Modeling of Synchronous Machines for System Studies

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of Electrical and Computer Engineering
UNIVERSITY OF TORONTO

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Abstract

This thesis proposes a new method for modeling synchronous machines for system studies and analysis. The new approach is based on machine dimensions and material properties. A sectoral model of the machine is developed. A linear reluctance matrix is used to relate the flux and magnetomotive force throughout the different sectors of the machine. The saturation effects are evaluated according to the iron sections' properties and dimensions, and then used to adjust the flux distribution. Skin-effect models are developed and used to describe the eddy current distribution and depth of penetration.

A new frequency-domain modeling approach is proposed and used to predict the operational inductances of the machine. The effect of magnetic saturation on the frequency response of the machine is investigated.

The results of the developed modeling approach are compared with measurements made for a large turboalternator. The model results are essentially as accurate as can be expected from the reported measurements.

The thesis also presents a new set of equivalent circuit models for the frequency-domain analysis of the synchronous machine. The new circuit models are thought to be more physically appropriate for describing the machine performance. The accuracy of the developed circuit models is comparable to that of the third order conventional circuit models, but with a fewer number of circuit parameters.
"This is of the bounty of my Lord, to test me as to whether I am grateful or ungrateful. He who is grateful is but grateful for his own good; and he who is ungrateful, verily, my Lord is self-sufficient, most generous..."

Prophet Solomon

Quran 27:40
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Chapter 1

Introduction

1.1 Background

Power systems consist of elements for generation, transmission, distribution, and loads. The synchronous machines are the main generating units of power systems. From the load side, synchronous motors are also used. This makes the synchronous machine one of the most important components of electric power systems.

The main overall objectives of power systems are security and reliability. Security of power systems means that the power systems are within their steady-state power flow constraints. Reliable operation of power systems refers to their ability to continuously supply the required electrical energy without interruption under abnormal operating conditions such as faults, switching, and load changes. In both modes of operation the power system behavior is dependent on the electrical and electromechanical processes of synchronous machines.

Therefore, modeling of synchronous machines is essential for power systems analysis and studies.

1.2 Motivation

This study is motivated by the increasing requirement for improved models of synchronous machines for system analysis and studies. This requirement is a consequence
Chapter 1. Introduction

of the continuous growth of the size of power systems and their operation near stability limits. Conventional, widely-used, synchronous machines models have following major limitations,

- Most analytical models for synchronous machines are based on Park’s transformation, which involves superposition of effects in the direct- and quadrature-axes. With saturation, the effects are nonlinear. Thus, superposition does not strictly apply. The orthogonal direct- and quadrature-axes are not independent magnetically. To obtain adequately accurate predictions from such conventional models, various and sometimes complex artifices have been employed.

An approach which avoids this problem is the use of finite element analysis of the magnetic structure of a one-pole machine sector. This typically requires 2000-10000 elements. Also, the analysis must be repeated for each time step. The resulting process is too complex, too time-consuming and too demanding on computer storage capacity to be attractive for most steady-state analysis.

- The structure and parameters of the currently used circuit models of the synchronous machines are inadequately related to the physical dimensions and the material properties of the machine. If there is a direct linkage between a circuit parameter and a section of the machine, there is a basis for improved insight into the explanation of the machine’s behavior. Also, fruitful communication between system analysts and machine designers is promoted when the models used in analysis and operation can be related directly to those quantities under the control of the designer. The approach of most system analysts seems to have been to accept the properties of the machine. Little attention has been devoted to communicating to the designer the properties which might be more desirable.

1.3 Objectives

The main objectives of this thesis are,
The development of an alternate approach for modeling of steady-state performance of synchronous machines including the effects of magnetic saturation. This approach should preserve the important property of finite element analysis that each model parameter is related to a physical section of the machine, its shape, and its material properties. It thus has a physical reality which is observed to be usually lacking in the conventional two-axis models.

The development of a frequency domain model for synchronous machines whose parameters are evaluated directly from dimensions and material properties of the machine. Developed models should use skin-effect elements dependent on frequency rather than constant parameters as conventional models.

1.4 Thesis Outlines

The currently used synchronous machine models for systems analysis are briefly explained in Chapter 2.

The mathematical formulation of the linear version of the proposed machine modeling approach is presented in Chapter 3.

In Chapter 4, modeling of the steady-state performance of synchronous machines including the effect of magnetic saturation is discussed. A new modeling scheme is proposed. Results from the proposed model are compared with measurements made for a wide range of load conditions on a 588-MVA turbogenerator.

Chapter 5 covers the frequency domain modeling of the synchronous machine. Alternative equivalent circuits for frequency domain modeling of the synchronous machine are presented. A new model for producing the frequency response of the synchronous machine from the machine's dimensions and material properties is also proposed. To verify the models, models' results are compared with measurements for the 588-MVA turbogenerator.

In Chapter 6, the modeling approaches introduced in Chapters 4 and 5 are used to investigate the possibility of using the low flux-level frequency response of the machine to model the machine's performance at its rated voltage.
Chapter 7 presents the thesis conclusions and major contributions. It also discusses suggestions for further research.
2.1 Introduction

It is stated in IEEE std 1110-1991 [1] that a complete synchronous machine model consists of a combination of a model structure and a set of parameter values. This definition can be broken into two interconnected distinct stages. The first stage is the construction of the model structure which is the basic form for machine representation. The second stage is the evaluation of the model parameters.

The model structure can be formed as lumped-parameter equivalent circuit, transfer function, differential equation representation, etc. The model structure includes, as well, the order number of the model. On the other hand, the model parameters can be evaluated either based on the manufacturers’ data, test results, or some other mathematical techniques.

This chapter presents the synchronous machine models which are currently used for systems studies and analyses. In Section 2.2 the mathematical formulation and theoretical backgrounds of currently used machine models are presented. The effect and modeling of saturation is discussed in Section 2.3. The basic categories of parameters calculation are presented in Section 2.4. Section 2.5 describes the available techniques for machine interface with the power systems. Chapter conclusions are

presented in Section 2.6.

2.2 Conventional Model Structure

The conventional electrical model of the synchronous machines for system studies is based on what is known as Two Reaction Theory, introduced by Blondel and Doherty [2] - [6]. In this modeling approach, the stator of the machine is considered as three windings 120° electrical degrees apart. The rotor structure has an excitation or field winding and one or more equivalent rotor body windings. The magnetic axis of the machine is defined as the direct axis (d-axis), and an orthogonal quadrature axis (q-axis) is located 90° electrical degrees ahead of the direct axis. The equivalent rotor windings are used to reflect the induced current paths in a round rotor iron body, or in the damper bars that are usually used in round rotor turbogenerators and in salient pole hydrogenerators. One group of the rotor equivalent circuits is aligned along the direct axis and another along the quadrature axis. This configuration is illustrated in Figure 2.1 [7].

2.2.1 Ideal Synchronous Machine

The modeling approach presented hereafter is carried out under the following assumptions [8]-[7],

1. Saturation effects are neglected. This allows the application of the superposition principles as the machine is then linear.

2. Stator winding currents are assumed to set up a magnetomotive force (mmf) sinusoidally distributed in space around the air gap. Therefore, the effect of space harmonics in the field distribution is neglected.

3. The mmf acting along the d-axis produces a sinusoidally distributed flux wave along that axis. The same applies for the q component of the mmf.
4. The damper bars can be represented by a number of short-circuited hypothetical windings along the two orthogonal axes d and q. The model order is then defined as the number of rotor circuits along either the d- or q-axis.

Under these assumptions, the machine is then known as an ideal synchronous machine.

### 2.2.2 Mathematical Formulation

By applying of Maxwell's equations to the configuration shown in Figure 2.1, the generator voltage $v_a$ for phase 'a' is

$$v_a = \frac{d}{dt} \Psi_a - r_a i_a$$  \hspace{1cm} (2.1)

where $\Psi_a$ is the flux linking phase a, $r_a$ is the winding resistance, and $i_a$ is the current
of phase a. The flux linking the three phases of the stator winding is a function of $\theta$, i.e., the angular displacement of the d-axis from phase a, as illustrated in Figure 2.1.

The machine model was further developed by Park who mathematically transformed the three-phase time-varying stator quantities (voltages, currents, and flux linkages) into time invariant direct- and quadrature-axis quantities under steady-state conditions [8]-[9].

The transformation matrix $[P]$ from the phase reference frame 'abc' to the rotor frame 'dq0' is given by

$$
[P] = \begin{bmatrix}
\cos \theta & \sin \theta & 1 \\
\cos(\theta - 120^\circ) & \sin(\theta - 120^\circ) & 1 \\
\cos(\theta + 120^\circ) & \sin(\theta + 120^\circ) & 1
\end{bmatrix}
$$

(2.2)

Using this transformation, all inductances used in modeling the machine are constants and neither time nor position-dependent. A shortcoming of Parks transformation is the power variance between the real machine and the rotor-referred machine.

This approach has been used extensively in developing different models. The basic differences between these models lie in their corrective approaches with respect to the above assumptions especially the inclusion of saturation effects and solid iron rotor body representation.

Crary and Concordia [11]-[14] extended Park's equation to include any symmetrical stationary network connected to the armature. They added the equations of the system to the equations of the machine model and transformed them simultaneously from the 'abc' axes to the 'dq' axes in the same manner as Park transformed the machine. This treatment suits a single machine connected to an infinite bus through a transmission line. But as the transmission system becomes more complicated, or as the number of machines exceeds one, the resulting equations become very complicated.

In [15], Kron introduced two orthogonal transformations of the stator quantities. The first transformation is from the phase reference 'abc' to the 'a$\beta_0$' frame through the following matrix

The second transformation is from \( \alpha\beta0 \) frame to the \( \text{dq}0 \) frame by

\[
[K_1] = \sqrt{\frac{2}{3}} \begin{bmatrix}
1 & 0 & \frac{1}{\sqrt{2}} \\
-\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\
-\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\] (2.3)

The second transformation is from \( \alpha\beta0 \) frame to the \( \text{dq}0 \) frame by

\[
[K_2] = \begin{bmatrix}
\cos \theta & \sin \theta \\
\sin \theta & -\cos \theta
\end{bmatrix}
\] (2.4)

These two orthogonal transformations show that a reference frame can be transformed to any other frame rotating at any speed. The stationary network connected to the stator of the machine can be transformed separately from the transformation of individual machines [16]-[17].

It is worth mentioning that some models are developed using the direct phase quantities rather than Parks transformation. Subramaniam and Malik presented this approach in [18] where the same assumptions of the ideal synchronous machine were used. In [19], Marti and Louie modified the direct phase model to include the saturation effect along each instantaneous direction of the resultant mmf in the air gap.

2.3 Saturation Representation

The implications of synchronous machine saturation have been reported and discussed extensively in the literature [20]-[27]. The saturation phenomenon affects significantly the internal excitation and load angle of the generator. Hence, accurate representation of saturation is important for accurate modeling of the generator. Many publications discuss this subject and investigate the difference in approach between salient pole and round rotor turbogenerators.

A majority of the published approaches have associated the saturation with the air gap flux or the voltage back of stator leakage reactance. While for the salient pole machines, the saturation is usually considered only in the direct axis, in solid iron ro-
tor turbogenerators, saturation is significant in both the direct and quadrature axes. Many approaches to account for this saturation phenomenon have been proposed. In [20], Shackshaft and Hensler assumed a constant difference between the saturated values of the d- and q-axes' mutual reactance to modify their values. This assumption was based on experimental tests. The evaluation of the q-axis saturation was also highlighted. Harley et al. [21] investigated different techniques to represent the saturation effect based on deriving correction factors to the unsaturated inductances. They demonstrated the effect of neglecting the q-axis saturation. They also investigated the effect of considering the total flux in calculating the saturation-level factors rather than the separate mmf components. They proved that the use of different saturation factors based on resolved components of flux linkage gives results similar to those of the model presented in [20]. An iteration-based model was developed by de Mello et al. where the saturation effect is first neglected to get an initial set of values for the currents, voltages, and mmfs [22]. These mmf values are then used in conjunction with the open circuit characteristic (OCC) to get the corresponding voltages and rotor position. The process continues until a tolerance criterion is met.

In [23]-[25], it was noted that saturation occurs in the stator teeth and yoke due to total stator flux, and it occurs in the rotor due to net rotor flux, but it does not occur in the linear air gap.

The idea of step-by-step computation of the magnetic field has been published as a direct attack to the saturation problem by Silvester and Chari in [26]-[27]. However, this method requires the field equations to be solved at each step, which requires a great number of calculations.

### 2.4 Model Parameters

As stated above, the models are constructed by determining the parameter values for the model elements such as reactances and resistances. This process can be based on either measurements or analytical approaches. Analytical methods usually employ data obtained from simulating the electromagnetic phenomena of the synchronous
A common example of such techniques is the finite-element method. This technique facilitates the representation of the details of magnetic flux distribution, including different effects such as saturation and eddy currents.

The following sections outline the different approaches for determination of synchronous machine parameters.

### 2.4.1 Parameters Obtained by Test Results

In reference [1], various tests are described to determine the synchronous machine electrical parameters. Machine parameters determined from these tests (mostly open-circuit and short-circuit tests) are direct-axis values only. This includes the unsaturated synchronous reactance $x_{du}$, the transient and subtransient reactances $x_{d}'$, $x_{d}''$, the transient and subtransient short circuit time constants $T_d'$, $T_d''$ and the transient and subtransient open circuit time constants $T_{do}'$, $T_{do}''$. It should be mentioned that this method of obtaining machine parameters presupposes a second-order circuit model of the synchronous machine.

### 2.4.2 Frequency Response Testing

The basic idea of this approach is to develop two port models of solid rotor turbo-generators, based upon measurements made at the generator terminals. These measurements are represented in the form of operational transfer functions which are defined in terms of a number of time constants. Mathematical expressions are then developed to express the machine parameters in terms of these time constants. This approach was acknowledged by Concordia in [28] as the likely future approach to the magnetic field calculation approaches considered at that time. Schultz et al. constructed an equivalent circuit with operational parameters whose real and imaginary components can be determined [29]. However, machine representation as a two-port network was first introduced by Umans et al. in [30]. They further investigated the necessary data-fitting algorithms to extract the machine parameters [31]. In [32], Dandeno and Poray obtained machine parameters by performing Standstill Frequency
Response (SSFR). The basic idea is to excite the machines stator or field while the machine is at standstill so that the exciting currents are quite low. The exciting signal has a frequency range from 0.001 Hz up to between 100 Hz and 200 Hz. Dandeno et al. gave additional experimental data using SSFR in the d- and q-axes in [33], where the machine models are validated by comparisons with different power system on-site simulation tests. Similar results are also reported by Schwenk in [34]. Some additional frequency response testing of machines when operating on line at load (OLFR) has also been investigated by Dandeno et al. in [35], where analytical adjustment of SSFR rotor equivalent circuit values to values reflecting on line conditions is also discussed. Similar results are also reported by Coults et al. in [36]. In this OLFR test, the generator field is excited over a range of frequencies when it is fully or partially loaded at normal voltage. The range of exciting frequencies for this test is between 0.1 Hz to about 5 Hz. A simpler form of testing known as open circuit frequency response (OCFR) is presented in [37]. In this test, the machine is open circuited, running at rated speed and with its stator terminal voltage ranges between 0.5 and 1.0 p.u. of normal voltage. The range of exciting frequency is from 0.01 Hz to 10 Hz. This is a broader range than that of the OLFR. Jack et al. emphasized the effect of the end rings on parameter determination using frequency response tests in [38].

A fourth-order model is developed by Chuanli and Ming to represent the rotor circuit [39]. The model is a state space model with the four state variables: load angle, slip, field flux, and quadrature circuit damping flux. In [40], Bacalao et al. present a frequency based model which considers both the saturation and the interface with an electromagnetic transient program. The model represents the machine equations in voltage form rather than current form. It uses the recursive convolution to obtain the time-domain solution.

### 2.4.3 Parameters Provided by Manufacturers

The machine parameters can be calculated by the manufacturers. IEEE std 1110-1991 [1] presents two cases of parameters derived by two manufacturers in the machine design state. Generally, manufacturers calculate constants for the direct axis by first
calculating parameter values for a circuit structure usually of the second order and then converting these circuit parameters to synchronous, transient and subtransient reactances and time constants.

### 2.5 Interfacing the Generator Model with the Transient Programs

Although the generator and the network to which it is connected can be solved as one system of equations, it is not easy to develop a general purpose computer program which can handle any network configuration.

When electric network components are connected together, certain boundary conditions must be met. These conditions are based on Kirchhoff's laws. That means the terminal voltages of the network and the generator should be the same at the connection points. Also, the output currents from one side (network or generator side) should be the inputs to the other.

