THE BEHAVIOR OF
REINFORCED CONCRETE KNEE JOINTS UNDER
EARTHQUAKE LOADS

by

Bill Angelakos

A Thesis submitted in conformity with the requirements
for the Degree of Doctor of Philosophy,
Department of Civil Engineering, in the
University of Toronto

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Bill Angelakos, Department of Civil Engineering, University of Toronto.

ABSTRACT

The poor performance of knee joint connections during recent earthquakes motivated a number of experimental investigations of knee joint behavior under reversed cyclic loading.

In this work the knee joint design problem is studied through a collective evaluation of the available experimental results and analytical modeling. The objective is to identify the critical response variables controlling the mechanics of knee joints under earthquake loads and to quantify the influence they have on the strength and deformation capacity of the joint.

A knee joint model is derived from simple mechanical constructs of equilibrium and compatibility. The parametric dependence of knee joint behavior is investigated for critical design parameters such as concrete strength, amounts and yield strengths of horizontal and vertical transverse reinforcement, and bond demand. Three different limiting equations are developed from the model limiting the joint shear resistance according with the three alternative modes of joint shear failure. These are: (i) yielding of horizontal and vertical transverse reinforcement, (ii) and (iii) yielding in either of the two principal reinforcing directions accompanied by crushing of the concrete in compression (here the softening influence of orthogonal tensile deformations is considered). For those test specimens from the experimental database that experienced a joint shear failure, the simple knee joint model predicts their joint shear capacity well.
Consistent with observations from interior connections it is shown that anchorage of the main reinforcement in the knee joint region prevails as the determining factor of the response of the joint panel. In addition, the same basic physical model that describes the source of resistance in interior connections also applies to knee joints; truss action, and diagonal strut action. By favorably anchoring the beam and column bars it is possible to develop the joint shear strength which is associated with one of the three failure modes described above. It is demonstrated by the experimental and analytical evidence (by finite element studies) that confinement in the vertical direction through the use of vertical stirrups is as important as that in the horizontal direction.

Based on the above experimental and analytical evidence, simple detailing guidelines are proposed for practical design of knee joints.
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<td>stirrup index of confinement</td>
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<td>$a_0$, $a_1$, $a_2$</td>
<td>coefficients related to the anchorage conditions of the transverse reinforcement</td>
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<td>$A_j$</td>
<td>effective cross sectional area for a plane within the joint parallel to the direction of shear</td>
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<td>$a_{rm_b}$</td>
<td>effective depth for the beam</td>
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<tr>
<td>$a_{rm_c}$</td>
<td>effective depth for the column</td>
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<td>$A_a$</td>
<td>area of the beam tension reinforcement</td>
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<td>$A_t$</td>
<td>cross-sectional area of transverse reinforcement</td>
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<td>$b_{j}$, $b$</td>
<td>effective width of the joint transverse to the direction of shear</td>
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<td>concrete index of confinement</td>
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<td>$b_b$</td>
<td>beam width transverse to the direction of shear</td>
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<td>column width transverse to the direction of shear</td>
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<td>joint core dimensions to centreline of perimeter hoop in both x and y-horizontal directions</td>
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<td>BF</td>
<td>beam member flexure contribution to total drift (%)</td>
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<td>BI</td>
<td>bond index $= d_b f_y / 4 t_v f'_c$</td>
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<td>BS</td>
<td>beam bar slip contribution to total drift (%)</td>
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<td>$b_{slip}$</td>
<td>beam bar slip at respective joint faces</td>
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<td>$C_{bl}$</td>
<td>compression force at the left beam member transmitted to the joint at the joint face</td>
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<td>$C_c$</td>
<td>compressive force in concrete</td>
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<td>CF</td>
<td>column member flexure contribution to total drift (%)</td>
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<td>d</td>
<td>effective depth of column</td>
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\[
\begin{align*}
\text{\(d_b\)} & = \text{tensile reinforcing bar diameter} \\
\text{\(d_{bt}\)} & = \text{transverse reinforcing bar diameter} \\
\text{\(d_{so}\)} & = \% \text{drift when post-peak resistance has decayed to 80\% of peak value} \\
\text{\(d_e\)} & = \text{distance from the extreme compression fibre to the centroid of the longitudinal tensile reinforcement} \\
\text{\(E_c\)} & = \text{concrete secant stiffness} \\
\text{\(E_o\)} & = \text{initial uncracked tangent modulus for concrete} \\
\text{\(E_{sh}\)} & = \text{strain hardening modulus for steel} \\
\text{\(F\)} & = \text{damage indicator} \\
\text{\(f_{bb}\)} & = \text{reinforcing bar stresses for the beam bottom reinforcing bar in the \(x\)-direction at the column centreline} \\
\text{\(f_{bt}\)} & = \text{reinforcing bar stresses for the beam top reinforcing bar in the \(x\)-direction at the column centreline} \\
\text{\(f'_c\)} & = \text{specified uniaxial concrete compressive strength} \\
\text{\(f'_{cc}\)} & = \text{peak compressive strength of confined cylinders} \\
\text{\(F_c\)} & = \text{compressive force acting on the joint boundaries} \\
\text{\(f_{ce}\)} & = \text{effective concrete strength of the square concrete prism} \\
\text{\(f_{ci}\)} & = \text{reinforcing bar stresses in the \(y\)-direction at the beam centreline for the column inner reinforcing bars} \\
\text{\(f_{cm}\)} & = \text{peak strength of concrete, crushing strength of concrete} \\
\text{\(f_{cmm}\)} & = \text{reinforcing bar stresses in the \(y\)-direction at the beam centreline for the column middle reinforcing bars} \\
\text{\(f_{co}\)} & = \text{reinforcing bar stresses in the \(y\)-direction at the beam centreline for the column outer reinforcing bars} \\
\text{\(f_{hx}, f_x\)} & = \text{average hoop bar stresses in the \(x\)-direction at the column centreline} \\
\text{\(f_{rx}, f_y\)} & = \text{average stirrup bar stresses in the \(y\)-direction at the beam centreline} \\
\text{\(f_{sp}\)} & = \text{peak splitting strength of concrete} \\
\text{\(F_r\)} & = \text{reinforcement stress} \\
\text{\(F_{s,opp}\)} & = \text{force in the beam top reinforcing bars} \\
\text{\(f_t\)} & = \text{tensile splitting strength of the square concrete prism} \\
\text{\(f_{xy}, f_{yh}\)} & = \text{hoop reinforcement yield stress}
\end{align*}
\]
\[ f_{ys}, f_{ys} = \text{stirrup reinforcement yield stress} \]
\[ f_{yr} = \text{yield strength for the beam and column tension reinforcement} \]
\[ h = \text{effective height of the column} \]
\[ h_c = \text{column thickness in the direction of loading} \]
\[ h_{cc} = \text{width of the concrete core measured to the outside of the hoops} \]
\[ JD = \text{joint distortion contribution to total drift (\%)} \]
\[ k_1, k_2, k_{ce} = \text{coefficients determined experimentally} \]
\[ k_e = \text{coefficient that accounts for the beneficial effects of confinement on strength} \]
\[ k_e = \text{geometric variable that characterizes the effectiveness of the specific arrangements of confining reinforcement} \]
\[ k_s = \text{reduction factor for the concrete strut} \]
\[ L, l_b = \text{beam length from inflection point to the joint centreline} \]
\[ l_{bf} = \text{beam length measured from the assumed inflection point to the joint face} \]
\[ l_c, H = \text{column length from inflection point to the joint centreline} \]
\[ l_{cf} = \text{column length measured from the assumed inflection point to the joint face} \]
\[ l_{dc} = \text{length of parabolic tensile stress distribution} \]
\[ l_t = \text{total anchorage length of the beam top or column outer reinforcement measured from their respective joint faces} \]
\[ M = \text{bending moment generated at the joint boundaries under opening action} \]
\[ M_{bf} = \text{maximum beam moment at the joint face} \]
\[ M_{cf} = \text{column moment at the joint face} \]
\[ M_{dc} = \text{moment at joint member interface at which diagonal cracking first occurs} \]
\[ n = \text{modular ratio (E_s/E_c)} \]
\[ n_t = \text{number of transverse reinforcing bar legs in the joint region} \]
\[ N_x = \text{beam axial force} \]
\[ N_y = \text{column axial force} \]
\[ P = \text{static horizontal load} \]
\[ R = \text{radius of bend of the tensile reinforcement} \]
\[ R/h_c = \text{radius of bend ratio} \]
\[ s = \text{centre to centre spacing of the hoops} \]
\[ s_r = \text{reinforcing bar slip} \]
\[ s' = \text{clear spacing between hoop reinforcement in the joint region} \]
\[ t_o = \text{tangent offset of the joint face from the assumed inflection point in the beam} \]
\[ T_{bl} = \text{tensile force in the left beam at the face of the column} \]
\[ T_{br} = \text{tension force at the right beam member transmitted to the joint} \]
\[ t_c = \text{tangent offset of the joint face from the assumed inflection point in the column} \]
\[ T_s = \text{probable tensile yield strength} \]
\[ u_0, u_1 = \text{coefficients determined experimentally} \]
\[ u_{av} = \text{average bond stress} \]
\[ u_L = \text{local bond stress} \]
\[ u_{m,o} = \text{maximum local bond stress for zero splitting crack width} \]
\[ u_{m,w} = \text{maximum local bond stress that can be obtained for a given splitting crack width} \]
\[ v = \text{poisson’s ratio for concrete} \]
\[ v_b = \text{average beam shear stress at the joint face} \]
\[ v_c = \text{average column shear stress at the joint face} \]
\[ V_c = \text{column shear force} \]
\[ V_{ct} = \text{column shear force acting on the joint boundary} \]
\[ v_f = \text{factored joint shear force} \]
\[ \overline{\nu}_{h,obs} = \text{observed horizontal joint shear stress} \]
\[ \nu_j = \text{joint shear stress} \]
\[ V_j = \text{joint shear force input} \]
\[ \overline{V_j} = \text{expected average shear stress} \]
\[ V_j^c = \text{horizontal joint shear input under closing action} \]
\[ V_{jh} = \text{horizontal joint shear force at the joint centre} \]
\( V_j^o \) = horizontal joint shear input under opening action
\( v_n \) = nominal joint shear stress
\( V_n \) = nominal joint shear strength
\( V_r \) = factored joint shear resistance
\( V_u \) = design shear force
\( v_{xy} \) = concrete joint shear stress
\( w \) = splitting crack width
\( w_i \) = clear spacing between longitudinal reinforcement
\( z \) = moment arm approximated to be 0.8\( d_e \)
\( Z \) = length of the hypotenuse before knee joint specimen drift
\( Z_d \) = length of the hypotenuse after knee joint specimen drift
\( z_m \) = slope of the post-peak branch of the stress-strain curve
\( \alpha \) = angle which defines the orientation of the principal tensile stress with respect to the horizontal axis
\( \alpha_r \) = shape factor characterizing the reinforcing bars
\( \alpha_s \) = angle between the concrete strut and the reinforcing tie
\( \alpha_{sm} \) = stress multiplier
\( \beta \) = ratio of average beam or column reinforcing bar stress to average stirrup or hoop stress at the column or beam centrelines, respectively.
\( \beta_1, \beta_2 \) = coefficient experimentally determined
\( \beta_{bb} \) = bond efficiency factor, beam bottom reinforcing bar layers
\( \beta_{bt} \) = bond efficiency factor, beam top reinforcing bar layers
\( \beta_{ci} \) = bond efficiency factor, column inner reinforcing bar layers
\( \beta_{cm} \) = bond efficiency factor, column middle reinforcing bar layers
\( \beta_{co} \) = bond efficiency factor, column outer reinforcing bar layers
\( \beta_G \) = bond efficiency factor, realizes stress gradients across the joint
\( \beta_m \) = material variable which can be approximated as \( \varepsilon_o \)
\( \beta_T \) = true bond efficiency factor, realizes effects of anchorage reinforcement slip and stress gradients across the joint
\( \gamma/2 \) = average concrete strain in the x-y plane at the joint centre
\( \gamma \) = shear distortion of the panel

xxiv
\( \gamma_1, \gamma_2 \quad = \quad \text{coefficients experimentally determined} \\
\gamma_c \quad = \quad \text{constant which depends on the joint classification} \\
\gamma_{xy} \quad = \quad \text{concrete joint shear strain} \\
\delta \quad = \quad \text{lateral displacement of the test specimen} \\
\epsilon'_{cc} \quad = \quad \text{concrete strain corresponding to peak concrete stress} \\
\epsilon_1 \quad = \quad \text{principal tensile strain} \\
\epsilon_2 \quad = \quad \text{principal compressive strain} \\
\epsilon_{2r} \quad = \quad \text{principal compressive strain at point of zero volumetric strain} \\
\epsilon_3 \quad = \quad \text{principal tensile strain in the out-of-plane direction} \\
\epsilon_A \quad = \quad \text{area strain} \\
\epsilon_{cm} \quad = \quad \text{strain at peak stress of confined concrete, crushing strain of confined concrete} \\
\epsilon_{cr} \quad = \quad \text{cracking strain of concrete} \\
\epsilon_o \quad = \quad \text{strain in the compressive strut at peak compressive stress, concrete strain at peak axial concrete strain} \\
\epsilon_s \quad = \quad \text{strain required in the tensile reinforcing tie} \\
\epsilon_v \quad = \quad \text{volumetric strain} \\
\epsilon_x \quad = \quad \text{average concrete longitudinal strains in the x-direction at the joint centre} \\
\epsilon_y \quad = \quad \text{average concrete longitudinal strains in the y-direction at the joint centre} \\
\epsilon_{yr} \quad = \quad \text{reinforcement yield strain} \\
\epsilon_z \quad = \quad \text{average concrete longitudinal strains in the z-direction at the joint centre} \\
\theta \quad = \quad \text{angle which defines the orientation of the principal tensile stress with respect to the horizontal (x) axis} \\
\lambda \quad = \quad \text{the effect of compression softening and confinement on the concrete strength in the principal compressive stress direction} \\
\lambda_1 \quad = \quad \text{factor to correct for the density of concrete} \\
\rho \quad = \quad \text{longitudinal reinforcement ratio} \\
\rho_b \quad = \quad \text{beam reinforcement ratio} 

xxv
\( \rho_{bb} = \) reinforcing bar ratio in the x-direction for the beam bottom reinforcement

\( \rho_{bt} = \) reinforcing bar ratio in the x-direction for the beam top reinforcement

\( \rho_{cc} = \) ratio of area of longitudinal steel to area of core of section

\( \rho_{ci} = \) reinforcing bar ratio in the y-direction for column inner reinforcing bars

\( \rho_{cm} = \) reinforcing bar ratio in the y-direction for column middle reinforcing bars

\( \rho_{co} = \) reinforcing bar ratio in the y-direction for column outer reinforcing bars

\( \rho_{h} = \) reinforcing bar ratio in the x-direction for the hoop reinforcement

\( \rho_{lc} = \) limiting longitudinal reinforcement ratio

\( \rho_{lo} = \) limiting longitudinal tension reinforcement ratio

\( \rho_{s} = \) reinforcing bar ratio in the y-direction for stirrup reinforcing bars

\( \rho_{v} = \) volumetric reinforcement ratio with reference to the concrete core measured to the outside of the hoops

\( \rho_{ve} = \) effective volumetric ratio for the transverse reinforcement

\( \rho_{vh} = \) hoop volumetric reinforcement ratio

\( \rho_{vs} = \) stirrup volumetric reinforcement ratio

\( \rho_{x} = \) per cent of total horizontal reinforcement at the column centreline

\( \rho_{y} = \) per cent of total vertical reinforcement at the beam centreline

\( \sigma_{1} = \) magnitude of the principal tensile stress in the concrete

\( \sigma_{2} = \) magnitude of the principal compressive stress in the concrete

\( \sigma_{\text{anch}} = \) anchorage reinforcement stress at joint centreline

\( \sigma_{\text{cto}} = \) residual tensile strength of cracked concrete when the splitting crack width just begins to grow

\( \sigma_{\text{lat}} = \) lateral confining pressure

\( \sigma_{n} = \) radial confinement stress

\( \sigma_{rc} = \) additional confining action provided by the cracked concrete

\( \sigma_{t} = \) stress developed in the transverse reinforcement crossing the crack
\( \sigma_{ta} = \) transverse reinforcement stress at joint centreline adjacent to anchorage reinforcement

\( \sigma_{tc} = \) stress in the transverse reinforcement at the joint centre

\( \sigma_x = \) average concrete normal stress in the x-direction at the joint centre

\( \sigma_y = \) average concrete normal stress in the y-direction at the joint centre

\( \sigma_z = \) average normal stress in the z-direction at the joint centre

\( \phi = \) strength reduction factor

\( \phi_c = \) material reduction factor for concrete

\( \omega = \) beam reinforcement index
CHAPTER 1
INTRODUCTION

1.1 Background

A reinforced concrete knee joint (knee joint) comprises one beam and one column connected at their ends, forming a 90° angle. Knee joints are used predominantly in two types of reinforced concrete structures: at roof tops of reinforced concrete buildings and in bridge bents of reinforced concrete bridges (Figure 1.1).

In the design of reinforced concrete bridges, redundancy is usually kept low to allow the structure some movement to accommodate thermal, creep and shrinkage effects. During an earthquake, knee joints in reinforced concrete bridges may experience many cycles in the inelastic range due to significant drift excursions. One consequence of having low redundancy in the bridge is that during a strong earthquake, failure of a knee joint connection may precipitate progressive collapse of the entire structure. The poor performance of knee-type connections in reinforced concrete bridges during the 1989 Loma Prieta earthquake (Housner 1990) motivated this study.

Knee joints are unique in that both the beam and column members terminate there as compared with other typical frame connections (CASE B - c.2, Figure 1.2). Most of the work done in the past to understand the behaviour of beam-column connections under earthquake-type loading, focussed on interior and exterior beam-column frame connections. The behaviour of such connections is understood fairly well, and relevant specifications have been incorporated in major design codes around the world. However, little work has been done towards understanding the behaviour of knee joints particularly under inelastic load reversals. A consistent mechanical model interpreting the workings of knee joints is essential for development of design guidelines for earthquake resistance of these connections.
1.1a) Knee Joints in Reinforced Concrete Frame Buildings.

1.1b) Knee Joints in Bridge Bents of Reinforced Concrete Bridges.

Figure 1.1 Typical Knee Joint Connections Found in Reinforced Concrete Structures.
**CASE A: Two columns framing into the joint**

(a) Interior

(b.1) Exterior

(c.1) Corner

(b.2) Exterior

(c.2) Corner

**CASE B: One column framing into the joint (roof connection)**

(a) Interior

(b.1) Exterior

(c.1) Corner

(b.2) Exterior

(c.2) Corner

Figure 1.2 Classification of Beam-Column Connections in Reinforced Concrete Structures (From Draft ACI352 - 1998).
1.2 Goals and Objectives

The scope of this study is to explore and interpret the mechanical behaviour of knee joints and to provide the necessary tools to quantify the resistance and deformability of these connections when subjected to earthquake type loading. Another objective is to contribute towards development of comprehensive design recommendations of these elements.

1.3 Code Development
1.3.1 Building Code Development

Most codes have adopted a capacity design approach for frame design assuming beam flexural hinging at the beam-column connections. The beam strength controls the shear input to the joint, and joint design is driven by its nominal shear capacity. Transverse reinforcement is placed within the joint to provide confinement for the joint core and to resist the joint shear. Codes specify additional detailing requirements to control cracking within the joint and bond deterioration of the main longitudinal reinforcement framing into the joint. A brief summary of the earthquake design requirements for joints in major design codes around the world follows to outline the state of practice and limitations with regards to knee-joints.


In North America, the American (ACI318(1991)) and Canadian (CSA(1994)) codes address the process of dimensioning the joint by specifying allowable stress limits. Depending on the type of joint (interior, exterior, etc.), the allowable stress is estimated as a multiple of the square root of the concrete strength. The proportionality constants were obtained by correlating the results of several tests on joints (Meinheit and Jirsa (1981)).

In ACI318-1991 the nominal joint shear strength must exceed the joint shear force input i.e.,

\[ \phi V_n \geq V_u \] \hspace{1cm} 1.1

where; \( V_u \) is the design shear force, \( V_n \) is the nominal joint shear strength, and \( \phi \) is a strength reduction factor equal to 0.85.
The nominal joint shear strength is given as:

$$V_n = \gamma_c \sqrt{f'_c \text{ (psi)}} b_j h_c$$  \hspace{1cm} 1.2

$$V_n = 0.083 \gamma_c \sqrt{f'_c \text{ (MPa)}} b_j h_c$$  \hspace{1cm} 1.3

where; $f'_c$ is the specified concrete compressive strength, $b_j$ is the effective joint width, $h_c$ is the column thickness in the direction of loading, and the constant $\gamma_c$ depends on the joint classification. $\gamma_c$ is equal to 12 for knee joints.

The effective joint width is given as:

$$b_j = \frac{(b_b + b_c)}{2} > b_b + \frac{1}{2} \text{ column depth } h_c \text{ on each side of the beam}$$  \hspace{1cm} 1.4

where; $b_b$ is the beam width, and $b_c$ is the column width transverse to the direction of loading.

A draft version of ACI352-1998 has recommended the constant $\gamma_c$ equal 8 under the new joint classification for knee joints (Case B - B.4, Figure 1.3). This recommendation came about as a result of the experimental work done on knee joints at Clarkson University (Cote and Wallace 1994, McConnell and Wallace 1995). In Figure 1.3 joint classifications are given for Type 2 Connections where members framing into the joint are designed to have sustained strength under deformation reversals into the inelastic range (Draft Version ACI352 - 1998).
TYPE 2 CONNECTIONS
CASE A: Two columns framing into the joint

A.1: $\gamma_c = 20$
A.2: $\gamma_c = 15$
A.3: $\gamma_c = 12$

CASE B: One column framing into the joint

B.1: $\gamma_c = 15$
B.2: $\gamma_c = 12$
B.3: $\gamma_c = 9$
B.4: $\gamma_c = 8$

Figure 1.3  $\gamma_c$ Values for Type 2 Connections (From Draft of ACI352-1998).
In CSA-1994, the factored joint shear resistance must exceed the factored joint shear force i.e.,

\[ V_r \geq V_f \]  \hspace{1cm} (1.5)

where; \( V_r \) is the joint shear resistance, and \( V_f \) is the factored joint shear force.

The joint shear resistance is given as:

\[ V_r = 1.5 \lambda_1 \phi_c \sqrt{f'_c} A_j \]  \hspace{1cm} (1.6)

where; \( \phi_c \) is the resistance factor for concrete and is equal to 0.6, \( \lambda_1 \) is a factor that accounts for low-density concrete, and \( A_j \) is the effective cross-sectional area for a plane within the joint parallel to the direction of shear.

The major difference in the North American Codes is that the Canadian Code (CSA - 1994) uses the Limit States Design approach whereas the American Code (ACI - 1991) uses the Strength Design Method.

In both North American codes, joint shear input is calculated on a horizontal plane at the joint centre and is obtained through free-body equilibrium of the joint panel subjected to boundary forces associated with yielding of the beam tension reinforcement at the column face as depicted in Figure 1.4 for an interior beam column connection. The horizontal joint shear input is obtained from satisfying horizontal equilibrium for the forces shown in the top portion of the free body diagram of the interior joint (Figure 1.5).

i.e.,

\[ V_{jh} = T_{br} + C_{bt} - V_{ct} \]  \hspace{1cm} (1.7)

where; \( V_{jh} \) is the horizontal joint shear force at the joint centre, \( T_{br} \) and \( C_{bt} \) are the tension and compression forces of the right and left beam member, respectively, transmitted to the joint and, \( V_{ct} \) is the column shear force acting on the joint boundary.
Figure 1.4  Idealized Forces Acting at Boundaries of Interior Beam-Column Connection Due to Earthquake-Type Loading (ACI352-1985).

Figure 1.5  Horizontal Forces at Boundaries of Interior Beam-Column Connection (ACI352-1985).
In the absence of beam axial forces, the tension and compression forces, on the critical beam sections are equal, therefore:

\[ V_{jh} = T_{br} + T_{bl} - V_{ct} \]  

1.8

where; \( T_{bl} \) is the tensile force in the left beam at the face of the column.

When calculating the tensile forces \( T_{br} \) and \( T_{bl} \) the yield strength of the beam reinforcement is considered 25 per cent greater than the nominal value. This increase accounts for the strain hardening which is expected to occur at large curvatures, in the plastic hinge regions.

As discussed above, the North American Codes require that the horizontal joint shear input obtained from Equation 1.8 does not exceed the horizontal joint shear resistance given by Equations 1.2, 1.3 and 1.6. Vertical joint shear is not checked explicitly. Instead, adequate vertical joint shear resistance is presumed to be provided by distributing the column reinforcing bars uniformly around the column perimeter (the joint is part of the column). Horizontal transverse reinforcement in the form of closed hoops is required within the joint region to help provide confinement of the joint core concrete. The column main bars and the horizontal stirrups form a "cage" which provides confinement for the joint core concrete. Until recently, vertical stirrups were not required in the North American codes. However, the draft version of ACI352-1998 requires that vertical stirrups should be provided through the full height of the joint for knee joint connections.

Using the ACI equilibrium model, the horizontal joint shear input to the knee joint can be obtained as follows. Figures 1.6 and 1.7 show the idealized forces acting at the joint boundaries for opening and closing actions. From Figure 1.7, satisfying horizontal equilibrium, the following expressions are obtained for the horizontal joint shear input (\( V_{jh} \)) under opening and closing actions (superscripts o and c), respectively.
Figure 1.6  Idealized Forces at Boundaries of Knee Joint Connection.

Figure 1.7  Horizontal Forces at Knee Joint Boundaries.
where; $T_s$, $C_s$, and $C_c$ are the force resultants in the tension and compression reinforcement and in the compression zone at the beam critical section, respectively, and $V_c$ is the column shear force.

New Zealand And Europe

Similar recommendations for seismic design of reinforced concrete beam-column joints are adopted by the CEB-FIP Model Code (1990) and the New Zealand Standard (NZS3101-1995).

The CEB-New Zealand Models (CEB-FIP Model Code (1990) and NZS3101 (1995)) consider horizontal and vertical shear input to the joint calculated from simple equilibrium considerations as in the North American design codes.

However, unlike the North American codes, which quantify the nominal joint shear resistance in an empirical manner, in the CEB-New Zealand models, joint shear resistance comprises contributions of two simultaneous self-equilibrating mechanisms: a diagonal compressive strut mechanism and a self-equilibrating truss mechanism (Paulay et al, 1978 and Paulay, 1986, 1989, Figure 1.8). Paulay hypothesized that unless high axial loads are present, the truss mechanism is the predominant source of resistance to the joint. It was further suggested that horizontal and vertical transverse reinforcement initially provide confinement for the joint core and then carry tensile forces in the truss mechanism, which dominates the response once significant bond deterioration occurs between the beam longitudinal reinforcement and the concrete within the joint. The nominal joint shear resistance for joints (including knee joints) designed for earthquake effects is $0.20 f'_c$. 

\[ V_j^o = T_s - V_c = C_c + C_s \]  
\[ V_j^c = T_s = C_c + C_s - V_c \]
1.8a) Diagonal Compressive Strut Mechanism in Joint Core of Interior Beam-Column Joint.

1.8b) Self-Equilibrating Truss Mechanism: Interior Beam-Column Joint.

Figure 1.8 Joint Shear Resisting Mechanisms for CEB-New Zealand Models (Paulay et al 1978).
Japan

The model Code of the Architectural Institute of Japan (Otani (1991)) also obtains joint shear input in a manner similar to the North American codes. Two mechanisms of shear resistance are postulated as those adopted by the CEB-New Zealand models. Unlike the CEB-New Zealand models, however, it is assumed that the main diagonal strut mechanism, not the truss mechanism, dominates in providing shear resistance within the joint. Transverse reinforcement is placed in the joint region to provide confinement to the joint core concrete. There is no special provision for the design of knee joints in this model.

1.3.2 Bridge Design Code Development

The capacity design philosophy is used in most bridge design codes (AASHTO (1994), CEN(1994), TNZ(1994)) with the exception of the Japan Bridge code (JRA(1990)) which expects elastic performance from all bridge components for the occurrence of the design earthquake.

Using the capacity design approach, flexural yielding in the columns is chosen as the source of inelastic behaviour in the bridge structure. The inelastic activity in the columns is usually located near the top, adjacent to the joint boundary or near the column foundation, such that it is easily detected and repaired in the event of damage resulting from a strong earthquake. For such an event, linear elastic behaviour is expected from the remaining components of the bridge. The United States bridge design code AASHTO (1994) modified its provisions to incorporate new insights into the behaviour of reinforced concrete bridges following a number of investigations of the damage to several bridges during the 1989 Loma Prieta earthquake.

A report from the Governor’s Board of Inquiry, Housner (1990), following an investigation of the damage sustained by several reinforced concrete bridges as a result of the Loma Prieta earthquake, found that these bridges were built at a time when the bridge design codes in the United States (Caltrans and AASHTO) had very low seismic requirements. In fact, seismic force requirements were so small that they were automatically satisfied for many of the reinforced concrete bridges that were designed for gravity and traffic loads.
As a consequence of this lack of emphasis on earthquake design, members of bridge structures were not detailed to allow for excursions in the inelastic range and thus responded in a brittle manner. It wasn’t until the 1971 San Fernando earthquake, when many buildings and bridges collapsed, that Caltrans adopted ductile detailing procedures in its bridge design standards. However, due to limited resources a low priority was placed on the seismic retrofitting of reinforced concrete bridge structures built prior to 1971 such as the Cypress Street Viaduct, which sustained significant damage and partial collapse twenty years later during the Loma Prieta earthquake.

It was also established from the Loma Prieta earthquake that, for cap (beam)-column connections in particular, there wasn’t adequate strength in the joint between the columns and bent caps in the bridge substructure under transverse dynamic loads. New provisions in AASHTO(1994) recognized the need to maintain the integrity of the connection regions so that the column members can attain their flexural capacity. This would be achieved for all subsequent new designs by extending the transverse confining reinforcement into the joint region and limiting the joint shear input of the joint to the nominal resistance \(V_n\) given by:

\[
V_n = 1.0 \, b_j \, h_c \, \sqrt{f'c} \, (\text{MPa}) \, \quad (\text{normal density aggregate concrete})
\]

1.11

The nominal shear resistance of the joint is the same as that given by ACI352 (1991) for type 2 joints (i.e. joints expected to experience several excursions in the inelastic region due to the presence of earthquake loading) classified as corner joints in reinforced concrete buildings.

It is evident from the above survey of current design codes that design criteria for knee joints are practically non-existent.

In the current study of knee joints it was important to identify a unique physical characteristic of knee joints that distinguishes them from other joints. The knee joint consists of only two members, one beam and one column, framing into each other as described above. Thus, the column does not extend above the joint and thus there is no axial restraint arising from gravity axial load or from structural continuity on the column unless it is applied externally. This implies that the beam and column share the same role in the behavioral
response of knee-type connections. In addition, the absence of the column above the joint may have certain implications on the confinement requirements of the joint core concrete.

1.4 Plan of Study

Published test data from monotonic and cyclic-load experiments on knee joints were studied in the first part of this thesis, to identify the parametric sensitivity of knee joint response to moment transfer at the connection. This review included a detailed account of an experimental program conducted at Clarkson University in Potsdam, New York. The purpose of the program was to study the response of knee joints to simulated earthquake action, and was done as a joint study between Clarkson University and the University of Toronto (Chapters 2 and 3).

It was a goal of this work to further the understanding of knee joint behaviour by establishing a mechanical model from first principles, such that the role of each important design parameter may be defined in a crisp and rational manner. To this end, after assessing the limitations of existing analytical models for joints (Chapter 2), a simplified mechanical model was developed based on average smeared stress-strain considerations (Chapter 4) and was correlated with the experimental trends (Chapters 5 and 6).

To help explore the behaviour of knee joints beyond the limits imposed by the range of values for the design parameters included in the experimental database, non-linear finite element analyses were also performed (Chapter 7).

The results compiled from the various components of this study were used as a basis in order to formulate a framework for practical design of knee joints (Chapter 8).

Chapter 9 summarizes the most important conclusions of the work and identifies areas in need of future research.
CHAPTER 2

REVIEW OF PREVIOUS WORK ON REINFORCED CONCRETE KNEE JOINTS

2.1 Introduction

Most of the tests performed on knee joints by researchers in the 1960s and 1970s were static load tests. It was not until significant damage and failure occurred, in knee joint connections in bridge bents of elevated bridge structures during the 1989 Loma Prieta Earthquake in California, that an investigation of their behaviour was undertaken.

Since many of the existing reinforced concrete bridge structures designed and constructed in the 1950s and 1960s in the United States are deficient, according to current seismic design standards, a number of investigations for possible retrofitting schemes for existing members in bridge bents, including knee joints, were also initiated following the Loma Prieta Earthquake (Seible and Priestley 1991, Bollo et al 1990, Housner 1990).

To date, very little work has been done on modeling the behaviour of knee joints. One approach that may be used to model knee joint behaviour under static loads is strut-and-tie idealization (STM). This is proposed by most prototype building codes around the world for treatment of D-regions for which no alternative simpler models have been developed (i.e. NZS3101(1995), ACI318(1991), CSA A23.3(1994), CEN(1994)). STM is used to approximate the load paths and force effects in highly nonlinear regions of structures where conventional methods of design are not appropriate. The solution of the internal forces is achieved by establishing equilibrium of a postulated admissible stress field in the D-region.

An alternative would satisfy both the equilibrium of forces and the compatibility of deformations. As discussed in Chapter 1, one of the primary goals of this research is to develop such a model for knee joint connections. A related model which takes a similar approach for interior connections has been proposed (Pantazopoulou and Bonacci, 1992) and will be discussed later on in this chapter.

In the sections that follow, results from tests performed on knee joints under either monotonic or reversed cyclic loadings are summarized. This review includes the performance of some possible retrofitting schemes for bridge bents of existing reinforced
concrete bridges that were tested following the Loma Prieta Earthquake. The final part of this chapter reviews the strut-and-tie approach, along with the model for interior connections mentioned in the preceding, as these would apply to knee joint connections.

It is assumed that under lateral sway, knee joint connections in frame structures are deformed such that points of inflection occur at the midspans and midheight of the adjacent beam and column. The structure shown in Figure 2.1 is an isolated knee joint connection between the inflection points. The system of forces represents the actions resisted by the connection under sway that either opens or closes the joint, respectively. From Figure 2.1 it follows that the beam and column members resist a net axial tension force under opening action and a net axial compression force under closing action. Figure 2.2 shows the free body diagram for the knee joint boundary under opening and closing actions.

Many of the tests performed on knee joints described below differ in the resultant forces at the knee joint boundary from those described above for the earthquake-type loading conditions. Regardless, these tests will be reviewed for the sake of completeness, and because the number of truly relevant tests on knee joints available in the literature is very limited.

2.2 Experimental Tests on Reinforced Concrete Knee Joints

2.2.1 Knee Joints Tested Under Monotonic Loading

Tests performed on knee joints in the 1960s and 1970s were motivated by knee-type connection failures at static loads much lower than the designed forces. Most of the failures occurred under opening actions, where the formation of a diagonal crack within the joint region resulted in a sudden brittle failure of the joint. At joint failure the adjacent beam or column members usually did not reach their design yield moments. Note that the detailing practices at the time were very different from current standards. The emphasis was placed more on securing that possible crack paths in the plane of action be crossed by reinforcement rather than being confined. For this reason, although intricate details had been used often for knee joints, seldom did these include closed stirrup reinforcement. These were identified by studying the stress distributions in the joint region obtained for uncracked concrete based on continuum mechanics analysis (i.e., using theory of elasticity, Nilsson 1973).
Figure 2.1  Free Body Diagram of a Model Knee Joint Member.

Figure 2.2  Free Body Diagram of an Idealized Knee Joint Region.
Many researchers in that period adopted a quantitative representation of the efficiency of the knee joint to describe its behavioral response. The efficiency was defined as the ratio of the moment attained at joint failure to the yield moment of the adjacent flexural member. The reference cross section in calculating the moment was the beam or column face adjacent to the joint. Efficiency values of greater than one were desired since that would signify sustained flexural yielding of the adjacent member, and hence, a more ductile response of the test specimen. Efficiency values lower than one were undesirable since a brittle knee joint failure would then govern the response. A review of the monotonic tests performed on knee joints follows.

Monotonic Tests of Knee Joints Under Opening Action

Nilsson (1973) and Nilsson and Losberg (1976) began an investigation on knee-type connections in 1965. This investigation was prompted by the failure of wing wall knee joints of bridge abutments in Sweden. The failure occurred as a result of large opening moments that were generated by earth pressure at the knee joint connection which connects the front wall of the abutment and the wing wall (see Figure 2.3).

Using the theory of elasticity, Nilsson obtained stress distributions of uncracked concrete within the joint region from a finite element analysis of the knee joint member loaded under opening action. This provided him with some guidance for the placement of reinforcement in tensile regions when detailing the knee joint region.

![Figure 2.3 Wing Wall Knee joint of Bridge Abutments.](image)
Figure 2.4 shows the joint stress distribution in the chosen x and y directions under opening action. From Figure 2.4, the two areas of weakness in the knee joint were identified as the inside and outside corners. The large bending stresses at the inside corner of the knee joint result in wide cracks in the concrete and high stresses in the reinforcement crossing the crack at low levels of loading ($\sigma_x$ in Figure 2.4 and Figure 2.5). The diagonal tensile stresses result in a major diagonal crack which tends to push out the outside corner of the knee joint ($\sigma_y$ in Figure 2.4 and Figure 2.5).

Nilsson considered the internal stresses acting within the knee joint region as a statically determinate force combination (see Figure 2.6). From Figure 2.6, $F_c$ is the compressive force acting at the joint boundaries, and $M$ is the bending moment generated at the joint boundaries under opening action.

The force system given in Figure 2.6 is a simplification of the highly nonlinear stress distribution within the knee joint. This simplification allowed Nilsson to isolate the performance of the reinforcement and concrete under these forces towards identifying possible failure modes for the knee joint.

Based on this rationale, Nilsson (1973) identified the following possible failure modes for knee joints:

1. Diagonal Tension Crack Failure, which occurs in the joint corner because tensile stresses generated by external moments are not resisted by reinforcement (Figure 2.5).

2. Splitting Crack Failure, which may occur when significant tensile stresses occur in the concrete perpendicular to the direction of the reinforcing bar. Such stresses may result for knee joints that are subjected to closing actions and have a high reinforcement percentage for the bent anchorage reinforcement.

3. Yielding of the reinforcement in the joint.

4. Anchorage Failure, which occurs as a consequence of bond deterioration between the concrete and reinforcement or local crushing of concrete under the bends in the reinforcement.

5. Concrete Compressive Failure, which occurs as a result of crushing of the concrete in the joint (Figure 2.6).
Figure 2.4 Joint Stress Distributions in X and Y Directions (Nilsson 1973).

Figure 2.5 Crack Patterns in Knee Joint: Opening Action.
Nilsson tested a number of U-shaped specimens under loading conditions which simulated those found in the actual wing wall structures (Figure 2.7). The test specimens were placed horizontally on the floor and were supported by roller bearings. Two hydraulic jacks placed back to back were then used to apply the static horizontal load (P) on the test specimens which resulted in opening moments at the knee joint connections (Figure 2.7).

A preliminary test investigation on three near full scale model knee joint specimens found that detailing schemes used for such knee joints in practice were deficient. The test specimens developed only between one third to one-half of the expected ultimate moment (i.e., efficiencies of \( \frac{1}{3} \) to \( \frac{1}{2} \), respectively). Figure 2.7 shows a typical knee joint detail commonly used in Sweden and the crack pattern at failure. Failure occurred suddenly as a diagonal crack formed in line with the bent reinforcement and subsequently pushed out the outside corner of the knee joint. This was a diagonal tension crack failure.

Due to the very sudden and brittle failure witnessed in this preliminary study, the focus of Nilsson’s work was to develop suitable reinforcement detailing schemes that would prevent this type of failure. After a considerable number of tests, Nilsson chose the detailing scheme shown in Figure 2.8 as being suitable. The diagonal tension failure is controlled by bending the main tension reinforcement for the beam and column from the inside tension face to the outside corner of the knee joint and back into the compression zone. The bend in the reinforcement prevents the outer corner from being pushed out from the remaining knee joint. The diagonal reinforcement provided in the joint region serves to strengthen and reduce crack widths of the inside corner of the knee joint (see Figure 2.8).
Figure 2.7 Specimen Details and Test Set-up (Nilsson 1973).

Figure 2.8 Selected Knee Joint Detailing Scheme (Nilsson 1973).
Nilsson determined experimentally that to ensure yielding of the main tensile reinforcement and in order for failure to occur outside the joint region, the area of the diagonal reinforcement should be about one-half of the area of the main tension reinforcement. Transverse reinforcement was not considered in the knee joint detailing schemes because of the difficulty encountered during placement.

Swann (1969) performed monotonic load tests on eighteen light-weight knee joint specimens. Thirteen specimens were tested under monotonic opening action and five specimens were tested under monotonic closing action. The detailing schemes considered and the test set-up are shown in Figure 2.9. Due to the loading condition shown in Figure 2.9 there is no beam axial load present under either opening or closing action. The specimen details chosen were those commonly used in actual knee joints. Additional tests were performed on test specimens with detailing schemes which included stirrups to resist the tensile forces in the concrete. A high reinforcement ratio of 3 per cent was used for the main tensile reinforcement. The test set up shown in Figure 2.9 is for conditions of closing action. Under opening action, the load cell and screw jack, seated on rollers, are thrust upward directly against the bottom of the beam member of the test specimen. Very low efficiencies, between 0.09 and 0.6 were obtained for the conventional detailing schemes under opening action. The test specimens which contained stirrups within the joint resulted in higher efficiencies, between 0.8 and 0.9.

Balint and Taylor (1972) continued the work done by Swann. In addition to testing knee joint specimens with common detailing schemes, they also investigated test specimens with two kinds of joint transverse reinforcement. Figure 2.10 shows the specimen detailing schemes and test set-up used in their study.

The joint transverse reinforcement used within the knee joint region of the test specimens consisted of either radial stirrups or a cage of mesh reinforcement. A select number of test specimens also contained a splay (haunch) of 100 mm or 150 mm and diagonal hairpin reinforcement. The reinforcement ratio for the main tensile reinforcement ranged between 0.42 per cent and 1.35 per cent.

The test specimens were bolted to the lab floor and were loaded by pushing up
Figure 2.9  Specimen Details and Test Setup (From, Swann 1969).
Figure 2.10 Specimen Details and Test Setup (From, Balint and Taylor 1972).
(opening action) or down (closing action) on the free end of the beam (Figure 2.10). Low efficiencies (between 0.22 and 0.6) were obtained for the conventional joint detailing schemes. Higher efficiencies were obtained for the test specimens that contained a haunch with diagonal hairpin reinforcement (efficiencies between 0.90 and 1.14). Similar efficiencies (between 0.85 and 1.04) were obtained for the test specimens that contained stirrup or mesh reinforcement in addition to the haunch and diagonal hairpin reinforcement.

Mayfield et al (1971, 1972) tested several light-weight knee joint specimens with a variety of detailing schemes under both opening and closing actions. The knee joint specimen details and test set-up are given in Figures 2.11 and 2.12. The main tensile reinforcement ratio ranged between 0.7 per cent and 1 per cent. An upward force was applied by a hydraulic jack to the underside of the beam to generate opening moments at the knee joint connection. Closing moments were generated by applying a downward force to the top of the beam.

