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CHARACTERISTICS OF JET IMPINGEMENT,
DRAINAGE AND COMPRESSION IN A FORMING ROLL
OF A TWIN-WIRE MACHINE

by

Jae Ho Jong

A thesis submitted in conformity with the requirements
for the Degree of Doctor of Philosophy,
Department of Mechanical and Industrial Engineering
University of Toronto

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0-612-41558-9
ABSTRACT

CHARACTERISTICS OF JET IMPINGEMENT, DRAINAGE AND COMPRESSION IN A FORMING ROLL OF A TWIN-WIRE MACHINE

Jae Ho Jong

Doctor of Philosophy, 1998

Department of Mechanical and Industrial Engineering

University of Toronto

This thesis involves analytical and experimental studies of the characteristics of jet impingement, the drainage mechanism and the compression process in the formation region of a twin-wire machine. The objective is to find the effects of operating parameters on flow and mat properties such as velocity, pressure and mat thickness, as well as their effects on sheet formation. The aim of the analytical work was to develop a physical flow model using hydrodynamic theory that could best describe the real flow behaviour in the wedge zone, the free-surface wedge zone, and the press zone. A set of governing equations was established and numerically solved for each zone. The change in wedge thickness was analytically predicted and compared with the measurements. The mean flow properties were calculated for a typical range of operating conditions. Experiments were conducted to study the resistance characteristics of fibre mat and screen. The results indicate the existence of three distinct regions in which the pressure drop through the fibre mat behaves differently. The mat resistance was estimated as a specific filtration resistance, which was compared with Darcy's law and the Kozeny-Carman equation. A
relationship was established between the mat resistance and the mass concentration. To study compression of the mat, experiments were performed using three types of pulp. It was demonstrated that the well-known empirical compressibility equation was valid over a wide range of compacting pressures, but only for compression over a long time. For compression over a short time, it was observed that when the mat was suddenly subjected to high pressure, it underwent a free-fall drop type of compression. This suggests that a saturated mat would behave more like a highly viscous fluid rather than a viscoelastic solid when subjected to high compression. Hence, the free-fall drop model was proposed to predict the consistency in the press zone and was found to be in agreement with the measurements. Video Beta Radiography and the Micro-Scanner method were used to analyze formation on the experimental samples. Using the formation diagram, it was determined that the Jet/Wire speed ratio had a significant effect on formation.
ACKNOWLEDGEMENTS

The author would like to thank many individuals and organizations for their help and advice throughout the progress of this thesis. Special guidance and frequent encouragement were given by Professor W. D. Baines and Professor I. G. Currie, his supervisors at the University of Toronto. The author greatly benefited from their support and invaluable knowledge. Also acknowledged is the contribution of Professor C. T. J. Dodson, who supported this project for the first three years, before moving to the University of Manchester Institute of Sciences and Technology.

Special thanks is also given to R. Gooding, J.D. McDonald, R. Daunais and A. Rompre of the Pulp and Paper Research Institute of Canada (PAPRICAN) for their technical support in trials of a pilot twin-wire machine. In particular, Dr. Gooding spent many hours discussing various aspects of the project. Important contributions came as well from R. Herzig, D. Johnson, J. Buchanan and G. Jackson at the Johnson Wire Industries; the idea of the flow-loop design is credited to JWI who provided several types of forming fabrics for the experiments. Another source of help came from J. Amini, originally at Domtar Inc. and now with International Paper. The technical support of M. Kalovski, L. Rooseman and D. Kalra of UTMIE was very much appreciated.

The person who should be given the most recognition for completion of this thesis is the author’s wife, Helen Hye Kyung Kim. She motivated the author and provided him with continuous encouragement, along with other family members. They all deserve praise and gratitude for their efforts.

Finally, the author wishes to thank NSERC, OGS and the University of Toronto Paper Forming and Performance Research Consortium for their financial support.
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<td>$A$</td>
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<td>$a$</td>
<td>Mat Resistance Coefficient [1/s]</td>
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<td>$a$</td>
<td>Inverse of the Elastic Modulus $I/E$</td>
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<tr>
<td>$BW$</td>
<td>Basis Weight [g/m$^2$ or gsm]</td>
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<td>Mass Concentration or Apparent Density [kg/m$^3$] or Biot's Consolidation Constant $K/(\mu a)$ in the Appendix</td>
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<td>$C$</td>
<td>Consistency [%], Coefficient or Damping Coefficient [N·s/m]</td>
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<td>$CSF$</td>
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<td>$CV$</td>
<td>Coefficient of Variation (%)</td>
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<td>$D$</td>
<td>Estimated Floc Diameter [mm]</td>
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<td>$E$</td>
<td>Elastic Modulus</td>
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<tr>
<td>$F$</td>
<td>Force [N] or Complex Potential Plane</td>
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<tr>
<td>$F(k, \phi)$</td>
<td>Incomplete Elliptic Integral of the first kind</td>
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<td>$f$</td>
<td>Function or Body Force with subscript</td>
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<td>$N$</td>
<td>Functional Coefficient</td>
</tr>
<tr>
<td>$n$</td>
<td>Number</td>
</tr>
<tr>
<td>$O$</td>
<td>Geometric Origin</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure [Pa]</td>
</tr>
<tr>
<td>$Q$</td>
<td>Volume Flow Rate [m$^3$/s] or Hodograph Plane</td>
</tr>
<tr>
<td>$q$</td>
<td>Magnitude of Free Jet Velocity</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of Curvature of a Forming Roll [m]</td>
</tr>
<tr>
<td>$R'$</td>
<td>Converted Radius of Curvature [m]</td>
</tr>
<tr>
<td>$r$</td>
<td>Representative Fibre Radius [m] or Constant in the Complex Plane</td>
</tr>
<tr>
<td>$S$</td>
<td>Specific Surface Area per Unit Volume of Solid Material [m$^2$/m$^3$] or Specific External Surface with Subscript [m$^2$/kg]</td>
</tr>
<tr>
<td>$SFR$</td>
<td>Specific Filtration Resistance [m/kg]</td>
</tr>
<tr>
<td>$s$</td>
<td>Solidity Ratio or Location of Jet Impingement in a Complex Plane</td>
</tr>
</tbody>
</table>
\( T \) — Wire Tension [N/m]
\( t \) — Time [s] or Jet Thickness [cm]
\( U \) — Velocity [m/s]
\( u \) — Velocity on x-axis [m/s]
\( V \) — Void Fraction
\( VBR \) — Video Beta Radiography
\( v \) — Vertical Velocity [m/s] or Fibre Specific Volume [m\(^3\)/kg]
\( W \) — Gravitational Force [N] or Face Velocity [m/s]
\( w \) — Displacement of Height Change in z-direction
\( y \) — Jet Thickness [cm or mm]
\( \alpha \) — Jet to Free Streamline Velocity Ratio
\( \varepsilon \) — Compressive Strain
\( \dot{\varepsilon} \) — Strain Rate [1/s]
\( \rho \) — Density [kg/m\(^3\)]
\( \mu \) — Dynamic Fluid Viscosity [Pa s]
\( \eta \) — Viscosity [Pa s]
\( \nu \) — Kinematic Fluid Viscosity [m\(^2\)/s]
\( \sigma \) — Compacting Stress [Pa]
\( \dot{\sigma} \) — Time-Dependent Stress
\( \phi \) — Porosity
\( \theta \) — Jet impingement Angle [°]
\( \zeta \) — Complex Potential Plane
\( \Omega \) — Summation Term in the Biot Analysis

Subscripts

\( 0 \) — initial
\( c \) — combined or critically uncompressed
\( d \) — damping
\( f \) — fraction
\( IM \) — immovable
\( MV \) — movable
\( m \) — mass
\( mat \) — mat
\( mo \) — uncompressed saturated mat
\( p \) — pressure or piston
\( s \) — screen, solid or suspension
\( T \) — total
\( t \) — terminal
\( v \) — void
\( w \) — wire
\( z \) — z-axis
CHAPTER 1

INTRODUCTION

1.1 Introductory Remarks

Ever since the invention of paper, which was credited to Ts'ra Lun of China in 105 A.D., papermaking technology has constantly evolved, making inroads around the world. In 1804, one of the most remarkable achievements in the industry was the development of the continuous papermaking machine, called the Fourdrinier machine. A brief history of the evolution of this papermaking machine is given in the handbook by Biermann (1993).

The inherent limitations of a conventional Fourdrinier machine have spurred the development of new dewatering techniques to achieve higher operating speeds and better product quality. This effort has resulted in the development of a second generation of papermaking machines, called twin-wire machines. The invention of the first modern twin-wire machine in 1953 is credited to Daniel Webster, who used the innovation of draining water through two converging wires. The first twin-wire former became commercially available in 1958. New machines using a twin-wire former are expected to dominate the industry as surveyed by Perrault (1990). Several types of twin-wire formers exist, but the operating principle is the same. An example of a twin-wire former, the Papriformer, is presented in Figure 1-1.
A jet of stock issues from a headbox and is intercepted by two converging wires. Drainage occurs through both wires. Captured fibres form a mat which travels around a forming roll and a couch roll before moving to the press section.

Little work has been done on drainage analysis around a forming roll in a twin-wire former. No detailed analysis has been carried out of the complete sheet build-up.
process for a twin-wire machine, despite attempts by Baines (1967), Meyer (1971), Hauptmann and Mardon (1973), Wahren (1987) and Miyanishi et al. (1989). This is because of the complexity of the simultaneous sheet growth and drainage mechanisms in a twin-wire machine. In contrast, the prediction of drainage at the wet-end pressing and the dry-end pressing is relatively well understood.

The lack of understanding of sheet growth and drainage is the motivation for this thesis, as discussed in the next section.

1.2 Motivation and Objective of This Thesis

A photograph of the jet impingement process in the forming roll region of the twin-wire former is presented in Figure 1-2, which shows the jet departing the headbox and impinging on the two converging wires.

Figure 1-2 Jet impingement in a twin-wire papermaking machine
A schematic diagram of the drainage process around the forming roll is shown in Figure 1-3. As suggested by Baines (1967), this region is divided into three zones: a free-jet zone, a wedge zone and a press zone, identified as ab, bc, and cd, respectively.

The pulp suspension leaves the headbox in a jet and travels through the air at a speed which may be as high as 1500 m/min. This is the free-jet zone.

As the jet impinges between the two converging wires, fibres in the jet are caught by the wires, and water begins to drain through both wires. This is the wedge zone.
Typically, the machine is operated in such a way that the jet speed is slightly higher than the wire speed. Impact of the jet results in a decrease in the fluid velocity. Consequently, the pressure should increase in the flow between the wires. This pressure increase pushes water through the wire and the forming mat. As the jet travels in the machine direction, more drainage takes place. When the forming mats from both wires make contact, mechanical compression from the wire tension causes more drainage. This is the press zone. Mat consistency increases in the press zone as a result of the decrease in mat thickness.

The focus of this research is the wedge zone and the press zone. It is believed that formation is largely determined in these regions in a twin-wire roll former. Formation is a measure of the weight uniformity of the sheet. It refers to the small-scale variation in the fibre mass distribution in the plane of the paper.

In order to improve formation, one must understand the flow-fibre interaction mechanism from the jet impingement point to the point where the formed mat leaves the forming roll. First, the governing parameters in the region should be identified. Second, the effect of each parameter should be studied either by experiment or by theory. In some cases, the governing parameters may greatly affect the flow properties, such as pressure and velocity, but may have no visible effect on overall formation. Nevertheless, it is important to analyze the mean flow properties and the homogeneous fibre mat build-up process before an attempt is made to understand fibre distribution and the formation mechanism. This analysis is conducted using reasonable approximations and the flow conservation equations in the wedge zone.
One of the great difficulties was the lack of knowledge of mat resistance and wire resistance. At present, these can be determined only by experiments. Wire resistance is defined using the concept introduced by Baines and Peterson (1951). To express mat resistance, the concept of specific filtration resistance, which is based on Darcy’s law and the Kozeny-Carman equation, is used. Another important issue is the characteristics of the fibre mat when subjected to high compression. Because the mat in the press zone undergoes compression on a short time scale, a model using the concept of free-fall drop is developed to predict consistency changes in the press zone.

1.3 Organization of This Thesis

This thesis consists of four main topics that are covered in six chapters: the estimation of screen and mat properties; the time-dependent compressible characteristics of the fibre mat; the development of flow-fibre equations in the wedge zone, the free-surface wedge zone and the press zone; and the formation analysis.

A literature review is presented in Chapter 2. The design of the experimental flow-loops and the experimental techniques used to evaluate the resistance properties of the screen and the fibre mat are described in Chapter 3. Chapter 4 covers the time-dependent compressibility analysis of the fibre mat. A free-fall drop model is proposed to predict consistency change in the press zone. Chapter 5 describes the use of Darcy’s law and the Kozeny-Carman equation to study the specific filtration resistance of the fibre mat. In Chapter 6, a set of governing equations is developed for jet impingement centred between two wires and jet impingement on one wire. Two analyses are performed for jet
impingement at the centre: i) the wedge shape determined from the measured data; ii) the wedge shape unknown. The governing equations with realistic physical approximations are numerically solved and compared with the measurements obtained from a pilot twin-wire machine. Formation analysis is carried out in Chapter 7 to access the effect of governing parameters on formation. Chapter 8 concludes the current study and suggests future research.
CHAPTER 2

LITERATURE REVIEW AND BACKGROUND INFORMATION

This chapter presents an overview of the relevant literature in the field and of current research.

2.1 Drainage Analysis

(a) Drainage  
(b) Oriented Shear  
(c) Turbulence

Figure 2-1 Hydrodynamic process affecting sheet formation [Parker (1972)]

To understand the fundamental physics of the drainage process in a papermaking machine, it is first necessary to visualize possible mechanisms for the fibre-water interaction and the drainage process. The classical study by Parker (1972) describes the basic sheet-forming process as a composite of three principal hydrodynamic processes, as depicted in Figure 2-1. According to his description, drainage is the flow through the screen and is characterized by a time-dependent flow velocity. Oriented shear is a distinct
process characterized by a mean velocity gradient. Turbulence is the random velocity fluctuation that exists in undrained suspensions. Parker then suggests that all these hydrodynamic processes contribute to the sheet forming process. Moreover, following the concept of Hisey (1956), drainage can take place through two mechanisms, filtration and thickening, as illustrated in Figure 2-2. Based on Parker’s definition, filtration occurs when the suspended fibres are free to move independently. Thickening occurs, on the other hand, when fibres are entangled and so form a coherent network. In the wedge zone, filtration is the main drainage mechanism, while thickening is predominant in the press zone. Other information on the mat forming process is found in the work of Wrist (1962), Norman et al. (1977) and Norman (1989).

![Figure 2-2 Possible mechanism of fibre deposition by drainage](image)

During the drainage process, water drains through the screen and the fibre mat. This process is analogous to the flow through two porous media, and flow resistance
changes throughout the process. Several experimental methods have been suggested to evaluate the resistance properties of the screen and the mat.

One of the first approaches was that of Collicutt (1947), who used a constant pressure filtration process applied to the pulp in the drainage tester. Later, Wahlstrom and O'Blenes (1962) developed a similar drainage tester and suggested that the concept of average mat resistance was not accurate because Darcy's law was invalid for the fibre mat owing to its nonhomogeneous structure. The experimental results of Clos and Edwards (1995) showed that any drainage system at constant pressure eventually reaches a limiting consistency. On the other hand, Ingmanson et al. (1961) and Sayegh and Gonzalez (1995) used a constant drainage rate test apparatus to investigate the effect of pressure and time on the resistance of the pulp mat.

Ramarao et al. (1994) investigated drainage and fine particle retention phenomena analytically and experimentally. They showed that drainage is not a constant pressure process, nor is it a constant filtration process. Also, fibre mats are compressible to a large degree, and the resistance varies even inside the mat. Their subsequent studies (1996) have provided a mathematical model for a compressible fibre mat. Sampson and Kropholler (1995, 1996) presented experimental and theoretical analyses of fibre mat buildup.

There are several studies in the literature of screen resistance. Estimates of screen resistance in this thesis are based on the study by Baines and Peterson (1951), who performed extensive tests to relate the screen resistance with the solidity ratio and the Reynolds number. Ingmanson et al. (1961) also developed a correlation by considering the viscous force and inertial force applied to the screen. Boadway and Gray (1963) used a similar approach to express the screen resistance.
Meyer (1969) suggested that some discrepancies between pressure drop measurements and predictions are caused by the hydrodynamic interaction between the fibre mat and the screen. This interaction causes the total pressure drop to be greater than the sum of the pressure drops through the fibre mat and the screen.

Johnson (1984) used flowing air in drainage experiments. He found that the nature of the sheet support provided by the forming fabric surface played a significant role in drainage, even at high mat weights. He developed a drainage index (DI) for fabrics based on the correlation of the pressure drop data across formed mats. Ng et al. (1990) also performed drainage analysis using air flow.

A summary of the various drainage analyses is presented in Table 2-1. Comparing the different drainage test results is difficult since test conditions as well as analysis techniques differ greatly, as noted by Kerekes and Harvey (1980).

2.1.1 Drainage in a Twin-Wire Former

The drainage process in a twin-wire former was initially investigated by Baines (1967), who used a one-dimensional approach to develop basic flow equations for the flow and the fibre mat. Subsequently, adopting a similar approach, Meyer (1971) developed governing equations using hydrodynamics, though he did not attempt to solve them. Hauptmann and Mardon (1973) studied the drainage process near the impingement region and emphasized the effect of backflow in the formation region.
Table 2-1 Brief summary of the drainage tester found in the literature

<table>
<thead>
<tr>
<th>Authors</th>
<th>Year</th>
<th>Theoretical Model</th>
<th>Method of Drainage</th>
<th>Main Parameters&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Measured Parameter&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Maximum Pressure Drop</th>
<th>Pulp or Screen</th>
<th>Main Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collicutt</td>
<td>1947</td>
<td>Modified Form of Poiseuille's Law for Capillary Flow</td>
<td>Constant Pressure</td>
<td>SFR vs. Vol. &amp; P</td>
<td>v, t</td>
<td>5.9 kPa</td>
<td>Groundwood</td>
<td>Relation between Filter Time and Filtrate Volume at Constant Pressure</td>
</tr>
<tr>
<td>Robertson and Mason</td>
<td>1949</td>
<td>Kozeny-Carman</td>
<td>Constant Pressure</td>
<td>SS, SV, Beating, Swelling</td>
<td>v, Mat Height</td>
<td>2 kPa</td>
<td>Sulfite, Kraft</td>
<td>Liquid Permeability Method</td>
</tr>
<tr>
<td>Ingmanson et al.</td>
<td>1961</td>
<td>Forchheimer type</td>
<td>Constant Velocity</td>
<td>P, v, Screen Geometry</td>
<td>P, v</td>
<td>211 kPa</td>
<td>Twill-Weave Screen</td>
<td>Pressure drop through screen at low &amp; high velocity</td>
</tr>
<tr>
<td>Wahlstrom and O'Blenes</td>
<td>1962</td>
<td>Empirical Equation</td>
<td>Constant Pressure</td>
<td>Pulp Type, t, P,</td>
<td>P, c, BW, t</td>
<td>60 kPa</td>
<td>Sulphite</td>
<td>Use of Average SFR is questionable.</td>
</tr>
<tr>
<td>Meadley</td>
<td>1963</td>
<td>Turbulence Model with Inertia Effect</td>
<td>Vacuum</td>
<td>P, SS, Stock Depth, t, t,</td>
<td>P, Stock Depth, t, t</td>
<td>60 kPa</td>
<td>Nylon Fibre, Groundwood, Unbleached Sulphite</td>
<td>Turbulence regime of Drainage</td>
</tr>
<tr>
<td>Gertjejansen</td>
<td>1964</td>
<td>Darcy, Kozeny-Carman</td>
<td>Constant Pressure</td>
<td>SFR, P, c, SS, SV</td>
<td>v, t, c</td>
<td>5.9 kPa</td>
<td>Bleached &amp; Unbleached Softwood, Hardwood</td>
<td>Agreement of SS, SV with other results</td>
</tr>
</tbody>
</table>

<sup>1</sup>v:velocity (m/s), t:time (s), P:pressure (Pa), c:consistency, SS:specific surface (m<sup>2</sup>/m<sup>3</sup>), SV:specific volume (m<sup>3</sup>/kg), BW:basis weight (kg/m<sup>2</sup>)
<table>
<thead>
<tr>
<th>Authors</th>
<th>Year</th>
<th>Theoretical Model</th>
<th>Method of Drainage</th>
<th>Main Parameters</th>
<th>Measured Parameter</th>
<th>Maximum Pressure Drop</th>
<th>Pulp or Screen</th>
<th>Main Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andrews and White</td>
<td>1969</td>
<td>Darcy, Kozeny-Carman</td>
<td>Constant Velocity</td>
<td>High Vel., SFR, P, c, BW</td>
<td>P, BW</td>
<td>100 kPa</td>
<td>West Coast Sulfite</td>
<td>High Speed tester</td>
</tr>
<tr>
<td>Ng et al.</td>
<td>1990</td>
<td>Channel Model, Drag Model</td>
<td>Const. Vel. by air</td>
<td>P, v, BW, Fibre length</td>
<td>P, BW, v, Fibre Length</td>
<td>14 kPa</td>
<td>Bleached Softwood Kraft (Black Spruce)</td>
<td>Drag Model by Air Flow</td>
</tr>
<tr>
<td>Mantar et al.</td>
<td>1995</td>
<td>Darcy, Kozeny-Carman</td>
<td>Constant Pressure</td>
<td>SFR, BW, c,</td>
<td>Filtrate Vol, t</td>
<td>16.9 kPa</td>
<td>Beaten &amp; Unbeaten Softwood, Hardwood</td>
<td>SFR = f(BW, c), Fine Content</td>
</tr>
<tr>
<td>Clos and Edwards</td>
<td>1995</td>
<td>Darcy, Kozeny-Carman</td>
<td>Constant Pressure</td>
<td>t, c, P, Surface Tension, Viscosity</td>
<td>c, t</td>
<td>1.38 kPa</td>
<td>Virgin Kraft</td>
<td>Limiting Consistency Model</td>
</tr>
<tr>
<td>Sayegh and Gonzalez</td>
<td>1995</td>
<td>Darcy, Kozeny-Carman</td>
<td>Constant Velocity</td>
<td>SFR, P, t</td>
<td>P, t</td>
<td>35 kPa</td>
<td>Groundwood, Bleached Softwood, Unbleached Hardwood</td>
<td>SFR = f(P, t), Maxwell based SFR model</td>
</tr>
<tr>
<td>Sampson and Kropholler</td>
<td>1995 1996</td>
<td>Kozeny-Carman</td>
<td>Modified Batch-Drainage Tester</td>
<td>c, t, Cumulative Mass</td>
<td>t, c, mass</td>
<td>gravity (N/A)</td>
<td>Chemical Pulp, Mechanical Pulp</td>
<td>Improved batch-Drainage Tester</td>
</tr>
<tr>
<td>This Study</td>
<td>1997</td>
<td>Darcy, Screen Equation</td>
<td>Constant Pressure</td>
<td>P, v, BW, SFR</td>
<td>P, v, BW</td>
<td>55 kPa</td>
<td>LWC, Newsprint, Unbleached Softwood</td>
<td>3 Pressure Drop Region by Basis Weight</td>
</tr>
</tbody>
</table>
A number of other studies can be found in the literature on the topic of water drainage in a twin-wire machine. Thorp and Barasch (1985) examined the drainage process and suggested that the jet impingement angle and the jet velocity were major factors affecting drainage. Wahren (1987) attempted a quantitative treatment of jet impingement around the forming roll based on the roll geometry. Miyanishi et al. (1989) used Poiseuille's law to develop a mathematical analysis of the drainage process.

2.1.2 Fibre Mat Resistance

The governing equations describing flow through a fibre mat can be derived from Darcy's law, which was originally developed from experiments on water flow through a bed of uniform sand. Scheidegger (1957), Collins (1961) and Bear (1972) studied Darcy's law, which states that "the rate of flow (volume per unit time) \( Q \) is proportional to the constant cross-sectional area \( A \) and is also proportional to the piezometric head \( \Delta P \), and inversely proportional to the height of a porous medium \( h \)". It can be formulated as

\[
v = \frac{Q}{A} = \frac{K \Delta P}{\mu h}, \tag{2-1}
\]

where \( v \) is the average flow velocity (m/s), \( Q \) is the volume flow rate (m\(^3\)/s), \( A \) is the cross-sectional area of the porous medium (m\(^2\)), \( h \) is the height of the porous media (m), \( \Delta P \) is pressure drop (Pa) and \( K \) is the permeability (m\(^2\)) of the porous matrix, which is solely dependent on properties of the solid matrix. The dynamic viscosity \( \mu \) ((N·s)/m\(^2\)) is the only fluid property in the formula.
For a fibre mat, the mat height $h$ is equal to the basis weight $BW$, divided by the mass concentration $c_m$. Hence

$$\Delta P = \frac{\mu}{K} \cdot \frac{v \cdot BW}{c_m} ,$$  \hspace{1cm} (2-2)$$

where $BW$ has units of kg/m$^2$ and $c_m$ has units of kg/m$^3$.

Here, $K$ and $c_m$ are determined by the properties of the fibre mat for a constant $BW$. In the literature, the term $1/(K \cdot c_m)$, which combines them, is typically referred to as the Specific Filtration Resistance (SFR). The mat resistance coefficient $a$, may be defined by combining the $SFR$ and $\mu$ into a single term for simplification, as shown by Jong et al. (1997):

$$\Delta P = \mu \cdot \left( \frac{1}{K \cdot c_m} \right) \cdot v \cdot BW = \mu \cdot SFR \cdot v \cdot BW = a \cdot v \cdot BW ,$$  \hspace{1cm} (2-3)$$

where the $SFR$ has units of m/kg and $a$ has units of l/s. If the working fluid is water and its viscosity is assumed to be constant at the operating temperature, then the $SFR$ is linearly proportional to the mat resistance coefficient $a$.

As shown by Jackson and James (1986), the permeability $K$ is characterized by certain properties of the porous medium: the porosity, the geometric shape and orientation of the solid particles, the surface area exposed to the working fluid, and the pore-size distribution. A large number of permeability relationships have been compared for different types of materials, and comprehensive reviews have been presented by a number
of researchers such as Scheidegger (1957), Collins (1961), Bear (1972) and Jackson and James (1986).

Only a few studies on randomly oriented compressible fibres are found in the literature. The most widely used approach has been to apply the Kozeny-Carman equation, which is based on the similarity between Darcy's law and Poiseuille's equation as originally studied by Carman (1937). Using the definition of a hydraulic radius, the Kozeny-Carman equation relates the permeability $K$ with the porosity $\phi$ and the specific surface area per unit volume of solid material $S$:

$$K = \frac{1}{k \cdot S^2} \cdot \frac{\phi^3}{(1-\phi)^2}, \quad (2-4)$$

where $k$ is the Kozeny constant which depends upon the shape of the pores, the particle orientation, and the pore-size distribution.

The validity of the Kozeny-Carman equation has been questioned due to the uncertainty of the Kozeny constant, as argued by Scheidegger (1962). He claims that the use of any permeability relationship other than Darcy's law would be unjustified. However, Darcy's law does not relate the permeability to the properties of the porous media. The permeability $K$ is defined simply as an experimentally determined proportionality coefficient. Therefore, it is possible to have the same permeability for a completely different structure of porous medium. Darcy's law is also not applicable to the analysis of a compacting porous medium. Therefore, the use of the Kozeny-Carman equation is pursued in this study with the criticisms borne in mind.
First, it is essential to determine the value of the Kozeny constant $k$ which is reasonable for wood fibres. Since wood pulp fibres have a highly irregular body structure, particularly after beating, direct measurement of the specific surface is not feasible and so is of no use in evaluating the Kozeny constant $k$ from permeability measurements.

The study by Fowler and Hertel (1940) was probably the first to suggest a meaningful value of $k$ applicable to a variety of fibrous materials similar to randomly packed wooden fibres. Their study was based on the air permeability method. Sullivan (1941, 1942) studied air flow through bundles of textile fibres to determine the value of the Kozeny constant. His study included a fibre orientation effect. Campbell (1947) used the value $k=5$ for a pulp mat; then, Robertson and Mason (1949) used the value $k=5.55$. At present, the Kozeny constant typically used is $k=5.55$. However, as pointed out by Ingmanson and Andrews (1959-a), this value of the Kozeny constant is valid only in the porosity range of $\phi<0.8$. In the porosity range of $\phi>0.8$, the Kozeny constant increases substantially. Actual consistency measurements by Gooding (1996) indicate that the porosity is greater than 0.8 in the jet impingement region of the PAPRICAN twin-wire former.

Another difficulty in using the Kozeny-Carman equation is determining the porosity $\phi$ and the specific surface $S$. The specific surface area $S$ must be determined by statistical or indirect means because the internal surface of any natural fibrous porous media is of extreme complexity. The preferred method for determining the specific surface area would be to assign a reasonable value to the Kozeny constant $k$ and calculate a relative value of the specific surface of the fibres. Brown (1950) performed air permeability tests to determine the specific surface area of pulp fibres. He found that the
specific surface area is somewhat dependent upon porosity when constant $k$ is assumed. As for the specific surface of pulp fibres, only a few data are available in the literature and the accuracy is uncertain. Moreover, a direct comparison of each set of data is difficult because of the various methods of pulp preparation and the different fibres used in the tests.

The determination of the porosity $\phi$ is not an easy task because of the swelling of fibres and the change in porosity under compaction. For non-swelling solid particles, the application of the Kozeny-Carman equation is straightforward since the porosity is directly obtained from the density of the bulk solid and the mass concentration of the solid in the medium. On the other hand, for wood fibres which swell, a considerable amount of water is contained inside the fibres, and this immovable water should be considered as part of the solid rather than the liquid. Since the swelling ratio is not known for a fibre, the porosity cannot be measured from the mass concentration. To overcome this, Robertson and Mason (1949) developed a method for calculating the porosity $\phi$ by defining the effective volume of the swollen fibres to be $v_s$ (m$^3$/kg). Then, at a mass concentration of $c_m$ (kg/m$^3$), the porosity $\phi$ becomes

$$\phi = 1 - v_s c_m.$$  \hspace{1cm} (2-5)

In this study, $v_s$ is called the fibre specific volume (m$^3$/kg) and is determined experimentally by the method described in Chapter 5.

Another permeability relationship for fibres which has been used by some researchers was developed by Davies (1952). He studied experiments of air flow through porous media and produced the empirical relationship
where $r$ is the representative fibre radius (m), and the Davies values for constants $k_1$ and $k_2$ were 4.0 and 56, respectively. Ingmanson and Andrews (1959-a) used this relationship for their permeability measurements through glass and nylon fibres. They confirmed the validity of the relationship and found that $k_1=3.5$ and $k_2=57$. The discrepancies in the constants were attributed to the differences in fibre orientations in the vertical direction.