There are basically two possible ways of interfacing a generator model with a network dynamic model. The first one is introduced by Brandwajn and Dommel [41],[42]. It is based on the calculating a three-phase Thevenin equivalent circuit of the network as seen from the generator stator terminals and solving it with the full set of generator equations. The final solution is then obtained by superimposing the voltage changes which result from the generator currents on the solution without the generator being connected. In [43], another approach is presented by the same authors. It is based on developing the Thevenin equivalent circuit for the generator in the form of a voltage source behind a time-invariant, symmetrical resistance matrix. The complete solution is then obtained by solving the network with the generator treated as one more voltage source.

These two approaches utilize the compensation theorem [44]-[47] to apply the superposition principle at each time step, where the system is considered linear during the transition from one time step to the next.
2.5.1 Representation of Magnetic Saturation in the Electromagnetic Transient Programs (EMTP)

Magnetic saturation in a synchronous machine affects not only the transient and steady state stability but also the electromagnetic transients in the corresponding power system.

Reference [48] shows that in case of steady-state analysis, it is usual to use an equivalent linear machine which gives correct answers at the particular operating point and approximate answers in the neighborhood. The open circuit characteristic (OCC) for this equivalent linear machine is represented by a straight line from the origin to the operating point, as shown in Figure 2.2. However, the concept of linear machine cannot be used for transient studies.

Figure 2.2: Linearization for steady state analysis.

Under transient conditions, the saturation effect appears both in the transformer- and speed-voltage components. For the speed-voltage terms, the saturation effect is considered by using linear correcting multiplier factors to convert the unsaturated flux linkage components to their saturated equivalencies. For the transformer-voltage, the saturation curve is approximated by a two-segment line as shown in Figure 2.3.
Figure 2.3: Linearization for transient analysis.

2.6 Conclusions

The models based on Parks transformation serve well for many applications, but since they invoke the principle of superposition, they are inherently incapable of including the effect of magnetic saturation adequately for all operating conditions. It should be mentioned, as well, that saturation representation is based on empirical formulas which are test- and machine-dependent.

For the finite-element based models, the electromagnetic field is typically divided into 2000-10000 elements. This increases the computations' complexity, analysis time, and computer storage requirements.

The rotor of the non-salient pole synchronous machine usually consists of a solid core which is not laminated in most cases. Therefore, damping currents can flow in the rotor body as well as the slot wedges and damper bars (if they exist). Consequently, an infinite number of current paths exist. So, it is not possible to model these elements with a simple lumped parameter model. In the most accurate models used currently, each of the direct- and quadrature-axis rotor circuits is represented by a third-order model. The disadvantage of this model is the difficulty in determining
its constants. The second-order model is rather easier in this aspect but it does not give acceptable results for many machines with damper windings. In other words, the required accuracy is dependent on the specific study being made on the system.

Also, the parameter determination in most of the models used is based on mathematical fitting for a set of measurements. This means the parameter values are not linked to the physical characteristics of the machine. It also makes the model development contingent on the existence of test results.

It is worth mentioning that the different techniques available for parameters determination are totally or partially dependent on some set of testing done under certain operating conditions. In order to make test results usable over a wider range, correcting factors should be used. This prevents model generality. Therefore, the implication of these factors is that available models for machine representation may not be used with confidence in simulating machine performance over a wide range of operating conditions especially those of the saturated machine.
Chapter 3

Linear Reluctance Model for Synchronous Machines

3.1 Introduction

This chapter presents the fundamentals and mathematical formulations of a recently developed modeling approach for synchronous machine [51]. The approach is based on developing an equivalent reluctance model from the dimensions and material properties of the machine. It provides inherent flexibility for representation of magnetic saturation and eddy currents.

This chapter focuses on developing the magnetic equivalent circuit for the round rotor machine, applicable in the linear regime, where the permeability of iron can be considered to be high. The rotor field is assumed to be non-time varying. This linear model will be validated by comparing its results with measurements available for a turboalternator. The effects of magnetic saturation will be introduced in Chapter 4, and the effects of rotor eddy currents will be introduced in Chapter 5.

The chapter is organized as follows: Model assumptions are listed in Section 3.2. Section 3.3 presents the conceptual idea of representing the machine as a number of sectionalized layers. Modeling of the different sections of the machine is then introduced in Sections 3.4 to 3.6. The mathematical formulation of the proposed reluctance model is shown in Section 3.7. In Section 3.8, the model validity is demonstrated by
applying it to an existing machine (Nanticoke #5) where the developed model is used to evaluate the basic machine parameters. The model results are then compared with those obtained from tests. The chapter summary is presented in Section 3.9.

3.2  Model Assumptions

To develop the linear model of the machine, the following assumptions are made.

1. The stator windings are assumed to be sinusoidally distributed. This assumption allows to neglect the effects of tooth ripples.

2. The permeability of the iron sections is high. Therefore, the reluctance of the flux path is composed solely of the air gap and leakage reluctances.

3. Magnetic saturation effects are negligible. With magnetic saturation neglected, the superposition principle is applicable. However, since the saturation effects are crucial to machine performance, accounting for these phenomena will be discussed in Chapter 4.

3.3  Machine Sectionalizing

In its radial direction, the machine can be regarded as three consecutive layers. The outer layer represents the stator windings, the second layer represents the air gap between the stator and the rotor, and the third layer is the rotor body.

In the circumferential direction, each one of these layers is sectored to a number of sectors per pole. The following sections describe how the rotor and the stator of the machine are sectionalized.

3.3.1  Rotor Sectionalizing

It is noted that the rotor of non-salient pole synchronous machine usually consists of a solid core, which is not laminated in most cases. Therefore, induced currents can
flow in the rotor body as well as the slot wedges and damper bars (if they exist). Therefore, without loss of generality, the developed modeling approach will be more concerned with machines with round rotor. Moreover, since one of the main objectives of this research project is to develop a better representation of the rotor body and its dynamics, the circumferential sectionalizing of the machine is dictated by the rotor configuration, i.e., rotor slotting, damper bars, and rotor teeth.

For uniformly slotted non-salient pole machines, the number of rotor sections is the same as the number of rotor slots. However, if the round rotor is not fully slotted, sectors of equal angles are used. The angular width of each section is equal to the rotor slot angle.

Figure (3.1) shows a cross section of a uniformly-slotted non-salient pole machine with its different sections.

3.3.2 Stator Sectionalizing

To have a compatible sectionalizing of the stator, the stator is first modeled by an equivalent current sheet, as will be explained in Section 3.4. Then, the stator is divided into the same number of sections which has been implied by the rotor configurations. The rotor and stator sections are linked together through a set of linear reluctances representing the air gap.

3.3.3 Sectors Numbering

The quadrature-axis is chosen as leading the direct-axis by ninety electrical degrees as usual [53]. The numbering starts at the d-axis and advances towards the q-axis. The three sections, which have the same angle with respect to the d-axis, are assigned the same number, i.e., the $k^{th}$ section of either the rotor, the stator, or the gap, is centered at angle $\theta_k$ measured from the d-axis. These three sections comprise the sector number $k$. This numbering scheme is illustrated in Figure 3.1.
Figure 3.1: Cross section of a uniformly-slotted 2-pole synchronous machine.
3.4 Stator Model

3.4.1 Stator Winding

The stator of the synchronous machine is wound with three sinusoidally distributed windings. Recalling Assumption 1 in Section 3.2, the effective sine-distributed number of turns of each phase $N_{se}$ is [54]

$$N_{se} = \frac{4}{\pi} K_w N_s$$ \hspace{1cm} (3.1)

where $N_s$ is the actual number of turns of each phase, and $K_w$ is the stator winding factor.

3.4.2 Stator Current

It is assumed that, zero sequence currents in the stator, if they occur, can be treated separately. Thus, for this condition

$$i_a + i_b + i_c = 0$$ \hspace{1cm} (3.2)

where $i_a$, $i_b$, and $i_c$ are the three phase currents. The total effect of a set of instantaneous currents in all three phase windings can be represented by the following single space vector [54]

$$\tilde{i}_s = i_a + j\frac{i_a + 2i_b}{\sqrt{3}} = \tilde{i}_s e^{j\alpha_s}$$ \hspace{1cm} (3.3)

where $\tilde{i}_s$ is the stator current space vector, and $\alpha_s$ is the angle between the space vector and the axis of phase $a$, as shown in Figure 3.2.

Assuming that the angle $\beta$ is the rotor position with respect to the stator current, the stator current referred to the rotor is

$$\tilde{i}'_s = \tilde{i}_s e^{j\beta}$$ \hspace{1cm} (3.4)

For a machine with $P$ poles, the stator winding can be represented as a revolving
current sheet $\vec{K}_s$ on the inner surface of the stator. This current sheet is given by [54]

$$\vec{K}_s = j\frac{3N_{se}}{2P} \vec{i}_s \quad A/\text{rad}$$  \hspace{1cm} (3.5)

This current sheet can be expressed referred to the rotor as [54]

$$\vec{K}_s' = j\frac{3N_{se}}{2P} \vec{i}_s' \quad A/\text{rad}$$  \hspace{1cm} (3.6)

### 3.4.3 Stator MMF

The sinusoidally distributed mmf sheet is replaced by a set of concentrated mmf sources which are equal to the integral of the mmf sheet over the appropriate arc. So for the $k^{th}$ sector of the stator winding, whose width is $\Delta \theta_k$ and centered at $\theta_k$, the sector's mmf $\varpi_{sk}$ is given by

$$\varpi_{sk} = \text{Re}(\vec{K}_s' \Delta \theta_k e^{-j\theta_k})$$  \hspace{1cm} (3.7)

### 3.4.4 Stator Slot Leakage

Figure 3.3 shows a typical stator slot for synchronous machines [57]. Assuming uniform current distribution over the slot, the permeance of the stator slot leakage $\varphi_{1s}$ is given by
\( \varphi_{ls} = \text{embedded length of slot} \times \text{slot permeance ratio} \times \mu_0 \)  

where the slot permeance ratio \( P_{ls} \) is the effective ratio of depth to width of the slot.

For the slot shown in Figure 3.3, the slot permeance ratio is given by [57]

\[
P_{ls} = \frac{k_s}{w}(d_3 + \frac{d_1}{3}) + \frac{d_1}{12w}(1 - k_s) - \frac{d_2}{4w}(k_s - \frac{2}{3})
\]  

and \( k_s \) is

\[
k_s = \frac{1 + \cos \theta}{2}
\]  

where \( \theta \) is the phase difference between currents in top and bottom coil sides.

Hence, the stator slot leakage reluctance of an arc of angular width \( \Delta \theta_k \) is given by

\[
\mathcal{R}_{lsk} = \frac{1}{\varphi_{ls}} \times \text{number of stator slots per sector}
\]

![Figure 3.3: A typical stator slot.](image)

### 3.4.5 Peripheral Leakage

There is a certain amount of flux which flows entirely in the peripheral rather than radial direction in the air gap of the machine. This peripheral leakage, over the arc \( \Delta \theta_k \), can be represented by the following reluctance
where \( g_k \) is the air gap length at angle \( \theta_k \), \( r_s \) is the stator radius, and \( l_s \) is the stator length.

Since the peripheral leakage flux exists through the whole gap length, it is more appropriate to share its effect on both the rotor and the stator. Therefore, the peripheral reluctance for each section is divided into two halves, where the first half is included in the stator model and the second one is considered with the rotor model.

### 3.4.6 Stator-Sector Model

Each sector of the stator can be represented by the two-reluctances model shown in Figure 3.4.

![Figure 3.4: The reluctance model of a stator section.](image)

### 3.5 Air Gap Model

For a uniform air gap, the reluctance is given by
where \( \mu_0 \) is the free space permeability, \( g_e \) is the effective gap length, and \( A \) is the area normal to the flux path.

For round rotor synchronous machine, the air gap between the rotor and the stator may not be uniform due to slotting. To account for the stator and rotor slotting effects, the developed air gap model should consider Carter's coefficient to get the effective air gap length. The total Carter coefficient is thus given by

\[
C = C_{rs}C_{ss}
\]

(3.14)

where \( C_{rs} \) and \( C_{ss} \) are the individual Carter factors for rotor slots, and stator slots respectively.

Assuming the field to be uniform over the width \( \Delta \theta_k \), and taking Carter's coefficients into consideration, the effective air gap length \( g_k \) of the \( k^{th} \) air gap section is

\[
g_k = g(\theta_k)C_k
\]

(3.15)

where \( g(\theta_k) \) is the air gap length at angle \( \theta_k \), and \( C_k \) is the total Carter's coefficient of sector \( k \). Thus, the air gap reluctance is

\[
R_{gk} = \frac{g_k}{\mu_0 l_{sr} \Delta \theta_k}.
\]

(3.16)

In the case of non-salient pole machines with a fully slotted rotor, the air gap reluctance will be the same for the different machine sectors.
3.6 Rotor Model

3.6.1 Rotor Slot Leakage Reluctance

The rotor slot can be visualized as shown in Figure 3.5. The field current is assumed to be uniformly distributed over the entire rotor slot. Also, the tooth top permeance is neglected. Under these assumptions, the primary slot permeance $\varphi_{rk}$ for the slot shown in Figure 3.5 can be given by

$$\varphi_{rk} = \mu_0 l \left[ \frac{y_1}{3x_1} \right]$$  \hspace{1cm} (3.17)

The permeance of the $k^{th}$ damper bar can be approximated by

$$\varphi_{dk} = \mu_0 l \left[ \frac{y_2}{x_2} \right]$$  \hspace{1cm} (3.18)

Figure 3.5: A typical rotor slot.
3.6.2 Field MMF

The mmf of each field slot is given by the product of the field current and the number of field turns in the considered slot, i.e., for slot number \( k \)

\[
\mathfrak{F}_k = N_{f_k} i_f \quad (3.19)
\]

where \( N_{f_k} \) is the number of field turns in slot number \( k \).

3.6.3 Rotor Slot Model

Figure 3.6 shows the main components of the flux linkage as seen by the rotor slot [7]. The rotor slot shown in Figure 3.5 can be modeled as the two-reluctances circuit shown in Figure 3.7, where both the bar leakage and the field leakage are represented by a single reluctance.

![Figure 3.6: Flux paths.](image)
Figure 3.7: The reluctance model of a rotor section.
3.7 Linear Steady State Reluctance Model

Figure 3.8 shows a section of The Reluctance Model of the machine with the resistance symbol used to represent the reluctances.

The stator leakage reluctance, peripheral leakage reluctance, air gap reluctance, field leakage permeance, and damper bar leakage permeance can be calculated using Equations (3.11,3.12,3.16,3.17,3.18) respectively.

Equation (3.7) gives the components of the equivalent distributed mmf of the stator referred to the rotor frame. The field mmf components are given by Equation (3.19).

![Diagram of Reluctance Model](image)

Figure 3.8: Section of The Reluctance Model of the synchronous machine.

The parallel reluctances shown in Figure 3.8 can be replaced by their parallel equivalent, as shown in Figure 3.9. In Figure 3.9, for sector number k, $R_{s_k}$ is the parallel equivalent of the leakage reluctance and half the circumferential leakage. $R_{g_k}$ is the air gap reluctance, and $R_{f_k}$ is the equivalent reluctance of half the perip-
Figure 3.9: Simplified section of The Reluctance Model of the synchronous machine.
eral leakage reluctance, the damper leakage reluctance, and the rotor slot leakage reluctance.

