0.30</td>
</tr>
<tr>
<td>Flow Width (cm)</td>
<td>6.8</td>
<td>8.8</td>
</tr>
</tbody>
</table>

7.4.4 The Darcy-Weisbach friction factor:

Friction factors ranged between 0.05 and 28.86 for scour chutes, with a mean of 0.52. Deposition zones were associated with friction factors ranging between 0.03 and 66.02 with a mean of 1.16. Figure 7.9 is a Moody diagram relating the Darcy-Weisbach friction factor to the Reynolds Number. For smooth surfaces, \(K\) (i.e. a relative roughness factor which relates \(f_f\) and \(Re\), \(f_f = \frac{K}{Re}\)) has been theoretically determined to be 96, but as roughness increases and slope angles steepen, \(K\) deviates substantially from 96 (Savat, 1980; Hodges, 1982; Roels, 1984). Bryan and Oostwoud-Wijdenes (1992) report average \(K\)-values of 1660 and 1500 associated with scour steps. Average \(K\)-values in this study were 1015, 1205 and 935 for all sites combined, deposition zones and scour chutes, respectively. These values show that deposition zones are on average rougher than the scour chutes and agree with results of the differences in particle size distributions between scour chutes and deposition zones.
Figure 7.9: Moody diagram relating the Darcy-Weisbach friction factor to Reynolds number.

7.4.5 Differences Between Scour Chutes and Deposition Zones:

Sections 7.4.1 through 7.4.4 indicate that some differences exist in the hydraulic and sediment transport characteristics of scour chutes and deposition zones. A Kruskal-Wallis ANOVA at the 0.05 significance level was performed on data from all experiments and used to determine whether significant differences existed between scour chutes and
deposition zones. The null and alternative hypotheses follow:

\[ H_0 : \mu_E = \mu_D \]
\[ H_A : \mu_E \neq \mu_D \]

where \( \mu \) is the mean value of the characteristic variable under investigation and the subscript denotes scour, \( E \), or deposition, \( D \). It is hypothesized that deposition zones with their relatively wider, shallower and divergent flow conditions, would be associated with:

1) smaller velocities; 2) smaller water and sediment fluxes; and 3) smaller shear velocities, shear stresses and unit stream powers. Deposition zones would also have smaller Reynolds and Froude numbers. As deposition zones are composed largely of coarser particle sizes, they should be associated with greater surface roughnesses, but this would also depend upon packing, bulk density and particle heterogeneity.

Table 7.4 gives the results of the Kruskal-Wallis ANOVA. The null hypothesis was rejected for width, slope gradient, unit discharge, velocity, shear velocity, shear stress, Reynolds and Froude numbers and unit stream power. Deposition zones were shown to have: 1) larger flow widths; 2) smaller slope gradients; 3) smaller unit discharges; 4) smaller velocities; 5) smaller shear velocities and shear stresses; 6) smaller Reynolds and Froude numbers; and 7) smaller unit stream powers.

Scour chutes had, on average, greater Reynolds and Froude numbers indicating greater potential for bed deformation. The greater shear velocities, unit stream powers, velocities, slopes and unit water discharges showed that scour chutes had conditions more conducive for sediment entrainment and transport than the deposition zones, as expected.
Table 7.4: Kruskal-Wallis ANOVA testing for differences between scour and deposition sites in terms of the characteristic variables (* denotes significant difference at the 0.05 significance level).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Scour chute and deposition zone means</th>
<th>Variable</th>
<th>Scour chute and deposition zone means</th>
</tr>
</thead>
<tbody>
<tr>
<td>width (cm)</td>
<td>$\mu_E = 6.8 \pm 3.9^*$</td>
<td>$u^*$ (cm s$^{-1}$)</td>
<td>$\mu_E = 4.6 \pm 0.8^*$</td>
</tr>
<tr>
<td></td>
<td>$\mu_D = 8.8 \pm 6.7^*$</td>
<td></td>
<td>$\mu_D = 4.4 \pm 0.9^*$</td>
</tr>
<tr>
<td>depth (mm)</td>
<td>$\mu_E = 3.1 \pm 1.0$</td>
<td>$\tau$ (Pa)</td>
<td>$\mu_E = 2.2 \pm 0.7^*$</td>
</tr>
<tr>
<td></td>
<td>$\mu_D = 3.0 \pm 1.0$</td>
<td></td>
<td>$\mu_D = 2.0 \pm 0.9^*$</td>
</tr>
<tr>
<td>slope (Sine)</td>
<td>$\mu_E = 0.081 \pm 0.01^*$</td>
<td>Re</td>
<td>$\mu_E = 2863 \pm 1122^*$</td>
</tr>
<tr>
<td></td>
<td>$\mu_D = 0.076 \pm 0.02^*$</td>
<td></td>
<td>$\mu_D = 2599 \pm 1148^*$</td>
</tr>
<tr>
<td>$q_w$ (cm$^3$ s$^{-1}$ cm$^{-1}$)</td>
<td>$\mu_E = 2.55 \pm 0.89^*$</td>
<td>Fr</td>
<td>$\mu_E = 1.7 \pm 0.5^*$</td>
</tr>
<tr>
<td></td>
<td>$\mu_D = 2.36 \pm 0.81^*$</td>
<td></td>
<td>$\mu_D = 1.6 \pm 0.6^*$</td>
</tr>
<tr>
<td>$q_s$ (g s$^{-1}$ cm$^{-1}$)</td>
<td>$\mu_E = 0.051 \pm 0.03$</td>
<td>Manning’s n</td>
<td>$\mu_E = 0.008 \pm 0.006$</td>
</tr>
<tr>
<td></td>
<td>$\mu_D = 0.057 \pm 0.03$</td>
<td></td>
<td>$\mu_D = 0.010 \pm 0.012$</td>
</tr>
<tr>
<td>$C_s$ (g l$^{-1}$)</td>
<td>$\mu_E = 29.8 \pm 21.1$</td>
<td>D-W ff</td>
<td>$\mu_E = 0.52 \pm 1.8$</td>
</tr>
<tr>
<td></td>
<td>$\mu_D = 32.1 \pm 24.5$</td>
<td></td>
<td>$\mu_D = 1.16 \pm 4.9$</td>
</tr>
<tr>
<td>Velocity (cm s$^{-1}$)</td>
<td>$\mu_E = 26.6 \pm 7.6^*$</td>
<td>Mean Stream Power</td>
<td>$\mu_E = 0.013 \pm 0.008$</td>
</tr>
<tr>
<td></td>
<td>$\mu_D = 24.6 \pm 8.5^*$</td>
<td>(W m$^{-2}$)</td>
<td>$\mu_D = 0.014 \pm 0.011$</td>
</tr>
<tr>
<td>R (cm)</td>
<td>$\mu_E = 0.27 \pm 0.08$</td>
<td>Stream Power per Unit Bed Area</td>
<td>$\mu_E = 0.57 \pm 0.57^*$</td>
</tr>
<tr>
<td></td>
<td>$\mu_D = 0.27 \pm 0.08$</td>
<td>(W m$^{-2}$ m$^{-2}$)</td>
<td>$\mu_D = 0.48 \pm 0.25^*$</td>
</tr>
</tbody>
</table>

Since deposition zones are associated with coarser particle sizes and shallower flows, it was expected that they would also have greater Darcy-Weisbach friction factors. As the aggregates protrude through the bed and create flow obstructions, deposition zones should have rougher surfaces and should sustain greater frictional losses. However, the null hypothesis was not rejected for the differences between scour and deposition with
regard to the Darcy-Weisbach friction factor. This does not support either the differences found between the particle size distributions of scour chutes and deposition zones or the inverse relationship found between MWD and slope.

As expected, the hydraulic and sediment transport conditions indicate that transport capacities in scour chutes are greater than deposition zones. Positive feedback mechanisms develop in which scour chutes and deposition zones work in tandem to redistribute sediment and transform the original profile. Because these two regions are so intricately related, this implies that they should not be treated separately.

7.5 Positive Feedback Mechanisms

Increases in upslope scouring enhances downslope deposition forming positive feedbacks and promoting deposition zone development. However, increases in upslope deposition enhance downslope scouring. These feedbacks can cause scour chutes to recede upslope and excavate sediments from upslope deposition zones. Figure 7.10 illustrates the effects of some positive feedback mechanisms which occurred in experiment 1. In sub-experiment 1B, a large deposition zone was observed between 2.75 and 4.5 m. By sub-experiment 1C, the end of the deposition zone receded roughly 0.5 m upslope, while its upslope portion widened (Figure 7.10). Similar effects occur between sub-experiments 2B and 2C with the deposition zone near 2.5 m, between sub-experiments 3B and 3C with the deposition zone near 3 m, and between sub-experiments 4B and 4C with the deposition zone near 4 m (See Appendix A).
Figure 7.10: Planimetric morphologies of experiment 1. Note the development of the deposition zone between 2.75 and 4.5 m.

Sub-Experiment 1A

Sub-Experiment 1B

Sub-Experiment 1C
7.6 Summary:

Initial microtopography played a key role in the initiation of scour chutes and deposition zones, but only in experiments which produced sequential scouring. Since the sequential scouring experiments were transport-limited and non-sequential scouring experiments were detachment-limited, initial microtopography only influenced the location and magnitude of scour and deposition under transport-limited conditions. Furthermore, since initial variations in microtopography were randomly located, this suggests that the variability in the wavelengths associated with sequential scouring may reflect the random nature of the initial slope profile.

Once initiated, scour chutes and deposition zones work in tandem to redistribute sediment and produce significant variations in the particle size along the longitudinal profile. This seems to be an important mechanism in the further development of scour chutes and deposition zones as it produces differences in surface roughness which act to self-enhance both scour and deposition. That is, deposition zones, with larger particle sizes and lower gradients, cause higher frictional losses which enhances further deposition. The smoother surfaces and steeper gradients in scour chutes promote further scour.

Upslope scour enhances downslope deposition and upslope deposition enhances downslope scour. This forms positive feedback mechanisms which enhance the development of sequential scouring. Positive feedbacks can also cause scour chutes to recede upslope and excavate material from upslope deposition zones.
8.0 Mathematical Model for Sequential Scouring:

8.1 Introduction:

A second goal of this research was to produce a simple, process-based, one-dimensional mathematical model for simulating the development of sequential scouring. The model was designed to predict the spatial and temporal changes in the longitudinal profile by simulating the processes of scour and deposition, as well as their influence on channel form.

The model is composed of three sub-models which are used to compute the magnitude of surface water discharge, the flow properties and the changes in surface elevation for a number of nodes along a downslope transect. These parameters are used to determine the water and sediment fluxes at each node. Through mass conservation, the differences between sediment input and output for each slope segment are used to determine the resultant changes of the surface at each node. These changes are used for the computations of the flow properties and the sediment and water fluxes for the next iteration.

Since no deep rilling, mass-wasting or soil creep were evident in any of the experiments and since runoff was not produced via rainfall, this model assumes only concentrated overland flow as the sole force responsible for sediment entrainment, transport and deposition. The model also assumes only transport-limited conditions. Although the universal applicability of the model is constrained by these assumptions, they fit well with observed data and substantially simplify the model's design. Including the effects of rainfall and other sediment transport processes, as well as detachment-limited conditions, is an objective for future research.
Lastly, the model was designed to be explanatory and predictive. Although the main function of the model is to predict the changes in the longitudinal profile associated with sequential scouring, it was also designed for assessing the impacts of individual system components on the rates of scour and deposition. Thus, the model can be used to assess the development of sequential scouring under a variety of conditions.

8.2 General Modelling Framework:

The model was constructed by coupling three sub-models (Figure 8.1): 1) a Runoff Sub-Model computes surface water discharge; 2) a Channel Hydraulics Sub-Model determines flow width, depth and velocity and the Darcy-Weisbach friction factor; and 3) changes in surface elevation are determined by a Sediment Transport Sub-Model. These sub-models compute the changes in the flow and surface conditions at one minute intervals for a number of fixed points along a transect. The numerical scheme was programmed in C++ (See Appendix B).

8.2.1 Initial and Boundary Conditions:

There are a great number of difficulties associated with any modelling exercise, such as scale issues and the use of empiricism, which constrain their universal applicability. However, many current complex models (e.g. CREAMS; Kinsel, 1980 and WEPP; Lane and Nearing, 1989) can be limited in their practical use solely by a need for large amounts of input data, some of which are not entirely or precisely known (Schmidt, 1991). Increasing the level of
Figure 8.1: Flow chart and iterative process for model of sequential scouring.
complexity in a model can both strengthen a model's performance while limiting its universal applicability. Therefore, it was decided to use the simplest possible approach which minimized the number of user inputs by considering only the most influential factors.