Meyer (1962) and Jönsson and Jönsson (1992) reviewed studies by Happel (1958, 1959, 1965), who developed an analysis to predict the flow resistance relative to the cylinder assembly in parallel or perpendicular configuration. Happel's expression for the permeability analysis is written in terms of the Kozeny constant for the configuration of flow perpendicular to the cylinders as follows:

$$
K = \frac{1}{r^2} \frac{1}{k_1(1-\phi)^{1.5} \left[1 + k_2(1-\phi)^3\right]},
$$

(2-6)

where $\phi$ is the porosity.

As stated, Darcy's law and the Kozeny-Carman equation have been used to represent the resistance characteristics of the fibre mat. There are also various criticisms of the simple hydraulic radius model which the Kozeny-Carman theory is based on. Nevertheless, the continued use of the Kozeny-Carman theory can be justified by the fact
that "its relatively simple analysis leads to intelligible results which can be related to experimental observations." [Ingmanson et al. (1959-b)].

2.2 Compression of the Fibre Mat

Campbell (1947) first acknowledged the importance of understanding mat compressibility in relation to the physics of water drainage. He set up a simple compression apparatus and reported that the mass concentration of the compressed pulp had a power law relation to the compacting pressure as shown in Eq. 2-8:

\[ c_m = \rho_{mat}C = \text{function}(\sigma_z) = K_1 \sigma_z^{K_2}, \quad (2-8) \]

where \( c_m \) is the mass concentration, or apparent density based on the fibre mat volume (kg/m\(^3\)), \( \rho_{mat} \) is the density of the wet mat (kg/m\(^3\)), \( C \) is the dimensionless mass-based consistency, \( \sigma_z \) is the compacting pressure (Pa), and \( K_1, K_2 \) are the constants to be determined from the experimental results.

Hisey (1956) studied the existence of several dewatering stages in a compressible mat, such as filtration and thickening. He observed that water was squeezed from within the mat due to compression under fluid drag forces. Subsequently, Ingmanson et al. (1959-b) measured the internal pressure drops within the fibre mat under fluid stress. They emphasized the importance of porosity distribution in the mat and using a knowledge of mat compressibility characteristics, correlated it with the internal pressure drop gradients. Gertjejansen (1964) tested a simple constant-pressure filtration method and collected
compressibility measurements. He established the reliability of test methods from the results of average specific filtration resistance and compressibility data.

To emphasize the time-dependent compression, Wilder (1960) investigated the creep properties of saturated fibre mats and correlated the results with an empirical compressibility equation. His experimental results confirmed the necessity of including the effect of time into the compressibility relationship. He attempted to formulate a simple empirical time-dependent compressibility equation. Jones (1963) made extensive compressibility studies of non-woven fibre beds to investigate fibre interaction mechanisms. His compression-recovery data showed the presence of hysteresis with pulp fibre. He attributed this non-recoverable compression-recovery deformation to fibre structural properties, mainly the fibre length-to-diameter ratio. Fibre repositioning under a load could be related to the initial loose formation of the mat.

In other engineering fields such as geophysics, another approach to the time-dependent analysis of compressible porous media has been widely used. Biot (1941) applied consolidation analysis to predict the time-dependent compression behaviour of soil. The governing equation for this process has the form of the heat conduction equation. The same idea, applied to the compression of the fibre mat, is described in Appendix C.

Emmons (1965) derived the fundamental equations of motion for pulp slurry under compression and filtration. He came up with governing equations similar to those given by Biot (1941). Recently, Jönsson and Jönsson (1992) presented a theoretical analysis of a flow model related to the pressure in compressible porous media.

Although several compressibility relationships are known that describe the compression of a fibre mat, as shown in Table 2-2, the fundamental structure of the
relationships is the same: the empirical relationship between a mass concentration and an applied compacting pressure, as shown in Eq. 2-8.

### Table 2-2 Typical compressibility models used in the literature

<table>
<thead>
<tr>
<th>Authors</th>
<th>Year</th>
<th>Compressibility Model$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Campbell</td>
<td>1947</td>
<td>$c_m = \rho_{mat}C = K_1 \sigma_z^{K_2}$</td>
</tr>
<tr>
<td>Ingmanson &amp; Whitney</td>
<td>1954</td>
<td>$c_m = \rho_{mat}C = K_1 \sigma_z^{K_2}$</td>
</tr>
<tr>
<td>Ingmanson &amp; Andrews</td>
<td>1959-a</td>
<td>$c_m = \rho_{mat}C = K_1 \sigma_z^{K_2}$</td>
</tr>
<tr>
<td>Ingmanson &amp; Andrews &amp; Johnson</td>
<td>1959-b</td>
<td>$c_m = \rho_{mat}C = K_1 \sigma_z^{K_2}$</td>
</tr>
<tr>
<td>Wilder</td>
<td>1960</td>
<td>$c_m - c_0 = (A + B \cdot \text{Log } t) \sigma_z^{K_2}$</td>
</tr>
<tr>
<td>Macklem</td>
<td>1961</td>
<td>$c_m = \rho_{mat}C = K_1 \sigma_z^{K_2}$</td>
</tr>
<tr>
<td>Meyer</td>
<td>1962</td>
<td>$c_m = c_1 + K_1 \sigma_z^{K_2}$ for $c_1 = c \text{(at mat surface)}$ when $\sigma_z = 0$</td>
</tr>
<tr>
<td>Nelson</td>
<td>1964</td>
<td>$c_m = c_0 + K_1 \sigma_z^{K_2}$</td>
</tr>
<tr>
<td>Emmons</td>
<td>1965</td>
<td>$\sigma - \sigma_1 = m(c^a - c^b)$</td>
</tr>
<tr>
<td>Andrew &amp; White</td>
<td>1969</td>
<td>$c_m = \rho_{mat}C = K_1 \sigma_z^{K_2}$</td>
</tr>
<tr>
<td>Ramarao &amp; Kumar</td>
<td>1996</td>
<td>$c_m = \rho_{mat}C = K_1 \sigma_z^{K_2}$</td>
</tr>
</tbody>
</table>

The empirical compressibility equation has been found to apply well to many types of pulp mat over a wide range of compacting pressures. Nevertheless, the equation has several shortcomings. First, it requires the mass concentration $c_m$ to be zero at zero compacting pressure. The difficult part of this problem is how to define accurately a zero compacting pressure applied to the mat and its corresponding mass concentration. One possible solution is to define the pressure as a summation of a compacting pressure $\sigma_z$ and a fluid pore pressure $p$, as suggested by Wilder (1960) and Wrist (1961), so that the total pressure $P_t$ never becomes zero. Then,

$^2$ Here, $A,B,K_1,K_2,a,b$ and $m$ are all empirical constants.
\[ P_t = \tau_t + p = \text{Constant}. \]  \hspace{1cm} (2-9)

In this case, for a water-saturated fibre mat, the total pressure is distributed between the water contained in voids and the solid fibre structure. The stress in pore water is hydrostatic and does not cause any appreciable compression of the solid fibre structure. However, this method has not been widely accepted. Second, no reasonable theoretical analysis has been developed to prove the fundamental physics of the empirical compressibility equation. Third, and most important, the empirical compressibility equation is obtained by applying compression to the fibre mat for a sufficient duration until the piston movement actually stops. In the drainage region around a forming roll of a twin-wire former, the fibre mat is subjected to high compression only for a very short time. The effect of time is totally neglected in the empirical compressibility equation, and this omission has led many researchers to overestimate the mat compression whenever the empirical compressibility equation is used. Meyer (1962), for example, suggested in his filtration theory that the empirical compressibility equation was sufficiently accurate and there was no reason to assume that the compressibility constant was a function of time. On the other hand, Wilder (1960) added the time effect into the empirical compressibility equation by including a logarithmic time factor. Recently, Ramarao and Kumar (1996) also avoided addressing the time-dependent compressibility effect in their drainage model, even though they acknowledged its significance to the fibre mat.

This study considers the importance of time-dependent compression on the mat formation process.
2.3 Formation Analysis

The ultimate goal of this analysis is formation improvement. Good formation reflects uniform fibres and filler distribution, while poor formation suggests the irregular distribution of fibres. An overview of formation analysis is given by Norman (1989) and Deng and Dodson (1994).

There is no universally accepted technique for measuring formation. Two common techniques are used in this study: video beta radiography and the light transmission technique. A beta radiography tester developed by Dodson et al. (1995) is used for formation measurements. To evaluate formation measurements, the method described by Farnood et al. (1995) is applied using a formation diagram.

Formation can also be measured using a light transmission method, as reported by Jordan and Nguyen (1986) and Lodjmark (1992). The former proposed the specific perimeter as a criterion for evaluating formation. Laleg and Nguyen (1995) studied the concept of the specific perimeter and the coefficient of variation and related them with the quality of paper formation.
CHAPTER 3

DRAINAGE FLOW LOOP ANALYSIS

This chapter describes a method for estimating screen properties and fibre mat resistance. Two types of drainage flow loop have been constructed for experiments: an open-loop and a closed-loop. The results reveal the existence of three distinct regions of the pressure drop through the fibre mat.

3.1 Objectives

When the jet impinges on the converging wires in a twin-wire former, there is a velocity decrease in the wedge that results in a pressure increase. The positive gauge pressure forces the water to drain through the forming mat and the screen. The amount of water drained depends on the resistance properties of the fibre mat and the screen.

The objective of this chapter is to present an experimental method for determining the resistance properties, and an analysis of the results. The results are used in the development of a physical flow model, presented in Chapter 6.

3.2 Concepts Underlying This Experimental Work

The pressure drop through a screen and fibre mat is assumed to be the sum of the pressure drops through each. The pressure drop through the screen can be expressed in
terms of a screen coefficient $k_s$ as discussed by Baines and Peterson (1951). The pressure
drop through the fibre mat may be written in terms of the specific filtration resistance,
$SFR$, as shown in Eq. 2-3. Thus

$$
\Delta P_{\text{total}} = \Delta P_{\text{screen}} + \Delta P_{\text{mat}}
= k_s \cdot \frac{\rho}{2} \cdot v^2 + \mu \cdot SFR \cdot v \cdot BW,
$$

(3-1)

where $k_s$ = Screen Coefficient

$SFR$ = Specific Filtration Resistance (m/kg)

$BW$ = Basis Weight (kg/m$^2$)

$v$ = face velocity of the fluid (m/s)

$\mu$ = dynamic viscosity (Pa·s)

The fibre mat is assumed to be incompressible.

3.3 Experimental Analysis for Light Weight Coated Pulp Suspension

3.3.1 Design of Experimental Flow Loop

Johnson Wire Industries suggested a design for the construction of open and closed experimental flow loops for pulp drainage testing. These loops, shown in Figures 3-1 and 3-2, are square plexiglas tubes with internal dimensions of 5.08 cm (2").
Figure 3-1 Schematic diagram of open flow loop

Figure 3-2 Schematic diagram of closed flow loop
The flow loops were designed in such a way that the resistance properties of the mat and the screen could be determined using Eq. 3-1. These setups are a rough simulation of the dynamic nature of the real formation process. The pressure buildup was simulated by the total head, and the vertical component of the jet velocity was the face velocity across the fibre mat and screen.

The open loop was used as the main drainage tester throughout the experiments. The advantage of the closed loop was the capability of generating higher pressure in the loop. However, the internal flow became very unstable due to the backflow phenomenon when the pump was stopped to collect the fibre mat sample. A second open loop (Open Loop 2) was also assembled to reduce the maximum overhead by about half.

The basic operating concept behind all the setups was to inject the pulp suspension quickly and uniformly into the flow loop so that a fibre mat was formed on a screen supported by flanges. A 1/2 hp sump pump was installed in the bottom reservoir to generate the circulating flow. The amount of pulp suspension injected into the loop could be controlled by a syringe to approximate the basis weight of several different paper grades such as lightweight tissue (≈20 g/m²), newsprint (≈49 g/m²), and linerboard (≈200 g/m²). The types of pulp suspensions used in the experiments were Light Weighted Coated (LWC), newsprint and Unbleached Chemical Softwood. Due to the availability of stock, LWC was used as the main pulp for testing. The stocks all had a fibre consistency of approximately 0.8%.

A differential pressure transducer was used to measure the pressure loss through the fibre mat and the screen. An orifice plate was installed below the screen to measure the average flow velocity inside the square tube. The calibration of the orifice plate was
performed by the falling water head test. The discharge coefficient of the orifice plate was determined from the measurements of the change in the water head and the corresponding discharge time.

From the pressure drop and the velocity measurements, the effect of the fibre mat resistance could be determined using Eq. 3-1. A gate valve was installed in the middle of the tube to control the overall flow rate. With the data acquisition system, the time history of the pressure loss increase and the flow velocity decrease could be related to the instantaneous accumulation of the fibre mat. Table 3-1 shows the specifications of the two screens used in the experiments. Screen A has a smaller open area than Screen B, as indicated by the mesh size in the MD (Machine Direction) and the CD (Cross-Machine Direction).

Table 3-1 Screen specifications¹

<table>
<thead>
<tr>
<th>Screen Type</th>
<th>Layer Type</th>
<th>Paper-Side Mesh [MDxCD]</th>
<th>Air Permeability [(cm³/s)/cm²]</th>
<th>Caliper [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Double</td>
<td>162x65</td>
<td>142.2</td>
<td>0.689 mm</td>
</tr>
<tr>
<td>B</td>
<td>Triple</td>
<td>82x70</td>
<td>223.5</td>
<td>0.689 mm</td>
</tr>
</tbody>
</table>

Water tests were conducted to investigate the properties of Screen A and Screen B. Figure 3-3 is a plot of the screen coefficient against the flow velocity when pure water was circulated in the loop. The figure, which is similar in concept to the Moody diagram, indicates that the flow is laminar at very low velocities, below 0.05 m/s, but becomes turbulent as the velocity increases. The same types of screens were used in tests on a pilot twin-wire machine at PAPRICAN, as discussed in Chapter 6.

¹ Courtesy of Johnson Wire Industries
Figure 3-3 Correlation of screen resistance $k_s$ and flow velocity in water: The flow is laminar below 0.05 m/s, corresponding to the Reynolds number of 2540 based on the internal tube diameter.

3.3.2 Experimental Method

For each trial, a clean screen was mounted between the flanges and sealed tightly with screws to prevent water leakage. Once the loop was completely filled with water, a certain volume of pulp suspension was injected by syringe into the tube entrance. As the injected fibres traveled down the tube and began to deposit on the screen, the pressure drop across the screen increased immediately due to the formation of the fibre mat. Consequently, the flow velocity decreased as more fibres were piled onto the screen. The changes in the pressure drop and the velocity were recorded in real time.
Figure 3-4 Typical example of data collection for measurements of pressure loss and flow velocity in the open loop ($\approx 20$ g/m$^2$ of LWC injection for Screen A): Initial flat region corresponds to the measurements for pure water circulation.

Figure 3-4 shows a typical example of the experimental results. In this trial, Screen A was used with an injection of 8 ml of LWC (Light Weighted Coated), equivalent to $\approx 20$ g/m$^2$. The initial flat regions in the figure correspond to the values of the pressure loss and the velocity for pure water in the loop. The random fluctuation of the signal in the initial flat region is a result of the turbulent flow regime. When the fibres were injected into the loop, there was a sudden change in the pressure loss and flow velocity.

Figure 3-5 shows typical open-loop results for pressure loss for LWC as the amount of pulp suspension injected into the loop is increased. The figure reveals two interesting trends. First, as shown by the dotted circle, a pressure spike was consistently observed for the cases where the amount of injected pulp suspension exceeded a volume of about 40 ml ($\approx 115$ g/m$^2$) for Screen B and about 50 ml ($\approx 140$ g/m$^2$) for Screen A. This
phenomenon has been attributed to a water hammer effect, caused by the sudden closure of the flow path as large numbers of fibres adhere quickly to the screen. This explanation is supported by the fact that no pressure spike existed when the same amount of fibres was added into the loop slowly.

Figure 3-5 Pressure loss variation resulting from different amounts of pulp suspension injection: Pressure peak in the dotted circle is attributed to the water hammer effect, while the reason for the small pressure increase indicated by the arrows is unclear.

Second, as indicated by the arrows in the figure, when the amount of pulp suspension is less than 10 ml (≈27 g/m²) for both screens, a continuous but relatively small increase in pressure loss at the plateau was observed for about 1-2 minutes after the fibre injection was completed, but then ceased. There are two possible explanations. There may have been a rearrangement of fibres on the screen for a short time after injection. Alternatively, after the fibres were deposited, fines in the water which would normally pass through the
screen may have been caught in the mat. This would reduce the permeability of the mat and increase the pressure drop slowly. The first possibility is discussed in the next section.

3.3.3 Analysis of Experimental Results

The trends of net pressure loss through the fibre mat for Open Loop, Open Loop 2 and Closed Loop are plotted against the basis weight in Figure 3-6. The graph is scaled to emphasize the net effect of pressure drop through the fibre mat by subtracting the pressure drop through the screen from the total drop.

![Diagram of Basis Weight vs. Net Pressure Loss through LWC fibre mat](image)

**Figure 3-6 Change in basis weight vs. net pressure loss through LWC fibre mat for open loop and closed loop:** The net pressure drop through the fibre mat is obtained by subtracting the pressure drop through the screen from the total drop. The results show the similar trends by having three distinctive regions. Only the maximum pressure drop changes according to the maximum possible head in each flow loop.
There are three interesting regions where the change in net pressure drop through the mat behaves distinctively when the basis weight is varied. The first, called the *linear region*, occurs for basis weight below 17-18 g/m². In this region, the pressure drop appears to increase almost linearly with the basis weight, as shown in Figure 3-7. The second, between 17-18 g/m² and 40 g/m², is called the *transient region*; here the pressure drop increases only slightly with an increase in basis weight. The third, called the *no-effect region*, occurs over 40 g/m², where no significant change in pressure drop is observed with the basis weight. This plateau indicates that the maximum head of the system has been reached.

![Figure 3-7](image_url)  
*Figure 3-7* Expanded view of the change of basis weight against net pressure loss through LWC fibre mat in the linear and transient regions in the open loop: The figure appears to show the linear increase of net pressure drop through the mat as the basis weight increases to approximately 18 gsm.
In the linear region, it is speculated that the fibres may move around easily inside the mat with smaller restriction from neighbouring fibres. They are likely influenced by the flow and may reposition themselves. In the transient region, the mat may become more compact and have less ability to move around. In the no-effect region, fibre movement is further restricted, especially near at the bottom of the fibre mat. Further addition of fibre should result in only a minor change in the pressure drop, which eventually reaches the maximum total head.

Fibre mobility inside the mat in the linear region may explain why the pressure loss still continues to grow at the plateau, as shown by the arrows in Figure 3-5. It is possible that fibre mobility allows the individual fibres to rearrange in such a way that the fibre distribution tends to become more uniform across the screen. While fibre rearrangement is taking place, the effective open area in the screen becomes smaller due to blockage by fibres. The reduction in open area during a certain time interval may generate a small increase in the pressure drop, which eventually levels off. As fibre mobility becomes less and more fibres are packed inside the mat, the rate of increase of the pressure drop also decreases, which is exactly what was observed as more fibres were added. This hypothetical explanation is supported by the observation that the formation of low basis weight paper samples tended to be more uniform than that of high basis weight samples.

In Figure 3-8 (a), the mat resistance, expressed by the specific filtration resistance \( SFR \), is plotted against the basis weight for all the flow loops. It is interesting to observe how the mat resistance behaves as a function of basis weight in the three regions of pressure drop. Figure 3-8 (b) shows the expanded view of the results in the linear and the transient regions for the open loop.
(a) $SFR$ vs. $BW$ for all data

(b) Expanded view of $SFR$ vs. $BW$ in the linear and transient regions for open loop tests: The figure shows that $SFR$ appears to increase linearly with $BW$ in the linear region. $SFR$ tends to remain constant in the transient region.

Figure 3-8 $SFR$ vs. $BW$ for open loop and closed loop tests
In the linear region, the mat resistance tends to increase with the basis weight, since more fibres should cause more resistance to the flow; the relationship appears to be linear. In the transient region and the no-effect region, however, the change in the mat resistance decreases with an increase in basis weight.

![Graph showing the relationship between basis weight and resulting flow velocity](image)

**Figure 3-9 Change in basis weight against the resulting flow velocity in the open loop**: The figure shows that, in the linear region, as the basis weight increases the resulting flow velocity rapidly decreases. But the flow is turbulent. In the no-effect region, the resulting flow becomes laminar, therefore, Darcy’s law becomes valid.

This behaviour does not appear to agree with the predictions. Nonetheless, it may be explained in terms of Darcy’s law, introduced in Eq. 3-1, as follows.
Figure 3-9 shows the relation between the basis weight of the injected fibres and the resulting flow velocity in the open loop. Consider the no-effect region, where the velocity remains \(< 0.1 \text{ m/s}\). This is the region where the flow shows the characteristics of laminar flow and the application of Darcy’s law remains valid, as shown by Bear (1972).

The experimental results show that the changes in pressure drop and flow velocity are very small for an increase in basis weight. These can be assumed to be almost constant. Then, based on Eq. 3-1, the mat resistance is a decreasing function of the basis weight in the no-effect region. This appears as a decrease in mat resistance with the basis weight.

One shortcoming of the current flow loop is that it is not capable of keeping a constant flow velocity while varying the basis weight, or vice versa. To achieve this would require a pressurized flow loop with feedback control and a variable speed motor.

### 3.3.4 Combined Pressure Drop Analysis

The analysis of the current results is based on the assumption that the pressure drop is caused by two separate mechanisms, the screen resistance and the mat resistance. Although this method of analysis seems appropriate, there are some unanswered questions arising from the physics of the process. One is the validity of Darcy’s law in a flow of high velocity. Darcy’s law may also be invalid in the high basis weight region where the mat may be subject to compression.

From a practical point of view, it would be simpler to analyze the total pressure drop by combining the effects of the screen and the mat, thereby considering them to be a single component of a porous medium.
Figure 3-10 Percent of pressure loss through the screen from the total loss in the open loop and closed loop: The figure shows that the screen effect is significant only in the linear region. In the transient and no-effect regions, the screen can be neglected in the analysis.

Figure 3-10 shows the percentage of the total pressure loss which occurs through the screen, for LWC open-loop and closed-loop tests. The graph shows that the presence of the screen has an effect on the total pressure drop for basis weight < 20 g/m², which falls mostly within the linear region. Once the basis weight exceeds 20 g/m², the screen effect becomes smaller, with a maximum associated error of less than 5%. Over 40 g/m², in the no-effect region, the pressure drop through the screen becomes less than 2% of the
total pressure drop. The results suggest that the fibre mat may be considered to be combined with a screen which has a higher pressure drop coefficient in the linear region. Therefore, it is possible to evaluate the total pressure drop in this region based only on the screen results. In the transient and the no-effect regions, the pressure drop through the screen may be neglected in the total pressure drop analysis. Then, the fibre mat may be considered to be the sole resistance mechanism for which Darcy’s law is satisfied.

### 3.4 Comparisons with Newsprint and Unbleached Softwood

Having established the characteristics of pressure drop and mat resistance for LWC, we now consider other pulps.

Different pulps show different drainage behaviours depending on the type of wood fibre, pulping process, beating time, stock preparation process, etc. The Canadian Standard Freeness (CSF) test is simple and reproducible, and it is customarily used for quick drainage analysis of a pulp. A schematic diagram of the CSF tester is shown in Figure 3-1. The CSF tester measures the drainage of a 1000 ml volume of 0.3% consistency pulp through a calibrated perforated screen plate. As the bottom lid is opened, the drained water fills the spreader cone. If drainage occurs fast, the water overflows to the side orifice and is collected in a cylinder. The CSF is defined as the volume of water that flows through the side orifice. Typically, for a mechanical pulp, the CSF is about 100-200 ml whereas unrefined softwood might reach as high as 700 ml, according to Biermann (1993). Fast drainage corresponds to a high freeness number. A more detailed description of the CSF tester and theoretical considerations can be found in the study by El-Hosseiny.
and Yan (1980). The CSF test was originally developed for groundwood pulps, and was not intended for chemical pulps. Nevertheless, it is the standard test method used throughout North America.

![Diagram of Canadian Standard Freeness tester in operation](image)

**Figure 3-11 Canadian Standard Freeness tester in operation [Biermann (1993)]**

Table 3-2 shows the fibre properties and CSF measurements for each pulp used in the Open-Loop experiments. The Op-Test measurements were obtained using an Op-Test Fibre Quality Analyzer in the lab, while the Kajaani measurements were provided by PAPRICAN. The CSF numbers for LWC and newsprint were provided by PAPRICAN.
Unbleached Softwood was tested using the CSF tester in the lab, and the CSF number was corrected for temperature using the correction equation supplied by Biermann (1993).

It is evident that unbleached chemical softwood has the fastest drainage behaviour while newsprint has the slowest. Despite the simplicity of the CSF tester, its use is limited since drainage is only indirectly related to the individual mat properties such as SFR, mat consistency and basis weight. This is the main reason many attempts have been made over the years to develop a practical drainage tester that would reflect the complex pulp characteristics.

Table 3-2 Fibre Properties of LWC, newsprint, and Unbleached Chemical Softwood tested by Op Test Fibre Quality Analyzer and Kajaani: The Kajaani measurements were carried out by PAPRican.

<table>
<thead>
<tr>
<th></th>
<th>LWC</th>
<th>Newsprint</th>
<th>Unbleached Softwood</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Op Test FQA</td>
<td>Kajaani (Paprican)</td>
<td>Op Test FQA</td>
</tr>
<tr>
<td>Fibre Mean Length (mm)</td>
<td>Arith*</td>
<td>0.52</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>LW*</td>
<td>1.59</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td>WW*</td>
<td>2.57</td>
<td>2.42</td>
</tr>
<tr>
<td>Fine Contents (&lt;&lt;0.2mm)</td>
<td>Arith</td>
<td>51.1%</td>
<td>46.03%</td>
</tr>
<tr>
<td></td>
<td>LW</td>
<td>13.3%</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>WW</td>
<td>0.064</td>
<td>N/A</td>
</tr>
<tr>
<td>Mean Curl</td>
<td>Arith</td>
<td>0.076</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>LW</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSF (ml)</td>
<td></td>
<td>210</td>
<td>102</td>
</tr>
</tbody>
</table>

* Arith: Arithmetic Weighted; LW: Length Weighted; WW: Weight Weighted

In the current experiments, LWC is compared with newsprint and unbleached chemical softwood. Figure 3-12 shows the change in net pressure drop with basis weight for all three pulps tested in the open loop. Newsprint appears to show behaviour similar to
that of LWC, having three distinctive pressure drop regions. The boundaries of these regions are similar to those for LWC. However, for unbleached softwood, behaviour of the pressure drop with increasing basis weight is somewhat different. Softwood pulp reaches the maximum pressure drop much more slowly than LWC or newsprint. Nevertheless, it also appears to have three distinctive pressure drop regions, where only the location of boundaries between regions has shifted relative to the other pulps.

![Diagram](image-url)

**Figure 3-12** Comparison of net pressure drop plotted against basis weight using LWC, newsprint and Unbleached Softwood: Unbleached Softwood shows a different behaviour of the pressure drop. It reaches the maximum loss much more slowly compared to LWC and newsprint. Nevertheless, it still shows the existence of three distinctive regions. The linear region is extended to approximately 40 gsm, while the transient region goes to approximately 80 gsm.
Comparisons of SFR against basis weight using LWC, newsprint and Unbleached Softwood in the open loop: Newsprint has higher resistance than LWC and Unbleached Softwood. SFR decreases in the high basis weight region for LWC and Newsprint while it remains almost constant for Unbleached Softwood.

The SFR is plotted against basis weight in Figure 3-13. The data were obtained from the open-loop experiments. As expected, newspaper shows higher resistance than LWC, because of its lower CSF number. The SFR for newsprint decreases with the basis weight in the no-effect region. Interestingly, unbleached softwood showed an almost constant value of SFR throughout the tested range of basis weights. In order to understand this behaviour, the thickness of the test samples was measured after the samples were collected and dried. The thickness was measured using a lab-scale electronic caliper device. Each paper sample was tested at least five times and the average was
obtained. The results of the caliper measurements are shown in Figure 3-14 with the corresponding basis weight. Due to sample collecting errors during the open-loop trials, some high basis weight samples had to be excluded from the caliper measurements.

![Figure 3-14](image)

**Figure 3-14** Dry mat height vs. basis weight of LWC, newsprint and Unbleached Softwood and linear regression line based on the data under 40 gsm: For the same basis weight, Unbleached Softwood compresses the least. All data points over 40 gsm show lower mat height than ideal incompressible mat height.

The linear regression lines were obtained based only on the data under 40 g/m² (i.e., excluding the data in the no-effect region for LWC and newsprint). For an incompressible porous medium where no compression is assumed to take place, a further increase in the basis weight in the no-effect region should result in a proportional increase
in the dry mat height, as indicated by the straight regression line. However, for LWC and newsprint, the mat height is smaller in the no-effect region than it would be for an ideal incompressible mat. The same trend was observed for unbleached softwood. For the same basis weight, unbleached softwood compresses the least.

The lower than extrapolated height of the dry mat in the no-effect region appears to be caused by compression of the mat. Fines also might have contributed to the height difference by filling voids in the fibre mat. Nevertheless, this suggests that Darcy's law in the no-effect region is inaccurate due to the compression behaviour of the fibre mat. Compression analysis of LWC, newsprint and unbleached softwood will be extensively discussed in the following chapter.
CHAPTER 4

COMPRESSIBILITY ANALYSIS OF FIBRE MAT

This chapter includes an experimental analysis of the time-dependent compression of a fibre mat. A detailed description of the apparatus is given as well as of the data collection setup. The results are analyzed for the time-dependent behaviour under constant load either by a piston or by a water head. A theoretical model is proposed based on the concept of free-fall drop, and the concept of a critically uncompressed consistency is introduced.

4.1 Objectives

The objective of this chapter is to identify the relationship among the parameters known to affect the compressibility behaviour of a fibre mat. Therefore, an analysis is carried out to describe the compression behaviour of a fibre mat as a function of time and compacting pressure.

In the experimental investigation, one-dimensional compression tests were performed to determine the compression characteristics of three different pulp types. Also, the change in mat height was analyzed and compared with theoretical predictions. From a theoretical viewpoint, the one-dimensional consolidation model based on Biot's study
(1941) is investigated in Appendix C under the assumption of a viscoelastic fibre mat. In this chapter, another form of compression model is suggested based on the concept of free-fall drop analysis. The model is applied by treating the mat as a highly viscous liquid when subjected to instant high compression. A theoretical analysis is performed to understand the physics.