The positive direction of the current (and mmf) is taken to be into the paper. Therefore, for the different sections of the machine, the flux-mmf relation, can be described as follows

- Stator

\[ \mathcal{f}_s = R_{s_k} (\phi_{s_k} - \phi_{g_k}) \]  

(3.20)

- Air gap

\[
0 = R_{s_k} (\phi_{g_k} - \phi_{s_k}) + R_{g_k} (\phi_{g_k} - \phi_{g_{k-1}}) + R_{f_k} (\phi_{g_k} - \phi_{f_k}) + R_{g_{k+1}} (\phi_{g_k} - \phi_{g_{k+1}})
\]

\[
0 = \phi_{g_k} (R_{s_k} + R_{g_k} + R_{f_k} + R_{g_{k+1}}) - \phi_{g_{k-1}} R_{g_k} - \phi_{s_k} R_{s_k} - \phi_{f_k} R_{f_k} - \phi_{g_{k+1}} R_{g_{k+1}}
\]

(3.21)

- Rotor

\[ \mathcal{f}_r = R_{f_k} (\phi_{f_k} - \phi_{g_k}) \]  

(3.22)

Setting these equations for each mesh, the overall Reluctance Model can be constructed as follows

\[
\begin{bmatrix}
\mathcal{f}_{s_1} \\
\mathcal{f}_{s_2} \\
\vdots \\
0
\end{bmatrix}
= 
\begin{bmatrix}
\text{Stator} & \text{Self} & \text{To Gap} & [0] & \Phi_{s_1} \\
\text{Self} & \text{Reluctance} & \text{Reluctance} & \vdots & \Phi_{s_2} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \text{Stator} & \text{To Gap} & \text{Self} & \Phi_{s_2} \\
0 & \text{To Gap} & \text{Reluctance} & \text{Reluctance} & \Phi_{s_2} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\mathcal{f}_{f_1} \\
\mathcal{f}_{f_2} \\
\vdots \\
0
\end{bmatrix}
\begin{bmatrix}
R_{s_k} \\
R_{g_k} \\
R_{f_k} \\
R_{g_{k+1}} \\
\vdots \\
\text{Rotor} & \text{To Gap} & \text{Self} & \Phi_{f_1} \\
\text{To Gap} & \text{Reluctance} & \text{Reluctance} & \Phi_{f_2} \\
\vdots & \vdots & \vdots & \vdots 
\end{bmatrix}
\]  

(3.23)
Or, simply

$$[\mathcal{A}] = [\mathcal{R}][\Phi]$$  \hspace{1cm} (3.24)

The reluctance matrix $[\mathcal{R}]$ is a square matrix whose dimensions are equal to the total number of meshes of the machine. In Equation (3.23) the suffixes $s$, $g$, and $f$ stand for stator, gap, and field respectively. The stator and rotor self-reluctance matrices are diagonal ones of the corresponding leakage reluctances. From Equations (3.20),(3.21), and (3.22), it can be shown that

$$\begin{bmatrix}
\text{Stator} \\
\text{To Gap} \\
\text{Reluctance}
\end{bmatrix} = - 
\begin{bmatrix}
\text{Stator} \\
\text{Self} \\
\text{Reluctance}
\end{bmatrix}$$  \hspace{1cm} (3.25)

$$\begin{bmatrix}
\text{Rotor} \\
\text{To Gap} \\
\text{Reluctance}
\end{bmatrix} = - 
\begin{bmatrix}
\text{Rotor} \\
\text{Self} \\
\text{Reluctance}
\end{bmatrix}$$  \hspace{1cm} (3.26)

### 3.7.1 Stator Flux Linkage

Assuming the stator and field currents are known, the mmf components of the left hand side of Equation (3.23) can be calculated using Equations (3.7) and (3.19). Therefore, the model can be represented in its inverse form referred to the rotor as

$$[\Phi] = [\mathcal{R}]^{-1}[\mathcal{A}]$$  \hspace{1cm} (3.27)

Thus, the flux component of each loop can be calculated. For a machine with ‘m’ sectors per pole, the flux components used in constructing the stator flux linkage are the ‘m’ flux components $\phi_{sk}$ from $k = \frac{m}{2}$ to $k = \frac{m}{2}$. To calculate the flux linkage due to these flux components, each sector flux is weighted by its sector width $\Delta \theta_k$. Each weighted element is then considered as a vector with angle $-\theta_k + \frac{\pi}{2}$, and the sum of these ‘m’ vectors is multiplied by the stator turns $N_{se}$. The result is the space vector $\bar{\Psi}'_s$ in the rotor reference frame.
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The stator flux linkage is then given in the stator reference frame by

$$\tilde{\Psi}_s = \sum_{k=-m/2}^{k=m/2} \frac{N_{se}}{P} \Phi_{sk} \Delta \theta_k e^{-j(\theta_k - \frac{\pi}{2})}$$

(3.28)

An additional flux linkage component should then be added to account for the end leakage of the stator winding. The literature provides a number of formulas to calculate the stator end leakage inductance $L_{sen}$ of the synchronous machine [55]-[57]. The total stator flux linkage is the sum of the flux linkage given by Equation (3.29) and the end leakage component given by $L_{sen}i_s$.

3.7.2 Stator Induced Voltage

The induced voltage in the stator $e_s$ is given by

$$\tilde{e}_s = \frac{d\tilde{\Psi}_s}{dt}$$

(3.30)

Thus the terminal voltage $\tilde{v}_s$ is

$$\tilde{v}_s = \tilde{e}_s - R_s \tilde{i}_s - L_{sen} \frac{d\tilde{i}_s}{dt}$$

(3.31)

where $R_s$ is the stator winding resistance per phase.

3.7.3 Field Flux Linkage

The construction of the field flux linkage is made by adding up the individual flux linkages of each field slot. The flux linkage of each field slot $\Psi_{f_k}$ is given by

$$\Psi_{f_k} = N_{f_k} \phi_{f_k}$$

(3.32)

The total field flux linkage $\tilde{\Psi}_f$ is then given by
\[ \bar{\psi}_f = \sum_{k=-m/2}^{k=m/2} \Psi_{f_k} \]  
\[ \tag{3.33} \]

The total field flux linkage is the sum of the flux linkage given by Equation (3.33), and the end leakage component given by \( L_{\text{end}} i_f \), where \( L_{\text{end}} \) is the end-winding leakage of the field windings.

### 3.7.4 Field Induced Voltage

The induced voltage in the field circuit \( \vec{e}_f \) is

\[ \vec{e}_f = \frac{d\bar{\psi}_f}{dt} \]  
\[ \tag{3.34} \]

Thus the terminal voltage of the field circuit is

\[ \vec{v}_f = \vec{e}_f - R_f \vec{i}_f - L_{\text{end}} \frac{di_f}{dt} \]  
\[ \tag{3.35} \]

where \( R_f \) is the field winding resistance.

### 3.8 Case Study

The validity of the linear model is demonstrated by applying it to the 588-MVA Nanticoke #5 machine. This machine has full length aluminum field winding wedges [38],[58]. The machine also has slots in the pole face, with copper strips running full length over the field winding and under the pole face slot wedges. These copper damper bars emerge from the rotor body and engage with the adjacent bars via overlapping flags.

#### 3.8.1 Machine Data

Appendix A summarizes the physical dimensions and ratings of this machine. The parameters needed for constructing The Linear Reluctance Model are evaluated in Appendix B.
3.8.2 Synchronous Machine Parameters

This section shows how *The Linear Reluctance Model* can be used to evaluate the fundamental or basic machine parameters. The d-q models employ these fundamental parameters to predict the machine electrical characteristics. The usual parameters are

- $L_d$, the direct-axis inductance, being the ratio of direct-axis flux linkage $\Psi_d$ to the d-axis current $i_d$, with the field circuit open. To use *The Linear Reluctance Model* for evaluating $L_d$, $i'_s$ is first set to be $i_d$. This is done by setting $\beta = 0^\circ$. Also, the field current $i_f$ is set to zero. The flux components are evaluated using Equation (3.27). The stator flux linkage referred to the rotor is then calculated using Equation (3.28). This flux linkage can be represented by

$$\bar{\Psi}'_s = \bar{\Psi}_d = L_d i_d$$  \hspace{1cm} (3.36)

Having $i_d$ and $\bar{\Psi}'_s$, the d-axis inductance $L_d$ is calculated.

- $L_q$, the quadrature-axis inductance, being the ratio of quadrature-axis flux linkage $\Psi_q$ to the q-axis current $i_q$. To use *The Linear Reluctance Model* for evaluating $L_q$, $i'_s$ is set to be $i_q$ by making $\beta = 90^\circ$. The flux components are evaluated using Equation (3.27). The stator flux linkage referred to the rotor is then calculated using Equation (3.28). The flux linkage in this case can be represented by

$$\bar{\Psi}'_s = \bar{\Psi}_q = L_q i_q$$  \hspace{1cm} (3.37)

- $L_{ff}$, the self inductance of the field circuit. With $i'_s$ equal to zero, $L_{ff}$ is the ratio between the field flux linkage and the field current. For a given field current, *The Linear Reluctance Model* can be used to get the field flux linkage $\bar{\Psi}_f$, then the field self inductance can be obtained from
\[ \tilde{\Psi}_f = L_{ff} i_f. \] (3.38)

- \( L_{md} \), the magnetizing inductance, which is the ratio between the stator flux linkage and the field current with zero stator current.

It should be mentioned that machine parameters in the field circuit can be referred to the stator reference frame by using the ratio \( n_e \), which is given by

\[ n_e = \frac{N_{fe}}{N_{se}} \] (3.39)

in which \( N_{fe} \) and \( N_{se} \) are the equivalent numbers of sinusoidally distributed turns in the field and in each stator phase winding. In each case, the equivalent sinusoidally distributed turns \( N_e \) are related to actual turns \( N \) by

\[ N_e = \frac{4}{\pi} k_d k_p N \] (3.40)

where \( k_d \) and \( k_p \) are the distribution and pitch factors for the windings.

For example, the actual field circuit resistance \( R_f \) referred to the stator becomes

\[ R_{fd} = R_f \frac{3}{2n_e^2} \] (3.41)

where the 3/2 factor links the 3-phase stator to the single phase rotor.

Table 3.1 shows the basic unsaturated parameters of the Nanticoke #5 as deduced from standard tests [1] together with corresponding values evaluated using the developed Linear Reluctance Model.

The results reported in Table 3.1 show that the standard linear parameters can be calculated with reasonable accuracy from machine dimensions and the manufacturer’s data using The Linear Reluctance Model. The close agreement between the two data sets validates the basic structure of the developed linear model.
Table 3.1: Machine Parameters in p.u.  
(Base impedance = 0.8228 Ω)

<table>
<thead>
<tr>
<th>Parameters in p.u.</th>
<th>Reluctance Model</th>
<th>IEEE results [1]</th>
<th>% Error based on IEEE results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_d$</td>
<td>2.347</td>
<td>2.348</td>
<td>0.05</td>
</tr>
<tr>
<td>$x_q$</td>
<td>2.314</td>
<td>2.313</td>
<td>0.017</td>
</tr>
<tr>
<td>Saliency</td>
<td>1.014</td>
<td>1.015</td>
<td>0.069</td>
</tr>
<tr>
<td>$x_{ls}$</td>
<td>0.196</td>
<td>0.19</td>
<td>3.16</td>
</tr>
<tr>
<td>$x_{md}$</td>
<td>2.151</td>
<td>2.158</td>
<td>0.3</td>
</tr>
</tbody>
</table>

3.9 Summary

In this chapter the approach of modeling the synchronous machine based on its configuration is presented. The secondary phenomena which inherently exist in the synchronous machine, such as slot effects, hysteresis, eddy currents, spatial harmonics of the mmf and flux, and iron saturation, are excluded at this stage. The idea of representing the stator's mmf as a set of concentrated mmf sources is explained. Dimension-based models for the stator slots, rotor slots, and air gap are developed. An equivalent Linear Reluctance Model is then developed to relate the flux and mmf distribution throughout the various sectors of the machine.

Using the developed model for the calculations of the basic parameters of the synchronous machine, such as direct and quadrature synchronous reactances, leakage reactance, magnetizing reactance, and field leakage reactance is described. Predicting the stator terminal voltage, field voltage, stator flux linkage, and rotor flux linkage is also explained.

To test the validity of the developed model, the model is used to calculate the basic parameters of an existing machine. The model results are compared with the measurement results. The comparison proves that the linear model can simulate the linear synchronous machines accurately. It should be mentioned that the model is developed in such a way that the different nonlinearities can be easily incorporated. In
the next chapter, the saturation effect is to be considered under steady-state operating conditions.
Chapter 4

Steady-State Nonlinear Model for Synchronous Machines

4.1 Introduction

In this chapter, The Linear Reluctance Model introduced in Chapter 3 is extended to include the saturation effect. A new approach for modeling of steady-state performance of synchronous machine including the effect of magnetic saturation is developed.

The chapter starts by introducing the modeling approach in Section 4.2. The mathematical formulations required for modeling the different iron sections of the machine are introduced in Section 4.3. Then, the development of the nonlinear steady-state model is shown in Sections 4.4 and 4.5. In Section 4.6, the model is applied to the test machine which was described in Chapter 3. In Section 4.7, the results of the developed model are compared with those from a conventional d-q model. Finally, the chapter summary is presented in Section 4.9.
4.2 Modeling Approach

4.2.1 Background

The conventional saturation models of synchronous machine are based on the assumption that saturation can be associated solely with the magnetizing branch $X_{ad}$. In the linear regime this reactance represents the reluctance of the air gap which is known to be linear.

The magnetic nonlinearity in the stator occurs in the teeth and yoke both of which carry the total flux linking the stator, i.e. the space vector sum of the assumed sinusoidally distributed air gap flux and the stator leakage flux [24]-[25]. (Strictly, there is a small component of the leakage flux in the end windings and due to space harmonics which does not occur in the teeth and yoke. Also, the contribution of leakage to tooth flux increases with depth of the tooth). The nonlinearity in the rotor occurs in its teeth and core both of which can be considered to carry the net rotor flux. Ignoring the space harmonics of the distributed rotor winding which can be partially accommodated as an external reactance, the net rotor flux is the space vector sum of the air gap flux and the rotor winding leakage flux.

4.2.2 Modeling of Nonlinear Elements

In Chapter 3, the sectionalizing concept was used to model the machine. Linear reluctances were used to model the air gap reluctance and the stator and rotor leakages. In developing The Linear Reluctance Model, it was assumed that the iron sections of both the stator and rotor were of negligible reluctance. In most cases, it would be inefficient to operate at maximum flux density levels, which would produce negligible iron reluctance [23]. Usually machines operate at flux densities in the stator and rotor for which the iron reluctances are significant.

A possible approach would be to assign a reluctance to each rotor tooth and to each circumferential rotor core section. For the stator, a reluctance could be assigned to the average tooth width in each sector and to the corresponding yoke arc as seen
from the rotor frame. The overall reluctance model could then be solved for sector fluxes given the stator and rotor mmfs. This approach has been rejected because the nonlinear reluctances would have to be evaluated and the resultant reluctance matrix inverted for each source mmf condition. Instead, a first solution for fluxes is obtained from an inverted matrix of the linear reluctances using the source mmfs. The mmf's required for these fluxes in the nonlinear elements are then evaluated and used as corrections to the source mmfs on the linear system. The process is repeated iteratively until an acceptable solution is reached. In this approach, the reluctance matrix needs be inverted only once.

As The Linear Reluctance Model gives flux densities in the sectors, it is appropriate to describe the iron reluctance by

\[ H = f(B) \]  \hspace{1cm} (4.1)

where \( H \) is the magnetic field intensity, and \( B \) is the magnetic flux density.

Throughout this chapter, the B-H characteristics of both the rotor and the stator of the machine are modeled by [23]

\[ H = aB + bB^n \]  \hspace{1cm} (4.2)

where \( a, b, \) and \( n \) are the saturation model parameters. An optimization program may be used to evaluate these parameters to produce an acceptable matching to the actual B-H characteristics.

### 4.3 Representation of Iron Sections

This section shows how the different iron sections of the machine such as stator yoke, stator teeth, iron teeth, and iron core are modeled according to their geometry. In the following analysis, it is assumed that the B-H curves for both the stator and the rotor are available. The following suffixes will be used in writing the equations for representing the iron sections of the machine,
4.3.1 Rotor Teeth

Figure 4.1 shows a typical section of the rotor of the machine. The width of the tooth is variable along its depth. Instead of dealing with each width separately, an average tooth width can be used. Also, the flux is not the same along the tooth due to the leakage flux component. However, these flux changes are assumed to be small enough such that the flux can be considered to be nearly constant along the tooth. Figure 4.2 shows the equivalent magnetic circuit. For the rotor tooth between loops \( k + 1 \) and \( k \), the flux \( \phi_{rtk+\frac{1}{2}} \) is

\[
\phi_{rtk+\frac{1}{2}} = \phi_{rk+1} - \phi_{rk}
\]  

(4.3)

where \( \phi_{rk+1} \) and \( \phi_{rk} \) are the flux components of rotor loops number \( k + 1 \) and \( k \). The flux density \( B_{rtk+\frac{1}{2}} \) of this tooth is

\[
B_{rtk+\frac{1}{2}} = \frac{\phi_{rtk+\frac{1}{2}}}{w_{rk+\frac{1}{2}} l_r}
\]

(4.4)

where \( w_{rk+\frac{1}{2}} \) is the average width of the rotor tooth between the two teeth \( k \) and \( k + 1 \), and \( l_r \) is the rotor length. The rotor B-H model is used to get \( H_{rtk+\frac{1}{2}} \). The corresponding tooth mmf \( \Im_{rtk+\frac{1}{2}} \) is

\[
\Im_{rtk+\frac{1}{2}} = H_{rtk+\frac{1}{2}} d_{rk+\frac{1}{2}}
\]

(4.5)

where \( d_{rk+\frac{1}{2}} \) is the length of the rotor tooth between sections \( k \) and \( k + 1 \) as shown in Figure 4.1.
Chapter 4. Steady-State Nonlinear Model for Synchronous Machines

Figure 4.1: Rotor section.
Both $r_{ck}$ and $r_{rk}$ are measured from the center of the machine.

