Adaptive and Robust Control of Flexible Joint Robots with Joint Torque Feedback

by

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A thesis submitted in conformity with the requirements for the Degree of

DOCTOR OF PHILOSOPHY

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Adaptive and Robust Control of Flexible Joint Robots with Joint Torque Feedback

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Abstract

This thesis focuses on the development of new motion and force control methods for flexible joint robots based on joint torque feedback.

Motion control issues of flexible joint robots are addressed first. A two-stage control scheme, consisting of a motion controller and a joint torque controller, is established in a systematic way for controlling flexible joint robots in the general n-link case. To deal with uncertainties in the robotic system, an adaptive and robust control algorithm is developed assuming that all system parameters, including the joint flexibility values, are unknown except for some of their bounds. The system stability is analyzed via the Lyapunov stability theory. Compared with published work, the result has the distinct feature that it does not require restrictions on joint flexibility, nor an exact knowledge of the parameters of the joint actuator subsystem.

Simultaneous motion and force control of flexible joint robots in constrained motion is then investigated. Three adaptive and robust control methods are developed to control both the motion and contact force simultaneously. The first two methods are based on a reduced constrained dynamic model; the first is an adaptive and robust saturation control method that accommodates parametric uncertainty in the system.
dynamics and guarantees the uniform ultimate boundedness of tracking errors: the second is an adaptive and sliding control algorithm that can cope with both parameter uncertainty and unknown additive bounded disturbance, while achieving global asymptotic stability. The third method, based on the derivation of a generalized transformed dynamic model, is a unified control scheme which controls flexible joint robots in terms of general coordinates, and ensures the global asymptotic convergence of motion, force and joint torque tracking errors, without requiring persistent excitation. Distinct from previous work reported in the literature, the proposed control schemes using a two-stage control strategy provide systematic approaches for motion and force control of flexible joint robots in the general n-link case, without requiring an exact knowledge of robot dynamics.

Coordinated control of multiple flexible joint robots is also addressed in this work. The proposed two-stage control scheme is extended to motion and force control of multiple flexible joint robots holding a common object. In the control scheme, new adaptive coordinated controllers are developed for both free and constrained motion. The controllers control the motion, internal force and contact force simultaneously, without requiring an exact knowledge of the dynamic parameters of the multirobot system. Asymptotic convergence of the control system is proved theoretically. Compared with previous work in the literature, the proposed control scheme provides a new approach for dealing with the uncertainty in the multiple flexible joint robot system.

Simulations have been conducted extensively to test the effectiveness of the proposed control methods. In addition, experimental implementation of the proposed adaptive and robust motion control scheme has been carried out on the IRIS robot manipulator. Based on the comparison with different control methods, the experimental results clearly illustrate the effects of joint flexibility on the control systems and verify that the proposed control scheme can control the flexible joint robot stably, in spite of dynamic uncertainty in the system. The theoretical developments are confirmed by simulations and experiments.
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Chapter 1

Introduction

1.1 Motivation

Most robot manipulators in real applications use transmission elements in their drive system to increase the torque capability, reduce the weight and bulk of actuators, achieve better mechanical structures, and improve mobility. While obtaining benefits in these aspects, robot manipulators usually have flexibility in their joints due to the elasticity in the transmission elements. In particular, robots equipped with harmonic drives would exhibit significant joint flexibility [66, 27].

Joint flexibility has a strong impact on system performance. It has been recognized that control of robots based on rigid-body dynamic formulation is inadequate in dealing with stringent operating conditions: joint flexibility should be taken into account in modeling and control design if high performance is to be achieved [74]. In some cases, joint flexibility can even lead to instability if it is neglected in the control design.

The existence of joint flexibility causes difficulties in modeling and control. First, flexibility introduces additional degrees of freedom. As the motor angle is no longer equivalent to the link angle, the order of the related dynamics becomes twice that of rigid robots. The resulting more complicated dynamic behavior of a physical system needs to be studied and evaluated in mathematical modeling. Second, the structural properties that facilitate the control of rigid robots, such as full actuation (i.e., an
independent control input for each degree of freedom) and passivity from input torque to link velocity, are lost when joint flexibility is included in the dynamic model. Flexibility in the joint induces fast dynamics. Without proper control, vibration could be excited. The control problems of flexible joint robots are further complicated if uncertainties are considered. In addition to the uncertainties in the link dynamics, such as unknown payload variations and external disturbances, the uncertainties in the drive dynamics, especially the uncertainty of joint stiffness values, have a significant effect on system performance.

Much effort has been devoted to dealing with the above issues. There is increasing interest in flexible joint robot control because of its wide application and theoretical challenge. However, as yet there is a lack of systematic approach. Existing control methods in the literature have some drawbacks, such as imposing restrictions on joint flexibility or requiring full knowledge of the system dynamics. Control methods that use joint torque measurements for controlling flexible joint robots do exist, but most of them are limited to the consideration of one single joint [54, 12].

To remedy these problems, control schemes using joint torque feedback techniques for controlling flexible joint robots in the general n-link case are developed in this research. The IRIS robot manipulator developed in the Robotics and Automation Laboratory at the University of Toronto, equipped with harmonic drivers in its joints, and with the capability of joint position and joint torque measurements, provides the experimental facility as well as the stimulation for this work.

Considering that robots in applications are required to perform mainly three types of tasks: i) maneuvering in an unconstrained space (free motion); ii) interaction with the environment (constrained motion); and iii) cooperative manipulation with other robots in a coordinated fashion, this research addresses control issues related to these three aspects and proposes new control schemes for motion control, motion/force control, and coordinated control of flexible joint robots, respectively. The proposed control schemes based on a two-stage control (or referred to as "cascade control") strategy, with an idea similar to the backstepping method, provide a systematic approach to the control of flexible joint robots. Adaptive and robust control methods are
developed to deal with the uncertainties in both link and drive dynamics, including the joint flexibility values. Stability and convergence properties of the whole system are analyzed via the Lyapunov stability theory. Real-time implementation issues are also addressed along with simulation and experimental verifications.

1.2 Literature Review

Traditionally control of robots has been based on the assumption of rigid body formulation. Under this assumption, most of the control methods developed assume that torque commands can be arbitrarily applied to each robot joint. In reality, however, these torque commands must be applied through the use of joint actuators. The actuators and the attached drive mechanisms possess non-negligible mechanical dynamics such as inertia, friction and compliance. As robots are required to perform more and more complicated tasks, control of robots based on rigid-body dynamics formulation has been recognized to be inadequate in dealing with more stringent operating conditions. Experimental and analytical evidence [74, 46] indicates that joint flexibility is a major problem due to the resulting higher system order and the introduction of resonant behavior. To ensure system stability and high system performance, joint flexibility should be taken into account in modeling and control designs. Because real robots usually have joint flexibility due to elasticity in the gear transmission, belts and drive shafts, etc., control of flexible joint robots has become an active research field. Extensive work on modeling and control of flexible joint robots has been reported in the literature. A brief review is presented in this section organized into three topics.

Motion Control of Flexible Joint Robots

Motion control is essential in all robot applications. The control issue is to design control systems that determine the control input necessary to ensure that the end-effector of the robot can reach a desired position or track a prescribed trajectory precisely. In the presence of joint flexibility, the motor angle is not necessarily equal
to the link angle. The control objective in terms of joint coordinates is equivalent to ensuring that the link angles follow a desired trajectory. Much effort has been devoted to achieving this goal. Various control methods using different techniques have been proposed. The main techniques used for modeling and control of flexible joint robots are singular perturbation, feedback linearization, integral manifold, adaptive and robust control approaches.

Marino and Nicosia [46] developed a control scheme using a singular perturbation technique to control flexible joint robots. Bortoff [3], Khorasani [24], and Spong [66, 67] et al. introduced two nonlinear control techniques, namely, the feedback linearization technique and the integral manifold technique. In the feedback linearization approach, the control algorithm requires the measurements of link position and velocity, rotor position and velocity, as well as link acceleration and jerk. The integral manifold technique is based on a reduced-order model of a flexible joint robot and requires full knowledge of the robot dynamics.

Adaptive control strategies have been proposed to deal with parametric uncertainty in flexible joint robots. A comprehensive study of adaptive control of flexible joint robots based on a singular perturbation approach was presented in [68]. Utilizing the concept of integral manifolds, Khorasani [25] developed corrected adaptive control laws. Relevant work was also conducted actively by Ghorbel and Spong [11, 10], etc.

Most control schemes based on the singular perturbation formulation are obtained under the assumption of weak joint flexibility (high rigidity). Chen and Fu [4] proposed an adaptive control scheme with a two-stage control algorithm, which does not require the restriction of weak joint flexibility but assumes an exact knowledge of the parameters in the rotor subsystem. A composite adaptive control of flexible joint robots was developed by Yuan and Stepanenko [82] which requires the joint flexibility values and parameters of the rotor subsystem. Lozano and Brogliato [43] proposed an adaptive control method for flexible joint robots with arbitrary joint flexibility. In their control algorithm, a desired motor angle trajectory is first computed from the robot dynamics and then used as an input to the controller. Asymptotic stability is obtained. However, further simulation or implementation of the control method is
required to assess its feasibility.

A robust controller for flexible joint robots was reported in [7]. Control methods such as simple PD control, decentralized velocity feedback control and robust $H_{\infty}$ control, have also been proposed by Tomei [76], Readman [58], and Moghaddam and Goldenberg [50], among others.

With the advent of robot technology, the use of joint torque measurement for the control of robot manipulators has been proposed by several authors [44, 54, 29, 12]. A brief literature review on the control of robot manipulators using joint torque feedback was given in [32]. However, concerning the control of flexible joint robots, control schemes using joint torque feedback techniques are still few, and most of them are limited to applications of a single joint [54, 12]. Pfeffer et al. [54] developed a joint torque sensor for the Puma manipulator, established a transfer function for modeling the flexible joint, and designed digital compensators for controlling joint torques. Hashimoto et al. [12] considered the effects of joint flexibility on the control design with joint torque positive feedback, and analyzed the system stability using the root locus technique. In [59], a high gain PD joint torque controller was designed. A computed-torque-like joint torque controller and a simple integral controller were developed in [33] without considering uncertainty in the robot system.

In order to develop control strategies with joint torque feedback for controlling flexible joint robots in the general n-link case, and to cope with uncertainty in both link and drive dynamics including the joint flexibility values, a two-stage adaptive and robust control scheme was proposed by Lin and Goldenberg [34]. The details are presented in Chapter 2.

**Motion and Force Control of Flexible Joint Robots**

Many advanced robot applications such as assembly and manufacturing require mechanical interaction of the robot manipulator with the environment. For successful execution of such tasks, it is desired that both motion and contact force be controlled simultaneously. Numerous approaches have been proposed to solve this problem. The typical force control schemes are explicit force control, hybrid position/force control
and impedance control [79, 57, 14]. The underlying control problem that has received extensive attention in the literature is the control of manipulators in constrained motion. During constrained motion, the end-effector of a robot manipulator is in contact with rigid frictionless surfaces which impose kinematic constraints on the robot motion. Solutions to this control problem can be found in Kankaanranta and Koivo [23], McClamroch and Wang [47], and Mills and Goldenberg [48], etc. In [23], a method for reducing the dimension of the dynamic model of constrained motion was presented, and a control method was proposed which leads to the exact decoupling of position and force controlled directions. Similar studies were conducted by McClamroch and Wang [47], and a general theoretical framework of constrained motion control was developed. In [48], descriptor theory was applied to constrained motion control and a linearized feedback controller was developed. Some adaptive control algorithms and robust control methods have also been proposed to deal with the case of parametric uncertainties [72, 81, 83].

However, in the works cited above, the robot manipulator is modeled as a completely rigid structure. So far, only a few studies have addressed the force control of flexible joint robots. Joint flexibility, introducing resonant modes that could be easily excited by the interaction of the robot manipulator with the environment, causes difficulties in control. In [69], Spong addressed the force control issue of flexible joint robots and derived a control algorithm for both hybrid control and impedance control methods. Mills [49] developed a control method using a composite control technique on a singular perturbation model. Jankowski and ElMaraghy [20] presented a nonlinear decoupling control method for constrained robots with flexible joints. Using the feedback linearization technique, Massoud and ElMaraghy [51] proposed two model-based control algorithms for motion and force control of flexible joint robots. Simulation and experimental results show that the tracking performance of the system is satisfactory. To overcome the effect of parametric uncertainty, an adaptive feedback control law, along with a sliding-mode observer that estimates the feedback states, was developed [9]. However, in their work, transforming the fourth derivative of the link position vector and the second derivative of the contact forces into the constraint
frame to obtain the decoupled system is required, and the computation of the control laws is very complicated. The coupling of the estimated state signals and the estimated parameters make the method very sensitive to signal noise. Besides, the uncertainty of joint flexibility values was not considered in the control scheme. An adaptive force control method for a single-link mechanism with joint flexibility was developed by Lian, Jean and Fu [31]. In the work, explicit force control approach is used, but constrained motion control is not considered, and the method can not be easily extended to the general n-link case.

In [35, 36, 37], new adaptive and robust control methods have been developed for the motion and force control of flexible joint robots in constrained motion. Distinct from previous work, the proposed control methods provide systematic approaches for motion and force control of flexible joint robots in the general n-link case, without requiring exact knowledge of robot dynamics including the joint flexibility values. The details are described in Chapter 3.

Coordinated Control of Multiple Flexible Joint Robots

There is a growing interest in the development of coordinated multi-manipulator systems. The potential applications of such systems cover a wide range, such as lifting a heavy object or assembling mating parts, and all other tasks recognized to be beyond the capability of a single manipulator. In all of these applications, multiple robot manipulators should be controlled in a coordinated fashion to handle complicated and dexterous tasks skillfully.

Numerous control methods have been proposed for the control of coordinated robots. These methods can be classified into two categories: i) master/slave control; and ii) hybrid position/force control. In master/slave control schemes [75, 45, 2], one or a group of arms plays the role of the master, and the rest of the arms form the slave group which moves in conjunction with the master. Once the trajectory of the master is planned, the trajectory of the slave is derived from a set of holonomic constraints. In hybrid position/force control approaches [13, 16, 77], the object motion and force trajectories are specified based on the overall dynamics of the multi-manipulator sys-
tem, and controlled by two explicit control laws, namely, motion control and force control laws. Considering that there are always uncertainties in the system dynamics, several adaptive and robust control schemes for coordinated robots have been proposed by Hu and Goldenberg [17], Jean and Fu [22], Song and Anderson [65], and Su and Stepanenko [73], etc.

In the control schemes mentioned above, the robot manipulators are assumed to be rigid. Compared with the large volume of literature available on the coordinated control of rigid robots, only a few studies so far have addressed the coordinated control of multiple flexible joint robots. Hu and Goldenberg [18] developed a motion and force control scheme for coordinated robot arms in the presence of joint flexibility with which asymptotic motion and force tracking is achieved. Jankowski and ElMaraghy [21] extended their nonlinear decoupling control law to multiple flexible joint robots. It was shown that the closed system has high-precision tracking capabilities. Ahmad [1] proposed a control method for multiple flexible joint robots by utilizing the constrained motion control technique. Their results rely on a full knowledge of the system dynamics.

1.3 Contributions of This Thesis

The contributions of this work are as follows:

Motion Control of Flexible Joint Robots with Joint Torque Feedback

A two-stage control scheme [34] consisting of a motion controller and a joint torque controller is developed for the control of flexible joint robots in the general n-link case. The stability of the control system and the uniform ultimate boundedness of the tracking error are proved under the assumption that all system parameters, including the joint flexibility values, are unknown except for some of their bounds. Compared with published work, the result has the distinct feature that it does not require restrictions on joint flexibility, nor an exact knowledge of the parameters of the actuator subsystem. The outcome is a useful framework for the generalization of
control methods previously developed for rigid robots.

**Motion and Force Control of Flexible Joint Robots in Constrained Motion**

Motion and force control problems of flexible joint robots are addressed in this research. Three adaptive and robust control methods have been developed to control both motion and contact force simultaneously.

1. Based on a reduced constrained dynamic model, an adaptive and robust saturation control method [35] is proposed to deal with parametric uncertainty in the system dynamics. It is proved that with the proposed control method, the tracking error of the closed-loop system is uniformly ultimately bounded (u.u.b.).

2. Considering that in addition to the uncertainty of the dynamic parameters, systems generally have some unmodeled dynamics, an adaptive and sliding control algorithm [36] is proposed to deal with both parameter uncertainty and unknown additive bounded disturbance. Asymptotic stability of the system is proved.

3. Based on the derivation of a generalized transformed dynamic model, a unified control scheme [37] is proposed. Composite motion and force tracking errors are utilized in the adaptive mechanism. The control scheme has two important features: i) it provides a unified approach to the motion and force control of flexible joint robots in general coordinate spaces (such as task space, joint space, etc.); and ii) the motion, force and joint torque tracking errors converge asymptotically to zero without persistent excitation conditions. Furthermore, friction force on the contact surface can be compensated for by the control scheme, and hence, the frictionless contact assumption is eliminated.

**Coordinated Control of Multiple Flexible Joint Robots Holding a Common Object**

The control of multiple robot systems is a challenging issue when flexibility in the robot joints and uncertainty in the system dynamics are considered. The proposed two-stage control scheme has been extended to the coordinated control of multiple flexible joint robots holding a common object. In the control scheme, new adaptive coordinated controllers have been developed for both free and constrained motion.
The controllers control the motion, internal force and contact force simultaneously, without requiring an exact knowledge of the dynamic parameters of the multirobot system. Asymptotic convergence of the control system has been proved theoretically. Simulation results illustrate the effectiveness of the proposed control methods. Compared with previous work in the literature, the proposed control scheme provides a new approach for dealing with uncertainty in multiple flexible joint robot systems.

**Experimental Implementation of the Proposed Control Scheme**

The proposed two-stage control scheme for the motion control of flexible joint robots has been analysed experimentally. Motion control experiments on single-joint-link and two-joint-link cases have been conducted to test the proposed control scheme. Comparisons of different control methods were carried out. The experimental results confirm the theoretical developments, illustrate clearly the effects of joint flexibility on the control systems, and show that the system with the proposed control method can track the desired trajectory stably.

The experiments have also revealed some important issues on control of flexible joint robots, such as hysteresis in the joint stiffness, noise sensitivity and nonlinear friction.

### 1.4 Organization of the Thesis

This thesis is composed of six chapters.

Chapter 1 introduces the historical perspective, presents the state of the art for the control of flexible joint robots, and summarizes the contributions of this research work.

Chapter 2 focuses on motion control of flexible joint robot manipulators with joint torque feedback. A two-stage control scheme consisting of a motion controller and a joint torque controller is established in a systematic way for the general n-link case. To deal with uncertainties in the robotic system, a robust adaptive control algorithm
is developed under the assumption that all system parameters, including the joint flexibility values, are unknown except for some of their bounds. The system stability is analyzed via the Lyapunov stability theory. Implementation issues of the proposed control method are also discussed in this chapter.

Chapter 3 addresses motion and force control problems of flexible joint robots in constrained motion. Three adaptive and robust control methods are described in detail. Based on the reduced constrained dynamic model, an adaptive and robust control scheme is developed which deals with parametric uncertainty in the system dynamics and guarantees uniform ultimate boundedness of the tracking errors. Also, an adaptive and sliding control algorithm is proposed to cope with both the parameter uncertainty and unknown additive bounded disturbance, and to achieve global asymptotic stability. Based on the derivation of a generalized transformed dynamic model, a unified control scheme is developed which provides a unified approach to motion and force control of flexible joint robots in general coordinate spaces and ensures the global asymptotic convergence of motion, force and joint torque tracking errors, without requiring persistent excitation.

Chapter 4 considers the coordinated control of multiple flexible joint robots holding a common object in free and constrained motion. The proposed two-stage control scheme is extended to the control of multiple flexible joint robots. In the control scheme, new adaptive coordinated controllers are developed to control the motion, internal force and contact force simultaneously. Asymptotic convergence of the control system is proved theoretically. Simulation results illustrate the effectiveness of the proposed control methods.

Chapter 5 describes the experimental implementation of the proposed control scheme on the IRIS robot manipulator. Motion control experiments on single-joint-link and two-joint-link cases are presented with comparisons of different control methods. The effects of joint flexibility are clearly illustrated in real experiments, and the stability of the proposed control system is verified experimentally.

Chapter 6 summarizes the results of this research work and presents recommendations for future studies.
Chapter 2

Adaptive and Robust Motion Control of Flexible Joint Robots with Joint Torque Feedback

2.1 Introduction

With the advent of robot technology, the use of joint torque measurement for controlling robot manipulators has been proposed by many authors [44, 54, 29, 12]. However, concerning the control of flexible joint robots, control schemes using joint torque feedback techniques are still few, and most of them are limited to applications of a single joint [54, 12]. Experimental and analytical evidence [74, 46, 67] has pointed out that joint flexibility causes difficulties in modeling and control. Control of robots based on rigid-body dynamics formulation is inadequate in dealing with more stringent operating conditions due to joint flexibility. In some cases, joint flexibility can lead to instability if it is neglected in the control design.

Much effort has been devoted to the control of flexible joint robots. Dynamic models and control methods based on different techniques have been proposed in the literature. As mentioned in Chapter 1, most control schemes using singular perturbation related techniques are restricted to weak joint flexibility (high rigidity). Chen and Fu [4] proposed an adaptive control scheme with a two-stage control algorithm
which does not require the restriction of weak joint flexibility but assumes the exact knowledge of parameters of the rotor subsystem.

To remedy these problems, Lin and Goldenberg [34] proposed a two-stage control scheme with joint torque feedback for motion control of flexible joint robots. The control scheme consisting of a motion controller and a joint torque controller is established in a systematic way for the general n-link case. To deal with uncertainties in the robotic system, an adaptive and robust control algorithm is developed under the assumption that all the system parameters including the joint flexibility values are unknown except for some of their bounds. The system stability is analyzed via the Lyapunov stability theory. The result compared with previous work has the distinct feature that no restriction of weak joint flexibility, nor an exact knowledge of the parameters of the rotor subsystem is required. The outcome is a useful framework for the generalization of control methods previously developed for rigid robots. The details of the proposed control scheme are presented in this chapter.

2.2 Dynamic Model of Flexible Joint Robots

Let us consider the standard dynamic equations of a flexible joint robot with joint torque measurements [66, 26]:

\[ D(q_l)\ddot{q}_l + C(q_l, \dot{q}_l)\dot{q}_l + G(q_l) = \tau_s \]  
(2.1)

\[ I_m\ddot{q}_m + B_m\dot{q}_m + \tau_s = u \]  
(2.2)

and

\[ \tau_s = K_s(q_m - q_l), \]  
(2.3)

where \( D(q_l) \) is the \( n \times n \) positive definite, symmetric inertia matrix; \( C(q_l, \dot{q}_l) \) is the \( n \times n \) matrix containing Coriolis and centripetal terms; \( G(q_l) \) is the \( n \times 1 \) vector of gravitational forces; \( I_m \) is the \( n \times n \) constant diagonal matrix with diagonal elements \( \gamma_i(\gamma_i + 1)J_{mi} \); \( J_{mi} \) is the inertia of the rotor/gear; \( \gamma_i \) is the gear ratio; \( B_m \) is the \( n \times n \) diagonal matrix of damping terms; and \( K_s \) is an \( n \times n \) diagonal matrix of the joint
torsional stiffness. Here $q_l$ and $q_m$ are $n$-dimensional vectors, representing the link angles and rotor angles, respectively; $u$ is the $n \times 1$ input torque control vector; and $\tau_s$ is the vector of joint torque measurements.

The dynamic model has the following fundamental properties [70], which can be exploited to facilitate the control system design and analysis:

**Property 2.1** The inertia matrix $D(q_l)$ is symmetric and positive definite. In addition, $D(q_l)$ is uniformly bounded\(^1\) above and below; that is, there exist $m_1, m_2$, and $0 < m_1 < m_2$, such that

$$m_1 \|x\|^2 \leq x^T D(q_l) x \leq m_2 \|x\|^2 \quad \forall q_l, \forall x \in \mathbb{R}^n. \quad (2.4)$$

**Property 2.2** All of the constant parameters, such as link masses, moments of inertias, etc., appear as coefficients of known functions of the generalized coordinates.

**Property 2.3** Using the Christoffel symbols, the matrix $\dot{D}(q_l) - 2C(q_l, \dot{q}_l)$ is a skew symmetric matrix.

### 2.3 Control Scheme with Joint Torque Feedback

For control design, we assume that the link angle $q_l$ and joint torque $\tau_s$ are measured and are available for feedback control. From Equation (2.3), we have

$$q_m = K_{\tau_s}^{-1} \tau_s + q_l.$$

Substituting this equality into Equation (2.2), using Equation (2.1) and Property 2.1, we get

$$I_{ts} \ddot{\tau}_s + B_{ts} \dot{\tau}_s + h_{ts}(q_l, \dot{q}_l, \tau_s) = u, \quad (2.5)$$

\(^1\)The boundedness of inertia matrix $D(q_l)$ requires, in general, that all joints of a robot be revolute. Hence, in this research, we consider only the control of revolute flexible joint robots.
where

\[ I_{ts} = I_m K_s^{-1} \]
\[ B_{ts} = B_m K_s^{-1} \]
\[ h_{ts} = (I_m D^{-1} + I)\tau_s + B_m \dot{q}_t - I_m D^{-1} C \dot{q}_t - I_m D^{-1} G. \]

Equations (2.1) and (2.5) constitute a dynamic representation of the flexible joint robot with the variables \((q_l, \tau_s)\).

We note that the link dynamics (2.1) has exactly the same formulation as the rigid dynamics if \(\tau_s\) is viewed as the "input control torque." Hence, in order to control the robot manipulator to track a desired trajectory, motion control methods developed for rigid robot control could be used to generate this "input control torque" which we will denote here as "desired joint torque" \(\tau_{sd}\) since joint flexibility exists. The next control objective is, naturally, to generate a suitable control torque \(u\) so that joint torque \(\tau_s\) can follow the desired joint torque \(\tau_{sd}\), and thus \(q_l\) will follow the desired trajectory \(q_d\). Based on this observation, a two-stage control scheme is developed. The control scheme is shown in Figure 2.1.

![Figure 2.1: The two-stage control scheme with joint torque feedback](image)

The proposed control scheme is similar to the idea of Backstepping control [28], that is, the control law for one subsystem is the reference signal for the next subsystem. In the following, it will be shown that, with joint torque controller, the proposed two-stage control scheme provides a systematic approach to the control of flexible joint robots. The control scheme has the advantage that it is intuitively sim-

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ple in concept and easy to implement. For clarity, we present the control design and analysis in known and unknown parameter cases.

### 2.3.1 Control scheme with all parameters known

In the case where all dynamic parameters are known exactly, the motion and joint torque controllers are designed under the following assumptions:

**A1** the signals \( q_l, \dot{q}_l, \tau_s, \) and \( \ddot{r}_s \) are available for feedback control;

**A2** the desired trajectory \( q_d(t) \in C^4 \) is bounded.

#### 1. Motion controller

Let us define tracking errors:

\[
e_t = q_l - q_d
\]
\[
e_t = \tau_s - \tau_{sd}
\]

and write

\[
v_l = \dot{q}_d - \Lambda(q_l - q_d) = \dot{q}_d - \Lambda e_l.
\]

The desired joint torque \( \tau_{sd} \) is generated using the Slotine and Li method [62]:

\[
\tau_{sd} = D(q_l)\ddot{v}_l + C(q_l, \dot{q}_l)v_l + G(q_l) - K_{ID}(\dot{q}_l - v_l).
\]  

where \( K_{ID} \) and \( \Lambda \) are diagonal matrices of positive gains.

Substituting (2.6) into (3.1) and defining \( r_l = \dot{q}_l - v_l \) leads to the following equation:

\[
D(q_l)\ddot{r}_l + C(q_l, \dot{q}_l)r_l + K_{ID}r_l = e_t.
\]  

#### 2. Joint torque controller

In order to control the actual joint torque to follow the desired joint torque \( \tau_{sd} \), a computed-torque-like control law is given as

\[
u = I_{ts}\dot{v}_l + B_{ts}v_l + h_{ts}(q_l, \dot{q}_l, \tau_s) - K_{ID}(\ddot{r}_s - v_t).
\]  

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Then, from Equation (2.5), we have

\[ I_t \dot{r}_t + B_t r_t + K_t D r_t = 0. \]

(2.9)

where \( v_t = \dot{\tau}_{sd} - \Lambda_t (\tau_s - \tau_{sd}) = \dot{\tau}_{sd} - \Lambda_t e_t, \quad r_t = \dot{\tau}_s - v_t, \) and \( K_t D \) and \( \Lambda_t \) are diagonal positive definite gain matrices.