Drawing on the concepts and ideas presented in previous chapters, those initial conditions deemed necessary through their effects on the overall system were incorporated into the modelling scheme. These are

1) initial bed elevation, \( z \), for nodes along the transect since initial microtopography was shown to influence the location and magnitude of scour and deposition;
2) the initial side-slope angle orthogonal to the downslope transect, \( \beta \), as it acts to concentrate surface discharge;
3) initial gradient, \( \alpha \), and header-tank discharge, \( Q_h \), through their influence on sediment transport; and
4) antecedent volumetric soil moisture, \( \theta \), and dry bulk density, \( \rho_b \), since they affect the timing and magnitude of surface discharge and local infiltration rates.

Furthermore, since the locations of local convexities and concavities in the surface were random, the inclusion of initial microtopography adds a stochastic element to the modelling approach. It is believed that this will allow for more accurate predictions of the wavelengths exhibited by sequential scouring. Variations in one or more of these initial conditions can be used to assess the effects of each component on the overall development of sequential scouring.

There are two boundary conditions used to constrain the development of the hillslope profile. First, the toe of the slope is considered as base-level and remains fixed. This
mimics the stationary weir. A second boundary condition is the rate of surface lowering of
the topmost node immediately below the header-tank outlet. Since no sediment entered the
system from the header-tank, the rate of change in elevation of the topmost node was
determined, using mass conservation, as a function of the sediment flux at the next
downslope node.

8.2.2 The Runoff Sub-Model:

Water discharge, \( Q_w \), has been derived using a variety methods. For example, Schmidt
(1991) and Band (1985a,b) determined \( Q_w \) as a function of rainfall intensity and distance
from the divide assuming a constant flow width. This method approximates water discharge
as linearly increasing downslope. Dunne et al. (1995) demonstrated the influence of
microtopography on disturbing this linear relationship by creating local flow convergence
and divergence. Other models assume water discharge to be a simple function of rainfall and
infiltration (e.g. Meyer and Wischmeier, 1969; Rose and Freebairn, 1985) and, thus, water
discharge is computed as a constant for all locations at each time increment. However,
through time, with this method, water discharge increases and tends toward a final
equilibrium value dependent on the infiltration capacity. Because of spatial heterogeneity in
soil characteristics and rainfall intensities, the ability to characterise the amount of runoff
with these approaches can be a limiting factor. Furthermore, in conditions of non-constant
flow width, describing runoff by a simple linear function can produce errors. Since it is
imperative to have a good estimation of this parameter, as it drives sediment transport in this
study, an alternative approach was taken.
As water discharge was produced via a constant-head tank mounted on the upslope end of the flume, runoff was modelled as a function of water entering the topmost element (See also Huang and Bradford, 1993). Iterating in a downslope fashion, the local flow width and soil conditions were used to determine the local infiltration losses. Mass balance was used to determine the magnitude of runoff at the next downslope node:

\[ Q^t_w = Q^t_{w_{x+1}} - [I^t_x \times w^t_{x+1} \times \Delta x] \]

where \( I^t_x \) is a time-dependent infiltration rate of the slope segment (cm min\(^{-1}\)), \( w^t_{x+1} \) is the flow width at the upper end of the slope segment (cm), and \( \Delta x \) is the length of the slope segment (cm). As the soil reaches saturation within each segment, \( Q^t_w \) tends toward the value chosen for the header-tank discharge. Infiltration rates were determined using constant head infiltrometers and characterised by the following formula:

\[ I^t_x = I_0 \times e^{-a\xi} \]

where \( a \) is an empirically derived exponent describing the shape of the curve and \( \xi \) is time. Note: \( \xi \) does not denote experimental time, \( t \). Rather, it is a counter variable which tracks the time since the onset of runoff in each segment. \( I_0 \) is the initial infiltration rate dependent on antecedent soil moisture and bulk density:

\[ I_0 = f(\rho_b, \theta) = a + b\theta + c\rho_b \]

where \( a, b \) and \( c \) were determined by linear regression (Figure 8.2). If the amount lost to infiltration exceeded the incoming water discharge, then runoff at the base of the segment was set to zero. If so, all successive downslope cells also had discharges equal to zero.
Figure 8.2: Infiltration rates for the soil mixture used in this study at five bulk densities and three volumetric moisture contents (A through E). Lines indicate inverse exponential fit for each test.

Non-linear regression to find $I$:

$$ I = I_0 e^{-bt} $$

A: $y = 1.847 e^{-0.273x}$; $r^2 = 0.91$
B: $y = 1.668 e^{-0.202x}$; $r^2 = 0.83$
C: $y = 1.443 e^{-0.214x}$; $r^2 = 0.78$
D: $y = 0.731 e^{-0.143x}$; $r^2 = 0.56$
E: $y = 0.321 e^{-0.117x}$; $r^2 = 0.23$

$\therefore b = -0.099 e^{(0.504 l_0)}$; $r^2 = 0.94$.

From linear regression:

$$ I_0 = a - (b \theta) - (c \rho_b) $$

$I_0 = 5.564 - 7.925 \theta - 2.447 \rho_b$; $r^2 = 0.90$
Since the soil mixture was placed above an impermeable perspex bed, this sub-model assumes an infiltration capacity of zero. Although this adequately describes the flume conditions, it has limited use in nature.

In summary, runoff is computed for successive downslope nodes beginning with the topmost node. First, the model computes the magnitude of runoff for the topmost node, then proceeds to the Channel Hydraulics Sub-Model and computes flow width, depth and velocity for the topmost node. The model returns to the Runoff Sub-Model and, using flow width and the local infiltration rate, calculates the magnitude of surface discharge for the next downslope node. The model proceeds until runoff, width, depth, velocity and unit water discharge are known for all nodes during the current time interval. The model then proceeds to the Sediment Transport Sub-Model and calculates the changes in surface elevation for the next iteration.

8.2.3 Sediment Transport Sub-Model:

Sediment transport, $q_s$, was modelled using a simple non-linear function:

$$q_{s_x} = k \left( q_{w_x} \right)^m \left( S_x \right)^n$$  \hspace{1cm} 8.4

where $k$, $m$, $n$ are empirically derived, $q_{w_x}$ is unit water discharge and $S$ is the local gradient. As $S$ is the tangent of the local slope angle, equation 8.4 can be rewritten so that:

$$q_{s_x} = k \left( q_{w_x} \right)^m \left( \frac{\partial z_x}{\partial x} \right)^n$$ \hspace{.5cm} 8.5
Furthermore, since sequential scouring is associated with downslope variations in flow width, the model must be modified to account for changes in flow width. Since \( q_w = \frac{Q_w}{w} \), then equation 8.5 becomes

\[
q_{tx}^t = k \left( \frac{Q_{wx}^t}{w_{x}^t} \right)^m \left( \frac{\partial z_x^t}{\partial x} \right)^n \quad 8.6
\]

Mass conservation is used to compute the change in bed elevation for each node:

\[
p_b \frac{\partial z_x^t}{\partial t} = - \frac{\partial q_{tx}^t}{\partial x} \quad 8.7
\]

Substituting equations 8.5 and 8.6 into equation 8.7 yields the following for determining changes in bed elevation for each time increment:

\[
p_b \frac{\partial z_x^t}{\partial t} = - \frac{\partial}{\partial x} \left[ k \left( \frac{Q_{wx}^t}{w_{x}^t} \right)^m \left( \frac{\partial z_x^t}{\partial x} \right)^n \right]
\]

\[
= -k \left( \frac{Q_{wx}^t}{w_{x}^t} \right)^m (w_x^t)^{-m} \left( \frac{\partial z_x^t}{\partial x} \right)^{n-1} \left( \frac{\partial^2 z_x^t}{\partial x^2} \right)
\]

\[
- k m \left( \frac{Q_{wx}^t}{w_{x}^t} \right)^{m-1} \left( \frac{\partial Q_x^t}{\partial x} \right) (w_x^t)^{-m} \left( \frac{\partial z_x^t}{\partial x} \right)^n
\]

\[
+ k \left( \frac{m (w_x^t)^{m-1}}{(w_x^t)^{2m}} \right) \left( \frac{\partial w_x^t}{\partial x} \right) \left( \frac{Q_{wx}^t}{w_{x}^t} \right)^m \left( \frac{\partial z_x^t}{\partial x} \right)^n .
\] 8.8

The three components of equation 8.8 calculate the contributions to sediment discharge as a result of changes in slope, water discharge and flow width, respectively. The third
component is an important, new approach and simulates the portion of sediment discharge which is mobilised or deposited as a result of converging and diverging flow conditions.

Equation 8.8 can be solved using a variety of techniques. Initially, an explicit finite difference form of equation 8.8 was introduced into the modelling scheme. An explicit methodology uses known values at time $t$ to compute the values at $t = t + 1$. However, the strong non-linearity of equation 8.8 produced massive numerical oscillations. Obviously, as it was the intention to reproduce the natural oscillations in bed height associated with sequential scouring, the explicit methodology could not be used to distinguish between real and artificial bed oscillations. Thus, the explicit approach was discarded in favour of an implicit methodology. This computes bed elevations at time $t = t + 1$ using both known values of bed elevation at $t$ and unknown “estimations” of the bed elevation at $t = t + 1$. This is iterated until the computed and “estimated” bed elevations at $t = t + 1$ converge. The convergent value becomes bed elevation at time $t = t + 1$. A Crank-Nicholson central differencing scheme coupled with a Newton-Raphson iterative approach (See Gerald and Wheatley, 1989) was used to solve equation 8.8 implicitly.

In summary, the Sediment Transport Sub-Model computes bed elevation for the next time increment. The changes in bed elevation are then used by the Channel Hydraulics Sub-Model to compute channel cross-sectional area and the flow properties.

**8.2.4 Channel Hydraulics Sub-Model:**

Many mathematical models which incorporate a component to simulate channel cross-section are limited by their inability to characterize flow width (Darby and Thornes, 1996).
This is further compounded by the fact that the channel response to a perturbation, such as increases in discharge, aggradation or degradation, is frequently dominated by changes in flow width (Darby and Thornes, 1996). Most models used for simulating hillslope profile development simply use constant width (e.g. Kirkby, 1971; Band, 1985a,b; Schmidt, 1991). Other models use simple empirical relationships for describing channel adjustment such as downstream hydraulic geometry relationships (See Richards, 1982; Knighton, 1984; Julien 1995; Julien and Wargadalam, 1995). As flow width alters the sediment and water fluxes and is a main cause of the positive feedback mechanisms between erosion and deposition, a method for incorporating variable flow width was necessary.

A simple method for determining flow width as a result of changes in bed elevation was devised. Note however that this method was intended only to describe flow width for “average” conditions. It is not suggested or intended that these hypothetical relationships represent an accurate account of flow width throughout complete experiments.

The general approach for determining flow width is fundamentally based on a combined form of a common discharge law,

\[ Q = A \cdot v \]  \hspace{1cm} (8.9)

and the Darcy-Weisbach equation (solved for velocity),

\[ v = \sqrt{\frac{8gRS}{ff}} \]  \hspace{1cm} (8.10)

where \( A \), \( v \), \( S \), \( g \) and \( ff \) are the cross-sectional area, velocity, slope, gravitational constant and the Darcy-Weisbach friction factor, respectively. \( R \) is the hydraulic radius which is equal
to the cross-sectional area divided by the wetted perimeter. Combining Equations 8.9 and 8.10 gives

\[ Q = A \sqrt{\left( \frac{8 g R S}{f f} \right)} \]  \hspace{1cm} 8.11

To solve Equation 8.11 for flow width, generalized forms of cross-sectional area and wetted perimeter must be known, or at least approximated. In the simple model used here, the channel cross-section is considered as a function of changes in bed height. There are three fundamental assumptions with regard to the shape of the channel cross-section. First, when \( \Delta z = 0 \), the channel shape maintains its original V-shaped form and side-slope angle, \( \beta \) (Figure 8.3a). Second, when \( \Delta z > 0 \), deposition occurs uniformly across the channel so that the vertex of the original V-shaped thalweg is buried (Figure 8.3b). This forms a trapezoidal channel with side-slope angles equal to the original. Third, when \( \Delta z < 0 \), erosion occurs producing a trapezoidal channel with a steeper side-slope angle, \( \beta_c \) (Figure 8.3c). These hypothetical representations of scour chute and deposition zone cross-sections fit well with observed data (See Figures 4.1b and 4.2b).