Figure 4.2: Equivalent magnetic circuit for a rotor section.
- a - Linear reluctance for slot leakage.
- b - Nonlinear rotor tooth reluctance.
- c - Nonlinear rotor core reluctance.
4.3.2 Rotor Core

It has been shown in [59] that the rotor circumferential flux can reasonably be assumed uniform over the width \( r_{ck} \), as defined in Figure 4.1. Knowing the mesh flux from The Linear Reluctance Model, the flux density \( B_{ck} \) of the \( k^{th} \) core section can be approximated by

\[
B_{ck} = \frac{\phi_{rk}}{r_{ck} l_r}
\]

(4.6)

From the B-H model of the rotor, the corresponding \( H_{ck} \) can be obtained. The required core mmf is assumed to be that along the curved path at radius \( r_{ck} \). Therefore, the core mmf \( \mathcal{Z}_{ck} \) is given by

\[
\mathcal{Z}_{ck} = r_{ck} \Delta \theta_k H_{ck}
\]

(4.7)

where \( \Delta \theta_k \) is the width of the \( k^{th} \) sector in radians.

In cases where there are no dots in the q-axis, \( r_{ck} \) in Equations (4.6) and (4.7) can be replaced by \( r_{rk} \).

4.3.3 Stator Yoke

Figure 4.3 shows a typical section of the stator of the machine. Figure 4.4 shows that the flux of a section of the yoke is the same as the flux of its pertaining mesh. Assuming uniform flux distribution over the yoke section, the following equation gives the flux density \( B_{yk} \) of the yoke section

\[
B_{yk} = \frac{\phi_{yk}}{d_y l_s}
\]

(4.8)

From the stator B-H model, the corresponding \( H_{yk} \) can be obtained. Considering a path at the mean radius of the yoke, the mmf \( \mathcal{Z}_{yk} \) of the \( k^{th} \) sector is given by

\[
\mathcal{Z}_{yk} = r_y \Delta \theta_k H_{yk}
\]

(4.9)
where $r_y$ is the mean radius of the stator yoke given by

$$\tilde{r}_y = r_y - \frac{d_y}{2} \tag{4.10}$$

where $r_y$ is the stator outer radius.

![Stator Section Diagram](image)

Figure 4.3: Stator section.
Both $r_y$ and $r_s$ are measured from the center of the machine.

### 4.3.4 Stator Teeth

As introduced in Chapter 3, the stator circumference is divided into a number of segments. Recalling that the sectionalizing of the stator depends on the rotor, each stator section is partly stator teeth and partly stator slots at any instant. Figure 4.3 shows a typical section of the stator.

Considering the iron section between loops $k$ and $k+1$ of the stator, its flux $\phi_{stk+\frac{1}{2}}$ is
Figure 4.4: Equivalent magnetic circuit for a stator section.

d - Linear reluctance for slot leakage.
e - Nonlinear stator tooth reluctance.
f - Nonlinear stator yoke reluctance.

\[ \phi_{stk+\frac{1}{2}} = \phi_{sk+1} - \phi_{sk} \] (4.11)

As shown in Figure 4.3, the stator slot is trapezoidal rather than rectangular. As mentioned in [55], the effective width of the tooth element is usually assumed to be at one third up the tooth depth \( d_s \). Therefore the arc width of each stator section is

\[
\text{Arc Width} = (r_s + \frac{d_s}{3}) \Delta \theta_k
\] (4.12)

where \( r_s \) is the stator radius, and \( d_s \) is the depth of the stator slot. Hence, the width of tooth in the considered sector, \( w_{sk} \), can be calculated by

\[
w_{sk} = \frac{w_{st}}{w_{st} + w_{ss}} \left( r_s + \frac{d_s}{3} \right) \Delta \theta_k
\] (4.13)

where \( w_{st} \) is the width of the stator tooth, and \( w_{ss} \) is the width of the stator slot, both at the bottom of the tooth as shown in Figure 4.3. The flux density \( B_{stk+\frac{1}{2}} \) of this equivalent tooth is

\[
B_{stk+\frac{1}{2}} = \frac{\phi_{stk+\frac{1}{2}}}{w_{sk} l_s}
\] (4.14)

Using the B-H model to get \( H_{stk+\frac{1}{2}} \), the corresponding mmf \( \Gamma_{stk+\frac{1}{2}} \) is
Chapter 4. Steady-State Nonlinear Model for Synchronous Machines

4.4 Nonlinear Model

In this section, the overall steady-state nonlinear model of the synchronous machine is developed. This section shows how to predict the flux components of the different sections of the machine for a given set of stator current, field current, and rotor angle, when the effect of the magnetic saturation is included.

Using the preset stator current $I_s$, and the angle $\beta$, the stator current can be transferred to the rotor frame to get $\bar{I}_s'$ by using Equation (3.4). The equivalent mmf sources of the stator sectors $\mathcal{F}_s$ can be calculated from Equation (3.7).

The field mmf $\mathcal{F}_{f_k}$ can be obtained by multiplying the field current by the number of field turns in each individual slot as given by Equation (3.19). The right hand side of The Linear Reluctance Model is thus constructed.

For a linear machine, the flux of each mesh of the machine can be calculated using the inverse form of The Linear Reluctance Model, i.e.

$$[\Phi] = [\mathcal{R}]^{-1} [\mathcal{F}]$$

(4.16)

To account for magnetic nonlinearities, these fluxes are then used in Equations (4.4,4.6,4.8,4.14) to obtain the flux density components associated with each iron section, i.e., $B_{rtk+\frac{1}{2}}$, $B_{ck}$, $B_{yk}$, and $B_{stk+\frac{1}{2}}$ respectively. Using the appropriate B-H models, the corresponding $H_{rtk+\frac{1}{2}}$, $H_{ck}$, $H_{yk}$, and $H_{stk+\frac{1}{2}}$ can be obtained. Equations (4.5,4.7,4.9,4.15) are then used to get $\mathcal{F}_{rtk+\frac{1}{2}}$, $\mathcal{F}_{ck}$, $\mathcal{F}_{yk}$, and $\mathcal{F}_{stk+\frac{1}{2}}$. Recalling Figures 4.2 and 4.4, these nonlinear mmfs introduce demagnetizing effects in the linear mmfs produced by the rotor and stator currents. To account for such demagnetizing effects, the nonlinear mmf components are used to correct the meshes mmf as follows:

$$\mathcal{F}_{rk new} = \mathcal{F}_{rk old} - \mathcal{F}_{ck} + \mathcal{F}_{rtk+\frac{1}{2}} - \mathcal{F}_{rtk-\frac{1}{2}}$$

(4.15)
\[ \mathcal{F}_{s_{\text{new}}} = \mathcal{F}_{s_{\text{old}}} - \mathcal{F}_{y_k} + \mathcal{F}_{s_{\text{m}+\frac{1}{2}}} - \mathcal{F}_{s_{\text{m}-\frac{1}{2}}} \]  

(4.17)

\( \mathcal{F}_{s_{\text{new}}} \) and \( \mathcal{F}_{r_{\text{new}}} \) are then used in the right hand side of Equation (4.16) for the following iteration cycle. This iterative process is continued until the sectors' fluxes converge. This process is referred to hereafter as "The Basic System". The block diagram of The Basic System is shown in Figure 4.5.

4.5 Typical Application

In this section, The Basic System introduced in Section 4.4 is used to predict the performance and flux distribution of the machine for any given steady-state operating condition. A typical case in power system analysis occurs when it is required to determine the field current and internal angle for a given stator voltage, current, and power factor.

A conventional lumped-parameters d-q model can be used to give an approximate initial set of values for the equivalent field current \( I_F \), and the internal angle \( \delta_i \). It is convenient at this stage to assume that the saliency effects can be ignored. Using the machine’s equivalent circuit shown in Figure 4.6, with the stator current \( I_s \), stator voltage \( V_s \), and power factor angle \( \alpha_s \) known, the internal voltage \( \mathcal{E}_s \) can be determined from
\[ \vec{E}_s = \vec{V}_s + \vec{I}_s (R_s + j\omega L_{se}) \]  

(4.18)

where \( L_{se} \) is any additional stator leakage inductance not included in the linear model, such as the end leakage of the windings. The magnetizing current \( \vec{I}_m \) can then be obtained from

\[ \vec{I}_m = \frac{\vec{E}_s}{j\omega L_s} \]  

(4.19)

where \( L_s \) is the saturated inductance of the machine. This parameter can be calculated from open- and short-circuit test data at the operating voltage of the machine as explained in [55]. As shown in Figure 4.6, the field current vector \( \vec{I}_F \) is

\[ \vec{I}_F = \vec{I}_m + \vec{I}_s \]  

(4.20)

Knowing the stator current, the angle \( \beta \) can be obtained from

\[ \beta = \angle \vec{I}_s - \angle \vec{I}_F \]  

(4.21)

as shown in Figure 4.7. The internal angle \( \delta_i \) can be calculated from

\[ \delta_i = \frac{\pi}{2} - \beta - \alpha_s \]  

(4.22)

![Equation 4.18](image_url)

Figure 4.6: Equivalent circuit of the synchronous machine.

Recalling Figure 4.5, The Basic System can now be used to predict the flux of
each mesh of the stator. Consequently, the flux linkage of the stator referred to the rotor $\bar{\psi}_s'$ can be calculated from Equation (3.28).

Under steady-state, the internal voltage can be calculated from

$$\bar{E}_s' = j\omega \bar{\psi}_s'$$

Therefore, the new stator terminal voltage is

$$\bar{V}_{s_{new}} = \bar{E}_{s_{new}} - (R_s + j\omega L_{se})\bar{I}_s'$$

This calculated stator voltage is compared with the desired (i.e. preset input) voltage. The difference in the voltage $\Delta \bar{V}_s$ given by

$$\Delta \bar{V}_s = \bar{V}_{s_{new}} - \bar{V}_s'$$

is used to get an approximate correction to the field current as follows

$$\Delta \bar{I}_F = \frac{\Delta \bar{V}_s}{j\omega L_s}$$

$L_s$ in Equation (4.26) is used as an iteration factor. A smaller value might be used to speed up the iteration or a larger one to avoid numerical oscillations. The new field
current is

\[ \vec{I}_{F_{new}} = \vec{I}_{F} + \Delta \vec{I}_{F} \]  

(4.27)

The phasor diagram shown in Figure 4.8 gives the phasor relationship between the new and old field currents. The actual value of the new field current is obtained by dividing the field current phasor given by Equation (4.27) by the equivalent current ratio \( N_i \) which is given by

\[ N_i = \frac{\text{Rated stator current}}{\text{Field current base}} \]  

(4.28)

where the field current base is the product of the field current required to produce rated voltage on the air-gap line times the unsaturated magnetizing inductance in p.u [60].

The iteration process continues until \( \Delta \vec{V}_s \) comes within an acceptable tolerance. Figure 4.9 illustrates the block diagram of the developed algorithm for this typical application.

Figure 4.8: Phasor diagram for predicting field current.
Figure 4.9: Overall nonlinear model of synchronous machines.
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4.6 Case Study

The machine described in Chapter 3 is used to demonstrate the validity of the developed approach for modeling machine saturation. Detailed drawings for the rotor and stator slots of this machine are used to get the necessary dimensions for the various sectors. Measured values of field current and internal angle, under different steady-state operating conditions, have been reported in [61] for this machine as shown in Table 4.1.

<table>
<thead>
<tr>
<th>Active Power (MW)</th>
<th>Reactive Power (MVAR)</th>
<th>Terminal Voltage ($KV_{L-L}$)</th>
<th>Stator Current (KA)</th>
<th>Field Current (A)</th>
<th>Internal Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>39</td>
<td>21.25</td>
<td>2.83</td>
<td>1365</td>
<td>16.0</td>
</tr>
<tr>
<td>200</td>
<td>95</td>
<td>21.63</td>
<td>5.77</td>
<td>1813</td>
<td>24.6</td>
</tr>
<tr>
<td>200</td>
<td>80</td>
<td>21.52</td>
<td>5.62</td>
<td>1739</td>
<td>25.6</td>
</tr>
<tr>
<td>200</td>
<td>60</td>
<td>21.36</td>
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<td>1643</td>
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</tr>
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<td>200</td>
<td>40</td>
<td>21.23</td>
<td>5.36</td>
<td>1558</td>
<td>28.9</td>
</tr>
<tr>
<td>201</td>
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<td>8.23</td>
<td>2059</td>
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<tr>
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<td>2003</td>
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</tr>
<tr>
<td>298</td>
<td>70</td>
<td>21.38</td>
<td>8.07</td>
<td>1943</td>
<td>36.0</td>
</tr>
<tr>
<td>299</td>
<td>63</td>
<td>21.30</td>
<td>8.08</td>
<td>1918</td>
<td>36.7</td>
</tr>
<tr>
<td>300</td>
<td>42</td>
<td>21.17</td>
<td>8.02</td>
<td>1835</td>
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<td>7.95</td>
<td>1791</td>
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<td>1750</td>
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<td>8.06</td>
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<td>2504</td>
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<td>38.6</td>
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<td>85</td>
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<td>2304</td>
<td>42.5</td>
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<td>159</td>
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<td>13.77</td>
<td>2858</td>
<td>42.5</td>
</tr>
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<td>499</td>
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<td>2800</td>
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<td>2753</td>
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<td>500</td>
<td>112</td>
<td>21.42</td>
<td>13.57</td>
<td>2699</td>
<td>46.1</td>
</tr>
</tbody>
</table>
In the following sections the model is first used to produce the open-circuit characteristic of the machine. Secondly, it is used to calculate the field current and internal angle of the machine for the given loading conditions.

4.6.1 Details of Stator Teeth

Figure 4.10 shows a section of the stator teeth of the Nanticoke #5 machine with the dimensions in mm. Since the dimensions of the notch along the stator wall are fairly small compared with the slot dimensions, it is quite convenient to neglect its effect on the uniformity of the flux distribution. Table 4.2 shows the different dimensions of the stator slot of the test machine as defined in Figure 4.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Length mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{st}$</td>
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</tr>
<tr>
<td>$w_{ss}$</td>
<td>45</td>
</tr>
<tr>
<td>$d_y$</td>
<td>478.8</td>
</tr>
<tr>
<td>$d_s$</td>
<td>196.9</td>
</tr>
<tr>
<td>$r_y$</td>
<td>1304</td>
</tr>
<tr>
<td>$r_s$</td>
<td>628.7</td>
</tr>
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</table>

Figure 4.10: Detailed stator teeth of Nanticoke #5.
### 4.6.2 Details of Rotor Teeth

A section of the rotor teeth is shown in Figure 4.11. The tooth width is not uniform along the slot. An average tooth width can be assumed as in the case of stator teeth. Table 4.3 shows the different dimensions of the rotor slot as defined in Figure 4.1. The average width of the iron of the rotor tooth is taken to be 25.4 mm.

#### Table 4.3: Nanticoke #5 rotor slot dimensions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Length mm</th>
</tr>
</thead>
<tbody>
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<tr>
<td>$r_r$</td>
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<tr>
<td>$d_{r,\text{field}}$</td>
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<td>$d_{r,\text{dummy}}$</td>
<td>131.6</td>
</tr>
</tbody>
</table>

Figure 4.11: Detailed rotor teeth Nanticoke #5.

### 4.6.3 Saturation Model

Figure 4.12 shows the B-H curves of both the stator and rotor iron of the Nanticoke #5 machine. The B-H characteristics of both the rotor and stator of the machine are modeled by Equation (4.2).
An optimization program is used to evaluate the parameters for the proposed saturation model to match the actual B-H curves.

Table 4.4 shows the parameters used in the modeling of the B-H curves for both the stator and rotor of the test machine.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator</td>
<td>563</td>
<td>86.9</td>
<td>7.76</td>
</tr>
<tr>
<td>Rotor</td>
<td>332</td>
<td>0.027</td>
<td>16.34</td>
</tr>
</tbody>
</table>
4.6.4 Calculation of Open Circuit Voltage

The nonlinear model introduced in Section 4.4 can be used to produce the open circuit characteristic of the machine. In this case it is required to predict the terminal voltage of the machine for the set of given values of the field current listed in Table 4.5 [61]. In order to achieve that, *The Basic System* is used with the stator current \( \bar{I}_s \) set to zero. The flux components of the stator meshes are evaluated through *The Basic System* of the model. Therefore, the flux linkage of the stator can be evaluated from Equation (3.28). Under steady-state operating conditions, the open circuit terminal voltage in the rotor reference frame \( \bar{E}_s' \) is given by

\[
\bar{E}_s' = j\omega \bar{\psi}_s'
\]  

(4.29)

This procedure is applied to the test machine, i.e., *Nanticoke #5* machine. Figure 4.13 shows the open circuit characteristic of the machine as calculated from the developed model as compared with the measurement values listed in Table 4.5. The maximum percentage error in the model results is 2%, where the percentage error of the results is defined as follows

\[
\text{Percentage Error} = \frac{\text{Model Result} - \text{Measurement}}{\text{Measurement}} \times 100
\]  

(4.30)
Table 4.5: *Nanticoke #5* open-circuit characteristic.