Convergence Analysis

Since separate subsystem stability in nonlinear control system does not always guarantee overall system stability [28], rigorous stability analysis should be provided for the whole system. In this section, the asymptotic stability and convergence properties of the entire control system are analyzed based on the Lyapunov stability theory. First, an important lemma is introduced and then Theorem 2.1 is proved.

**Lemma 2.1** [15] *Suppose that a symmetric matrix \( Q \) is partitioned as*

\[
Q = \begin{bmatrix}
Q_{11} & Q_{12} \\
Q_{12}^T & Q_{22}
\end{bmatrix},
\]

*where \( Q_{11} \) and \( Q_{22} \) are square. The matrix \( Q \) is positive definite if and only if \( Q_{11} \) is positive definite and \( Q_{22} > Q_{12}^T Q_{11}^{-1} Q_{12}. \)*

**Theorem 2.1** *Consider the robot dynamic system (2.1) and (2.5) satisfying assumptions A1 and A2. Using the motion controller (2.6) and joint torque controller (2.8), with control gains satisfying the requirement*

\[ \Lambda_t^T K_t D \Lambda_t > \frac{1}{4} K_t D^{-1}, \]

*the solution of the closed-loop system is globally asymptotically stable and tracking errors converge asymptotically to zero, i.e.,

\[ \lim_{t \to \infty} r_t = 0, \quad \lim_{t \to \infty} r_t = 0. \]

**Proof:** Substituting the control law (2.6) and (2.8) into the robot dynamic equations (2.1) and (2.5) yields two sets of dynamic error equations (2.7) and (2.9). Choosing
the Lyapunov function candidate as

$$V = \frac{1}{2} r_l^T D(q_l) r_l + \frac{1}{2} \dot{r}_l^T I_{ls} \dot{r}_l + \epsilon_l^T \Lambda_l^T K_{ld} \dot{e}_l$$  \hspace{1cm} (2.11)$$

which is positive definite as $D(q_l)$, $I_{ls}$, $\Lambda_l$ and $K_{ld}$ all are positive definite matrices. The derivative of $V$ is

$$\dot{V} = r_l^T D(q_l) \dot{r}_l + \frac{1}{2} \dot{r}_l^T D(q_l) \dot{r}_l + r_l^T I_{ls} \dot{r}_l + 2 \epsilon_l^T \Lambda_l^T K_{ld} \dot{e}_l$$

$$= r_l^T D(q_l) \dot{r}_l + \frac{1}{2} \dot{r}_l^T D(q_l) \dot{r}_l - r_l^T B_{ls} r_t - \epsilon_l^T K_{ld} \dot{e}_l - \epsilon_l^T \Lambda_l^T K_{ld} \dot{e}_l.$$  \hspace{1cm} (2.12)

From Equation (2.7) and the fact that $(\dot{D} - 2C)$ is skew symmetric, we have

$$\dot{V} = -r_l^T K_{ld} r_l + r_l^T \dot{e}_l - r_l^T B_{ls} r_t - \epsilon_l^T K_{ld} \dot{e}_l - \epsilon_l^T \Lambda_l^T K_{ld} \dot{e}_l$$

$$= - \left[ \begin{array}{c} r_l \\ \epsilon_l \end{array} \right] Q \left[ \begin{array}{c} r_l \\ \epsilon_l \end{array} \right] - r_l^T B_{ls} r_t - \epsilon_l^T K_{ld} \dot{e}_l$$  \hspace{1cm} (2.13)

with

$$Q = \begin{bmatrix} K_{ld} & -\frac{1}{2} I \\ -\frac{1}{2} I & \Lambda_l^T K_{ld} \Lambda_l \end{bmatrix}.$$  

By Lemma 2.1, matrix $Q$ is a positive definite matrix if the requirement (2.10) is satisfied. Since $K_{ld}$, $K_{ld}$ and $\Lambda_l$ are diagonal matrices of controller gains, there is no difficulty to choose them properly to meet the requirement, such that $Q > 0$, and $\dot{V}$ is negative definite.

Thus, the closed-loop system is globally asymptotically stable in the sense of Lyapunov. Since $\dot{V}$ is negative or zero, $V$ is bounded, $0 \leq V(t) \leq V(0)$. We have that $r_l$ and $r_t$ are bounded and belong to $L_2$. From Equations (2.7) and (2.9), $\dot{r}_l$ and $\dot{r}_t$ are bounded as well. This means that $r_l$ and $r_t$ are uniformly continuous functions, and so is $\dot{V}$. Therefore,

$$\lim_{t \to \infty} r_l = 0, \quad \lim_{t \to \infty} r_t = 0.$$  

These imply (according to Lemma A.4 in Appendix A) that $e_l \to 0$, $\dot{e}_l \to 0$, $e_t \to 0$, $\dot{e}_t \to 0$, $\dot{r}_l \to 0$, $\dot{r}_t \to 0$.  

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\[ \dot{\hat{t}} = 0 \text{ as } t \to \infty. \]

**Remark 1** From the control design and convergence analysis, it can be observed that, with the joint torque controller, motion control methods developed for rigid robots can be directly used in the motion controller of our control scheme for flexible joint robots. This will facilitate the control design of robot manipulators.

**Implementation Issues**

According to the control law, we note that in order to generate the desired joint torque \( \tau_{sd} \), we need only the signals \( q_l, \dot{q}_l \) and the desired trajectory \( q_d \) up to the second derivative, while the synthesis of the input control torque \( u \) requires the first and second order derivatives of \( \tau_{sd} \). From (2.6), we have

\[ \dot{\hat{t}}_{sd} = D(q_l)\ddot{q}_l + C(q_l, \dot{q}_l)\dot{q}_l + \dot{D}(q_l)\dot{q}_l + \dot{\hat{C}}(q_l, \dot{q}_l)q_l + \dot{\hat{G}}(q_l) - K_D(q_l - \dot{q}_l). \] (2.14)

This requires information on \( q_d^{(3)} \) and \( \ddot{q}_l \) in addition. But from the dynamic equation (2.1), \( \ddot{q}_l \) can be calculated from \( \tau_s, q_l \) and \( \dot{q}_l \), namely,

\[ \ddot{q}_l = D^{-1}(\tau_s - C(q_l, \dot{q}_l)\dot{q}_l - \hat{G}(q_l)) \]
\[ = f_1(q_l, \dot{q}_l, \tau_s). \]

Substituting it into (2.14), we have

\[ \dot{\hat{t}}_{sd} = f_2(q_d, \dot{q}_d, \ddot{q}_d, q_d^{(3)}, q_l, \dot{q}_l, \tau_s). \] (2.15)

Differentiating it again, we get

\[ \ddot{\hat{t}}_{sd} = f_3(q_d, \dot{q}_d, \ddot{q}_d, q_d^{(3)}, q_d^{(4)}, q_l, \dot{q}_l, \ddot{q}_l, \tau_s, \dot{\tau}_s). \]

\[ = f_4(q_d, \dot{q}_d, \ddot{q}_d, q_d^{(3)}, q_d^{(4)}, q_l, \dot{q}_l, \tau_s, \dot{\tau}_s). \] (2.16)
Thus, according to Equation (2.8),

$$u = f_5(q_d, \dot{q}_d, \ddot{q}_d, q^{(3)}_d, q^{(4)}_d, q_l, \dot{q}_l, \tau_s, \dot{\tau}_s)$$

which means that if the assumptions A1 to A2 are satisfied, the control method can be implemented.

In practice, $\dot{\tau}_s$ is not measured directly but is approximated by the first backward difference. Since the sampling rate can be high (1000Hz), this approximation is accurate enough for control purposes.

### 2.3.2 Robust adaptive control with all parameters unknown

In practice, there are uncertainties in the manipulator parameters. Here we assume that all parameters, both in link dynamics and drive system, including the joint flexibility values, are unknown except for some of their bounds. Based on the result of the previous section, a robust adaptive control algorithm is developed to deal with the uncertainties and to guarantee the stability and convergence of tracking errors.

#### 1. Adaptive motion controller

According to Property 2.2, we can rewrite Equation (2.1) as [62]

$$D(q_l)\ddot{q}_i + C(q_l, \dot{q}_i)\dot{q}_i + G(q_l) = Y(q_l, \dot{q}_l, \ddot{q}_l)P = \tau_s.$$  \hspace{1cm} (2.18)

where $Y(.)$ is an $n \times r$ matrix of known functions referred to as the regressor, and $P$ is an $r$-dimensional vector of parameters.

The adaptive control law is

$$\tau_{sd} = \hat{D}(q_l)\dot{v}_i + \hat{C}(q_l, \dot{q}_l)v_i + \hat{G}(q_l) - K_{ID}(\dot{q}_l - v_l)$$

$$= Y(q_l, \dot{q}_l, v_i, \dot{v}_i)\hat{P} - K_{ID}r_l,$$  \hspace{1cm} (2.19)

where $\hat{D}, \hat{C}, \hat{G}$ and $\hat{P}$ are estimated parameters. Substituting (2.19) into Equation
(2.1) gives

\[ D(q_l)\dot{r}_l + C(q_l, \dot{q}_l)r_l + K_lD\dot{r}_l = Y(q_l, \dot{q}_l, \dot{v}_l, \dot{w}_l)\hat{P} + e_t \]  

(2.20)

with \( \hat{P} = \hat{P} - P \). The parameter update law is given as

\[ \dot{\hat{P}} = -\Gamma^{-1}Y^T(q_l, \dot{q}_l, \dot{v}_l, \dot{w}_l)r_l. \]  

(2.21)

To prevent parameter drift [62] of the estimated parameters, parameter resetting method is used which will be detailed in the proof of Theorem 2.2 in this section.

2. Robust joint torque controller

We note that \( h_{ts} \) in Equation (2.5) contains the mixed uncertainties of the link dynamics and drive system. It would make the parameter update (if using the adaptive control method) or estimation of the uncertainty bounds (if using a robust control) somewhat difficult. However, by assuming that \( \bar{q}_l \) is available, we can write Equation (2.5) as follows:

\[ I_{ts}\ddot{\bar{r}}_s + B_{ts}\dot{\bar{r}}_s + I_m\ddot{\bar{q}}_l + B_m\dot{\bar{q}}_l + \tau_s = u. \]  

(2.22)

Thus \( h_{ts} \) is equal to \( I_m\ddot{\bar{q}}_l + B_m\dot{\bar{q}}_l + \tau_s \). Defining an \( n \times p \) matrix \( Y_t \) and a \( p \)-dimensional vector \( P_t \) as

\[ Y_t(\ddot{\bar{q}}_l, \ddot{\bar{q}}_l, \tau_s, \dot{\bar{r}}_s, \ddot{\bar{r}}_s) = [\text{diag}(\ddot{\bar{r}}_s), \text{diag}(\dot{\bar{r}}_s), \text{diag}(\ddot{\bar{q}}_l), \text{diag}(\dot{\bar{q}}_l), \tau_s] \]

\[ P_t = [..I_{tsi},..B_{tsi},..I_{mi},..B_{mi},1]^T, \]

Equation (2.22) becomes

\[ Y_t(\ddot{\bar{q}}_l, \ddot{\bar{q}}_l, \tau_s, \dot{\bar{r}}_s, \ddot{\bar{r}}_s)P_t = u. \]

We assume that the uncertainty of the parameters is bounded, that is, for the available parameters \( \hat{P}_t \), there exists \( \rho \in R_+ \) such that,

\[ \| \hat{P}_t \| = \| \hat{P}_t - P_t \| \leq \rho. \]  

(2.23)
Based on (2.8), using the control law given by

\[ u = \hat{I}_{ts} \dot{\hat{v}}_t + \hat{B}_{ts} v_t + \hat{h}_{ts} - K_{iD}(\hat{r}_s - v_t) + Y_i \Delta u, \quad (2.24) \]

we have

\[
\begin{align*}
I_{ts} \ddot{r}_t + B_{ts} \dot{r}_t + K_{iD} r_t &= (\hat{I}_{ts} - I_{ts}) \dot{v}_t + (\hat{B}_{ts} - B_{ts}) v_t \\
+ (\hat{h}_{ts} - h_{ts}) + Y_i \Delta u &= Y_i(\dot{q}_t, \ddot{q}_t, \tau_s, v_t, \dot{v}_t)(\dot{P}_t + \Delta u).
\end{align*}
\quad (2.25)
\]

where \( \Delta u \) is defined in (2.26).

**Theorem 2.2** Using the adaptive motion controller (2.19) with the parameter update law (2.21), and robust joint torque controller (2.24) with

\[
\Delta u = \begin{cases} 
-\rho \frac{Y^T_t \dot{r}_t}{\| Y^T_t \dot{r}_t \|} & \text{if } \| Y^T_t \dot{r}_t \| > \epsilon \\
-\frac{\rho}{\epsilon} Y^T_t \dot{r}_t & \text{if } \| Y^T_t \dot{r}_t \| \leq \epsilon
\end{cases}
\quad (2.26)
\]

for \( \epsilon > 0 \) and the control gains satisfying the requirement (2.10), the tracking errors of the closed-loop system (2.20) and (2.25) are uniformly ultimately bounded (u.u.b.).

**Proof:** Choose the Lyapunov function candidate as

\[
V = \frac{1}{2} r^T r + \frac{1}{2} \tilde{P}^T \Gamma \tilde{P} + \frac{1}{2} r^T r + e^T \Lambda \epsilon.
\quad (2.27)
\]

which is positive definite. Through a similar procedure as in the previous section, the derivative of \( V \) is

\[
\dot{V} = -\left[ r^T e^T \right] Q \left[ \begin{array}{c} r_i \\ e_t \end{array} \right] - r^T r - \dot{e}^T \dot{e} + \tilde{P}^T (Y^T r_i + \Gamma \dot{P}) + (Y^T r_i)^T (\dot{P} + \Delta u).
\quad (2.28)
\]

By properly choosing the control gains satisfying the requirement (2.10) so that \( Q > 0 \), with the parameter update law (2.21) and control \( \Delta u \) (2.26), we can show that
\( \dot{V} \) is negative as follows. Let us examine the last term in the equation above. If \( \| Y_t^T r_t \| > \varepsilon \), then

\[
(Y_t^T r_t)^T (\dot{P}_t + \Delta u) = (Y_t^T r_t)^T (\dot{P}_t - \frac{Y_t^T r_t}{\| Y_t^T r_t \|})
\leq \| Y_t^T r_t \| (\| \dot{P}_t \| - \rho) < 0.
\]

(2.29)

If \( \| Y_t^T r_t \| \leq \varepsilon \),

\[
(Y_t^T r_t)^T (\dot{P}_t + \Delta u) \leq (Y_t^T r_t)^T (\rho \frac{Y_t^T r_t}{\| Y_t^T r_t \|} + \Delta u)
\leq (Y_t^T r_t)^T (\rho \frac{Y_t^T r_t}{\| Y_t^T r_t \|} - \frac{\rho}{\varepsilon} Y_t^T r_t).
\]

(2.30)

This last term achieves a maximum value of \( \varepsilon \rho / 4 \) when \( \| Y_t^T r_t \| = \varepsilon / 2 \). Hence, we have

\[
\dot{V} \leq - \left[ \begin{array}{c} r_t^T e_t^T \\ \end{array} \right] Q \left[ \begin{array}{c} r_t^T e_t^T \\ \end{array} \right] - r_t^T B t s r_t - \dot{e}_t^T K_i D \dot{e}_t + \frac{\varepsilon \rho}{4}
\leq -\lambda_{\text{min}}(Q) \| (r_t^T, e_t^T) \|^2 - \text{min}(B_{\text{tsi}}) \| r_t \|^2
\leq -\text{min}(K_i D_i) \| \dot{e}_t \|^2 + \frac{\varepsilon \rho}{4}.
\]

(2.31)

If

\[
\lambda_{\text{min}}(Q) \| (r_t^T, e_t^T) \|^2 + \text{min}(B_{\text{tsi}}) \| r_t \|^2 + \text{min}(K_i D_i) \| \dot{e}_t \|^2 > \varepsilon \rho / 4,
\]

then \( \dot{V} < 0 \). \( V(t) \) decreases as long as (2.32) holds. The uniform ultimate boundedness of tracking errors follows from the results of Leitmann [30]. The details of the tracking error bounds are derived in Appendix B.1.

It should be noted that because (2.31) does not contain a quadratic term in \( \dot{P} \), the convergence of the tracking errors does not necessarily imply the convergence of the estimated parameters to their true values. In the absence of persistent excitation, parameter drift may occurs [62]. There are several methods to prevent parame-
ter drift, such as dead-zone method and composite adaptive control algorithm [63], etc. It should also be noted the fact that there is a difference between the control mechanisms of the rigid and flexible joint robots. In the rigid case, the estimated parameters generate the control input $u$ directly. In the flexible joint case, the estimated parameter $\hat{P}$ generates the desired torque command $\tau_{sd}$, and consequently affects the computation of $\dot{\tau}_{sd}$, $\ddot{\tau}_{sd}$, and control input $u$. As small variation in $\tau_{sd}$ may cause large changes in $\tau_{sd}$ and $\ddot{\tau}_{sd}$, preventing parameter drift is more demanding in the flexible joint case. The idea [6] to restrict the parameter estimations to within certain bounds, such that

$$l_i < P_i < h_i,$$

is adopted. The parameter update law (2.21) is thus augmented with the resetting conditions as follows:

$$\hat{P}_i = \begin{cases} 
  l_i & \text{if } \hat{P}_i(t) < l_i \\
  h_i & \text{if } \hat{P}_i(t) > h_i \\
  \hat{P}_i(t) & \text{otherwise}
\end{cases} \quad (2.33)$$

It can be shown that the addition of parameter resetting maintains the negativity of $\dot{V}$, i.e., Theorem 2.2 remains valid. The details of the proof is presented in Appendix B.2.

**Remark 2** It is noted that using a single bound $\rho$ to measure the parameter uncertainty may lead to an overly conservative design and limit the adjustable capability of the controller [71, 41]. To minimize this respect, $Y_i \Delta u$ can be partitioned as follows. Similar to [71], suppose we have the knowledge of the uncertainty bounds of each parameter component:

$$\| \hat{P}_{ti} \| \leq \rho_i \quad i = 1, 2, \ldots, p. \quad (2.34)$$

Partitioning

$$Y_i P_i = [Y_{i1}, \ldots, Y_{ip}] \begin{bmatrix} P_{i1} \\
  \vdots \\
  P_{ip} \end{bmatrix} = \sum_{i=1}^{p} Y_{ii} P_{ii} \quad (2.35)$$
Thus, our knowledge of the parameter uncertainty can be better utilized in control design as there is more freedom to adjust the control gains.

Remark 3 In the implementation, $\tau_{sd}$ can be calculated from Equation (2.14) except that $D, C$ and $G$ are now superseded by $\hat{D}, \hat{C}$ and $\hat{G}$. $\tau_{sd}$ may not necessarily be calculated via the dynamic equation, as the parameters are not known precisely. It can be obtained using a filtered signal or by the backward difference.

2.4 Simulation Examples

To verify the effectiveness of the control methods proposed in this chapter, numerical simulations are carried out based on the dynamic model of the IRIS robot established in the Robotics and Automation Laboratory at the University of Toronto [26]. Consider the motion control of the robot manipulator with an upper arm and a forearm located in a vertical plane, as shown in Figure 2.2.

The dynamic model of the rotor subsystems with joint torque sensors is given as follows:

$$\gamma_1(\gamma_1 + 1)J_{m_1}\ddot{q}_{m_1} + \gamma_1(\gamma_1 + 1)b_{m_1}\dot{q}_{m_1} + \tau_{s_1} = u_1$$

$$\gamma_2(\gamma_2 + 1)J_{m_2}\ddot{q}_{m_2} + \gamma_2(\gamma_2 + 1)b_{m_2}\dot{q}_{m_2} + \tau_{s_2} = u_2,$$
The values of the parameters are listed in Table 2.1.
Table 2.1 The parameter values of the flexible joint robot

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_m = 9.0 \times 10^{-6} kgm^2$</td>
<td>$J_m = 8.0 \times 10^{-6} kgm^2$</td>
</tr>
<tr>
<td>$b_m = 5.4 \times 10^{-4} Nm/(rad/sec)$</td>
<td>$b_m = 6.4 \times 10^{-4} Nm/(rad/sec)$</td>
</tr>
<tr>
<td>$\gamma_1 = 100$</td>
<td>$\gamma_2 = 50$</td>
</tr>
<tr>
<td>$K_{s1} = 10$</td>
<td>$K_{s2} = 8.5$</td>
</tr>
<tr>
<td>$a_1 = 0.1499$</td>
<td>$a_2 = 0.0311$</td>
</tr>
<tr>
<td>$a_3 = 0.0235$</td>
<td>$a_4 = 0.6762$</td>
</tr>
<tr>
<td>$a_5 = 0.1288$</td>
<td></td>
</tr>
</tbody>
</table>

In the robust adaptive control scheme, the matrix $Y$ is given as

$$Y(q_i, \dot{q}_i, v_i, \dot{v}_i) = \begin{bmatrix} \dot{\psi}_1, \dot{\psi}_1 + \dot{\psi}_2, Y_{13}, \cos(q_{l1}), Y_{15} \\ 0, \dot{\psi}_1 + \dot{\psi}_2, Y_{23}, 0, Y_{25} \end{bmatrix},$$

where

$$Y_{13} = \cos(q_{l2})(2\dot{\psi}_1 + \dot{\psi}_2) - \sin(q_{l2})(q_{l3}\dot{v}_{l2} + \dot{\psi}_1 v_{l2} + \dot{\psi}_2 v_{l1}),$$

$$Y_{23} = \cos(q_{l2})\dot{\psi}_1 + \sin(q_{l2})\dot{\psi}_1 v_{l1},$$

$$Y_{15} = Y_{25} = \cos(q_{l1} + q_{l2}).$$

and the resetting bound regions for the parameters of the link dynamics are set as follows:

$$a_1 \in (0.11, 0.4), a_2 \in (0.02, 0.1), a_3 \in (0.01, 0.04), a_4 \in (0, \infty) \text{ and } a_5 \in (0, \infty).$$

The resetting bounds for parameters $a_1, a_2$ and $a_3$ have to be carefully determined as these parameters affect the positive definition of the matrix $D$, while the restriction on $a_4$ and $a_5$ that are in the gravity terms could be quite loose. In the above, the restriction on $a_4$ and $a_5$ is only that they should be large than zero.

The uncertainty bounds for the parameters of the rotor subsystem, including the joint flexibility values, are assumed to be in intervals of $(1 \pm 20\%)$ of the true values, and the nominal parameters are simply chosen as the lower bound values.

The simulations are performed using Simnon. The control schemes are digitally implemented with a sampling rate of 1 msec. Two desired trajectories are used to test the performance of the control methods.

First, the desired trajectories are generated by fifth-order polynomials. Joint 1
Figure 2.3: The response of the system with known parameters

moves from 90 to 0 (degree), joint 2 moves from 0 to 60 (degree), both in the first second, and then remain constant afterwards. The responses of the system with known parameters and those of the adaptive and robust control system are shown in Figures 2.3 and 2.4. It is illustrated that excellent transient and steady-state performance are achieved by the proposed control scheme. The position tracking errors of the adaptive and robust control system are $e_{t1} = -2.895E - 5$ and $e_{t2} = 0.0002$ at time $t = 2(\text{sec})$, with $\epsilon$ being set as 5. The parameter estimation curves are plotted in Figure 2.4 as well. The initial values of the parameters are $a_1 = 0.11$, $a_2 = 0.025$, $a_3 = 0.015$, $a_4 = 0.6$ and $a_5 = 0.1$. It can be seen that some estimated parameters do not converge to their true values, and the resetting is necessary. As a
Figure 2.4: The response of the robust adaptive control system
comparison to the adaptive and robust control, the response of a non-adaptive-robust control system is plotted in Figure 2.5. Clearly, in the presence of the parameter uncertainty, the proposed adaptive and robust control method enhances remarkably the system tracking performance.

Second, the sinusoidal function is also used to specify the desired trajectory to test the dynamic performance of the control system:

\[ q_{d1} = 30 \sin(6.28t); \quad q_{d2} = 40 \cos(10.0t). \]

The results are shown in Figures 2.6 and 2.7. The tracking performance of the proposed control system is satisfactory.
Figure 2.6: The response of the system with known parameters

Figure 2.7: The response of the robust adaptive control system
2.5 Summary

In this chapter, motion control of flexible joint robots with joint torque feedback is addressed. A two-stage control scheme consisting of a motion controller and a joint torque controller is established in a systematic way for the general n-link case. To deal with uncertainties in the robotic system, an adaptive and robust control algorithm is developed assuming that all the system parameters including the joint flexibility values are unknown except for some of their bounds. The result compared with previous work in the literature has the distinct feature that it does not require restrictions on joint flexibility, nor an exact knowledge of the parameters of the rotor subsystem. The system stability is analyzed via the Lyapunov stability theory. Simulation results illustrate the effectiveness of the proposed control scheme. Experimental studies that further evaluate and verify the proposed control methods are presented in Chapter 5.

It is noted that with the joint torque controller, the methods developed for rigid robots can be used for the control design of flexible joint robots. In our control scheme, the motion controller is designed using the Slotine and Li method; the robust joint torque controller is developed in a similar way. It is obvious that other control methods, such as computed-torque control, composite adaptive control, and sliding control can also be incorporated in the control scheme. Hence, the control scheme developed in this chapter provides a useful framework for the control of flexible joint robots.
Chapter 3

Adaptive and Robust Motion and Force Control of Flexible Joint Robots with Joint Torque Feedback

3.1 Introduction

In the previous chapter, the control of flexible joint robots in free space motion was considered. Many advanced robot applications such as assembly and manufacturing require that robot arms or their payloads be in contact with the environment. The motion of the robot arm is thus constrained due to the fact that the robot arm is not free to move in certain directions, and there are contact forces arising between the robot arm and the environment. For the successful execution of tasks, it is desired that both the motion and the contact force be controlled simultaneously [79, 57, 14].

However, as mentioned in Chapter 1, only a few studies so far have addressed the motion and force control of flexible joint robots. Joint flexibility has a significant influence on robot dynamics, and should be taken into account in modeling and control design if high performance is to be achieved [69, 51]. When the robot end-effector
is in contact with the environment, a small deviation in position may cause a large force difference at the contact points; moreover, the interaction of the robot end-effector with the environment could excite the resonant modes due to joint flexibility and cause oscillation and instability. Hence, it is more critical to account for joint flexibility when dealing with motion/force control problems than with pure position control problems. The previous control methods proposed for the motion and force control of flexible joint robots are limited by either requiring the exact knowledge of the dynamic models [69, 49, 20, 51] or dealing only with a single-link case [31].

To remedy these problems, three adaptive and robust control methods have been developed by Lin and Goldenberg [35, 36, 37]. Based on the reduced constrained dynamic model, an adaptive and robust saturation control algorithm [35] and an adaptive and sliding control approach [36] were proposed for the motion and force control of flexible joint robots in constrained motion. The control schemes using a two-stage control strategy were established in a systematic way for flexible joint robots in the general n-link case, without requiring an exact knowledge of the robot manipulator parameters. Based on a generalized transformed dynamic model, a unified control method was developed [37] to further enhance system tracking performance and provide a unified approach for the motion and force control of flexible joint robots in a general coordinate frame (including task or joint spaces).

In this chapter, the three proposed control methods are presented. First, the two control methods based on the reduced constrained dynamic model are described. Then, the generalized transformed dynamic model is derived, and the unified control method is developed. The stability and convergence properties of the proposed control systems are rigorously analyzed via the Lyapunov stability theorem, followed by verification using simulations.

3.2 Constrained Dynamic Model of Flexible Joint Robots

Consider a general non-redundant n-link flexible joint robot moving in contact
with the environment as shown in Figure 3.1.

