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UMI
LATERAL MODE FREQUENCY LOCKING IN SEMICONDUCTOR LASERS

by

Reuven Gordon

A thesis submitted in conformity with the requirements for the degree of Master’s of Applied Science
Graduate Department of Electrical and Computer Engineering
University of Toronto

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Abstract

LATERAL MODE FREQUENCY LOCKING IN SEMICONDUCTOR LASERS
Reuven Gordon
Master of Applied Science
Department of Electrical and Computer Engineering
University of Toronto
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Frequency locking between the lateral modes of semiconductor lasers is investigated. This frequency locking leads to the phenomenon of beam-steering which is detrimental to fiber-pump telecommunication applications but potentially useful in the context of optical switching and free-space optical interconnects.

The experimental component of this thesis consists of a) making the first direct observation of higher order modes in these lasers, b) explaining the mechanism behind hysteresis in the far-field intensity shifts and c) analyzing radio-frequency modulations in the output intensity to extract all the relevant information about the interaction between the lateral modes.

Theoretical models were developed to explain the beam-steering phenomenon for linear coupling and nonlinear coupling associated with carrier population modulations. The work on linear coupling between the modes indicates that small asymmetries lead to significant regions of beam-steering. The dynamics associated with carrier modulation induced nonlinear coupling is sensitive to the carrier diffusion length. Regions of competitive behavior, frequency locking and chaos were identified with the nonlinear coupling model.
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Chapter 1

Introduction

1.1 Fiber-Pump Semiconductor Lasers and Beam-Steering

Erbium-doped fiber (EDF) amplifiers are currently used in the majority of long-haul telecommunication networks around the world [1] because they provide gain in a low-loss window of fiber-optic cable (around the wavelength of 1.55 μm). They can also be employed in fiber-laser systems. In both cases, the EDF’s energy state transition at 980 nm can be used to provide gain for longer wavelengths. For this reason, we require highly efficient sources that can reliably provide a stable, high-intensity pump beam at 980 nm. InGaAs-GaAs-AlGaAs or even aluminum-free GaAs-based semiconductor lasers are widely used for this purpose.

These fiber-pump lasers are generally designed with a ridge-waveguide structure that will ideally admit only the fundamental lateral mode (defined by the semiconductor layer structure in the vertical direction and by the ridge perturbation in the lateral direction). These lasers are required to produce high output powers, so it is favorable to have a wide ridge. This allows for a large modal gain, with a reduced chance of optically induced material damage due to large field intensities being confined to a narrow ridge. The larger ridge size also allows for reduced current densities. This reduces the carrier induced changes to the waveguiding structure.

Since the ridge is weakly index-guiding, higher-order modes can ‘creep’ in and even lase at high injection currents when temperature- and carrier-dependent refractive index changes become significant relative to the overall guiding properties. Spatial hole-burning in the carrier distribution becomes pronounced at high output powers. This is due to the stimulated emission from the fundamental mode. Consequently, the first-order mode could eventually see a large enough modal gain to lase [2].
Fig. 1.1. Observations of far-field intensity distribution at the onset of beam-steering.
The simultaneous lasing of multiple lateral modes has been associated with beam instabilities in weakly index-guided fiber-pump lasers. In this work, the question of how these instabilities come from interactions between the fundamental and first-order lateral modes will be addressed. Because these beam instabilities cause the far-field intensity distribution peak to shift $3 - 6^\circ$ in the lateral direction, the phenomenon is commonly referred to as 'beam-steering' and shown in Figure 1.1.\(^1\)

Beam-steering results in a sudden reduction in laser-fiber coupling of packaged fiber-pump lasers that are aligned to the fiber before the onset of beam instabilities. Even coupling between a broad-area photo-detector and the laser may be dramatically reduced. Consequently, beam-steering has been associated with kinks in the intensity-current curves obtained from these lasers [3].

1.2 Past Work on Beam Instabilities

Historically, fiber-pump semiconductor lasers have suffered from two major reliability problems. The first problem came from material damage due to operation at high optical powers. Improved fabrication techniques and refined designs have produced Al-free and AlGaAs lasers with reliable operation at large output powers over extended periods of time. However, the second problem of the beam instabilities remains. These instabilities still arise in weakly index-guided structures even at low injection currents.

In the early 1980's, numerical modeling of lateral mode behavior in semiconductor lasers suggested that beam instabilities might be caused by a lateral drifting of intensity profile of each of the modes [4]. However, this work assumed the intrinsic presence of large index asymmetries across the waveguide. This assumption is inconsistent with experimental evidence. The phase interference between the optical modes, which leads to crucial modal interaction effects, was also neglected.

In 1994, it was first speculated that the interference between the fundamental and first-order lateral waveguide modes in weakly index-guided structures [5], [3] leads

---

\(^1\)The term beam-steering is commonly associated with setting up phase fronts in antenna steering applications. For the case of beam steering from a single ridge laser, the propagation front of the two modes is not altered so much as the intensity maximum is shifted in the lateral direction.
to asymmetric far-field intensity distributions. This hypothesis was supported by several indirect observations.

In Reference [5], the back-plane electroluminescence emission showed stationary spatial oscillations along the axis of the waveguide. These were consistent with what would be expected from the interference between the two lowest order lateral modes. Spatial beating in the intensity of the two lateral modes produces a periodic spatial hole-burning pattern. This results in a ‘tell-tale’ lowered level of spontaneous emission that can be viewed from a window in the substrate contact. A schematic diagram of the optical interference patterns between the two lowest order lateral modes along the length of the cavity are shown in Figure 1.2 for realistic modal parameters.

The same conclusion was later drawn in Reference [3] by observing the periodicity in the maximum kink-free power (i.e. maximum power before beam steering) with changes in the cavity length. The periodicity in both cases follows the relation:

\[ L = \frac{2\pi}{k_0 - k_1} \]  

where \( L \) is the beat period length and \( k_m \) is the propagation constant for mode \( m \). Self-consistent numerical solutions have also confirmed the onset of higher order modes in these laser structures which agree well with Equation 1.1 [2].

Reference [3] noted changes in the beam-steering direction with variations in the cavity length. As the cavity is extended, additional half beat periods are added to the stable interference pattern between the lateral modes (see Figure 1.2). This creates an optical field distribution at the facets that alternates between being concentrated at one side of the ridge to the other. The resulting far-field distribution follows this variation by exhibiting beam-steering in alternating directions. Consequently, this observation is consistent with the modal interference hypothesis of [5].

In [5], the following question was raised: how can a stable far-field interference pattern emerge from two lateral modes of (generally) different frequencies? It was hypothesized that electro-optical nonlinearities could couple the modes and produce frequency locking between them. Frequency locking corresponds to having the relative
Fig. 1.2. Optical intensity pattern along laser cavity for fundamental and first-order modes of the same power.
phase between the lateral modes fixed at a constant value (i.e. they operate at the same frequency). However, no further investigation was attempted in [5].

It was later demonstrated that the lateral modes move (in the frequency spectrum) relative to one another [6]. This allows the cavity resonances of the two lateral modes to be separated by a wide range of frequencies. In [6], it was noted (indirectly) that differential changes in the effective indices of the two lateral modes could result from free-carrier absorption induced temperature variations. However, it is unlikely that the lateral modes remain at the same frequency over a range of operating conditions without some coupling between them.

It was shown by the author of [7] that facet-tilt coupling can lead to frequency locking between the two lateral modes. This approach is an example of how isolated geometrical asymmetries can create frequency-locked supermodes which consist of the two lateral modes. Because the facet-tilt geometry has two isolated coupling regions, a steady-state supermode approach was used to simplify the analysis. The formalism of this approach is presented for the first time in this thesis.

Previous works have addressed the possibility of frequency locking resulting from the interaction of the optical modes with the gain medium [8], [9]. In [8], the interaction of a gas laser's modes with the atomic polarization and population inversion was analyzed for its ability to produce stable asymmetric far-field intensities. In [8], it is suggested that Turing instabilities lead to stable structure formation of the field profile. This is not strictly true since only a single diffusion/diffraction term is present in the gas laser system, whereas Turing instabilities arise from two different diffusion terms operating on different time scales [10].

In semiconductor lasers, the optical diffraction and carrier diffusion operate on highly disparate time scales, and so spatially stable Turing structures are likely to occur. Lateral-mode frequency-locking in semiconductor lasers may be considered as a rare example of structure formation by means of Turing instabilities, which were not experimentally demonstrated until the late 1980's (more than thirty years after their theoretical discovery).
In Reference [9], intensity-beating interactions between the modes and the carriers are treated using a first-order perturbation approach. In addition, the carrier modulations were treated in a response function manner. This work was the first attempt at explaining frequency-locking between lateral modes in a ridge-waveguide structure. Some interesting phenomena, such as hysteresis in the beam-steering direction, can be explained with this model and are elaborated upon in this thesis.

1.3 Shortcomings of Past Work

The first work [4] on beam-instabilities omits the optical field interference between the lateral modes. This oversight results in an improper description of the lateral mode dynamics. However, the inclusion of asymmetries (introduced by fabrication imperfections) mentioned in [4] is interesting because they produce coupling between the lateral modes. Specifically, this coupling will appear at the facets, were the particular asymmetry is inverted by the beam's reflection. However, such coupling has not yet been adequately treated for regenerative lasing modes.

While the observations of back-plane emission patterns [5] and changes in the kink-power with laser cavity lengths [3] both suggest that lateral modes are interfering to produce beam-steering, direct observations of these modes in beam-steering lasers have not been reported. This leaves some uncertainty as to whether the interference between lateral modes is causing the observed beam-instabilities. In particular, how do we know that the interference pattern comes from two guided lateral modes? A single lateral mode may be deformed in a similar manner due to nonlinear interactions with the gain medium and/or coupling to free-space modes (that are perturbed by the ridge).

The past works are lacking in physically complete descriptions of the carrier-field interactions. They neglect the important features of carrier diffusion which govern the spatio-temporal structure formation processes. The fact that the lateral modes overlap with spatially distinct sets of carriers is also neglected by [9]. This is a serious oversight because it assumes that the spatially different optical modes are entirely dependent on a single set of carriers. As is shown in this thesis, such an assumption leads to unrealistic competitive behavior.
In addition, the approach of [9] does not capture the relevant dynamics because it uses a response function method to describe the effects of carrier modulation. Such an approach is relevant to phase-locking dynamics where the frequency difference between the interacting optical modes does not vary substantially. The response function approach is not adequate to describe the wide range of frequencies that exist during frequency-locking.

While the aforementioned improvements are required to model and understand the coupling between lateral modes, more innovative experimental techniques are also needed to observe the dynamics of these modes. These experimental techniques should be able to provide independent information about the type of coupling (e.g. linear, second-order nonlinear etc.), strength of coupling, strength of each oscillating mode and frequency difference between the modes.

So far, the measurements of back-plane emission, kink-power variations (with changes in the laser cavity length or duty-cycle of the injection current) and far-field profiles provide only static descriptions of the lateral-mode interaction, which is clearly inadequate.

1.4 Approach Taken

This work attempts to provide insight into beam-steering which will enable future works to control or eradicate the phenomenon. The major deficiency in our current understanding of these beam-instabilities can be summed up by the following question: how do the lateral modes in a semiconductor laser frequency lock?

In order to answer this question, the relevant coupling mechanisms between the lateral modes must be explored. Two coupling mechanisms are particularly relevant to semiconductor laser structures: linear coupling due to structural asymmetries in the laser and nonlinear carrier interactions. While former is introduced by imperfections in the laser structure, the latter is intrinsic to all semiconductor lasers.

Any lateral asymmetry in the laser structure will introduce linear coupling between the two lowest order lateral modes. For example, if the ridge guide is not perfectly aligned with the facets, or the wafer experiences some lateral strain, light will scatter
back-and-forth between the modes. The magnitude of this type of coupling may be reduced by increasing the precision of manufacturing processes. A crucial question arises: how precise must the fabrication processes be in order to ensure that frequency locking will not persist over any significant range of operation? This question motivates a linear steady-state model. The linear model isolates the effects of linear coupling from other possible interactions. The steady-state approach simplifies the analysis while retaining the clarity of the results.

Models attempting to capture the dynamics of optical-carrier interactions have suffered from several omissions in the past. This work includes the crucial elements of carrier diffusion and spatial distribution of the carriers which are essential to the laser dynamics. The dynamical model used here is most easily explored by numerical methods, but care is taken to ensure that a physical understanding of the governing processes is retained.

In order to fortify the link between beam-steering and lateral mode interactions, direct observations of the lateral modes in beam-steering lasers are required. These observations are clearly lacking from the past work on beam-steering semiconductor lasers. A high-resolution optical spectrum is the most definitive measurement of any laser's optical modes. Consequently, optical spectra measurements were made to observe directly the higher order modes.

The models developed in this thesis are also capable of explaining other curious features of beam-steering lasers. For example, although hysteresis in the beam-steering direction with changes in the injection current was observed more than 5 years ago, they have eluded explanation until the carrier-optical interaction model was developed. This work seeks to clearly explain curious features of beam-steering laser behavior by utilizing the newly developed model.

Experimental methods to obtain information about the interaction dynamics between the modes were also developed. These dynamics cannot be evaluated when the modes are frequency locked, because there is no dynamic variation in the light that is output. Therefore, it is favorable to look at the modes in the regions just outside of where they are frequency locked to observe how they interact with one another. The
most convenient dynamic measurement of a laser's dynamics is to detect the changes in the output-power.

In this work, we develop and demonstrate a theory that will allow for extraction of all relevant coupling parameters by looking at the modulation patterns in the output intensity. The strength of this theory is that it can provide information independently of all other observations. Although this theory is applied to the modulation dynamics of beam-steering lasers, it can be applied to a variety of other coupled oscillator systems.

1.5 Summary of New Results

The major novel contributions to the beam-steering problem from this work are:

1. Linear coupling model is developed for the practically relevant facet coupling geometry. The regenerative matrix approach can be extended to account for other coupling geometries. This model gives the range of frequency locking as a function of the amount of asymmetry that is introduced into the structure.

2. A nonlinear coupling model is developed which includes the relevant carrier dynamics for the first time. The result is a clear demonstration of the role the carriers play in determining the dynamics of the system. Three different regimes of interaction are discovered. These regimes are related to the carrier diffusion length and geometry of the system.

3. High resolution spectra confirm the emergence of higher order modes just before the onset of beam-steering. These higher order modes are not seen in lasers that do not exhibit beam-steering. This supports the claim that higher order lateral modes interfere to produce beam-steering.