Mayfield et al (1972) found that the most efficient detailing scheme would require, in addition to the appropriate main tensile longitudinal reinforcement, two types of diagonal reinforcement. Type 1, shown in details 5 through 8 in Figure 2.11 would provide the knee joint with strength, avoiding a diagonal tension crack failure by resisting the outside corner from being pushed away from the remaining knee joint. Type 2, shown in details 9 through 12 in Figure 2.11 helped reduce the crack width of the inner corner of the knee joint. On this premise, detail 26 of Figure 2.12 was chosen to be the most efficient detailing scheme for the knee joint based on the experimental results. The reasoning proposed by Mayfield et al (1972) was similar to that given by Nilsson (1973).

Skettrup et al (1984) tested several knee joint specimens with three different detailing schemes. Figure 2.13 shows the specimen details and test set-up. The two hydraulic jacks applied an equivalent upward force to create an opening moment at the knee joint connection. Efficiencies of greater than unity were obtained for some of the test specimens. However, the complexity of the specimen details would make it difficult for placement of the reinforcing bars.
Figure 2.11  Specimen Details: Opening and Closing Actions and Test Setup (From, Mayfield et al. 1971).
Figure 2.12  Test Specimen Details: Opening Action (From, Mayfield et al. 1972).
Figure 2.13 Specimen Details and Test Setup (From, Skettrup et al 1984).
More recently, Jackson (1995) tested five knee joint specimens that were detailed using intersecting U-bars. Figure 2.14 shows the specimen details and test set-up. The tests were displacement controlled with the loads being applied as shown in Figure 2.14 to create opening moments at the knee joint connections.

From the test results, Jackson found that the bar diameter of the main tension reinforcement appeared to be a more significant design parameter than the corresponding reinforcement ratio. When comparing results for specimens whose detailing differed only in the diameter of the main tensile reinforcement, the specimens with the larger diameter reinforcing bars experienced more distress in the knee joint region than those specimens with the smaller diameter reinforcing bars.

Jackson suggested that the onset of cracking within the knee joint region is not the primary cause of failure, but rather a sign that bond failure has occurred between the main tensile reinforcement and concrete.

Figure 2.14 Specimen Details and Test Setup (From, Jackson 1995).
Most researchers in the 1960's and 1970's detailed knee joints to prevent the outside corner of the knee joint from being pushed out and also to minimize the crack width at the inside corner. As described above, the integrity of the outside corner was maintained by either looping the main tensile reinforcement or by providing radial stirrups or mesh reinforcement, within the knee joint region. The crack width at the inside corner of the knee joint was controlled by providing diagonal hairpin reinforcement and/or a haunch at that location. Although the above detailing approaches for knee joints were adequate for conditions of monotonic loading, they may not be appropriate for conditions of cyclic loading. Current design philosophies regarding joint detailing for cyclic loading require transverse reinforcement within the joint region to resist the moment gradient through the joint. Transverse reinforcement serves the dual purpose of confining the joint core concrete and helping to carry the shear once diagonal cracking has occurred within the joint region.

Monotonic Tests of Knee Joints Under Closing Action

Kemp and Mukherjee (1968) studied the influence of the main tension reinforcement ratio on behaviour of the knee joint region. Figure 2.15 shows the specimen details and test set-up. Four portal frames and four L-shaped specimens were tested. Similar responses were observed in the knee joints for both types of specimens. Test results showed low efficiencies for specimens with high reinforcement ratios. Failures were sudden and brittle in nature, and occurred by concrete tensile splitting when a diagonal crack formed across the joint from the outside corner to the inside corner.

![Figure 2.15 Specimen Details and Test Setup (From, Kemp and Mukherjee 1968).](image-url)
As discussed above, Swann (1969) and Mayfield et al (1971) tested a number of knee joint specimens with detailing schemes commonly used in practice under closing action (see Figures 2.9 and 2.11, respectively). Some of the knee joint specimens were common to both studies with two major detailing differences. First, stirrups provided for the tests performed by Mayfield et al, were looped around all of the main longitudinal reinforcing bars, whereas stirrups were not looped around all of the main reinforcing bars in the joint region for the tests performed by Swann. Second, Swann used a higher reinforcement ratio for the main tensile reinforcement (about 3 per cent) when compared to that used by Mayfield et al (about 1 per cent).

Swann obtained efficiencies of between 0.75 and 1.0 for his knee joint test specimens. Mayfield et al obtained efficiencies of greater than 1.0 for almost all of their tests. The difference in efficiencies obtained by Swann and Mayfield et al may be attributed to the higher reinforcement ratio for the main tensile reinforcement in Swann’s tests. This higher reinforcement ratio may have precipitated a bond splitting failure between the concrete and main tensile reinforcing bars in the knee joint, resulting in a premature brittle failure.

Yuan et al (1982) studied the failure modes and cracking characteristics of knee joints in reinforced concrete frames. Figure 2.16 shows the three types of reinforcement detailing and the test set-up used in their study. Twelve specimens were tested under monotonic closing action, nine specimens without any initial flaws and three specimens with an initial crack at the outside corner of the knee joint. Of the three reinforcement detailing schemes, one detailing scheme involved splicing of the main tensile reinforcement within the joint region, a practice which is not allowed in current design codes. Results showed that once a diagonal crack occurred in the joint region, diagonal splitting tensile failure occurred under closing action.

More recent studies on knee joints were motivated by the significant damage that occurred to knee-type connections during the Loma-Prieta Earthquake in San Francisco in October 1989.
Figure 2.16  Specimen Details and Test Setup (From, Yuan et al 1982).
Zouzou and Haldane (1993) tested two large scale knee joint specimens, one with confining stirrups in the joint region and one without. Figure 2.17 gives the specimen reinforcement details and test set-up.

The tests were displacement controlled and the loading was applied to the top of the beam in a downward direction as shown in Figure 2.17. The specimen without confining stirrups experienced a brittle failure with spalling and crushing of the diagonal concrete strut, and considerable cracking in the joint region. In the specimen with confining stirrups, cracks were relocated outside the region of the confined diagonal strut with a majority of the cracks occurring in the adjoining member. This resulted in a more ductile failure of the specimen.

Luo et al (1994) tested twenty-seven full scale knee joint specimens. Figure 2.18 gives the specimen details and the test set up. Tests focused on the effects of tensile reinforcement ratio, radius of curvature for the main tensile reinforcement, the compressive strength of concrete, and splicing details, on knee joint behaviour. The column and beam flexural strengths were about the same for all test specimens. The beam top (tension-side) reinforcing bars, continued around the corner of the joint and were anchored in the column in all of the test specimens.

The test set-up consisted of a reaction frame in which the test specimens were supported on roller bearings. The static load was applied to the beam using a load jack, and the reaction force was measured using a load cell positioned between the column and the wall of the reaction frame (Figure 2.18).

A beam reinforcement index, \( \omega \), and radius of bend, \( R/h_c \), were used as the key parameters for gauging the performance of knee joints. \( \omega \) was defined as the ratio,

\[
\omega = \frac{A_s f_y}{b_j d_b f' c}
\]

where; \( A_s \) is the area of the beam tension reinforcement, \( f_y \) is the reinforcement yield strength, \( d_b \) is the tensile reinforcing bar diameter, and \( R \) is the radius of bend of the tensile reinforcement. Figure 2.19 shows a plot of \( \omega - R/h_c \) for all the experimental test data. Three failure regions are identified in the Figure as A, B, and C. Specimens within region A
Figure 2.17  Specimen Details and Test Setup (From, Zouzou and Haldane 1993).
Figure 2.18  Specimen Details and Test Setup (From, Luo et al 1994).

Figure 2.19  Reinforcement Index- Radius of Bend at Failure (From, Luo et al 1994).
displayed flexural type failure in the adjoining members. Specimens within region B displayed flexural failures in the weaker flexural member in addition to some splitting cracks within the joint region or spalling of cover concrete. Finally, specimens within region C displayed joint failure by concrete splitting.

Figure 2.19 shows that as the radius of bend decreases, the bearing stresses on the concrete reach a critical level. At this point, concrete splitting occurs perpendicular to the direction of the reinforcing bar. In addition, once the tensile reinforcement ratio exceeds a critical level bond splitting cracks form perpendicular to the direction of the reinforcing bar along its length.

Most knee joint specimen details tested by researchers in the mid-1960s and 1970s performed well under monotonic closing action. Joint failures for those specimens with low efficiencies were mostly due to either, concrete diagonal splitting tensile failure where a diagonal crack formed across the joint from the outside corner to the inside corner, or bond splitting failure between the concrete and main tensile reinforcement.

From the studies by Zouzou and Haldane (1993) and Luo et al (1994), it appears that the presence of joint confining reinforcement changed the failure mode of the specimen from that of diagonal splitting tensile failure, to that of flexural failure of the adjoining members or crushing of the diagonal compressive concrete strut.

With respect to bond splitting failure between the concrete and main tensile reinforcement, the presence of transverse reinforcement looped around the main tensile reinforcement may delay a bond splitting type failure from occurring or, more favorably, shift the failure to ductile flexural yielding of the adjoining members.

The importance of bond action between the concrete and the main tensile reinforcement within the joint region was apparent in the monotonic closing studies on knee joints discussed above. These results serve as an indicator of the important role that bond will play in understanding knee joint behaviour under conditions of cyclic loading where bond deterioration has been shown to be much more severe.
2.2.2 Knee Joints Subjected to Reversed Cyclic Loading

Very few cyclic loading tests had been performed on knee joints prior to the Loma Prieta Earthquake in 1989. Experimental studies on knee joints in the past consisted of test specimens with poorly detailed knee joint connections according to current earthquake design practice. In addition, lower strength reinforcing bars were used when compared to the high strength reinforcing bars that are available and used today. Two of the earlier tests on knee joint connections are discussed below.

Bertero and McClure (1964) tested five small scale, one-storey, single bay model frames to determine if the effects of reversed cyclic loading in the inelastic range for reinforced concrete frames could be ignored in design.

Figure 2.20 shows the specimen details and test set-up. The beam and column members were detailed with a tensile reinforcement ratio of 1 per cent at the connecting beam and column member for opening and closing actions. Haunches in the knee joint region were used to relocate plastic hinges away from the joint region and in the adjacent frame members. The beam and column longitudinal reinforcing bars within the joint region were anchored by welding their ends to bearing plates that were positioned on the two free faces of the knee joint. This was done to avoid anchorage failure of the longitudinal reinforcing bars under reversed cyclic loading conditions (Figure 2.20).

Two specimens were loaded monotonically to failure. The remaining three specimens were subjected to reversed cyclic loading in the inelastic range. When comparing the cyclically loaded specimens to the monotonically loaded specimens there was no evidence of deterioration in strength. It appears that the anchorage provided by welding the reinforcing bars to the end plates on the free faces of the knee joint was adequate enough to minimize the bond slip and to thereby help maintain the ultimate strength of the specimen. There was, however, a significant deterioration in the stiffness of the cyclically loaded specimens as the number of cycles increased, due to concrete splitting cracks which formed in the knee joint region parallel to the reinforcement.
2.20 a) Geometry and Detailing of Model Frames

2.20 b) Loading Setup

Figure 2.20 Specimen Details and Test Setup (From, Bertero and McClure 1964).
Beaufait and Williams (1968) tested seven one-quarter scale one-storey single bay reinforced concrete frames to reversed cyclic loading. Figure 2.21 shows the test specimen details. The three different detailing schemes used in the study would be totally unacceptable in current earthquake design codes because of splicing and inadequate anchorage of reinforcing bars in the joint region.

The test specimens were placed in a large load frame and subjected to a reversed cyclic loading. Failure occurred suddenly in the specimens once diagonal concrete splitting occurred in the joint.

![Figure 2.21 Specimen Details and Test Setup (From, Beaufait and Williams 1968).](image)
After significant damage and failure of knee-type connections during the Loma Prieta Earthquake in 1989, researchers have focused their work on understanding the cyclic behaviour of such connections.

An investigation was commissioned by the Governor’s Board of Inquiry in California to investigate the damage and failure sustained by many bridges during the Loma Prieta Earthquake. Findings revealed that many of the bridges that suffered extensive damage were built in the 1950s and 1960s and were designed with many hinges and joints to simplify analysis and to allow for displacements that arose from creep, shrinkage, temperature and prestressing effects (Housner 1990). These bridge structures lacked redundancy and thus were more susceptible to severe damage or even collapse in the event of a moderate to strong earthquake. In addition, as already mentioned in Chapter 1, these bridges were designed before such detailing procedures were adopted in the United States bridge design codes as to ensure ductility in overstressed members or connections. As a result, many brittle failures were witnessed in the bents of reinforced concrete bridge structures which lead to partial or even total collapse of bridge spans during the Loma Prieta earthquake.

The following upgrades provided by Caltrans were outlined in the report by the Governor’s Board of Inquiry, Housner (1990). To increase the strength and stiffness of the bridge bents the following retrofits were recommended: (i) steel jacketing of columns, joints and plastic hinge zones, (ii) post-tensioning of joints, columns and girders, and (iii) increasing footing sizes.

Steel jacketing of the rectangular columns enhanced the column flexural and shear strengths through confinement of the core concrete. In addition, it also prevented the buckling of the longitudinal column reinforcement. Similarly, steel jacketing or post-tensioning the joint regions enhanced the shear strengths of the joint through confinement of the core concrete. The anchorage of the girder reinforcement in the joint region was improved and possible yield penetration into the joint was reduced.

Cyclic load tests on knee joint connections that were motivated as a result of the brittle failures witnessed in such connections during the Loma Prieta Earthquake are reviewed below.
Bollo et al. (1990) conducted an experimental test study on undamaged segments of the Cypress Street Viaduct; an elevated freeway that experienced significant damage and collapse during the Loma Prieta Earthquake. Their aim was to determine why the structure collapsed, and also to identify possible retrofitting schemes for other similar existing elevated freeways.

Bollo et al. tested three different retrofitting schemes on existing bridge bents in an attempt to avoid the shear failures witnessed at the lower girder-to-column connections. The three retrofit schemes are described below.

The first retrofit scheme involved clamping a steel wide flange section to the exterior faces of the columns. In the second retrofit scheme, a steel jacket clamped to the lower joint was used to encase the pedestal. In addition, the column was reinforced and the beams were post tensioned longitudinally to provide additional strength and confinement to the joint, as well as to improve the anchorage of the beam and column reinforcement in the joint region. The third retrofit scheme focused on reinforcing the lower girder-to-column connection with steel rods that were either grouted or epoxied in holes drilled at an angle through the joint region.

In general, testing resulted in a strength and deformability increase in all three retrofitted bridge bents. However, there was no improvement in the lateral stiffness. An additional significant outcome from the retrofit tests was that the failure of the bridge bents was shifted from the lower girder-to-column connections, to the equally unfavourable location of the upper girder-to-column connections (knee joints). This result emphasized the importance of ensuring that failure does not occur at joints, where brittle shear failures govern the response, through a careful investigation of the strength and deformability of such connections.

Seible and Priestley (1991) conducted a study of reinforced concrete joints in multi-level bridge bents damaged during the 1989 Loma Prieta Earthquake. They selected for their investigation a single deck bridge bent (outrigger knee joint) from the Oakland Southbound Connector Freeway (I-980), which sustained severe damage to the knee joint connection.

Although the beam and column members were adequately confined with stirrups and interlocking spiral reinforcement, respectively, inadequate transverse reinforcement was
provided in the joint region.

Significant damage had occurred in the knee joint region of the actual structure. Many diagonal cracks had formed in the knee joint and there was evidence that concrete had spalled off the outside corner of the joint. In addition, a beam top reinforcing bar (#18, diameter = 57mm) had fractured at its bend.

Two current modeling philosophies for reinforced concrete were used by Seible and Priestley to develop design criteria to help gauge the performance of joints in multi-level reinforced concrete bridge structures during the Loma-Prieta Earthquake and to develop retrofitting designs for the repair of damaged joints in these structures. The modeling philosophies included analytical models, which study the mechanics of reinforced concrete through various limit states (Collins et al 1978), design models that link structural detailing and design (Schlaich et al 1987), and additional joint considerations suggested by Paulay et al (1978). The additional joint considerations required that the joint be stronger than the weaker of the adjoining members, the joint response remain elastic during the earthquake, and the overall performance of the structure not be influenced by joint deterioration.

A preliminary assessment by the authors of the San Francisco double deck bridge-bents and the selected single deck outrigger bent indicated that these structures did not meet the above-mentioned design criteria for an earthquake resistant ductile structure. A more detailed 2D nonlinear finite element analysis, based on the modified compression field model, was performed for the selected single deck outrigger bridge bent. The reinforcing bars were modeled discretely. The beam and column reinforcing bars were modeled as having a straight anchorage in the joint and a reduced yield level in the anchorage zone that decreased linearly from full yield based on ACI 318(1989) development length requirements to zero at the reinforcing bars tail end.

Results from the finite element analysis showed significant joint deterioration with yielding of both horizontal and vertical reinforcement and concrete crushing at the outside corner of the joint. Thus, as witnessed in the response of the actual structure, the knee joint controlled the overall capacity of the structure.

Since the knee joint was the weak link, the authors proposed the use of a concrete jacket with horizontal and vertical joint confining reinforcement to increase the stiffness of the joint.
Mazzoni et al (1991) tested two knee joint specimens designed according to the requirements of ACI352(1985). The motivation for the tests was the poor performance of the knee joint connections in outrigger knee joint bridge bents of the China Basin and I-980 freeways during the Loma Prieta earthquake.

The failure modes of the actual outrigger knee joint bridge bents suggested by Mazzoni et al was similar to that given by Seible and Priestley (1991), discussed above for the I-980 freeway bridge bents; inadequate anchorage of column reinforcement in the joint region and inadequate joint confining reinforcement.

Figure 2.22 shows the test set-up and specimen details. The knee joint specimens were tested horizontally on the lab floor. A single actuator was used to apply the cycles of opening and closing displacement. The two knee joint test specimens were designed such that the joint shear input under closing action was equal to the nominal strength of the knee-joint as given by ACI352(1985), i.e., $12 \sqrt{f'_c}$ (psi) or $1 \sqrt{f'_c}$ (MPa) (Chapter 1). To calculate the force in the beam top reinforcement in order to maintain the joint shear input, Mazzoni et al (1991) used the actual yield stress in the reinforcement.

Due to the uncertainty in concrete strength, the expected knee joint shear stress input ($V_j/ A_j \sqrt{f'_c}$, in psi) was about 10.2 and not the nominal value of 12.

The knee joint specimens were subjected to reversed cyclic loading until their load carrying capacity had deteriorated to one-half of the peak value. The observed joint shear stress input under closing action in the two-hoop and four-hoop specimens was $7.9 \sqrt{f'_c}$ (psi) and $8.0 \sqrt{f'_c}$, respectively. These results are significantly lower than the expected joint shear stress input of $10.2 \sqrt{f'_c}$ (psi).

The four hoop specimen was able to maintain its peak load carrying capacity for many cycles, whereas, the two-hoop specimen experienced significant deterioration of load carrying capacity with increasing cycles of loading.

In both test specimens the authors attributed the deterioration of strength to spalling of the concrete cover and subsequent loss of column reinforcement anchorage in the joint. The two-hoop test specimens exhibited wide diagonal cracks near the joint center under both opening and closing actions. A wide bond splitting crack was also observed at the level of the beam top reinforcement near the joint boundary.

The four-hoop test specimen did not sustain as much damage in the centre of the joint
Figure 2.22  Specimen Details and Test Setup (From, Mazzoni et al 1991).
region as did the less confined two-hoop specimen. It appeared that bond splitting leading to anchorage deterioration governed the response of the specimen because splitting cracks propagated along the length of the beam and column reinforcement that was anchored in the joint region.

In an effort to shift the failure from the joint to the adjoining beam member, Mazzoni et al retrofitted the four-hoop knee joint test specimen by placing two U-shaped reinforcing bars and diagonal reinforcement with an area of approximately one-half that of the main beam longitudinal reinforcement in the joint to improve confinement and also to increase the strength and stiffness of the inner corner of the joint under opening action, respectively.

Results from reversed cyclic load tests on the retrofitted knee joint specimen indicated a higher joint capacity than the original specimens, with beam flexural hinging governing the specimen response under opening action. Under closing action, the specimen was able to reach the expected joint shear input (10.2\sqrt{f'_{c}} (psi)) and to maintain this peak resistance over many cycles at drift levels about two to three times the drift levels achieved by the non-retrofitted four-hoop test specimen.

Kramer and Shahrooz (1994) investigated the use of mechanical anchorages in the knee joint region through composite construction as an alternative to conventional detailing which they believed would result in an overly congested joint region. The knee joint specimens were typical of knee joint connections found at the top level or at a set-back level of a multi-storey reinforced concrete frame building. Four knee joint specimens were tested. Three of the test specimens were of composite construction where the anchorage of the beam and column reinforcement in the joint region was by mechanical attachment to a structural steel section. One conventionally reinforced knee joint designed according to the ACI352 (1991) specifications was also tested for comparison. Figure 2.23 shows the test specimen details, the experimental test set up, and typical displacement history. The specimens were subjected to cyclic loading under opening and closing actions with increasing amplitudes of displacement.

The composite knee joint specimens did not experience cracking in the joint region. Most of the cracking occurred at or near the column face (in the beam) away from the joint. Beam flexural hinging governed the response of the composite specimens under closing
2.23a) Test Specimen Details.

2.23b) Test Setup.

Figure 2.23   Specimen Details and Test Setup (From, Kramer and Shahrooz 1994).
action. However, only two of the three composite specimens, i.e., those with shear studs or hoops in the joint region, were able to develop the flexural capacity of the beam under opening action.

The conventionally reinforced specimen experienced extensive cracking in the knee joint. The knee joint region governed the response of this test specimen with the outside corner of the joint being pushed off near the end of the test.

*Cote and Wallace* (1994) and *McConnel and Wallace* (1995) performed reverse cyclic load tests on knee joint specimens. The motivation for this study came from the poor performance of knee joints in the outrigger knee joint bridge bents during the 1989 Loma Prieta Earthquake and from the lack of cyclic load test data on such joints.

The goals of their experimental study were to determine the effects of design parameters such as joint shear input and amount of joint transverse reinforcement on knee joint performance and to develop recommendations for the earthquake design of knee joints by investigating the ACI352 (1991) limits for horizontal shear input to the joint.

The test specimens were half-scale models of a knee joint from a twenty-storey ductile moment resisting space frame subjected to a high seismic risk. The knee joints were subjected to displacement controlled reversed cyclic loading with at least two complete cycles at each drift level. The test specimen details, experimental test set-up and typical displacement history are shown in Figures 3.1 to 3.9 and Tables 3.1 and 3.2 in Chapter 3.

Test results indicated that the ACI352 (1991) joint specifications may be inadequate for the earthquake design of knee joints. The maximum horizontal shear input obtained from the experiments ranged between 5.55 - 9.68 \( \sqrt{f_c} \) psi (0.46 - 0.81 \( \sqrt{f_c} \) (MPa)) which is far less than the nominal shear strength of 12 \( \sqrt{f_c} \) (psi) specified by ACI352(1991). Based on these results, the authors recommended a nominal joint shear stress of 8 \( \sqrt{f_c} \) psi (0.66 \( \sqrt{f_c} \) (MPa)) for reinforced concrete knee joints designed for earthquake resistance. The experimental results from the Clarkson University test series will be studied in detail in Chapter 3.

*Thewalt and Stojadinovic* (1996) also investigated outrigger knee joint systems in reinforced concrete bridge structures. The goals of the study were to evaluate the behaviour
of existing outrigger knee joint structures under combined transverse (in-plane) and longitudinal (out-of-plane) loading, and to develop and verify by experiment retrofitting schemes and repair techniques for such structures (Figure 2.24). The specimens were tested under combined transverse and longitudinal loading in an attempt to better simulate the multi-directional shaking experienced by bridge structures during an earthquake.

Preliminary reversed cyclic loading tests were performed on two half-scale model specimens of existing outrigger knee joint systems, one with a long outrigger beam and the other with a short outrigger beam. Figure 2.24 shows the specimen details, test setup and the horizontal displacement patterns used in the experimental study. The length ratio of the long outrigger beam to the short outrigger beam was 2. The specimens were tested in an inverted position and the bridge deck of the actual structure was represented by a block of concrete which was used to anchor the outrigger beam to the lab floor and to approximate the fixity that would exist between the outrigger beam and bridge deck in the actual structure.

Results indicated that, in the transverse loading direction, (y-direction in Figure 2.24), failure occurred under closing action by splitting bond failure as the outside layer of column reinforcing bars split away from the joint at the location where they overlapped with the hook extensions of the beam top reinforcing bars. Diagonal cracking and spalling of concrete were also evident in the joint region.

In the longitudinal direction (x-direction in Figure 2.24) the torsional capacity of the outrigger beam governed the overall response of the long outrigger beam test specimen.

In an attempt to prevent splitting bond failure of the column reinforcement in the joint region, the long outrigger beam knee joint specimen was repaired by providing horizontal and vertical joint confining reinforcement. A more ductile response was obtained for the specimen and failure occurred by beam hinging.

Two additional upgrading schemes were considered to increase the knee joint capacity as well as the strength and stiffness of the outrigger beam for shear and torsion.

The two upgrading schemes considered were: (a) the "ductile" upgrade strategy where the deformability and damage is distributed throughout the structure as a result of column hinging from loading in the transverse direction and beam hinging from loading in the longitudinal direction, and (b) the "strong" upgrade strategy where the deformation and damage is limited only to the column. This increased the strength demand for the remaining
2.24a) Reinforcement Details: 
As-Built Short Outrigger Specimen.

2.24b) Reinforcement Details: 
As-Built Long Outrigger Specimen

2.24c) Test Setup and Loading Pattern.

Figure 2.24  Specimen Details, Test Setup and Horizontal Displacement History 
(From, Thewalt and Stojadinovic 1996).
structure when compared to the “ductile” upgrade strategy.

Experimental results showed that the strong upgrade strategy was more favorable. After investigating a number of different detailing strategies, it was determined that an adequate retrofitting scheme consisted of either a steel plate jacket or a post-tensioned concrete jacket for the member. Column hinging governed the response of the retrofitted outrigger knee joint specimen.

When compared to the structural deformability of the existing outrigger knee joint structure, the retrofitted structures had a drift capacity of about three times that of the existing structures. The drift was calculated with respect to the bottom edge of the outrigger beam in the test specimen (Figure 2.24). The drift capacity was defined as the measured drift when the specimen response had deteriorated below eighty per cent of the measured ultimate load. The load displacement hysteresis loops of the retrofitted structures were well rounded showing substantial energy dissipation and no pinching up to a drift ratio of 2.8 per cent. Thereafter, pinching was evident and was caused by sliding of the column across the flexural hinge zone (sliding shear). A drift ratio of 4.2 per cent was reached at the maximum drift capacity.


Figure 2.25 shows the specimen details and test set-up used by Megget and Ingham. In the first specimen, a standard hook was used for anchoring the beam and column main reinforcing bars in the joint region. Beam and column U-bars were used for anchorage in the joint region for the second specimen.

The test specimens were laid flat and secured to the strong floor at the columns end. A hydraulic jack was used to load the test specimens (Figure 2.25). Two complete cycles of opening and closing action were performed at 3/4 yield and displacement ductilities of 2, 4, 6, and 8. One final complete cycle of opening and closing actions was performed at a displacement ductility level of 10. Four D12 (12mm diameter) reinforcing bars were used
for the beam top and bottom reinforcement in both specimens. The expected joint shear input for both specimens under opening and closing actions was $4.6\sqrt{f'_c}$ (psi) ($0.38\sqrt{f'_c}$ MPa) and $5.7\sqrt{f'_c}$ (psi) ($0.47\sqrt{f'_c}$ MPa), respectively. These values were obtained using the nominal yield stress ($f_y = 300$ MPa) of the reinforcing bars.

Under closing action both specimens reached the expected joint shear stress. Thereafter, the resistance of the specimen with the conventional $90^\circ$ hook anchorages deteriorated rapidly. However, the specimen with the U-bars maintained its peak resistance until the second cycle at a displacement ductility of 6, at which time it showed a considerable drop in resistance.

Considerable spalling was noticed in the specimen with the conventional $90^\circ$ hook anchorages. It appears that once spalling occurred, the anchorage of the beam top reinforcing bars was compromised as the tail end of the reinforcing bars tended to push outwards.

This trend was not witnessed in the second test specimen where U-bars were used in the joint region. This may explain the improved performance of this specimen over the one with the conventional $90^\circ$ hook anchorages.

---

**Figure 2.25** Specimen Details and Test Setup (From, Megget and Ingham 1996).
Under opening action, the conventional 90° hook anchorage specimen did not reach the expected joint shear stress input. It reached its peak resistance at a displacement ductility of 6 and showed a decline in resistance in subsequent cycles. The specimen with U-bars in the joint region showed a similar response except that it managed to just reach the expected joint shear stress at a displacement ductility of 6 before it too showed a decline in resistance in subsequent cycles. One possible explanation for the poor performance under opening action is the damage caused to the compression zones due to bond deterioration as a result of several cycles of closing action. This diminishes the effective moment arm during opening action and thereby decreases the capacity of the section.

Megget and Ingham found that their test specimens failed at about one-half of the 0.2 $f'_c$ (MPa) limiting joint shear stress specified in the New Zealand Concrete Standards (NZS3101 (1995)). This is consistent with the results obtained by McConnell and Wallace (1995), Cote and Wallace (1994), and Mazzoni et al (1991).

Sexsmith et al (1997) tested a typical bridge bent of the forty year old Oak Street Bridge in Vancouver under cyclic loading to provide some insight as to some possible seismic retrofit schemes. As discussed earlier, the seismic design philosophy forty years ago is inadequate when compared with modern seismic design criteria. Structures designed in the past lacked proper anchorage of reinforcing bars and adequate joint reinforcement.

Figure 2.26 shows the prototype bridge bent and experimental test set-up Sexsmith et al used in their study. The test specimens were 0.45 scale models and represented the upper half of the prototype. The dead load on the specimen was simulated by applying vertical loads at the five bearing positions shown in Figure 2.26. The lateral load was applied horizontally by an actuator located above the beam, and was distributed equally to the beam at the two internal dead load bearing positions. The lateral loads were applied in this manner to simulate the load transfer from the bridge deck to the bridge bent (test specimen). The specimens were subjected to three cycles of opening and closing actions at each load or displacement level. The testing was load controlled up to 75 per cent yield and displacement controlled thereafter for displacement ductility levels of 1.5, 2, 3, 4, 6, 9, and 12 or up to failure if it occurred earlier.

The as-built specimen (OSB1) exhibited a brittle shear failure in the beam near the
Figure 2.26  Prototype Bridge Bent and Test Setup (From, Sexsmith et al 1997).

Figure 2.27  As-Built and Retrofit Specimen Details (From, Sexsmith et al 1997).
column face. This failure was evident due to the large diagonal cracks that formed in the beam adjacent to the joint. Bond splitting along the beam top reinforcing bars was also evident, and it occurred as a result of poor detailing, lack of proper confinement, and improperly located bar cut-off points in the beam. Since the beam did not develop its flexural strength, the column and knee joint were not tested to ultimate. Hence, there was very little damage to the knee joint and column. As a result of the poor performance of the as-built specimens, four retrofit schemes were tested. Figure 2.27 shows the specimen details for the as-built specimen (OSB1) and three of the four retrofit specimens (OSB3, OSB4, OSB5).

In an attempt to improve the shear and flexural capacity of the beam, retrofit specimen OSB2 consisted of post-tensioning the beam of the as-built specimen longitudinally. During testing of specimen OSB2 diagonal cracks formed in the beam near the joint as in the original specimen, however, they were not as wide. Flexural shear cracks also formed in the column which grew longer and wider at higher load cycles and resulted in shear failure of the column. Thus by strengthening the beam member the failure was shifted to the column.

Retrofit specimen OSB3 had a reinforced concrete beam cast to the underside of the existing beam in the as-built specimen, which was then vertically post-tensioned to the existing beam to enhance its shear and flexural capacity. In addition, circular steel jackets were grouted to the original rectangular columns to enhance their shear capacity. Failure occurred due to column flexure just above the circular steel jacket. Spalling of the concrete and column reinforcing bar buckling was evident at the failure location.

To preclude shear failure in the column, retrofit specimen OSB4 was designed to take advantage of the favorable results of retrofit specimen OSB3. Circular steel jackets were placed around the rectangular columns. In addition, the beam was post-tensioned longitudinally and vertically to resist possible shear failure in the beam near the joint region. This specimen failed in column flexure above the steel column jacket similar to retrofit specimen OSB3. Although there was some evidence of flexure and shear cracks in the beam, post-tensioning in both directions was enough to shift the damage and subsequent failure to the column. Diagonal cracks were also evident in the joint, but they did not get wider with increasing load cycles.
The final retrofit specimen, OSB5 utilized fibreglass wraps at critical locations of high shear in the beam and column members to enhance their shear capacity (Figure 2.27). In addition, the beam was post-tensioned longitudinally using external Dywidag bars to improve joint behaviour and to enhance the flexural strength of the beam.

The response of this specimen was more flexible than the previous ones. Many cracks formed in the joint region as well as in the beam and column members at the locations of the fiberglass wraps. However, these cracks did not grow in length or width. Column flexure governed the response of this specimen at the location of the inside faces of the column where spalling followed by buckling of the column reinforcing bars limited the column's capacity. One disadvantage of the fibreglass wraps is that it is sensitive from exposure to sunlight and it is vulnerable to external damage.

Several knee joint specimens were tested to reversed cyclic loads at Chongqing University of Architecture and Engineering in China (Bai et al 1994). A number of different anchorage detailing schemes for the beam and column longitudinal reinforcement were studied. Table 2.1 gives the test specimen reinforcement details. Figure 2.28 shows typical specimen details and the experimental test set-up. The knee joint specimens were subjected to displacement controlled reversed cyclic loads up to displacement ductility levels of 8 under both opening and closing actions, with two complete cycles at each level. The different joint anchorage detailing schemes for the beam and column longitudinal reinforcement consisted of:

**Column Reinforcing Bars**

- U-shaped, straight anchorage, or 90° hook anchored in joint or in adjacent beam-(column outer reinforcing bars only).

**Beam Reinforcing Bars**

- 90° hook anchored in joint or in adjacent column (beam top reinforcing bars only).

A number of specimens also contained a haunch and/or diagonal reinforcement in the joint. Transverse reinforcement was also provided in the joint region in the form of hoops and stirrups for some of the test specimens.

The observed horizontal joint shear stress in $\sqrt{f_c}$ (psi) ranged between 7.5 and 14.1 under closing action (0.63 - 1.18 $\sqrt{f_c}$ MPa).
Table 2.1 Test Specimen Reinforcement Details: Tests in China

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*1 Diagonal reinforcement in joint region
*2 Contained haunch
*3 Cross reinforcement in beam member, hinging away from joint
*4 Column inner reinforcement equals column outer reinforcement

A: All bars anchored in joint
B: Column outer bars anchored in beam, rest anchored in joint
C: u-column bars in joint, beam top bars anchored in column, beam bottom bars anchored in joint
D: Beam top bars anchored in column, rest anchored in joint
E: Beam top bars anchored in column, column outer bars anchored in beam, rest anchored in joint
F: Beam top bars anchored in column, straight anchorage for column outer bar, rest anchored in joint
G: Straight anchorage for beam top bar, column outer and inner bars, beam bottom anchored in joint
2.28a) Test Specimen Details.

2.28b) Test Setup.

Figure 2.28  Specimen Details and Experimental Test Setup (From, Bai et al 1994).
The specimens that contained U-shaped column reinforcing bars in the joint region and beam or column reinforcing bars with a 90° hook in the joint but anchored in the adjoining member performed much better than those specimens with beam and column reinforcing bars anchored with a 90° hook in the joint region.

When comparing the behaviour of test specimens with expected joint shear inputs larger than the nominal values given by ACI352-1985 ($12\sqrt{f_c}$ (psi)), the specimens listed above that had non-conventional anchorage of longitudinal reinforcement achieved yielding of the beam top reinforcing bars and reached drift levels of about 3.7 to 6.5 per cent. The drift values referenced here refer to the ratio of the horizontal displacement measured at the level of the actuator (beam centerline) to the column height from the column pin support to the beam centerline.

It is evident from the above survey that, by pertinent member retrofit schemes it is possible to remedy the deficient earthquake design procedures that were used in the 1950s and 1960s for reinforced concrete bridge structures. Steel jacketing or post-tensioning of potentially critical members will result in higher strengths and stiffnesses in these members during an earthquake. It is not enough, however, to blindly reinforce critical members without possessing a clear understanding of the strength and deformability of all members of the structural system. Without a thorough understanding of the strength and stiffness distribution within the structure, retrofitting a potentially critical member may result in the shifting of the failure mode during the event of an earthquake from one undesirable location in the structure to another equally undesirable location. This was evident in the tests done by Bollo et al (1990) where the failure mode shifted from shear failure in the lower girder-to-column connection to the equally unfavorable shear failure in the upper girder-to-column connection (knee joint) of the reinforced concrete bridge bent.
The above review of cyclic tests on reinforced concrete knee joint, edge joint, and connection specimens, indicated that in many of the tests bond splitting of the beam top reinforcing bars and the ensuing anchorage deterioration resulted in brittle responses of those specimens under closing action with failure occurring at joint shear inputs much lower than the nominal values specified by major design codes (i.e., ACI318-1991, NZS3101-1995). For specimens that had the beam top reinforcement anchored in the joint region bond splitting eventually resulted in an outward and upward movement of the reinforcing bars. This contributed to the poor performance of the test specimens by increasing bond slip of the beam top or column outer reinforcing bars.

Knee joint specimens that consisted of beam top reinforcement anchorage detailing procedures that prevented this movement performed much better. This was observed in specimens with U-shaped reinforcing bars in the joint region and those with the beam top or column outer reinforcing bars anchored in the adjacent column.

2.3 Modeling the Behaviour of Reinforced Concrete Knee Joints

A review of strut-and-tie models (STM) and their applicability to knee joints is presented here. In addition, an existing analytical model for interior connections which considers equilibrium of forces and compatibility of deformation is discussed with respect to its relevance for modeling knee joint mechanics.

2.3.1 Strut-and-Tie Models for Modeling the Behaviour of Reinforced Concrete Knee Joints

Strut-and-tie models are a generalization of the truss models used for the shear design of cracked reinforced concrete beams. They consist of a number of idealized tensile ties and compressive struts in the structure, interconnected by nodes. Solution of the internal forces is achieved by establishing equilibrium. For accuracy, it is important in this approach that the flow path of forces chosen and subsequent dimensioning of members be valid for the given loading and geometric constraints of the structure (Schlaich et al, 1987).

The structure may be divided into two regions, B-and D-regions. In B-regions (Bernoulli regions) the assumptions of plane sections are valid. In D-regions (Disturbed regions), the strain distributions across the sections are nonlinear and plane section theory.
is not applicable (Schlaich et al, 1987). Because of the highly nonlinear strain distribution in the knee joint, it is considered to be a D-region.

In the 1960s and 1970s researchers used truss models to idealize the forces in the knee joint as a means to address concerns that stemmed from the types of failures observed in these members.

Nilsson (1973) derived a relationship to limit the longitudinal tension reinforcement such that reinforcement yielding would preclude diagonal tension crack failure under opening action by using the truss model shown in Figure 2.29 for the forces within the knee joint. As shown in Figure 2.29, a parabolic tensile stress distribution of length $l_{dc}$ with a peak stress of $f_{sp}$ for the splitting strength of concrete is assumed. Using equilibrium of forces and assuming that the tension reinforcement has yielded, an expression for the limiting reinforcement ratio was obtained, i.e.,

$$\sqrt{2} \, T_s = \frac{2}{3} \, f_{sp} \, b \, l_{dc}$$

2.2

with

$$T_s = A_s \, f_y \, \text{and} \, \rho = \frac{A_s}{b d_e}$$

2.3

Figure 2.29  Idealized Truss Model for Knee Joint (Nilsson 1973).
By substitution into 2.2 the following can be obtained:

\[ \rho_{lo} = \frac{\sqrt{2} \ f_{sp} \ l_{dc}}{3 \ \sigma_{y} \ d_{e}} \]  \hspace{1cm} 2.4

where; \( d_{e} \) is the distance from the extreme compression fibre to the centroid of the longitudinal tensile reinforcement, \( \rho \) is the longitudinal reinforcement ratio, and \( \rho_{lo} \) is the limiting longitudinal tension reinforcement ratio such that reinforcing bar yielding precludes concrete splitting.

The moment at the joint member interface at which diagonal cracking first occurs, \( M_{dc} \) was also obtained as follows:

\[ M_{dc} = T_{z} \ z \]  \hspace{1cm} 2.5

where; \( z \) is the moment arm approximated to be \( 0.8d_{e} \). By substitution of 2.5 into 2.2 the diagonal cracking moment is given as:

\[ M_{dc} = 0.38 \ d_{e} \ b \ \theta_{dc} \ f_{sp} \]  \hspace{1cm} 2.6

Balint and Taylor (1972) found that better agreement is obtained with their experimental results when the right hand side of Equation 2.6 is multiplied by a "fudge" factor of 1.2.

Kemp and Mukharjee (1968) obtained an expression for the magnitude of the main tensile reinforcement ratio that would result in diagonal tensile splitting failure parallel to the diagonal compressive strut (i.e. from outside corner to insider corner) in the knee joint under closing action. Their formulation was based on the premise that the failure mechanism associated with diagonal tensile splitting failure of concrete in the joint was the same as that for tensile splitting of concrete in a split cylinder test. Results from split cylinder tests on square concrete prisms and idealized loading conditions for the knee joint under closing action were used to obtain the expression for the limiting reinforcement ratio such that the
flexural capacity of the adjoining member is reached prior to splitting parallel to diagonal compressive strut i.e.

\[ \rho_{te} = 1.36 \frac{f_t}{f_y} \]

where; \( \rho_{te} \) is the limiting longitudinal reinforcement ratio such that reinforcing bar yielding precludes splitting parallel to the diagonal compressive strut, and \( f_t \) is the tensile splitting strength of the square concrete prism.

Researchers have for some time recognized that joint boundary forces are resisted by the joint predominantly through a shear resisting mechanism. This results in inclined principal compressive and tensile stress in the joint region. Due to the low tensile strength of concrete, cracks form parallel to the direction of the principal compressive stress direction. The presence of these cracks weakens the resistance of the concrete in the principal compressive stress direction and hence the ability of the joint to resist shear.

**Application of STM to Knee joints**

Researchers have used strut-and-tie models (STM) to idealize the stresses in the joint region. Figure 2.30 shows examples of simple STM's used to approximate the force transfer in the joint region under opening and closing actions. The tensile forces in the reinforcement are modelled as tensile ties and the inclined principal compressive stress is modelled as a compressive strut. The intersection of these ties and strut are node regions and are labeled as A through B in Figure 2.30.

The STM's shown can be obtained, for example, from a linear elastic stress analysis where the direction for the compression strut of the STM is positioned in the direction of the principal compressive stresses obtained from the analysis. For the STM to be valid, the dimensions of the struts and ties should be such that their strengths are adequate to resist the forces. In addition, the nodes which interconnect the struts and ties should be adequate to allow for the load transfer between them.

The effective strength of the concrete struts used in dimensioning of the chosen STM depends on its multiaxial state of stress as well as any cracks or reinforcement crossing its path.
2.30a) Closing Action.

2.30b) Opening Action.

Figure 2.30 Simple Strut-And-Tie Models For Knee Joints.
An empirical approach was used by Schlaich et al (1987), based on experience, to obtain the effective concrete strength for the strut, i.e.

\[ f_{ce} = k_s f'_c \]  

where; \( f_{ce} \) is the effective concrete strength of the strut and \( k_s \) is the reduction factor for the strut.