Figure 3.1: Robot manipulator moving in contact with the environment

Let \( X \in \mathbb{R}^n \) denote the generalized position vector of the end-effector in Cartesian space, and \( q_l \in \mathbb{R}^n \) be the link angles in joint space. Assume that the environment constraints on the robot end-effector can be described by a set of algebraic equations:

\[
\Phi(X) = 0, \tag{3.1}
\]

where the mapping \( \Phi: \mathbb{R}^n \rightarrow \mathbb{R}^k \) is twice continuously differentiable. For a given robot manipulator, the vector \( X \) can be uniquely determined by the joint coordinate \( q_l \) with the kinematic relation

\[
X = H(q_l). \tag{3.2}
\]

Then the constraint equation (3.1) can be expressed in joint space as

\[
\Psi(q_l) = \Phi(H(q_l)) = 0. \tag{3.3}
\]

The Jacobian matrix of the above constraint equation is

\[
J_c(q_l) = \frac{\partial \Psi(q_l)}{\partial q_l} = \frac{\partial \Phi}{\partial X} \frac{\partial H(q_l)}{\partial q_l}. \tag{3.4}
\]

Since \( \Psi(q_l) = 0 \) is identically satisfied, it is evident that \( J_c(q_l)q_l = 0 \). In constrained motion, it is usually assumed that contact between the robot end-effector and the
environment is frictionless. Thus, when the end-effector is moving along the constraint surface, the generalized contact force $f \in \mathbb{R}^n$ in joint space is given by [47]

$$f = J^T_c(q_l)\lambda,$$  \hspace{1cm} (3.5)

where $\lambda \in \mathbb{R}^\kappa$ is the generalized Lagrange multiplier associated to the constraints and represents the independent normal contact force components.

The constrained dynamic equations of a general $n$-link flexible joint robot with joint torque measurements can then be described as follows [66, 26]:

$$D(q_l)\ddot{q}_l + C(q_l, \dot{q}_l)\dot{q}_l + G(q_l) = \tau_s + f = \tau_s + J^T_c(q_l)\lambda$$  \hspace{1cm} (3.6)

$$I_m\ddot{q}_m + B_m\dot{q}_m + \tau_s = u$$  \hspace{1cm} (3.7)

and

$$\tau_s = K_s(q_m - q_l),$$  \hspace{1cm} (3.8)

where all the terms, $D(q_l)$, $C(q_l, \dot{q}_l)$ and $G(q_l)$, and so on, are defined in Chapter 2, except $f$ and $\lambda$, which are defined in the above equation (3.5).

Since the presence of $\kappa$ constraints (3.3) causes the robot to lose $\kappa$ degrees of freedom in motion, $n - \kappa$ linearly independent position coordinates are sufficient to characterize the constrained motion. Let us partition the vector $q_l$ as

$$q_l = \begin{bmatrix} q_l^1 \\ q_l^2 \end{bmatrix},$$  \hspace{1cm} (3.9)

with

$$q_l^1 = [q_{1,1}^1, \ldots, q_{n-\kappa,1}^1]^T \in \mathbb{R}^{n-\kappa}$$

$$q_l^2 = [q_{1,2}^2, \ldots, q_{\kappa,2}]^T \in \mathbb{R}^\kappa.$$  

\[1\] Hereafter, we use "force" to mean a generalized force that could be force and/or torque.
According to the implicit function theorem, there always exists a function $\sigma$ which can be obtained from the constraint equation (3.3) [47], such that

$$q_i^2 = \sigma(q_i^1).$$

(3.10)

Thus, the joint space vector is expressed by $q_i^1$ as

$$q_i = \begin{bmatrix} q_i^1 \\ \sigma(q_i^1) \end{bmatrix}.$$  

Defining

$$L(q_i^1) = \begin{bmatrix} I_{n-x} \\ \frac{\partial \sigma(q_i^1)}{\partial q_i^2} \end{bmatrix},$$

(3.11)

we have

$$\dot{q}_i = \begin{bmatrix} \dot{q}_i^1 \\ \ddot{q}_i^2 \end{bmatrix} = L(q_i^1)\ddot{q}_i^1$$

$$\ddot{q}_i = L(q_i^1)\ddot{q}_i^1 + \dot{L}(q_i^1)\dot{q}_i^1.$$  

Substituting the above relations into (3.6), we get the reduced constrained dynamic equation

$$D(q_i^1)L(q_i^1)\ddot{q}_i^1 + B(q_i^1, \dot{q}_i^1)\dot{q}_i^1 + G(q_i^1) = \tau_s + J_c^T(q_i^1)\lambda,$$

(3.12)

where

$$B(q_i^1, \dot{q}_i^1) = D(q_i^1)\dot{L}(q_i^1) + C(q_i^1, \dot{q}_i^1)L(q_i^1).$$

It can be shown that the reduced constrained dynamic system has the following properties:

**Property 3.1** The LHS of Equation (3.12) can be expressed as [72]

$$D(q_i^1)L(q_i^1)\ddot{q}_i^1 + B(q_i^1, \dot{q}_i^1)\dot{q}_i^1 + G(q_i^1) = Y^1(q_i^1, \dot{q}_i^1, \ddot{q}_i^1)P,$$

(3.13)

where $Y^1(\cdot)$ is an $n \times r$ matrix of known functions, referred to as the regressor, and
$P$ is an $r$-dimensional vector of parameters.

**Property 3.2** Let $A(q^1_i) = L^T(q^1_i)D(q^1_i)L(q^1_i)$; then

$$\dot{A}(q^1_i) - 2L^T(q^1_i)B(q^1_i,q^1_i) = L^T(\dot{D} - 2C)L \quad (3.14)$$

is skew symmetric, as $(\dot{D} - 2C)$ is skew symmetric [62].

**Property 3.3** Since $J_c\dot{q}_l = 0$, i.e., $J_c(q^1_l)L(q^1_l)\dot{q}^1_l = 0$, and $q^1_l$ is linearly independent. we have

$$J_c(q^1_l)L(q^1_l) = 0, \quad L^T(q^1_l)J_c^T(q^1_l) = 0.$$  

Thus, multiplying equation (3.12) from left by $L^T$ yields

$$A(q^1_l)\ddot{q}^1_l + L^T(q^1_l)B(q^1_l,\dot{q}^1_l)\dot{q}^1_l + L^T(q^1_l)G(q^1_l) = L^T(q^1_l)\tau_s. \quad (3.15)$$

For controlling the system with joint torque feedback, we assume that the link angle $q_l$, joint torque $\tau_s$ and constraint force $\lambda$ are available for feedback control. As in Chapter 2, replacing $q_m$ in Equation (3.7) by $\tau_s$ and $q^1_l$, we get

$$I_{ts}\ddot{\tau}_s + B_{ts}\dot{\tau}_s + h_{ts}(q^1_l,\dot{q}^1_l,\ddot{q}^1_l,\tau_s) = u, \quad (3.16)$$

where

$$I_{ts} = I_mK_s^{-1}$$

$$B_{ts} = B_mK_s^{-1}$$

$$h_{ts} = I_m\ddot{q}_l + B_m\dot{q}_l + \tau_s = I_m\ddot{q}_l + (I_m\ddot{L} + B_mL)\dot{q}^1_l + \tau_s.$$  

Equations (3.12) and (3.16) constitute a dynamic representation for the flexible joint robot in constrained motion.
3.3 Control of Constrained Flexible Joint Robots

The objective of constrained robot control is to determine the input torque necessary to achieve trajectory tracking on the constrained surface with specified constraint forces. With the two-stage control strategy described in Chapter 2, control schemes are developed consisting of a constrained motion controller and a joint torque controller. The constrained motion controller is designed to generate the "desired joint torque" $\tau_{sd}$ for controlling motion and force simultaneously, and the joint torque controller computes the required control torque $u$ so that joint torque $\tau_s$ can track the desired joint torque $\tau_{sd}$, and thus the whole system achieves the control objective. The control scheme is shown schematically in Figure 3.2.

For clarity, let us consider first the control design and analysis with all parameters known.

3.3.1 Control scheme with all parameters known

In the case where all the dynamic parameters are known exactly, the constrained motion and joint torque controllers are designed under the following assumptions:

**A1:** the signals $q_l, \dot{q}_l, \tau_s, \dot{\tau}_s, \lambda$ and $\dot{\lambda}$ are available for feedback control;

**A2:** the desired trajectory $q_d(t) \in C^4$ and the desired force $\lambda_d \in C^2$ are bounded.

1. Constrained motion controller
Let us define the tracking errors:

\[ e_t = q_t - q_d \]
\[ e_f = \lambda - \lambda_d \]
\[ e_i = \tau_s - \tau_{sd} \]

and write

\[ v_t^1 = \dot{q}^1_d - \Lambda_t (q^1_t - q^1_d) \]
\[ r_t = \dot{q}^1_t - v_t^1 = (\dot{q}^1_t - \dot{q}^1_d) + \Lambda_t (q^1_t - q^1_d) \]
\[ F_c = J_c^T \lambda_d - J_c^T k_I \int_0^t (\lambda - \lambda_d) dt. \]

The desired joint torque \( \tau_{sd} \) is generated using a method similar to that of Slotine and Li [62]:

\[ \tau_{sd} = D(q_t^1) L(q_t^1) \dot{\psi}_t^1 + B(q_t^1, \dot{q}_t^1) v_t^1 + G(q_t^1) - K_{ID} L(q_t^1) r_t - F_c, \quad (3.17) \]

where \( K_{ID} \), \( k_I \) and \( \Lambda_t \) are diagonal matrices of positive gains.

Subtracting (3.17) from (3.12) and rearranging the terms leads to the following equations:

\[
D(q_t^1) L(q_t^1) \ddot{r}_t + B(q_t^1, \dot{q}_t^1) r_t + K_{ID} Lr_t = e_t + J_c^T [(\lambda - \lambda_d) + k_I \int_0^t (\lambda - \lambda_d) dt] \\
= e_t + J_c^T [e_f + k_I \int_0^t e_f dt] \quad (3.18)
\]

and

\[ A(q_t^1) \ddot{r}_t + L^T B(q_t^1, \dot{q}_t^1) r_t + L^T K_{ID} Lr_t = L^T e_t. \quad (3.19) \]

2. Joint torque controller

In order to control the actual joint torque to follow the desired joint torque \( \tau_{sd} \), the joint torque controller developed in Chapter 2 is adopted and rewritten as follows:

\[ u = I_{ts} \dot{v}_t + B_{ts} v_t + h_{ts} - K_{ID} r_t, \quad (3.20) \]

where \( v_t = \dot{\tau}_{sd} - \Lambda_t (\tau_s - \tau_{sd}) = \dot{\tau}_{sd} - \Lambda_t e_t, \ r_t = \dot{\tau}_s - v_t \), and \( K_{ID} \) and \( \Lambda_t \) are diagonal
positive definite gain matrices. Then, from (3.16) and (3.20), we have

\[ I_t r_t + B_t r_t + K_t D_t r_t = 0. \]  

(3.21)

3. Convergence Analysis

The stability and convergence properties of the closed-loop system are analyzed via the Lyapunov stability theory. The theorem is stated as follows.

**Theorem 3.1** Consider the robot dynamic system (3.12) and (3.16) satisfying assumptions A1 and A2. Using the constrained motion controller (3.17) and joint torque controller (3.20), with control gains satisfying the requirement

\[ \Lambda_t^T K_t D_t \Lambda_t > \frac{1}{\delta} K_t D_t^{-1}, \]  

(3.22)

the solution of the closed-loop system is globally asymptotically stable and tracking errors converge asymptotically to zero, i.e.,

\[ \lim_{t \to \infty} r_t = 0, \quad \lim_{t \to \infty} \dot{r}_t = 0 \]

and

\[ \lim_{t \to \infty} e_t = 0. \]

**Proof:** Choosing the Lyapunov function candidate as

\[ V = \frac{1}{2} r_t^T A(q_t^1) r_t + \frac{1}{2} r_t^T I_t r_t + e_t^T \Lambda_t^T K_t D_t e_t \geq 0, \]  

(3.23)

the derivative of V is

\[ \dot{V} = r_t^T A \dot{r}_t + \frac{1}{2} r_t^T \dot{A} r_t + r_t^T I_t \dot{r}_t + 2 e_t^T \Lambda_t^T K_t D_t \dot{e}_t \]

\[ = r_t^T A \dot{r}_t + \frac{1}{2} r_t^T \dot{A} r_t - r_t^T B_t \dot{r}_t - e_t^T \Lambda_t^T K_t D_t \dot{e}_t - e_t^T \Lambda_t^T K_t D_t \Lambda_t e_t. \]  

(3.24)

From Equation (3.19) and Property 3.2 that \((\dot{A} - 2 L_t^T B)\) is skew symmetric, we have

\[ \dot{V} = -r_t^T L_t K_t D_t \dot{L}_t r_t + r_t^T L_t^T e_t - r_t^T B_t \dot{r}_t - e_t^T K_t D_t \dot{e}_t - e_t^T \Lambda_t K_t D_t e_t \]

\[ = - \begin{bmatrix} (L_t r_t)^T & e_t^T \end{bmatrix} Q \begin{bmatrix} L_t r_t \\ e_t \end{bmatrix} - r_t^T B_t \dot{r}_t - e_t^T K_t D_t \dot{e}_t \]  

(3.25)
with
\[ Q = \begin{bmatrix} K_{iD} & -\frac{1}{2} I \\ -\frac{1}{2} I & \Lambda_t^T K_{iD} \Lambda_t \end{bmatrix}. \]

If the control gains satisfy the requirement (3.22), then \( Q > 0 \), and \( \dot{V} \leq 0 \). Thus, the system is globally asymptotically stable in the sense of Lyapunov, and we have
\[ \lim_{t \to \infty} r_l = 0, \quad \lim_{t \to \infty} r_t = 0. \]

Moreover, as \( t \to \infty \), the system converges to the equilibrium point, Equation (3.18) results in
\[ e_f + k_f \int_0^t e_f dt = 0. \]

Hence, we have \( \lim_{t \to \infty} e_f = 0 \) as well. This completes the proof of the theorem. \( \blacksquare \)

**Remark 1** Similar to the Chapter 2, \( \dot{q}_i^1 \) can be calculated using \( r_s \), \( q_i^1 \) and \( \dot{q}_i^1 \) from Equation (3.15), instead of Equation (3.6). This not only simplifies the computation, but also makes the computation possible in the case when the end-effector force is not available.

Some work on rigid robot control claims that motion and force control of robots in constrained motion can be achieved without the measurement of the end-effector force [72, 55]. In our control scheme, if we let \( F_c = J_c^T \lambda_d \), and follow the exact same steps as in the above convergence analysis, we can show that the system remains asymptotically stable without the end-effector force measurement, and as \( t \to \infty \), Equation (3.18) results in
\[ e_f = 0. \]

Therefore, we have the following statement.

**Corollary 3.1** Using the controllers (3.17) and (3.20), constrained motion control of flexible joint robots can be achieved without force measurement if \( F_c \) in the controller (3.17) is set to be \( J_c^T \lambda_d \), and the statements in Theorem 3.1 still hold.
For the adaptive and robust control developed in the following section, if \( F_c = J_c^T \lambda_d \) is used in the controllers, similar results (corollaries) can be obtained as well.

Although it is relevant to know that our controllers could also be used in the case where force measurement is not available, we will not elaborate on this issue. This is because in [78], it has been shown that the PI type controller is the most desirable for force control, and the P type control or force control without force feedback usually does not give satisfactory control performance in real applications.

### 3.3.2 Adaptive and robust control with all parameters unknown

Considering that, in practice, there are always uncertainties in the system dynamics, we assume a general situation where all parameters, both in the link dynamics and the drive system, including the joint flexibility values, are unknown except for some of their bounds. To overcome the effects of uncertainties on system performance, adaptive and robust control algorithms are developed as follows.

#### 1. Adaptive constrained motion controller

According to Property 3.1 and (3.17), the adaptive control law is derived as

\[
\tau_{sd} = \dot{D}(q^l_i)L(q^l_i)\dot{v}^l_i + \dot{B}(q^l_i, \dot{q}^l_i)v^l_i + \dot{G}(q^l_i) - K_{ID}L(q^l_i)r_i - F_c
\]

\[
= Y^1(q^l_i, \dot{q}^l_i, v^l_i, \dot{v}^l_i)\dot{P} - K_{ID}Lr_i - F_c. \tag{3.26}
\]

Subtracting (3.26) from (3.12) and rearranging the terms yields

\[
D(q^l_i)L(q^l_i)\dot{r}_i + B(q^l_i, \dot{q}^l_i)r_i + K_{ID}Lr_i
\]

\[
= (\dot{D} - D)L\dot{v}^l_i + (\dot{B} - B)v^l_i + (\dot{G} - G) + e_t + J_c^T[(\lambda - \lambda_d) + k_f \int_0^t (\lambda - \lambda_d) dt]
\]

\[
= Y^1(q^l_i, \dot{q}^l_i, v^l_i, \dot{v}^l_i)\dot{P} + e_t + J_c^T[e_f + k_f \int_0^t e_f dt]. \tag{3.27}
\]
with $\dot{P} = \dot{P} - P$. The parameter update law is given as

$$
\dot{P} = -\Gamma^{-1} Y^T(q_i^1, \ddot{q}_i^1, v_i^1, \dot{v}_i^1) L r_i. 
$$

(3.28)

As in Chapter 2, the parameter resetting method could be used to prevent parameter drift of the estimated parameters in the absence of persistent excitation.

2. Robust joint torque controller

In order to deal with the uncertainty in the drive subsystem, two robust control approaches are used. One is the robust saturation-type control algorithm developed in Chapter 2, which mainly concerns the uncertainty of the dynamic parameters, or so called, structured model uncertainty [42]; the other is the sliding mode control method, which is designed to deal with both structured and unstructured model uncertainties.

2.1 Joint torque saturation-type controller

To deal with the structured model uncertainty in the rotor subsystem, the robust joint torque controller developed in Chapter 2 is adopted. The formulas are the same as those in Chapter 2, except that $q_i$ is replaced by $q_i^1$.

The $n \times p$ matrix $Y_t$ and a $p$-dimensional vector of parameters $P_t$ are

$$
Y_t(q_i^1, \ddot{q}_i^1, \tau_s, \dot{\tau}_s, \tau_s) = \text{diag}(\dot{\tau}_s), \text{diag}(\hat{\tau}_s), \text{diag}(\hat{L}q_i^1 + \hat{\dot{L}}q_i^1), \text{diag}(\dot{L}q_i^1), \tau_s
$$

$$
P_t = [I_{tsi},...,B_{tsi},...,I_{mi},...,B_{mi},...,1]^T.
$$

The uncertainty of the parameters is assumed to be bounded, that is, for the available nominal parameters $\hat{P}_t$, there exists $\rho \in R_+$ such that

$$
\| \hat{P}_t \| = \| \hat{P}_t - P_t \| \leq \rho.
$$

(3.29)

The control law is

$$
u = \dot{I}_{ts} \dot{v}_t + \dot{B}_{ts} v_t + \dot{\hat{h}}_{ts} - K_{tD} r_t + Y_t \Delta u,
$$

(3.30)
where $\Delta u$ is defined as

$$
\Delta u = \begin{cases} 
-\rho \frac{Y_t^T r_t}{\| Y_t^T r_t \|} & \text{if } \| Y_t^T r_t \| > \epsilon \\
-\frac{\rho}{\epsilon} Y_t^T r_t & \text{if } \| Y_t^T r_t \| \leq \epsilon 
\end{cases}
$$

with $\epsilon > 0$. The error equation is

$$
I_{ts} \ddot{r}_t + B_{ts} \dot{r}_t + K_t D r_t = (\hat{I}_{ts} - I_{ts}) \dot{\tau}_t + (\hat{B}_{ts} - B_{ts}) \tau_t + (\hat{h}_{ts} - h_{ts}) + Y_t \Delta u \\
= Y_t(q_t^1, \ddot{q}_t^1, \tau, v_t, \dot{v}_t)(\hat{P}_t + \Delta u).
$$

2.2 Joint torque sliding controller

Consider that, in addition to the uncertainty of the dynamic parameters, systems generally have some unmodeled dynamics, measurement noise and external disturbance which can usually be represented as an unknown additive bounded disturbance. To deal with both parameter uncertainty and unknown additive bounded disturbance, we rewrite Equation (3.16) as follows:

$$
\hat{I}_{ts} \ddot{r}_s + \hat{B}_{ts} \dot{r}_s + \hat{h}_{ts}(q_t^1, \ddot{q}_t^1, \tau) + w = u.
$$

where $\hat{I}_{ts}, \hat{B}_{ts}$ and $\hat{h}_{ts}$ are the estimated nominal parameters; $w$ represents the dynamic uncertainty in the system: $w = (I_{ts} - \hat{I}_{ts}) \ddot{r}_s + (B_{ts} - \hat{B}_{ts}) \dot{r}_s + (h_{ts} - \hat{h}_{ts}) + d$, here $d$ denotes the unknown additive bounded disturbance; and $w$ is unknown but assumed to be bounded and can be expressed as

$$
\| w \| \leq \epsilon.
$$

Using the sliding control approach, a sliding surface should be set up so that when the system state trajectory is driven from its initial state onto the sliding surface, the tracking objective is equivalently achieved. For tracking the desired joint torque $\tau_{sd}$,
we define the sliding surface as

\[ s_t = r_t = \dot{r}_s - \dot{r}_{sd} + \Lambda_t(r_s - r_{sd}) = \dot{e}_t + \Lambda_t e_t. \quad (3.35) \]

Based on (3.20), the sliding control law is given as

\[ u = \dot{I}_{ts} \dot{v}_t + \dot{B}_{ts} v_t + \dot{h}_{ts} - K_{td} s_t - k_i sgn(s_t), \quad (3.36) \]

where \( k_i \) is a positive definite gain matrix with minimum eigenvalue \( \lambda_{k_i}^{\text{min}} \). Substituting this control law into (3.33) yields

\[
\begin{align*}
I_{ts} \dot{s}_t + B_{ts} s_t + K_{td} s_t &= (\dot{I}_{ts} - I_{ts}) \dot{v}_t + (\dot{B}_{ts} - B_{ts}) v_t + (\dot{h}_{ts} - h_{ts}) - d - k_i sgn(s_t) \\
&= -w(\dot{q}_l, \ddot{q}_l, \tau, v_t, \dot{v}_t) - k_i sgn(s_t). \quad (3.37)
\end{align*}
\]

3. Convergence analysis

For the purposes of classification, a control system consisting of an adaptive constrained motion controller (3.26) and a joint torque robust saturation-type controller (3.30) is called an adaptive and robust control system; and a control system consisting of an adaptive constrained motion controller (3.26) and a joint torque sliding mode controller (3.36) is called an adaptive and sliding control system. Based on the Lyapunov stability theory, two theorems are proved as follows.

**Theorem 3.2** Using the controllers (3.26), (3.30), the parameter update law (3.28) and \( \Delta u \) (3.31), with \( \epsilon > 0 \) and control gains satisfying the requirement (3.29), the tracking errors of the closed-loop system are uniformly ultimately bounded (u.u.b.).

**Proof:** Choosing the Lyapunov function candidate as

\[
V = \frac{1}{2} r_i^T A(q_i^1) r_i + \frac{1}{2} \dot{\gamma}^T \Gamma \dot{\gamma} + \frac{1}{2} r_i^T I_{ts} r_t + e_t^T \Lambda_t K_{td} e_t \geq 0, \quad (3.38)
\]

through a calculation similar to that in the previous section, the differentiation of \( V \)
If control gains satisfy the requirement (3.23), the matrix \( Q > 0 \). With the parameter update law (3.28) and control \( \Delta u \) (3.31), we show that \( \dot{V} \) is negative as follows. Let us examine the last term in the above. If \( \| Y^T r_t \| > \epsilon \), then

\[
(Y^T r_t)^T (\hat{P}_t + \Delta u) = (Y^T r_t)^T (\hat{P}_t - \rho \frac{Y^T r_t}{\| Y^T r_t \|}) 
\leq \| Y^T r_t \| (\| \hat{P}_t \| - \rho) < 0. \tag{3.40}
\]

If \( \| Y^T r_t \| \leq \epsilon \),

\[
(Y^T r_t)^T (\hat{P}_t + \Delta u) \leq (Y^T r_t)^T (\rho \frac{Y^T r_t}{\| Y^T r_t \|} + \Delta u) = (Y^T r_t)^T (\rho \frac{Y^T r_t}{\| Y^T r_t \|} - \frac{\rho}{\epsilon} Y^T r_t). \tag{3.41}
\]

This last term achieves a maximum value of \( \epsilon \rho / 4 \) when \( \| Y^T r_t \| = \epsilon / 2 \). That is,

\[
\dot{V} \leq - \left[ (Lr_t)^T e_t^T \right] Q \left[ \begin{array}{c} Lr_t \\ e_t \end{array} \right] = - \frac{\| B_t r_t \|}{\| Y^T r_t \|} - \frac{\| K_{tDi} \|}{\| Y^T r_t \|} + \frac{\epsilon \rho}{4} 
\leq -\lambda_{\text{min}}(Q) \| (Lr_t, e_t^T) \|^2 + \min(B_{tsi}) \| r_t \|^2 - \min(K_{tDi}) \| \dot{e}_t \|^2 + \frac{\epsilon \rho}{4}. \tag{3.42}
\]

If \( \lambda_{\text{min}}(Q) \| (Lr_t, e_t^T) \|^2 + \min(B_{tsi}) \| r_t \|^2 + \min(K_{tDi}) \| \dot{e}_t \|^2 \geq \epsilon \rho / 4 \), then \( \dot{V} < 0 \), the uniform ultimate boundedness of tracking errors follows using the results of Leitmann [30]. It can be shown that parameter update law augmented with the parameter resetting preserves the validation of Inequality (3.42). The details of the proof follow the same procedure as described in Appendix B.

The uniform ultimate boundedness of the tracking errors \( r_t \) and \( e_t \) guarantee the
boundedness of $e_f$. As the errors $r_t$ and $e_t$ can be made arbitrarily small by suitable selection of the coefficient $e_t$ from Equation (3.27), it can be seen that the force error $e_f$ will be affected mainly by the error in parameter estimation. The integral action in $F_c$ will reduce the error and make $e_f$ tend to zero.

**Remark 2** As in Chapter 2, $Y_t P_t$ and $Y_t \Delta u$ can be partitioned according to our knowledge of the uncertainty bounds of each parameter component. Correspondingly,

$$\Delta u = \begin{cases} -\frac{\rho_i}{\| Y_t^T r_t \|} & \text{if } \| Y_t^T r_t \| > \epsilon_i \\ -\frac{\rho_i}{\epsilon_i} Y_t^T r_t & \text{if } \| Y_t^T r_t \| \leq \epsilon_i \end{cases} \quad (3.43)$$

Thus, our knowledge of the parameter uncertainty can be better utilized in control design.

**Theorem 3.3** Using the controllers (3.26), (3.36) and the parameter update law (3.28), with control gains satisfying the requirement (3.22), the tracking error of the closed-loop system is asymptotically stable in the sense that $\lim_{t \to \infty} r_t = 0$, $\lim_{t \to \infty} e_t = 0$. And if the regressor signals $Y^1(q_d, q_d)$ are persistently exciting, the force tracking is achieved with $\lim_{t \to \infty} e_f = 0$.