4. Hysteresis and bistabilities are explained in terms of the optical-carrier interaction model.

5. A novel output intensity modulation method is developed to derive the frequency locking bandwidth, nature of the coupling, and intrinsic frequency difference between two lateral modes.
6. Radio-frequency (rf) spectra of the output intensity modulation is used to obtain information about the coupling between the lateral modes. Spectra consistent with chaotic regimes predicted by the nonlinear coupling model are also presented.

1.6 Applications to Other Systems

The results obtained in this work may also be of value to a broad range of phenomena which arise from the coupling between lateral modes in other laser systems [11]. Some of the results may be easily extended to spatially separated coupled lasers and external cavity laser systems (e.g. the rf intensity modulation pattern formalism of Chapter 5), phase-locking between evanescently overlapping beams in solid-state lasers and diode arrays (e.g. carrier beating model of Chapter 3), and filamentation in high-power broad-area lasers [11].

Understanding the frequency locking mechanisms underlying these beam instabilities may also prove useful in controlled steering applications, such as free space optical switching. Although, the ridge waveguide laser structure which inspired this work can only have a small shift in the lateral intensity maximum, laser arrays can give enhanced beam directioning. This is achieved by frequency locking the lateral modes with a particular phase relationship. The modeling of this work (particularly that of Chapter 3) provides insight into the interaction between the lateral modes and how to control their phase relationship.

1.7 Organization of Thesis

The aim of this work is to present an in-depth investigation into the origins of the beam-steering phenomenon. New models are presented which uncover the relevant modal interactions. These models are subsequently used to elucidate the experimental findings. In order to focus on a physical understanding of the results, mathematical derivations are moved to the appendices, while the final results and an outline of the methods used are given in the main chapters.
In Chapter 2, the facet-coupling model is presented as a pedagogical example of how linear coupling in a non-trivial coupling geometry can be treated with the regenerative matrix approach. This method searches for the steady-state supermode of the coupled mode system. The aim of this chapter is to derive the frequency locking bandwidth for a practically relevant linear coupling geometry.

The laser rate-equation model, which includes the relevant carrier dynamics, is treated next in Chapter 3. The aim of this chapter is to express the importance of including the diffusion, spatial localization and response dynamics of the carriers into the rate equations. A new understanding of the physical mechanisms behind frequency locking is obtained.

In Chapter 4, straightforward observations of beam-steering lasers are presented. Despite being fairly common, the results of this chapter supplement the current understanding by showing:

1. the emergence of higher order lateral modes before the onset of beam-steering, and
2. hysteresis and bistability in the beam-steering direction.

Understanding the features of hysteresis and bistability is aided by the theoretical foundation of Chapter 3.

In Chapter 5, the rf power spectrum analysis is presented. The observations of rf intensity modulation patterns in the lasers output power are presented. They are then explained in the context of the frequency locking equations (those derivable from Chapter 3 and even more general ones). One of the objectives of this chapter is to develop a new tool by which a variety of coupled oscillator systems may be analyzed.

In Chapter 6, a summary of the new results is presented. The relevance of these results to the beam-steering problem and coupled oscillator systems in general is highlighted. Several future research topics that follow naturally from this one are identified.
Chapter 2

Theory: Linear Coupling

2.1 Introduction

Previous works have suggested that beam-steering may originate from the interference between two frequency locked lateral modes semiconductor lasers. In order to control beam-steering, we need to achieve a better understanding of how these modes interact and ultimately come to operate at a single frequency.

In this chapter, we begin by looking at the possible coupling mechanisms that exist between the lateral modes. These are first evaluated qualitatively to determine which forms of coupling are dominant in the semiconductor laser system. Linear coupling resulting from optical scattering within the laser cavity is then discussed in more detail to establish which coupling geometries are expected to be most significant. Finally, a steady-state model is introduced to analyze the effects of introducing coupling at the facets.

The aim of this chapter is to identify the minimal amount of intermodal coupling that is required to produce frequency locking for a practically relevant and non-trivial coupling geometry. It will be shown that relatively minor perturbations to the ideal symmetric structure will allow for frequency locking.

2.2 Possible Coupling Mechanisms

Many coupling mechanisms could lead to frequency locking between the lateral modes of a semiconductor laser. These include intrinsic optical nonlinearities (including intra-band effects), carrier-sharing effects (coming from carrier induced gain and refractive index changes), local heating, random defect scattering and systematic geometric scattering. The significance of each remains to be evaluated.
While intrinsic optical nonlinearities are fast (~100 fs), they only provide minor perturbations when compared with inter-band carrier effects. Inter-band processes are not as fast (operating on the picosecond time scale), but are still relevant to lateral mode frequency locking where the interaction dynamics operates on 100 ps time scales. In the context of lateral mode coupling, inter-band processes are grouped under carrier-sharing effects and will be dealt with in the next chapter.

Local heating effects may be quite large, but suffer from slow relaxation times. Consequently, only for extremely small regions of frequency separation will these effects become significant.

Random defect and geometric scattering are due to perturbations to the ideal laser’s waveguide structure. These scattering locations can introduce a linear coupling between the lateral modes. However, they are extrinsic in the sense that improvements to fabrication processes can minimize the amount of scattering within the laser structure. While random scatterers will generally not give phase-coherent coupling between the modes, systematic structural asymmetries might provide enough coupling to enable frequency locking.

It is of practical interest to study the linear coupling introduced by structural asymmetries for several reasons. Firstly, only a portion of the lasers tested display beam instabilities at low injection currents. This is the case even though all the lasers tested nominally have the same wafer and structure. Therefore, they should display the same material and structural properties. Lasers produced close to one another on a particular wafer possess very similar beam-steering characteristics. This is indicative of the laser structure contributing to the beam-steering behaviour. Structural features which may be introduced by wafer strain, anisotropic etching of the ridge or etch mask-wafer misalignment, could cause the inter-modal coupling leading to beam-steering in some lasers but remain absent in others.

In addition, the steering direction (which depends on the relative phase between the frequency locked modes) is fairly deterministic upon the operating conditions in

---

1 For typical semiconductor laser geometries, the frequency separation between two lateral modes can be in the range from 0-25 GHz.
the lasers studied. Intensity dependent nonlinear coupling allows for two equally likely relative phase values (each corresponding to beam steering in a different direction since they are of opposite phase) while linear coupling allows for only a single relative phase between the modes. Consequently, if linear coupling is involved in the dynamics of these lasers, it would provide some preferred steering direction.

Finally, innovative design measures which seek to remove beam-steering should be wary of the amount of inter-modal coupling they introduce. For example, the introduction of facet-tilts to reduce the higher order mode reflectivity [12] (intended to increase their lasing threshold) may actually increase the inter-modal coupling and encourage beam instabilities.

2.3 Linear Coupling Between Lateral Modes

There are several discontinuities and asymmetries in semiconductor lasers which lead to inter-modal coupling. Among them are: (a) variations in the wafer's refractive index [4] due to strain or inhomogeneous growth layer thickness, (b) normal facet inter-modal coupling [13], (c) anisotropic etching of the guiding structure and (d) tilted facet coupling [14]. Of particular interest to coupling between the fundamental and first order lateral modes are (a), (c) and (d). These variations from the ideal structure break the lateral waveguiding symmetry and consequently provide coupling between modes of even and odd symmetry (such as the fundamental and first order modes).

If a lateral asymmetry exists along the length of the waveguide, then coupling between the first order and fundamental lateral modes will occur at the facets. This is because the propagation direction is flipped at the facets which results in an inversion of the lateral symmetry. In turn, the symmetry of the mode shapes will also be inverted and if the waveguide possesses odd lateral symmetry, the odd and even lateral waveguide modes will be coupled to one another. Misalignment of the waveguide ridge with the facet will also result in coupling at the facet. From this consideration, facet coupling is probably the most relevant form of coupling between the lateral waveguide modes.
2.4 Facet Coupling

In Reference [14], an approximate evaluation of the tilted facet inter-modal coupling is presented along with the resulting changes to the cavity eigen-modes from a single round trip analysis. The approach of [14] is not appropriate to discuss intermodal coupling of the oscillating modes because it only accounts for a single round-trip resonance. For weak coupling between the oscillating modes, a coupled oscillator approach should be used to include the regenerative nature of the problem.

Since the two modes share the same waveguide, the widely used rate-equation formalism for coupled lasers (as in [15]) is not easily adapted to this geometry. With such analyses, it is hard to distinguish whether the locking occurs due to carrier sharing effects (which will be dealt with in the next chapter) or tilted facet coupling. For this reason, a geometric analysis of the coupled oscillator problem similar to the one presented in [16] is used.

The geometric approach searches for a regenerative steady state of the coupled system by accounting for field propagation and reflection within the cavity. The lasing lateral modes are assumed to be operating close to their intrinsic cycles. In [16], the geometric approach was used for two coupled semiconductor lasers to get similar frequency locking bandwidth and phase relationship results as those derived by Adler [17] for the less complicated locking of an oscillator to an external signal. Here coupling at both facets between two lateral modes of a single ridge waveguide laser is considered.

2.5 Outline of Approach Taken

The facet coupling considered here consists of a waveguiding structure supporting two lateral modes with gain and inter-modal coupling as well as reflection at both facets. A schematic diagram of the coupling geometry is shown in Figure 2.1. The problem is complicated by the fact that there are two coupling paths (due to coupling at both

\textsuperscript{2}In the context of coupled waveguide modes, intrinsic refers to the state the modes would be in if the coupling was not present.
facets) between the two oscillating modes, so a steady state matrix approach is applied to obtain the region of frequency locking and phase relationship.

The geometrical analysis is performed for weak facet coupling between lasing modes. It is assumed that both modes are operating close to their intrinsic regenerative lasing states. Self-consistent simulations have shown that at injection levels where steering occurs both lateral modes can lase (due to spatial hole burning creating a better modal overlap for the first order mode with the gain profile) [5], which supports this assumption.

The coupling (normalized to the reflectivity of the facet) is assumed to be the same at each facet. This is done to simplify the analysis, but other cases can be handled with the same approach. A facet-tilt can intentionally or unintentionally be created by misalignment of the ridge with the facet plane. In such cases, the tilt angle (and therefore coupling for an identical facet structure) is the same at each facet. Asymmetries to the waveguiding structure which introduce lateral mode coupling at the facets can be treated with this approach as well.

2.6 Results of Analysis

The mathematical details of the matrix approach are given in Appendix A. The final result of this analysis for the structure outlined above is given by an expression for the required coupling at each facet, $\varepsilon$:

$$|\varepsilon|^2 \geq 2 \left( \frac{\pi L n}{c} \right)^2 \frac{\Delta \nu_0 \Delta \nu_1}{1 + (-1)^{N_0+N_1}}$$

(2.1)

where $L$ is the cavity length, $c/n$ is the speed of light in the medium and $\Delta \nu_m = \nu_m - \nu_{\text{locked}}$ is the frequency detuning from resonance of mode $m$, which is directly related to $\phi_m$ - the phase accrued in a single pass of the cavity modulus $2\pi$.

It is clear from Equation A.5 that the required coupling between the modes to allow for frequency locking increases as the frequency detuning between the modes increases. The denominator of Equation A.5 expresses restrictions of the symmetry of the
Fig. 2.1. Schematic of facet-tilt coupling geometry.
system on coupling between different longitudinal modes. This is feature of the analysis arises because the coupling at each facet is assumed to be equal.

In Figure 2.2, theoretical facet-tilt induced locking bandwidths (i.e. $2\sqrt{\Delta \nu_1 \Delta \nu_2}$) are shown as a function of the facet-tilt angle for a 980 nm high power laser with a 3 micron ridge. The tilt coupling from mode $m$ to mode $n$ is adapted from the approximate expression derived in [14]:

$$\epsilon_{mn} \approx i r_m b \sin(\theta) \left( 1 \pm \frac{\pi}{b^2} \right) \exp \left( - (b \sin(\theta))^2 / 2 \right)$$  \hspace{1cm} (2.2)

where $b = w/k_n$, $w$ is the width of the waveguide, $k$ is the propagation constant in a vacuum, $n$ is the effective index of the waveguide and $\theta$ is the tilt angle of the facet. In this case, $r_m$, the reflectivity of the facet at angle $\theta$ is normalized to the reflectivity for when there is no tilt. Although this expression was derived for the case of a single interface facet, it can be generalized to coated facets by calculating the facet reflectivity for the coating(s) to find the first order Taylor expansion reflectivity coefficient (which is the only significant parameter in the small angle derivation of Reference [14]).

It is apparent from Figure 2.2 that even a small angle tilt from the normal ($\sim 0.1^\circ$) will result in a large locking bandwidth of approximately 1 GHz for this particular laser structure. The spacing between longitudinal modes is $\sim 50$ GHz in a 700 $\mu$m laser. It follows that the maximum spacing between two sets of longitudinal mode combs (associated with the fundamental and first order lateral modes) is 25 GHz. Clearly, linear coupling introduced by small geometrical asymmetries can produce a substantial (when compared with the maximum frequency spacing between the modes) frequency locking bandwidth.

This coupling mechanism should be considered when introducing tilts to the facet to reduce higher order mode reflectivity in attempts to remove beam steering [12] and may be important to laser structures in general.
2.7 Summary

A steady-state geometric matrix approach was used to find the frequency locking bandwidth for the case of lateral mode coupling at both facets. Large locking bandwidths for lateral mode frequency locking can result from relatively small facet-tilts. Other similar geometrical asymmetries are expected to produce the same results. These asymmetries will be present with varying significance in different laser structures. In particular, lasers which have larger ridge misalignment, lateral etch anisotropy, or lateral strain will be subject to larger coupling.
Fig. 2.2. Facet-tilt induced coupling bandwidth.
Chapter 3

Theory: Carrier Induced Coupling

3.1 Introduction

In the previous chapter, linear coupling associated with built in structural asymmetry was analyzed for its ability to produce frequency locking between the lateral modes of a semiconductor laser. The strength of coupling was dependent upon the amount of asymmetry within the laser structure. In this chapter, we consider another coupling mechanism which is prominent in semiconductor lasers: interactions between the optical field intensity and the excited carriers. These interactions form the basis of an intrinsic nonlinear coupling between the lateral modes.

A rate equation model will be developed which describes the lateral mode - carrier interactions with a set of coupled differential equations. It includes critical features of the carrier dynamics which have previously been neglected. The different ways in which the lateral modes interact will be shown for the first time by numerical integration of the rate equations.

