FUNDAMENTAL STUDIES ON CONTRAST RESOLUTION OF ULTRASOUND B-MODE IMAGES

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
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and Institute of Biomaterials and Biomedical Engineering
University of Toronto

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0-612-45685-4
To my father
ABSTRACT

Fundamental Studies on Contrast Resolution of Ultrasound B-mode Images
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The primary objective of this thesis is to study the factors that affect the contrast resolution of ultrasound B-mode images.

A computer simulation model of B-mode image formation has been developed and validated. The model simulates beam profiles from linear, 1.5-D and curvilinear arrays. An initial study was performed to compare the accuracy and computational efficiency of different methods that have been used to calculate the field profile from linear arrays. It was concluded that the far-field approximation for the impulse response method yields the best trade-off between accuracy and efficiency.

The model was used to investigate the effects of transducer design, and tissue parameters on the image characteristics, specifically, texture and contrast resolution. It was shown that the scatterer distribution, as well as the transducer geometry, determines the textural appearance and the contrast of an image. It was also demonstrated that post-processing schemes can significantly alter the image appearance by changing the mean signal levels and speckle statistics. The simulation model can be used to predict optimum values for the parameters used in post-processing algorithms. Different measures of contrast resolution were examined, and it was concluded that the contrast-to-speckle ratio is the most accurate measure in defining an imaging system’s ability to detect low-contrast targets in background speckle noise.
In order to illustrate the clinical application of the model, a simple, approximate model of a carotid artery wall with fatty plaque was developed and a preliminary study was performed to analyze the B-mode image of this geometry. The simulated image indicates that the wall region is more echogenic than the plaque region as is the case in clinical images. The image also reveals sharp reflection signals at the wall-plaque, and plaque-lumen interfaces. It was shown that the signal levels at these interfaces depend on the backscattering coefficients and the distributions of the scatterers in the neighbouring media.
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<tr>
<td>1-D</td>
<td>One-dimensional</td>
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<tr>
<td>1.5-D</td>
<td>One and a half-dimensional</td>
</tr>
<tr>
<td>2-D</td>
<td>Two-dimensional</td>
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<tr>
<td>3-D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>$\alpha(f)$</td>
<td>Frequency-dependent attenuation function</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Frequency-independent attenuation constant</td>
</tr>
<tr>
<td>$\delta(t)$</td>
<td>Dirac delta function</td>
</tr>
<tr>
<td>$\frac{x^2}{\xi^2}$</td>
<td>Variance of the number of scatterers in a voxel</td>
</tr>
<tr>
<td>$\phi(\vec{r}, t)$</td>
<td>Velocity potential at field point $\vec{r}$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Compressibility</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of the medium</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>Differential backscattering cross-section</td>
</tr>
<tr>
<td>$\sigma_{in}$</td>
<td>Variance of the signal in the cyst</td>
</tr>
<tr>
<td>$\sigma_{out}$</td>
<td>Variance of the signal in the background</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Steering angle</td>
</tr>
<tr>
<td>$\theta_a$</td>
<td>Angle of the active aperture arc on a curvilinear array</td>
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<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Angular frequency for CW excitation</td>
</tr>
<tr>
<td>$\omega_{pRF}$</td>
<td>Pulse repetition angular frequency</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>$n$th harmonic of $\omega_{pRF}$ ($\omega_n=n\omega_{pRF}$)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength of incident ultrasound</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Wavelength at central frequency</td>
</tr>
<tr>
<td>$\lambda_n$</td>
<td>Wavelength at $n$th harmonic of $\omega_{pRF}$</td>
</tr>
<tr>
<td>$a$</td>
<td>Scatterer radius</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Apodization factor for $i$th element</td>
</tr>
<tr>
<td>AUBP</td>
<td>Area under beam profile</td>
</tr>
<tr>
<td>$b$</td>
<td>Scattering strength (relative to background) of scatterers in a cyst</td>
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Backscattering coefficient

$L(x)$
Lateral beam profile in $x$-direction

c
Speed of sound

c(f)
Frequency-dependent dispersion function

c_o
Speed of sound at the center frequency of transmit pulse

Contrast

$C_1$, $C_2$, $C_3$
Different definitions of contrast

$C_t$
Contrast measured by AUBP method

Contrast-to-speckle ratio

Continuous wave excitation

Center-to-center element spacing

Active aperture

Input excitation voltage

Output voltage

Impulse response of the transducer on transmission

Impulse response of the transducer on reception

Spatial Fourier transform,

Ratio of focal distance to active aperture, $D$

Full width at half maximum

Focal distance

Received force on transducer surface

Frequency

Central frequency for transmit pulse

Sampling frequency in time

Spatial sampling frequencies

Element height

Impulse response function

Impulse response function for an array

Impulse response function for an element

Impulse response function on reception
\( h_t(\vec{r},t) \)  \hspace{1cm} \text{Impulse response function on transmission}

\( I \)  \hspace{1cm} \text{In-phase component of RF signal}

\( k \)  \hspace{1cm} \text{Wave number, } k = \omega/c = 2\pi/\lambda.

\( L(\vec{r},\omega_c) \)  \hspace{1cm} \text{Lateral response of the transducer at field point } \vec{r}

\( L_A(\vec{r},\omega_c) \)  \hspace{1cm} \text{Array lateral response}

\( L_r(\vec{r},\omega_c) \)  \hspace{1cm} \text{Lateral response on reception}

\( L_t(\vec{r},\omega_c) \)  \hspace{1cm} \text{Lateral response on transmission}

\( M_r(\vec{r}_{fr},\omega) \)  \hspace{1cm} \text{Attenuation and dispersion transfer function on receive path}

\( M_t(\vec{r}_{ts},\omega) \)  \hspace{1cm} \text{Attenuation and dispersion transfer function on transmit path}

\( N \)  \hspace{1cm} \text{Number of array elements}

\( N_v \)  \hspace{1cm} \text{Number of scatterers per mm}^3

\( N_{vox} \)  \hspace{1cm} \text{Number of scatterers per voxel}

\( N_s \)  \hspace{1cm} \text{Number of segments in elevation direction for a 1.5-D array}

\( p(\vec{r},t) \)  \hspace{1cm} \text{Pressure at field point } \vec{r}

\( p_i(\vec{r}_s,t) \)  \hspace{1cm} \text{Incident pressure on a scatterer at } \vec{r}_s

\( p_s(\vec{r}_r,t) \)  \hspace{1cm} \text{Scattered pressure at } \vec{r}_r

\( p_0(t) \)  \hspace{1cm} \text{Pressure on the transducer surface}

\( PRF \)  \hspace{1cm} \text{Pulse repetition frequency}

\( Q \)  \hspace{1cm} \text{Quadrature-component of RF signal}

\( \vec{r} \)  \hspace{1cm} \text{Vector position of the field point with respect to the transducer}

\( r_o \)  \hspace{1cm} \text{Distance from the scatterer to a point on the transducer surface}

\( r_{oi} \)  \hspace{1cm} \text{Distance from the scatterer to the center of } i \text{th element}

\( \Delta \vec{r}_{avg} \)  \hspace{1cm} \text{Average vector position of scatterers within a voxel}

\( R \)  \hspace{1cm} \text{Distance between a point on transducer surface to the field point, } |\vec{r}|

\( R_c \)  \hspace{1cm} \text{Distance between the field point and the center of the transducer}

\( R_d \)  \hspace{1cm} \text{Distance } R \text{ in the denominator of the diffraction equation}

\( R_F \)  \hspace{1cm} \text{Focal distance}

\( R_i \)  \hspace{1cm} \text{Distance between the field point and the center of each element}

\( R_p \)  \hspace{1cm} \text{Distance } R \text{ in the exponent of the diffraction equation}
$R_C$  Radius of curvature

$R_L$  Lateral resolution

$R_x$  Receive focus in $x$-direction

$R_y$  Receive focus in $y$-direction

$\mathcal{R}_i$  Resolution calculated using method (i).

$\mathcal{R}_j$  Resolution associated with method $j$, $j=ii..vi$

$s$  Cyst size

$s_x$  Cyst size in $x$-direction

$s_z$  Cyst size in $z$-direction

$S$  Transducer surface

$S_i$  Surface of each element

$S_r$  Transducer surface on reception

$S_t$  Transducer surface on transmission

$S_{ex}$  Speckle size in $x$-direction

$S_{ez}$  Speckle size in $z$-direction

$S_{in}$  Mean signal in the cyst

$S_{out}$  Mean signal in the background

$SNR_{opt}$  Optimal signal-to-noise ratio

$t$  Time

$T_i$  Time delay applied to each element

$T_x$  Transmit focus in $x$-direction

$T_y$  Transmit focus in $y$-direction

$\Delta t$  Transmit pulse duration

$u$  Frequency dependence of attenuation , $\alpha(f)=\beta^u$

$v_o(t)$  Transducer surface velocity

$V(\omega)$  Transmitted pulse in frequency domain

$V_p(\omega)$  Shape of transmitted pulse in frequency domain

$\Delta V$  Voxel volume

$w$  Element width

$W_s(\bar{r}_{sr},\omega)$  Scattering transfer function
\( x_t(t) \)  
Transmit pulse

\( X_i \)  
Element location on x-axis

\( x_o \)  
\( x \)-coordinate of a point on transducer surface

\( x_s \)  
\( x \)-axis rotated by steering angle \( \theta \)

\( x_z \)  
\( x \)-coordinate of the field point

\( y_o \)  
\( y \)-coordinate of a point on transducer surface

\( y_z \)  
\( y \)-coordinate of the field point

\( z \)  
\( z \)-coordinate of the field point

\( z_z \)  
\( z \)-axis rotated by steering angle \( \theta \)
CHAPTER 1
INTRODUCTION

Ultrasound B-mode imaging is a widely used clinical diagnostic tool. An ultrasonic B-mode image can be obtained by sending ultrasonic pulses to the tissue, and successively receiving, processing and displaying the scattered waves. Key aspects of any B-mode imaging system are the spatial resolution, contrast resolution, and textural features.

Spatial resolution relates to the ability to distinguish two targets as the distance between them is diminished. The three components of spatial resolution are the axial, azimuthal, and elevation resolutions. Contrast resolution is a measure of how well differences in mean backscattered echo amplitude from different scattering media can be resolved. Contrast and spatial resolution are generally determined by the pulse length, frequency, and focusing properties of the transducer. Short pulses with a higher center frequency improve both the axial and lateral resolutions; wider aperture sizes used to focus ultrasound beams also provide better resolution in both lateral directions at the focal zone.

Texture (or speckle) is the granular appearance of an image resulting from the constructive and destructive interference of the ultrasound waves scattered by the tissue. Figure 1.1, which shows a B-mode image of the common carotid artery with atherosclerosis, demonstrates how the tissue parenchyma produces a finely textured speckle pattern. The presence of speckle may obscure small structures, such as lesions, by degrading the spatial resolution and contrast. The origin and characteristics of speckle are related to both the tissue properties and the characteristics of the imaging system.

In clinical B-mode imaging the ability to distinguish low-contrast targets that have acoustic properties very similar to those of the surrounding medium is of major importance. The visibility of such targets (e.g. lesions) depends on both the contrast and resolving power of the imaging system (Insana and Hall, 1994). In an effort to improve image quality, much work has been directed towards the design of better transducers. The complexities and high costs associated with building new transducers make it
especially important to develop simulation models that can predict beam profiles. An accurate and computationally efficient model provides a convenient mean for investigating the effects of various parameters (transducer geometry, frequency, etc.) on image quality, and enables improvements to be made prior to fabrication. Furthermore, a model provides an insight into the process of image formation which can aid in the interpretation of the clinical images.

![Image of common carotid artery with atherosclerosis](image)

Figure 1.1. Grey scale image of the common carotid artery with atherosclerosis (transverse and longitudinal views). The tissue parenchyma produces a finely textured echo pattern. The arrows indicate the plaque. (*Diagnostic Vascular Ultrasound*. London: E. Arnold, 1992).

### 1.1 Objectives

The primary objective of this work is to investigate the fundamental limits of B-mode imaging on contrast and spatial resolution. The underlying goal is to improve contrast and resolution of B-mode images so that small and/or diffuse abnormalities can be detected. To attain this end, B-mode images are simulated by a computer model and the relationship between the image characteristics (speckle pattern, contrast resolution) and a particular transducer geometry are investigated. In this work, the term *contrast resolution* refers to the resolving capability of an imaging system as a function of both size and contrast.
There are two secondary objectives of this work: to verify the modeling results with experimental data, and to provide a preliminary demonstration of the use of the computer model for simulating an image of an artery with atherosclerotic plaque.

1.2 Hypothesis

It is hypothesized that an accurate computer model of B-mode image formation will aid in investigating the effects of transducer design and tissue properties on contrast resolution.

1.3 Background

The computer model developed for simulating ultrasound B-mode images comprises two principal components: one for simulating the beam formation process, while the other models the scattering from the tissue. The following two sections provide an overview of these two components. The model was subsequently used to investigate the contrast resolution of B-mode images, and to simulate the image from a carotid artery with atherosclerotic plaque. A summary of relevant prior studies on contrast resolution is presented in Section 1.3.3, followed by a discussion of the success of ultrasonic imaging in characterizing atherosclerosis.

1.3.1 Transducer Geometry

The relationship between the tissue microstructure and the texture of B-mode images has been investigated by various groups (Foster et al., 1983; Wagner et al., 1983; Oostervald et al., 1985; Jacobs and Thijssen, 1991). It was found that transducer geometry plays an important role in determining the character of the speckle (Flax et al., 1981; Foster et al., 1983; Wagner et al., 1983). In fact, information about tissue microstructure can be deduced from the speckle pattern. Therefore, before a clear understanding of the relationship between texture and the tissue characteristics can be established, the effects of the imaging system should first be determined.

Linear phased array transducers have been widely used in medical ultrasonic imaging since they are capable of both electronically steering and focusing the ultrasound
beam. One problem associated with phased arrays is that steering and focusing can be achieved only in the plane of the transducer (azimuthal plane). An acoustic cylindrical lens is used to focus linear arrays in the perpendicular direction (elevation plane), but this only provides a fixed focus. Two dimensional (2-D) arrays have been proposed as a solution to the asymmetric focusing properties of linear phased arrays. These have the potential of focusing and steering the beam in both lateral directions, and thereby enabling a 3-D image to be obtained. Several groups have investigated the characteristics of 2-D arrays (Turnbull et al., 1991; Turnbull and Foster, 1991, 1992; Lu and Greenleaf, 1994; Lockwood and Foster, 1996; Brunke and Lockwood, 1997; Holm et al., 1997), but to date none are commercially available.

The major difficulty facing the development of 2-D arrays is the need to integrate a large number of small elements for both transmit and receive. The complexities and high costs involved in building these arrays have led to the development of the so-called 1.5-D array (Smith et al., 1990; Turnbull and Foster, 1992; Daft et al., 1994; Wildes et al., 1997). Such an array consists of a linear array of elements that are excited in successive groups (delays) to achieve focusing and steering in the azimuthal direction. Additional elements (the 0.5 of the 1.5-D) provide focusing in the elevation direction and enable the focal point to be varied.

1.3.2 Scattering Medium

Reflection and scattering of ultrasound waves occur as a result of a change in the acoustic properties (density, compressibility) of the medium. Specular reflections occur when the transmitted ultrasonic pulses interact with smooth tissue interfaces that have dimensions larger than the wavelength of the insonifying beam. These high level reflections, which generally originate from fluid-tissue or bone-tissue interfaces, produce bright boundaries in the image. On the other hand, diffuse scattering occurs when ultrasound interacts with changes in the acoustic properties of the medium over dimensions that are smaller than the insonating wavelength. This diffuse scattering contributes to the speckled texture features of the ultrasound image.

The first step in developing a model of B-scan image formation is to describe the scattering properties of the tissue. Two simplified theoretical models have generally been
used by various groups to simulate B-mode images – the particle model and the continuum model (Mo and Cobbold, 1993). More recently, a unified approach has been used to model the backscattered Doppler ultrasound from blood (Mo, 1990; Mo and Cobbold, 1992; Bascom and Cobbold, 1995; Bascom, 1995; Lim et al., 1996). In this approach, the tissue consists of elemental acoustic voxels. The voxels contain many scatterers that cannot be individually resolved by the transducer at the insonation frequency. Therefore, each voxel can be considered to act as single scattering unit with a scattering strength determined by the number of scatterers contained within it. The main advantage of the voxel model is computational efficiency.

1.3.3 Contrast Resolution

A common clinical problem in radiology and ultrasonic imaging is the detection of relatively small structures that differ slightly in contrast from their surroundings. In addressing this problem, researchers refer to a term, contrast resolution, which is defined as the minimum contrast required to detect an object of specific size in the presence of background noise.

One technique used in measuring the contrast resolution of an imaging system is the Contrast-Detail Analysis, which is an in-vitro method of evaluation. It has been used in radiography (Cohen et al., 1981), CT (Cohen and Bianca 1979), ultrasonic imaging (Smith et al., 1983; 1985) and non-medical optical imaging systems (Rose, 1948). This technique quantifies the ability of the total system (imaging plus observer), to detect, display and recognize objects that lie at the threshold of human vision in terms of contrast and size.

Contrast-Detail Analysis was first extended to ultrasound by Smith et al. (1983) with the aid of a new imaging phantom. The phantom has a tissue mimicking gel and a row of cones of varying contrasts (range ~20 dB relative to background). The diameter range for each cone is 1-18 mm. Smith et al. experimentally obtained contrast versus diameter curves by scanning the phantom, and noting the diameter of the cone at the threshold of detectability for each contrast level. They found that the curves had two regions: the high contrast region (> 4 dB), in which the detection capability of the system was weakly dependent on contrast and approached the spatial resolution of the system;
and the low contrast region (< 4 dB), in which the diameter for detection was strongly dependent on contrast. They performed a theoretical analysis of ultrasound contrast-detail detection in the presence of image noise based on stochastic signal and noise analysis. For a fixed level of ideal observer performance, they showed that contrast and diameter were inversely proportional, i.e., the log-log plot of contrast vs. diameter had a slope of -1. Because the performance of the human observer is less than ideal, their experimental results indicated lower slopes (-0.86 and -0.75). They concluded that by using certain image-processing techniques slopes approaching -1 might be achieved.

Contrast resolution of ultrasound imaging systems has also been studied by other researchers. Turnbull et al. (1992) simulated the images of a spherical cyst, and compared the contrast levels obtained with linear, 1.5-D and 2-D arrays. More recently, Johnson (1997) introduced a method called contrast response in which he also plotted contrast as a function of diameter. This method differs from the Contrast-Detail Analysis in that it plots the image contrast of an anechoic cyst as the cyst diameter is increased whereas the Contrast-Detail Analysis plots the minimum cyst size that can still be detected by a human/computer observer for a given cyst contrast. Furthermore, the latter is an experimental method, while the former employs computer simulations to predict the imaging performance.

Similar approaches to Johnson's have also been used previously by Vilkomerson et al. (1995) and, more recently, by Christopher (1997; 1998). Christopher (1997) simulated harmonic beam profiles as a function of radial distance and integrated the area under these profiles to predict relative contrast resolutions. Through simulations (1997) and experimental results (1998), he demonstrated that the contrast performance for higher harmonic imaging is better than that at the fundamental frequency.

1.3.4 Atherosclerosis and Ultrasonic Imaging

Stroke is a major cause of mortality, and 70% of all strokes are caused by atherosclerosis. Atherosclerosis is an arterial disease characterized by accumulation of lipids, cells and extracellular matrix in the intima producing the atheromatous plaques. The common carotid bifurcation and the adjacent segment of the internal carotid artery are common sites for atherosclerosis.
Chapter 1

The development and presence of a stenosis is not the only factor related with increased risk of stroke; the composition of plaque is also a great determinant of risk. Ultrasonic imaging has proved to be a useful, non-invasive tool in providing information about both stenosis and the nature of the plaque. Currently, there are two separate problems related with imaging carotid plaques:

1) B-mode imaging has been used for quantitative measurement of intima-media thickness (double-line) (Pignoli et al., 1986; Poli et al., 1988; O'Leary et al., 1991). The double-line is of clinical value to evaluate the severity of atherosclerosis and its progression. Better B-mode resolution would allow more precise measurements.

2) Ultrasonic imaging of plaques in patients with carotid disease has shown different levels of echogenecity (Landini et al., 1986; Bock and Lusby, 1992). The texture of an atherosclerotic deposit in the carotid arteries could be predictive of the chances that a patient will develop symptoms and/or stroke. Better contrast resolution would provide more precise definition of the structure, and perhaps, the internal contents of the lesion.

1.4 Thesis Overview

In order to understand the parameters that influence contrast and detail resolution, all the components of the imaging system should be accurately represented. In modeling the beam profiles from linear transducer arrays, a variety of simulation methods have been previously described. One important objective of this work is to compare these different techniques in terms of their accuracy and computation time. This analysis, which is presented in Chapter 2, proved to be very useful in choosing an accurate, efficient model for further work. Chapter 2 also discusses the extensions of the model to 1.5-D arrays and to arrays in which the elements lie on a curve.

While Chapter 2 focuses on the transmission and reception of the ultrasound beam by transducer arrays, Chapter 3 describes the interaction of the beam with the tissue medium. More specifically, it analyzes various theoretical models for the scattering and the frequency-dependent attenuation of the ultrasound beam.

The theoretical model presented in Chapter 2 was verified by comparing the simulation results with experimental data. Collaborative work with General Electric, (Milwaukee) helped attain this goal and the comparison is presented in Chapter 4.
Fundamental studies on contrast resolution were performed to investigate the effects of array design and tissue properties on imaging performance. The methods and results of these investigations are presented in Chapter 5.

Finally, a tissue scattering model was constructed and applied to the imaging of carotid artery with atherosclerotic plaque. Chapter 6 details the assumptions underlying the model and illustrates the carotid artery image results.

Chapter 7 summarizes the results of this thesis and the contributions made; it also discusses some thoughts on possible future work.
CHAPTER 2
PULSED RESPONSE OF TRANSDUCER ARRAYS

In order to simulate the B-mode images of a scattering medium, all the components of the system should carefully be modeled. Accordingly, simulation entails modeling the transmission of the ultrasound signal, frequency dependent attenuation and scattering, and the processes associated with reception and display of the image. The computer model considers linear transducer arrays, 1.5-D and curvilinear arrays, and 3-D targets imbedded in a scattering medium. This chapter describes the simulation of the transmitted pressure field from pulsed transducer arrays: two-way tissue effects are discussed in the next chapter.

2.1 Linear Arrays

Modern medical ultrasonic imaging systems generally employ linear phased arrays that enable the beam to be electronically steered and focused both on transmission and reception together with a fixed focus acoustic lens to focus in the elevation direction. A variety of simulation methods have previously been used to model the transient response from pulsed linear arrays. These commonly make use of different approximations, some of which lack adequate justification and whose influence on the accuracy have not been discussed.