**Initial Channel (\( \Sigma \Delta z = 0 \))**:

For \( \Sigma \Delta z = 0 \), the channel cross-sectional area is triangular where

\[ A = \frac{1}{2} w \times d \]  \hspace{1cm} 8.12
Figure 8.3: Assumed channel geometry when there are: a) no changes in bed elevation; b) positive changes in bed elevation (i.e. deposition); and c) negative changes in bed elevation (i.e. erosion).
and the wetted perimeter is

\[ P = 2\sqrt{\left(\frac{1}{2} w\right)^2 + d^2}. \]  

Equations 8.12 and 8.13 must be given in terms of flow width in order to solve Equation 8.11 for width. Since the side-slope angle, \( \beta \), is known, flow depth can be given as a function of flow width so that:

\[ d = \frac{1}{2} w \times \tan(\beta). \]  

Substituting Equation 8.14 into Equations 8.12 and 8.13, both \( A \) and \( P \) can be generalized as functions of flow width:

\[ A = \frac{1}{4} w^2 \tan(\beta) \]  

and

\[ P = w\sqrt{1 + \tan^2(\beta)}. \]

Substituting Equations 8.15 and 8.16 into Equation 8.11 and solving for flow width yields the following expression:

\[ w = \left( \frac{64 ff Q^2}{8 g S \left( \frac{(\tan(\beta))^3}{\sqrt{1 + (\tan(\beta))^2}} \right)} \right)^{0.2}. \]
Deposition ($\sum \Delta z > 0$):

For $\sum \Delta z > 0$, a trapezoidal shaped channel is formed maintaining the original side-slope angle, $\beta$, such that

$$A = \frac{1}{2}(w + w_b) \, d$$  \hspace{1cm} 8.18

and

$$P = w_b + 2 \sqrt{\left( d^2 + \frac{d^2}{\tan(\beta)^2} \right)}$$  \hspace{1cm} 8.19

where

$$d = \frac{1}{2} \, w \, \tan(\beta) - \sum \Delta z$$  \hspace{1cm} 8.20

and

$$w_b = \frac{2 \sum \Delta z}{\tan(\beta)}.$$  \hspace{1cm} 8.21

Substituting Equations 8.20 and 8.21 into Equations 8.18 and 8.19, a general expression of $A$ and $P$ in terms of flow width and cumulative changes in bed height can be determined:

$$A = C_1 w^2 - C_2$$  \hspace{1cm} 8.22

and
where

\[ P = C_3 + C_4 w \]  \hspace{1cm} 8.23

Substituting Equations 8.22 and 8.23 into Equation 8.11 and solving for flow width, the following general expression is obtained:

\[ C_1 = \frac{1}{4} \tan(\beta) , \]  \hspace{1cm} 8.24

\[ C_2 = \frac{(\sum \Delta z)^2}{\tan(\beta)} , \]  \hspace{1cm} 8.25

\[ C_3 = \sum \Delta z \left( \frac{2}{\tan(\beta)} - \sqrt{1 + \frac{1}{\tan(\beta)^2}} \right) \]  \hspace{1cm} 8.26

\[ C_4 = \sqrt{(\tan(\beta)^2 + 1)} . \]  \hspace{1cm} 8.27

Substituting Equations 8.22 and 8.23 into Equation 8.11 and solving for flow width, the following general expression is obtained:

\[ \left( \sum_{k=0}^{3} w^{2k} C_1^k C_2^{(-k+3)} \right) - FC_4 w = FC_3 \]  \hspace{1cm} 8.28

where

\[ F = \frac{ff Q^2}{8 g S} . \]  \hspace{1cm} 8.29
Erosion ($\sum \Delta z < 0$):

For $\sum \Delta z < 0$, a trapezoidal channel is formed with a steeper channel side-slope angle. Since a trapezoidal channel is formed, the general solutions of $A$ and $P$ for erosion are similar to those for deposition except for the formulation of $w_b$ (See Equation 8.21). Equation 8.21 becomes

$$w_b = 2 \left( \frac{2 \sum \Delta x}{\tan(90 - \beta_c)} + \sqrt{\left( \frac{2 \sum \Delta x}{\tan(90 - \beta_c)} \right)^2 + \left( \sum \Delta z \right)^2} \right)$$

8.30

here $\beta_c$ is the new side-slope angle. As such, Equations 8.23 - 8.26 become

$$C_1 = \frac{1}{4} \tan(\beta_c) ,$$

8.31

$$C_2 = \frac{1}{4} w_b \tan(\beta_c) ,$$

8.32

$$C_3 = w_b \left(1 - \sqrt{\left(\tan(\beta_c)\right)^2 + 1} \right)$$

8.33

and

$$C_4 = \sqrt{\left(\tan(\beta_c)\right)^2 + 1} .$$

8.34

The general solution for flow width when $\sum \Delta z < 0$ is identical to equation 8.28.
The Darcy-Weisbach Friction Factor:

The downslope variations in surface roughness resulting from scour and deposition seem to be important components for the development of sequential scouring. They were, therefore, incorporated into the Channel Hydraulics Sub-Model (See equation 8.17) by using the Darcy-Weisbach friction factor. The Darcy-Weisbach friction factor was modelled empirically as a function of the Reynolds number (See Figure 7.9):

$$\log(\nu) = 10.27 - 3.18 \log(Re); \quad n = 851; \quad r^2 = 0.3 .$$

8.3 Summary:

The Runoff, Sediment Transport and Channel Hydraulics Sub-Models were combined to produce a simple, dynamic, one-dimensional process-based model. The purpose of the model is to determine the changes in the longitudinal profiles, the planimetric morphologies and the local rates of scour and deposition associated with sequential scouring. The model can also be used to determine the local hydraulic conditions of scour chutes and deposition zones since it has the ability to characterise flow width, depth, velocity and flow resistance.

This modelling approach, with the specific use of flow width in all sub-models, provides a dynamic component to the model. Since channel cross-sectional area varies as a function of changes in bed elevation, different slope segments, although they may have similar incoming water discharges, can have different flow widths. This has a cascading effect in which all other subsequent computations are affected. For instance, this would lead to differential sediment discharges and variable changes in bed elevation along the transect.
This dynamic approach also inherently addresses the positive and negative feedback mechanisms in which: 1) each slope segment can be either self-preserving or self-destructive; and 2) each slope segment can influence their respective downslope segments. Thus, the model can simulate the interactions between scour and deposition; and, since the model allows the processes of scour and deposition to occur at a single location, but not simultaneously, it also addresses the transient nature of scour and deposition. This topic is rarely considered in modelling exercises (Band, 1990).
9.0 Model Validation, Sensitivity Analysis and Interpretation

9.1 Introduction:

Model validation, sensitivity analyses and the physical interpretation of model results are important components in modelling exercises. Validating models is essential to the modelling process as it provides a means to test how well simulated output agrees with actual results. Differences between simulated and measured results can be used to determine the validity of model assumptions and hypotheses, and suggest areas of improvement (Addiscott, 1993). Further insight to model efficacy is achieved from sensitivity analyses. Sensitivity analyses make two important contributions: 1) they can be used to define the optimal range of conditions in which a model properly functions; and 2) they can be used to identify parameters which may require further refinement.

9.2 Sensitivity Analysis:

By comparing the measured and modelled results for experiment 1, a sensitivity analysis was performed on the model by varying five input parameters and one internal parameter. Although only experiment 1 was used, the results of the sensitivity analysis should apply equally to other simulations. Header-tank discharge, dry bulk density, soil moisture, initial thalweg side-slope angle, initial microtopography and channel side-wall angle were varied to assess their affects on the modelled final longitudinal profile of experiment 1.
Sensitivity was assessed using an index which calculated the percent change in output versus the percent change in the parameter (See Addiscott, 1993). The percent change in input parameter was defined by the amount of change from a chosen baseline value. Baseline values were chosen to be similar to the recorded inputs for experiment 1. The sensitivity index was applied to all points on the hillslope. An index of zero implies there was no change in output per change in parameter, while large deviations from zero imply sensitivity.

Figure 9.1 illustrates the results of the sensitivity analysis. Header-tank discharge was varied by ±33% around a baseline of 1.5 t min⁻¹ (Figure 9.1a). These percentages reflect the experimental range of conditions. The output was found to be sensitive to changes in header-tank discharge as expected since water discharge is a major driving force for sediment transport within the model. Furthermore, the model operated without numerical instabilities with discharges between 0 and 3.0 t min⁻¹. Higher water discharge inputs created large water fluxes and lead to numerical instabilities within the Sediment Transport Sub-Model.

Antecedent dry bulk density was varied by 33% above and below 1.5 g cm⁻³. This range in dry bulk density (i.e. 1.0 – 2.0 g cm⁻³) is similar to natural environments. Figure 9.1b shows the output to be insensitive to changes in dry bulk density.

Three values of antecedent volumetric soil moisture were chosen for the sensitivity analysis: 0, 0.15 and 0.30 m³ m⁻³, or ±100% variations about a baseline of 0.15 m³ m⁻³ (Figure 9.1c). This range in soil moisture reflects dry to saturated conditions for the fine sand soil mixture. The output was found to be insensitive to changes in soil moisture.
Figure 9.1: Sensitivity index plots for percent changes in input parameters from chosen baseline values. Plots show the sensitivity of bed height at all locations to changes in input parameters: a) header-tank discharge; b) dry bulk density; c) volumetric soil moisture; d) thalweg side-slope angle; e) amplitude of microtopography; and f) channel side-wall angle.
Figure 9.1 continued.

(c) Baseline soil moisture: 0.15 ml/ml

(d) Baseline thalweg side-slope: 5 degrees
Figure 9.1 continued.

e) Baseline microtopography: 12.5 amplitude, 100 cm wavelength

Baseline microtopography: 75 degrees

f) Baseline channel angle: 75 degrees
The initial thalweg side-slope angle was initially varied by 1° increments between 1 and 10°. Numerical instabilities resulted below 1°. Below 1°, the model produced extremely large flow widths creating instabilities in flow width and depth calculations. Indeed, near 0°, the flow was infinitely wide and infinitely shallow. Within the 1 to 10° range, only locations near the header-tank were found to be sensitive. Figure 9.1d shows the sensitivity index for -80% and +100% variations from 5° (i.e. 1 and 10°).

Five hypothetical initial slope profiles were developed to reflect variations in microtopography. These were constructed by varying the amplitude of a sine curve with a wavelength of 100 cm. The five slopes simulated microtopography with amplitudes of 0, 6.25, 12.5, 25 and 50 cm. With an amplitude of 0 cm (i.e. smooth slope), since all local slopes were identical, sediment transport depended only on water discharge. Since water discharge eventually reaches a constant value for all nodes on the slope, all nodes degraded at an identical rate. Since the transport capacity of all nodes was identical, no deposition zones formed. With an amplitude of 50 cm (i.e. very rough slope), the local concavities had very steep gradients, while the convexities had gradients just above zero. Large quantities of sediment were removed from concavities and deposited immediately downslope on convexities. Through time, topographic high-points and low-points in the initial profile were removed and filled in, respectively, producing a smoother surface. Figure 9.1e shows ±50% variations in amplitude (i.e. 6.25 and 25 cm) about a baseline of 12.5 cm. Bed height was found to be very sensitive to changes in amplitude and numerically illustrates the influence of initial microtopography on location and magnitude of initial scour and deposition.
The channel side-wall angle, defined for scour chutes as a result of incision, was varied to determine the possible range of channel side-wall angles. Angles ranging from 10 to 90° were first simulated. Large numerical instabilities in the Channel Hydraulics Sub-Model resulted when values of channel side-wall angle were not between 65 and 80°. Although this limits the range of possible values for the channel side-wall angle, this range is representative of a non-vertical, steep side-wall corresponding to observations. Channel angles of 70 and 80° were compared to an angle of 75° (Figure 9.1f). Bed height was found to be relatively insensitive to channel side-wall angles between 70 and 80°.

The sensitivity analysis determined the optimal conditions under which the model can realistically operate. These are: 1) surface discharges between 0 and 3.0 l min⁻¹; 2) initial side-slope angles above 1°; and 3) channel side-wall angles ranging between 70 and 80°. The model operates as designed for any initial microtopography configuration, bulk density or volumetric moisture content.