3.2 Previous Work

A large body of work exists on carrier-induced nonlinearities and inter-modal coupling in semiconductor lasers. Longitudinal mode phase locking has been attributed to this type of interaction [18]. To model this interaction, a response function formalism is used to account for the carrier perturbations. Alternatively, the stability of the phase-locked state can be treated with a first-order perturbation method.

Instabilities in wide-area semiconductor lasers can also result from optical-carrier interactions. These instabilities have traditionally been treated with first-order perturbation techniques [11] or intensive numerical calculations [19].
Lateral mode frequency locking resulting from optically induced carrier perturbations for a ridge-waveguide geometry was first studied in Reference [9]. In this paper, we analyzed how frequency locking could result from self-induced carrier modulation (i.e. gain and refractive index) coupling between the lateral modes.

Reference [9] uses the perturbative methods developed for phase locking between longitudinal modes. As a result, the locking bandwidth between the modes was calculated as a function of the relevant laser parameters.

3.3 Problems with Previous Work on Lateral Mode Frequency Locking

In [9] some the vital features of the system were overlooked. In particular, a frequency response function was chosen to model the response of the carriers. This is the "textbook" approach [18] for phase locking between longitudinal modes of semiconductor lasers where the optical frequency difference between the modes varies minimally. The approach is not valid for the case of frequency locking where the frequency difference between the modes is chirped over a range of values from several Gigahertz to zero.

In addition, the approach of [9], being mainly perturbative, only considered a single background carrier distribution. Realistically, one must consider that the two modes are spatially distinct in the lateral direction. These modes overlap with different regions of the carrier population.

Finally, the effects of carrier diffusion have not been considered in previous attempts at describing this system. It is well known that in other dynamical systems diffusion is crucial to the formation of stable structures. In [9], the only reference to carrier diffusion is in its ability to wash out the carrier perturbations. This reduces inter-modal coupling and the chance of stable frequency locking.

3.4 Approach of this Work

A set of coupled rate equations that describe the dynamics of lateral mode frequency locking in a semiconductor laser will be derived in this Chapter. As in previous works, it will be shown that intensity interference patterns between the optical modes can
induce a periodic carrier perturbation. This perturbation acts like a grating to couple the two lateral modes.

As will be shown in what follows, the spatial distribution of the carriers must be included to prevent one mode from destroying the other (artificially). To include this feature, two separate carrier baths are introduced into our model. Each of these carrier baths represents carriers which are predominantly coupled to one of the lateral modes.

A set of carrier rate equations are used to model the carrier dynamics instead of using the response function method. This allows for the description of the relevant dynamics over a wide range of frequencies. The dynamics of the grating-like carrier perturbation and the two carrier baths (which account for the spatial localization of the optical modes) are represented with three rate equations.

Carrier diffusion is included into the model by introducing the relevant diffusive coupling and losses to all of the carrier baths. Our simulations will show that carrier diffusion is not only responsible for washing out the grating-like carrier perturbation (as previously speculated), but also serves a vital role in stabilizing the frequency locked modes.

The approach taken here only considers 2 longitudinal optical modes (each belonging to one of the lateral modes). The coupling to other longitudinal modes is very much reduced because the relevant interaction frequencies are near the carrier modulation bandedge (~ 50 GHz). It is postulated that phase locking between the longitudinal modes might result if additional modes were to be included.

3.5 Coupled Rate Equations

As is usually the case (e.g. [18]), we start with the Helmholtz wave equation and a widely used phenomenological carrier rate equation:

$$\nabla^2 E - \mu \varepsilon(N) \frac{\partial^2 E}{\partial t^2} = 0$$  \hspace{1cm} (3.1)

$$\frac{\partial N}{\partial t} = \frac{P}{\tau} - \frac{N}{\tau} + D \nabla^2 N - g(N) |E|^2$$  \hspace{1cm} (3.2)
Here $E$ is the electric field, $\mu$ is the magnetic permeability, $\varepsilon(N)$ is the carrier density dependent dielectric constant, $N$ is the excited carrier population, $\tau$ is the carrier lifetime, $D$ is the ambipolar diffusion constant and $g(N)$ is the carrier density dependent local gain (normalized along with the field strength to provide the correct carrier-intensity coupling).\(^1\) The phenomenological carrier rate equation is especially relevant here since the frequency spacing between the lateral modes is less than 25 GHz for a 750$\mu$m laser which suggests that inter-band processes will dominate. Intra-band carrier dynamics typically become relevant on much shorter time scales when phonon relaxation cannot restore the carrier energy distribution to equilibrium levels on the timescale of the laser's dynamics. The implicit reasoning here is that the beating between the closely spaced (in frequency) lateral modes will govern the pertinent dynamics of the system.

A set of first order (in time) differential equations are derived. The spatial dependence of the optical modes is removed by using perturbative methods.\(^2\) This is performed commonly in the analysis of a semiconductor lasers (for example see Reference [18]). However, the system under analysis here must contain the feature of two lateral modes having two different spatial distributions and consequently interacting with different (although overlapping) sets of carriers (or carrier baths). The details of the analysis is provided in Appendix B, but a quick enumeration of the recipe taken is given here:

1. Use the rotating wave approximation and first order perturbation methods to remove spatial dependence of the optical modes and reduce to first order differential equation.

2. Include a phenomenological loss term distributed evenly over the cavity.

---

1. The process of normalization of the second parameter with the first takes place naturally in the scaling of the carrier equations.

2. Only a first order perturbation is performed when removing the spatial dependence of the modes. Higher order perturbations will include changes to the optical mode shape. As can be seen from the steady state model of Reference [2], these changes are small for a typical ridge guiding laser structure.
3. Split the carriers into a sinusoidally varying (along the longitudinal dimension) grating-like carrier bath, $N_l$ and two additional carrier baths (each preferentially coupled to a given mode $- N_m$ and $N_n$).

4. Form the first order perturbation interaction with linear gain.\(^3\)

5. Integrate over the cavity to isolate the different modes (using the modal inner product definitions).

With these steps taken, the following set of equations can be found.

\[
\begin{align*}
\dot{E}_m &= -cE_m + ((1 - y)N_m + yN_n)E_m + fN_lE_n(\cos(\Psi) - \alpha \sin(\Psi))/2 \quad (3.3) \\
\dot{E}_n &= -cE_n + ((1 - y)N_n + yN_m)E_n + fN_lE_m(\cos(\Psi) + \alpha \sin(\Psi))/2 \quad (3.4) \\
\dot{\Psi} &= \Delta - \alpha(2y - 1)(N_n - N_m) - fN_l(\alpha(E_n/E_m - E_m/E_n)\cos(\Psi) \\
&\quad + (E_n/E_m + E_m/E_n)\sin(\Psi)) \\
\dot{N}_m &= P_m - bN_m - 2N_m((1 - y)E_m^2 + yE_n^2) - 2N_lE_mE_n\cos(\Psi) \quad (3.6) \\
&\quad + Dk_{mn}^2(N_n - N_m) \\
\dot{N}_n &= P_n - bN_n - 2N_n((1 - y)E_n^2 + yE_m^2) - 2N_lE_mE_n\cos(\Psi) \quad (3.7) \\
&\quad + Dk_{mn}^2(N_m - N_n) \\
\dot{N}_l &= -Dk_l^2N_l - bN_l - 2N_l(E_n^2 + E_m^2) - 4(N_m + N_n)E_mE_n\cos(\Psi) \quad (3.8)
\end{align*}
\]

where $E_m$ is the field in mode $m$, $y$ is the mode-carrier overlap fraction for a given mode with its own carrier bath (i.e. for mode $m$, the carrier bath is $N_m$), $f$ is the spatial overlap of the mode with the carrier perturbation, $N_l$, caused by the field intensity interference between the two modes, $\Psi$ is the relative phase between the two modes, $\alpha$ is the linewidth enhancement factor, $\Delta$ is the intrinsic frequency separation between the two modes (i.e. when there is no coupling present), $b$ is the carrier loss term (related to $\tau$) and $P_m$ is the rate at which the excited carriers are pumped into carrier bath $m$.

\(^3\)Although nonlinear gain is not considered explicitly, the effects of spatial hole burning are implicitly present in the carrier bath definitions.
$N_l$ refers to a sinusoidal carrier modulation and therefore it can be negative – this corresponds to it being out of phase with its own positive state. Physically, $N_l$ is formed by the perturbation of the intensity interference between the lateral modes.

In this set of equations, the parameters have been normalized to assist their manipulation. A schematic of the way in which the carrier baths may be separated is shown in Figure 3.1.

3.6 Preliminary Discussion of Coupled Rate Equations

Some important dynamical features, which will be discovered later, are apparent in this system of rate equations. Coupling exists between the fields of the two modes as a result of the carrier perturbation, $N_l$, which acts as a naturally phase-matched gain and refractive index grating. The field amplitude equations are coupled by means of the $N_l$ carrier grating. The relative phase between the modes is also influenced by $N_l$.

If we take the limit of the field amplitude and carrier amplitudes being essentially constant, the phase equation reduces to Adler's equation (also commonly referred to as the "phase-locking equation") for linear coupling between two oscillators. The implications of this result will be discussed in later chapters. The term involving $N_m - N_n$ in phase equation (3.4) represents an offset in the modal resonance due to carrier induced refractive index changes of modes $m$ and $n$.

The carrier baths that have no longitudinal variation, $N_m$ and $N_n$, are coupled to one another through carrier diffusion. The way in which these baths are defined is arbitrary, except that one should have a larger overlap with the first order lateral mode and the other should have a larger overlap with the fundamental lateral mode. This requirement separates the carrier baths so that one bath has the majority of its carriers localized at the center of the mode (the bath feeding the fundamental mode predominantly) and the other bath has the majority of its carriers at the edges of the waveguide (feeding the first order mode). Diffusion of the carriers acts against this spatial localization of the carriers and so the two baths are coupled to each other in a
Fig. 3.1. The carrier population is split up into 3 baths that: i) overlap predominantly with the fundamental mode (n), ii) overlap predominantly with the first order mode (m) and iii) are caused by the intensity beating term between the fundamental and first order modes (l).
way which tends to keep them uniform – that is:

\[ \frac{\partial N_m}{\partial t} \approx Dk_{mn}^2(N_n - N_m) \]  \hspace{1cm} (3.9)

Here \( k_{mn} \) is the \( k \)-vector of the length scale associated with the spatial separation of the modes and \( D \) is the diffusion constant.

This set of rate equations shows locking behavior, with a locking bandwidth which decreases as \( y \to 0.5 \) or the diffusion terms become large as was verified by a series of integrations. If the coupling due to diffusion between the two carrier baths is removed, the system becomes unstable. This is a relevant result: the structure formation in a nonlinear system is aided by the existence of diffusion which tends to couple the different modes of the system over space and act in a way that is opposite to the nonlinearities present. The nonlinearities "push" one way while diffusion pulls the other, which can lead to a stable balance. If the diffusion is not sufficient to counteract the nonlinearities, the system will become unstable.

The system defined by Equations (3.3)-(3.8) is rather complex for analytical methods (although not unthinkable). Instead, numerical methods were used to explore the parameter space and examine the evolution of the modes. Common laser parameters were employed for the variables \( (c, f, \alpha, D, b) \), a few different values were tried for parameters such as \( y, P_m, P_n, \Delta \), and \( k_{mn}, k_l \) to evaluate their sensitivity. \(|N_l|\) was also found to be relatively small (\( \sim 0.1 \)) in most cases as compared with the other carrier baths (\( \sim 1 \)), which supports the first order carrier perturbation approximations made in the derivation of this coupled mode treatment.

Reference [20] uses a similar set of equations to analyze the interaction between two longitudinal modes and five carrier baths (which take into account the different intensity beating terms). The diffusion lengths in 3 of these additional carrier perturbations are of the order of half the optical wavelength in the medium (\( \sim 150 \text{ nm} \)), so these perturbations have a lifetime which is 100 times shorter than \( N_l \). Consequently, these additional perturbations would be significantly smaller and are ignored in this analysis.
3.7 Longitudinal Mode Case

In Appendix B, the set of coupled rate equations for two interacting longitudinal modes was derived. It was also noted that this set was mathematically similar (with the exception of not resorting to a response function method) to the one derived in Reference [9]. Consequently, the results of [9] may be considered to describe the interaction between two longitudinal modes in a system with only a single lateral mode. As one would expect, the longitudinal mode set of rate equations can be derived from the lateral mode set of rate equations when the modal gain-overlap, $f$, is unity and the carrier sharing for the carrier baths, $y$, is 0.5.

3.7.1 Linear Stability Analysis

This set of coupled differential equations was analyzed with a first order stability analysis around its fixed points. This is done by finding steady state solutions to the system of equations, and noting the linear stability of small perturbations. In general, stable solutions will have eigenvalues lying exclusively in the left half of the complex plane (where the first order perturbations die away exponentially).

The steady state solutions were first solved and then the eigenvalues of the first order perturbative matrix were checked (using Maple V). Although this process would benefit from an analytical expression to test the stability, such as the Routh-Horowitz condition, the size of the problem makes finding such a condition impractical.

It was found, by exploring a wide range of parameters which encompass those expected of a semiconductor laser, that all fixed points which are physically real are dynamically unstable. These points will not present frequency locked solutions in the presence of perturbations from spontaneous emission and carrier shot noise. In addition, the fixed points will not be approached in the normal evolution.
3.7.2 Numerical Integration

By integrating the rate equations with a Runge-Kutta algorithm, more insight can be obtained into the expected dynamics of the system. It was found that the competition for the carriers always lead one mode to "kill" the other.

This behaviour is expected from an intuitive consideration of the the rate equations. For any relative phase between the two modes one mode will experience a larger gain than the other. This mode will subsequently pin the carrier density below the threshold of the other mode and cause it to die out.

Although previous works (e.g. [18]) allow for phase locking between longitudinal modes, all indications of this analysis is that frequency locking cannot be obtained between longitudinal modes of the same lateral and transverse waveguide mode.

3.8 Lateral Mode Case

From the previous section, it is clear that something is missing from the longitudinal mode set of rate equations that will not allow it to adequately describe the dynamics of the lateral modes. The elusive point is that the two lateral modes interact with spatially distinct sets of carriers as is shown in Figure 3.2. This prevents one of the modes from pinning the carrier bath of the other mode and causing it to die out.

These sets of carriers, or carrier baths, can be used to explain how spatial hole burning from the fundamental mode leads to the introduction of higher order lasing modes [2]. The fundamental mode pins its set of carriers at the location of the mode intensity maximum at the onset of lasing, while the population of carriers adjacent to this maximum continues to grow. The result is an increasing modal gain for the higher order lateral modes.