One objective of this work was to examine the accuracy and computational efficiency of different methods that have been employed to calculate the field profile from 1-D arrays (Crombie et al., 1997). These methods were divided into two broad categories: those that solve the problem in the spatial frequency domain (diffraction theory) and those that use a spatial impulse response technique.

In the past, diffraction theory was utilized extensively to calculate the lateral profiles of arrays. For continuous wave (CW) excitation, the time and spatial responses of the transducer are completely separable and, in the far-field region, the lateral response is simply the Fourier transform of the aperture function (Macovski, 1983). In the case of pulsed excitation, linear systems theory can be employed to calculate the wideband
diffraction pattern. The principal drawback of this approach is that it only provides a far-field solution. To obtain a solution closer to the transducer the less restrictive Fresnel approximation can be applied, but then the solution can no longer be expressed as the Fourier transform of the aperture.

The transient acoustic field can also be computed by determining the impulse response at all field points and convolving this with the excitation. This impulse response depends on the transducer geometry and on the field point position. Closed form expressions have been derived for a variety of geometries, e.g., circular transducers (Lockwood and Willette, 1973; Xue et al., 1996), rectangular pistons (Ullate and San Emeterio, 1992; San Emeterio and Ullate, 1992), and triangular surfaces (Jensen, 1996). Unfortunately, the derivative of the impulse response generally exhibits discontinuities, thus requiring the use of high sampling rates. Furthermore, the choice of the sampling frequency depends highly on the location of the observation point and the size of the aperture: higher sampling rates are needed for the axial field points in the far-field of small rectangular elements. Nonetheless, this method is exact within the restriction imposed by the Rayleigh integral.

A comparison of the different techniques and approximations used in determining the transient field of linear arrays is presented in Section 2.1.3. All methods are compared in terms of their accuracy and computation time to the exact, closed-form solution provided by San Emeterio and Ullate (1992). The theoretical basis for this comparison is explained below.

2.1.1 Theory

With reference to Figure 2.1, we consider a planar transducer surface $S$ imbedded in an infinite rigid baffle. The pressure field generated by this surface in a homogeneous, lossless medium can be calculated from the velocity potential $\phi(\mathbf{r}, t)$ using

$$p(\mathbf{r}, t) = \rho \frac{\partial \phi(\mathbf{r}, t)}{\partial t} \quad (2.1)$$

and the Rayleigh integral, yielding
Figure 2.1. Geometry and the coordinate system used in calculating the transient field. (a) The transducer plane is at \((x_0, y_0, 0)\) and the field point is at \((x_z, y_z, z)\). For the array simulated, \(w=150 \mu m\), \(d=200 \mu m\), \(D=6.35 \text{ mm}\) and \(H=10 \text{ mm}\). (b) The steering angle is \(\theta\) and the coordinates are \(x_s\) and \(z_s\).
where $\rho$ is the density of the medium, $c$ is the speed of sound, $R$ is the distance between the field point and elementary area, $ds$, on the transducer ($R = |\vec{r}|$), and the surface velocity, $v_0(t)$, is assumed to be uniform.

If the planar surface is imbedded in an infinite soft baffle, and if the pressure distribution $p_o(t)$ is defined over the surface, the transient field can be calculated from:

$$ p(\vec{r}, t) = \rho \frac{\partial}{\partial t} \int_S \frac{v_0(t - \frac{R}{c})}{2\pi R} ds ,$$

where the cosine of the angle between the field point and the normal vector, $\vec{n}$, to the transducer surface is often referred to as the obliquity factor (Macovski, 1979). For simplicity, most of the results presented in Section 2.1 were obtained by assuming a rigid baffle. The effect of soft baffle on resolution is presented in the paper by Crombie et al. (1997).

The Rayleigh integral may be solved in either the time or frequency domains. In the time domain, the field is determined by a temporal convolution between the excitation signal and the spatial impulse response. In the frequency domain, diffraction theory is used to determine a solution for the continuous wave (CW) radiation pattern at a single frequency. Broadband fields can then be obtained by superimposing the CW response for all of the excitation frequency components. These solutions and the various approximations involved in simplifying the calculations of the field profiles are explained below.

### A) Time Domain Solution

Equation (2.2) can be expressed in the form of a convolution in the time domain:

$$ p(\vec{r}, t) = \rho v_o(t) \frac{\partial h(\vec{r}, t)}{\partial t} ,$$

where
\[ h(\vec{r}, t) = \int_{S} \frac{\delta(t - \frac{R}{c})}{2\pi R c} ds \]

is the spatial impulse response, and \( \delta(t) \) is the Dirac delta function. For the linear array shown in Figure 2.1(a), consisting of \( N \) identical rectangular elements of width \( w \), height \( H \) and center-to-center spacing \( d \), the impulse response can be found by superimposing the impulse responses, \( h_e(\vec{r}, t) \), of the individual elements. If the array is located on the plane \( z = 0 \), then the array impulse response \( h_A(\vec{r}, t) \) at a field point \((x, y, z)\) can be expressed as:

\[
h_A(x, y, z, t) = \sum_{i=-N/2}^{N/2-1} A_i h_e(x - x_i, y, z, t - T_i), \tag{2.5}
\]

where \( x_i = (i + 0.5)d \) is the location of the center of the \( i \)th element on the \( x \)-axis, \( A_i \) is the apodization factor and \( T_i \) is the excitation delay.

To focus the array at an on-axis point \((x = 0, y = 0, z = R_F)\), the delay, \( T_i \), is varied quadratically with the position of the element center, i.e.,

\[
T_i = \left[ R_F - \left( R_F^2 + X_i^2 \right)^{1/2} \right]/c, \quad (i = -N/2...N/2-1). \tag{2.6}
\]

Additionally, if the beam is steered at an angle \( \theta \) (see Figure 2.1(b)), time delays of

\[
T_i = X_i \sin(\theta)/c, \tag{2.7}
\]

must be applied to each element. Consequently, if the beam is both steered and focused at a distance \( R_F \) along the steered axis \( z_s \), then the required time delays are given by:

\[
T_i = \left[ R_F - \left( R_F^2 + X_i^2 - 2R_F X_i \sin\theta \right)^{1/2} \right]/c, \tag{2.8}
\]

which, when substituted into (2.5), enables the array impulse response to be found.

The average pressure on the transducer surface on reception can be calculated by convolving the pressure at the field point, as given by (2.4), with the pressure impulse response \((dh_r(\vec{r}, t)/dt)\) of the receiver (Weight and Hayman, 1978):

\[
p_r(\vec{r}, t) = \frac{\rho}{c} v_o(t) * \frac{\partial h_t(\vec{r}, t)}{\partial t} * \frac{\partial h_r(\vec{r}, t)}{\partial t}, \tag{2.9}
\]

where the subscripts \( t \) and \( r \) indicate the transmit and receive modes, respectively. In
general the transmit and receive impulse responses are not identical since different focusing, steering and/or apodization strategies may be employed on transmit and receive. Indeed, in order to improve the resolution of the image without sacrificing the frame rate dynamic focusing is usually employed on reception.

*Exact Method:* By substituting (2.5) into (2.4) an exact solution for the transmit field profile of the linear array is obtained. Since a rectangular aperture lacks circular symmetry, multiple expressions are involved and their nature depends on the field point location. Closed form analytical expressions for the impulse response of a rectangular array element were given by San Emeterio and Ullate (1992) and these have been used as the "gold standard" in this comparison study.

*Approximations:* To reduce computational complexity many researchers have employed far-field and/or zero-height approximations. Stephanishen (1972), Turnbull and Foster (1991) and Jensen and Svendsen (1992) approximated the far-field impulse response of a rectangular element with a trapezoidal function. Jensen and Svendsen showed how this approach can be applied to an arbitrary-shaped transducer by dividing the shape into rectangles whose sizes were chosen to ensure that the far-field approximation was valid at each field point. To achieve this, the maximum dimension of each element, $a$, was chosen so that $a << 2\sqrt{\frac{\lambda}{l}}$, where $l$ is the distance to the field point and $\lambda$ is the wavelength associated with the highest excitation frequency. Turnbull and Foster used a hybrid approach in which they assumed a trapezoidal response for field points significantly off each element axis, but used the exact algorithm given by Lockwood and Willette (1973) for field points close to the axes. In contrast, Denisenko et al. (1985) ignored the height of the array and divided the line source into a small number of elements. For each element, they assumed that the impulse response was described by a rectangular function in the far-field.

**B) Frequency Domain Solution**

If the transducer surface velocity is sinusoidal at an angular frequency of $\omega_c$, i.e.
\( v_o(t) = v_o e^{i\omega_c t} \), (2.2) can be rewritten as:

\[
P_{CW}(\tilde{r}, t) = \frac{\rho}{2\pi} j\omega_c v_o e^{i\omega_c t} L(\tilde{r}, \omega_c),
\]

where

\[
L(\tilde{r}, \omega_c) = \int_\mathcal{S} \frac{e^{-j\omega_c \tilde{r}/c}}{R} ds.
\]

The integral equation, \( L(\tilde{r}, \omega_c) \), defines the lateral response of the transducer while the exponential term describes its temporal behaviour. Thus, the temporal and spatial responses of any planar transducer that vibrates with a harmonic velocity is separable. In the frequency domain, the transient pressure field can be calculated by taking the Fourier transform of (2.10):

\[
P_{CW}(\tilde{r}, \omega) = \frac{\rho}{2\pi} j\omega_c v_o \delta(\omega - \omega_c) L(\tilde{r}, \omega_c).
\]

If the transducer is excited by a train of short pulses with a pulse repetition angular frequency of \( \omega_{PRF} \), then the transmitted waveform can be expressed as:

\[
V(\omega) = V_p(\omega) \sum_n \delta(\omega - n\omega_{PRF}),
\]

where \( V_p(\omega) \) defines the shape of each transmitted pulse in the frequency domain. At each harmonic, \( \omega_n = n\omega_{PRF} \), the field can be calculated by substituting \( \omega_n \) for \( \omega_c \) and \( V_p(\omega_n) \) for \( v_o \) in (2.11). Hence, the transmit pressure field is given by:

\[
P(\tilde{r}, \omega) = \frac{\rho}{2\pi} \sum_n j\omega_n V_p(\omega_n) L(\tilde{r}, \omega_n).
\]

To compute the transmit-receive profile, the receiver lateral response \( L_r(\tilde{r}, \omega_n) \) must also be considered, enabling the pulse-echo response to be written as:

\[
P_r(\tilde{r}, \omega) = \frac{\rho}{2\pi} \sum_n -\omega_n^2 V_p(\omega_n) L_t(\tilde{r}, \omega_n) L_r(\tilde{r}, \omega_n),
\]

where \( L_t(\tilde{r}, \omega_n) \) is the transmit lateral response. The time domain solution can be obtained from this equation by taking the inverse Fourier transform.

For the linear array shown in Figure 2.1(a), the lateral response is given by:
where \( R = \sqrt{z^2 + (x_o - x_i)^2 + (y_o - y_i)^2} \), \( S_i \) defines the boundaries of the \( i \)th element, \( A_i \) is the apodization factor, and \( T_i \) is given by (2.8).

**Approximations:** The diffraction formula in (2.15) involves a double integral over the array’s aperture. To simplify the calculation of this integral, approximations are made to the distance \( R \) both in the exponent of (2.15) and in the denominator. In order to distinguish the effects of both approximations, we refer to the \( R \) in the exponent as \( R_p \), and \( R \) in the denominator as \( R_d \). To proceed, we consider the binomial expansion of \( R \):

\[
R = z \left[ 1 + \frac{1}{2} \left( \frac{x_o - x_i}{z} \right)^2 + \frac{1}{2} \left( \frac{y_o - y_i}{z} \right)^2 \right] - \\
\frac{1}{8} \left( \left( \frac{x_o - x_i}{z} \right)^2 + \left( \frac{y_o - y_i}{z} \right)^2 \right)^2 + \ldots 
\]

(2.16)

In the Fresnel region where \( \frac{\omega_n}{8c z^3} \max \ll 1 \) radian, \( R_p \) can be approximated by using the first two terms of (2.16):

\[
R_p \equiv z + \frac{(x_o - x_i)^2 + (y_o - y_i)^2}{2z}.
\]

(2.17)

Applying this approximation to (2.15) allows the lateral response to be expressed in terms of the two-dimensional spatial Fourier transform, \( \tilde{h} \) as:

\[
L_A(x_o, y_o, z, \omega_n) = e^{-j \omega_n \left( z + \frac{x_o^2 + y_o^2}{2z^2} \right)} \sum_{i=-N/2}^{N/2-1} A_i \frac{e^{-j \omega_n T_i}}{R_d} x \\
\tilde{h} \left\{ \text{rect} \left( \frac{x_o - x_i}{w} \right) \text{rect} \left( \frac{y_o}{H} \right) e^{-j \omega_n \left( \frac{x_o^2 + y_o^2}{2z^2} \right)} \right\},
\]

(2.18)

where \( \tilde{h} \) is evaluated at the spatial frequencies \( f_x = x_o / (\lambda_n z) \), \( f_y = y_o / (\lambda_n z) \), \( \text{rect}(x) \)
defines the aperture of each element, and $\lambda_n = 2\pi c/\omega_n$. If the Fourier transform is evaluated, a more computationally efficient form of (2.18) is

$$L_A(x_z, y_z, z, \omega_n) = \frac{jz\lambda_n}{4} e^{-j\frac{\omega_n}{c} z} \left\{ \text{erf}[\alpha_n(H/2 - y_z)] - \text{erf}[\alpha_n(-H/2 - y_z)] \right\}$$

$$+ \sum_{i=-N/2}^{N/2-1} \frac{A_i}{R_d} e^{-j\omega_n T_i} \left\{ \text{erf}[\alpha_n(X_i + w/2 - x_z)] - \text{erf}[\alpha_n(X_i - w/2 - x_z)] \right\},$$

(2.19)

where $\alpha_n = \sqrt{-j\pi/(\lambda_n z)}$ and erf(x) is the error function. A further simplification of this equation can be made by ignoring the height of the array, i.e. neglecting the $(y_z - y_o)^2$ and higher order terms in the binomial expansion of (2.16) to yield:

$$L_A(x_z, y_z, z, \omega_n) = \sqrt{\frac{jz\lambda_n}{2}} e^{-j\frac{\omega_n}{c} z}$$

$$+ \sum_{i=-N/2}^{N/2-1} \frac{A_i}{R_d} e^{-j\omega_n T_i} \left\{ \text{erf}[\alpha_n(X_i + w/2 - x_z)] - \text{erf}[\alpha_n(X_i - w/2 - x_z)] \right\}.$$ 

(2.20)

If the observation point can be assumed to be in the Fraunhofer region of each element, then the quadratic phase factor within the Fourier transform of (2.18) can be ignored enabling the lateral response to be expressed as:

$$L_A(x_z, y_z, z, \omega_n) = e^{-j\frac{\omega_n}{c}(z + \frac{x_z^2 + y_z^2}{2z})} H w \sin\left(\frac{y_z H}{\lambda_n z}\right) \sin\left(\frac{x_z w}{\lambda_n z}\right)$$

$$\times \sum_{i=-N/2}^{N/2-1} \frac{A_i}{R_d} e^{-j\omega_n T_i + \frac{x_z x_i}{zc}}.$$ 

(2.21)

If the height of the array is also ignored then (2.21) reduces to:

$$L_A(x_z, y_z, z, \omega_n) = e^{-j\frac{\omega_n}{c}(z + \frac{x_z^2}{2z})} w \sin\left(\frac{x_z w}{\lambda_n z}\right) \sum_{i=-N/2}^{N/2-1} \frac{A_i}{R_d} e^{-j\omega_n T_i + \frac{x_z x_i}{zc}}.$$ 

(2.22)

In (2.18)-(2.22), $R_d$ represents the approximation for $R$ in the denominator of (2.15). The various approximations used by different authors for $R_d$ are summarized in Table 2.1. For example, in the Fresnel region, Lu and Greenleaf (1994) used $R_d \approx R_i$, which is the distance from the center of each element to the field point.
2.1.2 Methods

Listed in Table 2.1 is a brief description of the approximations used by certain authors\(^1\) to calculate the transient field from a linear array. Each of the six methods shown were evaluated for an array geometry that is typical of clinical transducers. Specifically, the field profile from a 32 element segment of a linear array was calculated, using an element width \(w = 150\ \mu\text{m}\), height \(H = 10\ \text{mm}\), and center-to-center spacing \(d = 200\ \mu\text{m}\) (see Figure 2.1(a)). Further, a rigid baffle, no apodization \((A_i = 1)\) and \(c = 1540\ \text{m/s}\) were assumed. Finally, it was assumed that each element had a surface velocity waveform that was described by the Gaussian modulated cosine function shown in Figure 2.2, with a center frequency of 5 MHz, a 3 dB bandwidth of 2.35 MHz and unit peak amplitude.

<table>
<thead>
<tr>
<th>Method</th>
<th>Approximations</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Domain</td>
<td>(i) None</td>
<td>San Emeterio and Ullate (1992)</td>
</tr>
<tr>
<td>(ii) Trapezoidal (h_\phi(r,t)) and (H \neq 0)</td>
<td>Turnbull and Foster (1991)</td>
<td></td>
</tr>
<tr>
<td>(iii) Rectangular (h_\phi(r,t)) and (H = 0)</td>
<td>Denisenko et al. (1985)</td>
<td></td>
</tr>
<tr>
<td>Frequency Domain</td>
<td>(iv) Fresnel with (R_d = R_i), (H \neq 0)</td>
<td>Lu and Greenleaf (1994)</td>
</tr>
<tr>
<td>(v) Fresnel with (R_d = z + (X - X_i)^2 / 2z), (H = 0)</td>
<td>Jacovitti et al. (1985)</td>
<td></td>
</tr>
<tr>
<td>(vi) Fraunhofer with (R_d = ?), (H = 0) with (R_d = R_i) ((?), (H \neq 0)</td>
<td>Bardsley and Christensen (1981)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Durinckx (1981)</td>
</tr>
</tbody>
</table>

\(?\) denotes that the approximation could not be determined from the paper.

In Table 2.1, methods (i)-(iii) use the impulse response function of an element to calculate the field profile of the array by employing the convolution described by (2.4). This convolution was performed in the frequency domain by using a Fast Fourier

---

\(^1\) While the authors listed in the table used the approximations stated, their implementation of the algorithms to calculate the beam profile may have been different.
Figure 2.2. The Gaussian pulse with $f_c = 5$ MHz used in simulations. (a) The pulse in time domain, and (b) in frequency domain.
Transform algorithm. For the "gold standard" method (i) a sampling frequency of 4 GHz was used, which was sufficient to ensure that sampling errors were negligible. In method (ii), each element was broken up into 10 rectangles in the y-direction and the impulse response for each of these sub-elements was approximated using a trapezoidal function. The choice of 10 rectangles satisfied the far-field criterion previously described by Jensen and Svendsen (1992). For method (iii), the height was ignored ($H=0$), the elements were divided into 3 line segments in the x-direction, and a sampling frequency of 16 GHz was used. Figure 2.3 illustrates the impulse response of the array calculated at two different on-axis field points using methods (i)-(iii). Clearly, the choice of a trapezoidal function is a good approximation, whereas the failure of method (iii) to account for the array height generates the depicted discrepancy.

Methods (iv)-(vi) were simulated by substituting (2.19), (2.20) and (2.21) into (2.13). For all of the simulations, $R_d$ was taken to be the distance from the field point to the center of the array, $R_c$. Using this approximation rather than $R_i$, the distance to the element center, did not significantly change the results. To demonstrate how the Fresnel and Fraunhoffer approximations affect the results, the absolute peak pressure was calculated on-axis $(x_c=0, y_c=0)$ as a function of depth and these results are shown in Figure 2.4 for both the unfocussed and focused cases. For the transducer assumed in this study the Fresnel approximation shows excellent agreement with the exact method (i), on-axis at a normalized distance of $4\lambda_c z/DH > 0.1$, where $\lambda_c$ is the wavelength at an angular frequency $\omega_c$. However, as shown in the insets, this approximation breaks down closer to the transducer. The Fraunhoffer solution approaches the exact solution at much greater depths $(4\lambda_c z/DH > 3)$, though it converges less rapidly for a focused array.

An important aspect of field computation is the spatial resolution. The ability to distinguish between two adjacent points normal to the beam direction is governed by the beam profile, and consequently, the -6 dB beam width was used as a measure of the resolution in this direction. In the axial direction, the resolution is governed primarily by the bandwidth of the transmitted signal. In other words, over an axial resolution cell there is little change in the impulse signal. Since the waveforms at two adjacent points will be nearly identical in shape, but displaced in time, we have assumed that the axial
resolution at a specific location on the beam axis can be calculated from the -6 dB point of the envelope of the pressure waveform at that location. This was verified by comparing the beam width calculated from the envelope of $p(\bar{r}, t_0)$ with that calculated from the point spread function $p(\bar{r}_0, t)$. The results were almost identical and therefore to simplify the presentation of the results the following transformation was used to convert the time axis of $p(\bar{r}_0, t)$ to a virtual $z'$ axis

$$z' = \left[ 2 \left( \frac{z_0}{c} + \frac{\Delta T}{2} \right) - t - \frac{\Delta T}{2} \right] c \cos(\theta)$$

(2.23)
where $\Delta T$ is the duration of the transmitted burst. This transformation ensures that the peak of the point spread function occurs at the observation point $z_o$ and it converts the time axis to an equivalent spatial axis, thereby making interpretation of the graphs much simpler.

The aim of this study was not to calculate the variation in spatial resolution with position, but rather to compare the accuracy and computational efficiency of different methods for calculating the field profile from linear arrays. With this in mind, it should be noted that for most of this study the receiver was assumed to be a point located at the center of the array. The pressure impulse response for such a receiver is a Dirac delta function which reduces (2.9) to (2.4). Thus, these results represent the resolution as determined by the transmitter acting alone and do not represent the resolution of the imaging system. The influence of a receiver whose aperture is the same as the transmitter is also briefly examined in Section 2.1.3.

Using each of the algorithms listed in Table 2.1, the transmit pressure profile was computed at different field points $(x, y, z)$ and the resolution was determined from the logarithmic contour plots of these profiles. Methods (ii)-(vi) were all compared to the exact impulse response method (method (i)) both in terms of accuracy and computational speed. Specifically, the percentage error in the resolution for method $j$ was defined as:

$$
\%err = 100 \frac{R_i - R_j}{R_i}
$$

(2.24)

where $R_j$ is the resolution associated with that method and $R_i$ is the resolution calculated using method (i).