**Proof:** Choosing the Lyapunov function candidate as

$$V = \frac{1}{2} r_t^T A(q_d^t) r_t + \frac{1}{2} \tilde{P}^T \tilde{P} + \frac{1}{2} s_t^T I s_t + e_t^T A_t^T K_t D e_t \geq 0. \quad (3.44)$$

through a calculation similar to that in the previous section, we have the derivative of $V$ as

$$\dot{V} = -\left[ (L r_t^T e_t^T) Q \left[ \begin{array}{c} L r_t \\ e_t \end{array} \right] - s_t^T B s_t - \dot{e}_t^T K d \dot{e}_t \\ + \tilde{P}^T (Y^1 T L r_t + \Gamma \dot{P}) + s_t^T (-w - k_s \text{sgn}(s_t)). \right]. \quad (3.45)$$
The last term satisfies

\[ s_t^T(-w - k_t \text{sgn}(s_t)) \leq (\| s_t^T w \| - \| s_t^T k_t \|) \leq \| s_t \| (\varrho - \lambda_{k_t}^{\min}). \]  

(3.46)

With the parameter update law (3.28), we have

\[ \dot{V} \leq - \left( (Lr_t)^T e_t^T \right) Q \left[ \begin{array}{c} Lr_t \\ e_t \end{array} \right] - s_t^T B_t s_t - \dot{e}_t^T K_t \dot{e}_t + \| s_t \| (\varrho - \lambda_{k_t}^{\min}). \]

(3.47)

In order to meet the requirement of the sliding condition, the gain \( k_t \) is selected such that the minimum eigenvalue \( \lambda_{k_t}^{\min} \) satisfies

\[ \lambda_{k_t}^{\min} > \varrho + \epsilon. \]  

(3.48)

Then

\[ \dot{V} < - \left( (Lr_t)^T e_t^T \right) Q \left[ \begin{array}{c} Lr_t \\ e_t \end{array} \right] - s_t^T B_t s_t - \dot{e}_t^T K_t \dot{e}_t - \epsilon \| s_t \|. \]

(3.49)

Thus, with Condition (3.48), the adaptive and sliding control algorithm guarantees the existence of a sliding mode around the sliding surface \( s_t \). The tracking errors of the closed-loop system are asymptotically stable in the sense of Lyapunov, i.e.,

\[ \lim_{t \to \infty} r_t = 0, \quad \lim_{t \to \infty} e_t = 0. \]

And if the regressor signals \( Y^1(q_1^d, \dot{q}_d^1, \ddot{q}_d^1) \) are persistently exciting, the estimated parameters \( \hat{P} \) converge asymptotically to the true values \( P \) as shown in [64]. Then from Equation (3.27), we have

\[ \lim_{t \to \infty} e_f = 0. \]
Remark 3 Usually, the estimated parameters may not converge to their true values if the persistently exciting condition is not satisfied. But the error in parameter estimation is bounded, and so is the force tracking error. The integral action in $F_c$ can reduce the error.

Remark 4 The control law (3.36) contains a switching function $k_t \text{sgn}(s_t)$, which may lead to high frequency control chattering. To avoid this situation, a continuous function $\text{sat}(s_t)$ can be used to replace $\text{sgn}(s_t)$ [61].

As illustrated in the control design and stability analysis, the adaptive and sliding control method provides better theoretical results and leads to convergence of motion and force tracking errors to zero. But as $\text{sgn}(s_t)$ is replaced by $\text{sat}(s_t)$ in real implementation, the two proposed control systems have the same performance in the sense that they guarantee uniform ultimate boundedness of the tracking errors.

3.4 Simulation Examples

To verify the effectiveness of the proposed control methods, numerical simulations are carried out based on the dynamic model of the IRIS robot [26]. Consider the robot manipulator with an upper arm and a forearm located in a vertical plane, moving in contact with a circular path constraint, as shown in Figure 3.3.

The dynamic model of the rotor subsystems with joint torque sensors is given as follows:

$$
\gamma_1(\gamma_1 + 1)J_{m1}\ddot{q}_{m1} + \gamma_1(\gamma_1 + 1)b_{m1}\dot{q}_{m1} + \tau_{s1} = u_1
$$
$$
\gamma_2(\gamma_2 + 1)J_{m2}\ddot{q}_{m2} + \gamma_2(\gamma_2 + 1)b_{m2}\dot{q}_{m2} + \tau_{s2} = u_2,
$$

and the corresponding link dynamics is

$$
H_{11}\ddot{q}_{l1} + H_{12}\ddot{q}_{l2} - h\dot{q}_{l2}^2 - 2h\dot{q}_{l1}\dot{q}_{l2} + G_1 = \tau_{s1} + f_1
$$
$$
H_{21}\ddot{q}_{l1} + H_{22}\ddot{q}_{l2} + h\dot{q}_{l1}^2 + G_2 = \tau_{s2} + f_2,
$$
Figure 3.3: The schematic diagram of a robot manipulator and the circle constraint where each item and the parameter values are the same as defined in Chapter 2 and Table 2.1.

The constraint surface is expressed in the work space $X = [x, y]^T$ as

$$\Phi(X) = x^2 + y^2 - r^2 = 0$$

The kinematic relationship from the work space to joint space is given by

$$H(q_l) = \begin{bmatrix} l_1 \cos(q_{l1}) + l_2 \cos(q_{l1} + q_{l2}) \\ l_1 \sin(q_{l1}) + l_2 \sin(q_{l1} + q_{l2}) \end{bmatrix}.$$

Thus, in terms of joint space coordinates, the constraint can be expressed as

$$\Psi(q_l) = \Phi(H(q_l)) = l_1^2 + l_2^2 + 2l_1 l_2 \cos(q_{l2}) - r^2 = 0,$$
and the Jacobian matrix of (3.4) is

$$J_c^T(q_t) = \begin{bmatrix} 0 \\ -2l_1l_2 \sin(q_{t2}) \end{bmatrix}.$$  

Since $q_{t2}$ is constant on the constraint surface, let $q_t^1 = q_{t1}$, then the matrix defined in (3.11) is

$$L^T(q_t^1) = [1 \ 0].$$  

The constraint forces are

$$f_1 = 0$$

$$f_2 = -2l_1l_2 \sin(q_{t2}) \lambda.$$  

In the simulations, the desired trajectory $q_d^1$ is generated by a fifth-order polynomial (from $-90^\circ$ to $10^\circ$ in the first 2 seconds, then constant afterwards), and the desired $\lambda_d$ is chosen as a constant (10N). The simulation results are presented in Figures 3.4 to 3.9.

Figure 3.4 is the response of the system in the case where all the parameters are known precisely. It is illustrated that the tracking errors converge to zero. In the presence of parameter uncertainty, the adaptive and robust control scheme and adaptive and sliding control scheme are used. They are then compared with the non-adaptive-robust case. The results are plotted in Figures 3.5, 3.6 and 3.7 respectively. It is shown that the manipulator using the adaptive and robust control schemes can track the desired trajectories closely and have much better tracking performances than the non-adaptive-robust case. The two adaptive and robust control schemes have similar tracking performances; as shown in Figures 3.5 and 3.6, excellent transient and steady-state performance are achieved. While in Figure 3.7, it can be observed that without adaptive and robust control, the system performance is degraded, in particular, the deviation of the actual force from the desired value is significant in the presence of parameter uncertainty. The results show that the proposed adaptive and robust control methods are effective and practical as uncertainty is usually unavoidable in real applications.
Figure 3.4: The response of the system with parameters known precisely
Figure 3.5: The response of the adaptive and robust control system
I. Trajectory tracking curves
Figure 3.5: The response of the adaptive and robust control system
II. Parameter estimation curves
Figure 3.6: The response of the adaptive and sliding control system
Figure 3.7: The response of the non-adaptive-robust control system
The situation where the joint flexibility is neglected in the control design is also studied. Assuming that the parameters are known, by neglecting the joint flexibility, the dynamic model for control design is given as [66]

\[ \ddot{D}(q)\dot{q} + \bar{C}(q, \dot{q})\dot{q} + G(q) = u + f, \]

where \( \ddot{D} = [D(q) + I_m], \bar{C} = [C(q, \dot{q}) + B_m], \) and \( q = q_l = q_m. \) The control law as derived in (3.17) becomes

\[ u = \tau_{sd} = \ddot{D}(q^1)L(q^1)v^1 + \bar{B}(q^1, \dot{q}^1)v^1 + G(q^1) - K_{ID}L(q^1)r - F_c. \]

The simulation result plotted in Figure 3.8 shows that the flexible joint robot system using the rigid control method is unstable if the link angle is used in the feedback control. If the rotor angle is used instead, the system is stable, but it can be seen from Figure 3.9 that oscillations occur and the system performance is degraded.

The simulation results show clearly that joint flexibility should be taken into account in the system control design, and demonstrate the effectiveness of the proposed control scheme.
Figure 3.8: The response of the system using rigid control with link angle feedback

Figure 3.9: The response of the system using rigid control with rotor angle feedback
3.5 A Unified Approach to Motion and Force Control of Flexible Joint Robots

In the previous sections, two adaptive and robust control methods based on the reduced constrained dynamic model were developed. It is noted that control design based on a reduced dynamic model uses only the motion tracking error in its adaptive mechanism. Thus, only motion tracking is achieved and force tracking error can usually only be guaranteed to be bounded. The force error converges to zero if the persistent excitation condition is satisfied \[\text{[72, 35, 36]}\]. For rigid robots, Yao and Tomizuka [80] proposed an adaptive control algorithm to achieve both motion and force tracking. But the control structure is complicated, and the force control gain is restricted to a small value. Based on the derivation of a generalized transformed dynamic model, a new control method was proposed by Lin and Goldenberg [37]. It has two important features: i) it is proved theoretically that, with the proposed new control algorithm, motion, force and joint torque tracking errors converge asymptotically to zero without persistent excitation; and ii) the control scheme provides a unified approach to the motion and force control of flexible joint robots in general coordinate frames (including task space or joint space). Furthermore, friction force on the contact surface can also be compensated for by the control scheme, thus the frictionless contact assumption is eliminated. The details are presented in this section.

3.5.1 Reformulation of constrained dynamic model

Let us rewrite the dynamic equations of a non-redundant, general n-link flexible joint robot with joint torque measurements as follows:

\[
D(q_l)\ddot{q}_l + C(q_l, \dot{q}_l)\dot{q}_l + G(q_l) = \tau_s + f \tag{3.50}
\]

\[
I_m\dddot{q}_m + B_m\ddot{q}_m + \tau_s = u \tag{3.51}
\]

and

\[
\tau_s = K_s(q_m - q_l), \tag{3.52}
\]
where each symbol represents the same item previously defined.

In this section, we will not limit ourselves to joint space. Letting $F$ denote the contact force in task space, we have

$$f = J^T F,$$  \hspace{1cm} (3.53)

where $J \in \mathbb{R}^{nxn}$ is the Jacobian matrix, which is assumed to be nonsingular in the defined work space.

The environmental constraints are described by a set of algebraic equations rewritten in detail as follows:

$$\Phi(X) = 0, \quad \Phi(X) = [\phi_1(X), \ldots, \phi_\kappa(X)]^T, \quad \kappa \leq n,$$  \hspace{1cm} (3.54)

where the mapping $\Phi: \mathbb{R}^n \rightarrow \mathbb{R}^\kappa$ is twice continuously differentiable. The contact force $F$ in task space can be written as [80]

$$F = F_n + F_t = (\eta^T(X) + \xi^T(\mu, X, \dot{X}))\lambda,$$  \hspace{1cm} (3.55)

where $\lambda \in \mathbb{R}^\kappa$ is the generalized Lagrange multiplier;

$$F_n = \eta^T(X)\lambda \in \mathbb{R}^n, \quad \eta(X) = \frac{\partial \Phi}{\partial X} \in \mathbb{R}^{\kappa \times n}$$

represents the constraint force, i.e., the normal contact force in Cartesian space; and

$$F_t = \xi^T(\mu, X, \dot{X})\lambda \in \mathbb{R}^n, \quad \xi \in \mathbb{R}^{\kappa \times n}$$

is the friction force in the direction tangential to the surface. The magnitude of $F_t$ depends on the normal contact force $\lambda$ and the friction coefficient $\mu$, with the sign determined by the end-effector velocity. Function $\xi$ is linear in $\mu$. Under the assumption of frictionless contact surface, $F_t = 0$. For the constrained motion of robots, the constraint force $F_n$ need to be controlled.

Since the presence of $\kappa$ constraints causes the robot to lose $\kappa$ degrees of free-
dom, only \((n - \kappa)\) position coordinates can be specified independently, i.e., only \((n - \kappa)\) position coordinates of the robot need to be controlled in the constrained motion. Properly choosing \((n - \kappa)\) linearly independent curvilinear coordinates \(\Psi(X) = [\psi_1(X), \cdots, \psi_{n-\kappa}(X)]^T\), combining with the constraints (3.54), the motion of the robot is then uniquely determined. Let us define the general curvilinear coordinates as

\[
r = \begin{bmatrix} r_p \\ r_f \end{bmatrix}
\]  

(3.56)

with

\[
\begin{aligned}
r_p &= \Psi(X) = [\psi_1(X), \cdots, \psi_{n-\kappa}(X)]^T \in \mathbb{R}^{n-\kappa} \\
r_f &= \Phi(X) = [\phi_1(X), \cdots, \phi_{\kappa}(X)]^T = 0 \in \mathbb{R}^\kappa.
\end{aligned}
\]

Differentiating (3.56) yields

\[
\dot{r} = J_X \dot{X} = J_X J_q \dot{q}_l = J_q \dot{q}_l,
\]

(3.57)

where

\[
\begin{aligned}
J_X &= \frac{\partial r(X)}{\partial X} = \left[ \begin{array}{c} \frac{\partial \psi}{\partial X} \\ \frac{\partial \phi}{\partial X} \\ \eta(X) \end{array} \right] \in \mathbb{R}^{n \times n} \\
J_q &= \frac{\partial r(X(q_l))}{\partial q_l} = \frac{\partial r(X)}{\partial X} \frac{\partial X(q_l)}{\partial q_l} = J_X J \in \mathbb{R}^{n \times n}.
\end{aligned}
\]

Denoting a transformation matrix \(T(r) = J_q^{-1}\), we have

\[
\begin{aligned}
\dot{q}_l &= J_q^{-1} \dot{r} = T(r) \dot{r} \\
\ddot{q}_l &= T(r) \ddot{r} + \dot{T}(r) \dot{r}.
\end{aligned}
\]

Substituting the above relations into (3.50), we get

\[
D(r)T \ddot{r} + [D(r)\dot{T} + C(r, \dot{r})T] \dot{r} + G(r) = \tau_s + f.
\]

(3.58)
Premultiplying by $T^T$, Equation (3.58) can be rewritten in compact form as

$$A(r)\ddot{r} + B(r, \dot{r})\dot{r} + W(r) = T^T \tau_s + T^T f, \quad (3.59)$$

where

$$A(r) = T^T D(r) T$$
$$B(r, \dot{r}) = T^T [D(r) \dot{T} + C(r, \dot{r}) T] = T^T H(r, \dot{r})$$
$$W(r) = T^T G(r).$$

Thus, using the transformation matrix $T$, the transformed constrained dynamic equation is derived from the dynamic equation (3.50) with the constraints (3.54). This equation possesses the following properties:

**Property 3.4** By similarity transformation, matrix $A$ is still a bounded positive definite, symmetric matrix, and

$$\dot{A}(r) - 2B(r, \dot{r}) = T^T (\dot{D} - 2C) T \quad (3.60)$$

is skew symmetric.

**Property 3.5** The LHS of Equation (3.59) can be expressed as

$$A(r)\ddot{r} + B(r, \dot{r})\dot{r} + W(r) = Y_r(r, \dot{r}, \ddot{r}) P, \quad (3.61)$$

where $Y_r(\cdot)$ is an $n \times i$ matrix of known functions, referred to as the regressor, and $P$ is an $i$-dimensional vector of parameters.

**Property 3.6** Since $\Phi(X) = \Phi(X(q_l)) \equiv 0$, it is evident that $\eta(X)J(q_l)\dot{q}_l = 0$. If we partition $T = [L, M]$, then $\dot{q}_l = T\dot{r} = L\dot{r}_p$, and we have

$$\eta(X)J(q_l)L\dot{r}_p = 0.$$
As \( r_p \) are linearly independent coordinates, it follows that

\[
\eta(X)J(q_t)L = 0, \quad L^T J^T \eta^T(x) = 0
\]

and

\[
L^T J^T \eta^T(x)\lambda = L^T J^T F_n = 0
\]
or

\[
T^T J^T F_n = \begin{bmatrix} L^T \\ M^T \end{bmatrix} J^T F_n = \begin{bmatrix} 0 \\ M^T J^T \eta^T(x)\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ f_n \end{bmatrix}.
\]

This means that by similarity transformation, corresponding to \((n - \kappa)\) independent position coordinates \( r_p \), the constraint force \( F_n \) can always be transformed into \( \kappa \) independent components to be controlled. The \((n - \kappa)\) independent coordinates \( r_p \), combined with \( \kappa \) independent force components \( f_n = M^T J^T \eta^T(x)\lambda \), span the \( n \)-dimensional vector space.

The above properties are essential for designing the motion/force controller.

We write Equation (3.16) again here just for clarity:

\[
I_{ts}\ddot{r}_s + B_{ts}\dot{r}_s + h_{ts}(r, \dot{r}, \ddot{r}, \tau_s) = u, \quad (3.62)
\]

where

\[
I_{ts} = I_m K_s^{-1} \\
B_{ts} = B_m K_s^{-1} \\
h_{ts} = I_m \ddot{q}_t + B_m \dot{q}_t + \tau_s = I_m T\ddot{r} + (I_m \dot{T} + B_m T)\dot{r} + \tau_s.
\]

It should be noted that \( q_t^l, \dot{q}_t^l \) and \( \ddot{q}_t^l \) are replaced by \( r, \dot{r} \) and \( \ddot{r} \).

Equations (3.59) and (3.62) constitute a new dynamic representation of the flexible joint robot in constrained motion.

It is obvious that the derived transformed dynamic model is a general model.
because the choice of \( r_p = \Psi(X) \) is arbitrary. By properly choosing \( r_p \), the model can be expressed in generalized joint or task spaces. Hence, the dynamic model gives a unified expression for flexible joint robots in constrained motion, and consequently, the proposed control scheme developed in the following provides a unified approach for the motion and force control of flexible joint robots.

### 3.5.2 Adaptive constrained motion/force controller

The proposed motion/force controller is distinct from the previous constrained motion controller in that both motion and force tracking errors are used to construct the auxiliary tracking signal in the controller. Let us define the tracking errors:

\[
e_p = r_p - r_{pd} \]

\[
e_f = \lambda - \lambda_d \]

\[
e_t = \tau_s - \tau_{sd},
\]

where \( r_{pd} = \Psi(X(q_d(t))) \in R^{n-x} \) is the desired robot motion trajectory, and \( r_{pd} \in C^4 \) is bounded; \( \lambda_d \in R^c \) is the desired constraint force trajectory. According to Property 3.6, define an n-dimensional vector \( S \) as

\[
S = \begin{bmatrix} \hat{e}_p + k_p e_p \\ 0 \\ k_f \int_0^t e_f dt \end{bmatrix} + \begin{bmatrix} 0 \\ k_f \int_0^t e_f dt \end{bmatrix} = \begin{bmatrix} S_p \\ S_f \end{bmatrix}
\]  

(3.63)

and write

\[
v_r = \hat{\tau} - S = \begin{bmatrix} \dot{r}_{pd} - k_p e_p \\ -k_f \int_0^t e_f dt \end{bmatrix}. \quad (3.64)
\]

The following motion and force controller is proposed to generate the desired joint torque \( \tau_{sd} \):

\[
T^T \tau_{sd} = \dot{A}(r) \dot{u}_r + \dot{B}(r, \dot{r}) v_r + \dot{W} - T^T f - (T^T K_D T) S - K_{FD} \dot{S}_f
\]

(3.65)

or

\[
\tau_{sd} = T^{-T} (T^T \tau_{sd}) = \dot{D} T \dot{u}_r + \dot{H} v_r + \dot{G} - f - K_D TS - T^{-T} K_{FD} \dot{S}_f
\]

(3.66)
where $K_{FD} = [0, k_{FD}]^T$; $k_p$, $k_f$, $K_D$ and $k_{FD}$ are diagonal matrices of positive gains; and $\hat{A}$, $\hat{B}$ and $\hat{W}$ are matrices with the estimated parameters.

Substituting (3.65) into (3.59) leads to the following closed-loop dynamic equations:

$$
A \dot{S} + BS = T^T e_t + (\hat{A} - A) \dot{v}_r + (\hat{B} - B)v_r + (\hat{W} - W) - (T^T K_D T) S - K_{FD} \dot{S}_f
$$

$$
= T^T e_t + Y_r(r, \dot{r}, v_r, \dot{v}_r) \tilde{P} - (T^T K_D T) S - K_{FD} \dot{S}_f, \tag{3.67}
$$

with $\tilde{P} = \hat{P} - P$. The parameter update law is given as

$$
\dot{\tilde{P}} = -\Gamma^{-1} Y_r^T(r, \dot{r}, v_r, \dot{v}_r) S. \tag{3.68}
$$

It is noted that the parameter estimation is updated by both motion and force tracking errors $S$.

### 3.5.3 Robust joint torque sliding controller

To deal with the uncertainty in the system, the sliding controller (3.36) developed in Section 3.3.2 is used except that $q_i$, $\dot{q}_i$ and $\ddot{q}_i$ are now replaced by $r$, $\dot{r}$ and $\dddot{r}$.

### 3.5.4 Asymptotic convergence

The stability and convergence properties of the entire control system are analyzed based on the Lyapunov stability theory. With the proposed control scheme, we have the following theorem:

**Theorem 3.4** Consider the robot dynamic system (3.59) and (3.62). If control gains satisfy the requirement (3.22), then the adaptive constrained motion/force controller (3.66) with the parameter update law (3.68), and the joint torque sliding controller (3.36) guarantee that motion and force tracking errors of the closed-loop system converge asymptotically to zero, i.e.,

$$
\lim_{t \to \infty} e_p = 0, \quad \lim_{t \to \infty} e_f = 0 \quad \text{and} \quad \lim_{t \to \infty} e_t = 0.
$$
Proof: To prove the theorem, in addition to Lemma 2.1, the Lemmas cited in Appendix A are used to facilitate our subsequent analysis. Choosing the Lyapunov function candidate as

$$V = \frac{1}{2} S^T A(r)S + \frac{1}{2} \dot{P}^T \Gamma \dot{P} + \frac{1}{2} S_f^T k_fD S_f + \frac{1}{2} s_t^T I_{ts} s_t + e_t^T \Lambda_t^T K_{ID} e_t \geq 0,$$  \hspace{1cm} (3.69)

the differentiation of $V$ is

$$\dot{V} = S^T A \dot{S} + \frac{1}{2} S^T \dot{A}S + \dot{P}^T \Gamma \dot{P} + S_f^T k_fD \dot{S}_f + s_t^T I_{ts} \dot{s}_t + 2e_t^T \Lambda_t^T K_{ID} \dot{e}_t.$$  \hspace{1cm} (3.70)

Using the fact that $(\dot{A} - 2B)$ is skew symmetric and Equation (3.67),

$$S^T A \dot{S} + \frac{1}{2} S^T \dot{A}S = S^T (A \dot{S} + BS) = S^T (T^T e_t + Y_r \dot{P} - T^T K_D TS - K_fD \dot{S}_f),$$  \hspace{1cm} (3.71)

together with Equation (3.37), we have

$$\dot{V} = S^T T^T e_t - S^T T^T K_D TS - S^T K_fD \dot{S}_f + S_f^T k_fD \dot{S}_f - s_t^T B_{ts} s_t - e_t^T K_{ID} \dot{e}_t - e_t^T \Lambda_t^T K_{ID} \Lambda_t e_t + \dot{P}^T (Y_r^T S + \Gamma \dot{P}) + s_t^T (-w - k_t sgn(s_t)).$$  \hspace{1cm} (3.72)

With the parameter update law (3.68) and the equality $S^T K_fD \dot{S}_f = S_f^T k_fD \dot{S}_f$,

$$\dot{V} = -S^T T^T K_D TS + S^T T^T e_t - s_t^T B_{ts} s_t - e_t^T K_{ID} \dot{e}_t - e_t^T \Lambda_t^T K_{ID} \Lambda_t e_t + s_t^T (-w - k_t sgn(s_t))$$

$$= - \left[ (TS)^T e_t^T \right] Q \left[ \begin{array}{c} TS \\ e_t \end{array} \right] - s_t^T B_{ts} s_t - e_t^T K_{ID} \dot{e}_t + s_t^T (-w - k_t sgn(s_t)),$$  \hspace{1cm} (3.73)
where

\[ Q = \begin{bmatrix} \mathbf{K}_D & -\mathbf{I} \\ -\mathbf{I} & \Lambda_t^T \mathbf{K}_D \Lambda_t \end{bmatrix}. \]

By Lemma 2.1, matrix \( Q \) is positive definite if the requirement (3.22)

\[ \Lambda_t^T \mathbf{K}_D \Lambda_t > \frac{1}{4} \mathbf{K}_D^{-1} \]

is satisfied. Since \( \mathbf{K}_D, \Lambda_t \) and \( \mathbf{K}_D \) are diagonal matrices of controller gains, we can choose them properly to meet the requirement so that \( Q > 0 \). The last term in (3.73) is

\[ s_t^T (-w - k_t \text{sgn}(s_t)) \leq (\| s_t^T w \| - \| s_t^T k_t \|) \leq \| s_t \| (\varrho - \lambda_{k_t}^{\min}). \quad (3.74) \]

To meet the requirement of the sliding condition, the gain \( k_t \) is selected such that the minimum eigenvalue \( \lambda_{k_t}^{\min} \) satisfies

\[ \lambda_{k_t}^{\min} > \varrho + \epsilon. \quad (3.75) \]

Then

\[ \dot{V} < - \left[(T S)^T \ e_t^T \right] Q \left[ \begin{array}{c} TS \\ e_t \end{array} \right] - s_t^T B s_t - e_t^T \mathbf{K}_D \mathbf{e}_t - \epsilon \| s_t \|. \quad (3.76) \]

Thus, the entire system is stable in the sense of Lyapunov. We have \( \bar{P} \) bounded, \( S \in L_\infty \cap L_2 \), and \( s_t \in L_\infty \cap L_2 \). From Equation (3.35), it immediately follows that

\[ \lim_{t \to \infty} e_t = 0. \]

In Equation (3.67), we should note that \( \dot{v}_r = [\dot{v}_{\mathbf{r}_1}^T, -\dot{\mathbf{S}}_f^T]^T, \dot{v}_{\mathbf{r}_1} = \ddot{r}_p - k_p \dot{e}_p \). Rewriting
the equation as

\[
A \begin{bmatrix} \dot{S}_p \\ 0 \end{bmatrix} + A \begin{bmatrix} 0 \\ \dot{S}_f \end{bmatrix} + (\dot{A} - A) \begin{bmatrix} 0 \\ \dot{S}_f \end{bmatrix} + K_{FD} \dot{S}_f =
\]

\[-BS + (\dot{A} - A) \begin{bmatrix} \dot{v}_r \\ 0 \end{bmatrix} + (\dot{B} - B)v_r + (\dot{W} - W) + T^T e_t - (T^T K_{D} T) S, \quad (3.77)\]

partitioning \( A \) as \( A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \), where \( a_{11} \in R^{(n-\kappa)\times(n-\kappa)}, a_{22} \in R^{\kappa\times\kappa}, a_{12} \) and \( a_{21} \in R^{(n-\kappa)\times\kappa} \), and writing

\[
A_f = \begin{bmatrix} \dot{a}_{11} & \dot{a}_{12} \\ \dot{a}_{21} & \dot{a}_{22} + k_{FD} \end{bmatrix},
\]

\[
(\dot{A} - A) \begin{bmatrix} \dot{v}_r \\ 0 \end{bmatrix} + (\dot{B} - B)v_r + (\dot{W} - W) = Y_{r1}(r, \dot{r}, v_r, S_f, \dot{v}_r) \dot{P},
\]

we have

\[
A \begin{bmatrix} \dot{S}_p \\ 0 \end{bmatrix} + A_f \begin{bmatrix} 0 \\ \dot{S}_f \end{bmatrix} = -BS + Y_{r1} \dot{P} + T^T e_t - (T^T K_{D} T) S. \quad (3.78)
\]

The RHS of this equation is a bounded function (say, \( f_{\text{bounded}} \)) because \( S, \dot{P}, B \) and \( Y_{r1} \) are bounded and \( e_t \to 0 \). Matrices \( A \) and \( A_f \) are nonsingular and bounded. Premultiplying equation (3.78) by \([0, I]A^{-1}\), and writing \( A^{-1}A_f = \bar{A} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \),

we obtain

\[
\alpha_{22} \dot{S}_f = [0, I]A^{-1}f_{\text{bounded}}.
\]

Since \( \alpha_{22} \) is nonsingular as \( A^{-1}A_f \) is nonsingular, \( \dot{S}_f \) is bounded. Similarly, premultiplying equation (3.78) by \([I, 0]A^{-1}_f\), it can be shown that \( \dot{S}_p \) is bounded. Thus, \( \dot{S} \) is bounded. It implies that \( S \) is uniformly continuous, and \( S \to 0 \), which in turn implies \( e_p \to 0, \dot{e}_p \to 0 \) and \( S_f \to 0 \).