9.3 Model Validation:

It is imperative to have reliable data against which to test a model's performance (Addiscott, 1993). Without reliable data, a good model may be rejected or a bad model accepted. In this study, due to the variability in water and sediment discharge and the onslope hydraulic and sediment conditions, it is difficult to provide quantitative comparisons between measured and modelled results for model validation purposes. This variability resulted from both experimental design and the inability to accurately
characterize the on-slope hydraulic and sediment characteristics. Model validation will proceed by qualitatively examining the model's ability to simulate sequential scouring and the interactions between scour and deposition.

9.3.1 Modelled Longitudinal Profiles:

The modelled values of the initial, sub-experiment A and final longitudinal profiles for experiments 1 – 9 are shown in Figure 9.2. All remaining measured and modelled longitudinal profiles and cumulative changes in bed height for each sub-experiment are shown in Appendix C. Experiments 10 and 12 were excluded from the analysis. Water discharge did not reach the weir during the model run of experiment 10 due to large infiltration losses and, thus, the measured and modelled data differ substantially. Experiment 12 could not be assessed due to numerical instabilities in its model output. In experiment 12, the topmost node oscillated between scour and deposition. As it is physically impossible for deposition to occur at the topmost node, the model run for experiment 12 was considered unreasonable. Since the input values and the roughness of the initial surface were similar to those used in the other simulations, it is not known what caused the numerical instabilities.

The measured and modelled results were compared by two methods to determine the validity of the model proposed for sequential scouring. First, the modelled sub-experiment A profiles (Figure 9.2) were compared to their respective initial profiles to determine if correlations existed between the locations and magnitudes of initial scour and
Figure 9.2: The initial, sub-experiment A and final longitudinal profiles for model results.
Figure 9.2 continued.
Figure 9.2 continued.

9.2g) Experiment 7

9.2h) Experiment 8

9.2i) Experiment 9
deposition and local surface concavities and convexities. This test is designed to determine if the mechanism by which scour and deposition was initiated in the model was similar to that observed in experiments 1 – 9. Second, spatial autoregression (AR(2)) was used to determine whether the modelled final longitudinal profiles exhibited quasi-periodic behaviour and, if so, whether they exhibited similar wavelengths.

Positive changes in bed height, or deposition zones, in the modelled sub-experiment A longitudinal profiles tend to occur where the smallest local gradients existed in the initial profiles (e.g. Figure 9.2a at 3.5 m), while the greatest amount of scour tends to occur on the steepest gradients (e.g. Figure 9.2a at 1 m). Furthermore, there is a general tendency for the model to smooth out topographic high-points and fill in topographic low-points. These observations suggest that there is a direct relationship between positive changes in bed height and increasing convexity similar to measured results. The null hypothesis is that there is no correlation between local convexities and concavities in the initial surface and the modelled changes in bed height.

Histograms and coefficients of kurtosis and skewness indicated that the modelled changes in bed height (i.e. $\frac{\partial z}{\partial t}$) and rates of change in slope (i.e. $\frac{\partial^2 z}{\partial x^2}$) were normally distributed. Pearson correlation coefficients were used to determine if correlations existed between local convexities and concavities in the initial surface and the modelled changes in bed height. The correlation coefficients and their respective t statistics were used to test the null hypothesis at the 0.05 significance level (Table 9.1). Only locations between the weir and 7.0 m were used in the analysis to offset the influence of the header-tank.

The null hypothesis was rejected for modelled results of experiments 1 – 7 in
Table 9.1: Pearson correlation coefficients and corresponding t statistics for the relationship between initial local convexities and concavities and the modelled change in bed elevation at the end of sub-experiment A (* denotes significant at 0.05 significance level).

<table>
<thead>
<tr>
<th>Experiment #</th>
<th>Sequential Scouring</th>
<th>No alternating patterns of scour and deposition.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r</td>
<td>t</td>
</tr>
<tr>
<td>1</td>
<td>0.527</td>
<td>3.222*</td>
</tr>
<tr>
<td>2</td>
<td>0.773</td>
<td>6.331*</td>
</tr>
<tr>
<td>3</td>
<td>0.932</td>
<td>13.361*</td>
</tr>
<tr>
<td>4</td>
<td>0.705</td>
<td>5.165*</td>
</tr>
<tr>
<td>5</td>
<td>0.671</td>
<td>4.702*</td>
</tr>
<tr>
<td>6</td>
<td>0.431</td>
<td>2.482*</td>
</tr>
</tbody>
</table>

agreement with the analyses on the measured results (See Table 7.1). Furthermore, the null hypothesis for the measured and modelled results was rejected for all sequential scouring experiments and not rejected for most experiments in which sequential scouring did not occur. This suggests that the model adequately describes the influence of initial microtopography on the initiation of scour and deposition as found in experiments 1 – 9.

Spatial autoregression was used to determine if the final modelled longitudinal profiles of experiments 1 – 9 exhibited quasi-periodic behaviour. Table 9.2 shows that the modelled longitudinal profiles of experiments 1 and 3 – 9 exhibited quasi-periodic behaviour. Seven of the 10 comparisons in Table 9.2 agree with regard to exhibiting quasi-periodic behaviour. This suggests that the model is capable of simulating the quasi-periodic behaviour observed in the experiments although there were large differences in
Table 9.2: Wavelengths associated with the measured and modelled longitudinal profiles.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Measured longitudinal profiles</th>
<th>Modelled longitudinal profile</th>
<th>Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1</td>
<td>3.8</td>
<td>+45% difference</td>
</tr>
<tr>
<td>2</td>
<td>—</td>
<td>—</td>
<td>yes</td>
</tr>
<tr>
<td>3</td>
<td>5.6</td>
<td>4.1</td>
<td>-27% difference</td>
</tr>
<tr>
<td>4</td>
<td>4.0</td>
<td>3.7</td>
<td>-7.5% difference</td>
</tr>
<tr>
<td>5</td>
<td>—</td>
<td>3.8</td>
<td>no</td>
</tr>
<tr>
<td>6</td>
<td>1.8</td>
<td>4.1</td>
<td>56% difference</td>
</tr>
<tr>
<td>7</td>
<td>4.3</td>
<td>1.6</td>
<td>-59% difference</td>
</tr>
<tr>
<td>8</td>
<td>2.2</td>
<td>4.6</td>
<td>+52% difference</td>
</tr>
<tr>
<td>9</td>
<td>—</td>
<td>2.9</td>
<td>no</td>
</tr>
</tbody>
</table>

the wavelengths exhibited by these profiles. However, it was expected that the model should have produced quasi-periodic behaviour for experiment 2 since this experiment was associated with sequential scouring, and the model should not have produced quasi-periodic behaviour for experiment 9.

9.3.2 Modeled Planimetric Morphologies:

The measured and modelled final planimetric morphologies are shown in Figure 9.3. The results from spatial autoregression (AR(2)) shows that the model produces variations in form width for all experiments (Table 9.3), but it only produces quasi-periodic variations for experiments 1 – 5 and 8. It was expected that the model should have produced quasi-
Figure 9.3: The modelled planimetric morphologies of experiments 1 – 9.

9.3a) Experiment 1C

9.3b) Experiment 2C

9.3c) Experiment 3C
Figure 9.3 continued.

9.3d) Experiment 4C

9.3e) Experiment 5C

9.3f) Experiment 6C
Figure 9.3 continued.
Table 9.3: Wavelengths associated with the measured and modelled planimetric morphologies.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Measured planimetric morphology (m)</th>
<th>Modelled planimetric morphology (m)</th>
<th>Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.3</td>
<td>1.0</td>
<td>-77% difference</td>
</tr>
<tr>
<td>2</td>
<td>—</td>
<td>1.3</td>
<td>no</td>
</tr>
<tr>
<td>3</td>
<td>6.5</td>
<td>1.2</td>
<td>-82% difference</td>
</tr>
<tr>
<td>4</td>
<td>9.9</td>
<td>1.0</td>
<td>-90% difference</td>
</tr>
<tr>
<td>5</td>
<td>—</td>
<td>1.4</td>
<td>no</td>
</tr>
<tr>
<td>6</td>
<td>—</td>
<td>—</td>
<td>no</td>
</tr>
<tr>
<td>7</td>
<td>—</td>
<td>—</td>
<td>no</td>
</tr>
<tr>
<td>8</td>
<td>—</td>
<td>1.1</td>
<td>no</td>
</tr>
<tr>
<td>9</td>
<td>—</td>
<td>—</td>
<td>no</td>
</tr>
</tbody>
</table>

periodic behaviour for experiment 6 and not for experiment 8. There is no general agreement between the measured and modelled wavelengths in Table 9.3. However, the model does simulate quasi-periodic behaviour for 5 of the 6 sequential scouring experiments and produces no quasi-periodic behaviour for 2 of the 3 non-sequential scouring experiments.

The planimetric morphologies exhibit the positive feedback mechanisms associated with sequential scouring. For example, Figure 9.4 depicts two modelled planimetric morphologies during experiment 2. Between 30 and 45 minutes, the head of the deposition zone between 0 and 0.5 m migrated 25 cm upslope (i.e. negative feedback) and the
Figure 9.4: Two modelled planimetric morphologies during experiment 2 showing the positive feedback mechanisms associated with sequential scouring.

Deposition zone near 2.5 m migrated downslope 50 cm. Furthermore, the increased deposition between 2 and 2.5 m enhanced scouring downslope between 0.75 and 1.75 m. These examples show that the model is capable of reproducing both the positive feedback mechanisms associated with scour and deposition.

9.3.3 Flow Width and Depth:

Although the model simulates the general shapes of the longitudinal profiles, the measured and modelled cumulative changes in bed height differ by a factor of roughly 2.5 (See Appendix C). The model underestimates the positive and negative changes in bed height.
producing narrower deposition zones and wider scour chutes. In this case, the sediment fluxes associated with deposition and scour zones would increase and decrease, respectively. The smaller, modelled sediment fluxes of the scour chutes would lower the amount of sediment transported to deposition zones. In addition, the greater, modelled deposition zone sediment fluxes would increase the amount of sediment transported by deposition zones. This would cause less net removal from scour chutes and less net deposition in deposition zones and account for the underestimation of bed height changes. Although the model produces lower overall sediment removal, the higher transport capacities of the deposition zones allows more sediment to be transported to the weir.

Table 9.4 gives the ranges and averages velocities, widths and depths for both measured and modelled scour and deposition zones. The measured and modelled flow properties for the scour chutes are very similar with the exception of the range in scour chute widths. The model is incapable of reproducing the smallest measured scour chute flow widths. Furthermore, the model underestimates the averages and ranges of flow velocity and width for deposition zones. This shows that the model overestimates the smallest scour chute flow widths and substantially underestimates deposition zone flow widths. Thus, the model produces wider scour chutes and narrower deposition zones.

The measured and modelled weir sediment and water discharges were compared for experiment 1 (Figure 9.5). Although there is a 20 minute difference in the time for runoff to reach the weir, the model generally produces larger magnitude weir water and sediment discharges for time increments prior to reaching a runoff equilibrium. Thus, more sediment was transported to the weir and less remained on slope. In reality, although there
Table 9.4: Averages and ranges of measured and modelled form width, flow velocity and flow depth for experiment 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Averages</th>
<th>Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Modelled</td>
</tr>
<tr>
<td>Velocity (cm s⁻¹)</td>
<td>24.6</td>
<td>26.6</td>
</tr>
<tr>
<td>Depth (cm)</td>
<td>0.31</td>
<td>0.30</td>
</tr>
<tr>
<td>Width (cm)</td>
<td>7.4</td>
<td>26.5</td>
</tr>
</tbody>
</table>

Figure 9.5: Comparisons between measured and modelled weir sediment and water discharge for experiment 1.
were larger negative changes in bed height in scour chutes, much of the sediment was redeposited on slope in the deposition zones and less sediment was transported to the weir. Deviations between modelled and measured results can be explained by the problems encountered in modelling flow width. Modelled flow widths produce greater and smaller sediment fluxes in deposition zones and scour chutes, respectively. Thus, less sediment was removed from scour chutes and deposited in deposition zones. Future research should attempt to refine the method of computing flow width. A simple approach could be to empirically relate changes in flow width to changes in bed height. More complex methods could use downstream hydraulic geometry relationships defining flow width as a function of flow rate, resistance to flow, particle mobility and secondary flow (Julien and Waargadalam, 1995).