2.1.3 Results

Examples of acoustic wavefronts simulated using method (i) are shown in Figure 2.5 for the linear array specified in Figure 2.1 using the waveform shown in Figure 2.2. These results were generated assuming a zero steering angle and a focal distance of 25 mm. Figure 2.5(a) shows the pressure envelope amplitude in the central plane ($y=0$) as a function of azimuthal distance ($x$) and virtual depth ($z'$) when the main lobe has reached the focal point. Similar results are shown for the $y=5$ mm ($H/2$) plane
Figure 2.4. Comparison of the on-axis peak pressure profiles calculated using the exact method and the Fresnel and Fraunhofer approximations for (a) unfocused and (b) focused array ($R_f=25$ mm, $\theta=0$). The insets illustrate how the results vary close ($1 \leq z \leq 9$ mm) to the transducer surface.
Figure 2.5. Exact transmit pressure profiles calculated using method (i) for $R_c=25$ mm, $\theta=0$, when the main lobe reaches $z=25$ mm at: (a) central plane, $y_c=0$ and (b) $y_c=5$ mm. The transformation described by (2-23) was applied in order to present the results as a function of the virtual $z'$ axis. Shown is the envelope of the transient pressure and logarithmic plot showing the -6 dB, -20 dB and -40 dB contours. The contour lines were determined by normalizing with respect to the peak pressure in the central plane.

As expected the pressure amplitude diminishes off the central plane, however, the waveform originating from the edge at $y_c=-H/2$ separates from the main lobe resulting in the secondary peak illustrated at $z' \approx 23$ mm.

Comparisons of the $x-z'$ and $y-z'$ contours calculated using method (i) and method (iv) are shown in Figs. 2.6(a) and (b), respectively. The agreement between the two methods is excellent. From Figure 2.6(a), the axial and azimuthal ($x$) resolutions can be
calculated from the -6 dB contour at the focal point by taking cuts along the $x=0$ and $z=25$ mm lines. The lateral resolution in the $y$-direction (elevation) can be determined similarly from Figure 2.6(b). Consequently, this figure clearly indicates the shape of the 3-D resolution cell at the field point (0, 0, 25 mm).

In order to visualize how the lateral beam profile changes with depth, $x$-$y$ contours of constant pressure are plotted in Figure 2.7(a). Specifically, the -6 and -20 dB contours are shown at six different depths with the array focused at 25 mm in the $x$-direction and unfocused in the $y$-direction. The lateral resolution in both directions as well as the axial resolution were calculated as a function of depth and are shown in Figure 2.7(b). While there is little change in both the axial and elevation resolutions, the azimuthal resolution diminishes rapidly beyond the focus. It should also be noted that there is a natural focus in the elevation ($y$) plane at a depth of $H^2/(2\lambda_c)=162$ mm.

For the purposes of this work, the lateral resolution was determined only along the azimuthal plane. Figure 2.8(a) shows the percentage error in the azimuthal ($x$) resolution obtained using method (iv) at various depths using three different approximations for $R_d$. The results show that there is little difference between each approximation, and as a result, we chose $R_d=R_c$ for our subsequent simulations. In order to investigate whether these results can be applied to arrays of different dimensions but with the same $F$-number ($R_F/D$), two different aperture sizes ($D$) were simulated. Figure 2.8(b) reveals that the trends in % error are the same along the normalized depth ($4\lambda_c z/DH$), but that the errors are greater for the larger aperture size. This is expected, since the accuracy of approximation (2.17) is reduced for larger $D$. These results indicate that the trends observed from one array can be extended to arrays with different aperture sizes but identical $F$-numbers.

A) Accuracy of Different Methods

Figure 2.9 depicts the accuracy of the lateral and axial resolutions for methods (ii)-(vi) over a range of depths (10 mm - 155 mm). As in the previous figures, the array was unsteered and focused at 25 mm. The results for the trapezoidal approximation (method (ii)) are presented in Figure 2.9(a) which demonstrates that the errors in
Figure 2.6. Transmit field pressure contours (-6 and -20 dB) in: (a) $x$-$z'$ and (b) $y$-$z'$ planes for an array focus of 25 mm and no steering. The 6 dB resolution was determined by taking cuts along the dotted lines.
Figure 2.7. (a) Logarithmic $x$-$y$ contour plots of the field at increasing depths. The array is unfocused in the $y$-direction and focused in the $x$-direction at a depth of 25 mm (dotted line). (b) Azimuthal and elevation resolutions derived from the -6 dB contours shown in (a). The axial resolution is also shown.
Figure 2.8. The percentage error in lateral resolution between methods (i) and (iv) for (a) different approximations for $R_d$ and (b) different aperture sizes, $D$. For (b) the $F$-number ($R_f/D$) was kept constant for both apertures.
Figure 2.9. Accuracy associated with methods (ii)-(vi) as a function of depth (both normalized and un-normalized). The error in lateral and axial resolution was determined as the percentage difference between the resolution calculated using method (i) and: (a) method (ii), (b) method (iii) and (v), (c) method (iv), and (d) method (vi). The array was unsteered and focused at a depth of 25 mm ($R_c=25$ mm, $\theta=0$).
resolution are negligible. The accuracy of methods (iii) and (v) which ignore the array height, are shown in Fig 2.9(b). As illustrated, both the lateral and the axial resolution errors are less than 15%, both approximations have almost identical errors, and the errors decrease as the observation point approaches the far-field corresponding to the array height, i.e. \( z \gg H^2/4\lambda_c \approx 80 \text{ mm} \). The results for method (iv), which accounts for the array height, are presented in Fig 2.9(c). The errors are less than one percent in both lateral and axial resolution, indicating that the Fresnel approximation provides an accurate solution for this unsteered case when \( 4z\lambda_c/DH > 0.1 \). For method (vi) the field point is assumed to be in the Fraunhofer region of the element. From the results shown in Figure 2.9(d) it is apparent that the error in lateral resolution is unacceptable, especially when close to the focus. The explanation for this is as follows. At the focus the delay, \( T_i \), was introduced in order to cancel the quadratic phase term \( \omega n^2 \sigma_1^2/(2zc) \) at the center of each element in (2.18). However, when using the Fraunhofer approximation this quadratic phase term is ignored and this introduces the largest phase error at the focus. In contrast, for an unfocussed array, the error monotonically decreases with depth, as can be inferred from Figure 2.3(a). While the results shown in Figure 2.9(d) were calculated taking into account the height of the array, ignoring the height of the array has little effect on the magnitude of the errors in lateral resolution. For example, the mean error only increased from 59% to 63% when the height was ignored.

In summary, for an unsteered array, methods (ii) and (iv) were good approximations that yielded errors of <1%. Ignoring the height of the array, as in methods (iii) and (v), reduced the accuracy by as much as 15%. Unacceptable results were obtained when simulating a focused array using the CW Fraunhofer approximation \{method (vi)\}.

B) Effects of Steering

The results just presented (Figure 2.9) suggested that of the frequency domain methods, (iv) provides the most accurate solution when no beam steering is used. To investigate the effects of steering on the accuracy of method (iv) the array was steered at three different angles, \( \theta = 5^\circ, 10^\circ \) and \( 20^\circ \) while maintaining the same focal distance.
(R_\text{F}=25 \text{ mm}) along the steered axis \( z_s \) (see Figure 2.1(b)). This was accomplished by applying the delays described by (2.8) to each element and using method (i) to calculate the exact transmit profiles. Figure 2.10 illustrates the change in the transmit profile as the beam is steered away from the \( z \)-axis. These profiles were obtained when the main lobe of the transmit pulse reaches the focal point, i.e., at \( z_s=R_\text{F}=25 \text{ mm} \). It can be seen that with increasing steering angle, the -6 dB contour elongates along the \( x_s \) direction and gets narrower in the \( z_s \) direction.

The 6 dB axial and lateral resolutions were determined by taking cuts on the -6 dB contour along the \( x_s \) and \( z_s \) lines. Figure 2.11(a) indicates that, with increasing steering angle, the lateral resolution around and beyond the focal point deteriorates slightly. This behaviour can be explained by noting that the effective aperture size, \( D\cos \theta \) decreases with increasing \( \theta \), which increases the \( F \)-number. The effect of steering on the axial resolution is just the opposite. Along the \( z_s \) axis the effective pulse length decreases by \( \cos \theta \), causing the slight improvement in axial resolution shown in Figure 2.11(b).

The effect of steering on the accuracy of method (iv) is depicted in Figure 2.12, where it can be seen that the errors increase with steering angle. The lateral resolution error is less than 5% for \( \theta=5^0 \), 10% for \( \theta=10^0 \), but reaches a value of 50% for \( \theta=20^0 \). Minimum lateral resolution errors occur around the focal point, but unlike the unsteered case, these do not reduce with depth. Instead, they increase to a certain value and then remain constant. These characteristics are explained as follows: method (iv) employs the Fresnel approximation for \( R_p \) (see (2.17)), which assumes that

\[
\left[(x_z-x_o)^2 + (y_z-y_o)^2\right]^{2}_{\text{max}} < 8c\varepsilon^3/\omega_n.
\]

However, when the beam is steered, the field extends to regions where the value of \( (x_z-x_o)^2_{\text{max}} \) is comparable to \( z \), and hence, the approximation becomes poorer as the steering angle is increased. Moreover, as we go deeper into the medium, both \( x_z \) and \( z \) increase and therefore the approximation does not improve.
Figure 2.10. Contour lines (-6 dB and -20 dB) calculated using the exact method (i) at the focal distance \( z = z' = 25 \) mm for steering angles of: (a) \( \theta = 0^\circ \), (b) \( \theta = 5^\circ \), (c) \( \theta = 10^\circ \) and (d) \( \theta = 20^\circ \). The transformation described by (22) was used to present the results in terms of the virtual \( z' \) axis.
Figure 2.11. The effect of steering calculated using the exact method (i) on: (a) lateral resolution, and (b) axial resolution when the array was focused at 25 mm along the steered direction.
Figure 2.12. The effect of steering on the accuracy of method (iv) for: (a) the lateral and (b) the axial resolution.
C) Reception

The results presented in the previous sections assumed that the receiver was an ideal point located at the origin of the array coordinate system. However, in practice, the spatial impulse response of the array on reception, $h_r$, is not a simple Dirac delta function and consequently, the transmit-receive resolution is highly dependent on the choice of both $h_r$ and $h_t$.

The effects of $h_r$ on the resolution were investigated by assuming that in reception the array employed the same focusing and steering parameters as those used in transmission. In general, this does not represent the practical situation, since dynamic focusing is normally used on reception in order to maintain a constant $F$-number with depth, and thereby improve the system resolution. In order to simplify the calculations we assumed that the impulse response of the array on reception was identical to that on transmit i.e. $h_r = h_t$ in (2.9) and that $L_r = L_t$ in (2.14). Figure 2.13 shows contour maps for both the transmit and the transmit-receive response at the focal point 25 mm,

Figure 2.13. Comparison of the contour lines (-6 dB and -20 dB) in the $x-z'$ plane for the transmit and transmit-receive beam profile at the focal distance calculated using the exact method (i) for $z_s = R_f = 25$ mm, $\theta = 0$. 

calculated using method (i). It can be seen that on reception the lateral response gets narrower, whereas the on-axis axial profile becomes longer (-20 and -40 dB contours). In addition, the absence of the legs on the -40 dB contour line of the transmit-receive profile indicates suppression of the side lobes.

A comparison of the lateral and axial resolution for transmit and transmit-receive is shown in Figure 2.14(a). The results indicate that the lateral resolution is enhanced upon reception at the expense of axial resolution. Figure 2.14(b) illustrates how the errors associated with method (iv) vary with depth for both transmit and transmit/receive mode. The errors on reception are only slightly increased compared with those on transmit only.

Figure 2.14. The effect of transmit-receive on (a) lateral and axial resolution for the exact method (i), and (b) the errors in resolution associated with method (iv).
**D) Computation Time**

The computational burden of all the methods were determined by measuring the execution time for each method after each algorithm had been hand-optimized. All programs were run on a 486DX2-66 using the Watcom 32-bit compiler and the execution times were normalized with respect to method (i). The simulations were performed for 20x30 grid of field points over the range $0 \leq x \leq 10 \text{ mm}$ and $10 \leq z \leq 155 \text{ mm}$. When interpreting these results it is important to realize that the algorithms used to implement each method are not necessarily identical to those used by the authors listed in Table 2.1.

The computation time of the exact impulse response method is highly dependent on the sampling rate, $f_s$, which in turn is determined by the location of the field point as well as the size and geometry of the transducer. Higher rates are required for calculating the response of a small element at an on-axis location. The choice of a 4 GHz sampling frequency, that was used to obtain the previous presented “gold standard” results with method (i), is a major determinant of the execution time. It was found that when the sampling rate was increased to 8 GHz the results were changed by less than 0.1%. The same criterion was used for choosing the sampling rates for methods (ii) and (iii).

Figure 2.15 shows the trade-off between computation time and accuracy. The accuracy is represented by the percent mean and maximum errors in resolution over the entire calculated region. The gold standard for these results is method (i) with a 4 GHz sampling rate, a computation time of 1 and a mean and maximum error of 0. Using sampling rates of 2 GHz, 0.5 GHz and 0.25 GHz decreases the computation time, but increases the mean and (maximum) errors to approximately 0.3(1)%, 0.8(2)% and 2(5)%, respectively.

The same sampling rate, 4 GHz, was also employed for simulating method (ii) since the trapezoidal impulse response was assumed for all field points. The hybrid approach taken by Turnbull and Foster (1991) would decrease the computational burden by using lower $f_s$ for off-axis locations. Figure 2.15 indicates that the computation time for method (ii) is slightly greater than that for method (i). This is a direct consequence of dividing each array element into 10 sub-elements in the $y$-dimension. Using fewer sub-elements decreases the computation time at the expense of accuracy.
Figure 2.15. Accuracy vs. computational burden for all of the methods used in this paper. (a) The accuracy is represented by the percentage mean and maximum absolute errors in lateral resolution. (b) The computational burden is indicated by the execution times normalized with respect to the execution time of the exact method (i).
In a different implementation of the trapezoidal method, the area under the trapezoid was tracked, and the amplitude of the trapezoid was adjusted to match the integrated area over the time interval. This algorithm allowed lower sampling frequencies to be used while still maintaining an acceptable error: it also ensured that no energy was lost in the response. Figure 2.15 shows that, with a sampling frequency of 100 MHz, the mean and maximum error for method (ii) are only 0.55% and 3%, respectively. But the normalized calculation time (0.016) has been decreased by the very substantial factor of 63.

Method (iii) requires even higher rates since the array element, which is quite narrow in the x-direction (w=150 μm), is further divided into three smaller units requiring that a higher sampling rate be used. For the simulations presented, we used 16 GHz. Figure 2.15 shows that the computation time is increased because of this higher sampling rate.

In the frequency domain, the computation time is mainly determined by the bandwidth of the pulse and the pulse repetition frequency (PRF). For the Gaussian pulse used in our simulations frequencies from 0 - 11 MHz were considered. If we assume a PRF of 20 kHz, this frequency interval can be completely sampled using 2048 points. Increasing the PRF did not increase the error, but it decreased the computation time. Consequently, a PRF of 160 kHz (256 points) was chosen to sample the pulse in methods (iv)-(vi).

Figure 2.15 indicates that the computational time for method (iv) decreases by 70% compared with method (i) while the mean error is only 0.21%. In method (iv) most of the time was spent calculating the error functions associated with the array elements in (2.19). Ignoring the height of the array, as in method (v), did not significantly affect the computation time, but the mean error was increased to approximately 8%. For method (vi), there is a considerable reduction in computational time (0.06), at the expense of a much greater mean error (59%).

2.1.4 A Final Model for Linear Arrays

Based on the comparison study presented above, the trapezoidal method using a
100 MHz sampling rate was chosen to be used in further work since it provided the best trade-off between accuracy and efficiency. The computational efficiency was improved by tracking the area under each trapezoid. A similar approach was taken by Jensen and Svendsen (1992). Recently, D'hooge et al. (1997) also used similar low sampling frequencies for calculating a smoothed version of the impulse response function. Their method was based on the fact that the excitation pulse is band-limited, and therefore, the high-frequency components of the impulse function are irrelevant.

A) Elevation Focusing

An acoustic cylindrical lens is generally used in linear arrays to decrease the slice thickness in the focal region. The size of the focal region and the slice thickness at this region are determined by the wavelength, the focal distance and lens aperture (Shung et al., 1992). In general, a smaller F-number (focal distance/aperture size) results in a thinner slice thickness and a smaller depth of focus.

Although elevation focusing was not studied in the comparison presented above, its effects were subsequently considered. An ideal cylindrical lens was modeled by dividing the height of each array element into a sufficiently large number of sub-elements, and applying to each sub-element time delays that corresponded to the propagation delay through the lens.

A lens with a focal point of 2.5 cm and an aperture of 1.0 cm was chosen together with the linear array geometry defined in Figure 2.1. In Figure 2.16, which shows the effects of elevation focusing, it was assumed that the slice thickness corresponds to the -6 dB elevation resolution. Without the lens, the slice thickness is almost uniform throughout the depth, but with values that are comparable to the array height. With the simulated cylindrical lens the slice thickness is reduced considerably along the depth of focus\(^2\) (1.8 < z < 3.2 cm) at the expense of much increased values beyond this region.

\(^2\) Depth of focus can be defined as the distance between points where the field on axis is 3 dB less than that at the focal point (Kino, 1987).
Figure 2.16. Elevation focusing effects. Focusing with a fixed lens reduces the slice thickness around the focal region (1.8<z<3.2 cm).

**B) Dynamic Focusing and Apodization**

In B-mode imaging, the lateral resolution at a given depth is determined primarily by the wavelength corresponding to the pulse center frequency and the aperture sizes used both in transmit and receive. The lateral resolution at the focal point is given approximately by

\[ R_L = 1.2 F \lambda = 1.2f \lambda / D, \]

where \( F \) is the ratio of the focal depth \( f \) to the aperture \( D \). It is of course desirable to maintain uniform resolution throughout the axial image depth. But in transmission a fixed focus must be used; it is only during reception that the focal point can be changed. Dynamic focusing on reception can be achieved by adjusting the delays on each array element as a function of range (or time). In order to maintain a reasonably constant lateral resolution during reception the aperture should be increased with the focal depth so as to maintain a constant \( F \)-number.

Most imaging systems select one fixed focus and aperture on transmit, and continuously vary the aperture size on reception. This causes image degradation at distances far from the transmit focus. To reduce the effects of fixed transmit focus,

---

3 This is the Rayleigh resolution criterion, though it will be only approximately true for a linear array.
Freeman et al. (1995) used a retrospective filtering method. Specifically, they designed optimal filters that produce a new in-focus image by deconvolving the defocused transmit pattern. Through experimental results, they showed that this technique lengthens the depth of focus, and improves contrast resolution. Another method of maintaining image uniformity over the entire depth of interest is to use multiple transmit foci. Each focal zone, however, requires a separate ultrasound transmission and reception. Consequently, the addition of each focal zone slows down the image frame rate. Recent developments in digital B-mode imaging have allowed the use of multiple foci while maintaining a standard frame rate.

Of major concern in an imaging system is the presence of sidelobes. If the sidelobe level relative to the main beam is too large, signals from regions of object plane are imaged as though they came from the beam axis. This form of image distortion can be reduced through careful design of the array and also through the use of amplitude apodization (Maslak, 1985). Apodization reduces the sidelobe levels, thereby increasing the contrast resolution. The trade-off is poorer lateral resolution, since the main lobe width is slightly increased, however, the gain in the contrast response is more important in terms of the system performance (Johnson, 1997).

The results presented in Section 2.1.3 were obtained by assuming, for simplicity, the same fixed focal point and aperture for both transmission and reception. Even though the coefficients $A_i$ in (2.5) enable different apodization schemes to be investigated, uniform apodization was also used. The simulation model was subsequently modified to include dynamic focusing on reception. Figure 2.17 illustrates the effects of dynamic focusing on lateral resolution for the linear array described in Section 2.1.2. The solid line shows the resolution obtained when the same fixed aperture and focal distance were assumed both for transmission and reception (Tx=Rx), and this is identical to that shown in Figure 2.7. When the focal distance is continuously tracked during reception while maintaining the same aperture (dashed line), the resolution improves, but is not uniform throughout the depth. If the aperture is also dynamically adjusted to keep the $F$-number constant, the lateral resolution maintains low values throughout the image (dot-dash line).
2.1 Single Rx=Tx

Dyn. fcs. with Tx aperture

Dyn. fcs. with F-no=2

Figure 2.17. The effects of dynamic apodization. More uniform resolution is achieved when the focal point and the aperture are changed dynamically on reception, i.e. F-number is kept constant (F-no=2).

2.2 1.5-D Arrays

Several research groups have investigated the characteristics of 1.5-D arrays in the past few years (Smith et al., 1990; Turnbull and Foster, 1992; Daft et al., 1994) and in 1997, a new imaging system that employs a 1.5-D array was commercialized\(^4\). This new system and earlier research both confirm that by optimizing elevation focusing significant improvements in the contrast resolution can be achieved by using a 1.5-D array.

A schematic of the 1.5-D array\(^5\) modeled in this thesis is shown in Figure 2.18. This array has a total of five rows and three segments (\(N_y=3\)); two segments of elements on each side of the central segment. The boundaries of the segments are chosen as in a Fresnel zone plate, i.e. according to the criteria given by Turnbull and Foster (1991):

\(^4\) LOGIQ 700 MR, General Electric, Milwaukee.

\(^5\) The structure is believed to be close to that used in the G.E. Logiq system.
where $H$ is the total height of the array.

Variable time delays are applied to the central row for steering and/or focusing. Time delays are also used for the additional segments in the elevation direction in order to achieve dynamic focusing on reception. An acoustical lens is used to provide a fixed transmit focus as with linear arrays. The lens is required to provide a well-sampled focusing on transmit, since the small number of extra segments is insufficient to accomplish this.

The beam profile in the elevation direction from a linear array was compared with those from 1.5-D arrays with 3, 5 and 7 rows. The array elements were 350 microns wide, with a center-to-center spacing of 400 microns. For the 1.5-D array, the height, 15 mm, was divided into segments according to the relation given in (2.25) with $N_y = 2, 3$ and 4. In the azimuthal direction, the arrays were focused at 4 cm on transmit, and dynamically focused on receive with an $F$-number of 2. In the elevation direction, a cylindrical lens with a fixed focus at 4 cm was simulated. Time delays were applied to the extra rows of 1.5-D array to provide dynamic focusing on receive.