Integrating the lateral mode equations (3.3-3.9) gives insight into the effects of carrier diffusion on frequency locking. Typical (or reasonable) laser parameters, shown in Table 3.8, are used in the analysis.
Fig. 3.2 Interaction regions of carriers and optical intensities along the lateral dimension.
Table 3.1. Typical parameter values for fiber-pump ridge waveguide lasers in normalized and regular units, obtained from References [15] and [18].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (Regular Units)</th>
<th>Value (Normalized Units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = \frac{\partial \text{gain}}{\partial n}$</td>
<td>$1.1 \times 10^{-12} \text{m}^3/\text{s}$</td>
<td>1</td>
</tr>
<tr>
<td>$b = 1/\tau$</td>
<td>$0.5 \times 10^9 \text{ Hz}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>$\approx 1 \text{ GHz}$</td>
<td>$0.02$</td>
</tr>
<tr>
<td>$c = 1/\tau_p$, cavity loss rate</td>
<td>$0.5 \times 10^{12} \text{ Hz}$</td>
<td>1</td>
</tr>
<tr>
<td>$P$, at threshold</td>
<td>$\frac{\tau_p}{(a\tau)}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>$k$</td>
<td>$(2\pi/3)1/\mu\text{m}$</td>
<td>$0.02$</td>
</tr>
<tr>
<td>$y$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$f$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\approx 3$</td>
<td>$\approx 3$</td>
</tr>
<tr>
<td>$D$</td>
<td>$3.6 \mu\text{m}^2/\mu\text{s}$</td>
<td>72</td>
</tr>
</tbody>
</table>
3.8.1 Competitive Behavior

If the two flat carrier baths are strongly coupled by diffusion (i.e. they are in close proximity or have a large diffusion constant) they act as if they are a single carrier bath. The result is that one of the modes dies out as in the longitudinal mode case. The results of this integration for the field amplitudes are shown in Figure 3.3.

3.8.2 Unstable Behavior

If the diffusion coupling between the modes is too small, there exists no feedback to suppress nonlinear growth and the system becomes unstable. The nature of the instability is an attractor, as is apparent from Figures 3.4 and 3.5. These figures show that the evolution is bounded with points of attraction.

Figure 3.5 is of particular interest because it portrays an inversion symmetry in the perturbed carriers, $N_l$. Since $N_l$ is sinusoidally varying in the longitudinal dimension, its sign is indicative of its phase more than anything else. It is clear from this figure that two fixed point values of $N_l$ are the loci of the attractor. These fixed points represent two stable frequency locked stable states that have become unstable (at the bifurcation point) due to the lack of diffusive coupling between the carriers.

The motion on the attractor shows sensitive dependence on initial conditions (SDIC) which is a definitive characteristic of chaotic systems [21]. SDIC is demonstrated by noting an exponential separation of two paths of evolution from only slightly different initial conditions. The exponential separation is bounded by the diameter of the attractor, and so it rapidly saturates there. SDIC is shown in Figure 3.6 for a slight variation in the initial phase separation between the optical modes.

A quasi-continuous Fourier spectrum is another characteristic of chaotic systems. This is shown in Figure 3.7 for the perturbed carrier bath, $N_l$. The quasi-continuous spectrum indicates that the evolution is noisy, but generally contains some harmonic features.
Under strong diffusive coupling, the two carrier baths feeding the two lateral modes tend to act as a single carrier bath which is pinned by the mode with higher gain. This causes the lower gain mode to be below threshold and die out approximately exponentially. Shown here are the electric field amplitudes of the two modes which start out at similar strength but soon the decay of one and the growth of the other mode follows. (Normalized units are used for convenience).
Fig. 3.4. A three dimensional parametric plot of the phase, field amplitude in one of the modes, and population of the carrier bath feeding that mode. The same space confined attractor behavior is seen for such plots of all the other variables.
Fig. 3.5. A two dimensional parametric plot of the perturbed carrier's population shows an approximately symmetric pattern, indicating the two different attractor regions (one with a positive number of perturbed carriers and one negative) for the $N_I$ carrier distribution. The mirror symmetry in the plane of the perturbed carriers is indicative of the two fixed points of the laser’s evolution which become unstable with a decrease in the diffusive coupling between the carriers and consequently give rise to the attractor behaviour.
Fig. 3.6. By looking at the difference in evolutions of the relative phase between the two lateral modes for two slightly different initial phases ($\Delta \psi = 0.001$ rad), an approximately exponential behavior is seen until the phase separation reaches the boundary of the attractor. The exponential divergence of the phase difference with integration of the system (note this is a log plot) is indicative of a sensitive dependence on initial conditions (SDIC) which is a defining feature of chaotic systems. The noisy spikes are unimportant initial condition features, whereas the broad features are most relevant.
Fig. 3.7. A Fourier transform of the beating perturbed carriers is shown here in the unstable attractor regime. The result has a large amplitude over a wide range of frequencies with some periodic spikes corresponding to the periodicities of the attractor.
3.8.3 Frequency Locking

The third regime in parameter space occurs at an intermediate level of diffusive coupling. In this case, the carrier diffusion serves to stabilize the nonlinear coupling between the mode and the carrier baths and so frequency locking can result. The evolution to stable frequency locking is shown in Figures 3.8, and 3.9.

In Figure 3.8, the perturbed carriers, $N_l$, approach a fixed non-zero value. This indicates that coupling between the optical modes exists and approaches a constant value. Figure 3.9 clearly shows frequency locking results since the phase difference between the modes approaches a constant value.

When frequency locking occurs, the carrier baths $N_m$ and $N_n$ are pinned near threshold (one slightly above and one slightly below). In addition, the field amplitudes are approximately the same.

The frequency locking regime is approximately bounded by $k_{mn} = 0.15$ (strong coupling) and $k_{mn} = 0.01$ (weak coupling). The transition from the frequency locking regime to the strong coupling regime is a slow one (with changes in $k_{mn}$), in which stable states exist with a large discrepancy in the field intensity of each of the modes. The $k$ vector that most closely corresponds to the ($\sim 5\mu m$) ridge waveguide geometry of the lasers exhibiting frequency locking (i.e. those observed in this document), $k \approx 2\pi/3\mu m^{-1} = 0.02$ (normalized units), falls within the locking region, whereas other larger or smaller geometric configurations would not.\(^4\) It is interesting to note that this practical value is close to the region of chaotic behaviour and so unstable behaviour may be present for different parameter values.

The frequency detuning between the two modes, $\Delta$, was varied to determine the extent of the locking bandwidth. It was found that the locking bandwidth extends to a $\Delta \approx 0.15$ (or $\approx 7.5$ GHz) before one of the modes tends to die out (for the other parameters given in Table 3.8) as is the case for increasing $k_{mn}$. For larger values of $\Delta$, unstable two mode operation may be obtained.

\(^4\)The intensity beat term which causes the carrier perturbation, $N_l$, will have a lateral wavelength of the order of the ridge width for beating between the fundamental and first order modes. Consequently, the diffusion lengths that will determine the rate at which this perturbation gets washed out should be approximately equal to the width of the ridge.
Fig. 3.8. The perturbed carriers, $N_l$, reach a steady non-zero value when locking occurs.
Fig. 3.9. The relative dynamic phase, $\Psi$, between the two modes approaches a steady state, indicating that the two modes have the same frequency.
For any stable frequency locked state existing in the lateral mode rate equations, an isotropic state exists when $\Psi$ is replaced with $-\Psi$ and $N_l$ with $-N_l$. This is the expected bistability resulting from the nonlinear interaction between the carriers and the field. The same result has been previously expressed in Reference [9] where a modified form of Adler's equation was used to describe the relative phase between the modes for the locking phenomenon. This bistability will be further developed in the next Chapter to explain the observed bistability in the beam steering direction.

Clearly, a more thorough analysis can be given of the dynamics of this system, but the interesting result of frequency locking, (most important in this context of this beam steering problem) has been demonstrated as well as its bounding regimes characterized.

3.9 Summary

In this chapter, a set of coupled rate equations was derived to describe the dynamic interaction between lateral modes of a semiconductor laser. This set of equations includes significant features of the carrier dynamics which have been absent in previous models. Among these features are the spatial relation between the carriers and the optical modes and the effects of carrier diffusion.

It was shown that carrier diffusion and carrier induced coupling in a one dimensional system (without several lateral or transverse modes) does not result in frequency locking between the modes.

With the introduction of higher order lateral modes, frequency locking can occur when the lateral geometry of the structure falls within a specific length scale (determined by the diffusion length of the carriers - i.e. when diffusion balances the carrier induced nonlinearities). In addition, chaotic attractor and competitive (one mode dominating over the other and causing the latter to die out) behaviour was predicted and explained for extreme values of the diffusion parameter. The former occurs when carrier diffusion is too small to balance the carrier induced nonlinearities. The latter occurs when carrier diffusion couples the carriers together so strongly that they are pinned at the threshold value of one of the lateral modes. The discovery of these three regimes and the system of coupled lateral mode laser rate equations are new results of this thesis.
These results are especially pertinent in light of all the massively complex and impressively self-consistent modeling efforts that exist today to explain structure formation in semiconductor lasers (see for example reference [19] where inter-band and intra-band carrier and optical properties are taken into account in a non-perturbative manner). Such modeling often obscures the physical mechanisms behind the results they show by being too complex. Although 6 nonlinear coupled differential equations were required in this analysis, all of the features they represented were easily explained and the key dynamic regimes were demonstrated with variations to some of the relevant system parameters. It is clear that even this simple model, with only the complication of a higher order mode and carrier diffusion processes, is rich in dynamic properties.

Now that we have developed both linear and nonlinear coupling models to describe frequency locking, we can proceed to introduce and interpret the observed features of beam-steering lasers.
Chapter 4

Experiment:
Optical Spectra and Far-field Patterns

4.1 Introduction

This chapter will present new observations of semiconductor lasers that give additional insight into the beam-steering phenomenon. Experimental endeavors have focused on far-field intensity patterns, high resolution \((\Delta \lambda = 0.1 \text{ Å or } \Delta f = 3 \text{ GHz})\) optical spectra and power spectrum analysis of the intensity beating between the different waveguide modes. The former two experimental endeavors will be presented in this chapter and the more advanced power spectrum analysis will be discussed in the next chapter.

The observations shown here are particularly interesting because they are the first direct observations of multiple oscillating lateral modes in weakly index guided lasers just before the onset of beam steering. Special features in the far-field characteristics (i.e. bistability and hysteresis in the beam steering) suggests that nonlinear coupling mechanisms may be responsible for the beam-steering phenomenon.

4.2 Overview of Observed Characteristics

To give a feeling for the behavior of beam-steering lasers, an overview of the observed characteristics are briefly presented here.

Observations of beam-steering semiconductor lasers were made at a variety of operating conditions (i.e. injection currents and base temperatures). Some lasers exhibited beam-steering at the relatively low currents (well before expected optical damage) of \(130 - 210 \text{ mA}\). These lasers were nominally similar to others that did not exhibit beam instabilities until currents greater than \(300 \text{ mA}\). Slight variations in the ridge width, wafer thickness, strain and etch properties (i.e. any lateral asymmetries in the wafer structure) may have contributed to make these lasers exhibit beam-steering.
Bilateral steering and hysteresis of the steering direction with variations in the operating conditions have been observed. Lasers that did not produce beam instabilities until higher currents were less well behaved. They showed beam instabilities in the vicinity of a noisy output intensity modulation.

The optical spectra first show the onset of higher order lateral modes before beam-steering is observed. At this point, these modes are well separated in wavelength and can be easily distinguished from one another.

Modulation peaks in the output intensity spectrum of the laser under constant bias has been observed in the radio-frequency (rf) range after the onset of beam-steering. These peaks die away while the laser is leaving the region of beam-steering (i.e. when the injection current is increased). The occurrence of the beat spectra implies that some of the lateral mode pairs are fairly close in optical frequency (~GHz), but are not frequency locked.

4.3 Linking Modal Behavior and Observed Characteristics

Before embarking upon an in depth description of the observations, an outline linking the behavior of beam-steering lasers to the physical properties of the laser will be given.

4.3.1 Emergence of Higher Order Modes

Higher order lateral modes emerge in nominally single mode lasers. This is encouraged by carrier spatial hole burning in the center of the waveguide ridge (from the fundamental mode). As a result, the first order mode experiences increased modal gain until finally it lases.

Beam-steering does not have to occur when the higher order modes start to lase because the lateral modes may still be operating at different frequencies (i.e. they are unlocked). At larger current injection levels, either linear or nonlinear coupling between the modes causes them to lock. Frequency locking occurs when the intrinsic frequency separation of the modes enters the locking bandwidth provided by the inter-modal coupling.
The different lateral mode combs shift at different rates due to the effects of inhomogeneous spatial carrier hole burning and intensity induced heating [6]. Both of these effects increase with the injection current. The lateral modes may be degenerate or be separated in frequency, depending upon the operating conditions.

4.3.2 Frequency Locking and Beam-Steering

Lateral mode frequency locking is implied by a stationary asymmetric beam-steering pattern in the far-field. There rf-spectra of the intensity output does not show a gradual transition to frequency locking. This implies that frequency locking is a spontaneous process.

No direct evidence of nonlinear coupling (i.e. through carrier beating) has been observed. Certain observed features, such as hysteresis and bistability in the beam-steering direction, suggest that such nonlinear mechanisms are at play. The deterministic steering behavior of these lasers suggest that linear coupling mechanisms could also influence the frequency locking process.

4.3.3 Exiting the Region of Beam-Steering

The cavity resonances continue to shift as the injection current is increased. Some of the lateral modes obtain a large enough intrinsic frequency separation to leave the frequency locking bandwidth. Additionally, new modes may start to lase outside of the frequency locking bandwidth. These two events are marked by the far-field intensity pattern starting to return to its original center position (i.e. a reduction in the asymmetric component) and radio-frequency (rf) modulations in the laser's intensity output. The rf modulations are indicative of the temporal interference beating between lateral modes of different oscillation frequencies.

The rf modulations in the output intensity occur when the far-field distribution is still shifted to one side (but starting to return to the center). This implies that some of the modes are still frequency locked at this point. The remaining frequency locked modes set up a stationary grating in the carrier distribution by means of spatial hole burning.
The stable carrier-grating couples the unlocked modes. This coupling mechanism is linear because the carrier-grating is stationary. Observations of the rf-spectra, to be discussed in the next chapter, confirm the existence of linear coupling between the unlocked modes. These rf-spectra are not seen as the laser enters the region of beam-steering, which further suggests that the linear coupling is the result of linear coupling from a carrier-grating.