For example, if transverse strains cause cracking, parallel to the concrete strut, the effective concrete strength for the strut to be used in dimensioning the chosen STM is 80 per cent of the specified concrete strength (i.e., \( k_s = 0.8 \) in Equation 2.8 Schlaich et al (1987)).

Many researchers have defined the effective concrete strength for the strut in a similar form as that given in Equation 2.8 and they include MacGregor (1988), Alshegeir (1992), and Yun and Ramirez (1996).

The ACI-ASCE445 Shear and Torsion State of the Art Report (1993) proposed that the effective compressive strength in the strut be obtained by considering the deformation of the D-region using compatibility to obtain the principal tensile strain, based on the work done by Vecchio and Collins (1993), i.e.,

\[ f_{ce} = \frac{\lambda_1 \phi_c f_c}{0.8 + 170 \varepsilon_1} \leq 0.85 \phi_c f_c \]  

and,

\[ \varepsilon_1 = \varepsilon_x + (\varepsilon_o + \varepsilon_p) \cot^2 \alpha_s \]  

where; \( \varepsilon_x \) is the strain required in the tensile reinforcing tie, usually taken as the yield strain (0.002), \( \varepsilon_o \) is the strain in the compressive strut at peak compressive stress and equal to 0.002, \( \alpha_s \) is the angle between the concrete strut and the reinforcing tie, \( \phi_c \) is the material reduction factor for concrete, \( \lambda_1 \) is the factor to correct for the density of concrete, and \( \varepsilon_1 \) is the principle tensile strain.

With respect to the adequacy of the nodes in a STM, Yun and Ramirez (1996) found that the allowable concrete compressive stress in nodal zones depends on: (i) the confinement of the zone, (ii) the detrimental effects of tensile straining within the zone, and
(iii) splitting stresses, and hook bearing stresses resulting from anchorage of reinforcing bars or tension ties.

When considering the STM for the knee joint in Figure 2.30, it is likely that splitting cracks will form in the direction of the diagonal struts or at anchorage locations; here bends in the longitudinal reinforcement would result in deterioration of the specimen. Transverse reinforcement within the joint region would provide confinement to the joint and delay the onset of anchorage failure or crushing of the concrete strut. This behaviour was evident in the specimens tested by Zouzou and Haldane (1993) that contained transverse confining reinforcement for the concrete strut where a more ductile behaviour was observed when comparing these specimens with those without any joint confining reinforcement. The detailing of the specimen with the joint confining reinforcement was based on a modified strut-and-tie model originally proposed by Marti (1991).

2.3.2 Mechanical Model for Interior Connections

Pantazopoulou and Bonacci (1992) formulated a mechanical model for interior beam-column joints in laterally loaded concrete frame structures. The model establishes equilibrium of stress resultants, satisfies compatibility of deformations and incorporates models of material nonlinear behaviour. The main objective of their study was to gain a better understanding of the behaviour of interior beam-column joints to earthquake-type loading conditions. The following assumptions were made when formulating the interior joint model (Figure 2.31):

(i) the joint is considered to be a three dimensional solid, (ii) the ACI352 (1985) requirements for development lengths of the main longitudinal reinforcement have been satisfied, (iii) adequate crack control is provided in the joint region, (iv) lateral loads which idealize the earthquake loading are assumed parallel to one principal axis (x-axis Figure 2.31), (v) average stresses and strains are uniformly distributed over the entire joint, vi) shear stresses are applied to the joint through bond between the main longitudinal reinforcement and joint core concrete and through direct member actions, (vii) before yield, the principal stress and strain directions are assumed coincident, and (viii) bond is accounted for in the stress equations satisfying horizontal equilibrium using a variable $\beta$, which represents the ratio of average beam reinforcement stress to average hoop stress at the column centerline.
2.31a) Kinematic Assumptions.

2.31b) Equilibrium Requirements.

Figure 2.31 Kinematic and Equilibrium Considerations for Interior Connections (Pantazopoulou and Bonacci 1992).
For perfect bond, it is assumed that $\beta=0$. Physically, this means that the beam reinforcement stress has reduced to zero at the column centerline. For negligible bond, $\beta=1$, the beam reinforcement stress is assumed equal to the hoop stress at the column centerline. The actual bond conditions for the joint are between these two limits for $\beta$.

The critical point in the behaviour of the joint occurs at yielding of the joint confining reinforcement. Thereafter, the joint performance deteriorates rapidly. Pantazopoulou and Bonacci (1992) formulated their model to account for the significant milestone of hoop yielding.

A parametric investigation by Bonacci and Pantazopoulou (1993) showed the parametric dependence of the joint for the milestone of hoop yielding. It was found that increasing either of, the yield stress of the hoop reinforcement, the amount of hoop reinforcement or the confining axial stress in the beam increased the shear stress and shear strain at the joint centre at hoop yield. However, increasing either of the column compression axial stress, the concrete strength, or the amount of column longitudinal reinforcement resulted in higher shear stress but at a lower strain at hoop yield (i.e., stiffened the joint).

To aid in the formulation of a simplified mechanical model for knee joints in laterally loaded concrete structures a number of unique physical characteristics based on the geometry and loading of this type of member will be discussed. Figure 2.32 shows the idealized loading at the boundaries of an interior joint and a knee joint of a reinforced concrete frame structure subjected to lateral sway. From Figure 2.32(b), because both the beam and column terminate at the knee joint, the symmetric load patterns evident in the interior joint connections (see Figure 2.32 (a)) do not exist for the knee joint for opening or closing action.

For the interior joint model, due to the symmetry in loading and geometry, a uniform stress/strain distribution is assumed across the joint and a single factor $\beta$ is used for the beam top and bottom reinforcement. Due to its non-symmetric loading and geometry, the assumption of a uniform stress/strain distribution across the knee joint may not be appropriate. Further, the inclusion of only one variable $\beta$ in the model for the beam top and bottom reinforcement will not be able to account for the differing bond conditions at those locations. In addition, due to loading and geometry, the beam and column appear to play a similar role in the mechanics of the knee joint member under lateral sway. This implies
2.32a) Idealized Forces: Interior Joint.

2.32b) Idealized Forces: Knee Joint.

Figure 2.32   Idealized Forces for Interior joint and Knee Joint.
that bond conditions (and hence additional variables $\beta$) will need to be considered for the column longitudinal reinforcement as well.

An additional consideration for the knee joint mechanical model is that horizontal equilibrium of forces establishes that the axial force in the beam member is equal to the shear force in the column at the joint boundary. Similarly, from vertical equilibrium of forces, the axial force in the column member is equal to the shear force in the beam, at the joint boundary (see Figure 2.32b). In addition, the axial forces in the beam and column members are in compression under closing action and in tension under opening action.

The formulation of the simplified knee joint mechanical model is presented in Chapter 4.

In many of the test studies discussed in this Chapter the knee joint test specimens were not detailed according to current a-seismic design practices for joints (i.e., where confinement of joint core concrete and anchorage of main reinforcement are primary variables in determining the seismic design stress limits for joints).

For those test studies using current detailing procedures for earthquake design, whereas, some test specimens performed poorly (Clarkson tests) failing at shear stress levels far less than the nominal values given by current design codes, other test specimens met and even exceeded (China tests) those nominal values. To help explain this difference in knee joint behaviour, a good understanding of the inner workings of such connections is required. Once a clear understanding of knee joint behaviour has been achieved, comprehensive design recommendations for design of these elements can be proposed.

As was discussed in Chapter 1, this will be accomplished through a collective evaluation of the available experimental database and analytical modeling, which includes finite element modeling and simple mechanical constructs of equilibrium and compatibility.
CHAPTER 3

EXPERIMENTAL STUDY ON REINFORCED CONCRETE KNEE JOINTS:
CLARKSON UNIVERSITY

As was mentioned in Chapter 1, the knee joint study at Clarkson University was done in collaboration with the University of Toronto. The study consisted of two components: an experimental component where a number of knee joint specimens with various reinforcement detailing schemes were subjected to cyclic load tests simulating earthquake conditions, and an analytical modeling component, where the experimental data was analyzed to obtain valuable information into the behaviour of knee joints, and subsequently to contribute in part, to the development of comprehensive design recommendations for such connections. The experimental component of the study was performed at Clarkson University (the author was present and participated during the testing of three knee joint specimens). The reduction of the experimental results for modeling purposes was performed at the University of Toronto by the author in the course of this thesis and is presented here.

The main variables of the experimental study were: the magnitude of joint shear input, the amount of horizontal and vertical transverse reinforcement within the joint, and joint anchorage details of main beam or column reinforcement within the joint.

Since the column does not extend above the joint in knee joint connections, vertical transverse reinforcement (stirrups), in addition to the hoops, were included in the experimental study to determine the efficacy of confinement within the joint. A number of specimens were also tested to investigate alternative anchorage detailing schemes that were intended to alleviate the congestion of reinforcement anchored in the joint.

The effects of the above mentioned parameters on knee joint behaviour are investigated in this chapter.
3.1 Experimental Test Program

3.1.1 Test Specimen Details

The test specimens were approximately half scale two-dimensional models of the upper storey connections of a twenty storey typical moment resisting space frame in an area of high seismic risk. The prototype building was rectangular in plan with six nine-metre bays in one direction and four nine-metre bays in the other direction.

The test series was a parametric study with variables: the magnitude of shear input to the joint, the amount of horizontal and vertical joint confining reinforcement, and a variety of anchorage detailing schemes for the main reinforcement.

Sixteen specimens, divided into five groups, were tested in the experimental program. The overall dimensions of a typical test specimen and the cross-sectional geometry of the beam and column members are shown in Figures 3.1 and 3.2, respectively. For the first group of test specimens (KJ1 - KJ4) the beam member width was 230mm instead of the 280mm as shown in Figure 3.2.

Specimens of group one were tested first to investigate the effects of anchorage length of vertical u-shaped stirrups and the presence of diagonal hairpins within the joint on the performance of the knee joint specimen. The differences between test specimens of groups two through five were in the magnitude of the horizontal joint shear input under both opening and closing actions, and in the anchorage detailing schemes. The horizontal joint shear input was controlled by the amount and size of the beam top longitudinal reinforcing bars under closing action.

Within each group of test specimens the amount of horizontal and vertical transverse reinforcement was varied. Closed horizontal hoops with two additional cross ties at each hoop location were provided within the joint region (Figure 3.2). Inverted u-shaped vertical stirrups were also provided. Figure 3.3 shows typical reinforcement details for the transverse reinforcement in the joint region as well as the different anchorage detailing schemes used in the test study. Table 3.1 gives the identification of each specimen and the associated details of the beam, column and joint transverse reinforcement.
Figure 3.1 Specimen Dimensions.

Figure 3.2 Cross Section Details for Beam and Column Members.
3.3a) Specimen KJ11: 4 Hoops no Stirrups  
3.3b) Specimen KJ12: 2 Hoops 2 Stirrups

3.3c) Specimen KJ13: 4 Hoops 4 Stirrups  
3.3d) Specimen KJ14: Column Stub

Figure 3.3 Transverse and Anchorage Reinforcement Detailing Schemes.
3.3e) Specimen KJ15: Straight Anchorage of Column Rebars.

3.3f) Specimen KJ16: Headed Beam and Column Rebars.

Figure 3.3 (Cont’d) Transverse and Anchorage Reinforcement Detailing Schemes.

Beam and column main reinforcing bars terminated in the knee joint region with a standard 90° hook for all specimens with the exception of the group five test specimens, KJ14, KJ15 and KJ16. Specimen KJ14 had a 12" (300mm) column stub which allowed for straight anchorage of the column longitudinal reinforcing bars through the joint and into the stub. Specimen KJ15 had straight anchorage of the column reinforcing bars in the joint region. When compared with the column reinforcement in other specimens, specimen KJ15 had a larger number of smaller diameter reinforcing bars (Table 3.1). Both of specimens KJ14 and KJ15 had 90° hooks for anchoring the beam longitudinal reinforcement. Specimen KJ16 had headed bars as beam and column reinforcement with the T-head anchored within the joint region (Figure 3.3).

Concrete compressive strength was measured using 150mm x 300mm (6" x 12") concrete cylinders. Table 3.1 gives the concrete strengths for each specimen at the day of testing.

Grade 60 reinforcement was used for all test specimens. Table 3.2 gives the stress-strain properties for the longitudinal and transverse reinforcement. No 3 (9.5mm diameter) reinforcing bars were used throughout as transverse reinforcement.
Table 3.1 Test Specimen Reinforcement Details: Clarkson Test Study.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>$f_y$ (MPa)</th>
<th>Column Reinforcement</th>
<th>Beam Top Reinforcement</th>
<th>Beam Bottom Reinforcement</th>
<th>Joint Hoops</th>
<th>Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td>KJ1</td>
<td>45.7</td>
<td>4 #6 Corner</td>
<td>4 #5</td>
<td>2 #5</td>
<td>4 #3</td>
<td>4 #3¹</td>
</tr>
<tr>
<td>KJ2</td>
<td>49.7</td>
<td>&amp;</td>
<td>4 #5</td>
<td>2 #5</td>
<td>4 #3</td>
<td>4 #3¹</td>
</tr>
<tr>
<td>KJ3</td>
<td>45.0</td>
<td>4 #5 Middle</td>
<td>4 #5</td>
<td>2 #5</td>
<td>2 #3</td>
<td>2 #3</td>
</tr>
<tr>
<td>KJ4</td>
<td>45.6</td>
<td>4 #5</td>
<td>4 #5</td>
<td>2 #5</td>
<td>4 #3</td>
<td>4 #3</td>
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Group 2.

<table>
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<tr>
<th>Test Specimen</th>
<th>$f_y$ (MPa)</th>
<th>Column Reinforcement</th>
<th>Beam Top Reinforcement</th>
<th>Beam Bottom Reinforcement</th>
<th>Joint Hoops</th>
<th>Stirrups</th>
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</thead>
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<tr>
<td>KJ5</td>
<td>31.5</td>
<td>8 #7</td>
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<td>3 #6</td>
<td>4 #3</td>
<td>0</td>
</tr>
<tr>
<td>KJ6</td>
<td>33.0</td>
<td>8 #7</td>
<td>5 #6</td>
<td>3 #6</td>
<td>2 #3</td>
<td>2 #3</td>
</tr>
<tr>
<td>KJ7</td>
<td>32.9</td>
<td>8 #7</td>
<td>5 #6</td>
<td>3 #6</td>
<td>4 #3</td>
<td>4 #3</td>
</tr>
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</table>

Group 3.

<table>
<thead>
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<th>$f_y$ (MPa)</th>
<th>Column Reinforcement</th>
<th>Beam Top Reinforcement</th>
<th>Beam Bottom Reinforcement</th>
<th>Joint Hoops</th>
<th>Stirrups</th>
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<tr>
<td>KJ8</td>
<td>36.3</td>
<td>8 #6</td>
<td>4 #6</td>
<td>2 #6</td>
<td>4 #3</td>
<td>0</td>
</tr>
<tr>
<td>KJ9</td>
<td>38.5</td>
<td>8 #6</td>
<td>4 #6</td>
<td>2 #6</td>
<td>2 #3</td>
<td>2 #3</td>
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<tr>
<td>KJ10</td>
<td>37.9</td>
<td>8 #6</td>
<td>4 #6</td>
<td>4 #6</td>
<td>4 #3</td>
<td>4 #3</td>
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</table>

Group 4.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>$f_y$ (MPa)</th>
<th>Column Reinforcement</th>
<th>Beam Top Reinforcement</th>
<th>Beam Bottom Reinforcement</th>
<th>Joint Hoops</th>
<th>Stirrups</th>
</tr>
</thead>
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<tr>
<td>KJ11</td>
<td>35.0</td>
<td>8 #6</td>
<td>4 #6</td>
<td>4 #6</td>
<td>4 #3</td>
<td>0</td>
</tr>
<tr>
<td>KJ12</td>
<td>32.9</td>
<td>8 #6</td>
<td>4 #6</td>
<td>2 #6</td>
<td>2 #3</td>
<td>2 #3</td>
</tr>
<tr>
<td>KJ13</td>
<td>31.7</td>
<td>8 #6</td>
<td>4 #6</td>
<td>4 #6</td>
<td>4 #3</td>
<td>4 #3</td>
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Group 5.

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<th>$f_y$ (MPa)</th>
<th>Column Reinforcement</th>
<th>Beam Top Reinforcement</th>
<th>Beam Bottom Reinforcement</th>
<th>Joint Hoops</th>
<th>Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td>KJ14</td>
<td>33.6</td>
<td>8 #6</td>
<td>4 #6</td>
<td>4 #6</td>
<td>4 #3</td>
<td>0</td>
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<tr>
<td>KJ15</td>
<td>36.9</td>
<td>12 #4</td>
<td>4 #5</td>
<td>4 #5</td>
<td>4 #3</td>
<td>4 #3</td>
</tr>
<tr>
<td>KJ16</td>
<td>37.2</td>
<td>8-16mm dia.</td>
<td>4-16mm dia.</td>
<td>4-16mm dia.</td>
<td>4 #3</td>
<td>4 #3</td>
</tr>
</tbody>
</table>

¹ 135° anchorage for stirrups ² Two diagonal hairpins within joint region.

Table 3.2 Reinforcement Properties.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Reinforcing Bars</th>
<th>$f_y$ (MPa)</th>
<th>$\varepsilon_y$</th>
<th>$\varepsilon_{sh}$</th>
<th>$f_u$ (MPa)</th>
<th>$\varepsilon_u$</th>
<th>$f_r$ (MPa)</th>
</tr>
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<tbody>
<tr>
<td>KJ1 - KJ4</td>
<td>#3</td>
<td>445</td>
<td>0.0022</td>
<td>0.022</td>
<td>676</td>
<td>0.1435</td>
<td></td>
</tr>
<tr>
<td></td>
<td>#5</td>
<td>448</td>
<td>0.0022</td>
<td>0.013</td>
<td>703</td>
<td>0.1172</td>
<td></td>
</tr>
<tr>
<td></td>
<td>#6</td>
<td>452</td>
<td>0.0022</td>
<td>0.0144</td>
<td>714</td>
<td>0.1105</td>
<td></td>
</tr>
<tr>
<td>KJ5 - KJ13</td>
<td>#3</td>
<td>445</td>
<td>0.0022</td>
<td>0.022</td>
<td>676</td>
<td>0.1435</td>
<td>681</td>
</tr>
<tr>
<td></td>
<td>#6</td>
<td>461</td>
<td>0.0023</td>
<td>0.022</td>
<td>703</td>
<td>0.1435</td>
<td></td>
</tr>
<tr>
<td></td>
<td>#7</td>
<td>455</td>
<td>0.0023</td>
<td>0.022</td>
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</tr>
<tr>
<td>KJ14</td>
<td>#3</td>
<td>445</td>
<td>0.0022</td>
<td>0.022</td>
<td>676</td>
<td>0.1435</td>
<td>607</td>
</tr>
<tr>
<td></td>
<td>#6</td>
<td>448</td>
<td>0.0022</td>
<td>0.022</td>
<td>669</td>
<td>0.1435</td>
<td></td>
</tr>
<tr>
<td>KJ15</td>
<td>#3</td>
<td>445</td>
<td>0.0022</td>
<td>0.022</td>
<td>676</td>
<td>0.1435</td>
<td>700</td>
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<tr>
<td></td>
<td>#4</td>
<td>469</td>
<td>0.0022</td>
<td>0.022</td>
<td>724</td>
<td>0.1435</td>
<td></td>
</tr>
<tr>
<td></td>
<td>#5</td>
<td>434</td>
<td>0.0022</td>
<td>0.022</td>
<td>685</td>
<td>0.1435</td>
<td></td>
</tr>
<tr>
<td>KJ16</td>
<td>#3 16 mm dia.</td>
<td>445</td>
<td>0.0022</td>
<td>0.022</td>
<td>676</td>
<td>0.1435</td>
<td>538</td>
</tr>
</tbody>
</table>
3.1.2 Location of Instruments

During the tests the following variables were monitored: actuator load (beam axial load), specimen lateral displacement, joint cover expansion, joint core concrete strains, and longitudinal and transverse reinforcing bar strains.

The joint cover expansion was measured using the LVDT assembly shown in Figure 3.4. In specimens KJ11 through KJ14, the joint core concrete strains were also measured using strain gauges embedded in the joint centre (see Figure 3.4).

Figure 3.5 shows the locations of strain gauges used to measure the longitudinal and transverse reinforcing bar strains. The strain gauges were placed near the member-joint interface on beam and column longitudinal reinforcing bars. Strain gauges were also placed midway between the column reinforcing bars on the hoops and about 25mm (1") vertically down each leg of the stirrups.

Figure 3.6 shows the experimental setup, and the location of LVDT’s and wire potentiometers used to measure the lateral displacement and rotation of the test specimens, respectively.

Photographs of the embedded strain gauges and test set-up are given in Figures 3.7 and 3.8, respectively.

Figure 3.4 Cover and Embedded Concrete Gauges.
Figure 3.5  Reinforcement Strain Gauge Locations.

Figure 3.6  Experimental Test Set-Up.
Figure 3.7 Embedded Strain Gauges.

Figure 3.8 Test Set-Up.
3.1.3 Load History

A statically determinate test set-up was used in the experimental study (Figures 3.6 and 3.8). During testing a 560kN (125kip) hydraulic actuator pulled on the specimen under opening action and pushed on the specimen during closing action. The tests were displacement controlled. Figure 3.9 shows a typical drift history. Figures 3.10 and 3.11 show the equilibrating reactions on the specimen and knee joint boundary forces, respectively, under opening and closing actions.

The specimen drift (defined as the angle of rotation of the specimen from its base support) was calculated from:

\[
Drift = \frac{LVDTA - LVDTB}{915}
\]

where; 915mm (36") is the distance between the location of LVDTA and the pin connections at the end of the column, and LVDTA - LVDTB is the difference in displacement of LVDTA and LVDTB, respectively. This difference is taken to compensate for any horizontal movement of the column pin support (see Figure 3.6).

Figure 3.9  Drift History; Specimen KJ15.
Figure 3.10  Free Body Diagram of Loads Acting on Knee Joint Test Specimen.

Figure 3.11  Free Body Diagram of Loads Acting at Knee Joint Faces.
3.2 Evaluation of Experimental Results

In this section, the available experimental data is used to identify the milestones in the behavioral response of the knee joint specimens for improved modeling of the problem. For this purpose an array of experimental performance indicators are considered, including storey drift and horizontal joint shear input.

Drift

Drift is a familiar performance variable used in many earthquake design codes around the world. The equivalent storey drift was estimated from the displacement used in the Clarkson experimental study to control the actuator (Equation 3.1). This calculation accounts for the horizontal displacement at the column pin support, but it fails to account for the rotational flexibility of the column footing and the axial flexibility of the hollow structural steel member (Figure 3.12).

To avoid this problem, in this study the actual drift is calculated from the readings of two wire potentiometers (Figure 3.6). Thus, using the cosine law and the average value of displacement from the two wire potentiometers, the angle between the beam and column chords that arises as a result of opening and closing actions is (Figure 3.13):

\[ \alpha_d(rads) = \cos^{-1} \left( \frac{Z^2 - Z_d^2}{2 \ L \ H} \right) \]  

3.2

where; \( L \) and \( H \) are the beam and column lengths to the joint centerline, respectively, and \( Z \) and \( Z_d \) are the lengths of the hypotenuse before and after the deformation of the specimen, respectively.

The change in the 90° angle between the beam and column chords is calculated from,

\[ \theta_d(rads) = |\alpha_d(rads) - \frac{\pi}{2}| \]  

3.3
Figure 3.12  Built-In Flexibilities Due to Test Set-Up.

Figure 3.13  Knee Joint Specimen Drift Obtained From Wire Potentiometer Data.
and the actual drift of the test specimen is,

\[ \text{drift} = \frac{\delta}{H} \]  

where; \( \delta \) is the lateral displacement of the test specimen. For small values of the angle \( \theta_d \) (rads),

\[ \tan \theta_d = \theta_d = \frac{\delta}{H} = \text{drift} \]

The actual drift histories for the test specimens are given in Appendix A.

**Joint Shear Input**

The joint shear input in the Clarkson study was controlled under closing action by the amount and size of the beam top reinforcing bars (see Table 3.1). The horizontal joint shear input under closing action is given according to ACI352-85.

\[ V_j^e = T_s \]  

\[ T_s = \alpha_{sm} A_s f_y \]

where; \( T_s \) is the probable yield strength, and \( \alpha_{sm} \) is the stress multiplier and is equal to 1.25.

In terms of average shear stress (in \( \sqrt{f'_c} \)), Equation 3.6 reduces to,

\[ \bar{v}_j^e = \frac{T_s}{b_j h_c \sqrt{f'_c}} \]

where; \( v_j^e \) is the expected average shear stress (in \( \sqrt{f'_c} \)).
Therefore, the experimental joint shear stress (in $\sqrt{f_c^{'}}$) may be calculated from $F_{s,\text{top}}$, (where; $F_{s,\text{top}}$ is the force in the beam top reinforcing bars) from the following:

$$\bar{\nu}_{h,\text{obs}} = \frac{F_{s,\text{top}}}{b_j h \sqrt{f_c'}}$$ \hfill 3.9

In Equation 3.9 $\bar{\nu}_{h,\text{obs}}$ is the observed horizontal joint shear stress (in $\sqrt{f_c^{'}}$).

For purposes of comparison with the experimental results, the nominal shear strength of $1 \sqrt{f_c^{'}}$, MPa ($12 \sqrt{f_c^{'}}$ psi) given in ACI352-85 will be used here.
3.2.1 **Mechanism of Joint Resistance to Shear**

The joint resists the forces applied at its boundaries predominantly through a shear mechanism. This mechanism consists of a diagonal compression strut in the concrete and horizontal and vertical ties (i.e., tension forces in the reinforcement) to carry the principal compressive and tensile stresses within the joint. To illustrate the disposition of the stress field in the joint, this member is idealized as a panel loaded by shear at its boundary. The transverse reinforcement becomes necessary once the principal tensile stresses in the joint have exceeded the tensile cracking strength of the concrete and a diagonal crack has formed within the joint region. One consequence of shear is expansion of the core concrete. Hoops and stirrups within the joint region partially restrain the expansion. The principal tensile strains in the joint are $\varepsilon_1$ in the plane of shear action, and $\varepsilon_3$ in the orthogonal out-of-plane direction.

Figure 3.14 plots the history of residual hoop and stirrup strains for test specimens KJ12 and KJ13. Note that residual strain is the magnitude of strain at zero load between cycles of load reversal. It represents the permanent extension in the hoop or stirrup as a result of joint core concrete expansion. Generally, there is an increasing trend in the residual strain histories which supports the notion of a confining role for the hoop and stirrup within the joint region.

A measure of joint core concrete expansion is the volumetric strain in the joint, plotted in Figure 3.15 for a number of the Clarkson specimens. The x-axis in the plots represents the principal compressive strain in the joint. In general, the volumetric strain increases with increasing principal compressive strain.

Figure 3.16 shows the strain profile for the main beam top reinforcement for specimen KJ13. Two trends can be seen in the figure. Firstly, the strain in the reinforcement at the joint face is seen to drift towards the tensile region. Secondly, even under conditions where the reinforcement is under compressive forces at the knee joint boundary the reinforcing bars strain increases in tension (see drift levels 4O2, 4O3 and 6O1 in Figure 3.16). The above trends are common for both transverse reinforcement strains within the joint and longitudinal reinforcement strains at the joint boundaries for all the test specimens. The latter trend appears to be a direct consequence of shear in the joint region.
3.14a) Residual Hoop and Stirrup Strains: Specimen KJ12


Figure 3.14 Residual Hoop and Stirrup Strains: +ve Strain - Expansion.
3.15a) Specimen KJ7

3.15b) Specimen KJ9

3.15c) Specimen KJ12

3.15d) Specimen KJ13

3.15e) Specimen KJ15

3.15f) Specimen KJ16

Figure 3.15  Volumetric Strain- Principal Compressive Strain: +ve $\varepsilon_v$ - Expansion.
3.2.2 Joint Shear Input

The magnitude of shear sustained by the joint was one of the variables of the experimental program. Table 3.3 shows the expected and observed horizontal joint shear stress values, normalized in $\sqrt{f'_c}$, for the Clarkson series of test specimens as given by Equation 3.8 and 3.9, respectively. The expected horizontal joint shear stress values ranged from a minimum of $5.87\sqrt{f'_c}$ (psi) to a maximum of $12.63\sqrt{f'_c}$ (psi).

Group 2 test specimens were designed for the highest nominal horizontal shear stress input to the joint, approximately equal to the nominal shear stress given by ACI352-85, i.e., $1\sqrt{f'_c}$, MPa ($12\sqrt{f'_c}$ (psi)), for corner joints. When comparing the expected and observed horizontal joint shear stress values, the specimens of Group 2 attained values much lower than expected. In general, the beam top reinforcing bars did not reach yield for these specimens due to a premature bond splitting failure. Figure 3.17 shows a photograph of the joint region for specimen KJ7 at the final load stage. From Figure 3.17 very wide bond splitting cracks can be seen along both the beam top and column outer reinforcing bars.

When comparing expected and observed horizontal joint shear stress values for the specimens of the remaining groups, only specimens with smaller main bars (Specimens KJI to KJ4 and KJ15) were able to reach the expected shear stress values.

The joint shear stress input may also be calculated by considering the statics of the overall connection. Thus, the vertical joint shear stress input may be obtained from the
moment gradient through the joint (Table 3.3). From Figure 3.18 it follows that:

\[ \nu_v = \frac{M_{bf}}{h_c} \]  \hspace{1cm} 3.12

and thus,

\[ \bar{\nu}_{v,obs} = \left( \frac{M_{bf}}{h_c} \right) \left( \frac{1}{b_f h_c \sqrt{f'_c}} \right) \]  \hspace{1cm} 3.13

where; \( M_{bf} \) is the maximum beam moment at the face of the joint.

In the next two sections, a detailed investigation is made using the available knee joint data in order to interpret the mechanism of force transfer in the joint panel region.

Table 3.3 Joint Shear Input: Closing Action

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>( f'_c ) (ksi)</th>
<th>( T_s ) (kips)</th>
<th>( \nu'_j ) (( \sqrt{f'_c} ) psi)</th>
<th>( \nu_{v,obs} ) (( \sqrt{f'_c} ) psi)</th>
<th>( M_{bf} ) (in.kips)</th>
<th>( v_{c,obs} ) (( \sqrt{f'_c} ) psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1: ( b_i = 12.5''; h_c = 16'' ); ( f_y = 65.0 \text{ ksi} ); 4 #5 Beam Top Reinforcement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KJ1</td>
<td>6632</td>
<td>99.7</td>
<td>6.12</td>
<td>6.12</td>
<td>1841</td>
<td>7.06</td>
</tr>
<tr>
<td>KJ2</td>
<td>7215</td>
<td>99.7</td>
<td>5.87</td>
<td>5.87</td>
<td>1912</td>
<td>7.03</td>
</tr>
<tr>
<td>KJ3</td>
<td>6530</td>
<td>99.7</td>
<td>6.17</td>
<td>6.17</td>
<td>1770</td>
<td>6.85</td>
</tr>
<tr>
<td>KJ4</td>
<td>6610</td>
<td>99.7</td>
<td>6.13</td>
<td>6.13</td>
<td>1894</td>
<td>7.28</td>
</tr>
<tr>
<td>Group 2: ( b_i = 13.5''; h_c = 16'' ); ( f_y = 66.9 \text{ ksi} ); 5 #6 Beam Top Reinforcement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KJ5</td>
<td>4570</td>
<td>184.5</td>
<td>12.63</td>
<td>8.52</td>
<td>1956</td>
<td>8.37</td>
</tr>
<tr>
<td>KJ6</td>
<td>4780</td>
<td>184.5</td>
<td>12.35</td>
<td>7.91</td>
<td>1770</td>
<td>7.41</td>
</tr>
<tr>
<td>KJ7</td>
<td>4765</td>
<td>184.5</td>
<td>12.37</td>
<td>9.68</td>
<td>2124</td>
<td>8.90</td>
</tr>
<tr>
<td>Group 3: ( b_i = 13.5''; h_c = 16'' ); ( f_y = 66.9 \text{ ksi} ); 4 #6 Beam Top Reinforcement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KJ8</td>
<td>5270</td>
<td>147.6</td>
<td>9.41</td>
<td>7.54</td>
<td>1806</td>
<td>7.20</td>
</tr>
<tr>
<td>KJ9</td>
<td>5585</td>
<td>147.6</td>
<td>9.14</td>
<td>7.32</td>
<td>1938</td>
<td>7.51</td>
</tr>
<tr>
<td>KJ10</td>
<td>5500</td>
<td>147.6</td>
<td>9.21</td>
<td>8.23</td>
<td>1974</td>
<td>7.70</td>
</tr>
<tr>
<td>Group 4: ( b_i = 13.5''; h_c = 16'' ); ( f_y = 66.9 \text{ ksi} ); 4 #6 Beam Top Reinforcement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KJ11</td>
<td>5080</td>
<td>147.6</td>
<td>9.58</td>
<td>7.67</td>
<td>1859</td>
<td>7.55</td>
</tr>
<tr>
<td>KJ12</td>
<td>4775</td>
<td>147.6</td>
<td>9.89</td>
<td>7.21</td>
<td>1770</td>
<td>7.41</td>
</tr>
<tr>
<td>KJ13</td>
<td>4600</td>
<td>147.6</td>
<td>10.07</td>
<td>8.07</td>
<td>1903</td>
<td>8.12</td>
</tr>
<tr>
<td>Group 5: ( b_i = 13.5''; h_c = 16'' ); ( f_y = 65.0 \text{ ksi} ) (KJ14), ( f_y = 62.9 \text{ ksi} ) (KJ15), ( f_y = 70.6 \text{ ksi} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KJ14: 4 #6</td>
<td>4880</td>
<td>143.6</td>
<td>9.52</td>
<td>6.79</td>
<td>1682</td>
<td>6.97</td>
</tr>
<tr>
<td>KJ15: 4 #5</td>
<td>5350</td>
<td>96.6</td>
<td>6.11</td>
<td>6.11</td>
<td>1527</td>
<td>6.04</td>
</tr>
<tr>
<td>KJ16: 4 16M</td>
<td>5390</td>
<td>110.0</td>
<td>6.94</td>
<td>5.55</td>
<td>1301</td>
<td>5.13</td>
</tr>
</tbody>
</table>
Figure 3.17 Specimen KJ7: Final Load Stage

Figure 3.18 Vertical Joint Shear Stress.
3.2.3 Behaviour of Knee Joint Specimens: Opening Action

As outlined in Section 3.1.3, the knee joint tests were displacement controlled, and drift was used as the controlling parameter. Due to the inherent flexibilities in the test set-up, however, the specimen drift measured during the test was higher than the actual specimen drift obtained using the data from the two wire potentiometers. In order to maintain consistency with the observations during testing, the experimental specimen drift values will be used here as reference and the actual drift values will be presented where required.

(a) Observed Cracking Patterns

Based on the experimental reports (Cote and Wallace 1994 and McConnell and Wallace 1995) crack development within the joint region under opening action followed the sequence depicted in Figure 3.19. The front and side faces of the knee joint are unfolded in the Figure to show the crack patterns at the three drift levels considered.

At very low experimental drift levels (0.25 per cent to 0.5 per cent), flexural cracks appeared across the bottom and partly up the sides of the beam at the beam-joint interface. In addition splitting cracks also appeared extending half way across both faces of the joint, starting at the beam-joint interface, near the location of the beam bottom reinforcing bars.

![Crack Patterns Within Joint Region: Opening Action.](Figure 3.19)
At moderate experimental drift levels (0.75 per cent to 2.0 per cent), splitting cracks appeared within the joint region, usually at the levels of the hoops and at the level of beam top reinforcing bars. In addition, diagonal cracks formed near the inside corner of the joint region, particularly for specimens subjected to a higher shear input.

At higher experimental drift levels (3.0 - 6.0 per cent), additional diagonal cracks formed in the knee joint panel, predominantly near the outside corner. At this stage, previous splitting cracks that had formed at the level of the beam top reinforcing bars extended into the joint. Smaller diagonal cracks were also witnessed in some specimens near the inside corner of the joint. The widest cracks occurred along the level of the beam top reinforcing bar. A major diagonal crack also prevailed that initiated at the level of the beam top reinforcing bar near the beam-joint interface.

In general, more diagonal cracks developed in specimens with a higher shear input than those with a lower shear input. The diagonal cracks for the specimens with the lower shear input were located mostly near the outside corner of the joint faces, whereas the diagonal cracks appeared over a greater surface of the joint faces for the higher shear input specimens.

(b) Moment-Drift and Strain-Drift Relations

Figure 3.20 shows the moment-drift responses for the knee joint test specimens (see Table 3.1 for test specimen properties). From Figure 3.20 a plateau or near plateau is witnessed in the moment versus drift plots for most test specimens at approximately 1 per cent actual drift, coinciding with the formation of a diagonal crack. It appears that this crack formation is responsible for limiting the specimen’s load-carrying capacity. Figure 3.21 plots beam moment-drift and strain-drift (hoop gauge H1A-Figure 3.5) along with a schematic of the diagonal crack patterns of the joint faces for specimen KJ8. The labels 3O1 and 4O1 in Figure 3.21 refer to the first cycle at 3 per cent and 4 per cent experimental drift, respectively under opening action. A sharp increase in the hoop strain (H1A) is witnessed at 3 per cent experimental drift (about 1 per cent actual drift) coinciding with significant deterioration in the slope of the moment-drift plot and the formation of a large diagonal crack in the knee-joint region. Further examples showing this trend are found in Appendix B. With the exception of specimens KJ14 and KJ15, which showed a progressively steeper rate of hoop

3.20 c) Specimen KJ3: 2 Hoops 2 Stirrups
3.20d) Specimen KJ4: 4 Hoops 4 Stirrups

Figure 3.20 Moment-Drift Plots for Clarkson Test Specimens.
3.20 e) Specimen KJ5: 4 Hoops no Stirrups.  
3.20f) Specimen KJ8: 4 Hoops no Stirrups

3.20 g) Specimen KJ6: 2 Hoops 2 Stirrups.  
3.20h) Specimen KJ9: 2 Hoops 2 Stirrups

3.20 i) Specimen KJ7: 4 Hoops 4 Stirrups.  
3.20j) Specimen KJ10: 4 Hoops 4 Stirrups

Figure 3.20 (Cont’d) Moment-Drift Plots for Clarkson Test Specimens.
3.20 k) Specimen KJ11: 4 Hoops no Stirrups.

3.20 l) Specimen KJ14: 400 mm Column Stub

3.20 m) Specimen KJ12: 2 Hoops 2 Stirrups.

3.20 n) Specimen KJ15: Straight Anchorage of Column Bars.

3.20 o) Specimen KJ13: 4 Hoops 4 Stirrups.


Figure 3.20 (Cont’d) Moment-Drift Plots for Clarkson Test Specimens.
and stirrup strain increase with drift, all other specimens showed the opposite trend at higher drift levels. Strain reduction with increase in drift suggests reduction in the force input to the joint, and given this is a statically determinate system, this is a sign of strength degradation of the overall system. The better performance of KJ14 and KJ15, may be attributed to the presence of a column stub and smaller diameter longitudinal reinforcement, resulting in less damage in the joint region due to better confinement and less bond demand, respectively. Table 3.4 gives the actual drift levels at which major diagonal cracks occurred in the test specimens under opening action. It appears that diagonal cracks appeared at lower drift levels for specimens with higher shear inputs to the joint.

3.21a) Schematic of Joint Crack Patterns

3.21b) Strain-Drift Plot: Hoop Gauge (H1A).

3.21c) Moment-Drift Plot

Figure 3.21 Knee Joint Behaviour: Specimen KJ8 (3% and 4% Experimental Drift).
Table 3.4  Actual Drift Levels at Formation of Major Diagonal Cracks: Opening Action.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Actual Drift Level (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KJ8</td>
<td>1.05</td>
</tr>
<tr>
<td>KJ9</td>
<td>1.10</td>
</tr>
<tr>
<td>KJ10</td>
<td>1.10</td>
</tr>
<tr>
<td>KJ11</td>
<td>0.75, 1.20</td>
</tr>
<tr>
<td>KJ12</td>
<td>0.75, 1.30</td>
</tr>
<tr>
<td>KJ13</td>
<td>0.75, 1.20</td>
</tr>
<tr>
<td>KJ14</td>
<td>0.50, 0.83, 0.95</td>
</tr>
<tr>
<td>KJ15</td>
<td>0.95, 1.10, 1.80</td>
</tr>
<tr>
<td>KJ16</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Table 3.5 gives the drift levels at which the beam bottom reinforcing bars (BB) and column inner reinforcing bars (C14) yielded under opening action for each test specimen.

Drift values for those specimens that had faulty gauges, or for reinforcement that did not yield, were not reported. No definite trends are seen in Table 3.5.

Table 3.5 Actual Drift Levels at Yielding of Beam and Column Reinforcement: Opening Action.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Reinforcing Bar</th>
<th>Actual Drift Level (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KJ9</td>
<td>BB1</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>BB2</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>C4</td>
<td>1.80</td>
</tr>
<tr>
<td>KJ10</td>
<td>BB1</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td></td>
</tr>
<tr>
<td>KJ11</td>
<td>BB1</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>BB2</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>C4</td>
<td></td>
</tr>
<tr>
<td>KJ12</td>
<td>BB</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td>1.30</td>
</tr>
<tr>
<td>KJ13</td>
<td>BB</td>
<td>1.30</td>
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<tr>
<td></td>
<td>C14</td>
<td>1.40</td>
</tr>
<tr>
<td>KJ14</td>
<td>BB</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>C14</td>
<td>1.60</td>
</tr>
<tr>
<td>KJ15</td>
<td>BB</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>C14</td>
<td>0.40</td>
</tr>
<tr>
<td>KJ16</td>
<td>BB</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>C14</td>
<td>1.30</td>
</tr>
</tbody>
</table>
Specimens KJ6 and KJ11 to KJ13 showed premature strength loss under opening action starting at an actual drift level in the order of 2 per cent for specimen KJ12 and 3 per cent for specimens KJ6, KJ11 and KJ13 (Figure 3.20). This may be because specimens KJ11 to KJ13 were subjected to higher shear input under opening action than the remaining test specimens. Also, the presence of fewer hoops resulted in more deterioration under opening action (Figure 3.20: Specimen KJ12).

(c) Concrete Strains Within the Joint

Strain-drift plots for the embedded strain gauges (Figures 3.4 and 3.7) in the concrete are also considered in order to assess the condition of the joint core concrete.

Figure 3.22 plots transverse strains with imposed drift measured at the centre of joint core for specimens KJ11 to KJ13. Specimen KJ12, which had only two hoops and two stirrups within the joint region, developed the greatest amount of concrete expansion. Conversely, specimen KJ13, which had the greatest volumetric ratio of transverse reinforcement within the joint region consisting of four horizontal hoops and four vertical stirrups, showed the least joint core concrete expansion. When the joint core expansions of specimens KJ11 and KJ12 are compared, it appears that the two additional hoops in specimen KJ11 are more effective in confining the joint core than the additional two stirrups in specimen KJ12.