<table>
<thead>
<tr>
<th>Terminal Voltage (KV L-L)</th>
<th>Field Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.178</td>
<td>48</td>
</tr>
<tr>
<td>2.566</td>
<td>108</td>
</tr>
<tr>
<td>3.746</td>
<td>168</td>
</tr>
<tr>
<td>5.525</td>
<td>234</td>
</tr>
<tr>
<td>6.140</td>
<td>276</td>
</tr>
<tr>
<td>7.696</td>
<td>348</td>
</tr>
<tr>
<td>8.606</td>
<td>390</td>
</tr>
<tr>
<td>10.108</td>
<td>456</td>
</tr>
<tr>
<td>11.336</td>
<td>516</td>
</tr>
<tr>
<td>12.722</td>
<td>588</td>
</tr>
<tr>
<td>14.348</td>
<td>678</td>
</tr>
<tr>
<td>15.512</td>
<td>750</td>
</tr>
<tr>
<td>17.034</td>
<td>852</td>
</tr>
<tr>
<td>19.160</td>
<td>1002</td>
</tr>
<tr>
<td>20.064</td>
<td>1074</td>
</tr>
<tr>
<td>21.068</td>
<td>1158</td>
</tr>
<tr>
<td>21.984</td>
<td>1242</td>
</tr>
<tr>
<td>22.648</td>
<td>1314</td>
</tr>
<tr>
<td>23.220</td>
<td>1374</td>
</tr>
</tbody>
</table>
Figure 4.13: Open circuit characteristic.
4.6.5 Field Current and Internal Angle Prediction

Table 4.1 lists terminal voltage, stator current, field current, and internal angle for twenty four tested load values for the test machine. These test points are conducted at different levels of the active power loading so as to cover a wide range of the possible steady-state operating conditions of the machine.

In the following analysis the active power, reactive power, terminal voltage and stator current of Table 4.1 are used as input data to the model.

Using the circuit shown in Figure 4.6, the initial value of the field current and the rotor position are estimated. In this case the stator winding resistance $R_s$ is 0.001579Ω, the external inductance $L_{se}$ is 0.154 mH, and $L_{dsat}$ is taken to be 4.105 mH, as explained in Appendix B. The process explained in Section 4.5 is applied at each loading point.

Figures 4.15 and 4.16 show the results of the model against the field measurements. The maximum percentage error in the field current as defined by Equation (4.30) is 0.38%. The maximum absolute value of the error in internal angle is 0.567°. The error results in both the field currents and internal angle are shown in Figure 4.17.

The results shown in Figures 4.15 and 4.16 are in good agreement with the field measurements. This confirms the adequacy of the new model. It also verifies the developed algorithm used to predict the machine performance for given loading conditions.

4.7 Conventional Model

In this section, the conventional circuit model is used to evaluate the field current and internal angle for those operating conditions given in Table 4.1. Figure 4.14 shows a typical circuit model for the steady-state analysis of the synchronous machine [54].

The following model is used to represent the machine open-circuit characteristic

$$I_f = a_s E_m + b_s E_m^{n_s}$$  \hspace{1cm} (4.31)
where \( a_e, b_e, \) and \( n_e \) are the model parameters. The suffix \( e \) is used in Equation (4.31) to distinguish between these parameters and the parameters of the saturation model described by Equation (4.2). The parameter \( a_e \) is the slope of the air gap line. For the open-circuit characteristics shown in Figure 4.13, \( a_e \) is

\[
a_e = \frac{1001}{22000} \sqrt{3} = 0.079
\]

(4.32)

The parameters \( b_e \) and \( n_e \) are calculated by fitting the nonlinear portion of the open-circuit characteristics using an appropriate minimization technique. Table 4.6 lists the numeric values of the parameters used in Equation (4.31).

<table>
<thead>
<tr>
<th>( a_e )</th>
<th>( b_e )</th>
<th>( n_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.079</td>
<td>( 8.55 \times 10^{-14} )</td>
<td>3.77</td>
</tr>
</tbody>
</table>

(4.33)

Recalling the linear parameters of the Nanticoke #5 machine, where \( L_d \) is almost equal to \( L_q \), the saliency effects can then be neglected. Given the stator voltage \( \bar{V}_s \), and stator current \( \bar{I}_s \), the internal voltage \( \bar{E}_s \) can be calculated from

\[
\bar{E}_s = \bar{V}_s + \bar{I}_s (R_s + j\omega L_{ls})
\]

The magnitude of the magnetizing current \( \bar{I}_m \) is then obtained by using the open-circuit characteristic model given by Equation (4.31).
The angle of such magnetizing current lags behind that of the internal voltage by ninety degrees. The field current referred to the stator $I_F$ is the vectorial sum of the magnetizing and stator current. To get the actual field current, $I_F$ is divided by the equivalent current ratio $N_i$, as defined by Equation (4.28). For the *Nanticoke #5* machine,

$$N_i = \frac{15437}{1001 \times 2.15} = 7.105 \quad (4.34)$$

Also for the test machine, the stator leakage inductance $L_{ls}$ is assumed to be 0.19 p.u., i.e., $4.147 \times 10^{-4} \text{ mH}$. This process is carried out at each of the operating condition listed in Table 4.1. Figures 4.15 and 4.16 compare the performance of the developed and the conventional model together with the measurements.

The field currents predicted by both models are in close agreement with the measured values. However, the maximum errors in the field current produced by the developed model is 0.38%, which is less than that of the conventional model, which is 1.37%.

On the other side, while the maximum error in the internal angle calculated by the developed model is less than 0.57°, the error in the internal angle calculated by the conventional model can reach up to 6.6°.

Figure 4.17 shows the percentage error in the field current, and the error in rotor angle as produced from both the developed and the conventional model.

### 4.8 Model Convergence

To assure the convergence of the model, a conventional lumped-parameter d-q model is used to give an approximate set of initial values for the field current and the rotor position. Moreover, the value of the iteration factor $L_s$ of Equation (4.26) can be adjusted automatically between each two successive iteration steps to assure that the change of the field current is within the solution domain.

The results shown in Figures 4.13, 4.15 and 4.16 required between 2 to 4 iteration
cycles for the flux distributions to converge i.e. reach the steady state solution.
Figure 4.16: Internal angle.
Figure 4.17: Percentage error of developed and conventional model.
4.9 Summary

In this chapter, the modeling of the saturation phenomenon in synchronous machines under steady-state operating conditions, is presented. The model is developed in such a way as to preserve the spatial flux distribution amongst the different sections of the machine. This results in precise representation for different saturation levels throughout the iron portions of the machine. On the other hand, the technique used in modeling the saturation does not rely on the superposition theory, which should not be used in addressing nonlinear phenomena.

A first solution for fluxes is obtained from the inverted Linear Reluctances Model using the source mmfs. The mmf's required for these fluxes in the nonlinear elements are then evaluated and used as corrections to the source mmfs on the linear system. The process is repeated iteratively until an acceptable solution is reached. In this approach, the reluctance matrix needs be inverted only once.

The developed model is applied to the test machine Nanticoke #5. The open-circuit characteristic of the test machine was evaluated using the developed model. The developed model is also used to predict the field current and internal angle for a wide range of operating conditions.

The model results and the field measurements are almost identical, with maximum error less than 2%. This verifies the nonlinear model structure and validates the developed algorithm. Moreover, the results demonstrate that the developed model simulate the machine more precisely than the conventional model.
Chapter 5

Frequency-Domain Modeling of Synchronous Machines

5.1 Introduction

This chapter presents two new models for the frequency-domain analysis and representation of the synchronous machine. In the first model, new equivalent circuits for modeling the standstill frequency response (SSFR) of the synchronous machine are presented. The circuit parameters are determined by matching the measured data to the model frequency response using standard optimization methods.

In the second model, the sectionalized machine concept is used for producing a frequency-domain model of the synchronous machine. The developed model provides direct representation of the frequency-dependent skin-effect properties of the damper and rotor iron. The model does not rely on the existence of the SSFR measurements in order to be developed, i.e., it is not a mathematical fitting model. It is rather constructed from the machine’s dimensions, its material properties, and its open- and short-circuit tests.

The chapter starts by addressing the different currents induced in the rotor body in Section 5.2. The currently used frequency-domain models of the synchronous machines are presented in Section 5.3. In Section 5.4, an improved frequency-domain circuit model of the synchronous machine is presented. The frequency-domain model
of the sectionalized synchronous machine is developed in Section 5.5. The developed sectoral model is then used in Section 5.6 to predict the operational inductances of the test machine. The chapter summary is presented in Section 5.7.

5.2 Background

When a magnetic flux changes with time, an induced electric field is produced around the region of changing flux. This electric field establishes electromagnetic force (emf) in any winding encircling the magnetic path. This electric field is also produced within the magnetic material, and, if the material is a conductor, currents known as eddy currents are established [62].

Eddy currents have two major effects in cores subjected to alternating magnetic fields. First, the alternating flux tends to be forced toward the outer surface of the core. This phenomenon is called the magnetic skin effect. Second, the core material has resistance, and energy is dissipated as heat in the core due to the $Ri^2$ losses caused by the eddy currents.

Frequency-domain models of the synchronous machines should be able to represent these frequency-dependent phenomena adequately. They should provide an analytical approach to model eddy and bar currents.

The following section presents an overview of the currently used frequency models for synchronous machines. It also presents a critique of such models and provides the basis needed for development of improved frequency models.

5.3 Conventional Models

Figure 5.1 shows conventional transient flux-linkage/current circuits for the direct- and quadrature-axes. These are third-order models, the most complex usually employed. In these models, $\Psi_d$, $\Psi_f$, and $\Psi_q$ are the flux linkages of the stator direct-axis, the field, and the quadrature-axis respectively. $L_l$ is the leakage inductance associated with the stator winding, consisting of slot, harmonic, and end-winding components.
plus a portion of the air gap circumferential leakage. $L_{ad}$ and $L_{aq}$ represent mainly the linear air gap reluctances in the direct- and quadrature-axes. The quantity $n_e$ is the effective field/stator turns ratio and is given by

$$n_e = \frac{N_{fe}}{N_{se}}$$

in which $N_{fe}$ and $N_{se}$ are the equivalent numbers of sinusoidally distributed turns in the field and in each stator phase winding. In each case, the equivalent sinusoidally distributed turns $N_e$ are related to actual turns $N$ by

$$N_e = \frac{4}{\pi} k_d k_p N$$

where $k_d$ and $k_p$ are the distribution and pitch factors for the winding.

![Figure 5.1: Conventional frequency models for synchronous machines.](image)

(a) Third-order direct-axis circuit.
(b) Third-order quadrature-axis circuit.

### 5.3.1 A Simplification of Conventional Models

The linear direct axis circuit of Figure 5.1-a can be replaced by the simpler circuit of Figure 5.2 without any loss of information relating terminal quantities. This is accomplished by use of a simple transformation which was introduced for induction
machine analysis about three decades ago [64]-[65]. The result is now generally designated as a Γ-type circuit in contrast to the Τ-type circuit of Figure 5.1 because of its structure at the stator end [23]. This transformation has been reinvented frequently since that time in various forms. More recently, its validity for use with synchronous machines has been established independently by Kirtley in [66].

![Diagram of simplified Γ-form d-axis equivalent circuit](image)

Figure 5.2: Simplified Γ form d-axis equivalent circuit.

In this simple transformation, all the flux linkages to the right of $L_{ad}$ in Figure 5.1-a are multiplied by a factor $\gamma_d$ while all currents are divided by $\gamma_d$ where

$$\gamma_d = \frac{L_d}{L_{ad}} \quad (5.3)$$

and $L_d$ is the unsaturated direct axis inductance,

$$L_d = L_l + L_{ad} \quad (5.4)$$

In the new Γ-type circuit, the leakage parameter $L_{LD}$ to the right of $L_d$ is then given by

$$L_{LD} = \gamma_d L_l + \gamma_d^2 L_{f12d} \quad (5.5)$$

All other inductances and resistances parameters in Figure 5.2 are then made equal to those of Figure 5.1-a multiplied by $\gamma_d^2$. The new direct axis parameters are distinguished by use of the subscript $D$ rather than subscript $d$.

The equivalent transformation ratio $n_E$ for the new circuit is

$$n_E = \frac{n_e}{\gamma_d} \quad (5.6)$$
An immediate advantage of the circuit of Figure 5.2 is that it contains one fewer inductance parameter than does Figure 5.1-a, with no loss of ability to predict terminal information.

There has been considerable discussion in the literature about the nature of and need for the parameter $L_{f12d}$ in Figure 5.1-a [67]-[69]. Its presence in the conventional circuit has been shown to be essential in predicting torsional behavior [70]. The magnitude of this parameter is usually small, but in some optimizations and for some machines this parameter takes on a negative value. This is a result that may be mathematically useful but physically not realistic.

It is suggested that an explanation may follow from the almost universal practice of fixing the stator leakage inductance $L_l$ at a value obtained from the machine designer. This inductance certainly contains components arising from the stator slot leakage, the stator end leakage and the flux due to stator space harmonics. But it should be noted that the division of leakage inductance between stator and rotor is somewhat arbitrary in view of the air gap circumferential flux component, which is usually significant in large-gap turboalternators.

The unsaturated inductance $L_{ad}$ is physically and primarily identified with the reluctance of the air gap. If all or most of the circumferential leakage is included in the stator leakage inductance $L_l$, a negative value for $L_{f12d}$ may well be required on the rotor side to produce good match to SSFR test results. In the simplified circuit of Figure 5.2 all of the leakage components up to the surface of the rotor have been combined into the parameter $L_{LD}$. With magnetic linearity assumed, the division of leakage between stator and rotor is irrelevant in analyzing machine behavior and the anomaly of potentially negative inductance is avoided.

If the parameters of a $\Gamma$-type circuit of Figure 5.2 have been evaluated, the conventional circuit of Figure 5.1-a can be produced if considered necessary. Given a value for $L_l$ from the designer, the inductance $L_{ad}$ is found from Equation (5.4). $\gamma_d$ from Equation (5.3) and the inductance $L_{f12d}$ from Equation (5.5). All $D$-subscripts parameters to the right of $L_{LD}$ can be converted to their $d$-subscript values by dividing by $\gamma_d^2$. The effective turns ratio $n_e$ can be determined by use of Equation (5.6).
An additional advantage of the circuit of Figure 5.2 is that the inductance $L_d$ and the ratio $n_E$ can be obtained directly from steady-state open-circuit and short-circuit tests without any required input from design information. If, at field current $i_f$, the rms short-circuit current is $I_s$, and the unsaturated open circuit voltage (i.e., on the air gap line) is $V_s$, then

$$L_d = \frac{V_s}{\omega_s I_s} \quad (5.7)$$

and

$$n_E = \frac{3}{\sqrt{2}} \frac{I_s}{i_f} \quad (5.8)$$

The ratio $n_e$ in the conventional circuit is a well-defined quantity easily computed from knowledge of the numbers of stator and rotor turns, their distribution and their pitch. It may be argued that, if any quantity is to be requested from the manufacturer, it might preferably be this ratio. Given this quantity, the appropriate value of the factor $\gamma_d$ can be determined by comparison with the ratio $n_E$ obtained as in Equation (5.8) and used to obtain appropriate values of $L_l$ and $L_{ad}$.

### 5.3.2 Parameters of Conventional Circuits

The frequency modeling of synchronous machines has received extensive attention in the literature [1],[71]-[72]. Equivalent circuits are the currently preferred models. The parameters of these circuits are derived by parameter fitting to a set of results obtained by standstill frequency response tests (SSFR) [71]-[72]. The usual tests give

- $L_d(s)$, the direct-axis operational inductance, being the ratio of direct axis flux linkage $\Psi_d$ to the d-axis current $i_d$ with the field shorted,
- $L_q(s)$, the ratio of $\Psi_q$ to $i_q$,
- $sG(s)$, the ratio of $i_f$ to $i_s$ with field shorted, and
• $L_{af0}(s)$, the ratio of $\Psi_f$ to $i_d$ with field open.

All of these are expressed as Laplace transforms, usually graphed for magnitude and phase over a frequency range which may extend from $10^{-3}$ to $10^3$ Hertz. Model structures are chosen for direct- and quadrature-axes, such as the third-order circuit of Figure 5.1, or the second-order models which result from deleting $L_{2d}$, $R_{2d}$, $L_{3q}$ and $R_{3q}$ from Figure 5.1 [73]. The parameters are iterated mathematically to obtain a best fit typically using a least-squares method. In developing a direct-axis model, greatest weight is usually assigned to the function $L_d(s)$. In general, the fitting results improve as the order and complexity of the model is increased. In some studies, fourth- or higher order models have been tried in order to obtain a more accurate fit [74].

The shunt branches in the conventional circuits, such as those shown in Figure 5.1, represent the best compromise of linear parameters available for the order chosen. It is not appropriate to associate any one branch with the physical damper winding in that axis.