4.2 Experimental Configuration

4.2.1 Experimental Apparatus and Methods for Compression By Piston

![Diagram of compression experimental apparatus]

Figure 4-1 Schematic of a compression experimental apparatus

An experimental apparatus was set up to investigate the characteristics of the compressive response of a fibre mat, as shown in Figure 4-1. A dead weight was placed on
a permeable piston which descended on a saturated fibre mat by gravity. The bottom of the cylinder was blocked with a rubber plug. Water escaped from the mat surface through the piston, and a syringe was inserted through the cylinder wall to remove the drained water on the top of the piston. The apparatus was made of acrylic so that the process could be recorded by a CCD camera. Two pistons with either 5.6% or 41.7% porosity were used for the experiments. These were prepared by drilling tiny holes (about 1.5 mm in diameter) with equal spacing. The piston with 5.6% porosity was used mainly for the time-dependent compression tests, while the other one was used for the verification of the empirical compressibility equation.

![Figure 4-2 Schematic diagram of overall compression experimental setup](image)

Figure 4-2 shows a schematic diagram of the overall experimental setup for data collection and video analysis. A CCD camera was connected to a VCR and a digital timer to record the experimental process. A frame grabber was installed in a computer to acquire a clean image which showed the location of the piston and the time.
4.2.2 Preparation of a Saturated Mat

The compressibility experiments were performed on three different types of pulp suspensions: LWC, newsprint and unbleached softwood.

To prepare a saturated fibre mat, the sample pulp suspension was well shaken to keep it uniformly mixed. Then, the pulp suspension was quickly weighed and poured into the cylinder. The bottom plug was taken away, and the water started draining immediately through the screen located at the bottom of the cylinder. The drained water was collected in a beaker located on top of a digital scale, and the change in weight was constantly monitored. As drainage progressed, the fibres started piling up and formed a mat on the screen. When the decreasing water level was almost at the same height as the forming fibre mat, the bottom opening was blocked with the plug.

Finally, the mat left inside the cylinder was assumed to be saturated. The most difficult part of the preparation process was determining the exact moment and location when the water level descended to the top surface of the mat height. This was only possible through visual observation. Nevertheless, once the right location was set, it became easy to replicate conditions for the saturated mat since the drainage time and the amount of the drained water were consistent for the same pulp.

Figure 4-3 shows the drainage behaviour of the three tested pulps. For 500ml of pulp suspension, unbleached softwood was the quickest to be drained. The saturated level was reached at approximately 90 seconds, while it took about 17 minutes and 91 minutes for LWC and newsprint, respectively. This was consistent with the measurements of the CSF number of each pulp suspension, as mentioned in Chapter 3.
Figure 4-3 Drainage characteristics of LWC, newsprint and Unbleached Softwood under gravity: The tests were performed using the apparatus shown in Figure 4-1 without the piston. For 500 ml of pulp, Unbleached Softwood has the fastest drainage rate.

4.2.3 Calculation of Fibre Mat Consistency

Visual observation of the compression test indicated the rate of change of the fibre mat height. The measured height was converted into an equivalent mat consistency using the technique described below.

Initially, the total mass of the saturated mat $M_{sm}$ was obtained by subtracting the mass of the drained water $M_{dw}$ from that of the original suspension $M_{os}$ poured into the cylinder. Here, it was assumed for simplicity that only the water was allowed to drain; fines leaving the suspension were considered to be part of the drained water:
While the mat was undergoing compression, the amount of water escaping through the top surface $M_{\text{we}}$ (kg) was assumed to be equivalent to the change in mat volume $\Delta Vol_m$ ($m^3$). Again, the fines and small fibres were not accounted for:

$$\Delta Vol_m = \text{Volume of Mat before compression} - \text{Volume of Mat after compression} \quad (4-2)$$

$$M_{\text{we}} = \rho_w \cdot \Delta Vol_m \quad , \quad (4-3)$$

where $\rho_w$ is the density of water (kg/m$^3$).

Then, the mass of the compressed mat $M_{\text{cm}}$ (kg) becomes

$$M_{\text{cm}} = M_{\text{sm}} - M_{\text{we}} \quad . \quad (4-4)$$

When each compression test was completed, the compressed fibre mat was removed and weighed. Then, it was put into a drying oven. After the fibre mat was completely dried, the mass of the dry mat was measured and used to calculate the wet mat consistency $C$:

$$C = \frac{\text{Mass of Dry Mat (kg)}}{\text{Mass of Compressed Mat (kg)}} . \quad (4-5)$$

Also, the mass concentration of the mat $c_m$ (kg/m$^3$) is

$$c_m = \rho_m \cdot C = \frac{\text{Mass of Compressed Mat (kg)}}{\text{Volume of Compressed Mat (m}^3) \cdot C \quad . \quad (4-6)$$
4.3 Experimental Results

4.3.1 Compression over a Long Time under Constant Piston Load

The compression results for LWC are shown in Figure 4-4, which plots the mass concentration $c_m$ as a function of increasing compacting pressure $\sigma_z$ for several trials under the same conditions.

![Figure 4-4 Compression of LWC mat over a long time under static loads](image)

Figure 4-4 Compression of LWC mat over a long time under static loads: The figure represents the data from five trials under the same condition. The power law type of curve fit is used to evaluate the compressibility constants as shown in the empirical compressibility equation.

For each trial, a saturated fibre mat was prepared as described in the previous section and was subjected to a static load for a sufficient duration until the change in mat height
became insignificant and no additional water drained through the surface. A static load was applied by placing a brass weight on the permeable piston. A gradual increase in weight decreased the mat height. The compacting pressure was calculated from the total weight applied on the fibre mat, including the weight of the piston. As indicated by the curve fit in Figure 4-4, the results confirm the validity of the empirical compressibility equation defined in Eq. 2-8.

![Compacting Pressure vs Mass Concentration Graph](image)

**Figure 4-5 Compression of LWC, newsprint and Unbleached Softwood over a long time under static loads:** The results show that all three pulps follow the empirical compressibility equation when subjected to compression over a long time.

Tests should be repeated for different pulp mats because of the different fibre mat structure and composition mechanism. For purposes of comparison, the same compression tests were conducted for the two other pulp types. Only two trials were carried out for
each pulp mat. As shown in Figure 4-5, the results indicate that all three pulps follow the empirical compressibility equation.

Based on the results, newsprint is less likely to be compressed under the same compacting pressure than LWC and unbleached softwood. The reason for this is unclear. It was anticipated that the drainage rate, typically indicated by the CSF number, would play an important role in compressibility. Under the assumption that a high drainage rate is the product of a large opening area or void ratio, it is expected that a pulp mat with a high drainage rate might be compressed more easily under the same compression load. This is true for newsprint, which has the lowest CSF number and less compressibility. However, it is not obvious for LWC and unbleached softwood. The compressibility mechanism is too complex to attribute the reason to a single mat property. It is possible that other properties such average fibre length and fibre wall thickness may affect compression.

Figure 4-6 illustrates the compressive stress and strain relation of a saturated LWC mat after a long time has elapsed. The stress here is the compacting pressure applied on the mat, whereas the compressive strain $\varepsilon$ is defined as

$$
\varepsilon = \frac{h_o - h}{h_o},
$$

(4-7)

where $h_o$ and $h$ are the initial and measured mat height, respectively.

It is clear that the compressive stress-strain relationship is not linear; it can, however, be linearized by dividing it into three linear regions as indicated by the dotted lines.
Figure 4-6 Stress-strain curve for LWC fibre mat: The figure shows a non-linear compressive stress-strain behaviour. It is possible to divide into three linearized zones.

Figure 4-7 Average void fraction vs. compacting pressure using LWC, newsprint and Unbleached Softwood
Figure 4-7 illustrates the decrease in the average void fraction as the compacting pressure increases. Void fraction $V_f$ is defined as the ratio of volume occupied by void $V_v$ per total volume, which is the sum of void volume $V_v$ and solid volume $V_s$:

$$V_f = \frac{V_v}{V_v + V_s}.$$  

(4-8)

Here, the void volume $V_v$ was assumed to be equivalent to the total volume occupied by the water. It is important to distinguish the difference between the void fraction $V_f$ and the porosity $\phi$. When the water is absorbed inside fibres and cannot be easily removed, it should be considered as part of the solid volume since it acts as a resistance mechanism to the moving water. Therefore, the porosity $\phi$ is defined as

$$\phi = \frac{V_{MV}}{V_{MV} + V_{IM}}.$$  

(4-9)

where $V_{MV}$ is the volume occupied by the movable water, and $V_{IM}$ is the volume occupied by the solid and the immovable water. Therefore, the average porosity of the mat should be smaller than the average void fraction, as shown in Figure 4-7. In some references, the void fraction is referred to as the absolute porosity, and the porosity as the effective porosity.

Figure 4-8 compares the results of this study with those of previous work. The figure shows that the empirical compressibility equation is valid over a wide range of compacting pressures for a variety of pulps.
Figure 4-8 Comparison of various results from compression over a long time of the fibre mat: The figure shows that the empirical compressibility equation is applicable to the various types of pulp in a wide range of compacting pressure.

4.3.2 Time-Dependent Creep and Recovery under Constant Piston Load

In order to identify the effect of time dependency on compression, several experiments were carried out with a saturated fibre mat, which was subjected to a wide range of compacting pressures; LWC was used as the test sample for compression-recovery tests. All the tests were conducted with the 5.6% porosity piston.

Recovery tests were performed immediately following the compression tests on the same fibre mat. These were carried out by removing the piston and then adding water
without disturbing the compressed mat. The water level was maintained at a height of 0.5 cm water above the compressed mat surface. The mat was allowed to absorb the water until it stopped expanding. Hysteresis was observed. Only a small portion of the compressed strain was recovered.

An interesting phenomenon can be observed regarding the strain behaviour, as seen in Figure 4-9. The results demonstrate the time-dependent behaviour of the compressive strain under four different conditions of static load applied to the saturated fibre mat. The figure indicates the same strain results on two different scales: a linear scale and a logarithmic scale.

The first graph in Figure 4-9 (a) implies that with long duration, the strain of the mat under different loads tends to change similarly and eventually reaches a limiting value. Overall, the long-term behaviour obeys the following relationship:

$$
\varepsilon = K_1 \left(1 - \exp\left(-K_2 \cdot t\right)\right),
$$

(4-10)

where $\varepsilon$ is the compressive strain, $t$ is the time in seconds, and $K_1$ and $K_2$ are constants.

The second graph in Figure 4-9 (b) emphasizes the initial change in strain. As indicated by the dotted circle, the strain under a high static load showed a sudden increase for time $< 5$ seconds. Such a rapid increase was intriguing. The reason for it was unclear at the time, but a reasonable explanation was suggested, which is explained in the next section in conjunction with the free-fall experiments.
Figure 4-9 Strain vs. time under four static loads applied to the saturated LWC fibre mat plotted on linear and logarithmic scales: The rapid increase of strain in (b) under the compression of 9453 Pa is indicated by the dotted circle. A reasonable explanation for this intriguing behaviour is given in relation to the free-fall drop analysis.
Figure 4-10 Change in consistency plotted against time in compression-recovery tests: The compressed mat eventually reaches a limiting consistency that can be determined from the empirical compressibility equation. It can only recover a small portion of the compressed region as shown in the recovery tests.

Figure 4-10 shows a number of interesting results from the compression and recovery analysis. The compression results indicate that as the applied load is increased the general trend in change in consistency is somewhat similar. After a sufficient time has passed, the compressed mat reaches a limiting consistency that corresponds to the compacting pressure determined by the empirical compressibility equation. The recovery results show that the process includes a highly non-recoverable deformation stage. Based on the degree of mat recovery, the compressed mat is capable of recovering only a small portion of the compressed region. This is probably because the fibre mat undergoes a fibre
rearrangement process until the fibres are fully aligned horizontally. Only then does recoverable deformation start to take place through individual fibre compression and bending. Jones (1963) attributed the existence of this non-recoverable region to the fibre repositioning which takes place under compression. He also suggested that the length-to-diameter ratio is important and affects the compressive response, which depends on the fibre bending characteristics. Accordingly, non-recoverable deformation would not be expected for infinitely long fibres because fibre repositioning would be difficult.

4.4 Free-Fall Experiments

In the previous section, it was found that compression over a short time under high pressure was different from that under low pressure. The reason was unclear, so similar compressibility experiments were carried out by dropping pistons of different weights into pure water and also into 0.8% consistency LWC suspension. Three different weights were used for the tests, and these were converted into the corresponding static pressures.

4.4.1 Basic Concepts

It is important to understand the physics involved in the free-fall study. When the permeable piston is suddenly dropped in pure water or a low consistency suspension, it will reach a terminal velocity $v_t$ within a very short time. In such a case the force balance is determined by Newton's second law. Three forces are present: the gravitational force, the
resistance force by drag, and the small buoyancy force. Here, the buoyancy force can be neglected because of the relatively small volume of the piston.

As the piston accelerates, the resistance force increases. Soon the acceleration becomes zero and the forces are balanced as the piston reaches a constant terminal velocity. In a typical fluid mechanics problem, the short time an object takes to reach a terminal velocity is usually ignored since the time interval is very small. However, when a fibre mat is subjected to high compression in the press zone, the time interval is also very small. This is the case in a high-speed papermaking machine.

This section analyzes the initial behaviour in which the piston accelerates before reaching a terminal velocity. Figure 4-11 presents a model for predicting the time scale in the free-fall experiments.

\[ W = M g \]

![Schematic of a free-fall drop model](image)

Figure 4-11 Schematic of a free-fall drop model
The governing equations for the model can be written as follows, assuming the resisting force varies linearly with velocity:

\[
M \cdot \frac{d^2 z}{dt^2} + C_d \cdot \frac{dz}{dt} = W = M \cdot g,
\]

where \( M \) is the mass of the piston, \( C_d \) is the damping coefficient, and \( W \) is the gravitational weight. Since \( v = \frac{dz}{dt} \), the above differential equation can be simplified to

\[
M \cdot \frac{dv}{dt} + C_d \cdot v = W = M \cdot g,
\]

where \( v \) is the velocity.

This equation is a first-order differential equation. Solving it for \( v \) with the initial condition of \( v=0 \) at \( t=0 \) gives

\[
\frac{dz}{dt} = v = \frac{W}{C_d} \left(1 - \exp\left[-\frac{C_d}{M} \cdot t\right]\right).
\]

By integrating this to find \( z \) with the initial condition of \( z=0 \) at \( t=0 \), the equation can be solved for \( z \) as a function of \( t \) as follows:

\[
z = \frac{W}{C_d} \left(t + \frac{M}{C_d} \exp\left[-\frac{C_d}{M} \cdot t\right]\right) - \frac{W \cdot M}{C_d^2}.
\]
Since $W$ is a constant gravitational force applied instantaneously on the piston as a step function when $t=0$, the velocity and the distance traveled by the piston with respect to time are shown in Figure 4-12. The piston quickly reaches a terminal velocity $v_t$, depending on $W$ and $C_d$ as follows:

$$v_t = \frac{W}{C_d}. \tag{4-15}$$

![Diagram of velocity vs. time and distance vs. time](image)

(a) Velocity vs. time  
(b) Distance vs. time

**Figure 4-12** Predicted movement of a piston as a function of time in viscous liquid: The plot of Eq. 4-13 is shown in (a), while the result of Eq. 4-14 is presented in (b).
4.4.2 Analysis of Free-fall Experimental Results

Figures 4-13 to 4-15 show the results of free-fall experiments conducted both in water and in LWC of 0.8% consistency. The top graph in each pair shows the change in piston velocity versus time under a constant load, while the bottom graph represents the distance traveled by the piston versus time. When the piston is dropped in pure water, the trends in the changes in velocity and distance seem to follow the predictions given in Figure 4-16. For all three free-fall experiments in water, the piston approached the terminal velocity within 0.3 second, while it took close to 0.5 second to reach a terminal velocity with the 0.8% LWC suspension. Also, the trials using 0.8% LWC produced smaller terminal velocities than the trials using water.

![Graphs showing changes in velocity and distance for water and 0.8% LWC](image)

(a) In water  
(b) In 0.8% LWC

Figure 4-13 Static pressure 852 Pa
Figure 4-14 Static pressure 9453 Pa

(a) In water

(b) In 0.8% LWC

Figure 4-15 Static pressure 24743 Pa

(a) In water

(b) In 0.8% LWC
An intriguing behaviour was observed for the case of (b) in Figure 4-15, where the light piston was dropped into 0.8% LWC. Although the suspension was well shaken to ensure the uniform distribution of fibres, the piston did not maintain a constant velocity, showing a significant decrease in velocity as the time progressed. The reason for the slowdown can be attributed to the effect of individual fibres blocking the holes and reducing the actual opening area, increasing the resisting force. Here, the gravitational force was not strong enough to force the water through the holes and clean out the blocking fibres. It was observed that once the piston began to slow down, no more visible fibres came out through the holes.

Figure 4-16 shows the plot of the velocity change over a short time for the free-fall trials. It is clear that the terminal velocity is reached quickly in pure water, but the fibres in 0.8% LWC increase the resistance to the piston movement. The figure indicates that the time for the piston to reach terminal velocity is less than 0.5 sec. It is hard to determine the exact time based on the results. On the other hand, the time for the mat compression in the twin-wire forming roll is on the order of 0.02-0.05 seconds, which is less than the time needed to reach terminal velocity. Calculation of the drainage analysis applied to the twin-wire former using the free-fall drop model is carried out in Chapter 6. Also included in Figure 4-16 is the drag resistance of the piston $k_p$, which was calculated using the observed values of the terminal velocity. The piston resistance is based on the same concept of screen resistance $k_s$. As expected, the piston resistance is higher in 0.8% LWC than in water. During the trials, it was observed that the fibres and fines escaped through the holes of the piston. This produced extra shear and caused the effective viscosity to increase.
Figure 4-16 Piston velocity versus time under different static loads in water or 0.8% consistency LWC mat: The figure shows that the terminal velocity is reached within 0.1-0.5 sec. It seems that the time scale for the mat compression in the forming roll is one order of magnitude smaller and is in the range of 0.02-0.05 sec.

Figure 4-17 shows the piston velocity as a function of time at different static loads applied to the saturated fibre mat. The graph indicates a possible reason a rapid increase in strain was observed previously in Figure 4-9 (b) for the high static pressure of 9453 Pa. The velocity tends to decrease linearly on the log scale for the cases of 943 Pa, 2540 Pa, and 5159 Pa.
Figure 4-17 Piston velocity applied to the saturated LWC fibre mat under four different static loads: The figure show the existence of a constant velocity region for the compression of 9453 Pa. The constant velocity can be regarded as a terminal velocity.

For the case of 9453 Pa, the initial velocity remains relatively constant, until it starts decreasing log-linearly with time. This constant velocity can be regarded as the terminal velocity, as described in the free-fall experiments. When a heavy static load is suddenly applied to the saturated fibre mat, initially the piston drops like a free-fall object. This is because the top part of the mat provides very weak resistance to the permeable piston until the fibres are compressed enough.

As the fibre mat is compressed further, the resistance force should increase and the velocity should decrease. Experimental observation also confirms that under a heavy load,
the piston moves initially at a high constant speed for a certain time. While the piston was moving at a constant speed, the breakup of the fibre mat was visible at the top of the mat. The fibres were breaking apart, and some of them were passing through the piston holes. Then, as the piston started slowing down, no more fibres escaped through the holes, which indicates stronger fibre bonding under further compression.

This suggests that the rapid increase in compressive strain under high static load can be attributed to the existence of a terminal velocity, which arises from the lack of resisting force in the fibres at the top of the mat. In other words, the mat has the weak fibre network at the top.

### 4.4.3 Existence of a Critically Uncompressed Consistency, $C_c$

It is now logical to assume that when a fibre mat is suddenly subjected to high pressure, the top fibre network within the structure may not have enough strength to withstand it. This leads to the suggestion that a saturated fibre mat may behave like a highly viscous fluid or a viscoelastic solid, depending on the initial consistency of the mat. This means that when a saturated mat is compressed, there has to be a critically uncompressed consistency $C_c$ at which the mat behaviour changes from that of a fluid phase to that of a solid phase. The critical consistency presumably depends on the applied pressure: it should increase as the applied pressure increases because the fibres require a higher resistance to balance the higher compression, and this comes from a higher consistency mat. This concept is illustrated in Figure 4-18.
Figure 4-18 Concept of a critically uncompressed consistency: The figure indicates that a critically uncompressed consistency $C_e$ would exist when a saturated mat is compressed. The critical consistency depends on the magnitude of applied pressure. It should rise as the applied pressure increases. In the nonrecoverable region, the randomly oriented fibres are repositioned and stacked up against each other until all the fibres are fully rearranged horizontally. Then, the mat would undergo a recoverable compression process. When fully compressed, the mat follows the empirical compressibility equation.

Region A corresponds to a fully compressed fibre mat, while Region B corresponds to the mat where the liquid phase characteristic still dominates and free-fall drop can occur for high pressure. Possible resistance forces in the mechanism consist either of a viscous shear force from a water-fibre mixture or a resisting force from fibres. During free-fall, the resisting force from fibres would be very weak, and the fibre mat can be considered to be a highly viscous suspension until enough water exits the mat.
Figure 4-19 Graphical representation of possible compression mechanism: The figure shows that when the saturated mat is prepared in (a) the top part of the mat would be more dilute than the bottom, since the water is drained through the bottom of the cylinder. In (b), the free-fall drop occurs initially until the mat becomes critically uncompressed. Then, the mat goes through nonrecoverable compression until fibres are fully arranged horizontally. The mat reaches the recoverable boundary in (c). Finally, the mat undergoes recoverable compression and becomes fully compressed in (d).
When the mat reaches a critically uncompressed consistency $C_c$, it would undergo non-recoverable compression, that is, the randomly oriented fibres are repositioned and stacked up against each other until all the fibres are fully rearranged horizontally. During this compression, the resisting force due to the fibres would increase and slow down the rate of change of the mat height. After the fibres are repositioned, the fibre mat would undergo recoverable compression. The degree of recoverable compression depends on fibre stiffness and flexibility.

Once the fibre mat is fully compressed, it would reach a final consistency which follows the previously defined empirical compressibility equation. Based on this concept, Figure 4-19 shows a graphical representation of a possible compression mechanism observed in current experiments. As shown in (a), when the saturated mat is initially prepared, the top part of the mat would be more dilute than the bottom part, since the water was drained through the bottom of the cylinder. Therefore, the top part would be subjected to a faster compression under load. Also, the follow-up compression processes can be graphically depicted as shown in (b) to (d), respectively. In order to consolidate the concept of a critically uncompressed consistency, several experiments have been performed by changing the initial consistency of the prepared mat.

Five experimental results are compared as shown in Table 4-1. Tests #1 and #3 were conducted with an initial consistency of about 3.7% in a partially saturated mat. Test #2 is the default case of a fully saturated mat. Tests #4 and #5 were performed with an initial consistency of 2.39% and 2.89%.
Table 4-1 Various conditions of the initial fibre mat prepared for compression under 9453 Pa

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Mass of the 0.8% Suspension (g)</th>
<th>Initial Mat Height (cm)</th>
<th>Initial Mat Consistency (%)</th>
<th>Saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>492 g</td>
<td>8.3</td>
<td>3.74</td>
<td>Partially</td>
</tr>
<tr>
<td>2)</td>
<td>490 g</td>
<td>13.5</td>
<td>2.13</td>
<td>Fully</td>
</tr>
<tr>
<td>3)</td>
<td>246 g</td>
<td>4.5</td>
<td>3.70</td>
<td>Partially</td>
</tr>
<tr>
<td>4)</td>
<td>246 g</td>
<td>6.4</td>
<td>2.39</td>
<td>Fully</td>
</tr>
<tr>
<td>5)</td>
<td>246 g</td>
<td>5.4</td>
<td>2.87</td>
<td>Partially</td>
</tr>
</tbody>
</table>

The final results of the strain, the distance traveled, and the corresponding velocity against time are shown in Figure 4-20 (a), (b) and (c), respectively.

(a) Strain vs. time

Figure 4-20 Compressive response of fibre mats with different initial consistencies under a static pressure of 9453 Pa (Cont’d)
(b) Distance traveled vs. time: The figure shows that the height decrease of trial 1 is twice that of trial 3. This implies that when the initial consistency of the uncompressed mat is sufficiently high (higher than \( C_r \)) the mat exhibits a viscoelastic behaviour under compression. When the initial consistency is low (lower than \( C_r \)), as indicated by trials 4 and 5, then the mat behaves like a highly viscous fluid and the piston free-falls in the mat.

(c) Velocity vs. time: The velocity of piston initially reaches a terminal velocity when the saturated mat has a relatively low consistency (cases 2, 4 and 5). This occurs because the resistance force is very weak compared to the applied force. Therefore, the piston drops like a free-fall object in the mat until the mat produces appreciable resisting force.

Figure 4-20 Compressive response of fibre mats with different initial consistencies under a static pressure of 9453 Pa
In Figure 4-20 (a), it is interesting to observe how curves #1 and #3 show the same strain in the long run, but this is not the case initially. For the absolute change in mat height, graph (b) shows that the height decrease of curve #1 is twice that of curve #3. This confirms that when the initial consistency of the uncompressed mat is sufficiently high (higher than \( C_c \)), then the mat behaves like a viscoelastic solid under compression. On the other hand, as indicated by curves #4 and #5, when the initial consistency is low (lower than \( C_c \)), then the mat behaves like a highly viscous fluid. This is similar to the free-fall case since the velocity reaches constant.

Based on these results, it is reasonable to introduce a free-fall drop region when high compressive pressure is applied to the mat over a very short time. This approach is compared with the Biot model in Appendix C, which considers the mat to be a viscoelastic medium combining a spring and a damper.
CHAPTER 5

ANALYSIS OF FIBRE MAT RESISTANCE

This chapter deals with the characteristics of the specific filtration resistance (SFR) based on Darcy’s law. The effect of the Kozeny constant is reviewed, and a relation is established between the specific filtration resistance and the mass concentration.

5.1 Objectives

Certain properties of the fibre mat, such as the specific surface and specific volume, can be related to the SFR using Darcy’s law and the Kozeny-Carman equation. Finding the mat properties enables the SFR to be expressed as a function of the change in mass concentration. Therefore, the objective of this chapter is to use several methods to determine the properties of the mat and relate them to the mat resistance.

5.2 Calculation of Specific Surface and Specific Volume

Three methods have been used for the evaluation of the specific surface and the specific volume of the fibre mat:
(a) the method of Robertson-Mason (1949) based on the Kozeny-Carman equation

(b) the method of Ingmanson-Whitney (1954) based on the Kozeny-Carman equation and the empirical compressibility equation

(c) the method of Davies (1952) based on an empirical relationship

5.2.1 The Method of Robertson-Mason

The method of Robertson-Mason is applied by substituting Eq. 2-5 into Eq. 2-4:

\[
K = \frac{1}{k \cdot S^2} \cdot \frac{(1 - v_s c_m)^3}{v_s^2 c_m^2},
\]

(5-1)

where \( S \) is the specific surface area per unit volume of solid material (m\(^2\)/m\(^3\)), and \( v_s \) is the specific volume (m\(^3\)/kg). By defining the specific external surface \( S_v \) (m\(^3\)/kg) as \( S_v = S \cdot v_s \) and using \( k = 5.55 \), we get

\[
K = \frac{1}{5.55 \cdot S_v^2} \cdot \frac{(1 - v_s c_m)^3}{c_m^2}.
\]

(5-2)

Transposing the equation, it can be written as

\[
(K \cdot c_m^2)^{1/3} = \left( \frac{1}{5.55 \cdot S_v^2} \right)^{1/3} \cdot (1 - v_s c_m),
\]

(5-3)

where \( S_v \) is the specific external surface area (m\(^2\)/kg).
Figure 5-1 Permeability and SFR against time in compression tests for LWC: The figures show the changes in permeability and specific filtration resistance as a function of time under constant loads. The free-fall drop region exists in the initial compressive behavior of the fibre mat under 9453 Pa. During initial few seconds of constant velocity, both $K$ and $SFR$ remain relatively constant. Then, $K$ decreases and reaches the minimum value when the mat is fully compressed, while $SFR$ increases and reaches a maximum value.
Robertson and Mason (1949) stated that if permeability measurements are made for several different values of the mass concentration $c_m$, a plot of $\left(K \cdot c_m^2\right)^{1/3}$ against $c_m$ would yield a straight line. Therefore, the specific external surface area $S_e$ can be calculated from the intercept on the vertical axis where $c_m = 0$ and the linear regression of the data, and the specific volume $v$, from the intercept on the horizontal axis, where $\left(K \cdot c_m^2\right)^{1/3} = 0$.

Figure 5-1 (a) and (b) show the permeability and the specific filtration resistance, respectively, as functions of time under constant compression. The graphs are plotted on logarithmic scales under four different compressive loads.

Eq. 2-1 was modified to evaluate the permeability $K$ with time as follows:

\[
K = \frac{\Delta V / \Delta t \cdot \mu \cdot h}{A \cdot \Delta P},
\]

where $\Delta V / \Delta t$ is the change in water volume leaving the fibre mat per the change of time (m$^3$/s), $A$ is the area of fibre mat (m$^2$), $h$ is the mat height (m), and $\Delta P$ is the compacting pressure (Pa).

The mat consistency increases as the saturated fibre mat undergoes compression under a constant load. Consequently the mat height decreases, assuming the basis weight remains constant. Therefore, the permeability of the fibre mat decreases, as shown in Figure 5-1 (a). The specific filtration resistance $SFR$ is inversely proportional to the permeability $K$. Figure 5-1 (b) presents the results. The $SFR$ follows a logarithmic relationship with time. Again, the free-fall drop region is observed in the initial
compressive behavior of the fibre mat for a high compacting pressure of 9453 Pa. During the initial few seconds of constant velocity, both the permeability and the SFR remain relatively constant. Then, as expected, the permeability $K$ decreases and reaches the minimum value for all four cases when the mat is fully compressed. On the other hand, the SFR increases and reaches a maximum value for a fully compressed mat.

Figure 5-2 shows the results of the modified Kozeny-Carman plot obtained from Eq. 5-3. A linear fit was used to find the intersection of the straight line with the axes. The specific volume $v_s$ is found to be 0.002735 m$^3$/kg, and the specific external surface $S_v$ is calculated as 3078.4 m$^2$/kg. These values are in reasonable agreement with those obtained by Robertson and Mason (1949).

