UNDERSTANDING TEACHING FOR UNDERSTANDING IN THE MATHEMATICS CLASSROOM

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Education
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Abstract

North American students’ poor performance in mathematics and the controversy regarding how mathematics should be taught were the basis for this interpretive study on teaching for understanding. The study inquired into teachers’ practice, explored their beliefs and views, and examined their content knowledge in order to understand what teachers understand about teaching for understanding in the context of mathematics. Factors within the context in which they teach were also considered. Two comprehensive cases were developed within the framework of the study. The first focuses on a grade seven teacher who was at the beginning of his teaching career. The second concentrates on an experienced grade five teacher who had taught for twenty five years.

Data in the form of field notes, observation, tape recordings of daily classroom instruction and audio taped interviews were collected over a four month period for each teacher. Key themes that emerged from the data, teachers’ beliefs and views about the teaching and learning of mathematics, teachers’ content knowledge, teachers’ perspective of understanding, how teachers promote understanding, how they perceive their role as teacher, and their style of teaching were discussed in order to develop detailed portraits of the participants. Factors of context that mitigate against teaching for understanding were also addressed.

Similar to other closely related studies in the literature, this study revealed that teachers’ views of understanding may vary and may not necessarily be congruent with those of the research literature and curriculum developers. The study also demonstrated that teachers may not have well formed images of what it means to teach for understanding. In addition, factors within the context that teachers work add to the challenge of teaching for understanding. Unlike other studies in the literature, this study demonstrated that language arts is an important referent for understanding what teachers understand about teaching for understanding.
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Chapter 1  Introduction

At the heart of the concern about students' performance in mathematics in North America lies a controversy regarding how mathematics should be taught. Advocates of reform favor an approach that promotes meaning and understanding in students' learning of mathematics. Traditionalists argue for a back-to-basics approach with its emphasis on proficiency with basic operations and the ability to apply rules. These differing views regarding the nature of instruction and the implication for students' learning were the basis for this interpretive study which looks at elementary teachers' mathematical practice. The study inquires into teachers' understanding of teaching for understanding in mathematics and the implications for their practice. It also considers factors that affect that understanding and the realities of teaching in this way in the current educational climate. As participant observer I spent a period of four months, almost daily, in each of two elementary teachers' classrooms. The cases I developed within the framework of this study portray these teachers' unique views and understandings of what it means to teach for understanding in mathematics. In discussing issues related to the teaching and learning of mathematics this chapter sets a context for the study. It also provides an overview of the content and structure of subsequent chapters in the thesis.

Setting of The Problem

Mathematics education in North America is frequently characterized by inadequate student achievement and declining participation in mathematics courses (Silver and Stein, 1996). The public perception is that North American students do not measure up to their Japanese and Chinese counterparts. Much discussion is focused on students' poor results, particularly with higher level cognitive skills involving understanding and problem solving (Carpenter, Matthews, Lindquist, and Silver, 1984). Results from the second international assessment of educational progress in mathematics, The Second International Mathematics
Study revealed that Canadian students ranked midway between other participating countries of the world even though Canada was at the top for spending (Lewington, 1992). Canadians trailed Taiwan and South Korea (Japan was not part of these surveys), but were ahead of the United States. In comparison to provinces such as Quebec, British Columbia, and Alberta, Ontario students scored the lowest in mathematics in the international survey. Results indicated that students scored reasonably well in basic mathematics tasks, but not in sophisticated problem solving. Findings from The Third International Mathematics and Science Study, the largest of its kind in the world, confirmed that Japan, China and Korea are still ahead of Canada, and Ontario students ranked the lowest compared to their provincial counterparts (Small, 1996). Among the five participating provinces British Columbia students scored the highest, then Alberta and Newfoundland, followed by New Brunswick and Ontario (Robitaille, Taylor, Orpwood (1996). Despite the limitations of international surveys (Silver and Kenney, 1995), public perception is that North American students are not measuring up to their Japanese and Chinese counterparts.

As noted, Ontario students’ test results in mathematics have been a concern for the advocates of reform and those who favor a more traditional approach to education. Both interest groups attribute students’ poor performance to the curriculum and to the nature of classroom instruction and are calling for change. The traditionalists, such as the Coalition of Parents and People for Education and other critics of education favor a back-to-basics approach, not only for mathematics, but for all subject areas. As Nelson, Carlson, and Palonsky (1993) suggest, "Not only do we need to reemphasize the fundamental skills in elementary school; we need to insist upon rigorous evaluation of the skills before we allow students to continue in school" (p. 158). Many believe that success in mathematics is measured by students’ proficiency with the basic operations and their ability to apply rules. The growth of remedial learning centers such as Kumon mathematics, which promote this kind of learning, is testimony to their belief. Yet, a widely used form of instruction which
relies on explanation and practice, evident in a majority of classrooms, does not necessarily foster mathematics achievement (Lo, Wheately, and Smith, 1994).

Closely tied to concerns regarding the content and how it should be delivered is the issue of assessment. Proponents of the back-to-basics approach to learning believe that the absence of system-wide testing in Ontario for the past twenty years has been a factor in the decline of students’ achievement. They argue for uniformity of teaching and standardization of content that this type of testing is believed to improve. However, as Landsberg (1995) points out, "Standardized tests have nothing to do with improved learning or intellectual achievement. (In fact they can consume enormous and futile amounts of time, effort and money.)" (p.K 1). According to Stake (1995), standardized test results tell very little about students’ thought processes and how their learning is progressing. It is also suggested that teachers’ practice tends to be influenced by their perceptions of the content of externally mandated tests, especially when they view that the test results have important consequences for them and their students (Romberg, Zarinnia, and Williams, 1989). Moreover, standardized tests are quite limited as guides for instructional decision making (Silver and Kenney, 1995).

As a voice of the reform, The National Council of Teachers of Mathematics recommends that the long-standing tradition of school mathematics be replaced by an approach that emphasizes teaching for understanding (Prawat, Remillard, Putnam, and Heaton, 1992). However, this way of teaching, which stresses "doing", rather than "knowing that" (National Council of Teachers of Mathematics, 1989), is extremely complex (Hiebert, 1986) and difficult to achieve, as this study demonstrates. In Gardner’s (1993) view, teaching for understanding is challenging for schooling and for society. For the teacher, it requires teaching in more intricate ways than simply transmitting information. For the learner, it suggests a need for flexibility and thoughtful engagement, rather than skill development and memorization.
Most teachers believe that they teach for understanding (Byers and Herscovics, 1977) and this may be problematic as the following illustrates. Firstly, there is a lack of consensus regarding what it means to teach for understanding. As Cohen and Ball (1990a, b), Gardner (1993), and Prawat (1989), point out, teachers vary in their perceptions of what it means to teach for understanding. Moreover, their views may not necessarily be congruent with those of the research community and curriculum developers. Secondly, many teachers are steeped in their traditional views of teaching and learning mathematics (Cohen and Ball, 1990a, 1990b). The narrow perspective and limited understanding of mathematics that teachers have developed, as a result of their early classroom learning (Lortie, 1975; Zeichner and Gore, 1990), have implications for their practice. My experiences with teachers as a consultant and teacher educator are congruent with these views of the literature (Cohen and Ball 1990a, 1990b; Lortie, 1975; Zeichner and Gore, 1990). In a majority of classrooms, often I have seen mathematics presented as a collection of facts and procedures (Gregg, 1995; Romberg, 1992) where memorization rather than construction of knowledge is stressed.

Projects such as the Summer Math for Teachers Program at Mount Holyoke College (Ball, 1996) further demonstrate how teachers’ early experiences affect their practice in mathematics. Many teachers who participated in the project, particularly those who taught elementary children, reported that their experiences as students were so negative that they chose not to study mathematics beyond their middle years of high school. The teachers also talked about how unprepared they felt and how they lacked the necessary resources to help their own students understand or do mathematics.

Many of the preservice teachers that I have taught frequently talked about their anxieties and the confidence they lacked in their ability to teach mathematics. In some instances, students who were mathematics majors were afraid to teach elementary children. The story of Tania is particularly compelling. Prior to coming to the faculty of education, she had studied calculus for two years. When faced with the prospect of teaching a lesson
on multiplication of fractions to a grade eight class, she was overcome with fear. All she could think about were the difficulties with fractions and decimals she had experienced as an elementary student.

In summary, I have focussed on concerns regarding the teaching and learning of mathematics for education today. The following section provides an overview which maps out the organization of a study that inquires into teachers' practice in mathematics.

The Study

Rogers (1995) suggests that "Any inquiry is set within a context that gives shape and meaning to its questions and interpretations" (p. 6). In the interpretive tradition, this study is designed to inquire into what it is that two teachers understand about teaching for understanding in mathematics. In so doing, it takes an in-depth look at their classroom practice and explores their views on teaching for understanding in mathematics. The intricate connection of beliefs and content knowledge and their effect on teaching for understanding, along with issues of context, are also examined.

The study focuses on two elementary teachers whom I came to know through my work as a consultant and teacher educator. Their varied professional experience and their comfort level with my presence in their classrooms were factors that motivated me to invite these two teachers to participate in this study. One teacher was an experienced pedagogue, who had spent twenty years in the classroom working with both adults and children. At the time of the study she was teaching grade five and had been for the previous seven years. The other teacher, who was just beginning his career, was teaching grade seven for the second time. I spent four months, almost daily, observing in each teacher's classroom so that I might come to learn about their practice and their understanding of teaching for understanding. I will talk further about these teachers as the study unfolds. In addition, I will describe my role as participant observer (Hammersley and Atkinson, 1992) and discuss the methodology I employed to gather and interpret the necessary data for the study.
Because this study seeks to understand and interpret how two teachers "construct their world around them" (Glesne and Peshkin, 1992, p.6), an interpretive approach to inquiry seemed appropriate. The open and emergent nature of such an approach "sets the stage for discovery" (Glesne and Peshkin, 1992, p.6) of what it is that teachers understand about the complexities of teaching for understanding and the implications for their practice. It also allows for multiple means of data collection. The thick, rich description characteristic of interpretive inquiry is well suited for capturing the meaning perspectives of participants in particular situations. To preserve the unique character, life history, professional experience, and work context of participants, separate case studies are developed. As Goode and Hatt (1952) suggest, "The case study... is a way of organizing social data so as to preserve the unitary character of the social object being studied" (p.331). However, for the sake of consistency a similar structure and format were employed for the data gathering and analysis phases, and for the final write up.

Like other cases in the literature that focus on teaching for understanding in mathematics, this study also inquires into teachers' practice and their understanding of this complex phenomenon. However, it differs in that it provides an in-depth portrait of two elementary teachers' practice and their understanding of teaching for understanding. As I noted earlier, I spent four months, almost daily, in each teacher's classroom. The study also looks at factors of context that both support and mitigate against teaching for understanding. A detailed discussion of comparable cases in the review of the literature uncovers further differences that exist. The discussion also highlights that there is a paucity of comprehensive cases that focus exclusively on what teachers understand about teaching for understanding. This study then, as one small piece of the research on understanding, will add to the body of knowledge regarding cognitions and context related to teaching for understanding. Moreover, it will help fill the need for more "schools-based" research in mathematics (Goldin, 1993) and "feed the development of increasingly useful theories of mathematics teaching and learning" (Hiebert and Carpenter, 1992). The study, which is
based in teachers' practice, will provide two more cases on teaching for understanding that may be used by other researchers and practitioners in the research and educational community.

Overview of the Study

The thesis is organized so that in Chapter Two a review of the literature looks at relevant research that has been conducted on teaching for understanding (Alkin, 1992; Cohen, McLaughlin, and Talbert, 1993; Hiebert and Carpenter, 1992; Roulet, 1998). It also provides evidence of the need for more comprehensive case studies that concentrate on the problem of what teachers understand about teaching for understanding and the context for that teaching. According to Gardner and Boix-Mansilla (1994), "There have only been scattered attempts to define what is meant by this phrase [teaching for understanding] and set up programs that explicitly address this goal" (p. 200).

Chapter Three provides a detailed discussion of the methodological features of the study and the research approach. Selection of the participants, setting, role of the researcher, data collection, data analysis and ethical considerations are discussed. Measures to ensure the credibility (Janesick, 1992) of the study are also addressed in the chapter. As I previously suggested, an interpretive approach was selected as it allowed for an in-depth look into a phenomenon as complex and challenging as teaching for understanding and provided for the thick, rich description necessary for capturing the meaning perspectives of the participants as they constructed the world around them. It is from an interpretive perspective then that I developed the following questions which frame this study.

a) What does it mean to teach for understanding in the context of the mathematics classroom?

b) What are the implications of teachers' perceptions of what it means to teach for understanding?
c) What is the nature of the context in which teachers create their practice in mathematics?

These three questions provide a basis for the development of this study which affords the reader an opportunity to learn about what it means to teach for understanding in mathematics.

Prior to the presentation and interpretation of the data in Chapters Five and Six, a detailed picture of the two teachers is presented in Chapter Four. Included in the chapter is a preliminary discussion of these teachers’ views of the concept of understanding. The presentation and interpretation of the data in Chapters Five and Six are intended to address the first two questions that guide this study. The comprehensive portraits that emerge in these two chapters allow the reader to make a connection with the participants while coming to understand what it means to teach for understanding in the context of mathematics. To answer the third question, the final chapter concentrates on issues of context that both mitigate and support teachers’ practice. A discussion of the problematic nature of teaching for understanding and the reality of teaching in this way in today’s educational climate is included. Personal reflections and ideas for future research in this area of teacher education are addressed as I conclude the thesis.
Chapter 2  Literature Review

This interpretive study, which is based in teachers' practice, seeks to understand what it means to teach for understanding in mathematics. As suggested in Chapter One, teaching for understanding is complex, challenging, and open to broad interpretation, which is reflected in the diverse views of the literature. The purpose of this chapter is to present these views. The chapter also discusses factors that affect teachers' understanding about what it means to teach for understanding. These factors, teachers' beliefs about teaching and learning, teachers' content knowledge, and the context in which teachers create their practice are important as they are the key concepts that frame this study. Studies of others who have conducted research on this complex phenomenon are also addressed. These studies, which are closely tied to this research, provide a context for this inquiry. They also demonstrate that there is a paucity of comprehensive case studies that focus exclusively on what teachers understand about teaching for understanding.

In order to do justice to this review of the literature, I first need to talk about the concept of understanding as it is at the heart of what it means to teach for understanding. Like teaching for understanding, it too is a complex idea, and "What is understanding?" is a question that is not easily answered (Perkins, 1993).

What is Understanding?

In education as in everyday life the term "understanding" is often liberally used to represent a range of ideas. For example, in a discussion regarding what it means to understand in mathematics, Byers (1980) suggests that understanding will have different meanings depending on teachers' use. Some teachers in speaking about the progress of their students will say "The student doesn't understand." What they really mean is that "The student does not comprehend the question" (Byers, 1980). There are other teachers who will use the term as a way of describing students' problem solving ability. They will
say that a student who is able to take a key step in solving a non-routine problem, or who is able to grasp quickly the main idea of a piece of mathematics, understands. Sierpinska (1994) points out that in the context of mathematics understanding may be used to talk more generally about understanding 'mathematical concepts', and more specifically about understanding concepts such as number, quantity, volume, function, limit of sequence, linear independence of vectors. She adds that other things such as patterns and phenomena are also objects of understanding.

A study which explored the nature of scientific understanding in "in-school" and "out-of-school" contexts provides further evidence. In this study Anderson and Roth (1989) noted that a discrepancy existed between researchers' and students' interpretation of the concept of understanding. According to students, understanding meant being prepared to answer recall questions. For the researchers it was a more complex and intellectually active process of conceptualization. The study also revealed that students' "in-school" understanding of science did not fit with "out-of-school" usage of scientific knowledge. Anderson and Roth (1989) argue that, in fact, many students claimed to "understand" scientific topics when they actually did not. My experience with children in classrooms, in the area of mathematics, is congruent with the findings of this science based study. Like Anderson and Roth (1989), I have found that students' (if not teachers') views of what it means to understand in mathematics differed from mine. Many viewed understanding in mathematics as the ability to reduce knowledge to a list of facts to be memorized in order to complete a particular task. For me, understanding is a more complex and conceptually challenging process as the following demonstrates. When elementary students successfully perform operations with money, teachers assume that they "understand" its part-whole relationship. I believe that students know how to figure out calculations with money because of its importance and use in real life, but many lack a deep understanding. When I suggest that teachers ask students to express different amounts such as $1.09 or $0.62 in fractional form to show their understanding of its part-whole relationship, students generally
are unable to do so. They fail to understand that one dollar is the same as a hundred hundredths and that a part of a dollar can be expressed as a fraction out of one hundred.

What is Understanding in Mathematics?

Early in the nineteenth century, in debates about school mathematics, Warren Colburn defended understanding and discovery as the avenue to learning arithmetic in his book, First Lessons in Arithmetic on the Plan of Pestalozzi, with Some Improvements (1821). His conception of understanding emphasized the importance of mental discipline, the emerging theory of that time. Colburn argued that “students would understand arithmetic if they were allowed to discover its rules” (Sztajn, 1995, p. 377). He believed that through a series of specially chosen examples purposefully arranged in a sequence of increasing difficulty, students would develop their own methods of solving problems. The rules that govern arithmetic would then emerge for students from these methods. Toward the end of the century, McLellan and Dewey (1895) advocated for the need to develop understanding rather than skill in mathematics. They argued that mathematics was not a disjointed enterprise and would be best understood if it was seen as a subject in relation to other fields of activity (Brown, Cooney, and Jones, 1990). In so doing they designed a curriculum for mathematics to improve students’ level of understanding beyond that which existed in classrooms at the time (Hiebert and Lefevre, 1986). For Poincaré (1956) understanding in mathematics went beyond logical reasoning and was determined by one’s ability to grasp a mathematical argument in one global idea. As with other mathematicians at the turn of the century, Poincaré viewed understanding in mathematics as an “all-or-nothing” phenomenon (Byers, 1980), devoid of partial understanding.

Brownell (1935) also argued for the need for understanding in mathematics. He opposed the idea of instruction which saw learning as the acquisition of a series of isolated skills or “the mechanistic mastery of procedures” (Brown et al., 1990, p.640). According to Brownell, “meaning” was the key in children’s learning of arithmetic as it allowed them to see the sense in what they learn. For example, children would see why four times five is
twenty based on what is known about the meaning of those numbers and on the nature of multiplication as repeated addition (Brown et al.). In Brownell’s view, meaning was derived from within the connectedness of mathematics itself and encouraged understanding. The goal of instruction then was “to make arithmetic less a challenge to the pupil’s memory and more a challenge to his intelligence” (Brownell, 1935, p.31).

More recent theories have advanced beyond the “all-or-nothing” phenomenon (Byers, 1980; Skemp, 1987), acknowledging that there are different levels (Byers, 1980) and degrees (Backhouse, 1978) of understanding. Holt (1964) and Van Engen (1953) viewed understanding as a continuum which accounted for students’ partial understanding of concepts and ideas. To help teachers learn what students really know as opposed to what they might give the appearance of knowing, Holt (1964) developed the following seven point list that teachers could use in planning instruction and assessing learning.

I feel I understand something if and when I can do some, at least, of the following: (1) state it in my own words; (2) give examples of it; (3) recognize it in various guises and circumstances; (4) see connections between it and other facts or ideas; (5) make use of it in various ways; (6) foresee some of its consequences; (7) state its opposite or converse.

(p. 36-37)

According to Holt this list was only a beginning in assessing students’ understanding.

Van Engen’s (1953) approach is different. In a discussion regarding the development of concepts Van Engen talks about the “meaning” of understanding. He points out that what is meant by understanding is different from the “meaning” of meaning. According to Van Engen, understanding is “more nearly a process of integrating concepts, placing them in a certain knowledge according to a set of criteria ... Understanding is an organizational process” (p. 76). Van Engen notes, when a student says “I know what you mean but I do not understand it,” the student might know what to do, but not know why it should be done. Skemp’s (1976) two tiered model also corroborates that understanding is not uni-dimensional. His model (Skemp, 1976), which
applies both to learning and teaching (Byers, 1980), delineates two categories of understanding: instrumental and relational. Instrumental understanding is based on a student’s ability “to apply rules [of mathematics] without reason” (Byers, 1980). Relational understanding, which according to Skemp (1976) was deeper than instrumental understanding, was based on a student’s ability “to deduce rules from more general mathematical relationships” (Byers and Herscovics, 1977, p. 24). Students who demonstrated relational understanding would know “what to do and why”. Byers and Herscovics (1977) extended Skemp’s (1976) classification system and added two more levels of understanding, intuitive and formal understanding. According to the tetrahedral model they developed, a student might have at any one time a mixture of these different types of understanding of a particular topic. Like Skemp, Davis (1978) proposed a model which postulated levels of understanding. His model was based on the moves that teachers make in the process of instruction. Davis (1978) notes that moves teachers make when teaching mathematical concepts such as place value, exponent, or triangle are different from those they make when teaching children to become skilled at carrying out a procedure such as long division, bisecting a line, or measuring an angle. These moves could be used as a model for planning instruction and assessing students’ knowledge. Byers (1980) suggests that Davis’ model is important because of its ability to evaluate systematically a student’s understanding of a mathematical topic.

According to Hiebert and Carpenter (1992), the idea of understanding as making cognitive connections is a long standing theme in mathematics. Haylock (1982) explains that models based on this theme allow for a broad spectrum of degrees or levels of understanding of mathematical concepts. In his view, “The more connections the learner can make between the new experience and previous experiences, the greater and consequently the more useful the understanding” (Haylock, 1982, p. 54). Similarly, Hiebert and Carpenter (1992) suggest that “mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is
determined by the number and strength of the connections" (p. 67). Good, McCaslin and Reys (1992) also conceptualize understanding in mathematics in terms of a student's ability to make connections. In their view, "To achieve understanding, students must learn not only the individual elements in a network of related content but also the connections between them, so that they can explain them in their own words and can access and apply it appropriately to solve problems" (p. 128). Carey, Fennema, Carpenter, and Franke (1995) add that there are universals, irrespective of culture, regarding how people come to understand certain basic arithmetic ideas.

Pirie and Kieren (1989) depict understanding in mathematics as a growing process which is levelled and recursive. It is levelled in the sense that each level is not the same as the previous level, and recursive in that each level is not defined in terms of itself. The following is a summary of Pirie's and Kieren's (1989) theory of transcendent recursion.

Mathematical understanding can be characterized as levelled but non-linear. It is a recursive phenomenon and recursion is seen to occur when thinking moves between levels of sophistication.... Indeed each level of understanding is contained within succeeding levels. Any particular level is dependent on the forms and processes within and, further, is constrained by those without. (p. 8)

Gardner (1993) defines understanding as "having sufficient grasp of concepts, principles, or skills so that you can bring them to bear on new problems and situations" (p. 21). He notes that there are three kinds of understanding: intuitive understanding of the young child, understanding where the learner is only able to apply knowledge in a very explicit context, and genuine understanding which constitutes expertise. The following is a description of genuine understanding. "The capacity to use current knowledge, concepts, and skills to illuminate new problems or unanticipated issues" (Gardner and Boix-Mansilla, 1994, p. 200). It is suggested that "genuine understanding has been achieved if an individual proves able to apply knowledge in new situations, without applying such knowledge erroneously or inappropriately; and if he or she can do so spontaneously, without specific instruction to do so" (Gardner and Boix-Mansilla, 1994, p. 200). This performance perspective of understanding is at the core of an integrated curriculum project.
Gardner (1993) and his colleagues designed to promote understanding within and beyond the disciplines. Perkins and Blythe (1994) pointed out that from a "performance perspective" understanding was "a matter of being able to do a variety of thought-demanding things with a topic - like explaining, finding evidence and examples, generalizing, applying concepts, analogizing and representing the topic in a new way" (p. 5).