9.5 Summary of Model Performance:

The model produces greater flow widths, smaller flow depths and smaller velocities for deposition in comparison to scour chutes in agreement with the experimental observations (Table 9.4). Furthermore, the model results show that deposition zones have smaller local gradients and greater surface roughnesses than scour chutes. The differences in width and slope produce greater and smaller sediment transport capacities in scour chutes and deposition zones, respectively. This alters the downslope sediment and water fluxes and produces the positive feedback mechanisms observed between scour and deposition zones.

Table 9.5 is a summary of the similarities and differences found between the
measured and modelled results. Comparisons between the measured and modelled longitudinal profiles indicate that 7 of the 9 simulations agree with respect to quasi-periodic behaviour, while 5 of the 9 comparisons between the measured and modelled planimetric morphologies agree. Overall, these results suggest that the model produced sequential scouring in the model runs of experiments 1, 3, 4, 5 and 8 as indicated by both oscillations in the longitudinal profiles and planimetric morphologies. Thus, since sequential scouring was observed in experiments 1 – 6 and not observed in experiments 7 – 9, the model reproduced sequential scouring correctly in 75% of the cases and reproduced non-sequential scouring correctly in 67% of the cases.

Table 9.5: Overall similarities and differences between measured and modelled results.

<table>
<thead>
<tr>
<th>Experiment #</th>
<th>Quasi-periodic behaviour in the longitudinal profile</th>
<th>Quasi-periodic behaviour in form width</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Modelled</td>
</tr>
<tr>
<td>1</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>2</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>3</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>4</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>5</td>
<td>no*</td>
<td>yes*</td>
</tr>
<tr>
<td>6</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>7</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>8</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>9</td>
<td>no*</td>
<td>yes*</td>
</tr>
</tbody>
</table>

* denotes difference
Both the measured and modelled results show that the initiation of scour and deposition was influenced by local convexities and concavities in the initial surface. Furthermore, both the modelled and measured results showed that initial microtopography played a stronger role in the sequential scouring experiments in agreement with experimental observations.

Although the model underestimates both positive and negative changes in bed height, it simulates the general shape of the longitudinal profiles reasonably well for all experiments (See Appendix C). Furthermore, the model simulates the interactions between scour and deposition and the positive feedback mechanisms observed between scour chutes and deposition zones.

Combining all aspects of model validation, the model proposed for sequential scouring performs reasonably well for its intended purpose. Future research should attempt to redefine the relationships between changes in bed height and its effects on flow width, and those between bed materials and surface roughness. These two relationships were found to have strong impacts on the initiation and development of sequential scouring.
10.0 Conclusions:

The combined effects of erosion and deposition commonly produce distinctive spatial patterns of topography, soil type and vegetation. Alternating patterns of erosion and deposition frequently develop in arid and semi-arid regions on gentle hillslopes, while scour and fill processes in natural alluvial rivers commonly produce riffle-pool sequences. A smaller scale sequence of erosion and deposition was investigated by this study.

10.1 Initiation of Sequential Scouring:

The initiation of scour chutes and deposition zones was shown to be influenced by local microtopographic variations in the initial longitudinal profile which created local surface concavities and convexities. Scour and deposition, or negative and positive changes in bed height, were found to be correlated with initial concavities and convexities, respectively. The influence of microtopography played a strong role only in experiments which produced sequential scouring. These experiments were also found to be operating under transport-limited conditions. Experiments in which sequential scouring did not occur were associated with detachment-limited conditions and no relationships were found between initial microtopography and the location of scour and deposition. This suggests that the influence of initial microtopography on the initiation of scour and deposition is only important under transport-limited conditions.

Results suggest that under transport-limited conditions, initial microtopographic
convexities and concavities in the surface alter local transport capacities and dominate the initiation of scour and deposition. Under detachment-limited conditions, scouring is controlled by soil entrainment resistance and initial scour is not necessarily controlled by local gradients. Although deposition may be controlled by local decreases in gradient, the amount of sediment available for deposition is limited. Thus, under detachment-limited conditions, initial scour and deposition were not well correlated with microtopography.

10.2 Longitudinal Sediment Sorting:

Longitudinal sediment sorting associated with sequential scouring appeared to be a critical component for enhancing scour and deposition. Deposition zones, with lower slope gradients, were dominated by coarser particle sizes. Scour chutes, with greater slope gradients, were characterized by a relative lack of aggregates. The rougher surfaces in the deposition zones reduced flow velocities and increased flow divergence, while scour chutes, with smoother surfaces, promoted flow convergence and greater velocities. Thus, the longitudinal sediment sorting associated with scour and deposition accentuated further deposition in the deposition zones and enhanced further erosion in scour chutes.

An inverse relationship was found between slope and particle size. This finding is in contrast to many studies that report direct relationships between grain-size and gradient. The large proportion of aggregates in the soil mixture may explain this apparent contradiction. Soil aggregates, with lower densities, are more easily mobilised by surface wash than solid particles of comparable size. Aggregates were probably preferentially
deposited on the convex portion of depressions (i.e. lower gradients) since convexities cause flow divergence and thinning. Local shear stresses were decreased and aggregates were no longer transported. Surface convexities contributed to sediment sorting and the formation of isolated deposition zones. Through continued scouring, more aggregates were progressively deposited in deposition zones, local gradients decreased further and an inverse relationship developed between particle size and slope.

The longitudinal sediment sorting observed with sequential scouring may reflect entrainment, transport or deposition selectivity. The selectivity and distribution of processes on hillslopes can produce a heterogeneous distribution of bed material from homogeneous initial conditions. The coarser, lower density aggregates in this study were the first to be mobilised and redeposited short distances downslope. Initially, flow conditions favoured entrainment and transport of aggregates. As the supply of loose surface aggregates declined, surface wash progressively favoured movement of finer fractions. It appeared that the flow was capable of transporting most fines through the deposition zones. Thus, aggregates were transported shorter distances than finer particles.

10.3 Interactions Between Scour and Deposition:

The results of this research concur with the few other studies which have focussed on the interaction between scour and deposition. It was found that, scour chutes had narrower and deeper flows, greater velocities, smoother surfaces and steeper slope gradients than deposition zones. These conditions increased the unit stream powers and water and
sediment fluxes in scour chutes. Deposition zones had lower unit stream powers and water and sediment fluxes. Thus, more sediment was transported from scour chutes to deposition zones than could be transported by through deposition zones, enhancing further deposition. As less sediment exited deposition zones, scouring immediately downslope was enhanced. The differences between scour and deposition zones set up positive feedback mechanisms where upslope scour enhanced downslope deposition and upslope deposition enhanced downslope scouring. These feedbacks also caused scour chutes to recede headward and excavate materials from upslope deposition zones. Deposition at the head of deposition zones and sediment removal on the leeside caused some deposition zones to migrate upslope.

10.4 Spatial and Temporal Variability of Sequential Scouring:

Sequential scouring was spatially and temporally variable, leading to differences in the final longitudinal profiles and planimetric morphologies of experiments 1 – 6 despite similar initial conditions. Variability of sequential scouring was attributed to: 1) random variations in initial microtopography; 2) differences in soil moisture between experiments; and 3) the transient nature of the flow properties and their effects on the water and sediment fluxes. Random variations in microtopography caused the locations of initial scour and deposition to vary between experiments. Antecedent soil moisture differences affected soil erodibility and the amount of runoff prior to soil saturation. Microtopography and soil moisture affected both the amount of sediment transported and local transport
capacities altering the rates of scour and deposition. The transience of the flow properties caused many locations to experience both scour and deposition. This caused the re-excavation of deposition zones and sediment infilling of scour chutes.

The variability of sequential scouring shows that the evolution of longitudinal profiles is complex and reacts sensitively to initial conditions, to interactions between dominant processes, to changes in surface material characteristics and to downslope variations in water and sediment fluxes. It has frequently been assumed that characteristic hillslope profiles which reflect dominant processes develop independent of initial slope form (Kirkby, 1971). Although all experiments which produced sequential scouring were associated with similar processes and initial conditions, an equilibrium form did not develop. Furthermore, the final profile was influenced by the initial slope profile.

10.5 Wavelengths of Sequential Scouring:

Sequential scouring exhibited wavelengths ranging between 2 and 6 m. Since these wavelengths developed under similar stream power and bed material conditions, the wavelengths are believed to reflect the slope, discharge and dominant grain-size conditions used in this study. Smaller particles move more frequently and travel further than larger particles with mean travel distances increasing with increasing discharge. If the wavelengths of sequential scouring are dependent on slope, discharge and grain-size, it follows that all experiments should have produced sequential scouring. Kirkby (1991) argued that shorter mean travel distances occur under transport-limited conditions and
longer ones under detachment-limited conditions. This may explain the lack of sequential scouring in experiments 7 – 10. It is possible that the flume length was shorter than the mean travel distance.

10.6 Optimal Conditions for Sequential Scouring:

Sequential scouring occurred in the laboratory environments with relatively low stream powers between ~0.15 and 0.25 W m\(^2\) and moderate sediment discharges under transport-limited conditions. Since these conditions apply to artificial runoff experiments, the development of sequential scouring under rainfall conditions needs to be addressed.

For example, medium and coarse particles, in the silt-sand range, are more vulnerable to rainsplash detachment. Finer grain-sizes tend to travel further with travel distances dependent upon slope and rainsplash kinetic energy. Rainsplash is a selective process which can produce natural variations in surface soil texture. These textural variations can lead to differences in surface roughness and cause flow divergence and convergence, which is critical in the development of sequential scouring.

10.7 Mathematical Model for Sequential Scouring:

A simple, one dimensional mathematical model was developed for simulating sequential scouring. Observations were used to develop a model to predict the longitudinal profile and planimetric morphologies of sequential scouring. The model adequately simulated:
1) the processes observed with the development of sequential scouring and the influence of microtopography; and

2) the positive feedbacks associated with sequential scouring.

The model produced sequential scouring under conditions similar to those used in the laboratory when sequential scouring occurred.

The model simulates the feedbacks between slope form, surface characteristics and active processes, and their effects on the water and sediment fluxes. Alterations in any one of these components can be used to determine their effects on sediment redistribution and the overall evolution of sequential scouring. Thus, the model can be used as an explanatory tool to shed light on the interactions between form and process.

10.8 Implications of Sequential Scouring:

The dominant processes associated with sequential scouring and its morphology share some similarities to other natural forms which behave periodically such as riffle-pool sequences, scour steps and cyclic rills. The development of sequential scouring reflects:

1) considerable variations in the properties of surface material caused by longitudinal sediment sorting;

2) strong feedback mechanisms between flow properties and slope form; and

3) alternations in downslope water and sediment fluxes.

This study has made several important contributions to soil erosion research. First, it provided insight to the relationship between initial microtopography and the initiation of
scour and deposition. Second, the selective nature of entrainment, transport and deposition processes was investigated. Third, this study showed that dominant processes should not be analysed in isolation. Rather, studies concerning general slope development must focus on the interactions between the dominant processes. Fourth, under assumptions of uniform initial conditions, sequential scouring was associated with significant spatial and temporal variability. This demonstrated the potential errors associated with assumptions of uniformity. Lastly, a mathematical model was developed and shown to adequately simulate sequential scouring.