![Graph](image)

**Figure 5-2** Modified Kozeny-Carman plot obtained from Eq. 5-3: $K$ was evaluated at 1000 seconds for each compacting pressure as shown in Figure 5-1 (a). A linear fit was used to find the intersection of the straight regression line with the $x$ and $y$ axes.
It is necessary to justify the choice of the value $k=5.55$ for the Kozeny constant, since $k$ changes considerably over a range of porosity $\phi > 0.8$. Using Eq. 5-5, the approximate range of porosity for a fully compressed mat can be calculated. It turns out that the porosity range is below 0.8 except for the case of 943 Pa, where the porosity was calculated to be 0.86. Therefore, as a first approximation, a Kozeny constant of 5.55 is used for a fully compressed mat.

5.2.2 The Method of Ingmanson-Whitney

The method of Ingmanson-Whitney is based on the concept that the average SFR of a porous mat is related to its fibre properties. The method involves the use of the Kozeny-Carman equation and the substitution of Eq. 2-5 for the porosity. The modified form is

$$SFR = \frac{k \cdot S_v^2 \Delta P_{mat}}{\int_0^{\Delta P_{mat}} \left[ \left(1 - v_s c_m \right)^3 / c_m \right] dp},$$

where $k$ is the Kozeny constant of 5.55, $\Delta P_{mat}$ is the pressure drop through the mat, and $p$ is the mechanical compacting pressure developed at a point in the mat. The equation is combined with the empirical compressibility equation

$$c_m = K_1 \sigma_z^{K_2},$$

where $K_1$ and $K_2$ are constants. Then, by substituting and integrating, the $SFR$ can be expressed in the following form:
Therefore, Ingmanson and Whitney (1954) suggested that Eq. 5-7 can be used as a working relation to determine the specific surface $S_v$ and the specific volume $v_s$, provided that the compressibility constants $K_3$ and the SFR at two or more values of the pressure drop are known. The method is limited by the application of the empirical compressibility equation whose shortcomings have been pointed out previously. Using this method, the specific volume and the specific external surface are calculated to be $0.004494 \text{ m}^3/\text{kg}$ and $4482.2 \text{ m}^2/\text{kg}$, respectively.

5.2.3 Method Based on the Permeability Relationship of Davies

Another method is introduced based on the permeability relationship initially suggested by Davies (1952), as described in Eq. 2-6. The values used for the constants $k_1$ and $k_2$ are those given by Ingmanson et al. (1959-a, b) based on their experimental results.

According to Meyer (1962), the Davies equation would be preferable over the Kozeny-Carman equation because it can account for the change in porosity within the fibre mat. Using the definition of the hydraulic radius $r$ for non-circular and irregular fibres, the following relationship can be given:

$$\frac{1}{S} = \frac{r}{2},$$

(5-8)
where $S$ is the specific surface per unit volume of a swollen fibre ($m^2/m^3$) and $r$ is the hydraulic radius of a fibre (m). Then, the Davies relationship becomes

$$K = \frac{1}{k_1S^2(v_r c_m)^{1.5} \left[1 + k_2(v_r c_m)^3\right]},$$

(5-9)

where $k_1 = 3.5$ and $k_2 = 57$, as given by Ingrmanson et al. (1959-a, b). Therefore, if two or more values of the permeability $K$ are known, the specific surface and the specific volume can be estimated.

Table 5-1 shows the combined results for the specific volume and the specific external surface of LWC obtained from all three methods.

**Table 5-1  Comparison of the fibre properties for LWC using three different methods:** Accurate values of specific surface and specific volume are not known. Therefore, the average values of all three different methods are calculated for analysis.

<table>
<thead>
<tr>
<th>Method</th>
<th>Year</th>
<th>CSF (ml)</th>
<th>$v_r$ (m$^3$/kg)</th>
<th>$S_v$ (m$^2$/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robertson-Mason</td>
<td>1949</td>
<td>210</td>
<td>0.002735</td>
<td>3078.4</td>
</tr>
<tr>
<td>Ingrmanson-Whitney</td>
<td>1954</td>
<td>210</td>
<td>0.004494</td>
<td>4482.2</td>
</tr>
<tr>
<td>Davies</td>
<td>1952</td>
<td>210</td>
<td>0.003347</td>
<td>2742.5</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>—</td>
<td>210</td>
<td><strong>0.003525</strong></td>
<td><strong>3434.4</strong></td>
</tr>
</tbody>
</table>

Each method tends to give different values for the specific surface and the specific volume, which can deviate up to ± 30% from the average value. Since no accurate values of the specific surface and the specific volume are known, the average values of all three methods are used: 3434.4 m$^2$/kg and 0.003525 m$^3$/kg for the specific external surface and the specific volume, respectively.
5.3 Specific Filtration Resistance and the Kozeny Constant

Using the relationship of the Kozeny-Carman equation and the concept of SFR, the following relationship is found:

\[
SFR = \frac{k \cdot S_v^2 \cdot c_m}{(1 - \nu_s \cdot c_m)^3}.
\]  \hspace{1cm} (5-10)

Once the properties of a pulp mat, such as \(S_v\) and \(\nu_s\), are known, the relationship between the SFR and \(c_m\) (or Consistency \(C\) (%)) can be determined provided that the Kozeny constant \(k\) is also known.

As explained, the Kozeny constant \(k = 5.55\) is valid only in the porosity range of \(\phi < 0.8\). A number of expressions for the Kozeny constant over a wide range of porosity have appeared in the literature, as shown in Table 5-2.

In several of these references, the formula for the permeability relationship is written in terms of a drag force \(F\) per unit length of a cylinder. Therefore, the formula was converted to the equivalent Kozeny constant \(k\) using the following relationship, derived by Skartsis et al. (1992):

\[
F = \frac{\pi \mu U r^2}{K(1-\phi)^2},
\]  \hspace{1cm} (5-11)

where \(F\) is the drag force per unit length of a cylinder (N/m), \(\mu\) is the dynamic viscosity (Pa s), \(U\) is the superficial velocity (m/s), \(r\) is the representative cylinder radius (m), \(K\) is
the permeability (m²) and \( \phi \) is the porosity. Then, using Eq. 2-4 and Eq. 5-8, the equivalent Kozeny constant becomes

\[
k = \frac{F}{4\pi \mu U} \left( \frac{\phi^3}{1 - \phi} \right).
\]

\[ (5-12) \]

**Table 5-2 Comparison of the Kozeny constant predictions**

<table>
<thead>
<tr>
<th>Authors</th>
<th>Expression for Kozeny Constant ( k )</th>
<th>( \phi )</th>
</tr>
</thead>
</table>
| Davies (1952) with Ingrnanson’s Constants | \[
\frac{3.5\phi^3}{(1 - \phi)^{0.5}} \left[ 1 + 56(1 - \phi)^3 \right]
\] | 0.5 < \( \phi \) |
| Happel (1959) for Perpendicular Cylinders | \[
\frac{2\phi^3}{(1 - \phi)^2 \left[ \ln \left( \frac{1}{1 - \phi} \right) - \frac{1 - (1 - \phi)^2}{1 + (1 - \phi)^2} \right]}
\] | 0.4 < \( \phi \) |
| Happel (1959) for Parallel Cylinders | \[
\frac{2\phi^3}{(1 - \phi)^2 \left[ 2 \ln \left( \frac{1}{1 - \phi} \right) - 3 + 4(1 - \phi) - (1 - \phi)^2 \right]}
\] | 0.4 < \( \phi \) |
| Drummond & Tahir (1984) for Square Arrays | \[
\frac{2\phi^3}{(1 - \phi)^2 \left[ \ln \left( \frac{1}{1 - \phi} \right) - 1.476 + \frac{2(1 - \phi) - 0.796(1 - \phi)^2}{1 + 0.489(1 - \phi) - 1.605(1 - \phi)^2} \right]}
\] | 0.65 < \( \phi \) |
| Drummond & Tahir (1984) for Triangular Arrays | \[
\frac{2\phi^3}{(1 - \phi)^2 \left[ \ln \left( \frac{1}{1 - \phi} \right) - 1.497 + 2(1 - \phi) - \frac{(1 - \phi)^2}{2} \right]}
\] | 0.4 < \( \phi \) |
| Jackson & James (1986) for 3-D Arrays | \[
\frac{5\phi^3}{3(1 - \phi)^{2} \left[ - \ln(1 - \phi) - 0.931 \right]}
\] | 0.7 < \( \phi \) |
| Skartsis et al. (1992) for Random Fibres | \[
\frac{\phi^3}{(1 - \phi)^2 \left[ \ln \left( \frac{1}{1 - \phi} \right) - 1.536 + 2(1 - \phi) - \frac{(1 - \phi)^2}{2} \right]}
\] | 0.68 < \( \phi \) |

Based on the formulas in Table 5-2, the prediction of the Kozeny constant \( k \) for each relationship is plotted in Figure 5-3. The permeability relationship of Davies (1952) is used with the experimental constants obtained by Ingrmanson et al. (1959-a, b).
One of the parameters known to affect the Kozeny constant but neglected is the aspect ratio of a fibre in the cross-sectional shape. According to the study by Labrecque (1968), the Kozeny constant $k$ was found to increase significantly beyond an aspect ratio
of 3:1. Since most of the experiments to determine the Kozeny constant have been performed with circular cylinders, the validity of the Kozeny constant could be questioned when it is applied to fibres with a non-circular cross-sectional shape. Fortunately, most wood fibres have cross-sectional aspect ratios of less than 3.5:1 until after drying, according to Farrar's unpublished work (1964), which is referred to by Labrecque (1968). Therefore, it is assumed in this study that the Kozeny constant is not affected by the cross-sectional shape of the fibres. An expression for the SFR is developed using the relationship given by Davies (1952) with Ingmanson's values for the Kozeny constant. This gives

\[
SFR = \frac{k_1 \cdot S_v^{-2} \cdot c_m^{0.5}}{v_s^{0.5}} \left[1 + k_2 (v_s \cdot c_m)^3 \right], \tag{5-13}
\]

where \(k_1=3.5\) and \(k_2=56\). The formula for the SFR in Eq. 5-13 is plotted as a function of \(c_m\) in Figure 5-4, with the values of the specific external surface \(S_v\) and the specific volume \(v_s\) obtained from the three methods described previously and the average values. Four data points refer to the values of SFR for the mat subjected to four different compression loads. Also the selected values of the SFR found in the related literature are compared.

The results of Mantar et al. (1995) show that as the mass concentration increases, the SFR rises at first and then drops. They suggested that the initial rise in the SFR was due to the increasing amount of fines. As the mass concentration increased, larger flocs were formed, and consequently the mat became more porous and the SFR dropped.

Other results obtained by Robertson and Mason (1949) and Gertjejansen (1964) also show the values of SFR over a reasonable range. It seems that Eq. 5-13 is a reasonable prediction of the change in SFR with mass concentration.
Experimental Results for LWC (210 ml CSF)

Prediction with Values Given By Robertson-Mason's Method
Prediction with Ingmanson-Whitney's Method
Prediction with the Method by Davies and Ingmanson's Constants
Prediction with Average Values

Robertson & Mason (1949): Bleached Sulphite (665 ml CSF)
Pires (1989): Groundwood Pulp (350 ml CSF) for 32.2 kPa
Mantar (1995): Beaten Bleached Northern Softwood Kraft (400 ml CSF) 150 gsm
Mantar (1995): ThermoMechanical Pulp (120 ml) for BW of 150 gsm
Gertjejansen (1964): Unbleached Douglas-fir Kraft (390 ml CSF)
Gertjejansen (1964): Bleached Hardwood Kraft (400 ml CSF)
Gertjejansen (1964): Bleached Hardwood Kraft (600 ml CSF)

Figure 5-4 Comparison of various results of SFR plotted against $c_m$: Four current data points refer to the SFR values obtained from the mat subjected to four different loads.
CHAPTER 6

THEORETICAL AND PRACTICAL ANALYSIS OF JET IMPINGEMENT, DRAINAGE AND COMPRESSION IN THE FORMATION REGION OF A TWIN-WIRE FORMER

This chapter develops a physical flow model in the region where the suspension jet impinges on the wire in a twin-wire former. A full set of governing equations is found for jet impingement at the centre or on the bottom wire. Two cases are considered for jet impingement at the centre: i) the wedge shape determined from measurement and ii) the wedge shape unknown. The equations are numerically solved by using reasonable approximations for typical operating conditions. The free-fall drop model is applied in the press zone to represent a time-dependent compression of the low consistency mat. The analytical results are compared with the measured data and the practical significance is discussed.

6.1 Objectives

This chapter is based on the classical work of Baines (1967), who proposed a set of governing equations for the theoretical analysis of the suspension jet in the formation region of a twin-wire machine. The equations were derived based on conservation...
principles, but they were not solved due to their nonlinearity and an insufficient knowledge of fibre properties.

Table 6-1 Comparison of current analysis to the study by Baines (1967)

<table>
<thead>
<tr>
<th>Analysis in the free-jet zone</th>
<th>Baines (1967)</th>
<th>Current analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet expansion near impingement using Continuity &amp; Bernoulli</td>
<td>Equal jet thickness approach</td>
<td></td>
</tr>
</tbody>
</table>

Governing equations in the wedge zone

<table>
<thead>
<tr>
<th>Governing equations in the wedge zone</th>
<th>Baines (1967)</th>
<th>Current analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuity of water Mass conservation of fibres Bernoulli equation Darcy’s law</td>
<td>Continuity of water Mass conservation of fibres Bernoulli equation A modified Darcy’s law &amp; Kozeny-Carman equation Screen equation</td>
<td></td>
</tr>
</tbody>
</table>

Wedge shape

<table>
<thead>
<tr>
<th>Wedge shape</th>
<th>Baines (1967)</th>
<th>Current analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Found by assuming a reasonable pressure profile &amp; using the constant tension requirement</td>
<td>Case i) assumed known from PAPRICAN Data using constant $R'$ Case ii) numerically predicted assuming reasonable $k_s$ and SFR that are found experimentally</td>
<td></td>
</tr>
</tbody>
</table>

Assumptions in the wedge zone

<table>
<thead>
<tr>
<th>Assumptions in the wedge zone</th>
<th>Baines (1967)</th>
<th>Current analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasonable pressure change from 0 to $T/R$</td>
<td>Linear decrease of horizontal velocity from $U_{jet}$ to $U_{wire}$</td>
<td></td>
</tr>
</tbody>
</table>

Governing equations in the press zone

<table>
<thead>
<tr>
<th>Governing equations in the press zone</th>
<th>Baines (1967)</th>
<th>Current analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darcy’s law</td>
<td>Free-fall drop model</td>
<td></td>
</tr>
</tbody>
</table>

Solutions

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Baines (1967)</th>
<th>Current analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not obtained</td>
<td>Numerical solutions obtained by the software, Mathematica for two cases where wedge shape is known &amp; unknown</td>
<td></td>
</tr>
</tbody>
</table>

Experimental comparison

<table>
<thead>
<tr>
<th>Experimental comparison</th>
<th>Baines (1967)</th>
<th>Current analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not done</td>
<td>Compared with PAPRICAN data</td>
<td></td>
</tr>
</tbody>
</table>

Impingement location

<table>
<thead>
<tr>
<th>Impingement location</th>
<th>Baines (1967)</th>
<th>Current analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 case</td>
<td>2 Cases</td>
<td></td>
</tr>
<tr>
<td>Impingement at the Centre</td>
<td>Impingement at the centre</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Impingement on the bottom wire</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, the first objective of this chapter is the redevelopment of the governing equations in the formation region on the basis of work by Baines (1967). Several modifications have been made, and some new ideas have been implemented to improve the analysis. A detailed comparison appears in Table 6-1. The second objective is to derive
numerical solutions with reasonable assumptions for a typical machine geometry and a range of operating conditions. The solutions are compared with the measurements.

6.2 Physical Models

The focus of the analysis is the flow pattern in the formation region where the jet from the slice impinges at the centre or on the wire. Formation in this region is the result of a complex process, but it can be simplified by introducing reasonable approximations to define a physical flow model based on flow and fibre conservation equations. Here the suspension jet is assumed to be a uniform mixture of water and fibres which is free of random fluctuation. Also the consistency of the jet is low so that the jet is assumed to have the properties of water. The effect of individual fibre orientation is beyond the scope of the current analysis. It is also assumed that there is symmetrical drainage and fibre deposition through both wires in the wedge zone.

(a) Impingement at the centre
Figure 6-1 Two cases of jet impingement mechanisms (Cont’d)
Figure 6-1 illustrates two extreme cases of the jet impingement mechanism in a twin-wire machine. For the case of jet impingement at the centre in (a), the geometry of the region is divided into three zones: a free-jet zone, a wedge zone, and a press zone, as introduced by Baines (1967). The free-jet zone covers a free-surface jet issuing from a headbox slice to the point of impact. In this zone, no restraining walls exist, and the shear stress can be neglected. Therefore, the flow is governed by ideal flow equations.

The wedge zone, confined by two wires, extends from the impingement point to the point where the two forming mats make contact. This zone is assumed to be divided into a liquid wedge and two growing mats. The effect of shear along the screen is negligible due to very thin boundary layers, as described in Appendix-B. Therefore, the flow in the wedge zone is also governed by ideal flow equations. The deposition of the fibre mat is described by conservation of mass.

Once the forming mats come into contact, water is removed by mechanical compression imposed by wire tension. The corresponding region is called the press zone and extends from the end of the wedge zone to the end of the forming roll. Here, the mat
thickness decreases as the water is drained, and the consistency of the mat increases accordingly, which increases the specific filtration resistance. Typically, the mat in the press zone has been treated as a visco-elastic medium subjected to pure compression. In this study, however, the mat is treated as a highly viscous liquid and the new concept of a free-fall drop model is applied to predict the time-dependent compression of the mat.

For the case of jet impingement on the bottom wire in Figure 6-1 (b), the region is divided into four zones: a free-jet zone, a free-surface wedge zone, a wedge zone and a press zone. The difference is the addition of a free-surface wedge zone where the jet impinges on the bottom wire first. Drainage occurs through the bottom wire while the top surface is open to the atmosphere. After the stock has partially drained, the free surface comes into contact with the top wire. In the free-surface wedge zone, the shape of the free-streamline is not known, but the pressure along the free-streamline is atmospheric. Newton’s second law is used to describe the flow in this zone. In the subsequent wedge zone, non-symmetrical mats develop on each side. For simplicity, if the screen resistance is assumed to be dominant in this zone, the symmetrical approach would be reasonable. Then, the same ideal flow equations are used as in the wedge zone in Figure 6-1 (a).

6.3 Free-Jet Zone

In the free-jet zone, the jet is open to atmospheric pressure and is not restricted by solid walls. Along the free surface, no viscosity effect exists, and the pressure is constant. There are two possible approaches for explaining the jet behaviour in the free-jet zone.
The first is to assume that the jet has equal thickness throughout the free-jet zone, which means that the contraction of the jet at the vena contracta and the expansion of the jet at impingement are both neglected. This simple approach is chosen in this study: we assume that the jet thickness at the impingement point is the same as the jet thickness at the slice exit.

The second approach, suggested by Baines (1961), assumes that jet expansion takes place near the impingement point. The expansion is caused by the Jet/Wire speed ratio and the screen resistance. Baines (1967) developed an equation for jet expansion using the continuity and Bernoulli equations.

It is difficult to determine the exact expansion of the jet thickness since it depends on the downstream conditions. In practice, the jet expansion would be very small.

6.4 Wedge Zone: Impingement at the Centre

The flow in the wedge zone is represented as a symmetrical jet in a porous boundary. Two detailed cases are considered. In the first case, the wedge shape is determined from PAPRICAN measurements using a constant curvature. Test conditions of PAPRICAN trials are summarized in Appendix-D. In the second case, the wedge shape is unknown and analytically determined. For both cases, the governing equations are set up, simplified and numerically solved using mathematical software, Mathematica 2.2. Mathematical procedures are shown in Appendix-G. Mean flow properties are calculated for a range of operating and fibre variables given in Table 6-2. Realistic values of the
screen coefficient $k_s$ and the mat resistance $SFR$ are based on the experimental results discussed in Chapter 3.

<table>
<thead>
<tr>
<th>Operating conditions used for analytical calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet Speed</td>
</tr>
<tr>
<td>Wire Speed</td>
</tr>
<tr>
<td>Headbox Consistency</td>
</tr>
<tr>
<td>Impinging Angle</td>
</tr>
<tr>
<td>Default Fabric Tension</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

6.4.1. Geometric Definitions of Impingement Point and Nip

(a) Location of the nip when there is no jet

(b) Location of the impingement point and the nip when there is a jet

Figure 6-2  Defining the location of the nip and the jet impingement point
It is necessary to understand the geometric definitions of an impingement point and a nip as used in this chapter. As shown in Figure 6-2, the impingement point is where the jet touches the wire. Using the symmetrical approach, the top and bottom wire impingement points are assumed to be in the same line. The nip is defined as the point where the two wires make contact when there is no jet, as shown in (a). When the jet enters the wedge zone, the wire shape changes. The location of the nip was used by PAPRICAN as a reference point for wedge thickness measurement.

6.4.2 Symmetrical Jet Approach

In this approach, the jet impingement is assumed to be symmetrical about the central stream line of the jet, as described by Baines (1967). This assumption is based on the fact that the forming region, including the wedge zone and the press zone, is long relative to the jet and mat thickness and also the centrifugal force is small. Based on PAPRICAN results, the wedge zone length is in the range of 20-30 cm, while the wedge thickness decreases from 1 cm to approximately 1 mm in the wedge zone.

The actual geometry is shown in Figure 6-3 (a). It is converted into a symmetrical geometry by keeping the half jet thickness constant as shown in (b). In fact, the exact geometric conversion would have produced an arc length longer than \( L \), but the difference is small for a given geometry and so is neglected in the analysis.

Since the forming roll radius \( R \) is also known from the machine geometry and \( t_1, t_2 \) and \( L_1+L_2=L \) are known from PAPRICAN results, the relationship for the shape of the outer wire \( R' \) can be determined, given that \( R' \) is assumed to be constant.
Figure 6-3  Conversion from real to symmetrical geometry: The jet around the forming roll in (a) is converted to a symmetrical geometry in (b) by keeping the half jet thickness constant. The exact geometric conversion would produce the length longer than $L = L_1 - L_2$, but the difference is very small for a given geometry, therefore neglected.
In the second case, \( R' \) is unknown and is determined by a pressure balance. Governing equations with reasonable assumptions of screen and mat properties determine the shape of \( R' \) in the wedge zone.

![Diagram](image)

**Figure 6-4 Flow pattern in a symmetrical porous boundary shape:** The figure shows the symmetrical half jet to represent the case in Figure 6-1 (a). Flow through the mat is normal to the surface, therefore, the face velocity \( W \) is used from the geometrical interpretation which relates the stream function to volume flow through the control surface of unit depth.

In both cases, if the central stream line can be replaced by the imaginary solid line of the \( x \)-axis and an inviscid flow assumption is made, the case in Figure 6-1 (a) becomes equivalent to solving for the case shown in Figure 6-4. In other words, the fibre mat growth on one wire can approximately represent the whole flow behaviour.

#### 6.4.3 Wedge Shape Determined From the PAPRICAN data

In the first case, the shape of the screen porous boundary \( y(x) + h(x) \) is determined from the curvature \( R' \), taking the jet thickness \( t_1 \) at the nip, the mat thickness \( t_2 \) and the wedge length \( L = L_1 + L_2 \) from the PAPRICAN data:
where $L$ is the distance of the wedge zone (m) and $R'$ is the equivalent radius of curvature of the forming roll (m). When the jet impinges on the wire, the gap thickness at the nip is measured to give $t_i$; $L_1$ was evaluated using the PAPRICAN pressure profile assuming that the pressure starts to increase at the impingement point; $L_2$ was approximated from the inter-fabric gap measurements and the drainage profile assuming that the gap thickness tends to flatten out in the press zone and the drainage rate becomes relatively low and constant. This corresponds to Zone 4, as defined by Gooding (1996).

Several trial cases were chosen to represent the geometric shape of the screen based on the PAPRICAN inter-fabric thickness measurements, as shown in Table 6-3. Since the initial jet thickness leaving the headbox is 9 mm, it is assumed that the jet thickness at the impingement point $x=0$ is the same. The PAPRICAN thickness measurements were performed starting from the nip at $x=L_i$. As a default case, this study considers an ideal case with an initial jet thickness of 9 mm and an arbitrary wedge length of 15 cm for the water jet or 20 cm for the suspension jet in the wedge zone.

The converted curvature $R'$ was calculated based on the total wedge length $L=L_1+L_2$. Then, the jet thickness $2t_i$ at $x=L_1$ was predicted from the calculated value of $R'$. It was compared with the measured value of $2t_i$ at $x=L_1$ and found to be in good agreement, as shown in Table 6-3. Therefore, the current method of determining $R'$ seems reasonable.
Table 6-3 Calculation of the wedge zone geometry based on PAPRICAN data: The table compares the measured jet thickness at the nip with the predicted one using the constant radius of curvature.

<table>
<thead>
<tr>
<th>Case</th>
<th>Measured 2t₁ at x=L₁ (NIP)</th>
<th>2t₂ at x=L₂ (cm)</th>
<th>L₁ (cm)</th>
<th>L₂ (cm)</th>
<th>L (cm)</th>
<th>R' (m)</th>
<th>Predicted 2t₁ at x=L₁ (NIP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal Case</td>
<td>N/A</td>
<td>1 mm</td>
<td>N/A</td>
<td>N/A</td>
<td>15 or</td>
<td>2.813</td>
<td>N/A</td>
</tr>
<tr>
<td>Default Case</td>
<td>9 mm</td>
<td>1.8 mm</td>
<td>8</td>
<td>0.4 mm</td>
<td>6</td>
<td>14</td>
<td>2.279</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
<td>1 mm</td>
<td>N/A</td>
<td>N/A</td>
<td>15 or</td>
<td>2.813</td>
<td>N/A</td>
</tr>
<tr>
<td>Water Only</td>
<td>483-1</td>
<td>3.8 mm</td>
<td>8</td>
<td>1 mm</td>
<td>13</td>
<td>21</td>
<td>5.513</td>
</tr>
<tr>
<td></td>
<td>9 mm</td>
<td>1.8 mm</td>
<td>8</td>
<td>0.4 mm</td>
<td>6</td>
<td>14</td>
<td>2.279</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
<td>1 mm</td>
<td>N/A</td>
<td>N/A</td>
<td>15 or</td>
<td>2.813</td>
<td>N/A</td>
</tr>
<tr>
<td>Base Case</td>
<td>471-01</td>
<td>3.8 mm</td>
<td>8</td>
<td>1 mm</td>
<td>13</td>
<td>21</td>
<td>5.513</td>
</tr>
<tr>
<td>473-01</td>
<td>9 mm</td>
<td>3.8 mm</td>
<td>8</td>
<td>1 mm</td>
<td>13</td>
<td>21</td>
<td>5.513</td>
</tr>
<tr>
<td>474-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>800 m/min</td>
<td>474-2</td>
<td>4.6 mm</td>
<td>7</td>
<td>1.4 mm</td>
<td>15</td>
<td>22</td>
<td>6.368</td>
</tr>
<tr>
<td></td>
<td>9 mm</td>
<td>5.2 mm</td>
<td>7</td>
<td>2 mm</td>
<td>15</td>
<td>22</td>
<td>6.914</td>
</tr>
<tr>
<td>1000 m/min</td>
<td>477-4</td>
<td>5.2 mm</td>
<td>7</td>
<td>2 mm</td>
<td>15</td>
<td>22</td>
<td>6.914</td>
</tr>
</tbody>
</table>

For the case shown in Figure 6-4, the continuity equation is written as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (6-2)
\]

Assuming that the horizontal velocity \( U₂(x) \) is uniformly distributed within a very thin gap, to define the velocity at the surface of the mat, Eq. 6-2 is linearly integrated along the \( y \)-axis direction from zero to \( y \). Then,

\[
y \cdot \frac{dU₂}{dx} + v = 0. \quad (6-3)
\]

Another assumption is that flow through the mat is normal to the surface. The face velocity \( W \) is found from the geometric interpretation which relates the stream function to the volume flow through the control surface of unit depth as shown by Baines (1967):
\[ W = v - U_2 \cdot \frac{dy}{dx}. \] (6-4)

For the pressure drop analysis through the screen and the mat it is necessary to use the face velocity \( W \) rather than the vertical velocity \( v \).

6.4.3.a Water Jet

First, consider the simple case of a pure water jet, as illustrated in Figure 6-5.

![Figure 6-5 Geometry of the water jet in a symmetrical porous boundary shape](image)

There are four unknowns: the pressure \( P(x) \), the jet thickness \( y(x) \), the horizontal velocity component \( U_2(x) \) and the vertical velocity component \( v(x) \). There are three
governing equations known a priori: the continuity equation in Eq. 6-3, the porous boundary shape in Eq. 6-1, and the Bernoulli equation, which describes the momentum conservation between the jet and the flow inside the wedge assuming a negligible height difference:

\[ \frac{1}{2} \rho \cdot U_1^2 = P(x) + \frac{1}{2} \rho \cdot [U_2(x)^2 + v(x)^2]. \]  

(6-5)

For simplification, another assumption is introduced about the reduction of the horizontal flow velocity \( U_2(x) \) from \( U_1 \) at the impingement point to the wire velocity \( U_w \) at the end of the wedge zone: the decrease of \( U_2(x) \) is assumed to be linear as a first approximation. Then, the above equations are sufficient to find the vertical velocity profile \( v(x) \) and the pressure distribution \( P(x) \) along the x-axis.

Under the assumption of a linear decrease of \( U_2(x) \) in the wedge zone, the following relationship is found:

\[ U_2(x) = \frac{U_w - U_1}{L} \cdot x + U_1. \]  

(6-6)

Next, the governing equations are simplified. The resulting pressure distribution and face velocity have the following form:

\[ P(x) = \frac{\rho}{2} \left[ U_1^2 - \left( \frac{U_w - U_1}{L} \cdot x + U_1 \right)^2 - \left( \frac{U_w - U_1}{L} \right)^2 \cdot \left( \frac{(L-x)^2}{2R'} + t_2 \right)^2 \right], \]  

(6-7)

\[ W(x) = -\frac{d}{dx} \left[ \frac{(L-x)^2}{2R'} + t_2 \right] \cdot \left( \frac{U_w - U_1}{L} \cdot x + U_1 \right). \]  

(6-8)
(a) Vertical velocity $v(x)$ vs. distance for the ideal case of $L=15cm$

(b) Pressure $P(x)$ change vs. distance for the ideal case of $L=15cm$

(c) Face velocity $W(x)$ vs. the distance for the ideal case of $L=15cm$

Figure 6-6 Predicted flow properties from the impingement point for the ideal case of $L=15cm$: The figures in (b) and (c) are the solution of Eqs. 6-7 and 6-8. Although they appear to be straight lines, they are curves in polynomial form.
The equations were solved for the ideal case. With parameters listed in Table 6-2. The results of vertical velocity, pressure and face velocity are plotted in Figure 6-6.