It is evident from the previous discussion that "What is understanding?" is not easy to answer. Views of the research reflect its varied and broad interpretation. Some models conceptualize understanding as an "all-or-nothing" dichotomy. Others use multiple categories. There are models of understanding which are based on theories of instruction. Others focus on theories of learning, which Byers (1980) believes are more plausible. Sometimes the concept of understanding is developed within the framework of a subject specific project. In other instances a more global view of understanding is promoted as in Project Zero (Gardner, 1993). The literature also reveals that understanding could also be classified according to students' ability to make cognitive connections. My view of understanding is congruent with the multiple categories theory which postulates that there are different levels (Byers, 1980) and degrees (Backhouse, 1978) of understanding. It is also consistent with Byer's (1980) suggestion that models which focus on theories of learning are more credible. Moreover, the idea of understanding as making cognitive connections (Hiebert and Carpenter, 1992) fits with my interpretation of what it means to understand not only in mathematics, but for learning generally. In the table that follows I provide a summary of previously discussed theories that define understanding according to categories. This table serves as a useful organizer for connecting views of the literature with a discussion of the two participants in Chapters Four, Five and Six. Now that I have focused on the concept of understanding and features which highlight its problematic nature, I move to a discussion of teaching for understanding.
Table 1  **Summary of the Perspectives on Understanding**

<table>
<thead>
<tr>
<th>Author</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holt (1964)</td>
<td>• Applies to teaching and learning.</td>
</tr>
<tr>
<td></td>
<td>• Provides seven point list for assessing student learning and planning instruction.</td>
</tr>
<tr>
<td>Van Engen (1953)</td>
<td>• Applies to learning.</td>
</tr>
<tr>
<td></td>
<td>• Defines understanding as a process of integrating and organizing concepts according to a set of criteria which are unidentified.</td>
</tr>
<tr>
<td></td>
<td>• Understanding is differentiated from meaning.</td>
</tr>
<tr>
<td>Skemp (1976)</td>
<td>• Applies to teaching and learning.</td>
</tr>
<tr>
<td></td>
<td>• Distinguishes two types of understanding.</td>
</tr>
<tr>
<td></td>
<td>• Instrumental understanding - The ability to apply an appropriate remembered rule to the solution of a problem without knowing why</td>
</tr>
<tr>
<td></td>
<td>the rule works.</td>
</tr>
<tr>
<td></td>
<td>• Relational Understanding - The ability to deduce rules or procedures from more general mathematical relationships - what to do and why.</td>
</tr>
<tr>
<td>Byers and Herscovics (1977)</td>
<td>• Applies to teaching and learning.</td>
</tr>
<tr>
<td></td>
<td>• Describe four levels of understanding.</td>
</tr>
<tr>
<td></td>
<td><strong>Instrumental Understanding</strong></td>
</tr>
<tr>
<td></td>
<td>The ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works.</td>
</tr>
<tr>
<td></td>
<td><strong>Relational Understanding</strong></td>
</tr>
<tr>
<td></td>
<td>The ability to deduce rules or procedures from more general mathematical relationships - what to do and why.</td>
</tr>
<tr>
<td></td>
<td><strong>Intuitive Understanding</strong></td>
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<td></td>
<td>The ability to solve a problem without prior analysis of the problem.</td>
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<td></td>
<td><strong>Formal Understanding</strong></td>
</tr>
<tr>
<td></td>
<td>The ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of</td>
</tr>
<tr>
<td></td>
<td>reasoning.</td>
</tr>
<tr>
<td>Davis (1978)</td>
<td>• Applies to teaching.</td>
</tr>
<tr>
<td></td>
<td>• Describes understanding as a process of organizing and integrating knowledge according to a set of &quot;moves&quot; teachers make in the</td>
</tr>
<tr>
<td></td>
<td>process of instruction.</td>
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<tr>
<td></td>
<td>• Moves are the logical things teachers do in teaching mathematics.</td>
</tr>
<tr>
<td></td>
<td>• Classifies understanding according to a taxonomy of concepts, generalizations, procedures, and number facts.</td>
</tr>
<tr>
<td>Haylock (1982)</td>
<td>• Applies to learning</td>
</tr>
<tr>
<td></td>
<td>• Understanding is the ability to make cognitive connections. The more connections the learner can make between the new experience</td>
</tr>
<tr>
<td></td>
<td>and the previous experience, the deeper the understanding.</td>
</tr>
<tr>
<td>Hiebert and Carpenter (1992)</td>
<td>• Applies to learning.</td>
</tr>
<tr>
<td></td>
<td>• Mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is</td>
</tr>
<tr>
<td></td>
<td>determined by the number and strength of the connections.</td>
</tr>
<tr>
<td>Good, McCaslin and Reys (1992)</td>
<td>• Applies to learning.</td>
</tr>
<tr>
<td></td>
<td>• Understanding is determined by a student's ability to make connections and apply connected ideas in one's own words to problem</td>
</tr>
<tr>
<td></td>
<td>solving.</td>
</tr>
<tr>
<td>Author</td>
<td>Model</td>
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<td>------------------------</td>
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</table>
* Describes understanding as a growing process that is levelled and recursive. It is *levelled* in the sense that each level is not the same as the previous one and *recursive* in that each level is not defined in terms of itself. |
* Defines understanding as having sufficient grasp of concepts, principles, or skills so that they can be applied to new situations or problems.  
* Describes 3 kinds of understanding:  
  a) Intuitive understanding of the young child who applies theories about the world promiscuously.  
  b) Context specific understanding of the student who can use knowledge in a very limited way.  
  c) Genuine understanding in which knowledge is applied correctly and spontaneously in new situations. |

**Teaching for Understanding**

In a discussion about what it means to teach for understanding, Alkin (1992) suggests that teachers need to do more than just present information. He points out that if true understanding is to be achieved, “teachers need to induce conceptual change in students, not simply infuse knowledge into a vacuum” (Alkin, 1992, p. 1383). In so doing, teachers need to insure that students’ misconceptions are corrected in order to prevent the distortion and fragmentation of new learning. Similarly, Anderson and Roth (1989) point out that understanding in science involves a process of conceptual change since students frequently enter science instruction with ways of understanding that are substantially different from true scientific understanding. In order to promote meaningful understanding, Anderson and Roth (1989) suggest that appropriate academic goals and well designed instruction are needed. They add that instruction should take the form of scaffolded dialogue and academic work in a community of learners. Good et al. (1992) also agree that teaching for understanding “involves inducing conceptual change in students” (p. 130). In their view, interactive discourse and thoughtful discussion between teacher and student and among students are important for students’ construction of meaning that will lead to clear understanding. They recommend that teachers allow sufficient time for this complex
process. Because understanding involves linking new knowledge to existing knowledge, teachers also need to pay attention to what students know at different stages of the learning process (Fennema, Carpenter, and Peterson, 1989). Good and Brophy (1991) emphasize that “depth” rather than “breadth” is important when teaching for understanding. From the perspective of teachers’ subject matter knowledge, Ball (1991a) suggests the following regarding what is needed to teach for understanding.

Teaching for understanding entails keeping wide range of considerations in mind: about the substance of the content, about the ways in which the nature and discourse of mathematics are represented, and about social and cultural aspects of mathematics. Teachers’ capacity and inclination to weave together these different considerations is a critical part of teaching mathematics for understanding, one that goes beyond simply knowing or being aware of certain things. (p. 81)

In an interim report for a project on educating for understanding, Gardner and Boix-Mansilla (1994) noted that teaching for understanding was grounded in four essential elements that interact dynamically with each other. The first element is essential questions or generative issues. According to Gardner and Boix-Mansilla (1994) these questions or issues need to be rich and engaging topics of interest for students and non students. The second element is the goals of understanding. Any course, lecture, or exercise needs to have one or more of these goals that specifies what it is that teachers want students to understand as a result of their participation. The third element is the performances of understanding which “specify just what students need to do in order to demonstrate that they have in fact achieved requisite understanding” (p. 213). The last element is ongoing assessments. Performances of understanding need to continuously assess students’ understanding, rather than being “one-shot” tests.

These views of the research on teaching for understanding that I have just presented are consistent with recommendations and perspectives of professional organizations such as The National Council of Teachers of Mathematics. They are also consistent with my views on what it means to teach for understanding. In guidelines such as The Curriculum and
Evaluation Standards for School Mathematics (NCTM, 1989) and The Professional Standards for Teaching Mathematics (NCTM, 1991), The National Council of Teachers of Mathematics encourages teaching for understanding and higher order applications. They advocate a kind of instruction that emphasizes "doing" rather than "knowing that", as I do. The guidelines also recommend that teachers provide opportunities for students to explore concepts and ideas, and to create knowledge in the course of active engagement with the discipline. Perspectives of the research also fit with policy reform efforts such as the Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve (CSDE, 1985) undertaken to affect change in mathematics instruction, which emphasizes not only the "how" but the "why" of mathematics (Peterson, 1990a). According to the vision of the Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve (CSDE, 1985) teaching for understanding emphasizes "the relationships among mathematical skills and concepts and leads students to approach mathematics with a common sense attitude, understanding not only how but also why skills are applied" (p. 12). The twelve guiding principles of the Mathematics Frameworks (CSDE, 1985) encourage teachers to facilitate and support students as they become active learners and construct their own understanding of mathematics.

The previous discussion has provided views of the literature on what it means to teach for understanding. These views, which are congruent with my interpretation of teaching for understanding, highlight the complexity and challenges for teaching in this way. It is evident that teaching for understanding is not a simple undertaking. As Gardner (1993) points out, "In practice, it's really quite difficult" (p. 21). In the following section I talk about factors that affect teaching for understanding. The discussion further illuminates its complex nature.
Factors that Affect Teaching for Understanding

It is suggested that teachers' beliefs and views about teaching and learning (Prawat, 1989), teachers' content knowledge about the subject domain (Prawat, 1989) and dimensions of context (Anderson and Roth, 1989; Cohen and Ball, 1990a, 1990b; Gardner, 1993; Good et al., 1992; Prawat, 1989; Schoenfeld, 1988) are factors that affect teachers' understanding of what it means to teach for understanding and their practice. Although research on the importance of resources such as knowledge, skills and beliefs that teachers bring to the learning (Heaton, 1992; Prawat, 1992; Putnam, 1992; Remillard, 1992), and my own extensive interactions with preservice and inservice teachers provide evidence that teachers' beliefs and knowledge (Peterson, Fennema and Carpenter, 1991; Thompson, 1992) are complexly intertwined, I have chosen to discuss each of these factors separately for analytical clarity.

Teachers' Beliefs and Views

According to Schoenfeld (1992), beliefs may be interpreted as "an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior..." (p. 358). He points out that teachers' beliefs about the nature of mathematics determine the environment of the classroom, which in turn shapes students' mathematical understandings. Thompson (1992) adds that, "teachers' beliefs appear to act as filters through which teachers interpret and ascribe meanings to their experiences as they interact with children and the subject matter" (p. 139). It is suggested that beliefs may have such a powerful influence on teachers' practice that teachers who possess the same conceptual understanding may teach differently as a result of their beliefs about teaching and learning (Prawat, 1989).

Current views of the reform movement, which are reflected in documents of the National Council of Teachers of Mathematics (NCTM, 1989, 1991), suggest that many teachers have a narrow view of mathematics which influences their teaching and students'
learning. According to Marshall (1992) teachers perceive learning mathematics as "the transmission of discrete elements from a static body of knowledge to passive students" (p. 2). Brown et al. (1990), and Romberg (1992) suggest that teachers do not appreciate the generative possibilities that the discipline offers, nor do they see the learning of mathematics as problematic. In fact, understanding of mathematics is tied only to students' success on tests or homework (Brown et al.). In many instances, learning is seen as "stamping in" (Marshall, 1992, p.8) correct responses. Teachers are frequently blind to the fact that mathematics is man made and is the product of human ability (Romberg, 1992). In my work with both preservice and inservice teachers I have found the above views about teachers and student teachers to be true. I recall the student teacher who disagreed with my suggestion that, as teachers, we need to provide opportunities for children to learn for understanding rather than for rote. She challenged me when I recommended that the concept of equivalent fractions be taught so students understand it as multiplying by a form of one because it would be more meaningful. The following were my reasons. Multiplying by a form of one connected to students' earlier learning that multiplying and dividing by a form of one resulted in the same number. Also, the concept of one is an important anchor in mathematics. Her response was, "If it was good enough for me to learn that you make equivalent fractions by multiplying the top by a number and doing the same to the bottom, and I came this far in school, then it's good enough for my students to learn it in that way too".

According to Brown et al. (1990), and Thompson (1986), teachers form some of their beliefs and conceptions about mathematics and its teaching even before they begin their teacher education programs. Lampert (1990) suggests that "Beliefs about how to do mathematics and what it means to know in school are acquired through years of watching, listening, and practicing" (p. 32). As learners, teachers have spent many hours in classrooms observing teaching practice, a phenomenon which Lortie (1975) describes as the "apprenticeship of observation." It is these early experiences that strongly influence how
teachers come to know in mathematics (Zeichner and Gore, 1990) and the beliefs that they develop about the subject matter. Schoenfeld (1989) points out that research in mathematics education assumes that the development of beliefs about mathematics is strongly influenced by the cultural setting of the classroom. According to Feinman-Nemser and Buchmann (1986), classroom experiences may not be the best teacher, but they are an extremely influential teacher. The problem then becomes that “teacher beliefs tend to come home to roost in successive generations of teachers, in what for the most part may be a vicious pedagogical/epistemological circle” (Schoenfeld, 1992, p. 360). The repeating cycle is not easy to change. Also, the long standing tradition of classroom mathematics does not necessarily fit with the complex nature of teaching for understanding, which requires a more conceptually oriented view and a broader based vision of mathematics.

**Teachers' Knowledge**

The role played by subject matter knowledge in teaching is complex (Ball, 1991b; Wineburg and Wilson, 1991). As Fennema and Franke (1992) point out, “there is no consensus on what critical knowledge is necessary to ensure that students learn mathematics” (p. 147). However, studies conducted with teachers in different subject areas (Ball, 1991b; Hashweh, 1986; Wineburg and Wilson, 1991) support the view that teachers who possess a rich, integrated knowledge of the subject can influence instruction in a positive way (Fennema and Franke, 1992). Hashweh (1986) suggests that teachers who are more knowledgeable about a particular subject are able to weave together their understandings of different aspects of the discipline. They are also able to recognize and deal with students’ misconceptions. According to Prawat (1989), teachers' knowledge of content and the depth of understanding they are able to promote in their students are clearly related. From the perspective of Fennema and Franke (1992), "what a teacher knows is
one of the most important influences on what is done in classrooms and ultimately on what
students learn" (p. 147).

Ball (1991a) argues that teachers' subject knowledge “matters” to instruction and
students’ learning. In a discussion regarding the connection between teachers’ knowledge
of mathematics and concerns for improving its teaching and learning, Ball outlines the kind
of subject matter knowledge that is needed in order to teach for understanding. In so doing
Ball talks about four dimensions of subject matter understanding that she believes teachers
need to have: knowledge of the substance of mathematics, knowledge about the nature and
discourse of mathematics, knowledge about mathematics in culture and society, and the
capacity for pedagogical reasoning. Ball further delineates the kind of knowledge that
teachers need to have. They need to know such things as, our number system is based on
tens and division by zero is indefinable. They need to be cognizant of the knowledge
students need to have at different grade levels. Also, they need to have an understanding of
the mathematical meanings underlying concepts and procedures- to know “why” they are
doing what they do. Finally, teachers need to understand that mathematics consists of a
network of connections, rather than a series of disparate ideas and procedures.

In Ball’s (1991a) view, teachers who have a sense of the complex nature and
discourse of mathematics know what counts as an answer in mathematics, and what counts
as doing mathematics. They are also aware that mathematical knowledge is based on
convention and logic and are able to distinguish the difference between the two ideas.
Moreover, teachers who possess knowledge of the role played by mathematics in culture
and society are able to connect mathematics to real life in meaningful ways. They also
possess the ability to develop students’ awareness of the evolution of mathematical ideas
across history and in different cultures. Along with the aforementioned subject matter
knowledge, teachers also need to have “a rich repertoire of representations” (Ball, 1991a,
p. 80) in order to teach from a more conceptually oriented view.
Closely related to content knowledge is a different kind of knowledge which allows teachers to transform what they know into "something meaningful for students" (Prawat, 1989). This special knowledge, which Shulman (1986) defines as pedagogical content knowledge, is needed to teach for understanding (McLauglin and Talbert, 1993). According to Shulman's (1986) conception, pedagogical content knowledge includes:

- the most useful forms of representation of those [content] ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-
- in a word, the ways of representing and formulating the subject to make it comprehensible to others.... an understanding of what makes the learning of specific topics easy or difficult, the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons.

(p. 9)

Fennema, Carpenter, and Peterson (1989) point out that teachers' pedagogical content knowledge is a critical component in children's learning of mathematics with understanding. Teachers need this kind of knowledge in order to make informed decisions for effective instruction in mathematics (Borko et al., 1992).

The previous two sections have dealt with teachers' beliefs and teachers' knowledge, two concepts which frame this study. As suggested, these concepts are factors that affect what teachers understand about teaching for understanding and their practice. In the section that follows, I discuss the third factor which affects teaching for understanding, the context in which teachers create their practice.

The Context in Which Teachers Create Their Practice

Because teachers' cognitions do not exist in a vacuum, the influence of context on teachers' thinking and practice is important in unpacking the idea of teaching for understanding. Talbert and McLaughlin (1993) point out that context has the capacity to significantly shape practice. A survey of the literature on context effects on educational
outcomes and their own research lead Talbert and McLaughlin (1993) to form a view of teaching which is permeated by the multi layers of context. According to Talbert and McLaughlin (1993), these layers of context range from students in a classroom, subject matter, colleagues, school administrators and school organization to broader environments beyond school systems such as parents, the community, and institutions such as government.

From an alternative perspective Gardner (1993) talks about obstacles that stand between schools and educating for understanding. In a discussion on educating for understanding he identifies the following as obstacles, the test-text context phenomenon, the correct-answer phenomenon, pressure for coverage, cognitive Freudianism, institutional constraints, and disciplinary constraints. Gardner (1993) notes that pressure for coverage is the greatest enemy of understanding. Good et al. (1992) point out that few teachers have been trained to teach for understanding. Most teachers are limited by too little time for exploring instructional models, curriculum, and ways for assessing student performance that are new. Good et al. suggest that there are three factors which inhibit curriculum reform: external pressures for simple measures to determine students' success, schools that discourage teachers from making professional decisions, and limited knowledge about how to structure classroom settings that foster mathematical understanding.

According to Anderson and Roth (1989) research in science, which in my view easily applies to mathematics, reveals that students themselves are obstacles to teaching for understanding as a result of the "task environments" to which they have become conditioned in science classrooms. Anderson and Roth (1989) add that textbooks and other materials, management and curricular demands are issues of context that mitigate against the development of classroom environments that contribute to the kind of instruction that promotes understanding. These findings are also consistent with Newman (1988) who reported that external pressures on social studies teachers to cover prescribed amounts of curriculum, as well as tests, district guidelines, and textbooks were major obstacles to
teaching for understanding. My experiences in the area of mathematics also fit with these findings. Many of the teachers with whom I have worked as a consultant and teacher educator often expressed the urgency they felt to cover the long list of curriculum outcomes in order to meet school board expectations. They also talked about the need to ensure students' success on board and provincial standardized testing in mathematics.

There are studies within the literature that illustrate how the interplay of context and classroom environments affect teachers' practice. In an experimental study with a second grade teacher, Wood, Cobb, and Yackel (1990) concluded that she was able to change her practice in mathematics to a way of teaching that promoted meaning and understanding in students' learning because of the ongoing support that she received from the researchers. However, in reading, where she received no support from the researchers and was constrained in instruction by her obligations to administrators, she continued to maintain her usual approach of moving children through the instructional materials at a set pace to prepare them for mastery skill tests. Wood et al. also noted the following:

"schools in which the teachers' goal is to direct students through instructional activities that emphasize discrete skills at a pace that will ensure complete coverage of the curriculum and then to evaluate students' ability to provide correct answers... mitigate against learning that involves constructing mathematical relationships and structures. (p. 510)"

In some environments teachers are compelled to deal with conflicting messages about classroom instruction. Eisenhardt et al. (1993) noted in a study conducted with preservice teachers that practicing teachers in the placement schools were presented with conflicting messages by central administrators. At the same time that these administrators expressed a commitment to teaching for conceptual knowledge, they mandated implementation of a curriculum and testing programs that emphasized procedural knowledge. Eisenhardt et al. add that in some instances school level administrators conveyed a comparable message to practicing teachers. In my work with classroom teachers I have found the same to hold true. On the one hand, central administrators expect
teachers to instruct from a more conceptually based perspective. On the other hand, teachers are expected to follow curriculum guidelines which divide mathematics into a set of learning expectations to be covered in a prescribed fashion within a limited time frame. For example, teachers must ensure that they cover expectations from each of the five strands of Number Sense and Numeration, Measurement, Geometry and Spatial Sense, Patterning and Algebra, and Data Management and Probability monthly. Failure to address expectations from each of these strands in their teaching becomes problematic for a teacher. Formal reporting is set up so that teachers must address students' progress in each of the strands.

Board administered tests also include items from these five different areas.

There are instances in the literature in which the reverse occurs, and teachers influence broad policy that is intended to bring about change in practice to a way of teaching that promotes understanding. They inhibit or modify the goals of teaching for understanding to suit their preexisting practice, beliefs, and knowledge. Studies (Cohen and Ball 1990a, 1990b) which looked at California's state wide effort, The Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve (CSDE, 1985), to change teachers' mathematical practice provide insight into the problem. Teachers were equipped with curriculum materials and revised textbooks designed to promote understanding in children's learning. However, in many instances they modified the way that they used the materials. For example, some teachers implemented an element of the new curriculum in their lessons, but the content was organized within the existing structure of traditional school mathematics. Others embraced the use of visuals and concrete materials provided by the new policy, but their use was filtered through the teachers' established practice.

The previous discussion has focused on factors that affect teachers' understanding of what it means to teaching for understanding and their practice. In this last section of the chapter I now shift to a discussion of studies in the literature that are closely related to the questions that this inquiry seeks to understand.
Related Studies

This last section looks at studies that are closely related to this study. These studies are important as they provide an important context for situating this study as part of the larger phenomena in the research that looks at teaching for understanding. The studies highlight the complex nature of teaching for understanding and the challenges it presents for teachers. They also provide evidence that a paucity of comprehensive case studies that focus exclusively on teachers’ understanding of what it means to teach for understanding in mathematics exists in the literature. In the following section I examine these studies.

A number of case studies of second and fifth grade teachers’ experiences (Prawat, Remillard, Putnam, and Heaton, 1992) were developed within the framework of The California Study of Elementary Mathematics (Peterson, 1990a) to examine the effects of policy reform on classroom teaching and learning, and to gain an understanding of their perspectives on teaching mathematics for understanding. These case studies demonstrate that teachers generally believe that they teach for understanding. Yet, their views of understanding are often apprehended “through lenses of older policies” that stress didactic teaching, memorizing procedures, learning facts and recitation (Cohen and Ball, 1990a, 1990b). The studies also illustrate that teachers’ perceptions of what it means to teach for understanding vary and their views may not necessarily fit with researchers’ and the vision of curriculum developers (Prawat et al.). The following is a review of eight cases discussed in the literature that are a subset of the larger body of cases developed within The California Study of Elementary Mathematics (Peterson, 1990a).

The case of Carol Turner illustrates how one second grade teacher “who was disposed to teach in ways that seemed consistent with the California Mathematics Curriculum Frameworks” (Ball, 1990, p.263), interpreted teaching for understanding in mathematics. According to Ball, Carol believed that she taught mathematics for
understanding. However, Ball raises the question if this was the case. As Ball points out, Carol viewed mathematics as a set of tools with which she needed to equip her students. In the following Ball describes Carol’s practice.

Carol’s Practice: “Teaching Without Understanding Is Not Teaching”

Carol demands that her students understand. She makes sure that they attend and she believes that the mark of a successful teaching moment is that “every eye was on [me].” Syllogistically, she reasons: When the children are looking at her, they are probably engaged, then they will understand. They will understand because she carefully structures their activities so that they will develop correct understandings: giving them templates for explaining procedures, guiding their work with manipulatives, leading them with questions. (p.264)

The case of Mark Black (Wilson, 1990) provides an alternate perspective on what it means to teach for understanding. Wilson notes that this fifth grade teacher talked about different types of understanding. Although Mark emphasized the acquisition and mastery of algorithmic knowledge, he did suggest that there was a distinction between understanding for which he aimed and understanding that was required to know “why” an algorithm worked. Wilson suggests he possessed a “building block” notion of understanding, devoid of intuition, where concepts rested on a foundation of mathematical rules and procedures.

As with the previous studies, cases developed by Cohen (1990), Jennings Wiemers (1990), and Peterson (1990) provide evidence that teachers vary in their perceptions of what it means to teach for understanding. In addition, their perceptions may not necessarily fit with those of researchers and curriculum developers. According to Cohen, Mrs. Oublier, the teacher in the case study on which he reported, believed that changes she had instituted in her teaching practice promoted understanding in students’ learning. Cohen suggests that this teacher was “teaching math for understanding”, but the question was how well her students understood. In Cohen’s view, her teaching reflected a lack of in-depth knowledge of mathematical concepts necessary for probing students’ answers, and for initiating the
kind of classroom discourse that leads to deep understanding. In fact, he believed that she conducted classes in ways that discouraged exploration of students’ understanding.

The case of Joe Scott (Jennings Wiemer, 1990) illustrates yet another view of what it means to teach for understanding. According to Jennings Wiemer, Joe Scott viewed mathematics as a fixed body of procedures that students needed to learn. The procedures were intended as tools for students “to pack into their mathematical tool chests”. For Joe, teaching for understanding meant insuring that students understood how to apply traditional school mathematics learning to real life situations. Jennings Wiemer points out that teaching in this way satisfied his “traditional approach to implementing state-wide mathematics reform” (p. 297).

Cathy Swift, a grade two teacher that Peterson (1990) observed, is a case in contrast with the other teachers previously discussed. According to Peterson,

Cathy Swift is a teacher who stands out because she readily admits that she doesn’t know if she is teaching for understanding and she is unsure of how to teach children to solve mathematical problems and how to think. (p. 295)

Cathy herself admitted that she felt less knowledgeable and confident teaching mathematics than she did teaching literature. In her view mathematics was less “creative” and open-ended than literature. Peterson points out that Cathy viewed mathematics as a fixed body of content to be covered, and it was “a lot more sequential than literature”. Cathy felt she had the freedom to teach literature in the way that she wanted, a feeling which mathematics did not afford her. However, she did point out that she was not sure if she would have changed the way that she taught mathematics, even if she felt that she had the license to do so.