4.4 Observations of Optical Spectra and Far-field Patterns

4.4.1 Multi-mode Optical Spectra

As mentioned in the introduction, previous works observed features which suggest the emergence of higher order lateral modes contributing to beam steering. A more direct observation of the emergence of these higher order modes can be made by monitoring the optical spectrum at various injection currents and temperatures. This technique can also provide quantitative information about the wavelength separation of the relevant modes and their evolution patterns.

As is shown in Figure 4.1, a higher order lateral mode comb starts to emerge at \( \sim 100 \) mA with 10 GHz separation from the nearest fundamental lateral mode resonance at approximately 973 nm. The comb is indicative of several lasing longitudinal modes. This observation uses a high resolution (\( \sim 0.1 \) Å) single grating monochromator to resolve the separate mode peaks. Longitudinal modes of the 750 \( \mu \)m Fabry-Perot lasers used in our experiments have a spacing of 50 GHz at 980 nm.

Many features of the higher order modes can be deduced from these spectra. The higher order lateral mode comb peaks emerge at longer wavelengths than those of the fundamental mode. This represents a lower temperature (i.e. closer to the energy band-edges) carrier distribution overlapping with the first order mode. This interpretation makes sense since the higher order lateral modes are not as susceptible to heating at the center of the ridge (from the field intensity maximum of the fundamental mode) and consequently are pumped by carriers of lower energy (i.e. longer wavelength).
Fig. 4.1. Emergence of first order lateral mode comb.
The separation between the peaks of the fundamental and first order mode combs corresponds to the frequency detuning between the oscillating modes which is an important parameter in characterizing the locking phenomenon. In addition, noisy features appear in the background of optical spectra close (within 10 mA) to regions of beam steering (shown in Figure 4.2). These features reflect the possibility of dynamic the interactions between closely spaced waveguide modes in the presence of coupling. Similar noisy features in the optical spectra of gain coupled transverse modes have been observed in Reference [22].

Observations on lasers that have the same structure (nominally), but do not steer (at least up to injection currents of 300 mA), show no higher order lateral mode combs in the optical spectra.

4.4.2 Bilateral Steering and Hysteresis

Far-field emission patterns corresponding to those expected from the interference of the first order and fundamental lateral waveguide modes have been observed (see Figure 4.3 for theoretical plots and Chapter 1 for experimental observed far-fields). The steering direction of the far-field depends on the relative phase between the modes. As shown in Figure 4.4, bilateral steering has been observed in these semiconductor lasers.

The bilateral steering corresponds to two different stable frequency locked states with opposite phase. When linear coupling is concerned, only one of these states is stable, however with nonlinear coupling (of the type discussed in Chapter 3) both steering states are stable.

The bilateral patterns shown in Figure 4.4 are particularly interesting because both steering directions were observed at the same operating conditions. The only difference in obtaining two steering directions is that for steering in one direction, the current was increased from smaller values, while for steering in the other direction, the current was decreased from higher values. It is clear that these lasers exhibit hysteresis in their beam steering direction with variations in current.

Switching between the two stable steering direction states was also observed by viewing the far-field pattern with an IR vidicon camera. For certain operating conditions,
Fig. 4.2. Noisy features in optical spectra.
Fig. 4.3. A shift in the far-field intensity is observed in beam steering semiconductor lasers. The shift is consistent with one expected from the interference of a fundamental and first order mode, as is illustrated here for two Hermite-Gaussian modes.
Fig. 4.4. Hysteresis in far-field steering direction.
at the onset of steering, each steering direction would appear stable for \( \sim 1 \) s, then a rapid switching would occur to the other direction.

4.4.3 Discussion of Bilateral Steering

The observed features of bilateral steering, hysteresis and switching between bistable states can be explained in terms of the coupling between the lateral waveguide modes. The relative phase between the lateral waveguide modes determines their steering direction. If a certain phase, \( \phi \), corresponds to steering to the left, then \( \phi + \pi \) corresponds to steering to the right.

In the case of linear coupling, the dynamics of the relative phase between the modes is given by an expression of the form:

\[
\frac{d\phi}{dt} = \Delta \omega + \Gamma \sin(\phi + \Theta)
\]  

(4.1)

where \( \Delta \omega \) is the intrinsic frequency separation between the modes (i.e. if no coupling existed), \( \Gamma \) is the locking bandwidth which is dependent upon the coupling strength and intensity of the modes, \( \phi \) is the dynamic phase difference between the modes and \( \Theta \) is another phase term dependent upon the mode properties (see [23]). This equation is found commonly in systems when external signals are used to frequency lock oscillators or a collection of oscillators mutually lock to one another and is usually referred to as Adler’s equation. [17]

Adler’s equation as presented above is easily derivable from the Master Equations of in Chapter 3 (esp. Equation (3.5)) for the case of linear coupling (i.e. \( N_1 \) is constant) when \( E_m, E_n, N_m \) and \( N_n \) are all considered to be constant.

The frequency locked state corresponds to the condition that \( d\phi/dt = \dot{\phi} = 0 \), and is only stable for values of \( \phi \) where \( d\phi/d\phi < 0 \). Equation 4.1 can give at most one unique stable value (mod(2\(\pi\))) for the relative phase between the modes. Consequently, only one steering direction is allowed in the case of linear coupling for a given set of operating conditions. A single continuous range of \( \Delta \phi = \pi \) is allowed for variations in \( \Delta \omega \) or \( \Gamma \). The physical reason for having only a single locked state solution is that linear coupling is sensitive to the phase between the modes over the entire \( 2\pi \) cycle.
For the case of second order nonlinear coupling resulting from intensity dependent carrier perturbations, the coupling repeats itself once over a cycle of the relative phase, so some phase information is lost and two stable states separated by $\pi$ exist. It was found in Reference [9] that the relative phase equation for the case of nonlinear coupling changes to:

$$\frac{d\phi}{dt} = \Delta \omega + \Gamma \sin(2\phi + \Theta)$$  \hspace{1cm} (4.2)$$

where the only difference from Equation 4.1 is an extra factor of two in the sine term. Although this equation is strictly not valid to model the dynamics of carrier induced intermodal coupling, it is acceptable for identifying the fixed points of the evolution (corresponding to frequency locked states). This modified phase rate equation has two physically distinguishable stable frequency locked states, where one solution corresponds to steering in one direction and the other in the opposite direction. Consequently, this model allows for the observed bilateral steering at a single state of operation.

If $N_I$ responds fast enough to changes in the phase, $N_I \sim \cos(\phi + \zeta)$ in the analysis of Chapter 3. In addition, if $E_m, E_n, N_m$ and $N_n$ are all considered to be constant, then the modified form of Adler's equation above results from Equation 3.5.

By using the modified form of Adler's Equation (4.1), one can explain the observed hysteresis and switching between steering directions. Each of the stable states of Equation 4.2 has a region of attraction. The region of attraction that the modes falls into when they first enter the locking bandwidth determines their final frequency locked state's relative phase. The regions of attraction are outlined in the phase plot of Equation 4.2 shown in Figure 4.5.

If the relative phase between the modes falls within one region of attraction when the current is increased until $|\Delta \omega| = |\Gamma|$ (i.e. the point at which the intrinsic frequency detuning enters into the locking bandwidth), steering to one direction will result. Alternatively, if the relative phase falls into another region of attraction, another frequency locked state may result, leading to steering in the opposite direction. This situation is consistent with the observed hysteresis and is depicted schematically by the arrows in Figure 4.6(a). To determine exactly how the initial relative phase between the modes is
dependent upon the operating conditions, a more thorough analysis of the rate equations governing the modal dynamics is required.

If $|\Delta \omega| \cong |\Gamma|$, slight noise perturbations from the frequency locked relative phase between the modes can result in $\phi$ being bumped into the adjacent region of attraction (see Figure 4.6(b)). This noisy jumping between the stable states explains the observed switching mentioned above.

It should be noted that although linear coupling allows for bilateral steering, since the phase of the coupling is arbitrary, it doesn’t allow for bistable operation given a fixed set of operating conditions. The deterministic behavior in most of the semiconductor lasers analyzed suggests that there is a substantial component of linear coupling component which favors a particular locked state (i.e. steering direction). The steering direction was found to alternate with changes in cavity length in Reference [3]. This further suggests that a fixed phase linear coupling is present between the lateral modes.

### 4.5 Range of Steering

Adler’s equations suggest that linear coupling allows for beam-steering over a continuous range of $\pi$ and nonlinear coupling is limited to a range of $\pi/2$. Observing the range of steering in the far-field is one possible way to determine which type of coupling is dominant. Since the lasers observed often had several longitudinal modes, it is a difficult procedure to make a clear observation of the phase change.

In addition, the simulations of Chapter 3 suggest that nonlinear coupling only allows for a limited range in the relative phase between the modes (around 1.5 radians) for a wide variation in the operating conditions. The dynamics in this case (where the grating enhances the coupling which in turn contributes to creating the grating) has a limited range of steering which is much less than $\pi/2$. Outside of this range, a large disparity exists in the respective intensities of the lateral modes and so steering might not be as visible, even if the modes are frequency locked. Consequently, the results predicted by Adler’s equation might not be valid for practical observations of the steering range.
Fig. 4.5. Plot showing region of attraction for two different fixed points that represent two separate steering directions at a given set of operating conditions.
Fig. 4.6. In (a), two different steering directions result depending upon whether the locking bandwidth is approached from above or below. (b) Perturbations for frequency locking close to the bandedge lead to steering back and forth.
4.6 Summary

In this chapter, we obtained insight into the beam-steering phenomenon by taking a closer look at commonly used characterization techniques. High-resolution optical spectra showed the emergence of higher-order lateral modes just before the onset of beam-steering. This was seen exclusively in beam-steering lasers. This optical spectra measurement is the first direct observation indicating that higher order lateral modes contribute to beam-steering.

Changes in the far-field intensity distribution with variations in the injection current were observed. Previously observed features of bistability and hysteresis were explained for the first time in the context of the phase-locking (Adler's) equation. These features suggest that nonlinear coupling mechanisms contribute to beam-steering. Dynamic instabilities in the steering direction were also explained with the phase-locking equation.

The work of this chapter was predominantly based on static observations of beam-steering lasers. To determine more about the interaction dynamics that lead to beam-steering, new methods are required which observe the lasers in a time varying situation.
Chapter 5

Experiment:
Field Intensity Radio-Frequency Beating Analysis

5.1 Introduction

In this chapter, observations of radio-frequency (rf) modulations in the intensity output of beam-steering lasers will be presented. A coupled oscillator formalism will be used to explain the observed features in terms of the coupling between the two lateral modes. The facility of this formalism to derive the relevant parameters of the coupled oscillator system will be shown. This is the first set of observations that will give information about the interaction dynamics between the lateral modes.

Consider a DC-biased laser is operating with a single lateral mode and several longitudinal modes. Fluctuations in the output intensity for highly coherent longitudinal modes may be observable with a beat frequency corresponding to the longitudinal mode comb spacing. This beat frequency would be about 50 GHz for the lasers observed in this thesis. Since there is only one lateral mode, the entire output will beat homogeneously.

Alternatively, the frequency spacing between different lateral modes (each having a different longitudinal mode) may be quite small, or even zero. When the two lowest order lateral modes are present, their interference pattern produces an asymmetric far-field distribution. This pattern oscillates back and forth in the lateral direction at the beat frequency between the two lateral modes. Stable beam-steering results when the two modes are frequency locked.

The two lateral modes interact with one another in a way that can affect their relative phase and ultimately produce frequency locking. Even when the interactions are not strong enough to produce frequency locking, interesting information about the coupling between the modes is present in their dynamics. By observing the rf modulations in the laser's output intensity, information about the interactions between the modes
can be deduced. It is the observation and interpretation of this intensity output beating that will be presented in this chapter.

5.2 Past Work

Output intensity beating between the lateral modes of a gas laser has been observed in Reference [24]. This work presented beat patterns in the rf-spectrum of lasers exhibiting asymmetric far-field patterns (i.e. beam-steering). The beat peaks are attributed to nonlinearities in the gain medium causing even harmonics to form in the frequency spectrum. Nonlinearities are commonly suspected to produce harmonic beating. In contrast to this hypothesis, it will be shown that the interaction between two coupled oscillators, which is at the heart of the frequency locking problem under consideration here, will produce strong beating which agrees well with the observed characteristics. Other features, such as a period doubling route to chaos and broad-band spectra have been observed in the gas laser system [24].

5.3 Experimental Setup and Detected Signal

The beat spectra were obtained by focusing the output from a semiconductor laser onto a high-speed photo-diode (with a 20 ps response time). This setup is shown in Figure 5.1. Since unlocked lateral modes are expected to swing back and forth as well as experience amplitude modulation, both of these variations are present in the spectra obtained.

By translating the photo-diode in the lateral dimension across the far-field, a maximum, minimum and subsequent maximum in the modulation signal could be obtained. The maxima correspond to regions where the beating component between the first order and fundamental lateral modes undergoes the maximum variance. The minimum occurs when the detector's overlap with the spatial beating component is minimized. At this point, beating oscillations in the far-field will not be detected by the photo-diode.
A mathematical description of the different beating components can be given by the following:

\[ I(\vec{r}, t) \propto (E_0(\vec{r}, t) + E_1(\vec{r}, t))^2 \]

\[ = E_0^2 + 2E_0E_1 + E_1^2 \tag{5.1} \]

where \( E_m(\vec{r}, t) \) is the temporally and spatially varying field in lateral mode \( m \). Since the detector has a response time of approximately 20 ps, only the slowest beating and amplitude modulation terms due to the modal interaction of equation (5.1) will be represented. In addition, the detector's small area means that not all of the spatial contribution of the field can be detected. If this were not the case, the beating between the fundamental and first order lateral modes, which is anti-symmetric in space, would give no contribution to the spatially averaged radio-frequency signal.

The detected signal is given by:

\[ S(t) = \int_{\text{detector plane}} g(\vec{r}) \left( A_0^2(\vec{r}, t) + A_1^2(\vec{r}, t) + 2A_1(\vec{r}, t)A_0(\vec{r}, t)\cos(\phi(t)) \right) d\vec{r} \tag{5.2} \]
where \( A_m \) is the slowly varying field amplitude of mode \( m \) and \( g(\vec{r}) \) is the detector efficiency at position \( \vec{r} \).