Figure 3.23 plots strain-drift measurements in the diagonal of the joint panel, recorded at the joint centre, for specimens KJ11, KJ13 and KJ14. Placing four stirrups within the joint region, in addition to the four hoops (KJ13), provided better confinement to the joint core concrete. This can be seen when joint core concrete expansions are compared for specimens KJ11 and KJ13. When comparing the same variable for specimens KJ11, KJ13 and KJ14, results show that providing the column stub (KJ14) along with hoops within the joint region is more effective in reducing the joint core concrete expansions than providing only hoops or hoops and stirrups for confinement. The enhanced performance seen in specimen KJ14 can be attributed to a larger hysteretic area for the strain versus drift response when compared to specimens KJ11 and KJ13.
3.22 a) Specimen KJ11 (4 #3 Hoops, no Stirrups).

3.22 b) Specimen KJ12 (2 #3 Hoops, 2 #3 Stirrups).

3.22 c) Specimen KJ13 (4 #3 Hoops, 4 #3 Stirrups).

Figure 3.22  Transverse Concrete Strains at Joint Core Centre - Effects of Transverse Reinforcement.
3.23 a) Specimen KJ11 (4 #3 Hoops, no Stirrups, no Stub).

3.23 b) Specimen KJ13 (4 #3 Hoops, 42 #3 Stirrups, no Stub).

3.23 c) Specimen KJ14 (4 #3 Hoops, no Stirrups, 400 mm Column Stub).

Figure 3.23  Diagonal Concrete Strains at Joint Core Centre - Effects of Transverse Reinforcement and Column Stub.
From Figures 3.21 through 3.23 it also appears that the deterioration seen in the moment-drift plots (Figure 3.20) under opening action for specimens KJ6 and KJ11 to KJ13 are marked by simultaneous expansion of the joint core. From the strain record it is evident that this expansion is a consequence of diagonal cracking within the joint region and is caused by joint shear action. Therefore, confinement delays deterioration caused by joint shear in knee joints, in a manner similar to what is familiar with interior beam-column connections.

3.2.4 Behaviour of Knee joint Specimens: Closing Action

a) Observed Cracking Patterns.

The sequence of crack development in the joint panel under closing action is depicted in Figure 3.24. The back, top and side faces of the knee joint are unfolded in the Figure to show the crack patterns for the three different drift levels.

At low experimental drift levels, (0.25 per cent to 0.5 per cent experimental drift), flexural cracks appeared across the top and part of the way down both sides of the beam at the beam joint interface.

Figure 3.24 Crack Patterns Within Joint Region: Closing Action.
Flexural cracks also appeared across the back of the joint, and splitting cracks extended about half way across both faces of the joint starting from the back of the column, near the location of the beam bottom reinforcement.

At moderate experimental drift levels (0.75 per cent to 2.0 per cent experimental drift), a number of diagonal cracks appeared, in most cases near the joint corners. The first diagonal cracks appeared at experimental drift levels of 1 per cent to 1.5 per cent. Flexural cracks appear starting at the back of the column and extending as splitting cracks on both faces of the joint at or near the locations of the hoop reinforcement. Additional flexural cracks also appeared on the back and top faces of the column.

At higher experimental drift levels (3.0 per cent to 6.0 per cent), additional diagonal cracks formed within the joint region concentrating mostly near the joint centre. Additional flexural cracks appeared on the back and top of the joint.

As done for opening action, plots of the test data will be used as reference to help interpret the observed response of the knee joint specimens under closing action.

b) Moment-Drift and Strain-Drift Relations.

The first significant inelastic response in most of the test specimens under closing action is seen in the moment-drift plot at the first cycle of 3 per cent experimental drift (3C1) (Figure 3.20).

With the exception of specimens KJ14, KJ15, and KJ1 to KJ4, which had smaller diameter beam and column longitudinal reinforcing bars or a column stub, significant deterioration in the load carrying capacity under closing action in the moment-drift plots occurred at 4 per cent closing experimental drift (between 1.5 per cent and 2.5 per cent actual drift, see moment-drift Plots). Pinching was evident at 3 per cent and was pronounced at 4 per cent experimental drifts.

Specimens that showed considerable deterioration in their load-carrying capacity sustained high shear input in the joint, either under closing action (5 #6 beam top reinforcing bars), such as specimens KJ5 to KJ7, or under opening action (4 #6 bottom beam reinforcing bars), for example specimens KJ11 to KJ13. Specimen KJ16 which contained headed beam and column reinforcement within the joint region also showed considerable deterioration in
load-carrying capacity under closing action. Similar strain histories were observed in the beam top reinforcing bars and column outer reinforcing bars. In all cases, reinforcement strain reached a peak at experimental drift levels of 2 per cent to 3 per cent (between 1 per cent and 1.5 per cent actual drift) followed by a steady drop for higher drift levels.

Specimens KJ5 to KJ7 developed extensive cracking in the joint region following the line of beam top reinforcing bars, column outer reinforcing bars and column inner reinforcing bars (these cracks suggest splitting due to bond failure). By the end of the test significant spalling had occurred in the joint of these specimens at the top, front and back faces. It appears that bond deterioration played a significant role in the performance of these specimens.

Figure 3.25(b) plots the experimental moment-drift relationship for specimen KJ8. Strain-drift plots for selected strain gauges on hoops (H2A, H3A, and H4A- Figure 3.5) are also provided in Figure 3.25 along with a schematic of the diagonal crack patterns on the joint faces for the drift level considered.

Sharp increases in strain were recorded at about 1 per cent actual drift, which also represents the initiation of strength loss in the moment-drift curve. This hoop strain increase is a measure of joint core concrete expansion due to the formation of diagonal cracks. Similar observations can be made for other test specimens under closing action (see Appendix B). As in the case of opening action, core expansion was directly linked to loss of shear resistance, unless passive confining mechanisms could be mobilized.

Drift levels associated with yielding of beam top reinforcing bars (TB) and column outer reinforcing bars (C23) are listed in Table 3.6. Once again, drift levels are not recorded for test specimens with faulty gauges or for reinforcing bars that did not yield. From Table 3.6 it can be seen that most of the reinforcement yielded at actual drift levels between 0.80 per cent and 1.00 per cent.

c) Concrete Strains within Joint.

Transverse strain readings within the joint, plotted in Figure 3.22 for specimens KJ11 and KJ12 indicate that joint core concrete growth is increasing at higher experimental drift levels (3 per cent to 6 per cent) under closing action.
3.25 a) Schematic of Joint Crack Patterns at 3% Nominal Drift.

3.25 b) Moment-Drift Plot.

3.25 c) Strain-Drift: Hoop Gauge (H2A).

3.25 d) Strain-Drift: Hoop Gauge (H3A)  

3.25 e) Strain-Drift: Hoop Gauge (H4A).

Figure 3.25 Knee Joint Behaviour: Specimen KJ8 (3% Experimental Drift).
Table 3.6 Actual Drift Levels at Yielding of Beam and Column Reinforcement: Closing Action.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Reinforcing Bar</th>
<th>Actual Drift Level (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KJ8</td>
<td>TB1</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>TB2</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>C23</td>
<td>0.90</td>
</tr>
<tr>
<td>KJ9</td>
<td>TB</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>C23</td>
<td>0.80</td>
</tr>
<tr>
<td>KJ10</td>
<td>TB</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C23</td>
<td>0.90</td>
</tr>
<tr>
<td>KJ11</td>
<td>TB</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>C23</td>
<td>0.95</td>
</tr>
<tr>
<td>KJ12</td>
<td>TB</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>C23</td>
<td>0.95</td>
</tr>
<tr>
<td>KJ13</td>
<td>TB</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>C23</td>
<td>0.95</td>
</tr>
<tr>
<td>KJ14</td>
<td>TB</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>C23</td>
<td></td>
</tr>
<tr>
<td>KJ15</td>
<td>TB</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>C23</td>
<td>0.70</td>
</tr>
<tr>
<td>KJ16</td>
<td>TB1</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>TB2</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>C23</td>
<td>0.90</td>
</tr>
</tbody>
</table>
3.2.5 Typical Modes of Failure Observed in Tests

In this study, failure is defined by the drift level associated with a drop of 20 per cent from the peak load-carrying capacity of the specimen. In order to identify all possible failure modes for such connections under opening and closing actions, a list of events which may have limited the load-carrying capacity of the available specimens is compiled.

Formation of Plastic Hinge for Longitudinal Reinforcing bars at Member-Joint Interfaces

Under opening action, the beam bottom reinforcing bars and the column inner reinforcing bars yielded adjacent to the joint (plastic hinge zones). Alternatively, the beam top reinforcing bars and column outer reinforcing bars yielded under closing action. Spalling of the surrounding concrete occurred at these locations after several inelastic excursions. This results in a reduction of the effective moment arm and consequently a reduction in the beam and column moment capacities at the joint faces (Cote and Wallace 1994 and McConell and Wallace 1995).

Joint Core Concrete Expansion

The magnitude of shear input was significant or poor confinement was provided, which resulted in diagonal cracks that formed within the joint region under both opening and closing action. These cracks grew in length, width and number (in well-detailed joints).

One consequence of the formation of diagonal cracks due to shear input to the joint was joint core concrete expansion. This resulted in higher strains in the hoop and stirrup reinforcing bars in the vicinity of the cracks. As shown in Sections 3.2.3 and 3.2.4, the formation of diagonal cracks within the joint region resulted in deterioration of the moment capacities and stiffness of the connections.

Slip due to Bond Deterioration of Longitudinal Reinforcing Bars Within Joint Region

When large bars were used or inadequate confinement was provided in the joint, slip due to bond deterioration occurred under both opening and closing actions. Signs of bond deterioration were splitting cracks that formed along the longitudinal reinforcing bars in the joint region and local yielding of the transverse reinforcement that looped around those bars.
Slip due to Anchorage Deterioration Under Closing Action

Based on the geometry of the knee joint specimens and the formation of cracks due to reverse cyclic loading, anchorage deterioration may have occurred under closing action. One consequence of anchorage deterioration under closing action was the upward and outward movements of the top and back portions of the joint, respectively. This leads to spalling of the concrete cover at the outside corner of the joint.

Fracture of Longitudinal Reinforcing bars Within Joint Region

The beam and column reinforcing bars fractured at the location of the bends within the joint region.

Splitting of Joint Core Concrete

The joint core concrete split due to the presence of large diameter reinforcing bars within the joint region.

Failure Modes of Knee joint Specimens: Opening Action

Group #1 and #5: Specimens KJ1 to KJ4 and KJ14 to KJ16

Specimens in Groups #1 and #5 did not fail under opening action. Their response was governed predominantly by flexural hinging associated with yielding of the beam bottom reinforcing bars. This is evident from the considerable spalling of the concrete in the beam at the location of the beam bottom reinforcing bars near the column face. The capacity of these specimens was limited from deterioration of the joint due to expansion of the core concrete and by the reduced moment arm of the adjoining members that resulted from considerable spalling.

Group #2, #3, #4: Specimens KJ5 to KJ13

The behavioural responses of the specimens in Groups #2 through #4 were governed by flexural hinging, loss of the compression zone at the joint face, and to some extent joint core concrete expansions due to the formation and growth of diagonal cracks.

Pinching was evident in the moment-drift plots under opening action especially for the Group #3 test specimens.
Specimens of this group (KJ11 to KJ13) contained the highest tensile reinforcement ratio for the beam anchorage reinforcement. As a result, significantly more deterioration occurred in the beam compression zone near the joint face where previous splitting cracks at the level of the beam top reinforcement had formed during cycles of closing action. This precipitated the decline in the effective moment arm under opening action.

In general, the specimens with only two hoops within the joint region did not perform as well as those with four hoops. The importance of hoops and stirrups within the joint region can be seen from the history of transverse concrete strains recorded by the embedded strain gauges (Figure 3.22). From the plots in Figure 3.22, it is clear that significant growth of the joint core concrete occurs in the transverse direction under opening action. Providing hoops in the joint region restraints the joint core concrete expansion, which, if it were allowed to develop it would ultimately lead to joint shear failure because of reduced concrete strength in the principal compressive direction.

Failure Modes of Knee joint Specimens: Closing Action

The behaviour of the Group #1 test specimens was governed by flexural hinging of the beams and anchorage or bond deterioration of the beam top reinforcing bars.

Specimens in Group #2, #3, and #4, and specimen KJ16 in Group #5, failed under closing action.

Specimens in Group #2 experienced significant spalling in the front, back and top of the joint region. It appears that the failure mechanism was anchorage deterioration of the beam top reinforcing bars and column outer reinforcing bars.

The behaviour of specimens in Groups #3 and #4 was governed by bond splitting along the main tensile reinforcement of the beam (beam top bars).

Specimen KJ16, with headed beam and column reinforcing bars, failed due to anchorage deterioration of the beam top reinforcing bars. Significant spalling had occurred on the joint face near the beam top reinforcing bars making the welded plates at the ends of those bars visible by the end of the test (Figure 3.26).

The behaviour of specimen KJ14 was governed by column flexural hinging. Wide cracks were evident on the back of the joint near the beam face under closing action. There
were very few cracks visible near the location of the beam top reinforcing bars.

It appears that the behaviour of specimen KJ15 was governed predominantly by flexural hinging of the beam and column adjacent to the joint.

Comparing specimens KJ11 and KJ14, it was concluded that KJ14 performed much better under closing than KJ11, even though both specimens had the same shear input under closing action. As discussed earlier, it appears that the presence of the column stub in specimen KJ14 prevented the bond or anchorage deterioration at the location of the beam top reinforcing bars. This was evident from the absence of cracks at that location during testing.

Specimens of Group #4 performed worse than their counterparts from Group #3. Both groups of specimens had the same shear input to the joint under closing action. However, the group #4 specimens had a higher shear input under opening action than the Group #3 specimens. The higher shear input for the Group #4 specimens under opening action might, (as a result of more diagonal cracking within the joint), have accelerated the bond deterioration of these reinforcing bars under closing action.

In both Groups #3 and #4, specimens with four stirrups within the joint region, specimens KJ10 and KJ13, performed best in their respective groups under closing action. The additional stirrups within the joint region may have served to confine the beam top reinforcing bars thereby delaying bond deterioration of those bars.

Figure 3.26 Specimen KJ16: Final - Side Face.
CHAPTER 4

FORMULATION OF A KNEE JOINT MODEL

In this chapter, a mechanical model is formulated that describes the workings of knee joint connections, and can be used to assess quantitatively the resistance and deformability of these members. The model satisfies equilibrium and compatibility requirements and incorporates previously developed stress-strain relations describing the non linear material behaviour under mechanical load. The proposed formulation is suitable for conditions of monotonic loading on the knee joint, however, the required extensions of the model to deal with earthquake conditions would require including degrading properties in the stress-strain relationship for concrete so as to better represent the reversed cyclic response of members.

4.1 Definition of the Problem

The knee joint resists the forces at its boundary predominantly through a shear resisting mechanism. Concrete resists the principal tensile and compressive stresses generated through shear until the principal tensile stress exceeds the cracking stress of concrete. At that point, a crack forms perpendicular to the direction of principal tensile stress. Thereafter, redistribution of tensile stresses to the transverse reinforcement relieves concrete stresses in the principal tensile direction. The transverse reinforcement present within the knee joint region confines the concrete and resists the joint growth that would result from damage buildup. Evidence of joint core concrete expansion was observed in the Clarkson experimental test results (Chapter 3).

Experimental evidence (Chapters 2 and 3) has shown that bond splitting along the beam and column longitudinal reinforcing bars within the highly congested knee joint region is also typical.

A consequence of bond splitting is that additional resistance needs to be provided from the transverse reinforcement within the knee joint region to prevent the splitting crack from widening. Significant splitting crack widths would result in excessive slip of the main reinforcement and ultimately, premature bond failure of the knee joint connection.

An additional aspect of knee joint behaviour is the kinematic response of bent
longitudinal reinforcing bars anchored in the joint when bond resistance in the lead-in length is negligible. This results in additional stress to be resisted by the transverse reinforcement as the bent bar kicks outward and upwards (Figure 4.1). This condition is most likely to develop under conditions of cyclic loading when significant bond deterioration occurs along the lead-in length of the longitudinal reinforcement due to bond splitting and concrete crushing in front of the reinforcing bar lugs. This problem was alleviated in the tests performed in China (Chapter 2) when the main beam and column reinforcement was extended and anchored in the adjacent column and beam members, respectively.

Figure 4.1  Kinematics of Beam Top Reinforcing Bar: Negligible Bond Along Lead-in Length.

4.1.1  Idealization of Knee Joint Member

To reproduce the observed mechanical response of knee joints, all of the above features need be considered in formulating a behavioral model.

Figure 4.2 shows the deflected shapes of typical reinforced concrete structures subject to lateral forces that simulate earthquake loading. The behaviour of the part of the structure within the dotted circle, depicted in Figure 4.2a), was the object of the companion experimental study and the main focus of this work.
4.2a) Deflected Shape of a Reinforced Concrete Frame Building: Definition of Knee Joint Region.

4.2b) Deflected Shape of Reinforced Concrete Bridge Structures.

Figure 4.2 Deflected Shapes of Reinforced Concrete Structures Subjected to Earthquake Loads.
The knee joint connection of Figure 4.3 is a two-dimensional idealization. Also shown is an approximation of the actual pattern of in-plane loads acting on it during lateral sway. The pin and roller boundary conditions at points A and B, respectively, represent points of inflection in the actual frame structure (Figure 4.2). The location of the inflection points, usually assumed to occur near the member's midspan during lateral sway, may shift during loading due to load redistributions, particularly when considering the different degree of restraint provided to the two ends of the top-storey column. Specifically, the location of these inflection points will be different for knee joints found in reinforced concrete frame buildings compared to those found in bridge bents of multi-span bridges. The beam axial force \( N_x \), in Figure 4.3 is generated as a result of the earthquake loading from free body equilibrium.

4.1.2 Idealization of Knee Joint Connection

For the in-plane loading considered here, the knee joint is assumed to act as a two-dimensional panel loaded with shear and normal forces along its boundaries. The idealized knee joint panel with the applied boundary forces under opening and closing actions is given in Figure 4.4. In the formulation developed here, only first order effects are considered.

Unlike the interior connections, which have shear and normal forces acting on all four faces, knee joint connections have two surfaces free of external load (Figure 4.4). This detail turns out to be very significant from a modelling perspective, because in order to acknowledge this point, it is necessary to adopt a non-uniform distribution of stress within the joint panel. This renders the model much more complicated than similar formulations that have been proposed for interior joints.

4.1.3 Bond Stress Between Concrete and Longitudinal Reinforcing bars

The loads acting on the knee joint boundary described above are transferred to the joint through shear by direct member action and more predominantly by bond stresses generated along the beam and column longitudinal reinforcing bars.
Figure 4.3  Free Body Diagram of a Model Knee Joint Member.

b: Beam
c: Column
f: Face

a) Opening Action  b) Closing Action

Figure 4.4  Free Body Diagram of an Idealized Knee Joint Region.
The approach adopted here to calculate joint shear input is that specified by ACI352 (ACI352-(1985)). As discussed in Chapter 1, the horizontal joint shear force input is calculated at the joint centre by satisfying horizontal equilibrium of the forces acting at the joint boundaries (Figure 4.5). The following expressions are obtained for joint shear input under opening and closing actions, respectively:

\[
V_j^o = \bar{T}_s - V_c = C_c + C_s \\
V_j^c = \bar{T}_s = C_c + C_s - V_c
\]

In an interior joint, even if bond deterioration along tensioned straight-anchored longitudinal bars within the joint region is severe, anchorage can still be achieved on the other side of the joint in the adjacent beam member which is usually a flexural compression zone. However, when beam and column reinforcing bars are anchored within a knee joint, forces at the knee joint boundary, generated by the adjacent beam and column members, are resisted entirely by actions within the joint panel. Consequently, bond is an important consideration for these connections because in case of severe bond deterioration within the joint region, load transfer to the joint would not be possible and the knee joint connection would fail. When beam and column reinforcing bars are not anchored within the joint region, but rather extend and are anchored within the adjacent column and beam, the effects of bond deterioration within the joint region may not be as significant since load transfer may
be achieved through bond in the adjacent members.

As discussed in Section 2.2.2, Pantazopoulou and Bonacci (1992) accounted for bond in a model of interior joint mechanics by including a bond efficiency factor, $\beta$, in the horizontal equilibrium equation. With reference to Figure 4.6 (Figure 2.33(b) repeated) the following equation is obtained from satisfying equilibrium requirements of horizontal forces:

$$\sigma_x - \rho_x f_x - \frac{N_x}{bh} = 0$$  \hspace{1cm} 4.3

and,

$$\rho_x f_x = \rho_h f_h + \rho_b f_h$$  \hspace{1cm} 4.4

where; $\sigma_x$ is the average concrete stress in the x-direction, $N_x$ is the beam axial load, $f_x$ is the average horizontal reinforcement stress, $f_h$ is the average beam reinforcement stress, $f_h$ is the average hoop stress, $\rho_x$ is the percentage of total horizontal reinforcement at the column centerline (includes hoop and beam reinforcement), $\rho_h$ is the hoop reinforcement ratio, and $\rho_b$ is the beam reinforcement ratio. These ratios ($\rho_x$, $\rho_h$, and $\rho_b$), are area ratios referenced to the joint core concrete.

Recall that the bond efficiency factor relates the stresses in the beam longitudinal reinforcement and hoop reinforcement at the column centerline. Thus, introducing $\beta$ in equation 4.4 results in the following:

$$\rho_x f_x = \rho_h f_h + \beta \rho_b f_h \hspace{1cm} \text{where; } \rho_x = \rho_h + \beta \rho_b \hspace{1cm} \text{and } f_x = f_h$$  \hspace{1cm} 4.5

A bond efficiency factor will be used here to account for the effects of bond in the proposed model. Unlike the interior joint model, which due to the inherent symmetry in the bond conditions of the beam top and bottom longitudinal reinforcements, incorporated only one bond efficiency factor, the knee joint model requires different bond efficiency factors for each layer of beam and column reinforcing bars under both opening and closing actions. Note that this correlation is incorporated in both horizontal and vertical panel equilibrium equations.
4.1.4 **Relationship Between Joint Shear Force \( V_j \) and Internal Beam Axial Load \( N_x \)**

In interior beam column connections of statically indeterminate structures, incremental internal axial forces in the beam members are generated when adjacent columns restrain the expansion of the beam elements. These expansions are a result of deformations in the joint and beam flexural hinge regions due to the applied earthquake loading. Thus, there is no direct link between the internal beam axial force and the horizontal joint shear input.

For knee joint connections, there is a direct relationship between the internal beam axial force and the horizontal joint shear input because the magnitude of the internal beam axial force is directly related to the earthquake loading that causes lateral swaying. From a modeling perspective, it would be beneficial if the joint shear input could be computed directly from the beam axial force.

In this thesis, such a relationship is obtained through simple algebraic manipulations of the global and sectional equilibrium equations of the knee joint panel for monotonic loading. For earthquake loading, however, a relationship between the horizontal joint shear input and beam axial force is difficult to obtain since the states of stress in the beam reinforcing bars depend on their load-deformation history.
Additional equilibrium requirements that are unique to knee joints are that the beam axial force is equal to the column shear force and the column axial force is equal to the beam shear force (see Figure 4.4). These will be useful in the model formulation.

4.1.5 Stresses and Strains Within Knee Joint Region

Assuming that the reinforcing bar layout within the joint region provides adequate crack control, average stresses and strains can be used in the formulation of the knee joint model to represent the states of stress and strain. Average strains are also used in the formulation to satisfy the requirements of compatibility (Collins 1978) and hence enable the use of the general laws of coordinate transformation for a second order tensor.

4.1.6 Coordinate System to Investigate Average Stresses and Strains

Principal tensile and compressive directions of the material stress tensor are chosen to investigate the average stresses and average strains within the knee joint region.

The principal axes are chosen as the coordinate system because the computed principal stress magnitudes can be directly compared with material strengths obtained from simple laboratory tests.

In addition, a direct relationship between the average strains and average stresses for concrete can be established in the principal axes coordinate system using appropriate models describing the nonlinear behaviour of concrete (Collins 1978).

4.1.7 Effects of Transverse Reinforcement Within Knee Joint Region

Transverse reinforcement within the knee joint region is assumed here to serve the dual role of resisting shear parallel to its orientation within the joint and helping to maintain the strength of the diagonal compressive strut by passively confining the joint core concrete.
4.2 Mechanics of Knee Joints

4.2.1 Equilibrium Requirements

The relationship between the joint shear force input ($V_j$) and the beam axial force ($N_x$) is obtained from requirements of global and sectional equilibrium for the idealized knee joint member. The relationship between the beam axial force and column axial force is obtained from the global equilibrium requirements. The remaining equilibrium expressions required are obtained from the equilibrium requirements of the knee joint panel.

4.2.1.1 Global Equilibrium Requirements

An expression relating the column axial force to the beam axial force is obtained from Figure 4.3 by summing the moments about point A and requiring that the moment residual be zero. Hence, the axial force in the column is related to that of the beam by:

$$N_y = \frac{l_c}{l_b} N_x$$

where; $l_c$ and $l_b$ are the beam and column lengths, respectively, from the inflection points to the joint centerline.

The moments in the beam and column at the corresponding joint interfaces are computed as (Figures 4.3 and 4.4):

For the beam member moment:

$$M_{bf} = l_{bf} N_y$$

By substituting Equation 4.6, Equation 4.7 is written in terms of the beam axial force:

$$M_{bf} = \frac{l_c}{l_b} l_{bf} N_x$$

For the column member moment:

$$M_{cf} = l_{cf} N_x$$
where; $l_{br}$ and $l_{cf}$ are the beam and column lengths measured from the assumed inflection points to the beam-joint interface and column-joint interface, respectively, and $M_{cf}$ is the column moment at the column-joint interface.

### 4.2.1.2 Sectional Equilibrium Requirements

The horizontal joint shear input under opening or closing action for a given axial load can be obtained as follows:

First, given the beam member axial load, the corresponding moment at the joint face can be obtained through global equilibrium requirements (Equation 4.8). Next, a beam sectional analysis for the given combination of axial load and moment is performed to obtain the corresponding beam reinforcing bar strains and stresses. Finally, Equations 4.1 or 4.2 are used to calculate the joint shear input under opening or closing actions, respectively.

**Stress-Strain Models**

1) **Reinforcement.**

   A bi-linear stress-strain relationship is used to model the longitudinal beam and column reinforcing bars (figure 4.7) and is given by:

   $f_{sr} = E_s \varepsilon_s ; \quad |\varepsilon_s| \leq |\varepsilon_{yr}|$ \hspace{1cm} 4.10

   $f_{sr} = f_y + E_{sh} (\varepsilon_s - \varepsilon_{yr}) ; \quad |\varepsilon_s| > |\varepsilon_{yr}|$  \hspace{1cm} 4.11

   where; $f_{sr}$ is the reinforcement stress, $f_y$ is the reinforcement yield, $E_{sh}$ is the strain hardening modulus, and $\varepsilon_{yr}$ is the reinforcement yield strain.

2) **Concrete in compression.**

   The two approaches considered when modeling the stress-strain behaviour of confined concrete were from Sheikh and Uzumeri (1982), and Park et al (1982).

   The modified Kent and Park model (Park et al 1982) is used to model the stress strain behaviour for unconfined and confined concrete in compression because of the ease of its implementation in the knee joint model (Figure 4.8). Tension stiffening effects and compression softening effects are ignored.
Figure 4.7  Stress-Strain Relationship for Reinforcing Bars in Tension and Compression.

Figure 4.8  Modified Kent-Park Stress-Strain Relationship for Confined Concrete.
For concrete compressive strains up to and including strain at peak concrete strength, i.e.,

\[ \varepsilon_c \leq k_c \varepsilon_o \]  \hspace{1cm} 4.12

\[ f_c = k_c f'_c \left[ \frac{2\varepsilon_c}{k_c \varepsilon_o} - \left( \frac{\varepsilon_c}{k_c \varepsilon_o} \right)^2 \right] \]  \hspace{1cm} 4.13

where; \( \varepsilon_o \) is the concrete strain at peak uniaxial concrete strain, \( k_c \) is a coefficient that accounts for the beneficial effects of confinement on strength, and is defined by,

\[ k_c = 1 + k_e \rho_{vh} \left| \frac{f_{yh}}{f'_c} \right| \]  \hspace{1cm} 4.14

and, \( \varepsilon_c \) is the concrete compressive strain, \( \rho_{vh} \) is the volumetric ratio of hoop reinforcement with reference to the concrete core measured to the outside of the hoop, and \( f_{yh} \) is the hoop reinforcing bar yield stress. Parameter \( k_c \) is a geometric variable that characterizes the effectiveness of the specific arrangement of confining reinforcement and is given by (Mander et al 1988):

\[ k_c = \left( 1 - \sum_{i=1}^{n} \frac{(w_i)^2}{6b_{jc}d_{jc}} \right) \left( 1 - \frac{s'}{2b_{jc}} \right) \left( 1 - \frac{s'}{2d_{jc}} \right) \]  \hspace{1cm} 4.15

where; \( w_i \) is the clear spacing between longitudinal reinforcement, \( b_{jc} \) and \( d_{jc} \) are the joint core dimensions to centerlines of perimeter hoop in the plane of the core cross section, respectively, \( s' \) is the clear spacing between the hoop reinforcement in the joint region, and \( \rho_{cc} \) is the ratio of area of longitudinal steel to area of core of section.
For concrete strains greater than the strain at peak concrete strength, i.e.,

\[ \varepsilon_c > k_c \varepsilon_o \]  

\[ f_c = k_c f'_c \left[ 1 - z_m (\varepsilon_c - k_c \varepsilon_o) \right] \geq 0.2 k f'_c \]

where; \( z_m \) represents the slope of the post-peak branch of the stress-strain curve, and is calculated by the geometry of the curve (Figure 4.7) as:

\[ z_m = \frac{0.5}{\varepsilon_{50u} + \varepsilon_{50h} - k_c \varepsilon_o} \]

\[ \varepsilon_{50u} = \frac{3 + \varepsilon_o f'_c}{f'_c - 1000} \]

\[ \varepsilon_{50h} = 0.75 \rho_{vh} \sqrt{\frac{h_{cc}}{s}} \]

where; \( h_{cc} \) is the width of the concrete core measured to the outside of the hoops, \( s \) is the centre to centre spacing of the hoops. Concrete within the hoops or stirrups is assumed confined while concrete outside this volume is assumed unconfined.

To obtain the joint shear input that arises from conditions of cyclic loading, the beam axial force and deformation history of the beam reinforcing bars at the joint face are required. An appropriate stress-strain model is used to obtain the stress in the reinforcing bars for their current deformation state. Equations 4.1 and 4.2 are then used to calculate the joint shear inputs under opening and closing actions, respectively.
4.2.1.3 Equilibrium Requirements of Knee joint Panel

The joint geometry used in the model formulation is defined in Figure 4.9. The assumed stress distributions for the knee joint are shown in Figure 4.10. Assumed stresses are shown for vertical and horizontal cuts through the joint centre. The beam and column shear stresses ($v_b$ and $v_c$) are assumed to be uniformly distributed across the knee joint panel boundaries. The concrete normal stresses and joint shear stress are assumed to vary linearly within the knee joint panel in both the horizontal and vertical directions.

As done in the model for interior joints (Pantazopoulou and Bonacci, 1992), the stresses in the hoop and beam reinforcing bars ($x$-direction) at the joint centre, are coupled in the horizontal equilibrium Equation of the knee joint model. Here, the stresses in the stirrup and column reinforcing bars ($y$-direction) at the joint centre are also coupled in the vertical equilibrium equation of the knee joint model. The average hoop and stirrup stresses are taken as the values occurring at the joint centre in the horizontal and vertical directions, respectively.

The assumed distribution of stresses are a departure from the interior joint model. It was mentioned earlier that this is primarily because of the non-symmetrical loading conditions within the knee joint, and specifically because the internal stress state must meet the equilibrium requirements at the loaded faces, and vanish at the free (top and back) faces of the joint. Naturally, this may only be achieved if very steep stress gradients develop.

Figure 4.9 Knee joint Geometry.
4.10a) Internal Stresses in Knee Joint Panel for Vertical Cut at Joint Centre (Vertical Stresses not Shown).

4.10b) Internal Stresses in Knee Joint Panel for Horizontal Cut at Joint Centre (Horizontal stresses not Shown).

Figure 4.10  Free Body Diagram of Internal Stresses in Knee Joint Panel.
within the relatively small and geometrically complex region of the joint. Due to the limitations of the knee joint model, the stress gradients assumed across the knee joint can not be accounted for in the formulation.

**Horizontal Equilibrium of Internal Stresses in Knee Joint Panel**

Horizontal equilibrium of the internal stress (shown in Figure 4.10a) is as follows:

\[ \rho_{bt} f_{bt} + \rho_{bb} f_{bb} + \rho_h f_h + \sigma_x - v_c \frac{d}{2h} = 0 \]  \[ \text{4.21} \]

\[ N_x = - v_c b d \]  \[ \text{4.22} \]

where; \( \rho_{bt}, \rho_{bb}, \) and \( \rho_h \) are the reinforcing bar ratios in the x direction for the beam top, beam bottom, and hoop reinforcement, respectively, \( f_{bt}, f_{bb}\) and \( f_h \) are the average reinforcing bar stresses in the x direction at the column centerline for the beam top, beam bottom and hoop reinforcing bars, respectively, \( \sigma_x \) is the average concrete normal stress in the x direction at the joint centre and \( v_c \) is the average column shear stress.

**Vertical Equilibrium of Internal Stresses in Knee Joint Panel**

Vertical equilibrium of the internal stresses shown in Figure 4.10b is given by:

\[ \rho_{co} f_{co} + \rho_{ci} f_{ci} + \rho_{cm} f_{cmm} + \rho_s f_s + \sigma_y - v_b \frac{h}{2d} = 0 \]  \[ \text{4.23} \]

\[ N_y = - v_b b h \]  \[ \text{4.24} \]

where; \( \rho_{co}, \rho_{ci}, \rho_{cm} \) and \( \rho_s \) are the reinforcing bar ratios in the y direction for column outer, column inner, column middle and stirrup reinforcing bars, respectively, \( f_{co}, f_{ci}, f_{cmm}, \) and \( f_s \) are the average reinforcing bar stresses in the y direction at the beam centerline for the column outer, column inner, column middle and stirrup reinforcing bars, respectively, \( \sigma_y \) is
the average concrete normal strain in the y-direction at the joint centre, and \( v_y \) is the average beam shear stress.

The above equilibrium expressions relate average stresses for concrete and reinforcing bars in the vertical and horizontal directions at the joint centre. The average concrete stresses at the joint centre represent a second-order tensor, and hence, by coordinate transformation stress equilibrium equations may be written directly in the system of principal coordinates. The second-order stress tensor with reference to the global system of coordinates, \( x, y, z \), is given below:

\[
\sigma = \begin{bmatrix}
\sigma_x & v_y & 0 \\
v_y & \sigma_y & 0 \\
0 & 0 & \sigma_z
\end{bmatrix}
\]

where; \( \sigma_z \) is the average normal stress in the z direction (i.e. normal to the plane of action) at the joint centre. As discussed earlier, loading is assumed to act in the x-y plane. It is also assumed that the concrete has no residual tensile strength and thus the maximum principal stress does not exceed zero, i.e.,

\[
\sigma_1 = 0
\]

This requires that,

\[
\sigma_y = -\frac{v_y}{\tan\theta}
\]

\[
\sigma_x = -v_y \tan\theta
\]

\[
\sigma_2 = \sigma_x + \sigma_y = -v_y \left(\tan\theta + \frac{1}{\tan\theta}\right)
\]
where; \( \theta \) is the angle which defines the orientation of the principal tensile stress with respect to the horizontal (x) axis, \( \sigma_1 \) is the magnitude of the principal tensile stress in the concrete, and \( \sigma_2 \) is the magnitude of the principal compressive stress in the concrete.

### 4.2.1.4 Modelling the Effects of Bond

The objective of this formulation is to describe the mechanics of knee joint behaviour using a simple conceptual idealization. To maintain a compatible level of complexity throughout the model, the stress transfer occurring over the bar from the critical section to the joint centre was represented here by an averaging constant, \( \beta \).

A similar approach was used in the interior joint model (Pantazopoulou and Bonacci 1992). Parameter \( \beta \), referred to in the remainder as the bond efficiency factor, is incorporated in the equilibrium equations. As described earlier, \( \beta \) is the ratio of average beam or column reinforcing bar stress to average stirrup or hoop stress at the column or beam centerlines, respectively. Accordingly, Equations 4.21 and 4.23 for horizontal and vertical stress equilibrium in the joint may be rewritten as:

**Horizontal Equilibrium**

\[
\rho_x f_x + \sigma_x - \nu_c \frac{d}{2h} = 0
\]

where;

\[
\rho_x = \rho_h + \beta_{bt} \rho_{bt} + \beta_{bb} \rho_{bb} \quad \text{and} \quad f_x = f_h
\]

**Vertical Equilibrium**

\[
\rho_y f_y + \sigma_y - \nu_b \frac{h}{2d} = 0
\]

where;

\[
\rho_y = \rho_s + \beta_{co} \rho_{co} + \beta_{ct} \rho_{ct} + \beta_{cm} \rho_{cm} \quad \text{and} \quad f_y = f_s
\]
\( \rho_y \) is the per cent of total vertical reinforcement at the joint centre (includes stirrup and column reinforcement).

Subscripts bt, bb, co, ci and cm in Equations 4.31 and 4.33 refer to beam top, beam bottom, column outer, column inner and column middle reinforcing bar layers, respectively. \( \beta_{cm} \) in Equation 4.30 is taken as the average of \( \beta_{co} \) and \( \beta_{ci} \).

Because the loading conditions for knee joints are non-symmetric, the four bond efficiency factors (\( \beta_{bt}, \beta_{bb}, \beta_{co} \) and \( \beta_{ci} \)) need to be defined under both opening and closing actions. As in the interior joint model (Pantazopoulos and Bonacci, 1992), a value of 0 for \( \beta \) will represent perfect bond. A value of 1 for \( \beta \) will indicate poor bond.

### 4.2.2 Kinematic Assumptions

The assumed deflected shape for the knee joint after loading is shown in Figure 4.11. Average values are assumed for horizontal strain (\( \epsilon_x \)), vertical strain (\( \epsilon_y \)), and shear distortion (\( \gamma \)).

The average deformation components for the kinematics of the knee joint represent a second-order tensor given by:

\[
\varepsilon = \begin{bmatrix}
\epsilon_x & \frac{\gamma}{2} & 0 \\
\frac{\gamma}{2} & \epsilon_y & 0 \\
0 & 0 & \epsilon_z
\end{bmatrix}
\]

where; \( \epsilon_x, \epsilon_y \), and \( \epsilon_z \) are the average concrete longitudinal strains in the x, y and z directions respectively, at the joint centre, \( \gamma/2 \) is the average concrete shear strain in the x-y plane at the joint centre, and \( \gamma \) is the shear distortion of the panel. The equations of coordinate transformation for a second order tensor are used to relate the concrete shear and normal strains in the x, y and z coordinate system to the principal strains \( \epsilon_1 \) and \( \epsilon_2 \) (notation here: \( \epsilon_1 > \epsilon_2 \)).

i.e.,
\[ \varepsilon_1 + \varepsilon_2 = \varepsilon_y + \varepsilon_x \]  

\[ \gamma = \frac{2 (\varepsilon_1 - \varepsilon_x)}{\tan \alpha} = 2 (\varepsilon_1 - \varepsilon_y) \tan \alpha \]  

\[ \tan^2 \alpha = \frac{\varepsilon_1 - \varepsilon_x}{\varepsilon_2 - \varepsilon_y} = \frac{\varepsilon_2 - \varepsilon_y}{\varepsilon_1 - \varepsilon_x} \]  

where; \( \alpha \) is the angle which defines the orientation of the principal tensile strain with respect to the horizontal axis.

Figure 4.11  Knee Joint Kinematics.
4.2.3 Non linear Models of Stress-Strain Response for Steel and Concrete

4.2.3.1 Stress-Strain Relationships for Hoop and Stirrup Reinforcing bars

The stress-strain relationship for the hoop and stirrup reinforcing bars is assumed as elastic-perfectly plastic in tension and compression (see Figure 4.12). For reinforcing bars in the $x$ (hoop) and $y$ (stirrup) directions the stress-strain relationship is given as:

$$f_s = E_s \varepsilon_s, \text{ and } |f_s| \leq |f_{sy}|$$

4.38

Figure 4.12  Stress-Strain Relationship for Hoop and Stirrup Reinforcing Bars in Tension and Compression.
4.2.3.2 Stress-Strain Relationships for Concrete in Principal Coordinates

Two approaches are considered for modelling the stress-strain relationships for concrete in the principal axes coordinate system. The difference in each approach stems from how each model deals with compression softening in the principal compressive stress direction.

In the first approach, the tensile strength of concrete in the principal tensile stress direction ($\sigma_1$) is assumed to be zero, i.e.,

$$\sigma_1 = 0$$  \hspace{1cm} 4.39

The concrete principal compressive stress, $\sigma_2$, is limited by the crushing strength of concrete. The non linear stress-strain relationship for the joint core concrete is modelled by a compression softening model (Vecchio and Collins, 1986), which accounts for the detrimental effects of the principal tensile strains on the concrete compressive strength in the principal compressive stress direction, and a confinement model (Kent and Park, 1982) which accounts for the beneficial effects of passive confinement provided by hoops and stirrups on the concrete strength in the principal compressive strut within the joint.

The relationship between stress and strain in the principal compressive direction is taken as (Figure 4.13):

$$\sigma_2 = E_c \varepsilon_2$$  \hspace{1cm} 4.40

and,

$$\sigma_2 = f_{cm} \left[ 2 \left( \frac{\varepsilon_2}{\varepsilon_{cm}} \right) - \left( \frac{\varepsilon_2}{\varepsilon_{cm}} \right)^2 \right]$$  \hspace{1cm} 4.41
where:

\[ f_{cm} = \lambda f'_c \] 

4.41a

\[ \varepsilon_{cm} = \lambda \varepsilon_o \] 

4.41b

\[
\lambda = \frac{1 + k_e \rho_{vh} \left| \frac{f_{yh}}{f'_c} \right|}{0.8 - 0.34 \left( \frac{\varepsilon_1}{\varepsilon_0} \right)} \quad \text{and} \quad 0.8 - 0.34 \left( \frac{\varepsilon_1}{\varepsilon_0} \right) \geq 1
\]

4.41c

In Equation 4.41a, \( \lambda \) accounts for the effects of compression softening and confinement on the concrete compressive strength in the principal compressive stress direction, \( \varepsilon_{cm} \) is the strain at peak stress of confined concrete, and \( f_{cm} \) is the peak strength of concrete. The numerator in Equation 4.41c accounts for the beneficial effects of confinement on concrete strength and was defined in Equation 4.14 (Section 4.2.1.2). The denominator in Equation 4.41c accounts for the detrimental effects of principal tensile strain (\( \varepsilon_1 \)) on concrete strength.

![Figure 4.13](image)

Figure 4.13   Stress-Strain Relationship for Concrete in Principal Compressive Stress Direction.
The concrete strength in the principal compressive stress direction is reached when the principal compressive strain ($\varepsilon_2$) has attained the crushing strain value, i.e.,

$$\varepsilon_2 = \varepsilon_{cm} \quad 4.42$$

The same model above is considered with the exception that the weakening influence of the out-of-plane tensile strain ($\varepsilon_3$) is taken into account in the compression softening model (Kirschner and Collins, 1986). This is done by replacing the principal tensile strain ($\varepsilon_1$) in Equation 4.39c with:

$$\varepsilon_T = \sqrt{\varepsilon_1^2 + \varepsilon_3^2} \quad 4.43$$

The second approach considered for modeling the stress-strain relationship for concrete in the principal axes coordinate system, is from the work of Pantazopoulou and Mills (1994). Pantazopoulou and Mills (1994) derived a simplified constitutive model which accounts for the influence of the physical properties of the concrete material microstructure on the concrete's mechanical response.