One of the most important issues to surface in case studies on which Putnam, Heaton, Prawat, and Remillard (1992) reported as part of The California Study of Elementary Mathematics (Peterson, 1990a) was the complex interrelationships between teachers’ beliefs and knowledge and their mathematical practice. Putnam et al. point out
that the kind of teaching advocated by The Mathematics Frameworks (CSDE, 1985) and The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) relies heavily on teachers’ beliefs and knowledge. The case of Valerie Taft, a fifth grade teacher (Putnam, 1992) reveals how a teacher’s beliefs and knowledge can shape the way that she teaches mathematics. According to Putnam, Valerie viewed mathematics “as a set of useful procedures” whose ultimate importance was determined by how effectively they could be applied to everyday life. Because she believed it was more important to teach students the “how” of mathematics, rather than the “why”, Putnam points out that her teaching focused on completing the steps for procedures. In addition, her lack of subject matter knowledge prevented her from realizing that she had treated the content incorrectly. Putnam is quick to point out that The Mathematics Frameworks (CSDE, 1985) recommends that, in order to teach for understanding, teachers require richly developed content knowledge and an approach that allows for exploration and sense making.

Although subject matter knowledge is not the sole factor in teaching for understanding, it is critical (Heaton, 1992). The case of Sandra Stein raises questions about the role of subject matter knowledge in teaching for understanding. It also illustrates what happens when a teacher who has the best of intentions tries to teach a concept which she does not understand (Heaton, 1992). According to Heaton, Sandra believed in the importance of making mathematics fun and engaging for her students. Heaton suggests that it seemed as if Sandra had a clear sense of how children could be helped to learn. To achieve her goals and to make mathematics meaningful, Sandra relied less on the textbook and focused her efforts on involving students in practical problems with a real life context. Yet when Heaton observed Sandra teach, she demonstrated a limited understanding of the mathematics concepts that she taught. She misrepresented the content and failed to attend to important mathematical ideas that were at the heart of activities that she was developing in her lessons. Heaton explains that in the two lessons that Sandra taught “her efforts to teach for understanding may have lead to misunderstanding” (p. 160).
The case of Karen Hill (Prawat, 1992) demonstrates that teachers may make important changes in their views about teaching mathematics and begin to appreciate the merits of an approach that promotes meaning and understanding. However, these changes may not necessarily affect their practice. Prawat suggests that this teacher’s views about the new curriculum had changed “a bit” in the three month period between the first and second times that she interviewed her. Yet, she continued to express very traditional ideas about mathematics which are reflected in the following statement.

Mathematics learning proceeds hierarchically, with children first needing to master the “basics”, certain math facts and procedures learned in rote fashion, before getting into problem solving and other application sorts of activities. (p. 199)

The teacher also explained that she always provided students with reasons for the rules and procedures she taught, and she would illustrate their real world application. In this sense the teacher believed that she was providing students with understanding. Yet Prawat indicates that her teaching focused on the procedural, which was reflected in the question and answer format she regularly employed. She also made limited use of concrete representations, because in her view they “added” to understanding rather than were “at the heart” of it.

In addition to demonstrating the effect of teachers’ beliefs and knowledge on their practice, the case of Jim Green (Remillard, 1992), albeit in a small way, addresses the influence of context on teachers’ practice. This case is different from the previous three cases (Heaton, 1992; Prawat, 1992; Putnam, 1992) in that the teacher appeared to understand mathematical concepts and was comfortable teaching them. As Remillard suggests, the teacher seemed to have flexible and detailed knowledge about the mathematical concepts that he taught and was able to articulate them. For Jim, teaching for understanding meant making mathematics applicable to daily life. Yet, at the same time that he wanted his students to realize the relevance of mathematics to their lives, it was important for him to
teach rules and formulas. He believed that through procedural learning children would automatically develop conceptual understanding, as teaching rules and procedures went "hand in hand" with teaching for understanding. The following description of Jim's views provides added insight.

Jim stands behind the Framework's proposition that children should understand and be able to apply the underlying concepts of mathematical rules and procedures, and he believes that he teaches concepts to his students. This stance does not belittle the importance of mastering rules and procedures. Jim most certainly believes his students should become proficient with the mechanical elements of mathematics, but he also expects that they will understand why these procedures work. (p. 181)

Based on her observations of Jim's teaching, Remillard (1992) points out that the integration of the conceptual aspects of mathematics into his procedural approach to lessons was somewhat sporadic. Remillard also notes that in looking at the case of Jim Green it is important to consider the effects of context on his teaching. On the one hand the state of California had mandated through its policy initiatives that teachers change the way that they teach mathematics to a more conceptually oriented approach. Yet, the school district in which he taught expected teachers to follow a carefully paced mastery learning of basic skills program in order to address the problem of low standardized test results of a large minority population. This mastery learning program was structured so that teachers would teach the skill, practice it, apply it, test for mastery. For students who did not achieve the required level of mastery on the first test, the teacher would reteach and retest. According to Remillard, "Such an approach seemed at odds with the Framework's focus on understanding and on the development of skills including computational mastery within meaningful contexts" (p. 192).

Putnam et al. (1992) provide the following summary regarding the cases discussed above.

The cases suggest clearly that teaching mathematics for understanding cannot be reduced to using the right textbook, having students work in groups, using manipulatives, or using mathematics activities in real-worlds
settings. All these may serve as important resources or tools for good mathematics teaching... It is how teachers use these resources or tools, however, that shapes the learning environment for students. (p. 214)

In their view, teaching that promotes meaningful learning should not simply ban telling and explaining to the exclusion of allowing students to construct knowledge. Teachers need to make responsible decisions as to how to effectively integrate these ideas.

As suggested earlier, these cases are closely related to this study on teaching for understanding in that they provide a necessary guide for understanding what elementary teachers understand about teaching for understanding. Similar to this study, these studies examine teachers’ beliefs and knowledge and the effect on their practice, but they do so in the context of changes in instruction mandated by the Mathematics Frameworks (CSDE, 1985). The present study was designed to inquire exclusively into the phenomenon of teaching for understanding in mathematics. Although brief mention is made of the influence of context in the case of Jim Green (Remillard, 1992), the cases reviewed previously do not attend to factors of context that mitigate and support teaching for understanding, as this study does. Furthermore, the cases do not provide complete portraits or analyses of teachers’ practice. As Prawat et al. (1992) point out:

We did not spend enough time with the teachers to develop comprehensive cases of them as teachers. The stories of these teachers, however, do illustrate issues that we saw as important for other teachers we visited. We also believe that the cases are useful, more generally, for any teachers making changes in their teaching practice. (p. 151)

Researchers observed and interviewed teachers only twice in four months. Each teacher was observed teaching two thirty to ninety minute mathematics lessons during each week that a researcher visited a school. Interviews, which lasted for a duration of approximately three hours, were intended to inquire into teachers’ goals for mathematics teaching, how they viewed teaching that subject for understanding, how they assessed student learning, and other factors which influenced their teaching of mathematics (Prawat et al., 1992). In this
study I spent four months, almost daily, observing each teacher in the classroom so that I might collect copious data regarding their views, beliefs and content knowledge on teaching for understanding. Extensive interviewing was carried out. Formal interviews were conducted three times a month, while informal interviews were conducted after most lessons to gain further insight into teachers' thinking and to better understand their actions and behaviors.

Ideas and practices for teaching both procedural and conceptual knowledge (Hiebert and Lefevre, 1986) are two interrelated aspects of learning to teach for understanding (Eisenhardt et al., 1993). These terms, procedural and conceptual, are used to denote a distinction made between two forms of mathematical knowing (Hiebert and Lefevre, 1986). According to Eisenhardt et al.,

Procedural knowledge refers to mastery of computational skills and knowledge of procedures for identifying mathematical components, algorithms and definitions... Conceptual knowledge refers to knowledge of the underlying structure of mathematics— the relationships and interconnections of ideas that explain and give meaning to mathematical procedures. (p. 9)

A two year study in which Eisenhardt et al. (1993) examined novice teachers' knowledge, beliefs, thinking, and actions regarding the teaching of mathematics revealed some of the tensions and pressures that these teachers experienced as they attempted to move beyond teaching for procedural knowledge and teach for conceptual knowledge. The study of Ms. Daniels (Eisenhardt et al.) is one case in point which reports on student teachers' views and beliefs and their efforts to teach for procedural and conceptual knowledge. This study shows that the teacher knew the difference between the two kinds of knowledge and believed that both were necessary for understanding in mathematics. Yet, she had difficulty articulating her ideas about how to teach for conceptual understanding. Observations of her teaching indicated that the value she placed on learning algorithms and procedural skills until they were well entrenched in students' minds was evident in a
majority of her lessons. They also illustrated her less frequent and unsuccessful attempts to teach for conceptual understanding. Eisenhardt et al. point out that limitations in her knowledge base led, on some occasions, to her focusing exclusively on procedural knowledge. In addition, there were other factors such as the type of lesson she was expected to teach, the pressure of coverage and time, and the difference in expectations of the university course work and placement schools that needed to be taken into account, in looking at the tension between her teaching for procedural and conceptual knowledge. Eisenhardt et al. further note that there were few opportunities for Ms. Daniels, as with other novice teachers, to observe teaching for conceptual knowledge in their host schools, or to receive encouragement for teaching in this way.

Although Eisenhardt et al. (1993) inquired into teachers' understanding of teaching for understanding, as the present study does, the authors were particularly interested in participants' knowledge of mathematical concepts and their understanding of the connections among concepts and procedures. The present study goes beyond and takes an in-depth look at teachers' beliefs and perceptions of teaching for understanding in relation to their practice in mathematics. It also considers more than the process of learning to teach for conceptual knowledge in mathematics. Unlike the teachers in Eisenhardt’s study, who were preservice teachers in their final year of a teacher education program, the two participants in this study were both practicing teachers.

An extensive study on teaching for understanding which Roulet (1998) conducted demonstrates that teachers who choose to teach from a conceptually based perspective may be confronted with obstacles which mitigate against their attempts to teach in this way. In the study, Roulet (1998) examined each of two high school teachers' conceptions and practice in mathematics over a four month period in order to look at how their images of subject translated into practice. Roulet (1998) points out that, even though a majority of high school teachers are unwilling to endorse instructional strategies recommended by The Professional Standards for Teaching Mathematics (NCTM, 1991), these two teachers were
working to implement mathematics education reform in their classroom practice. One teacher "with a well developed social constructivist image of mathematics" (Roulet, 1998, p.iii) managed to put his personal philosophy into practice in spite of the opposition encountered from students, their parents, and the administration. However, the second teacher, "with a more mixed subject conception" that was in the process of transition and reconstruction turned to traditional transmissive modes of instruction when confronted with the forces of opposition.

There are a number of similarities between the present study and Roulet’s (1998). Like this study, Roulet’s study examines teachers’ beliefs and knowledge and their intimate connection with teaching for understanding. It too considers obstacles which confront teachers in their efforts to teach for understanding. In so doing, Roulet also develops in-depth portraits of two teachers’ practice in mathematics. Unlike this study, which inquires into elementary teachers’ understanding of teaching for understanding, Roulet’s study focuses on two high school teachers’ mathematical practice and their efforts to implement mathematics education reform in their classroom.

In summary, this review of the literature has provided an in-depth view of the research on understanding and teaching for understanding, key concepts that underpin this study. It has also provided a context for this study and demonstrated why a study such as this is needed to help fill the gap that exists in the literature. Evidence suggests that teaching for understanding is complex and open to interpretation as this study demonstrates. Most teachers believe that they teach for understanding as Gardner (1993) readily points out, “Nearly every teacher I know -myself included- would claim to be teaching for understanding” (p. 21). The problem is that teachers, as this study will demonstrate, have different understandings of what it means to teach in this way (Cohen and Ball, 1990a, 1990b; Gardner, 1993; Prawat, 1989). To conduct research into a phenomenon as complex and challenging as teaching for understanding warrants an approach which is able to capture the subtleties of its multi-dimensional nature, and to weave together the network of evidence
regarding how teachers' beliefs, views and content knowledge affect this type of teaching. In the chapter that follows I discuss the research approach selected for the study and aspects of methodology.
Chapter 3  Method

In the previous chapter I examined the research on teaching for understanding and factors that affect that understanding. To inquire into a phenomenon as complex and challenging as teaching for understanding warrants an approach that is able to capture its multidimensional nature and the subtleties of teachers' understanding. This chapter discusses the research approach I selected for the study and provides a rationale for my choice. Methods for selection of participants and the setting, role of the researcher, data collection and data analysis procedures, and measures that were taken to ensure trustworthiness of the research are also addressed.

The Research Approach - A Rational

Morgan and Smircich (1980) suggest that the appropriateness of the research methodology is contingent on the nature of the phenomenon to be studied. As demonstrated, teaching for understanding is complex and not easily characterized by a single category or idea. Most teachers believe that they teach for understanding and their views may not necessarily be congruent with those of the research and professional community. Teachers' understanding is both constrained and supported by their beliefs and content knowledge, which themselves are complex concepts. Moreover, teachers do not carry out their practice in a vacuum and are affected by the multi-layered context in which they teach. To capture the complexities of teachers' understanding and the intricacies of their beliefs and content knowledge of mathematics, an approach with the openness of interpretive inquiry seemed appropriate.

An approach of this nature allows researchers to carry out inquiry in the natural setting of participants' lived experiences. In this way researchers may come to understand how participants make sense of their world and their experiences in that world (Glesne and Peshkin, 1992; Merriam, 1998). As Erickson (1986) suggests, interpretive methods are
useful when one needs to know more about the meaning-perspectives of participants in particular events. Interpretive research also defines the nature of data collection and data analysis procedures (Merriam, 1988) and provides for the thick description that Denzin (1994) suggests, “gives the context of an experience, states the intentions and meanings, and reveals the experience as a process” (p. 505).

Frequent use of interpretive methodologies is evident in current research on teacher education (Cooney, 1994). This research reveals a growing interest in studies on teaching for understanding (Alkin, 1992) and the use of case study as a common way of describing teachers’ meaning-making processes in mathematics education (Cooney, 1994). In this inquiry case study allows for the in-depth examination of how teachers make sense of the world and their experiences in that world. Each case in its particularity and complexity “tells a story” (Stake, 1994) of teaching for understanding in mathematics. At the same time that the case focuses on a single actor, it is also part of a larger phenomenon in the research that looks at teaching for understanding in mathematics. As Denzin and Lincoln (1994) suggest, every instance of a case bears the stamp of the general class of phenomena to which it belongs. The detailed description and in-depth view in each case is intended to inform the reader so that personal experience may be connected with that of the participant. As Merriam (1998) suggests, it enables readers to compare the “fit” with their own situations.

Case study is not a methodological choice for this inquiry, rather a choice of object to be studied (Stake, 1994). It is defined by the nature of the final report. Consistent with this view, Merriam (1998) points out that case studies do not claim any particular methods for data collection and data analysis. Therefore, techniques characteristic of the interpretive approach selected for the study are employed for gathering and analyzing data in each case. These techniques are discussed in the following section on methods for selection of participants and the setting, role of the researcher, data collection and data analysis procedures, and measures that were taken to ensure trustworthiness of the research.
Selection of Participants

As noted earlier, in any social setting researchers need to gain access to the varied perspectives of participants so that they come to understand their actions. According to Erickson (1986) gaining valid insights into participants’ point of view is important to the success of the research enterprise. Access rests in part on participants selected for the study and the trust and rapport a researcher is able to establish with them. Purposeful sampling (Bogdan and Biklen, 1992, p. 71) and attention to Morse’s (1994) criteria for a good participant guided the selection of participants for this study. According to Morse (1994), a good participant has the knowledge and experience the researcher requires, has the ability to reflect, is articulate, has the time to be interviewed, and is willing to participate in the study.

Of the many teachers with whom I had worked as a consultant and teacher educator, I selected two that stood out for me, as they fit Morse’s (1994) criteria for a “good informant”. Regular conversations revealed their willingness to talk openly about their practice. They also expressed concern about making mathematics meaningful for students. My invitation to participate in this study was greeted with enthusiasm. Yet, one of my choices resulted in a false start. Despite my earlier work as a consultant with this teacher and others on her team, she confessed after three weeks in her classroom that my presence as a researcher caused her considerable discomfort. The research was aborted, and I continued with the second teacher as I cautiously sought out and interviewed other candidates until an appropriate replacement was found.

My deliberate effort to select two teachers with different life histories and professional experience resulted in the participation of Eric and Melany, who are at the center of this study. Eric, one of the teachers with whom I had initially worked, was in his third year of teaching. At the time of this study he was teaching grade seven in a small urban school in a working class neighborhood. Eric often talked about making quick
strides in order to become an administrator. Already, he was looking for a new teaching assignment that would afford him increased responsibility in his quest to achieve his goal. Melany was an experienced pedagogue who was recognized by her peers for her teaching expertise. She had taught for twenty years in a variety of settings. At the time of this study she was teaching grade five in the same urban school that she had taught for nine years. The population of the school was predominantly ethnic and working class. Based on my previous acquaintance with Melany and Eric, it was my sense that they would be good participants (Morse, 1994).

In the year prior to this study I had worked on a regular basis, almost monthly, with Eric and other teachers in his school, both in workshops and in their classrooms. As I noted above, Eric was easily able to converse about a variety of educational issues. He frequently talked about the need to provide students with experiences that were grounded in real life learning and his role in doing so. He also suggested that he would be interested in participating in research, as this was “something” he had never done before. In his view being part of a research project would be a valuable learning experience. Also, it would add to his professional repertoire. His comfort level with my presence in his classroom was an incentive.

Like Eric, Melany had a facility with language and could easily express her thoughts and ideas. When Melany talked about issues of teaching, she saw them as more problematic in their complexity than Eric. Her participation in this study was more about making change in her mathematics program than it was about having another experience to add to her professional repertoire. For the past five years Melany had been working on making mathematics more like her language arts program, which had also undergone major changes. Melany viewed her involvement as an opportunity to gain new ideas in mathematics and said that I, as the “math expert”, would be a good resource. The departure of a colleague who had provided Melany with fresh ideas for teaching
mathematics had created a void for her. In addition, I would serve as a positive role model, particularly for her female students.

My previous relationship with both Melany and Eric minimized the challenge that researchers often experience in their effort to gain access and connect with potential participants in the field (Hammersley and Atkinson, 1992). Our relationship also facilitated the development of rapport and trust building (Erickson, 1986). The credibility I had established in Eric's school from my earlier work with his peers was beneficial. It reduced the challenge of initial rapport building with the main gatekeepers. Not only was the principal supportive, he thought it was a good idea for Eric to participate. Yet, at the same time that familiarity allows for ease of access and rapport building, suspending one's preconceptions and maintaining marginality may be more challenging (Hammersley and Atkinson, 1992). As Powdemaker (1966) suggests, a researcher needs to be intellectually poised between "familiarity" and "strangeness" so as not to lose one's objectivity (Glesne and Peshkin, 1992). Regular reflection and discussion with individuals outside this study assisted in maintaining a balanced analytical perspective.

Melany offered a different situation. Although we had known each other prior to the research endeavor, I had never been to her school. Rapport building with the main gatekeepers began with a meeting with her principal to talk about the study and to provide him with a written description. The early stages of fieldwork in the school were particularly focused on gaining access and building trust. I took time to engage in social interaction for "social and pragmatic reasons" (Hammersley and Atkinson, 1992). At the same time, I also made certain to maintain marginality and to stay committed to my purpose for being there. Basic ethical principles and measures to protect participants were also ensured. At the outset of my work, each participant was informed of the purposes and the activities of the research both orally and in writing. They were also made aware of the option to withdraw at any time. The additional burden that participating in this study would involve
was outlined. Ethical reviews were completed for and approved by the research committee of the particular boards of education.

Role of the Researcher

Hammersley and Atkinson (1992) suggest, “Rather than engaging in futile attempts to eliminate the effects of the researcher, we should set about understanding them” (p. 17). In any interpretive study there are varying degrees of participation (Glesne and Peshkin, 1992; Hammersley and Atkinson, 1992). As researcher, I was neither complete observer (Hammersley and Atkinson, 1992), nor complete participant (Hammersley and Atkinson, 1992). My role fluctuated between participation and observation. In Eric’s classroom, most days I sat at the back observing the pattern of his teaching and events of the day. Even though I had offered to be an “extra pair of hands” in return for allowing me to come into his classroom to conduct research, I tended to be more of an observer than a participant. It was only toward the end of my stay that Eric provided me with an opportunity to teach a unit on integers. He wanted me “to experience” what it meant to teach mathematics when little time was available for anything other than teaching to the prescribed outcomes of the curriculum. This opportunity allowed me “to pay back” Eric for the research I conducted in his classroom.

The situation was different for Melany. As I suggested earlier, Melany agreed to allow me to conduct research in her classroom in exchange for new ideas for her mathematics program. As Glesne and Peshkin (1992) suggest, researchers need to reciprocate in return for a teacher’s consent to spend numerous hours at the back of their classroom or in interview sessions. As with Eric, I would sit most days observing in her classroom as she taught. However, there were times when I was more involved with Melany in my role as researcher. At her request, I provided input into her mathematics program and helped her plan activities for the units she taught on area and structures. In some instances in conducting research it may appear that “overparticipation” (Bogdan and
Biklen, 1992, p. 88) has occurred. However, “As the task of field work is to become more and more reflectively aware of the frames of interpretation of those we observe” (Erickson, 1986, p. 140), this opportunity fit within “the participant/observer continuum” (Bogdan and Biklen, 1992, p. 88), and the goals of this study.

The Setting

Researchers engage in observational fieldwork because they are concerned with context and its effect on the actions of participants (Bogdan and Biklen, 1992). To do justice to the phenomenon under study I spent almost every day of a four month period observing in each teacher’s classroom. I would also go with the teacher and their students to activities outside the classroom. I would go with Eric to the gym when he taught physical education and to the shop for design and technology. With Melany I went to the library and class trips outside the school. The majority of my days began before students arrived for class in the morning and lasted till well after dismissal time at the end of the day.

Although the focus of my observation was on the teaching of mathematics, I wanted to observe other areas of the curriculum. My intention was to examine and compare each teacher’s practice in mathematics with other areas that they taught. I also wanted to vary the pattern of my observation in the classroom, “to ensure as full and representative a range of coverage as possible, not just to identify and single out the superficially ‘interesting’ events” (Hammersley and Atkinson, 1992, p. 48). The extended time that I spent with each of these teachers afforded me an in-depth view of their practice. It also provided me with valuable information culled from conversations whose spontaneity supplemented field note information and audio taped data from scheduled interviews and daily classroom proceedings. Discussions with other participants in the setting were also a useful source of data. Even though our discussions were social and pragmatic in nature, I was always vigilant of the importance of my role as researcher in the gathering of data for analysis.

The previous two sections have provided a brief introduction to the participants, the setting, and some of the early research work carried out for this study. The next chapter
extends this introduction to develop a more complete picture of Eric and Melany and begins to unpack their views on the concept of understanding.

Data Collection

Data collection is a time when progressive problem solving (Hammersley and Atkinson, 1992), theory building and hypothesis testing take place - when order and understanding emerge (Morse, 1994). Researchers use a variety of procedures to gain first hand accounts of the situation under investigation. These procedures are generally defined by the research approach (Merriam, 1988). For this study on teaching for understanding the following methods characteristic of an interpretive approach were employed: participant observation in the form of field notes, audio taping of teachers' classroom practice, collection of documents such as teachers' lesson plans, curriculum materials, student tests and textbook materials, and formal and informal interviews. The use of multiple sources of information was intended to cross-check and validate results of data collection (Merriam, 1998). These methods also allowed for the full range of variation in social organizational arrangements and meaning-perspectives, and for the collection of recurrent instances across a wide range of events (Erickson, 1986). Because the research for this study was conducted as two independent cases, data collection for each teacher was done separately. I began with Eric as he was one of the first two teachers that I had invited to participate in this study. In retrospect I believe that I pressed him more for his views on understanding than I did Melany during the data gathering period. Perhaps it was my inexperience as a researcher and Eric's willingness to respond.

Field Notes

As noted earlier, the majority of my daily visits to each teacher's classroom in the four month period consisted of full days of observation. This routine allowed me to observe a teacher's practice in mathematics in the context of other content areas of the
The results of my observations were recorded in extensive field notes. Consistent with Hammersley’s and Atkinson’s (1992) recommendation, these field notes consist of concrete descriptions of social processes and their contexts. They include detailed descriptions of important happenings during the day and recordings of informal conversations and interviews that frequently occurred. These written accounts captured events “in their integrity” (Hammersley and Atkinson, 1992, p. 145) for later analysis and interpretation. Field notes were also used to verify and supplement data collected from audio tapes which recorded the ebb and flow of daily classroom life.

Analytic notes, which were included either within the body of the field notes or appended at the end of a day’s written notes, helped to narrow and direct the focus of progressive data collection. They include reflections on conversations and events, hunches and other thoughts about the phenomenon under investigation. The following is a sample of field notes with accompanying analytic notes I collected for Melany. They provide insight into the observational work carried out for the study and its relationship with the final report.

Sample from the Field Notes

September 24- Afternoon Measurement Activity

I believe that this math activity is supposed to provide the students with practice so they will be prepared for measuring the playground during the integrated unit for social and environmental studies. Melany explains the steps of the activity to the students and models what they are to do. The activity requires that students kick a rolled up kleenex, a cotton ball, and an eraser with their heel, estimate the distance they kicked the object, then record their measurement in a chart. (see Figure 1). The rule is that first they must estimate the distance the object was kicked and record the estimate. Then they measure the distance accurately with a meter stick and record the measurement.
Figure 1. Chart in Which Students Recorded Their Measurements

HOW FAR DID YOU KICK IT?