Investigation of a wider range of slope angles and lengths, discharges and grain-size distributions is an important topic for future research. This could be used to further define: 1) optimal conditions under which sequential scouring occurs; 2) relationships between longitudinal sediment sorting and the wavelengths exhibited by sequential scouring; and 3) feedbacks between slope form and surface characteristics with their effects on sediment transport.
11.0 References:


Zingg, A.W., (1940), Degree and length of land slope as it affects soil loss in runoff. Agricultural Engineering, vol. 21, no. 2, 59-64.
APPENDIX A:
The Planimetric Morphologies of Experiments 1 – 12
Experiment 1:

Sub-Experiment 1A

Sub-Experiment 1B

Sub-Experiment 1C
Experiment 2:

Sub-Experiment 2A

Sub-Experiment 2B

Sub-Experiment 2C
Experiment 3:

Sub-Experiment 3A

Sub-Experiment 3B

Sub-Experiment 3C
Experiment 4:

**Sub-Experiment 4A**

![Graph showing form width (cm) vs. distance from weir (m)]

**Sub-Experiment 4B**

![Graph showing form width (cm) vs. distance from weir (m)]

**Sub-Experiment 4C**

![Graph showing form width (cm) vs. distance from weir (m)]
Experiment 5:

Sub-Experiment 5A

Sub-Experiment 5B

Sub-Experiment 5C
Experiment 6:

Sub-Experiment 6A

Sub-Experiment 6B

Sub-Experiment 6C
Experiment 7:

Sub-Experiment 7A

Sub-Experiment 7B

Sub-Experiment 7C
Experiment 8:

Sub-Experiment 8A

Sub-Experiment 8B
Experiment 9:

Sub-Experiment 9A

Sub-Experiment 9B

Sub-Experiment 9C
Experiment 10:

Sub-Experiment 10A

Sub-Experiment 10B

Experiment 11:
Experiment 12:

Sub-Experiment 12A

Sub-Experiment 12B
APPENDIX B:

Mathematical Model for Sequential Scouring.
Hillslope Model

Partial requirements for Ph.D. Thesis

Soren Erik Brun

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This program is a dynamic one-dimensional model for simulating sequential scouring

Compile as Standard DOS with BGI graphics using Borland C++

#include <iostream.h>
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <conio.h>
#include <ctype.h>
#include <graphics.h>
#include <dos.h>
#include <new.h>
#include "bgipath.h"

void far initial_data(float &id, float &bd, float &mc, float &al, float &le, int &dur, int &seg, int &beta, int &th, float &seg_len);

void far get_initial_z(int seg);
void far flow_dimension.dep(int t, int x, int beta);
void far flow_dimension.ero(int t, int x, int beta, int th);
void far water_discharge(int t, int y, int x, float id, float mc, float bd, int seg, float seg_len);

void far Slope(int t, float al, int seg, float seg_len);
void far hillslope change(int t, float bd, int seg, float seg_len);
void far ff(int t, int x, int seg);
void far unit_discharge(int t, int seg);
void far save_file(void);
FILE &output_file(int &reply);
void far SwapArrays(int y, int t, int seg);
void far DynamicArrays(int size);
void far print_to_file(FILE *out, int t, int x, float al, int seg, float seg_len);
void far FreeArrays(void);
void far draw_screen(int x, int y, float len, int seg);
void far plot_width(int j, int x, int y, int seg);
void far pause(void);
void far plot_z(int x, int y, int seg);
void far startgraphmode(void);
void far animate(int j, int x, int y, float len, int dur, int seg);
char ExitRoutine(void);
float CalcWidth(float a, float b, float c, float d, float e);
void far clear_kb(void);
float huge *width[2]; // subscript is a portion of time dimension
float huge *depth[2]; // space dimension is dynamically allocated later
float huge *Qw[2]; // the rest of the time dimension is created
float huge *tan_B[2]; // virtually by swapping between arrays
float huge *z[3]; // before values are swapped they are saved to file
float huge *initial_z;
float huge *qw[2];
float huge *slope[2];
float huge *dz[3];
float huge *velocity[2];
double huge *friction[2];
double huge *reynolds[2];
float huge *F[15];
int huge *I[2];
main()
{
  FILE *output;
  int t = 0, TimeCounter = 0;
  float discharge, density, moisture, length, alpha, segment_length;
  int duration, segments, beta, theta, reply, horiz, vert;
  initial_data(discharge, density, moisture, alpha, length, duration,
               segments, beta, theta, segment_length);
  DynamicArrays((segments + 1)); // creates dynamic arrays
  get_initial_z(segments); // gets initial slope elevations
  output = &output_file(reply); // gets output file on prompt
  startgraphmode(); // initialize dos graphics
  horiz = getmaxx();
  vert = getmaxy(); // screen resolution
  draw_screen(horiz, vert, length, segments); // draws initial screen
  while (TimeCounter <= duration){ // runs model for "duration" minutes
    Slope(t, alpha, segments, segment_length); // calcs slope
    for (int x = segments; x >= 0; x--){
      water_discharge(t, TimeCounter, x, discharge, // Runoff SubModel
                      moisture, density, segments, segment_length);
      if (dz[t][x] <= 0) {
        ff(t, x, segments); // friction factor for erosion
        flow_dimension_ero(t, x, beta, theta); // Chan Hyd. SubMod.
      }
      else{
        ff(t, x, segments); // friction factor for deposition
        flow_dimension_dep(t, x, beta); // Chan Hyd. SubModel
      }
  }
unit_discharge(t, segments); //calc unit discharge
hillslope_change(t, density, segments, segment_length);
plot_width(TimeCounter, horiz, vert, segments); //plots width graphics
plot_z(horiz, vert, segments); //plots change in z graphics
animate(TimeCounter, horiz, vert, length, duration, segments);
if (reply == 1)
    print_to_file(output, TimeCounter, t, alpha, segments,
                     segment_length);
SwapArrays(TimeCounter, t, segments); //swaps dynamic arrays
t = 1;
TimeCounter++;}
};
pause();
closegraph();
cout << "
Have a nice day"
; FreeArrays(); //frees arrays space
fclose(output);
return 0;
/*-------------------------------------------*/
+ Obtains initial data and references back to main +
+-----------------------------------------------*/
void far initial_data(float &id, float &bd, float &mc, float &al, float &len,
                      int &dur, int &seg, int &beta, int &th, float &seg_len)
{
cout << "This model calculates hydraulic conditions and changes\n" << "in bed height for any number of evenly spaced nodes on a\n" << "slope for any length of time (< 32,768 minutes).\n" << "You are required to supply a few initial inputs\n" << "Enter the length of a slope segment, or the distance between\n" << "slope nodes, in metres.\n";
cin >> seg_len;
cout << "Enter slope length in metres. Note: the slope length must\n" << "be evenly divisible by segment length.\n";
cin >> len;
cout << "Enter the storm duration (Integer) in minutes\n" << "Maximum = 32,768 mins. (22.75 days).\n";
cin >> dur;
cout << "Enter the initial discharge (Integer) in cm3/min.\n";
cin >> id;
cout << "Enter the initial soil bulk density in g/cm3.\n";
cin >> bd;
cout << "Enter antecedent volumetric moisture content cm3/cm3.\n";
cin >> mc;
cout << "Enter side-slope angle (Integer):" << endl;
cin >> beta;
cout << "Enter overall hillslope angle:" << endl;
cin >> al;
cout << "Enter channel side-slope angle (Integer) between 71 and 89:" << endl;
cin >> th;
seg = len / seg_len; /* calculates segment #s */
puts("---------------------------------------------------------------");
cout << "Slope Length = " << len << " metres" << endl;
cout << "Storm Duration = " << dur << " minutes" << endl;
cout << "Number of Slope Segments (Iterations) = " << (seg + 1) << endl;
cout << "You will require the elevation of " << (seg + 1) << " nodes" << endl;
cout << "from 0 to " << len << " metres." << endl;
puts("---------------------------------------------------------------");
} /*---------------------------------------------*/

+ Creates dynamic array for slope location dimension
++
+ Swaparray function creates virtual dynamic array for time dimension
+++---------------------------------------------*/

void far DynamicArrays(int size){
    int j;
    initial_z = (float *)calloc(size, sizeof(float));
    for (j = 0; j < 2; j++){
        width[j] = (float *)calloc(size, sizeof(float));
        depth[j] = (float *)calloc(size, sizeof(float));
        Qw[j] = (float *)calloc(size, sizeof(float));
        tan_B[j] = (float *)calloc(size, sizeof(float));
        qw[j] = (float *)calloc(size, sizeof(float));
        slope[j] = (float *)calloc(size, sizeof(float));
        velocity[j] = (float *)calloc(size, sizeof(float));
        if[j] = (int *)calloc(size, sizeof(int));
        friction[j] = (double *)calloc(size, sizeof(double));
        reynolds[j] = (double *)calloc(size, sizeof(double));
            puts( "n1 Out of Memory, Try a shorter slope length" );
            exit(1); //If no room for arrays, exit model
        }
    }
    for (j = 0; j < 3; j++){
        z[j] = (float *)calloc(size, sizeof(float));
        dz[j] = (float *)calloc(size, sizeof(float));
    }
if(!z[j] && !dz[j]){
    puts("n2 Out of Memory, Try a shorter slope length");
    exit(1);
}
for (j = 0; j < 15; j++){
    F[j] = (float *)calloc(size, sizeof(float));
    if(!F[j]){
        puts("n3 Out of Memory, Try a shorter slope length");
        exit(1);
    }
}
/*---------------------------------------------------------*/
+ Gets initial z elevations for hillslope positions +
+---------------------------------------------------------*/
void far get_initial_z(int seg){
    FILE *fp;
    char filename[40];
    int answer; /* conditions for switch cases */
    cout << "The next item to enter is z, or initial surface elevation, \n"
        << "of successive slope nodes starting at the weir. \n"
        << "There are two ways to input this data: \n"
        << "1) you can enter them from a file; or \n"
        << "2) you can enter them manually\n"
        << "Which would you like? \n"
        << "Enter 1 to open a file. Enter 2 for keyboard input. \n";
    scanf("%d", &answer);
    clear_kb();
    switch (answer){
        case 1:{ // case 1 for file input
            cout << "Enter a filename (with extension): \n";
            gets( filename );
            if ( fp = fopen( filename, "r")) != NULL ){
                cout << "Successful opening: " << filename << endl;
                for (int x = 0; x <= seg; x++){
                    fscan(fp, "%f", &initial_z[x]);
                    z[0][x] = initial_z[x];
                }
            }
            fclose(fp);
            if (ExitRoutine() == 'X')
                break;
        }
else
    continue;
}
else{
    cout << "\nError opening file " << filename << ".\n"
    "You must have initial height values\n"
    "If you do not, please exit the program\n"
    "Type x for exit, <ENTER> to continue\n";
    char ch = toupper(getchar());
    if (ch == 'X')
        exit(1);
    else
        continue;
}
break;
}

//case 2 for prompt input
cout << "This routine allows you to input your z values manually.\n"
    "The number of values you need is determined by:\n"
    "Slope Length divided by the length of the slope segments.\n"
    "Currently, the length of slope segments is set to 25 cm\n"
    "For example, if you have a 5 metre slope length, then you\n"
    "will need (5/0.25) + 1 = 21 values"
    "where the first value is\n"
for (int x = 0; x < seg + 1; x++){
    cout << "\nLocation " << x << "\nEnter value (cm): ";
    cin >> initial_z[x];
    z[0][x] = initial_z[x];
}
break;
}
default:
    cout << "Invalid choice, try again.\n";
}
for (int x = seg; x >= 0; x--)
    dz[0][x] = 0; //initial change in thalweg is zero
}/*---------------------------------------------------------*/
+ Generic Routine to exit fileopen functions +
+---------------------------------------------------------*/
char ExitRoutine(void){
    char ch;
cout << "Enter x to exit fileopen routine"
    << ", any other to continue.\n";
cin >> ch;
return toupper(ch);
}

<@
+ Routine to open output file with a reply reference back to main +
+--------------------------------------------------------------------------*/

FILE &output_file(int &reply){ //reply used in main to enable file printing
    FILE *out;
    char filename[40];
    cout << "\nWould you like to save the output in a separate file?\n"
        << "This will save all variables for all locations through time\n"
        << "Enter 1 for 'yes' and 2 for 'no'. If you think you have\n"
        << "insufficient hard disk space choose 'no'\n";
cin >> reply;
switch ( reply ){
    case 1: {//case 1 for saving data to file
        while(1){
            cout << "Enter a filename (with extension)\n";
            gets( filename );
            if ( (out = fopen( filename, "w")) ! = NULL ){
                /*file header*/fprintf(out, "time, location, discharge, width, 
                depth, velocity, z, Reynolds, dz, slope, friction\n");
                cout << "Successful opening " << filename << endl;
                if ( ExitRoutine() == 'X')
                    break;
                else
                    continue;
            }
        }
    }
    else {//error routine for inability to write to file
        cout << "Error opening file " << filename << "\n";
        if (ExitRoutine() == 'X')
            break;
        else
            continue;
    }
}
break;
}

case 2: //case 2 for not saving data to file
break;
}

default://bad key input
cout << "Invalid choice, try again. \n";
}
return *out;
}

/**
 * Initializes BGI graphics mode
 */
void far startgraphmode(void){
  int gdriver = DETECT, gmode, gerr;
  initgraph(&gdriver, &gmode, BGIPATH);   // get driver and mode
  gerr = graphresult();
  if (gerr != grOk){
    cout << "BGI error: " << grapherrormsg(gerr) << endl;
    exit(gerr);
  }
}