A strain-dependent estimate of the material stiffness, represented by the secant modulus of elasticity, is used in the model to evaluate the compressive stress ($\sigma_2$) at a given compressive strain ($\varepsilon_2$).

The secant stiffness ($E_s$) represents the resistance to mechanical loading in concrete and depends on the physical properties of the concrete compressive strut. The difference in the initial secant modulus of uncracked concrete for different concretes is measured by the varying degrees of porosity of each concrete. Porosity is a good measure of resistance because it is an indicator of the voids present in the solid mass of concrete which ultimately affects its mechanical properties ($E_c$). For example, $E_c$ decreases with an increase in the volume of voids in the microstructure. Voids result naturally from the properties of the concrete mix (such as capillary pores) or are induced mechanically from loading (damage build-up). In the model it is assumed that all types of voids have the same influence on the
mechanical properties of concrete regardless of whether they were generated naturally or mechanically.

Based on the model, concrete stress $\sigma_2$ in the principal direction is given as:

$$\sigma_2 = E_c \varepsilon_2$$  \hspace{1cm} (4.44)

and,

$$E_c = E_o F$$  \hspace{1cm} (4.45)

where; $E_o$ is the initial uncracked tangent modulus for concrete, $F$ is a damage indicator and is given as a function of the capillary porosity, which is an indicator of the initial area of voids before mechanical loading, and the area strain ($\varepsilon_A$), which represents the increase in voids due to mechanical loading in the cross section supporting the load.

The expression for the damage indicator $F$, is given as:

$$F = \frac{1}{1 + \frac{\varepsilon_A}{\beta_m}}$$  \hspace{1cm} (4.46)

where; $\beta_m$ is a material variable which can be approximated as $\varepsilon_o$ from a simple calibration of tests.

To illustrate this definition of area strain, volumetric strain ($\varepsilon_v$) is plotted against principal compressive strain ($\varepsilon_2$) for an axially loaded concrete cylinder with and without confinement in Figure 4.14. Negative volumetric strain indicates volumetric contraction whereas positive volumetric strain indicates volumetric expansion. Reversal from volumetric contraction to volumetric expansion occurs at the post peak conditions of the stress-strain response of the concrete cylinder. This is a critical milestone in the behaviour of the concrete since it corresponds to a significant drop in the resistance ($f_c$) of the concrete and is identified by uncontrolled crack propagation and growth. As can be seen in the figure, the beneficial effects of confining the concrete are displayed by a delay in the occurrence of unstable crack propagation as well as a higher resistance and deformability in the concrete.
Figure 4.14  Volumetric Strain-Principal Compressive Strain (Pantazopoulou and Mills 1994).

The area strain, which quantifies the extent of damage in concrete, is defined geometrically by the coordinate bound between the $\varepsilon_\nu - \varepsilon_2$ curve and 1:1 line. The line $\varepsilon_\nu = (1-2\nu)\varepsilon_2$ represents the linear elastic response of the concrete, and $\nu$ is concrete's poisson ratio. From the figure, the confined curve is seen to follow the linear elastic response up until $\varepsilon_{2l}$ (onset of cracking in concrete), where the curve begins to depart from the linear elastic response. Thereafter, the area strain increases at a quicker rate. $\varepsilon_{2r}$ in Figure 4.14 represents the principal compressive strain at the point of zero volumetric strain.

The following relationships are used to define the principal tensile strain in the out-of-plane direction ($\varepsilon_3$). First, the beneficial effect of lateral confinement on the uniaxial strength of concrete are estimated using the following empirical equation (Richart et al 1928):

$$f'_{cc} = f'_c + 4.1\sigma_{lat} \tag{4.47}$$

where; $f'_{cc}$ is the peak compressive strength of confined cylinders, and $\sigma_{lat}$ is the lateral confining pressure.

For the reinforced concrete knee joint considered here the transverse reinforcement provides passive confinement to the joint core, defined as:
\[ \sigma_{\text{lat}} = \frac{k_e f_y A_t n_t}{bs_i} \]

(4.48)

where; \( A_t \) is the area of the transverse reinforcement, \( n_t \) is the number of transverse reinforcing bar legs in the joint region, and \( k_e \) was defined in Equation 4.15 in Section 4.2.1.2.

Richart et al (1928) gave the following relationship for the peak strength of the confined concrete cylinders, \( f_{cc} \) and corresponding strain, \( \varepsilon'_{cc} \):

\[ \varepsilon'_{cc} = \varepsilon_0 \left( \frac{f'_{cc}}{f'_c} - \zeta_2 \right) \]

(4.49)

where; \( \varepsilon'_{cc} \) is the concrete strain corresponding to peak concrete stress and \( \zeta_1 \) and \( \zeta_2 \) are empirical coefficients.

Imran and Pantazopoulou (1996) found from tests on cylinders subjected to triaxial states of stress that values of \( \zeta_1 = 6 \) and \( \zeta_2 = 0.83 \) result in good correlation of their results with Equation 4.49. They also obtained the following empirical equation which relates the concrete volumetric strain \( (\varepsilon_v) \) to the principal compressive strain \( (\varepsilon_2) \) and lateral confining pressure \( (\sigma_{\text{lat}}) \):

\[ \varepsilon_v = (1 - 2v) \frac{2\sigma_{\text{lat}}}{E_c} + (1 - 2v) \varepsilon_2 - \varepsilon'_{cc} \left[ \frac{\varepsilon_2 - \varepsilon_{cr}}{\varepsilon'_{cc} - \varepsilon_{cr}} \right]^2 \]

(4.50)

where; \( \varepsilon_{cr} \) is the cracking strain of concrete and \( v \) is the poisson's ratio for concrete and varies between 0.15 and 0.20.

To obtain a relationship for the out-of-plane principal strain \( (\varepsilon_3) \) Equations 4.47 through 4.49 are substituted into Equation 4.50 which is then substituted into 4.51 below given for the definition of area strain \( (\varepsilon_A) \). i.e.,

\[ \varepsilon_A = \varepsilon_v - \varepsilon_2 = \varepsilon_1 + \varepsilon_3 = 2\varepsilon_3 \quad \rightarrow \quad \varepsilon_3 = \frac{\varepsilon_v - \varepsilon_2}{2} \]

(4.51)
4.2.4 Summary

The equilibrium compatibility and stress-strain equations discussed in Sections 4.2.1, 4.2.2, and 4.2.3, respectively, collectively describe the mechanical behaviour of the knee joint panel. Simple algebraic manipulations will be performed on these equations to describe the behavioural response of knee joints at critical milestones.

In the interior joint model, which is used here as basic reference, the critical milestones for the joint were identified as: hoop yielding, column reinforcing bar yielding, and principal diagonal compressive strut crushing (Pantazopoulou and Bonacci (1992)). The formulation of the model was based on expressions describing the critical milestones. The possible failure mechanisms of the interior joint, precluding any bond type failures were: hoop yielding followed by either column reinforcing bar yielding or concrete crushing, column reinforcing bar yielding followed by either hoop yielding or concrete crushing, and concrete crushing.

Precluding any bond type failures, the critical milestones for the knee joint can be identified as: hoop yielding, stirrup yielding, and principal diagonal compressive strut crushing. Possible failure modes for the knee joint are: hoop yielding followed by either stirrup (or column reinforcement if stirrups are not present) yielding or concrete crushing, stirrup yielding followed by either hoop yielding or concrete crushing. As in the interior joint model, the critical milestone of hoop or stirrup yielding will be the basis for the formulation of the knee joint model.
4.2.5 Knee Joint Behaviour Before Yielding of Hoops or Stirrups

Because of the assumptions of smeared stress and strain in concrete, it is possible to calculate the orientation of principal axes with reference to an orthogonal system x-y using the tensor transformation relations. Furthermore, assuming that the directions of principal stress and strain coincide up to the point of reinforcement yielding, i.e.,

$$\theta = \alpha$$  \hspace{1cm} 4.52

the directions of the two systems of principal axes, namely that of stress, $\theta$, and that of strain, $\alpha$, is calculated from:

$$\tan^2 \theta = \tan^2 \alpha = \frac{(\varepsilon_1 - \varepsilon_x)}{(\varepsilon_1 - \varepsilon_y)} = \frac{(\varepsilon_2 - \varepsilon_y)}{(\varepsilon_2 - \varepsilon_x)}$$  \hspace{1cm} 4.53

For behaviour prior to yielding of steel, and assuming orthogonally reinforced panels, it follows that:

$$\tan^2 \theta = \left( \frac{\sigma_2}{E_c} \frac{1}{(\varepsilon)} - \frac{f_y}{E_s} \right) \left( \frac{\sigma_2}{E_c} \frac{1}{(\varepsilon)} - \frac{f_x}{E_s} \right)$$  \hspace{1cm} 4.54

In arriving at Equation 4.54, it was assumed that the concrete strains and transverse reinforcing bar strains at the joint centre were equal in the horizontal and vertical directions.

Expressions for the hoop and stirrup reinforcing bar stresses in the x and y direction, $f_x$ and $f_y$ respectively, found in Equation 4.54 are obtained in terms of the joint shear stress and angle of principal stresses as follows:

By substituting Equations 4.22 and 4.28, Equation 4.30 for the average hoop reinforcing bar stress in the x-direction can be written as:

$$f_x = \frac{1}{\rho_x} (v_j \tan \theta - \frac{N_x}{2bh})$$  \hspace{1cm} 4.55
Similarly, by substituting Equations 4.6, 4.24, and 4.27, Equation 4.32 for the average stirrup reinforcing bar stress in the y-direction, can be written as:

\[ f_y = \frac{1}{\rho_y} \left( \frac{v_j}{\tan \theta} - \frac{N_x}{2bd \ l_b} \right) \]  

Equation 4.32

Equations 4.55 and 4.56, are then solved for the joint shear stress:

\[ v_j = \frac{1}{\tan \theta} \left( \rho_x f_x + \frac{N_x}{2bh} \right) \]  

Equation 4.57

\[ v_j = \tan \theta \left( \rho_y f_y + \frac{N_x}{2bd \ l_b} \right) \]  

Equation 4.58

The objective is to evaluate the angle \( \theta \) in terms of deformation, so as to enable numerical evaluation of the relationship between imposed joint distortion and shear stress, in analogy to the previous work. To achieve this, a series of algebraic manipulations are carried out between the terms of the equilibrium and compatibility relations and the tensorial properties given in the preceding. Hence, Equations 4.55, 4.56 and 4.29 are substituted into Equation 4.54. The resulting expression for \( \tan^2 \theta \) is expressed in terms of the joint shear stress. Next, Equation 4.57 is substituted for the joint shear stress. After some simple algebraic manipulations a fourth order polynomial for \( \tan \theta \) is obtained:

\[
\tan^4 \theta \left[ \frac{1 + \frac{1}{n \rho_x} \left( \frac{N_x}{2bhE_x} \right)}{1 + \frac{1}{n \rho_y}} \right] + \tan^2 \theta \left[ \frac{\left( \frac{N_x}{2bd} \right) \left( \frac{\nu_c}{\nu_b} \right) \left( \frac{1}{E_y \rho_y} \right)}{\left( \frac{n \rho_x \nu_x}{2bhE_c} + \frac{N_x}{2bhE_c} \right) \left( 1 + \frac{1}{n \rho_y} \right)} \right] - 1 = 0
\]  

Equation 4.59

where; \( n \) is the modular ratio \( (E_y/E_x) \).

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4.2.5.1 Solution Strategy: Before Yielding of Hoop or Stirrup Reinforcement.

The following solution strategy is used to obtain the important knee joint parameters which describe the behaviour of the joint prior to hoop or stirrup yielding. To facilitate this process, the calculations are performed using a spreadsheet program. Given the geometry of the knee joint and the mechanical properties of the reinforcement and concrete the solution strategy is as follows:

1. Select a value for the hoop strain ($\varepsilon_h$)
   
   The starting value should be small (corresponding to pre-yielding response).
   
   In addition, small increments are selected for $\varepsilon_x$ to avoid iteration for the concrete secant modulus ($E_s$). The value of the tangent modulus of concrete ($E_t = 2f_c/\varepsilon_0$) is assigned to $E_c$ for the first run.

2. Assume a value for the beam axial force ($N_x$).
   
   The positive convention for the beam axial force is shown in Figure 4.10 ($N_x$ is positive, i.e., compression, under closing action; and $N_x$ is negative, i.e., tensile under opening action).

   
   The convention for $\theta$ is positive under closing action and negative under opening action. Values of $\theta$ are guessed until the residual of the polynomial for $\tan\theta$ equals zero (Equation 4.59).

4. Calculate the joint shear stress ($v_j$) using Equations 4.38 and 4.57. i.e.,

$$v_j = \frac{1}{\tan\theta} \left( \rho_x E_s \varepsilon_x + \frac{N_x}{2bh} \right)$$

The convention for $v$ is positive under closing action, and negative under opening action.
5. Perform a beam sectional analysis at the joint face to establish the normal forces associated with flexure which are needed to obtain the joint shear input according to the ACI352(1985) approach.

   For the assumed value of $N_x$ in Step 2, calculate the beam member moment at the joint face using Equation 4.7. Next, perform a beam sectional analysis for the given combination of axial load and moment to obtain the beam reinforcing bar strains and stresses. The mechanical properties used for the concrete and reinforcement were described in Section 4.2.1.2. Finally, Equations 4.1 or 4.2 are used to obtain the joint shear input under opening or closing actions, respectively. The joint shear stress is obtained by dividing the joint shear force input by the effective joint area.

6. If the joint shear stress obtained from Step 5 does not equal that obtained from Step 4, a new estimate of the beam member axial force is made (Step 2) and Steps 3 through 5 are repeated. An acceptable value for the shear stress in Step 6 is obtained if it varies by less than or equal to plus or minus 5 per cent of the joint shear stress obtained in Step 4.

7. Calculate the stirrup strain ($\varepsilon_y$) using Equations 4.38 and 4.56. i.e.,

\[
\varepsilon_y = \frac{1}{E_s\rho_y} \left( \frac{v_j}{\tan \theta} - \frac{N_x l_c}{2bdl_c} \right)
\]

4.61

Check that the stirrup strain is less than or equal to the yield strain. If the yield strain has been exceeded, a lower value of $\varepsilon_y$ is selected (Step 1) and Steps 2 through 7 are repeated.
8. Calculate the principal tensile and compressive strains, $\varepsilon_1$ and $\varepsilon_2$, and the joint shear distortion ($\gamma$).

From Equations 4.53, 4.60 and 4.61, $\varepsilon_1$ and $\varepsilon_2$ are evaluated as:

$$\varepsilon_1 = \frac{1}{E_s(1 - \tan^2\theta)} \left[ \nu \tan \left( \frac{\rho_y - \rho_x}{\rho_x \rho_y} \right) \frac{N_x}{2bh\rho_x} + \frac{N_x}{2bd\rho_y l_b} \right]$$  \hspace{1cm} 4.62

$$\varepsilon_2 = \frac{1}{E_s(1 - \tan^2\theta)} \left[ \nu \left( \frac{1}{\rho_y \tan \theta} - \frac{\tan^3 \theta}{\rho_x} \right) - \frac{N_x}{2bd \rho_y l_b} \frac{l_c}{2bh \rho_x} \right]$$  \hspace{1cm} 4.63

$\gamma$ is evaluated as, from Equations 4.36, 4.60 and 4.62:

$$\gamma = \frac{2}{E_s(1 - \tan^2\theta)} \left[ \nu \frac{1}{\rho_x \rho_y} (\rho_y \tan^2 \theta - \rho_x) + N_x \tan \theta \left( \frac{1}{2bd \rho_y l_b} - \frac{1}{2bh \rho_x} \right) \right]$$  \hspace{1cm} 4.64

9. Calculate $\lambda$ from Equations 4.41c and 4.62. i.e.,

$$\lambda = \frac{1 + k_e \rho_{vh} \left| \frac{f_yh}{f_c} \right|}{0.8 - 0.34 \left( \frac{\varepsilon_1}{\varepsilon_o} \right)} \text{ and } 0.8 - 0.34 \left( \frac{\varepsilon_1}{\varepsilon_o} \right) \geq 1$$  \hspace{1cm} 4.65

Calculating $\lambda$ as shown above is for the first alternative stress-strain model for concrete in compression, as discussed in Section 4.2.3.2. Including the influence in the out-of-plane direction, $\varepsilon_1$ in Equation 4.59 above is replaced by Equation 4.43. Equations 4.47 through 4.51 are used to calculate the
principal tensile strain in the out-of-plane direction, and Equation 4.62 is used to calculate the principal tensile strain (in-plane). Those values are then used to calculate $\lambda$. When using the second compression softening model for concrete, the denominator of Equation 4.65 is replaced by Equation 4.46 for the damage indicator, $F$, and Equations 4.47 through 4.51 are used again to solve for $\lambda$.

10. Check if concrete crushing has occurred in the principal compressive stress direction. Calculate $\lambda \varepsilon_0$. If this value is less than or equal to $\varepsilon_2$ obtained from Equation 4.63 then crushing has occurred.

11. Calculate the concrete secant modulus ($E_c$) for the current strain state from Equations 4.40 and 4.41. i.e.,

$$E_c = \frac{\sigma_2}{\varepsilon_2} = \frac{\lambda f_c}{\varepsilon_2} \left[2 \left(\frac{\varepsilon_2}{\lambda \varepsilon_0}\right) - \left(\frac{\varepsilon_2}{\lambda \varepsilon_0}\right)^2\right]$$

4.66

If the value of $E_c$ is much less than the assumed value then a smaller increment value for $\varepsilon_x$ needs to be assumed (step 1).

12. Repeat Steps 1 through 11 for higher values of $\varepsilon_x$ until yielding of the hoops, yielding of the stirrups, or concrete crushing occurs. For each value of $\varepsilon_x$ the value of $E_c$ obtained using Equation 4.66 from the previous run is used.
4.2.6 Knee joint Behaviour After Yielding of Hoops or Stirrups

Once the transverse reinforcing bars have yielded in one direction, the next milestone in the knee joint's behaviour is either bar yielding in the orthogonal direction, or crushing of the principal diagonal compressive concrete strut at which point the joint has reached its ultimate capacity. The relationships for the knee joint parameters that describe the mechanics of the knee joint for the above mentioned critical milestones are presented below.

As discussed in Section 4.2.5, the angles of the principal stresses and principal strains are assumed to coincide up to first yielding of the transverse reinforcement in either direction. Thereafter, Equation 4.52 is no longer valid. For this condition, the transverse bar stress is set to its yield value. Next, the joint shear stress, \( v \), is solved for prior to calculating the angle of the principal stresses, \( \theta \).

4.2.6.1 Behaviour After Yielding of Hoop Reinforcement (x-direction)

The objective here, as in the derivations above for conditions prior to transverse reinforcement yielding in the joint, is to evaluate \( \theta \) in terms of imposed deformation. This will be achieved, once again, by selecting the hoop strain as the controlling variable.

Equation 4.37 gives the hoop strain as:

\[
\varepsilon_x = \varepsilon_2(1 - \frac{1}{\tan^2\theta}) + \frac{\varepsilon_y}{\tan^2\theta}
\]

From Equations 4.38 and 4.40, the hoop strain may be expressed in terms of stresses. i.e.,

\[
\varepsilon_x = \frac{\sigma_2}{E_c}(1 - \frac{1}{\tan^2\theta}) + \frac{f_y}{E_s \tan^2\theta}
\]

From Equation 4.29, the principal compressive stress is given as:

\[
\sigma_2 = \sigma_x + \sigma_y
\]
The concrete stress in the horizontal direction is given by Equations 4.28 and 4.30 with the hoop stress assigned the yield value, i.e.,

\[ \sigma_x = - \rho_x f_{xy} - \frac{N_x}{2bh} = - v_j \tan \theta \quad 4.70 \]

where; \( f_{xy} \) is the hoop reinforcement yield stress.

Rearranging Equation 4.70, the expression for \( \tan \theta \) is obtained as:

\[ \tan \theta = \frac{(\rho_x f_{xy} + \frac{N_x}{2bh})}{v_j} \quad 4.71 \]

The concrete stress in the vertical direction is obtained from Equations 4.6, 4.24, 4.27, 4.32, and 4.71 as:

\[ \sigma_y = - \rho_y f_y - \frac{N_x}{2bd} \frac{l_c}{l_b} = - \frac{v_j^2}{(\rho_x f_{xy} + \frac{N_x}{2bh})} \quad 4.72 \]

Combining Equations 4.69, 4.70 and 4.72, \( \sigma_2 \) is expressed as:

\[ \sigma_2 = - \rho_x f_{xy} - \frac{N_x}{2bh} - \frac{v_j^2}{(\rho_x f_{xy} + \frac{N_x}{2bh})} \quad 4.73 \]

The stirrup stress (y-direction) is given by Equation 4.56. i.e.,

\[ f_y = \frac{1}{\rho_y} \left( \frac{v_j}{\tan \theta} - \frac{N_x}{2bd \ l_b} \right) \quad 4.74 \]

A quadratic expression for the joint shear stress, \( v_j \), is obtained by substituting Equations 4.71, 4.73 and 4.74 into Equation 4.68, i.e.,
Solution Strategy: Knee Joint Behaviour After Hoop Reinforcement Yielding.

1. Select a value for the hoop strain ($\varepsilon_x$).

   This value should be greater than the hoop yield strain. Once again small increments are selected for the hoop strain to avoid iteration for the concrete secant modulus.

2. Guess a value for the beam axial force ($N_b$).

3. Obtain the value for the joint shear stress, $\nu_j$, using Equation 4.75.

4. Calculate the value of $\theta$ using Equation 4.71.

   Steps 5 through 12 are the same as in Section 4.2.5.1. Stirrup yielding and concrete crushing are checked for each hoop strain increment. Once either of these two milestones has been reached, the joint has reached its ultimate shear capacity.

   Expressions establishing these upper limits for shear capacity can be obtained as follows:

Hoop Reinforcement Yielding Followed By Stirrup Reinforcement Yielding.

From Equation 4.72, the stirrup reinforcement stress is assigned its yield value ($f_{sy}$) to give:

$$\nu_n = \sqrt{\left(\rho_y f_{sy} + \frac{N_x}{2bd} \frac{l_c}{l_b}\right)\left(\rho_x f_{xy} + \frac{N_x}{2bh}\right)}$$  \hspace{1cm} 4.76
Hoop Reinforcement Yielding Followed By Concrete Crushing.

From Equation 4.73, the principal compressive stress, $\sigma_2$, is assigned the crushing strength of concrete, $f_{cm}$ (see Equation 4.41), to give:

$$v_n = \sqrt{(|f_{cm}| - \rho_x f_{xy} - \frac{N_x}{2bh} \rho_x f_{xy} + \frac{N_x}{2bh})}$$  \hspace{1cm} 4.77

where; $v_n$ is the nominal joint shear stress.

4.2.6.2 Behaviour After Yielding of Stirrup Reinforcement (y-direction).

The derivations here are similar to those in the previous section. The main difference is that there is a reversal in the “x” and “y” variables in the expressions. In addition, the stirrup strain replaces the hoop strain as the controlling displacement parameter.

The following expressions are obtained for $\tan \theta$ and the quadratic expression for the joint shear stress:

$$\tan \theta = \frac{v_j}{\rho_y f_{yy} + \frac{N_x}{2bd} \frac{l_c}{l_b}}$$  \hspace{1cm} 4.78

$$\varepsilon_y = \left[ -\rho_y f_{yy} - \frac{N_x}{2bd} \frac{l_c}{E_c} \right] - \left[ \frac{N_x}{2bh \rho_x E_s} \frac{1}{\left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{l_c}{l_b} \right)^2} \right] v_j^2 + \left[ \frac{1 + \frac{1}{n \rho_x}}{E_c \left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{l_c}{l_b} \right)^3} \right] v_j^4$$  \hspace{1cm} 4.79
Solution Strategy: Knee Joint Behaviour After Stirrup Reinforcement Yielding.

The solution strategy here is similar to that for knee joint behaviour after hoop reinforcement yielding. Here, the stirrup strain is used as the controlling displacement parameter in Step 1. In addition, Equations 4.79 and 4.78 are used in Steps 3 and 4 above to obtain the joint shear stress and \( \theta \), respectively. The remaining steps (Steps 5 through 12) are the same as those given above.

Expressions for the upper limits of joint shear capacity can be obtained here as was done above following the milestone of hoop reinforcement yielding.

Stirrup Reinforcement Yielding Followed By Hoop Reinforcement Yielding.

\[
\nu_n = \sqrt{\left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{I_c}{I_b} \right) \left( \rho_y f_{yy} + \frac{N_x}{2bh} \right)}
\]

4.80

As expected, this expression is identical to that obtained above for hoop reinforcement yielding followed by stirrup reinforcement yielding.

Stirrup Reinforcement Yielding Followed By Concrete Crushing.

\[
\nu_n = \sqrt{\left( |f_{cm}| - \rho_y f_{yy} - \frac{N_x}{2bd} \frac{I_c}{I_b} \right) \left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{I_c}{I_b} \right)}
\]

4.81

Equations 4.76, 4.77, 4.80 and 4.81 will be investigated in Chapter 8 towards arriving at consistent design recommendations for the design of knee joints.
4.3 Performance of Model: Sensitivity Study of Model Parameters

In this section, a sensitivity study is done of the model parameters and the algebraic combinations given in Equations 4.1 through 4.81. The parameters included in the study are: bond efficiency factors ($\beta_{ci}$, $\beta_{co}$, $\beta_{bb}$, and $\beta_{cb}$), hoop strain increment, compression softening model used, influence of amounts of confinement, assigned value for $\varepsilon_{ac}$, angle of principal tensile stress direction, $\theta$, and beam axial force, $N_x$.

The formulation of the mechanical model described in this chapter is founded on the premise that joint shear failure (i.e., hoop and stirrup yielding and/or concrete crushing) dominates the response of the connections. The objective was to select a test specimen from the experimental database of knee joint tests, which displayed a predominantly joint shear failure to use as a reference in performing the sensitivity study. Specimen U17 from the tests performed in China (Bai et al 1994) displayed such a failure and for this reason was selected for further study. The experimental test set-up was discussed in Chapter 2 and is shown in Figure 2.28. Figure 4.15 provides the specimen details. The beam bottom (2-20mm dia.) and column inner (2-25mm and 2-22mm dia.) reinforcement consisted of 90° bends, anchored in the joint region. The beam top (2-25mm and 1-22mm dia.) and column outer (2-25mm and 2-22 mm dia.) reinforcement consisted of 90° bends which were anchored in the adjacent column and beam members, respectively, as shown in the figure. Horizontal transverse reinforcement in the joint was 6 No. 10 hoops.

Figure 4.15 Test Specimen Details: Specimen U17 (Bai et al 1994).
To perform the sensitivity study, the procedures discussed in section 4.2.5.1 (before yielding of hoop or stirrup reinforcement) and sections 4.2.6.1, or 4.2.6.2 (behaviour after yielding of hoop or stirrup reinforcement, respectively) are followed.

The required model and design parameters for specimen U17 are given in Table 4.1. In performing the sensitivity study for the model parameters, in order to study a wider range of the model parameters under opening action, the beam bottom reinforcement ratio was increased from 0.00598 to 0.01905 (\(A_{\text{bo}} = 2000 \text{mm}^2\)), and the beam top reinforcement ratio was adjusted from 0.01297 down to 0.00598 (\(A_{\text{bt}} = 630 \text{mm}^2\)), such that the joint, and not the beam, governed the response. These values are indicated in brackets in Table 4.1, and will remain fixed as the selected model parameter is varied.

Table 4.2 lists the above mentioned model parameters in the study, as well as the range of values to be considered. To limit the number of test cases for the bond efficiency factors, the same \(\beta\) values will be assigned to both the column inner and beam bottom reinforcement (i.e., \(\beta_{ci} = \beta_{bo}\)), and the column outer and beam top reinforcement (i.e., \(\beta_{co} = \beta_{bt}\)). In doing so, it is assumed that these reinforcing bars behaved similarly under opening and closing actions, respectively. To illustrate the use of the model, sample calculations are provided in Appendix C for loading conditions of both closing and opening actions.

Table 4.1 Model and Design Parameter Values for Specimen U17

<table>
<thead>
<tr>
<th>(b) ((\text{mm}))</th>
<th>(d) ((\text{mm}))</th>
<th>(h) ((\text{mm}))</th>
<th>(E_s) ((\text{MPa}))</th>
<th>(f'_c) ((\text{MPa}))</th>
<th>(\varepsilon_o)</th>
<th>(E_m) ((\text{MPa}))</th>
<th>(n)</th>
<th>(f_{by}) ((\text{MPa}))</th>
<th>(f_{ny}) ((\text{MPa}))</th>
<th>(D) ((\text{N}))</th>
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<tr>
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<td>350</td>
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<td>25.1</td>
<td>-0.002</td>
<td>25,100</td>
<td>8.53</td>
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<td>400</td>
<td>200,000</td>
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<table>
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<tr>
<th>(\rho_h)</th>
<th>(\rho_s)</th>
<th>(\rho_{bo})</th>
<th>(\rho_{bt})</th>
<th>(\rho_{co})</th>
<th>(\rho_{ct})</th>
<th>(\rho_c)</th>
<th>(\rho_m)</th>
<th>(\rho_y)</th>
<th>(K_c(\text{joint})) (\text{(eqn.4.15)})</th>
<th>(K_c(\text{beam})) (\text{(eqn.4.15)})</th>
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</thead>
<tbody>
<tr>
<td>0.00897</td>
<td>0</td>
<td>0.00598</td>
<td>0.01297</td>
<td>0.01659</td>
<td>0.01659</td>
<td>0</td>
<td>0.01667</td>
<td>0.535</td>
<td>0.141</td>
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<table>
<thead>
<tr>
<th>(\beta_{bo})</th>
<th>(\beta_{bt})</th>
<th>(\rho_s)</th>
<th>(\beta_{co})</th>
<th>(\beta_{ct})</th>
<th>(\beta_c)</th>
<th>(\rho_y)</th>
</tr>
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<td>0.5</td>
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<td>0.3</td>
<td>0.4</td>
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Table 4.2 Parameter Range for Sensitivity Study.

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<th>Model Parameter</th>
<th>Nominal Value</th>
<th>Range Considered</th>
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<tr>
<td>$\beta_{ci} = \beta_{bb}$</td>
<td>closing 0.5 / opening 0.3</td>
<td>0.2 - 1.0</td>
</tr>
<tr>
<td>$\beta_{co} = \beta_{bt}$</td>
<td>closing 0.3 / opening 0.5</td>
<td>0.2 - 1.0</td>
</tr>
<tr>
<td>$\varepsilon_n$ increment</td>
<td>0.0001</td>
<td>0.00005 - 0.0016</td>
</tr>
<tr>
<td>Amount of Confinement (k_c)</td>
<td>1.114</td>
<td>1 - 1.5</td>
</tr>
<tr>
<td>Compression Softening Model</td>
<td>Vecchio/Collins</td>
<td>Vecchio/ Collins</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Vecchio/Collins &amp; $\varepsilon_3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pantazopoulos/Mills</td>
</tr>
<tr>
<td>$\varepsilon_o$</td>
<td>-0.002</td>
<td>(-0.0015) - (-0.0035)</td>
</tr>
<tr>
<td>$\theta, N_x$</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

* Investigate sensitivity of $v_j$ and $\gamma$ to variations from the converged values of $\theta$ and $N_x$

4.3.1 Influence of $\beta$

Figures 4.16 and 4.17 show plots of joint shear stress ($v$) and joint shear strain ($\gamma$) under closing and opening actions, respectively, for pairs of combinations of the bond efficiency ratio $\beta$. The critical milestones of hoop yielding, stirrup yielding (or column reinforcing bar yielding if stirrups not present), or concrete crushing, are marked on the figures.

The following observations were made from Figures 4.16 and 4.17: as $\beta$ increases, the joint shear stress corresponding to both critical milestones of hoop yielding and stirrup (or column reinforcement) yielding increased, while the value of the joint shear strain corresponding to the first critical milestone of either hoop yielding or stirrup yielding was about the same for all values of $\beta$, and increased with increasing values of $\beta$ for yielding in the other direction (second milestone).

For closing action, by varying the values for $\beta$, values obtained for the joint shear stress from the model were comparable with those obtained from the experimental results (Figure 4.16). A horizontal line is indicated in Figure 4.16 at $v = 6.2$ MPa instead of a point $(v, \gamma)$ because the shear strain value at joint capacity was not determined from the test.
Figure 4.16  Effect of β on Joint Shear Stress ($\nu_j$) and Joint Shear Strain ($\gamma$): Closing Action.
Figure 4.17  Effect of $\beta$ on Joint Shear Stress ($v_j$) and Joint Shear Strain ($\gamma$): Opening Action.
4.3.2 Influence of Hoop Strain Increment Value

Figure 4.18 shows the $v$-$\gamma$ response when the hoop strain increment varies from 0.00005 to 0.0016 (i.e., hoop yield strain), for closing and opening actions. There is no effect on $v$, and very little effect on $\gamma$ unless the hoop yield strain is applied in one step. This insensitivity can be attributed to the low stress levels developed by the joint core concrete.

4.3.3 Influence of Amounts of Confinement

To investigate the effects of confinement, a range of values between 1 and 1.5 is investigated for $k_c$ (Equation 4.14). Figure 4.19 b) shows that the $v$-$\gamma$ response for opening action varies very little with changes in the value for $k_c$ at the critical milestone of hoop yield. The joint for the selected specimen was underreinforced and as a result low stress levels developed in the concrete. To investigate the influence of confinement on the $v$-$\gamma$ response for the joint, the joint core concrete was overreinforced such that it may be subjected to significant stress levels. This resulted in the beam controlling the response for both opening and closing actions. A further attempt was made to shift the failure to concrete crushing in the joint by decreasing the concrete strength and increasing the beam tensile reinforcement ratio (higher joint shear input). Once again the beam controlled the response under opening action. However, under closing action joint core crushing governed the response. Figure 4.19a) shows that joint shear stress and strain at concrete crushing increases with higher values of joint confinement

4.3.4 Influence of Compression Softening Model

Plots of the $v$-$\gamma$ response for both compression softening models considered in this study (Section 4.2.2.2) are given in Figure 4.20 for both closing and opening actions. Under closing and opening actions, there is very little difference between the Vecchio/Collins model and the Pantazopoulou/Mills model when the nominal values are used (Figures 4.20a and 4.20c). Hoop yielding occurs first, followed by column reinforcement yielding. To allow for a comparison between the two models the nominal values were adjusted as was done in Section 4.3.3 to shift the failure to concrete crushing in the joint. Once again the beam controlled the response under opening action. Under closing action concrete crushing in the joint governed the response for both models with the
4.18a) Closing Action.  

4.18b) Opening Action.

Figure 4.18 Effect of Hoop Strain Increment on $v_j$ and $\gamma$: (Nominal Values).

4.19a) Closing Action: $f'_c = 20$ MPa.  

4.19b) Opening Action (Nominal Values).  

$\rho_{bi} = 0.01905$

Figure 4.19 Effect of Joint Confinement on $v_j$ and $\gamma$.  

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4.20a) Closing Action: $f'_c = 25.1$ MPa  
4.20b) Closing Action: $f'_c = 20$ MPa 
(Nominal Values). 
$\beta's = 0.8$, $\rho_{bt} = 0.01905$

4.20c) Opening Action: Nominal Values.

**Figure 4.20** Effect of Compression Softening Model on $v_j$ and $\gamma$.

4.21a) Closing Action.  
4.21b) Opening Action.

**Figure 4.21** Effect of Value for $\varepsilon_c$ on $v_j$ and $\gamma$ (Nominal Values).
Vecchio/ Collins model predicting higher values for joint shear stress and strain than the Pantazopoulou/Mills model. When the \( v-\gamma \) responses were compared for the Vecchio/Collins model with and without the inclusion of the principal tensile strain in the out-of-plane direction negligible differences were seen.

4.3.5 Influence of \( \varepsilon_0 \) (Concrete Strain at Peak Stress)

Values considered for \( \varepsilon_0 \) range between 0.0015 and 0.0035. Figure 4.21 shows the \( v-\gamma \) plots for this range of values and for closing and opening actions. There is no effect on \( v \) for both milestones of hoop yielding and column reinforcement yielding. However, \( \gamma \) increases as \( \varepsilon_0 \) increases for both events of hoop yielding and column reinforcement yielding for this test specimen.

4.3.6 Influence of \( \theta \) and \( N_x \)

As outlined in Section 4.1.5.1, input to the model is the hoop strain. A value for \( N_x \) (beam axial force) is assumed and the fourth order polynomial expression for \( \tan \theta \) (Equation 4.59) is used to calculate \( \theta \) (angle of the principal tensile stress referenced from the positive x-axis). Next, the joint shear input is calculated using two alternative methods, namely (a) Equation 4.60 according to the model derivation and (b), a beam sectional analysis is performed at the joint face. Should there be a significant discrepancy between these two values for the joint shear stress input, a different value of \( N_x \) is assumed and a new value for \( \theta \) is obtained. This procedure is repeated until the joint shear stress input obtained from the model and section analysis converge.

In this section, the effect on \( v \) and \( \gamma \) resulting from deviations in the values \( \theta \) and \( N_x \) from the converged values are investigated. Conditions prior to hoop or stirrup yielding are of primary interest because past the critical milestone of either hoop or stirrup yielding rapid deterioration occurs in the joint.

To investigate the effects on \( v \) and \( \gamma \) of a given deviation in \( \theta \) from the converged value, plots of variations in \( v \) and \( \gamma \) at each level of hoop strain are investigated. Figure 4.22 shows the \( \gamma-\varepsilon_h \) responses under both closing and opening actions for a deviation of \( \theta \) of +/- 0.005 degrees. The largest change in \( \gamma \) occurs at a hoop strain of 0.0008 (about 50 per cent of hoop yield). This change in \( \gamma \) is significantly more than at other hoop strain values. To
4.22a) Closing Action.  

4.22b) Opening Action.  

Figure 4.22  Effect on $\gamma$ from Varying True Value of $\theta$ by $+/-$ 0.005°.

4.23a) $\Delta \theta = +/- 0.005°$  

4.23b) $\Delta \theta = +/- 1°$  

Figure 4.23  Effect on $v$ from Varying True Value of $\theta$: Closing Action.
explain this trend, the equations used to calculate the joint shear strain are investigated.

From Equation 4.53, the principal tensile strain can be expressed as:

\[
\varepsilon_1 = \frac{\varepsilon_x - \varepsilon_y \tan^2 \theta}{\sqrt{1 - \tan^2 \theta}}
\]  

4.82

The expression for \( \gamma \) is given by Equation 4.36 and 4.52 as:

\[
\gamma = \frac{2(\varepsilon_1 - \varepsilon_x)}{\tan \theta}
\]  

4.83

If \( \varepsilon_x \) (hoop strain) is approximately equal to \( \varepsilon_y \) (stirrup strain), and \( \theta \) is approximately equal to 45\(^\circ\), an investigation of Equation 4.82 shows that very small changes in \( \theta \) will result in significant changes in \( \varepsilon_1 \) and consequently \( \gamma \). An investigation of the output from the model shows that for this test case, at a hoop strain (\( \varepsilon_y \)) of 0.0008, the stirrup strain (here column reinforcement strain of \( \varepsilon_y \)) is approximately the same and the value of \( \theta \) is very close to 45\(^\circ\). Thus, the largest change in \( \gamma \) at a hoop strain of 0.0008 is consistent with Equations 4.82 and 4.83 (Figure 4.22).

A similar plot for the joint shear stress does not show the same singularity trend. Figure 4.23 shows the \( \nu - \varepsilon_h \) response under closing action for deviations in \( \theta \) of 0.005 degrees and 1 degree (a similar trend is seen under opening action). An increasing trend in the change in \( \nu \) is seen only after \( \theta \) is increased to +/- 1 degree. This trend can be explained by looking at the equation used to calculate \( \nu \) in the model. \( \nu \) is obtained from Equation 4.60 as:

\[
\nu_j = \frac{1}{\tan \theta} \left( \rho_s E_s \varepsilon_x + \frac{N_{x-}}{2bh} \right)
\]  

4.84

From the model results, \(|\theta|\) increases for increasing values of hoop strain (\( \varepsilon_y \)). At higher values of \( \theta \), \( \nu \) is more sensitive to changes in \( \theta \).

From the above discussion, it is evident that \( \gamma \) is significantly more sensitive to changes in \( \theta \) than \( \nu \) is. Because of this, the focus of the remainder of the study will be on \( \gamma \).
To further investigate the sensitivity of $\gamma$ to variations in $\theta$, plots of $\gamma - \Delta \theta$ (Figure 4.24) and plots of per cent error in $\gamma$ (based on selected $\Delta \theta$) - corresponding residual from Equation 4.59 (Figure 4.25) are compared for closing and opening actions. From the above Figures it appears that $\gamma$ is more sensitive to variations in $\theta$ under closing action. To ensure an error of less than $\pm 5$ per cent, the variation in $\theta$ must be less than $\pm 0.001$ degrees, and the corresponding residual from Equation 4.59 must be less than $\pm 0.001$.

Plots of the residual from Equation 4.59 versus $\theta$ for hoop strain of 0.0008 (50 per cent of hoop yield) and 0.0016 (hoop yield) show sharp increases in the residual for values of $\theta$ greater than the actual value (here = $45^\circ$) for both conditions of closing and opening action (Figure 4.26). For values less than the actual value of $\theta$ a stabilizing trend is seen in the residual for both closing and opening actions.

To investigate the effects of variations of $N_x$ on $\gamma$, plots of $\gamma - \Delta N_x$ (Figure 4.27) and plots of per cent error in $\gamma$ (based on selected $\Delta N_x$) - corresponding residual from Equation 4.59 (Figure 4.28) are compared for closing and opening actions. Similar trends in sensitivity for $\gamma$ were seen for variations in $N_x$ as were seen for variations for $\theta$.

To ensure an error of less than $\pm 5$ per cent, the variation in $N_x$ must be less than $0.1\text{kN}$ ($\sim 1/15000$ of beam members squash load) and the corresponding residual from Equation 4.59 must be less than $\pm 0.00004$.

![Graphs showing residual vs $\theta$ for closing and opening actions.](4.26a) Closing Action. 4.26b) Opening Action.

Figure 4.26 Effects on Residual from Equation 4.59 by Varying $\theta$ from Converged Value. ($\sim 45$ Degrees).
4.24a) Closing Action.  

Figure 4.24  Effect on γ from Varying True Value of θ.

4.24b) Opening Action.

4.25a) Closing Action.  

Figure 4.25  Per cent Error in γ and Resulting Residual from Varying True Value of θ.

4.25b) Opening Action.

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4.27a) Closing Action.  
4.27b) Opening Action.

Figure 4.27  Effect on $\gamma$ from Varying True Value of $N_x$.

4.28a) Closing Action.  
4.28b) Opening Action.

Figure 4.28  Per cent Error in $\gamma$ and Resulting Residual from Varying True Value of $N_x$. 

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4.4 Model Predictions of Sustained Joint Shear: Chongqing University Tests

The model was used to predict the sustained joint shear for those knee joint specimens tested in China that failed in joint shear. The assumed values of $\beta$ used in the analyses were: $\beta_{bt} = \beta_{co} = 0.5$; and $\beta_{bb} = \beta_{ci} = 0.1$.

Table 4.3 compares the experimental and model results for sustained joint shear of the nine knee joint test specimens. With the exception of specimens u12 and u13 the simple knee joint model predicted the joint shear capacity very well. As the chosen values for $\beta$ are not directly linked with the concrete strength, the lower concrete strength values for specimens u12 and u13, when compared to the remaining specimens, may be responsible for the poorer predictions for joint shear capacity.