<table>
<thead>
<tr>
<th>ITEM</th>
<th>MY ESTIMATE</th>
<th>ACTUAL DISTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper tissue</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cotton ball</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eraser</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A discussion ensues as to what constitutes an estimate. On the board Melany writes the following words.

as
about
over 1m
more than 1m
less than 1m.

(I notice that these words connect with the assignment that I saw on the board the day before.)

Using a student’s measurement of 1m 22 cm from the indoor recess activity as her example, Melany tries to explain how to round when the amount is not exact. Sensing their confusion, Melany changes her approach and provides pairs of students with meter sticks. She then has them find 1m 22 cm on their meter sticks. As they do, she draws their attention to the end point on the meter stick and explains that the first marking is counted as zero not one (good that she notes this important fact). After this brief explanation, students are then asked to find other amounts on their meter sticks. It is obvious from Melany’s
comments as she circles the room observing students’ strategies that her explanation appears “to have fallen on deaf ears”. Most students neglected to move one unit to the right before they began to count and are still identifying the marking at the end point as one. I have often observed students make this error when measuring with a ruler, and Melany seems to be aware of it. At this point she stops the class and proceeds to teach a lesson “off the cuff” on how to count units correctly. As she does, she explains why it is necessary to move a distance of one unit before a value of one can be assigned. She also notes that the distance between any two long markings on the meter stick is always equal to one unit of measure. I am most impressed that Melany is cognizant of the problems that children frequently encounter when they measure with meter sticks or other measuring tools of this nature. It is not often that I have seen teachers engage in this practice to ensure that students understand how to measure correctly. Melany seems to possess a combination of deep understanding of content, and knowledge of the appropriate pedagogy that teachers need to teach for understanding. I need to check out and compare my observations with my audio tape recording of the lesson because it is important information for future analysis and writing of my thesis. I notice that once students are actively engaged in measuring Melany checks that every estimate and actual measure for each object is recorded with the correct unit of measure in the charts.

Analytic Notes and Reflections on Field Notes

September 24- Afternoon Measurement Activity

My interview with Melany today revealed some very interesting aspects about her understanding of mathematics. Melany’s ability to recognize the difficulty that children experienced counting units on a meter stick, and her way of dealing with it are classic evidence of what Shulman identifies as pedagogical content knowledge. Her ability to weave together a lesson from the homework assignment on two, three, and four digit numbers and connect it to the textbook was also impressive. It is also fascinating that
Melany feels so differently about mathematics than she does about language arts-classic of the literature. She says that teaching mathematics is not the same as teaching language arts. This is something that I need to pursue further with her, and on which I need to focus. Why does she feel this way? Where do these thoughts come from? Why does Melany feel so differently about teaching mathematics than she does about language arts?

**Documents**

Hammersley and Atkinson (1992) suggest that it would be hard to conceive of an ethnographic account without some reference to documentary material in context. Teachers' weekly lesson plans, tests, professional support materials, teachers' manuals and textbooks were examined to gain a better understanding of each case. Weekly lesson plans provided an overview of the content to be covered in different subject areas during a week. They also revealed the time of day that a subject was taught and the amount of time the teacher devoted to teaching it. Tests, text books, and other materials also provided insight into a teacher's view of the particular subject area. Melany rarely used the textbook in mathematics or any subject area because she felt that textbook teaching did not foster learning. Eric, on the other hand, used a textbook that leaned more to algorithmic learning than to promoting depth of understanding of concepts. However, he did supplement this text with materials he created.

**Interviews**

Patton (1990) points out that the interviewer's task is first and foremost to gather data. In this study both informal and formal interviews were conducted regularly and used to support observational data collected through field notes. Informal interviews were usually conducted after a lesson so that I might gain further insight into teachers' thinking in order to better understand their actions and behavior. These interviews usually occurred about three to four times a week and were held at some point in the day that was convenient.
for the teacher. Because of the spontaneous nature of these interviews, audio taping was not always an option. When this occurred I was forced to commit to memory the information and to record it as soon as the discussion ended. Interviews of this type were also held with other informants in the setting.

Formal interviews were conducted differently. They were held three times a month and planned in advance to fit a teacher’s schedule. Questions for these interviews were designed to uncover specific aspects of teachers' understanding of teaching for understanding in mathematics often prompted by events I observed in class. There were times when these questions focused on issues that were not answered in informal interviews and were still puzzling. All formal interviews were audio taped and the results transcribed for later data analysis, as were the results of audio taped informal interviews.

Data Analysis

Data analysis does not necessarily follow in a linear fashion and occurs simultaneously with data collection. However, intense data analysis takes place after the data collection phase is completed (Merriam, 1988). It is the stage when analytic coding of the data is carried out in preparation for writing of the final report (Glesne and Peshkin, 1992). Data for each teacher was analyzed separately. Unlike the data collection stage, this time I began with Melany. My hunch was that analysis of the data for Melany would provide me with a more comprehensive coding scheme that I could also use with the data I collected for Eric.

Intense analysis began with careful reading and rereading of field notes and analytic notes to familiarize myself fully with the data in preparation for the process of analytic coding. Further reading of the data was then carried out in order to develop a scheme for organizing and coding (Glesne and Peshkin, 1992; Hammersley and Atkinson, 1992). After several readings patterns began to emerge in the field notes and particular concepts and ideas became apparent. The following are examples of the forty or more concepts and
ideas that emerged as I read through the field notes and analytical notes: time constraints, Melany’s view of her role as a teacher, Melany’s view of herself versus her peers’ view, children’s knowledge and beliefs about mathematics, teacher’s views of mathematics versus views of language arts, structure of the class, making connections, pedagogical content knowledge, learning for understanding. Sorting and refining of these concepts and ideas led to the elimination of those that were less relevant. Criteria I employed to determine which concepts and ideas I would keep and those that I would discard were: the frequency with which they appeared, their relevancy to the goals of the study, and their link with key concepts in the literature such as factors that affect teaching for understanding. Those concepts and ideas that I kept were listed and assigned a colored symbol. I then went back through the data to color code it according to the symbols. Once relevant segments of the data were noted, they were then recorded on index cards. Searching of the data was also intended to identify disconfirming evidence, an activity for improving the probability that findings and interpretation will be found credible (Erickson, 1986; Glesne and Peshkin, 1992; Lincoln and Guba, 1985). I also read and coded transcriptions of interviews and audio taped data of classroom lessons as a way of checking results of the classifying and categorizing of the field notes and analytic notes. This process of cross-checking was also intended to corroborate the dependability of the data. Concepts that remained such as making connections, teacher’s beliefs, time constraints, pedagogical content knowledge, views of mathematics versus views of language arts, assessment, teacher’s content knowledge, curriculum documents, learning for understanding, children and classrooms were then grouped by index card into more general categories according to identified phenomena. These categories were then assigned names which were more inclusive and abstract than those used for the concepts within them. The following are the categories: factors of context that affect (inhibit) teaching for understanding, factors of context that promote teaching for understanding, teachers’ views and beliefs about the discipline, teachers’ views about students, teachers’ views about their roles as teachers, effectiveness of
the teacher, reactivity, access, promoting understanding, content knowledge of the teacher, perspective on understanding, changing perspective. Grouping of concepts within categories revealed key linkages of concepts across the categories. In some instances the same concepts appeared in different categories confirming the consistency and dependability of the data (Merriam, 1998). Once the coding for Melany, which resulted in a comprehensive scheme, was completed, I then went back and repeated the process for Eric. I found that analysis of the data for Melany provided a frame of reference for my work with Eric. However, I was vigilant to ensure that the data were analyzed on their own merit. On completion, comparison of results revealed similarities in my findings.

Further investigation demonstrated that categories which emerged from the coding process matched with the key concepts that frame this study - teachers’ beliefs, teachers’ content knowledge, and factors of context. These categories gave rise to the central themes, teachers’ beliefs about the teaching and learning of mathematics, teachers’ content knowledge, teachers’ perspective of understanding, how teachers promote understanding, how they regard their role as teacher, and their style of teaching, which served as scaffolding for my writing in Chapters Five and Six. Further scrutiny of these themes revealed their natural fit and close link with the three questions that guide this inquiry.

**Trustworthiness of the Data**

Evidence of measures taken to ensure trustworthiness of the research such as triangulation (Glesne and Peshkin, 1992; Hammersley and Atkinson, 1992; Lincoln and Guba, 1989; Merriam, 1998), searching for negative cases (Erickson, 1986; Glesne and Peshkin, 1992; Lincoln and Guba, 1985) have been addressed as part of the discussion on procedures for data collection and analysis in previous sections. The following are intended as further evidence to support these ideas regarding trustworthiness of the data. Morse (1994) suggests that it is the amount of data collected, rather than the number of subjects which contributes to the rigor of a study. A dilemma with which I was confronted in this
inquiry was how to deal with the extensive amount of data I had collected. After spending eight months of research almost daily from early morning until well after students were dismissed, I needed to figure out how best to present this data. Prolonged time researching in a setting is more likely to contribute to credible findings and interpretation (Lincoln and Guba, 1985; Glesne and Peshkin, 1992). Time at the research site, time spent interviewing and time building relationships all contributed to the trustworthiness of the inquiry.

**Presentation of the Data-Writing**

Presentation of the data is a time when "the author attempts to weave a text that recreates for the reader the real world that was studied" (Denzin, 1994, p. 507). This stage of the research process is filled with new insights, intuition, and decision making (Glesne and Peshkin, 1992). Before I began writing I went back and carefully reread the data a number of times to determine how "to capture the lived experience of participants" (Janesick, 1994, p. 218). Because the first two questions of the study focused on two different, but connected aspects of teachers’ understanding of teaching for understanding, my initial response was to address them separately in my writing. Sometimes it makes sense to talk about what we believe and what we do as two separate ideas. After deliberation and discussion with my thesis supervisor, I decided that there are times when such an approach does not do justice to the phenomenon under study. Empirical and practical considerations and the research on the interrelationship between thinking and action (Butt, 1983; Kilbourn, 1988; Schon, 1983) provided powerful evidence to suggest that I combine thinking with action in my writing. In so doing I addressed both question one which inquires into teachers’ thinking, and question two which looks at teachers’ practice as they relate to Melany in Chapter Five, and Eric in Six. My decision to change the order in my writing and discuss Melany before Eric was influenced by images that emerged from analysis of the data, and other data sources. The structure that I designed for writing Chapter Five served as a model for Chapter Six.
Presentation of the data took the form of episodes. Of the ten episodes that I recall for Melany, I selected the three that best portrayed her perspective on teaching for understanding. To guide my writing, I used themes, outlined in the data analysis section, that emerged from restructuring of the twelve categories. Those themes that fit with question one, which inquired into teachers' thinking, were teachers' content knowledge, teachers' beliefs about teaching and learning mathematics, and teachers' perspective of understanding. Those that connected with question two, which looked at teachers' practice, were how teachers promote understanding, how they regard their role as teacher, and their style of teaching. The following key ideas, subsumed by these themes, were also included in developing the episodes: the teacher's perspective of the discipline, resources used, mathematics as compared to language arts, knowledge of mathematical concepts, and instructional strategies. These key ideas came from the categories that gave rise to the themes. On completion of writing Chapter Five, I repeated the process for Eric in Chapter Six.

In summary this chapter has discussed the methodology employed for this study. The next chapter has two purposes. First, it extends the description presented in this chapter and provides a more detailed view of each participant. Secondly, it begins to unpack Melany's and Eric's perspectives on understanding. In subsequent chapters I continue the development to complete in-depth portraits of the two participants.
Chapter 4  Preview of Eric and Melany

This chapter offers a more complete picture of Eric and Melany. In so doing it describes their perceptions of themselves as teachers and talks about how others regarded them. It also describes Eric's and Melany's views on the teaching of language arts and mathematics. The discussion on language arts is important to this study as it provides a useful referent for understanding what it means to teach for understanding in mathematics. The chapter also unveils Eric's and Melany's views on the concept of understanding. This preview of Eric and Melany and their views on understanding set the stage for the detailed portraits that are developed in Chapters Five and Six. In keeping with the format of the previous chapter, I begin with Eric. I want to point out that I have remained faithful to the quoted material from the data as I presented participants' perceptions and views in this chapter; only when it interfered with meaning or readability were minor changes made. I have continued this practice in subsequent chapters.

Overview of Eric

As a beginning teacher Eric seemed to have “a natural talent” and sailed through his first year of teaching to achieve permanent teaching status. During the time that I observed Eric, his classroom always ran smoothly and student behavior was never an issue. He managed his students with ease and maintained an excellent rapport with them. They willingly abided by the class rules and readily followed routines. It was easy to sense the mutual respect that they had for each other. His principal and vice principal lauded his ability as a teacher, and his colleagues held him in high esteem. At the board level, Eric served on a variety of committees, and he regularly presented workshops to administrators and principals on the computer tracking system that he had developed.
When Eric taught a lesson he carefully laid out his expectations and modeled procedures. I recall the lesson he taught on the science fair project assignment. To demonstrate for students what he expected, he presented his version of a project. First he displayed a project which was illustrated on heavy cardboard so that it would stand freely. All students were expected to use these boards for their projects. Eric then followed with a ten minute talk on the project, which in this instance was solutions and suspensions. When he finished, he modeled how students were to answer questions about their topics from their peers. Eric ended the lesson by carefully reviewing each step that he had modeled.

In Eric’s view the learning he provided for his students was not the same as other teachers. Even though he believed that it was not possible to run a totally experiential program at the grade seven level, he suggested that the type of learning that he set up for his students was more open ended, and based more in experience than the "average teacher provides" (interview, April 9, p. 9). It was different from just giving them information or using the textbook.

For instance, one type of learning he provided was how to conduct your own research. As Eric suggested, “students were always working on some independent project or topic in a subject, solving and sifting through information" (interview, April 9, p. 12). In order to do the research students had to collect information on a topic, organize it, and decide how to structure the final submission. He emphasized that the finished product was always their decision. One example of a research project that he described was the Native Peoples Project in history. For this project students had to gather information, synthesize their findings, and hand in a written package to him. Eric explained that he would give them a one page sheet of instructions which laid out how to read through a set of materials and how to organize the information according to the categories that he had set out for them.

A second type of learning involved the use of concrete materials. In geography he would use compasses to teach students how to figure out angular bearings. Once students knew how to read a compass, he would take them outdoors to practice. In mathematics he
would use manipulatives such as fraction sets and geoboards to help students "see" the concepts they were learning. The use of portfolios and writing folders also played an important role.

A third type of learning which Eric provided was intended to simulate real life situations. He particularly believed that students required the ability to respond to the need of the moment in their jobs as adults, whether it was business, or teaching, or whatever they would be doing. As I noted in Chapter Three, Eric believed that students had to know how to cope in the real world when they were adults, and his responsibility was to teach them the skills.

I think there's tons of times in the real world, in all aspects of life, that you are asked to perform on a moment's notice. Like maybe you're an account manager, or maybe you're a bond trader, or maybe you're a whatever. It's sort of taken for granted. You've got all these skills, or be able to do all these things. So the boss walks in and he says, "Okay I got this stuff. I hate to tell you this but I need it done by lunch time. Can you get it for me?"
You don't have the night to go home and prepare and get ready. You've got to sit down and get it ready....
I think that they've [the students] got to understand what that feels like.
(interview, April 9, p.4)

According to Eric these skills were more important than the content he taught. The following incident that I observed provides evidence of Eric's belief that it was important to prepare his students for adulthood and the world of work.

Eric was always very diligent about making certain that his students were well informed about upcoming tests and quizzes. He explained that he would always review the necessary subject matter for a test or quiz and would tell students of the appointed date. However, this day in particular students were not told previously about the quiz on comparing and ordering fractions that he was about to give them. According to Eric this was the first time that he had ever surprised these students with a test or quiz. They received no advance warning. As they entered the room he greeted them with the instructions that
they were to take their seats and put away their books; they were having a mathematics quiz. A look of disbelief registered on their faces. Most students appeared to cope with the situation. There were some who appeared stressed and anxious; one student became very excited and overwrought. Later that day when I talked with Eric about the experience, he explained to me that his intention that morning was to simulate the kind of work situation which required dealing with the unexpected with which students, as adults, might be faced. In his view, the experience was beneficial for them, rather than detrimental. As Eric suggested, the life skills that he was developing in his students were the kind that the board promoted in their exit outcomes and that employers wanted for their employees.

I would now like to turn to a discussion about Eric's views on language arts. For the purpose of this inquiry language arts provides a useful referent for understanding what it means to teach for understanding in mathematics. Eric's perspective on language arts was different from his view of mathematics. According to Eric the language arts curriculum was less structured and permitted teachers the flexibility that he thought was necessary to teach effectively. For example, if students were working on a writing assignment and they were experiencing difficulty with the correct use of adjectives and adverbs, he explained that he could easily stop, deal with the problem, and still cover the outcomes. If he chose to "flip around" the order in which ideas were presented during language arts and he taught bias one week instead of creative writing, that was all right. He also believed that the curriculum was sufficiently broad-based to allow him to choose his own novels rather than teach those outlined in the guidelines. In Eric's view he had the license to select whatever "vehicle" he wanted for presenting concepts. If he were teaching bias or prejudice he could easily use the newspaper, a unit on forests or ecosystems rather than follow the expectations of the curriculum. According to Eric, "Language arts has latitude and transcends all boundaries. It comes from all over, and kids are involved in language outside of school. But in math they don't think about, and have math learning outside of school" (interview, April 23, p. 133).
Mathematics was different from language arts for Eric. It had a more logical progression than language arts. He said that, "it doesn't lend itself to spinning all over the place" (interview, March 25, p. 8). Often in our conversations, as is common, Eric tended to be colloquial. What he probably meant by the previous comment was that mathematics did not provide him with the flexibility in his planning and teaching that language did, as noted in the previous section. In mathematics Eric maintained a tight schedule that fit the curriculum outcomes. He never veered from the schedule or went beyond the boundaries as he did in language arts. He taught only what was expected. Nevertheless, he did point out that he taught mathematics differently from the way he taught other subjects. According to Eric, he "would pay lip service to what they [the board] wanted done [in the other areas] but not in math" (field notes, April 22, p. 88).

Even though Eric provided students with research and "hands-on" experiences in mathematics, he explained that he was more heavily into Socratic teaching. This kind of teaching included "a lot of telling, imparting information and dispensing ideas so they [students] could perform a skill or complete a task" (interview, April 9, p. 13). Eric suggested that eighty percent of his time in mathematics was devoted to Socratic teaching. The other twenty percent of the time students worked with partners in "hands on" investigations to figure out ideas on their own. According to Eric these percentages also fit with his style of teaching in history. However, for language arts, science, and design and technology the percentages were reversed. In the following excerpt from the data he provides an explanation for his choice.
Well I'd say, I'm more heavily into Socratic teaching in math, and history, some type of social studies, history/geography. I'm more open in science, language arts, design and tech ... Yeah, I think in math. I think in math and probably in social studies and what not, there seems to be more of a laid out, this is what you should be teaching. It's like a checklist. There it is. [He pointed to the year's outline for mathematics that he had posted on the wall by his desk.] I highlight things. It's more time efficient [in mathematics] to tell than to allow them to discover. (interview April 9, p. 14)

In the overview of Eric that I have presented I have talked about Eric's perception of himself as a teacher and how others saw him. I have also discussed his perspective on language arts and mathematics. Before I talk about Eric's views on the concept of understanding, I want to briefly sketch out what it meant for Eric to be a beginning teacher. In one conversation in particular I recall Eric saying that he was still young and impressionable and that he had not taught long enough to develop all of his own ideas. He said that he had not figured it all out yet so he was constantly modifying, reviewing, and revising his lessons and assignments in order to meet the needs of his students. He was willing to try out other teachers' ideas, and if they worked well he would incorporate them into his repertoire. It also meant that he was prepared to make modifications in his program to accommodate the needs of his students. For example, the changes he made to his language arts program this year were motivated by problems his students had encountered last year. The assignments he designed for the novel studies at that time were far too difficult and above the level of the students. From his perspective it was important to do what was best for students. I now shift the discussion to Eric's views on understanding, particularly his views on understanding in mathematics.

**Eric's Views of Understanding**

I found that Eric was more willing early in our relationship to talk about his views on understanding than was Melany. I believe that my familiarity with Eric was a factor. As I noted in the previous chapter, in the year prior to this study I had come to know Eric quite
well professionally as a result of our work together. I am certain that the comfort level I had established with Eric made me more relentless than I was with Melany in my pursuit to learn about his views on understanding. However, on those occasions that I did press Eric he would explain, as Perkins (1993) advises, that it was quite difficult to articulate a definition of understanding.

...You know, like, I don’t know necessarily what understanding is. I’m sure that I’ll see a kid that can manipulate it [an idea or concept], and work it around, and wheel it around, and they can apply different strategies. And sure I’d say, yes, that kid understands. But to define what understanding means, it beats me. (interview, March 28, p. 1)

Yet, he did suggest that understanding was more of an innate or gut feeling and he could tell when students understood, especially when he examined their work. The longer I spent in Eric’s classroom the more I came to learn about his views on what it means to understand.

The following discussion serves as evidence. As I observed Eric teaching, I began regularly to notice that he would say to his students, I want you to understand. When I asked him what he meant by this statement, he explained that the implied message was that he wanted his students to have a well thought out reason for their answer rather than just copying down someone else’s ideas. He wanted them to have some reasonable explanation for their written work even if the answer was not correct, as he states in the following comment.

I want you [the student] to explain why you put the answer there and not because Matthew told me to put it there. I want you to feel confident if you put that answer there, right or wrong, you can explain to me why you put it there. (interview, March 26, p. 14)

In Eric’s opinion, a student’s completion of an assignment was not necessarily a guarantee that the student understood.

According to Eric, when students truly understood concepts and ideas they would be able to do two things. They would be able to make connections (Good, McCaslin and Reys,
1992; Hiebert and Carpenter, 1992), and they would be able to apply the knowledge and the skills that he imparted in a new way (Gardner, 1993). To describe what he meant, he used an example from his work on the meaning of fractions. Eric thought that if his students could apply their knowledge about halves and quarters to thirds and sixths, then that showed him that they understood the concept of fractions. He also suggested, "When they [students] formed the pathways and connections themselves to the end product and they're able to say, Wow, okay, I got it; now I understand" (interview, March 25, p. 15), that demonstrated true understanding. Eric emphasized that students who truly understood ideas or concepts did not have to mimic the teacher. In a later conversation that I had with Eric, he explained.

The kid will show me he understands ten years from now when I run into them in the store, in the shopping mall, and they start rambling off that they are a math specialist in the University of Toronto. And they still never had anybody explain it quite well, and clearly enough, about how to divide fractions as I did in grade seven. I'll say, wow, okay, you must have really got it. (field notes, April 1, p. 88)

Eric also noted that true understanding was dependent on a number of factors. First, it was important that a teacher set up activities that had sufficient depth, so that students would be able to "sow it [ideas] around, and mix it around, and reinforce it" (interview, March 25, p. 15). Secondly, a teacher needed to have sufficient time for these kinds of activities. Eric pointed out that he did not think that he had the time to teach in this way. For this reason he hoped that his students would be able to make some of the fundamental connections of concepts on their own. Thirdly, the learning environment that the teacher created was important. He believed that certain learning environments generated more understanding than others. A discovery learning environment was more conducive to promoting learning for understanding.

According to Eric teachers needed to have a good understanding of the content in order to effectively teach mathematics or any subject for understanding. Studies conducted
with teachers in different subject areas (Ball, 1991b; Hashweh, 1986; Wineburg and Wilson, 1991) support the belief that teachers who possess a rich integrated knowledge of the subject can influence instruction in a positive way (Fennema and Franke, 1992). Eric also believed that, if a teacher was not able to see the connections that existed among concepts, then there was little likelihood that the teacher could structure learning so it was meaningful. From my observations it was evident that Eric possessed characteristics of a teacher who teaches for understanding. In many of our conversations Eric demonstrated his knowledge of the discrete understandings of mathematical concepts and his comfort level with the discipline. I want to add that as a student he had always been successful in mathematics in high school and in university, Eric had a deep understanding of the concepts that he taught. One conversation that stands out for me was a discussion on the topic of division of fractions. Eric suggested that to teach division of fractions so it was meaningful, it was important to connect the concept to division with whole numbers. This comment is significant for two reasons. Firstly, it suggests that Eric was aware of the need to draw students’ attention to the many connections that exist in mathematics. In Haylock’s (1982) view, the more connections the learner can make, the deeper the understanding. Secondly, it implies that Eric knew that the measurement model for division with whole numbers could be used to teach division of fractions to help students “see” why the quotient ends up greater than the dividend. From my experience, many elementary teachers are not aware of how the measurement model can be used to teach division of fractions. However, Eric was. He convinced me of his understanding when he explained that in order for students to answer the question, divide four by one eighth, they needed to think about the problem as how many groups of one eighth are there in four. He pointed out that if students were able to conceptualize the problem in this way, it suggested they understood the idea of what it means to divide by a fraction. This approach to dividing fractions is more meaningful than memorizing the rule, to divide by a fraction, invert and multiply.
Eric also believed in the idea that there were different levels or degrees of understanding in any classroom, a notion that is formally discussed in the literature (Backhouse, 1978; Byers, 1980). The following excerpt provides evidence of Eric’s views.

All the kids will understand it [the concept of mixed numbers and improper fractions] at different levels. How about that? Every kid in this room will understand something about that sheet. Some will understand it better than others. Some of them don’t understand the instructions, let alone the answers. (interview, March 26, p. 16)

For Eric, it was not possible to teach for one hundred percent understanding, because, as he said, “You can’t make all kids understand all the things that you’re teaching” (interview, March 28, p. 1). Moreover, not many students in his class had what he considered to be true understanding. In his view, there might only be three or four students who truly understood when a new concept or idea was taught.