![Diagram](image)

Figure 2.18. Schematic of the geometrical arrangement of transducer elements for a 1.5-D ultrasonic array with 5 rows.
Figure 2.19(a) depicts the beam profile in the elevation direction at z=2.5 cm for the linear array and the 1.5-D array with 3, 5 and 7 rows. As illustrated, the beam width decreases as the number of rows is increased. Figure 2.19(b) shows the slice thickness as a function of depth (2<z<6 cm). The figure indicates that, as the number of rows is increased, the size of the focal region is also increased providing improved and more uniform resolution in the elevation direction.

2.3 Curvilinear Arrays

In B-mode imaging, the image quality is limited by the relationship between the maximum frame rate (number of images per second), the depth of field and the number of scan lines. For real-time images (high frame rates), there is generally a trade-off between depth and field of view. Therefore, high-quality real-time images are difficult to obtain for deep organs, such as the liver, which require a large field of view.

Curvilinear arrays provide an increased field of view without requiring beam steering which inevitably results in some loss in performance. A schematic for a curvilinear array is presented in Figure 2.20. The elements lie on a curved surface, and are activated in successive groups to scan the field, and hence the name, *curvilinear*. The focal points (RF) lie on an arc, and the field of view increases with depth. Typical parameters for such an array are: 128 elements, width=350 μm, center-to-center spacing=400 μm (~5cm long array), height=10 mm, and radius of curvature=4-6 cm.

The field profiles from linear and curvilinear arrays were compared using the above parameters for both geometries. Figure 2.21 shows the lateral resolution versus depth for two different F-numbers of 1 (strong focusing) and 3 (weak focusing). It is clear from these plots that the lateral resolution follows the same pattern for both arrays, but that the values are higher (poorer resolution) for the curvilinear array. Note also that the difference between resolution values is less for F=3. The curvilinear array-to-linear array ratio of the resolution at the transmit focus (Tx) is plotted as a function of the F-number in Figure 2.22 (Tx=4 cm for F=1, and 10 cm for F=3). The figure indicates that the ratio approaches unity as F is increased. The F-number is increased by increasing the focal distance (RF) and/or decreasing the active aperture (θa). The curvilinear array
Figure 2.19 (a) The beam profiles, and (b) the resolution in the elevation direction for a linear array and 1.5-D arrays with 3, 5 and 7 rows.
Figure 2.20. Curvilinear array with elements of width $w$, spacing $d$, and radius of curvature $R_C$. The first two scan lines, which are obtained by firing only four consecutive elements for illustrative purposes, are indicated with dotted lines. The angle of the active aperture arc is represented by $\theta_a$. 
Figure 2.21. Lateral resolution curves for linear and curvilinear arrays. (a) Strong focusing (F-number=1), and (b) weak focusing (F-number=3).
Figure 2.22. The ratio of the curvi-linear array resolution to linear array resolution as a function of F-number. The same number of elements were used in both arrays.

behaves like a linear array under both these circumstances.

In conclusion, the lateral resolution from curvilinear arrays is degraded in comparison to that obtained from a linear array (see Figure 2.21). The primary advantage of the curvilinear array is that it enables a larger field of view to be achieved without the need for beam steering and as a result maintains the same lateral resolution over all angles, which should be contrasted to the performance of a linear array whose resolution is degraded with steering angle (see Section 2.1.3).

2.4 Chapter Summary

Various time-domain and frequency-domain methods for calculating the transient field generated by pulsed linear arrays were presented in Section 2.1.1. Their accuracy and efficiency were compared to the exact convolution-impulse response method using a sufficiently high sampling rate (4 GHz) to ensure negligible errors. A reduction of computational time without increasing the error may be achieved by reducing the sampling rate to match the expected impulse response characteristics at different field
points.

It was shown that the trapezoidal far-field approximation (method (ii)) yielded accurate results with about the same computational time as the exact method when same sampling frequency was used for both methods. This method also allowed for much smaller sampling frequencies to be employed without compromising accuracy, thereby reducing the simulation time considerably. In method (iii), which uses a rectangular impulse response, the height of the array is ignored, and this degrades the accuracy by as much as 15%. Furthermore, the computational burden is increased owing to the higher sampling rate required for the smaller sub-elements. In the frequency domain, the Fresnel approximation (method (iv)) provides an accurate solution for the unsteered array, and is generally more efficient than method (i). Employing the Fraunhoffer approximation (method (vi)) further improves the computational efficiency, but the errors are unacceptable (>50%).

When the array is steered, the accuracy of method (iv) is highly dependent on the steering angle. For example, if \( \theta=20^\circ \), the lateral resolution error can reach 50%; however, for small steering angles \( \theta<10^\circ \) the CW solution stills provides accurate results with mean errors of <7%. The measured accuracy of the CW approach can also be influenced by the choice of receiver. However, when the array is used as the receiver with the same focusing/steering parameters as in transmit mode, the errors in resolution are almost the same as those obtained with an ideal point receiver.

In conclusion, methods (i), (ii) and (iv) yield the most accurate solutions. Moreover these methods account for the height of the array, and hence, can compute the field at locations off the center plane \( y\neq 0 \). The primary drawback associated with the exact impulse response method (methods (i)) is that the accuracy and the computation time are highly dependent on the sampling frequency used, which is in turn determined by the array geometry and the field point location. In order to improve the computational efficiency while providing an exact solution with reasonable accuracy (~5%) the pressure field domain could be divided into regions and varying sampling rates could be used in each. No such problem exists for method (iv) which provides an efficient and simple algorithm that is independent of the field point location and the element size. The
primary advantage of method (i) is that it yields an exact solution for the near-field region of the source, whereas methods (ii) and (iv) employ the far-field and par-axial approximations, respectively. Furthermore, method (iv) is only accurate (errors <7%) for steering angles that are <10°. Therefore, for higher steering angles method (i) or (ii) should be used if an accurate solution is to be obtained.

It should be noted that the effects of such frequency-dependent factors as attenuation, scattering and the transducer characteristics were not incorporated in this comparison study. More recently, Berkhoff et al. (1996) demonstrated that attenuation has a significant influence on both axial and lateral resolution. Methods (i), (ii) and (iv) can be easily modified to include these frequency dependent factors and can, in addition, be used to simulate the transient response from 2-D arrays. Moreover, since the height of the element is typically much smaller for a 2-D array than that used in our simulations (Turnbull et al., 1992), the Fresnel approximation will be more accurate for each element. These 2-D arrays, which are the focus of considerable current research, should result in significant improvements in the elevation resolution.

Using the trapezoidal method {method (ii)} with a sampling frequency of 100 MHz, the simulation model was revised to include the effects of elevation focusing and dynamic focusing. The model was subsequently modified to simulate beam profiles from 1.5-D and curvilinear arrays. It was shown that the addition of extra rows of elements in the 1.5-D array improves the elevation resolution as compared with the linear array. The simulation results also indicated that the azimuthal resolution from a curvilinear array is slightly degraded compared with that from a linear array. However, curvilinear arrays are more desirable when imaging large targets since they provide uniform performance throughout their field of view.
3.1 Linear Systems Theory for Modeling B-mode Images

In modeling the B-mode image formation, we shall assume that the transducer and the propagation medium behave in a linear manner (Norton and Macovski, 1978; Stephanishen, 1981; Jacovitti et al., 1985; Fish and Cope, 1991). This assumption enables linear systems theory to be used for analyzing the imaging process illustrated in Figure 3.1. The process is modeled by a cascade of linear operations, where the time functions are shown in lower case and their Fourier transforms in upper case.

The impulse response of the transmit transducer \( e_m(t) \), enables the velocity on the transducer surface \( v_o(t) \), to be related to the input excitation voltage \( e_i(t) \), while the output voltage \( e_o(t) \) on receive transducer is related to the received force \( f_s(t) \), by its impulse response \( e_m(t) \). Mathematically, these two relations are expressed by:

\[
\begin{align*}
  v_o(t) &= e_i(t) * e_m(t), \\
  e_o(t) &= f_s(t) * e_m(t). 
\end{align*}
\]  

(3.1)

In clinical B-mode imaging although the transmit and receive transducers are the same, the aperture generally differs on transmission and reception as shown in Figure 3.2. Given the normal velocity profile \( v_o(t) \) on the transmitter's surface, \( S_n \), the incident pressure on the scatterer at \( \vec{r}_s \) is given by

\[
p_i(\vec{r}_s, t) = p \frac{dv_o(t)}{dt} * h_t(\vec{r}_{ts}, t)
\]  

(3.2)

where \( h_t(\vec{r}_{ts}, t) \) is the spatial impulse response on transmit as described in Chapter 2. The scattered pressure at \( \vec{r}_r \) caused by the scatterer at \( \vec{r}_s \) is related to the incident pressure by

\[
p_s(\vec{r}_r, t) = p_i(\vec{r}_s, t) * w_s(\vec{r}_{sr}, t) * m_t(\vec{r}_{ts}, t) * m_r(\vec{r}_{sr}, t),
\]  

(3.3)

and in the frequency domain by

\[
P_s(\vec{r}_r, \omega) = P_i(\vec{r}_s, \omega) * W_s(\vec{r}_{sr}, \omega) * M_t(\vec{r}_{ts}, \omega) * M_r(\vec{r}_{sr}, \omega),
\]  

(3.4)
Figure 3.1. Block diagram illustrating ultrasound wave propagation, scattering and attenuation as a cascade of linear operators.

Figure 3.2. The coordinate system used for analyzing the response due to a scatterer. In general the aperture used in transmission differs from that used in reception.
where $w$, is the pressure-to-pressure impulse response of the scatterer, $W$, is the scattering transfer function (Stephanishen, 1981), and $M_t$ and $M_r$ account for the effects of attenuation and dispersion in the transmit and receive paths, respectively. The force acting on the receiving surface area $S_r$ can therefore be found by integrating the scattered pressure over the surface, i.e.,

$$f_S(t) = K \int_{S_r} P_s(\vec{r}_r, t) \, ds_r,$$

(3.5)

where the constant $K=2$ for a perfect reflector.

Substituting (3.1), (3.2) and (3.3) into (3.5), the input and output voltages are related by

$$e_o(t) = \rho K \left( e_i(t) \ast em_i(t) \ast em_o(t) \ast \frac{\partial h_t(\vec{r}_{is}, t)}{\partial t} \ast m_i(\vec{r}_{is}, t) \ast \int_{S_r} m_r(\vec{r}_{sr}, t) \ast w_s(\vec{r}_{sr}, t) \, ds_r \right),$$

(3.6)

where $em_i(t) \ast em_o(t)$ is the two-way electro-mechanical impulse response of the transducer.

If the electro-mechanical characteristics of the transducer are ignored, then radiation-coupling of a transducer-target system is a linear filter that defines the relationship between $v_o(t)$ and $f_S(t)$. Rhyne (1977) found an exact solution for the radiation coupling of a disk radiating to, and receiving from, a planar target using the impulse response method. Fung et al. (1992) also derived this function for a disk to planar target, as well as for disk to monopole and dipole scatterers. They, however, used the angular spectrum method to determine the radiation-coupling functions.

The impulse response of the transducer, $h_t(\vec{r}_{is}, t)$, in (3.6) was discussed in Chapter 2. In this chapter, the effects of scattering and two-way attenuation and dispersion are analyzed. More specifically, section 3.2 describes the model for two-way attenuation and dispersion. Section 3.3 presents the theory for the scattering function for small spherical and cylindrical targets. Finally, the acoustic voxel method for modeling scattering is described in section 3.4.
3.2 Frequency-dependent Attenuation and Dispersion

The attenuation in tissue is generally frequency-dependent process, i.e. higher frequency components of the pulse get more attenuated, resulting in a change in the pulse shape as the wave travels deeper into the medium. Berkhoff et al. (1996) have shown that frequency-dependent attenuation has a large influence on the quality of ultrasound images.

Frequency-dependent attenuation is accompanied by velocity dispersion, which refers to the fact that different frequencies propagate with different speeds in the medium, also resulting in a change in pulse shape. Attenuation and dispersion are linked by the Kramer-Kronig relationship (O'Donnell et al., 1981).

Different models have been used to describe the dispersive medium (Gurumurthy and Arthur, 1982; Fish and Cope, 1991; Jensen et al., 1993; Berkhoff et al., 1996). Most assume that the attenuation and dispersion transfer function is given by

\[ M(r, f) = \exp[-\alpha(f)r] \exp[-2j\pi f/c(f)], \tag{3.7} \]

where \( \alpha(f) \) can be expressed by the power-law relationship

\[ \alpha(f) = \beta f^{-u}. \tag{3.8} \]

Attenuation and phase velocity are related by (Fish and Cope, 1991):

\[ \alpha(f) = \frac{\pi^2 f^2 c(f)}{c_o^2} \frac{dc(f)}{df}, \tag{3.9} \]

where \( c_o \) is the speed of sound at the center frequency, \( f_c \). Substituting (3.8) into (3.9), and solving the differential equation with the condition \( c(f_c) = c_o \), the phase velocity is given as:

\[ c(f) = c_o + \frac{c_o^2 \beta}{\pi^2} (\ln f - \ln f_o) \quad u = 1 \tag{3.10} \]

\[ c(f) = c_o + \frac{c_o^2 \beta}{\pi^2} f^{u-1} - f_o^{u-1} \quad u \neq 1 \]

A linear dependence of attenuation on frequency \((u=1)\) has been assumed for our computations and the effects of attenuation and dispersion have been incorporated by using (3.7), (3.8) and (3.10).
3.3 Scattering of Ultrasound

The scattering characteristics of an object depend on its shape, size and acoustic properties (density and compressibility) and is generally frequency and angle-dependent. The scattering function has been obtained for some simple geometries such as spheres (Morse and Ingard, 1968; Ziomek, 1995) and cylinders (Morse and Ingard, 1968; Flax, 1980; Li and Ueda, 1989; Ziomek, 1995).

3.3.1 Scattering from a Small Sphere

In modeling the B-mode image formation from a tissue-mimicking imaging phantom, most researchers assume that the scattering volume consists of a collection of point targets positioned randomly in the medium. When a plane wave is incident upon a very small \((ka<<1)\) sphere of radius \(a\), it scatters the wave equally in all directions, and hence, behaves like a monopolar point source. The scattering function in the far-field \((kr >> 1)\) of a sphere whose density is identical to the medium in which it is immersed is given in the frequency domain as (Morse and Ingard, 1978)

\[
W_s(kr_o) = \frac{1}{3} \left( \frac{\kappa - \kappa_s}{\kappa} \right) k^2 a^3 \frac{e^{ikr_o}}{r_o},
\]

where \(r_o = |\vec{r} - \vec{r}_s|\), \(\vec{r}_s\) is the vector from the origin of coordinates to the sphere, and \(\kappa\) and \(\kappa_s\) are the compressibilities of the medium and the scatterer, respectively.

Since the above equation only holds for a planar incident wave, it is not possible to directly insert (3.11) into (3.6) to solve for the radiation coupling of a transducer array to a point target system. However, this solution can be extended for a radial wave by using the plane-wave angular spectrum method (Fung et al., 1992). This method assumes that any wave pattern can be written in terms of an angular distribution of its plane-wave components. The relationship between the impulse response and angular spectrum methods have been described by Stephanishen (1991).

Using the angular-spectrum method (Fung et al., 1992), and the spherical coordinate system, the velocity potential \(\Phi(\vec{r})\) is given by

\[
\Phi(\vec{r}) = \int_0^\infty \int_{-\pi}^{\pi} A(\theta,\phi) e^{ik\cdot\vec{r}} d\phi d\theta,
\]
where $C$ is the integration contour for the complex polar angle $\theta$, and $A$ is the angular spectral density function:

$$A(\theta, \varphi) = \frac{k^2}{4\pi^2} \sin \theta \cos \theta S(k \sin \theta \cos \varphi, k \sin \theta \sin \varphi),$$

(3.13)

in which the spectral density function $S$ for a velocity distribution $v_o$ on the transducer surface is written in Cartesian co-ordinates as:

$$S(k_x, k_y) = \frac{j}{k_z} \int \int v_o(x, y, 0) e^{-j(k_x x + k_y y)} dx dy.$$  

(3.14)

The total force on the receiver surface caused by a scatterer at $\vec{r}_s$ can be found by integrating the pressure over the receiving transducer area $S_r$, i.e.

$$F_S(\omega) = 2 \int \int_{S_r} P_s(\vec{r}) dx dy.$$

But $P_s(\vec{r}) = P_t(\vec{r}_s) W_s(\vec{r} - \vec{r}_s)$ and $P_t = j\omega \rho \Phi$, so that from (3.12) the total force can be expressed as

$$F_S(\omega) = -2 jkpc \int_{S_r} dx dy \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} A(\theta, \varphi) e^{jk\cdot\vec{r}_s} W_s(\vec{k}, \vec{r} - \vec{r}_s) d\theta d\varphi.$$  

(3.15)

In general, the scattering function is complex, and consequently closed-form analytical expression cannot be obtained for the force. However, for a small sphere a closed-form solution can be obtained by inserting (3.11) into (3.15):

$$F_S(\omega) = \frac{-j4\pi\rho c (ka)^3}{3} \frac{\kappa - \kappa_s}{\kappa} \int_{-\pi}^{\pi} \int_{C} A(\theta, \varphi) e^{jk\cdot\vec{r}_s} d\theta d\varphi \int_{S_r} \frac{e^{jk\omega}}{2\pi r_o} dx dy.$$  

(3.16)

Noting that the first integral defines the velocity potential on transmit, and the second integral is the transfer function on reception, (3.16) can be rewritten as:

$$F_S(\omega) = \frac{4\pi\rho (j\omega)^3}{3c^2} \frac{\kappa - \kappa_s}{\kappa} \Phi(\vec{r}_s, \omega) \times H_r(\vec{r}_o, \omega).$$  

(3.17)

By using the convolution theorem, this equation can be written in time domain as:

$$f_S(t) = \frac{4\pi\rho a^3}{3c^2} \frac{\kappa - \kappa_s}{\kappa} \frac{\partial^2 \Phi(\vec{r}_s, t)}{\partial t^2} \times h_r(\vec{r}_o, t).$$  

(3.18)

Now the velocity potential can be expressed as
\[ \phi(\vec{r}, t) = v_o(t) \ast h_t(\vec{r}, t), \] (3.19)

and, hence, the total force on the transducer can be calculated using the impulse response method introduced in Chapter 2:

\[ f_S(t) = \frac{4\pi \rho a^3}{3c^2} \frac{(\kappa - \kappa_S)}{\kappa} \frac{\partial^3 v_o(t)}{\partial t^3} \ast h_t(\vec{r}_s, t) \ast h_r(\vec{r}_o, t). \] (3.20)

This final form for the radiation coupling between a transducer-point target system is in agreement with the expressions derived by Stephanishen (1981), Jensen (1991), and Fung et al. (1992).

### 3.3.2 Scattering from a Cylinder

A common method of measuring the transient pulse-echo field of a transducer system is to scan a small spherical target. However, the echo amplitude from such a small target is generally not always satisfactory. Recently, cylindrical targets in the form of long wires with diameters smaller than the wavelength have been used to perform such measurements (Raum and O'Brien, 1997). Such targets give a much higher echo signal.

When a plane wave with angular frequency \( \omega \) \((k = \omega/c)\) is incident at an angle \( \theta \) on an infinite cylinder of radius \( a \), the scattering function in the far-field of the cylinder \((kr \cos \theta \gg 1)\) can be expressed as (Li and Ueda, 1989):

\[ W_s(kr_o) = \left( \frac{2}{\pi k \cos \theta r_o} \right)^{1/2} \sum_{n=0}^{\infty} e_n \sin \eta_n \exp(j\eta_n) \cos \gamma, \] (3.21)

where \( r_o \) is the radial distance between the cylinder and the vector position \( \vec{r} \), and \( \gamma \) is the angle in the cylindrical co-ordinate system \((r_o, \gamma, z)\) for the target. The expressions for \( e_n \) and \( \eta_n \) are given in Li and Ueda's paper (1989). When this expression is inserted in (3.15), a closed-form analytical solution cannot be derived, and the complex integral has to be solved numerically. However, further approximations enable a closed-form expression to be obtained.

If the target is sufficiently far from the transducer surface, the plane-wave spectrum can be approximated by a single incident wave orthogonal to the cylinder axis (Fung et al., 1992). In this case \((\theta = 0)\), the scattering function for an infinite small-
diameter \((ka << 1)\) cylinder in the far-field \((kr >> 1)\) is given by (Morse and Ingard, 1968):

\[
W_s(kr_o) = \sqrt{\frac{\pi}{8}} k^{1.5} a^2 \frac{e^{ikr_o}}{\sqrt{r_o}} \left( \frac{\kappa_s - \kappa}{\kappa} + \frac{2\rho_s - 2\rho}{\rho_s + \rho} \cos \gamma \right),
\]

where \(\rho\) and \(\rho_s\) are the densities of the medium and the scatterer, respectively. To determine the radiation-coupling between a transducer and a cylindrical target, this equation should be substituted into (3.15):

\[
F_S(\vec{r}_s, \omega) = -j4\pi \rho ck^{2.5} a^2 \sqrt{\frac{\pi}{8}} \int_0^{\pi} \int_0^{\pi} A(\Theta, \Phi) e^{jK \cdot \vec{r}} d\Theta d\Phi \times \\
\int \int_{S_r} e^{jkr_o} \frac{\sqrt{r_o}}{2\pi r_o} \sqrt{r_o} \left( \frac{\kappa_s - \kappa}{\kappa} + \frac{2\rho_s - 2\rho}{\rho_s + \rho} \cos \gamma \right) dxdy.
\]

For a linear array, we can assume that \(\cos \gamma\) and \(\sqrt{r_o}\) do not vary significantly over the surface of each array element, so that (3.23) can be rewritten as:

\[
F_S(\vec{r}_s, \omega) = -\sqrt{\frac{\pi}{8}} j4\pi \rho ck^{2.5} a^2 V_o(\omega) \times H_t(\vec{r}_s, \omega) \times H'_r(r_o, \omega),
\]

where

\[
H'_r(r_o, \omega) = \sum_{i=-N/2}^{N/2-1} A_i \sqrt{r_{oi}} \left( \frac{\kappa_s - \kappa}{\kappa} + \frac{2\rho_s - 2\rho}{\rho_s + \rho} \cos \gamma_i \right) e^{-j\omega T_i} H_{re}(r_{oi}, \omega),
\]

is the modified array transfer function on receive, \(H_{re}\) is the transfer function for a single element on receive, \(r_{oi} = |\vec{r}_s - \vec{r}_i|\) is the radial distance (in cylindrical co-ordinates) between the scattering target and the element center, and \(\gamma_i\) is the angle between the scatterer and the element as shown in Figure 3.3. The total force in time domain can be determined by taking the inverse transform of (3.24).