### 5.4 Discussion of Possible Beat Spectra

The power (frequency) spectrum of the detected output of two unlocked, but interacting lateral modes depends on the detector efficiency function and the manner in which the modes are coupled. The beat term found in Equation 5.2 has a modulation depth of \( 4 \int g A_0 A_1 \, d\vec{r} \). This has the potential of being large provided that the overlap integral is also large. Less severe modulations in \( A_m \) are also expected to be present. The basis of this analysis is that the coupling between the two modes will be betrayed by the dynamic evolution of \( \phi(t) \) and \( A_m(t) \).

The beat spectra associated with \( \phi(t) \) can give a large power spectrum signal and may be solved analytically (with some approximations). To characterize the effect of intermodal coupling outside the frequency locking band, we look to a generalized form of Adler's equation [17] for the relative dynamic phase evolution between the two modes (introduced in the last chapter).

This can be written as:

\[
\frac{d\phi}{dt} = \Delta \omega + \Gamma f(\phi) \tag{5.3}
\]

where \( \phi \) is the dynamic phase separation between the two oscillators (in this case, the two lateral modes).

For linear coupling, the common result is that \( f(\phi) = \sin(\phi + \theta) \) [17], where \( \theta \) is the phase angle resulting from oscillator amplitude mismatch or complex coupling (associated with gain or phase lags). An analogous result can be found for mutual coupling between two oscillators, provided it is assumed that the amplitude modulation is small with respect to the phase variation [15]. This leads to a single unique solution for the relative phase of the locked state.

---

1A generalization is made here to emphasize that the coupling between the modes may be of an arbitrary form. Consequently, this approach can be applied to a variety of other systems.
With nonlinear coupling mechanisms, some of the phase information may be discarded. Since the evolution of the intensity for the combined modes is essentially the same at two points in one period of the phase evolution, a \( \tau \) periodicity in \( f(\phi) \) is expected for intensity dependent mechanisms. In particular, for the cases of carrier modulation induced index perturbations, \( f(\phi) = \cos(2\phi + \theta) \) (see Chapter 3 and Reference [9]).

The component of the power spectrum coming from the beating term will then be the Fourier transform of \( \cos(\phi(t)) \), where \( \phi(t) \) is governed by Equation 5.3 (see Figure 5.2 for the linear and nonlinear cases mentioned above). While all harmonics (including the zeroth harmonic) of the first peak are found for the linear coupling case, only odd harmonics are present for the case of 2nd order nonlinear coupling. The respective power beating spectra are shown in Figure 5.2. Clearly, the nature of coupling is readily evident from inspection of the power beat spectra. The result represented in Figure 5.2 can be shown to be true for all \( f(\phi) \) that are periodic in \( 2\pi \) and \( \pi \) respectively (see Appendix C).

Other important results can be derived from the structure of the observed beat spectra. The relationship between the relative peak heights and the factor \( \Gamma / \Delta \omega \) is shown in Appendix D. Using this information, along with the fact that the spacing between the peaks is equal to \( \sqrt{(\Delta \omega)^2 - \Gamma^2} \) (and not simply \( \Delta \omega \)), both the locking bandwidth and the intrinsic frequency separation between the mode can be uniquely and independently determined at each point of operation.

As hinted at above, practical use of the beat spectra is not that easy since modulation to the modal amplitudes is also present. Possible beat spectra are shown in Figure 5.3, with different amounts of cross term modulation (\( 2A_0A_1 \cos(\phi(t)) \)) and amplitude modulation. These spectra were calculated using the rate equations of Chapter 3 for nonlinear carrier induced coupling, except that the carrier coupling was replaced with linear coupling to simulate the effect of a linear coupler (e.g. a carrier grating enforced by other frequency locked modes or some facet tilt). The amplitude modulation portion typically gives a higher second beat peak, which makes the sum of the contributing terms have a higher second peak than presented in the analysis above. For the remainder of

\footnote{Other orders of nonlinearity give distinct spectra as well.
Fig. 5.2. Theoretical power beating spectra for a) linear and b) nonlinear forms of Adler's equation.
this chapter, amplitude modulation of each of the individual modes is neglected in favor of phase modulation.

5.5 Observed Spectra and Discussion

5.5.1 Sample Beat Spectra

A sample of the observed beat spectra in the power output when the laser starts returning to the center from its steering position (at ~ 162 mA for this particular laser at 20°C) is shown in Figure 5.4. The spacing between the peaks decreases as the injection current increases (or temperature decreases), until the peaks disappear at around 100 MHz spacing. These peaks in the rf power spectrum are indicative of the beating between the two lateral modes. As described above, the comb of beat peaks is indicative of the interaction between the lateral modes, caused by the coupling that is present between them.

This observation of beating may be the result of an emerging set of modes at a longer wavelengths (due to the gain peak shifting with heating of the carriers) that are not frequency locked, but coupled through a linear grating set up in the carriers by frequency locked modes. This speculation is validated by the fact that the beating occurs as the far-field returns from its steering position, which suggests that many of the modes contributing to the far-field are not locked. Coarse resolution optical wavelength spectra taken on this laser show a shift in the lasing peak location as the beam steers back. In addition, all harmonics of the beat peaks were present with monotonically descending amplitude which suggests that the coupling between the modes is linear (in contrast to a strong odd harmonic favouring of 2nd order nonlinear coupling as shown in Figure 5.2, or the maximum second peak characteristic of 3rd order nonlinear coupling).

No beating was observed at the onset of steering which implies that locking is a spontaneous phenomenon. This is more conducive to nonlinear phenomenon, since the formation of the carrier grating leads to increased coupling in a self-enhancing path towards frequency locking. Numerical simulations of the nonlinear carrier induced coupling of Chapter 3 showed that there is a region of competition at the edge of the frequency
Fig. 5.3. Possible power beating spectra using linear coupling dynamics with different detection efficiency of the self-beating and cross-beating terms. Cross-beating terms were assigned a 2 order of magnitude smaller detection efficiency.
Fig. 5.4. Observed intensity beat spectra for different currents.
locking band, which will cause one mode to be favored over the other and consequently no beating will be seen. As mentioned earlier, it is suspected that although nonlinear coupling contributes to the initial event of frequency locking, linear coupling is present due to carrier-gratings which are formed by additional frequency locked modes.

5.5.2 Slit Measurements

Minor attempts were made to determine the spatial inhomogeneity of the detected signal by placing a 1 mm slit at the far-field of the laser (4 cm away from the facet) between the laser and the focussing lens. This was done to determine how the beat peaks evolve when different portions of the far-field are focussed onto the detector. A variation in the relative amplitudes of the beat peaks was observed. This variation can be attributed to different relative contributions of amplitude and beat term fluctuations. Two sample spectra are shown (in Figure 5.5) for when the slit was placed at the maximum intensity of the far-field and when it was at the edge of the far-field. The beating at the edge of the far-field has a large relative beat height that descends fairly homogeneously, whereas spectra taken from close to the center of the far-field show less reduction of the second beat peak.

This phenomenon can be explained in terms of the spatial dependence of the beating component in the far-field. Towards the edge of the far-field, the cross beating term is of a single sign over space and oscillates between \(-2A_0A_1\) and \(2A_0A_1\), which is bound to be larger than the oscillations in \(A_m\) alone for similar modal amplitudes. Towards the center of the far-field, the cross beating term is of both signs over space and so it partially cancels itself out. This leads to the amplitude modulation \(A_m(t)\) having more of an effect. In addition, the amplitudes are larger towards the center of the far-field.

Even though the amount of each component present at any given time has not been quantified, the fact that their is variation in the relative peak heights implies that both components are present.
Fig. 5.5. Beat spectra obtained with a blocking slit near the center and at the edge of the far-field.
5.5.3 Tracking the Beat Spectra

By noting the changes in the peak positions and heights with variations in temperature and current, we now have a means of tracking the frequency locking parameters over a variety of conditions. This is a new result and in the time available, was only done for straight-forward coupling of light into the photodetector and only with changes in the operating current, so the results are rather limited. The changes in the locking bandwidth and the intrinsic frequency separation with variations in current are presented in Figures 5.6 and 5.7.

It can be seen that the change in intrinsic frequency separation between the modes with variations in current is linear (with a mean difference from the linear fit of 27.5 MHz over the 800 MHz range). The slope of 42.3 MHz/mA gives a fairly slowly varying intrinsic frequency change versus current. Since steering occurs for approximately 50 mA in these lasers, the locking bandwidth is approximately 1 GHz. Two other lasers tested gave slopes of 44 (produced from same wafer) and 158 MHz/mA (different wafer).

A possible basis for theoretical comparison probably comes from the fully self consistent numerical simulations of Reference [25], in which the modal index difference between the modes is plotted for several current values. This paper suggests that \( \frac{(\Delta n_1 - n_0)}{(\Delta I)} \approx 7 \times 10^{-6} \) is the appropriate slope for the index difference change with current variations. Assuming that the first order approximation:

\[
\frac{\partial \Delta f}{\partial I} = \frac{f (\Delta (n_1 - n_0))}{n (\Delta I)}
\]

is valid, it leads to a 600 MHz/mA intrinsic frequency with current slope. Although the discrepancy between these two values is just under an order of magnitude, it should be noted that the laser structure in Reference [25] is different from those used here and that the higher order mode was not allowed to lase in the simulations of the aforementioned reference. It can be argued that since the traveling wave model of [25] does not consider the longitudinal variation of the mode, results for the relative effective propagation constants might be overestimated by this approach.
Fig. 5.6. Locking bandwidth changes with variations in current.
Fig. 5.7. Intrinsic frequency separation changes with variations in current.
In Reference [3], variations in the length of the laser were linked to changes in the kink power (presumably where the laser steers). The results from this report indicate that going from one locked state the next (i.e. a relative ‘motion’ of approximately 50 GHz between the two combs) is accompanied by a 60 mW variation in the output power. This implies that a 0.3 mW/mA efficiency is required for agreement with our results.

The thesis of J. Guthrie [26], showed hops in the beam steering direction for current changes of approximately 100 mA. This suggests a 500 MHz/mA which is closer to the theoretical work of [25] associated with this thesis. Discrepancies between these results and the ones found using the beat spectra may result from non-uniform variation in the modal effective index difference with changes in injection current. However, variations in going from structure to structure may be enough to account for the discrepancies alone.

Aside from determining which beating components are contributing to the observed beat spectra, other difficulties exist in making useful observations. In particular, as can be seen from the optical spectra, many longitudinal modes contribute to the intensity output, which complicates the analysis. Some rf-spectra show two beat combs (perhaps coming from two sets of lateral modes - see Figure 5.8). It is suspected that these combs are usually superimposed due to phase locking (for example see Reference [18]) resulting from carrier beating.

Hysteresis also manifested itself in this experiment with the appearance and disappearance of the beat peaks with variations in current. When the current is increased from below the onset of beam steering, the beat spectra disappear at higher currents than they appear when the current is decreased from above towards the onset of beam steering again. Hysteresis is a common phenomenon in nonlinear systems where the path to a given state is governed by the influence past states had on the system. By observing the optical spectra with different current paths, hysteresis in the spectral features are also observable.
5.5.4 Chaotic Interactions

A chaotic attractor regime of operation was found to exist in the carrier interaction modeling of Chapter 3. This regime showed broad power spectrum features, which have also been observed in the beat spectra of this laser at higher injection currents (\sim 270 mA) (see Figure 5.9). At these currents, the lasers start to experience efficiency roll off, the carrier bath broadens (due to increased pumping, carrier non-pinning, temperature and diffusion) and there is a reduced spectral purity (i.e. multiple interacting lasing modes), so the interaction between two lateral modes may be lowered and the diffusion lengths increased. It is precisely this weak coupling regime that was shown to give rise to chaotic behavior in the aforementioned analysis.

5.6 Applications to Other Systems

Other systems may be more favorable for the analysis in this chapter because they possess a more uniform output intensity and the effects of different types of coupling mechanisms can be easily identified. For example, the interaction between two coupled lasers which have a single lateral mode will give a predictable beat component in the intensity output. Y-laser configurations [27] used to obtain over 50 nm of tunability and phase-locked solid-state lasers (with a large frequency gap between them) are examples of practical systems that might benefit from this type of analysis.

In addition, other types of coupling that rely on different nonlinearities or experience modulation can be expressed with the Adler's equation formalism and consequently, are candidates for the analysis presented in this chapter.

5.7 Summary

In this chapter, observations of the output intensity were linked to the interactions between the lateral modes. This is the first attempt to observe the dynamic lateral mode interactions. The formalism presented allows for simple extraction of all the relevant modal interaction parameters (i.e. locking bandwidth, intrinsic frequency separation and the nature of coupling) that lead to frequency locking. This is done independently
of any other observations. Observations of rf modulation patterns in the intensity output of a beam-steering laser were analyzed with this formalism.
Fig. 5.8. Radio-frequency beat spectra from the intensity output which show the interaction of multiple longitudinal resonances (which occupy the two lateral modes).
Fig. 5.9. Broad power spectra characteristic of a chaotic attractor.
Chapter 6

Conclusions and Outlook

6.1 Beam-Steering Revisited

The major source of unreliable behaviour in fiber-pump semiconductor lasers is beam-steering. These lasers are a crucial component of long-haul fiber communication systems. This beam instability is characterized by a lateral (spatial) shift of the far-field intensity pattern. Beam-steering can start at injection currents well below the operating maximum. The result of beam-steering is unstable coupling of the pump laser to the fiber system.

Prior to this work, beam-steering has been attributed to the interference between the fundamental and first-order lateral modes. Until now, only indirect observations have been made to support this hypothesis. Even in the first work on beam-steering, [5], it was realized that the lateral modes must be frequency locked to produce the stable beam-steering pattern. However, little has been done to uncover the physical processes that lead to frequency locking.

6.2 A New Understanding of Lateral-Mode Coupling

This work represents the first in-depth study into the origins of beam-steering in semiconductor lasers. The main results are: (a) a physical understanding of the different processes that serve to frequency lock the lateral modes, (b) experimental observations of higher order modes in beam-steering lasers and beam-steering far-field patterns and (c) the development and utilization of a new experimental method that can be used to characterized coupled oscillator systems (including this one).
6.2.1 Physical Understanding of Coupling Mechanisms

There are two prevalent coupling mechanisms which can serve to frequency lock the lateral modes. The first is linear coupling resulting from asymmetries in the laser structure. These asymmetries are systematically (but mostly unintentionally) introduced in fabrication. In lasers with wafer strain, asymmetrically etched waveguiding ridges, and/or ridge-facet misalignment, coupling is introduced at the facets where the waveguide symmetry is inverted.