Overview of Melany

Even though Melany had struggled early in her career, her teaching now could easily be described as a "portrait of expertise" (Shulman, 1987). Melany’s close colleague, Sherri, described her as someone who “always knows how things are going to work and what the outcome will be” (field notes, November 11, p. 150). Other teachers in the school lauded her superior teaching ability and provided me with unsolicited accounts of her achievements. Her peers in the teaching community had honored her with a special award from The National Reading Association for excellence in teaching language arts.

From my observations and conversations with Melany and from comments that her colleagues would casually offer about her teaching, I came to see her as a teacher who had, in Shulman’s (1987) terms, “... the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive
to the variations in ability and background presented to the students" (p. 15). There were many occasions when I observed this unique ability about which Shulman (1987) talks.

I recall how she effectively facilitated students' learning and skillfully orchestrated successes that they might not otherwise attain. One situation stands out for me. Her students had to design a questionnaire for the playground study which was part of an integrated unit she had developed with a colleague. Each student had to develop questions that would elicit from peers in a neighboring school detailed information about the positive and negative characteristics of their playground. It was a difficult task even for an intermediate student. However, Melany knew how to inject the appropriate blend of support and independence as her students worked through the task. She would ask just the right question at the opportune moment to stimulate her students' thinking. I recall when they were struggling with the content of the questions Melany suggested that they think back to concerns they had with their own school playground. This advice seemed to be the jump start that they needed. The model questions that she wrote on the chalkboard to guide them also worked effectively. It was evident from their work at the end of the lesson that all students had successfully completed the task with impressive questions for grade five children. The following are samples: What are some of the problems on the play structures at your school? What places are safe to play? What places are unsafe on the playground? Why? What kinds of activities do you play on the field of your school? What do people play when the field is closed? What kinds of activities do boys play? What kinds of activities do girls play?

I should add that discipline was never an issue for Melany while I observed in her classroom. An effectively worded comment or a discerning glance at the right moment seemed to diffuse any situation before it became a problem. Melany, like Eric, believed that she offered her students learning experiences that other teachers did not. Most teachers subscribe to this belief, Melany demonstrated that she actually took this statement to heart. For example, to provide students with practice for finding area Melany had them complete
an investigation designed by Marilyn Burns, rather than have them find how much carpeting was needed for a house. In her view, not many grade five students ever bought carpeting for their house, and there really was no point in teaching students ideas that had no meaning for them. Moreover, Burns' investigations engaged students in inquiry that connected with their interests and knowledge base. Melany also regularly integrated mathematics concepts with other areas of the curriculum in order to make learning meaningful for her students.

For the present overview I have included three examples to illustrate what integration meant for Melany. These examples also demonstrate how she began from students' knowledge base. The first example is the integrated social and environmental studies unit which extended over much of the time that I spent observing in Melany's classroom. This unit, which Melany and Sherri developed, was designed to integrate mathematics with environmental studies. A key feature of the unit was to engage students in solving real life problems. A major activity for students was to conduct a study of the school playground to find out how effectively the playground met the needs of the school population. The task involved interviewing teachers and students throughout the school to determine their views on whether the playground was well used. Students also had to observe the play area every day for one week at peak times such as recess, before school began in the morning, and in the afternoon to determine the pattern and frequency of use of the equipment. The data they collected from their observations was analyzed and used as the basis for deciding if additional equipment was needed. A class forum was held, and after much discussion a decision was made that the school needed to buy more equipment. Students then worked together to figure out the total cost of equipment from catalogue prices paying close attention to budget constraints, as Melany advised. The class was also responsible for measuring the school yard in order to determine the availability of space for the new equipment. When all this work was completed, a report was drafted and sent to the school principal for approval.
The second example involves the Structures Unit in science. In this unit Melany integrated mathematics with science. The unit began with three dimensional geometry activities which were intended to familiarize students with the names and characteristics of the solids. These activities were followed by an activity in which students built structures out of newspaper. The purpose of structures activity was to provide students with an opportunity to figure out which geometric shapes offered the most stability in preparation for bridge building. The unit then culminated with the design, building, and testing of their bridges for sturdiness.

The third example concentrates on the teaching of French, which was not Melany's favorite subject. Even in this subject Melany made certain that school learning was connected to students' real life interests. To help students learn the names of different nouns, Melany had them conduct surveys in French to find out which sports were favored by the class. Once the data was collected and analyzed, students were then required to graph their results. This French activity demonstrated yet another way in which mathematics was integrated with different areas of the curriculum.

For Melany it was important to provide students with the structure that she thought that they needed and wanted. Each morning she would post the day's schedule on the board before her students arrived in the classroom. The schedule outlined the subjects she would be teaching, the activity for each subject, the homework assignments that would be taken up that day, and the homework for that evening. She always made certain that students were well acquainted with the rules and routines of the classroom and posted them, along with the names of monitors for the week, in a strategic location in the classroom. In Melany's view, when a teacher was responsible and consistent with students, it would "pay off" in their classroom behavior. As I suggested earlier, discipline was never a problem in her classroom.

I want to shift now and talk about Melany's language arts program. The discussion is important for two reasons. First, Melany's persona as a teacher can not be portrayed
adequately without addressing her language arts program. It defined who she was as a teacher, and it was the source of her images and views about teaching. Secondly, as I suggested earlier, with respect to this inquiry, language arts provides an important referent for understanding what it means to teach for understanding in mathematics. From my many conversations with Melany I learned that language arts had a preferred status for her. Over the years she had spent considerable time reading and researching in the area of language arts. The support she had received from colleagues and board personnel facilitated the development of the program that she now had in place. The program focused on reading, writing and discussion. From my view, it was comprehensive and rigorous yet sufficiently flexible to accommodate the diverse backgrounds and levels of ability of her students. For most activities, students worked at their own pace on individualized assignments with the occasional whole class assignment.

Melany reserved mornings in her schedule for language arts so she could devote considerable time to it each day. There were times when she would extend language arts into the afternoon after the lunch break. Her reason for choosing mornings was that it afforded more time for teaching than afternoons. Also, she did not want her ESL students, who were withdrawn for upgrading in the afternoon, to miss it. Most mornings began with Melany reading to her students from a variety of authors which included different styles of writing. She wanted to expose students to different genres so that she might develop their appreciation of literature. Morning readings were usually followed with a class discussion on the reading. Then students were given instructions for the upcoming writing activity. Because Melany’s writing program was individualized, she and Sherri, the resource teacher for language arts, would meet regularly with students, either in small groups or one-on-one, to discuss their work. Sherri’s role was to provide support for Melany’s language arts program. While students worked on their stories and poems, she and Melany judiciously offered suggestions. As the stories and poems began to emerge, public readings known as "audiences" were held. This activity was unique in that it provided a forum for students to
read their stories or poems to classmates who would then offer advice in a constructive manner. During the time that I was present, students were comfortable with the process and favorably accepted critiquing and suggestions. Another technique that Melany used with her students for editing poems and stories was partner-work. For this activity students would work with friends in pairs and take turns providing each other with ideas for improving their writing.

Reading was also an important component of Melany’s language arts program. It too was individualized. During this activity students would read their favorite library book independently for about twenty minutes each day. In addition, students read once a week to children in the younger grades. An important ritual during language arts was status-of-the-class. For this activity, either Melany or Sherri would stop the class at strategic intervals to survey students in order to determine their progress on the variety of assignments on which they were working. This tracking activity insured that Melany’s finger was always on the pulse of her language arts program.

I have just given a quick overview of Melany’s language arts program to illustrate its complex nature and the extensive time that she devoted to it. The situation was not the same for mathematics. Similar to other teachers on her team, Melany taught mathematics for 45 minutes in the afternoon. However, less than half way through this inquiry, Melany decided to change her schedule and move language arts to the afternoon. As a result, mathematics was switched to the morning so that more time could be devoted to it. According to Melany, a better balance was needed between language and mathematics in her overall program. She also wanted to accommodate her ESL students in mathematics because the ESL teacher had moved language classes to the afternoon. Even though Melany suggested that mathematics lacked some of the important features of her language program, she did not want these students to be “short changed”.

Melany’s mathematics program was not as individualized and diverse in its approach as was her language arts program. According to Melany, it did not provide
students with the “opportunity for independent thinking” (interview, November 26, p. 40) in the same way that language arts did. Yet, she did suggest that the experiences she provided for students in mathematics were different from those prescribed by the board or that most teachers offered.

Melany was most emphatic when she told me that she would only teach “something [in mathematics], if I could teach it to make it meaningful” (interview, November 26, p. 45), which was her motivation for using Marilyn Burns’ materials. Burns (1987) uses a holistic approach to teaching mathematics. The investigations she develops are designed to promote meaning and understanding in children’s learning. Burns also sets up the investigations so that students work both individually and in small groups to problem solve, to inquire into ideas, and to experience real life mathematics. Discussion, journal writing and the use of concrete materials are important features of the investigations. Melany started to use these investigations when she “began changing the way she thought about mathematics” (field notes, September 27, p. 45). She selected them on the basis of their fit with concepts in the prescribed mathematics curriculum and her level of confidence in teaching them. She liked Marilyn Burns’ ideas. In Melany’s view, the narrative style that Burns uses to describe investigations, and the description of children’s responses provided Melany with the detail that she needed to construct the kind of activities in which she wanted her students engaged.

Another consideration for Melany in deciding which concepts to teach was, “how well they fit together” (interview, November 26, p. 39). According to Melany, it made more sense to teach area and perimeter together, rather than to teach area one month and five months later teach perimeter. Pedagogically and conceptually this was a sound decision. When area and perimeter are taught as two connected ideas rather than as isolated concepts, it reduces the probability of confusion that students often experience. Instead of finding perimeter when they need to find area or vice versa, or using the wrong unit of measure when solving a problem, students are more inclined to carry out these tasks correctly. The
added benefit of teaching in this way is that students may come to appreciate some of the less obvious relationships between perimeter and area such as four-sided figures with the same area can have different shapes, or the closer the shape of these figures approximates a square, the greater their surface area. To make learning of decimals more meaningful for her students, Melany said that she always taught them after fractions. In Melany’s view the purpose of teaching them in this way was to help students realize that decimal numbers were yet another notation for fractions.

From our many conversations I came to learn that Melany did not think that textbooks were useful for teaching new concepts. She suggested that "with textbook work you don’t need to think" (field notes, November 29, p. 227). However, there were instances when the textbook did play a role in her program. She used the textbook when she thought that she did not have any other options for interesting ways to teach a concept, or when she needed a break from the intensity of teaching. When I first observed her in the classroom, she was using a textbook to teach standard and expanded forms of numbers and to teach multiples of ten. She explained at the time that even if it was boring for her to teach in this way, she did not feel that she had any other options. She had used Baratta-Lorton’s (1976) approach the previous year to teach place value and taught other bases such as three, four, and five before teaching base ten. Baratta-Lorton sets up learning so that students explore groupings in other bases in order to facilitate their understanding of place value with base ten. From Melany’s point of view, it was more meaningful than the textbook method, but it was confusing for her students because they were not accustomed to working in other bases. So this year she reverted to the conventional textbook method. It was less work for her as she only needed to focus on base ten, and it saved her from preparing all the extra materials. The textbook was also reserved for students to practice their skills, and for days when supply teachers replaced her due to absence.
These, then, are impressions of Melany that I developed as I observed in her classroom over a period of four months. Let me turn to Melany’s views on understanding, particularly those views connected to mathematics.

Melany’s Views of Understanding

To fully capture Melany’s views on understanding I have included in this section her general comments regarding the phenomenon, her views on teaching for understanding and learning for understanding. According to Melany, “understanding was not something that you could talk about in the abstract” (interview, October 17, p. 39). It had to be tied to a specific content area because it was not “a generic concept which was inclusive to all disciplines” (field notes, October 7, p. 66). Each content area or concept had its own set of criteria that determined what it meant to understand.

You know when we were working on area, then I can tell you what I think it means to understand area at this point. And when it has to do with writing, it means, you know, I would have to look at what they’re writing now in terms of poetry, and what they’re writing heading into fiction writing, whatever.... (interview, October 17, p. 40)

For Melany, teaching for understanding meant “giving kids experiences where they can start coming up with their own, expressing and clarifying their own understanding of concepts” (interview, November 20, p. 13). In her view, neither transmission teaching or textbook work provided students with this kind of learning. According to Melany it was a teacher’s responsibility to motivate students to think, to make generalizations. The teacher also needed to provide students with the opportunity to talk about their thinking, and what they were thinking, as Good, McCaslin and Reys (1992) advise. For Melany, “being able to articulate what you think, is a very high level of understanding” (interview, November 29, p. 19). It was also important to extend students’ thinking, and making learning relevant as she notes in this brief excerpt from the data.
How can students be expected to understand what large numbers are when I myself don't. When do we ever really get to see what five thousand or twenty five hundred looks like in real life.

(field notes, September 18, p. 11)

In order for students to learn for understanding, Melany suggested that they had to be able to connect new concepts in different ways- to their reality, to other mathematical concepts and to other disciplines. The more connections that students made, the more meaningful the learning would be for them and the greater the understanding. As I noted earlier, the idea of making connections to increase understanding in mathematics is a well discussed theme in the literature (Haylock, 1982; Hiebert and Carpenter, 1992). The following is an example of important connections needed for making the learning of area more meaningful: that area and perimeter are related, that square units are used for measuring area, that area concerns the amount of surface, that area is important when building houses or buying land, and so on.

According to Melany, the greatest understanding occurred when learning was "needs generated" (field notes, November 27, p. 222) either by the student, or by the task at hand. An example of what she meant by “needs generated” can be seen in the measurement activity which I discussed in Chapter Three. When Melany realized that students were using their meter sticks incorrectly to measure the distances, she stopped the class. At that point she taught a lesson on the concept of a linear unit and how to measure correctly with a meter stick. When it was obvious to her that most students could measure correctly, she had them resume the original activity. Later, in conversation about this incident she explained that this was an instance where teaching evolved from students’ needs.

Like Eric, Melany also subscribed to the belief that in any class there are degrees of understanding, "[a] wide spectrum of different levels or degrees of understanding", as Haylock (1982, p. 54) suggests. According to Melany, the indicators of these degrees of
understanding might be the strategies that a student employed to work through an idea, the nature of a student’s communication, and the nature of the reflection in a student’s writing. Students who could talk about an idea and explain it in their own words demonstrated understanding. Moreover, if they had the ability to express themselves effectively in writing, that constituted real understanding.

A discussion that Melany had with her students, before she set them to work on an activity that focused on the relationship between perimeter and area of four-sided figures, demonstrates how levels of understanding might vary in a classroom. For this activity students first had to outline their foot with a marker on one-centimeter graph paper. Then they had to lay string around the outline of their foot in order to find its perimeter. At this point they were asked to predict if the piece of string that was laid on the graph paper to make a square would have the same or different area measure from their foot. An additional condition was that the square had to include as many complete squares on the graph paper as possible. I want to point out that this activity, which also considered the relationship between perimeter and area, was similar to one on which students had worked the previous week.

During the discussion on the foot activity in which students talked about their predictions, I observed that there were some who demonstrated that they clearly understood that the square would have the greater surface area. These students were able to show with a series of diagrams that as the rectangle became “longer and thinner” and closer to the shape of their foot, its area diminished. They were also able to demonstrate the reverse— that the area of the rectangle increased as it approached the shape of a square. There were students who sensed that the square would have the greater surface area. However, they were unable to provide an explanation. Other students said that they thought the areas would be different, but their reasoning was faulty as the following sample responses demonstrate.
Student One: You don't count the little parts of the squares in the foot and that would make up for the difference.

Student Two: The areas would be different because the foot goes in and out and the square doesn't.

Student Three: The areas would be different because the foot has curves in it, but the square has straight lines.

Student Four: The areas would be different because it's flatter.

(field notes, November 7, p. 129)

There were those students who appeared to have no idea at all. Perhaps they were not prepared to risk presenting their ideas to their peers. Some quietly urged me to tell them what I thought the correct answer might be. As Backhouse (1978) points out, “Not all pupils will be equally successful; there are degrees of understanding...” (p. 40). To complete this brief discussion on Melany’s views I want to add that, for her, understanding was a changing and evolving process. Because of its dynamic nature, students’ understanding never stayed the same and was affected by different experiences. Citing her own situation as an example, Melany noted that her understanding in mathematics was also changing, and it would probably undergo further change by the time that I was finished my research in her classroom.

In summary, this chapter reveals that Eric was a teacher who had a “natural talent” for teaching. According to Eric, he provided students with learning that was different from the type of learning that other teachers offered their students. In his view it was more open-ended and based more in experience, and different from just giving students information or using the text book. Although it was not easy for Eric to articulate what understanding meant for him, he did suggest that it was more of an innate or gut feeling and he could tell when students understood, especially when he examined their work. According to Eric, when students truly understood concepts and ideas they would be able to make connections as Good, McCaslin and Reys (1992) and Hiebert and Carpenter (1992) suggest, and they would be able to apply the knowledge and the skills that he imparted in a new way (Gardner, 1993).
It is also evident that Melany was a teacher who “always knew how things were going to work and what the outcomes would be” (field notes, November 11, p.150). As she suggests, she would only teach concepts and ideas if she could make them meaningful for her students. Moreover, Melany had the capacity to transform the content knowledge she possessed into forms that were pedagogically powerful and adaptive to the variations in ability and background presented to her students (Shulman, 1987). In her view, “understanding was not some thing that you could talk about in the abstract” (interview, October, 17, p.39). It had to be tied to a specific content area because it was not “a generic concept which was inclusive to all disciplines” (field notes, October 7, p.66). For Melany, the greatest understanding occurred when learning was “needs generated” (field notes, November, 27, p.222). Like Eric, she believed that in any class there are degrees of understanding, “[a] wide spectrum of different levels or degrees of understanding” (Haylock, 1982, p. 54).

The next two chapters further develop the pictures of Eric and Melany I have presented. However, the nature of the writing varies in order to accommodate the presentation and interpretation of the data. Also the order has been changed, as I noted in Chapter Three, in which I talk about Melany and Eric. This decision was influenced by images that emerged from analysis of the data and conversations with others about my research.
Chapter 5  A Portrait of Melany’s Life in the Classroom

The purpose of this chapter is to address the first two questions that guide this study as they relate to Melany.

1) What does it mean to teach for understanding in the context of the mathematics classroom?

2) What are the implications of teachers’ perceptions of what it means to teach for understanding?

The chapter begins by briefly revisiting Melany’s views about language arts; those are significant because they define Melany’s image of herself as a teacher, and they serve as a useful benchmark for understanding her perspective on mathematics. A discussion of her approach to mathematics follows and sets the stage for the three teaching episodes that are the focus of the chapter.

As I explained in Chapter Three, initially I intended to address the first two questions separately. However, empirical and practical considerations and the research on the interrelationship between thinking and action (Butt, 1983; Kilbourn, 1989; Schon, 1983) convinced me to deal with both concepts together. The following key themes which emerged from the data provide the necessary scaffolding for what I have to say: teachers’ beliefs about the teaching and learning of mathematics, teachers’ content knowledge, teachers’ perspective of understanding, how teachers promote understanding, how they regard their role as teacher, and their style of teaching. The first three themes (teachers’ beliefs about the teaching and learning of mathematics, teachers’ content knowledge, teachers’ perspective of understanding) are tied to teachers’ thinking, which is at the heart of question one. The latter three (how teachers promote understanding, how they regard their role as teacher, and their style of teaching), which are practice-related, fit with question number two. The following key ideas subsumed by these themes are also addressed: the teacher’s perspective of the discipline, resources used, mathematics as compared to language
arts, knowledge of mathematical concepts, and instructional strategies. My interpretive commentaries are integrated with the descriptive passages in each episode.

Melany’s Approach to Language Arts

Melany’s approach to language arts reflected her view of herself as a facilitator of learning and a picture of what it means to teach for understanding. She explained that she only resorted to telling when she had no other strategies upon which to draw. Her extensive repertoire in this area of the curriculum allowed her to fill any “time-lag” in her day with a language activity - something she would not do with mathematics. According to Melany, the process nature of her language arts program “accommodated a wide breadth of understanding” (field notes, November 15, p. 156). There were few large group assignments in language arts during the time that I observed in Melany’s classroom. Most activities were individualized to meet the diverse needs of student and to allow them to work at their own level of ability. At the same time that she encouraged students to take responsibility for their learning, Melany also tried to provide them with the support she thought they needed:

Kids aren’t being told what to do, when to do it, how to do it. Well maybe, how to, but they’re not being told what to write, how long to write it, how long it has to be. Like there’s all kinds of things that all of a sudden aren’t cut and dried for them, so it’s a little disconcerting for them. They’re responsible to choose their own topic. They’re responsible to decide when it’s done and when it needs to be revised, all that kind of stuff. I mean I’m there to help them out and give them suggestions, but it’s their, basically their responsibility. (interview, October 9, p. 2)

Students chose their own books for independent reading, acted as critics during the ritual of public audience, and conferred with each other as they worked. Assignments reflected Melany’s effort to make learning meaningful and connect it to students’ real life experiences and interests. One that stands out was a story writing assignment in which students were asked to describe an encounter with a favorite television or movie character.
The enjoyment that students experienced was reflected in their enthusiasm for the task and the quality of the stories they produced. In summary I suggest that Melany believed that in language arts she provided for a "breadth of understanding", made learning meaningful for her students and promoted understanding. Also, the process nature of her program accommodated her facilitative style of teaching.

Melany’s Approach to Mathematics

In the following paragraphs I briefly discuss Melany’s early experiences with mathematics; then I talk about her approach to mathematics. The discussion regarding her early experiences is important because it acts as a backdrop for understanding changes she was making in her approach to teaching mathematics. It also provides a frame of reference for understanding her views of teaching for understanding. Melany pointed out that, at the beginning of her career, she regarded mathematics as something she was obliged to teach, not something that she really wanted to teach. She gave students drills, the very drills that she "flunked out on", and she focused on knowing facts and timetables. She said that she never considered asking students what they were thinking; neither did she consider that there might be more than one way to get an answer. She based instruction on the progression in the curriculum guidelines, and she used the textbook for ideas on how to teach the concepts. She claimed that her approach to teaching had been colored by her early negative experiences in elementary school. Research suggests that teachers’ early classroom experiences often shape the way that they think about mathematics (Brown, Cooney, and Jones, 1990; Feinman-Nemser and Buchmann, 1986; Lortie, 1975; Schoenfeld, 1992; Thompson, 1986; Zeichner and Gore, 1990).

Although Melany had always been the best student in her class and could do mathematics, she believed that she was not very good at it because she failed basic skills testing as early as grade two. The problem was that she could not do her number facts, particularly addition and subtraction, quickly enough. According to Melany mathematics
was dry, dull, and boring in elementary school. It was rules and drills with only one right answer. It was not until high school that mathematics became fascinating for Melany; then it became more like doing a wonderful puzzle, especially algebra and geometry. It was the time when Melany was most successful in school, and her highest marks were in mathematics. However, there were still “things that I didn’t really know how they worked till I got to teach it” (interview, October 4, p. 35). I want to add that Melany chose not to study mathematics beyond high school, and the professional development in which she engaged as a teacher was limited to special activity days mandated by the school board.

For Melany, mathematics was now “a topic that I am becoming more and more enamored with and find very fascinating” (interview, October 4, 1996, p. 27). She was aware that “there’s very many more ways to get these answers and in fact to find out from kids how they get their answers was fascinating” (interview, October 4, p. 34). Also, she was spending more time in mathematics “having kids talk and relate to each other, having more kids explaining in a class discussion what they thought and why” (interview, November 27, p. 27). In Melany’s view, her approach to mathematics was becoming more like language arts as she suggests in the following.

So that’s [my mathematics program] becoming more like my language program where I base my teaching on what their needs are, whereas in the past it would have been according to what was in the textbook. I shouldn’t have said what’s in the textbook. It would have been according to the curriculum guidelines and then going to the textbook and seeing what they would teach. (interview, October 4, p. 34)

Melany pointed out that she had been working at changing her mathematics program for about the past five years. (She indicated that she had experienced similar changes in language arts earlier in her career.) Even though her mathematics program was changing she doubted if it could ever be exactly the same as her language arts program. From her point of view, mathematics was still more structured and answer-oriented than language arts. Also, not all the teaching techniques that she used for language arts could be used for mathematics. For example, it was too difficult to set up mathematics so that all students
worked at their own pace. However, she did insist that her mathematics program offered students experiences that other teachers wouldn’t or couldn’t as noted in this comment. “It is my impression that not too many people give kids the kind of richness of experiences in math that I do” (interview, December 16, p. 41).

In the following paragraphs I talk about features of Melany’s mathematics program that she believed made it different from other teachers. The “hands-on” problem solving investigations in which her students were regularly engaged was one feature. These investigations were taken from Marilyn Burns’ resources. Melany favored Burns’ approach to teaching mathematics because it made mathematics meaningful for students, and the investigations fit with topics in the curriculum. As I noted in Chapter Four, Burns presents mathematics differently from most textbook authors. According to Melany, the narrative style that Burns uses “almost scripted” the lessons for the teacher and it was similar to the language arts resources that she liked to use. The following is an example of an excerpt of a lesson taken from Marilyn Burns (1987) in which students explore multiplication with rectangles.