Table 4.3 Model Predictions for Sustained Joint Shear Under Closing Action:
Tests in China.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$\Gamma*_{e}$ (MPa)</th>
<th>$\sqrt{\Gamma*_{e}}$ (MPa)</th>
<th>Yield 1st Direction</th>
<th>Yield 2nd Direction</th>
<th>Experiment</th>
<th>Ratio $\gamma_{e,mode}$/ $\gamma_{e,exp}$ x 100 %</th>
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<tr>
<td></td>
<td>$\psi_{r}$</td>
<td>$\gamma$</td>
<td>$E/E_{m}$</td>
<td>$\psi_{r}$</td>
<td>$\gamma$</td>
<td>$E/E_{m}$</td>
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<td>----------</td>
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<td>----------------</td>
<td>----------------</td>
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</tbody>
</table>
CHAPTER 5

EVALUATION OF KNEE JOINT BEHAVIOUR: CLOSING ACTION

5.1 Knee Joint Experimental Database

Experimental evidence used to evaluate knee joint behaviour is compiled primarily from cyclic tests performed at Chongqing University (China), and at Clarkson University (N.Y).

The main difference between the two test programs was in the anchorage detailing used for the longitudinal reinforcement of the beam and column. In the Clarkson Study, standard 90° hook anchorages in the knee joint were used for both the beam and column longitudinal reinforcement. This detailing approach is common in North American practice. In the China Study, 90° hook anchorages were also used for the beam and column longitudinal reinforcement. However, in most test specimens in the China study, the beam top and column outer reinforcement were anchored in the adjacent member following the 90° bend and not in the joint region. Table 5.1 (repeated from Table 2.1) summarizes the specimen details for the test series in China. Tables 5.2 and 5.3 (repeated from Tables 3.1 and 3.2) summarize the specimen details for the Clarkson test study.

5.2 Discussion of Experimental Results

From the available experimental evidence, three design parameters appear to play a significant role in knee joint performance: bond, which can be characterized by a bond index, amounts of joint confinement, and joint concrete strength.

To help describe the influence of the above design parameters on knee joint behaviour, two additional parameters are introduced, namely: (a) shear sustained by the joint ($v_j$), and (b) an index of drift capacity, $d_{80}$, defined as the drift when post-peak resistance had decayed to 80 per cent of the peak value. The joint shear sustained is obtained using the ACI352-85 approach discussed earlier (Equation 4.2, Chapter 4) and is used as an indicator of the intensity of shear that the joint must resist. $d_{80}$ is obtained by tracing the envelope over the peak value of load at each drift cycle. The resulting curve which envelopes the hysteretic response is used to determine the drift level corresponding to post-peak resistance of 80 per cent of the peak load.
<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>$f'_e$ (MPa)</th>
<th>$f_e$ (MPa)</th>
<th>Anchorage</th>
<th>Column $^*$</th>
<th>Beam Top Reinforcement</th>
<th>Beam Bottom Reinforcement</th>
<th>Joint</th>
</tr>
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<tbody>
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<td></td>
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<td>Type</td>
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<td>Beam Top Anchorage</td>
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<td>D</td>
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<td>3φ20</td>
<td>2φ16</td>
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<td>16-366</td>
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<td>28-362</td>
<td>A</td>
<td>2φ28, 2φ22</td>
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<td>u8</td>
<td>27.6</td>
<td>16-475</td>
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<td>20-468</td>
<td>E</td>
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<td>4φ8</td>
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<tr>
<td>u13</td>
<td>28.3</td>
<td>25-410</td>
<td>G</td>
<td>2φ25, 2φ22</td>
<td>2φ25, 1φ22</td>
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<td>6φ10</td>
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<tr>
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<td>22-393</td>
<td>B</td>
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<td>10-320</td>
<td>E</td>
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<td>2φ25, 1φ22</td>
<td>2φ20</td>
<td>6φ10</td>
</tr>
<tr>
<td>u17</td>
<td>29.6</td>
<td>25-365</td>
<td>C</td>
<td>2φ25, 2φ22</td>
<td>2φ25, 1φ22</td>
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<td>6φ8</td>
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<td>25.7</td>
<td>22-466</td>
<td>C</td>
<td>2φ25, 2φ22</td>
<td>2φ25, 1φ22</td>
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<td>6φ8</td>
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<td>20-405</td>
<td>C</td>
<td>4φ20</td>
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<td>2φ16</td>
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<td>C</td>
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<td>8-460</td>
<td>C</td>
<td>3φ16, 3φ14</td>
<td>4φ16</td>
<td>2φ16</td>
<td>6φ8</td>
</tr>
</tbody>
</table>

$^*$1 Diagonal reinforcement in joint region
$^*$2 Contained haunch
$^*$3 Cross reinforcement in beam member, hinging away from joint
$^*$4 Column inner reinforcement equals column outer reinforcement

A All bars anchored in joint
B Column outer bars anchored in beam, rest anchored in joint
C u-column bars in joint, beam top bars anchored in column, beam bottom bars anchored in joint
D Beam top bars anchored in column, rest anchored in joint
E Beam top bars anchored in column, column outer bars anchored in beam, rest anchored in joint
F Beam top bars anchored in column, straight anchorage for column outer bar, rest anchored in joint
G Straight anchorage for beam top bar, column outer and inner bars, beam bottom anchored in joint
Table 5.2 Test Specimen Reinforcement Details: Clarkson Test Study.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>$f'_{c}$ (MPa)</th>
<th>Column Reinforcement</th>
<th>Beam Top Reinforcement</th>
<th>Beam Bottom Reinforcement</th>
<th>Joint</th>
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<td></td>
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<td></td>
<td></td>
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<td>KJ1</td>
<td>45.7</td>
<td>4 #6 Corner</td>
<td>4 #5</td>
<td>2 #5</td>
<td>4 #3</td>
</tr>
<tr>
<td>KJ2</td>
<td>49.7</td>
<td>&amp;</td>
<td>4 #5</td>
<td>2 #5</td>
<td>4 #3</td>
</tr>
<tr>
<td>KJ3</td>
<td>45.0</td>
<td>4 #5 Middle</td>
<td>4 #5</td>
<td>2 #5</td>
<td>2 #3</td>
</tr>
<tr>
<td>KJ4</td>
<td>45.6</td>
<td></td>
<td>4 #5</td>
<td>2 #5</td>
<td>4 #3</td>
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<td></td>
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<td></td>
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<td>3 #6</td>
<td>4 #3</td>
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<td>2 #3</td>
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<td>4 #3</td>
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<td>4 #6</td>
<td>2 #6</td>
<td>4 #3</td>
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<td>4 #3</td>
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<td>4 #6</td>
<td>4 #6</td>
<td>4 #3</td>
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<tr>
<td>KJ16</td>
<td>37.2</td>
<td>8-16mm dia.</td>
<td>4-16mm dia.</td>
<td>4-16mm dia.</td>
<td>4 #3</td>
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</table>

1 135° anchorage for stirrups  
2 Two diagonal hairpins within joint region.

Table 5.3 Reinforcement Properties.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Reinforcing Bars</th>
<th>$f'_{c}$ (MPa)</th>
<th>$\varepsilon_{y}$</th>
<th>$\varepsilon_{ah}$</th>
<th>$f_{y}$ (MPa)</th>
<th>$f_{u}$ (MPa)</th>
<th>$f_{v}$ (MPa)</th>
<th>$f_{t}$ (MPa)</th>
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<tbody>
<tr>
<td>KJ1 - KJ4</td>
<td>#3</td>
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<td>0.022</td>
<td>676</td>
<td>0.1435</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>#5</td>
<td>448</td>
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<td>0.0144</td>
<td>703</td>
<td>0.1172</td>
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<tr>
<td></td>
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<td>0.022</td>
<td>676</td>
<td>0.1435</td>
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<td>700</td>
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<td>#5</td>
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<td>676</td>
<td>0.1435</td>
<td>538</td>
<td>0.1435</td>
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</table>

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**Bond Index**

Bond index is defined as the average bond stress \( u_{av} \) normalized by the square root of the concrete strength. Under closing action, the bond stress depends on the yield stress of the beam or column outer reinforcement at the respective joint faces, i.e.,

\[
u_{av} = \frac{A_s f_y}{\pi d_b l_i} = \frac{d_b f_y}{4l_i^2}
\]

where; \( u_{av} \) is the average bond stress over the entire length of the anchored reinforcing bar, \( l_i \) is the total anchorage length of the beam top or column outer reinforcement measured from their respective joint faces and includes the lead-in-length, hook and tail-end length. Thus,

\[
Bond index = BI = \frac{d_b f_y}{4l_i \sqrt{f'_c}}
\]

**Amount of Confinement**

Hoops and stirrups are assumed equally effective in confining the knee joint core concrete. When both hoops and stirrups are present in the joint region the effective volumetric reinforcement ratio is assumed to be given as:

\[
\rho_{ve} = \sqrt{\rho_{vh}^2 + \rho_{vs}^2}
\]

where; \( \rho_{ve} \) is the effective volumetric ratio for the transverse reinforcement, and \( \rho_{vh} \) and \( \rho_{vs} \) are the hoop and stirrup volumetric reinforcement ratios, respectively.

Table 5.4 summarizes the results of tests performed in China and Clarkson University. The two tests performed by Mazzoni et al (1991) from the University of California at Berkeley are also included in the table. The shear sustained by the joint is given in column 11 in Table 5.4. The joint shear sustained was calculated from the moment gradient through the joint and is listed in columns 12 through 13 (horizontal joint shear stress was obtained from the column moment gradient whereas vertical joint shear stress was obtained from the beam moment gradient, respectively), i.e.,
Table 5.4 Summary of Experimental Knee Joint Database: Closing Action.

<table>
<thead>
<tr>
<th>Spec. b</th>
<th>d</th>
<th>h</th>
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<th>f_&lt;sub&gt;_c&lt;/sub&gt;</th>
<th>hoops</th>
<th>stirrups</th>
<th>bond index</th>
<th>bond index</th>
<th>v_{reinf}</th>
<th>v_{reinf}</th>
<th>horiz. v</th>
<th>vertic. v</th>
<th>drift</th>
<th>failure</th>
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<tr>
<td>ident. (mm)</td>
<td>(mm)</td>
<td>(mm)</td>
<td>(psi)</td>
<td>(MPa)</td>
<td>vol. ratio</td>
<td>vol. ratio</td>
<td>in (\sqrt{f'_c})</td>
<td>in (\sqrt{f'_c})</td>
<td>Reinf force in (\sqrt{f'_c})</td>
<td>Reinf force in (\sqrt{f'_c})</td>
<td>force in (\sqrt{f'_c})</td>
<td>force in (\sqrt{f'_c})</td>
<td>at 80%</td>
<td>mode</td>
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<td>(4)</td>
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<tr>
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<sup>*</sup>1: u5 may have contained 1% fibre volumetric ratio,  <sup>**</sup>2: u6 may have contained 2% fibre volumetric ratio,  nd: load did not degenerate to 80% of peak  
<sup>**</sup>3: The sustained joint shear according to ACI352-1985: Column (11) x d ln; where, \( h_b = 400 \) mm  
Failure Modes: 1 Beam Hinging, 2 Column Hinging, 3 Joint Shear, 4 Beam Anchorage, 5 Column Anchorage, 6 Away From Joint
Table 5.4 (cont’d) Summary of Experimental Knee Joint Database: Closing Action.

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<th>h</th>
<th>$f'_c$ (psi)</th>
<th>$f'_s$ (MPa)</th>
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<th>stirrups</th>
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<th>bond index column ((\sqrt{f'_s}))</th>
<th>$v_{\text{reinf}}$ reinf AC1' (\psi)</th>
<th>$v_{\text{reinf}}$ reinf (\psi)</th>
<th>horiz. $v_j$ from col. (\sqrt{f'_c})</th>
<th>vertic. $v_j$ from beam (\sqrt{f'_r})</th>
<th>drift at 80% postpeak %</th>
<th>failure mode</th>
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Tests by Mazzoni et al.

|          |    |    |    |    |    |    |    |    |    |    |    |    |    | |
| 1        | 250| 265| 185| 6100| 42.1| 0.01672| 0.00635| 0.0998| 0.0998| 0.790| 0.691| 10.23| 12.01| 1.94| 4, 5 |
| 2        | 250| 265| 185| 6100| 42.1| 0.03345| 0.00800| 0.0998| 0.0998| 0.800| 0.700| 10.34| 12.14| 2.95| 4, 5 |

*1: u5 may have contained 1% fibre volumetric ratio, *2: u6 may have contained 2% fibre volumetric ratio, nd: load did not degenerate to 80% of peak

*For sustained joint shear according to ACI352-1985: Column (11) x d/h; where, h, = 400 mm

Failure Modes: 1 Beam Hinging, 2 Column Hinging, 3 Joint Shear, 4 Beam Anchorage, 5 Column Anchorage, 6 Away From Joint
Fairly good agreement is seen when comparing the joint shear sustained obtained as the beam or column moment gradient (assuming maximum moment at the hoop centroid near the joint interface and zero moment at the hoop centroid near the free face of the joint for the beam moment gradient; assuming maximum moment at the centroid of the beam bottom bars and zero moment at the centroid of the beam top bars for the column moment gradient) to those obtained using the yield strength of the beam or column tensile reinforcement (columns 11 through 13 Table 5.4).

One important observation that can be made regarding the shear sustained by the joint from Table 5.4 is that much higher values are obtained in the China test specimens, exceeding the nominal value of \(12\sqrt{f'_{c}}\) (psi) given by ACI352-85, when compared to the Clarkson test specimens or those tested by Mazzoni et al. Those specimens tested at Clarkson University that were expected to reach the nominal capacity given by ACI352-85 (i.e. specimens KJ5-KJ7) did not perform well. Soon after the beam or column reinforcement reached yield or near yield, the strain levels dropped as splitting bond failure governed the response. In contrast, the specimens tested in China were able to reach, and in many cases exceed, the nominal shear capacity of \(12\sqrt{f'_{c}}\) (psi) with one or both of the beam and column members hinging at their respective joint faces.

Table 5.4 gives the failure modes in the sequence they occurred (column 15 in table) established here for each test specimen. These failure mode types were discussed in Chapter 3 and are summarized in the table. It is clear that anchorage failure governed the response in the Clarkson test specimens which had an expected joint shear input of \(6.5\sqrt{f'_{c}}\) (psi), or greater, with the exception of specimen KJ14 which had a 400mm column stub. It is believed that the presence of the column stub helped prevent or delay splitting bond failure

\[ v_{s} = \frac{M_{s}}{h} \frac{1}{bd} \frac{1}{\sqrt{f'_{c}}} \]

\[ v_{e} = \frac{M_{y}}{d} \frac{1}{bh} \frac{1}{\sqrt{f'_{c}}} \]
at the level of the beam top reinforcement by providing adequate confinement of these reinforcing bars.

5.2.1 Effects of Bond Index

To investigate the effects of bond index on knee joint behaviour, Figure 5.1 shows plots of bond index in beam top reinforcement versus bond index in column outer reinforcement for the experimental database. The diagonal line shown in Figure 5.1a) corresponds to equal bond indices for the beam and column reinforcement. This is typically the case for the Clarkson test specimens which contained standard 90° hook anchorages of both beam and column reinforcement in the joint. Those specimens that lie up above and towards the left from the diagonal line, usually have standard 90° hook anchorages for the beam top reinforcement, whereas the column outer reinforcement is usually anchored in the adjacent beam or is U-shaped. Those specimens that lie down below and to the right of the diagonal line usually have standard 90° hook anchorages for the column outer reinforcement and beam top reinforcement that is anchored in the adjacent column.

Figure 5.1b) is the same plot as Figure 5.1a), except here the dominating failure modes are displayed. It appears from the plot that the failure modes with higher strain energy i.e., member hinging followed by joint shear failure or anchorage failure, dominate for bond indices below a limiting value of \(6\sqrt{f'_c}\) (psi). The bond index of the column outer reinforcement may be as high as \(9\sqrt{f'_c}\) (psi) but the specimen may still achieve the higher energy modes of failure without prior anchorage failure. However, a conservative approach requires that the bond indexes for the beam and column be limited to \(6\sqrt{f'_c}\). Thus, based on the above discussion, a suitable knee-joint detailing scheme may be anchorage of the beam top and column outer reinforcement in the adjacent column and beam members, respectively, provided the bond indices are within \(6\sqrt{f'_c}\) (psi).

Figure 5.2 shows plots of bond index versus \(d_{80}\). Figure 5.2a) plots maximum bond index on the y-axis, while Figure 5.2b) plots the average of the beam and column reinforcement bond indices. The plots show that the drift at 80 per cent peak load increases as the bond index decreases. This trend is more pronounced in Figure 5.2b).
5.1a) Experimental Test Program

5.1b) Failure Modes

Figure 5.1  Effects of Bond Index on Knee Joint Behaviour: Closing Action.

5.2a) Maximum Bond index.

5.2b) Average Bond Index

Figure 5.2  Bond Index Versus $d_{80}$ (%).
5.2.2 Effects of Amounts of Confinement

Figure 5.3 shows plots of joint shear sustained (multiples of $\sqrt{f'*}$, in psi) versus effective transverse reinforcement volumetric ratio (Equation 5.3) for those specimens with response dominated by joint shear failure. It appears that there is an increasing trend in the shear sustained by the joint as the effective volumetric ratio increases.

A plot of $d_{eb}$ versus effective transverse reinforcement volumetric ratio (Figure 5.4) does not show any appreciable trend. This is expected since shear failure did not appear to control the displacement of the test specimens. The major source of deformation occurred in the plastic hinge regions of the beam and column members which had formed earlier (prior to joint shear failure) in the response of the specimens (Table 5.4, Column 15).

5.2.3 Effects of Concrete Strength

Figure 5.5 shows plots of joint shear sustained versus concrete strength (in psi) for those specimens with response dominated by joint shear failure. When the sustained joint shear stress was plotted in terms of $\sqrt{f'^*}$ a decreasing trend occurred as the concrete strength increased (Figure 5.5a). However, when the sustained joint shear stress was plotted in psi units no trends were apparent in the plot (Figure 5.5b).

Figure 5.3  Sustained Joint Shear ($\sqrt{f'*}$, in psi) Versus Effective Transverse Reinforcement Volumetric Ratio.
Figure 5.4  Effective Transverse Reinforcement Volumetric Ratio Versus $d_{60}(\%)$.

5.5a) Joint Shear Stress in $\sqrt{f'_c}$ (psi)  
5.5b) Joint Shear Stress in psi.

Figure 5.5  Sustained Joint Shear Stress Versus Concrete Strength.
5.3 Parametric Investigation of Knee Joint Model: Influence of Design Variables

In this section, a parametric study is performed on the knee joint model's design variables. These variables consist of: concrete strength, hoop and stirrup area reinforcement ratios, hoop and stirrup reinforcement yield strengths, and beam span length. The input values for the model and design variables are those for specimen U17 from the China test program and are given in Table 4.1. The Vecchio/Collins model is used to account for concrete compression softening effects. A beam top reinforcement ratio of 0.01905 is used in place of 0.01297, shown in Table 4.1 to preclude a beam failure and thus to allow for a broader range of study of the design variables. Calculations for the parameter study were done using the same approach as that outlined in the example problem in Appendix C. The values for β were arbitrarily chosen as $\beta_{bt} = \beta_{ce} = 0.3$ and $\beta_{bs} = \beta_{ct} = 0.5$.

Figure 5.6 shows the effect that each of the design variables had on shear stress and shear strain at the milestone of hoop and/or stirrup yielding. For the specimens used in this study, hoop yielding was always reached first. It can be observed from Figure 5.6 that increasing either the hoop reinforcement ratio, hoop yield stress, or stirrup yield stress resulted in hoops yielding at higher values of shear stress and shear strain. Increasing the concrete strength, however, resulted in lower values in shear stress and strain at hoop yield. Increasing the stirrup reinforcement ratio resulted in a mixed response with respect to joint shear resistance. An increasing trend was noticed in the joint shear stress, whereas a decreasing trend was seen in the joint deformation. Varying the beam span length (which effectively corresponds to changing the magnitude of the beam axial load) had very little effect on either the joint shear stress or joint shear strain. Figure 5.7 shows a plot of shear stress versus shear strain summarizing the parameter investigation of the design variables. The arrows indicate the increasing direction of the design variable. The common point in the plot indicates shear stress and shear strain at nominal conditions (i.e., Specimen U17: Test study in China).

Before comparing the design variable trends obtained from the model calculations to those obtained from the reduction of the available experimental database two issues need to be addressed.

First, the reported values of sustained joint shear stress from the experimental results
Figure 5.6 Parametric Tendencies at Hoop and/or Stirrup Yield: Closing Action.
Figure 5.7  Summary of Parametric Investigation: Closing Action.

were obtained at joint failure. Joint failure is defined here as conditions in the joint where: transverse reinforcement yielding has occurred in both horizontal and vertical directions, transverse reinforcement yielding has occurred in one direction and is followed by concrete crushing, or concrete crushing has occurred prior to transverse reinforcement yielding.

Second, when increases in volumetric ratios were related to increases in area ratios for the transverse reinforcement there was a one-to-one relationship between the two.

When the experimental and model trends for amounts of joint transverse reinforcement on joint performance were compared, it appears that increasing the area (or volumetric) ratio of the joint transverse reinforcement results in a higher sustained joint shear (Figures 5.6, 5.7, and 5.3).

When the experimental and model trends for joint concrete strength on joint performance were compared, a decreasing trend was seen in the sustained joint shear (in \( \sqrt{f'_{c}} \)) for increasing concrete strength (Figures 5.6 and 5.5a). However, a plot of sustained joint shear not normalized by \( \sqrt{f'_{c}} \) versus concrete strength from the experimental results did not show any apparent trends (Figure 5.5b).
CHAPTER 6

EVALUATION OF KNEE JOINT BEHAVIOUR: OPENING ACTION

6.1 Knee Joint Experimental Database

In this chapter, the knee joint behaviour is examined under opening action for the same database of experiments that was considered for closing action in Chapter 5.

6.2 Discussion of Experimental Database

Sustained joint shear and per cent drift at 80 per cent of peak load \(d_{80}\) are also used here to help describe the influences that the design parameters (bond index, amount of joint confinement and joint concrete strength) have on knee joint behaviour.

Equations 5.2 and 5.3 are used to calculate the bond index and the effective transverse reinforcement volumetric ratio, respectively. Bond indices reported here are for the beam bottom reinforcement and column inner reinforcement. Table 6.1 summarizes the knee joint cyclic test results for opening action.

Comparing column 11 and columns 12 and 13 in the table indicates fairly good agreement between the observed joint shear sustained (Equation 4.1, Chapter 4) and that obtained as the column or beam moment gradient through the joint (Equations 5.3 and 5.4 in Chapter 5, respectively).

Beam bottom bar hinging dominated the response of many of the test specimens (Column 15, Table 6.1). Where deterioration could be detected in the load-displacement curves under opening action, typically beam hinging was followed by the loss of the beam compression zone at the joint face. Anchorage deterioration of the beam or column reinforcement and diagonal crack formation in the joint also contributed to the loss in strength and stiffness of the test specimens.
## Table 6.1 Summary of Experimental Knee-Joint Database: Opening Action

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<th>h (mm)</th>
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*:* u5 may have contained 1% fibre volumetric ratio,  \*:* u6 may have contained 2% fibre volumetric ratio,  nd : load did not degenerate to 80% of peak  
\*:* For sustained joint shear according to ACI352-1985: Column (11) x dhc, where, hc = 400 mm  
Failure Modes: 1 Beam Hinging, 2 Column Hinging, 3 Joint Shear, 4 Beam Anchorage, 5 Column Anchorage, 6 Away From Joint
Table 6.1 (cont’d) Summary of Experimental Knee Joint Database: Opening Action

| Spec. b d h | f’_s (psi) | f’_c (MPa) | vol. ratio | hoops | stirrups | bond index beam (V'_{ct}) | bond index column (V'_{ct}) | V_{ext} (psi) | horiz. vі | vertic. vі | drift at 80% postpeak | failure mode |
|------------|--|-----|-----|-----|-----|----------------|----------------|----------------|--------|--------|----------------|----------------|-----------|
| (1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11) (12) (13) (14) (15) |
| Clarkson Tests |
| KJ1 340 350 320 | 6628 | 45.7 | 0.01700 | 0.00687 | 4.82 | 5.62 | 2.14 | 1.44 | 1.64 | nd | 1 |
| KJ2 340 350 320 | 7208 | 49.7 | 0.01700 | 0.00687 | 4.62 | 5.39 | 1.90 | 1.69 | 1.93 | nd | 1 |
| KJ3 340 350 320 | 6526 | 45.0 | 0.00848 | 0.00344 | 4.86 | 5.66 | 2.04 | 1.68 | 1.92 | nd | 1 |
| KJ4 340 350 320 | 6613 | 45.6 | 0.01700 | 0.00687 | 4.83 | 5.63 | 1.98 | 1.76 | 2.01 | nd | 1 |
| KJ5 340 350 320 | 4569 | 31.5 | 0.01572 | 0.00000 | 7.17 | 7.95 | 5.85 | 2.70 | 3.08 | nd | 1 |
| KJ6 340 350 320 | 4786 | 33.0 | 0.00766 | 0.00348 | 7.00 | 7.77 | 5.66 | 2.75 | 3.14 | 1.74 | 1 |
| KJ7 340 350 320 | 4772 | 32.9 | 0.01572 | 0.00696 | 7.01 | 7.78 | 5.81 | 2.46 | 2.80 | nd | 1 |
| KJ8 340 350 320 | 5265 | 36.3 | 0.01572 | 0.00000 | 6.68 | 6.43 | 3.61 | 1.73 | 1.97 | nd | 1 |
| KJ9 340 350 320 | 5584 | 38.5 | 0.00786 | 0.00348 | 6.48 | 6.24 | 3.45 | 1.78 | 2.03 | nd | 1 |
| KJ10 340 350 320 | 5497 | 37.9 | 0.01572 | 0.00696 | 6.53 | 6.29 | 3.55 | 1.65 | 1.89 | nd | 1 |
| KJ11 340 350 320 | 5076 | 35.0 | 0.01572 | 0.00000 | 6.80 | 6.55 | 5.50 | 3.85 | 4.39 | 2.81 | 1,4,5 |
| KJ12 340 350 320 | 4772 | 32.9 | 0.00786 | 0.00348 | 7.01 | 6.75 | 5.61 | 4.15 | 4.74 | 2.08 | 1,4,5 |
| KJ13 340 350 320 | 4598 | 31.7 | 0.01572 | 0.00696 | 7.14 | 6.88 | 5.53 | 4.81 | 5.49 | 2.93 | 1,4,5 |
| KJ14 340 350 320 | 4873 | 33.6 | 0.01572 | 0.00000 | 6.74 | 5.59 | 5.48 | 3.74 | 4.26 | nd | 1 |
| KJ15 340 350 320 | 5352 | 36.9 | 0.01572 | 0.00696 | 5.20 | 7.64 | 2.92 | 3.21 | 3.66 | nd | 1 |
| KJ16 340 350 320 | 5395 | 37.2 | 0.01572 | 0.00696 | 10.09 | 10.95 | 3.90 | 3.02 | 3.44 | nd | 1 |

Tests by Mazzoni et al.

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*1: u5 may have contained 1% fibre volumetric ratio, *2: u6 may have contained 2% fibre volumetric ratio, nd: load did not degenerate to 80% of peak
*3: For sustained joint shear according to ACI352-1985: Column (11) x d/h_v; where, h_v = 400 mm

Failure Modes: 1 Beam Hinging, 2 Column Hinging, 3 Joint Shear, 4 Beam Anchorage, 5 Column Anchorage, 6 Away From Joint
6.2.1 Effects of Bond Index

Figure 6.1 shows the distribution of values for the bond index of beam bottom bars plotted against the bond index of column inner bars. Beam hinging occurred first in all of the test specimens. Figure 6.1b) displays the dominating failure modes. No definite trends are seen, but bond indices for the beam and column rebars below 8 result in the higher energy dissipating mechanism of beam hinging. Figure 6.2 plots bond index versus $d_{\text{so}}$. Figure 6.2a) plots the maximum bond index ($\sqrt{f'_c}$ psi) on the y-axis while Figure 6.2b) plots the average of the beam and column bond indices in the y-axis. No trend is evident in these plots, which may suggest the knee joint performance is insensitive to bond index under opening action, whereas some other unrelated mechanism controls the response.

6.2.2 Effects of Amounts of Confinement

Figure 6.3 shows plots of joint shear stress input ($\sqrt{f'_c}$ psi) against effective volumetric ratio of transverse reinforcement (Equation 5.3, Chapter 5) for the specimens where joint shear failure occurred subsequent to beam hinging. No definite trends are seen.

No apparent trends were evident from a plot of $d_{\text{so}}$ versus effective volumetric ratio (Figure 6.4). As was the case for closing action, the major source of deformation under opening action occurred in the plastic hinge regions of the beam and column members and not in the joint (Table 6.1, Column 15).

Figure 6.3 Sustained Joint Shear ($\sqrt{f'_c}$ in psi) Versus Effective Transverse Reinforcement Volumetric ratio.
6.1a) Experimental Test Program

6.1b) Failure Modes

Figure 6.1  Effects of Bond Index on Knee Joint Behaviour: Opening Action.

6.2a) Maximum Bond index.

6.2b) Average Bond Index

Figure 6.2  Bond Index Versus \( d_{80}(\%) \).
Figure 6.4  Effective Transverse Reinforcement Volumetric Ratio Versus $d_{60}$ (%).

6.5a) Joint Shear Stress in $\sqrt{f'_c}$ (psi)  
6.5b) Joint Shear Stress in psi

Figure 6.5  Sustained Joint Shear Versus Concrete Strength.
6.2.3 Effects of Concrete Strength

Figure 6.5 shows the plots of joint shear sustained (in multiples of $\sqrt{f_c}$ in psi) versus concrete strength (psi). It appears that sustained joint shear is insensitive to concrete strength.

6.3 Parametric Investigation of Knee Joint Model: Influence of Design Variables

A parametric study is performed here on the knee joint model design variables similar to that conducted in Section 5.3 for closing action. The same reference input values are used for the model and the design variables; i.e., those corresponding to specimen U17 from the China test program given in Table 4.1. The beam bottom and top reinforcement ratios used here are those shown in brackets in Table 4.1. Once again, this precludes a beam failure from governing the response, and allows a broader range of values for the design variables to be studied. Values selected for $\beta$ were $\beta_{bb} = \beta_{ci} = 0.3$ and $\beta_{bt} = \beta_{co} = 0.5$.

Figure 6.6 shows the effect that each of the design variables had on shear stress and shear strain at the milestone of hoop yielding. From Figure 6.6 it can be seen that increasing either the hoop reinforcement ratio or hoop or stirrup yield stress resulted in the hoops yielding at higher values for shear stress and strain. Increasing the concrete strength resulted in lower values for shear stress and strain at hoop yield. Increasing the stirrup reinforcement ratio had a mixed effect on joint shear resistance. An increasing trend was noticed in the joint shear stress, whereas a decreasing trend was seen in the joint deformation. Varying the beam span had little effect on the joint shear resistance.

The parametric trends are similar to those obtained in conditions of closing action in Chapter 5. Figure 6.7 shows a plot of shear stress versus shear strain summarizing the parameter investigation of the design variables. Once again, the arrows indicate the increasing direction of the design variable and the common point in the plot represents the value of shear stress and strain at nominal conditions.

Whereas decreasing trends are seen for joint shear stress and strain with increasing concrete strength in Figure 6.6, Figure 6.7 shows a mixed result influence on joint shear resistance. In Figure 6.7 joint shear stress increases for increasing concrete strength, whereas
Figure 6.6  Parametric Tendencies at Hoop and/or Stirrup Yield: Opening Action.
joint deformation decreases. Similar trends were seen for the interior joint model described earlier. There was no evidence of any trends in the design variables from the reduction of the available test results.
CHAPTER 7

FINITE ELEMENT MODELLING OF REINFORCED CONCRETE KNEE JOINTS

It was illustrated in the preceding discussion that several parameters influence the behaviour of knee joints subjected to earthquake type loading. Experimental test programs studying knee joint behaviour have been limited to investigating the influence of a small range of values for the important variables because of the cost of large scale testing.

Nevertheless, an extensive parametric investigation of the mechanics of knee joints is needed to quantify the significance of the various design decisions. To help achieve this objective, a finite element study of knee joint connections was performed so as to complete and corroborate the results of the experimental database compilation.

7.1 Parameter Study

Using the available experimental database, it was possible to identify those parameters that most affect the knee joint behaviour. The magnitude of the effects on knee joint behaviour resulting from changes in these parameters is investigated by means of a finite element sensitivity analysis.

In the Clarkson experimental test study, bond deterioration played a significant role in the behaviour of the knee joint specimens, to the extent that the mode of failure and strength were controlled by this effect.

The above results are the driving force behind the investigation performed here on knee joint behaviour for different anchorages of beam and column longitudinal reinforcement. Other parameters affecting knee joint behaviour that are also investigated include, concrete strength, hoop and stirrup reinforcement ratio, and, hoop and stirrup yield strengths.

Experimental test studies on knee joints have been performed with different loading conditions, which resulted in different stress combinations at the knee joint boundaries. To model the test conditions, the behaviour of the knee joint will be investigated for different combinations of shear, moment, and axial load at the boundaries of the joint panel. This will be achieved by varying the test specimen dimensions (i.e., beam length).
7.2 Description of Computer Programs Used in the Finite Element Study

In order for the calculated responses to be relevant with the experimental data, the computer program used in the finite element study should support modeling the non-linear stress-strain behaviour of reinforced concrete. In addition, it should allow for discrete modeling of reinforcing bars so that the influence of bond-slip between the concrete and reinforcement on the knee joint’s performance may be idealized and studied by special elements.

With the above criteria in mind, Stan2D (Atrach 1992), a non-iterative two dimensional non-linear finite element analysis program, that uses secant stiffness iterations with small load increments was used in the sensitivity study. The element library of the program includes general shaped four-noded plane stress elements, two-noded truss elements, and two-noded contact elements. Prior to the work by Atrach (1992), similar efforts were made at the University of Toronto towards implementing an appropriate constitutive model (Modified Compression Field Model) for reinforced concrete finite element analyses (Stevens et al 1991).

To help analyse the output, a post-processor program TRIXpost was employed (Bentz 1998). Specifically, TRIXpost was used to perform equilibrium checks, plot deflected shapes, and summarize data later used in a spreadsheet program to produce sectional plots for concrete stresses and strains.

7.2.1 Modeling of Reinforcement

Truss elements were used in the study to model the reinforcing bars. They consist of two nodal points with three translational degrees of freedom per node. For this study, the degrees of freedom allowed for the truss element were along the axes x and y of the global Cartesian co-ordinate system (Figure 7.1) whereas translation in the out-of-plane direction (z) was restrained. A trilinear model was used to define the stress-strain relationship for each steel material type (Figure 7.2).

7.2.2 Modeling Bond Between Reinforcement and Concrete

Bond slip between the concrete and reinforcing bars was modelled by the contact elements. They were used to link the nodes of the four-noded plane stress element with those
Figure 7.1  Truss Element in Local and Global Coordinate Systems.

Figure 7.2  Reinforcing Bar Stress-Strain Properties
of the truss elements (Figure 7.3).

In many of the tests from the experimental database, bond-splitting type failures dominated the knee joint response under closing action. The bond-slip model that will be adopted for this finite element study should represent closely actual bond conditions in knee joint response.

A group of pullout tests that approximate bond conditions experienced in knee joint tests, were carried out by Gambarova et al (1989). These tests consisted of a short deformed reinforcing bar (18 mm dia.) embedded in concrete with a preformed splitting crack in the plane passing through the bar longitudinal axis. The splitting crack widths were kept relatively constant throughout the pullout tests by applying a confining force. The pullout tests were performed at a constant slip rate (0.05 mm/hour) for specimens with different preformed splitting crack widths (0.0 mm, 0.1 mm, 0.2 mm, 0.3 mm).

Giuriani et al (1991) used the above test results to obtain empirical equations which relate the key parameters associated with bond between the reinforcing bar and concrete after bond splitting of the cover concrete. These parameters were identified as local bond stress, splitting crack width, longitudinal reinforcing bar slip, and radial confinement stress. This radial confining action consists of two components: the confining action arising from the residual concrete tensile stress transmitted by the crack faces; and, the confining action provided by transverse reinforcement crossing the plane of the splitting crack.
The following expression was obtained for the local bond stress as a function of reinforcing bar slip and splitting crack width:

\[ u_L = u_{m,w} \left( 1 - e^{-\left( \beta_1 \cdot \beta_2 \frac{w}{d_b} \right)} \right), \quad \text{where,} \quad u_{m,w} = u_{m,o} \left( 1 - \gamma_1 \frac{w}{d_b} \right) \]  

7.1

where; \( u_L \) is the local bond stress, \( u_{m,w} \) is the maximum local bond stress that can be obtained for a given splitting crack width, \( u_{m,o} \) is the maximum local bond stress for zero splitting crack width, \( w \) is the splitting crack width, \( s_i \) is the reinforcing bar slip and \( \gamma_1, \beta_1, \beta_2 \) are coefficients that are experimentally determined (all in consistent units).

The expression for local bond stress as a function of splitting crack width and radial confinement stress is given by:

\[ u_L = u_o \left( \frac{1}{1 + k_1 \frac{w}{d_b}} \right) + u_1 \left( \frac{1}{1 + k_2 \frac{w}{d_b}} \right) \sigma_n; \quad \text{where,} \quad \sigma_n = A \sigma_t + B \sigma_{rc} \]  

7.2

where; \( \sigma_n \) is the radial confinement stress, \( \sigma_t \) is the stress developed in the transverse reinforcement crossing the crack, \( \sigma_{rc} \) is additional confining action provided by the cracked concrete, \( A \) is the stirrup index of confinement given by the ratio of the total area of transverse reinforcement crossing the splitting crack, divided by the area of longitudinal reinforcement in the splitting plane, \( B \) is the concrete confinement index, given by the ratio of the net area of concrete in the splitting plane to the area of the layer of longitudinal reinforcing bar in the splitting plane, and \( u_o, u_1, k_1, k_2 \) are coefficients determined experimentally.

\( u_L \) in the above equation is limited to the maximum local bond stress obtained from Equation 7.1. As mentioned above, the radial confinement stress consists of a confinement contribution from the transverse reinforcing bars (\( \sigma_t \)) and residual tensile stresses from the faces of the cracked concrete (\( \sigma_{rc} \)). Expressions for these contributions follow.
Confinement contribution from the transverse reinforcement:

\[ \sigma_t = E_s \left( \frac{a_2}{d_b} \left( \frac{w}{d_b} \right)^2 + \frac{a_1}{\left( \frac{d_b}{d_b} \right)^2} \left( \frac{w}{d_b} \right) + a_0 \right) \]  

where; \( d_b \) is the transverse reinforcing bar diameter, \( \alpha_r \) is a shape factor characterizing the reinforcing bars, and \( a_0, a_1, a_2 \), are coefficients related to the anchorage conditions of the transverse reinforcement.

Contribution to Confinement Stress from Cracked Concrete:

\[ \sigma_{rc} = \frac{\sigma_{clo}}{\left( k_{cc} \frac{d_b}{d_a} \frac{w}{d_b} + 1 \right)} \]  

where; \( \sigma_{clo} \) is the maximum residual tensile strength (a function of \( \sqrt{f''} \)) of cracked concrete when the splitting crack width just begins to grow, \( d_a \) is the aggregate size, and \( k_{cc} \) is a coefficient determined experimentally.

Since the above expressions are obtained from test results using only one reinforcing bar diameter (i.e. \( d_b = 18 \text{mm} \)) Giuriani et al adopted a similarity criterion to make the expression applicable to all reinforcing bar diameters. To achieve this, they implemented the ratios \( s/d_a \) and \( w/d_b \) in the expressions. As a result, the experimentally determined coefficients are independent of the reinforcing bar diameter. A negative aspect of the empirical coefficients used in the expression obtained by Giuriani et al (1991) was that they were obtained specifically for the experiments performed by Gambarova et al (1989).

The bond constitutive model derived by Giuriani et al (1991) appears to adequately describe the bond-splitting type behaviour witnessed in the knee joint specimens and will be used here.

The values assigned to the constant in Equations 7.1 to 7.4 by Giuriani et al (1991) are given in Table 7.1.

To arrive at a suitable bond-slip model for the finite element study, Equations 7.1
to 7.4 are used to plot the bond stress-slip responses for confinement conditions ranging from no confinement to heavy confinement in the knee joint (see Figure 7.4).

The plots in Figure 7.4 are used as a reference to arrive at the two bond slip models proposed for this study (Figure 7.5). The peak bond strength shown for the "poor" bond model (Figure 7.5b) is consistent with results obtained by other researchers for anchorages with minimal or no confinement (Sozen and Moehle 1990, Tepfers 1979). The peak bond strength shown for the "regular" bond model (Figure 7.5a) lies between that for the "poor" bond model and that proposed by Eligehausen et al (1983) for the generally higher local bond strength for pullout test specimens (see Figure 7.6).

<table>
<thead>
<tr>
<th>Equation</th>
<th>Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation 7.1: $u_{m,0} = 15$ MPa; $\beta_1 = 60; \beta_2 = 0; \gamma_1 = 70; \gamma_2 = 0.8$</td>
<td></td>
</tr>
<tr>
<td>Equation 7.2: $u_o = 1.8$ MPa; $u_1 = 0.8; k_1 = 115; k_2 = 35$</td>
<td></td>
</tr>
<tr>
<td>$a_0 = \frac{a_1^2 \frac{\nu_{12}}{\nu_{21}}}{4a_2 \left( \frac{u_{12}}{u_{21}} - 1 \right)}$; $a_1 = 8 \frac{u_{o2}}{E_s}$; $a_2 = \frac{4u_{12}}{E_s} d_b$; $\alpha_r = 2$</td>
<td></td>
</tr>
<tr>
<td>$u_{12}/u_{21} = 0.3$; $u_{o2} = 2.5$ MPa; $u_{12} d_b = 500$ MPa</td>
<td></td>
</tr>
<tr>
<td>Equation 7.4: $\sigma_{eio} = 1.2$ MPa; $k_{cc} = 100$</td>
<td></td>
</tr>
</tbody>
</table>

196
7.4a) No Confinement.  

7.4b) Light Confinement.  

7.4c) Typical Confinement.  

7.4d) Heavy Confinement.  

Figure 7.4 Bond Stress-Slip Relationships.
7.5a) Regular Bond Model

7.5b) Poor Bond Model

7.5c) Poor Bond Models: Finite Element Model Predictions of Peak Load.

Figure 7.5 Proposed Bond Models for Finite Element Analyses

Figure 7.6 Bond-Slip Constitutive Model Proposed by Eligehausen et al. (1983).
7.2.3 Modeling of Concrete

The concrete in the knee joint specimens was modelled using the four-noded general shaped plane stress element defined in Figure 7.7. The plane stress element contains only corner nodes with two degrees of freedom, namely the displacements along the axes x and y of the global Cartesian coordinate system. The notation, \( f \) and \( u \) at each node for the given x or y- direction in Figure 7.7 refers to the nodal forces and displacements, respectively.

The nonlinear inelastic behaviour of concrete is modelled using empirical relationships based on the modified compression field theory (Vecchio and Collins, 1986). The stress-strain relationship for concrete in the principal compressive and tensile directions is shown in Figure 7.8. The behaviour of concrete in compression can be modelled as confined or unconfined (Figure 7.8). The notation \( f_p \) and \( \varepsilon_p \) in Figure 7.8 denotes the peak concrete stress and strain, respectively, for the unconfined and confined concrete models.