In order to show that \( e_f \to 0 \), we need to show \( \dot{S}_f \to 0 \), or \( \dot{S} \to 0 \). Differentiating
Equation (3.78) gives

\begin{align*}
A \begin{bmatrix} \ddot{S}_p \\ 0 \end{bmatrix} + A_f \begin{bmatrix} 0 \\ \ddot{S}_f \end{bmatrix} = \dot{A} \begin{bmatrix} \dot{S}_p \\ 0 \end{bmatrix} + \dot{A}_f \begin{bmatrix} 0 \\ \dot{S}_f \end{bmatrix} \\
-\dot{B} \dot{S} - \dot{B} S + Y_{r1} \dot{P} + Y_{r1} \ddot{P} + T^T \dot{e}_t + \dot{T}^T e_t - (T^T K_D T) \dot{S} - 2T^T K_D \dot{T} S.
\end{align*}

(3.79)

Since \( S, e_t \) and \( \dot{P} \) converge to zero, \( B, T, \dot{P} \) and \( \dot{S} \) are bounded, which in turn implies that \( r, \dot{r}, \ddot{r} \) are bounded because the desired trajectory is bounded and \( r_{pd} \in C^3 \). \( \dot{Y}_{r1}, \dot{A}, \) and \( \dot{A}_f \) are bounded. Hence, the RHS of the equation is bounded. Using a similar process as above, it can be shown that \( \ddot{S}_f \) and \( \ddot{S}_p \), i.e., \( \ddot{S} \), are bounded, so that \( \dot{S} \) is uniformly continuous and \( \dot{S} \to 0 \). The proof of the theorem is thus completed.

In the proof, it has been shown that motion and force tracking errors can converge simultaneously to zero without persistent excitation conditions, which are usually difficult to satisfy in practice. The nonsingularity of \( A_f \) can be ensured by modifying the parameter update law with the resetting method [34]. Distinct from the control method of [80], the proposed control scheme has a relatively simple formulation with PI type force feedback control function, and the restriction that the force feedback gain must be very small is not required. Friction force on the contact surface can also be compensated for by the control scheme (the term \( T^T f \) in (3.65)), so that the frictionless contact assumption is no longer needed.

As mentioned in the previous section, \( \text{sgn}(s_t) \) can be replaced by \( \text{sat}(s_t) \) to avoid control chattering, with the cost that the tracking errors become to be u.u.b.

### 3.5.5 Simulation examples

Consider the two-link flexible joint robot manipulator in a vertical plane moving along a horizontal line, as shown in Figure 3.10.

The dynamic equations used in simulations are the same as the previous ones. For simplicity, assume \( l_1 = l_2 = 0.25m \).
Figure 3.10: The schematic diagram of a robot manipulator and the constraint

The Jacobian matrix \( J \) is

\[
J = \begin{bmatrix}
-l_1 \sin(q_{11}) - l_2 \sin(q_{11} + q_{12}) & -l_2 \sin(q_{11} + q_{12}) \\
l_1 \cos(q_{11}) + l_2 \cos(q_{11} + q_{12}) & l_2 \cos(q_{11} + q_{12})
\end{bmatrix}.
\]

The constraint is expressed in task space \( X = [x, y]^T \) as

\[
\Phi(X) = y = 0.
\]

With the proposed control scheme, if we choose \( r_p = x \) and \( r_f = \Phi(X) = y = 0 \), then

\[
J_q = J X J = J \quad \text{as} \quad J X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I,
\]

and we have \( T = J_q^{-1} = J^{-1} \). The contact force can be determined to be

\[
F_n = [0, 1]^T \lambda, \quad F_t = [1, 0]^T \mu \text{sgn}(\dot{x}) \lambda,
\]

and just as indicated by Property 3.6,

\[
T^T J^T F_n = [0, 1]^T \lambda.
\]

The constraint can also be expressed in the generalized joint space as

\[
\Phi(X(q_1)) = q_{12} + 2q_{11} = 0.
\]
If we choose \( r_p = q_1 \) and \( r_f = \Phi(q) = q_{12} + 2q_1 = 0 \), then

\[
J_q = \frac{\partial r(X(q_1))}{\partial q_i} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad T = J_q^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix},
\]

and

\[
T^T J^T F_n = \begin{bmatrix} 0 \\ l_2 \cos(q_1 + q_{12}) \end{bmatrix} \lambda.
\]

The proposed control method provides a unified approach for the control the robot manipulator in any suitably chosen coordinate space. Numerical simulations for the above two choices of coordinate \( r \) have been conducted.

For \( r = [x, 0]^T \), the desired trajectories are set to be

\[
\begin{align*}
    r_{pd} &= 2l_1 \cos(40 + 20 \cos(0.5 \pi t)) \text{ (meter)} \\
    \lambda_d &= 15 \text{ (N)}.
\end{align*}
\]

For \( r = [q_{11}, 0]^T \), the desired trajectories are given as

\[
\begin{align*}
    r_{pd} &= 40 + 20 \cos(0.5 \pi t) \text{ (degree)} \\
    \lambda_d &= 20 + 10 \cos \pi t \text{ (N)}.
\end{align*}
\]

The results are plotted in Figures 3.11 and 3.12 respectively; they illustrate that the system is stable and has very good tracking performance. The controller can successfully control the robot in generalized coordinate space (either task space or joint space).

### 3.6 Summary

In this chapter, the motion and force control of flexible joint robots in constrained motion is considered, and three adaptive and robust control methods are introduced. Based on the reduced constrained dynamic model, the adaptive and robust saturation control scheme ensures the uniform ultimate boundedness of the tracking errors; the adaptive and sliding control scheme can deal with more general uncertainty cases and achieves global asymptotic convergence of tracking errors. Distinct from previous work, the proposed control schemes using a two-stage control strategy provide a
Figure 3.11: The response of the control system in task space
Figure 3.12: The response of the control system in joint space
systematic approach for the motion and force control of flexible joint robots in the general n-link case, and do not require an exact knowledge of robot dynamics by assuming that all system parameters including the joint flexibility values are unknown except for some of their bounds. Furthermore, a generalized transformed dynamic model is derived with three important properties. Based on this model, a unified control method is proposed, which provides a unified approach for the motion and force control of flexible joint robots in a general coordinate frame (including task space or joint space). It has been shown that with the proposed control scheme, motion, force and joint torque tracking errors converge simultaneously to zero without requiring persistent excitation. To our knowledge, this has not previously been proved for flexible joint robots. Simulation results confirm the effectiveness of the proposed control methods.
Chapter 4

Adaptive and Robust Control of Multiple Flexible Joint Robots Holding a Common Object

4.1 Introduction

As the application of robots is extended from simple tasks, such as painting and welding, to more sophisticated missions, such as assembling mechanical parts and environmental material handling, there is a growing interest in the development of coordinated multi-manipulator systems. Multi-manipulator systems can perform tasks recognized to be beyond the capability of a single manipulator. Handling a heavy and bulky object is a typical example.

The control issues of multi-manipulator systems are more complex than that of a single robot manipulator as coordinated manipulation is required. When multiple robot manipulators hold a common object in cooperative manipulation, closed kinematic chains are formed. These impose kinematic and dynamic constraints and result in internal forces (stress) on the object and robots. If the manipulators are not carefully coordinated, conflicting motions among them may cause excessive stress or even damage to the object or robots. The control issue is further complicated if the manipulated object is required to be in contact with the environment. Force
arising from contact with the environment has a significant impact on the object and robotic system, and should be controlled simultaneously with the motion as well as the internal forces. These problems have attracted much attention from researchers in recent years. However, as mentioned in Chapter 1, so far only a few studies have addressed the coordinated control of multiple flexible joint robots [18, 21, 1]. Besides, the methods developed were based on a full knowledge of the system dynamics.

In this chapter, adaptive and robust control methods for coordinated multiple flexible joint robot manipulators [40, 39] are presented. Control issues of multiple flexible joint robots holding a common object in free motion and constrained motion are both considered. Concise dynamic models of multiple flexible joint robot manipulators holding a common object are derived to facilitate the control design. Combined adaptive and robust control methods are developed under the assumption that all system parameters including the joint flexibility values are unknown except for some of their bounds. Stability analysis shows that with the proposed control methods, simultaneous control of motion, internal force and contact force is achieved.

4.2 Dynamic Model of a Multiple Flexible Joint Robot System

4.2.1 Multiple flexible joint robots holding a common object in free motion

Let us first consider the case where multiple flexible joint robots hold a common object in free motion. As shown in Figure 4.1, there are \( \gamma \) (\( \gamma \geq 2 \)) robot manipulators. All robot end-effectors hold the same object at \( \gamma \) specified contact points and move in a coordinated fashion to transport the object along a pre-specified trajectory. In establishing the dynamic model of the system, the following assumptions are made:

- Each manipulator is nonredundant; hence all manipulators have the same number of joints, say \( n \).
All the end-effectors of the manipulators are firmly holding the object so that there is no relative motion at the contact points and there is perfect force transmission between the robots and object.

Figure 4.1: Multiple flexible joint robots holding an object

To describe the system geometry, kinematics and dynamics, coordinate frames are located as shown in Figure 4.1 so that the positions and orientations of the robots and object can be determined. The frames $O_j\{X_j, Y_j, Z_j\}$, $j = 1, ..., \gamma$ represent the end-effector frames of the robots; the frame $O_t\{X_t, Y_t, Z_t\}$ is fixed at the object's center of mass; and a world coordinated frame $O\{X, Y, Z\}$ is established on a base, to which all end-effectors of the robot manipulators and object are referenced.

The overall system dynamics consists of the dynamics of each robot manipulator and the common object. The understanding of motion and force transformation between different frames is essential in deriving the system dynamic models. In order to help in this respect, a schematic mapping diagram is plotted in Figure 4.2 to demonstrate the motion and force transformation relationship among joint space, end-effector frames and object frame. (Each quantity in the diagram will be defined in the following sections when it appears in an equation.)
Dynamic Equations of Multiple Flexible Joint Robots

The dynamic equations of multiple flexible joint robots with joint torque measurements can be described as follows [66, 26]:

\[
D_j(q_{i,j})\ddot{q}_{i,j} + C_j(q_{i,j}, \dot{q}_{i,j})\dot{q}_{i,j} + G_j(q_{i,j}) = \tau_{s,j} - J_j^T F_j
\]

\[
I_{m,j}\ddot{\theta}_{m,j} + B_{m,j}\dot{\theta}_{m,j} + \tau_{s,j} = u_j,
\]

and

\[
\tau_{s,j} = K_{s,j}(q_{m,j} - q_{i,j}),
\]

where the subscript \(j\) denotes the robot manipulator number, \(j = 1, 2, ..., \gamma\); \(D_j(q_{i,j})\) is the \(n \times n\) positive definite, symmetric inertia matrix of robot \(j\); \(C_j(q_{i,j}, \dot{q}_{i,j})\) is the \(n \times n\) matrix containing coriolis, and centripetal terms; \(G_j(q_{i,j})\) is the \(n \times 1\) vector...
of gravitational forces; $I_{m,j}$ is the $n \times n$ constant diagonal matrix representing the inertia of the rotors and gears; $B_{m,j}$ is the $n \times n$ diagonal matrix of damping terms; and $K_{s,j}$ is an $n \times n$ diagonal matrix of the joint torsional stiffness. The $n$-dimensional vectors $q_{i,j}$ and $q_{m,j}$ represent the link angles and rotor angles, respectively; $u_j$ is the input torque control vector; $\tau_{s,j}$ is the vector of joint torque measurements; and $F_j$ is the vector of end-effector force of robot $j$. Finally $J_j$ is the $n \times n$ Jacobian matrix of robot $j$ from joint space to the end-effector frame.

For simplicity, let

$$D = \text{block diag}(D_1, \ldots, D_\gamma) \in \mathbb{R}^{n\gamma \times n\gamma}$$
$$C = \text{block diag}(C_1, \ldots, C_\gamma) \in \mathbb{R}^{n\gamma \times n\gamma}$$
$$G = [G_1^T, \ldots, G_\gamma^T]^T \in \mathbb{R}^{n\gamma}$$
$$I_m = \text{block diag}(I_{m,1}, \ldots, I_{m,\gamma}) \in \mathbb{R}^{n\gamma \times n\gamma}$$
$$B_m = \text{block diag}(B_{m,1}, \ldots, B_{m,\gamma}) \in \mathbb{R}^{n\gamma \times n\gamma}$$
$$q_l = [q_{l,1}^T, \ldots, q_{l,\gamma}^T]^T \in \mathbb{R}^{n\gamma}$$
$$q_m = [q_{m,1}^T, \ldots, q_{m,\gamma}^T]^T \in \mathbb{R}^{n\gamma}$$
$$\tau_s = [\tau_{s,1}^T, \ldots, \tau_{s,\gamma}^T]^T \in \mathbb{R}^{n\gamma}$$
$$u = [u_{1}^T, \ldots, u_{\gamma}^T]^T \in \mathbb{R}^{n\gamma}$$
$$F = [F_1^T, \ldots, F_\gamma^T]^T \in \mathbb{R}^{n\gamma}$$
$$J = \text{block diag}(J_1, \ldots, J_\gamma) \in \mathbb{R}^{n\gamma \times n\gamma}.$$

The dynamic equations of the multiple flexible joint robots can be expressed more concisely as

$$D(q_l)\ddot{q}_l + C(q_l, \dot{q}_l)\dot{q}_l + G(q_l) = \tau_s - J^TF$$ \hspace{1cm} (4.4)
$$I_m\ddot{q}_m + B_m\dot{q}_m + \tau_s = u$$ \hspace{1cm} (4.5)

and

$$\tau_s = K_s(q_m - q_l).$$ \hspace{1cm} (4.6)
With joint torque measurements $\tau_s$, from Equation (4.6), we have

$$q_m = K_s^{-1} \tau_s + q_l.$$  

Substituting this equality into Equation (4.5), we get

$$I_{ts} \ddot{\tau}_s + B_{ts} \dot{\tau}_s + h_{ts}(r, \dot{r}, \ddot{r}, \tau_s) = u,$$  

where

$$I_{ts} = I_m K_s^{-1}$$
$$B_{ts} = B_m K_s^{-1}$$
$$h_{ts} = I_m \ddot{q}_l + B_m \dot{q}_l + \tau_s.$$

Dynamic Equation of the Object

The dynamic equation for the common object can be derived by the Newton-Euler formulation.

$$M_o(X_o) \ddot{X}_o + N_o(X_o, \dot{X}_o) \dot{X}_o + G_o(X_o) = \sum_{j=1}^{\gamma} J_{oj}^T F_j = J_o^T F,$$  

where $M_o$ is the inertia matrix of the object, $N_o$ is the Coriolis and centrifugal terms, and $G_o$ is the vector of gravitational forces. Here $X_o$ is an $n$-dimensional vector denoting the Cartesian coordinate of the mass center of the object; $J_{oj}$ is the $n \times n$ Jacobian matrix from the object frame to the $j^{th}$ end-effector frame, $J_o = [J_{o1}^T, \ldots, J_{o\gamma}^T]^T \in \mathbb{R}^{m \times n}$.

Similar to the robot dynamics, the object dynamics has the property that the matrix $(\dot{M}_o - 2N_o)$ is skew symmetric (See Appendix C for details).

The Combined Robot/Object Dynamics

Since each robot manipulator holds the object firmly at the contact point that
forms the close kinematic chain, we have the following relationship:

\[ \dot{X}_e = J \dot{q}_l = J_o \dot{X}_o, \]  

(4.9)

where \( \dot{X}_e \) is the velocity of the end-effectors of the robots. If all robot manipulators work in a non-singular work space, \( J \) is a non-singular matrix. We then have

\[ \dot{q}_l = J^{-1} J_o \dot{X}_o \]  

(4.10)

\[ \ddot{q}_l = J^{-1} J_o \ddot{X}_o + \frac{d}{dt} (J^{-1} J_o) \dot{X}_o \]  

(4.11)

\[ = J^{-1} J_o \ddot{X}_o + (J^{-1} \dot{J}_o - J^{-1} \dot{J} J^{-1} J_o) \dot{X}_o. \]

The matrix \( J_o \) is full rank, but it is not square. Thus, for a given trajectory \( X_o(t) \), the applied force \( F \) is not unique and can be expressed in the form as

\[ F = J_o^+ \{ M_o(X_o) \dot{X}_o + N_o(X_o, \dot{X}_o) \ddot{X}_o + G_o(X_o) \} + F_l = F_{mc} + F_l, \]  

(4.12)

where \( J_o^+ = J_o (J_o^T J_o)^{-1} \). Here \( F_l \) represents an internal force (stress within the system), that produces zero generalized force and does not affect the motion of the object. It can be any vector in the null space of \( J_o^T \). The values of \( F_l \) are usually specified according to the desired squeezing force on the object [52] and load distribution among the robot arms [16, 17]. Thus, the applied force \( F \) is divided into two parts: \( F_{mc} \), which contributes directly to the object dynamics (motion and force), and \( F_l \), which does not affect the object dynamics.

With the above relations, substituting Equations (4.10), (4.11) and (4.12) into (4.4), we get

\[ M_l \ddot{X}_o + N_l \dot{X}_o + G_l = \tau_s - J^T F_l \]  

(4.13)

with

\[ M_l = (DJ^{-1}J_o + J^T J_o^+ M_o) \]

\[ N_l = D \frac{d}{dt} (J^{-1}J_o) + CJ^{-1}J_o + J^T J_o^+ N_o \]
\[ G_l = G + J^T J_o^+ G_o. \]

Premultiplying both sides of (4.13) by \( J_o^T J^{-T} \) and noting that \( J_o^T J^{-T} J^T F_I = J_o^T F_I = 0 \), we have

\[ M \ddot{X}_o + N \dot{X}_o + W = J_o^T J^{-T} \tau, \tag{4.14} \]

with

\[
M = J_o^T J^{-T} D J^{-1} J_o + M_o \\
N = J_o^T J^{-T} D \frac{d}{dt}(J^{-1} J_o) + J_o^T J^{-T} C J^{-1} J_o + N_o \\
W = J_o^T J^{-T} G + G_o.
\]

The combined robot/object dynamics equations possess the following properties:

**Property 4.1** The matrix \( M \) is a bounded positive definite, symmetric matrix, and

\[
\dot{M} - 2N = J_o^T J^{-T} \dot{D} J^{-1} J_o + \dot{M}_o + 2(J_o^T J^{-T})D \frac{d}{dt}(J^{-1} J_o) \\
-2(J_o^T J^{-T})D \frac{d}{dt}(J^{-1} J_o) - 2J_o^T J^{-T} C J^{-1} J_o - 2N_o \\
= J_o^T J^{-T} (\dot{D} - 2C) J^{-1} J_o + (\dot{M}_o - 2N_o) \tag{4.15}
\]

is skew symmetric.

**Property 4.2** The LHS of Equations (4.13) and (4.14) can be expressed as

\[
M_l \ddot{X}_o + N_l \dot{X}_o + G_l = Y_l(X_o, \dot{X}_o, \ddot{X}_o) P \tag{4.16} \\
M \ddot{X}_o + N \dot{X}_o + W = Y(X_o, \dot{X}_o, \ddot{X}_o) P, \tag{4.17}
\]

where \( Y_l(.) \) and \( Y(.) \) are matrices of known functions, referred to as regressors, and \( P \) is a vector of parameters.
4.2.2 Multiple flexible joint robots holding a common object in constrained motion

In many applications, it is required that the manipulated object be in contact with the environment and exert a specified contact force on the environment. Let us consider $\gamma$ robot arms holding a rigid object moving along a rigid surface in a coordinated fashion, as shown in Figure 4.3, under the same assumptions as in the previous section. To derive the dynamic model of the system in this case, the corresponding mapping diagram of the motion and force transformation relationship is plotted in Figure 4.4.

![Figure 4.3: Multiple flexible joint robots holding an object in constrained motion](image)

**Dynamic Equation of Multiple Flexible Joint Robots**

Because the multiple robot manipulators holding a common object are already in a closed chain, the end-effectors of all the robots are constrained whether the object is in free motion or constrained motion, and hence the dynamic equations of multiple flexible joint robots remain the same as those in the previous section.
Dynamic Equation of the Constrained Object

The dynamic equation for the constrained object needs to be modified to reflect the contact force that arises from contact with the environment:

\[ M_o(X_o) \ddot{X}_o + N_o(X_o, \dot{X}_o) \dot{X}_o + G_o(X_o) = J_o^TF - F_{oc}, \]

where \( M_o, N_o \) and \( G_o \) are defined as in the previous section, and \( F_{oc} \) is the contact force exerted by the environment on the object.

The environmental constraint is described as a holonomic smooth manifold and can be expressed in terms of \( X_o \) as

\[ \Psi(X_o) = 0, \]

where the mapping \( \Psi: R^n \rightarrow R^\kappa, n > \kappa \), is twice continuously differentiable. The Jacobian matrix of the above constraint equation is

\[ J_c(X_o) = \frac{\partial \Psi(X_o)}{\partial X_o}. \]

Since \( \Psi(X_o) = 0 \) is identically satisfied, it is evident that \( J_c(X_o) \dot{X}_o = 0 \). When the object is moving along the constrained surface, the constraint force on the object is
given by [47]

\[ F_{oc} = J_z^T(X_o)\lambda, \]  

(4.21)

where \( \lambda \in \mathbb{R}^\kappa \) is the associated Lagrange multiplier.

**The Combined Robot/Object Dynamics**

With the relations of (4.10), (4.11) and the following slight modification of Equation (4.12)

\[ F = J_o^+ \{ M_o(X_o)\ddot{X}_o + N_o(X_o, \dot{X}_o)\dot{X}_o + G_o(X_o) + F_{oc} \} + F_I = F_{mc} + F_I \]  

(4.22)

we substitute them into (4.4) and get

\[ M_I \ddot{X}_o + N_I \dot{X}_o + G_I = \tau_s - J^T J_o^+ F_{oc} - J^T F_I. \]  

(4.23)

Premultiplying both sides by \( J_o^T J^{-T} \), we have

\[ M \ddot{X}_o + N \dot{X}_o + W = J_o^T J^{-T} \tau_s - F_{oc}, \]  

(4.24)

where \( M_I, N_I, G_I, M, N \) and \( W \) are defined as in the previous section. The equation possesses the same properties as mentioned in the previous section.

**Reduced Dynamic Model**

Since the presence of \( \kappa \) constraints causes the multi-robot system to lose \( \kappa \) degrees of freedom, \( n - \kappa \) linearly independent coordinates are sufficient to characterize the constrained motion of the object. Let us partition the vector \( X_o \) as

\[ X_o = \begin{bmatrix} X_o^1 \\ X_o^2 \end{bmatrix} \]  

(4.25)

with

\[ X_o^1 = [X_{o1}^1, \ldots, X_{o(n-\kappa)}^1]^T \in \mathbb{R}^{n-\kappa} \]
\[ X_o^2 = [X_{o1}^2, \ldots, X_{on}^2]^T \in R^\kappa. \]

According to the implicit function theory, from the constraint equation (4.19), \( X_o^2 \) can be expressed as \( X_o^2 = \sigma(X_o^1) \), [47].

Defining
\[
L(X_o^1) = \begin{bmatrix}
I_{n-\kappa} \\
\frac{\partial \sigma(X_o^1)}{\partial X_o^1}
\end{bmatrix},
\]
we have
\[
\dot{X}_o = \begin{bmatrix}
\dot{X}_o^1 \\
\dot{X}_o^2
\end{bmatrix} = L(X_o^1)\dot{X}_o^1
\]
\[
\dot{X}_o = L(X_o^1)\dot{X}_o^1 + \dot{L}(X_o^1)\dot{X}_o^1.
\]

Substituting the above relations into (4.23) and (4.24), we get
\[
M_i L(X_o^1)\ddot{X}_o^1 + H_i \dot{X}_o^1 + G_i = \tau_s - J^T J^T F_{oc} - J^T F_l \tag{4.27}
\]
\[
ML(X_o^1)\ddot{X}_o^1 + B(X_o^1, \dot{X}_o^1)\dot{X}_o^1 + W(X_o^1) = J_o^T J^T \tau_s - J_e^T \lambda \tag{4.28}
\]
where
\[
H_i(X_o^1, \dot{X}_o^1) = M_i \dot{L} + N_i L
\]
\[
B(X_o^1, \dot{X}_o^1) = M \dot{L}(X_o^1) + N(X_o^1, \dot{X}_o^1)L(X_o^1) = J_o^T J^T H_i(X_o^1, \dot{X}_o^1).
\]

It can be shown that the reduced dynamic equations have the following properties:

**Property 4.3** The LHS of Equations (4.27) and (4.28) can be expressed as
\[
M_i L(X_o^1)\ddot{X}_o^1 + H_i \dot{X}_o^1 + G_i = Y_i^1(X_o^1, \dot{X}_o^1, \ddot{X}_o^1)P \tag{4.29}
\]
\[
ML(X_o^1)\ddot{X}_o^1 + B(X_o^1, \dot{X}_o^1)\dot{X}_o^1 + W(X_o^1) = Y^1(X_o^1, \dot{X}_o^1, \ddot{X}_o^1)P, \tag{4.30}
\]
where \( Y^1(.) \) is an \( n \times r \) matrix of known functions, referred to as the regressor, and \( P \) is an \( r \)-dimensional vector of parameters.

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Property 4.4 Let \( A(X_o^1) = L^T(X_o^1)ML(X_o^1) \), then \( A \) is a bounded positive definite, symmetric matrix, and

\[
\dot{A}(X_o^1) - 2L^T(X_o^1)B(X_o^1, \dot{X}_o^1) = L^T(M - 2N)L
\]  

(4.31)

is skew symmetric.

Property 4.5 Since \( J_c \dot{X}_o = 0 \), i.e., \( J_c(X_o^1)L(X_o^1)\dot{X}_o^1 = 0 \), and \( X_o^1 \) is linearly independent, we have

\[
J_cL(X_o^1) = 0, \quad L^T(X_o^1)J_c^T = 0.
\]

Thus, multiplying Equation (4.28) from left by \( L^T \) yields

\[
A(X_o^1)\ddot{X}_o^1 + L^T B(X_o^1, \dot{X}_o^1)\dot{X}_o^1 + L^TW(X_o^1) = L^TJ_o^TJ^{-T}\tau_s.
\]

(4.32)

With the above derived dynamic equations and properties, we are now ready to design the adaptive and robust control laws.

### 4.3 Adaptive and Robust Control Scheme

The successful coordinated operation of multiple robots transporting an object along a pre-specified trajectory requires the simultaneous control of both motion and contact force, as well as precise internal force regulation. The objective of flexible joint robot control is to determine the input torque to meet these requirements. Based on the two-stage control strategy [36], a control scheme consisting of a coordinated controller and a joint torque controller is developed in such a way that the coordinated controller is designed to generate the "desired joint torque" \( \tau_{sd} \), and the joint torque controller computes the required control torque \( u \) so that joint torque \( \tau_s \) can follow the desired joint torque \( \tau_{sd} \). Thus the whole system achieves the control objective.

In practice, it is usually impossible to obtain the exact dynamic model of a multiple flexible joint robot system. To deal with the parameter uncertainty in the multiple robot system, adaptive and robust controllers are developed assuming that all system
parameters including the joint flexibility values are unknown except for some of their bounds.

4.3.1 Adaptive coordinated controller

Coordinated controllers for both free motion and constrained motion are presented in this section.