In this lesson the students investigate multiplication from a different approach—through a geometric investigation of rectangular arrays. Having children look at multiplication from a geometric perspective contributes to their concept development. Too often children’s multiplication instruction focuses heavily on learning the rules and procedures for performing multiplication calculations at the expense of learning underlying concepts. Teaching mathematical understanding must emphasize underlying concepts with procedures....

DAY ONE: MAKING THE RECTANGLES

For this part of the lesson, I used one-inch square tiles, paper ruled into one-half-inch squares, and scissors. The children were seated in small groups; there were six groups of four children each and one group of three. I had enough paper so that I could give twenty-five to each group and I had duplicated enough paper so that each group had four sheets. I put an extra stack of paper on the supply table. There were enough scissors so each student could have a pair.

I began by introducing rectangular arrays. I explained, “There are twenty-five tiles for your group. I’d like you to work with partners for this first task.” (p. 72)
Another component of Melany’s program that she thought made it different was the ideas from Barb, her former colleague. Barb had introduced Melany to Marilyn Burns. She credited Barb with “getting her into another approach to teaching mathematics” (interview, November 27, p. 27). According to Melany, Barb was “[the] one teacher at the elementary level in my twenty years of teaching who had a sense of teaching... with something of interest in math” (interview, October 4, p. 35). Barb, like Marilyn Burns, believed in the importance of a holistic approach to the teaching and learning of mathematics.

Melany pointed out that the strategies she used to make mathematics more meaningful for her students also made her program different from other teachers. One strategy, about which she talked, was the use of concrete materials. In Melany’s view the materials provided students with an opportunity “to see” ideas, and helped to build their understanding. Research (Cruickshank and Sheffield, 1988; Labinowicz, 1985) suggests that the appropriate use of concrete materials contributes to children’s understanding of mathematical concepts. Another strategy of Melany’s was to start with a basic concept when teaching new ideas and gradually move to concepts that were more complex. One example Melany used to explain what she meant was the concept of area, a topic on which she was working at the time. In Melany’s view, the basic concept for area was “covering-the-surface”. With this idea in mind, she would begin a lesson by illustrating a closed figure on the overhead. Then, she would demonstrate, for example, how to cover the surface area with standard units of measure using centimeters. Once students were comfortable measuring with centimeters, she would move to working with meters. Eventually, students would solve area problems using larger units of measure. A second example was the concept of place value, on which she was also working. For place value, the basic approach was to teach grouping by tens and hundreds using the base ten materials\(^1\). Then, she would move to teaching thousands, ten thousands, and hundred thousands. I was able to observe Melany use these strategies in her teaching during the time I spent in her classroom.
The above introduction sets the stage for understanding the three episodes of teaching that follow. As I explained in Chapter Three, I spent most days of a four month period in Melany's classroom from early morning until well after students were dismissed for home. From the ten episodes that stand out as markers, I have selected three. I have chosen these episodes because first they show how Melany implemented new ways of teaching that made her mathematics program more like her language program. Second, the episodes illustrate how Melany promoted understanding in her teaching. Third, they indicate Melany's depth of knowledge and deep understanding of mathematical concepts. Finally, the episodes reveal Melany's beliefs about the teaching and learning of mathematics and her views on understanding.

A Different Approach to Area

This first episode describes an approach to teaching the concept of area that is different from what Melany and other teachers I have observed generally use. This approach was implemented as part of the integrated social and environmental studies unit Melany designed with her colleague, Sherri. Previously I pointed out that Melany believed in the importance of making learning relevant and connecting new ideas to students' life experience. Unless a need was created for learning a particular concept or idea, Melany believed that it was best not to teach it. According to Melany the playground study, which I described in the previous chapter, was an ideal place to teach area because a need was created. To determine the most effective use of the playground, it was important for students to know the area and the perimeter of the school yard. They had to ensure that there was still sufficient space for younger students to play once the new equipment was in place. I move now to a discussion of Melany's approach to teaching area.

As we had initially agreed, I helped Melany plan the activities for area. This opportunity to work with Melany provided me with valuable learning about her attitude to mathematics. I also discovered that, unlike language arts, teachers on the grade five team
usually planned their own mathematics units. Typically, teachers follow the progression laid out in textbooks and curriculum documents when they plan measurement units. They begin with perimeter, then follow with area. Melany, on the other hand began with area and followed with perimeter. Her reasoning was that the investigations on perimeter and area that she liked to use from Marilyn Burns were laid out differently from curriculum documents.

Melany’s decision to begin with standard units of measurement was influenced by a belief that students would already have some basic ideas about area and perimeter from grade four. Also, the curriculum guidelines recommended that teachers begin with standard measurement. Her plan was to begin with an activity in which students were to find the area of a number of different classroom objects using square centimeter paper and the flats\(^2\) from the base ten materials. Then they would engage in an activity with square meters. In Melany’s view, working with square meters would prepare the students for finding the area of the playing field. By the end of the unit, she wanted students “to understand that length times width gives you area” (field notes, September 25, p. 27).

At this juncture I found myself in somewhat of a dilemma. From my experience, students at this grade level need the experience of measuring with nonstandard units before they work with standard measurement. In so doing it allows them to focus on the process of measuring, rather than being distracted by the standard units. It also provides students with an opportunity to come to understand that the basic unit of measure for area is a square, which is one unit by one unit, not a “centimeter squared” - a term that students and teachers frequently use incorrectly. Also, they would learn that area is more than just multiplying length by width, rather it is the sum of all the squares that make up the surface of the interior of a closed figure. It is also the product of the number of square units in one row by the number in one column. Holmes (1995) points out that:
Using nonstandard units, children can think about the numerical aspects of a measurement and not be concerned with learning standard units. Nonstandard units also help children be attentive to the attribute being measured and the process of measuring and contributes to the development of an intuitive understanding of measurement. (p. 364)

Melany said that she always began a new topic in mathematics with a basic concept. Based on this comment I assumed that she would start with nonstandard measurement. However, she wanted to begin with activities that focused on standard measurement. Even though Melany was a teacher, who in Shulman's (1987) terms had that special amalgam of content and pedagogy, in my view she seemed too ready to “jump in” to teach standard measurement before she found out what students actually knew about area, and before she taught nonstandard measurement. Moreover, Melany was cognizant of the difficulties that students often experience with measurement (Fennema, Carpenter and Peterson, 1989).

But I always found that [getting the right unit of measure] difficult, because they can't see that a square centimeter is different than a centimeter. They really don’t. I'd always get these answers and I'd always have to say that area is in square centimeters. And when you have to say that, I guess what that means is that they don't really know what it means. (interview, September 27, p. 13)

With these thoughts in mind, I proposed that Melany first find out what students knew about the concept of area. A teacher would need to know what knowledge students bring to the learning in order to teach for understanding (Fennema, Carpenter and Peterson, 1989). I also suggested that she start with nonstandard units. Melany was somewhat taken aback by the idea that she ask her students what they knew about area, because it was not the way that she usually taught mathematics. She usually began units in language arts by first finding out what students knew about the topic. Her response was, "Well that's not my style, but I could try anyway" (interview, September 25, p. 29). She confessed that, “If Marilyn Burns says it in her book to ask them what they know about multiplication then I'd do it" (interview, September 25, p. 29). Melany agreed to try out my suggestions and incorporated them into her first lesson.
I also want to point out that on a number of occasions Melany challenged me to show her how learning in mathematics could be individualized as it was for language arts. By individualized Melany meant that, "students could work on a task and I could wander around and see what they're doing, instead of always sort of saying, do this, do this, do this" (interview, September 25, p.32). In Melany's view teaching for understanding was not possible unless learning could be individualized. However, in her mind it could not be done: "[ I ] couldn't imagine how this could happen in mathematics" (field notes, September 25, p. 32). She also talked about the ease with which she could change direction in a language lesson as the need arose. I recall the day she approached me during language arts to point out that it was easy for her to "break up the pacing" of a language lesson with an impromptu writing assignment. She could do it even if the children were writing at different levels. However, she could not envision doing the same for mathematics: "I can't just say oh let's go look at page ten in the textbook, because I think, geez, it's going to be impossible for some of those kids to do it" (interview, September 24, p. 2).

In spite of her doubts about my suggestions for teaching the unit on area, Melany consented to try. She began the first area lesson with the question, "What does area mean?" The first child to respond was one of her brightest students. First he drew a rectangle on the board with sides that represented 2m by 5m, then he proceeded to talk about how to find "the area". He actually described how to find the perimeter. He began by writing \(2 \times 5 \, \text{m} = 10 \, \text{m}\) and pointed out that this number sentence represented the sum of one pair of sides. Then, he wrote \(4 \, \text{m}\) below the \(10 \, \text{m}\) and said that it represented the sum of the other two sides of the rectangle. To illustrate how he arrived at the two sums and explain what he meant by area he wrote the following on the board,

\[
P = l \times w = 2+2
\]

\[= 4 = 5+5 = 10 = 10.\]
At that point Melany, as other students, suggested that he was wrong and proceeded to explain why his explanation was faulty. She noted that he had confused perimeter with area and illustrated on the board that he should have written the following for finding perimeter.

\[ P = w + w + l + l = 14 \text{m} \]

A second student suggested that to find the area he needed to multiply length times width. The student followed with the formula, \( A = lw \), then he wrote it on the board. With further convincing, the first student finally accepted that he had erred. Melany then moved to questioning other students about their ideas. The following excerpt is a sample of the interchange. It illustrates the variation in students’ ideas about area and some of their confusion about the concept. It also demonstrates how Melany effectively provided the appropriate blend of support and independence as she “keeps a wide range of considerations in mind” (Ball, 1991a).

1. Melany: What does area mean? What are we talking about?
2. Second Student: Draw a box. [At this point Melany drew a box on the board.]
5. Second Student: Then you multiply them.
6. Melany: [Alluding to the first student’s incorrect explanation of area.] Okay, so that’s what he meant when he said area equals length times width.
7. Second Student: No, how you do that is length times width times height.
8. Melany: That would give you volume.
10. Melany: This works for flat. So what are we figuring out? [She pointed to the diagram on the blackboard.] Like what does this stand for?
11. Third Student: We’re figuring out this part inside here. [She approached the board and touched the inside of the rectangle.]
12. Melany: Did you hear what she said? Say it again.

13. Third Student: You figure out the inside. You put little boxes inside. [Melany divided the interior of the rectangle into small squares.]

Note the variation in students' understanding of the concept of area that is revealed in the dialogue above and in the section that follows.

14. Melany: Okay, so we’re figuring out what’s inside here.

15. First Student: Yah, that’s what I was thinking of, volume.

16. Melany: Okay, So were figuring out what’s inside here. Yes.

17. Fourth Student: Is it like if it was a mansion. Like let’s say it was a mansion.

18. Melany: If it was what?

Even though Melany was completely caught off guard by this student’s comment (17) she reserved judgment and continued with her questioning in order to uncover his thinking.


21. Fourth Student: And it was like 13 000 square feet or something like that.

22. Melany: Okay so he’s saying the area. We talked about you say something square. Where’s my eraser? Just a minute. You guys know a lot about this. So he said that it would be like a mansion is 13 000 square feet. So we measure area in squares. See these little squares. [She pointed to the little squares she had drawn in the rectangle that was illustrated on the board.] What else do you know about area?

It is from a comment like this (22) that I came to view Melany as a teacher who recognizes that students have varied interpretations of mathematical concepts that count as knowledge.

23. Fifth Student: How ugh. How many squares are inside?

24. Melany: Okay, like for example if I were to ask you which desk has the most area, your desk top or my desk top?

25. Sixth Student: Yours.
26. Melany: My desk top is much bigger than your desk top. What, the top of what?

27. Seventh Student: The top of the coat rack. [The coat rack to which the student alluded was a free standing open closet which stretched about two thirds the length of the room. The closet served as a divider between Melany’s and her colleague’s classroom in the open area where they taught.]

28. Melany: Right. So the area of the top of the coat rack is bigger than the area of the top of my desk. So we’re figuring out how much is filling in this space. Anything else?

29. Third Student: The difference between area and volume. Area, like if you were to put boxes in this part of the room, like from there to there and all around. [She pointed to the part of the room where they were sitting between the window and the closet where the students hung their coats.] For area, we would only put the squares on the bottom. For volume, you put them from bottom to top.

30. Melany: Yeah. We would fill it right up.

(audio tapes, September 27, p. 19)

In summary, the dialogue reveals Melany’s ability to effectively facilitate “interactive discourse and thoughtful discussion between teacher and student and among students... that will lead to clear understanding” (Good, McCaslin and Reys, 1992). As a teacher who keeps a “wide range of considerations in mind” (Ball, 1991a), Melany recognizes that, what counts as knowledge in mathematics is open to interpretation, which in my view is a necessary prerequisite for teaching for understanding. The following excerpt taken from a conversation I had with Melany after the discussion provides added insight.
Oh, okay. I guess I didn’t think that they would have so many ideas about area. They had some interesting ideas about area and they were coming up with the logarithm [sic], you know of length times width. The first student went into great lengths to show how you do length times width and he had done length plus width to do perimeter. And I don’t think that he really liked me suggesting that he hadn’t done any time at all, that he had done all adding. So any ways, the only kid, the first kid who got it was the kid who said that it’s all that part in there and used their hand to show the part, you know, inside the lines. So then I knew that some of them understood it, but the whole idea of length times width gives you area was something that didn’t seem to make sense to them. It was something that they had obviously been taught and they memorized it, but they really didn’t understand why it worked. So that was really interesting for me.

interview, September 27, p. 12

I want to shift now to a discussion of the activity on nonstandard measurement that followed. It was the first time that Melany had used nonstandard units in teaching measurement. For this activity students had the choice of working with a partner in a small group, or on their own. The objective was to find the area of three classroom objects using three different nonstandard units of measure which the teacher provided. The units consisted of two different size and color squares cut out of construction paper, and the flats from the base ten materials. The use of the flats resulted in some unanticipated outcomes.

Melany began the activity with a demonstration on the board of how to record the area measurement for each object in a chart. The sequence students were to follow was first estimate the number of squares needed to cover the surface area of the object, then record the estimate. Secondly, they were to lay down the squares on the object and find the actual measure of the area. This measurement was also recorded in the chart. Each object was to be measured three times using a different size square for each trial. The purpose of having students measure the same object with different size squares was to help them see that the area measure of a figure is determined by the size of the unit. In other words, the larger the unit the smaller the measure that represents the area. The reverse occurs as the unit decreases in size. After checking to ensure that students knew what to do, Melany set them to work.
As Melany and I walked about the room observing and conferring with students, we were fascinated by the range of responses. To provide a framework for discussion of these responses, I have adapted Davis' (1978) model of understanding based on moves that students make that demonstrate their understanding of procedures (see Table 2) in order to classify our observations.

Table 2  Adaptation of Davis' (1978) Model of Understanding

<table>
<thead>
<tr>
<th>Level 1:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding how the procedure works</td>
</tr>
<tr>
<td>Children understand the procedure to the extent that they can make the following moves:</td>
</tr>
<tr>
<td>1. Accurately carry out the procedure as instructed</td>
</tr>
<tr>
<td>2. Execute the procedure accurately in more than one way</td>
</tr>
<tr>
<td>3. Show another student how to carry out the procedure</td>
</tr>
<tr>
<td>4. Explain the procedure step by step</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding why the procedure works</td>
</tr>
<tr>
<td>5. Give a plausible explanation or proof justifying the procedure</td>
</tr>
<tr>
<td>6. Recognize the applicability of the procedure in new contexts</td>
</tr>
</tbody>
</table>

There were some students who, as instructed, used one size square at a time and accurately counted partial squares to find the surface area of an object. In my view, their ability to carry out the procedure as instructed would qualify them as a Level 1: (1) in their understanding of how the procedure works. Of this group of students, there were some who realized that if they laid down one row of squares in the width and another in the
length, they could easily find the area. Because they were able to execute the procedure in more than one way and to provide justification I would classify them at a Level 2: (5) in understanding. There were some who covered the surface area and counted all the squares to find the area. Even though they laid the squares down correctly, they did not count accurately. They assigned partial squares a value of one. Following Davis’ (1978) adapted model, I would classify these students as close to Level 1 in understanding, however they still have not achieved this plateau. In my view, students who laid down different size squares on the same object because they wanted to ensure that “everything fit evenly” (field notes, October 21 p. 81) had very little understanding of the process. At the lowest end of the range of abilities, there was one student who had no idea that to accurately measure area it was important that the squares were aligned so that there were no spaces in between. Some students, though few, appeared to have such limited understanding that they failed to realize that the squares were the units of measure that determined the surface area of the objects. At the other extreme there was one student who surprised Melany with his sophisticated use of the flats to find area. He “proved able to apply knowledge in new situations, without applying such knowledge erroneously or inappropriately... spontaneously, without specific instruction” (Gardner and Boix-Mansilla, 1994, p. 200). The following incident provides evidence.

When Melany selected the flats as one possibility for the activity, she assumed that students would use them for measuring in the same way that the construction paper squares would be used. However, this student did not. He reasoned that each flat was equal in area to one hundred square centimeters. To cover the surface of the object that he was measuring it took four whole and two partial flats. Each partial flat was equal in value to 70/100, and each whole one to 100/100. With this idea in mind he wrote the following sentence to represent the surface area of the book, noting that the measurement was in square centimeters:

\[ 2 + 2 \cdot \frac{70}{100} + \frac{70}{100} = 5.40 \text{ cm}. \]
In conversation with Melany later that day she said that she would never have realized his exceptional ability in mathematics if she had not done this activity. From my experience, few grade five students could demonstrate the understanding of fractions and decimals that he displayed. Moreover, not many students at that level would have been able to apply these ideas to the process of measuring as spontaneously as he did. In my view this student demonstrated genuine understanding (Gardner and Boix-Mansilla, 1994). I should add that Melany also noted that he possessed a similar ability in language arts.

As a follow up students were asked to reflect in writing about what they had learned from the activity about measuring with nonstandard units. Although this kind of writing was a routine occurrence for language arts, it was the first time that she had used it for mathematics. Students’ responses provide further insight into their understanding of the concept of area. The following are examples.

1. First Student:
Area
I measured my Mathquest book. I used 4 flats and 8 small. I used the 4 flats because it covers most of the book. I didn’t use the same objects because they didn’t fit. I used different sizes because you are supposed to make it fit right.

It is obvious from this reflective piece that the student had minimal understanding of the procedure. In order to correctly measure the surface area of an object, a student would need to use only one size unit. In my view, this student has not achieved the first level of Davis’ (1978) adapted model of understanding of a procedure.

2. Second Student:
I measured my math quest. I used tiles. I made two rows. One along the width and one along the length. Then I counted both of the rows and then I multiplied then my answer was 80. I picked the tiles because when I finished my math quest would be colorful.

It appears that this student was able to carry out the procedure for finding the area in nonstandard units in other than the prescribed way. From my view, the student does not necessarily provide a plausible proof which suggests a Level 1: (4) understanding.
3. Third Student:
I measured my chair top. With one big orange square and 5 small yellow squares because all of them fitted on perfectly and if I used two big orange squares than it wouldn’t fit.

On the basis of this student’s writing it is obvious that there is little understanding of the concept of measuring with square units in order to find the area of an object.

4. Fourth Student:
I measured my ruler with 12 colored squares, because if I used 12 yellow squares than it deffentley wouldn’t fit perfectley.

I suggest that this student’s attempt at reflective writing demonstrates very little understanding of the concept of measuring with square units to find the area of an object. As I noted earlier Melany had several ESL students in her class. From the spelling it appears that this student was a second language learner.

5. Fifth Student:
I measured my math quest cover. I used orange flats. First I put the flats across. Then I put them down and there was 4 but there was 6 tens more so I put down 4.60. I used the same size squares because if I used different squares it would be a different number that I got.

It is evident from this student’s reflections that the flats were used for measuring. The student demonstrates an awareness of the part whole relationship of decimal numbers represented by the flats. However, the student incorrectly uses the term tens when it should be tenths because .60 is the same as six tenths. This student would probably classify as a Level 2 learner.

6. Sixth Student:
Aria
I chose small sqwars becuase it was easier for me I like when I chuse esear ways for myself. I measured my desk top. I measured it with small sqrs. I used all of the same sizes. I used et becaus it was isier.

The writing suggests that this student was another of Melany’s ESL students who was in the process of developing his English skills. The student had the right idea about
how to carry out the measuring, but did not have a plausible explanation suggesting a Level 1 (1) understanding of the procedure.

(field notes September 27, p. 44a)

In the following excerpt Melany offers her reflections on the activity. As I noted earlier, in Melany's view, unless learning could be individualized and students had the opportunity to talk about their ideas then they would not be learning for understanding.

What I like about the approach that I did in math is that it does really give those kids an opportunity to show you the extent of their ability to think in their thinking. The other way I would have taught area would never have afforded them the opportunity to show me that. I think I gave them some basics. I think I gave them a bit of structure. You know like we're talking about. Then, I provided them with some materials that are appropriate to the activity. Like for poetry I don't give them novels, you know. And I, you know, gave them a format to sort of get them focused. Like you know they had to make a little table in their notebook because I just could have visions of thirty two kids all running to the table... So I think I just give them a few; it's kind of similar in a sense that I've given some structure and a purpose and an appealing activity to do. I mean I didn't see anybody not doing it. I mean they were all quite engaged in the task, and they all had different ways of approaching it. So, it was quite easy. It was quite individualized and it was quite easy. Remember I was saying to you, I challenge you to come up with something where I can do something individualized in math. I'm always looking for it, right! And in this case I was conferencing with kids with different situations with what they were doing. Yeah, it was quite similar to my way of doing language, very similar, yeah.

(interview, September 27, p. 18)

It was at this point that Melany decided to switch from nonstandard units to standard measurement. She was anxious to make certain that students knew how to find area with standard units. I was not as convinced as she that students were ready to make this transition. Nevertheless, she had tried out new ideas that made mathematics more like language arts for her, even when the strategies were not at her fingertips.

Yeah. This is definitely a new approach for me and I liked it because they come up with things, like it's totally unpredictable. So for me, as a teacher, it keeps it interesting and fresh for me. Oh, now what am I going to do with this idea? I mean because I know what area is. It's not like I'm shaky with the content. So I'm quite happy to see what they come up with. And to tell
you the truth, when I think about how I might have approached it another way. I'm thinking, I would have stood up in front of the class and eventually would have got them to see length times width equals area. And I would have thought, oh that's great they're getting it. And I wouldn't have ever really. I don't think I would have realized how much they weren't getting it. This way, I understand where they are at any ways and who knows. The interesting thing is to see how can I extend their understanding. And maybe, we should have a discussion at the end about it. See what happens.

(interview, September 29, p. 12)

In summary, the episode I have just discussed offers insight into Melany's beliefs and views about teaching mathematics and her content knowledge - factors that affect a teacher's understanding of teaching for understanding (Prawat, 1989). Comments such as, "I couldn't imagine how this (individualizing her program) could happen in mathematics" (field notes, September 25, p. 32) provides evidence of Melany's belief that mathematics could never be taught like language arts. Melany's ability to uncover students' understanding of concepts, in my view, suggests she is a teacher who possesses a deep understanding and knowledge of the discipline.

I now move to the second episode which focuses on strategies that Melany employed to develop students' understanding of place value. Although this episode is not as extensive as the other two that I selected, I chose it because place value is a topic with broad application and importance in students' learning of a variety of mathematical concepts. Frequently students' difficulties with number concepts and procedures can be traced to their weak understanding of place value (Cruickshank and Sheffield, 1988). I want to add that place value is an area of mathematics about which teachers themselves have fuzzy understandings (Graeber, Tirosh, and Glover, 1989). Also, the episode provides further insight into Melany's understanding and knowledge of mathematical concepts, particularly the importance of grouping in students' understanding of place value.
What Is Place Value About?

Place value is the basis for work that students do with whole numbers, fractions, and decimals. From my experience, students who do not understand place value frequently have difficulty with trading of whole numbers and decimals (Cruickshank and Sheffield, 1988), changing fractions to percent and vice versa, correctly placing the decimal in multiplication and division, percent, ratio, proportion, and probability. As a characteristic of number, place value determines the value of a digit in relation to its position in a number. Students' knowledge of grouping is fundamental to their work with place value.

As most elementary teachers, Melany taught place value in the first term. When I arrived in her classroom she was working on multiples of ten and one hundred, standard and expanded notation, and large numbers. From my observations, there was a particular routine to Melany's lessons. She began by taking up a brain teaser or problem that had been assigned in the morning when students arrived for class. A short introduction to the lesson, which often included a discussion about a question on the chalk board, followed. One question I recall asked students to prove how a set of numbers are multiples of ten. At the grade five level a question such as this suggests that students show how these numbers can be divided evenly by ten.

How can you prove that these numbers are multiples of ten?
   a) 70
   b) 80
   c) 100
   d) 500
   e) 1000

As students discussed the question, Melany would record their input on the chalkboard. When the exercise was completed, students were assigned practice questions to answer in their notebooks. As they worked, Melany and Janet, her educational assistant, would move about the room observing and conferring with the students to ensure that they
were able to answer the questions. When Melany was confident that they understood what to do, she would assign questions from the textbook which were either completed in class, or finished at home for the following day.

On the particular day of this episode, Melany followed her usual routine. First she took up the homework, which in this instance focused on standard and expanded forms of numbers. I noticed that some students had difficulty figuring out the place value of the digits in numbers such as 5089. One student suggested that the expanded form was 5000 + 800 + 90 + 9. Another thought that it should be 5000 + 0 + 8 + 9. However, Melany effectively dealt with the problem with the use of concrete materials and pictorial images.