7.2.3.1 Failure Modes

Concrete crushing is defined in the program for both the tension-compression and compression-compression regions as:

**Tension-Compression Region:** \((\sigma_1, \sigma_2 < 0 \text{ and } \sigma_1 + \sigma_2 \leq 0)\)

\[
\frac{\sigma_2}{f'_c} \leq -1
\]

**Compression-Compression Region:** \((\sigma_1, \sigma_2 < 0)\)

\[
\left(\frac{\sigma_2}{f'_c} + \frac{\sigma_1}{f'_c}\right)^2 + 3.65 \frac{\sigma_1}{f'_c} \geq -\frac{\sigma_2}{f'_c}
\]

The program calculates the element stiffness matrix by using the two-point (two x two) Gauss quadrature rule to evaluate the integral. In the first load step, the element stiffness matrices are obtained assuming isotropic material behaviour. Thereafter, the element stiffness matrices are referenced to the principal axes.

Given the nodal displacements resulting from each load iteration, the element strains and stresses are calculated at each Gauss point location. Further details can be obtained from Atrach (1992).
Figure 7.7  Plane-Stress Element in Local and Global Coordinate Systems

Figure 7.8  Stress-Strain Relationship for Concrete in the Principal Tensile and Compressive Directions (Atrach 1992)
7.3 Program Verification

The performance of the program was gauged using the experimental results of specimen KJ13 from the Clarkson test study (Chapter 3). This specimen was selected to test the program because of its poor performance under closing action. Failure was caused by bond splitting at the level of the beam top reinforcement in the joint region. Figure 7.9 shows a sketch of the reinforcement details for specimen KJ13. A summary of the specimen details and steel and concrete material properties used in the analysis is given in Table 7.2.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>$f_c$ (MPa)</th>
<th>Column Reinforcement</th>
<th>Beam Top Reinforcement</th>
<th>Beam Bottom Reinforcement</th>
<th>Joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>KJ13</td>
<td>31.7</td>
<td>8 #6</td>
<td>4 #6</td>
<td>4 #6</td>
<td>4 #3 at 80mm 4 #3 at 80mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reinforcing Bar</th>
<th>$f_y$ (MPa)</th>
<th>$\varepsilon_y$</th>
<th>$\varepsilon_{sh}$</th>
<th>$E_{sh}$</th>
<th>$\varepsilon_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#3</td>
<td>445</td>
<td>0.022</td>
<td>0.022</td>
<td>2000</td>
<td>0.23</td>
</tr>
<tr>
<td>#6</td>
<td>461</td>
<td>0.023</td>
<td>0.023</td>
<td>2000</td>
<td>0.1435</td>
</tr>
</tbody>
</table>

Figure 7.10 shows the mesh used in the analysis. A 40mm x 40mm grid is used in the joint region and in the adjoining beam and column members up to a distance of 400mm away from the joint. Thereafter, the grid is 40mm x 80mm. The grid size of the mesh chosen for the analyses was governed by the requirements of the bond stress-slip model. The bond stress-slip models used in the analyses are only valid over small lengths of reinforcing bar diameter (i.e., 1 to 3 bar diameters). One mesh was chosen for this study because it was prohibitively expensive with respect to time to consider a second finer mesh. Although the same trends would persist with a second finer mesh, there may be a sensitivity with the magnitude of the results.

The boundary conditions for the knee joint model are shown in Figure 7.10. The knee joint specimen is pin connected at the column support, and pin connected at the beam end to the relatively stiff contact element, and to a very stiff truss element which extends vertically downward from the beam to another pin support.

As discussed in Section 7.2, two different bond models (Figure 7.5) were used in the
Figure 7.9 Test Specimen KJ13 Details: Clarkson Study
Figure 7.10a) Element Numbering for Knee Joint Mesh

Figure 7.10  Finite Element Mesh for Knee Joint Study.
Figure 7.10b) Node Numbering for Knee Joint Mesh

Figure 7.10(cont’d) Finite Element Mesh for Knee Joint Study.
analysis to simulate bond between the longitudinal reinforcement and the concrete; namely the regular bond model and the poor bond model. The goal is to compare knee joint behaviour for the two cases. Contact elements were also attached to the hoops and stirrups in the joint region only. At the hoop and stirrup ends a stiff bond-slip model was used in both the horizontal and vertical directions to simulate the connection with the main beam and column reinforcement. The bond-slip model used for the hoop and stirrup interior nodes was that proposed by Elghihanen (1983) (Figure 7.6). This was based on the assumption of the existence of good bond in the concrete and transverse reinforcement at those locations.

The program does not account for an increase in concrete strength due to the benefits of confinement from hoop and stirrup reinforcement in the joint, in the concrete material stress-strain relationship under compression. Using the Kent and Park (1982) confinement model discussed in Chapter 4 a strength increase of approximately 5 MPa (i.e., 31.7 MPa to 36.5 MPa) for the joint core concrete is obtained for this specimen.

To help decide on an appropriate load increment value for the finite element runs, a comparison was made between the values of the concrete secant stiffnesses in the principal compressive and tensile directions, for the previous and current load steps, based on that chosen load value.

Plots showing the ratio of the concrete secant stiffness of the current and previous load steps for both the compressive and tensile directions were used to determine the error associated with a selected load increment value.

Figure 7.11 shows such plots, which are typical of all the finite element runs done under opening and closing actions. From Figure 7.11, good convergence is seen in the secant stiffness under both closing and opening actions. The specimen was loaded by applying horizontal displacements at the beam's end (node 693, Figure 7.10b). A contact element with a high bond stiffness (k=250,000 N/mm) was used. A total of 180 load stages were needed at load increments of 46000N (to the left) under closing action and 67000 N (to the right) under opening action to achieve the desired drift levels (i.e., for closing action; drift =δ/920 where; δ =33 mm; required load increment = 46000 = [(33)(250000)]/180).

The analytical results of the knee joint response are compared with the experimental results for both closing and opening actions. The analytical results described in the following paragraphs are with the core concrete modelled as confined, except where noted.
7.11a) Opening Action

Figure 7.11  Ratio of Current to Previous Concrete Secant Stiffnesses.
Table 7.3  Execution Times for Selected Machines

<table>
<thead>
<tr>
<th>Machine</th>
<th>CPU Speed</th>
<th>Memory</th>
<th>Max. Bandwidth</th>
<th>Time/ Load Step, seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pentium II 233</td>
<td>233 MHz</td>
<td>64 MB</td>
<td>494</td>
<td>60</td>
</tr>
<tr>
<td>Pentium Pro 200</td>
<td>200 MHz</td>
<td>32 MB</td>
<td>494</td>
<td>75</td>
</tr>
<tr>
<td>Pentium 120</td>
<td>120 MHz</td>
<td>48 MB</td>
<td>494</td>
<td>215</td>
</tr>
<tr>
<td>Pentium II 300</td>
<td>300 MHz</td>
<td>64 MB</td>
<td>1240</td>
<td>300</td>
</tr>
<tr>
<td>Pentium II 233</td>
<td>233 MHz</td>
<td>64 MB</td>
<td>1240</td>
<td>360</td>
</tr>
<tr>
<td>Pentium 200</td>
<td>200 MHz</td>
<td>32 MB</td>
<td>1240</td>
<td>1800</td>
</tr>
</tbody>
</table>

The program was run on a number of different computers. Table 7.3 gives the time required per load step, the maximum bandwidth, the corresponding CPU speed, and available memory for the chosen machine. Figures 7.12 and 7.13 show typical deflected shapes for the knee joint under closing and opening actions, respectively.

Figure 7.14 shows a comparison of the beam axial load versus drift response for the finite element results using the regular and poor bond-slip models and the envelope of the experimental results under closing action. Drift is calculated as the horizontal displacement of the centre node in the joint, divided by the column length (920mm). Good agreement was seen between the experimental and analytical results for the ascending part of the load-drift response. Thereafter, the response of the “regular bond” specimen showed an increasing trend until yielding of the beam top and column inner reinforcement which occurred at their respective joint faces, at a drift level of about 1.6 per cent. Beam hinging governed knee joint response beyond this point. Peak resistance in the experimental results occurred at a drift level of about 1.3 per cent, at which point the beam and column main reinforcement strains at their respective joint faces were near yield or had just yielded. Next, there was a steady decline in the bar strains as bond-splitting failure mechanism dominated the response. There was no significant damage in the joint with the hoop and stirrup strains well below yield. The "poor bond" specimen correlated fairly well with the experimental response. Peak resistance occurred at about 1.7 per cent when the column outer reinforcement at the joint face just yielded. Thereafter, the beam and column tensile reinforcing bar strain dropped very quickly as bond-splitting failure occurred. A comparison of the distribution of hoop stresses along a vertical cross-section at the joint centre for both the experimental and “poor
Figure 7.12  Knee Joint Deflected Shape: Closing Action.

Figure 7.13  Knee Joint Deflected Shape: Opening Action.
bond” analytical model results at peak response, indicates fairly good agreement. Also included in Figure 7.14 is the response of the “regular bond” specimen with the concrete modelled as unconfined; that response is also very poor and is governed by concrete crushing in the joint. A comparison of the load versus displacement responses for both the regular-confined and regular-unconfined models indicates that assuming a horizontal plateau beyond peak conditions in the stress versus strain response for the confined concrete model (Figure 7.8) may be unrealistic.

The appropriateness of the finite element model to approximate poor bond in knee joints with standard 90° hook anchorages for the main reinforcement, was tested by comparing its results with the experimental results obtained from the Clarkson test specimens (KJ5 through KJ13) that experienced bond or anchorage type failures. Figure 7.5c) shows the bond models used for the analyses.

Table 7.4 compares the peak beam axial load values obtained from the experimental results to those obtained from the finite element results for each specimen. Good agreement is seen between the experimental and analytical results.
Table 7.4  Finite Element Model Predictions for Peak Beam Axial Load Under Closing Action: Clarkson Tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Finite Element Results</th>
<th>Experimental Results</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_{FE\text{-}max}$</td>
<td>$N_{EXP\text{-}max}$</td>
<td>$(N_{FE}\div N_{EXP}) \times 100%$</td>
</tr>
<tr>
<td>KJ5</td>
<td>249.1 (kN)</td>
<td>281.8 (kN)</td>
<td>88.4</td>
</tr>
<tr>
<td>KJ6</td>
<td>250.4 (kN)</td>
<td>256.2 (kN)</td>
<td>97.7</td>
</tr>
<tr>
<td>KJ7</td>
<td>295.0 (kN)</td>
<td>307.5 (kN)</td>
<td>95.9</td>
</tr>
<tr>
<td>KJ8</td>
<td>252.9 (kN)</td>
<td>261.3 (kN)</td>
<td>96.8</td>
</tr>
<tr>
<td>KJ9</td>
<td>254.5 (kN)</td>
<td>280.8 (kN)</td>
<td>90.6</td>
</tr>
<tr>
<td>KJ10</td>
<td>287.5 (kN)</td>
<td>288.5 (kN)</td>
<td>99.7</td>
</tr>
<tr>
<td>KJ11</td>
<td>251.7 (kN)</td>
<td>266.5 (kN)</td>
<td>94.4</td>
</tr>
<tr>
<td>KJ12</td>
<td>247.1 (kN)</td>
<td>256.2 (kN)</td>
<td>96.4</td>
</tr>
<tr>
<td>KJ13</td>
<td>284.4 (kN)</td>
<td>277.2 (kN)</td>
<td>102.6</td>
</tr>
</tbody>
</table>

Figure 7.15 compares the beam axial load-drift response of the analytical results for the "regular bond" specimen and the envelope of the experimental results under opening action. Three cases are considered for the "regular bond" specimens. Two of the three cases involve treating all the concrete as confined, while the third treats concrete as unconfined.

For one of the confined concrete runs, the concrete cover was modelled as having spalled off at the extreme compression fibres of the joint-member interface. To achieve this, the concrete strength was assigned a very small value for elements 131 and 141, and elements 240 and 250 at the column and beam member-joint interfaces, respectively (Figure 7.10a).
Figure 7.15  Comparison of Analytical Models to Experimental Results: Opening Action

Figure 7.15 indicates good agreement in the experimental and analytical results for the ascending portion of the experimental response. Model runs using the confined concrete reach a peak value and then continue into a plateau, whereas the experimental result shows a steady decline in its capacity beyond a drift level of about 2.5 per cent. The unconfined model follows the post peak response of the experimental curve somewhat better than the confined concrete models. However, it fails suddenly due to concrete crushing in the joint at about 3.1 per cent drift.

The beam bottom reinforcement yielded at about 1.2 per cent drift in the experimental test specimen, compared to a drift of about 1.5 per cent for the analytical specimen results.

Fairly good agreement is seen in the hoop stress distribution along a vertical section at the joint centre when the experimental and analytical results using confined concrete are compared at the instance of beam reinforcement yielding.
7.4 Investigation of Anchorage Detailing of Longitudinal Reinforcement on Knee-Joint Behaviour

To investigate the effect anchorage detailing has on knee joint behaviour, three different anchorage schemes were modelled, in addition to the standard 90° hook anchorage of beam and column longitudinal reinforcement in the joint region.

The three anchorage schemes involve the following changes to the “standard” model: anchorage of beam top reinforcing bars in the adjacent column (“BT” model), anchorage of column outer reinforcement in the adjacent beam (“CO” model), and anchorage of both beam top and column outer reinforcement in the adjacent column and beam, respectively (“BTCO” model).

All anchorages extended a distance of 400mm (d=h=400mm) into the adjacent members, and were modeled by properly extending the beam top or the outer column reinforcing bars of the “standard” model. When modeling the knee joint specimens with bars that extended into the adjacent member, the end of anchorage was located at the same position as the tension reinforcement of that member. From a practical standpoint, this modeling approach was considered unrealistic. As a result of the close proximity of the longitudinal reinforcement, significant bond-splitting (Orangun et al 1977) would occur at the joint faces, and the tensile reinforcement would not be able to attain its yield strength. To model the extremely poor bond conditions that would result, the bond model indicated by the thick solid line in Figure 7.5b) was used. Given that the bars extending into the adjacent member and the existing tension reinforcement of that member are all placed at the same level assuming a value of zero for the stirrup index of confinement (i.e., $A = 0$; Absence of confinement) is reasonable. This finite element model will be referred to as BTCO-SP in the following discussion.

Actual placement of bars that extend into the adjacent members are offset to avoid the bond related problems described above. Thus to model actual conditions more realistically, the beam top reinforcement was anchored over a distance of 400mm in the adjoining column member and 80mm on the inside of the column outer reinforcement. Similarly, the column outer reinforcement was anchored over a distance of 400mm in the adjoining beam member and 80mm below the beam top reinforcement (Figure 7.16). This finite element model with the offset anchorage detail will be referred to as BTCO-2 in the
Finite element analyses were performed using the three anchorage schemes for the regular bond and poor bond models.

**Load Displacement Plots**

Figure 7.17 shows the beam axial load-drift responses of the analytical models for both the regular and poor bond models under closing action. Also included in the figure are an envelope of the actual experimental results for Specimen KJ13, which contained a standard $90^\circ$ hook anchorage of beam and column reinforcement into the joint. Results indicate from Figure 7.17a), with the exception of BTCO model, that, provided “adequate” bond conditions exist in the knee joint, the anchorage detailing scheme used does not affect the knee joint behaviour. In reality, however, the performance of knee joints with standard anchorage detailing in existing structures and in the lab environment under earthquake-type loading, has been very poor. The poor behaviour is attributed to splitting-bond failure of the longitudinal reinforcement in the joint. To investigate the effects of anchorage on knee joint performance under bond conditions that are representative of true conditions, the “poor bond” model was used in the knee joint analytical models with the different anchorage schemes (Figure 7.17b).

From Figure 7.17b) it appears that providing anchorage of either the column or beam
7.17a) Beam Axial Load Versus Drift: Regular Bond - Closing

7.17b) Beam Axial Load Versus Drift: Poor Bond - Closing

Figure 7.17 Beam Axial load Versus Drift Plots for Different Anchorage Detailing Schemes.
longitudinal reinforcement in the adjoining members results in improved performance of the knee joint specimen. If the anchorage detail is provided for both column and beam reinforcement, then, the knee joint performance is significantly enhanced.

Little difference is seen in the performances of the BTCO and BTCO-2 models. However, when accounting for the extremely poor bond conditions that result from the close proximity of the longitudinal reinforcement (BTCO-SP model), a much better performance is seen in the BTCO-2 model where the extending bars are offset from the tension reinforcement at the member - joint face. The performance of the BTCO-SP model was governed by bond splitting of the beam top and column outer reinforcement. These bars did not yield at or away from their respective joint faces.

In both the BTCO and BTCO-2 models, the tensile reinforcement (beam top bars and column outer bars) yield away from the joint faces prior to yielding at the joint. The above behaviour, in addition to the better anchorage conditions, may explain the enhanced knee-joint behaviour seen in the BTCO model over the other analytical models. Yielding away from the joint faces results in a lower shear input to the joint. In addition, although the bars may ultimately yield at the joint faces extensive member hinging will not occur there and hence anchorage deterioration will be minimized. An investigation of the stress plots and drift contributions of the knee joint components might help explain the behaviour.

Reinforcing Bar Stress Plots

To provide further insight into the observed differences in behaviour in the above anchorage schemes, plots of reinforcing bar stresses were made for the four analytical models.

Figures 7.18 and 7.19 show the stress profiles for both column and beam longitudinal anchorage reinforcement for the standard model and BTCO model. Stress plots for each longitudinal reinforcing bar are for the regular bond and poor bond models. In addition, each plot contains the reinforcing bar stress plots for three different load stages, namely at (i) low load (elastic condition), (ii) load at yielding of longitudinal reinforcement at the joint faces (or load stage of significant stiffness deterioration), and (iii) ultimate or near ultimate conditions. Stress plots for the anchorage reinforcement of the CO and BT model specimens under closing action and for the standard model specimen under opening action, are provided.
7.18a) Beam Top Reinforcing Bar Stresses: a) Regular Bond  b) Poor Bond

Figure 7.18  Anchorage Reinforcing Bar Stress Profiles: Standard Anchorage.
7.18b) Column Outer Reinforcing Bar Stresses: a) Regular Bond b) Poor Bond

Figure 7.18 (Cont’d) Anchorage Reinforcing Bar Stress Profiles: Standard Anchorage.
7.19a) Beam Top Reinforcing Bar Stresses: a) Regular Bond b) Poor Bond

Figure 7.19 Anchorage Reinforcing Bar Stress Profiles: BT Anchorage.
7.19b) Column Outer Reinforcing Bar Stresses:  a) Regular Bond  b) Poor Bond

Figure 7.19 (Cont'd)  Anchorage Reinforcing Bar Stress Profiles:  BT Anchorage.
in Appendix D. For the finite element analyses using the “regular bond” model, member hinging governed the response. For those runs that used the “poor bond” model, with the exception of BTCO and BTCO-2 anchorage models, bond degradation resulted in deterioration in the specimen’s response at the higher load steps. Thus, ultimate conditions are defined here as the load stage where the peak load capacity is attained, or near the final load stage where the load-drift curve plateaus.

In general, the stress in the reinforcing bars at the joint face attenuates quicker along the anchored length when comparing the “regular bond” model specimens to the “poor bond” model specimens. A second, and perhaps more significant observation, is that the entire anchorage bar length is active in bond transfer (Figures 7.18 and 7.19). This may explain the better performance of the specimens with anchored beam or column reinforcement in the adjoining members, which is consistent with the better performance seen in the China test specimens over those tested in the Clarkson series of specimens. In addition, this observation may validate utilizing the bond index as defined and used in Chapters 5 and 6 in design recommendations for reinforced concrete knee joints.

Drift Contribution of Knee joint Components

To obtain further insight into the effects of different anchorage detailing on knee joint behaviour, the drift contribution of each knee joint component is investigated. The knee joint components believed to contribute to specimen drift are: beam member flexure (BF), column member flexure (CF), beam slip (BS), column slip (CS), and, joint distortion (JD).

Drift contributions from beam and column member flexure are given as:

\[ BF = \frac{t_b}{l_b} \]  \hspace{1cm} 7.7

\[ CF = \frac{t_c}{l_c} \]  \hspace{1cm} 7.8

where; \( t_b \) and \( t_c \) are the tangent offsets of the joint faces from the beam and column supports, respectively. The tangent offsets were obtained using the moment - area method. The curvatures were calculated at sections along the member’s length using the strain readings.
from the outermost layers of reinforcement at that section. Using this approach it is assumed that the beam and column members behave as cantilevers with the angle between the member - joint faces being maintained.

Drift contributions from beam and column slip are given as:

\[ CS = \frac{c_{\text{slip}}}{\text{arm}_c} \left(1 - \frac{h_b}{2l_c}\right) \]  \hspace{1cm} (7.9)

\[ BS = \frac{b_{\text{slip}}}{\text{arm}_b} \left(1 - \frac{h_c}{2l_b}\right) \]  \hspace{1cm} (7.10)

where; \( b_{\text{slip}} \) and \( c_{\text{slip}} \) are the beam and column bar slip values at the respective joint faces, \( \text{arm}_c \) is the effective depth for the column (i.e., 0.7 \( h_c \)) and \( \text{arm}_b \) is the effective depth for the beam (i.e., 0.7 \( h_b \)). The beam and column bar slip values are obtained by accumulating the slip over their entire anchorage lengths.

Drift contribution from joint distortion is given as:

\[ JD = \gamma \left(1 - \frac{h_c}{2l_b} - \frac{h_b}{2l_c}\right) \]  \hspace{1cm} (7.11)

where; \( \gamma \) is the joint distortion obtained as the difference in strain (\( \varepsilon_{45} - \varepsilon_{135} \)) from two lines in the joint. These lines are the two diagonals of a grid of 4 points (2x2) that form a square in the joint core.

Figures 7.20 and 7.21 show drift contributions under opening action, for the standard anchorage-regular bond model and, for KJ13 from the Clarkson experimental test study, respectively. From Figure 7.20, the contributions from beam and column slip provide the majority of the knee joint's total drift at the lower drift levels. At higher drift levels, the beam flexure contribution to total drift begins to increase. From Figure 7.21, the contributions from beam and column flexure dominate at the low drift levels and show a small increasing trend beyond 2 per cent total drift. Drift contributions from beam and
Figure 7.20  Knee Joint Component Drift Contributions: Opening Action, Standard Anchorage-Regular Bond Model.

Figure 7.21  Knee Joint Component Drift Contributions: Opening Action, Experimental Results (Specimen KJ13).
column slip dominated in terms of contribution to the knee joint's total drift beyond about 1 per cent total drift.

Figures 7.22 and 7.23 show drift contributions for the four anchorage models being considered under closing action assuming "regular" and "poor bond" conditions, respectively. Using the "regular bond" anchorage model, Figure 7.22 shows that beam and column slip provided a major portion of the total drift at the lower drift levels of specimen response, whereas the flexure contribution to drift increased at higher drift levels.

From Figures 7.22b) and 7.22c), it appears that the member with the longer anchorage of tensile reinforcement provided higher flexure and slip contributions to total drift when compared to their counterparts with the standard 90° hook anchorages (similar trends are seen in Figures 7.23b and 7.23c). These trends observed above for the drift contributions also depend on the demand placed on the tensile reinforcement at the joint face as well as the current strain level in that reinforcement.

It is also evident from Figure 7.22 that providing the longer anchorage lengths for both beam and column tensile reinforcement results in a higher joint contribution to drift and a lower contribution to drift from beam and column slip. This may explain the better axial load-drift response seen in the BTO anchorage model when compared to the other models.

When the poor bond anchorage model is used, different trends are seen in the four anchorage models. Figure 7.23a) indicates that for the standard anchorage model, beam slip provided the highest contribution to total drift. This may explain the poor performance seen in its axial load versus drift response (Figure 7.17b).

The drift contributions for the BT and CO anchorage models, Figures 7.23b and 7.23c, respectively, show that beam and column contributions to total drift were highest at low and high (especially BT model) drift levels with a small increase in the total drift contribution from beam and column flexure occurring at about 2 per cent total drift due to reinforcement yielding.

The beam and column slip contributions to total drift for the BTO model (7.23d) were their highest until about 2 per cent drift when contributions to total drift from beam and column flexure began to increase. It is evident from Figures 7.23 and 7.17b) that by extending one or both of the beam and column tensile reinforcement beyond the joint, and
Figure 7.22 Knee Joint Component Drift Contributions: Closing Action, Regular Bond Model.
Figure 7.23  Knee Joint Component Drift Contributions: Closing Action, Poor Bond Model.
anchoring them in the adjacent member will result in improved specimen performance. Beam and/or column slip is reduced and the flexural capacity of the beam and column members is allowed to be more fully developed when compared to the specimen with standard 90° hook anchorages.

Figure 7.24 shows the drift contributions under closing action obtained from the experimental results of specimen KJ13. From Figure 7.24, drift contributions from beam and column flexure dominate at low drift levels, whereas beam and slip contributions to total drift dominate beyond the 1% total drift level.

The knee joint component drift contributions using the experimental results from KJ13 were also obtained from Equations 7.7 through 7.11.

The joint distortion $\gamma$ was obtained from the difference in strain readings ($\varepsilon_{45°} - \varepsilon_{135°}$) taken from the embedded strain gauges in specimen KJ13 (Figure 3.7).

The beam and column slip values were obtained using the experimental results as follows:

![Figure 7.24 Knee Joint Component Drift Contributions: Closing Action, Experimental Results (Specimen KJ13).](image-url)
Given the strain histories for the tensile reinforcement at the beam and column member faces, an appropriate algorithm that models the stress versus strain behaviour for reinforcement subjected to reversed cyclic loads was needed. The Menegotto-Pinto Model (Menegotto and Pinto 1977) was used here to obtain the bar stresses at the joint face for the given strain history and specifically for the drift level of interest. Next, a constant bond stress assumption was made for bond along the entire length of the anchorage bar. Bond stress values of 8 MPa and 4 MPa were chosen under opening and closing actions, respectively (Figures 7.5a and 7.5b). This together with the reinforcing bar stress at the joint face were used to obtain the distribution of stresses and corresponding strains along the bar’s anchorage length. The beam and column slip values were obtained by integrating the strains along the entire anchorage length of the bar.

The tangent offsets of the joint faces from the beam and column supports were obtained using the moment-area method as was done for the finite element results. However, strain readings were not taken along the entire length of the beam and column members in the experimental study. To help obtain the curvature distribution along the members length the following assumptions were made in addition to those above: Flexural hinging was assumed to occur at the member-joint faces; the hinge length depended on the length of the member, and the ratio of the moment at the member - joint face obtained from the statics of the system to the yield moment of that member; the remaining length of the member (outside hinge zone) was assumed to behave elastically.
7.5 Evaluation of Model Assumptions

7.5.1 Assumptions of Strain Gradient in Knee Joint

In formulating the mechanical model in Chapter 4, a number of assumptions were made regarding concrete stress and strain in the joint region. It was assumed that the concrete joint shear stress (and shear strain) and normal strains varied linearly in the knee joint panel for cross sections in both the horizontal and vertical directions. Model restrictions, however, resulted in average values for the above variables at the joint centre.

To investigate the assumptions postulated above, section plots for concrete joint shear stress and strain ($\tau_{xy}$ and $\gamma_{xy}$) and concrete normal strain in the $x$ ($\varepsilon_{x}$) and $y$ ($\varepsilon_{y}$) directions are given for the standard anchorage model specimen for closing action (regular and poor bond models) and opening action (regular bond model) in Figures 7.25 to 7.27.

Each variable investigated ($\varepsilon_{x}$, $\varepsilon_{y}$, $\varepsilon_{xy}$, $\gamma_{xy}$) is plotted for vertical and horizontal sections at and near the joint centre. Variable stress-strain plots are drawn at each section for four different load stages, namely, LS1 for isotropic behaviour of concrete, LS2 for the connection assumed still in the elastic range, LS3 at yield of longitudinal reinforcement at the joint faces (or when significant stiffness deterioration is witnessed), LS4 at ultimate or near ultimate conditions.

**Normal Concrete Strains: $\varepsilon_{x}$, $\varepsilon_{y}$**

In general, $\varepsilon_{x}$ and $\varepsilon_{y}$ are linear only for the isotropic response (LS1). Thereafter, at higher load steps, a roughly parabolic shaped distribution in most cases skewed is seen to develop (Figures 7.25a, 7.26a, and 7.27a).

**Joint Shear Stress and Shear Strain**

The joint shear stress is parabolic for the load stage where concrete properties are isotropic (LS1). At higher load stages, a roughly parabolic shape is seen for the shear stress (Figures 7.25b, 7.26b, and 7.27b).

The joint shear strain also results in a roughly parabolic distribution for the higher load stages. This trend is more pronounced for closing action and for shear strains across horizontal sections.
It appears under closing action, in many instances, assuming average values for concrete joint shear stress and normal concrete strain at the joint centre may be adequate to describe the strain or stress distributions in the joint (see Figures 7.25 and 7.26).

Under opening action, a linear approximation may be adequate for the normal strains ($\varepsilon_x$, and $\varepsilon_y$). As was done for closing action, assuming average values for the concrete joint shear stress at the joint centre adequately describe conditions in the joint.

Stress and strain sectional plots for the CO and BTCO anchorage model specimens are provided in Appendix E. Similar trends are seen when the concrete strain and stress distributions for the different anchorage models are compared.
7.25a) Normal Concrete Strains, $\varepsilon_x$ and $\varepsilon_y$: Standard Anchorage Model - Regular Bond

Figure 7.25   Section Plots of Concrete Strains and Stresses: Closing Action.
7.25b) Concrete Joint Shear Stress, $v_{xy}$: Standard Anchorage Model - Regular Bond

Figure 7.25  Section Plots of Concrete Strains and Stresses: Closing Action.
7.25c) Concrete Shear Strains, \( \gamma_{xy} \): Standard Anchorage Model - Regular Bond

Figure 7.25  Section Plots of Concrete Strains and Stresses: Closing Action.
7.26a) Normal Concrete Strains, $\varepsilon_x$ and $\varepsilon_y$: Standard Anchorage Model - Poor Bond

Figure 7.26  Section Plots of Concrete Strains and Stresses: Closing Action.
7.26b) Concrete Joint Shear Stress, $v_{xy}$: Standard Anchorage Model - Poor Bond

Figure 7.26  Section Plots of Concrete Strains and Stresses: Closing Action.
7.26c) Concrete Shear Strains, $\gamma_{xy}$: Standard Anchorage Model - Poor Bond

Figure 7.26  Section Plots of Concrete Strains and Stresses: Closing Action.
7.27a) Normal Concrete Strains, $\varepsilon_x$ and $\varepsilon_y$: Standard Anchorage Model - Regular Bond

Figure 7.27 Section Plots of Concrete Strains and Stresses: Opening Action.
7.27b) Concrete Joint Shear Stress, $v_{xy}$: Standard Anchorage Model - Regular Bond

Figure 7.27  Section Plots of Concrete Strains and Stresses: Opening Action.
Figure 7.27  
Section Plots of Concrete Stains and Stresses: Opening Action.

7.27c) Concrete Shear Stains, Yx: Standard Anchorage Model - Regular Bond
7.5.2 Assumptions for Moment Gradient in Knee Joint Region

In formulating the model, it was assumed that the stresses at the respective joint faces for the beam and column reinforcement attenuate within the joint as a result of bond transfer. The model parameter $\beta_T$ is used to quantify the joint shear input in the knee joint model. The assumptions for $\beta_T$ are discussed in the following section. In this section, the plausibility of defining the joint shear input for knee joints as the moment gradient across the joint as done for interior joints, is investigated for both the horizontal (column moment gradient across joint) and vertical (beam moment gradient across joint) directions. The joint depths used to calculate the moment gradient are determined as was done in Chapter 5.

The joint shear values obtained from the beam and column moment gradient described above are compared with the corresponding average joint shear stress values at the joint midheight in Table 7.5. Results show good agreement between the average joint shear values at the joint midheight and those obtained as the moment gradient through the joint.

7.5.3 Assumptions for Model Parameter $\beta_T$

$\beta_T$ represents the ratio of the longitudinal to transverse reinforcing bar stresses at the joint centre. To gain further insight of the magnitudes for $\beta_T$, Figures 7.28 to 7.30 show plots of reinforcing bar stresses for horizontal and vertical sections taken at the midheight of the joint. Plots of the horizontal and transverse reinforcement are shown for three load stages: LS1, low (elastic or near elastic behaviour), LS2, yield of longitudinal tensile reinforcement at joint face, and, LS3, at ultimate or near ultimate conditions.

Figure 7.28 shows reinforcement stress plots for the standard anchorage and regular bond models under opening action.
Table 7.5  Investigation of Moment Gradient in Joint.

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Z'P8 1

L'ESE

1Z'Z

9'1 9C

S'OEI

28'0

O'LZI

(NY

(varu)

ZPI'I
101'1
ZSO' 1

OPO' I

PSO' I

-

(NA)

pla!A-08


It appears that the stress distributions are approximately linear for horizontal (Figure 7.28a) and vertical (Figure 7.28b) sections at the midheight of the joint. This may imply a bending component in addition to the shear component contributing to joint resistance. Thus, there appears to be a stress gradient across both horizontal and vertical sections at the joint’s midheight under opening action.

The bond efficiency factors, $\beta_T$'s, used in the mechanical knee joint model need to take into account the bond slip along the lead-in length of the anchorage reinforcement (represented by $\beta$) as well as the stress gradient across the joint (represented by $\beta_G$). From the finite element results (Figure 7.28) the ratio between the stresses in the anchorage reinforcement to those in the transverse reinforcement at the joint centre gives the true value of the bond efficiency factor ($\beta_T$). The $\beta_T$ value contains the effects, with regards to shear input to the joint, of both the bond slip along the lead-in length of the anchorage reinforcement and the stress gradient across the joint.

Based on the above discussion the bond efficiency factor, $\beta_T$ (used in the mechanical knee joint model) may be obtained as follows:
\[ \beta_T = \beta \cdot \beta_G = \left( \frac{\sigma_{\text{anch}}}{\sigma_{\text{ta}}} \right) \left( \frac{\sigma_{\text{ta}}}{\sigma_{\text{tc}}} \right) = \left( \frac{\sigma_{\text{anch}}}{\sigma_{\text{tc}}} \right) \]

where; \( \sigma_{\text{anch}} \) is the anchorage reinforcement stress at the joint centerline, \( \sigma_{\text{ta}} \) is the stress in the transverse reinforcement at the joint centre, and \( \sigma_{\text{ta}} \) is the transverse reinforcement stress at the joint centerline adjacent to the anchorage reinforcement. To obtain the value of \( \beta \) in the absence of a transverse reinforcing bar near the anchorage reinforcement at the joint centerline, the concrete strain (Figures 7.25-7.27) may be used to calculate the transverse reinforcement strain. Using this approach assumes that there is perfect bond between the concrete and transverse reinforcement.

Recall, that for the interior joint model (Chapter 2) only the bond slip along the lead-in length of the anchorage reinforcement needs to be accounted for. This was done using the bond efficiency factor \( \beta \). The value of \( \beta \) varied between 0 (perfect bond) and 1 (poor bond). As discussed above, the knee joint model needs to account for both the bond slip (\( \beta \)'s) and stress gradient (\( \beta_G \)'s) across the joint (Equation 7.12). Similar to the interior joint model the \( \beta \) values in the knee joint model should also be between 0 and 1.

For example, from the horizontal joint section, Figure 7.28a) for load stage 3 (LS3), the ratio of reinforcement stresses for the column inner bars (\( \sigma_{\text{ci}} \)) and adjacent stirrup bars (\( \sigma_{\text{si}} \)) is 0.89 (i.e., \( \beta_{\text{ci}} = \sigma_{\text{ci}} / \sigma_{\text{si}} = 330 / 370 = 0.89 \)). In the same manner, a \( \beta \) value of 0.74 is obtained for the ratio of reinforcement stresses given by the beam bottom bars at the joint centerline and the adjacent hoop bars (H4) (i.e., \( \beta_{\text{bb}} = \sigma_{\text{bb}} / \sigma_{\text{h4}} = 325 / 440 = 0.74 \)).

Figures 7.29 and 7.30 give plots of reinforcing bar stresses at the joint midheight for the standard anchorage model for conditions of "regular bond" and "poor bond", respectively, under closing action. Results indicate that stresses appear to be more uniformly distributed than for conditions under opening action. Higher stresses developed in the tensile longitudinal reinforcement whereas lower stresses were seen in the transverse reinforcement when comparing results from the "poor bond" model to those of the "regular bond" model (Figures 7.29a, 7.30a, and Figures 7.29b, 7.30b).
7.29a) Horizontal Section

7.29b) Vertical Section

Figure 7.29  Reinforcement Stress Distributions at Joint Midheight- STD Anchorage, Regular Bond Model: Closing Action.

7.30a) Horizontal Section

7.30b) Vertical Section

Figure 7.30  Reinforcement Stress Distributions at Joint Midheight- STD Anchorage, Poor Bond Model: Closing Action.
Table 7.6 gives the calculated $\beta$ values for the standard anchorage - regular bond model under both opening and closing actions. From Table 7.6 a broad range is seen for the $\beta$ values. However, in most cases the values are reasonably close to the range of 0 to 1 defined in the mechanical knee joint model. When the influence of the stress gradient across the joint is accounted for, values for the bond efficiency factors ($\beta_T$'s) range between approximately 0 for the main compression reinforcement, to equal to or greater than 1 for the main tension reinforcement, under both opening and closing actions (Figures 7.28 to 7.30).

Similar trends were seen in the reinforcement stress distributions under closing action for the different anchorage conditions (i.e., CO, BT, BTCO) considered in this study. The results are given in Appendix F.

Table 7.6 Calculated $\beta$ Values

<table>
<thead>
<tr>
<th>Load Stage</th>
<th>$\beta$ Values</th>
<th>Standard Anchorage Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_{bt}$</td>
<td>Opening Action</td>
</tr>
<tr>
<td>LS1</td>
<td>$\beta_{co}$</td>
<td>Regular Bond</td>
</tr>
<tr>
<td>(Elastic)</td>
<td>$\beta_{bb}$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\beta_{ci}$</td>
<td>-0.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.68, 0.48 *</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.71, 4 *</td>
</tr>
<tr>
<td>LS2</td>
<td>$\beta_{bt}$</td>
<td>-3.05</td>
</tr>
<tr>
<td>(Reinforcement Yield)</td>
<td>$\beta_{co}$</td>
<td>-------------------------</td>
</tr>
<tr>
<td></td>
<td>$\beta_{bb}$</td>
<td>0.61, 0.59 *</td>
</tr>
<tr>
<td></td>
<td>$\beta_{ci}$</td>
<td>0.66, 1.16 *</td>
</tr>
<tr>
<td>LS3</td>
<td>$\beta_{bt}$</td>
<td>-0.51</td>
</tr>
<tr>
<td>(Ultimate)</td>
<td>$\beta_{co}$</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>$\beta_{bb}$</td>
<td>0.73, 0.74 *</td>
</tr>
<tr>
<td></td>
<td>$\beta_{ci}$</td>
<td>0.72, 0.89 *</td>
</tr>
</tbody>
</table>

* Using transverse bar stress
7.5.4 Assumptions for Principal Stress/Strain Direction

The knee joint model formulated in Chapter 4 calculates the principal stress direction at the joint centre for each load stage considered. It is assumed in the model that this value is a good representation of the principal stress direction for the knee joint core region where resistance to shear occurs. To investigate the validity of the model’s assumptions, Figures 7.31 and 7.32 show the principal strain directions of the standard anchorage - "regular bond" specimen under opening and closing actions, respectively, for the conditions of: longitudinal reinforcement yield at the joint face, and, at or near ultimate conditions.

Results show that the principal strain directions are approximately equal for different regions within the joint core. In addition, when comparing principal strain orientations for the two load stages considered, there is a change in the magnitude but there is very little change in the orientation of the principal strains.
7.31a) $\varepsilon_1$: At Reinforcement Yield (Joint Face).

7.31b) $\varepsilon_1$: At or Near Ultimate.

7.31c) $\varepsilon_2$: At Reinforcement Yield (Joint Face).

7.31d) $\varepsilon_2$: At or Near Ultimate.

Figure 7.31 Orientation of Principal Strains- STD Anchorage, Regular Bond Model: Opening Action.
7.32a) $\varepsilon_1$: At Reinforcement Yield (Joint Face).

7.32b) $\varepsilon_1$: At or Near Ultimate.

7.32c) $\varepsilon_2$: At Reinforcement Yield (Joint Face).

7.32d) $\varepsilon_2$: At or Near Ultimate.

Figure 7.32 Orientation of Principal Strains- STD Anchorage, Regular Bond Model: Closing Action.
7.6 Parametric Study of Knee Joint Mechanics

As was done in Chapters 5 and 6, where a parametric study of the knee joint design parameters was performed using the knee joint model (Chapter 4), a parametric study of these variables using the finite element knee joint model is also performed here to verify the variable trends given by the knee joint model.

Tables 7.7 and 7.8 provide a summary of the range of knee joint design variables investigated for closing and opening actions, respectively. The standard anchorage-regular bond finite element model is used here (Sections 7.3 and 7.4) for closing and opening actions. As discussed earlier, this model corresponds to specimen KJI3 for the Clarkson test study (Table 7.2 and Figure 7.9).

The reinforcement properties given in Table 7.2 were used for the parametric study under opening action. For closing action, the areas of the beam and column longitudinal reinforcement were increased to allow for a higher joint shear input. The areas for the beam top and bottom reinforcement were increased from 1140mm$^2$ (4#6 rebar) to 1425mm$^2$ (5#6 rebar), whereas the areas for the column outer and inner reinforcement were increased from 855mm$^2$ (3#6 rebar) to 1140mm$^2$ (4#6 rebar). The two #6 column middle reinforcing bars were unchanged. The transverse reinforcement yield strength was changed from 440MPa to 400MPa. As before, the specimens were loaded by applying horizontal displacement at the beam end. For each run, the milestone of hoop or stirrup yield was recorded.

Table 7.7 Range of Design Variables: Parametric Study - Closing Action

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Nominal Values</th>
<th>Range Considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{yh}$, $f_{ys}$ MPA</td>
<td>400</td>
<td>300, 400, 500</td>
</tr>
<tr>
<td>$f'_{ce}$ MPA</td>
<td>31.7</td>
<td>20, 31.7, 40, 60</td>
</tr>
<tr>
<td>$\rho_h$</td>
<td>.00786 (4#3 rebar)</td>
<td>*1 (2#3, 4#3, 4#4)</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>.00480 (4#3 rebar)</td>
<td>*2 (2#3, 4#3, 4#4)</td>
</tr>
<tr>
<td>$\ell_b$, fraction of column depth (400mm)</td>
<td>3.3</td>
<td>3.3, 4.3, 5.3</td>
</tr>
</tbody>
</table>

*1 .00393, .00786, .01397, *2 .00262, .00524, .00931350
Table 7.8 Range of Design Variables: Parametric Study - Opening Action

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Nominal Values</th>
<th>Range Considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{yh}$; $f_{ys}$, MPa</td>
<td>450</td>
<td>300, 450, 550</td>
</tr>
<tr>
<td>$f'_{c}$, MPa</td>
<td>31.7</td>
<td>20, 31.7, 40, 50</td>
</tr>
<tr>
<td>$\rho_h$; $\rho_s$</td>
<td>.00786, .00524 (4#3 hoops &amp; stirrups)</td>
<td>*(^{1}) (2#3, 4#3, 4#4)</td>
</tr>
<tr>
<td>$l_b$, fraction of column depth (400mm)</td>
<td>3.3</td>
<td>3.3, 4.3, 5.3</td>
</tr>
</tbody>
</table>

*\(^{1}\) .00393; .00262; .00786; .00524; .01397; .0093135

7.6.1 Closing Action

Figure 7.33 shows the beam axial load-drift response for each of the design variables and ranges considered. With the exception of the variables $f'_{c}$ and $l_b$, the ranges considered for each of the remaining variables did not result in significant variations in the load-drift responses.