The predicted pressure distribution in (b) and the face velocity in (c) appear to be solid straight lines, but are curves in polynomial form, as given in Eqs. 6-7 and 6-8.

To express the effect of the screen and to find the local velocity through the screen, the concept of the solidity ratio \( s \) can be used. The value of the solidity ratio is approximately determined from the screen specification, and \( s = 0.65 \) is used for the calculation of the screen coefficient. Chapter 3 also discussed the use of screen resistance \( k_s \) to express the effect of the screen:

\[
k_s = \frac{\Delta P}{\rho \cdot W^2} = f(W, s) ,
\]

(6-9)

where \( W \) is the face velocity normal to the screen (m/s). Solutions of the governing equations for \( P(x) \) and \( W(x) \) can also be used to find the change in \( k_s \) along the \( x \) direction.

**Figure 6-7** Screen coefficient \( k_s \) plotted against distance for the ideal case: The screen coefficient is defined in Eq. 6-9. The change occurs due to the change in face velocity as shown in Figure 6-6.
Figure 6-7 shows $k_s$ plotted against the distance from the impingement point. It is obvious that the screen resistance varies rapidly, and the assumption of a constant value of $k_s$ in the wedge zone is questionable. Nevertheless, no other alternative seems to exist for the analysis of pressure drop other than assuming a reasonable value of $k_s$.

Using $W(x)$, the drainage rate profiles for three different cases are compared in Figure 6-8. The solid line is the curve predicted for case 483-1, assuming that the horizontal velocity $U_2(x)$ decreases to $U_w$ within 14 cm. The prediction is approximately twice as high as the actual measurements. This deviation is probably due to the assumption that the water jet velocity decreases linearly.

Despite its shortcomings, the approach shows the possibility that the flow properties can be predicted in the wedge zone.

![Figure 6-8](image)

**Figure 6-8** Comparison of drainage rate predictions to the PAPRICAN measurements for water jet only: Current model predicts approximately twice higher drainage rate than the measurements.
6.4.3.b Suspension Jet

In this section, an ideal fibre suspension jet is considered. Since the wire shape \( y(x) + h(x) \) in Figure 6-4 is assumed to be known, there exist four unknown variables: \( P(x) \), \( h(x) \), \( U_2(x) \) and \( v(x) \).

In this case, one more governing equation can be written using the conservation of mass for the fibre mat. Also, the other governing equations are modified to include the effect of suspension consistency. However, this gives rise to another unknown parameter, the mat consistency \( C_{\text{mat}} \), which is difficult to determine. Nevertheless, \( C_{\text{mat}} \) can be evaluated from the boundary condition. Since the mat has to come in contact with the other mat at the end of the wedge zone, it gives rise to another condition that the governing equations have to satisfy. Thus, the modified equations are, in addition to the Bernoulli equation in Eq. 6-5, the conservation of mass for water and for fibres:

\[
\begin{align*}
v(x) &= -y(x) \cdot \frac{dU_2(x)}{dx} \cdot \frac{(1 - C_s)}{(1 - C_{\text{mat}})}, \\
\rho_s \cdot C_s \cdot U_1 \cdot t_1 &= \rho_s \cdot C_s \cdot U_2(x) \cdot y(x) + \rho_{\text{mat}} \cdot C_{\text{mat}} \cdot U_w \cdot h(x).
\end{align*}
\]

Several assumptions have to be made to simplify this problem. First, the density of suspension \( \rho_s \) is assumed to be almost the same as the mat density \( \rho_{\text{mat}} \). Second, no fibres are assumed to escape through the wire. Third, it is assumed that the horizontal velocity \( U_2(x) \) decreases linearly.

Figures 6-9 and 6-10 show the predicted change of the mat thickness and the related flow properties. Analytical predictions such as the growth of the mat thickness cannot be verified from machine observations. Nevertheless, the analysis shows that the
A complex problem can be simplified using reasonable assumptions. The results in the figures were obtained by solving the equations numerically using Mathematica 2.2. The mathematical procedure is shown in Appendix G.1.1.

Figure 6-9 Prediction of jet and mat thickness from the impingement point for the ideal case

Figure 6-10 Prediction of flow properties $v(x)$, $P(x)$ and $W(x)$ from the impingement point for the ideal case: $P(x)$ and $W(x)$ do not change linearly as previously explained.
Also the mat contact boundary condition makes it possible to estimate the value of mat consistency $C_{mat}$ by treating it as an unknown. Since the mat height $h(x)$ has to meet the other side of the mat at the end of wedge zone $x=L$, $C_{mat}$ was found numerically to be 0.077 for the ideal case. Figure 6-11 shows the estimate of the average mat consistency based on the mat contact boundary conditions under different operating conditions. Also shown is the consistency profile for the base case from PAPRICAN measurements.

![Consistency Profile](image)

**Figure 6-11 Comparison of consistency profiles in the wedge zone:** Current model assumes the consistency to be the same throughout the wedge zone. Mat contact boundary condition gives rise to the evaluation of the mat consistency for the ideal case, the base case, trial 474-2 and trial 477-4.

The pressure loss relationship through the screen and the fibre mat are related to the face velocity $W(x)$ as follows:
\[ P(x) - P_0 = \Delta P_{\text{screen}} + \Delta P_{\text{mat}} \]
\[ = \frac{k_x}{2} \rho_g W(x)^2 + \rho_m \cdot C_{\text{mat}} \cdot \mu \cdot SFR \cdot h(x) \cdot W(x) \] (6-12)

One major obstacle arises in the evaluation of the fibre mat resistance \( SFR \). As discussed in Chapter 5, the \( SFR \) changes as the mat thickness \( h(x) \) increases. Also the screen coefficient \( k_x \) varies as the face velocity \( W(x) \) changes along the wire.

![Figure 6-12](image)

**Figure 6-12 Pressure drop through the screen and the mat, respectively:** The figure shows that the screen resistance is dominant first, but the mat resistance becomes significant towards the end of wedge zone.

To approximate the effect of the pressure drop through the screen compared to the total pressure drop in the wedge zone, an attempt was made to solve Eq. 6-12. For the ideal case, \( W(x) \) and \( h(x) \) were found from the governing equations. Assuming the values
of \( k \) and \( SFR \) given in Table 6-2, the percent of the pressure drop through the screen in the wedge zone was evaluated and shown in Figure 6-12 with \( k_s = 30; SFR = 5\times10^6 \text{ m/kg} \). As the mat approaches the end of the wedge zone, the pressure drop through the mat becomes dominant.

Figure 6-13 compares the predicted drainage rates with PAPRICAN measurements. The predictions seem to be approximately twice as high as the measurements. Nevertheless, the trends are similar since they are monotonically decreasing.

![Figure 6-13](image)

**Figure 6-13** Comparison of the predicted drainage rates to the measurements for the suspension jet

**6.4.3.c Pressure Distribution in the Wedge Zone**

The pressure \( P(x) \) obtained from the governing equations is the effect of the dynamic liquid pressure. Also the pressure induced by the wire tension \( T/R \) influences the
pressure in the wedge zone. Although $T/R$ is a controlling factor in the press zone, the magnitude is unknown in the wedge zone. It seems reasonable to assume that towards the end of the wedge zone, $T/R$ would have a greater effect on the pressure, and at the end of the wedge zone, the pressure would reach a value of $T/R$. (In fact, the PAPRICAN data showed that the pressure in the press zone was only about half of $T/R$. No reasonable explanation has been found for this.)

Baines (1967) proposed that the total pressure in the wedge zone can be assumed to change from zero at the impingement point to the maximum of $T/R$ at the end of the wedge zone in the following form:

\[ P_r(x) = \frac{T}{R} \cdot f\left(\frac{x}{L}\right), \]  

(6-13)

where $f\left(\frac{x}{L}\right)$ can have a specific form. He tried two types of equations;

\[ f\left(\frac{x}{L}\right) = \frac{x}{L} \quad \text{or} \quad \left(\frac{x}{L}\right)^n. \]  

(6-14)

In this study using a similar concept, the total pressure change in the wedge zone is assumed to be the following:

\[ P_r(x) = \frac{P(x)}{P(L)} \cdot \frac{T}{R}, \]  

(6-15)

where $P(x)$ is the hydrodynamic pressure and $P(L)$ is the maximum value at the end.
(a) **Comparison with T = 5 kN/m:** The model result follows the PAPRICAN measurements of 470-01, while Eq. 6-15 suits the results from the pretrial.

(b) **Comparison with base case of T = 7 kN/m:** Eq. 6-15 follows the PAPRICAN measurements in the wedge zone.

**Figure 6-14 Comparison of pressure distributions in the wedge zone:** The figures show the repeatability problems in the measured data.
Figure 6-14 compares the pressure distributions $P(x)$ and $P(x)$ with the actual pressure measurements recorded by PAPRICAN. The measurements show a repeatability problem, as demonstrated by the inconsistency between the pretrial case and 470-01. Other pressure results reported by Gooding (1996) also showed similar repeatability problems under the same operating conditions. This is understandable given the fact that the experiment is difficult to perform and the pressure measuring technique is still in the development stage.

The pretrial measurement shows an additional increase in pressure towards the end of the wedge zone. This is probably due to additional pressure generated by mechanical compression. However, it is difficult to explain why the maximum pressure in Figure 6-14 (a) is somewhat greater than the wire tension pressure $P=T/R$. This contrasts with the measurements of Hergert and Sanford (1984), which did not show a pressure higher than $T/R$.

The measurement for case 470-01 shows good agreement with the predicted hydrodynamic pressure, but the effect of $T/R$ is not represented in the measurement. Figure 6-14 (b) shows that the total pressure prediction seems to be in better agreement with the measured pressure than with the hydrodynamic pressure. However, the pressure in the press zone drops to nearly half of $T/R$.

Overall, the pressure profiles measured by Gooding (1996) showed a similar characteristic pulse shape. That is, the pressure rises from the impingement point to a maximum peak near the end of the wedge zone and then descends to a constant value which is significantly less than $T/R$. 
6.4.4 Wedge Shape Unknown

6.4.4.a Water Jet

In the case where the wedge thickness is unknown for a water jet, the equilibrium of pressure \( P(x) \), face velocity \( W(x) \), and screen resistance \( k_s \) would determine the corresponding wedge shape \( y(x) \) shown in Figure 6-5. The screen equation was defined in Eq. 6-9. By combining it with the continuity equation of water in Eq. 6-3, the Bernoulli equation in Eq. 6-5, and the face velocity in Eq. 6-4, the equations are simplified to the following nonlinear differential equation:

\[
k_s \cdot U_2(x)^2 \cdot [y'(x)]^2 + 2k_s \cdot U_2(x) \cdot U_2'(x) \cdot y'(x) \cdot y(x)
+ (k_s + 1) \cdot [U_2'(x)]^2 \cdot y(x)^2 - U_1^2 + U_2(x)^2 = 0
\]  \( \text{(6-16)} \)

The mathematical manipulation is shown in Appendix G.1.2. Again by assuming that the horizontal velocity \( U_2(x) \) decreases linearly from \( U_l \) to \( U_w \) over some distance \( L \), the nonlinear equation can be solved numerically with the boundary condition \( y(0) = t_0 \), using Mathematica 2.2. A length \( L \) of 14 cm is used to be consistent with the PAPRICAN results of water jet trial case 483-1.

The equation produces two solutions for \( y(x) \), but \( y(x) \) has to decrease along the \( x \)-direction. Therefore, only one solution is accepted. Figure 6-15 shows the analytical prediction of the half jet thickness \( y(x) \) for \( k_s = 10, 30, 60 \) and 100. As \( k_s \) increases, less water escapes through the screen, and the jet thickness decreases slowly. That is, the drainage length increases.
Figure 6-15  Effect of screen resistance on the change in jet thickness: The figure indicates that as $k_s$ increases the drainage length increases for a water jet.

Figure 6-16  Effect of jet and wire velocity on jet thickness: The figure shows that as the jet and wire speeds increase the drainage length increases for a water jet.
Figure 6-16 also shows the effect of the increase in jet and wire velocity on jet thickness for $k_s = 30$. As the jet speed increases, the drainage length increases. It also shows that the change in the jet speed from 640 m/min to 1040 m/min has produced a smaller increase in the drainage length than the one produced by the change in the screen resistance from 10 to 100.

![Graph showing jet thickness and drainage length](image)

**Figure 6-17 Prediction of wire shapes in a forming roll geometry:** The figure shows the model results using various values of $k_s$. The proper value of $k_s$ appears to be between 10 and 30.

The half jet thickness in Figure 6-15 is converted into the full jet thickness in a real forming roll geometry, and the resulting plot is shown in Figure 6-17. The figure is produced by converting the actual jet thickness of $2y(x)$ to the outer wire shape. The
PAPRICAN measurements of water jet trial case 483-1 are also plotted for comparison. It appears that \( k_s = 30 \) or 60 results in a jet thickness somewhat higher than the measured value.

This discrepancy is related to the following shortcomings of the current analytical method: i) the assumption that \( k_s \) is constant, ii) the assumption that the velocity decreases linearly within the distance \( L \) and iii) the assumption that the centrifugal force is neglected.

### 6.4.4.b Suspension Jet

The pressure drop through the screen and the mat was defined in Eq. 6-12. Other governing equations are the continuity equation for water, Eq. 6-10, the mass conservation of fibres, Eq. 6-11, the Bernoulli equation, Eq. 6-5, and the face velocity relation, Eq. 6-4. Also under the assumption of a linear velocity decrease of \( U_2(x) \) in Eq. 6-6, six equations are established. These equations are solved to give numerical values of the six unknowns, \( y(x), h(x), v(x), U_2(x), W(x) \) and \( P(x) \). As shown in Appendix G.1.3, the combined equations can be simplified into the following nonlinear differential equation:

\[
f_1(x) \cdot [y'(x)]^2 + [f_2(x) \cdot y(x) - f_3(x)] \cdot y'(x) + f_4(x) \cdot y(x)^2 - f_5(x) \cdot y(x) + f_6(x) = 0, \quad (6-17)
\]

where \( f_1(x) = C_s (C_1 x + U_i)^2 \)

\[ f_2(x) = (C_1 x + U_i) (2C_s C_6 + C_7 C_4 C_3) \]

\[ f_3(x) = C_7 C_3 (C_1 x + U_i) / C_5 \]
Then, the equation is numerically solved for $y(x)$ with the boundary condition of $y(0)=t_0$. A reasonable value of mat consistency $C_{\text{mat}}=3\%$, as shown in Table 6-2, is used for the calculation. To find the effect of screen resistance and mat resistance, the values of $k_s$ and $SFR$ were varied to predict the resulting wedge thickness.

Figures 6-18 and 6-19 show the resulting shape of the wedge thickness and the outer wire shape for $k_s=30$ and $SFR=10^8$ m/kg. The outer wire shape is compared with the PAPRICAN measurements for the base case. Figures 6-20 and 6-21 show corresponding plots for $k_s=30$ and $SFR=5\times10^8$ m/kg. The analytical predictions are compared with the measurements from PAPRICAN.

\[
\begin{align*}
  f_4(x) &= C_7C_4\frac{C_6}{C_5} + C_8C_6^2 + C_5^2 + \rho^2/2 \\
  f_3(x) &= C_7C_3\frac{C_6}{C_5} \\
  f_6(x) &= (C_1^2x^2 + 2C_1\cdot U_1\cdot x) \cdot \rho^2/2, \quad \text{and} \\
  C_1 &= (U_w-U_I)/L \\
  C_2 &= (1-C_4)/(1-C_{\text{mat}}) \\
  C_3 &= \rho C_4 U_1 \cdot t_0 \\
  C_4 &= \rho C_4 (C_1 \cdot x + U_1) \\
  C_5 &= \rho C_{\text{mat}} U_w \\
  C_6 &= C_1 \cdot C_2 \\
  C_7 &= \rho C_{\text{mat}} \cdot \mu \cdot SFR \\
  C_8 &= k_s \cdot \rho^2/2.
\end{align*}
\]
Figure 6-18  Prediction of the half jet wedge thickness for $k_s=30$ and $SFR=10^8 \text{ m/kg}$: The dotted line is the shape of the screen. The solid line shows the accumulation of the fibre mat below the screen predicted by the current model.

Figure 6-19  Prediction of outer wire shape for $k_s=30$ and $SFR=10^8 \text{ m/kg}$: The figure shows the comparison of the predicted jet thickness to the measured one.
Figure 6-20 Prediction of the half jet wedge thickness for $k_s=30$ and $SFR=5\times10^8$ m/kg

Figure 6-21 Prediction of outer wire shape for $k_s=30$ and $SFR=5\times10^8$ m/kg
Figure 6-22 Prediction of the half jet wedge thickness for different values of $SFR$ for a constant screen coefficient: The figure shows that the drainage length increases as $SFR$ increases in the wedge zone.

Figure 6-23 Prediction of the half jet wedge thickness for different values of $k$, for a constant $SFR$: The figure shows that the drainage length increases as $k$, increases.
Figure 6-22 shows the effect of varying SFR for $k_s = 30$. Figure 6-23 shows the case of varying $k_s$ with $SFR = 10^8$ m/kg. In both cases, increasing the mat resistance or the screen resistance increases the length of the drainage zone. Based on the analytical results only, it is hard to determine which resistance mechanism would be more dominant in the wedge zone. It is evident at least that a change in $k_s$ has a greater effect in the initial drainage zone before the nip, while an increase in SFR tends to lengthen the drainage zone towards the end of wedge zone after the nip.

![Graph showing pressure profiles for different Jet/Wire speed ratios](image)

**Figure 6-24 Pressure profiles for different Jet/Wire speed ratios:** The figure shows that the pressure change is not linear for a small Jet/Wire ratio in the wedge zone.

Figure 6-24 shows the pressure profile in the wedge zone. Because the default case showed a pressure increase that appeared to be linear, for verification, the same governing
equations were solved with different Jet/Wire speed ratios while maintaining the same jet speed. The results clearly show that the pressure change is not linear, as indicated by the high Jet/Wire speed ratio.

![Graph showing Wire Shapes Comparison](image)

**Figure 6-25 Comparison of outer wire shapes:** The figure compares the current model result with the other model result by Baines and the PAPRICAN measurements. For the current model, $k_s$ and $SFR$ were varied to closely match the measured wire shape.

The plots in Figure 6-25 compares the outer wire shapes for the current analytical result, the PAPRICAN measurements for the Base Case, and the prediction using the Baines method with PAPRICAN data. The current result is plotted using $k_s=8$ and $SFR=2\times10^8$ m/kg. Here, the method of Baines (1967) is based on a constant tension
requirement along the wire: it predicts the wedge thickness assuming that the pressure
distribution would change from 0 at the impingement point to $T/R$ at the end of the wedge
zone. An initial jet thickness of 9 mm, a final wedge thickness of 1 mm, and a wedge
length of 21 cm were used for the calculation. The weakness of the Baines prediction is
the approximation of the pressure distribution, which is directly related to all other
parameters such as the jet speed, the screen resistance and the mat resistance. Therefore,
for an assumed pressure distribution, the wedge shape would be the same for any set of
parameters.

6.4.5 Discussion of Wedge Shapes

For two cases, i) wedge shape determined from the measured data and ii)
wedge shape unknown, there are certain differences in assumptions and in the resulting
wedge shapes. In the first case, the wedge shape is predicted assuming that the wedge
inlet thickness is equal to the free-jet thickness, the wedge thickness at the end is known,
the curvature is constant and that it faces inwards towards the centre line, as shown in
Figure 6-26. The resulting wedge shape agrees well with the PAPRICAN measurements.
On the other hand, in the second case, the wedge thickness predicted by solving the
governing equations shows that the resulting shape faces outwards from the centre line
when a linear or quadratic decrease of horizontal velocity is assumed in the wedge zone.
This means that it would take longer for the water to drain out, suggesting that there may
be other drainage mechanisms which could be included in the theoretical analysis to
improve the flow model. The likely candidate is the centrifugal force, which may induce slightly more drainage.

Figure 6-26 Comparison of the predicted half wedge thickness to the measured one

6.5 Free-Surface Wedge Zone: Impingement on the Wire

6.5.1 Water Jet

In analyzing the free-surface wedge zone as shown in Figure 6-1 (b), the initial impingement zone is magnified to emphasize the determination of the free stream line boundary, as depicted in Figure 6-27. The main difference between this case and the porous boundary case is the existence of a free streamline $y(x)$. First, the simple case of pure water jet impingement onto the wire is discussed.
In order to simplify the problem, the assumptions are discussed in Appendix F. A simple application of Newton's second law in the direction normal to the wire defines the pressure distribution,

$$\frac{d\{\rho \cdot v(x) \cdot y(x)\}}{dt} = \rho \cdot g \cdot y(x) - \Delta P_{\text{screen}} = \rho \cdot g \cdot y(x) - k_s \cdot \frac{P}{2} \cdot v(x)^2. \quad (6-18)$$

The continuity equation for water gives

$$\frac{d}{dt} \{y(x)\} = -v(x). \quad (6-19)$$

Then, a numerical solution was obtained by solving the equations as shown in Appendix G.2.1. The results are illustrated in Figure 6-28 for the change of water jet thickness $y(x)$ when the angle or $k_s$ are varied.
Figure 6-28 Water jet height for different $k_s$ and $\theta$. The figure shows the effect of $k_s$ and $\theta$ on the drainage rate. As $k_s$ increases, the drainage rate decreases significantly. Also as $\theta$ increases, the impinging jet produces a faster drainage.
6.5.2 Suspension Jet

When fibre mat deposition is considered, the impingement phenomenon is sketched in Figure 6-29 and a longitudinal element is shown in Figure 6-30.

Figure 6-29 Schematic of the mat deposition in the free-surface wedge zone

Figure 6-30 Sketch of formation region in the direction normal to the wire
Newton’s second law is applied in the differential form of the governing equation based on the same assumptions described in Appendix F. For a small impinging angle, the horizontal velocity is assumed to be constant at $U_l$. Also the mat velocity is assumed to be $U_w$, which is assumed to be the same as $U_l$ in the analysis. Therefore, it is again identical to the constant motion of a cylinder of length $y(x) = y_s(x) + h(x)$, which impinges on a screen at rest, with time taken for impact at $t = x/U_l$.

Assuming that the shape of the free streamline is $y(x) = y_s(x) + h(x)$, the governing equations are

1) Newton’s Second Law

$$
\frac{d}{dt} \left[ \rho_s \cdot v(x) \cdot y(x) \right] = \rho_s \cdot g \cdot y(x) - \frac{k_s}{2} \cdot \rho_s \cdot v(x)^2 - \rho_s \cdot \mu \cdot SFR \cdot h(x) \cdot C_{mat} \cdot v(x),
$$

(6-20)

2) Continuity

$$
\frac{dy(x)}{dt} = -v(x),
$$

(6-21)

3) Conservation of Mass for Fibre Mat

$$
\rho \cdot (C_{mat} - C_s) \cdot \frac{dh(x)}{dt} = \rho \cdot C_s \cdot v(x).
$$

(6-22)

By simplifying the equations and solving them numerically with proper boundary conditions, $v(0) = U_l \cdot \sin \theta$ and $h(0) = 0$, the shape of the free surface streamline $y(x) = y_s(x) + h(x)$ and the fibre mat growth $h(x)$ can be estimated. A full description of the mathematical procedure is included in Appendix G.2.2.
Figure 6-31 $y(x)$ and $h(x)$ for different $k$, for $SFR = 5E5$ m/kg and $\theta=8^\circ$

Figure 6-32 $y(x)$ and $h(x)$ for different $k$, for $SFR = 5E8$ m/kg and $\theta=8^\circ$
Figure 6-31 and Figure 6-32 present examples of the fibre mat accumulation and the free surface for two values of SFR and three values of \( k_s \) for \( \theta=8^\circ \). No experimental verification is available. The results in both figures were obtained under typical machine operating conditions. To simplify the calculation, the initial vertical jet thickness \( t_i \) was assumed to be 1 cm at impact. The horizontal axis shows the position of the wire from the point of jet impact. The vertical axis is the height of the free surface \( y(x) \) and the fibre mat thickness \( h(x) \). It is evident that the effect of the screen is significant in defining the drainage rate.

Figure 6-33 Vertical velocity \( v(x) \) using different \( k_s \) for \( SFR = 5E5 \) m/kg and \( \theta=8^\circ \):
The initial vertical jet velocity at the impingement point is determined from the boundary condition of \( v(0) = U_1 \cdot \sin \theta = 640 \) m/min \( \cdot \sin 8^\circ = 1.48 \) m/s.
Figure 6-33 shows the vertical velocity profile $v(x)$ along the wire length.

When the specific filtration resistance of the mat is varied as indicated in Figure 6-34, its effect on drainage is also significant.

![Diagram showing free stream line $y(x)$ and mat height $h(x)$ for different SFRs using $k_t=10$ and $\theta=8^\circ$. As the mat resistance increases, the drainage rate decreases.]

**Figure 6-34** Free surface $y(x)$ and mat height $h(x)$ for different SFRs using $k_t=10$ and $\theta=8^\circ$: As the mat resistance increases, the drainage rate decreases.

### 6.5.3 Effect of Wire Tension $T$

A form of Newton's second law was used to define the pressure distribution, which also deflects the wire, as shown in Figure 6-35. In order to verify the effect of wire
tension, the following equation is considered in the pressure term in Newton’s law and the governing equations are solved numerically:

\[ \Delta P_w = \frac{T}{R} = T \cdot \frac{d^2 y_w}{dx^2}. \]  \hspace{1cm} (6-23)

where \( T \): Wire Tension (N/m), \( R \): Radius of Curvature (m) and \( y_w \): Wire Coordinate (m).

![Figure 6-35 Effect of wire tension](image)

Then, the governing equation is changed to

\[ P = \frac{d}{dt} \left[ \rho \cdot y(x) \cdot \left( v(x) + \frac{dy_w}{dx} \right) \right]. \]  \hspace{1cm} (6-24)

The above equation is replaced with the left hand side of Eq. 6-20. Then, the modified equation is solved with Eq. 6-21 and Eq. 6-22 to show the effect of tension. The mathematical procedure is shown in Appendix G.2.3.

The three plots in Figure 6-36, (a), (b) and (c), show the fibre mat growth and the free surface drop for a realistic wire tension of 5 kN/m and very large wire tensions of 50
kN/m and 500 kN/m. A screen coefficient \( k_s \) of 10 and a mat resistance \( SFR \) of \( 5 \times 10^8 \) m/kg were used. The positions of the wire, the top of the formed sheet, and the free surface are plotted against the distance from the point of jet impact. The horizontal axis is a line tangent to the wire at the point of impingement.

The plot in Figure 6-36 (a) shows that the wire shape curves upwards. The total deflection is less than 1° at the end of the wire length. In Figure 6-36 (c), the wire tension is so high that the wire can be assumed to be rigid. This case corresponds to the one illustrated in Figure 6-32, where the wire tension was not a variable.

\[
k_s = 10
\]
\[
SFR = 5 \times 10^8 \text{ m/kg}
\]
\[
T = 5 \text{ kN/m}
\]

**Figure 6-36** Free surface, fibre mat and wire elevation for different tensions (Cont’d)
(b) For large tension $T = 50 \text{ kN/m}$

$k_s = 10$

$SFR = 5 \times 10^8 \text{ m/kg}$

$T = 50 \text{ kN/m}$

(c) For very large tension $T = 500 \text{ kN/m}$

$k_s = 10$

$SFR = 5 \times 10^8 \text{ m/kg}$

$T = 500 \text{ kN/m}$

Figure 6-36 Free surface, fibre mat and wire elevation for different tensions
6.6 Free-Surface Wedge Zone and Wedge Zone for Impingement on the Wire

Once the jet impinges on the wire and travels some distance in the free-surface wedge zone, the top free surface should make contact with the forming roll, and drainage will start to take place through both sides. The concept of the wedge zone is the same here as in the previous section. However, the resistance through each side is not the same, since the resistance mechanism is different. On the forming roll side wire, the screen is the only resistance, but on the outer wire side, the resistance comes from the screen and the formed mat developed in the free-surface wedge zone. This requires a unsymmetrical drainage analysis.

Figure 6-37 Prediction of jet thickness for impingement on the wire: The figure shows that the free jet impinges on the bottom wire with an angle $\theta=8^\circ$. The free-surface jet travels on the wire until the top surface hits the wire on the forming roll. Then, the mats grow on both sides of the screen. The mat growth is unsymmetrical and eventually both mats will meet.
Nevertheless, a simplified numerical analysis can be performed if it is assumed that the screen resistance is dominant over the mat resistance entering the wedge zone. Then, the symmetrical wedge zone approach can be used to predict the wedge thickness around the forming roll. Figure 6-37 includes the free-surface jet zone, the free-surface wedge zone, and the wedge zone, using the analysis applied to each zone. A free jet, whose thickness is assumed to be 0.9 cm, impinges on the wire with jet angle $\theta=8^\circ$. The wire has $k_s = 30$. A mat is developed on the bottom wire, and the mat resistance $SFR$ is assumed to be $10^8$ m/kg. The free-surface jet travels 5 cm on the wire until the top surface hits the top wire on the forming roll side. Then, the mat starts developing on the top wire, while the mat continues to grow on the outer wire side. The mat growth will be unsymmetrical, and eventually both mats will make contact at some point where the press zone starts.

6.7 Press Zone Analysis

Once the increasing fibre mats from both wires make contact at the end of the wedge zone, water drainage occurs because of the mechanical pressure imposed by the wire tension $T$. The drainage process involved in the press zone can be considered to be pure compression. This study includes the numerical prediction of mat height change and consistency change in the press zone before the mat leaves the forming roll. The predicted results are compared with the measured values obtained in the PAPRICAN trials. The models considered for the prediction of the consistency change are Biot’s time-dependent consolidation model and the free-fall drop model, as discussed in Chapter 4. The analysis
based on the Biot model is found in Appendix C, since the model failed to produce reasonable results.

Realistic values for the properties are taken from the flow-loop experiment and the compressibility test with the parameters of the PAPRICAN trials. These are tabulated in Table 6-4.