/**
 * Partitions and draws initial screen
 */
void far draw_screen(int x, int y, float len, int seg){
  char s[5];
  int xscale = (x/seg);
  int yscale = (y / 20);
  setcolor(RD);  //screen partition
  rectangle(0, 0, x, y);
  line(0, y/2, x, y/2);
  setcolor(LIGHTGRAY);
  setlinestyle(DOTTED_LINE, 0, NORM_WIDTH);
  for (int i = 1; i <= 4; i++)//draws grid lines
    line((xscale * (seg / 4) * i), 1, (xscale * (seg / 4) * i), (y - 1));
  setlinestyle(SOLID_LINE, 0, NORM_WIDTH);
  for (i = 1; i <= seg; i++){
    if (((i % 2) == 0) //draws tick marks on axes
      line((xscale * i), 1, (xscale * i), 5);
    line((xscale * i), ((y / 2) - 2), (xscale * i), ((y / 2) + 2));
    line((xscale * i), (y - 1), (xscale * i), (y - 5));
  }
  settextjust(LT, CENT);
  for (i = 1; i < 10; i++)//draws y-axis scale for width partition
    sprintf(s, "%d", (100 - (i * 10)));
  outtextxy(6, (yscale * i) + 4, s);
setcolor(LIGHTGRAY);
line(1, (yscale * i) + 4, 4, (yscale * i) + 4);
}
setlinestyle(DOTTED_LINE, 0, NORM_WIDTH);
line(10, (0.75 * y), x, (0.75 * y));
setcolor(WHITE);//draws y-axis scale for changes in z partition
moveto(4, (0.625 * y));
outtext("+");
moveto(4, 0.75 * y);
outtext("0");
moveto(4, (0.875 * y));
outtext("-");
settextjustify(CENTER_TEXT, BOTTOM_TEXT);
for (i = 1; i <= 4; i++) //draws x-axis scale
  sprintf(s, "%.1f m", (i * (seg / (4 * len))));
  outtextxy((xscale * (seg / 4) * i), (y - 2), s);
}
moveto((x - 120), 15);
outtext("Time = "); //prints time of each iterations

/*+ Calculates local slope for each segment +*/
void far Slope(int t, float al, int seg, float seg_len){
  for (int x = 0; x <= seg; x++){
    if (x == seg) //local slope of topmost segment
      slope[t][x] = sin(0.01748 * al);
    else // ^--radians conversion
      //local slope for rest of segments
      if ((z[t][x + 1] - z[t][x]) >= 0)
        slope[t][x] = (z[t][x + 1] - z[t][x]) / (seg_len * 100);
      else
        slope[t][x] = 0;
  }
}

/*+ Calculates water discharge for each segment for each iterations +*/
void far water_discharge(int t, int y, int x, float id, float mc,
  float bd, int seg, float seg_len){
  float Io, b;
  Io = 5.564 - (7.925 * mc) - (2.447 * bd); //infiltration curve
if (Io <= 0)
    Io = 0.321; //lowest possible Io determined from infiltration columns
if (x == seg) {
    Qw[t][x] = id; //discharge entering topmost node is constant
    I[t][x] = y; //time counter to determine time since flow at cell
}
else {
    if (y == 0) { //initial loss of discharge from infiltration
        if (Qw[t][x + 1] > 0) {
            Qw[t][x] = Qw[t][x + 1] - (Io * (seg_len * 100) * width[t][x + 1]);
            if (Qw[t][x] < 0) Qw[t][x] = 0;
        }
        else {
            Qw[t][x] = 0;
            I[t][x] = 0; //set time counter
        }
    }
    else { //subsequent losses to infiltration
        b = -(0.099 * exp(0.504 * Io));
        if (Qw[t - 1][x] > 0) I[t][x] = I[t - 1][x] + 1; //time counter
        else I[t][x] = I[t - 1][x];
        if (Qw[t][x + 1] > 0 && width[t - 1][x] > 0) {
            Qw[t][x] = Qw[t][x + 1] - ((Io * exp(I[t][x] * b)) * (seg_len * 100) * width[t - 1][x]);
            if (Qw[t][x] < 0) Qw[t][x] = 0;
        }
        else if (Qw[t][x + 1] > 0 && width[t - 1][x] <= 0) {
            Qw[t][x] = Qw[t][x + 1] - ((Io * exp(I[t][x] * b)) * (seg_len * 100) * width[t][x + 1]);
            if (Qw[t][x] < 0) Qw[t][x] = 0;
        }
        else
            Qw[t][x] = 0;
    }
}

/*---------------------------------------------------------------+
   Darcy-Weisbach friction factor
   +---------------------------------------------------------------*/

void far ff(int t, int x, int seg) {
    if (x == seg)
        friction[t][x] = 0.85; //dummy value for first iteration
    else {

if (t > 0 && reynolds[t-1][x] > 0)
    friction[t][x] = pow(10,((-3.18*log10(reynolds[t-1][x]) + 10.27)));
else
    friction[t][x] = 0.2;
}

/*------------------------+                      +
 * Channel hydraulics sub-model for deposition cases
 +------------------------*/
void far flow_dimension_dep( int t, int x, int beta){
    float F1, F2, C1, C2, C3, C4;
    float g, q, side_slope;
    const float radians_conv = 0.01748;
    side_slope = tan( radians_conv * beta);
    g = 9.8;
    q = (Qw[t][x] / 60) * 0.000001;
    C1 = side_slope * 0.25;
    C2 = pow( (dz[t][x] / 100), 2) / side_slope;
    C3 = (2 * dz[t][x] / 100) * ((1 / side_slope)-(sqrt( 1 + (1 / pow(side_slope, 2)))));
    C4 = sqrt(pow(side_slope, 2) + 1);
    if (slope[t][x] <= 0) slope[t][x] = 0.001;
    F1 = (friction[t][x] * pow( q, 2)) / (8 * g * slope[t][x]);
    F2 = F1 * C3;
    if (Qw[t][x] > 0){
        width[t][x] = 2 * CalcWidth(F1, F2, C1, C2, C4) * 100;
        depth[t][x] = (0.5 * (width[t][x] / 2) * side_slope) - (dz[t][x]);
        velocity[t][x] = (Qw[t][x] / 60) / (0.5 * ((width[t][x] / 2)
            + (2 * dz[t][x] / side_slope)) * depth[t][x]);
        reynolds[t][x] = (4 * velocity[t][x] * depth[t][x]) / (1.146e-6 * 10000);
    }
    else{
        width[t][x] = 0;
        depth[t][x] = 0;
        velocity[t][x] = 0;
        reynolds[t][x] = 0;
    }
}

/*-----------------------------------+        +
 * Iterates and return width for erosion and deposition
 +-----------------------------------*/
float far CalcWidth(float a, float b, float c, float d, float e){
    float dummy = -0.001, w = 0.001;
    while (dummy < b){//while loop to converge at a width value
dumy = ((pow(w, 6) * pow(c, 3)) - (3 * pow(w, 4) * pow(c, 2) * d) + (3 * pow(w, 2) * c * pow(d, 2)) - pow(d, 3)) - (a * e * w);

w += 0.001;
}
return (w - 0.001);

/*-----------------------------------------------+
+ Channel Hydraulics SubModle for erosion cases +
+-----------------------------------------------*/

void far flow_dimension_ero(int t, int x, int beta, int th){
    float g, a, q, side_slope, C1, C2, C3, C4, F1, F2;
    float wb, A, B, C, channel_slope;
    const float radians_conv = 0.01748;
    channel_slope = tan(radians_conv * (90 - th));
    g = 9.8;
    side_slope = tan((radians_conv * beta));
    if(dz[t][x] == 0 || Qw[t][x] > 0){
        C1 = pow(side_slope, 3) / sqrt(1 + pow(side_slope, 2));
        q = (Qw[t][x] / 60) * 0.000001;
        if (t == 0) friction[t][x] = 0.85;
        a = (64 * friction[t][x] * q * q) / (8 * g * slope[t][x] * C1);
        width[t][x] = (pow(a, 0.2)) * 100;
        depth[t][x] = 0.5 * width[t][x] * side_slope;
        velocity[t][x] = (Qw[t][x] / (0.5 * width[t][x] * depth[t][x])) / 60;
        reynolds[t][x] = (4 * velocity[t][x] * depth[t][x]) / (1.146e-6 * 10000);
    }
    else if(Qw[t][x] > 0){
        A = 0.25;
        B = (-2 * fabs(dz[t][x]) / tan(radians_conv * th));
        C = -pow(dz[t][x], 2);
        wb = (-B + sqrt(pow(B, 2) - (4 * A * C))) / 0.5;
        C1 = 0.25 * channel_slope;
        C2 = 0.25 * pow((wb / 100), 2) * channel_slope;
        C3 = (wb / 100) * (1 - (sqrt(1 + (pow(channel_slope, 2)))))
        C4 = sqrt((pow(channel_slope, 2)) + 1);
        q = (Qw[t][x] / 60) * 0.000001;
        if (slope[t][x] <= 0) slope[t][x] = 0.001;
        F1 = (friction[t][x] * pow(q, 2)) / (8 * g * slope[t][x]);
        F2 = F1 * C3;
        if (Qw[t][x] > 0){
            width[t][x] = 2 * CalcWidth(F1, F2, C1, C2, C4) * 100;
            depth[t][x] = (0.25 * width[t][x] * side_slope) - dz[t][x];
            velocity[t][x] = (Qw[t][x] / 60) / (0.5 * ((0.5 * width[t][x])
            )});
        }
    }
+((0.5*width[t][x])+(2*depth[t][x]*channel_slope)))*depth[t][x]);
reynolds[t][x]=(4*velocity[t][x]*depth[t][x])/(1.146e-6*10000);
}

else{
  width[t][x] = 0;
  depth[t][x] = 0;
  velocity[t][x] = 0;
  reynolds[t][x] = 0;
}

/*---------------------------------------------+ 
+ Calculates water discharge per unit width. NOTE: calculates qw to the mth + 
+ power for the sediment transport Sub-Model (i.e. hillslope_change) + 
+--------------------------------------------------------------------*/

void far unit_discharge(int t, int seg){
  float m = 2.3453; //from sediment transport equation
  for (int x = 0; x <= seg; x++){
    if(Qw[t][x] > 0 && width[t][x] > 0)
      qw[t][x] = Qw[t][x] / width[t][x];
    else
      qw[t][x] = 0;
    if(qw[t][x] > 0) qw[t][x] = pow(qw[t][x], m);
    else
      qw[t][x] = 0;
  }
}

/*---------------------------------------------+ 
+ Sediment transport sub-model Crank-Nicolson approach using + 
+ Newton-Raphson Iteration and + 
+--------------------------------------------------------------------*/