5.3.3 Commentary on The Parameters of Conventional Circuits

The major limitation of these conventional models is that the structure and parameters of these circuits are inadequately related to the physical dimensions and the material properties of the synchronous machine. If there is a direct linkage between a circuit parameter and a section of the machine, there is a basis for improved insight into the explanation of the machine's behavior. Also, fruitful communication between system analysts and machine designers is promoted when the models used in analysis and operation can be related directly to those quantities under the control of the designer. The approach of most system analysts seems to have been to accept the properties of the machine. Little attention has been devoted to communicating to the designer the properties which might be more desirable.
5.4 An Improved Frequency Domain Model

A sketch of a 90-degree cross-section of the rotor of the test machine *Nanticoke #5* is shown in Figure 5.3. This rotor has uniformly distributed aluminum damper bars in both direct and quadrature axes, the 3 bars near the direct axis being somewhat smaller than the 6 bars near the quadrature axis. The slots below the smaller bars are filled with a non-conducting material [73].

![Diagram of rotor](image)

Figure 5.3: A 90-degree cross-section of the rotor of the test machine.

5.4.1 Scope of Analysis

Much of the discussion associated with frequency models has been directed at the accuracy with which the models match the *SSFR* test data over the measured frequency range. Little attention is given to specifying the accuracy and the frequency range that is to be considered adequate for various types of studies. As an example, the highest frequency considered to be significant for torsional vibration studies in 60-Hz turbo alternators is considered to be about 55 Hz. In stability studies, the frequency range of greatest interest is usually considered to be 0.1 to 5 Hz [72].

Ideally, the models used for a study should be the simplest ones that gives adequate accuracy for that type of study. In this chapter, attention will be focused on a consistent modeling of $L_d(s)$ and $L_q(s)$ over the frequency range of $10^{-3}$ to 60 Hz.
This focus on these particular stator functions is due to the fact that, interaction of the stator with the attached system is of greatest importance in most stability, hunting and resonance studies.

### 5.4.2 Steady-State Parameters

The following parameters were obtained from open- and short-circuit test data listed in Appendix B

\[
L_d = \frac{15430}{6572} = 2.35 \text{ pu}
\]

and

\[
n_E = \frac{3}{\sqrt{2}} \frac{6572}{1001} = 13.93
\]

The measured function \(L_d(s)\) is shown in Figure 5.4. The parameter \(L_d\) may be derived from the measured magnitude \(|L_d|\) as frequency approaches zero. This value of \(L_d = 2.15 \text{ pu}\) is somewhat lower than that obtained from the open/short circuit tests in Equation (5.9). This is usually attributed to the low permeability of the stator and rotor iron at the low values of flux density used in SSFR tests. Finite element analysis has shown that this relative permeability can be as low as 150 for SSFR tests [59]. Considering that the flux path length in iron is about 30 times the gap length and that the iron permeability for the open/short-circuit tests would be an order of magnitude higher, the difference of about 9% between the two values of \(L_d\) is readily explained.

### 5.4.3 First-Order Model

In the direct-axis, the field winding dominates the transient behavior, particularly at low frequency. Let us first consider the extent to which the simple first-order model shown in Figure 5.5 can represent the measured function \(L_d\) shown in Figure 5.4.

If a first-order model applies, the value of the field leakage inductance \(L_F\) can
be derived by noting the minimum value of the angle $\alpha_{d_{\text{min}}}$ of $L_d(s)$ which is $45^\circ$ in Figure 5.4 for this machine. Given the magnitude of the angle

$$\frac{L_d}{L_F} = \frac{2\sin|\alpha_{d_{\text{min}}}|}{1 - \sin|\alpha_{d_{\text{min}}}|} = 4.83$$

from which $L_F = \frac{2.45}{4.83} = 0.445$ pu. The field time constant may now be estimated by noting the frequency $f_{\text{min}}$ at which this minimum angle occurs. For this machine $f_{\text{min}} = 0.059$ Hz. Then,

$$\tau_F = \frac{L_F}{R_F} = \frac{\tan\left[45^\circ - \frac{|\alpha_{d_{\text{min}}}|}{2}\right]}{2\pi f_{\text{min}}} = 1.12 \text{ s}$$

where $R_F$ is the field circuit resistance. If the base angular frequency $\omega_b$ is $377$ rad./s. then the equivalent field resistance $R_{fd}$ is
Figure 5.5: First-order direct-axis model.

\[ R_{fd} = \frac{L_F}{\tau_F \omega_b} = 0.00105 \text{ pu} \quad (5.13) \]

This may be compared with the value calculated from the field resistance \( R_f \) using the measured current ratio \( n_E \) to transfer it to the stator reference turns. From Equation (3.41)

\[ R_{fd} = \frac{3R_f}{2n_E^2 Z_{sb}} = 0.001 \text{ pu} \quad (5.14) \]

It is seen from Figure 5.6 that this first order model for \( L_d(s) \) is reasonably accurate up to about 0.1 Hz. For higher frequency, the effects of damper and rotor iron conductivities must be included.

### 5.4.4 Damper Modeling

At low frequency, the current distribution in each bar can be considered uniform. Noting that the fields for each axis can be considered as sinusoidally distributed about that axis, the effective low-frequency damper resistances reflected into the stator reference frame can be determined by weighting the resistance of each bar by the value of the sine function appropriate to its bar position. These resistances will also include provision for the end-winding resistances.

With excitation from the stator, as frequency increases, the current in each bar is forced toward the surface of the bar near the air gap. The effective penetration or skin depth in the bar \( \delta_b \) is [75]
where \( \sigma_b \) is the conductivity of the bar material and \( f \) is the excitation frequency.

With such skin effect, the damper bars can be modeled approximately as an impedance. A model is required that accommodates various configurations of damper bars. The two common configurations of the damper bars are circular and rectangular.

For circular bars, the first rigorous model was presented in [76]. This analysis accounted for two-dimensional variation of current density in the bar cross-section. In [77], the conductor was modeled as a group of parallel filamentary conductors of incremental cross-section area. Results obtained from the two methods show no significant difference in the bar resistance or reactance [78].

Similarly, several models for the impedance of rectangular bars are reported in the literature [57],[77]-[80]. In [57], it was assumed that the leakage flux crosses the
slot at right angle to the slot wall over the entire depth of the slot. It is also assumed that the current density in the bar does not vary with position across its width. This approach is most applicable for bars in open slots. It reduces the field problem to a one-dimensional solution of the diffusion equation. However, this approximation is violated near the top of the slot and also in case of semi-closed slots. An improved analytic solution had been developed in [79] by solving the diffusion equation for the current density in the bar, but retaining the two-dimensional variation in density as well as accounting for the semi-closed slot above the bar. This approach was employed in [80] for the T-shaped conductors in semi-closed slots. A more exact numerical model was introduced in [77].

Therefore, the damper bars of depth $d$ can be modeled approximately in each of the axes as an impedance of the form [79]

$$Z_b = R_b \frac{k_b}{2} \left[ \frac{\sinh(k_b) + \sin(k_b)}{\cosh(k_b) - \cos(k_b)} \right] (1 + j) \text{ ohm} \quad (5.16)$$

where $k_b = 2d_k/\delta_b$ and $R_b$ is the bar resistance. For flux linkage models, the corresponding operational inductance $L_b(s)$ is

$$L_b(s) = \frac{Z_b}{j\omega} = \frac{R_b k_b}{4\pi f} \left[ \frac{\sinh(k_b) + \sin(k_b)}{\cosh(k_b) - \cos(k_b)} \right] (1 - j) \text{ H} \quad (5.17)$$

The damper model requires only two parameters, $R_{bD}$ and $k_{bD}$ in the direct-axis, and $R_{bQ}$ and $k_{bQ}$ in the quadrature-axis. In this modeling, the variations in the width of each bar are ignored.

### 5.4.5 Iron Modeling

In the iron of the rotor, high frequency currents will be forced to near the rotor surface. At lower frequencies, the iron currents may extend down the teeth and into the rotor core.

In order to obtain a reasonably simple model, the variations with depth in the width of the available current paths are ignored. A simple approach is used, based on the surface impedance $Z_s$ of a deep conducting plane, which is given by [75]
Chapter 5. Frequency-Domain Modeling of Synchronous Machines

\[ Z_i = \sqrt{\frac{j\omega \mu}{\sigma}} = \sqrt{\frac{2\pi f \mu_i}{\sigma_i}} L^{45^\circ} \Omega/\text{square} \quad (5.18) \]

As an element in a flux linkage model, the surface impedance becomes an inductance,

\[ L_i(s) = \frac{Z_i(s)}{j2\pi f} \text{ H/square} \quad (5.19) \]

An approximate model for the iron can be expressed in terms of a single constant \( k_i \) which lumps together the results of Equation (5.18) and Equation (5.19), the proportion of the rotor periphery in iron and transferred to the stator reference frame

\[ L_i(s) = \frac{k_i}{\sqrt{f}} (1 - j) \quad (5.20) \]

with separate values of \( k_{iD} \) for the direct-axis and \( k_{iQ} \) for the quadrature-axis.

5.4.6 Model Structure

The proposed structure for direct- and quadrature-axis models is shown in Figure 5.7. With stator excitation at high values of frequency, the currents in both the damper bars and the rotor iron are forced to near the rotor surface. Since these currents are side by side around the periphery, their model elements in Figure 5.7 are properly connected in parallel. In these \( \Gamma \)-type circuits, the parameters \( L_{LD} \) and \( L_{LQ} \) include the stator leakage and all of the air gap circumferential leakage up to the rotor surface. In the direct-axis, the parameter \( L_{FD} \) includes the leakage around the field winding plus the leakage across the damper bars.

As frequency is reduced, the mean levels of the induced currents in the damper bars and the iron will be increasingly below the rotor surface. It may be argued that the appropriate point of connection of the damper and iron elements should be moved somewhat toward the field terminals as frequency is reduced. For simplicity, this refinement will be ignored. However, this dependence of structure on frequency provides insight into the reason for the ladder type structure which characterizes some
higher order circuits such as that shown in Figure 5.1.

### 5.4.7 Evaluation of Parameters

An optimization routine was used to evaluate parameters for the proposed circuits to best match the measured SSFR results for the functions $L_d(s)$ and $L_q(s)$. The inductances $L_d$ and $L_q$ were taken from the asymptotic values as frequency approached zero, leaving 6 parameters to be evaluated for the direct axis and 4 for the quadrature axis.

Each optimization was based on minimizing the sum of the squares of the magnitudes of the differences between the measured and model vectors divided by the magnitude of the measured vector as in the general expression

$$
\sum \left( \frac{|L(s)_{\text{measured}} - L(s)_{\text{model}}|}{|L(s)_{\text{measured}}|} \right)^2
$$

(5.21)
This optimization gives equal weight to a given percentage error, thus allowing for the considerable variation in the magnitude of the function with frequency.

The results of this optimization are shown in Figure 5.8 and Figure 5.9. It is observed that the accuracy of the models is quite good, particularly over the upper frequency part of the spectrum. For the $L_d(s)$ function, the largest magnitude error is about 2.7% at 0.14 Hz, and the largest angle error is about $1.4^\circ$ at 0.08 Hz. In the frequency range 0.5–50 Hz, of major significance for stability and resonance studies, the error is generally below 0.8% in magnitude and 0.6$^\circ$ in angle. For the $L_q(s)$ function, the largest magnitude error is about 3% at 0.13 Hz, and the largest angle error is about $1.8^\circ$ at 0.35 Hz.

The parameters obtained from the optimizations are listed in Table 5.1. It is noted that the value of the field resistance $R_{FD}$ is approximately equal to the values derived earlier by Equation (5.14). The sum of the inductance parameters $L_{LD}$ and $L_{FD}$ is approximately equal to the value 0.445 pu derived for the first order model in Section 5.4.3. The damper resistances $R_D$ and $R_Q$ are approximately equal to the values calculated from the dimensions of the bars and end rings. The constants $k$ for the two axes differ by an amount which is consistent with the differences in bar and iron tooth dimensions.

Table 5.1: Machine parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>pu value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{LD}$</td>
<td>0.2147</td>
</tr>
<tr>
<td>$L_{LQ}$</td>
<td>0.2180</td>
</tr>
<tr>
<td>$k_{iD}$</td>
<td>0.3247</td>
</tr>
<tr>
<td>$k_{iQ}$</td>
<td>0.2262</td>
</tr>
<tr>
<td>$R_D$</td>
<td>0.0017</td>
</tr>
<tr>
<td>$R_Q$</td>
<td>0.0027</td>
</tr>
<tr>
<td>$k_{bD}$</td>
<td>0.6275</td>
</tr>
<tr>
<td>$k_{bQ}$</td>
<td>0.4391</td>
</tr>
<tr>
<td>$R_{FD}$</td>
<td>0.0013</td>
</tr>
<tr>
<td>$L_{FD}$</td>
<td>0.2432</td>
</tr>
</tbody>
</table>
5.4.8 Model Validation

The accuracy of modeling is comparable to that obtained with third-order conventional circuits. The number of parameters to be evaluated in the optimization is 6 for the direct axis and 4 for the quadrature axis. These may be compared with 7 for each of the direct and quadrature axes in the conventional third-order models.

Each parameter of the models can be evaluated directly from the dimensions and material properties of the generator and thus provide a direct communication link to the machine designer. The models are directly applicable to frequency response analysis.
Figure 5.8: Frequency plots of $L_d(s)$ using developed model.
Figure 5.9: Frequency plots of $L_q(s)$ using developed model.
5.5 Frequency-Domain Model of Sectionalized Machine

In conventional frequency-domain circuit models, circuit parameters are obtained by mathematically fitting the frequency measurements of the machine to a pre-defined circuit. This results in the value of the machine parameters are dependent on the model order, e.g., the numeric value of $L_{fd}$ for a second-order model is not the same as for a third-order model, even though both are for the same machine. Moreover, the determination of the parameters of the circuit model is based on the existence of the frequency measurements.

In the following sections, a new frequency-domain model for the synchronous machine is developed. The developed model is dependent on machine dimensions and material properties. It does not need the SSFR data in order to be developed.

5.5.1 Rotor Slot - Revisited

The steady-state model for the rotor was introduced in Section 3.6. The model was carried out in two steps. While the first step was modeling the rotor slots, the second step was concerned with the field currents representation.

Each rotor slot was represented by three reluctances, as explained in Section 3.6. The field current was assumed to be uniformly distributed over each field slot. This current was then modeled as a single concentrated current source in each field slot. Dummy slots were regarded as slots with zero current source. Therefore, the steady-state rotor slot model consisted of a single current source with three parallel reluctances. This model is re-illustrated in Figure 5.10.

As the frequency of the stator current increases, the $emf$ induced in the rotor body will be more significant. Therefore, the resulting eddy currents may no longer be negligible. The following section discusses the frequency-dependent induced currents in both the bars and rotor iron.
5.5.1.1 Iron Induced Currents

In the case of the Nanticoke #5 machine, the conductivity of the cast steel of the rotor is about $3 \times 10^6$ mho/m. The relative permeability of the steel is estimated to be about 200 for the low values of flux density that apply during SSFR tests. These values result in a penetration depth of approximately $\frac{20}{\sqrt{f}}$ mm. This suggests a penetration depth into a rotor tooth equal to the depth of the damper bars (3.91 cm) at a frequency of about 0.25 Hz and a penetration depth to the bottom of the quadrature axis rotor slots (13.16 cm) at a frequency of about 0.02 Hz. From this it is concluded that skin effect is a major factor in representing the iron over most of the frequency range of interest in stability and resonance studies [73].

Using Equation (5.18), the impedance of each iron sector is then given by

$$Z_{ik}(s) = \sqrt{\frac{2\pi \mu_i f}{\sigma_i}} \frac{l_r}{w_{ik}} 45^\circ$$

(5.22)

where $l_r$ is the rotor length, and $w_{ik}$ is the width of the $k^{th}$ iron section.

5.5.1.2 Damper Bars Currents

For the duraluminum alloy of the bars in Nanticoke #5, with conductivity of about $2 \times 10^7$ mho/m, the penetration depth is approximately $\frac{115}{\sqrt{f}}$ mm. Regarded as rectangular
bars, Equation (5.16) is used to model the impedance of each bar as follows

\[ Z_{bk} = R_{bk} \frac{k_{bk}}{2} \left[ \frac{\sinh(k_{bk}) + \sin(k_{bk})}{\cosh(k_{bk}) - \cos(k_{bk})} \right] (1 + j) \text{ohm/bar} \]  

\[(5.23)\]

where \( k_{bk} = \frac{2d_k}{\delta_{bk}} \), \( R_{bk} \) is the resistance of bar number \( k \), and \( d_k \) is the \( k^{th} \) bar depth. The corresponding operational inductance \( L_{bk}(s) \) is

\[ L_{bk}(s) = \frac{Z_{bk}}{j\omega} = \frac{R_{bk}k_{bk}}{4\pi f} \left[ \frac{\sinh(k_{bk}) + \sin(k_{bk})}{\cosh(k_{bk}) - \cos(k_{bk})} \right] (1 - j) \text{H/bar} \]  

\[(5.24)\]

5.5.1.3 Model Assumptions

The two induced currents explained in Sections 5.5.1.1 and 5.5.1.2 have a major effect on the rotor model developed in Chapter 3. The current can no longer be assumed a single-valued uniformly distributed current over the rotor slot. The existence of damper and eddy currents means a single current source cannot fully represent the rotor slot under high frequency. As a result, the three reluctances representing the rotor slot can no longer be replaced by their parallel equivalent.