<table>
<thead>
<tr>
<th>Property</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target BW (kg/m³)</td>
<td>0.048</td>
</tr>
<tr>
<td>$h_0$ (m) at start</td>
<td>0.001</td>
</tr>
<tr>
<td>$c_{mo}$ (kg/m³) at start</td>
<td>48</td>
</tr>
<tr>
<td>Wire Tension $T$ (N/m)</td>
<td>5000</td>
</tr>
<tr>
<td>Approximated Applied Pressure $\sigma_z$ (Pa)</td>
<td>13000</td>
</tr>
<tr>
<td>Estimated Length of Press Zone (cm)</td>
<td>25</td>
</tr>
<tr>
<td>Contact Time $t$ (sec) for Base Cases</td>
<td>0.025</td>
</tr>
</tbody>
</table>

6.7.1 Calculation with Free-fall Drop Model

The free-fall drop model is introduced based on the idea that the low-consistency mat may behave like a highly viscous liquid when it is suddenly subjected to a high pressure. The governing equations were already introduced in Eqs. 4-13, 4-14 and 4-15. Figure 4-21 suggests that there exists a constant velocity region for 9453 Pa where a mass $M$ of 1.099 kg was used. This can be interpreted to mean that there is a terminal velocity $v_t$ of 1.25 cm/s. Using Eq. 4-15, the damping coefficient $C_d$ is calculated to be 862.5 (N·s/m). Then, from Eq. 4-14, the distance traveled by the wire is found to be 0.297 mm on one side. Since water removal occurs from both sides of the mat through the top and
bottom wires, the actual distance traveled by both wires is 0.594 mm. Because the initial mat height was assumed to be 1 mm, the final mat thickness, by the time the mat leaves the forming roll, is 0.406 mm.

Figure 6-38 Prediction of jet and mat thickness in the wedge and press zones: In the wedge zone, the prediction is based on the method using the wedge shape determined from the PAPRICAN data. In the press zone, the free-fall drop model predicts the shape of the total mat thickness.

Figure 6-38 shows the mat thickness in the press zone predicted using a free-fall drop model along with the PAPRICAN measurements. The change in thickness in the press zone is equivalent to a change in mass concentration from 48 kg/m$^3$ to 118 kg/m$^3$, as
shown in Figure 6-39. The height measurement by Gooding (1996) showed that the mat thickness changed from 1 mm at the start of the press zone to around 0.4 mm at the end of the forming roll. These are equivalent to changes in the mass concentration from 48 kg/m$^3$ to 120 kg/m$^3$. Therefore, the prediction by the free-fall drop model shown in Figure 6-39 is very good. It appears that the free-fall drop model could reasonably be expected to predict the mat thickness and the consistency.

![Graph showing mass concentration and mat consistency in the press zone.](image)

**Figure 6-39 Predicted mass concentration and mat consistency in the press zone using the free-fall drop model:** It is assumed that the mat density $\rho_m$ is approximately 1150 kg/m$^3$ to convert the mass concentration to the mass-based consistency.

Finally, based on the free-fall drop model, the mat resistance $SFR$ in the press zone can be predicted using Eq. 5-13, which was developed based on the Kozeny-Carman equation. Figure 6-40 shows the $SFR$ in the press zone using the free-fall drop model.
Figure 6-40 Predicted SFR in the press zone based on Eq. 5-13: The fibre properties $S_v$ and $v_s$ are obtained from the average values in Table 5-1.

There are several sources of error associated with the current analysis. First, the calculation was performed based on an experimental compressive pressure of 9453 Pa as opposed to a wire pressure of 13000 Pa. Second, the pulp used for the compression test was LWC, while newsprint was used for the PAPRICAN trials. However, the effect of using LWC should be insignificant, as shown in Chapter 4. Third, the resistance property of the actual forming fabric opening may be somewhat different from that of the permeable piston. Fourth, the length of the press zone and the contact time are approximate. Overall, despite some drawbacks, the free-fall drop model seems to be an effective way to predict the change in consistency in the press zone.
CHAPTER 7

FORMATION ANALYSIS OF EXPERIMENTAL SAMPLES

This chapter describes the formation analysis of samples from flow loop experiments and pilot machine trials. Two methods are used to measure formation: Video Beta Radiography and the Micro-Scanner method. The concept of the formation diagram is detailed.

7.1 Objectives

Current knowledge of forming and drainage analysis is limited. Hence, it is difficult to envision any direct analytical theory linking the governing parameters, the fluid and mat properties with formation. A direct way to monitor the effect of individual governing parameters on formation is to vary each parameter separately and measure the resulting formation.

Therefore, the objective of this chapter is to relate the effects of the operating parameters and the flow properties to paper formation by performing a formation analysis. The paper samples were obtained from the twin-wire former trials and the open loop trials. For simplicity, the term “PAPRICAN sample” refers to the paper samples obtained from the trials of the PAPRICAN twin-wire pilot machine.
7.2 Formation Measurement Methods

Two methods are used to measure formation: video beta radiography (VBR), and the Micro-Scanner method.

The VBR method involves beta radiography, image processing and data analysis. A prepared paper sample and a calibration wedge are exposed to a beta radiation source, and the image is captured on film. The film is developed, and the image is digitized and processed for computer data analysis. The VBR method is explained in more detail by Dodson et al. (1995). Figure 7-1 shows a schematic diagram of VBR.

![Schematic diagram of VBR](image)

Figure 7-1 Schematic diagram of VBR [Ng and Dodson (1995)]

The Micro-Scanner method is performed by light scanning, image processing and data analysis. The main difference between VBR and the Micro-Scanner method is the
source of transmission: VBR uses C¹⁴-based Beta radiation as a source, while the Micro-Scanner method uses light transmission.

Typically it would take 30 to 40 minutes to carry out one formation test using VBR. Developing the film, image analysis and data processing could take up to another 20 to 30 minutes. The Micro-Scanner method is quick and convenient; it takes around 5 minutes to complete the formation analysis of each sample.

7.3 Formation Results of Samples from Twin-Wire Former Trials

The samples were collected during jet impingement trials conducted at PAPRICAN. To ensure that the results were comparable, the paper samples used in the VBR analysis were taken to PAPRICAN for formation analysis by the Micro-Scanner method.

7.3.1 Analysis of the Split Sheet Samples

In order to study the possible effects due to each of the two converging wires, the PAPRICAN samples were split into two sections of approximately equal thickness by the Sheet Split Technique¹. Since the split samples were coated with a fairly thick sheet of Mylar, beta radiation was not able to penetrate them. Therefore, only the Micro-Scanner method was used for the formation analysis.

¹ The sheet split-up was carried out by JWI Limited.
Table 7-1 shows the results for the split samples obtained using the Micro-Scanner method. As defined by Jordan and Nguyen (1986), the specific perimeter is the total length of the median density contour per unit of viewed area. The contrast intensity is the coefficient of variation of picture point intensity calibrated by mass. The formation index is calculated by dividing the average Specific Perimeter by the average standard deviation of the variation. The formation index can be used as a simple index to compare formation: a higher formation index means better formation. More detailed studies have been done by Jordan and Nguyen (1986) and Lodjmark (1992).

Table 7-1 Results obtained using the Micro-Scanner method for split samples

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Formation Index</th>
<th>Contrast Intensity(%)</th>
<th>Specific Perimeter (mm⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>470-1 Top</td>
<td>55</td>
<td>22.09±7.80</td>
<td>3.00±0.09</td>
</tr>
<tr>
<td>470-1 Bottom</td>
<td>56</td>
<td>22.98±8.91</td>
<td>3.02±0.11</td>
</tr>
<tr>
<td>472-01 Top</td>
<td>58</td>
<td>23.21±2.61</td>
<td>3.26±0.12</td>
</tr>
<tr>
<td>472-01 Bottom</td>
<td>60</td>
<td>19.59±2.03</td>
<td>3.10±0.09</td>
</tr>
<tr>
<td>472-02 Top</td>
<td>59</td>
<td>21.23±3.90</td>
<td>3.15±0.15</td>
</tr>
<tr>
<td>472-02 Bottom</td>
<td>61</td>
<td>17.47±2.65</td>
<td>3.14±0.15</td>
</tr>
<tr>
<td>472-03 Top</td>
<td>50</td>
<td>23.00±2.98</td>
<td>3.08±0.07</td>
</tr>
<tr>
<td>472-03 Bottom</td>
<td>46</td>
<td>23.55±0.80</td>
<td>2.84±0.04</td>
</tr>
<tr>
<td>477-01 Top</td>
<td>47</td>
<td>27.49±10.01</td>
<td>2.97±0.19</td>
</tr>
<tr>
<td>477-01 Bottom</td>
<td>43</td>
<td>25.20±3.82</td>
<td>2.81±0.10</td>
</tr>
<tr>
<td>479-1 Top</td>
<td>54</td>
<td>21.98±3.69</td>
<td>3.04±0.05</td>
</tr>
<tr>
<td>479-1 Bottom</td>
<td>50</td>
<td>23.25±6.79</td>
<td>2.91±0.05</td>
</tr>
<tr>
<td>480-1 Top</td>
<td>56</td>
<td>21.55±5.50</td>
<td>3.05±0.08</td>
</tr>
<tr>
<td>480-1 Bottom</td>
<td>51</td>
<td>22.55±4.49</td>
<td>2.99±0.15</td>
</tr>
<tr>
<td>481-01 Top</td>
<td>59</td>
<td>21.44±2.94</td>
<td>3.13±0.07</td>
</tr>
<tr>
<td>481-01 Bottom</td>
<td>52</td>
<td>24.75±6.38</td>
<td>3.03±0.08</td>
</tr>
<tr>
<td>484-01 Top</td>
<td>54</td>
<td>22.18±7.91</td>
<td>3.04±0.17</td>
</tr>
<tr>
<td>484-01 Bottom</td>
<td>51</td>
<td>24.13±7.68</td>
<td>2.86±0.10</td>
</tr>
</tbody>
</table>
Figure 7-2 is plotted to emphasize the comparison between the top and bottom layers of the samples. The “top” section refers to the wire rolling from the breast roll and converging to the outer edge of the forming roll. The “bottom” section refers to the wire wrapping around the forming roll inside. Henceforth, the top will be referred to as the outer wire side, and the bottom as the inner wire side. Also the sample number is specified in the same notion, as shown in Table C-1 in Appendix C.

![Graph showing formation comparison]

**Figure 7-2 Formation comparison of various split samples between the top and the bottom sections**

Based on the results, the following two trends can be noted:

1) When the wire tension remains constant (from 472-02 to 484-01), all samples except one (472-02) showed that formation was better in the top layers (outer wire side) than in the bottom layers (inner wire side). This behaviour might be explained as follows. Turbulence generated inside a headbox is known to be one of the main factors causing fibre flocs to break. The turbulence is carried along by the jet. Due to the inertia of the
jet, more drainage would occur through the outer wire near the jet impingement region within the wedge zone. Thus, the fibres, which are more uniformly distributed by turbulence in the headbox, would be deposited on the outer wire side. Since it takes only a short time (~20 milli-seconds in the trials) for the jet to reach the wire from the headbox, turbulence decay would be negligible.

2) For the trials in which wire tensions were varied, the mat deposited on the inner wire had almost the same or slightly improved formation compared with the mat on the outer wire. The reason for this is unclear. If statement (1) is true, the same principle should apply to these cases. Possibly, the change in wire tension affected the rate of mat deposition so that mat formation was significantly changed in the early stage of the press zone, where the shear would be the main mechanism breaking flocculation. More experimental samples are required to provide a reasonable explanation.

7.3.2 Formation Comparison between the Micro-Scanner Method and VBR

Table 7-2 shows the results from the Micro-Scanner measurements. When the results are compared with the measurements by Gooding (1996), they are almost the same, the difference being less than 4%. Therefore, the reproducibility of the Micro-Scanner method is verified.

Figure 7-3 shows comparison of the formation analysis using the Micro-Scanner method. For comparison between the two methods, the formation diagram obtained from the VBR method is illustrated in Figure 7-4. It tracks the effects of different operating
conditions in a single graph by plotting the mean floc grammage $G$ against the mean floc diameter $D$.

**Table 7-2 Micro-Scanner measurements (on 97/01/27):** Comparing the Formation Index between the current measurement and the one by Gooding shows a good repeatability.

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Formation Index</th>
<th>Formation Index by Gooding (1996)</th>
<th>Contrast Intensity(%)</th>
<th>Specific Perimeter (mm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>470-01</td>
<td>50</td>
<td>49</td>
<td>22.82±0.78</td>
<td>3.13±0.06</td>
</tr>
<tr>
<td>471-01</td>
<td>45</td>
<td>45</td>
<td>25.83±1.17</td>
<td>3.07±0.16</td>
</tr>
<tr>
<td>472-01</td>
<td>48</td>
<td>50</td>
<td>24.43±1.96</td>
<td>3.12±0.12</td>
</tr>
<tr>
<td>472-02</td>
<td>47</td>
<td>49</td>
<td>24.96±0.74</td>
<td>3.12±0.06</td>
</tr>
<tr>
<td>472-03</td>
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<td>43</td>
<td>25.97±1.04</td>
<td>3.12±0.11</td>
</tr>
<tr>
<td>473-01</td>
<td>44</td>
<td>44</td>
<td>25.36±0.56</td>
<td>2.95±0.08</td>
</tr>
<tr>
<td>474-1</td>
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<td>25.45±1.14</td>
<td>3.01±0.11</td>
</tr>
<tr>
<td>474-2</td>
<td>45</td>
<td>44</td>
<td>25.69±1.10</td>
<td>3.03±0.10</td>
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<tr>
<td>477-01</td>
<td>39</td>
<td>38</td>
<td>27.72±0.75</td>
<td>2.89±0.14</td>
</tr>
<tr>
<td>479-01</td>
<td>45</td>
<td>46</td>
<td>25.27±1.42</td>
<td>3.02±0.08</td>
</tr>
<tr>
<td>480-1</td>
<td>47</td>
<td>45</td>
<td>24.49±0.92</td>
<td>3.04±0.05</td>
</tr>
<tr>
<td>481-01</td>
<td>43</td>
<td>N/A</td>
<td>26.29±1.66</td>
<td>3.03±0.09</td>
</tr>
<tr>
<td>481-02</td>
<td>44</td>
<td>42</td>
<td>25.87±0.93</td>
<td>2.99±0.09</td>
</tr>
<tr>
<td>484-01</td>
<td>46</td>
<td>45</td>
<td>23.34±4.45</td>
<td>3.02±0.03</td>
</tr>
</tbody>
</table>

**Figure 7-3** Formation comparison of the samples using the Micro-Scanner method
As indicated in Figure 7-4, the smaller $G$ and $D$, the better the formation tends to be, as shown by Farnood et al. (1995).

![Figure 7-4 Formation diagram of PAPRICAN samples using the VBR method](image)

Figure 7-4 Formation diagram of PAPRICAN samples using the VBR method: The figure shows that the Jet/Wire speed ratio is important in paper formation. The sample with Jet/Wire ratio less than one in the case of 472-03 has poor formation. Also, the case of 477-01 having 840/800 for J/W ratio has poor formation compared to others. However, the case of 474-2 which has the same operating condition shows better formation. For Jet/Wire ratio of one in the case of 472-02, formation is the best. This is in contradiction with the conventional wisdom, since there is no shear to deflocculate fibres when the Jet/Wire ratio is one.

Despite the differences in the two techniques, both indicate similar formation results, as shown in Figures 7-3 and 7-4. Using either method, case 472-02 has good formation, and case 477-01 has very poor formation. Using VBR, case 472-03 is found to have very poor formation, but using the Micro-Scanner method formation is only relatively poor. Figure 7-4 indicates the important effect of the Jet/Wire speed ratio on paper formation. For example, the sample with a Jet/Wire ratio of less than 1 (case 472-
03) has very poor formation. Also, case 477-01 with a J/W ratio of 840/800, has poor formation compared with the other cases. However, case 474-2, which has the same operating conditions as case 477-01, seems to have better than average formation results. This inconsistency suggests that more trials are required to draw meaningful conclusions.

Nevertheless, these findings suggest the importance of the J/W speed ratio difference as a significant factor affecting paper formation. The J/W speed ratio tends to have a greater effect on the shear forces, which are also known to be a significant factor affecting paper formation. These forces are produced in a twin-wire machine which exerts high pressures between the two sides of the fibre mat. Hardman and Cole (1960) explain that these shear forces can greatly enhance paper formation and improve the overall quality of the paper by generating deflocculation. This suggests that a Jet/Wire speed ratio of 1 should produce no shear on the fibres and should result in poor formation. However, the formation results show that better formation was produced with a J/W speed ratio of 1 than with a higher ratio; very poor formation was found when the J/W speed ratio was below 1.

Nordstrom and Norman (1995) also found that the Jet/Wire speed ratio is an important factor affecting formation. However, their results indicated that optimal formation in a twin-wire roll former was observed at a Jet/Wire speed difference of +20 m/min. Since the current results show that good formation was observed with a Jet/Wire speed ratio of 1, there appears to be a contradiction, indicating that more complicated mechanisms are involved in the process.

Figure 7-5 shows the Coefficient of Variation (CV) plotted against the Zone Size. The convention is to choose a Zone Size of 1 mm and compare the corresponding CV.
values; the higher the $CV$ value, the worse the formation. In case 472-03, formation was very poor. This is similar to the results found in the formation diagram.

![Graph of Coefficient of Variation vs Zone Size]

**Figure 7-5 Coefficient of variation plotted against zone size:** The higher CV is, the worse the formation becomes. The trial of 472-02 shows the best formation while 472-03 shows the worst.

Anisotropy in paper is another factor to be considered, although it is independent of paper formation. According to Scharcanski and Dodson (1995), one of the main reasons for it is that fibre orientations are affected by speed differences between the suspension jet and the forming fabric. Since the Jet/Wire speed ratio difference seems to be a significant factor in formation analysis, based on PAPRICAN results, it is worthwhile investigating the effect of fibre orientation resulting from a change in Jet/Wire speed ratio.

Figure 7-6 shows polar plots of local gradient orientations using a zone size of 0.2 mm. A perfect circle would imply the isotropy of fibre orientation. The paper sample
becomes more anisotropic as the probability of the occurrence of different angles become non-uniform, which appears in a polar plot as eccentricity. An ellipse with a different angle would show the fibre orientations in a preferred direction.

![Polar plots](image.png)

**Figure 7-6 Polar plots of anisotropy for selected samples:** The figure, obtained from VBR, shows that the fibre orientation may not be as much affected by the Jet/Wire speed ratio as expected. It is possible that the fabric type may be important. Nevertheless, it is too early to draw any conclusion.

However, all the PAPRICAN samples showed similar characteristics of anisotropy with the exception of case 484-01, which was produced with a more permeable fabric, QFINE ET. This suggests that the fibre orientation is not affected as much as expected by the Jet/Wire speed ratio difference; it seems to be more dependent on the forming fabric type than on the other parameters considered. However, more test results are required.
7.4 Formation Results of Samples from the Experimental Flow Loop

![Formation Comparison Chart](image)

**Figure 7-7** Formation comparison of the samples from the open flow loop tests to those from a gap former and a handsheet process

VBR was used to analyze the formation of the selected paper samples collected from the open flow loop experiment discussed in Chapter 3. Figure 7-7 compares the results for the open loop experiment with those for a gap former and for handsheet trials. The formation results for the samples from the PAPRICAN trials are also shown.

The experimental paper samples from the open loop trials show good formation results compared with the machine-made sheets and much better formation than the handsheets. In general, the handsheet samples tend to show the worst formation, probably because there is no dispersing mechanism to deflocculate fibres during the handsheet-making process. The machine-made samples are subjected to strong turbulence generation in the headbox and shear along the wire, which tend to deflocculate the fibres and keep the
forming mat as uniform as possible. Therefore, the machine-made samples are expected to have better formation than the handsheet samples under normal circumstances.

![Diagram: BW vs. D for G ≈ constant for the samples obtained from the open flow loop tests](image)

**Figure 7-8 BW vs. D for G ≈ constant for the samples obtained from the open flow loop tests:** The figure shows that the basis weight of the samples varied. To compare the formation from different papermaking methods, it is reasonable to use the samples having the similar range of basis weight.

What is most interesting is the formation results for the open-loop samples. These are widely scattered in the estimated floc diameter $D$ (horizontal direction), when compared with the other results. This is because the target basis weight of the open loop samples varied widely from 5 gsm to 200 gsm, as shown in Figure 7-8. If formation is compared only among samples in a similar target basis weight range, for example, newsprint samples from the twin-wire former, which are all in the range of 45-50 gsm, then it is found that the open loop samples have a similar range of $D$. Nevertheless, the results from the estimated floc grammage $G$ of the open-loop samples indicate that more deflocculation occurred during the open-loop experiments than during the machine trials. This is difficult
to understand, since turbulence is the only dispersing mechanism in the open-loop experiments. There are two possible explanations for this behaviour.

The first may be related to the repositioning of fibres in the open loop experiments as discussed in Chapter 3. In a twin-wire former, once the fibres deposit on the moving wires, the drainage process starts immediately, and fibre mats are not strongly subjected to further dispersing. However, in the open-loop experiment, even after the fibre mat is formed, it is still affected by the flow movement until the loop is shut off. This continuous movement may have been a factor in arranging fibres more uniformly.

The second may have to do with the way fibres are injected from a syringe. In each trial, it was not possible to release all the fibres at once manually. In some high basis weight cases, it took as much as one full second to release them. The early-released fibres were deposited on the screen at a time when the last-released fibres were still some distance upstream. This means that the actual fibre deposition rate in the open-loop experiment does not reflect the real target basis weight used in a papermaking machine. For example, consider a target basis weight of 50 gsm in a twin-wire former, and then in an open-loop experiment. In a twin-wire former, 50 gsm of fibres should be actually contained across the jet so that when the jet impinges on a moving wire, the same amount of fibre can be delivered continuously. In contrast, in the open-loop experiment, 50 gsm of fibres are deposited over a certain time interval depending on how fast the fibres are injected from the syringe so that the consistency of the suspension across the plane direction should be a lot lower (more dilute) than in the twin-wire papermaking process for a comparison with the same basis weight. This may be why the open-loop samples showed relatively consistent floc grammage $G$ although the floc diameter $D$ was scattered.
CHAPTER 8

CONCLUSIONS

8.1 Summary and Concluding Remarks

The goal of this study was to improve the understanding of fundamental sheet forming, drainage and compression processes in the jet impingement region around the forming roll of a twin-wire papermaking machine. Extensive analyses were carried out to correlate the effects of operating parameters on fluid and mat properties with the eventual goal of improving sheet formation. A brief summary of the current study is presented in Figure 8-1.

For an analysis of flow and mat properties, reasonable physical approximations were made to investigate the mean flow behaviour in the draining jet. Using hydrodynamics in the wedge zone and the free-surface wedge zone, a set of governing equations was developed for the cases of symmetric and asymmetric jet impingement.

For the symmetric case, two cases were considered: i) wedge shape determined from the measured data; and ii) wedge shape unknown. The governing equations with realistic physical approximations were numerically solved and compared with the measurements. The analytical results indicate that the drainage length increased as the screen or mat resistance increased.
When the wedge shape was unknown, the results indicated a slower drainage rate. It seems that the analysis can be improved by including extra drainage mechanisms such as centrifugal force. When the wedge shape was determined from the PAPRICAN data, the prediction of the drainage rate in the wedge zone was approximately twice that of the measured value, suggesting room for improvement. The predicted pressure profile and the measurement in the wedge zone showed a similar increasing trend, but the pressure peak observed in the PAPRICAN data could not be explained adequately. Also it could not be...
explained why the measured pressure was higher than $T/R$ in some trials. However, the hydrodynamic approach had limited success since the complex jet impingement behaviour was described in terms of a simplified physical model.

To estimate the properties of the screen and the fibre mat, numerous experiments were conducted using a drainage tester designed to acquire real time measurements of the pressure drop and the average velocity. The analysis of the fibre mat resistance as a function of basis weight and flow velocity verified the unique characteristics of the three distinct regions of pressure drop defined in this study. It was found that the use of Darcy’s law was questionable due to the inertial effect of high-speed flow and the compressible characteristics of the porous mat.

The use of the Kozeny-Carman equation enabled the mat resistance to be expressed in terms of several fibre mat properties: the specific surface, the specific volume and the mass concentration. Several different models were compared to relate the effect of the porosity to the Kozeny constant. The variation of the specific filtration resistance with the mass concentration was analyzed. It was proved that the Kozeny-Carman approach was a reasonable method for analyzing the fibre mat resistance.

The time-dependent compressibility analysis demonstrated that the present technique of treating mat compression in the press zone as a combination of spring-damper systems would be inaccurate. When subjected to high compression, the mat in the press zone would likely behave as a highly viscous liquid rather than as an elastic or a viscoelastic porous medium. This concept was validated by comparing the actual change in the mat thickness in the press zone with the prediction of the free-fall drop model.
Formation analysis of the experimental samples was performed using two measuring techniques: VBR, based on beta radiation transmission; and the Micro-Scanner method, based on light transmission. The results revealed that the governing parameters affect final formation. Nevertheless, a more in-depth analysis and more data are required to obtain clear answers.

8.2 Short-Term Improvement and Long-Term Direction of Future Research

Although this study has contributed to the knowledge of sheet formation and the drainage process in the forming roll region of a twin-wire former, there are still many questions to be answered.

In the long run, several important characteristics of the headbox suspension jet will have to be included in the analysis: the fibre orientation effect, the fibre flocculation effect, and the turbulent effect. These factors would make the current analysis complex, but ultimately they are likely candidates for affecting formation.

To achieve short-term improvement, several suggestions can be made. First, more experiments are required in a pilot twin-wire former to gather more data. Second, the governing equations for a physical flow model in the wedge zone need to be modified to reflect non-symmetrical drainage behaviour in the forming roll. Third, the design of the current flow loop needs to be modified so that the fibre deposition process on the screen can be visually recorded in real time. This will enable the direct measurement of the specific filtration resistance as well as compression measurements over time. It would help
if other pulp types could be tested for comparison. The first candidate should be incompressible glass fibres since the resistance is likely to be closer to Darcy’s concept. Last, more compressibility experiments using different pulp types are needed to verify the free-fall drop model. These should be accompanied by trials using the same pulp on a pilot machine.

It is also suggested that the results obtained in this study be further extended to Fourdrinier machines. This will not be difficult since the concept of the Fourdrinier jet from a headbox is similar to the free streamline jet analysis carried out in this study.
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APPENDIX-A

JET IMPINGEMENT ANALYSIS ON A SOLID FLAT PLATE BY CONFORMAL MAPPING TECHNIQUE

A.1 Introduction

The hydrodynamics of jet impingement is an important aspect of current research in many practical industrial applications. Examples include the design of headboxes, stock jet flow and the coating process in the pulp and paper industry; and Vertical Take Off & Landing aircraft in connection with ejectors, jet engines and air-cushion vehicle studies in the aerospace industry. An analytical flow model is developed as a fundamental basis to understand the flow characteristics of jet impingement. This is accomplished by introducing three major assumptions as follows:

1) The flow may be adequately represented by a 2-D potential flow model.
2) The fibre particles have no significant effect on the flow characteristics.
3) The jet may be considered to impinge on a solid flat plate of infinite extent.

Therefore, no screen effect is included in the analysis.

With these assumptions, an oblique impinging jet analysis is performed with potential flow theory using the conformal mapping method. Pressure distribution along the solid flat plate is predicted for two jet cases as follows:
1) jet coming from an infinite distance from the plate,

2) jet coming from a headbox nozzle located at a finite distance from the plate.

A.2 Development of Exact Solutions

A.2.1 Jet from an Infinite Distance

A two-dimensional jet issued from an infinite distance is first considered by using a conformal mapping method, specifically the Schwarz-Christoffel transformation. As shown in Figure A.1 (a), the flow situation is applicable to the classical case of the Helmholtz-Kirchhoff streamline analysis, as detailed by Milne-Thomson (1962) and Robertson (1965), since the jet is bounded by free streamlines and rigid boundaries.

(a) The free streamline jet configuration

(b) Conformal mapping planes

Figure A-1 Infinite jet impinging on a solid flat plate
Considering these boundaries, the physical plane (z-plane) may be transformed into a complex potential plane (F-plane), as shown in Figure A.1 (b). Along the boundaries BC and EF, the stream function $\psi$ is constant with the jet widths, $t_1$ and $t_2$, being dependent on the impinging angle to satisfy the continuity, Bernoulli and the momentum equations. Also, the complex velocity, $v$, is transformed into a polygonal area of the logarithmic hodograph Q-plane by Kirchhoff's method;

$$v = u - iv = qe^{-i\theta} \quad (A-1)$$

$$Q = \ln\left(\frac{U_0}{v}\right) = \ln \frac{U_0}{q} + i\theta \quad (A-2)$$

In the Q-plane, the magnitude of free jet velocity, $q$, is known along the boundaries of BC and EF. Also known is the flow angle, $\theta$, along the solid boundary originating from the jet impinging point, DD', in both directions. Then, both the F-plane and the Q-plane are mapped onto the real axis of the $\zeta$-plane using the Schwarz-Christoffel transformation with their corner conditions corresponding to the real axis points in the $\zeta$-plane. It is also noteworthy that the stagnation point, DD', is a floating point which may vary between EE and C'C depending on the impinging angle. By mapping the F-plane onto the $\zeta$-plane and using the complex potentials for a source and a sink to find unknown constants, the following is found:

$$\frac{dF}{d\zeta} = \frac{U_0}{\pi} \cdot \frac{\zeta - s}{\zeta^2 - 1} \quad (A-3)$$
Also from the Q-plane transformation onto the \( \zeta \)-plane and by the corner conditions,

\[
Q = \cosh^{-1} \frac{1 - s \zeta}{\zeta - s}
\]  \hspace{1cm} (A-4)

From the above equations, the geometric details of the physical \( z \)-plane may be found in terms of the mapping parameters in the \( \zeta \)-plane. By the definition and by mathematical manipulation, the following can be found:

\[
dz = \frac{1}{\nu} \cdot dF = \frac{e^q}{u_o} \cdot \frac{dF}{d\zeta} \cdot d\zeta
\]  \hspace{1cm} (A-5)

By integration, the physical domain information is known as a function of the \( \zeta \)-plane condition.

\[
z(\zeta) = \frac{r}{\pi} \left\{ \frac{s - 1}{2} \ln(1 - \zeta) + \frac{s + 1}{2} \ln(1 + \zeta) + \sqrt{1 - s^2} \cdot \sin^{-1} \zeta \right\}
\]  \hspace{1cm} (A-6)

Also, \( \zeta \) is found as a function of the pressure coefficient \( Cp \) from Bernoulli’s equation to find the pressure distribution along the solid boundary. The location of the jet impinging point, \( s \), is found with respect to the geometric origin, \( O \), from the momentum equation. Figure A-2 shows the shapes of free streamlines for different impinging angles. The results show that for a 10 degree impinging angle, 99.24 percent of the total flow will move toward flow direction.
Figure A-2 Shapes of free streamlines for the impinging angles of 10°, 60° and 90°: Origin is the jet impingement point.