Once the homework was taken up, students were asked to put away all textbooks. This instruction was greeted with a loud sigh of relief from students as it signaled no further textbook work for that afternoon. Melany then walked about the room distributing two or three handfuls of macaroni to pairs of students. On completion of the distribution, students were instructed to work with their partner to find a way for figuring out, without counting each piece separately, how many pieces they had on their desks.

I'm going to give you some macaroni. I'm going to give you a pile of macaroni and I'd like you to keep them on your desk for starters. What I'd like you to do is figure out how many pieces there are. I'd also like you to do it in such a way, like say you say to me, there are sixty four here. I'd like you to somehow put them on your desk so that I can quickly check to see that you are right, that I don't have to go through and count each one.

(audio tapes, September 18, p. 5)

Melany was careful not to tell students how to organize the macaroni, only that the strategy they used had to be obvious to her. Even when students requested further direction, Melany offered her support without leading them— a technique she frequently used in language arts. I sensed that she wanted to see how students would work through the task of grouping the macaroni to show the numbers they chose to represent. I should add that earlier that day Melany had talked briefly about her concerns regarding students' learning
of place value. In her view place value seemed to be more of a rote process, and students needed to have “some understanding”. According to Melany, it was important for students to “see” what numbers looked liked, especially large numbers, and she wanted to provide them with activities that would improve their understanding.

The following dialogue reveals how students went about the task of organizing the macaroni. It demonstrates the way in which Melany facilitated “interactive discourse and thoughtful discussion between teacher and student and among students... that will lead to clear understanding” (Good, McCaslin and Reys, 1992, p. 130). It also shows how Melany provided a balance of support and independence.

1. Melany: Yes.
2. First Student: Can you put them in groups of ten?
3. Melany: Yeah. You can group them by ten.
4. Second Student: By twenty?
5. Melany: You decide how you want to group them because grouping is the key to it all. Yes?
6. Third Student: What if you have one less?
7. Melany: So then you're going to have to figure out what happened to that. Okay. You can't throw it away. It has to stay on your desk. Remember that first rule I gave you. It has to stay on your desk. Don't put these on your lips please because you don't know who's been touching them. You don't want to be putting them in your mouth. You don't know who's been touching them. So they're staying on your desk.

Once Melany set the students to work she walked about the room examining the different representations. After every student had completed at least one example, Melany stopped the class to discuss their representations as the following sequel of the dialogue illustrates. I want to draw attention to the way in which Melany focuses on the idea of grouping to elicit students' knowledge of number and place value.
8. Melany: What kind of groupings did you use? All of you finally figured out that you could group them and easily figure out. So how many did you put in a group?


10. Melany: Okay, so then how many groups do you have there?

11. Fourth Student: I have ten.

12. Melany: [Looking at the student’s representation] You have ten groups? You have eleven groups.


14. Melany: You have eleven groups. She has eleven groups of ten. So how much is that?

15. Fifth Student: One hundred and ten.

16. Melany: Do you agree?

17. Sixth Student: Yeah.

18. Melany: How did you group yours?

19. Sixth Student: Groups of five. We have one hundred and seventy five.

20. Melany: You have one hundred seventy five altogether. So you counted five, ten, and were quickly able to figure out how many you had.

21. Sixth Student: Yeah.

22. Melany: You girls, how did you group yours?

23. Seventh Student: First we divided them into fives and then we put them into tens.

It is evident from the previous dialogue that Melany is a teacher who does more than “just present information” (Alkin, 1992). The following section demonstrates how she connects ideas as she continues to probe students’ understanding of number concepts.

24. Melany: How many groups of ten do you have?

25. Seventh Student: Fifteen.

26. Melany: How many groups altogether?

27. Seventh Student: Fifteen.
28. Melany: Fifteen groups of ten and anything left over?
29. Seventh Student: Five.
30. Melany: So how much is fifteen groups of ten and five left over?
31. Eight Student: Twenty.
32. Melany: Twenty would be two groups of ten. Fifteen groups. I’m going to use the short form of ten. [She proceeded to write this amount on the board.] How much is that?
33. Ninth Student: One hundred and fifty.
34. Melany: And five left over, right? So how do you get one fifty so quickly? How did you get this was one hundred fifty and five more? How did you do that?
35. Ninth Student: Well if I take away five, and it’s ten times ten is one hundred and ten times five is fifty.
36. Melany: Does anyone have another way of getting one hundred fifty so quickly?

(audio tapes, September 18, p. 6)

The dialogue that I have just presented reveals that Melany is a teacher who knows how to ask the right questions in order to uncover students’ knowledge of mathematical concepts. She also shows how she effectively uses concrete materials for doing so. As she suggested in a conversation that I had with her after the lesson, “It’s just nice to have concrete proof that they understand grouping” (interview, September 18, p. 12). To finish the lesson Melany followed with a brief discussion of ten and its importance in our decimal number system. From there she moved to activities which focused on grouping by hundreds, then thousands.

In summary, Melany’s teaching in this episode is a contrast in perspectives. At the same time that she taught lessons that were “tied to the textbook”, she also provided opportunities for students to explore concepts and ideas in order to create knowledge (NCTM, 1989, 1991) as the grouping activity demonstrated. It is also evident that Melany was a teacher who understood that students needed to have a clear grasp of the concept of
grouping “in order to be quick at their math and be able to show that they know what they are doing” (interview, September 18, p.12). Comments such as “grouping is the key to it all” (audio tapes, September 18, p.5) in section (5) of the dialogue further demonstrates that Melany recognized the importance of students’ understanding of grouping as a prerequisite for working with number and place value. As research (Fennema and Franke, 1992) suggests, “... what a teacher knows is one of the most important influences on what is done in classrooms, and ultimately on what students learn” (p. 147).

To complete the portrait of Melany’s life in the classroom I move to the third episode of this chapter which describes Melany’s approach to teaching three dimensional geometry. As Cathcart, Pothier and Vance (1994) suggest, “... geometry learning for elementary school students should be informal, involve explorations, discovery, guessing, and problem solving” (p. 100). The work on geometry, as I noted in Chapter Four, was part of a science unit on structures. Similar to the first episode, Melany was trying out new ideas that would make mathematics more like language arts. In so doing, these ideas were intended to provide opportunities for students to explore concepts and ideas in order to create knowledge in the course of active engagement with the discipline (NCTM, 1989, 1991).

**Science as Mathematics, Mathematics as Science**

From my view, geometry, particularly 3-dimensional geometry, is an area of the curriculum that teachers often find troublesome to teach. Either they leave it to the end of the year, or they skip over it completely. For some, it is the content knowledge that is challenging, for others it is having the appropriate strategies. In some instances, it is both. The availability of concrete materials is also a problem. Although Melany taught 3-dimensional geometry as part of a science unit on structures, she limited her teaching to one activity in which students identified the faces, edges and vertices of the structures. It was not the content knowledge or a lack of confidence that was a problem for Melany, it was
having suitable teaching resources in the style of Marilyn Burns. “And I mean there might be some content [in geometry] that I’m not familiar with, but I can easily figure it out.... It’s the pedagogy” (interview, December 31, p. 49).

As an aside, in my judgment as an educator who has worked with students in mathematics classrooms, one activity would not suffice to develop their knowledge of 3-dimensional geometry at the grade five level. Neither would it meet the objectives of the curriculum. Research (Fuys, Geddes, and Tischler, 1988) suggests that students need a variety of learning experiences in order to develop their knowledge of geometric concepts. I suggested that Melany expand the geometry component of the science unit to include different sorting and classifying activities to increase students’ knowledge and understanding of the solids. Melany agreed on condition that I help her with the planning. As I pointed out in Chapter Three, in some instances in conducting research it may appear that “overparticipation” (Bogdan and Biklen, 1992, p. 88) has occurred. However, the nature of my participation fit within “the participant/observer continuum” (Bogdan and Biklen, 1992, p. 88) and the goals of this study.

I worked with Melany to design a series of 3-dimensional geometry activities as part of the structures unit. One goal of the activities was to ensure that students were able to identify the solids by name and describe their features. Another was for students to develop their knowledge and understanding of the solids. For example, they would be able to distinguish between those solids that were polyhedra and those that were not and provide reasons for their choices. In a similar manner they would be able to distinguish between pyramids and prisms. Moreover, they would know that a triangular prism could not be classified as a pyramid because it does not have a minimum of three triangular faces that meet at the apex, and so on. Also, they would come to understand some of the less obvious aspects of solids such as why a cube could be classified as a rectangular prism.

There were five activities in all, however I have chosen to focus on two as they provide valuable insight into Melany’s beliefs and practice in mathematics and her views on
understanding. The first activity was introduced in science class. For this activity students were divided into groups of four and given a collection of the following solids to sort: cylinders, cones, rectangular prisms, triangular prisms, hexagonal prisms, cubes, spheres, square base pyramids, tetrahedra, and pentagonal base pyramids. Rather than telling students how to group the solids, Melany encouraged them to choose their own categories. In this way she could use the activity to assess their knowledge. Also, as I noted earlier, Melany preferred to allow students to develop their own understanding of ideas and concepts rather than telling them. In her view, telling was not as effective for promoting learning as the following suggests. “I don’t like telling them. Like what’s the point in telling them, right. The kids who already know it, know it. The kids who don’t know it won’t get it from me telling them.” (interview, November 8, p. 21)

The following dialogue, which occurred at the beginning of the lesson, illustrates how Melany encouraged students to take responsibility for their learning and construct their own knowledge of concepts. Yet, at the same time she ensured that they understood the instructions – a technique she frequently used in language arts.

1. Melany: Each group is going to be getting a collection of solids. They’re called 3-D solids. They not only have length as a dimension, and width as a dimension; they have length and width and.

2. First Student: And height.

3. Melany: Or maybe I should say they have length, they have width, and they have depth or height. So they’re called 3-D, 3-dimensional solids. They’re solid. Now you’re going to be given a set of these at your table, at your group. We always have to work in groups of four. Okay here’s your instructions. [At this point she turned on the overhead projector to display the instructions.] Okay it says sort. What does the word sort mean to you?

4. Second Student: Sort them out like.

5. Melany: Who can help him?

6. Third Student: Fixing them up.
7. Melany: Fixing them up?
8. Fourth Student: Organize them.
10. Fifth Student: You have a scattered pile. If you have a scattered pile of objects...

[At this point Melany asked the student to speak up because the class could not hear him.]

11. Fifth Student: If you have a pile of objects, sorting out means you take the same objects and put them into different groups.
12. Melany: Okay so you have the same objects and you put them in groups. Anybody want to agree or disagree with him?
13. Sixth Student: I agree with him.
15. Eighth Student: I agree with him because it means putting them into groups.
16. Melany: So you're going to be putting them into groups and they have to have something the same in their groups. So, for example. No I'm not going to give you an example. Read question one.

[A student read question one.]
17. Seventh Student: What happens if I say that this goes in this group and someone else says it goes in a different group?

Note below (18) how Melany encourages students to keep an open mind to the variety of possibilities that they could use for sorting.

18. Melany: Yes. So if one person says, "I think that this one goes in this group". And someone else says, "No, I think it goes in this group". So the first person has to say, "Well this is my rule for putting it in this group because all these shapes have the same whatever". Then, the second person disagrees and says, "I think that it should go in this group because all these shapes have this". So, if you have a disagreement, then you can sort it in two different ways. Sort it with the first person's rule, then with the second person's rule.

19. Ninth Student: What about judges?
20. Melany: No. You don’t have to have judges. There can be more than one way to sort them. Now number two is where you have to do some work. Read number two.

21. Second Student: “Record the reasons for sorting the solids the way that you did.”

22. Melany: Record means write. You write down in your notebook. That’s why you have it there. You can write down all the shapes in this group go together because, and you write down your reason. And if one person has one reason and another person has a different reason, you can write down two different ways. Does anyone have any questions about that?

(audio tapes, November 8, p. 7)

I want to draw attention to Melany’s use of the term “dimensions” at the beginning of the dialogue. In my view it is important that teachers introduce this term in mathematics because it provides students with a more flexible view of what constitutes length, width or height respectively of any figure or 3-dimensional object. Often students, as do their teachers, assume that the length of any figure or 3-dimensional object must always be the longest side. However, this assumption may not necessarily hold true. Once the figure or object is rotated, the longest side may no longer be the length. In fact, it may be the width or the height.

During the activity Melany walked about the room observing and talking with students about their work. Only when she thought it necessary, or when a student requested, did she intervene to offer assistance. Otherwise, she allowed the natural ebb and flow of learning to prevail as they worked through the task. On completion, students reported on the results of their sorting. Their responses demonstrated their varied ability and knowledge of the solids. Some groups sorted the solids in very rudimentary ways, such as those that rolled and those that did not. Other groups sorted them in more sophisticated ways. For example, one group of students “put all cubes and rectangles together because they had six faces, all pyramids and rectangles together because they had five faces, and all hexagonal and octagonal prisms together” (field notes, November 8, p.
Another group said that they put "all cylinders together because they were round and they rolled; triangular and hexagonal prisms together because they were flat" (field notes, November 8, p. 132).

In discussion with Melany later that day she noted that the activity provided her with valuable learning about students' knowledge of the solids. In her view, students who could only classify the solids according to whether they rolled or not had a very limited understanding. Students who could categorize the solids in more than one way, such as by the shape and number of faces, demonstrated a higher level of understanding. Students who sorted and classified the solids in more than two ways, with added detail, had an even higher level of understanding. This category scheme for assessing students' performance marked the first occasion in which Melany openly talked about what understanding in mathematics meant for her. As I noted in Chapter Four, Melany was reluctant to talk about understanding "in the abstract" (interview, October, 17, p. 39) because it was not "a generic concept which was inclusive to all disciplines" (field notes, October 7, p. 66). For Melany, understanding had to be connected to a particular concept or content area.

There were four more activities which afforded students the opportunity to apply previous learning to the task at hand as they sorted and classified the solids in new ways. In so doing learning evolved from the general to the specific. Once students could classify the solids according to those that were polyhedra and those that were not and give reasons for their choices, they moved to the next stage. At this level they worked only with those solids that were polyhedra to determine if they were prisms or pyramids, and providing reasons for their choices. Further activities focused exclusively on pyramids, then prisms, and so on. Each activity culminated in a public audience, a language arts ritual that was now a regular routine for mathematics.

I want to talk now about the fifth activity. It was the culmination and final development of the work on the solids. It occurred the day before the quiz. The activity stands out because of the level of discussion that occurred just before students were to
create their definitions of the prisms and pyramids. Although students had spent considerable time working with the solids the previous day, Melany was concerned because she was not sure how deeply they understood prisms and pyramids. Like any teacher, Melany preferred to have her finger on the pulse of the class and know exactly what was happening.

Melany gathered the students on the carpet at the back of the room. When they were ready, she placed different types of pyramids on the carpet and posed the question, "What is a pyramid?". The students easily talked about the pyramids naming them and discussing their distinguishing features. Just as they were finishing the discussion about pyramids, Melany shifted to prisms. The dialogue that follows portrays the varied levels of understanding of the solids that emerged from Melany's discussion with the students. It also reveals Melany's skill as a facilitator, and her ability to uncover students' understanding which in my view are characteristics of a teacher who teaches for understanding. As the dialogue unfolds I unpack the different levels of understanding according to an adapted version of Davis' (1978) model of understanding based on moves students make that demonstrate their understanding of a concept (see Table 3) is used.

| Table 3  Adaptation of Davis’ Model of Understanding |
|-----------------|---------------------------------|
| Level 1:        | Children understand a concept to the extent that they can make the following moves: |
|                 | 1. Give or identify examples of the concept |
|                 | 2. Defend choices of examples of the concept |
|                 | 3. Give or identify nonexamples of the concept |
|                 | 4. Defend choices of nonexamples of the concept |
| Level 2:        | Characteristics of the concept |
|                 | 5. Identify things that are necessarily true about examples of the concept |
|                 | 6. Determine properties sufficient to make something an example of the concept |
|                 | 7. Tell how one concept is like (or unlike) another concept |
|                 | 8. Define the concept |
|                 | 9. Recognize the applicability of the concept in unfamiliar contexts |
The dialogue begins just as the class finished talking about whether a pyramid could have more than one apex.

1. First Student: So the bottom of the triangular faces [of a pyramid] meet at the base. At the top the triangular faces stop at the apex.

2. Melany: Okay, so there is only one apex. Now the other question that I have for you is, what is a prism? We have some shapes here. [Melany held up some cylinders and different shape prisms.] I want you to tell us, which one doesn't belong and why. Which one doesn’t belong?

3. Second Student: The one with the rounded sides.

4. Melany: Tell me what it’s called.

5. Second Student: A cylinder.

6. Melany: Why do you say that the cylinder doesn’t belong?

7. Second Student: (pointing to the cylinder) That one of them is kind of like a circle. It goes around. It’s kind of circular and the others [the prisms] don’t roll.

Although the student (7) knew that a cylinder was not a prism, her reason for not including it suggests a limited understanding of the criteria that determine a prism. I suggest that the student was possibly at a Level 1: (3) in her understanding of the concept.

[Melany acknowledges a third student.]

8. Melany: Okay, what do you say to that?

9. Third Student: I think it does belong, because aren’t those faces? [He pointed to the two opposite ends.]

It appears on first glance from the student’s comment (9) that she was correct in suggesting that a cylinder could be classified as a prism because the opposite ends of a prism are the faces. However, on closer examination the response is problematic. A cylinder can not be classified as a prism because the opposite ends are not joined by rectangular faces. I suggest that this student had not achieved Level 1 in understanding of the concept.
10. Melany: That’s a good question. What is a face?

11. Fourth Student: [pointing to the base of the cylinder] The circle part on the side. It’s got two faces.

12. Melany: This part here. [Melany pointed to the base and its opposite face.] Okay. So, it’s got two faces. Why is this not a face [running her finger across the curved surface of the cylinder]?

13. Fourth Student: Because it goes around and around.

Melany’s ability to effectively probe students’ ideas to elicit their understanding of the solids provides evidence that she is a teacher who possessed the pedagogical content knowledge that teachers need in order to teach for teaching for understanding (Fennema, Carpenter and Peterson, 1989). As Alkin (1992) points out, teaching for understanding involves more than presenting information.

14. Melany: Okay. It goes around. How else might you say that this is not a face?

15. Fifth Student: Okay the center part isn’t a face because the faces can stabilize a shape. The center part is not a face. Like for instance, it’ll sit right like that. [The student pointed to the triangular face of a prism.]

In my view, this student’s (15) explanation, albeit unconventional, suggests a deep understanding of the structure of prisms. I would easily classify this student’s understanding at the upper end of Level 2 of the table.

16. Melany: Another reason that this is not a face [pointing to the curved surface of the cylinder].

17. Sixth Student: Because, all faces are flat, and that’s round.

18. Melany: Say that again.

[The student repeated his comment.]
19. Melany: So they have to be flat, and this is not flat. [running her finger along the curved surface of the cylinder]. Okay. So now this does not belong. What is this called?


[Melany removed the cylinder and continued with the prisms.]

21. Melany: [pointing to the prisms] Now what are these shapes called?

22. Eighth Student: Solids

23. Melany: Solids, that’s true. But, also pyramids are solids. These are something even more special than that.


As I suggested earlier in the dialogue, this student (24) was able to identify prisms, but appeared not to possess an in-depth understanding of the criteria that qualify a solid as a prism.

25. Melany: Okay, they’re prisms. So, let’s take a look at them. What is a prism? [pointing to the prisms] So these are all prisms. Can you tell me what a prism is. Who would like to try?

26. Eighth Student: I think it’s a shape that is sort of stretched. Like someone took it and stretched it and it is 3-D.

The previous student’s (26) description of a prism reveals his depth of understanding of the structure of solids, which suggests a Level 2: (5) understanding of the concept. Note in the section that follows how Melany uncovers these students’ understanding of the solids.

27. Melany: But a cylinder is not a prism, and it’s stretched.

28. Eighth Student: It’s like the real shape of a prism is 2-D. It’s really a 2-D shape, and when you stretch it, it’s called a prism.

29. Melany: So you’re making a 2-D shape into a 3-D sort of thing. What can you tell us about the faces on each end of a prism? Look on the faces at the end of the prism.
30. Ninth Student: [Speaking out before others had a chance to put their hands up] They're the same.

31. Melany: Would you let people think for themselves. Does everyone agree that the faces on each end of a prism are the same?

32. Tenth Student: Yeah.

33. Melany: Now if you look at the faces on the side they are all a particular shape. What is the shape of this face? The face on the side, what shape is it?

34. Tenth Student: Rectangle.

35. Melany: Do you agree with him that all of these faces are rectangles?

36. Eleventh Student: Yeah.

37. Melany: Okay. This is another prism.

(audio tapes, November 11, p. 30)

The discussion continued until all the different prisms had been covered. Then Melany had the students return to their seats to develop their own definitions for pyramids and prisms. In her view, if students could write ideas in their own words, it demonstrated understanding. Also, writing required a higher level of understanding than articulating ideas, as she suggests in the following comment. “I do think it is more difficult to be able to write it in your own words so it’s clear and correct” (interview, December 16, p. 9).

According to Melany, in her experience, this discussion was the most outstanding example of students’ ability to express their mathematical knowledge. For her, it was the level of thinking that students demonstrated, as she notes in the following comment. “A child who is able to express ideas clearly in their own words rather than regurgitate something she’d read somewhere, understands” (interview, December 16, p. 8). In her view, the students were so skillful in their ability to communicate with each other, they “bypassed her”. Teaching this unit on geometry was a unique experience for her. She explained, ”I think I got a lot more out of that [unit] than I ever have before in math” (interview, December 16, p. 7). In her view, mathematics had become even more like language arts. Students were discussing ideas and learning from each other. They were
doing more than "just memorizing terms"; they were learning concepts and they were
"explaining what they thought and why" as the following reveals.

Students are responsible for figuring out their own process. They’re responsible for figuring out their own way to come to the answer. They’re responsible to solve the problem in a way that makes sense to them. They’re responsible to articulate how they got that, and they’re responsible to explain how they got that. (interview, December 9, p. 3)

In summary, this episode further illustrates Melany’s comfort level with teaching mathematics, particularly 3-dimensional geometry. It also reveals that Melany was a teacher who employed the appropriate blend of content knowledge with pedagogy in order to integrate mathematics with science in order to teach for understanding. Lastly, the episode shows that Melany understood what counted as knowledge in students’ learning of mathematics.

During that whole unit on 3-D geometry, there was a whole lot of thinking going on.... Yeah, it was great. I really enjoyed it, and I was just amazed at their thinking. And I was amazed at who was coming up with definitions, you know. Like Becky just knows; she understands math concepts. I would have thought that, looking at her language. No, I shouldn’t say in language, because that is language, but in reading and writing.

(interview, December 16, p. 7)

I would now like to turn to the concluding part of this chapter in which I briefly talk about Shulman’s (1986, 1987) conception of pedagogical content knowledge to which I alluded in Chapter Four. As I noted in Chapter Two, Fennema, Carpenter and Peterson (1989) point out that teachers’ pedagogical content knowledge is a critical component in children’s learning of mathematics with understanding. Borko et al. (1992) suggest that teachers need this kind of knowledge in order to make informed decisions for effective instruction in mathematics. Although there were several instances in which Melany demonstrated her ability to relate subject matter knowledge with pedagogical knowledge, this incident particularly stands out.
Pedagogical Content Knowledge

In a discussion of pedagogical content knowledge Shulman (1986) suggests that a teacher needs to have knowledge of the content, an understanding of the knowledge and background that students bring to the learning, and an awareness of the stages they will pass through in developing mastery. The teacher must also be able to transform content knowledge so that it is comprehensible to students. There were many instances when Melany demonstrated these traits, not only in language arts, but also in mathematics. It was as if she had a "sixth sense" when it came to making the right move at the strategic moment in a lesson, as she demonstrates in this particular incident. She was particularly aware of students' varied levels of understanding of concepts and was able to accommodate these in her teaching.

The incident on which I wish to focus occurred in the second geometry activity during the science unit on structures. Earlier in the day Melany had discussed her plans for the activity with me. She decided that first students would sort and group the solids according to those that had only flat surfaces and those that did not. The purpose of this activity was to differentiate solids that were polyhedra from those that were not. The activity was to be followed by a discussion which focused on the sorting process and the different ways that the solids had been grouped. The succeeding day was reserved for making certain that students knew the appropriate names for the solids and how to write them correctly. Yet, I noticed that once students began to talk about their different groupings after the sorting had been completed, Melany stopped the lesson and proceeded to draw a chart on the chalkboard. She divided the chart into two columns. At the top of one column she wrote the heading, polyhedra. In the second, she wrote the heading, not polyhedra. Then she continued the discussion. Each time a student identified and named a solid, Melany would make a particular effort to emphasize the correct name and spelling of the solid, particularly when a student erred. Then she would record the name of the solid in the appropriate column on the chart. For example, when a student incorrectly called an
octagonal prism an octagon Melany corrected the student and made certain that the appropriate name was recorded in the chart. When I asked her later that day why she decided to include the chart with the names of the solids at that point in the lesson, she explained:

Part of it was a management thing. Like what are these kids going to do sitting there with nothing to record. And I don’t know if somebody said something or not, but I just knew that I didn’t have to wait for more experience. They wanted to know what these things [the solids] are called. Some kids were coming up and asking me if this is called this and this is called this, and all that had happened. I just thought go right into it. So those were the two things. (interview, November 8, p. 15)

There are many teachers who might not have recognized the problem, or might not have been able to shift direction on a moment’s notice during a mathematics lesson, because they lacked the necessary content knowledge or pedagogical skill. Yet, for Melany it seemed effortless. Even when she thought that students were sufficiently knowledgeable to talk about the solids, she was easily able to make the necessary adjustment when she realized there was a problem. The incident reflects one of the several instances in which Melany demonstrated her pedagogical content knowledge, a critical component of children’s’ learning of mathematics for understanding (Fennema, Carpenter and Peterson, 1989).