Adaptive Coordinated Controller I (free motion control)

In the case where the manipulated object is moving freely in a work space, let $X_d(t)$ and $F_{ld}(t)$ denote the desired motion of the object and the desired internal force, respectively, and define the tracking errors

$$e_x = X_o - X_d$$
$$e_{lf} = F_l - F_{ld}$$
$$e_t = r_s - r_{sd}$$

and the auxiliary signals

$$v_l = \dot{X}_d - \Lambda_l(X_o - X_d)$$
$$\tau_l = \dot{X}_o - v_l = (\dot{X}_o - \dot{X}_d) + \Lambda_l(X_o - X_d).$$

The proposed coordinated controller is designed with two parts: a motion controller $\tau_{mc}$ and an internal force controller $\tau_{lf}$

$$\tau_{mc} = \hat{M}_l\ddot{v}_l + \hat{N}_l v_l + \hat{G}_l - K_{lD}J^{-1}J_o \tau_l$$
$$\tau_{lf} = J^T[F_{ld} + K_{IP}(F_{ld} - F_l) + K_{II} \int_0^t (F_{ld} - F_l)dt],$$

where $K_{lD}$, $K_{IP}$, $K_{II}$ and $\Lambda_l$ are diagonal matrices of positive gains.

The desired joint torque $\tau_{sd}$ is then generated as

$$\tau_{sd} = \tau_{mc} + \tau_{lf}.$$  \hspace{1cm} (4.35)

Substituting (4.35) into (4.13) and rearranging the terms leads to the following
equations:

\[ M\ddot{r}_l + N\dot{r}_l + K_{lD}J^{-1}J_0\dot{r}_l = (\dot{M}_l - M_l)\dot{v}_l + (\dot{N}_l - N_l)v_l + (\dot{G}_l - G_l) \]

\[ + e_t + J^T[(I + K_{lP})(F_{Id} - F_l) + K_{II} \int_0^t (F_{Id} - F_l)dt] \]

\[ = Y_l(X_o, \dot{X}_o, v_l, \dot{v}_l)\ddot{P} + e_t - J^T[(I + K_{lP})e_{If} + K_{II} \int_0^t e_{If}dt], \quad (4.36) \]

and

\[ M\ddot{r}_l + N\dot{r}_l + J_o^TJ^{-T}K_{lD}J^{-1}J_0\dot{r}_l = Y(X_o, \dot{X}_o, v_l, \dot{v}_l)\ddot{P} + J_o^TJ^{-T}e_t, \quad (4.37) \]

with \( \dot{P} = \dot{P} - P \). The parameter update law is given as

\[ \dot{P} = -\Gamma^{-1}Y^T\tau_l = -\Gamma^{-1}Y_l^T(J^{-1}J_0)\tau_l. \quad (4.38) \]

Adaptive Coordinated Controller II (constrained motion control)

For constrained motion of the object, we define the tracking errors

\[ e_x = X_o^d - X_o^1 \]
\[ e_f = \lambda - \lambda_d \]
\[ e_{If} = F_l - F_{Id} \]
\[ e_t = \tau_s - \tau_{sd} \]

and the auxiliary signals

\[ v_l^1 = \dot{X}_o^1 - \Lambda_l(X_o^d - X_o^1) \]
\[ r_l = \dot{X}_o^1 - v_l^1 = (\dot{X}_o^1 - \dot{X}_d^1) + \Lambda_l(X_o^1 - X_o^d) \],

where \( X_o^d(t) \), \( \lambda_d \) and \( F_{Id}(t) \) denote the desired motion of the object, the desired contact force and the desired internal force, respectively.

The proposed coordinated controller consists explicitly of three components, namely, a motion controller \( \tau_{mc} \), a contact force controller \( \tau_{fc} \), and an internal force controller \( \tau_{If} \).

\[ \tau_{mc} = \dot{M}_l\dot{v}_l^1 + \dot{H}_l v_l^1 + \dot{G}_l - K_{lD}J^{-1}J_0\dot{r}_l \quad (4.39) \]
\[
\tau_{fc} = J^T J^+ [J^T \lambda_d + J^T k_{FP} (\lambda_d - \lambda) + J^T k_{FI} \int_0^t (\lambda_d - \lambda) dt]
\]
(4.40)
\[
\tau_{IF} = J^T [F_{ld} + K_{IP} (F_{ld} - F_I) + K_{II} \int_0^t (F_{ld} - F_I) dt],
\]
(4.41)

where \( K_{ID}, k_{FP}, k_{FI}, K_{IP}, K_{ID} \) and \( \Lambda_l \) are diagonal matrices of positive gains.

The desired joint torque \( \tau_{sd} \) is given as
\[
\tau_{sd} = \tau_{mc} + \tau_{fc} + \tau_{IF}.
\]
(4.42)

Substituting (4.42) into (4.27) and rearranging the terms leads to the following equations:
\[
M_l \ddot{\mathbf{q}}_l + H_l \dot{\mathbf{q}}_l + K_{ID} J^{-1} J_0 L r_l = Y_1^T (\mathbf{x}_1^l, \dot{\mathbf{x}}_1^l, v_1^l, \dot{v}_1^l) \mathbf{P} + \mathbf{e}_t
\]
\[
- J^T J^+ [J^T (I + k_{FP}) \mathbf{e}_f + J^T k_{FI} \int_0^t \mathbf{e}_f dt]
\]
\[
- J^T [(I + K_{IP}) \mathbf{e}_{IF} + K_{II} \int_0^t \mathbf{e}_{IF} dt],
\]
(4.43)
\[
M \ddot{\mathbf{q}}_l + B \dot{\mathbf{q}}_l + J_0^T J^{-T} K_{ID} J^{-1} J_0 L r_l = Y_1^T (\mathbf{x}_1^l, \dot{\mathbf{x}}_1^l, v_1^l, \dot{v}_1^l) \mathbf{P}
\]
\[
+ J_0^T J^{-T} \dot{\mathbf{e}}_l - J_0^T [(I + k_{FP}) \mathbf{e}_f + k_{FI} \int_0^t \mathbf{e}_f dt],
\]
(4.44)

and
\[
A \ddot{\mathbf{r}}_l + L^T B \dot{\mathbf{r}}_l + L^T J_0^T J^{-T} K_{ID} J^{-1} J_0 L \mathbf{r}_l
\]
\[
= L^T Y_1^T (\mathbf{x}_1^l, \dot{\mathbf{x}}_1^l, v_1^l, \dot{v}_1^l) \mathbf{P} + L^T J_0^T J^{-T} \dot{\mathbf{e}}_l
\]
(4.45)

with \( \mathbf{P} = \dot{\mathbf{P}} - \mathbf{P} \), and the parameter update law as
\[
\dot{\mathbf{P}} = -\Gamma^{-1} Y_1^T L r_l = -\Gamma^{-1} Y_1^T (J^{-1} J_0) L r_l.
\]
(4.46)

**Remark 1** The proposed adaptive coordinated controllers consisting explicitly of motion, internal force and contact force control components, have clear physical meaning and concise formulation. Comparing controllers I and II, it could be easily observed
that controller I can be regarded as a special case of controller II. If we set \( \tau_{fe} = 0 \), \( L = I \) and \( X^1 = X \), controller II reduces to controller I.

**Remark 2** This new adaptive coordinated controller can be used directly for the control of rigid joint robots; it possesses the advantages of controlling the motion, internal force and contact force simultaneously without requiring an exact knowledge of the dynamic parameters of the multirobot system. Compared with [17], the proposed controller has a more concise and simpler formulation and does not require: i) the inverses of inertia matrices; and ii) the derivative of the end-effector force.

### 4.3.2 Robust joint torque controller

In the presence of joint flexibility, the joint torque controller is designed to drive the actual joint torque to follow the desired joint torque \( \tau_{sd} \). To deal with the uncertainty in the rotor subsystem, a sliding mode control approach is used. We define the sliding surface as

\[
s_t = \dot{\tau}_s - \dot{\tau}_{sd} + \Lambda_t (\tau_s - \tau_{sd}) = \dot{e}_t + \Lambda_t e_t,
\]

and write

\[
v_t = \dot{\tau}_s - s_t = \dot{\tau}_{sd} - \Lambda_t e_t.
\]

For tracking the desired joint torque \( \tau_{sd} \), the joint torque sliding control law developed in Chapter 3 [36] is adopted:

\[
u = \dot{I}_{ts} \dot{v}_t + \dot{\hat{B}}_{ts} v_t + \dot{\hat{h}}_{ts} - K_{tD} s_t - k_t \text{sgn}(s_t),
\]

where \( \dot{I}_{ts}, \dot{\hat{B}}_{ts} \) and \( \dot{\hat{h}}_{ts} \) are the estimated nominal parameters; \( K_{tD} \) and \( \Lambda_t \) are diagonal positive definite gain matrices; and \( k_t \) is a positive definite gain matrix with minimum eigenvalue \( \lambda_{k_t} \).

Substituting this control law into (4.7), yields

\[I_{ts} \dot{s}_t + B_{ts} s_t + K_{tD} s_t = w(\dot{r}, \ddot{r}, \tau_s, \dot{v}_t, \dot{u}_t) - k_t \text{sgn}(s_t),\]
where \( w = (\hat{I}_{ts} - I_{ts})\dot{\psi}_t + (\hat{B}_{ts} - B_{ts})\psi_t + (\hat{h}_{ts} - h_{ts}) + d \) represents the dynamic uncertainty in the rotor subsystem. As the uncertainty is bounded, it could be expressed as

\[
\| w \| \leq \rho. \tag{4.50}
\]

### 4.3.3 Stability analysis

The stability and convergence properties of the entire control system are analyzed based on the Lyapunov stability theory. With the proposed control scheme, we have the following theorems under the condition that control gains satisfy the requirement

\[
\Lambda_t^T K_{iD} \Lambda_t > \frac{1}{4} K_{iD}^{-1}. \tag{4.51}
\]

**Theorem 4.1** Consider the multiple flexible joint robot system holding a common object in free motion. Using the adaptive coordinated controller I (4.35) with the parameter update law (4.38) and the joint torque sliding controller (4.48), the tracking error of the closed-loop system is asymptotically stable in the sense that

\[
\lim_{t \to \infty} e_x = 0, \quad \text{and} \quad \lim_{t \to \infty} e_t = 0.
\]

And if the regressor signals \( Y(X_d, \dot{X}_d, \ddot{X}_d) \) are persistently exciting, the internal force tracking is achieved with

\[
\lim_{t \to \infty} e_{if} = 0.
\]

**Theorem 4.2** Consider the multiple flexible joint robot system holding a constrained object. Using the adaptive coordinated controller II (4.42) with the parameter update law (4.46) and the joint torque sliding controller (4.48), the tracking error of the closed-loop system is asymptotically stable in the sense that

\[
\lim_{t \to \infty} e_x = 0, \quad \text{and} \quad \lim_{t \to \infty} e_t = 0.
\]

And if the regressor signals \( Y^1(X_d^1, \dot{X}_d^1, \ddot{X}_d^1) \) are persistently exciting, the contact force and internal force tracking are achieved with

\[
\lim_{t \to \infty} e_f = 0, \quad \text{and} \quad \lim_{t \to \infty} e_{if} = 0.
\]
We prove Theorem 4.2 below. Theorem 4.1 can be proved in the same way. In fact, as pointed out in Remark 1, if $\tau_f = 0$, $L = I$, and $X^1 = X$ in the coordinated controller II and $F_{oc} = 0$ in the dynamic model, the proof of Theorem 4.1 follows the proof of Theorem 4.2.

**Proof of Theorem 4.2:** Choosing the Lyapunov function candidate as

$$V = \frac{1}{2} r_l^T A r_l + \frac{1}{2} \dot{P}^T \Gamma \dot{P} + \frac{1}{2} s_t^T I_{ts} s_t + e_t^T \Lambda_t^T K_{tD} e_t > 0,$$

(4.52)

the differentiation of $V$ yields

$$\dot{V} = r_l^T A \dot{r}_l + \frac{1}{2} r_l^T A \dot{r}_l + \dot{P}^T \Gamma \dot{P} + s_t^T I_{ts} \dot{s}_t + 2 e_t^T \Lambda_t^T K_{tD} \dot{e}_t.$$

(4.53)

Using the fact that $(\dot{A} - 2L^T B)$ is skew symmetric and Equation (4.45),

$$r_l^T A \dot{r}_l + \frac{1}{2} r_l^T A \dot{r}_l = -L^T J_o^T J^{-T} K_{tD} J^{-1} J_o L r_l$$

$$+ L^T J_o^T J^{-T} e_t + L^T Y^1 (X_o^1, \dot{X}_o^1, v_1, \dot{v}_1) \dot{P}).$$

(4.54)

Together with Equation (4.49), we have

$$\dot{V} = -r_l^T L^T J_o^T J^{-T} K_{tD} J^{-1} J_o L r_l + r_l^T L^T J_o^T J^{-T} e_t$$

$$- s_t^T B_{ts} s_t - \dot{e}_t^T K_{tD} e_t - \dot{e}_t^T \Lambda_t^T K_{tD} \Lambda_t e_t$$

$$+ \dot{P}^T (Y^1 L r_l + \Gamma \dot{P}) + s_t^T (w - k_t \text{sgn}(s_t)).$$

(4.55)

With the parameter update law (4.46), it becomes

$$\dot{V} = - \left[ (J^{-1} J_o L r_l)^T e_t^T \right] Q \left[ J^{-1} J_o L r_l \right] - s_t^T B_{ts} s_t$$

$$- \dot{e}_t^T K_{tD} e_t + s_t^T (w - k_t \text{sgn}(s_t)),$$

(4.56)
where

\[
Q = \begin{bmatrix}
K_lD & -\frac{1}{2}I \\
-\frac{1}{2}I & \Lambda_t^T K_lD \Lambda_t
\end{bmatrix}.
\]

Since \( K_lD, \Lambda_t \) and \( K_lD \) are diagonal matrices of controller gains, it is possible to choose them properly to meet the requirement (4.51), so that \( Q > 0 \). The last term in (4.56) is

\[
s_t^T (w - k_t \text{sgn}(s_t)) \leq (\| s_t^T w \| - \| s_t^T k_t \|)
\leq \| s_t \| (\varrho - \lambda_{k_t}^{\text{min}}). \tag{4.57}
\]

To meet the requirement of the sliding condition, the gain \( k_t \) is selected such that the minimum eigenvalue \( \lambda_{k_t}^{\text{min}} \) satisfies

\[
\lambda_{k_t}^{\text{min}} > \varrho + \epsilon \tag{4.58}
\]

Then

\[
\dot{V} < - \left[ (J^{-1}J_o L r_t)^T e_t^T \right] Q \left[ J^{-1}J_o L r_t \varepsilon_t \right] - s_t^T B_t s_t - \hat{c}_t^T K_l D \hat{c}_t - \epsilon \| s_t \| \tag{4.59}
\]

The tracking errors of the closed-loop system are asymptotically stable in the sense of Lyapunov, i.e.,

\[
\lim_{t \to \infty} r_t = 0, \quad \lim_{t \to \infty} e_t = 0.
\]

And if the regressor signals \( Y^1(X_d^1, \dot{X}_d^1, \ddot{X}_d^1) \) are persistently exciting, the estimated parameters \( \hat{P} \) converge asymptotically to the true values \( P \). Then from Equations (4.44) and (4.43), we have

\[
\lim_{t \to \infty} e_f = 0, \quad \lim_{t \to \infty} e_{ff} = 0.
\]
Usually, the estimated parameters may not converge to their true values as the persistently exciting condition is not satisfied. But the error of the parameter estimation is bounded. From Equations (4.44) and (4.43), it is clear that the force tracking errors are also bounded. The integral action and the proportional gain \((I + k_{IP})\) and \((I + K_{IP})\) in \(\tau_f c\) and \(\tau_{IF}\) can reduce the errors and make them as small as required.

4.4 Simulation Examples

To verify the proposed control methods, the control schemes are applied to the coordinated manipulation of two planar robot manipulators. For simplicity, we assume that the two robots are identical: each has three joint-links, and the coordinate frames are located as shown in Figure 4.5. The position of the object moving in a horizontal plane is determined by the coordinates \(x_o, y_o\) and the orientation \(\theta_o\).

Based on the configuration, the Jacobian matrices are derived as follows:

\[
J_{o,j} = \begin{bmatrix}
1 & 0 & S_{123,j}\tau_j \\
0 & 1 & -C_{123,j}\tau_j \\
0 & 0 & 1
\end{bmatrix}, \quad j = 1, 2
\]
\[
J_j = \begin{bmatrix}
-l_1 \sin(q_{11,j}) - l_2 S_{12,j} - l_3 S_{123,j} & -l_2 S_{12,j} - l_3 S_{123,j} & -l_3 S_{123,j} \\
l_1 \cos(q_{11,j}) + l_2 C_{12,j} + l_3 C_{123,j} & l_2 C_{12,j} + l_3 C_{123,j} & l_3 C_{123,j} \\
1 & 1 & 1
\end{bmatrix}, \quad j = 1, 2
\]

where
\[
S_{12,j} = \sin(q_{11,j} + q_{12,j})
\]
\[
C_{12,j} = \cos(q_{11,j} + q_{12,j})
\]
\[
S_{123,j} = \sin(q_{11,j} + q_{12,j} + q_{13,j})
\]
\[
C_{123,j} = \cos(q_{11,j} + q_{12,j} + q_{13,j}).
\]

with \( j = 1, 2 \).

The system parameters are given in Table 4.1.

<table>
<thead>
<tr>
<th>( m_o = 3kg )</th>
<th>( r_1 = 0.25m )</th>
<th>( r_2 = 0.25m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 = 1kg )</td>
<td>( l_1 = 1m )</td>
<td>( l_{c1} = 0.5m )</td>
</tr>
<tr>
<td>( m_2 = 1kg )</td>
<td>( l_2 = 1m )</td>
<td>( l_{c2} = 0.5m )</td>
</tr>
<tr>
<td>( m_3 = 0.25kg )</td>
<td>( l_3 = 0.25m )</td>
<td>( l_{c3} = 0.125m )</td>
</tr>
<tr>
<td>( I_{m1} = 0.02kgm^2 )</td>
<td>( B_{m1} = 5.0 \frac{Nm}{(rad/s)} )</td>
<td>( K_{s1} = 1000(Nm/rad) )</td>
</tr>
<tr>
<td>( I_{m2} = 0.015kgm^2 )</td>
<td>( B_{m2} = 4.0 \frac{Nm}{(rad/s)} )</td>
<td>( K_{s2} = 800(Nm/rad) )</td>
</tr>
<tr>
<td>( I_{m3} = 0.005kgm^2 )</td>
<td>( B_{m3} = 2.0 \frac{Nm}{(rad/s)} )</td>
<td>( K_{s3} = 600(Nm/rad) )</td>
</tr>
</tbody>
</table>

The system dynamics is established with
\[
D_j = \begin{bmatrix}
d_{11,j} & d_{12,j} & d_{13,j} \\
d_{21,j} & d_{22,j} & d_{23,j} \\
d_{31,j} & d_{32,j} & d_{33,j}
\end{bmatrix}, \quad C_j = \begin{bmatrix}
c_{11,j} & c_{12,j} & c_{13,j} \\
c_{21,j} & c_{22,j} & c_{23,j} \\
c_{31,j} & c_{32,j} & c_{33,j}
\end{bmatrix}, \quad j = 1, 2
\]

and
\[
M_o = \text{diag}\{m_o, m_o, I_o\}, \quad N_o = 0.
\]

The details of the dynamic equations of the system are derived in Appendix D.
Simulations are performed using Simnon. The control scheme is digitally implemented with a sampling rate of 1 msec.

4.4.1 Free motion control

Suppose that the robots are required to move the object in a circle without changing the object's orientation, and to maintain a constant "push" force (internal force) against each other. The desired motion and internal force trajectories are given as

\[
\begin{align*}
x_d &= 0.5 + 0.25\cos(t) \\
y_d &= 1.0 + 0.25\sin(t) \\
\theta_d &= 0 \\
F_{td} &= [25 \ 0 \ 0 \ -25 \ 0 \ 0]^T.
\end{align*}
\]

The simulation results are plotted in Figures 4.6 and 4.7. Figure 4.6 is the response of the system in the case where all parameters are known precisely. We use this simulation just for reference. It can be seen that the system with the proposed control scheme has excellent tracking performance. In the presence of the parameter uncertainty, the proposed adaptive and robust control scheme is compared with the non-adaptive-robust case. The results are presented in Figure 4.7. It is illustrated that the adaptive and robust control system has much better tracking performance than the non-adaptive-robust control system.

4.4.2 Constrained motion control

Consider the situation in which the object is required to move along a straight line in contact with the constraint surface and exert a constant force on the surface. Meanwhile, the robots are required to keep a constant "push" force (internal force) against each other.
Fig. 4.6 (a)

Motion trajectory in x coordinate, desired , actual ---

Tracking error in x coordinate

Fig. 4.6 (b)

Motion trajectory in y coordinate, desired , actual ---

Tracking error in y coordinate
Figure 4.6: The response of the system with parameters known precisely, (a), (b) and (c) Motion tracking trajectory; (d) Internal force tracking trajectory
Fig. 4.7 (a) Motion trajectory in x coordinate

Fig. 4.7 (b) Motion trajectory in y coordinate

Fig. 4.7 (a) Tracking error in x coordinate

Fig. 4.7 (b) Tracking error in y coordinate
Figure 4.7: Comparison between the adaptive/robust control system and the non-adaptive-robust control system. (solid line: adaptive and robust control; dashed line: non-adaptive-robust control; dotted line: desired)
The constraint is expressed in task space as

$$\Psi(X_o) = y_o - 1.0 = 0,$$

and the constraint force is

$$F_{oc} = [0 \ 1 \ 0]^T \lambda.$$

The desired motion, internal force and contact force trajectories are assumed to be

$$x_d = 0.5 + 0.25 \cos(t)(m)$$
$$\theta_d = 0 (rad)$$
$$F_{cd} = [0 \ 1 \ 0]^T \lambda_d, \quad \lambda_d = 15 (N)$$
$$F_{I_d} = [25 \ 0 \ 0 \ -25 \ 0 \ 0]^T.$$

The motion, contact force and internal force tracking trajectories are plotted in Figures 4.8, 4.9 and 4.10, respectively. It is illustrated that the proposed control system has very good tracking performance. Simulation of the system neglecting joint flexibility in the control design has also been performed. Vibration occurs in the response (see Figure 4.11) and the system is unstable.

4.5 Summary

Control of multiple robot systems is a critical issue in the success of coordinated manipulation. The control issue becomes more challenging when flexibility in the robot joints and uncertainty in the system dynamics have to be considered.

This chapter presents an adaptive and robust control scheme for the control of multiple flexible joint robots. The proposed control scheme can control the motion, internal force, contact force and joint torque simultaneously. Stability analysis of the control system shows that with the proposed control scheme, asymptotic motion, force and joint torque tracking are achieved. The control scheme has the attractive feature that it does not require restrictions on joint flexibility, nor an exact knowledge of the
Figure 4.8: Motion tracking trajectory of the proposed control system

dynamic parameters in the system. Compared with previous work in the literature, the proposed control scheme provides a new approach to dealing with uncertainty in multiple flexible joint robot systems.

Simulation results illustrate the effectiveness of the proposed control methods. The control scheme could be used in industrial robots with actuators that are coupled to high gear-ratio drive mechanisms, such as harmonic drives etc., and applied to perform tasks such as assembly, manufacturing and hazardous material handling.
Figure 4.9: Contact force tracking trajectory of the proposed control system

Figure 4.10: Internal force tracking trajectory of the proposed control system
Figure 4.11: Response of the system neglecting joint flexibility in the control design
Chapter 5

Experimental Implementations

5.1 Introduction

Various control methods have been proposed for controlling flexible joint robots, but experimental evaluation of the proposed control methods is still seldom performed. Many factors make the experimental verification a challenging issue. These include high sensitivity to noise in the measured signals due to the high order of control algorithms and dynamic models of flexible joint robots, unmodeled dynamics in flexible joints, and many other practical considerations, such as system time delay and actuator saturation. Reported experimental results on verifying proposed control methods were usually conducted on specially constructed experimental devices with built-in torsional springs [10, 51]. In [7], experiments on a two-link robot with harmonic drives were carried out. However, due to the noise in the measured signal, only standard PD control was implemented, and the effect of joint flexibility on the system was not clearly demonstrated in the experiments.

In this chapter, we present experimental results obtained on the IRIS robot manipulator. Using joint torque feedback and digital filters, the proposed two-stage control scheme for motion control of flexible joint robots was successfully implemented in real experiments. Based on the comparison of step responses of the system using different control methods under the same control gains, the experimental results illustrate the effects of joint flexibility on system performance. Control systems neglecting joint
flexibility perform poorly or even become unstable. The proposed control scheme can control the robot manipulator stably. Trajectory tracking experiments were conducted to further verify the effectiveness of the proposed control scheme.

In the following, the IRIS robot facility is briefly described. Then the experimental setup and implementation are presented, and the experimental results are reported in detail afterwards.

### 5.2 Description of the IRIS Robot Facility

The IRIS (Institute of Robotic and Intelligent Systems) facility [19] is a modular, reconfigurable and expandable robot system. It is designed to be easily disassembled and reassembled to provide a multitude of configurations. The baseline layout of the facility is two manipulators with four rotary joints for each manipulator, as shown in Figure 5.1. The joint module is composed of a frameless DC brushless motor, a harmonic drive, an optical rotary encoder to measure motor displacement, and a custom-designed torque sensor to measure the load torque. A schematic diagram of the joint is plotted in Figure 5.2(a).

![Figure 5.1: The baseline layout of the IRIS facility](image)
The harmonic drive is a special type of gear mechanism which consists of three parts: a wave-generator, a flexspline and a circular spline (see Figure 5.2(b)). The features of harmonic drives are high transmission ratio, low weight, small size and virtually zero backlash. However, the harmonic drive introduces flexibility in the joint and multiplies the effects of friction [66, 26]. Experiments [27] show that the harmonic drive is the major source of joint flexibility in IRIS robots.

Photograph of a harmonic drive (HD Systems Inc.)

Figure 5.2: Structure of IRIS joint and harmonic drive components

The IRIS robots are controlled by a distributed computer system based on RISC processor nodes able to provide up to 80 MFLOPS per node, and a fast I/O system
associated with each node allowing a sampling rate of up to 5 KHz. Control designs are implemented on the IRIS-joint via a computer node built around a 50 MHz EISA bus based IBM-PC compatible host computer. A RISC coprocessor board (AMD-29050) is attached to the PC bus. It has a powerful RISC processor with a built-in floating-point unit and its own memory (several MBytes). The memory access time and the execution speed of standard computation are very fast. Details about the RISC board can be found in the YARC systems reference manual.

The control system hardware includes the following I/O boards attached to the host bus: i) ADC boards, ii) DAC boards, and iii) Digital I/O boards. These boards are connected to the low-level control system that includes interfaces to the optical encoders, signal-conditioning circuits and the power amplifiers. The IRIS hardware organization is shown in Figure 5.3.

![Figure 5.3: The IRIS hardware organization](image)

The host and RISC computers, while each executes its own code, are running in parallel with a specified communication mechanism. The codes are written in C and High C. Control algorithms are digitally implemented with a sampling rate of 1 msec, and data is stored into a data file at a sampling rate of 10 msec.

### 5.3 Experimental Setup and Implementation

**Experimental setup**

To evaluate control methods for flexible joint robots in real experiments, sensors
that measure link positions are indispensable because link position $q_l$ is not the same as rotor position $q_m$ in the presence of joint flexibility, whether or not $q_l$ is used in the control law. For this reason, in our experimental setup, an external encoders is mounted on each of joints 2 and 4 of IRIS manipulator #2, as shown in Figure 5.4. The external encoder is HEDS-5010, the same as the internal encoder installed already on the motor side, so that the existing circuit channels for joints 1 and 3 can be directly used to connect the external encoders to the computer system.

![Figure 5.4: The newly mounted external encoders](image)

The reconfigurability of the IRIS facility made it possible to carry out the experiment first on a single-joint-link, and then on a two-joint-link. Also, the robot arm was located in both a horizontal and a vertical plane.