A.2.2 Jet from a Finite Distance

(a) The free streamline jet configuration

(b) Conformal mapping planes

Figure A-3 Finite jet impinging on a solid flat plate

The main difference for the case of a jet from a finite distance is the introduction of the headbox nozzle into the geometry, as shown in Figure A.3 (a), similar to the study by Ehrich (1961). This introduces two more governing parameters to the analysis; jet/free
stream line velocity ratio \((\alpha = U_j/U_o)\) and nozzle-end normal distance from the plate \((y_1, y_2)\).

Jet velocity and free stream line velocity would become noticeably different when the nozzle gets very close to the solid surface. The corresponding mapping planes are shown in Figure A-3 (b). The physical boundaries of D'E'F'G'B'C'D are transformed into the F-plane and Q-plane. With \(Q\) being the same in Eq. A-4, \(dF/d\zeta\) becomes

\[
\frac{dF}{d\zeta} = \frac{U_j t i}{\pi(M + N)} \cdot \frac{\zeta - s}{(\zeta^2 - 1)\sqrt{(\zeta - r_1)(\zeta - r_2)}} \quad (A-7)
\]

where \(M\) and \(N\) are functions of \(r_1\) and \(r_2\).

The geometric details as a function of \(\zeta\) may be found by using the same method given in Eq. A-5. Integration of \(z\) with respect to \(\zeta\) is performed analytically. It is required to make a fractional linear transformation to reduce the given integral as a form of incomplete elliptic integral of the first kind. By following the table of integral summarized by Gradsheteyn (1980), \(z\) is found as a function of \(\zeta\). The following shows \(z(\zeta)\) in a simplified form.

\[
z(\zeta) = \frac{t}{\pi} \cdot \alpha \cdot \frac{i}{(M + N)} \{ - M \cdot \sin^{-1} f_1[\zeta, \eta, r_2] + N \cdot \sin^{-1} f_2[\zeta, \eta, r_2] + \sqrt{1 - s^2} \cdot P \cdot F(k, \phi) \} \quad (A-8)
\]

where \(M, N, P\) and \(k\) are functions of \(r_1\) and \(r_2\) and \(F(k, \phi)\) is an incomplete elliptic integral of the first kind with \(\phi\) as a function of \(r_1, r_2\) and \(\zeta\).
Finally, when the location of the headbox \((y_1, y_2)\), the slice opening gap length \((t)\) and the jet impinging angle \((\theta)\) are known, the relationships among the governing parameters \((r_1, r_2, \alpha, s, t_1 \text{ and } t_2)\) may be found from the given geometric details for \(y_1/t\) and \(y_2/t\), the continuity and the momentum equations, and the relative strength ratio of the sinks. Integration along the exit leads to jet velocity definition in terms of the mapping parameters. For the case of \(\alpha=1\), Figure 6-15 in Chapter 6 already showed the pressure distribution along the solid plate. The results found in the analysis suggest that the location of the headbox is not a main governing parameter which affects the pressure distribution along the surface, provided that the headbox is located sufficiently far from the wire, which is the typical case found in a papermaking machine. The analysis show that if the headbox is located at least two jet diameters away from the plate, the effect of the backflow is negligible.
Questions regarding the possible effects of a boundary layer on sheet formation have been raised on several occasions during the current study. The approach taken to verify these possible effects is to use a simple boundary layer analysis with a constant suction velocity applied at the bottom, as shown in Figure B-1. The mathematical treatment is described by Schlichting (1968) who used the continuity and momentum equations to predict the boundary layer thickness. It may not entirely represent the same physical model taking place in the forming roll region, but the results give some idea of the effects of a boundary layer in terms of its order of magnitude, since the change of suction...
velocity is not large with respect to the jet velocity within a short distance from the point of jet impact.

The analysis shows that the boundary layer thickness $\delta$ becomes a direct function of the kinematic viscosity $\nu$ of the flow and the suction velocity $v_0$ in the following form:

$$\delta = 2\frac{\nu}{v_0} \quad \text{(B-1)}$$

For a typical pulp suspension of 0.5% consistency at 50°C inside a headbox assuming that $\nu$ is approximately $0.5 \times 10^{-6}$ m$^2$/s (same as the water property) and the vertical velocity is approximately 0.5 m/s, the thickness of the boundary layer is around 2 $\mu$m, which is very small compared to the average fibre length of 1-3 mm. Therefore, it is hard for an individual fibre to suddenly change its orientation near the boundary layer. It is safe to say that the boundary layer effect is negligible in the wedge zone.
APPENDIX-C

TIME-DEPENDENT COMPRESSIBILITY ANALYSIS USING THE CONCEPT OF BIOT MODEL FOR VISCOELASTIC FIBRE MAT

C.1 Analytical Approach in Compressibility Analysis

In this approach, a fibre mat is assumed as a visco-elastic material. The motivation for the theoretical modeling of the fibre mat compressibility is based on the classical work by Biot (1941), which is briefly discussed later. He derived a general analysis of 3-D consolidation theory by assuming the whole process to be identical with squeezing the water out of an elastic porous medium.

If the same idea can be used for the pulp fibre mat, though fibres are not elastic, it is possible to model the compressibility behaviour of the fibre mat from a theoretical point of view with reasonable modifications. This would include the effect of visco-elasticity, the effect of strain and consistency of the mat with respect to time under a constant load. When Biot’s theory is applied to the present case of fibre mat compressibility, a number of original assumptions should be redefined to suit the current needs. Table C-1 explains the assumptions imposed by Biot (1941) and how they are modified in this study.
Table C-1 Comparison of assumptions for Biot's analysis and this study

<table>
<thead>
<tr>
<th>Biot (1941)</th>
<th>Actual Fibre Mat Behaviour</th>
<th>Approach Taken Here</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotropy</td>
<td>Anisotropy</td>
<td>1-D anisotropy</td>
</tr>
<tr>
<td>Reversible σ-ε</td>
<td>Non-recoverable ε involved</td>
<td>Only compressive ε</td>
</tr>
<tr>
<td>Linear σ-ε</td>
<td>Nonlinear σ-ε</td>
<td>Linearized σ-ε</td>
</tr>
<tr>
<td>Small Strains</td>
<td>Relatively large strain</td>
<td>Relatively large strain</td>
</tr>
<tr>
<td>Elastic medium</td>
<td>Visco-elastic medium</td>
<td>Voigt or Standard Solid Model</td>
</tr>
</tbody>
</table>

C.1.1 Viscoelastic Models

The term "visco-elasticity" is a generalization of elasticity and viscosity. Current work is based on the hypothesis that the microscopic structure of a viscoelastic material such as a fibre mat is mechanically equivalent to a network of viscous and elastic elements. The ideal elastic element and viscous element can be represented by a spring and a damper, respectively. These basic elements are combined in a number of different configurations either in series or in parallel to represent a visco-elastic medium. In this study, some of the simpler visco-elastic models are considered to identify which model would best describe the behaviour of a pulp fibre mat under a constant compressive load. Therefore, the following analysis discusses the suitability of three well-known configurations of visco-elastic models for current applications. More detailed information on the topic of viscoelasticity is available from the book by Christensen (1971).
1) Maxwell Model

As shown in Figure C-1 (a), the Maxwell model consists of a spring and a dashpot placed in series. Mathematically, it can be expressed in a differential form as;

\[ \dot{\varepsilon}(t) = \frac{\dot{\sigma}(t)}{E} + \frac{\sigma(t)}{\eta} \quad (C-1) \]

A general solution for the above first order differential equation may be determined when \( \sigma(t) \) or \( \varepsilon(t) \) is given. To determine the behaviour of the Maxwell Model in creep test, the constant stress is applied; \( \sigma(t) = \sigma_0 H(t) \) where \( H(t) \) is the Heaviside unit function.

Then, the resulting strain as a function of time becomes

\[ \varepsilon(t) = \sigma_0 \left\{ \frac{1}{E} + \frac{t - \tau_o}{\eta} \right\} H(t - \tau_o) \quad (C-2) \]
As shown in Figure C-1 (b), deformations grow steadily as a function of time. Therefore, this behaviour would not be suitable to express the fibre mat compression behaviour observed in the experiment.

2) Voigt Model

Figure C-2 Voigt model

The combination of the elastic and viscous elements in parallel is typically known as a Voigt model. Its configuration is graphically illustrated in Figure C-2 (a). The model is mathematically given as

\[ \sigma(t) = E \varepsilon(t) + \eta \dot{\varepsilon}(t) \]  

(C-3)

\(^{1}\) also known as a Kelvin model
For the case of constant stress, $\sigma_0$, the strain is found as

$$\varepsilon(t) = \frac{\sigma_0}{E} \left(1 - \exp\left(-\frac{E}{\eta}(t - \tau_0)\right)\right) H(t - \tau_0)$$

(C-4)

Since the current experimental results show the exponential change of compressive strain with time as indicated by Figure C-2 (b), the Voigt model may be a simple and reasonable representation of fibre mat compression analysis.

3) **Standard Solid Model**

(a) Representations

(b) Strain vs. Time

Figure C-3 Standard solid model

Two previously described models are convenient in visualizing linear visco-elastic behaviour in a simple manner, but they do not represent the realistic behaviour of real
materials, where relations are more complex and difficult for mathematical manipulations. For better approximations of the visco-elastic behaviour, slightly more complex models can be incorporated with more springs and dampers used in the combinations. This generalized model may give a more accurate fit to any required degree of approximation. It, however, generates a more complicated mathematical expression. One way to simplify this phenomenon is by lumping together Voigt elements into a single element with an average retardation time. For a particular case, if the response to stress is of interest for only a particular time duration, the elements with shorter retardation times compared to this can be considered "fully deformed" and can be represented with an isolated spring. The elements with retardation times in the time range of interest can be lumped as a single Voigt element. Therefore, an approximate model known as a standard solid model can be represented by a Voigt model in series with a spring, as shown in Figure C-3 (a). Its mathematical representation is expressed as

\[
\left( \frac{1}{E_1} + \frac{1}{E_2} \right) \sigma(t) + \frac{\eta}{E_1 E_2} \dot{\sigma}(t) = \varepsilon(t) + \frac{\eta}{E_1} \dot{\varepsilon}(t) \tag{C-5}
\]

The strain may be expressed as follows by using an operational calculus

\[
\varepsilon(t) = \sigma_o \left[ \frac{(E_1 + E_2) + \eta \frac{\partial}{\partial t}}{E_1 E_2 + E_2 \eta \frac{\partial}{\partial t}} \right] H(t - \tau_o) \left[ 1 - \text{Exp}\left( \frac{-E_1 \cdot (t - \tau_o)}{\eta} \right) \right] H(t - \tau_o) \tag{C-6}
\]
Depending on the selected viscoelastic model of the fibre mat, one may reach different analysis for the distribution of stresses and the settlement of fibres with time. Therefore, a careful choice should be made in selecting a viscoelastic model which provides the best accord between the experimental observations and the particular analytical solution.

**C.1.2 Governing Equations for 1-D Compressibility of Saturated Fibre Mat**

The analysis of the fibre mat compressibility and water removal can be expressed from the work done by Biot (1941). The general 3-D governing equations for the saturated elastic medium are derived in his work. For simplicity, the current compression test can be simulated with a one-dimensional approximation in z direction. Several assumptions, similar to Biot’s but less restricted, have to be made for current 1-D theoretical compression analysis of a saturated fibre mat,

1] Isotropy of the mat in plane direction
2] Viscoelastic fibre mat
3] Non-recoverable compression, No recovery analysis
4] Non-linear stress-strain relation
5] Initial height of a saturated mat at zero compacting pressure as defined previously in the preparation of the saturated mat

Biot’s 1-D consolidation model based on elastic deformation of media is represented in Figure C-4. Here, an elastic modulus, $E$, of the deformable medium is assumed to be constant, whereas in reality it is not.
Assuming the fibres to be an elastic medium saturated with water, the Biot’s 1-D governing equations become

\[
\frac{1}{a} \frac{\partial^2 w}{\partial z^2} - \frac{\partial p}{\partial x} = 0 \quad \text{(C-7)}
\]

\[
\frac{\partial^2 p}{\partial x^2} = \frac{1}{c} \frac{\partial^2 p}{\partial t^2} \quad \text{(C-8)}
\]

where, \( w \) is the displacement of height change in the \( z \) direction, \( a = 1/E \) and \( c = K/(\mu \alpha) \). Here, \( E \) is the modulus as a function of time, \( K \) is the Darcy’s permeability and \( \mu \) is the dynamic viscosity. Then, the solution for \( w \) can be found with proper boundary conditions and initial conditions as follows:

\[
w = h_0 \left[ 1 - 8 \pi^2 a \sigma_z \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \left( 1 - \exp \left( -\left( \frac{(2n+1)\pi}{2h_0} \right)^2 ct \right) \right) \right] \quad \text{(C-9)}
\]

where \( h_0 \) is the initial mat height. By taking into account that
\[ \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8} \]  

(C-10)

Then, for \( t=\infty \), the expression can be simplified to

\[ w(t = \infty) = h_n[1 - a\sigma_b] \]  

(C-11)

Biot's analysis can be further extended to include the viscoelastic characteristics of the fibre mat by using the concept of correspondence principle, which brings out the same structure between theories of elasticity and viscoelasticity. The correspondence principle, as studied by Biot (1961), states that equations valid for the linear theory of elasticity with linear boundary conditions and time independent constraints can immediately be extended to viscoelasticity by the substitution of time operators for the elastic coefficients.

![Figure C-5 Strain as a function of time for 953 Pa](image)

\[ \varepsilon(t) = 0.545 - 0.51 \exp(-0.0043t) \]
In this experiment, because no lateral expansion is allowed and water can only escape through the top surface, the governing equations can be rewritten simply by substituting the elastic modulus $E$ with $E(t)$ and $c$ with $c(t)$. Also, the effect of $c(t)$ becomes of secondary importance since it is included inside a decaying exponential term. Therefore, the change of mat height would be a direct function of the modulus $E$ which is very much time dependent. An example of the results is studied to correlate the strain, the modulus and the time as shown in Figure C-5. Once the compressive strain results are found with respect to time change, the data can be curvefitted by using the strain and time relationship for a suitable viscoelastic model. In this case, either the Voigt model or the standard solid model would be well suited. For better approximation, the standard solid model would be preferred although it is more complicated. The relationship inside the figure was obtained by curvefitting the standard solid model. This method of determining the time-dependent modulus needs to be performed for each compressive load and for each pulp specimen. The job may become tedious, but it is required since no analytical expression could be easily found for determining the time-dependent coefficients.

A careful analysis is also required for the effect of $c(t)$ on the change of mat height. As described earlier, $c$ is a function of $K$ and $E$. Although the permeability $K$ was assumed to be constant for the original Biot's analysis, it would be realistic to assume a decreasing permeability under compression. Based on Darcy's law, the permeability can be related as a function of time under constant load by measuring the drained water volume out per given time and the change of mat height. Nevertheless, the effect of $c(t)$ becomes secondary and it would only have a small effect in the long time solution.
Figure C-6 shows a comparison of the mat height change based on actual experimental results and predicted results for decreasing and constant $c(t)$. When the constant $c(t)$ is assumed, the prediction seems to be in good agreement with the experimental results for both short and long time behaviour of the fibre mat compression. However, when the decreasing $c(t)$ is used, the prediction seems to underestimate the degree of compression by some degree as the time progresses. This range of error for long time behaviour was within 4% for 953 Pa and increased to around 10% for 5159 Pa.

![Figure C-6 Predictions of mat height against experimental data under 953 Pa](image-url)
C.2 Proposed Time-Dependent Compressibility Equation Based On Biot’s Consolidation Model

The Biot model is converted to the appropriate form of equation to express the time-dependent compression of a viscoelastic fibre mat.

Let’s consider the mat height, $h$, or the mat concentration, $c_{mat}$, as a function of time, $t$, and applied compacting pressure, $\sigma_z$, as follows:

$$h = \text{function}(t, \sigma_z)$$  \hspace{1cm} (C-12)

$$c_{mat} = \text{function}(t, \sigma_z)$$

while the relationship between the mat height and the mass concentration is known to follow the assumption that only the water is allowed to leave the mat surface.

$$c_{mat} \cdot h = BW$$  \hspace{1cm} (C-13)

where $BW$ is the dry basis weight.

For an uncompressed saturated fibre mat, the initial consistency, $c_{mo}$, is

$$c_{mo}(t = 0, \ \sigma_z = 0) = \frac{BW}{h_0(t = 0, \ \sigma_z = 0)}$$  \hspace{1cm} (C-14)

Then, Eq. C-9 can be rewritten in terms of consistency for a constant $\sigma_z$ as

$$c_{mat}(t) = c_{mo} \cdot \frac{1}{\frac{\sigma_z}{E(t)} \cdot \Omega(t)}$$  \hspace{1cm} (C-15)
where $\Omega(t) = \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \left[ 1 - \exp \left( - \frac{(2n+1)\pi}{2 \cdot \frac{BW}{c_{mo}} c t} \right)^2 \right]$

Here, the complexity of the problem lies in determining $\frac{\sigma_z}{E(t)}$ experimentally, which actually is the strain, $\varepsilon(t)$, because the relation between the stress, $\sigma_z$, and the strain at infinite time, $\varepsilon(\infty)$, is non-linear as previously described in Figure 4-6. Whenever the value of the compacting pressure is changed, the resulting time-dependent strain needs to be reevaluated without any obvious relationship. This changes the experimentally determined constants, $K_1$ and $K_2$, in Eq. 4-7, whenever $\sigma_z$ is varied for a particular type of pulp mat. It is possible that there exists an empirical relationship between two constants and the compacting pressure. However, it would become very time-consuming to determine. Henceforth, its possible existence is only acknowledged here. One way of handling the job is by dividing the non-linear region into several linear regions, the method that is widely used in such non-linear problems. This is beyond the scope of current analysis. Further study could be carried out on that matter.

There are still some shortcomings in the time-dependent compressibility equation based on Biot’s idea. Firstly, it is only valid while the compacting pressure remains constant. Although attempts have been made to include the varying $\sigma_z$ into the compressibility equation, these have not been successful. Further study is required in this regard. At least, it could be used along with the known empirical compressibility equation to cover wider ranges of compacting pressures. In a practical situation near the jet impingement region, the applied pressure varies along the wire. In this case, the average
pressure may be used to avoid the difficulty. Secondly, it cannot properly predict the consistency changes in free-fall drop region. In order to cover that region, the time-dependent strain relation in Eq. 4-10 needs to be modified to include more constants with more exponential decay terms. This means the use of more complicated viscoelastic models.

C.3 Calculation Applied to the Press Zone

As discussed in the previous section, the Biot model is based on a simplified assumption that the mat compression would be the process of squeezing water out of an elastic porous medium. To use the modified Biot equation in Eq. C-15, calculations are required to determine the time-dependent elastic modulus $E(t)$ and the term $\Omega(t)$ including the consolidation coefficient $c(t)$.

First, the results in Figure 4-9 are used to find the change of strain $\varepsilon(t)$ and consequently the elastic modulus $E(t)$. Assuming for simplicity that the long time strain behaviour follows Eq. 4-10, the constants $K_1$ and $K_2$ are found by curve-fitting the results. These are tabulated in Table C-2.

<table>
<thead>
<tr>
<th>Applied Pressure (Pa)</th>
<th>$K_1$</th>
<th>$K_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>943</td>
<td>0.5441</td>
<td>0.0022</td>
</tr>
</tbody>
</table>
Figure C-7 shows the comparisons of curvefit with actual results for long time strain behaviour. It is obvious that Eq. 4-7 is too simple to express the change of strain although it shows the overall characteristics of compressive strain after a long time. Moreover, it introduces non-trivial errors in estimating short time strain behaviour especially for the case of 9453 Pa where the higher compressive strain occurred due to free-fall drop in short time. For more accurate strain expression, though not pursued any further in this study, more complex relationships could be used. For example,

\[
\varepsilon(t) = K_1 - K_2 \cdot \exp(-K_3 \cdot t)
\]

\[
\varepsilon(t) = K_1 \left(1 - \exp(-K_2 \cdot t)\right) + K_3 \left(1 - \exp(-K_4 \cdot t)\right)
\]

Despite the simple relationship expressed for long term strain behaviour, it would not be of much practical use in the press zone for the estimation of short time strain behaviour due to non-trivial error. Therefore, to find a short time strain behaviour, a power law type of new relationship was introduced for time duration of less than 10 seconds based on Figure 4-13 (b) where the strain tends to increase linearly in logarithmic scale for short time. Figure C-8 compares the actual data with power law curve for short time.
Figure C-7 Comparison of curvefit with the results for compression over a long time

Figure C-8 Comparison of curvefit with the results for initial time behaviour
Table C-3 Elastic modulus $E(t)$ for short time and long time behaviour

<table>
<thead>
<tr>
<th>Pressure (Pa)</th>
<th>Short Time $E(t)$</th>
<th>Long Time $E(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>943</td>
<td>$76048 t^{-0.5031}$</td>
<td>$1773[1-\text{Exp}(-0.0022 t)]^t$</td>
</tr>
<tr>
<td>2540</td>
<td>$224778 t^{-0.6428}$</td>
<td>$3523[1-\text{Exp}(-0.0028 t)]^t$</td>
</tr>
<tr>
<td>5159</td>
<td>$234500 t^{-0.6136}$</td>
<td>$6718[1-\text{Exp}(-0.005 t)]^t$</td>
</tr>
<tr>
<td>9453</td>
<td>$110303 t^{-0.9231}$</td>
<td>$10898[1-\text{Exp}(-0.012 t)]^t$</td>
</tr>
</tbody>
</table>

Then, the elastic modulus for short time and long time behaviour can be expressed in simple form as tabulated in Table C-3. An attempt was made to correlate the elastic modulus $E(t)$ with respect to pressure. However, no obvious relationship was found to exist for the short time $E(t)$. On the other hand, possible linear relationships seemed to exist among the numerical coefficients for long time $E(t)$. This gives the possibility of having a simplified relation for long time elastic modulus for any compacting pressure applied for a particular type of pulp. However, further study would be required to confirm the possibility. For calculation purpose applied in the press zone, the approach taken in this chapter is to use an expression for short time $E(t)$ of 9453 Pa in place of $E(t)$ for 13 kPa applied by wire tension, since no measurement was made at 13 kPa. This certainly would result in a little underestimation of the elastic modulus for 13 kPa, but this is now the best that can be done.

Next, it is required to evaluate the consolidation coefficient $c(t)$. As previously defined in Eq. 4-20, the consolidation coefficient is a function of Darcy’s permeability and
the elastic modulus. Although $c(t)$ changes with time, the effect of changing $c(t)$ is negligible overall as proved in Figure C-6. This is due to the fact that $c(t)$ is included in the decaying exponential term and becomes of secondary importance. The value of $c(t)$ is calculated from the results of compressibility experiments for the contact time of 0.025 second. It is found to be approximately 0.6645. The corresponding $\mathcal{A}(t=0.025 \text{ sec})$ is found to be 0.9978.

Finally, the calculation for mass concentration $c_m$ at $t=0.025 \text{ sec}$ shows that $c_m$ changes from 48 kg/m$^3$ to 48.2 kg/m$^3$ for that time duration. This is only a 0.4% change of $c_m$ for the time duration of 0.025 sec. Since the measurement by Gooding (1996) showed that $c_m$ changes from 48 kg/m$^3$ to 120 kg/m$^3$ in the press zone at PAPRICAN trial, the prediction by Biot model is totally unrealistic.

The most likely reason for the failure by Biot model is the assumption of the porous medium being an elastic solid. The mat with the consistency being around 2-4% does not behave as an elastic solid when compressed as found in Chapter 4. It is expected that the Biot’s consolidation model would be more applicable once the mat consistency has increased sufficiently. Therefore, when the mat with medium or high consistency is rolled into press rolls in the dry section, the Biot model may give rise to a useful tool by including time-dependent consolidation effect. However, it is beyond the scope of current study, but worthwhile pursuing for the development of compression model in the press roll in the future.
APPENDIX-D

TRIAL CONDITIONS FROM PAPRICAN TWIN-WIRE PILOT MACHINE

Full scale experiments were carried out by PAPRICAN using the twin-wire pilot machine in conjunction with this work. The scale and the cost involved in these tests were large, and author’s contributions to the trials were limited to:

- Initiation of the jet impingement project proposal
- Outline of the proposal including what parameters to be investigated
- Evaluation of the forming fabric properties and pulp properties using experimental flow-loop in Chapter 3 and the compressibility analysis in Chapter 4
- Continuous feedback to the trial methods and operating conditions
- Participation in several trials
- Formation analysis of the samples collected in the trials in Chapter 7

It should be acknowledged that most of the trials were performed by PAPRICAN including the preparation of the trials, data collection and data analysis. This appendix covers the brief summary of the PAPRICAN trial conditions, that are required to compare with theoretical prediction. Fully-detailed analysis of the result is available from the work by Gooding (1996). The analysis is currently ongoing and is subject to the restriction of
publication at the time of writing. To summarize the PAPRICAN trial conditions, Table D-1 is presented.

Table D-1 PAPRICAN trial conditions [Gooding (1996)]

<table>
<thead>
<tr>
<th>Date</th>
<th>Run Number</th>
<th>Machine Speed (m/min)</th>
<th>Jet Rush (m/min)</th>
<th>J/W Speed Ratio</th>
<th>Tension (kN/m)</th>
<th>Fabric Type</th>
<th>Impinging Location</th>
</tr>
</thead>
<tbody>
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<td>96/02/07</td>
<td>Pretrial 1</td>
<td>600</td>
<td>40</td>
<td>1.067</td>
<td>5</td>
<td>Albany UltraTex</td>
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<td>96/02/29</td>
<td>Pretrial 2a</td>
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<td>40</td>
<td>1.067</td>
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<td>Albany UltraTex</td>
<td>NIP</td>
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<td>Pretrial 2b</td>
<td>600</td>
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<td>1</td>
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<td>1.067</td>
<td>5</td>
<td>JWI 2000</td>
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<td>471-01</td>
<td>600</td>
<td>40</td>
<td>1.067</td>
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<tr>
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<td>600</td>
<td>40</td>
<td>1.067</td>
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<td>JWI 2000</td>
<td>NIP</td>
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<td>96/05/09</td>
<td>472-02</td>
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<tr>
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<td>QFINE ET</td>
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</tr>
</tbody>
</table>
This appendix covers the effects of several operating parameters considered to affect flow mechanism and paper formation. The results of formation analysis was fully discussed in Chapter 7, and the effect of individual parameters on formation is further discussed in this appendix based on formation results of PAPRICAN samples. The parameters investigated are:

1) Fabric Property and Mat Resistance  
2) Impingement Location  
3) Jet/Wire Speed Ratio  
4) Impingement Angle, $\theta$

E.1 Effect of Fabric Property, $k_r$ and Fibre Mat Resistance, SFR on Formation

The VBR formation results showed that some benefits on formation have been achieved by changing the forming fabrics, which resulted in different screen coefficient $k_r$. This is in contradiction to the discussion by Gooding (1996) who suggested no appreciable change in formation based on the Micro-Scanner results.

It appears that the VBR results are worthwhile pursuing for a reasonable explanation. By using more permeable forming fabric, a slightly better formation by the
VBR method was attained in a final sample. Since the drainage rate in the wedge zone is the parameter much likely to be affected by changing the forming fabrics, it is speculated that faster drainage in the wedge zone may result in better formation.

This speculation may be explained by the fact that when the jet suspension leaves the headbox, fibres tend to be most uniformly mixed and distributed due to the turbulence generated in the headbox. Since the turbulence can only decay in the free jet zone and the wedge zone, the best chance of fibres depositing uniformly on moving wires should take place closer to the jet impingement region assuming that the shear forces are not a dominant deflocculating mechanism. Faster drainage signifies more and quicker fibre deposition in less time to be subjected to flocculation. This phenomenon should lead to a better formation of paper. On the other hand, a major drawback also exists. More fines would escape through more permeable screen. Therefore, the fine contents would decrease. Fine retention ratio is an important element to maintain overall integrity of the paper. Definitely, it is necessary to keep the balance on the control of certain properties for optimal papermaking process.

The effect of the mat resistance property on formation is still unknown and very speculative. It is only certain that as the mat resistance increases the drainage rate should decrease. However, the PAPRICAN results do not indicate any direct linkage between the mat resistance and the formation. No literature has been found on any possible relationship. It can only be speculated that in the wedge zone the effect of wire resistance property is much greater than the fibre mat property, as found in Chapter 3. If one can assume that the main portion of formation is greatly determined in the wedge zone and early part of press zone, then the effect of mat resistance property on formation would be
negligible in a twin-wire former. However, it is also possible in a speculative manner that the shear, which is another deflocculation mechanism other than turbulence, would be affected by the mat resistance property. That is, for high mat resistance, less water would drain and lower consistency would be dominant. Then, the lower consistency mat should behave more liquid-likely and be subjected to the shear. Whether or not this speculation is feasible has not been determined, but at least it could serve as a certain guideline for future study.

**E.2 Jet Impingement Location**

This study suggests the possible effect of different jet impingement location as depicted in Figure 6-1 of Chapter 6. According to the original design of a twin-wire machine, as described by De Montigny (1967) and Baines (1967), the stock jet was to hit the middle of the two converging wires as shown in Figure 6-1 (a). Over the last three decades, this concept has been the basis of actual machine operation in industry. However, based on close observations of several machines and extensive discussions with many industry experts, Figure 6-1 (b) more likely represents the actual operating conditions currently in use.

PAPRICAN trial results indicate that no major effect on formation has been observed by changing the location of jet impingement. The case of hitting the wire side first produced a slight advantage in formation than the case of hitting the roll side first. However, the difference is very small and could be negligible. The change of the impingement location seems to result in no visible formation improvement. It seems that
the impingement location is a main factor affecting only the fluid properties for an analytical purpose, but not necessarily the actual paper formation.

**E.3 Jet/Wire Speed Ratio**

When the jet speed exceeds or lags behind the fibre mat speed, a pressure change can be produced by the difference of Jet/Wire speed ratio. The $x$-momentum equation can be simplified, as follows, by assuming the mat along with the inside water moves at the same speed of the wire. Also, the mat velocity and the stock velocity outside the mat are assumed to be constant respectively. Obviously, the discontinuity at the boundary between the mat and suspension should be assumed, and this in fact is physically impossible.

\[
\rho \left( \frac{\partial u}{\partial t} + u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \cdot \nabla^2 u + \rho \cdot f_x
\]  

(E-1)

Then,

\[
\Delta P_{JW} = -\rho \left( U_1 \cos \theta - U_w \right) \int \frac{v(x)}{h(x)} \, dx
\]

(E-2)

Despite the importance, the effect of Jet/Wire speed ratio is not theoretically considered anymore in the study because of the conflict of assumptions. Because the Newton's 2nd law is used with the assumption of flow and fibre mat moving at the same speed, the introduction of Jet/Wire speed ratio would invalidate the approach. Definitely, some assumptions have to be improvised for the development of a reasonable model. This is left for future work.
E.4 Impingement Angle, $\theta$

For an assumption of jet symmetry, a jet impingement angle can not be a factor in the wedge zone. But for a free stream jet, it is probably an important factor affecting drainage characteristics based on the change of fluid properties. However, its effect on paper formation is still very much unknown. The restriction of fixed headbox position of the twin-wire pilot machine in PAPRICAN trial prevented the further study of the effect of angle change on paper formation.