A steady-state model was developed to analyze the non-trivial linear coupling geometry. This model includes the regenerative nature of the lasing modes for the first time. The result of this model is a quantitative description of how easily the lateral modes frequency lock as a function of the amount of asymmetry that is introduced into the laser structure.

The second source of coupling comes from the interaction between the optical modes and the carriers. This type of coupling is intrinsic to semiconductor lasers and as such, has received much attention in the past (e.g. [4], [11], [19] and [9]). However, no previous work has adequately described the interaction dynamics between the lateral modes in a narrow ridge-waveguide laser structure.

For the first time, we include the spatial distribution of the carriers, carrier diffusion, optical mode intensity interference patterns and the nonlinear carrier dynamics in a single model. The model was investigated numerically to uncover three different dynamic regimes which depend critically on carrier diffusion. Even though the investigation was numerical, the underlying physics of the system was not obscured because of the clarity of the model.

When carrier diffusion was dominant over the nonlinear optical-carrier interactions, competition between the modes was high. Consequently, only one of the modes was dominant. However, if the carrier diffusion was too small to suppress the nonlinear

---

1 Other coupling mechanisms were dismissed in Chapter 2 because either they did not serve to couple lateral modes of odd and even symmetry or they were strong or fast enough to significantly affect the modal dynamics.
interactions, chaotic behaviour was observed. Intermediate to these two extremes carrier diffusion balanced the nonlinearities. In this region frequency locking resulted.

Clearly, this model shows the existence of three radically different regimes of operation. Such results could not be hoped for from models that unjustifiably discard important features of the carrier dynamics.

6.2.2 Observing Higher Order Modes and Far-Field Patterns

The investigations of this work rest entirely on the hypothesis that higher order lateral modes interact with the fundamental mode to produce beam-steering. Although previous observations have suggested that this is the case, no-one has made a direct measurement of simultaneous lasing of these higher order lateral modes.

To fill this void, high-resolution optical spectra were taken of the output from beam-steering lasers. These clearly show the emergence of a higher order lateral mode just before the onset of beam-steering. In addition, similar lasers that did not exhibit beam-steering showed only a single lateral mode in their optical spectrum.

Far-field intensity patterns of the laser output have been used to characterize the performance of beam-steering lasers for many years. However, curious features such as hysteresis in the beam-steering direction with changes in the laser's injection current, have eluded explanation prior to this work.

Models of the nonlinear laser dynamics developed in this work were used to explain the features of hysteresis, bi-stability and unstable switching in the laser's far-field intensity distribution.

6.2.3 Intensity Modulation Analysis

In the past, experimental investigations of beam-steering semiconductor lasers have been limited to static observations (i.e. far-field intensity patterns, imaging the spontaneous emission from active region). Very little information can be obtained about the dynamic interactions between the lateral modes with isolated snapshots of the modal interaction.
In this work, a new experimental method for analyzing the interplay between lateral modes was developed. The lasers were observed close to (but not within) the regions of frequency locking. Although the modes are not frequency locked at these points, they still interact with one another. This interaction provides insight to the dynamics of the coupled modes.

It was shown that all the required information about this interaction is accessible by observing the output intensity modulation of the laser. The technique developed and demonstrated in this work allows for a complete characterization of the coupling between the lateral modes. In addition, the results obtained are independent from all other observations.

Observations of the output intensity modulation patterns also showed the expected (from the nonlinear model of Chapter 3) chaotic features of the modal interactions.

6.3 Future Work

There are several possible extensions of the work presented in this thesis:

- Tracking the relative movement of the lateral mode combs optically (by means of the monochromator), with changes in the operating conditions (i.e. the injection current or temperature). This will allow for the first direct measurement of the frequency separation between the lateral modes. However, movements in the gain peak make this measurement far from trivial.

- Combine the beat spectra measurements with the monochromator on a low resolution setting to isolate the interaction of a single particular first order – fundamental mode pair. This will allow for a better characterization of the interaction between the two modes. However, maintaining a large enough signal to the high-speed photo-diode is one major challenge for this method.

- Apply the beat spectra measurements and analysis to two coupled lasers (before the onset of higher order lateral modes). Coupled semiconductor lasers have received much attention for their complex dynamic behavior. By utilizing the methods of
Chapter 5 on this system, the interaction between the coupled lasers can be fully characterized.

- Extend the analysis of Chapter 3 to include linear coupling and use analytic methods to present a general description of the possible dynamics. Alternatively, the analysis of Chapter 3 could be made more quantitative by resorting to a particular modal expansion of the carrier concentration (e.g. a Hermite-Gaussian expansion). These extensions both seek to make the analysis more quantitative which would be useful for application to a very specific laser structure.

- Compare nominally similar laser structures that both do and do not exhibit beam steering. (Speculations have been made that the role of the threshold carrier concentrations in these lasers is relevant in determining their behaviour [9]).

6.4 Outlook: Laser Structure Designs

6.4.1 Beam-Steering Inhibition

Beam-steering can be dependent on errors in the fabrication processes. This source of beam-steering can be controlled by improving the fabrication tolerances. From the analysis of Chapter 2, it is now possible to gauge the fabrication precision required to obtain minimal regions of beam-steering.

To control beam-steering resulting from intrinsic carrier-optical field interactions, more drastic measures are required. Several strategies have been undertaken to try and eliminate beam steering from fiber-pump semiconductor lasers. Most of these strategies consist of removing the higher order modes or increasing their threshold to such an extreme that they will never be able to lase.

Vertical current guiding structures have been utilized by a number of research groups as a means of reducing the current spreading and thereby reducing the pumping of higher order modes which have their modal maxima away from the center of the ridge. These guiding structures are introduced by the growth and selective area etching
to produce reverse p-n regions. They are more easily implementable than buried heterostructure lasers, although the latter provide better control over the allowed modes with stronger index guiding.

Other more creative strategies, such as the introduction of facet tilts (see Reference [12]), increase the outcoupling losses of higher order modes and prevent them from lasing have been suggested. These do not require costly regrowths, but may introduce increased intermodal coupling at the facets which could prove to be hazardous, as was discussed in Chapter 2.

The alterations to the laser structure outlined above all seek to reduce the chance of lasing in higher order modes. They only draw on the understanding that interactions between the fundamental and higher order lateral modes lead to beam steering. With the physical understanding of the carrier diffusion processes developed in this work, future design efforts can tackle the beam-steering problem with more arsenal. For example, it was noted in Chapter 3 that for short diffusion lengths extremely competitive behaviour resulted. Consequently, only one of the lateral modes was allowed to lase. This suggests that decreasing the ridge width is favorable for the inhibition of beam-steering.

6.4.2 Novel Laser and Laser System Design

This work also introduces tools for evaluating the interaction between the modes (i.e. the modulation spectra analysis of Chapter 5). Such tools are generally applicable to a variety of other laser systems where lasing modes interact with each other due to either nonlinear or linear coupling mechanisms. Consequently, more insight can be obtained into the operation of novel devices and device functionality. Existing devices, such as cleaved coupled-cavity lasers, coupled lasers for communication with chaotic signals, two wavelength solid-state lasers and waveguide arrays may also benefit from the characterization presented in Chapter 5.

By understanding the way in which coupled lasing modes operate, one can propose schemes such as waveguide arrays that steer in a controllable manner (by varying the relative phase of the output from under each of the closely spaced ridges in the array). In
addition, selectively coupled structures that utilize the enhanced gain of coupled modes can be implemented with increased tunability.
Appendix A

Matrix Method for Facet Tilt Frequency Locking

In this Appendix, an expression will be derived that relates the amount of coupling that two lateral modes of a laser have at each facet to the maximum frequency detuning between the two modes that will allow for frequency locking.

The geometry of this system, as described in Chapter 2, is that of two lateral modes sharing a waveguide and being coupled to each other equally at both facets. In this analysis, a matrix approach is used to determine the steady state resonances of the mutually coupled regenerative oscillator system.

The light coupled from mode 0 into mode 1 given a steady state injection at both facets can be written in the matrix form:

\[
\begin{pmatrix}
A_1 \\
B_1
\end{pmatrix}
= \frac{\epsilon_0 \kappa_0^2}{1 - \kappa_0^2}
\begin{pmatrix}
1 & C/\kappa_0 \\
1/C\kappa_0 & 1
\end{pmatrix}
\begin{pmatrix}
A_0 \\
B_0
\end{pmatrix}
\]

(A.1)

where \( A_m \) is the complex field in mode \( m \) at facet a and \( B_m \) is the complex field in mode \( m \) at facet b, \( \kappa_m^2 = r_a r_b g_m^2 \) is the round trip field gain of the cavity with \( r_\mu \) being the complex modal reflectivity at facet \( \mu \) and \( g_m \) being the single pass gain and phase accrual of mode \( m \), \( \epsilon_{mn} \) is the inter-modal facet coupling from mode \( m \) into mode \( n \), and \( C = \sqrt{r_a/r_b} \). For an illustration of the coupling geometry, see Chapter 2.

This light is then coupled back into mode 0 in a similar manner, so the regenerative round trip field equation can be written as:

\[
\begin{pmatrix}
A'_0 \\
B'_0
\end{pmatrix}
= \frac{\epsilon_0 \kappa_0^2 \epsilon_1 \kappa_1^2}{(1 - \kappa_0^2)(1 - \kappa_1^2)}
\begin{pmatrix}
1 + 1/(\kappa_0 \kappa_1) & C/\kappa_0 + C/\kappa_1 \\
1/C\kappa_0 + 1/C\kappa_1 & 1 + 1/(\kappa_0 \kappa_1)
\end{pmatrix}
\begin{pmatrix}
A_0 \\
B_0
\end{pmatrix}
\]

(A.2)
with the primes referring to the field coupled back into the modes. The threshold requirement for this configuration is that:

\[
\begin{pmatrix}
A_0 \\
B_0
\end{pmatrix} = \begin{pmatrix}
A_0' \\
B_0'
\end{pmatrix} \tag{A.3}
\]

An expression for the magnitude of the coupling product can then be formulated for a small detuning from resonance using equation (A.2) with the threshold condition (A.3):

\[
|\epsilon|^2 = \frac{\left( (A_0^2 - 1)^2 + A_0^2 \sin^2(\phi_0) \right) \left( (A_1^2 - 1)^2 + A_1^2 \sin^2(\phi_1) \right)}{\left( A_0 A_1 \right)^2 \left( (-1)^{N_0 + N_1} 4 A_0 A_1 + (-1)^{N_0} 2 A_0 (1 + A_1^2) + (-1)^{N_1} 2 A_1 (1 + A_0^2) + A_0^2 + A_1^2 + 2 \right)} \tag{A.4}
\]

where \( \kappa_m = A_m \exp(i(\phi_m + N_m \pi)) \) has been used, \( \phi_m \) is single pass phase accrual (which is small enough to make the approximation \( \cos(\phi_m) \approx 1 \) valid) and \( N_m \) is the longitudinal mode number corresponding to the lateral mode \( m \) under consideration. The denominator of (A.4) shows how intertwined the two modes are due to the existence of two coupling regions which makes it difficult to obtain analogous results to those obtained by Adler [17] as was done in [16], but the essential features of the locking bandwidth and phase equations remain.

By asserting, as is realistic with regenerative laser oscillators, that each of the coupled oscillating modes are operating near its independent lasing thresholds (i.e. \( |\tau a_{\nu m} g_m| \equiv A_m \approx 1 \)), (A.4) can be reformulated (for the case when both modes are away from resonance so that \( A_m^2 - 1 \ll A_m^2 \sin^2(\phi_m) \)), as:

\[
|\epsilon|^2 \geq 2 \left( \frac{\pi L n}{c} \right)^2 \frac{\Delta \nu_0 \Delta \nu_1}{1 + (-1)^{N_0 + N_1}} \tag{A.5}
\]

where \( L \) is the cavity length, \( c/\nu \) is the speed of light in the medium and \( \Delta \nu_m = \nu_m - \nu_{\text{locked}} \) is the frequency detuning from resonance of mode \( m \), which is directly related to \( \phi_m \).
It is interesting to note that the expression in (A.5) only allows for locking when both even or both odd longitudinal modes are participating. For the case when one of the longitudinal modes is odd and the other is even, the coupling bandwidth has to be re-evaluated to include higher order terms in the detuning. Since these terms are small, this results in a much smaller locking bandwidth. The physical reasoning for this result is that in the case of even-odd longitudinal mode coupling the coupling at one facet is cancelled by that of the other facet. For asymmetric cases in which one facet has more coupling than the other (e.g. if the tilt is greater at one end or the facets are coated differently), only part of the coupling at one facet will be cancelled by that of the other facet and a situation similar to the single coupling path case of [16] results. In either case, the facet coupling mechanism does not exclude the possibility of the observed bilateral steering which depends only on the relative phase between the modes at a given facet.

The phase relationship between the two oscillators is another interesting feature of such locked systems. The usual result is that the oscillators with higher intrinsic frequency lead in phase while those of lower intrinsic frequency lag. This system has the interesting feature of being doubly coupled so the results of the phase calculation also provides some insight into such systems.

By calculating the modes for the system described by Equations (A.1) and (A.2), the relative phase of the oscillators can be found. The phase accrued by the regenerative traversals of mode 0, \( \theta_0 \), (after neglecting the coupling phase) is:

\[
\theta_0 = \arg \left( \frac{\kappa_0^2 (1 \pm \kappa_0)}{1 - \kappa_0^2} \right) \approx \frac{\phi_0 + \pi}{2} \tag{A.6}
\]

where the approximation applies for a small detuning, \( \phi_0 \), so the oscillator is operating close to its regenerative unity gain as was used in deriving Equation (A.4). The same expression holds for the phase of the first order mode by replacing the 0 subscripts with 1.
Using the condition that the round-trip phase accrual in the coupled oscillator system must be a multiple of $2\pi$, the phase difference between the modes is found to be:

$$\theta_1 - \theta_0 = \frac{\phi_1 - \phi_0}{2} = \frac{\pi n L}{c} (\nu_1 - \nu_0)$$

(A.7)

which is consistent with the common result for locked coupled oscillators: the oscillator with the higher intrinsic frequency leads in phase while the other lags.
Appendix B

Derivation of the Lateral Mode Field-Carrier Rate Equations

In this appendix, the coupled rate equations used to describe the evolution of two lateral modes when carrier beating effects are taken into account will be derived. The approach will use a first order perturbation to remove the spatial degrees of freedom from the system. Similarly, to reduce the complexity of the carrier equations, but retain the important features of carrier diffusion and different modal overlap with the carriers, separate carrier baths are defined in a phenomenological fashion.