To include these two induced currents in The Reluctance Model, the following assumptions are made

1. Each induced current can be represented by a single-valued concentrated current source.

2. The approximate locations of these induced currents are as shown in Figure 5.11.

In the following sections, the technique used to develop the frequency domain model is demonstrated.

5.5.2 Overall Frequency Domain Model

Figure 5.12 shows the frequency-dependent model of the rotor slot. A second current source is used to represent the currents induced in the rotor body. This results in increasing the number of meshes representing the machine by \( m \) meshes. Therefore, the size of the reluctance matrix becomes \( 4m \). The reluctance \( R_a \) shown in Figure 5.12
Figure 5.11: Frequency-domain model for the synchronous machine.

is the parallel combination of both the peripheral reluctance and half of the bar reluctance. Similarly, the reluctance $R_b$ is the parallel combination of the field slot reluctance and the other half of the bar reluctance. The overall Reluctance Model becomes

$$
\begin{bmatrix}
\mathbf{Z}_s \\
0 \\
\mathbf{Z}_b \\
\mathbf{Z}_f
\end{bmatrix} =
\begin{bmatrix}
R_{ss} & R_{sg} & 0 & 0 \\
R_{gs} & R_{gg} & R_{gb} & 0 \\
0 & R_{bg} & R_{bb} & R_{bf} \\
0 & 0 & R_{fb} & R_{ff}
\end{bmatrix}
\begin{bmatrix}
\Phi_s \\
\Phi_g \\
\Phi_b \\
\Phi_f
\end{bmatrix}
$$

(5.25)

In Equation (5.25), each element of this extended reluctance matrix is an $m \times m$ matrix. Also, each element of either the mmf or the flux columns is an $m \times 1$ vector. The suffixes $s$, $g$, $b$, and $f$ stand for stator, gap, damper bars, and field respectively. This equation can be written as

$$
[\mathbf{Z}_d] = [\mathbf{R}_d][\Phi_d]
$$

(5.26)
The suffix 4 is used in Equation (5.26) to distinguish between this extended model and the steady-state linear model represented by Equation (3.24).

![Image of frequency-domain model of synchronous machine](image)

**Figure 5.12**: Frequency dependent model of the rotor slot.

Figure 5.13 shows the block diagram of the overall frequency domain model of the synchronous machine. Starting by *The Extended Reluctance Model* in its inverse form, the flux linkage of each damper bar $\Psi_{bk}$ can be obtained. Multiplying the flux linking each bar by $j\omega$ gives the voltage induced in that bar, $e_{bk}$. The bar currents can then be obtained from

$$i_{bk} = \frac{e_{bk}}{Z_{bk}}$$  \hspace{1cm} (5.27)

where $Z_{bk}$ is the impedance of bar number $k$ as given by Equation (5.23). Since the equivalent number of turns for each bar equals one, the bar currents are used in *The Extended Reluctance Model* as bar mmfs.

The flux linking the iron section between rotor sectors $k$ and $k+1$ is assumed to be

$$\Psi_{i_{k+\frac{1}{2}}} = \frac{\Psi_{bk} + \Psi_{bk+1}}{2}$$  \hspace{1cm} (5.28)

where $\Psi_{bk}$ and $\Psi_{bk+1}$ are the flux linkages for damper bars $k$ and $k+1$ respectively. The
voltages induced in each iron section are then evaluated. The eddy current induced by this \( \Psi_{i_{k+\frac{1}{2}}} \) is obtained by

\[
i_{i_{k+\frac{1}{2}}} = \frac{e_{i_{k+\frac{1}{2}}}}{Z_{i_{k+\frac{1}{2}}}}
\]  

(5.29)

where \( Z_{i_{k+\frac{1}{2}}} \) is the iron impedance as defined in Equation (5.22).

For sector number \( k \) of the rotor, the iron current source is given by

\[
i_{i_{k}} = \frac{i_{i_{k+\frac{1}{2}}} + i_{i_{k-\frac{1}{2}}}}{2}
\]  

(5.30)

This current is added to the bar current of sector \( k \), which was given by Equation (5.27). The result of the summation is the equivalent induced current, which is then used in the mmf vector of The Extended Reluctance Model. The block diagram of this iterative process is shown in Figure 5.13.

Figure 5.13: Block diagram of the overall frequency domain model for the synchronous machine.

### 5.6 Model Verification

The model introduced in Section 5.5.2 is used to calculate the operational inductances \( L_d(s) \) and \( L_q(s) \) of the synchronous machine, as defined in Section 5.3.2. Initially the
field current is assumed to be zero for the first iterative step. The flux linkage compo-
nents for each machine section can be evaluated using *The Extended Reluctance Model* in its inverse form. The rotor flux linkages are used to predict the field, damper and iron current components as illustrated in Figure 5.14. The iterative process continues until there are no significant changes in the calculated currents between two successive iteration steps. When this condition is satisfied, the stator flux linkage components, i.e., $\Psi_d$ and $\Psi_q$, are divided by the corresponding stator current components $i_d$ and $i_q$ to give the required $L_d(s)$ and $L_q(s)$. This algorithm is used at each frequency point in the specified frequency range.

![Diagram](image_url)

*Figure 5.14: Using the developed frequency domain model to calculate the operational inductances.*

This algorithm is applied to the test machine *Nanticoke #5* to predict its operational inductances $L_d(s)$ and $L_q(s)$.

The results of this model are shown in Figure 5.15 and Figure 5.16. It is observed that the accuracy of the model is quite good particularly over the upper frequency part of the spectrum. For the $L_d(s)$ function, the largest magnitude error is about 2.7% at 0.08 Hz, and the largest angle error is about 1.4° at 0.08 Hz. In the frequency range 0.5-50 Hz, of major significance for stability and resonance studies, the error is generally below 1.0% in magnitude and 0.6° in angle. For the $L_q(s)$ function, the
largest magnitude error is about 3% at 0.02 Hz and the largest angle error is about 1.8° at 0.05 Hz.

Figure 5.15: Frequency plots of $L_d(s)$ using developed model.
Figure 5.16: Frequency plots of $L_q(s)$ using developed model.
5.7 Summary

In this chapter an alternate set of equivalent circuit models has been presented to represent the frequency response of turbogenerators as seen from its stator terminals. Skin-effect elements dependent on frequency are used instead of the constant parameters of conventional models. The number of parameters to be evaluated in the optimization is 6 for the direct-axis and 4 for the quadrature-axis. This may be compared with 7 for each of the direct and quadrature axes in the conventional third-order model.

Also, a novel model for representing the sectionalized synchronous machine in the frequency domain is represented. The model parameters are evaluated directly from dimensions and material properties of the generator. Opposite to conventional models, the developed model does not need the SSFR data for model development. The technique developed is potentially useful as it allows for the prediction of the SSFR of the machine with a high degree of accuracy.
Chapter 6

Nonlinear Frequency-Domain Modeling of Synchronous Machines

6.1 Introduction

In the standstill frequency response (SSFR) techniques, the machine is disconnected from the system. The stator is excited by a low level (± 60 A, ± 20 V) source over the range of frequencies from 1 mHz to 1 kHz [7]. Models are then derived from SSFR tests and used to predict the machine behavior under load and transient conditions. Since the tests are conducted at such low flux levels, the SSFR tests may not be used with confidence in simulating machine performance over a wide range of operating conditions [35].

To overcome such a limitation, some machines were tested under running conditions but over a restricted operating range. This test is known as the On-line frequency response (OLFR) [36]-[37]. The disadvantage of this test is that it requires testing on an operating unit connected to the system, possibly under special system or machine conditions.

A simpler form of the OLFR test is obtained by running the machine with the main field of the machine excited and the stator terminals open-circuited. This procedure is referred to as the Open circuit frequency response (OCFR) [37]. Since this test is exclusively a direct-axis test, it provides no firm basis on how to model the q-axis
In this chapter, the small-signal analysis concept is used to develop a frequency-domain model of the saturated synchronous machine. The chapter examines the possibility of using the nonlinear steady-state model developed in Chapter 4 together with the linear frequency-domain model presented in Chapter 5 to determine the effect of saturation on the frequency response of the machine. The proposed model can be used to assess the response of the machine to frequency components imposed on the machine by the systems to which the machine is connected.

The chapter starts by introducing the modeling approach in Section 6.2. The model development steps are explained in Section 6.3. In Section 6.4, the developed model is applied to the test machine. The chapter summary is presented in Section 6.5.

6.2 Modeling Approach

Small-signal analysis assumes linear response of a generator to perturbations in the generator armature and field currents. This means that the permeability describing the behavior of the generator iron can be considered constant and independent of the magnitude of the small-signal swings in flux density. However, as a result of the hysteresis nature of iron, the B-H path for small disturbances is different from the path followed for steady-state operation [1]. Therefore, the incremental reluctances are used instead of the normal reluctances, where the normal reluctance is defined as the ratio between the mmf and the flux at any point of the \( S - \phi \) curve, and the incremental reluctance is defined as the slope of the \( S - \phi \) curve at a specific point. Figure 6.1 shows the difference between the normal and incremental reluctances.

6.3 Model Development

In Chapter 3, the linear model is set up to give the fluxes in the teeth, yoke and core sectors, including the effect of leakages without considering the saturation effect.
In Chapter 4 the model was extended to include the saturation effects. This was done by evaluating the mmf drops throughout the iron sections of the machine. These demagnetizing mmfs were used to correct the meshes mmf produced by the linear model. The corrected mmfs were then used to evaluate the flux distribution in the different meshes of the machine. The process is repeated in an iterative manner until steady-state flux and mmf distribution is reached. This algorithm predicts the flux and mmf distributions for any steady-state operating conditions.

The SSFR model developed in Chapter 5 includes frequency-dependent parameters. The model is best suited to predict the response of the machine to on-line frequency perturbations from the system on the machine, such as in case of hunting, and in case of harmonics generated by power electronics components. In these studies, the most important operational inductances are $L_d(s)$ and $L_q(s)$ under saturation.

The proposed model is carried out in two steps,

1. For a given steady-state operating condition, i.e., pre-specified stator voltage, stator current, and power factor, the nonlinear steady-state model developed in Chapter 4 is used to predict the steady-state flux and mmf distribution in the
different sections of the machine. The incremental reluctances of the different iron sections of the machine are then evaluated. This is discussed in detail in Section 6.3.1.

2. The extended reluctance model presented in Chapter 5 is modified to include the incremental reluctances. This overall model is used to model the machine performance under varying frequency small-signal perturbations. This is explained in Section 6.3.2.

### 6.3.1 Incremental Reluctances

In the steady-state model of Chapter 4, at the final stage of the iteration, the flux density components $B_{r}, B_{c}, B_{s}$, and $B_{y}$ are evaluated. Using the saturation model represented by

$$ H = aB + bB^n, $$

(6.1)

the incremental reciprocal permeability can be expressed as

$$ \nu = \frac{dH}{dB} = a + bnB^{n-1} $$

(6.2)

for each element. Thus, the corresponding incremental reluctance $R_{inc}$ is

$$ R_{inc} = \frac{d}{wl_s} \left( \frac{dH}{dB} \right) $$

(6.3)

where $d$, $w$ and $l_s$ are the depth, width, and length respectively for each iron section.

### 6.3.2 Modified Frequency Model

The frequency model introduced in Chapter 5 is used to predict the machine performance at different frequencies. Figure 6.2 shows a section of the nonlinear reluctance model of the synchronous machine. An oblique stroke through the element is used to denote nonlinearity. In Figure 6.2 the letters $a$, $b$, $c$, and $d$ are used to desig-
nate incremental reluctance for the yoke, stator tooth, rotor tooth, and rotor core respectively.

\[ [\Phi_4] = [\mathbf{R}_4]^{-1}[\mathbf{S}_4] \]  

(6.4)

The damper mmf components are determined iteratively using the model shown in Figure 5.14. Using the mmf components of the different sections of the machine, the corresponding flux linkage components and consequently the incremental operational
inductances are evaluated.

![Diagram](image)

Figure 6.3: Overall linearized frequency model of the saturated synchronous machine.

### 6.4 Case Study

In this section the developed model is applied on Nanticoke #5 to predict its frequency response at rated voltage. The machine is assumed to be working at its rated operating conditions as described in Appendix A. The nonlinear model shown in Figure 4.9 is used to predict the incremental reluctances, field current and rotor position for the steady-state rated operating conditions. The extended reluctance matrix described in Section 5.5.2 is modified to include these incremental reluctances. This modified extended reluctance is then used in the frequency-domain model shown in Figure 5.14 to predict the operational inductances of the test machine at rated voltage.

The results of this model are shown in Figures 6.4 and 6.5 together with the SSFR measurements.

Figures 6.4 and 6.5 show that, under low frequency (up to 0.01 Hz), the magnitude of the incremental operational inductances is 70% of their SSFR values.

Other than the low-frequency response (below 1.0 Hz for the direct-axis and 2.0 Hz for the quadrature-axis), there is not much difference between the rated voltage results and the standstill values. A similar conclusion was reported in [1].
Figure 6.4: Comparison of the direct-axis operational inductance at rated voltage and at standstill.
Figure 6.5: Comparison of the quadrature-axis operational inductance at rated voltage and at standstill.
6.5 Summary

This chapter proposes the use of a new model for frequency-domain analysis of saturated synchronous machines. In contrast with the conventional models in which OLFR results are used to modify the SSFR-based models, the developed model is based on physical assessments of the machines dimensions and material properties at the specific operating conditions. It uses frequency-dependent models in describing the frequency-dependent phenomena rather than constant parameters, as in the case of conventional circuits models. This makes the new model more physically appropriate.

Results of the developed models are shown. Unfortunately no measurements were available to confirm the advantages of the developed model. However, the accuracy achieved by the main two components of the developed models (steady-state nonlinear model and the linear frequency-domain model) and the physical principles behind the modeling approach are strong reasons to believe in the adequacy of the developed model.

The model results prove that the saturation effects are crucial for system analysis below the 2 Hz limit such as is the case in system hunting. However, for a higher frequency range operation at rated voltage, the SSFR results might be used without affecting the machine performance significantly.
Chapter 7

Conclusions

7.1 Summary

This work details a new approach to the modeling of synchronous machines for systems studies. In Chapter 2 a brief review of the currently used machine models is introduced. The common techniques used to represent machine saturation and eddy currents in systems analysis are also reviewed. The review leads to the conclusion that, the available models may not be used with enough confidence to model the machine under different operating conditions. Chapter 3 exploits the idea of a sectionalized machine, introduced in [51], to derive a linear reluctance model for the synchronous machine. The linear model preserves a direct relationship between machine dimensions and materials and model parameters. The basic parameters of a 588-MVA turboalternator are evaluated using the developed linear model. Model results are shown to be in a close agreement with measurements. In Chapter 4, an iterative-based approach is developed to include the machine saturation for steady-state operation. The linear model is first used to predict an initial flux and mmf distributions. Saturation effects are then included to modify the flux distribution. Different iron sections of the machine are represented by their corresponding materials' B-H curves. The adequacy and accuracy of the nonlinear model are potentially checked by predicting the open-circuit characteristic of the 588-MVA turboalternator. The nonlinear model is also used to predict the steady-state field current and load...
angle of the turboalternator. The model results closely agree with the measurement results.

Chapter 5 deals with the frequency-domain modeling of the synchronous machine. A frequency-dependent skin-effect model is presented and is used to develop a new set of equivalent circuits for the synchronous machine. The validity of both the skin-effect model and the developed circuit models are demonstrated by using the developed model to determine the frequency response of the test machine. Model results are then compared with measurements of the frequency response on the stationary machine. The skin-effect model is then used with the sectionalized machine model to predict the frequency response of the machine. The developed model is used to predict the operational inductances of the 588-MVA turboalternator over a wide range of loading conditions. The maximum percentage error in the model results is less than 2%.

In Chapter 6, the small-signal analysis concept is used to develop a frequency-domain model of the saturated synchronous machine. The nonlinear steady-state model developed in Chapter 4, and the linear frequency-domain model presented in Chapter 5, are integrated to determine the effect of saturation on the frequency response of the machine. The developed model is applied to the test machine. The model results agree with the expected performance.

### 7.2 Contributions

The contributions of this thesis are,

- The mathematical formulation for the conceptual framework of the sectionalized machine introduced in [51] is developed and applied to a large turboalternator.

- Development of a new approach for modeling of steady-state performance of synchronous machines including the effect of saturation. The developed approach is based on developing a sectoral model of the machines and then solving it iteratively. The technique avoids the limitations of the superposition-based models. The computational efforts needed for the developed model are significantly less
than that needed for a finite-element based model. The model can be used to correctly predict the steady-state field current and load angle when the machine is operating higher than its rated voltage, or when the machine is operating at low power factor and high field current.