In this analytical study for a free-stream jet, it was shown that some changes of fluid properties (not necessarily formation) could be found when the impinging angles are varied as previously investigated in Figure 6-29 (b). The height of free surface jet decreased only 20 % more for 16° angle of impingement than 4°. This suggests that the jet impingement angle $\theta$ may not be as critical as anticipated. It seems that the same amount of drainage could be achieved by choosing the screen with more open area.

Another way to investigate this effect is to consider a simple but extreme case of jet impingement. Suppose that the ideal flow jet impinges on a long flat plate with solidity ratio of 1 at certain angle. Exact solutions could be found using potential flow theory and a conformal mapping method as described in Appendix-A. The results illustrated the size of jet deflection region as well as the change of fluid properties. Figure E-1 shows the change of pressure along the boundary as a function of impingement angle. $C_p$ is the pressure coefficient, $t$ is the jet thickness and $\lambda/t$ is the multiple of jet thickness. For easy understanding, the figure is drawn in such a way that the location of the peak follows the centre of pressure with a jet originating from the right plane.
Figure E-1 Effect of impingement angle on pressure along the boundary
APPENDIX-F

ASSUMPTIONS OF THE JET IN THE FREE-SURFACE WEDGE ZONE

Consider the water jet impingement case shown in Figure 6-27. First, the jet impingement angle $\theta$ is assumed to be very small ($\theta \leq 8^\circ$). Then, consider a uniform screen with a resistance of $k_s$ as defined in Eq. 6-9. For a small $\theta$, $U_2(x) \approx U_1 \cos \theta \approx U_1$ and the longitudinal velocity is effectively constant with $x$. It is assumed that the vertical motion is thus convected at the speed $U_2(x)$. Then, it is identical to the motion of a cylinder of length $y(x)$ which impinges on a screen at rest as shown in Figure F-1. Here, the time since impact is $t=x/U_2(x)$.

![Figure F-1 Vertical motion of the jet impingement](image)

To verify that the above assumptions are reasonable, consider the flow velocities in terms of the face velocity $v$ and let $U_2$ be the longitudinal velocity component immediately above the screen as shown in Figure F-2.
Assuming that the jet is a potential flow,

\[
\frac{1}{2} \rho \cdot U_1^2 = \Delta P + \frac{1}{2} \rho \cdot [U_2^2 + v^2]
\]

(F-1)

By using the screen equation, the relationship becomes

\[
U_2^2 = U_1^2 - (k_s + 1) \cdot v^2
\]

(F-2)

\[
\frac{U_2}{U_1} = \sqrt{1 - (k_s + 1) \cdot \left(\frac{v}{U_1}\right)^2}
\]

(F-3)

At the impingement, the boundary condition shows that \( w/U_1 \approx \sin \theta \).

For example, if \( \theta = 8^\circ \), then \( \cos 8^\circ = 0.990 \) and \( \sin 8^\circ = 0.139 \). And let \( k_s = 10 \).

Then, from Eq. F-3

\[
\frac{U_2}{U_1} = \sqrt{1 - (10 + 1) \cdot (0.139)^2} = 0.887
\]

At the free stream surface, \( U_2 = U_1 \cdot \cos \theta = 0.990 \ U_1 \).
Assuming that the longitudinal flow rate remains unchanged vertically along the y direction and the velocity profile is linear for simplicity, then

\[ \frac{y}{2} (0.990 \cdot U_1 + 0.990 \cdot U_1) = \frac{y_1}{2} (0.990 \cdot U_1 + 0.887 \cdot U_1) \]  

(F-4)

Then,

\[ \frac{y_1}{y} = 1.05 \]

Therefore, the maximum error in assuming the longitudinal velocity constant is approximately 5% underestimate of the jet thickness at impact. And as the flow develops, the error will be less.

Newton’s 2nd law is applied for the body of length y and is shown in Eq. 6-18. For the jet impingement of a suspension in section 6.5.2, the same assumptions are applied. One additional assumption is that the wire speed \( U_w \) is the same as the jet speed to achieve a constant vertical motion. Therefore, Eq. 6-20 is found.
APPENDIX-G

MATHEMATICAL PROCEDURES

G.1 Wedge Zone

G.1.1 Solutions For Wedge Shape Determined From PAPRICAN Data

In section 6.4.3.b, the governing equations were introduced for the suspension jet: Eqs. 6-1, 6-4, 6-5, 6-6, 6-10 and 6-11. There are six unknowns; \( y(x) \), \( v(x) \), \( U_2(x) \), \( W(x) \), \( h(x) \), \( P(x) \). Let \( \frac{U_w - U_1}{L} = \alpha \) and \( \frac{1 - C_z}{1 - C_{mat}} = \beta \).

From Eq. 6-11,

\[
h(x) = \frac{C_s \cdot U_1 \cdot t_1 - C_s \cdot U_2 \cdot y(x)}{C_m \cdot U_w}
\]  

(G-1)

Then, Eq. G-1 is inserted into Eq. 6-1 along with Eq. 6-6, then

\[
y(x) = \frac{(L - x)^2 + t_2 - \frac{C_s \cdot U_1 \cdot t_1}{C_m \cdot U_w}}{2R'} - \frac{C_s}{C_m \cdot U_w} \cdot (\alpha x + U_1)
\]  

(G-2)

Therefore, \( y(x) \) is found as a function of \( x \). Then, \( h(x) \) is found from Eq. G-1, \( v(x) \) is found from Eq. 6-10. \( P(x) \) is found from Eq. 6-5. \( W(x) \) is found from Eq. 6-4. The resulting solutions are plotted in Figures 6-9 and 6-10.
In section 6.4.4.a, the governing equations are from Eqs. 6-3, 6-4, 6-5, 6-6 and 6-9.

First, let \( \frac{U_w - U_1}{L} = C_1 \).

By substituting Eq. 6-6 into Eq. 6-3,

\[
\nu(x) = -y(x) \cdot C_1
\]

By putting Eq. 6-6 and Eq. 6-3 into Eq. 6-5,

\[
P(x) = \frac{1}{2} \rho \{-C_1^2 \cdot x^2 - 2C_1 \cdot U_1 \cdot x - C_1^2 \cdot y(x)^2\}
\]

Also from Eq. 6-4,

\[
W(x) = -C_1 \cdot y(x) - (C_1 x + U_1) \cdot y'(x)
\]

By equating Eq. 6-5 with Eq. 6-9, the following non-linear differential equation is found

\[
k_1 \cdot (C_1 x + U_1)^2 \cdot [y'(x)]^2 + 2k_2 \cdot C_1 \cdot (C_1 x + U_1) \cdot y(x) \cdot y'(x) + (k_2 + 1) \cdot C_1^2 \cdot y(x)^3 + 2C_1 \cdot U_1 \cdot x + C_1^2 \cdot x^2 = 0
\]

where the boundary condition is \( y(0) = t_1 \).

The simplified equation is in the form of

\[
[y'(x)]^2 + A_1(x) \cdot y(x) \cdot y'(x) + A_2(x) = 0
\]

where \( A_1 \) and \( A_2 \) are function of \( x \). Also, the screen coefficient \( k_1 \) determines the solution.

Then, the equation is solved numerically by Mathematica. It generates two resulting
solutions for \( y(x) \). Since \( y(x) \) has to decrease, only one solution is chosen. The results are shown in Figures 6-15, 6-16 and 6-17.

**G.1.3 Solutions For Suspension Jet With Wedge Shape Unknown**

In section 6.4.4.b, the governing equations are given by Eqs. 6-4, 6-5, 6-6, 6-10, 6-11 and 6-12.

First, let 
\[
C_1 = \frac{U_w - U_1}{L}, \quad C_2 = \frac{1 - C_s}{1 - C_{mat}}, \quad C_3 = \rho_s \cdot C_s \cdot U_1 \cdot t_1, \quad C_4 = \rho_s \cdot C_s \cdot (C_1 x + U_1) \quad \text{and} \\
C_5 = \rho_m \cdot C_m \cdot U_w.
\]

Then Eq. 6-10 becomes
\[
v(x) = -y(x) \cdot C_1 \cdot C_2 = -y(x) \cdot C_6
\]
where \( C_6 = C_1 \cdot C_2 \).

Also from Eq. 6-11,
\[
h(x) = \frac{C_3 - C_4 \cdot y(x)}{C_5} \tag{G-8}
\]
And Eq. 6-5 becomes
\[
P(x) = \frac{1}{2} \rho \left[ U_1^2 - (C_1 x + U_1)^2 - C_6^2 \cdot y(x)^2 \right] \\
= \frac{1}{2} \rho \left[ -C_1^2 x^2 - 2C_1 U_1 x - C_6^2 \cdot y'(x) \right] \tag{G-9}
\]
Also Eq. 6-4 becomes
\[
W(x) = -C_6 \cdot y(x) - (C_1 x + U_1) \cdot y'(x) \tag{G-10}
\]
Let \( C_7 = \rho_m \cdot C_{mat} \cdot \mu \cdot SFR \) and \( C_8 = k_s \cdot \frac{\rho}{2} \).

Put Eq. G-10 into Eq. 6-12.

\[
P(x) = C_8 \cdot (C_1 x + U_1) \cdot [y'(x)]^2 + [2C_8 \cdot C_6 \cdot (C_1 x = U_1) \cdot y(x) - \frac{C_7 \cdot C_3}{C_5} (C_1 x + U_1)
\]

\[
+ \frac{C_7 \cdot C_4}{C_5} (C_1 x + U_1) y(x) \cdot y'(x) + \left[ \frac{C_7 \cdot C_4 \cdot C_6}{C_5} + C_8 \cdot C_6^2 \right] y(x)^2
\]

\[
- \frac{C_7 \cdot C_3 \cdot C_6}{C_5} \cdot y(x)
\]

From Eq. G-9 and Eq. G-11, the equation is simplified to the form given in Eq. 6-17.

The boundary condition is \( y(0) = t_0 \). The resulting solutions vary as the screen coefficient \( k_s \) and the mat resistance \( SFR \) change. The results are shown from Figures 6-18 to 6-23.

**G.2 Free-Surface Wedge Zone**

**G.2.1 Solutions for Water Jet**

Using Newton 2nd law and Continuity, the governing equations are shown in Eqs. 6-18 and 6-19.

By non-dimensionalizing the variables

\[
t = \frac{x}{U_1}, \quad \frac{gy_0}{U_1^2} = \frac{1}{Fr^2}, \quad \bar{v} = \frac{v}{U_1}, \quad \bar{y} = \frac{y}{y_0}, \quad \bar{x} = \frac{x}{y_0}
\]

where \( Fr \) is the Froude Number.

Eq. 6-19 becomes
Also, Eq. 6-18 becomes

\[
\frac{d(\rho \cdot U_1 \cdot \bar{v} \cdot y_0 \cdot \bar{y})}{y_0 \cdot \frac{d\bar{v}}{dx}} = \rho \cdot g \cdot y_0 \cdot \bar{y} - \frac{k_s}{2} \cdot \rho \cdot U_1^2 \cdot \bar{v}^2
\]

Then, Eq. G-13 becomes

\[
\frac{U_1^2 \cdot \rho \cdot y_0 \cdot d(\bar{v} \cdot \bar{y})}{y_0 \cdot \frac{d\bar{v}}{dx}} = \rho \cdot g \cdot y_0 \cdot \bar{y} - \frac{k_s}{2} \cdot \rho \cdot U_1^2 \cdot \bar{v}^2
\]

Therefore,

\[
\frac{\bar{v}}{\bar{y}} \frac{d\bar{v}}{dx} + \frac{d\bar{y}}{dx} = \frac{g \cdot y_0}{U_1^2} \cdot \bar{y} - \frac{k_s}{2} \cdot \bar{v}^2
\]  \hspace{1cm} (G-13)

Using Eq. G-12, then Eq. G-13 becomes

\[
\frac{d\bar{v}}{dx} = \frac{1}{F_r^2} \left( \frac{k_s}{2} - 1 \right) \frac{\bar{v}^2}{\bar{y}}
\]  \hspace{1cm} (G-14)

For typical operating conditions, \( \frac{1}{F_r^2} \) is very small compared to other term and is negligible.

For simplicity, non-dimensionalized variables are written without bar subsequently.

By differentiating Eq. G-12 with respect to \( x \), then \( y'' = -\frac{dv}{dx} \). Therefore,

\[
y \cdot \frac{dv}{dx} = -\left( \frac{k_s}{2} - 1 \right) \bar{v}^2
\]

\[
y \cdot y''(x) = -\left( \frac{k_s}{2} - 1 \right) [y']^2
\]

\[
y \cdot y'' = [y']^2 \cdot \left( \frac{k_s}{2} - 1 \right)
\]  \hspace{1cm} (G-15)

There are two boundary conditions that exist.

i) \( y(0) = 1 \)
Finally, Eq. G-15 is numerically solved to obtain the solutions for \( y(x) \). Then, \( v(x) \) is found as well. They are functions of the screen coefficient \( k_s \) and the impingement angle \( \theta \) only.

To validate the numerical solutions computed using Mathematica, Eq. G-15 is analytically solved and compared with the numerical results in Appendix I.

G.2.2 Solutions for Suspension Jet

The governing equations are given in Eqs. 6-29, 6-21 and 6-22.

From Eq. 6-20,

\[
y \frac{dv}{dt} + v \frac{dy}{dt} = g \cdot y - \frac{k_s}{2} \cdot v^2 - \mu \cdot SFR \cdot C_{mat} \cdot h \cdot v
\]

Since \( \frac{dy}{dt} = -v \), the equation becomes

\[
\frac{dv}{dt} = g - \left( \frac{k_s}{2} - 1 \right) \frac{v^2}{y} - \mu \cdot SFR \cdot C_{mat} \cdot \frac{h}{y} \cdot v \quad (G-16)
\]

Also let \( C_i = C_m - C_s \). Then,

\[
\frac{dh}{dt} = \frac{C_s}{C_i} \cdot v \quad (G-17)
\]

By non-dimensionalizing the variables

\[
t = \frac{x}{U_1}, \quad \frac{gy_0}{U_1^2} = \frac{1}{Fr^2}, \quad \bar{v} = \frac{v}{U_1}, \quad \bar{y} = \frac{y}{y_0}, \quad \bar{x} = \frac{x}{y_0}, \quad \bar{h} = \frac{h}{y_0}
\]
where $F_r$ is the Froude Number.

Then, Eq. G-16, Eq. G-17 and Eq. 6-22 become

$$
\frac{d\bar{v}}{dx} = \frac{1}{F_r^2} - \left( \frac{k_s}{2} - 1 \right) \frac{\bar{v}^2}{\bar{y}} - \mu \cdot SFR \cdot C_{mat} \cdot \frac{\bar{h}}{\bar{y}} \cdot \bar{v} \cdot \frac{y_0}{U_1}
$$

(G-18)

$$
\frac{d\bar{h}}{dx} = \frac{C_1}{C_s} \cdot \bar{v}
$$

(G-19)

$$
\frac{d\bar{y}}{dx} = -\bar{v}
$$

(G-20)

For typical operating conditions, $\frac{1}{F_r^2}$ is very small compared to other terms and is negligible.

For simplicity, again non-dimensionalized variables are written without bar subsequently.

From Eqs. G-19 and G-20,

$$
\frac{dy}{dx} = -\frac{C_1}{C_s} \cdot \frac{dh}{dx}
$$

$$
y = -\frac{C_1}{C_s} \cdot h + I_1
$$

To find $I_1$, the boundary conditions are $y = 1$ and $h = 0$ at $x = 0$.

Therefore,

$$
y = -\frac{C_1}{C_s} \cdot h + 1
$$

(G-21)

Also, from Eq. G-18,

$$
\frac{d\bar{v}}{dx} = \frac{C_1}{C_s} \cdot \frac{d^2 h}{dx^2}
$$

(G-22)

By putting Eqs. G-19, G-21 and G-22 into G-18,
\[ h'' = -\left(\frac{k_s}{2} - 1\right) \cdot \frac{C_1}{C_s} \cdot \frac{[h']^2}{(-\frac{C_1}{C_s} \cdot h + 1)} - \mu \cdot SFR \cdot C_{mat} \cdot \frac{y_o}{U_1} \cdot \frac{h \cdot h'}{[-\frac{C_1}{C_s} \cdot h + 1]} \]

Let \( A_2 = \left(\frac{k_s}{2} - 1\right) \cdot \frac{C_1}{C_s} \) and \( A_3 = \mu \cdot SFR \cdot C_{mat} \cdot \frac{y_o}{U_1} \), then

\[ h''(-\frac{C_1}{C_s} \cdot h + 1) = -A_2 \cdot [h']^2 - A_3 \cdot h' \cdot h \]

Let \( A_4 = -\frac{C_1}{C_s} \). Then,

\[ h''(A_4 \cdot h + 1) + A_2 [h']^2 + A_3 \cdot h' \cdot h = 0 \]  

(G-23)

To solve the above equation numerically, two boundary equations are required.

At \( x = 0, \ h = 0 \). Also \( h' = \frac{C_s}{C_1} \cdot v(0) = \frac{C_s}{C_1} \cdot \sin \theta \). Then, solution for \( h(x) \) is numerically obtained. Alternatively similar to Appendix I, Eq. G-23 can be further simplified.

Let \( P = \frac{dh}{dx} = h' \). Then \( \frac{dP}{dx} = h'' = \frac{dP}{dh} \cdot \frac{dh}{dx} = P' \cdot P \).

Therefore, Eq. G-23 becomes

\[ P' \cdot P \cdot (A_4 \cdot h + 1) + A_2 \cdot P^2 + A_3 \cdot P \cdot h = 0 \]

\[ P' + \frac{A_2}{A_4 \cdot h + 1} P = -\frac{A_3}{A_4 \cdot h + 1} \]  

(G-24)

Eq. G-24 is the 1st-order differential equation.

Using the method of integration by parts, it becomes

\[ h' = P = \frac{-A_3}{A_2(A_2 + A_4)}(A_2 \cdot h - 1) + I_2 \cdot (A_4 \cdot h + 1) \frac{dh}{A_4} \]  

(G-25)

Where \( I \) is the integration coefficient found from the same boundary conditions.
\[ I_2 = \frac{C_s}{C_1} \cdot \sin \theta - \frac{A_3}{A_2 (A_2 + A_4)} \]

Eq. G-25 is solved for \( h(x) \) and generates the same numerical solutions given by Eq. G-22.

The resulting solutions are plotted in from Figures 6-31 to 6-34.

### G.2.3 Solution for Suspension Jet with Wire Tension \( T \)

The governing equations are

\[ P_1 = \frac{d}{dt} \left( \rho \cdot y(x) \cdot \left( v(x) + \frac{dy_w}{dx} \right) \right) \]  

\[ P_2 = \rho_0 \cdot g \cdot y(x) - \frac{k_s}{2} \cdot \rho \cdot v(x)^2 - \rho_0 \cdot \mu \cdot SFR \cdot h(x) \cdot C_{mat} \cdot v(x) \]  

And let \( s = \frac{dy_w}{dx} \) and \( C_l = C_{mat} \cdot C_s \).

\[ P_3 = \frac{T}{R} = T \cdot \frac{d^2 y_w}{dx^2} = T \cdot \frac{ds}{dx} \]  

Other governing equations are Eqs. 6-21, 6-22.

From Eq. 6-24a,

\[ P_1 = \frac{d}{dt} (\rho y v) + \frac{d}{dt} (\rho y s) = P_2 \]

Then,

\[ \frac{d}{dt} [\rho y v] = -\frac{k_s}{2} \cdot \rho v^2 - \rho \cdot \mu \cdot SFR \cdot C_m \cdot h \cdot v - \rho \cdot \frac{d}{dt} [y \cdot s] \]

\[ \frac{d}{dt} [y \cdot v] = -\frac{k_s}{2} \cdot v^2 - \mu \cdot SFR \cdot C_m \cdot h \cdot v - y \cdot \frac{ds}{dt} - s \cdot \frac{dy}{dt} \]

From Eq. 6-21,
\[ \frac{d}{dt}[y \cdot v] = -\frac{k_s}{2} \cdot v^2 - (\mu \cdot SFR \cdot C_m \cdot h - s)v - y \cdot \frac{ds}{dt} \]

Here \( s(\mu \cdot SFR \cdot C_m \cdot h) \), therefore

\[ \frac{d}{dt}[y \cdot v] = -\frac{k_s}{2} \cdot v^2 - \mu \cdot SFR \cdot C_m \cdot h \cdot v - y \cdot \frac{ds}{dt} \quad \text{(G-26)} \]

Also, from Eq. 6-20a and 6-23a, \( P_2 = -P_3 \). Then,

\[ \frac{ds}{dx} = \frac{1}{T} \cdot \left[ \frac{k_s}{2} \cdot \rho \cdot v^2 + \rho \cdot \mu \cdot SFR \cdot C_m \cdot h \cdot v \right] \quad \text{(G-27)} \]

Also, put Eq. G-27 into Eq. G-26 and non-dimensionalize equation similar to the method used in the section G.2.2, the resulting equation becomes

\[ \frac{dv}{dx} = \left[ -\frac{k_s}{2} \cdot \frac{v^2}{y} - \mu \cdot SFR \cdot C_{mat} \cdot \frac{y_0}{U_1} \cdot \frac{h}{y} \right] - \frac{\rho \cdot U_1^2}{T \cdot y_0} \cdot \frac{k_s}{2} \cdot \frac{v^2 + \mu \cdot SFR \cdot C_{mat} \cdot \frac{y_0}{U_1} \cdot v \cdot h}{y} \quad \text{(G-28)} \]

From Eq. G-21 and Eq. G-22,

\[ \frac{C_1}{C_s} \cdot h'' = -\frac{k_s}{2} \cdot \left[ \frac{C_1}{C_s} \right]^2 \cdot \frac{[h']^2}{(-\frac{C_1}{C_s} \cdot h + 1)} - \mu \cdot SFR \cdot C_m \cdot \frac{y_0}{U_1} \cdot \frac{C_1}{C_s} \cdot h' \cdot h \]

\[ -\frac{\rho \cdot U_1^2}{T \cdot y_0} \cdot \frac{k_s}{2} \cdot \left( \frac{C_1}{C_s} \right)^2 \cdot (h')^2 + \frac{\mu \cdot SFR \cdot C_m \cdot y_0}{U_1} \cdot h' \cdot h \]

Here, let \( A_2 = -\frac{k_s}{2} \cdot \frac{C_1}{C_s} \), \( A_3 = \mu \cdot SFR \cdot C_m \cdot \frac{y_0}{U_1} \), \( A_4 = -\frac{C_1}{C_s} \)

\[ B_1 = \frac{\rho \cdot U_1^2}{T \cdot y_0}, \quad B_2 = \mu \cdot SFR \cdot C_m \cdot \frac{y_0}{U_1} \]

Then Eq. G-29 becomes
\[ h''(A_4 \cdot h + 1) + [A_2 - B_1 \cdot B_3 \cdot A_4 \cdot (A_4 \cdot h + 1)] \cdot (h')^2 \]
\[ + [A_3 + B_1 \cdot B_2 \cdot (A_4 \cdot h + 1)] \cdot h' \cdot h = 0 \]  \hspace{1cm} (G-30)

Boundary conditions required are the same as the ones given in the section G.2.2. Finally, Eq. G-30 is numerically solved for \( h(x) \). The solution can be verified by further simplifying Eq. G-30 using the same method shown in the section G.2.2.
APPENDIX-H

CALIBRATION OF ORIFICE PLATE

Figure H-1  A simple sketch of head change in the open loop in Figure 3-1

With the change of water head from 1 to 2 in Figure H-1, the discharge rate continuously varies.

1) Continuity

\[ d(Vol) = (Q_i - Q_o)dt \]

where \( d(Vol) = A_z \cdot dZ \).

\[ A_z \cdot \frac{dZ}{dt} = Q_o \]  \hspace{1cm} \text{for } Q_i = 0

\[ dt = \frac{A_z}{Q_o} \cdot dZ \]

\[ t = \int_{z_2}^{z_1} \frac{A_z}{Q_o} \cdot dZ \]  \hspace{1cm} (H-1)

2) Energy Equation with head loss \( h_{e,l} \)
\[
\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + Z_2 + h_t
\]

where \( h_t = K_{\text{entrance}} \cdot \frac{v^2}{2g} + K_{\text{exit}} \cdot \frac{v^2}{2g} + K_{\text{screen}} \cdot \frac{v^2}{2g} + f \cdot \frac{L}{D} \cdot \frac{v^2}{2g} \)

Then, since \( P_1 = P_2 = v_1 = 0 \),

\[
Z = (K_{\text{entrance}} + K_{\text{exit}} + K_{\text{screen}} + f \cdot \frac{L}{D}) \cdot \frac{v^2}{2g}
\]

Therefore,

\[
v = \sqrt{\frac{2g \cdot Z}{K_{\text{entrance}} + K_{\text{exit}} + K_{\text{screen}} + f \cdot \frac{L}{D}}} = C_1 \cdot \sqrt{Z} \quad \text{(H-2)}
\]

\[
\therefore v = f(Z)
\]

By putting Eq. H-2 into Eq. H-1,

\[
t = \int_{Z_2}^{Z_1} A \cdot v \cdot dZ = \int_{Z_2}^{Z_1} A \cdot \frac{dZ}{C_1 \cdot \sqrt{Z}} = \frac{A}{C_1} \cdot [\frac{1}{\frac{1}{2} + 1}]_{Z_2}^{Z_1} = \frac{2A}{C_1} \cdot (\sqrt{Z_1} - \sqrt{Z_2})
\]

\[
\therefore Z_1, Z_2, t \text{ are measured. Therefore, } C_t \text{ is calculated. Also the velocity is calculated. For a constant head, the velocity remains constant. The resulting pressure drop in the orifice plate is calibrated against the known velocity.}
\]
APPENDIX-I

VALIDATION OF NUMERICAL SOLUTIONS COMPUTED USING MATHEMATICA

To validate the numerical solutions computed using Mathematica, one simplified governing equation in Appendix G is chosen and solved analytically. Then, the numerical solution obtained by Mathematica is compared with the analytical solution for the same differential equation.

In Appendix G.2.1, the numerical solution was obtained for the simplified governing equation in Eq. G-15 in the case of water jet in the free-surface wedge zone.

\[ y \cdot y'' = [y']^2 \cdot \left( \frac{k}{2} - 1 \right) \]  \hspace{1cm} (G-15)

Eq. G-15 is a non-linear differential equation. It has the similar form compared to the examples given in the handbook by Fogiel (1994). Using the similar solution technique, first let \( P = \frac{dy}{dx} = y'(x) \), then \( \frac{dP}{dx} = y''(x) \). Also by chain rule, \( \frac{dP}{dx} = \frac{dP}{dy} \cdot \frac{dy}{dx} = P' \cdot P \).

Then, Eq. G-15 becomes

\[ y \cdot P' \cdot P = P^2 \cdot \left( \frac{k}{2} - 1 \right) \]  \hspace{1cm} (I-1)

\[ P' = \frac{P \cdot \left( \frac{k}{2} - 1 \right)}{y} \]
Eq. I-1 is a first-order differential equation.

\[
\frac{dP}{dy} = \frac{P}{y} \left( \frac{k_x}{2} - 1 \right)
\]

\[
\frac{dP}{P} = \frac{dy}{y} \left( \frac{k_x}{2} - 1 \right)
\]

\[
y' = P = C_1 \cdot y^{\left(\frac{k_x}{2} - 1\right)}
\]  \hspace{1cm} (I-2)

Let \( a = k_x/2 - 1 \) and assume \( k_x > 2 \).

\[
\frac{dy}{dx} = C_1 \cdot y^a
\]

\[
y^{-a} \cdot dy = C_1 \cdot dx
\]

Integrating the above equation,

\[
\frac{1}{-a + 1} \cdot y^{-a + 1} = C_1 \cdot x + C_2
\]  \hspace{1cm} (I-3)

From two boundary conditions, i) \( y(0) = 1 \) and ii) \( y'(0) = -\sin \theta \), the two coefficients are found from Eqs. I-2 and I3.

\[
C_1 = -\sin \theta, \quad C_2 = \frac{1}{-a + 1}
\]

Therefore, Eq. I-4 is the required solution.

\[
y = \left[ (-a + 1) \cdot (C_1 \cdot x + C_2) \right]^{-\frac{1}{a-1}}
\]  \hspace{1cm} (I-4)

Plot of the analytical solution given in Eq. I-4 is compared to the numerical solution computed by Mathematica in Figure I-1. Table I-1 also shows the values for comparison. Both results are almost identical up to the fifth decimal point.
Table I-1 Comparison of numerical solution to analytical one

<table>
<thead>
<tr>
<th>$x/y_0$</th>
<th>Analytical Solution in Eq. I-4 for the governing Eq. G-15</th>
<th>Numerical Solution By Mathematica for Eq. G-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
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<td>0.923619</td>
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<td>0.888964</td>
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<td>3</td>
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<td>0.866645</td>
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<td>4</td>
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<td>5</td>
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<td>0.837379</td>
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<tr>
<td>6</td>
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<td>0.826779</td>
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<tr>
<td>7</td>
<td>0.817798</td>
<td>0.817795</td>
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<tr>
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<td>0.81001</td>
</tr>
<tr>
<td>9</td>
<td>0.803153</td>
<td>0.803149</td>
</tr>
<tr>
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<td>0.797022</td>
</tr>
<tr>
<td>11</td>
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<td>0.79149</td>
</tr>
<tr>
<td>12</td>
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<td>0.786452</td>
</tr>
<tr>
<td>13</td>
<td>0.781832</td>
<td>0.781828</td>
</tr>
<tr>
<td>14</td>
<td>0.777562</td>
<td>0.777559</td>
</tr>
<tr>
<td>15</td>
<td>0.773597</td>
<td>0.773594</td>
</tr>
</tbody>
</table>

For water jet in free-surface wedge zone with $k_s = 30$.

Figure I-1 Comparison of numerical solution of the non-linear differential equation in Eq. G-15 to the analytical solution given in Eq. I-4: The figure shows that both results produced almost identical plots for water jet in the free-surface wedge zone. $k_s$ of 30 was used.
REFERENCES FOR APPENDICES