Equation (3.24) indicates that the radiation-coupling between a linear array and a small-radius cylinder lying orthogonal to the wave of propagation can be approximately solved in the far-field by using the impulse response method.
3.3.3 Scattering from a Homogeneous Medium

Two different theoretical models are used in this thesis to simulate scattering from a homogeneous medium: the particle model and the voxel model. The particle model assumes that:

1) the medium is composed of a large collection of randomly distributed, point sized particles;
2) the scattered pressure is much smaller than the incident pressure so that the effects of multiple scattering can be ignored, and
3) superposition is applied to sum the scattered amplitudes from individual scatterers.

When modeling scattering from realistic tissue volumes, the computational burden can be large since the particle densities are quite high: for example, there are approximately 4.5 million red blood cells in a 1 mm$^3$ of 40% hematocrit blood. The approximate voxel approach suggested by Mo and Cobbold (1992) not only offers a highly accurate solution, but also reduces the computation time considerably (Lim et al., 1996).

The voxel approach differs from the particle method by dividing the medium into small volumes (acoustic voxels) each of which contain many scatterers that cannot be
individually resolved by the transducer at the incident wavelength. The number of point scatterers in each voxel will be determined by the statistics governing the scatter distribution. Consequently, each voxel can be considered to act as a single scatterer whose strength depends on the number of point scatterers it contains. Superposition can then be used to find the total response due to a given volume of the medium. One can also consider the variation in the scattering strengths of each voxel to model the fluctuations in acoustic properties of the medium, thereby providing a link between the particle and continuum approaches in the form of a unified theory. Moreover, depending on the selection of voxel size, computational efficiency can be greatly increased relative to the "exact" particle approach. Lim et al. (1996) compared the two approaches in terms of accuracy and efficiency, and showed that a voxel size of \( \lambda/20 \) produces very accurate results, with mean squared errors of 0.05-0.15, while reducing the computations by a factor of about 100 at physiological hematocrits.

For a wideband system (Bascom and Cobbold, 1995), the backscattered signal, \( x(t) \), from a signal voxel can be expressed as,

\[
x(t) = \frac{\sqrt{\sigma_b N_{\text{vox}}}}{R} \chi_T \left( t - 2 \frac{\mathbf{k} \cdot (\bar{r} + \Delta \bar{r}_{\text{avg}})}{kc} \right),
\]

(3.25)

where \( \sigma_b \) is the differential backscattering cross-section, \( N_{\text{vox}} \) is the number of particles in the voxel, \( \bar{r} (R = |\bar{r}|) \) is the vector position of the voxel center, \( \Delta \bar{r}_{\text{avg}} \) is the average vector position of the scatterers within the voxel with respect to the voxel center, and the function \( \chi_T(.) \) represents the transmitted signal.

### 3.4 Chapter Summary

The focus of this chapter was on the interaction of the signal with the tissue medium. By assuming that all the sub-processes involved in B-mode ultrasound imaging are linear, it has been shown that the overall process can be modeled by a set of linear operators. In order to account for the effects of attenuation, it has been assumed that the attenuation increases linearly with frequency. In addition, using the Kramers-Kronig relations, the effects of dispersion have been accounted for. The scattering functions for
small spheres and cylinders were introduced, and using the far-field approximation, closed-form solutions for the received force were derived using the angular-spectrum method. Finally, the voxel method for modeling the scattering from a homogeneous medium has been discussed.
CHAPTER 4
EXPERIMENTAL VERIFICATION OF
THE COMPUTER MODEL

In order to verify the computer model of ultrasound B-mode image formation, simulation results were compared to experimental data provided by General Electric (GE), Milwaukee. This data was obtained using a linear transducer array and an imaging phantom with nylon fibers. The apodization, waveform sequence etc. were all set to non-imaging-quality values for purpose of comparison with simulations.

Three sets of data were received from GE: each set consisting of the I/Q (in-phase and quadrature components of the RF signal) data. The data was obtained by scanning an imaging phantom with a linear array (probe #547L, 5.0 MHz, 47 mm long array), and consisted of 195 scan lines. Each scan line of data consisted of 1024 points that were obtained using a sampling frequency of 5.0 MHz. The three different data sets were:

1) I/Q data with demodulation frequency ($f_c$) of 5.0 MHz, and transmit focus (Tx) at 11.7 cm,
2) I/Q data with $f_c$=5.0 MHz and Tx=3.4 cm,
3) I/Q data with $f_c$=3.75 MHz and Tx=11.8 cm.

4.1 Methods

4.1.1 The Imaging System

The array consisted of 192 elements as follows: width=88 μm, center-to-center spacing=246.3 μm, height=10.1 mm. The focal length of the lens in elevation was 6.9 cm in a water medium at 20 C. On transmission, 128 elements were used for deep focusing (11.7-11.8 cm) and 70 elements were used for shallow focusing (3.4 cm). On reception, dynamic focusing was employed to yield $F$-number=2. The same digital pulse sequence was applied to each element of the array but with appropriate delays being introduced to achieve focusing.
The two-way impulse response of a standard 547L probe had been determined experimentally, though the response for the specific transducer used to obtain the three sets of data was not available. The impulse response data (sampled at 35.97 MHz) is shown in Figure 4.1(a). In addition, the transmit pulse sequences for $f_c=5.0$ and 3.75 MHz (sampled at 40 MHz) are indicated in Figure 4.1(b). Thus, for a perfect reflector placed in the transmission/reception path the transducer output signal will be the convolution of these sequences with the two-way impulse response and the results are given in Figure 4.1(c) and (d). It should be noted that the I/Q data from which the received signal was computed had been subjected to internal Time-Gain-Compensation (TGC curve) which took into account many factors such as attenuation, diffraction, electronics etc. Although the shape of this curve was not provided, the necessary information could be recovered from the image data.

The imaging phantom was filled with a hypoechoic liquid medium containing graphite powder that had an attenuation coefficient of 0.5 dB/MHz-cm. It also contained seven nylon fibers aligned horizontally at the center of the phantom and located at depths of: $z=2, 4, 6, 8, 10$ and 14 cm. These fibers were 100 μm in diameter and lay orthogonal to the image plane.

### 4.1.2 Method of Comparison

The received (RF) signal envelopes for the three sets of data were computed from $(I^2+Q^2)^{1/2}$, and the images were displayed using MATLAB. Figure 4.2(a) shows the image for $f_c=5.0$ MHz, and $Tx=3.4$ cm. The speckle pattern that can be seen in the background is a result of the weak scattering caused by the graphite powder. It is also clear that the seven nylon fibers lie at a small angle, perhaps due to slight experimental misalignment of the transducer on the phantom surface. As measured from the image, this angle was found to be approximately $0.7^\circ$. Also apparent on the right side of this image are some additional targets that cause strong scattering, but information about these was not provided and hence, they were not considered in this comparison study. Figure 4.2(b) shows the same image but log-compressed so that the background is mostly suppressed and the nylon fibers are more clearly delineated.
Figure 4.1. (a) The two-way impulse response for the transducer. (b) The digital input pulse sequences. (c) The convolution of the two-way impulse response with the input pulse sequence for $f_c=5.0$ MHz, and (d) for $f_c=3.75$ MHz.
The number concentration of the graphite powder and its scattering strength relative to the fibers determine the echogencity of the background. Since this information was also not available, the background was assumed to be devoid of scatterers, i.e., it was anechoic. The seven fibers were assumed to be aligned at an angle of $0.7^\circ$ to the center. The images were then simulated using the computer model described in Chapter 3. Initially, the fibers were modeled as "line sources" by simulating a sequence of closely spaced ($<<\lambda$) point scatterers over a distance of 1 cm.

All the simulated images were computed using a sampling frequency of 100 MHz. In order to enable the simulated and experimental images to be compared, the convolution of the experimentally determined two-way impulse response and the input pulse were first upsampled to this frequency. The envelopes were then downsampled to the 5 MHz to comply with the experimental images. The envelope amplitudes were also corrected for the internal TGC curve. This was accomplished by measuring the seven peak envelope signal along the center scan line from both the experimental ($P_e$) and the simulation ($P_s$) images, and dividing the former by the latter ($P_e/P_s$). The complete TGC curve was determined by interpolating $P_e/P_s$ for all the 1024 points along the depth axis. The simulation data along each scan line was then multiplied with this TGC function.

4.2 Results and Discussion

Figures 4.3 and 4.4 compare the simulation and experimentally obtained logcompressed images for $f_c=5.0$ MHz, $Tx=3.4$ cm and $f_c=3.75$ MHz, $Tx=11.8$ cm. A qualitative study of both sets of images indicate a good qualitative agreement between the results, except for the appearance of the butterfly shaped side-lobes in the simulation results. In addition, the lateral and axial resolutions also appear to be slightly better in the simulation results.

In order to provide a quantitative comparison, the pressure envelopes along the central scan line were plotted and correlated. Figures 4.5 and 4.6 show the results for two cases, and Table 4.1 lists the correlation coefficients for all three sets. These results

---

1 The radiation coupling function for a linear array-cylindrical scatterer, as described in Chapter 3, was not available at the time the comparisons were made.
Figure 4.2. The experimental image for $f_c=5.0$ MHz and $Tx=3.4$ cm. (a) The raw image with the speckle pattern. (b) The log-compressed image.
Figure 4.3. Log-compressed images for $f_c=5.0$ MHz and $T_x=3.4$ cm: (a) simulated and (b) experimental.
Figure 4.4. Log-compressed images for $f_c=3.75$ MHz and $Tx=11.8$ cm: (a) simulated and (b) experimental.
Figure 4.5. The pressure envelopes along the center scan line for $f_c=5.0$ MHz, $T_x=3.4$ cm. (a) Fibers at 2, 4, 6 cm. (b) Fibers at 8 and 10 cm. (c) Fibers at 12 and 14 cm.

Figure 4.6. The pressure envelopes along the center scan line for $f_c=3.75$ MHz, $T_x=11.8$ cm. (a) Fibers at 2, 4, 6 cm. (b) Fibers at 8 and 10 cm. (c) Fibers at 12 and 14 cm.
indicate good agreement between the experimental and simulation results along the central scan line with correlation coefficients of > 0.85). It should also be noted that the simulated envelopes appear later in time. This is due to the fact that the model assumed that the fibers were placed at distances 2, 4, 6...14 cm along the 0.7° line. The experimental data, however, indicates that they were located closer to the transducer: approximately at 1.7, 3.7, 5.7...13.7 cm.

Table 4.1
Correlation coefficients between the model and the experiment along the center scan line

<table>
<thead>
<tr>
<th>Fiber depth</th>
<th>5.0 MHz, Tx=3.4cm</th>
<th>5.0 MHz, Tx=11.7cm</th>
<th>3.75 MHz, Tx=11.8cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 cm</td>
<td>0.9012</td>
<td>0.9395</td>
<td>0.9546</td>
</tr>
<tr>
<td>4 cm</td>
<td>0.8779</td>
<td>0.9108</td>
<td>0.9909</td>
</tr>
<tr>
<td>6 cm</td>
<td>0.9822</td>
<td>0.9808</td>
<td>0.9754</td>
</tr>
<tr>
<td>8 cm</td>
<td>0.9701</td>
<td>0.9888</td>
<td>0.9936</td>
</tr>
<tr>
<td>10 cm</td>
<td>0.9433</td>
<td>0.9886</td>
<td>0.9901</td>
</tr>
<tr>
<td>12 cm</td>
<td>0.9271</td>
<td>0.9738</td>
<td>0.9857</td>
</tr>
<tr>
<td>14 cm</td>
<td>0.9153</td>
<td>0.9349</td>
<td>0.9627</td>
</tr>
</tbody>
</table>

There are four possible reasons for the discrepancy between the experimental images and the simulation results:

1. The two-way transducer impulse response was measured for a transducer of the same type but differed from that used to obtain the image data. Although the response was assumed to be the same for all transducers of this type, there might well be variations. This might affect the beam shape and width, which in turn would affect the resolution.

2. There was a slight chance that the transducer apodization function was left on during the measurements. Information about the apodization function was not provided due to confidentiality issues. Applying amplitude apodization can smear out the image by lowering the side-lobes and degrading the lateral resolution.
(3) The cylinders (nylon fibers) were modeled as "line sources" by superimposing the responses from closely spaced scatterers aligned along the axis. This approximation probably affects the frequency behaviour of the scattering, and hence the shape of the beam profile on reception. A more accurate model of the scattering from the cylinders might possibly yield smearing out effect in the simulation images.

(4) The simulation model assumed that the transducer consisted of identical elements whereas imperfections in the actual composite array may be present.

Following this initial comparison study the effects of the assumption that the nylon fibers could be modelled by a line of point scatterers was studied. Specifically, the radiation coupling function for a linear array-cylindrical target system was obtained. As described in Section 3.3, the solution was determined in closed-form for the limiting case in which the cylinder is in the far-field of the array and its diameter is much less than the insonating wavelength. In fact, these are very rough approximations for the conditions used in the experiments \((ka \approx 1)\). Figure 4.7 shows the simulation images \((f_c=5.0 \text{ MHz}, \text{ and } T_x=3.4 \text{ cm})\) obtained by modeling the fibers by cylinders and a sequence of point scatterers. There is good agreement between the images qualitatively. The envelopes at two different depths are plotted in Figure 4.8, which also indicates that the two models yield approximately the same results. Figure 4.9 depicts the peak envelopes at the seven different depths for both models. The peak values are similar at depths where the far-field approximation holds, but differ at distances closer to the transducer where both solutions employ the far-field approximation.

As noted above, the closed-form solution for the radiation coupling function was derived by assuming the cylinder diameter was such that \(ka<<1\) . In fact, the experimental fibers were close to \(ka=1\), so that it was decided to also investigate the other limiting case where \(ka>>1\). For this case the solution is independent of the frequency (Morse and Ingard, 1968). Figure 4.10 compares the solutions for both cases. It will be noted that axial resolution is increased, and in addition, there is a slight smearing out effect in the lateral direction for \(ka>>1\). However, the side lobes can still be observed, and thus, it seems unlikely that the model used for the fiber scatterers is the source of the discrepancy between the images shown in Figures 4.3 and 4.4.
Figure 4.7. Simulated images from the nylon fibers modeled as (a) infinite cylinders and (b) a sequence of point scatterers ($f_c=5.0$ MHz and $T_x=3.4$ cm.).

The results presented up to this point suggest that the most likely cause for the absence of the side-lobes in the experimental images is amplitude apodization. This was investigated by assuming a simple Gaussian apodization, and investigating its effects on the simulated images. Figure 4.11 shows the image for the cylinder at $z=6$ cm. Uniform apodization ($A_i=1$) was assumed for 4.11(a) as before, but Gaussian apodization was simulated on transmit for 4.11(b), and on transmit-receive for 4.11(c). Assuming unity length for the transducer array, the Gaussian apodization function had standard deviation $\sigma=0.1$. The images indicate that the sidelobes are greatly reduced while the lateral resolution is degraded when apodization is employed.
Figure 4.8. Simulated pressure envelopes of the signals arriving from the fibers at (a) 4 cm, and (b) 14 cm. The dotted line represents the fiber modeled as an infinite cylinder whereas the solid line is for a sequence of point scatterers.

Figure 4.9. Peak envelope plotted as a function of fiber number. This shows that the discrepancy between the point scatterer and the cylinder model is very small except for the first fiber which is closest to the transducer array.
Figure 4.10. Simulated images from the nylon fibers modeled as infinite cylinders with (a) \( ka >> 1 \) and (b) \( ka << 1 \) (\( f_c = 5.0 \) MHz and \( Tx = 3.4 \) cm.).

### 4.3 Chapter Summary

Chapter 4 uses experimentally obtained data to verify the model of B-mode image formation. The data consists of images of a phantom with cylindrical nylon fibers. Although there is a good qualitative agreement between experimental and simulation images, the computer model yields side-lobes which are absent in the original images, and narrower axial beam profiles.

When modeling the scattering from the cylindrical targets, it has been assumed that the insonating wavelength was larger than the radius of the cylinder \( (ka << 1) \). Also, it has been assumed that uniform apodization was employed when scanning the phantom with a linear array. These assumptions may explain the discrepancy between the experimental images and simulation results. It has been shown that different
approximations of the scattering function from a cylinder \((ka<<1\) or \(ka>>1\)) affect the beam shape significantly in the axial direction. Also, a gaussian apodization function has been simulated to demonstrate how the side-lobes can significantly be reduced with amplitude apodization.

Figure 4.11. Effects of apodization on simulated images (fiber at \(z=6\) cm., \(f_c=5\) MHz, \(T_x=3.4\) cm. (a) Uniform apodization on transmit-receive, (b) Gaussian apodization on transmit only, and (c) Gaussian apodization on transmit-receive.
CHAPTER 5
CONTRAST RESOLUTION

5.1 Effects of Scatterer Density on B-mode Images

In clinical B-mode imaging, the detection of low-contrast targets, such as focal lesions, against the background created by tissue, is of major importance. Because the ability of an imaging system to identify such lesions depends on its contrast resolution performance, it is important to develop experimental methods for measuring the performance. One such method has been described by Smith et al., (1985) in which a "contrast-detail" phantom is scanned. Such phantoms have tissue-mimicking material as background, and targets of varying dimensions and contrast relative to the background. Evidently, in order to achieve realistic results, the background should produce a fine B-scan texture similar to that from tissue.

The "contrast-detail" phantom designed by Smith et al. (1985) contains seven cone-shaped targets imbedded in background medium. The particle scatterers are identical (same size, density and compressibility) in each target and the background, but vary from one target to the other. The concentration of the particles in the background and targets also vary, and range from 1.74 to 152.67 per mm$^3$. Turnbull et al. (1992) simulated images from a medium containing spherical cysts in order to compare the contrast performance of 2-D and linear arrays in the elevation direction. They chose a number concentration of 15 per mm$^3$ for their medium, and reported that the image statistics did not vary significantly for their assumed simulated imaging system as the scattering density was varied from 15 to 333 per mm$^3$.

The presence of speckle in B-mode images decreases the visibility of small lesions by degrading the contrast resolution. The speckle pattern is a result of both the imaging system parameters and the acoustic properties of tissue. First and second-order statistics of B-mode images have been employed extensively for quantitative assessment of speckle (Flax et al., 1981; Wagner et al., 1983; Foster et al., 1983; Smith and Wagner, 1984; Oosterveld et al., 1985; Jacobs and Thijssen, 1991; Moltehen et al., 1995). It has been shown that for a medium having randomly distributed identical scatterers, the
magnitude of the image signal is Rayleigh distributed, and the average speckle size for large scatterer densities depends only on the resolution cell size of the imaging system.

Before performing basic studies on the contrast resolution of the linear array imaging system described in previous chapters, it was necessary to choose a suitable scatterer density, \( N_r \). As stated above, densities in the range 10-100 per mm\(^3\) have been used in previous simulations and experimental studies. However, the concentrations in tissue are considerably larger, for example, in blood with a hematocrit of 40%, there are \( \sim 5 \times 10^6 \) red blood cells per mm\(^3\). Therefore, it was necessary to derive a speckle size versus \( N_r \) plot similar to that presented by Oostervald et al. (1985).

### 5.1.1 Effects on Average Speckle Size

The speckle size as a function of \( N_r \) was computed for two linear array designs whose parameters are listed in Table 5.1 together with the excitation pulse waveform. The scattering medium was 9×3×5 mm\(^3\) (in x-y-z directions, respectively). Figure 5.1 shows the log-compressed B-mode images for a medium with \( N_r = 100 \) mm\(^3\). It can be observed that the two systems yield distinctly different speckle patterns. The much coarser pattern from Array 2 is in part due to the longer pulse width in the axial direction, and the larger center-to-center spacing \( (d) \) in the azimuthal direction.

<table>
<thead>
<tr>
<th>w, d, H</th>
<th>ARRAY 1</th>
<th>ARRAY 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>75 ( \mu )m, 150 ( \mu )m, 15 mm</td>
<td>88 ( \mu )m, 246.3 ( \mu )m, 10.1 mm</td>
<td></td>
</tr>
</tbody>
</table>

The average speckle size was calculated by computing the 2-D autocovariance function for each of the B-mode images, and then finding the full-width-at-half-maximum (FWHM) along the axial and lateral directions. For example, Figure 5.2(a) shows the
autocovariance function of the B-mode image obtained with Array 2 by scanning a medium having \( N_v = 200 \text{ mm}^{-3} \).

Figure 5.3 shows the axial and lateral speckle size as a function of \( N_v \) for both arrays. It should be noted that the speckle size was determined from a single realization for each \( N_v \) since the computation time is long for \( N_v > 10 \) (approximately a month for \( N_v = 5000 \)). Consequently, the standard deviations (SDs) could not be shown on these plots. However, six realizations were simulated for \( N_v = 10 \) in order to give an indication of the range at this density. Figure 5.3(a) indicates that the axial speckle size at first decreases and then tends towards a more constant value for \( N_v \geq 100 \text{ mm}^{-3} \) for both array systems. The lateral speckle size follows the same pattern as the axial size for Array 1, on the other hand, the lateral speckle size of Array 2 first decreases, after which it seems to increase \((p<0.05)\). Oostervald et al. (1985) concluded that the lateral speckle size reaches a constant value as the scatterer density is increased, though they only considered densities up to 20 mm\(^{-3}\). The slight increase at higher values of \( N_v \) might be explained as follows. As the density is increased, the average distance between scatterers decreases. When this becomes less than the insonating wavelength, the contribution from closely neighbouring scatterers may behave more like a single scattering unit thereby causing an apparent reduction in the scatterer density. For example, the speckle size for Array 2 with \( N_v = 5000 \) is approximately the same as that with \( N_v = 50 \).

### 5.1.2 Effects on Image Contrast

Figure 5.3 indicates that the speckle pattern (size and shape) from an imaging phantom with \( N_v = 10 \) should be similar to that from tissue with \( N_v \geq 10^5 \), assuming that both mediums have randomly distributed, identical small scatterers. However, this does not imply that the B-mode images of these mediums with imbedded cysts will look the same. Oostervald et al. (1985) and Smith et al. (1985) have shown that the mean echo signal level increases as a function of \( \sqrt{N_v} \).

In order to investigate the effects of scatterer concentration on image contrast, a homogeneous medium with a box-shaped anechoic (no scattering) cyst was scanned
Figure 5.1. Speckle pattern in a small volume around the focal point from (a) Array 1, and (b) Array 2. The parameters for the two arrays are listed in Table 5.1.
Figure 5.2.(a) 2-D autocovariance function of the image shown in Figure 5.1(a). (b) Lateral cut through \( z=0 \) and (c) axial cut through \( x=0 \).
Figure 5.3. Speckle size as a function of $N_v$. (a) Axial speckle size, and (b) lateral speckle size. The error bars at $N_v=10$ indicate the range from 6 measurements.
using Array 1 (see Table 5.1). The dimensions of the medium and the cyst were 4.5×3×3 mm³ and 1.2×1.2×0.9 mm³, respectively, and three different scatterer densities were simulated: \( N_v = 10^2 \), \( 10^3 \), and \( 10^4 \). The schematic shown below in Figure 5.4 illustrates the rectangular cyst embedded in the medium.