- Development of an alternate set of equivalent circuit models to represent the frequency response of the synchronous machine as seen from its stator terminals. Skin-effect elements are used instead of the constant parameters of conventional models. The accuracy of the model is comparable to that obtained with third-order conventional circuit models but with a fewer number of parameters to be evaluated through the optimization process.

- Development of a novel technique for frequency-domain modeling of the machine. The developed model is solely dependent on the machine dimensions, material properties, and open- and short-circuit tests of the machine. The model is not a mathematical-fitting model and does not depend on a particular circuit order. The developed model can be used to produce the standstill frequency response of the machine with high accuracy.

- Development of a new model to represent the frequency response of the saturated synchronous machine. The proposed model can be used to predict the response of the machine to frequency components imposed on the machine by the systems to which the machine is connected.

### 7.3 Further Study

Further study may be directed to the following,

- The considerable difference between the results of the steady-state circuit model and the measurements might be attributed to the assumption of a sinusoidal field distribution. The effect of the field harmonics needs to be investigated and analysed. The developed sectoral model can be used to predict the machine performance with sinusoidal and non-sinusoidal field winding distribution.
• In the developed frequency-domain model, it is assumed that the high frequency current is forced to near the rotor surface. With high frequency currents in the field there will also be skin effect in the iron around the walls of the field slots and there may be also skin effect at the bottom of the damper bars. The developed model can be extended to include an additional skin-effect element to accommodate this effect. Such a model can be used to produce the transfer function $sG(s)$ adequately.

• The development of the mathematical equations for the representation of the sectoral model in the transient regime. A further extension will be interfacing the time domain model with an electromagnetic transients program such as The EMTP or The EMTDC. This will allow the use of the developed model for analyses of systems transients.

• Investigating the possibility of using the approach used in this thesis for modeling the internal faults of the machine such as short-circuits of stator windings or field windings.

• Expanding the use of the sectionalized machine concept to machines other than synchronous machines such as the Written Pole Machines (WPM).
Appendix A

Data for Test Machine *Nanticoke #5*

The test machine (*Nanticoke #5*) is a two pole 588 MVA non-salient pole machine. The machine ratings are given in Table A.1. The stator and rotor data of the test machine are given in Table A.2 and (A.3) respectively.

<table>
<thead>
<tr>
<th>Table A.1: Machine ratings.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of poles</strong></td>
</tr>
<tr>
<td><strong>Rated MVA</strong></td>
</tr>
<tr>
<td><strong>Rated power factor</strong></td>
</tr>
<tr>
<td><strong>Rated MW</strong></td>
</tr>
<tr>
<td><strong>Rated voltage(L-L)</strong></td>
</tr>
<tr>
<td><strong>Synchronous reactance</strong></td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
</tr>
<tr>
<td><strong>Rated speed</strong></td>
</tr>
</tbody>
</table>
Table A.2: Stator data.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator length</td>
<td>6934 mm</td>
</tr>
<tr>
<td>Stator effective length</td>
<td>6380 mm</td>
</tr>
<tr>
<td>Core outside radius</td>
<td>1304.3 mm</td>
</tr>
<tr>
<td>Core inside radius (bore)</td>
<td>628.65 mm</td>
</tr>
<tr>
<td>Armature slot depth</td>
<td>196.85 mm</td>
</tr>
<tr>
<td>Armature slot width</td>
<td>43.94 mm</td>
</tr>
<tr>
<td>Stator slots</td>
<td>42</td>
</tr>
<tr>
<td>Stator leakage</td>
<td>0.19 pu</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>0.001579 ohm @25°C</td>
</tr>
</tbody>
</table>

Table A.3: Rotor data.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Slots per pole</td>
<td>18</td>
</tr>
<tr>
<td>Rotor-length of pole face</td>
<td>6875.8 mm</td>
</tr>
<tr>
<td>Rotor-radius</td>
<td>549.9 mm</td>
</tr>
<tr>
<td>Field slots</td>
<td>24</td>
</tr>
<tr>
<td>Field turns per pole</td>
<td>120</td>
</tr>
<tr>
<td>Field resistance</td>
<td>0.1021 ohm @25°C</td>
</tr>
<tr>
<td>Air-gap length</td>
<td>78.74 mm</td>
</tr>
<tr>
<td>Rated field current</td>
<td>1001 A d.c.</td>
</tr>
</tbody>
</table>
Appendix B

Parameters for The Linear Reluctance Model

To construct the reluctance matrix, the machine is divided to 36 sections per layer. hence the total number of meshes is $36 \times 3 = 108$. The sector width is taken to be $10^\circ$. The following sections present the numerical values for the different parameters of the test machine.

B.1 Base Quantities

The base MVA $S_b = 588MVA$, and the base phase voltage is the rated phase voltage, i.e.,

$$V_b = \frac{22}{\sqrt{3}} = 12.7KV$$  \hspace{1cm} (B.1)

The stator base impedance is

$$Z_{b_s} = \frac{KV_L^2 - L_{base}}{MVA_{base}} = \frac{22^2}{588} = 0.823\Omega$$  \hspace{1cm} (B.2)

The base current is

$$I_b = \frac{V_b}{Z_b} = 15.43KA$$  \hspace{1cm} (B.3)
B.2 Unsaturated Synchronous Reactance

The open- and short-circuit curves of the machine can be used to determine the unsaturated synchronous reactance $X_{du}$. From the open circuit test, the field current required to produce machine's rated voltage on the air gap line is 1001 A. From the short-circuit test, the short circuit stator current with 1001 A field current is 6572 A.

Therefore, $X_{du}$ is

$$X_{du} = \frac{\text{Rated stator current}}{\text{Short circuit stator current at rated field current}}$$

$$= \frac{15.43}{6.572}$$

$$= 2.348 \text{ pu}$$

which is 1.932 Ω.

B.3 Field Windings

The Nanticoke #5 machine has 36 rotor slots. The number of field slots is 24, with 10 conductors in each field slot. Therefore, the slot angle $\alpha_f = 10^\circ$. The field winding distribution factor $k_{df}$ is given by

$$k_{df} = \frac{\sin(\frac{c_f \alpha_f}{2})}{c_f \sin(\frac{\alpha_f}{2})} = 0.828$$

where $c_f$ is the number of slots per pole per phase of the field winding which equals 12.

The field windings are full pitch, i.e., $k_{pf} = 1$. Therefore, the field winding factor is

$$k_{wf} = 0.828$$

The effective field turns $N_{fe}$ is
$N_{fe} = \frac{4}{\pi} k_{wf} N_f = 126.5$  \hfill (B.7)

### B.4 Stator Windings

The number of stator slots $S$ is 42. Therefore, the slot angle of the stator $\alpha_s$ is

$$\alpha_s = \frac{360}{42} = 8.57^\circ$$  \hfill (B.8)

The number of slots per pole per phase of the stator $c_s$ is 7. The distribution factor of the stator $k_{ds}$ is

$$k_{ds} = \frac{\sin\left(\frac{\alpha_s}{2}\right)}{c_s \sin\left(\frac{\alpha_s}{2}\right)} = 0.956$$  \hfill (B.9)

The windings of the stator are short pitch. The coil span is 19 slots. This yields to a pitch factor of

$$k_{df} = \sin\left(\frac{19\alpha_s}{2}\right) = 0.989$$  \hfill (B.10)

The effective stator turns $N_{se}$ is

$$N_{se} = \frac{4}{\pi} k_{ws} N_s = 8.423$$  \hfill (B.11)

### B.5 Turns Ratio

The effective turns ratio $n_e$ is

$$n_e = \frac{N_{fe}}{N_{se}} = \frac{126.5}{8.423} = 15.02$$  \hfill (B.12)
Appendix B. Parameters for The Linear Reluctance Model

B.6 Carter Coefficients

Carter coefficients are calculated for both the stator and the rotor slots based on the width of the slot, the width of the teeth, and the air gap length, as explained in [57]. In case of Nanticoke #5, different Carter coefficients are assigned to the rotor slots according to their dimensions. For the stator slots $C_s = 1.06$, for the field slots $C_q = 1.06$, and for the dummy slots $C_d = 1.03$. Therefore the average effective air gap length can be taken to be 1.1 times the actual quoted length, i.e.,

$$g_e = 1.1 \times 0.079 = 0.0869 \text{ m}.$$

B.7 Magnetizing Inductance

The unsaturated magnetizing inductance of the machine can be given by [57]

$$X_m = 2\pi f \frac{3\pi}{2} \left( \frac{N_{se}}{F} \right)^2 \mu_0 \frac{l_{sr_s}}{g_e} = 1.83 \text{ } \Omega$$

(B.13)

With base impedance of $Z_{b_s}$, the magnetizing inductance is 2.22 pu.

B.8 Stator Slot Reluctance

Considering the stator slot shown in Figure B.1, the slot permeance ratio $\varphi_{s1}$ is given by

$$\varphi_{s1} = \varphi_A + \varphi_B + 2\varphi_{AB}$$

(B.14)

where $\varphi_A$ and $\varphi_B$ are the permeance ratio for the upper and lower coil sides respectively, and $\varphi_{AB}$ is the mutual permeance ratio. These different permeance ratios can be given in terms of slot dimensions as follows

$$\varphi_A = \frac{d_1}{6w} + \frac{d_2}{w_1} + \frac{d_3}{w}$$

(B.15)
where $\theta$ is the phase shift between the two coil sides in the same slot. For the dimensions given in Table B.1, the slot permeance ratio for Nanticoke is $\varphi_{s1} = 7.28$.

![Figure B.1: Section of the Nanticoke # 5 stator slot.](image)

### Table B.1: Nanticoke #5 stator slot dimensions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Length mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>43.9</td>
</tr>
<tr>
<td>$w_1$</td>
<td>54.0</td>
</tr>
<tr>
<td>$d_1$</td>
<td>162.0</td>
</tr>
<tr>
<td>$d_2$</td>
<td>19.0</td>
</tr>
<tr>
<td>$d_3$</td>
<td>15.6</td>
</tr>
</tbody>
</table>

The slot reluctance is thus

$$R_{s1} = \frac{1}{\mu_0 l_s \varphi_{s1}} = 15759.44 \text{ AT/Wb} \quad (B.16)$$

The corresponding stator slot inductance $L_{s1}$ is
Appendix B. Parameters for The Linear Reluctance Model

\[ L_{s1} = \mu_0 l_s \frac{N_s^2}{P_c_s} \Phi_{s1} = 222 \mu \text{ H} \]  
\[ \text{(B.17)} \]

With the base inductance \( L_{base} = 2183 \mu \text{H} \), the stator slot inductance is

\[ L_{s1} = \frac{222}{2183} = 0.102 \text{ pu} \]  
\[ \text{(B.18)} \]

### B.9 Peripheral Stator Leakage

The ratio of the peripheral air gap leakage to the magnetizing inductance is [57]

\[ \frac{X_p}{X_m} = \frac{g_e^2 P^2}{\pi^2 r_s^2} \]  
\[ \text{(B.19)} \]

With \( X_m = 2.22 \text{ pu} \), the peripheral leakage \( X_p \) is 0.017 pu.

### B.10 Stator End Leakage

The stator end leakage reactance can be approximated to [57]

\[ X_e = 2\pi f 16 \frac{N_s^2}{P^2} \mu_0 r_s = 0.058 \Omega \]  
\[ \text{(B.20)} \]

which is 0.071 pu.

### B.11 Air Gap Reluctance

Since the machine is a non-salient pole machine with uniform air gap, the air gap reluctance can be calculated directly from Equation (3.13) at the average machine diameter. The air-gap reluctance for each sector is

\[ R_{\text{gap}} = \frac{g_e}{\mu_0 r \Delta \theta l_s} = 105388 \text{ AT/Wb} \]  
\[ \text{(B.21)} \]
B.12 Field Slot Reluctance

The field slot can be simplified as shown in Figure B.2 with corresponding dimensions in Table B.2. The slot permeance ratio is given by

\[ \varphi_f = \frac{d_1}{3w_1} = 0.77 \]  \hspace{1cm} (B.22)

Hence, the field slot reluctance \( R_f \) is

\[ R_f = \frac{1}{\mu_0 l_r \varphi_f} = 149640.5 \text{ At/Wb} \]  \hspace{1cm} (B.23)

Table B.3 shows the dimensions of the dummy slot. The slot reluctance for this slot is 155135 At/Wb.

![Figure B.2: Section of the Nanticoke #5 approximate rotor slot.](image)
### Appendix B. Parameters for The Linear Reluctance Model

#### Table B.2: Nanticoke #5 field slot dimensions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>58.7</td>
</tr>
<tr>
<td>$w_2$</td>
<td>70.5</td>
</tr>
<tr>
<td>$d_1$</td>
<td>136.2</td>
</tr>
<tr>
<td>$d_2$</td>
<td>39.9</td>
</tr>
</tbody>
</table>

#### Table B.3: Nanticoke #5 dummy slot dimensions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>39.9</td>
</tr>
<tr>
<td>$w_2$</td>
<td>54.0</td>
</tr>
<tr>
<td>$d_1$</td>
<td>89.3</td>
</tr>
<tr>
<td>$d_2$</td>
<td>42.3</td>
</tr>
</tbody>
</table>
Appendix C

Sensitivity Analysis

C.1 Introduction

The cross-section and axial length dimensions of the machine, which are needed to develop the linear distributed model, may not be accurately determined. Under such conditions, an estimated set of dimensions may be used, instead, to construct the distributed machine model. The accuracy of the obtained model-results should, then, be assessed.

In this appendix, the sensitivity of the distributed-model-performance to small changes in machine dimensions is discussed. The effect of probable measurement-noise on the accuracy of the developed model is also addressed.

C.2 Linear Model

The number of independent parameters in the linear model of Figure 3.1 is five, which are: air gap length in the direct axis, air gap length in the quadrature axis, stator leakage per sector, rotor leakage per field slot, and rotor leakage per dummy slot. Under linear assumptions, any change in these five parameters is expected to affect linearly the corresponding machine parameters. To verify this statement, the effect of changing the effective air gap length on the magnetizing inductance is investigated. The linear model developed in Chapter 3 is used to evaluate the
magnetizing inductance of the machine for a varying air gap length in the range of ± 5% of the quoted air gap length, while keeping the other machine dimensions and parameters unchanged. Results are plotted in Figure C.1. These results show that the magnetizing inductance is linearly proportional to the effective air gap length. Such results are expected under linear assumptions.

![Linear model sensitivity analysis](image)

Figure C.1: Changes in the magnetizing inductance with air gap length.

Similarly, it can be shown that the stator leakages is linearly dependent on the reluctance of the stator slot. Same conclusion applies for the rotor.

C.3 Nonlinear Model

The objective of this section is to investigate the sensitivity of the nonlinear steady-state distributed model to small changes in the dimensions of the iron sections. The
open-circuit characteristic (OCC) of the test machine is evaluated for the two following cases:

- Using the given dimensions of the rotor slot-tooth combinations to evaluate the rotor-tooth flux density and magnetic field. This means Equations (4.4) and (4.5) are evaluated for the five different cross sections along the rotor tooth.

- Considering the rotor tooth to be of a rectangular shape, with an approximate average width over the entire depth of the tooth.

Although the difference between the assumed average width and the exact dimension of the tooth varies in the range of ± 10% of the exact dimensions, the obtained OCCs are within 1% difference.

This can lead to the conclusion that the sensitivity of the nonlinear steady-state distributed model to small changes in the machine dimensions is almost negligible.

C.4 Effect of Measurements Noise

The parameters evaluated from short- and open-circuit tests will be linearly proportional to the measurements i.e. 1% error in any measurement will cause 1% error in the accuracy of the evaluated parameters.

The SSFR tests provide the direct and quadrature operational impedances i.e. \( Z_d(s) \) and \( Z_q(s) \), respectively. The operational inductances, \( L_d(s) \) and \( L_q(s) \) are then determined from:

\[
\begin{align*}
L_d(s) &= \frac{Z_d(s) - R_a}{s} \\
L_q(s) &= \frac{Z_q(s) - R_a}{s}
\end{align*}
\]
At the low frequency end, the effect of $R_a$ is dominant, which makes the operational inductances very sensitive to the noise.

Moreover, the SSFR tests are conducted near zero flux density in the rotor. Consequently, the rotor iron permeability is considerably lower than that under open or short circuit tests conditions. This explains the noticeable difference between $L_d(0)$ determined from SSFR tests, and $L_{du}$ determined from open and short circuit tests.

Therefore, in the developed distributed frequency-domain model, the convergence of the model results to the low frequency end measurements was not relevant.

However, over higher frequency range, the effect of the operational inductances becomes more dominant than that of $R_a$. Hence, the matching between results of the developed model and the SSFR measurements can be taken as a measure of accuracy of the developed model.
Bibliography


[34] H.R. Schwenk, “Deriving Synchronous Machine Models from frequency Response Data”, IEEE Symposium Publication 83THO 101-6-PWR.