**Kinematic and dynamic parameters of the system**

The nominal values of the kinematic and dynamic parameters of the system are listed in Tables 5.1 and 5.2.
Table 5.1

<table>
<thead>
<tr>
<th></th>
<th>Joint 2</th>
<th>Joint 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>gear ratio (rotor side)</td>
<td>101</td>
<td>51</td>
</tr>
<tr>
<td>gear ratio (link side)</td>
<td>17.1</td>
<td>11.75</td>
</tr>
<tr>
<td>subscript i</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>link length $l_i$</td>
<td>0.255m</td>
<td>0.245m</td>
</tr>
<tr>
<td>$I_{mi}$</td>
<td>0.1 $kgm^2$</td>
<td>0.02 $kgm^2$</td>
</tr>
<tr>
<td>$B_{mi}$</td>
<td>1.5 $Nm/(rad/sec)$</td>
<td>2.2 $Nm/(rad/sec)$</td>
</tr>
<tr>
<td>$K_{si}$</td>
<td>10 $Nm/degree$</td>
<td>20 $Nm/degree$</td>
</tr>
<tr>
<td>$I_i$ [without load//with load]</td>
<td>[0.04//0.1]$kgm^2$</td>
<td>[0.02//0.076]$kgm^2$</td>
</tr>
<tr>
<td>$B_i$</td>
<td>2.11 $Nm/(rad/sec)$</td>
<td>1.0 $Nm/(rad/sec)$</td>
</tr>
<tr>
<td>$g_i$ [without load//with load]</td>
<td>[1.3//3.57]$Nm$</td>
<td>[1.28//3.45]$Nm$</td>
</tr>
</tbody>
</table>

Table 5.2

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>without load</td>
<td>0.2038</td>
<td>0.0208</td>
<td>0.0125</td>
<td>0.813</td>
<td>0.097</td>
</tr>
<tr>
<td>with 1 kg load</td>
<td>0.2624</td>
<td>0.0748</td>
<td>0.0687</td>
<td>1.0425</td>
<td>0.3175</td>
</tr>
</tbody>
</table>

The dynamic model of the rotor subsystems is expressed as:

$$I_{mi}\ddot{q}_{mi} + B_{mi}\dot{q}_{mi} + \tau_{si} = u_i, \quad i = 1, 2.$$ 

The link dynamics in the single-joint-link case is given as:

$$I_i\ddot{q}_{l1} + B_i\dot{q}_{l1} + g_i = \tau_{si}, \quad i = 1, 2.$$ 

and the dynamic equations for the two-joint-link case are:

$$H_{11}\ddot{q}_{l1} + H_{12}\ddot{q}_{l2} - h\ddot{q}_{l2}^2 - 2h\dot{q}_{l1}\dot{q}_{l2} + G_1 = \tau_{s1}$$

$$H_{21}\ddot{q}_{l1} + H_{22}\ddot{q}_{l2} + h\ddot{q}_{l1}^2 + G_2 = \tau_{s2}.$$
Uncertainty in the system dynamics

In real robot applications, there is uncertainty in the system dynamics due to payload variations, friction and unmodeled characteristics. The real dynamic parameter values are not available for the IRIS robot system. The nominal values were obtained from the mechanical design and experimental estimation. The analysis of friction in the drive system of IRIS joints is very complicated: the coefficient of the damping terms can vary with temperature, payload and configuration; in addition, there is Coulomb friction [26]. The joint stiffness $K_i$ is also an unknown parameter. Experiments were conducted to identify its nominal value. The measured joint torque via joint torsion $e = q_m - q_i$ plotted in Figure 5.5 shows that there is hysteresis in the joint stiffness, and the joint stiffness value is a non-linear function of joint torque and joint torsion. The linear model $\tau_s = k_s(q_m - q_i)$ is a rough approximation to the real relationship. By neglecting the hysteresis and using a segment linearization approach, it is possible to express the relationship as follows:

$$
\tau_s = \tilde{k}_s(q_m - q_i) + \Delta\tau = (\tilde{k}_s + \Delta k)(q_m - q_i),
$$

Figure 5.5: Joint stiffness $K_s$
where $k_s$ is a constant nominal value in the given linearization region, and $\Delta k$ is an unknown variable but bounded.

A detailed description and modeling of joint stiffness, hysteresis and soft windup feature, including asymmetric Coulomb friction, can be found in [27].

Noise in measured signals

Joint positions are measured by optical encoders, and velocity signals are obtained by differentiating position signals. The differentiation process can introduce derivative noise in the velocity signal, especially at low speed (see Figure 5.6 (a)).

![Figure 5.6: Noise in velocity signal](image)

Noisy signals cause serious problems in control implementations. To partially remedy these problems, a first-order filter $bs/(s+b) = 100s/(s+100)$ is used to get the velocity signal from the position measurements. The function of the filter is to reject the high frequency components resulting from the derivative noise. The bandwidth of the filter is determined by the value of $b$. Considering that the highest frequency of the robot motion is $\omega = 10 \text{rad}/\text{sec} \approx 1.59 \text{Hz}$, and the sampling rate of the IRIS control system is $1000 \text{Hz}$, $b$ is chosen to be $10 \cdot \omega = 100$. The filter is discretized by bilinear transformation and digitally implemented in the control system. (In Matlab,
the discretization can be done by the command: $bilinear([100, 0], [1, 100], 1000)$). As shown in Figure 5.6 (b), the noise in the velocity signal is reduced remarkably by the filter.

We used this filter in the control system for obtaining derivative signals whenever required.

The measurement noise in the joint torque sensor signal is quite large. This issue has been noted in [27]. A second-order Butterworth filter with a cut-off frequency of 20Hz was used and verified experimentally. Figure 5.7 shows the noisy signal and the filtered signal. As this filter is available in the system software code, it is adopted in our experiments. In general, a lowpass filter introduces stable poles in the system, acts as if a damping term was added to the system dynamics, and will retain the system's stability [56].

![Figure 5.7: Noise in joint torque sensor signal](image)

**Experimental implementation**

Considering that the true values of the system dynamics are unknown and there is unmodeled dynamics such as friction and hysteresis phenomena in the drive system, the proposed adaptive and robust control method is implemented in the experiments.
According to the control laws (2.19) and (2.24), to generate the desired joint torque $\tau_{sd}$ in the adaptive controller, only the measured signals $q_i, \dot{q}_i$ and $\tau_s$ are required; to compute control torque $u$ in the joint torque controller, $\dot{\tau}_{sd}, \ddot{\tau}_{sd}$ and $\dddot{\tau}_s$ are needed. Considering that there is uncertainty in the system dynamics and that the measured signals are contaminated by noise, $\dot{\tau}_{sd}$ is not calculated from Formula (2.14), but is obtained using the filter $100s/(s + 100)$. The same derivative filter is applied to $\dot{\tau}_{sd}$ to get $\ddot{\tau}_{sd}$, and applied to $e_r$ to obtain $\dot{e}_r$. Since the sampling rate in the system is high, using derivative filters to obtain the derivative signals can not only reduce the computational burden, but also be more accurate. This is because direct calculation has fewer accumulated errors than calculation from the formula in the presence of system uncertainty and signal noise.

To evaluate the effectiveness of the proposed control method, comparisons with the PD controller were made in all the experiments. The PD controller is recognized as a powerful control method in practical uses, and it has been shown [70, 76] that PD control with rotor angle feedback can control flexible joint robots stably with good tracking performance.

Experiments on PD control using link angle feedback and adaptive control neglecting joint flexibility have also been conducted to evaluate the effects of joint flexibility. As is known, if joint flexibility is not negligible, a system using these control laws may be unstable and may experience oscillation in its responses. These experiments are limited to be only conducted in the single-joint-link case just for evaluating the joint flexibility effects, and will not be conducted in the two-joint-link case as to prevent the robot system from possible damage.

### 5.4 Experimental Results and Evaluation

#### 5.4.1 Step-response of the system

To evaluate the effects of joint flexibility, simple step inputs are used to examine the responses of different control systems. Experiments were conducted in the single-
joint-link case.

For joint 2, the step input is set to be \( q_d = 15(\text{degree}) \). For comparison, the control gains in PD control with rotor angle feedback and PD control with link angle feedback are set to be the same: \((K_p, K_d) = (4.5, 0.18) \times 180/\pi; \) the control gains for the proposed control method and adaptive control neglecting joint flexibility are set to be \((45, 1.6)\). The responses of the individual control systems are plotted in Figures 5.8 to 5.12, respectively.

The results show that using the same control gains as their partners, PD control with link angle feedback and adaptive control neglecting joint flexibility are unstable. (In the experiments, it is also observed that if the control gains decrease to a certain extent, the systems become stable with large tracking errors. This is consistent with the theoretical and simulation results [70, 32]).

For joint 4, the step input is set to be \( q_d = 25(\text{degree}) \). Similar results are obtained, as shown in Figure 5.13. It can be seen that due to the low flexibility (high stiffness) of the joint, the torsion between the rotor and link angles is small, as expected.

The experimental results clearly illustrate the existence of joint flexibility and its effects on the control systems. The proposed control scheme and the PD controller with rotor angle feedback can control the flexible joint robot stably with good tracking performance.

5.4.2 Trajectory tracking control of the system

To evaluate the tracking performance of the proposed control method, sinusoidal functions are used to generate desired trajectories. The responses of the proposed control system are compared with the responses of the PD controller with rotor angle feedback. The experiments are conducted first in the single-joint-link case, and then in the two-joint-link case.

Trajectory tracking in single-joint-link case

For joint 2, \( q_d = 40 \sin 3t(\text{degree}) \). The joint moves in a horizontal plane along the specified trajectory. The responses of the control systems are presented in Figures
Figure 5.8: Response of the PD controller with rotor angle feedback

Figure 5.9: Response of the PD controller with link angle feedback

Figure 5.10: Response of the proposed control scheme
Figure 5.11: The velocity, torque and parameter estimation curves of the proposed control scheme

Figure 5.12: Response of adaptive control neglecting joint flexibility
Proposed control scheme

Adaptive control neglecting joint flexibility

Figure 5.13: Response of the control systems at Joint 4

5.14 and 5.15. The tracking errors, torque profiles and parameter estimation curves are plotted in Figures 5.16 to 5.19.

Joint 4 was commanded to move along the desired trajectory $q_d = 45 \sin 2t (\text{degree})$ in a vertical plane. The results are shown in Figures 5.20 to 5.25.

The experimental results show that the proposed control scheme has much better tracking performances than the PD controller. However, it can be observed that the torque signals of the proposed control method are more 'noisy' than that of the PD controller; in other words, there is chattering in the torque signals. The chattering is considered to be caused by two sources. One is the control algorithm itself, as
Figure 5.14: Trajectory tracking of the PD controller with rotor angle feedback (Joint 2)

Figure 5.15: Trajectory tracking of the proposed control scheme (Joint 2)
Figure 5.16: Comparison of the tracking errors (Joint 2)

Figure 5.17: Torque profiles of the PD controller (Joint 2)
Figure 5.18: Torque profiles of the proposed control scheme (Joint 2)

Figure 5.19: Parameter estimation curves of the proposed control scheme (Joint 2)
Figure 5.20: Trajectory tracking of the PD controller with rotor angle feedback (Joint 4)

Figure 5.21: Trajectory tracking of the proposed control scheme (Joint 4)
Figure 5.22: Comparison of the tracking errors (Joint 4)

Figure 5.23: Torque profiles of the PD controller (Joint 4)
Figure 5.24: Torque profiles of the proposed control scheme (Joint 4)

Figure 5.25: Parameter estimation curves of the proposed control scheme (Joint 4)
adaptive and robust control may generate some chattering. The other is the noise in the measured signals, the link velocity and joint torque, because a controller using noisy signals as inputs will reflect the noise, more or less, in the outputs. It is also noted that the parameter estimation does not converge to constant values, and the parameter resetting is well utilized in the experiments.

**Trajectory tracking in two-joint-link case**

Experiments in two-joint-link case were carried out both in horizontal and vertical planes. Joints 2 and 4 were required to move simultaneously with the desired trajectories:

\[ q_{d1} = 40 \sin 3t \quad \text{(Joint 2)} \]
\[ q_{d2} = 45 \sin 2.5t \quad \text{(Joint 4)}. \]

The experimental results are shown\(^1\) in Figures 5.26 and 5.27. The dynamic tracking performance of the proposed control method is better than that obtained with the PD controller, but is not so superior as expected. The torque signals become more 'noisy' than in the single-joint-link case. For comparison, the torque profile of joint 2 with two joint-links moving in a horizontal plane is plotted in Figure 5.28 as compared with Figure 5.18; the torque profile of joint 4 with the two joint-links moving in a vertical plane is plotted in Figure 5.29 as compared with Figure 5.24. The results indicate that dynamic coupling is a cause of this heavier chattering. The custom-designed torque sensors might be too sensitive to coupling motions; as described in [27] even motor power on/off can cause large noise in the torque sensors. Besides, due to joint flexibility, high frequency modes can easily be stirred by coupling motions. Therefore, experiments on higher DOF flexible joint robots are more difficult. Our experimental work is just a preliminary attempt. The results show that with the proposed control method, the system can track the desired trajectory stably.

One relevant issue is that in all sinusoidal trajectory tracking experiments, the

---

\(^1\)In all the figures, time is the time for recording data, not the time in the trajectory equations.
Joint 2

Joint 4

PD control with rotor angle feedback

PD control with rotor angle feedback

Proposed control scheme

Proposed control scheme

Comparison of the tracking errors

Comparison of the tracking errors

Figure 5.26: Experiment on the two-joint-link robot moving in a horizontal plane
 PD control with rotor angle feedback

Proposed control scheme

Comparison of the tracking errors

Figure 5.27: Experiment on the two-joint-link robot moving in a vertical plane
Torque profiles of the PD controller (Joint 2)

Torque profiles of the proposed control scheme (Joint 2)

Figure 5.28: Joint 2 torque signals when the two-joint-link robot is moving horizontally
Torque profiles of the PD controller (Joint 4)

Torque profiles of the proposed control scheme (Joint 4)

Figure 5.29: Joint 4 torque signals when the two-joint-link robot is moving in a vertical plane
tracking errors of the proposed control method in the neighborhood of the trajectory peaks are relatively large. Coulomb friction and hysteresis in the joint torque are major causes of these tracking errors because in the neighborhood of each peak point, the speed approaches zero and the robot is changing the motion direction.

5.5 Summary

The experimental implementation of the proposed two-stage control scheme for the motion control of flexible joint robots is described in this chapter. Motion control experiments in single-joint-link and two-joint-link cases have been conducted to test the proposed control scheme. The experimental results confirm the theoretical developments. Based on the comparison of step responses of the system using different control methods under the same control gains, the experimental results clearly illustrate the effects of joint flexibility on the control systems. Control systems neglecting joint flexibility perform poorly or even become unstable, but the PD controller with rotor angle feedback and the proposed control scheme can control the flexible joint robot successfully. The stability of the proposed control system is verified experimentally.

The experiments also revealed some important issues regarding the control of flexible joint robots, such as hysteresis in the joint stiffness, noise sensitivity and nonlinear friction. These issues are open problems demanding further studies.
Chapter 6

Conclusions

6.1 Conclusions

In this thesis, control issues of flexible joint robots have been studied systematically. Three important aspects relevant to robot applications have been addressed: i) the control of flexible joint robots maneuvering in an unconstrained space; ii) the motion and force control of flexible joint robots in constrained motion; and iii) the coordinated control of multiple flexible joint robots in cooperative manipulation. New adaptive and robust control methods have been developed for each case, analyzed theoretically, and verified by simulations and experiments.

A two-stage control scheme using a joint torque feedback technique have been developed in Chapter 2 for the motion control of flexible joint robots in the general n-link case. The control scheme is formulated based on a standard flexible joint robot model, and is composed of a motion controller and a joint torque controller. To deal with uncertainties in the robotic system, an adaptive and robust control algorithm has been developed assuming that all system parameters including the joint flexibility values are unknown except for some of their bounds. The system stability is analyzed via the Lyapunov stability theory. The outcome is a useful framework for the generalization of control methods previously developed for rigid robots. The result has the distinct feature that it does not require restrictions on joint flexibility, nor an exact knowledge of the parameters of the rotor subsystem.
Implementation issues of the proposed control scheme have also been discussed in Section 2.3. To prevent parameter drift and to enhance system robustness, the parameter update law is augmented with the resetting conditions expressed in (2.33). Further, the uncertainty bounds needed to derive the robust control law in the control scheme depend only on the parameters of the drive system. It is easy to partition $Y_i \Delta u$ into different sizes of components as shown in Equation (2.37). Thus, our knowledge of the parameter uncertainty can be better utilized in control design.

Motion and force control issues of flexible joint robots in constrained motion have been investigated extensively. Three proposed adaptive and robust control methods were presented in Chapter 3. Based on the reduced constrained dynamic model, an adaptive and robust control scheme was developed which can deal with parametric uncertainty in the system dynamics and guarantee uniform ultimate boundedness of the tracking errors; an adaptive and sliding control algorithm was proposed to cope with both the parameter uncertainty and unknown additive bounded disturbance, and to achieve global asymptotic convergence of tracking errors. Distinct from previous work in the literature, the proposed control schemes using a two-stage control strategy provide a systematic approach for the motion and force control of flexible joint robots in the general n-link case, and do not require an exact knowledge of robot dynamics.

Furthermore, a generalized transformed dynamic model was derived with three important properties. Based on this model, a new control method was proposed which provides a unified approach for the motion and force control of flexible joint robots in general coordinate spaces (such as task space or joint space). It has been shown that, with the proposed control scheme, motion, force and joint torque tracking errors converge asymptotically to zero without persistent excitation conditions. The main idea is quite simple in hindsight. We transform the dynamic model into a suitably chosen n-dimensional space spanned by independent motion and force vectors. The composite motion and force tracking error is then utilized to construct the auxiliary vector $\mathbf{S}$ as formulated in (3.63). Thus, distinct from the reduced order dynamic model, the motion and force tracking errors are both used in the adaptive mechanism, and a significant theoretical result is obtained.
Digital simulations verified the effectiveness of the three proposed control methods.

New results have also been obtained on the coordinated control of multiple flexible joint robots. The two-stage control scheme developed in Chapters 2 and 3 has been extended to the coordinated control of multiple flexible joint robots holding a common object. In the control scheme, new adaptive coordinated controllers have been developed for both free and constrained motion. When multiple robot manipulators hold a common object in cooperative manipulation, closed kinematic chains are formed. These impose kinematic and dynamic constraints. In terms of the object coordinates $X_0$, we formulated a concise dynamic model for the system. When the object is in contact with the environment, a reduced dynamic model is derived in the same way as in Chapter 3, except that it is expressed in task space. Based on the system dynamic model, the adaptive and robust control scheme was designed, and analyzed via the Lyapunov stability theory. It has been shown that the proposed control scheme can simultaneously control the motion of the object, the internal force among the robots, the contact force between the object and the environment, and the joint torques. Asymptotic motion and force tracking is achieved without requiring an exact knowledge of the system dynamics. Simulations were carried out on two planar robot manipulators, each having three joint-links. The results illustrated the effectiveness of the proposed control methods.

The proposed two-stage control scheme for the motion control of flexible joint robots has been also successfully implemented on the IRIS reconfigurable robot manipulator. Motion control experiments in the single-joint-link and two-joint-link cases have been conducted to test the proposed control scheme. Comparisons were carried out with PD control and the control method neglecting joint flexibility. The experimental results confirmed the theoretical developments. They clearly illustrated the effects of joint flexibility on the control systems, and showed that with the proposed control method, the system could track the desired trajectory stably.

The research advances the knowledge in the field of the control of flexible joint robots. Analytical and experimental results demonstrate that the proposed control methods can be used in real applications for controlling industrial robots whose ac-
tuators are usually coupled to high gear-ratio drive mechanisms, such as harmonic drives, and for performing tasks such as assembly, manufacturing and hazardous material handling.

6.2 Future Work

The control of flexible joint robots is a comprehensive field, where many open problems require further studies. The following relevant research directions are suggested based on this thesis work.

- Usually, direct adaptive approach does not guarantee that each estimated parameter will converge to its true value [70, 62]. Methods comprising direct and indirect adaptive algorithms have been proposed [63], and robust adaptive methods have been suggested to achieve the convergence in the presence of external disturbances including noise [42]. In our work, the parameter resetting was used which could be regarded as a kind of robust adaptive strategy — parameter projection. Further research in this direction could be conducted to enhance system performance by developing and evaluating different robust adaptive laws applied in the framework of the proposed control scheme.

- Most motion and force control methods developed for robots in constrained motion assume that the environment with which the robot end-effector interacts is known. This could be true in most industrial manufacture cases. However, in some other applications, such as environmental exploration and waste material handling, the environment is unknown or only partially known. Flexible joint robots would be the desired candidates to be used in these cases due to their inherent compliance with external objects in cases of accidental collision, etc. The motion and force control of flexible joint robots in constrained motion with unknown environments is therefore an important and challenging topic from the practical and theoretical points of view.

- The proposed unified control method in Chapter 3 could be extended to the
coordinated control of multiple flexible joint robots holding a common object. A generalized coordinate frame must first be defined, a dynamic model should then be formulated, and controllers could be developed to control multiple flexible joint robots with respect to any suitably selected generalized coordinates.

- Experimental implementation of the motion and force control schemes for flexible joint robots is an important part of future research. The adaptive and robust control methods developed in Chapters 3 and 4 have been analyzed theoretically and tested extensively by simulation. However, since inherent factors, such as unmodeled dynamics in the joints and measurement noise, etc., are not explicitly considered in the theoretical analysis, experimental evaluation is needed to further evaluate the effectiveness.

- The proposed two-stage control scheme with joint torque feedback could be extended to the control of redundant flexible joint robots.
References


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Appendix A

Stability Definitions and Important Lemmas

The stability definitions and important lemmas used in the thesis are cited in this Appendix. They can be found in Sastry and Bodson [60], Slotine and Li [64], Leitmann [30] and Desoer and Vidyasagar [8].

A.1 Stability Definitions

Consider a dynamic system described by the vector differential equation

\[ \dot{x} = f(t, x), \quad x(t_0) = x_0 \]  

(A.1)

where \( x \in \mathbb{R}^n, t \geq 0, t_0 \geq 0 \), and \( f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n \) is continuous. It is assumed that the equation has a unique solution \( x(t, x_0, t_0) \) corresponding to the initial condition \( x(t_0) = x_0 \), for all \( t \geq t_0 \).

\( x = x_e \) is called an equilibrium point or equilibrium state of (A.1), if \( f(t, x_e) = 0 \) for all \( t \geq 0 \).

In the following definitions, we consider that \( x = 0 \) is the equilibrium state of (A.1), i.e. \( f(t, 0) = 0 \) for all \( t \geq 0 \).

Definition A.1 Stability, The equilibrium state 0 of the system (A.1) is stable, if
for all $t_0 \geq 0$ and any positive $\varepsilon$, there exists a positive scalar $\delta(\varepsilon, t_0)$, such that

$$\| x(t_0) \| < \delta(\varepsilon, t_0) \Rightarrow \| x(t) \| < \varepsilon, \quad \text{for all } t \geq t_0 \quad (A.2)$$

**Definition A.2** *Asymptotic Stability*, The equilibrium state $0$ of the system (A.1) is asymptotically stable, if

(i) it is stable,

(ii) there exists a positive scalar $\delta(t_0)$ such that

$$\| x(t_0) \| < \delta(t_0) \Rightarrow \lim_{t \to -\infty} \| x(t) \| = 0. \quad (A.3)$$

**Definition A.3** *Global Asymptotic Stability*, The equilibrium state $0$ of the system (A.1) is globally asymptotically stable, if

(i) it is asymptotically stable,

(ii) for all $x_0 \in \mathbb{R}^n$,

$$\lim_{t \to -\infty} \| x(t) \| = 0. \quad (A.4)$$

**Definition A.4** *Uniform Stability*, The equilibrium state $0$ of the system (A.1) is uniformly stable, if in the definition A.1 $\delta$ can be chosen independent of $t_0$.

Asymptotically uniformly stable and globally asymptotically uniformly stable are defined likewise.

**Definition A.5** *Uniform Boundedness*, Given a solution $x(.) : [t_0, t_1] \rightarrow \mathbb{R}^n$, of (A.1), it is said to be uniformly bounded with respect to the set $S$ if there is a positive constant $d(x_0) < \infty$, possibly dependent on $x_0$ but not on $t_0$, such that

$$\| x(t) \| \leq d(x_0), \quad \text{for all } t \in [t_0, t_1]$$

**Definition A.6** *Uniform Ultimate Boundedness*, Given a solution $x(.) : [t_0, \infty) \rightarrow \mathbb{R}^n$, of (A.1), it is said to be uniformly ultimately bounded (u.u.b.) with respect to the set $S$ if there is a non-negative constant $T(x_0, S) < \infty$, possibly dependent on $x_0$
but not on $t_0$, such that

$$x(t) \in S, \quad \text{for all } t \geq t_0 + T$$

Uniform ultimate boundedness says that the solution trajectory of (A.1) beginning from $x_0$ at time $t_0$ will eventually become bounded, that is, will enter and remain within the set $S$.

### A.2 Important Lemmas

In addition to lemma 2.1 introduced in Chapter 2, the following important lemmas are utilized in this thesis.

**Lemma A.1** If function $g(t) \in L_p$, $1 \leq p < \infty$, and $g$ is uniformly continuous, then $|g(t)| \to 0$ as $t \to \infty$.

**Lemma A.2** Barbalat's Lemma If a differentiable function $g(t)$ has a finite limit as $t \to \infty$, and if $\dot{g}(t)$ is uniformly continuous, then $\dot{g}(t) \to 0$, as $t \to \infty$.

**Lemma A.3** A function $g$ is uniformly continuous, if its derivative $\dot{g}(t)$ is bounded.

**Lemma A.4** Let

$$e = H(s)r$$

(A.5)

where $H(s)$ is an $n \times m$ strictly proper, exponentially stable transfer function. Then $r \in L_2^m$ implies that $e \in L_2^n \cap L_\infty^n$, $\dot{e} \in L_2^n$, $e$ is continuous, and $e \to 0$ as $t \to \infty$. If, in addition, $r \to 0$ as $t \to \infty$ then $\dot{e} \to 0$ as $t \to \infty$. 