![Figure 5.4. Schematic for the 2-D view of a box-shaped anechoic cyst embedded in a homogeneous background with randomly distributed identical point scatterers.](image)

Figures 5.5(a), (b) and (c) show the simulated B-mode images of these three mediums. It can be observed that the speckle pattern is similar in all the images, whereas the contrast depends on the choice of \( N_v \). More specifically, the image contrast increases as the number density is increased from 100 mm\(^{-3}\) to 1000 mm\(^{-3}\), but decreases when \( N_v \) is further increased to 10,000 mm\(^{-3}\). At this concentration, there is strong constructive interference effects around the borders that lie orthogonal to the beam propagation (see the Appendix). There is also strong destructive interference in the regions between the borders. Therefore, even though the total mean signal level has increased in proportion to \( \sqrt{N_v} \), most of the signal has shifted to the edges, and hence the decrease in contrast. The decrease might also be caused by the slight increase in the lateral speckle size as \( N_v \) is increased from 1000 to 10,000.

A quantitative measure of the contrast between the cyst and the background is (Smith et al., 1983):

\[
C = \frac{S_{out} - S_{in}}{\sqrt{S_{out}^2 + S_{in}^2}},
\]  

(5.1)
where $S_{in}$ is the mean signal measured inside a region of the cyst, and $S_{out}$ is the average signal measured from the same sized regions outside the cyst. The region dimension was chosen to be the cyst size minus the resolution in the axial and lateral directions (FWHM in the $z$- and $x$-directions), as used by Turnbull et al., (1992). The signal outside was measured from eight different regions and averaged. In the case of Figure 5.5(c), the selected background regions were chosen to be free of the edge effects. The average contrast values for $N_v=100$ and 1000 were found to be $0.73 \pm 0.07$ and $0.82 \pm 0.04$, respectively. When $N_v$ was increased to $10^4$, the cyst contrast was found to decrease to $0.58 \pm 0.09$ ($p<0.001$). These measurements indicate that image contrast is influenced by the scatterer densities in tissue, but does not vary linearly with $N_v$.

Figure 5.5. Box-shaped anechoic cyst in a medium with (a) $N_v=100$. 
Figure 5.5. Box-shaped anechoic cyst in a medium with (b) $N_r=1000$, and (c) $N_r=10,000$. 

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Chapter 5
5.2 Effects of Cyst Shape on B-mode Images

Contrast resolution phantoms generally contain cystic targets that generate a circular area in the scan plane of the imaging phantom. In the phantom designed by Smith et al. (1985), the cysts are circular cones, whereas the commercial “contrast resolution phantom” (ATS Laborities, Inc., Model 512) has cylindrical targets. In an effort to compare the contrast performance of 1-D versus 2-D arrays, Turnbull et al. (1992) simulated spherical cysts, which also yield circular areas in the scan plane. Even though cones and cylinders are sufficient for investigating the performance of 1-D arrays in the lateral direction, spherical targets are necessary to measure the performance of 1-D and 2-D arrays in the elevation direction.

Box-shaped cysts were generally simulated in this work. There were two reasons for this:

1. The resolution of an imaging system is different in all three directions, $x$, $y$ and $z$. When spherical cysts are simulated, the measured image contrast is a result of the system performance in all these directions. The aim in this study was to separate the performance in the lateral direction from that in the axial and elevation directions, and to investigate each individually. For example this can be achieved by simulating
a box cyst with dimensions much larger than the resolution in y- and z-directions, and varying the x-direction size.

2. In order to accurately measure the image contrast from a cyst, the background medium should be large enough to account for the grating lobe and side-lobe effects, and also to include neighbouring areas having the same dimensions as the cyst. If a spherical cyst is simulated, the size of the medium has to be increased in all three directions as the cyst-size is increased. However, increasing the size of a box-shaped cyst in the lateral direction would only require an increase in the x-dimension of the medium. Consequently, less computation time would be required than if a spherical cyst was used, whose diameter was equal to the x-dimension of the box.

The effects of cyst shape on B-mode images were investigated by considering both spherical and box-shaped anechoic cysts in a background medium with $N_r = 10^4$. The dimensions of the medium and the box hole are given in Section 5.1.2. The spherical cyst had the same volume (diameter = 1.352 mm) as the box cyst. Figure 5.5(d) indicates that the edge effects that can be observed around the box cyst are greatly diminished for the spherical cyst. The reason for this is that the border between the box hole and the medium lies orthogonal to the beam axis, while the spherical cyst surface has only a small area that can be regarded as orthogonal.

From the above results it is clear that the B-mode image of a target is affected by both its shape and orientation. Not all cysts or lesions in human body reveal a spherical orientation. An artery wall with plaque, for example, has approximately a cylindrical shape, and images of this geometry also display edge effects. The "double-line" pattern (Pignoli et al., 1986) observed in arterial walls is a result of the intima-blood and adventitia-media interfaces.

### 5.3 Contrast Resolution Analysis

A prime objective of this work is to quantify contrast resolution of B-mode images, and to investigate how it is affected by the transducer design and the medium properties. The results presented in Section 5.1 indicate that both affect the contrast resolution.
One widely used quantitative measure of resolution is the beam full width at half-maximum amplitude (FWHM). It is also referred to as the -6 dB resolution. If two identical point targets were separated by one FWHM, they would, by definition, be resolved. This analysis, however, ignores the effects of the side-lobes and speckle. In reality, the scanned tissue consists of targets embedded in backscattering medium rather than isolated objects in non-backscattering background. Therefore, as will be demonstrated in the next section, the -6 dB resolution criterion fails to measure and compare the performances of different transducer designs.

Wildes et al. (1997) have compared the elevation performance of linear and 1.5D arrays by simulating beam profiles in the elevation (y) direction. They have referred to the -6 dB point as the detail resolution, and the -20 dB point as an indicator of the width of the side-lobes and, therefore a measure of the contrast resolution.

As mentioned in Chapter 1, contrast-detail phantoms have also been used to quantify contrast resolution of existing imaging systems. However, this method cannot predict the performance of a proposed system prior to fabrication. One of the goals of this thesis is to obtain a method of contrast resolution analysis, based on theory and simulations, that will serve as a means of predicting the performance. We have chosen to plot simulated cyst resolution by assuming a cyst of dimension $s$ in a background medium, and plotting image contrast, $C$, as a function of $s$. Two different simulation methods were used to measure the contrast: the "area under the beam profile" (AUBP) technique, and contrast measurements which are directly derived from simulated B-mode images of the cyst.

5.3.1 Methods

A) Area Under the Beam Profile

Consider a one-dimensional cyst of length $s=2a$ in the lateral $x$-direction and suppose that its backscattering strength relative to the background is denoted by $b$. The image contrast for this cyst can be determined using eqn. (5.1) in the following manner.

If the beam is sufficiently far displaced from the cyst the background signal will be proportional to the total area under the lateral beam profile, $B_p(x)$, i.e.,
However, when the beam axis passes through the cyst center, the signal will contain components from both inside and outside the cyst such that

\[ S_{\text{out}} \propto \int_{-\infty}^{+\infty} B_P(x) dx. \] (5.2)

where the first term denotes the signal coming from inside the cyst, and the bracketed represents the background signal.

The image contrast can then be determined using eqns. (5.1) to (5.3) and plotted as a function of cyst size. However, it should be noted that the contrast obtained in this manner is not the true contrast, but only an indicator and, thus, is denoted by \( C_f \) in order to distinguish it from the true contrast, \( C \). The reason for the discrepancy is two-fold. This beam area technique is a 1-D analysis, i.e. it ignores the imaging performance in the \( y \)- and \( z \)-directions by assuming that the cyst size is much larger than the resolution in these two directions. The second reason is that the method also ignores the effects of speckle. The advantage of this method over the use of the FWHM resolution measure is that it accounts for the signals picked up from the background by the side-lobes and grating-lobes.

B) Simulations of B-mode Images

A computer model for B-mode image formation was developed to perform fundamental studies on contrast resolution. This model, which is explained in detail in Chapters 2 and 3, simulates images from linear, phased, 1.5-D and curvilinear arrays. It assumes a 3-D medium with many identical scatterers, thereby accounting for the imaging performance in all directions as well as for speckle.

The simulation model was used to image box-shaped cysts embedded in background medium and to obtain \( C \) vs. \( s \) plots in which \( s \) denotes the cyst size in the direction of interest \( (x, y \text{ or } z) \). The size in the other two directions was chosen to be much larger than the resolution in order to minimize the contribution of the background.
signal to the cyst in these directions. This makes it possible to separate out the influence of the resolution in each direction.

Identical point scatterers with relative scattering strengths of \( b \) and unity \( (b=1) \) were randomly distributed in the cyst and the background medium, respectively. The dimensions of the background medium were chosen to be large enough in order to allow for signals arriving from grating lobes and side-lobes. The images were plotted using \(^\circ\)MATLAB, and the contrast was calculated as explained in Section 5.1.

**5.3.2 Effects of Array Geometry on Contrast Resolution**

A) Linear Arrays

Two linear arrays having the same FWHM were simulated in order to demonstrate that FWHM is not an accurate indicator of imaging performance. It will be shown that the grating lobes are the main determinants of contrast resolution. The FWHM is mainly determined by \( F \)-number, while the location and amplitude of grating lobes are determined by element spacing. As indicated in Table 5.2, arrays 1 and 2 had the same input parameters but differed in their element size and spacing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ARRAY 1</th>
<th>ARRAY 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w, d, H )</td>
<td>75 ( \mu )m, 150 ( \mu )m, 15 mm</td>
<td>300 ( \mu )m, 450 ( \mu )m, 15 mm</td>
</tr>
<tr>
<td>( T_x = T_y = R_y )</td>
<td>3 cm</td>
<td>3 cm</td>
</tr>
<tr>
<td>( F )-number on ( R_x )</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Pulse shape</td>
<td>see Fig. 2.2</td>
<td>see Fig. 2.2</td>
</tr>
</tbody>
</table>

Figure 5.6 shows the simulated beam profiles of these arrays at the focus \( (z = 3 \) cm) in the \( x \)-direction. It should be noted that the arrays have the same \(-6 \) dB (FWHM) and \(-20 \) dB points, and therefore, would have identical imaging performance according to the criteria set by Wildes et al. (1997). If these arrays were used for imaging strong reflectors such as pins, the images should indeed appear identical.
The anechoic cyst (b=0) images in Figure 5.7 were obtained by assuming a 6×1×1.5 cm³ medium with N₁=10. The cyst size in the x-direction was set at 2 mm (s=2), and 5 mm in the z-direction, i.e. much larger than the resolution in this direction. The size in the y-direction was chosen to be infinite in order to minimize the effects of imaging performance in the elevation direction. As shown, the images obtained with the two arrays have different contrast levels. The reason for this discrepancy with the performance predicted from a consideration of the FWHM is that the beam profile of Array 2 reveals a -50 dB grating lobe, while for Array 1, which has a spacing of λ/2, grating lobes are absent (see Figure 5.6).

Figure 5.8 shows the image contrast versus the cyst dimension s for the two arrays as obtained from the B-mode images (b=0). It can be seen that Array 1 has, on average, 13% higher contrast than Array 2. The contrast (C₁) as indicated by the AUBP method, follow the same trend as those in Figure 5.8(a), but the values are slightly different as this method ignores the effects of speckle and the performance in the y- and z-directions.

![Figure 5.6. Transmit-receive beam profiles at the focus (z = 3 cm) in the x-direction for the two arrays described in Table 5.2.](image-url)
Figure 5.7. Simulated image of a s=2 mm cyst with (a) Array 1, and (b) Array 2. The images are shown using the same intensity scale.
Figure 5.8. Contrast as determined from simulations (C) and the AUBP method (C_i) plotted as a function of cyst size for two linear arrays. The array input parameters are described in Table 5.2.

B) 1.5-D Arrays

With 1.5-D arrays the additional rows of elements in the elevation direction can be used to dynamically focus the beam on reception, thereby decreasing the slice thickness and enhancing the contrast resolution. Ultrasound images from linear and 1.5-D arrays were simulated in order to compare their performance in the elevation direction. The input parameters used for the arrays are listed in Table 5.3, from which it will be noted that total array height was the same for both array types: for the 1.5-D arrays it was divided into 5 rows, either with equal sub-heights, or Fresnel heights [see eqn. (2.25)].

Figure 5.9 shows the simulated beam profiles of all three arrays at z=6 cm in the y-direction. The arrays have identical beam profiles at the transmit focus (z=Tx), but differ at z=6 cm. According to the criteria used by Wildes et al. (1997), Array 1 has the worst detail and contrast resolution (-6 dB and -20 dB points, respectively); Array 2 has slightly better detail resolution but slightly worse contrast resolution when compared with Array 3.

Anechoic box-shaped cysts were placed in a homogeneous medium (N_v=10) in order to derive the contrast versus cyst length characteristics. The dimensions of the
medium were 1x4x1.5 cm; the cyst size in the y-direction was varied, 5 mm in the z-direction, and infinite in the x-direction in order to minimize the effects of imaging performance in these two directions.

Table 5.3 Linear and 1.5-D array parameters used for examining the contrast resolution.

<table>
<thead>
<tr>
<th>ARRAY 1</th>
<th>ARRAY 2</th>
<th>ARRAY 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Array</td>
<td>1.5-D Array</td>
<td>1.5-D Array</td>
</tr>
<tr>
<td><strong>H</strong></td>
<td>15 mm</td>
<td>15 mm with 5 rows</td>
</tr>
<tr>
<td><strong>w, d</strong></td>
<td>350 μm, 400 μm</td>
<td>350 μm, 400 μm</td>
</tr>
<tr>
<td><strong>Tx = Ty</strong></td>
<td>4 cm</td>
<td>4 cm</td>
</tr>
<tr>
<td><strong>F-number on Rx</strong></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td><strong>F-number on Ry</strong></td>
<td>N/A</td>
<td>2</td>
</tr>
<tr>
<td><strong>Pulse shape</strong></td>
<td>see Fig. 2.2</td>
<td>see Fig. 2.2</td>
</tr>
</tbody>
</table>

1. See equation (2.25).
2. Wildes et al., 1997.

Figure 5.9. Beam profiles in the elevation direction for a 1-D and two 1.5-D arrays (Arrays 2 and 3) as described in Table 5.3.
From Figure 5.10(a) it can be seen that for small cyst dimensions ($s<6$ mm), the contrast performance is as predicted by the criterion set by Wildes et al., i.e. contrast from Array 3 is the best while that from Array 1 is the poorest. However, as $s$ increases further, the contrast levels from all three arrays are almost identical; in fact, Array 1 and 2 yield even slightly higher contrast levels than Array 3. A similar trend is seen for the AUBP method in Figure 5.10(b), though the values are slightly different.

The effect of the number of elevation rows on contrast resolution was investigated by dividing the height dimension of the 1.5-D array ($H=15$ mm) into 7 rows with Fresnel heights and comparing its performance to that of Array 2 (5 rows). The beam profile with 7 rows have lower amplitude side-lobes than that with 5 rows. Consequently, the contrast resolution should be better with 7 rows and this is supported by the results shown in Figure 5.11. The greater the number of rows, the better the focusing characteristics will be in the elevation direction, and hence performance will be improved.

![Figure 5.10](image)

**Figure 5.10.** Contrast as a function of cyst size for a linear and two 1.5-D arrays whose parameters are listed in Table 5.3. (a) Contrast as determined from simulations.
Figure 5.10. Contrast as a function of cyst size for a linear and two 1.5-D arrays whose parameters are listed in Table 5.3. (b) Contrast as determined from AUBP method.

Figure 5.11. Contrast as determined from simulations ($C$), and from the AUBP method ($C_f$) as a function of cyst size for two 1.5-D arrays.
C) Curvilinear Arrays

As mentioned in Chapter 2, curvilinear arrays cover a larger field of view than linear arrays. Furthermore, the lateral resolution remains constant over different angles, in contrast to the changing resolution from phased arrays. Here, our objective was to compare the contrast performance at different angles of curvilinear arrays and linear phased arrays that used elements with the same size ($w=150 \, \mu m$, $d=200 \, \mu m$, and $H=10 \, mm$).

![Figure 5.12. Geometry for the phased and curvi-linear arrays used for scanning two 3.0 mm diameter cysts located at $z=r=3.0 \, cm$, $\theta=0^\circ$ and $z=2.48 \, cm$, $\theta=45^\circ$.](image)

Both arrays were focused at 3 cm on transmit, and dynamically focused with an $F$-number of 3 on receive. The excitation pulse was a Gaussian wave with a center frequency of 3 MHz. The curvilinear array had a $R_c=3 \, cm$. Two spherical 3.0 mm diameter anechoic cysts embedded in a $6\times1\times1.5 \, cm^3$ medium with $N_v=10$ were scanned with the array systems (see Figure 5.12 for further details).

Figure 5.13 shows the B-mode images simulated using the linear phased array. The contrast of the 3.0 mm cyst is $0.89\pm0.04$ when it is placed on the axis of the transducer, but it drops to $0.63\pm0.10$ at $\theta=45^\circ$. This is expected since the performance of a linear phased array degrades as the steering angle is increased. In contrast, as demonstrated by the images presented in Figure 5.14, the curvilinear array yields a
uniform contrast over the entire depth of field. Specifically, the contrast for both cysts are approximately the same (~0.85±0.03). These results indicate that the curvilinear array provides better performance at high steering angles though there is some degradation with no steering (0.89±0.04 versus 0.85±0.03).

Figure 5.13. B-mode images of a 3.0 mm spherical cyst scanned by the linear phased array. (a) Cyst at z=3.0 cm, θ=0°, and (b) cyst at z=2.48 cm, θ=45°.
Figure 5.14. B-mode images of a 3.0 mm spherical cyst scanned by the curvi-linear array. (a) Cyst at \( z=3.0 \text{ cm}, \theta=0^\circ \), and (b) cyst at \( z=2.48 \text{ cm}, \theta=45^\circ \).
5.3.3 Effects of Scatterer Density on Contrast Resolution

Although the effects of scatterer concentration on the speckle size was discussed in Section 5.1, its effect on the contrast resolution was not considered. This was studied by using linear Array 1 (Table 5.2) to simulate the images from a volume of $2 \times 0.2 \times 0.5$ cm$^3$, which is smaller than that previously used ($4 \times 1 \times 1.5$ cm$^3$). With this volume scatterer densities of up to 10,000 mm$^{-3}$ could be simulated within our computational burden limit. The size of the cyst was 2.0 mm in the $x$-direction, 1.5 mm in the $z$-direction, and infinite in the $y$-direction. Figure 5.15 indicates that the contrast increases as $N_v$ is increased from 10 to 100 ($p<0.05$). The contrast seems to decrease slightly for $N_v>100$, but the results are not statistically significant.

![Figure 5.15. Contrast versus scatterer density.](image)

It should be recalled that the image contrast for the smaller cyst (1.2 mm in the $x$-direction) of Section 5.1.2 decreased as $N_v$ was increased from $10^3$ to $10^4$. This observation suggests that the change in contrast as a function of $N_v$ also depends on the cyst size. This is further illustrated in Figure 5.16, from which it can be observed that the contrast for $N_v=100$ is higher only for small cysts ($s \leq 2$ mm). This is due to the fact that
larger cysts contain less speckle noise ($S_{in} \rightarrow 0$), and hence, contrast levels approach unity regardless of the speckle size in the background.

![Graph showing cyst size in x-direction (mm) vs. contrast](image)

Figure 5.16. Showing the effect of cyst size on the contrast for $N_v = 10$, and 100.

### 5.3.4 Hyper-echoic and Hypo-echoic Cysts

Up to this point all the cysts have been assumed to be anechoic. It is therefore reasonable to enquire how the presence of hyper- and hypo-echoic cysts might affect the contrast resolution. This was achieved by setting $b=4$ and $b=0.25$, respectively.

Array 1 of Table 5.2 was modeled to simulate the images of hyper- and hypo-echoic cysts. The parameters for the medium and the cyst were the same as those used to obtain Figure 5.7(a). In Figure 5.17, which shows the images of 2.0 mm diameter hypo-and hyper-echoic cysts, it will be noted that the background echogenecity is the same for both, as expected. The contrasts calculated by eqn. (5.1) are $0.64 \pm 0.06$ for the hyper-echoic cyst and $0.77 \pm 0.02$ for the hypo-echoic cyst. However, the reverse seems to be the case as revealed by eye\(^1\). In fact, other measures of contrast have been previously

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\(^1\)When the images of Fig. 5.17 were displayed on a monitor, 10 independent observers stated that the hyper-echoic cyst image exhibited a higher contrast relative to the background.
proposed of which two are of particular interest. Denoting the contrast of eqn. (5.1) as $C_1$, the first of these definitions (Patterson and Foster, 1983) can be written as

$$C_2 = \frac{S_{\text{out}} - S_{\text{in}}}{S_{\text{out}}}, \quad (5.4)$$

and the second (Turnbull et al., 1992) is given by

$$C_3 = 20 \log \left( \frac{S_{\text{in}}}{S_{\text{out}}} \right). \quad (5.5)$$

Figure 5.18 and 5.19 show the dependence of all three contrast values ($C_1, C_2, C_3$) on the $x$-direction cyst size for hyper-echoic and hypo-echoic cysts. The trends for all three contrast measures are the same though the values are quite different. Only definition $C_2$ yields a higher contrast for hyper-echoic cysts, which suggests that this is more accurate measure of the visually observed contrast. Additional support for this is given in Figure 5.20 which displays the mean signal versus the off-axis distance for anechoic, hypo- and hyper-echoic 6 mm cysts all in identical background distributions. The mean signal was calculated by averaging the envelope of the RF signal in the axial direction over the cyst length for each scan line in the image. It will be noted that outside the cyst regions the mean signal levels are identical. The hyper-echoic cyst shows the largest change relative to the background, which supports the use of the contrast definition given by eqn. (5.4).

It should also be noted that even though the contrast for anechoic cysts was calculated using eqn. (5.1) in the previous sections, the difference between $C_1$ and $C_2$ values are insignificant for anechoic cysts, and hence, these measurements were not repeated. For example, for the plot shown in Figure 5.8, recalculating the contrast using eqn. (5.4) increases the values by a maximum (for 1 mm cyst) of 0.45%. This is due to the fact that in the denominator of eqn. (5.1) $S_{\text{in}} \ll S_{\text{out}}$ for anechoic cysts.