void far hillslope_change( int t, float bd, int seg, float seg_len){
  int ck, count = 0;
  float a, b, c, k = 0.001, n = 1.72, m = 2.1453;
  c = - k / (bd * (2 * pow((seg_len * 100), (n + 1))));
  for (int x = 0; x <= seg; x++) z[t + 1][x] = z[t][x]; /*initial guess for z[t + 1][x]*/
  for (x = 0; x < seg; x++)
    F[count][x] = 0;
  count++;
  x = 0;
  ck = 0; // Convergence criteria, False, diff in F > 0.0001
do{
  if (t == 0) {
    for (x = 0; x <= seg; x++) {
      if (x == 0) {
        z[t + 1][x] = initial_z[x];
        dz[t + 1][x] = (z[t + 1][x] - initial_z[x]);
      } else if (x > 0 && x < seg) {
        if (width[t][x] > 0 && width[t][x + 1] > 0)
          a = (qw[t][x] * n * pow((-z[t][x] - z[t][x + 1]), n - 1)
               * (z[t][x + 1] + (2 * z[t][x]) - z[t][x - 1])
               * pow(Qw[t][x], m - 1))
               * (pow(width[t][x], -m))
               * pow((-z[t][x] - z[t][x + 1]), n))
          + (pow(Qw[t][x], m) * (-m * pow(width[t][x], -m - 1))
               * (pow(width[t][x], -2 * m) * (width[t][x + 1] - width[t][x]))
               * pow((-z[t][x] - z[t][x + 1]), n));
        b = (qw[t][x] * n * pow((-z[t + 1][x] - z[t + 1][x + 1]), n - 1)
             * (-z[t + 1][x + 1] + (2 * z[t + 1][x]) - z[t + 1][x - 1])
             + ((m * pow(Qw[t][x], m - 1))
                 * pow((-z[t + 1][x] - z[t + 1][x + 1]), n))
             + (pow(Qw[t][x], m) * (-m * pow(width[t][x], -m - 1))
                 * (pow(width[t][x], -2 * m) * (width[t][x + 1] - width[t][x]))
                 * pow((-z[t + 1][x] - z[t + 1][x + 1]), n));
      } else {
        a = (qw[t][x] * n * pow((-z[t][x] - z[t][x + 1]), n - 1)
             * (-z[t][x + 1] + (2 * z[t][x]) - z[t][x - 1])
             - qw[t][x])
             * pow((-z[t][x] - z[t][x + 1]), n));
        b = (qw[t][x] * n * pow((-z[t + 1][x] - z[t + 1][x + 1]), n - 1)
             * (-z[t + 1][x + 1] + (2 * z[t + 1][x]) - z[t + 1][x - 1])
             + ((qw[t][x + 1] - qw[t][x]) * pow((-z[t + 1][x]
                 - z[t + 1][x + 1]), n));
      }
      z[t + 1][x] = z[t][x] + (c * (a + b));
      dz[t + 1][x] = (z[t + 1][x] - initial_z[x]);
    }
  } else {
    z[t + 1][x] = z[t][x] + ((2 * (z[t + 1][x] - z[t][x - 1])
                               * width[t][x - 1]) / (2 * width[t][x]));
    dz[t + 1][x] = (z[t + 1][x] - initial_z[x]);
  }
}
count++;
for (x = 0; x < seg; x++)
    \[F[count][x] = \text{fabs}(z[t+1][x] - z[t][x])\];
else{
    for (x = 0; x <= seg; x++)
        if (x == 0)
            \[z[t+1][x] = \text{initial}_z[x] \]
        \[dz[t+1][x] = (z[t+1][x] - \text{initial}_z[x])\];
    else if (x > 0 && x < seg){
        if (width[t][x] > 0 && width[t-1][x] > 0 && width[t][x + 1] > 0 &&
            width[t-1][x+1] > 0 && Qw[t][x] > 0 && Qw[t - 1][x] > 0
        && Qw[t][x + 1] && Qw[t - 1][x + 1] > 0){
            a = (qw[t - 1][x] * n * pow((-z[t][x] - z[t][x + 1]), n - 1))
            * (-z[t][x + 1] + (2 * z[t][x]) - z[t][x - 1]))
            + ((m * pow(Qw[t][x], m - 1)) * (Qw[t - 1][x + 1] - Qw[t - 1][x])
            * (pow(width[t - 1][x], m)) * pow((-z[t][x] - z[t][x + 1]), n))
            + (pow(Qw[t - 1][x], m) * (-m * pow(width[t-1][x], -m - 1)))
            * (pow(width[t-1][x], -2 * m)) * (width[t-1][x+1] - width[t-1][x])
            * pow((-z[t][x] - z[t][x + 1]), n));
        b = (qw[t][x] * n * pow((-z[t+1][x] - z[t + 1][x + 1]), n - 1))
            * (-z[t+1][x+1] + (2 * z[t+1][x]) - z[t+1][x - 1]))
            + ((m * pow(Qw[t][x], m - 1)) * (Qw[t][x] + 1) - Qw[t][x])
            * (pow(width[t][x], m)) * pow((-z[t+1][x] - z[t + 1][x + 1]), n))
            + (pow(Qw[t][x], m) * (-m * pow(width[t][x], -m - 1)))
            * (pow(width[t][x], -2 * m)) * (width[t][x+1] - width[t][x])
            * pow((-z[t+1][x] - z[t + 1][x + 1]), n));
    else{
        a = (qw[t-1][x] * n * pow((-z[t][x] - z[t][x + 1]), n - 1))
            * (-z[t][x + 1] + (2 * z[t][x]) - z[t][x - 1]))
            + ((qw[t - 1][x] + 1) - qw[t - 1][x]) * pow((-z[t][x]
            - z[t][x + 1]), n));
        b = (qw[t][x] * n * pow((-z[t+1][x] - z[t + 1][x + 1]), n - 1))
            * (-z[t+1][x + 1] + (2 * z[t+1][x]) - z[t+1][x - 1]))
            + ((qw[t][x] + 1) - qw[t][x]) * pow((-z[t+1][x] - z[t + 1][x + 1]), n));
    }
    \[z[t+1][x] = z[t][x] + (c * (a + b))\]
    \[dz[t+1][x] = (z[t+1][x] - \text{initial}_z[x])\];
else{
}
\[ z[t+1][x] = z[t][x] + \frac{(2*(z[t+1][x] - z[t][x]))* \text{width}[t][x - 1])}{(2 * \text{width}[t][x])}; \]

\[ dz[t+1][x] = (z[t+1][x] - \text{initial}_z[x]); \]

```
count++;  
for (x = 0; x < seg; x++)  
    F[count][x] = fabs(z[t+1][x] - z[t][x]);
}
x = 1;  /*baselevel never changes don't need to check it*/
ck = 1;  /*initialize before the loop*/
while(ck && x < seg)  /*turns Crank-Nicolson Loop On or Off*/
    if (fabs(F[count][x] - F[count - 1][x]) < 0.0001)  
        ck = 1  /*True, no diff in F at this node, check next node*/;
    else  
        ck = 0;  /*diff F >= 0.001 at this node do another iteration*/
        x++;
}
while (!ck && count < 15)  /*end loop if all node yield ck = 1*/
    if (!ck)  /*If any one of the nodes doesn't convergence end simulation*/
        cout << "No Convergence\n";
        exit(1);
}
/*--------------------------------------------------------------------------------*/
+ Draws width per time increment on screen  +
+--------------------------------------------------------------------------------*/
void far plot_width(int j, int x, int y, int seg){
    int yscale = (y/2)/100;
    int xscale = (x/seg);
    char s[10];
    setcolor(CYAN);
    setlinestyle(SOLID_LINE, 0, NORM_WIDTH);
    for (int i = 0; i < seg; i++){
        line(((xscale * i) + 1), ((y/4) - (width[0][i] * yscale)),
            (xscale * (i + 1)), ((y/4) - (width[0][i + 1] * yscale)));
        line(((xscale * i) + 1), ((y/4) + (width[0][i] * yscale)),
            (xscale * (i + 1)), ((y/4) + (width[0][i + 1] * yscale)));
    }
    setcolor(WHITE);
    sprintf(s, "%d", j);
    moveto(x - 80, 15);
    outtext(s);
```
void far plot_z(int x, int y, int seg) {
    int yscale = 100;
    int xscale = (x/seg);
    setcolor(BROWN);
    setlinestyle(SOLID_LINE, 0, NORM_WIDTH);
    for (int i = 0; i < seg; i++) {
        line(((xscale * i) + 1), ((0.75 * y) - (dz[1][i] * yscale)),
             (xscale * (i + 1)), ((0.75 * y) - (dz[1][i + 1] * yscale)));
    }
    delay(30);
}

void far animate(int j, int x, int y, float len, int dur, int seg) {
    int yscaledz = 100;
    int xscaledz = (x/seg);
    int yscalew = (y/2)/100;
    int xscalew = (x/seg);
    char s[10];
    setcolor(BLACK);
    if (j < dur) {
        for (int i = 0; i < seg; i++) {
            line(((xscaledz * i) + 1), ((y/4) - (width[0][i] * yscalew)),
                 (xscaledz * (i + 1)), ((y/4) - (width[0][i + 1] * yscalew)));
            line(((xscalew * i) + 1), ((y/4) + (width[0][i] * yscalew)),
                 (xscaledz * (i + 1)), ((y/4) + (width[0][i + 1] * yscalew)));
        }
        sprintf(s, "%d", j);
        moveto(x - 80, 15);
        outtext(s);
        for (i = 0; i < seg; i++) {
            line(((xscaledz * i) + 1), ((0.75 * y) - (dz[1][i] * yscaledz)),
                 (xscaledz * (i + 1)), ((0.75 * y) - (dz[1][i + 1] * yscaledz)));
        }
        draw_screen(x, y, len, seg);
    }
}
void far print_to_file(FILE *out, int y, int t, float al, int seg, float seg_len) {
    for (int x = 0; x < seg + 1; x++) {
        fprintf(out, "%d,%.2f,%.2f,%.2f,%.2f,%.2f,%.2f,%.4f,%.2f\n",
                y, (x * 0.25), Qw[t][x], width[t][x], depth[t][x], velocity[t][x],
                (initial_z[x] + dz[t][x]) - (tan(0.01748 * al) * (x * (seg_len*100))),
                reynolds[t][x], dz[t][x], slope[t][x], friction[t][x]);
    }
} /*---------------------------------------------------------------------------------------
+ Swaps arrays effectively creating a virtual time dimension for arrays
+---------------------------------------------------------------------------------------*/

void far SwapArrays(int y, int t, int seg) {
    if (y > 1) {//writes arrays to file before swapping(see main)
        for(int x = 0; x <= seg; x++) {//makes t - 1 = t
            width[t - 1][x] = width[t][x];
            depth[t - 1][x] = depth[t][x];
            Qw[t - 1][x] = Qw[t][x];
            tan_B[t - 1][x] = tan_B[t][x];
            z[t][x] = z[t + 1][x];
            qw[t - 1][x] = qw[t][x];
            slope[t - 1][x] = slope[t][x];
            dz[t][x] = dz[t + 1][x];
            velocity[t - 1][x] = velocity[t][x];
            I[t - 1][x] = I[t][x];
            friction[t - 1][x] = friction[t][x];
            reynolds[t - 1][x] = reynolds[t][x];
        }
    }
} /*---------------------------------------------------------------------------------------
+ pauses screen to give viewer chance to see output
+---------------------------------------------------------------------------------------*/

void far pause(void) {
    cout << "Press <Spacebar> to continue";
    while (getch() != ' ');
} /*---------------------------------------------------------------------------------------
+ frees space created for the dynamic arrays
+---------------------------------------------------------------------------------------*/

void far FreeArrays(void) {
    int j;
    free(initial_z);
}
for (j = 0; j < 2; j++){
    free(width[j]);
    free(depth[j]);
    free(Qw[j]);
    free(tan_B[j]);
    free(qw[j]);
    free(slope[j]);
    free(velocity[j]);
    free(I[j]);
    free(friction[j]);
    free(reynolds[j]);
}
for (j = 0; j < 3; j++){
    free(z[j]);
    free(dz[j]);
}
for (j = 0; j < 15; j++)
    free(F[j]);

/*-----------------------------------------------------------------------------------------+
+    Clears keyboard buffer
+-----------------------------------------------------------------------------------------*/

void far clear_kb(void){
    char junk[80];
    gets(junk);
}
Appendix C:

Plots of measured and modelled bed height and changes in bed height for experiments 1 – 9.
Experiment 1:

- Modelling changes in bed height (cm)
- Measured changes in bed height (cm)

---

- Modelled bed height (cm) -- Measured bed height (cm)
Experiment 2:

Experiment 2A

Experiment 2B

Experiment 2C

- Modelled changes in bed height (cm)
- Measured changes in bed height (cm)

- Modelled bed height (cm) — Measured bed height (cm)
Experiment 3:

Experiment 3A

Experiment 3B

Experiment 3C
Experiment 4:

---

**Experiment 4A**

- Modelled changes in bed height (cm)
- Measured changes in bed height (cm)

---

**Experiment 4B**

- Modelled changes in bed height (cm)
- Measured changes in bed height (cm)

---

**Experiment 4C**

- Modelled changes in bed height (cm)
- Measured changes in bed height (cm)
Experiment 6:

- Modelled changes in bed height (cm)
- Measured changes in bed height (cm)
Experiment 7:

---

**Experiment 7A**

- Modelled changes in bed height (cm)
- Measured changes in bed height (cm)

**Experiment 7B**

- Modelled changes in bed height (cm)
- Measured changes in bed height (cm)

**Experiment 7C**

- Modelled changes in bed height (cm)
- Measured changes in bed height (cm)
Experiment 8:

- Modelling changes in bed height (cm)
- Measured changes in bed height (cm)
Experiment 9:

- Experiment 9A
  - Modelling changes in bed height (cm)
  - Measured changes in bed height (cm)

- Experiment 9B
  - Modelling changes in bed height (cm)
  - Measured changes in bed height (cm)

- Experiment 9C
  - Modelling changes in bed height (cm)
  - Measured changes in bed height (cm)