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Appendix B

Proof of Theorem 2.2: the Tracking Error Bounds and Parameter Resetting

B.1 Derivation of the Tracking Error Bounds

In this section, the tracking error bounds in Theorem 2.2 are derived in details. Let us write the Lyapunov function (2.27) as follows

\[ V = \frac{1}{2} r_t^T D(q_t) r_t + \frac{1}{2} \hat{P}^T \Gamma \hat{P} + \frac{1}{2} r_t^T I_{ts} r_t + e_t^T \Lambda_t^T K_{ID} e_t \]

\[ = \frac{1}{2} \begin{bmatrix} r_t & e_t & \dot{e}_t \end{bmatrix} Q_v \begin{bmatrix} r_t \\ e_t \\ \dot{e}_t \end{bmatrix} + \frac{1}{2} \hat{P}^T \Gamma \hat{P} \]  

(B.1)

where

\[ Q_v = \begin{bmatrix} D(q_t) & 0 & 0 \\ 0 & \Lambda_t^T I_{ts} \Lambda_t + 2 \Lambda_t^T K_{ID} & \Lambda_t^T I_{ts} \\ 0 & I_{ts} \Lambda_t & I_{ts} \end{bmatrix} \]
As $V$ is positive definite, $Q_v > 0$. Denoting $m_B = \lambda_{\min}(Q_v) > 0$, $M_B = \lambda_{\max}(Q_v) > 0$, and $f_P(t) = \tilde{P}^T \Gamma \tilde{P} \geq 0$, the Lyapunov function satisfies

$$\frac{1}{2} m_B \| (r_t^T, e_t^T, \dot{e}_t^T) \|^2 + \frac{f_P(t)}{2} \leq V(t) \leq \frac{1}{2} M_B \| (r_t^T, e_t^T, \dot{e}_t^T) \|^2 + \frac{f_P(t)}{2}. \quad (B.2)$$

The derivative of $V$ in (2.31) can also be rewritten as

$$\dot{V} \leq - \begin{bmatrix} r_t^T & e_t^T \end{bmatrix} Q \begin{bmatrix} r_t \\ e_t \end{bmatrix} - r_t^T B_{ts} r_t - \dot{e}_t^T K_{tD} \dot{e}_t + \frac{\epsilon_p}{4}$$

$$\leq - \begin{bmatrix} r_t^T & e_t^T & \dot{e}_t^T \end{bmatrix} Q_v \begin{bmatrix} r_t \\ e_t \\ \dot{e}_t \end{bmatrix} + \frac{\epsilon_p}{4}, \quad (B.3)$$

where

$$Q_v = \begin{bmatrix} K_{tD} & -\frac{1}{2} I & 0 \\ -\frac{1}{2} I & \Lambda_t^T K_{tD} \Lambda_t + \Lambda_t^T B_{ts} \Lambda_t & \Lambda_t^T B_{ts} \\ 0 & B_{ts} \Lambda_t & B_{ts} + K_{tD} \end{bmatrix}.$$  

Denoting the minimum eigenvalue of matrix $Q_v$ by $\lambda_{\min}(Q_v)$, we have

$$\dot{V} \leq -\lambda_{\min}(Q_v) \| (r_t^T, e_t^T, \dot{e}_t^T) \|^2 + \frac{\epsilon_p}{4} \quad (B.4)$$

$$\dot{V} \leq -\frac{\lambda_{\min}(Q_v)}{M_B} (M_B \| (r_t^T, e_t^T, \dot{e}_t^T) \|^2 + f_P) + \frac{\lambda_{\min}(Q_v)}{M_B} f_P + \frac{\epsilon_p}{4} \quad (B.5)$$

$$\dot{V} \leq -\frac{\lambda_{\min}(Q_v)}{M_B} (2V) + \frac{\lambda_{\min}(Q_v)}{M_B} f_P + \frac{\epsilon_p}{4} \quad (B.6)$$

Letting $\alpha = \frac{2\lambda_{\min}(Q_v)}{M_B}$, multiplying both sides of Inequality (B.6) by $e^{\alpha t}$, integrating, and then multiplying by $e^{-\alpha t}$, we get

$$V(t) \leq V(0)e^{-\alpha t} + \left( \int_0^t \frac{\alpha f_P}{2} e^{\alpha t} dt \right) e^{-\alpha t} + \frac{\epsilon_p}{4\alpha} (1 - e^{-\alpha t}), \quad (B.7)$$

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hence,

\[
\frac{1}{2} m_B \| (r_i^T, e_i^T, \dot{e}_i^T) \|^2 + \frac{\dot{f}_P(t)}{2} \leq V(0)e^{-\alpha t} + \left( \int_0^t \frac{\alpha f_P}{2} e^{\alpha t} dt \right)e^{-\alpha t} + \frac{M_B}{\lambda_{\min}(Q_0)} \frac{\epsilon \rho}{8} (1 - e^{-\alpha t}). \tag{B.8}
\]

The limitation as \( t \to \infty \) is

\[
\lim_{t\to\infty} \left( \frac{1}{2} m_B \| (r_i^T, e_i^T, \dot{e}_i^T) \|^2 + \frac{\dot{f}_P(t)}{2} \right) \leq \lim_{t\to\infty} \left( \int_0^t \frac{\alpha f_P}{2} e^{\alpha t} dt \right)e^{-\alpha t} + \frac{M_B}{\lambda_{\min}(Q_0)} \frac{\epsilon \rho}{8} \tag{B.9}
\]

By l'Hospital law,

\[
\lim_{t\to\infty} \left( \int_0^t \frac{\alpha f_P}{2} e^{\alpha t} dt \right)e^{-\alpha t} = \lim_{t\to\infty} \frac{\alpha f_P e^{\alpha t}}{\alpha e^{\alpha t}} = \lim_{t\to\infty} \frac{f_P(t)}{2}
\]

Therefore, we get

\[
\lim_{t\to\infty} \| (r_i^T, e_i^T, \dot{e}_i^T) \|^2 \leq \frac{M_B}{4m_B} \frac{\rho}{\lambda_{\min}(Q_0)} \epsilon \tag{B.10}
\]

The tracking errors are bounded, and as \( \epsilon \to 0 \), the tracking errors converge to zero.

### B.2 Proof of Theorem 2.2 with Parameter Resetting

In the adaptive and robust control algorithm proposed in Chapter 2, the parameter update law (2.21)

\[
\dot{P} = -\Gamma^{-1} Y^T(q_i, \dot{q}_i, v_i, \dot{v}_i) r_i \tag{B.11}
\]

is augmented with the resetting conditions (2.33)

\[
\dot{P}_i = \begin{cases} 
  l_i & \text{if } \hat{P}_i(t) < l_i \\
  h_i & \text{if } \hat{P}_i(t) > h_i \\
  \hat{P}_i(t) & \text{otherwise} 
\end{cases} \tag{B.12}
\]
to constrain the parameter estimates within certain bounds about the true parameters, so as to ensure the convergence and stability of the control system, and enhance system robustness. It can be shown that the addition of parameter resetting maintains the negativity of \( \dot{V} \), such that the statement of Theorem 2.2 remains valid, and the uniform stability of the system is ensured as parameter drift is prevented.

The parameter update law with resetting condition can be expressed as

\[
\dot{P}_i = \begin{cases} 
- \left[ \Gamma^{-1} Y^T r_i \right]_i & \text{if } l_i < \hat{P}_i(t) < h_i \text{ or } \\
& \text{if } \hat{P}_i(t) = l_i \text{ and } [Y^T r_i]_i < 0 \text{ or } \\
& \text{if } \hat{P}_i(t) = h_i \text{ and } [Y^T r_i]_i > 0; \\
0 & \text{if } \hat{P}_i(t) = l_i \text{ and } [Y^T r_i]_i > 0 \text{ or } \\
& \text{if } \hat{P}_i(t) = h_i \text{ and } [Y^T r_i]_i < 0. 
\end{cases} \tag{B.13}
\]

For convenience, we use \([\cdot]_i\) to mean the \(i^{th}\) element in the vector \([\cdot]\). Note that if \(\hat{P}_i(t) = l_i\), then \(\tilde{P}_i < 0\), and if \(\hat{P}_i(t) = h_i\), then \(\tilde{P}_i > 0\), we have equivalently

\[
\dot{P}_i = \begin{cases} 
- \left[ \Gamma^{-1} Y^T r_i \right]_i & \text{if } l_i < \hat{P}_i(t) < h_i \text{ or } \\
& \text{if } (\hat{P}_i(t) = l_i \text{ or } \hat{P}_i(t) = h_i) \text{ and } \tilde{P}_i^T [Y^T r_i]_i > 0; \\
0 & \text{if } (\hat{P}_i(t) = l_i \text{ or } \hat{P}_i(t) = h_i) \text{ and } \tilde{P}_i^T [Y^T r_i]_i < 0. 
\end{cases} \tag{B.14}
\]

Using the Lyapunov function as given in (2.27)

\[
V = \frac{1}{2} r_i^T D(q) r_i + \frac{1}{2} \tilde{P}^T \Gamma \tilde{P} + \frac{1}{2} r_i^T I_{tr} r_i + e_i^T \Lambda_t^T K_t D \hat{e}_t > 0, \tag{B.15}
\]

the differentiation of \(V\) yields

\[
\dot{V} = - \left[ \begin{array}{c} r_i \\
e_i 
\end{array} \right] Q \left[ \begin{array}{c} r_i \\
e_i 
\end{array} \right]^T - r_i^T B_{tr} r_i - \dot{e}_i^T K_t D \hat{e}_t \\
+ \tilde{P}^T (Y^T r_i + \Gamma \tilde{P}) + (Y_t^T r_i)^T (\tilde{P}_t + \Delta u). \tag{B.16}
\]
Let us denote

\[ \varpi = - \begin{bmatrix} r_t^T & e_t^T \end{bmatrix} \begin{bmatrix} r_t \\ e_t \end{bmatrix} - r_t^T B_t s r_t - \dot{e}_t^T K_{tD} \dot{e}_t + (Y^T r_l)^T (\tilde{P}_t + \Delta u), \]

then

\[
\dot{V} = \varpi + \tilde{P}_t^T (Y^T r_l + \Gamma \dot{P}). \tag{B.17}
\]

Applying the parameter update law (B.14), Equation (B.17) yields

\[
\dot{V} = \begin{cases} 
\varpi & \text{if } l_i < \hat{P}_i(t) < h_i \text{ or } \\
\varpi + \tilde{P}_i^T [Y^T r_l]_i & \text{if } (\hat{P}_i(t) = l_i \text{ or } \hat{P}_i(t) = h_i) \text{ and } \tilde{P}_i^T [Y^T r_l]_i > 0; \\
\varpi + \tilde{P}_i^T [Y^T r_l]_i & \text{if } (\hat{P}_i(t) = l_i \text{ or } \hat{P}_i(t) = h_i) \text{ and } \tilde{P}_i^T [Y^T r_l]_i < 0.
\end{cases} \tag{B.18}
\]

Because \( \tilde{P}_i^T [Y^T r_l]_i < 0 \) in \( \varpi + \tilde{P}_i^T [Y^T r_l]_i \), \( \dot{V} \leq \varpi \). Thus, through the same derivation as developed in Chapter 2, we have

\[
\dot{V} \leq \varpi \leq - \begin{bmatrix} r_t^T & e_t^T \end{bmatrix} \begin{bmatrix} r_t \\ e_t \end{bmatrix} - r_t^T B_t s r_t - \dot{e}_t^T K_{tD} \dot{e}_t + \frac{\epsilon^p}{4} \\
\leq -\lambda_{\min}(Q) \| (r_t^T, e_t^T) \|^2 - \min(B_{tst}) \| r_t \|^2 \\
- \min(K_{tD,e}) \| \dot{e}_t \|^2 + \frac{\epsilon^p}{4}. \tag{B.19}
\]

That is the same inequality as (2.31). The convergence of the tracking errors remains as stated in Theorem 2.2. In addition, as estimated parameters are restricted within pre-specified bounds, the uniform stability and uniform ultimate boundedness of the solution (the tracking trajectory and estimated parameters) of the system are ensured.
Appendix C

Object Dynamics and Property

The dynamic equation of the common object presented in Chapter 4 is derived in this Appendix, followed by the proof of the important property that the matrix \((\dot{M}_o - 2\dot{N}_o)\) is skew symmetric.

To derive the object dynamics, let us analyze the motion and forces on the object. The coordinate frames are located as shown in Figure 4.1. The vectors of \(X_o\) and \(X_{e,j}\) can be written in detail as

\[
X_o = [r_o^T, \theta_o^T]^T = [x_o, y_o, z_o, \theta_{o_1}, \theta_{o_2}]^T
\]

\[
X_{e,j} = [r_{e,j}^T, \theta_{e,j}^T]^T = [x_{e,j}, y_{e,j}, z_{e,j}, \theta_{e_1,j}, \theta_{e_2,j}]^T
\]

and their derivatives are

\[
\dot{X}_o = [\dot{r}_o^T, \omega_o^T]^T
\]

\[
\dot{X}_{e,j} = [\dot{r}_{e,j}^T, \omega_{e,j}^T]^T
\]

where \(r_o\) is the position vector from the origin of the base frame to the origin of the object frame, \(r_{e,j}\) is the position vector of the end-effector of the \(j^{th}\) robot manipulator, \(\theta_o\) is the orientation vector of the object frame measured in base coordinates, and \(\theta_{e,j}\) is the orientation vector of the end-effector of the \(j^{th}\) robot manipulator. \(\omega_o\) and \(\omega_{e,j}\) are the corresponding angular velocity vectors.
When multi-manipulators grasp the object firmly (i.e., Assumption 2), there is no relative motion between the object and the end-effectors. The following relations are valid:

\[ \omega_{e,j} = \omega_o \quad (C.1) \]

and

\[ r_{e,j} = r_o + R_o r_{oe,j} \quad (C.2) \]

where \( R_o \in \mathbb{R}^{3\times3} \) is an orthogonal rotation transform matrix that maps \( r_{oe,j} \) measured in \( O_t \) coordinates into a base coordinate representation.

Differentiating equation (C.2) yields

\[ \dot{r}_{e,j} = \dot{r}_o + \omega_o \times (R_o r_{oe,j}) = \dot{r}_o - (R_o r_{oe,j}) \times \omega_o = \dot{r}_o - S(R_o r_{oe,j}) \omega_o, \quad (C.3) \]

where \( S(.) \in \mathbb{R}^{3\times3} \) is a skew symmetric matrix operating on a vector \([70]\). If \( a = (a_x, a_y, a_z)^T \), then

\[ S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}, \]

and for any vector \( P = (p_x, p_y, p_z)^T \), \( a \times P = S(a)P \).

From (C.1) and (C.3), we have

\[ \begin{bmatrix} \dot{r}_{e,j} \\ \omega_{e,j} \end{bmatrix} = \begin{bmatrix} I_3 & -S(R_o r_{oe,j}) \\ 0 & I_3 \end{bmatrix} \begin{bmatrix} \dot{r}_o \\ \omega_o \end{bmatrix}, \quad (C.4) \]

that is,

\[ \dot{X}_{e,j} = J_{oj} \dot{X}_o \quad (C.5) \]

with

\[ J_{oj} = \begin{bmatrix} I_3 & -S(R_o r_{oe,j}) \\ 0 & I_3 \end{bmatrix}, \quad J_{oj}^T = \begin{bmatrix} I_3 & 0 \\ S(R_o r_{oe,j}) & I_3 \end{bmatrix}. \quad (C.6) \]

Obviously, the Jacobian matrix \( J_{oj} \) is always of full rank.
Also, the generalized force vectors of \( F_o \) and \( F_j \) can be partitioned as

\[
F_o = [f_o^T, n_o^T]^T \\
F_j = [f_{e,j}^T, n_{e,j}^T]^T
\]

where \( f_o \) and \( f_{e,j} \) are force vectors; \( n_o \) and \( n_{e,j} \) are torque (or, moment) vectors. When multi-manipulators grasp firmly on the object, the forces and moments applied by the multi-manipulators on the object can be mapped equivalently at the object frame as

\[
f_o = f_{e1} + f_{e2} + \cdots + f_{e\gamma}, \quad (C.7)
\]
\[
n_o = \sum_{j=1}^{\gamma} (n_{e,j} + (R_o r_{oe,j}) \times f_{e,j}). \quad (C.8)
\]

We have then

\[
F_o = \begin{bmatrix} f_0 \\ n_0 \end{bmatrix} = \sum_{j=1}^{\gamma} \begin{bmatrix} I_3 & 0 \\ S(R_o r_{oe,j}) & I_3 \end{bmatrix} \begin{bmatrix} f_{e,j} \\ n_{e,j} \end{bmatrix} = \sum_{j=1}^{\gamma} J_{o,j}^T F_j. \quad (C.9)
\]

With the above motion and force relationships, the dynamic equation of the object is obtained by using the Newton-Euler formulation as

\[
m_o \ddot{r}_o + m_o \ddot{g} = f_o \quad (C.10)
\]
\[
R_o I_o R_o^T \dot{\omega}_o + \dot{\omega}_o \times R_o I_o R_o^T \omega_o = n_o \quad (C.11)
\]

where \( \ddot{g} = [0, 0, 9.81]^T \).

Denoting

\[
M_o = \begin{bmatrix} m I_3 & 0 \\ 0 & R_o I_o R_o^T \end{bmatrix}, \quad N_o = \begin{bmatrix} 0 & 0 \\ 0 & S(\omega_o)R_o I_o R_o^T \end{bmatrix}, \quad G_o = \begin{bmatrix} m_o \ddot{g} \\ 0 \end{bmatrix}, \quad (C.12)
\]

we get

\[
M_o(X_o) \ddot{X}_o + N_o(X_o, \dot{X}_o) \dot{X}_o + G_o(X_o) = \sum_{j=1}^{\gamma} J_{o,j}^T F_j = J_o^T F. \quad (C.13)
\]
The dynamic equation of the object has the following important property:

**Property** The matrix \((\dot{M}_o - 2N_o)\) is skew symmetric.

**Proof:** Because \(\dot{R}_o = S(\omega_o)R_o\) [70], the differentiation of the matrix \(M_o\) yields

\[
\dot{M}_o = \begin{bmatrix}
0 & 0 \\
0 & S(\omega_o)R_oI_oR^T_o + R_oI_oR^T_oS^T(\omega_o)
\end{bmatrix}.
\]  
(C.14)

Thus,

\[
\dot{M}_o - 2N_o = \begin{bmatrix}
0 & 0 \\
0 & R_oI_oR^T_oS^T(\omega_o) - S(\omega_o)R_oI_oR^T_o
\end{bmatrix}.
\]  
(C.15)

Let \(A = R_oI_oR^T_oS^T(\omega_o) - S(\omega_o)R_oI_oR^T_o\), we have

\[
A + A^T = [R_oI_oR^T_oS^T(\omega_o) - S(\omega_o)R_oI_oR^T_o] + [S(\omega_o)R_oI_oR^T_o - R_oI_oR^T_oS^T(\omega_o)] = 0.
\]  
(C.16)

This shows that matrix \((\dot{M}_o - 2N_o)\) is a skew symmetric matrix.
Appendix D

Dual-Arm System Description

D.1 Dynamics of Robot Manipulators

The dual-arm system is plotted in Figure D.1. For simplicity, the two robot manipulators are assumed to be identical. Each has three joint-links. The coordinate frames are located as shown in the figure. By virtue of this identity, only the dynamic equation for one of the two manipulators needs to be developed. In order to simplify the notation, the subscript \( j \) used to designate a particular robot will be dropped in the following development unless it is necessary otherwise.

Using the Lagrangian formulation approach [70] with the same assumptions as [66], the dynamic equation of a planar flexible joint robot manipulator with three
revolute joints can be derived as

\[ \begin{align*}
D(q_l) \ddot{q}_l + C(q_l, \dot{q}_l) \dot{q}_l + G(q_l) &= \tau_s \\
I_m \ddot{\theta}_m + B_m \dot{\theta}_m + \tau_s &= u
\end{align*} \]  \hspace{1cm} (D.1)

and

\[ \tau_s = K_s(q_m - q_l), \]  \hspace{1cm} (D.3)

where

\[ D(q_l) = \begin{bmatrix}
d_{11} & d_{12} & d_{13} \\
d_{21} & d_{22} & d_{23} \\
d_{31} & d_{32} & d_{33}
\end{bmatrix} \]  \hspace{1cm} (D.4)

with

\[ \begin{align*}
d_{11} &= m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} C_2) \\
&\quad + m_3 (l_1^2 + l_2^2 + l_{c3}^2 + 2l_1 l_2 C_2 + 2l_1 l_{c3} C_{23} + 2l_2 l_{c3} C_3) + I_1 + I_2 + I_3 \\
d_{12} &= m_2 (l_{c2}^2 + l_1 l_{c2} C_2) + m_3 (l_2^2 + l_{c3}^2 + l_1 l_2 C_2 + l_1 l_{c3} C_{23} + 2l_2 l_{c3} C_3) + I_2 + I_3 \\
d_{13} &= m_3 (l_{c3}^2 + l_1 l_{c3} C_{23} + l_2 l_{c3} C_3) + I_3 \\
d_{21} &= d_{12} \\
d_{22} &= m_2 l_{c2}^2 + m_3 (l_2^2 + l_{c3}^2 + 2l_2 l_{c3} C_3) + I_2 + I_3 \\
d_{23} &= m_3 (l_{c3}^2 + l_2 l_{c3} C_3) + I_3 \\
d_{31} &= d_{13} \\
d_{32} &= d_{23} \\
d_{33} &= m_3 l_{c3}^2 + I_3,
\end{align*} \]

\[ C(q_{l,j}, \dot{q}_{l,j}) = \begin{bmatrix}
c_{11} & c_{12} & c_{13} \\
c_{21} & c_{22} & c_{23} \\
c_{31} & c_{32} & c_{33}
\end{bmatrix} \]  \hspace{1cm} (D.5)
with
\[
\begin{align*}
c_{11} &= -[m_2l_1l_2S_2 + m_3(l_1l_2S_2 + l_1l_3S_23)]\dot{q}_{12} - m_3(l_1l_3S_{23} + l_2l_3S_3)\dot{q}_{13} \\
c_{12} &= -[m_2l_1l_2S_2 + m_3(l_1l_2S_2 + l_1l_3S_23)](\dot{q}_{11} + \dot{q}_{12}) - m_3(l_1l_3S_{23} + l_2l_3S_3)\dot{q}_{13} \\
c_{13} &= -m_3(l_1l_3S_{23} + l_2l_3S_3)(\dot{q}_{11} + \dot{q}_{12} + \dot{q}_{13}) \\
c_{21} &= [m_2l_1l_2S_2 + m_3(l_1l_2S_2 + l_1l_3S_23)]\dot{q}_{11} - m_3l_2l_3S_3\dot{q}_{13} \\
c_{22} &= -m_3l_2l_3S_3\dot{q}_{13} \\
c_{23} &= -m_3l_2l_3S_3(\dot{q}_{11} + \dot{q}_{12} + \dot{q}_{13}) \\
c_{31} &= m_3l_1l_3S_{23}\dot{q}_{11} + m_3l_2l_3S_3(\dot{q}_{11} + \dot{q}_{12}) \\
c_{32} &= m_3l_2l_3S_3(\dot{q}_{11} + \dot{q}_{12}) \\
c_{33} &= 0,
\end{align*}
\]

and
\[
G(q_{1,j}) = [g_1, g_2, g_3]^T \quad (D.6)
\]

with
\[
\begin{align*}
g_1 &= [m_1l_c C_1 + m_2(l_1 C_1 + l_2C_{12}) + m_3(l_1C_1 + l_2C_{12} + l_3C_{123})]g \\
g_2 &= [m_2l_2C_{12} + m_3(l_2C_{12} + l_3C_{123})]g \\
g_3 &= m_3l_3C_{123}g.
\end{align*}
\]

Here \(g = 9.81\). In a horizontal plane \(G(q_{1,j}) = 0\).

In the above formula, the following trigonometric representations are used for notational simplification. (Subscript \(j\) is not omitted here so as to distinguish the difference between the joint angles of the two robots).
\[
\begin{align*}
S_{1,j} &= \sin(q_{11,j}) \\
C_{1,j} &= \cos(q_{11,j}) \\
S_{12,j} &= \sin(q_{11,j} + q_{12,j}) \\
C_{12,j} &= \cos(q_{11,j} + q_{12,j})
\end{align*}
\]
In Equations (D.2) and (D.3), $I_m$, $B_m$, and $K_s$ are diagonal matrices. Once the corresponding parameters are given, they are uniquely determined. All the parameters of the system are listed in Table 4.1 in Chapter 4.

D.2  Dynamics of Common Object

As derived in Appendix C, the dynamic equation of the object has the formula

$$ M_o(X_o) \ddot{X}_o + N_o(X_o, \dot{X}_o)\dot{X}_o + G_o(X_o) = \sum_{j=1}^{\gamma} J_{oj}^T F_j = J_o^T F. \tag{D.7} $$

For the planar system, the coordinate vector $X_o$ can be expressed as $X_o = [x_o, y_o, \theta_o]^T$, where $x_o$ and $y_o$ are the position coordinates, and $\theta_o$ is the orientation.

The rotation matrix $R_o$ can be derived as

$$ R_o = \begin{bmatrix} \cos(\theta_o) & -\sin(\theta_o) & 0 \\ \sin(\theta_o) & \cos(\theta_o) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{D.8} $$

It should be noted from the system configuration that

$$ \theta_o = q_{11.1} + q_{12.1} + q_{13.1} = q_{11.2} + q_{12.2} + q_{13.2} - 180^\circ, $$

and as measured in $O_i$ coordinates, $r_{oe,1} = [-r_1, 0, 0]^T$ and $r_{oe,2} = [r_2, 0, 0]^T$. Therefore, using formula (C.6), we have

$$ J_{oj} = \begin{bmatrix} 1 & 0 & S_{123,j} r_j \\ 0 & 1 & -C_{123,j} r_j \\ 0 & 0 & 1 \end{bmatrix}, \quad j = 1, 2 \tag{D.9} $$
From formula (C.12), we can also obtain

\[ M_o = \text{diag}\{m_o, m_o, I_o\}, \quad N_o = 0. \]

In a horizontal plane \( G_o = 0 \).

## D.3 Kinematics of the System

According to the coordinate frame given in Figure D.1, the kinematics of the system can be described. The task space coordinates representing the position and orientation of the robot end-effector with respect to the base are given by \( X_{e,j} = [x_{e,j}, y_{e,j}, \theta_{e,j}]^T \). The robot manipulator forward kinematics can be expressed as

\[
X_{e,1} = \begin{bmatrix} x_{e,1} \\ y_{e,1} \\ \theta_{e,1} \end{bmatrix} = \begin{bmatrix} l_1C_{1,1} + l_2C_{12,1} + l_3C_{123,1} \\ l_1S_{1,1} + l_2S_{12,1} + l_3S_{123,1} \\ q_{1,1} + q_{2,1} + q_{3,1} \end{bmatrix} \quad (D.10)
\]

\[
X_{e,2} = \begin{bmatrix} x_{e,2} \\ y_{e,2} \\ \theta_{e,2} \end{bmatrix} = \begin{bmatrix} l_1C_{1,2} + l_2C_{12,2} + l_3C_{123,2} + d \\ l_1S_{1,2} + l_2S_{12,2} + l_3S_{123,2} \\ q_{1,2} + q_{2,2} + q_{3,2} \end{bmatrix} \quad (D.11)
\]

The robot manipulator Jacobian matrices can be derived as

\[
J_j = \begin{bmatrix}
-l_1S_{1,j} - l_2S_{12,j} - l_3S_{123,j} & -l_2S_{12,j} - l_3S_{123,j} & -l_3S_{123,j} \\
l_1C_{1,j} + l_2C_{12,j} + l_3C_{123,j} & l_2C_{12,j} + l_3C_{123,j} & l_3C_{123,j} \\
1 & 1 & 1
\end{bmatrix}, \quad j = 1, 2
\]

with the relationship:

\[
\dot{X}_{e,j} = J_j \dot{q}_j, \quad j = 1, 2 \quad (D.12)
\]

Denoting \( X_e = [X_{e,1}^T, X_{e,1}^T]^T \), \( q_l = [q_{l,1}, q_{l,2}]^T \) and \( J = \text{block diag}(J_1, J_2) \), we have

\[
\dot{X}_e = J \dot{q}_l. \quad (D.13)
\]

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For this specific system, it is also straightforward to present the coordinate of the object in terms of the joint coordinates.

\[
X_o = \begin{bmatrix}
  l_1C_{1,1} + l_2C_{12,1} + (l_3 + r_1)C_{123,1} \\
  l_1S_{1,1} + l_2S_{12,1} + (l_3 + r_1)S_{123,1} \\
  q_{11,1} + q_{12,1} + q_{13,1}
\end{bmatrix}
= \begin{bmatrix}
  l_1C_{1,2} + l_2C_{12,2} + (l_3 + r_2)C_{123,2} + d \\
  l_1S_{1,2} + l_2S_{12,2} + (l_3 + r_2)S_{123,2} \\
  q_{11,2} + q_{12,2} + q_{13,2} - 180
\end{bmatrix}
\]  

(D.14)

If we denote \( J_{oq,j} \) the Jacobian matrices from joint space to the coordinate of the object, then we have the relation

\[
\dot{X}_o = J_{oq,j} \dot{q}_{l,j}, \quad j = 1, 2
\]  

(D.15)

where

\[
J_{oq,j} = \begin{bmatrix}
  -l_1S_{1,j} - l_2S_{12,j} - (l_3 + r_j)S_{123,j} & -l_2S_{12,j} - (l_3 + r_j)S_{123,j} & -(l_3 + r_j)S_{123,j} \\
  l_1C_{1,j} + l_2C_{12,j} + (l_3 + r_j)C_{123,j} & l_2C_{12,j} + (l_3 + r_j)C_{123,j} & (l_3 + r_j)C_{123,j} \\
  1 & 1 & 1
\end{bmatrix}.
\]

It could easily be verified that

\[
J_j = J_{o,j} J_{oq,j}, \quad j = 1, 2
\]  

(D.16)

As Jacobian matrices are critical in the coordinated control system, equality (D.16) also provides one more checkup for the correctness of the derived Jacobian matrices.

With the above derived kinematic relationships, the joint coordinates \( q_l \) can be expressed in terms of the task space coordinates \( X_e \) or \( X_o \) by inverse kinematics.