\footnote{It should be recalled that for the 2-D images shown in Figs. 5.7(a) and 5.17 it was concluded that the hyper-echoic cyst as perceived by eye showed the highest increase/decrease relative to the background.}
Figure 5.17. B-mode images of a 2.0 mm (a) hyper-echoic cyst, and (b) hypo-echoic cyst.
Figure 5.18. Contrast versus hyper-echoic cyst size for three different contrast definitions (a) $|C_1|$, (b) $|C_2|$, and (c) $|C_3|$. 
Figure 5.19. Contrast versus hypo-echoic cyst size for three different contrast definitions (a) $C_1$, (b) $C_2$, and (c) $C_3$.

Another interesting point to note is that even though the contrast increases as a function of cyst size for anechoic and hyper-echoic cysts, Figure 5.19 indicates that it decreases for hypo-echoic cysts. This finding is in disagreement with the results of Vilkomerson et al. (1995) who showed theoretically that the contrast increases with cyst size for hypo-echoic cysts. It is likely that this is because they failed to consider the effects of speckle.
Figure 5.20. Normalized mean signal versus off-axis distance (x-distance measured from the line through the center of the cyst) for three types of 6.0 mm diameter cysts.

Detectability

Detectability

Contrast is not necessarily a measure of target detectability. When an observer looks at an envelope detected ultrasound image and tries to determine if a target is present among background speckle noise, a measure of the ability to detect its presence is called the detectability. Thus, the quality of an ultrasound image can also be based on the performance of an ideal observer to detect a specific target type. A measure of the performance has been given by Smith et al. (1983; 1985), and is referred to as the optimal signal-to-noise ratio, $SNR_{opt}$:

$$SNR_{opt} = C \sqrt{M} = C \sqrt{A/A_c},$$  \hspace{1cm} (5.6)

where $C$ denotes the contrast, $A$ is the target area, $A_c$ is the area of the speckle cell, and $M$ is the number of speckle cells within the target area. Assuming that the speckle cell shape is elliptical, and that the target is rectangular in the scan plane with dimensions $s_x$ and $s_z$, eqn. (5.6) can be rewritten as:

$$SNR_{opt} = C s_x \left( \frac{\sqrt{s_x/s_z}}{\sqrt{\frac{\pi S_{cx} S_{cz}}{4}}} \right),$$  \hspace{1cm} (5.7)
Figure 5.21. Contrast and detectability versus cyst size for anechoic, hypo-echoic and hyper-echoic cysts. (a) Contrast, (b) Detectability
where \( S_{cx} \) and \( S_{cz} \) are the speckle cell sizes in the lateral (\( x \)) and axial (\( z \)) directions. Equation (5.7) indicates that the detection performance is a function of both contrast and size. For the cysts considered above, the term in the brackets is a constant, and therefore, \( \text{SNR}_{\text{opt}} \approx C \sqrt{s_x} \). Figure 5.21 shows \( C_2 \) and \( C_2 \sqrt{s_x} \) versus \( s_x \) for anechoic, hyper- and hypo-echoic cysts. Note that even though the \( C_2 \) contrast of the 1.5 mm hypo-echoic cyst is slightly higher than that of the 6 mm cyst\(^3\), the detectability of the larger cyst is higher.

### 5.3.5 Effects of Post-Processing on Contrast Resolution

The block diagram for typical post-processing performed in digital B-mode imaging systems is displayed in Figure 5.22(a). The envelope-detected RF signal is first compressed using a simple logarithmic transfer function. The output, generally an 8-bit signal, is further processed using various algorithms (high-pass/low-pass/band-pass filtering, adaptive filtering, edge-enhancement, etc.). The signal is then mapped into grey-scale and displayed on a monitor. The filters, the grey-scale mapping function and the gamma function of the monitor all affect the final image appearance.

#### A) Log-Compression

Clinical ultrasound imaging systems generally employ logarithmic compression to compress the wide dynamic range input signal and thereby enhance the detectability of weak targets using a display with limited dynamic range and an observer that typically may only be able to discriminate 15 to 20 grey scale levels. This type of non-linear processing greatly influences the image contrast.

All the B-mode images presented in this chapter were compressed using the sequence of functions developed by Jensen\(^4\) and which are shown in Figure 5.22(b). The output signal from this sequence was displayed using a grey-scale range of 64. While the scheme used is somewhat different from that shown in Figure 5.22(a), it does combine the effects of pure log-compression and grey-scale mapping.

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\(^3\) This statement is more clearly supported by the graph of Figure 5.19(b) which also includes the SD’s.

\(^4\) FIELD: available at: http://www.it.dtu.dk/~jaj/field/field.html
The image contrast values given by eqns. (5.1), (5.4) and (5.5) were all calculated using the uncompressed envelope signal, $y_1$. Applying log-compression actually reduces the image contrast since it decreases the discrepancy between strong and weak signals. Furthermore, log-compression also alters the second-order statistics in the image, i.e. the average speckle size (Thijssen et al., 1988). The overall effect is that the lesion detectability as given by $\text{SNR}_{\text{opt}}$ (see equation (5.7)) is highly influenced by the log-compression algorithm.

Patterson and Foster (1983) suggested yet another quantitative measure of our ability to perceive anechoic cysts against speckle noise. They defined the contrast-to-speckle ratio (CSR) as

$$CSR = \frac{S_{\text{out}} - S_{\text{in}}}{\sqrt{\sigma_{\text{out}}^2 + \sigma_{\text{in}}^2}},$$

(5.8)

where $S_{\text{out}}$ and $S_{\text{in}}$ denote the average signal outside and inside the cyst, and $\sigma_{\text{out}}^2$ and $\sigma_{\text{in}}^2$ are the variances. The contrast-to-speckle ratio, which is also referred to as the lesion signal-to-noise-ratio, has also been used by others (Thijssen et al., 1988; Chen et al., 1996; Rownd et al., 1997; Stetson et al., 1997) to analyze image quality. In fact, Smith and Wagner (1984) demonstrated that CSR is analogous to $\text{SNR}_{\text{opt}}$ by showing that, for a spherical cyst, both parameters increase linearly with the same slope as the lesion diameter is increased.

The choice of the constant $c$ in the log function affects both the image contrast and the CSR. The higher this constant, the larger the discrepancy between weak and strong signals, and hence, the higher the contrast. However, $c$ also alters $\text{SNR}_{\text{opt}}$ (CSR) by changing the speckle size and hence the variance of the average signal. In order to investigate the effects of $c$ on CSR, 8 different images of a 2 mm anechoic cyst were simulated. Measurements were made from equal size areas of $S_{\text{out}}$ and $S_{\text{in}}$ together with the variances $\sigma_{\text{out}}^2$ and $\sigma_{\text{in}}^2$, and the CSR values for all the images were estimated from eqn. (5.8).

Figure 5.23 shows the output signal, $y_\gamma$, plotted against the envelope signal, $y_1$, for different values of $c$. Note that the weaker signals are much more enhanced with lower values of $c$. Figure 5.24 displays the B-mode images of the cyst with three
different values of $c$. The image contrast $C_2$ as calculated from eqn. (5.4) using the envelope signal ($y_1$) was found to be 0.85, and the CSR was 6.86. When the measurements are made after log-compression, i.e. using $y_7$, the following ($C_2$, $c$) paired values were obtained (0.30,0), (0.71,0.03) and (0.84,2.0). This indicates that image contrast is degraded after log-compression, and that this effect is more profound for lower values of $c$. The CSR values, on the other hand, are (6.89,0), (10.27,0.03) and (8.86,2.0).

In other words, the ability to perceive the cyst increases with log-compression, and there is an optimum value of $c$ that yields the best performance. Figure 5.25 plots $C_2$ and CSR as a function of $c$. Even though the image contrast increases linearly with $c$, the CSR reaches a maximum at $c = 0.03$.

**B) Adaptive-Filtering for Speckle Reduction**

The presence of speckle in B-mode images degrades the contrast resolution and hence, the object detectability. Spatial (Shankar and Newhouse, 1985; Shankar, 1986; Trahey et al., 1986) and frequency (Magnin et al., 1982) compounding techniques have been proposed in the past to reduce the speckle noise. By smoothing the images, these methods effectively reduce the variance of the speckle in the parenchyma, but they also cause blurring of anatomical features.

Adaptive speckle reduction (ASR) techniques selectively smooth the images to suppress regions of fully developed speckle while preserving resolvable object structure. These signal-processing techniques first use local image statistics to differentiate regions of speckle noise from regions containing highly structured components. Subsequently, they employ a low-pass filter to smooth regions of speckle. Various ASR algorithms have been implemented both in software (Bamber and Daft, 1986), and in hardware (Bamber and Phelps, 1991). More recently, Dutt and Greanleaf (1996) proposed a new ASR filter for log-compressed B-mode images. Their method uses the statistics of log-compressed echo images to derive a parameter that quantifies the extent of speckle formation. This is then used as a parameter for a filter that selectively smoothes speckle.
Figure 5.22. (a) Typical block diagram for post-processing performed in digital ultrasound imaging systems. (b) Processing block diagram assumed by Jensen and used in this thesis.
Figure 5.23. The output signal, $y_\gamma$, versus the envelope, $y_1$, for different values of $c$.

Figure 5.24. B-mode image of a 2 mm, anechoic cyst with (a) $c=0.0$. 
Figure 5.24. B-mode image of the 2-mm, anechoic cyst with (b) $c=0.03$, and (c) $c=2.0$. 
Figure 5.25. (a) $C_2$ as a function of $c$ for the 2-mm anechoic cyst. (b) CSR versus $c$. 
Figure 5.26. (a) B-mode image of the 2 mm, anechoic cyst with $c=0.0$ and (b) the same image after adaptive filtering.
Chapter 5

The effects of adaptive filtering on lesion contrast and detectability were investigated by applying the above-mentioned ASR filter to the log-compressed images of a 2 mm anechoic cyst. The original envelope signals from the eight different images were first compressed using $c=0$ (Dutt and Greanleaf's method requires pure log-compression). One such log-compressed image is shown in Figure 5.26(a), and the corresponding filtered image is displayed in 5.26(b). The image contrast and CSR values were determined for both sets: $C_z=0.37$, CSR=5.11 for the unfiltered images and $C_z=0.36$, CSR=5.20 for the filtered set. These values indicate that although the image contrast has degraded slightly with ASR filtering, the lesion detectability in turn has improved. This is a direct result of the fact that adaptive filtering decreases speckle variance in images.

5.4 Chapter Summary

In this chapter we have investigated the effects of various factors on contrast resolution of ultrasound B-mode images. First, by simulating images of a homogeneous medium with scatterer density $N_s$, it has been shown that $N_s$ affects the speckle size in both the axial and lateral directions. As $N_s$ is increased the axial speckle size decreases and approaches a constant value set by the resolution cell size of the transducer. On the other hand, there is evidence that the lateral speckle size, although showing a similar initial trend, reaches a minimum and subsequently increases.

In order to demonstrate how the image contrast is influenced by $N_s$, an anechoic cyst was simulated. The contrast increases as $N_s$ is increased until a certain point after which it seems to decline. This trend might be a direct consequence of the change in the lateral speckle size as explained above: i.e., a larger speckle size results in an encroachment into the cyst area resulting in poorer contrast.

Different array geometries have been simulated to investigate the effects of transducer design on contrast resolution. By simulating two linear arrays with same $F$-numbers (same resolution) but different center-to-center spacing (different grating lobe levels), it has been shown that grating lobes degrade the contrast resolution significantly. Linear arrays were also compared to 1.5-D and curvilinear arrays. The results indicate that, by focusing the extra rows of elements in the elevation direction, significantly higher contrast levels can be obtained from 1.5-D arrays. We have also demonstrated that
curvilinear arrays exhibit uniform contrast performance throughout their field of view, whereas the contrast from a phased linear array degrades with increasing steering angles.

Different definitions of contrast and detectability have been examined. It was concluded that CSR, the contrast-speckle ratio, is the best measure of an imaging system's ability to identify low-contrast targets among background speckle noise. Finally, it has been shown that different post-processing schemes such as log-compression and adaptive filtering may alter CSR by changing the contrast and/or speckle noise in the images.
CHAPTER 6
B-MODE IMAGE SIMULATION OF A CAROTID ARTERY WITH PLAQUE

Atherosclerosis is the underlying cause of most cardiovascular diseases such as myocardial infarctions and cerebrovascular insufficiency. It is a common form of arterial disease in which deposits of yellowish plaques containing cholesterol and lipids are formed within the intima and inner media of large and medium-sized arteries. The common carotid bifurcation and the adjacent segment of the internal carotid artery are the most common sites for atherosclerosis.

Advanced atherosclerotic lesions were classified as follows by Stary et al. (1995). The fatty plaque (type IV lesion) is the initial stage in which extracellular lipids accumulate into the intima. Prominent new fibrous tissue forms in type V lesions. Fibrofatty plaques (type Va) have a superficial cap over the lipid core; the cap is composed of smooth muscle cells and dense fibrous tissue. Calcified plaques (type Vb) are identified by calcium salt deposition into the intima. Further progression of the disease may lead to complete replacement of the lipid core by connective tissue producing the fibrous plaque (type Vc). Complications of the type IV or V lesions include ulceration, hemorrhage and thrombosis, and result in type VI lesions.

Ultrasound B-mode imaging has been used extensively to evaluate both the degree of intima-media thickness (Pignoli et al., 1986; Poli et al., 1988; Craven et al., 1990; O'Leary et al., 1991; Selzer et al., 1994) and the plaque morphology (Landini et al., 1986; Bock and Lusby, 1992). These two parameters are an indication of the severity of atherosclerosis and its progression. The imaging systems used in these studies have center frequencies of 7-8 MHz. More recently, high frequency intravascular imaging has been the focus of major research and development (Lockwood et al., 1992; Tobis et al., 1994). This technique uses a catheter-tip transducer which operates in the frequency range 20-40 MHz. Higher frequencies yields better resolution and hence, the different layers of the artery wall and the internal contents of the plaque can better be assessed.
The final objective of this thesis is to demonstrate the application of the computer model for simulating a clinically relevant situation. As stated above, one clinical problem associated with imaging carotid plaques is analyzing the plaque composition. Ultrasonic imaging of plaques has shown different levels of echogenicity which could be indicative of the severity of the disease. In this chapter, the B-mode image of a carotid artery wall with fatty plaque was simulated in an attempt to investigate the factors that determine the contrast and texture in the images of an atherosclerotic artery. However, the scattering model employed many approximations and assumptions, hence, this work should only be regarded as a preliminary feasibility study. A more refined simulation requires considerably more detailed knowledge of the ultrasonic properties of the artery and plaque.

6.1 Ultrasonic properties of the artery wall and plaque

The different layers of an arterial wall is schematically illustrated in Figure 6.1(a). Lockwood et al. (1991) compared the ultrasonic appearance of muscular (femoral) and elastic (carotid) arteries to corresponding histological findings. They observed four different regions of scattering, and concluded that the ultrasonic appearance of an artery was a result of scattering from different concentrations of elastin and collagen.

1) Media: In muscular arteries, media consists of tightly packed smooth muscle cells. The relatively low content of elastin and collagen causes an echolucent appearance. On the other hand, media in elastic arteries consists of elastic membranes arranged in concentric circular layers (in aorta, membranes are 6-18 μm apart, and 2.5 μm thick). Spaces between these layers are occupied by smooth muscle cells, collagen fibrils and elastic fibers. High levels of scattering occur as a result of this elastic content.

2) Adventitia: In muscular arteries, adventitia consists of bundles of collagen fibrils and elastin: consequently, it has a brighter appearance than the muscular media. The fibrous tissue is more loosely packed in the elastic arteries, and, hence, the scattering from adventitia is slightly less or equal to that from elastic media.

3) Elastic Lamina: Muscular arteries have very distinct internal elastic laminae and less distinct external laminae. The laminae, with an average thickness of 3 μm, are too small
Figure 6.1. (a) Schematic representation of the different layers of an arterial wall. (b) A cross-sectional view of a carotid artery sample obtained at autopsy. This sample was taken as the basis of the simulation model.
to be resolved, but they form sheets that give strong reflections. These high reflections contrast with the echolucent muscular media, giving the muscular arteries a three-ringed appearance. In contrast, the laminae are much less distinct in the elastic arteries and since the elastic media has high scattering, it is difficult to locate a boundary between media and adventitia in ultrasonic images.

4) Intima: The normal intima, only a few cell layers thick, is not seen in ultrasound images. It is composed of smooth muscle cells, elastic fibers, collagen fibrils and an outer layer of endothelial cells bordering the lumen. Fibrous thickening of intimal layer occurs with age. The thickened intima can be distinguished from the internal elastic lamina once the thickness exceeds the resolution limit of the imaging system.

Using a high frequency ultrasound backscatter microscope, Lockwood et al. (1991) measured the ultrasonic properties of femoral and carotid arteries in the high (35-36 MHz) frequency range. The scattering measurements were made at two different angles of incidence. The “axial” measurements were made along the axis of the artery (at 0° to the interfaces between layers of adventitia, media, etc.) whereas the “radial” measurements were performed at 90° to the interfaces. Their findings indicate that scattering depends not only on frequency, but also on the direction of the ultrasound beam. There is a significant increase in scattering measured in the radial direction compared to that measured in the axial direction. The reason for this increase is that, in the radial direction, there is more scattering from the concentric layer of elastic tissue in the arterial wall.

Landini et al. (1986) performed backscatter measurements in the frequency range 4-15 MHz in normal and four groups of atherosclerotic aortic specimens (fibrous, fatty, fibrofatty and calcified). They concluded that calcium salts and connective tissue (in fibrous plaque) increase the value of the backscattering coefficient (BSC)\(^1\) whereas fatty tissue has a low coefficient since it is a weak reflector.

Picano et al. (1985) measured the angle dependence of backscattering from these five type of aortic specimens at 15 MHz. Their results indicates that as the beam is

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\(^1\) Average power backscattered per steradian from a unit volume of scatterers for an incident plane wave of unit amplitude.
moved away from the normal incidence, the BSC values drop in normal, fibrous, fibrofatty and calcified specimens. The explanation for this angle dependence was given as follows: In normal specimens, the main scatterers might be the elastic layers in the media and oriented normally to the incident beam; this would cause directional scattering. In fibrous and calcified specimens, the scatterers might be the thick collagen bundles and calcium laminae which are also arranged in a perpendicular way to the beam. In the fatty plaques, however, the scatterers (cholesterol crystals) are randomly distributed and comparable in size to the wavelength. As a result of these two factors, the backscattering is independent of the angle of incidence.

6.2 Simulation Methods

6.2.1 Modeling Scattering from the Artery Wall with Plaque

In order to study the geometry and the contents of the arterial wall with plaque, a sample was obtained from the Toronto General Hospital. The sample, which is shown in Figure 6.1(b), was a cross-sectional cut from a right carotid artery that was removed at autopsy. Using a high-power microscope, the sample was examined for gross and microscopic details that were required for the scattering model. The model parameters and approximations are explained in the next two sections, and are summarized in Table 6.1.

A) Summary of Right Carotid Artery-Plaque Measurements

- distance from skin surface to the outer wall of the carotid artery (measurement from 5 normal subjects) - 1-1.5 cm
- maximum carotid artery outer diameter = 1.0 cm
- plaque dimension at the thickest site = 2.85 mm
- average media thickness = 0.78 mm
- average adventitia thickness = 0.4 mm
- thickened intima (measured from far wall) = 0.38 mm
- average lipid number density = $230 \times 10^3$ per mm$^3$ (10 measurements: range $+40 \times 10^3$, $-35 \times 10^3$. Area of each measurement $\approx 0.06 \text{ mm}^2$.)
- average smooth muscle cell density = $1,750 \times 10^3$ per mm$^3$ (12 measurements: range $+390 \times 10^3$, $-460 \times 10^3$. Area of each measurement $\approx 0.01$ mm$^2$.)

Table 6.1 Arterial wall model parameters.

<table>
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<th>Parameter</th>
<th>Value</th>
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</thead>
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<tr>
<td>Outer wall diameter</td>
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<td>Wall thickness</td>
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<tr>
<td>Distance from skin surface to wall</td>
<td>15 mm</td>
</tr>
<tr>
<td>Average SMC density</td>
<td>$1750 \times 10^3$ per mm$^3$</td>
</tr>
<tr>
<td>Average lipid density</td>
<td>$230 \times 10^3$ per mm$^3$</td>
</tr>
<tr>
<td>BSC for SMCs</td>
<td>$7 \times 10^{-5} f^{1.5}$ cm$^{-1}$ Sr$^{-1}$</td>
</tr>
<tr>
<td>BSC for lipids</td>
<td>$3.7 \times 10^{-5} f^{1.6}$ cm$^{-1}$ Sr$^{-1}$</td>
</tr>
<tr>
<td>Voxel size</td>
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</tr>
<tr>
<td>Medium size</td>
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</tr>
<tr>
<td>Average speed of sound</td>
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</tr>
<tr>
<td>Attenuation coefficient</td>
<td>1 dB/cm-MHz (Goss et al., 1978)</td>
</tr>
</tbody>
</table>

Figure 6.2 A schematic of the arterial wall geometry modeled in this thesis. A longitudinal view was simulated.
B) Summary of Approximations for the Scattering Model

First and foremost, the artery wall and plaque geometries were assumed to be cylindrical for simplicity as shown in Figure 6.2. The radius of the outer wall was set at 5 mm whereas that of the inner wall was 3.44 mm (wall thickness = 0.78+0.4+0.38 = 1.56 mm).

It is difficult to resolve the different layers of an artery wall with ultrasound broadband systems operating at frequencies below 15 MHz (Lockwood et al., 1991; Lu, 1994). Therefore, the entire wall was assumed to consist of media-like matter since media is the thickest layer. The main scatterers in the wall were assumed to be smooth muscle cells (SMC) (Lu, 1994), and those in the fatty plaque, lipids (Landini et al., 1986).

The scattering from blood in the lumen was ignored since it is well known that the backscattered power from blood is much less than that from an artery wall. The scattering from the intervening tissue between the skin and the artery wall was also ignored since the focus of this study was the contrast between the wall and the plaque.

The scattering from the wall and plaque was modeled by using the acoustic voxel approach discussed in Chapter 3. This model assumes that (1) the average scatterer (SMC and lipid) separation is small compared to \( \lambda \); (2) the scatterers are identical and their size is less than \( \lambda \); (3) the acoustic properties of each scatterer are close to that of the surroundings so that scattering is weak and multiple scattering can be ignored, and (4) the particle concentration is Gaussian distributed.

When modeling the scattering medium, the tissue was divided into elemental acoustic voxels of dimension \( \Delta x=\Delta y=\Delta z \ll \lambda \). The voxel size determines both the computation time for simulations and the accuracy of the voxel method as compared to the exact particle approach (Lim et al., 1996): the computation time decreases as the voxel size is increased at the expense of decreased accuracy. A voxel size of \( \lambda/10 \) was chosen for modeling the artery wall.