To start, a set of rate equations is derived for the interaction between two longitudinal modes with the same lateral and transverse modes. This is then phenomenologically extended to the case of two different lateral modes.

The field within the cavity is described by the wave equation:

$$\nabla^2 E - \mu \epsilon(N) \frac{\partial^2 E}{\partial t^2} = 0. \quad (B.1)$$

where $E$ corresponds to the field and $\epsilon$ changes with carrier concentration, $N$, in the cavity as:

$$\epsilon(N) = \epsilon_0 + \frac{\partial \epsilon}{\partial N} N. \quad (B.2)$$

We define the modes of the resonator cavity when material transparency has been achieved, which is set to correspond to the condition $N = 0$ throughout the cavity (with suitable modifications made to the carrier equations), as:

$$\nabla^2 A_m + \omega_m^2 \mu \epsilon_0 A_m = 0. \quad (B.3)$$
Any field within the unpumped cavity can be made up by a sum of these modes with a given relative phase and field amplitude:

\[ E = \sum_{m} \frac{1}{2} (\xi_m(t) A_m(\mathbf{r}) \exp(i\omega_m t) + c.c.) \]  (B.4)

Under the condition that, \[ |\frac{\partial \xi_m}{\partial N}| \ll \epsilon_o \], the field within the cavity can still be accurately approximated by an expansion of these modes. In addition, the (very good) slowly time varying envelope approximation is made that:

\[ \omega_m \xi_m \gg \frac{\partial \xi_m}{\partial t} \]  (B.5)

and

\[ \omega_m^2 \xi_m \gg \frac{\partial^2 \xi_m}{\partial^2 t}. \]  (B.6)

Using the above approximations in equation B.1, the following rate equations for each mode can be formed:

\[ \frac{\partial \xi_m}{\partial t} = -i\omega \frac{\partial \xi}{\partial N} \sum_n \xi_n \langle NA_m | A_n \rangle \]  (B.7)

where \( \langle A|B \rangle \) is the spatial inner product between \( A \) and \( B \). By expressing \( \xi_m \) in terms of its magnitude and phase, \( \xi_m = E_m e^{i \phi_m} \), phase and field amplitude equations can be formed:

\[ \frac{\partial E_m}{\partial t} = \sum_n E_n \langle NA_m | A_n \rangle (\sin (\phi_n - \phi_m) + \alpha \cos (\phi_n - \phi_m)) - \frac{E_m}{\tau} \]  (B.8)

\[ \frac{\partial \phi_m}{\partial t} = \sum_n \frac{E_n}{E_m} \langle NA_m | A_n \rangle (\alpha \cos (\phi_n - \phi_m) + \sin (\phi_n - \phi_m)) \]  (B.9)

where the carriers have been normalized so that \( \omega \frac{\partial \xi}{\partial N} = i + \alpha \) and an exponential cavity loss time constant, \( \tau \) has been incorporated.

At this point, a particularly simplistic geometry is introduced, to derive a set of coupled mode equations easily without losing sight of the essential physical processes.
The geometry consists of having perfect electric conductor walls which pin the TE modes to zero (permitting sine modes) and do not admit the flow of carriers (i.e. $\frac{\partial N}{\partial x} = \frac{\partial N}{\partial z} = 0$ at the boundaries and cosine modes are permitted).

The first case considered will be that of two longitudinal modes with degenerate transverse and lateral modes, so the problem is essentially reduced to one-dimension. In this case, the modes considered will be:

$$A_m = \sin \left( \frac{m\pi z}{L} \right)$$

(B.10)

where $L$ is the longitudinal length of the cavity.

To simplify the analysis, once again, the macroscopic carrier rate equation will be used, which is given by:

$$\frac{\partial N}{\partial t} = P - \frac{N}{\tau_N} + D \nabla^2 N - 2N |E|^2$$

(B.11)

where $E$ has been suitably normalized to remove all prefactors and $N$ refers to the carrier density above transparency. $P$ is offset by the transparency pumping rate, so that $P = P_0 - N_0/\tau_N$ where $N_0$ is the transparency carrier concentration and $P_0$ is the conventional pumping rate. The term containing $|E|^2$ has five different spatially varying components, with $k$ vectors of magnitude $k_m - k_n$, $0$, $k_m + k_n$, $2k_n$ and $2k_m$ (assuming that the perturbation to the flat carrier density $N_0$ is relatively small - i.e. taking the first order in the perturbation). A further approximation is allowed since $k_m - k_n \ll 2k_m \approx 2k_n$ in the systems that we are interested in, which means that the carrier perturbations of the order of $k_m$ will be washed out by diffusion and only $N_0 = N$ and $N_{k_m - k_n} = N_i$ need be accounted for (see Reference [20]).
By using equations B.8, B.10, B.11 and cosine carrier distributions for the carrier densities, the following set of equations can be formed:

\[
\frac{\partial E_m}{\partial t} = -cE_m + NE_m + fN_lE_n\cos(\Psi) - \alpha\sin(\Psi) / 2 \tag{B.12}
\]

\[
\frac{\partial E_n}{\partial t} = -cE_n + NE_n + fN_lE_m\cos(\Psi) + \alpha\sin(\Psi) / 2 \tag{B.13}
\]

\[
\frac{\partial \Psi}{\partial t} = \Delta - fN_l(\alpha(E_m/E_m - E_m/E_n)\cos(\Psi) + (E_n/E_m + E_m/E_n) \sin(\Psi)) \tag{B.14}
\]

\[
\frac{\partial N}{\partial t} = P - bN - 2N(E_n^2 + E_m^2) - 2N_lE_mE_n\cos(\Psi) \tag{B.15}
\]

\[
\frac{\partial N_l}{\partial t} = -Dk^2N_l - bN_l - 2N_l(E_n^2 + E_m^2) - 4(N)E_mE_n\cos(\Psi) \tag{B.16}
\]

where \( \Psi = \phi_m - \phi_n \). This model is mathematically closest to that of Reference [9]. It has the advantage of taking into account the dynamics of the flat and perturbed carrier distributions explicitly with a rate equation, whereas Reference [9] doesn’t consider the flat carrier’s response and uses a response function for the perturbed carriers.

This formulation, however, doesn’t allow for stable locked states with both modes lasing, since the competition for the flat carrier bath causes one mode to kill the other one (i.e. the mode with the higher gain). The inadequacy of this longitudinal mode model to allow for frequency locking is indicative of requiring two carrier baths, each feeding one mode predominantly. The act of splitting up the carriers is actually justifiable for a geometry in which two lateral modes exist, so their spatial overlap with the carriers in the lateral direction is distinct.

Consequently, we perform the operation of splitting up the carrier distribution into two baths that feed each mode preferably. This process will not be justified by a transform analysis, because such a task is not performed easily or elegantly. The results are consistent with the PEC cavity described above if all higher order carrier distributions are disregarded (which is not entirely valid since \( k_z \ll k_x \) and so the lateral sum perturbations may not be sufficiently washed out by carrier diffusion).

Although the operation is phenomenological in nature (but can be performed more rigourously by an expansion of the carrier distribution using spatially separated bins or
some orthogonal expansion series), splitting up the "flat" carriers into two separate baths requires self consistent changes to the field and phase equations. These have been accounted for by using a common modal overlap for the carrier and field rate equations and reintroducing terms (particularly to the phase equation) that drop out of the longitudinal mode analysis. The diffusive coupling between the two split carrier baths is assumed to be linear, by virtue of allowing for an arbitrary (not necessarily orthogonal in any sense) division of the carriers.

Consequently, the following set of equations is found:

\[
\begin{align*}
\dot{E}_m &= -cE_m + ((1 - y)N_m + yN_n)E_m + fN_lE_n(\cos(\Psi) - \alpha \sin(\Psi))/2 \quad (B.17) \\
\dot{E}_n &= -cE_n + ((1 - y)N_n + yN_m)E_n + fN_lE_m(\cos(\Psi) + \alpha \sin(\Psi))/2 \quad (B.18) \\
\dot{\Psi} &= \Delta - \alpha(2y - 1)(N_n - N_m) - fN_l(\alpha(E_n/E_m - E_m/E_n) \cos(\Psi) \\
&+ (E_n/E_m + E_m/E_n) \sin(\Psi)) \quad (B.19) \\
\dot{N}_m &= P_m - bN_m - 2N_m((1 - y)E_m^2 + yE_n^2) - 2N_lE_mE_n \cos(\Psi) \quad (B.20) \\
&+ Dk_{mn}^2(N_n - N_m) \\
\dot{N}_n &= P_n - bN_n - 2N_n((1 - y)E_n^2 + yE_m^2) - 2N_lE_mE_n \cos(\Psi) \quad (B.21) \\
&+ Dk_{mn}^2(N_m - N_n) \\
\dot{N}_l &= -Dk_l^2N_l - bN_l - 2N_l(E_n^2 + E_m^2) - 4(N_m + N_n)E_mE_n \cos(\Psi) \quad (B.22)
\end{align*}
\]

where \( f \) is the cross modal overlap with the beating carrier distribution and \( y \) is the fraction of carriers that each bath feeds. In the case of \( y = 0.5 \) and \( f = 1 \), the same equations as those shown for the coupling between longitudinal modes result.

The results of the simulations show for the laser parameters relevant to ridge waveguide semiconductor lasers, \( N_m, N_n \gg N_l \), which suggests that the approximation of not considering higher order spatial perturbations to the carriers or the field distribution is valid.
Appendix C

Intensity Beating in Coupled Lasers

In this appendix, the periodicity of the rf beat spectra will be proven.

C.1 Beat Peak Frequency Spacing

General form of Alder’s Equation:

\[ \dot{\phi} = a + b f(\phi) \quad |f(\phi)| \leq 1 \]  

(C.1)

If \(|a| > |b|\) (i.e. the oscillators are outside the locking bandwidth), \(\phi(t)\) increases \((a > 0)\) or decreases \((a < 0)\) monotonically.

Assuming that \(f(\phi)\) is periodic in \(\phi\) with period that is a multiple of \(\pi \rightarrow f(\phi) = f(\phi + n\pi)\).

Case of \(n = 2\) (e.g. \(f(\phi) = \sin(\phi)\)):

\[ \phi(t) = \phi(t + T) - 2\pi \] where \(T\) is the time that it takes to go from \(\phi = \alpha\) to \(\phi = \alpha + 2\pi\) (since the phase evolution given by the generalized form of Adler’s equation is the same for \(\alpha\) and \(\alpha + 2\pi\)).

\(I_{AC} \sim \sin[\phi(t)] = \sin[\phi(t + T) - 2\pi] = \sin[\phi(t + T)]\) so \(\sin[\phi(t)]\) is periodic in \(T\) so all harmonics of \(1/T\) can be expressed because:

\[
\text{F.T. of } I_{AC} \sim \int I_{AC} \exp(-i\omega t) \, dt \\
\sim \int \sin[\phi(t)] \exp(-i\omega t) \, dt \\
= \int \sin[\phi(t + T)] \exp(-i\omega t) \, dt \\
= \int \sin[\phi(t)] \exp(-i\omega(t + T)) \, dt
\]
\[ \omega = \frac{2\pi N}{T} \] where \( N \) is an integer, so all harmonics are allowed.

**Case of** \( n = 1 \) (e.g. \( f(\phi) = \sin(2\phi) \)):

\[ \phi(t) = \phi(t+T) - \pi \] where \( T \) is the time that it takes to go from \( \phi = \alpha \) to \( \phi = \alpha + \pi \) (since the phase evolution given by the generalized form of Adler’s equation is the same for \( \alpha \) and \( \alpha + \pi \)).

\[ I_{AC} \sim \sin[\phi(t)] = \sin[\phi(t + T) - \pi] = -\sin[\phi(t + T)] \] so \( \sin[\phi(t)] \) is periodic in \( T \) so only odd harmonics of \( 1/2T \) can be expressed because:

\[
\text{F.T. of } I_{AC} \sim \int I_{AC} \exp(-i\omega t) \, dt \\
\sim \int \sin[\phi(t)] \exp(-i\omega t) \, dt \\
= -\int \sin[\phi(t + T)] \exp(-i\omega t) \, dt \\
= -\int \sin[\phi(t)] \exp(-i\omega(t + T)) \, dt
\]

\[ \omega = \frac{\pi(2N + 1)}{T} \] where \( N \) is an integer, so only odd harmonics are allowed.

**General Case**

Can use the same arguments, but will have distinct cases for different periodicities of the general form of Adler’s equation. These can be quickly identified experimentally.
Appendix D

Relative Peak Heights

Information about the locking bandwidth and intrinsic frequency detuning can also be obtained from the relative peak heights in the spectrum. In fact, by knowing the frequency spacing between the beat peaks, the relative peak heights and the form of Adler’s equation that governs the dynamics of the two modes, the locking bandwidth and intrinsic frequency separation can both be uniquely determined.

In this Appendix, it will be shown how the relative peak heights allow you to extract information about the system. For simplicity, the basic form of Adler’s equation, given by:

\[
\frac{\partial \phi}{\partial t} = \Delta \omega + \Gamma \sin(\phi)
\] (D.1)

will be used in this analysis. This equation describes the relative phase between two linearly coupled oscillators, where \( \Gamma \) is the locking bandwidth and \( \Delta \omega \) is the intrinsic frequency detuning between the two oscillators.

Since the beat term between the two oscillating modes evolves in time with a \( \sin(\phi) \) dependence, the beat peaks in the Fourier spectrum will correspond to the Fourier amplitudes the function \( f = \sin(\phi) \). Consequently, we seek an expression for \( f \).

By integrating Adler’s equation by parts, it can be shown that:

\[
f = \sin(\phi) = \frac{a - \sin(bt)}{a \sin(bt) - 1}
\] (D.2)

where \( a = \Gamma / \Delta \omega \) and \( b = \sqrt{(\Delta \omega)^2 - \Gamma^2} \). An infinite expansion of the denominator was attempted to obtain an analytical expression of the Fourier components (which are scaled delta functions at the beat frequencies). This has not yet lead to a final result.
Using 2 different numerical methods (both the FFT and a numerical integration routine) on Equation D.2, the same results for the relative peak heights as a function of $\Delta \omega / \Gamma$ were found for all adjacent peaks in the case of linear coupling. This is shown in Figure D.1.
NOTE TO USERS

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References