Figures 7.34 and 7.35 plot $\sqrt{f'_{c}}$ (MPa units) and $\nu$ (in MPa units) versus $\gamma$ (rads), respectively. The $\nu$-$\gamma$ response for the design variable beam length ($l_b$) are not plotted because very little change was seen in these values from varying the beam length. In fact, only small changes were seen in the $\nu$-$\gamma$ response at the milestone of hoop yielding for the remaining variables. The joint shear stress was obtained using the ACI352-85 approach. In general, first yield occurred in the hoop (H2) near the joint centre.

Comparing the parametric trends obtained here, Figures 7.34 and 7.35, using the knee joint finite element model with those obtained using the mechanical knee joint model, (Figures 5.6 and 5.7 respectively), good agreement is seen in all cases but for the effect of $f'_{c}$ (discrepancy occurs in comparison of $\nu$(MPa) versus $\gamma$ plot) on the $\nu$-$\gamma$ plots.

Further analyses are needed for conditions of closing action and opening action (Section 7.6.2) to produce a broader spectrum of $\nu$-$\gamma$ responses for each design variable to help confirm the above trends.
7.33a) Transverse Reinf. Yield Strength, $f_y$.

7.33b) Concrete Strength, $f'_c$.

7.33c) Hoop Reinforcement Ratio, $\rho_h$.

7.33d) Stirrup Reinforcement Ratio, $\rho_s$.

7.33e) Beam Member Length, $l_b$.

Figure 7.33  Beam Axial Load Versus Drift Responses: Design Parameters- Closing Action.
Figure 7.34 Joint Shear Stress ($\sqrt{f_c}$, MPa) Versus Joint Shear Strain: Closing Action.

Figure 7.35 Joint Shear Stress (MPa) Versus Joint Shear Strain: Closing Action.
7.6.2 Opening Action

Figure 7.36 shows the beam axial load-drift response for the design variables under opening action. Figures 7.37 and 7.38 plot $v$ in $\sqrt{f'_c}$ (MPa units) and $v$ (in MPa units) versus $\gamma$ (rads), respectively. In general, first yield occurred in the bottom hoop (H4) near the joint centre. When the parameter trends obtained using the finite element model (Figures 7.37 and 7.38) are compared with those obtained using the mechanical knee joint model (Figures 6.6 and 6.7) respectively, similar trends are seen in all cases.

Figure 7.36  Beam Axial load-Drift Responses: Design Parameters - Opening Action.
Figure 7.37 Joint Shear Stress ($\sqrt{f_c}$, MPa) Versus Joint Shear Strain: Opening Action.

Figure 7.38 Joint Shear Stress (MPa) Versus Joint Shear Strain: Opening Action.
CHAPTER 8

DESIGN RECOMMENDATIONS FOR REINFORCED CONCRETE KNEE JOINTS

8.1 Bond Index: Limits for Nominal Joint Shear Stress, Closing Action.

8.1.1 Knee Joints Detailed with Standard 90° hook anchorage for Beam and Column Reinforcement: Clarkson Experimental Test Study.

Knee joint connections detailed with standard 90° hook anchorages for the beam and column reinforcement showed distress at joint shear input levels between 7 - 9 $\sqrt{f'_c}$ psi (0.58 - 0.75 $\sqrt{f'_c}$ MPa). Significant stiffness degradation and strength loss in the specimens occurred at drift levels between 1.5 and 2.5 per cent.

These specimens typically had a Bond Index between 6.5 and 8. The low drift levels listed above raise concerns for knee joint connections in bridge bents of multispans bridges which may undergo several cycles at these drift levels during earthquake conditions.

Thus, for knee joints detailed with the standard 90° hook anchorage described above, that have Bond Indices between 6 and 8 for the beam and column tension reinforcement, the current draft ACI352 recommendations of 8 $\sqrt{f'_c}$ psi (0.66 $\sqrt{f'_c}$ MPa) for the nominal joint shear stress may be unconservative. Instead, based on the findings here, a value of 6 $\sqrt{f'_c}$ psi (0.5 $\sqrt{f'_c}$ MPa) should be used.

8.1.2 Knee Joints Detailed with Extended Beam and Column Reinforcement Anchorage: Tests Performed in China.

Specimens with Bond Indices less than 6 for both beam and column reinforcement, were capable of sustaining joint shear stress levels between 11 and 13 $\sqrt{f'_c}$ psi (0.83 - 1.25 $\sqrt{f'_c}$ MPa) ranging at 3.5 per cent to 6 per cent drift.

The anchorage detail of these specimens was based on extending beam and column tension reinforcement past the 90° hook into the adjacent members. Alternative details were continuous looping of the column reinforcement from the inside corner to the outside corner of the joint.
Thus, for knee joints detailed with extended anchorage for the beam and column reinforcement, with Bond Indices of less than 6 for the beam and column tension reinforcement, it is recommended raising the nominal joint shear strength to $10\sqrt{f'_{c}}$ psi (0.83 $\sqrt{f'_{c}}$ MPa). One consequence of extending both beam and column reinforcement beyond the hook, and anchoring them in the adjacent column and beam members, respectively is that member hinging may occur away from the joint faces. This was witnessed in both the finite element study results performed here (BTCO and BTCO-2 models) as well as in the experimental test study on knee joints performed in China.

As mentioned in Chapter 1, flexural yielding in the columns is the preferred source of inelastic behavior in bridge structures to allow for the easy detection and repair of damage that may occur during a large earthquake. Should damage occur in the beam member away from the face of the joint as a result of beam flexural hinging, the detection and repair of that member would be difficult. The above consequences should be considered when detailing bents of reinforced concrete bridge structures.

### 8.2 Nominal Joint Shear Strength: Derivations from Model

The nominal joint shear strength for reinforced concrete knee joints may also be obtained from the knee joint model expressions (Equations 4.76, 4.77 and 4.81, Chapter 4). The equations are given below. Equations 8.1 and 8.2 give the nominal joint shear capacity corresponding to yielding of hoop and stirrup reinforcement.

#### 8.1

$$v_{n} = \sqrt{\left(\rho_{y}f_{yy} + \frac{N_{x}}{2bd} \frac{l_{c}}{l_{b}}\right) \left(\rho_{x}f_{xy} + \frac{N_{y}}{2bh}\right)}$$

#### 8.2

$$v_{n} = \sqrt{\left(\rho_{y}f_{yy} + \frac{N_{x}}{2bd} \frac{l_{c}}{l_{b}} + \frac{D}{4bd}\right) \left(\rho_{x}f_{xy} + \frac{N_{x}}{2bh}\right)}$$

Equation 8.1 considers loading by lateral sway, such as would occur under earthquake action and neglects possible simultaneous presence of dead loads in the beam span. Instead, Equation 8.2 accounts for a downward dead load, $D$, acting at the beams midspan.
To obtain the nominal joint shear stress using Equations 8.1 and 8.2, an appropriate value for the beam axial load (N_J) is needed. For a well detailed knee joint, the beam and column tensile reinforcement should be able to undergo several cycles in the inelastic range at the respective joint faces during the event of an earthquake. Accounting for an overstrength in the beam tensile reinforcement, a beam sectional analysis at the joint face and the overall statics of the knee joint system can be used to evaluate the corresponding beam axial load.

Equation 8.3 gives the nominal joint shear strength corresponding to hoop reinforcement yielding followed by concrete crushing. It accounts for the presence or absence of a simultaneous dead load (D) acting at the beams midspan.

\[
\nu_n = \sqrt{\left( f_{cm} - \rho_x f_{xy} - \frac{N_x}{2bh} \right) \left( \rho_x f_{xy} + \frac{N_x}{2bh} \right)} \tag{8.3}
\]

Equations 8.4 and 8.5 give the nominal joint shear strength for stirrup yielding followed by concrete crushing for the presence and absence of a dead load (D) acting at the beams midspan, respectively.

\[
\nu_n = \sqrt{\left( f_{cm} - \rho_y f_{yy} - \frac{N_x}{2bd} \frac{l_c}{l_b} \right) \left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{l_c}{l_b} \right)} \tag{8.4}
\]

\[
\nu_n = \sqrt{\left( f_{cm} - \rho_y f_{yy} - \frac{N_x}{2bd} \frac{l_c}{l_b} - \frac{D}{4bd} \right) \left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{l_c}{l_b} + \frac{D}{4bd} \right)} \tag{8.5}
\]

In Equations 8.3 through 8.5 appropriate values are needed for both beam axial load (N_J) and for the effective concrete strength (f_cm).

Many researchers have published alternative definitions for the effective concrete
strength. Usually, the effective concrete strength \( f_{\text{em}} \) is calculated as a fraction of the uniaxial concrete cylinder compressive strength (i.e., Yun and Ramirez (1996), MacGregor (1988), and Schlaich et al, (1987)).

It appears that for knee joints an effective concrete strength in the range of 0.6 \( f'c \) to 0.85 \( f'c \) is reasonable. Effective concrete strength values of 0.85 \( f'c \) would be typical for instances where the joint concrete strut is not significantly strained. This may occur for cases of low joint shear input to the joint, or alternatively, when the joint is under-reinforced.

Effective concrete strength values of 0.65 \( f'c \) would be typical for a heavily worked joint. This may occur for cases of high joint shear input with high transverse reinforcement ratios for the hoops and stirrups in the joint.

Examples showing the use of Equations 8.2, 8.3 and 8.5 under conditions of both closing and opening actions can be found in Appendix C.

A significant outcome from this work is that the physical model that describes interior connections also holds true for knee joints. A truss mechanism works initially while bond along the lead-in length of the anchorage reinforcement can be relied on. When the bond has degenerated along the lead-in length of the bar diagonal strut action governs knee joint response (Figure 8.1).

Figure 8.1 Physical Model for Reinforced Concrete Knee Joints.
CHAPTER 9

CONCLUSIONS AND DISCUSSION

9.1 Conclusions

1. A mechanistic knee joint model suitable for conditions of monotonic loading has been developed. The formulation satisfies equilibrium and compatibility requirements and incorporates existing models of nonlinear material behaviour.

2. The model allows for the numerical evaluation of the relationship between imposed joint deformation and shear stress sustained by the joint. Due to the direct relationship between beam axial load and horizontal joint shear stress, an additional requirement is that the joint shear stress obtained from the knee joint model must converge with that obtained from the statics of the overall structural system.

3. The relationship between imposed joint deformation and stresses within the joint that is inherent in the model, allows for the evaluation of shear capacity in addition to the identification of the possible failure modes for the knee joint. Possible failure modes in the joint include bond splitting failure of the anchorage reinforcement, crushing of the diagonal compressive strut, yielding of hoop and/or stirrup reinforcement followed by crushing of the diagonal compressive strut and yielding of both hoop and stirrup reinforcement.

4. The knee joint resists the forces applied at its boundaries predominantly through a shear resisting mechanism. A diagonal concrete strut and transverse tension ties supplied by the transverse reinforcement in the horizontal and vertical directions resist the principal compressive and tensile stresses within the joint, respectively. Transverse reinforcement becomes necessary once principal tensile stresses exceed the tensile cracking strength of concrete and a diagonal crack has formed within the joint region.
5. The physical model that describes interior connections also holds true for knee joints. A truss mechanism works initially while bond along the lead-in length of the anchorage reinforcement can be relied on. When the bond has degenerated along the lead-in length of the bar diagonal strut action governs knee joint response.

6. An additional consequence of shear in the joint is expansion of the joint core concrete. The role of hoops and stirrups in the joint region, or the presence of a concrete column stub, is to partially restrain the expansion of the concrete through passive confinement of the joint core.

7. In general, knee joint specimens with standard 90° hook anchorages failed under closing action as a result of bond splitting failure of the beam top reinforcement. The presence of hoops and stirrups in the joint region helped delay this failure by confining the rebar. In addition, providing a column stub appears to be more effective, than using hoops and stirrups alone, in delaying and perhaps mitigating bond splitting failure of the beam top reinforcement.

8. In general, knee joint specimens with standard 90° hook anchorages were limited in strength under opening action due to a decrease in the effective moment arm and hence in the beam member flexural capacity at the joint face. The decrease in the effective moment arm was due to damage build up as a result of bond splitting, at the location of the beam top reinforcement that occurred during cycles of closing action in the concrete. Shear in the joint also limited the response under opening action. Providing a concrete column stub resulted in better performance under opening action by confining the beam top reinforcement such that very little damage build-up occurred to the concrete at the joint face. Failure under opening action is more likely to be associated with flexural hinging in the adjacent members, since these members are in net axial tension.

9. Knee joint behaviour recorded from experimental and finite element analyses was in general corroborated by the knee joint model. Further study is needed to resolve
some of the discrepancies that have surfaced in the model's prediction of knee joint
behaviour particularly under opening action. The sensitivities of joint shear stress
\( \nu \) (in \( \sqrt{F'} \)) and the joint shear strain, \( \gamma \), at the instance of transverse reinforcement
yielding to an array of important knee joint design variables is outlined below. Increasing either the hoop reinforcement ratio or hoop or stirrup yield stresses,
results in higher values for shear stress and strain corresponding to transverse reinforcement yielding. Increasing the concrete strength results in lower values for
those response indices. Increasing the stirrup reinforcement ratio has a mixed effect
on joint shear resistance. An increasing trend is noted in the joint shear stress at
yield, whereas a decreasing trend is seen in the corresponding joint deformation.

10. The knee joint model developed here allows for a direct link between the shear
deformations in the joint and the overall displacement of the structure. This would
enable a performance based evaluation of the knee joint for a particular structural
system.

10. The suitability of the model for conditions of earthquake-type loading would require
including degrading properties in the stress-strain relationship for concrete so as to
better represent the reversed cyclic response of members.

11. Extending one, or both, of the beam and column tensile reinforcement beyond the
joints and anchoring them into the adjacent column and beam members, respectively,
improved the specimen performance as witnessed when compared to the response of
specimens with longitudinal reinforcement in the joint anchored by standard 90°
hooks. Anchoring the reinforcement in the adjacent member eliminates the bond
deterioration due to the kinematic tendency of the bent rebar to kick upwards and
outwards as it is pulled under tension which is typical in the reinforcement with
standard 90° hook anchorages. In specimens with the extended anchorages
(especially for both beam and column reinforcement) beam and/or column slip is
reduced and the flexural capacity of the beam and column members is allowed to be
more fully developed. This results in a higher beam and column flexure contribution
to drift and consequently a higher joint contribution to drift as well. The above comments also apply to specimens with column reinforcement that is u-shaped in the joint region.

12. The entire length of an anchored reinforcing bar whose anchorage extends beyond the 90° hook into the adjacent member appears to be active in bond transfer.

13. When detailing knee joint connections that consist of standard 90° hook anchorages for the beam and column longitudinal reinforcement, the recommended value of $8\sqrt{f'_c}$ psi ($0.66\sqrt{f'_c}$ MPa) for the nominal joint shear strength in the current ACI352 draft is unconservative. For the standard anchorage detail where the Bond Index is greater than 6, (typically Bond Index is between 6 and 8 for standard 90° hook anchorages), a value of $6\sqrt{f'_c}$ psi ($0.5 \sqrt{f'_c}$ MPa) should be used for the nominal joint shear strength.

14. When detailing knee joint connections where both beam and column reinforcement are extended beyond the 90° hook, and anchored in the adjacent column and beam members, respectively, provided the Bond Indices for both beam and column reinforcement are less than 6, a value of $10\sqrt{f'_c}$ psi ($0.83 \sqrt{f'_c}$ MPa) can be used for the nominal joint shear strength. The above should also apply for column reinforcement that is u-shaped in the joint. However, as discussed in Section 8.1.2 care must be taken when detailing such connections to avoid damage in undesirable locations that may result from the event of a large earthquake.

15. The nominal joint shear strength can be calculated directly using the five expressions below derived from the mechanistic knee joint model. Precluding bond failure, and for the occurrence of transverse reinforcement yielding prior to concrete crushing, the nominal joint shear strength is calculated for hoop and stirrup yielding, hoop yielding followed by concrete crushing; and stirrup yielding followed by concrete crushing. To perform the calculations, appropriate values must be assigned for the beam axial load and effective concrete strength corresponding to this stage of behaviour.
Hoop and Stirrup Yielding (Absence of Downward Load at Beam's Midspan).

\[ v_n = \sqrt{\left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{l_c}{l_b} \right) \left( \rho_x f_{xy} + \frac{N_x}{2bh} \right)} \]  

9.1

Hoop and Stirrup Yielding (Downward Load Acting at Beam's Midspan).

\[ v_n = \sqrt{\left( \rho_y \rho_{xy} + \frac{N_x}{2bd} \frac{l_c}{l_b} + \frac{D}{4bd} \right) \left( \rho_x f_{xy} + \frac{N_x}{2bh} \right)} \]  

9.2

Hoop Yielding Followed by Concrete Crushing.

\[ v_n = \sqrt{\left( |f_{cm}| - \rho_x f_{xy} - \frac{N_x}{2bh} \right) \left( \rho_x f_{xy} + \frac{N_x}{2bh} \right)} \]  

9.3

Stirrup Yielding Followed by Concrete Crushing.

\[ v_n = \sqrt{\left( |f_{cm}| - \rho_y f_{yy} - \frac{N_x}{2bd} \frac{l_c}{l_b} \right) \left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{l_c}{l_b} \right)} \]  

9.4

Stirrup Yielding Followed by Concrete Crushing (Downward Load Acting at Beam's Midspan).

\[ v_n = \sqrt{\left( |f_{cm}| - \rho_y f_{yy} - \frac{N_x}{2bd} \frac{l_c}{l_b} - \frac{D}{4bd} \right) \left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{l_c}{l_b} + \frac{D}{4bd} \right)} \]  

9.5
16. Results from a finite element analysis of knee joints indicate that normal strains ($e_x$ and $e_y$) in the knee joint core are linear for the isotropic material response. When nonlinear stress-strain relationships for concrete and reinforcement are considered a roughly parabolic shaped distribution is obtained for both normal strains. The joint shear stress ($\nu$) and strain ($\gamma$) also follow a parabolic distribution in the joint for the isotropic material response. This persists in the nonlinear range of material response. The above trends are consistent for both opening and closing actions. The distribution of strains and stresses in the knee joint core indicate that using average values for the strain and stress variables at the joint centre will, in most cases, be adequate.

17. It appears that the joint shear input for knee joints may be defined as the moment gradient across the joint. Good agreement is seen between the average joint shear values at the joint midheight to those obtained as the moment gradient through the joint.

18. Principal strain orientations appear to remain relatively constant within the joint core region over the entire range of loading. Further finite element and experimental studies of the knee joint should be performed to verify this trend.
9.2 Points Deserving Further Notice:

1. Failure of knee joints connections in bridge bents of multispan bridges has prompted the urgency of developing comprehensive design recommendations for such connections. In addition to the design recommendations stated earlier, the knee joint mechanical model developed here should be applied in an overall performance based evaluation and design procedure for reinforced concrete bridge structures.

2. One aspect that needs to be further investigated is the influence of transverse loads, which are acting on the beam span (perpendicular to the beam’s longitudinal axis), on the mechanics of joint failure.
REFERENCES


2. ACI-ASCE Committee 318, "Building Code Requirements for Reinforced Concrete and Commentary (ACI318-91/ACI318R-91)". American Concrete Institute, Detroit, 1991, pp. 353.


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52. Schlaich, J., Schafer, K., and Jennewein, M., "Toward a Consistent Design of Structural Concrete", Journal of the Prestressed Concrete Institute, V.32, No. 3, May-June 1987, pp.74-150.


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APPENDIX A

DRIFT HISTORIES:

CLARKSON TEST SPECIMENS.
Figure A.1 Drift Histories for Clarkson Test Specimens.

A.1 a) Specimen KJ1

A.1 b) Specimen KJ2

A.1 c) Specimen KJ3

A.1 d) Specimen KJ4
A.1 e) Specimen KJ5

A.1 f) Specimen KJ8

A.1 g) Specimen KJ6

A.1 h) Specimen KJ9

A.1 i) Specimen KJ7

A.1 j) Specimen KJ10

Figure A.1 (Cont’d) Drift Histories for Clarkson Test Specimens.
Figure A.1 (Cont’d) Drift Histories for Clarkson Knee Joint Test Specimens.
APPENDIX B

KNEE JOINT BEHAVIOUR
DUE TO
DIAGONAL CRACK FORMATION
OPENING ACTION
Figure B.1 Knee Joint Behaviour: Specimen KJ11 (3% Experimental Drift).

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B.2 a) Joint Diagonal Crack Pattern.

B.2 b) Moment-Drift plot

B.2 c) Strain-Drift Plot: Beam Top Bar Gauge.  
B.2d) Strain-Drift Plot: Embedded Gauge(DIN)

Figure B.2 Knee Joint Behaviour: Specimen KJ13 (2% Experimental Drift).
Figure B.2 (Cont'd) Knee Joint Behaviour: Specimen KJ13 (2% Experimental Drift).
CLOSING ACTION
Figure B.3 Knee-Joint Behaviour: Specimen KJ13 (3% Experimental Drift - No Visual).
B.4 a) Joint Diagonal Crack Pattern.

B.4 b) Moment-Drift Plot.

B.4 c) Strain-Drift Plot: Column Inner Bar Gauge.

B.4 d) Strain-Drift Plot: Beam Bottom Bar Gauge.

Figure B.4 Knee Joint Behaviour: Specimen KJ16 (1% Experimental Drift).
B.4 e) Strain-Drift Plot: Hoop Gauge (H2).

B.4 f) Strain-Drift Plot: Hoop Gauge (H3).

B.4 g) Strain-Drift Plot: Stirrup Gauge (V1).

B.4 h) Strain-drift Plot: Stirrup Gauge (V2).

Figure B.4 (Cont’d) Knee Joint Behaviour: Specimen KJ16 (1% Experimental Drift).
APPENDIX C

SAMPLE CALCULATIONS FOR KNEE JOINT MODEL FORMULATION.
Sample Calculations for Knee Joint Model Formulation.

Specimen U17 tested in China is analyzed for conditions of closing and opening actions based on the knee-joint model formulation.

**Joint properties:** joint geometry and reinforcement detailing are shown in Figure 4.15.

**Joint dimensions:**

- b = thickness = (beam thickness and column thickness)/2 (ACI352-1985) = 250mm
- h = distance between centroids of beam top and beam bottom reinforcement = 420mm
- d = distance between centroids of hoop reinforcement along column member width = 350mm
- D = Downward load at beam's midspan = 200 kN

**Beam reinforcement:**

- Beam top reinforcement yield stress $f_{ybt} = 400$ MPa
- Total area ratio beam top reinforcement $\rho_{bt} = A_{bt}/bh = 1362/((250)(420)) = 0.01297$
- Bond efficiency factor, $\beta_{bt} = 0.5$ closing (assumed)
  
  0.1 opening (assumed)

- Beam bottom reinforcement yield stress $f_{ybb} = 367$ MPa
- Total area ratio beam bottom reinforcement $\rho_{bb} = A_{bb}/bh = 628/((250)(420)) = 0.00598$
- Bond efficiency factor, $\beta_{bb} = 0.1$ closing (assumed)
  
  0.5 opening (assumed)

- Volumetric ratio for stirrups in beam member, $\rho_{vs} = \text{volume of stirrups} / \text{volume of core} = 94200 \text{ mm}^3 / 5737500 \text{ mm}^3 = 0.016$

**Column reinforcement:**

- Column outer and inner reinforcement yield stress $f_{yc0} = f_{yci} = 400$ MPa
- Total area ratio column outer and inner reinforcement $\rho_{co} = \rho_{ci} = A_{co}/bd = A_{ci}/bd = 1742/((250)(350)) = 0.01991$
Column reinforcement (cont’d)

Bond efficiency factor, $\beta_{co} = 0.5$ closing (assumed)  
0.1 opening (assumed)  
$\beta_{ci} = 0.1$ closing (assumed)  
0.5 opening (assumed)

Joint reinforcement:

Hoop reinforcement yield stress $f_{yh} = 320$ MPa

Total area ratio $A_t/bh = 942 \text{mm}^2 / ((250)(420)) = 0.008976$

Volume ratio $p_v = 6$ hoops x 1300 mm length/hoop x $78.5 \text{mm}^2$/hoop/(350 x 250 x 420) mm

$ = 0.01667$

Effective total horizontal requirement ($\rho_x$)

$\rho_x = \beta_{bt} \rho_{bt} + \beta_{bb} \rho_{bb} + \rho_h = 0.01606$

$f_{yx} = (\beta_{bt} \rho_{bt} f_{ybt} + \beta_{bb} \rho_{bb} f_{ybt} + \rho_h f_{yh}) / \rho_x = 354$ MPa

Effective total vertical requirement ($\rho_y$)

$\rho_y = \beta_{co} \rho_{co} + \beta_{ci} \rho_{ci} + \rho_s = 0.01195$

$f_{yy} = (\beta_{co} \rho_{co} f_{yc} + \beta_{ci} \rho_{ci} f_{yci} + \rho_s f_{ys}) / \rho_y = 400$ MPa

Concrete properties:

$f'_c = 25.1$ MPa

$\varepsilon_o = 0.002$ (strain at $f'_c$)

$E_{ci} = 2f'_c / \varepsilon_o = 25100$ MPa

Confinement coefficient, $k_e$ (Equation 4.15)

$$k_e = \frac{(1 - \sum_{i=1}^{n} (wi)^2 / 6bd)(1 - s')/(2b)(1 - s')/(2d)}{1 - \rho_{cc}}$$
\( k_e : \text{Joint} \)

\[
\rho_{ce} = \frac{A_{ci}}{bd} = \frac{3484 \text{mm}^2}{(350)(250)} = 0.03982
\]

\[
\sum \frac{w_i^2}{6bd} = \left( 6 \frac{(50)^2}{6} + 2 \frac{(290)^2}{6} \right) \frac{1}{(350)(250)} = 0.34895
\]

\[
k_e = \frac{(1 - 0.34895)(1 - 65/2(250))(1 - 65/2(350))}{1 - 0.03982} = 0.535
\]

\( k_e : \text{Beam} \)

\[
\rho_{cc} = \frac{A_b}{bd} = \frac{(1362 + 625)}{(150)(450)} = 0.02948
\]

\[
\sum \frac{w_i^2}{6bd} = \left( 2 \frac{(395)^2}{6} + \frac{100^2}{6} + 2 \frac{35^2}{6} \right) \frac{1}{(150)(450)} = 0.80123
\]

\[
k_e = \frac{(1 - 0.80123)(1 - \frac{75}{2(450)})(1 - \frac{75}{2(150)})}{1 - 0.02948} = 0.14074
\]
Closing Action

Before yielding of Hoops or Stirrups (this example, column reinforcement):

Step 1: Select a value for the hoop strain, $\varepsilon_x = 0.0001$

Step 2: Guess the beam axial load, $N_x = 36,000\text{N}$

Step 3: Solve for the angle to principal plane, $\theta$ (Equation 4.59)

$$a_1 \tan^2 \theta + a_2 \tan^4 \theta - 1 = 0$$

Recall, Equation 4.59 is derived based on the knee-joint model given in Figure 4.3.

For the tests performed in China, however, a downward load, $D (200 \text{kN})$ was applied at the beam midspan (Figure 2.28).

In light of this, a small modification is needed for coefficient $a_1$, i.e.:

$$a_1 = \frac{\left( \frac{1}{E_s \rho_y} \right) \left( \frac{0.5D + \frac{N_x l_c}{l_b}}{2bd} \right)}{\left( n \rho_x \varepsilon_x + \frac{N_x}{2bhE_c} \right) \left( 1 + \frac{1}{n \rho_y} \right)} = 1.27$$

$$a_2 = \frac{1 + \frac{1}{np_x} + \frac{N_x}{2bhE_c \varepsilon_x}}{1 + \frac{1}{np_y}} = 0.53$$

Solve for $\theta = 38.3^\circ$.

Step 4: Solve for the joint shear stress $\nu$, (Equation 4.60)

$$\nu_j = \frac{1}{\tan \theta} \left( \rho_x E_s \varepsilon_x + \frac{N_x}{2bh} \right) = 0.62 \text{MPa}$$
Step 5: Perform Sectional Analysis at the Joint Face to obtain the joint shear input according to the ACI352-85 approach.

The stress-strain model for concrete and reinforcing bars are given in section 4.2.1.2. Concrete is assumed to be confined within the centroid of the stirrup reinforcement. Outside this region, the concrete is unconfined and it is further assumed here that the concrete spalls off when the extreme compressive strain exceeds 0.0035.

The beam sectional analysis is performed on a spreadsheet. The moment-curvature response for the beam section, is obtained for two or three beam axial loads based on chosen increments of the beam tensile reinforcement strain. From Equation 4.2, joint shear force is calculated at each level of tensile reinforcement strain. Dividing by the effective joint area (bh) results in the joint shear stress. Next, given the assumed beam axial load from Step 2, statics is used to obtain the beam moment at the joint face.

For the test performed in China, (Figure 2.30) the beam moment at the joint face is given as:

\[ M_b = 100D - 1297 N_x \]

all units are in N and mm.

Linear interpolation is then used to obtain the joint shear stress for the calculated beam moment and beam axial load.

From performing the above vertical analysis, \( v_j = 0.62 \text{MPa} \), which is equal to that obtained in Step 4.

Step 6: Since \( v_j \) (Step 6) = \( v_j \) (Step 5), estimate for the beam axial force is okay.

Proceed to Step 7.

Step 7: Calculate the stirrup strain (Equation 4.61).

Equation 4.61 is modified to account for the midspan beam load.

\[
\varepsilon_y = \frac{1}{E_x \rho_y} \left( \frac{v_j}{\tan \theta} - \frac{0.5D + N_x l_c}{\frac{I_c}{l_b}} \right) = 0.00004
\]
Check if stirrups (or column reinforcement have yielded) $\varepsilon_y < \varepsilon_{\text{yield}} (0.002)$; ok

Step 8: Calculate the principal tensile (Equation 4.53) and compressive (Equation 4.35) strains, and the joint shear strain (Equations 4.36 and 4.52).

$$\varepsilon_1 = \frac{(\varepsilon_x - \varepsilon_y \tan^2 \theta)}{1 - \tan^2 \theta} = 0.000195$$

$$\varepsilon_2 = \varepsilon_y + \varepsilon_x - \varepsilon_1 = -0.00052$$

$$\gamma = \frac{2(\varepsilon_1 - \varepsilon_x)}{\tan \theta} = 0.00024$$

Step 9: Calculate $\lambda$ (Equation 4.65)

$$\lambda = \frac{1 + k_e \rho_{f_v}}{f_{f_v}} \left| \frac{f_{y_h}}{f_c} \right| \text{ and } 0.8 - 0.34 \left( \frac{\varepsilon_1}{\varepsilon_o} \right) \geq 1$$

$$= 1.114$$

Step 10: Check if concrete crushing has occurred:

$$\lambda \varepsilon_o = (1.114)(-0.002) = -0.0022 > \varepsilon_2$$

crushing has not occurred.

Step 11: Check the concrete secant modulus for the current strain state (Equation 4.66)

$$E_c = \frac{\lambda f'}{\varepsilon_2} \left[ 2\left( \frac{\varepsilon_2}{\lambda \varepsilon_o} \right) - \left( \frac{\varepsilon_2}{\lambda \varepsilon_o} \right)^2 \right] = 24,800 \text{ MPa}$$

This is 95% of the initial concrete test modulus ($E_{ci}$)
Step 12: Steps 1 through 11 are repeated for higher values of hoop strain.

Table A1 below, summarizes the output. From the Table, it can be seen (second-last line) that hoop yielding ($\varepsilon_x=0.0016$) is the first critical milestone that is reached. At this stage, the concrete secant modulus is about 81% of the initial concrete tangent modulus and the stirrup (column here) reinforcement is at about 90% of yield ($\varepsilon_y=0.002$).

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<th>$v$ (psi)</th>
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<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
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Behaviour after Yielding of Hoop Reinforcement

The steps outlined in section 4.2.6.1 are followed to obtain the knee joint response after hoop reinforcement yielding.

Step 1: Select the hoop strain value, $\varepsilon_x$, $\varepsilon_x = 0.00195$

(Here chose case where column reinforcement yielding occurs).

Step 2: Guess a value for the beam axial load, $N_x = 204,270$ N

Step 3: Calculate the joint shear stress, $v_j$ (Equation 4.75)

$$\varepsilon_x = b_0 + b_1 v^2 + b_2 v^4$$
Once again, a slight modification is required in coefficient $b_1$ from Equation 4.75 to account for the downward load at the beam midspan, i.e.

$$b_o = \frac{-\rho_x f_{xy} - \frac{N_x}{2bh}}{E_c} = -308.54 \times 10^{-6}$$

$$b_1 = \frac{0.5D + \frac{1}{l_c} N_x}{\frac{1}{l_b} 2bd\rho_y E_s} = -13.78 \times 10^{-6}$$

$$b_2 = \frac{1 + \frac{1}{n\rho_y}}{E_c (\rho_x f_{xy} + \frac{N_x}{2bh})^3} = 2.06 \times 10^{-6}$$

Solving gives $v_j = 6.055 \text{ MPa}$

Step 4: Calculate the angle to the principal plane, $\theta$ (Equation 4.71)

$$\tan \theta = \frac{\rho_x f_{xy} + \frac{N_x}{2bh}}{v_j}, \quad \theta = 45.3^\circ$$

Steps 5 through 12 are the same as before. The results are given in the last line of Table A1. For this test case, the model predicts that column reinforcement yielding ($\varepsilon_y = \varepsilon_{xy}$) follows hoop yielding.
Calculation of Nominal Joint Shear Strength for Hoop and Stirrup Yielding: Closing Action.

The nominal joint shear stress for both hoops and stirrup yielding obtained using the knee joint model (see last line, v, Table A1), can be obtained directly using the following equation:

\[ \nu_n = \sqrt{\left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{\ell_c}{\ell_b} + \frac{D}{4bd} \right) \left( \rho_x f_{xy} + \frac{N_x}{2bh} \right)} \]

where; \( N_x = 204273 \text{ N} \) (Table A1)

\[ \nu_n = \left\{ \left[ (0.01195)(400) + \left( \frac{204273}{2(250)(350)} \right) \left( \frac{1410}{2500} \right) + \left( \frac{200000}{4(250)(350)} \right) \right] \right\}^{\frac{1}{2}} \]

\[ = 6.060 \text{ MPa} \]

The value obtained here is approximately the same as that obtained using the step by step approach in Table A1.
Calculation of Nominal Joint Shear Strength for Hoop Yielding Followed by Concrete Crushing: Closing Action.

The nominal joint shear strength for hoop yielding followed by concrete crushing is given by:

\[ v_n = \sqrt{\left( f_{cm} \right) - \rho_x f_{xy} - \frac{N_x}{2bh} \left( \rho_x f_{xy} + \frac{N_x}{2bh} \right)} \]

where;

\[ v_n = 7.90 \text{ MPa} \quad \text{for} \quad f_{cm} = 0.65 f'_c \]
\[ v_n = 9.65 \text{ MPa} \quad \text{for} \quad f_{cm} = 0.85 f'_c \]

Calculation of Nominal Joint Shear Strength for Stirrup Yielding Followed by Concrete Crushing: Closing Action.

The nominal joint shear strength for stirrup yielding followed by concrete crushing is given by:

\[ v_n = \sqrt{\left( f_{cm} \right) - \rho_y f_{yy} - \frac{N_x l_c}{2bd l_b} - \frac{D}{4bd} \left( \rho_y f_{yy} + \frac{N_x l_c}{2bd l_b} + \frac{D}{4bd} \right)} \]

where;

\[ v_n = 7.87 \text{ MPa} \quad \text{for} \quad f_{cm} = 0.65 f'_c \]
\[ v_n = 9.60 \text{ MPa} \quad \text{for} \quad f_{cm} = 0.85 f'_c \]

Thus, failure in the knee joint will be governed by hoop and stirrup yielding.
Opening Action

The same solution strategy is used for conditions of opening action as was done for closing action. Recall, based on sign convention, beam axial load \( \left( N_J \right) \), joint shear stress \( \left( \nu_j \right) \), angle to principal plane \( \theta \), and joint shear strain \( \gamma \) have negative values under opening action.

Table A2 shows the results for the knee joint reinforcement under opening action. Once again, the critical milestones are hoop yielding followed by column reinforcement yielding. The results in Table A2 are based on different values. For the beam top and bottom reinforcement ratios, \( \rho_{bb} \) and \( \rho_{bc} \), respectively (see Table 4.1). This was done to study a wider range of the model parameter under opening action (see Section 4.3).

<table>
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<th>( \nu ) (MPa)</th>
<th>( \nu(\sqrt{\nu_j}) ) (psi)</th>
<th>( \varepsilon_0 )</th>
<th>( \varepsilon_1 )</th>
<th>( \varepsilon_2 ) (rads)</th>
<th>( \gamma )</th>
<th>( \lambda \varepsilon_0 ) (MPa)</th>
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Calculation of Nominal Joint Shear Stress for Hoops and Stirrup Yielding: Opening Action.

As was done for closing action, the nominal joint shear stress under opening action is obtained directly from the following equation:

\[ v_n = \sqrt{\left( \rho_y f_{yy} + \frac{N_x}{2bd} \frac{\ell_c}{\ell_b} \frac{D}{4bd} \right) \left( \rho_x f_{xy} + \frac{N_x}{2bh} \right)} \]

where; \( N_x = -138713 \) N (Table A2)

\[ \rho_x = \beta_{bb} \rho_{bb} + \beta_{bt} \rho_{bt} \rho_h \]
\[ = (0.5)(0.01905) + (0.1)(0.00598) + 0.008976 \]
\[ = 0.01909 \]

\[ \rho_y = \beta_{ci} \rho_{ci} + \beta_{co} \rho_{co} \]
\[ = (0.5)(0.01991) + (0.1)(0.01991) \]
\[ = 0.01195 \]

\[ v_n = \left[ \left(0.01195\right)(400) + \frac{-138713}{(2)(250)(350)} \left( \frac{1410}{2500} + \frac{200000}{(4)(250)(350)} \right) \right] \]
\[ \left[ \left(0.01909\right)(320) + \frac{-138713}{(2)(250)(420)} \right] \]
\[ = 5.17 \text{ MPa} \]

The values obtained here is the same as that obtained using the step-by-step approach in Table A2.
Calculation of Nominal Joint Shear Strength for Hoop Yielding Followed by Concrete Crushing: Opening Action.

The nominal joint shear strength for hoop yielding followed by concrete crushing is given by:

\[ v_n = \sqrt{\left( \left| f_{cm} \right| - \rho_x f_{xy} \frac{N_x}{2bh} \right) \left( \rho_x f_{xy} + \frac{N_x}{2bh} \right)} \]

where;

\[ v_n = 7.69 \text{ MPa} \quad \text{for } f_{cm} = 0.65 f_c \]
\[ v_n = 9.30 \text{ MPa} \quad \text{for } f_{cm} = 0.85 f_c \]

Calculation of Nominal Joint Shear Strength for Stirrup Yielding Followed by Concrete Crushing: Opening Action.

The nominal joint shear strength for stirrup yielding followed by concrete crushing is given by:

\[ v_n = \sqrt{\left( \left| f_{cm} \right| - \rho_y f_{yy} \frac{N_x l_c}{2bd l_b} - \frac{D}{4bd} \right) \left( \rho_y f_{yy} + \frac{N_x l_c}{2bd l_b} + \frac{D}{4bd} \right)} \]

where;

\[ v_n = 7.48 \text{ MPa} \quad \text{for } f_{cm} = 0.65 f_c \]
\[ v_n = 8.98 \text{ MPa} \quad \text{for } f_{cm} = 0.85 f_c \]

Thus, failure in the knee joint will be governed by hoop and stirrup yielding.
APPENDIX D

ANCHORAGE REINFORCEMENT STRESS PROFILES.
Figure D.1  Anchorage Reinforcement Stress Profiles: Closing Action
CO Anchorage Model - Beam Top Reinforcing Bar.
a) Regular Bond Model     b) Poor Bond Model

Figure D.2  Anchorage Reinforcement Stress Profiles: Closing Action
CO Anchorage Model - Column Outer Reinforcement.
Figure D.3  Anchorage Reinforcement Stress Profiles: Closing Action. BTOC Anchorage Model - Beam Top Reinforcing Bar.
a) Regular Bond Model  
(b) Poor Bond Model

Figure D.4  Anchorage Reinforcement Stress Profiles: Closing Action  
BTCO Anchorage Model - Column Outer Reinforcing Bar.
Figure D.5  Anchorage Reinforcement Stress Profiles: Opening Action - Regular Bond Model
Standard Anchorage Model - Beam Bottom Reinforcing Bar.
Figure D.6  Anchorage Reinforcement Stress Profiles: Opening Action - Regular Bond Model
Standard Anchorage Model - Column Inner Reinforcing Bar.
APPENDIX E

SECTIONAL PLOTS OF CONCRETE STRAINS AND STRESSES IN JOINT.
E.1 a) Normal Concrete Strains, $\varepsilon_x$ and $\varepsilon_y$.

Figure E.1  Section Plots of Concrete Strains and Stresses: Closing Action
CO Anchorage Model - Poor Bond.
E.1 b) Concrete Joint Shear Stress, $v_{xy}$.

Figure E.1 (Cont’d)  Section Plots of Concrete Strains and Stresses: Closing Action
CO Anchorage Model - Poor Bond
E.1 c) Concrete Shear Strains, $\gamma_{xy}$

Figure E.1 (Cont’d) Section Plots of Concrete Strains and Stresses: Closing Action CO Anchorage Model - Poor Bond.
E.2 a) Normal Concrete Strains, $\varepsilon_x$ and $\varepsilon_y$.

Figure E.2  Section Plots of Concrete Strains and Stresses: Closing Action
BTCO Anchorage Model - Regular Bond.
3.2 b) Concrete Joint Shear Stress, $v_{xy}$.

Figure E.2 (Cont'd)  Section Plots of Concrete Strains and Stresses: Closing BTCO Anchorage Model - Regular Bond.
3.2 c) Concrete Shear Strains, $\gamma_{xy}$.

Figure E.2 (Cont'd)  Section Plots of Concrete Strains and Stresses: Closing BTCO Anchorage Model - Regular Bond.
APPENDIX F

REINFORCEMENT STRESS DISTRIBUTIONS AT JOINT MIDHEIGHT.
Figure F.1  Reinforcement Stress Distributions at Joint Midheight- CO Anchorage, Regular Bond Model: Closing Action.

Figure F.2  Reinforcement Stress Distributions at Joint Midheight- CO Anchorage, Poor Bond Model: Closing Action.
Figure F.3  Reinforcement Stress Distributions at Joint Midheight- BT Anchorage, Regular Bond Model: Closing Action.

Figure F.4  Reinforcement Stress Distributions at Joint Midheight- BT Anchorage, Poor Bond Model: Closing Action.
Figure F.5 Reinforcement Stress Distributions at Joint Midheight- BTCO Anchorage, Regular Bond Model: Closing Action.

Figure F.6 Reinforcement Stress Distributions at Joint Midheight- BTCO Anchorage, Poor Bond Model: Closing Action.