The tissue-dependent modeling parameters required to simulate the backscattered signal from a voxel are \( \sigma_b \), the backscattering cross-section and \( N_{\text{vox}} \), the number of scatterers within the voxel (see eqn. (3.25)). The backscattering cross-section is related to BSC by (Bascom and Cobbold, 1995):
\[
\text{BSC} = \frac{\sigma_b \bar{\xi}^2}{\Delta V},
\]
where \(\bar{\xi}^2\) is the variance of the number of scatterers in a voxel and \(\Delta V\) is the voxel volume. When simulating scattering from blood, with red blood cells as the scatterers, \(\sigma_b\) is constant whereas \(\bar{\xi}^2\) varies with flow conditions, and therefore, BSC is a function of \(\bar{\xi}^2\). On the other hand, soft tissue, like the artery wall, is a fixed structure with closely spaced scatterers, and \(\bar{\xi}^2\) is not expected to vary much. Hence, as a first approximation, BSC is assumed to vary directly with \(\sigma_b\).

As mentioned in Section 6.1, Landini et al. (1986) measured BSC as a function of frequency in the range 4-15 MHz for normal and atherosclerotic aortic walls. Their estimated value for the normal artery wall is \(7 \times 10^{-5} f^{1.5} \text{ cm}^{-1} \text{ sr}^{-1}\) and that for the wall with fatty plaque is \(3.7 \times 10^{-5} f^{1.6} \text{ cm}^{-1} \text{ sr}^{-1}\). In this study we assumed that backscattering from the carotid artery is similar since it is also an elastic artery. Consequently, we used the above values to represent the backscattering coefficients from SMCs and lipids in our carotid artery wall model. The number density values \(N_{\nu}\) for SMCs and lipids were measured from the sample, and are listed in the previous section. The voxel scattering density values \(N_{\text{vox}}\) for the plaque and wall regions were determined from these \(N_{\nu}\) values.

Finally, the speed of sound was assumed to be constant, 1540 m/sec, in the arterial wall, the plaque and the lumen.

6.2.2 The Imaging System

In order to accurately simulate the B-mode image of the carotid artery sample, the dimensions of the tissue model should be chosen carefully. It should be sufficiently large so that it incorporates the majority of the sidelobes and grating lobes in the lateral beam profiles. On the other hand, since the medium will be divided into acoustic voxels, its dimensions are constrained by the available computer memory size and simulation time. Because of these considerations, only half the vessel geometry, namely, that extending
from the outer wall to the lumen center, was incorporated in the simulations. Thus, the axial dimension of the tissue was set to 5 mm.

Even though it is recognized that clinical vascular imaging systems generally employ frequencies in the 7-8 MHz range, a center frequency of 5 MHz was chosen for the transducer array model. This enabled the voxel size to be set to $\lambda/10=30 \, \mu m$. Although a smaller voxel size (e.g. $\lambda/20$) and a higher center frequency would have been desirable this was not practical with the available computing power.

A 1.5-D array was modeled in order to maintain relatively uniform and thin slice thickness throughout the medium. The array had seven rows of elements, each with a height of 2.1 mm, giving a total elevation direction height of 14.7 mm. To avoid grating lobes, the center-to-center spacing between elements was chosen to be $\lambda/2=150 \, \mu m$. The array was focused in both the azimuthal and elevation directions at $z=1.75 \, cm$ on transmission, and dynamically focused with $F$-number of 1.0 on reception over the depth range from 1.5 to 2.0 cm. Hamming apodization was used in the azimuthal direction to reduce the side-lobe levels. Figure 6.3 shows the azimuthal beam pulse-echo profiles at $z=1.5, 1.75$ and $2 \, cm$, both with and without apodization. It will be noted that with apodization the response falls 60 dB below that of the on-axis response for lateral distances of $>1.5 \, mm$. Consequently, it was assumed that for the computations on a single scan line an azimuthal ($x$) width of 3 mm should suffice. Because a total of 20 scan lines were simulated, the total medium azimuthal ($x$) width was set to 6 mm (200 voxels). Figure 6.4, which shows the elevation beam profiles at $z=1.5, 1.75$ and $2 \, cm$, indicates that the response falls 60 dB below the on-axis response for $y>4.5 \, mm$, and hence, the medium size was set at 9 mm (300 voxels) in the elevation ($y$) direction.

<table>
<thead>
<tr>
<th>Table 6.2 Imaging array parameters.</th>
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<tbody>
<tr>
<td>Element width and spacing</td>
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<tr>
<td>Element height</td>
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<td>Number of rows</td>
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<td>$Tx = Ty = Ry$</td>
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<tr>
<td>$F$-number on Rx</td>
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<td>Pulse shape</td>
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</table>
Figure 6.3. Beam profiles in the azimuthal direction at \( z = 1.5, 1.75 \) and 2.0 cm (a) without, and (b) with apodization.
Figure 6.4. Beam profiles in the elevation direction at $z=1.5$, 1.75 and 2.0 cm.

6.3 Results and Discussion

Figure 6.5 shows the simulated B-mode image of the arterial wall sample. It can be seen that the mean signal level from the wall is approximately twice the mean signal from the fatty plaque: a finding that is in accord with the results from clinical studies (Bock and Lusby, 1992).

The second observation is the appearance of three bright borders at the "soft tissue"-outer wall, inner wall-plaque and plaque-lumen interfaces. These arise from the effects of constructive interference that occurs when the incident pulse moves between regions of differing acoustic properties. As discussed in the Appendix, edge effects occur when the ultrasound beam is orthogonal to the interface between two differing scattering media. As mentioned earlier, the scattering from soft tissue was set to zero. Consequently, the strong reflection at $z=15$ mm would probably be greatly reduced if the tissue medium adjacent to the outer wall had been correctly modeled. Similarly, the strong reflection from the plaque-lumen interface ($z=19.4$ mm) arises from the assumption that the scattering from blood was negligible. However, the scattering from
blood is much less than that from the arterial wall, so that in this case the assumption is realistic. In fact, a bright border line is seen in clinical images (Pignoli et al., 1986).

Figure 6.5. (a) Simulated B-mode image of the carotid artery wall with fatty plaque. (b) Normalized mean envelope signal.
The edge effect observed between the inner wall and plaque at $z=16.56$ mm is of particular significance and merits more detailed examination. Figure 6.6(a), showing the envelope of the RF signal received from the central scan line, indicates that the signal coming from the wall-plaque interface is higher than the plaque/blood interface. However, this is likely due to the differing backscattering functions assumed for the two media. In order to demonstrate this dependence we assumed that both media could be modeled by Rayleigh scatterers ($\text{BSC} \propto f^4$) so that the frequency dependence was much stronger than that previously assumed ($\sim f^{1.55}$). In addition it was assumed that the ratio of the two backscattering coefficients remained the same. Figure 6.6(b) shows the RF signal envelope from the central scan line for this new pair of values. In comparing these results with (a) it is clear that the wall signal from $z=16.56$ has been significantly reduced in comparison to that from $z=19.4$. This suggests that the frequency dependence of
scattering is of major importance in governing the wall signal amplitudes, which emphasizes the need for careful BSC measurements in order to arrive at realistic B-mode simulations. In the Appendix it is shown through a simple 1-D model that the level of constructive interference at an interface is also dependent on the scatterer distributions (density and separation). Consequently, the density \((N_r)\) values for smooth muscle cells and lipids should also be carefully measured.

### 6.4 Chapter Summary

The B-mode image of a carotid artery wall with fatty plaque was simulated with an approximate model in order to gain an understanding of the factors that determine the image appearance. It should be noted that an accurate simulation model requires a detailed knowledge of the density and backscattering properties of all the scatterers in the different layers of the arterial wall and the plaque.

As in clinical images of fatty plaque, the wall region appeared to be brighter than the plaque region in the simulated image. Strong reflection signals were observed at the wall-plaque and plaque-lumen interfaces. It was demonstrated that the level of constructive interference at an interface depends on the two modeling parameters, BSC and \(N_r\). It was therefore concluded that these parameters should accurately be determined for future studies.
CHAPTER 7
SUMMARY AND CONCLUSIONS

A computer model was developed to simulate ultrasound B-mode image formation, and to perform fundamental studies on contrast resolution for 1-D and 1.5-D imaging arrays. The model consisted of two major parts; simulating the beam formation from transducer arrays, and modeling the interaction between the ultrasound beam and the medium (scattering and attenuation). After comparing the model predictions with experimental measurements it was employed to investigate the effects of various factors (transducer geometry, medium properties etc.) on contrast resolution. Finally, it was applied to the imaging of artery wall with plaque.

7.1 Detailed Summary

Various theoretical methods have been used in the past to model the beam formation from linear arrays. A comparison study was performed to determine the “best” method in terms of its accuracy and computational efficiency. The study, presented in Chapter 2, indicates that the trapezoidal method, which is an approximation to the exact impulse response method, provides the most efficient and accurate results. The primary disadvantage of the exact impulse response method (method (i)) is that the accuracy and the computation time are highly dependent on the sampling frequency used, which is determined by the array geometry and the field point location. The approximate trapezoidal solution, on the other hand, could be revised to allow for lower sampling rates by tracking the energy, and therefore yields the best trade-off between accuracy and efficiency. The CW solution (method (iv)) also provides accurate results, and it is computationally more simple and efficient. However, the accuracy of method (iv) is highly dependent on the steering angle, $\theta$; it deteriorates for $\theta>10^\circ$.

Using the trapezoidal method, the model for the linear array was first improved to include elevation focusing, dynamic focusing and apodization. It was then modified to simulate the beam profiles from 1.5-D and curvilinear arrays. It was demonstrated that 1.5-D arrays provide better and more uniform slice thickness when compared to linear
arrays. Furthermore, the results indicate that the resolution is improved as the number of extra rows in the elevation direction is increased. It was also shown that the lateral resolution from curvilinear arrays is slightly degraded when compared to that obtained from a linear array. However, there are two advantages with curvilinear arrays: they have a wider field of view, and the lateral resolution remains constant over this field. This is in contrast to the performance of flat linear arrays whose resolution degrades when steered to cover the same field.

Chapter 3 focuses on developing the model to simulate the interaction of the ultrasound beam with a tissue medium. Based on the Kramers-Kronig relations, frequency dependent attenuation and dispersion effects in tissue were incorporated. The effects of scattering was accounted for, first by considering the scattering by small spheres. Based on this model, the scattering function for cylindrical targets was derived for the limiting case where the radius of the cylinder is much smaller than the wavelength, and the target is in the far field. Finally, the acoustic voxel method for modeling scattering from a homogeneous medium was reviewed.

The simulation model results were compared to data obtained from B-mode imaging experiments performed by GE Medical Systems in Milwaukee. A linear array with different center frequency and focusing schemes was employed to collect three sets of data. The imaging phantom contained seven nylon fibers aligned in the central scan line. Using the computer model with the cylindrical target-scattering function derived in Chapter 3, images of this phantom were simulated. Results presented in Chapter 4 demonstrate a good agreement for the beam profiles in the axial direction. However, the presence of side-lobes in the simulation images which are absent in the original images indicated either that the model did not accurately simulate the array, or that the experimental conditions were not accurately known. Further investigations indicated that the discrepancy probably arose from the use of a proprietary and confidential apodization function during the experimental measurements. Because the details of this function could not be revealed, simulations were conducted by using a simple Gaussian apodization function. These demonstrated that the side-lobes could be suppressed in the simulated images thereby supporting the hypothesis that the discrepancies arose form the use of apodization during the experiments.
In Chapter 5, images of a homogeneous medium were simulated by using the computer model in order to obtain an understanding of the factors that can determine the contrast resolution of a B-mode imaging system. Using the “particle approach”, the scattering properties of the medium were simulated by assuming identical point scatterers with an average density of \( N_s \). The effects of \( N_s \) on the speckle size were investigated first. It was shown that at low densities, the speckle size was dependent on both \( N_s \) and the transducer properties: for a given transducer, the lower the density, the larger the speckle size. At higher densities, the axial speckle size reached a constant value that was mainly determined by the transmitted pulse length. The lateral speckle size also decreased, reached a value set by the transducer geometry, and then exhibited a gradual increase at very high densities. This observation was statistically significant for only one of the array geometries studied and hence, needs to be further investigated by determining the speckle size at even higher densities (\( N_s > 5000 \)).

Anechoic box-shaped cysts were simulated in the medium to investigate the effects of various transducer geometries on the contrast resolution. In order to avoid long computation times a low scattering density of 10 mm\(^{-3}\) was chosen for these studies. Even at this density the computation time was in the order of a day. At this density the speckle pattern was fully formed and consequently, the results can be expected to be representative of those at more realistic densities. First, it was shown that two linear arrays with different element spacing, but with identical F-numbers, (hence, same lateral resolution) yield different contrast resolution. The element spacing determines the level of grating lobes presence of which significantly degrades the contrast. Secondly, the performance of linear arrays and 1.5-D arrays with different geometries were compared. It was demonstrated that for small cysts, the contrast achieved by 1.5-D arrays is superior due to the lower slice thickness achieved by focusing the extra rows of elements. Dividing the height of the array into equally-sized rows also yields better contrast resolution as compared to that obtained by using Fresnel heights. Increasing the number of additional rows was found to decrease the side-lobe levels resulting in improved contrast resolution. Finally, images from curvilinear and phased arrays were simulated and compared. The results indicated that curvilinear arrays have a uniform contrast
performance over the whole field of view whereas the contrast from phased arrays degrades as the steering angle is increased.

The effects of the medium properties on contrast resolution were also investigated. First, the scatterer density was increased to study the relationship between $N_v$ and contrast resolution. It was shown for small cysts ($s<2$ mm) that as $N_v$ was increased, the contrast increased to a peak value after which it seemed to decrease. This effect was linked to the effect of $N_v$ on speckle size: a larger speckle size resulted in more encroachment into the cyst area and hence degraded the contrast. The contrast resolution associated with both hyper- and hypo-echoic cysts were also investigated. These two types of cysts, together with anechoic cysts, were compared in terms of their contrast and target detectability. For a given imaging system (speckle size), the detectability was found to vary directly with contrast and size. The results indicated that the detectability of a (+12 dB) hyper-echoic cyst is higher than that of an anechoic cyst (0 dB), which in turn is higher than that of a (−12 dB) hypo-echoic cyst.

The final part of the contrast resolution analysis involved studying the effects of post-processing on lesion detectability. It was shown that logarithmic compression, which is extensively used in clinical B-mode imaging, greatly influences the visibility of a target by altering both the image contrast and the speckle size. The log-compression function modeled in this work contained a constant parameter, $c$, the choice of which had different effects on contrast and detectability. The image contrast increases as $c$ is increased, but the detectability does not exhibit such a linear relationship: it first increases to a maximum value after which it decreases. The effects of adaptive filtering on images were also investigated, and it was shown that filtering improves the lesion visibility by decreasing the speckle noise in the images.

The final part of this thesis was designed to demonstrate the application of the model for simulating a clinically relevant situation. Chapter 6 describes the simulation of a B-mode image of a carotid artery wall with fatty plaque. It should be noted that many approximations and assumptions were employed when modeling the scattering from this tissue specimen. The plaque region appeared to be less echogenic than the wall in the simulated image. The image also revealed a strong reflection signal from the wall-plaque
interface. It was demonstrated that the strength of this signal is determined by the frequency behaviour and the distribution of the scatterers in the two neighbouring media.

### 7.2 Contributions

#### 7.2.1 Primary Contributions
- A comparison study was performed to determine the accuracy and computational efficiency of various methods used to model the field profile from pulsed linear arrays. It was concluded that the trapezoidal method yields the best trade-off between accuracy and efficiency.
- The field profile from curvilinear arrays was simulated for the first time. It was demonstrated that the contrast resolution from a curvilinear array is uniform throughout the field of view.
- Various quantitative measures are currently used to evaluate an imaging system’s performance. A comparison of the most commonly used measures (-6 and -20 dB resolution, image contrast, CSR) indicated that the Contrast-to-Speckle Ratio (CSR) is the best measure in terms of defining an imaging system’s ability to detect low-contrast lesions against background noise.

#### 7.2.2 Secondary Contributions
- The radiation-coupling function for a linear array-to-cylindrical target system was theoretically derived for cases when the diameter of the cylinder is much smaller than the wavelength and the target is in the far-field of the array.
- The effects of number density of scatterers on B-mode images were studied at higher densities than those previously used in literature. The results suggest that the lateral speckle size is not only a result of the specific array design, but might also depend on the number of scatterers in the medium.
- The effect of cyst shape on B-mode images was investigated. It was shown that edge effects, which are not prominent in images of spherical cysts, can be quite pronounced in images of box-shaped or cylindrical lesions.
- The significant influence of array geometry on contrast resolution was demonstrated by simulating images of cysts scanned by linear, 1.5-D and curvilinear arrays.

- The effects of post-processing schemes on contrast resolution were investigated. It was shown that the simulation model can be used to predict optimum values for parameters used in post-processing algorithms.

- A preliminary model of arterial wall with plaque was developed, and the B-mode image of this geometry was simulated in order to gain a better understanding of the textural appearance of plaque. It was shown that the strong border signals at the wall-plaque and plaque-lumen interfaces are a result of constructive interference, and the level of these signals is dependent on the backscattering coefficients of the scatterers in the neighbouring media.

### 7.3 Future Investigations

The transducer array geometries considered in this work include linear, 1.5-D and curvilinear arrays. The 2-D array is yet another type of array geometry that has been widely investigated but which is not yet commercially available. The complexities involved with 2-D arrays are the requirement of a large number of small transducer elements, and the problems of integrating these elements for both transmit and receive modes. The computer model developed in this study could easily be revised to simulate images from 2-D arrays, which would enable the contrast resolution of 2-D arrays to be studied.

Another important aspect concerns the effects of non-linearities and the use of higher harmonics for harmonic imaging. As mentioned in Chapter 3, tissue is almost always assumed to behave linearly when modeling the B-mode image formation. This assumption is made since the signals used in clinical ultrasound imaging systems are small (Gurumurthy and Arthur, 1982). In fact, all finite-amplitude waves experience a degree of non-linear distortion as they travel through tissue, but this effect is generally treated as being small enough to be ignored at current diagnostic signal levels.

The effect of non-linearity can be observed in the frequency domain with the presence of higher harmonics - these harmonics occur at integer multiples of the original
frequency. This non-linear behaviour of tissue is important for two reasons: (1) It can provide a new measure for tissue characterization since the degree of non-linearity might vary depending on tissue structure; and (2) If clinical imaging systems produce large enough signal amplitude to cause significant non-linear effects, images with better contrast resolution might be attained. The feasibility of this non-linear harmonic imaging has been demonstrated by Ward et al. (1997), who have shown that the higher harmonics have narrower beamwidths (better resolution) and lower side-lobe levels (better contrast). There is considerable interest in applying these concepts to commercially available systems. It would therefore be of great interest to revise the computer model to include non-linear imaging and to use the model for studying the improvements to the contrast resolution that can be achieved.

Finally, it should be noted that the arterial wall model simulated in this work is an approximate model and needs much improvement. First and foremost, the backscattering coefficients and the densities of all the scatterers (collagen fibrils, elastic fibers, smooth muscle cells and lipids) in the different layers of the wall, and the plaque should accurately be determined. Fibrous plaques as well as the fatty ones could then be modeled, and the textural appearance of the images could be compared. Secondly, higher computing power would enable higher frequencies to be used. The model could then be employed to measure the media-intima thickness.

In vitro tissue experiments could also be performed to verify the results of the atherosclerotic carotid artery model. Specimens of carotid arteries removed at autopsy would first be scanned with the imaging system. After ultrasound interrogation, the arteries would be cut opened for histological examination. B-mode images of these arteries would then be simulated by using the histological findings, and the two sets of results could be compared.
REFERENCES


References


APPENDIX

Edge Effects

When an interface between two different scattering media lies orthogonal to ultrasound beam propagation, "edge effects" can be present in the B-mode images. This phenomenon was observed around the box-shaped cysts of Chapter 5 and at the arterial wall-plaque interface of Chapter 6. The objective of this appendix is to provide an explanation for the edge effects using the 1-D case.

Let us assume that a line of point scatterers are distributed along a 2 mm distance around z=4.0 cm on the axis of the transducer array, Array 1 of Table 5.1. We will consider three different distributions to demonstrate how edge effects can occur under certain conditions.

First, let us assume that 600 scatterers are randomly placed between z=3.9 cm and z=4.1 cm. Figure A.1(a) shows the envelope of the resultant RF signal, and indicates that the echo signals add up constructively and destructively in a total random manner. In the second case where the scatterer density is increased (6000 scatters), the constructive interference at the edges (z=3.9 cm and z=4.1 cm) increases since the scatterer spacing is significantly decreased. This results in strong signals at the borders as illustrated in Figure A.1(b). Finally, let us consider the case where 600 scatterers are distributed regularly along the axis with a spacing of approximately \( \lambda/100 \). The resultant envelope signal shown in Figure A.1(c) also reveals edge effects; the echo signals add up constructively at the interfaces since the scatterer separation is much smaller than the wavelength. It should also be noted that the signal level in between the edges is lower than that observed in Figure A.1(a). This indicates that there is also strong destructive interference arising from closely, regularly spaced scatterers.

The constructive interference phenomenon can also be demonstrated by simulating a simpler case where the transmit pulse is a one-cycle sinusoid with a frequency of 5 MHz. Let us assume that 8 scatterers are placed along the transducer axis with a spacing much smaller than the wavelength. When the scatterers are randomly distributed as shown in the left-hand column of Figure A.2(a), the sinusoids add up
constructively at the edges resulting in strong signals arriving from the borders. Figure A.2(b) illustrates the sine waves arriving from each scatterer; the absolute value of the sum of these signals is shown in Figure A.2(c). When the scatterers are regularly spaced with a separation of 37.5 µm (right-hand column), there is again strong constructive interference at the edges, but also, the sinusoids cancel each other in the space between resulting in total destructive interference.

It is apparent from the results presented in this appendix that the scatterer distribution (density and separation) greatly influences B-mode image appearance, and hence should carefully be modeled.

Figure A.1. Envelope of the RF signal received from a line of scatterers distributed along 3.9<z<4.1 cm. (a) Randomly distributed 600 scatterers.
Figure A.1. Envelope of the RF signal received from a line of scatterers distributed along $3.9 < z < 4.1$ cm. (B) Randomly distributed 6000 scatterers and (c) regularly spaced 600 scatterers.
Figure A.2. The edge effects for randomly distributed (left-hand column) and regularly spaced (right-hand column) scatterers. (a) The distribution the scatterers. (b) The signals received from each scatterer. (c) The absolute value of the total received signal.