In summary, I have presented three episodes in order to address the first two questions that guide this study. Each episode has portrayed different aspects of Melany’s thinking on what it means to teach for understanding in the context of mathematics and has considered the implications for her practice. In so doing, I have focused on Melany’s beliefs about the teaching and learning of mathematics, her content knowledge and her perspective of understanding, themes which are tied to question one. I have also discussed how Melany promoted understanding in her teaching, how she regarded her role as teacher, and her style of teaching, themes which fit with question two. As I presented different aspects of her practice, I have also focused on three key areas of the mathematics curriculum
and their underlying concepts: area, place value and three dimensional geometry. In the next chapter I deal with Eric.
Footnotes

1. Base ten materials are mathematical manipulatives that are used for teaching place value and number operations. They are designed to illustrate the different powers of tens such as hundreds, thousands, etc. They may be commercially produced such as Dienes blocks, abacuses, money, etc., or constructed by students. The materials which Melany used were Dienes blocks. These materials are designed so that they are proportional in size to the values they represent. For example, one is represented by a single plastic cube which is one cubic centimeter in size. Ten is represented by ten single cubes joined on one face only to make a rectangular prism. The dimensions of the prism would be 1x1x10. One hundred is represented by a hundred single cubes joined to form a 10x10 array. One thousand is represented by a thousand individual cubes which are joined to form a cube with dimensions, 10x10x10.

2. Flat is the name given to the 10x10 array which is part of the set of Dienes blocks.

3. Geometric solids are manipulatives such as cylinders, cones, spheres, pyramids and prisms that are usually constructed out of wood.
Chapter 6  A Portrait of Eric’s Life in the Classroom

In this chapter I present a portrait of Eric’s life in the classroom. The chapter begins with a brief introduction in which I revisit Eric’s views on teaching. Then I talk about his approach to mathematics. The discussion sets a context for the three episodes that are the focus of the chapter. The first episode concentrates on Eric’s approach to teaching the beginning concepts of a unit on fractions he developed. The second episode describes his approach to teaching multiplication of fractions. I have chosen this particular episode because it contrasts with Eric’s approach in the first episode. The third episode describes an introductory lesson on mixed numbers and improper fractions Eric taught. This episode effectively illustrates his depth of understanding and knowledge of mathematical concepts. My interpretive commentaries are integrated with the descriptive passages in each episode.

The chapter follows the structure and style of the previous chapter as it addresses the first two questions of the study. However, there is one striking difference between this chapter and the previous one that emerges as the chapter unfolds. Acts of teaching are conceptualized in more straight forward terms in this chapter than in the one prior. When Melany talked about issues of teaching they were more problematic and multifaceted than they were for Eric. In her view mathematics was more about finding new ways to make it more like her language arts program. Although it was important to Eric that his students were successful in mathematics, he seemed more concerned with covering the curriculum. In the following I revisit Eric’s views on teaching.
Eric’s Views on Teaching

In conversation with Eric I learned that he viewed his teaching as a combination of the traditional and the nontraditional. For Eric, traditional teaching was teaching by rote, telling, and using the textbook. Teaching in this way meant giving students knowledge and from Eric’s perspective, “when you get knowledge from someone else, this is learning” (field notes, April 9, p. 99). His only “strong memories” of mathematics from elementary and high school were of teachers who taught in this way. Eric pointed out that he favored a traditional approach to teaching mathematics because it was more efficient, it took less time to tell than to discover, and it allowed for practice of new skills. As he suggests, “I am a strong advocate of... [the idea that] the human brain learns things through repetition” (interview, February 26, p. 12).

Over the four month period in which I spent mostly full days in Eric’s classroom, there were many instances when I observed him “giving students knowledge”. One occasion that stands out is a lesson Eric taught on lowest-terms fractions. This lesson took place after he had taught the principle of equivalent fractions. The occasion is particularly striking because, at the same time that Eric employed a traditional textbook approach to teach converting fractions to lowest terms, he incorporated ideas that promoted conceptual understanding.

Eric introduced the lesson with, “Today I’m going to explain to you like the textbook does so it will help you” (field notes, March 4, p. 41). On the overhead projector was a transparency (see Figure 2) that was partially exposed. It contained the steps for converting a fraction to lowest terms; only the following was visible.
Figure 2. Converting a Fraction to Lowest Terms

Lowest Terms Fractions
1. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
   
   example: Is 9/12 in lowest terms?
   
   factors of 9 = 1

He proceeded to walk students through the steps for ascertaining if a fraction was in lowest terms. First he talked about how to determine if a number was a factor of another number. While doing so, he uncovered the following section of the transparency revealing numbers that were factors of the numerator.

Figure 2. Converting a Fraction to Lowest Terms

Lowest Terms Fractions
1. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
   
   example: Is 9/12 in lowest terms?
   
   factors of 9 = 1

2. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
   
   example: Is 9/12 in lowest terms?
   
   factors of 9 = 1, 3
Then he uncovered more of the transparency and talked about the denominator and its factors.

**Figure 2.** Converting a Fraction to Lowest Terms

**Lowest Terms Fractions**

1. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
   
   **example:** Is $9/12$ in lowest terms?
   
   factors of $9 = 1$

2. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
   
   **example:** Is $9/12$ in lowest terms?
   
   factors of $9 = 1, 3$

3. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
   
   **example:** Is $9/12$ in lowest terms?
   
   - factors of $9 = 1, 3$
   
   - factors of $12 = 1, 2, 3, 4, 6, 12$.

He followed by drawing students' attention to the number three noting that it was the common factor of the numerator and the denominator, which confirmed that $9/12$ was not in lowest terms. Then he uncovered more of the transparency to display the following:
Figure 2. Converting a Fraction to Lowest Terms

<table>
<thead>
<tr>
<th>Lowest Terms Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.</td>
</tr>
<tr>
<td>example: Is 9/12 in lowest terms?</td>
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<tr>
<td>factors of 9 = 1</td>
</tr>
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<td>2. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.</td>
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</tr>
<tr>
<td>-factors of 9 = 1, 3</td>
</tr>
<tr>
<td>-factors of 12 = 1, 2, 3, 4, 6, 12.</td>
</tr>
<tr>
<td>4. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.</td>
</tr>
<tr>
<td>example: Is 9/12 in lowest terms?</td>
</tr>
<tr>
<td>-factors of 9 = 1, [3]</td>
</tr>
<tr>
<td>-factors of 12 = 1, 2, [3], 4, 6, 12.</td>
</tr>
<tr>
<td>3 is the G.C.F., therefore 9/12 is not in lowest terms.</td>
</tr>
</tbody>
</table>

As Eric uncovered the following question, "What is 9/12 in lowest terms?", he began to talk about how to change a fraction to lowest terms.
Figure 2. Converting a Fraction to Lowest Terms

Lowest Terms Fractions

1. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
   example: Is 9/12 in lowest terms?
   factors of 9 = 1

2. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
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   factors of 9 = 1, 3

3. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
   example: Is 9/12 in lowest terms?
   -factors of 9 = 1, 3
   -factors of 12 = 1, 2, 3, 4, 6, 12.

4. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
   example: Is 9/12 in lowest terms?
   -factors of 9 = 1, [3]
   -factors of 12 = 1, 2, [3], 4, 6, 12.
   3 is the G.C.F., therefore 9/12 is not in lowest terms.

5. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.
   example: Is 9/12 in lowest terms?
   -factors of 9 = 1, [3]
   -factors of 12 = 1, 2, [3], 4, 6, 12.
   3 is the G.C.F., therefore 9/12 is not in lowest terms.
   example: What is 9/12 in lowest terms?

Eric explained that in order to convert a fraction to lowest terms, it was necessary to divide by a form of one. The form of one that he used was 3/3, as shown.
**Figure 2.** Converting a Fraction to Lowest Terms

<table>
<thead>
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  **example:** Is 9/12 in lowest terms?  
  factors of 9 = 1 |
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  - factors of 9 = 1, [3]  
  - factors of 12 = 1, 2, [3], 4, 6, 12. |
| 5. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.  
  **example:** Is 9/12 in lowest terms?  
  - factors of 9 = 1, [3]  
  - factors of 12 = 1, 2, [3], 4, 6, 12. |
| 6. A fraction is in lowest terms when the greatest common factor of the numerator and denominator is 1.  
  **example:** Is 9/12 in lowest terms?  
  - factors of 9 = 1, [3]  
  - factors of 12 = 1, 2, [3], 4, 6, 12. |

3 is the G.C.F., therefore 9/12 is not in lowest terms.

**Example:** What is 9/12 in lowest terms?  
3 is the G.C.F., therefore 9/12 is not in lowest terms.
example: What is $\frac{9}{12}$ in lowest terms?

\[
\frac{9}{12} = \frac{3}{3} \\
\frac{9}{12} \div \frac{3}{3} = \frac{3}{4}
\]

This is a form of "1".

To check our answer:
- factors of 3 = 1, 3
- factors of 4 = 1, 2, 4

G.C.F. is "1"

Given the nature of this type of research, there are different interpretations that one could place on Eric's approach to reducing fractions in order to convert to lowest terms. In my view, when a teacher presents the concept of equivalent fractions as dividing (or multiplying) by a form of one, even if telling is the teacher's strategy, it is conceptually more meaningful for students than saying, do the same to the numerator as you do to the denominator. An approach such as this, which is based on the principle of one, helps students appreciate that equivalent fractions are just different representations for the same number concept. In other words, they differ in form, not in value, as their appearance might suggest. (Note: The principle of one is based on the idea that multiplying or dividing any number by one or a form of one never changes the value of the number. Students learn this principle as it applies to whole numbers as early as grade three.) This approach to making fractions equivalent on which the first episode concentrates is generally not used by teachers. Yet, Eric appeared comfortable using it. I turn now to a brief discussion of Eric's views about nontraditional teaching.

From Eric's perspective nontraditional teaching meant learning through experience and for him, "learning by experience was the most valid kind of learning" (field notes, April 9 p. 99). Previously in chapter four I talked in detail about the different kinds of experiential learning that Eric provided for his students. These were: doing independent research, engaging in "hands-on" learning, and learning skills for dealing with the
unexpected in life. He suggested that these three types of learning were intended to develop both the content and the skills that his students would need both in and out of school. Eric also described non-traditional teaching as "just sort of traveling with them [the students], just sort of discovering things along the way with them, not really telling them a whole lot yet" (interview, February 26, p. 13). In his view, nontraditional teaching was "doing a lot of the spatial stuff" to develop understanding and helping students see concepts better. I interpreted Eric’s colloquial way of speaking as meaning that he had his students engage in experiences where they manipulated concrete objects to build visual images of concepts. He also suggested that nontraditional teaching included having students keep portfolios and writing folders.

In the following paragraphs I focus on Eric’s views about mathematics. This discussion is important because it is a basis for understanding the moves he makes in his teaching in the three episodes that follow.

Eric’s Views About Mathematics

According to Eric, mathematics was sequential and ideas built, not only from one year to the next, but from one week to the next. He cited the following example to explain what he meant. Before students were able to learn about place value with decimals and fractions, they first needed to know about place value with whole numbers. They also needed to know how to group by tens or tenths before they could work with hundreds or hundredths respectively. In his view, mathematics was straightforward and did not provide the flexibility he needed to try out varied ways of approaching ideas that language arts did. As I noted previously, when Eric talked about mathematics he often used language arts as a basis for comparison. He pointed out that mathematics, unlike language arts, had specific outcomes and an outline which he felt obliged to follow over the year.

For Eric there was a logical progression to teaching mathematics, whereas in language arts it was different. For example, in mathematics equivalent fractions had to be
taught before addition of fractions; otherwise, students would not be able to solve exercises with unlike denominators. However, in language arts, it was easy to teach a creative writing activity one week, bias the next week, or “flip it around”, and it would not matter. In Eric’s view, mathematics was rules and conventions and it was important to follow them. The following excerpt in which Eric was taking up questions on how to convert mixed numbers to improper fractions is an example. I want to draw attention to his explanation regarding why mixed numbers or answers to questions should not be left as improper fractions.

Some of you might have done this right. [At this point he wrote on the blackboard $17/8 = 1 \frac{9}{8}$.] You’re not wrong. You’re right. It is right. But the convention, the standard way again, a mixed number isn’t written or shouldn’t be left written with an improper fraction in it. Because this, $1 \frac{9}{8}$, $9/8$ is still more than one whole, isn’t it. $8/8$ is one whole and you’ve got $9/8$. You generally don’t write with an improper fraction... It’s just not the convention; the way that things are written. What I mean by convention is you can still understand what’s going on, but it’s not the proper way that things are done. It’s not the proper way that were used to dealing with things, and math is international. So it’s important that we keep using the same standard conventions everywhere; so every one understands exactly what we are doing. (audio tapes, March 28, p. 13)

Eric also believed that mathematics lacked the latitude that language arts provided. Even though there were different techniques he could use to teach a concept in mathematics such as using a geoboard or fraction sets to teach fractions, it did not offer him the scope that language arts did. In language arts he could teach any novel he wished. He could use the newspaper, a short story, or poetry to teach concepts such as bias and courage. In mathematics, he could not do the same. He was more willing to “take different turns and directions...” (interview, March 25, p. 8) in language arts than he was in mathematics. For Eric, mathematics was closely tied to the classroom, and he did not think that students ever thought about it outside, as they might with language arts. However, he did point out, as I noted in Chapter Four, that he taught mathematics differently from most teachers. Nevertheless, he did suggest that the majority of his teaching in mathematics was tied to teaching by telling, rather than experiential learning where students discovered on their own.
The above overview of Eric’s perspective on teaching and his beliefs about mathematics sets the scene for understanding the following three episodes. As I said in Chapter Three, I spent most days of a four month period in Eric’s classroom. From ten episodes I have selected the three that follow for these reasons. Firstly, they illustrate the nature of his understanding and depth of knowledge of mathematical concepts. Secondly, they demonstrate the dominant strategies that he used in his practice. Thirdly, they reveal his beliefs about the teaching and learning of mathematics. They also demonstrate how he promoted understanding of mathematical concepts. Finally, they illustrate how he viewed his role as teacher.

A “Hands On” Approach to Fractions - The Power of One

This episode focuses on the introductory work to a unit on fractions that Eric taught his students. It particularly concentrates on his unique approach to teaching the concept of equivalent fractions. Although Eric favored an approach to mathematics that was based on rote “because it took less time to tell than to discover”, the episode reveals how he taught from a more conceptually oriented view to promote meaning in students’ learning. He believed that if he devoted additional time to develop visual images of concepts in the early part of the unit it would build students’ understanding of concepts. According to Eric, it would prepare them for moving through ideas more quickly in the latter part of the unit when “we’re going to start jamming through the numbers, and we’re going to be doing some rote paper and pencil repetition” (interview, February 26, p. 13). The episode also reveals Eric’s depth of knowledge of mathematics, and his awareness of the misconceptions and partial understandings that students generally bring to the learning of fractions, factors that affect teaching for understanding.

Research (Bezuk and Cramer, 1989; Carpenter, Corbitt, Kepner, Lindquist, and Reys, 1981; Piel and Green, 1994) suggests that elementary, as well as secondary students demonstrate a lack of understanding of and have misconceptions about fractions.
Frequently, students add denominators rather than just the numerators when operating with fractions. They incorrectly estimate fractional values suggesting that the sum of fractions such as $12/13 + 7/8$ are 19 or 21. Students often mistakenly apply the whole number notion that multiplication makes larger and division smaller to operations with fractions, and so on. I want to add that, in some instances, teachers have these same misconceptions (Thipkong and Davis, 1991).

It was evident from my conversations with Eric that based on his teaching of grade seven the previous year and pretests this year, he was aware of the difficulties that students experience with fractions. According to Eric, “many of them [the students] did fractions mechanically and could do them, but they had no understanding of what they are doing” (field notes, February 16, p. 4). In Eric’s opinion students needed to understand fractions so they could apply them to different situations. For example, they had to be able to take what he had told them about the relationship of halves, quarters or eighths to one whole and apply it to fifths. If students could see in their heads all the ways that fractions could be represented visually, “When they are being asked to crunch them, they’re going to be better at it” (field notes, February 19, p. 18). He suggested when students couldn’t visualize fractional concepts, or didn’t have a firm grasp on them, that is when he would lose them.

Eric thought that if students understood fractions, they would know that fractions were less than one whole and that the fractional parts that made up the whole had to be equal in size. They would understand that the process of making one fraction equivalent to another changed only the numerical representation of the second fraction, not its value. They would know that the principle of one, an important anchor in mathematics, was at the heart of the principle of equivalent fractions. Also, they would understand division of fractions rather than memorizing a procedure. By this he meant, they would understand that division meant how many times $1/4$ fits inside $1/2$.

In the time that I observed Eric teaching mathematics, he taught fractions. First he focused on the meaning of fractions. Then he taught the principle of equivalent fractions,
making fractions equivalent, comparing fractions, ordering fractions, reducing fractions to lowest terms, changing fractions to mixed numbers and vice versa. Then he concentrated on operations with fractions, and connecting fractions to decimals. I was struck by the difference in his approach to teaching concepts in the first part of the unit in compared to the latter part. The contrast becomes apparent as the chapter unfolds.

Before I talk about Eric’s approach to teaching the concept of equivalency, I want to briefly describe his lesson on the meaning of fractions. This “hands-on” lesson, which was the basis for his work on equivalency, incorporated the use of visual images to build understanding. Eric began the lesson by illustrating a 4x4 square on a dot paper transparency on the overhead. First he named the square as one whole, then he invited a student to divide the square into two equal parts. A second student followed by shading each part of the square with a different color marker to show the two parts. Eric then recorded the fraction 1/2 under the illustration. Students were instructed to do the same on their dot paper, but were given an added challenge. They were asked to divide the square in such a way that the two halves were not obvious. This meant that they could not divide the square with a horizontal, vertical, or diagonal line. Although the task was challenging for students, the majority were able to figure out at least one way to solve the problem. Once students finished their patterns on dot paper, they replicated them on their geoboards.

Students then took turns presenting their geoboard patterns in small groups to the class. As they displayed their work, Eric talked about the different configurations. Each time students presented, he would poll the class to find out if they thought that the patterns represented one half. There were some patterns where it was obvious that the two parts were congruent. It was evident that these students understood the concept to the extent that they could go beyond the simplest level of representing the idea. For others it was necessary to count the small squares on the geoboard to confirm that the two parts were equal in size. Students who created these patterns demonstrated a more sophisticated level of understanding. Their patterns prompted a discussion about the idea of congruency.
From my observations Eric appeared easily able to facilitate such a discussion without reference to a teacher's resource book. On completion of the discussion of the concept of 1/2 Eric repeated the process for 1/4, 1/8, and 1/16. In a conversation I had with him later that day, I asked why he had chosen the fractions 1/2, 1/4, 1/8, and 1/16. He said:

My understanding in math tells me I don't want to use the examples of fifths when I'm showing the kids how to do this kind of stuff, because fifths don't lend themselves very well to equivalent fractions with things that kids will understand. If I go with halves, quarters, eighths, they all just build on one another, and you can ... identify a very easy pattern to that. You can work with it and then you can start branching out.

(interview, February 16, p. 7)

The previous description of Eric's lesson on the meaning of fractions, albeit brief, demonstrates how Eric used visual images to develop the concept of fractions for students. In order to teach from a more conceptually oriented view, teachers need to have the content knowledge and repertoire to do so (Ball, 1991a). The strategies Eric employed helped students "see" concepts and provided them with the opportunity to engage in "hands-on" learning. Alkin (1992) suggests that teachers need to do more than just present information in order to teach for understanding. Eric's teaching of the concept of equivalency, which follows, provides further evidence.

As I move to a discussion of Eric's teaching of the concept of equivalency I should note its importance in mathematics as a principle. Before students are able to compare and order fractions with unlike denominators, they need to know how to make fractions equivalent. They also need to have knowledge of the concept of equivalency in order to add and subtract fractions with unlike denominators. To convert fractions to decimal numbers and vice-versa, students need to understand the principle of equivalent fractions. Also, knowledge of the concept of equivalency is a prerequisite for work with percent, rate, ratio, proportion, and probability-ideas that all build on the concept of a fraction. It was apparent from my conversations and observations that Eric was aware of these ideas. To teach the
concept of equivalency teachers usually tell students that to make fractions equivalent, they should do the same to the top as they do to the bottom. In other words, if you multiply or divide the top by three, then multiply or divide the bottom by three. As commonly taught, this strategy for fractions emphasizes the algorithmic method and attends minimally to meaning or understanding. The method emphasizes how, but not why. Eric’s approach was different.

The following paragraphs describe Eric’s approach to the concept of equivalency. It was divided into three parts. In the first part Eric laid the ground work for the development of the concept. In so doing he established the importance of one whole with the aid of pictorial illustrations and connected it to its fractional representations. He began by displaying a series of squares on a dot paper transparency (see Figure 3) on the overhead. Each square was five units by five units and covered an area of twenty five dots.

Figure 3. A Series of Squares on the Dot Paper Transparency

He identified the first square as one whole and proceeded to talk about the meaning of this concept.

The first square, I’m going to label it one whole in brackets. I’ve put the number one. So what I’m telling you is, if you say one whole, the number one is the numerical representation of what I just said. I said one whole and the number that represents one whole is this. [He pointed to the number one that he had just written in brackets below the square.] The symbol that represents one whole is this symbol right here- the number one.

Now if I want to take my pen and you want to take your pencil, I want to represent to you, I want to show you with a picture, I want to use this square.
How can I visually show you that I understand what one whole is? How much one whole is? How can I show you that visually, so you could see it in a picture? (audio tapes, February 19, p. 15)

At this point a student suggested that Eric shade in the whole square, which he did. He followed by joining the dots in the interior of the square to make sixteen smaller squares and talked about how one could be represented in fractional form, in this instance \(\frac{16}{16}\).

1. Eric: I'm going to try to think of another way that I could write one whole as a fraction. That's a fraction. In the first example, this is one whole. What symbol or what number did I use to represent one whole?

2. First Student: One.

3. Eric: One. In the second example, the middle square, here, that's still one whole. I'm still showing you one whole, but I've written it in a different way. How did I write one whole, in the second example? What was the different method that I used?

4. Second Student: \(\frac{16}{16}\).

The following segment of the dialogue (5-10) illustrates Eric's efforts to ensure that students understood the purpose of the numerator and the denominator in any fraction. Note (11-13) how he reinforces the connection between one whole and its fractional equivalent, \(\frac{16}{16}\).

5. Eric: \(\frac{16}{16}\). Good. Sixteen pieces out of a possible sixteen. The bottom number in the fraction tells us that the whole was divided into sixteen equal size pieces. What does the top number tell us? What information does it tell us?

6. Third Student: That there was sixteen out of a possible sixteen.

7. Eric: What do you mean that there were sixteen out of a possible sixteen?

8. Third Student: There was sixteen ...ummm, sixteen boxes.

9. Eric: Okay. The bottom number tells us that we took the whole and we divided it into sixteen pieces. The top number tells us?

10. Fourth Student: We shaded in sixteen.
11. Eric: Yah, we shaded in sixteen or we were working with, we took sixteen of those squares. How do you know by looking at these two illustrations? How do you know that both one and 16/16 are the same amount? Can you tell me? How do you know that?

12. Fifth Student: The whole larger box, the boxes are shaded in.

13. Eric: The entire larger box here is shaded in. And even though we shaded in sixteen smaller boxes, when you add all of the sixteen smaller boxes together, the result is the larger box. The entire box or square is shaded in. So they’re both the same. They’re both the same?

14. Sixth Student: Size.

In the section that follows Eric repeats the process for 2/2. I want to draw attention to the manner in which he continues to make connections for students. First, he relates 2/2 to one whole (19-26). Then, he ties it to other fractional representations (27-30).

15. Eric: Size. The same amount. Let’s take a look. Let me get my other pen out here. Let’s take a look this time. Take the third square. I’m going to divide it, like this. [He divided it into two equal parts.] I’m still going to concern myself with one whole. This time I’ve taken the large square and I’m still telling you. I want to worry about a whole. I still want to talk about a whole. But the first thing I want to recognize is, how many equal size pieces did I divide the whole into? In this case, how many equal size pieces did I divide the whole into?

16. Seventh Student: Two.

17. Eric: What kind of shading should I do to represent to you that I’m working with the whole, the whole thing? What kind of shading should I do? Can you describe for me what I should shade.

18. Eight Student: All of it.

19. Eric: All of it. We’re still talking about the whole aren’t we? We’re still talking about the whole thing. How am I going to represent numerically, with a number, that out of two equal size pieces, we’re working with or concerned with both of them? How am I going to represent this with a number?


21. Eric: Where am I going to put the two?
22. Ninth Student: On top of the other two.

23. Eric: By putting that two up top here. I’ve said that out of the two equal size pieces that the whole is divided into, I’m working with, or I have two of them. Umm, how would you say that number there?

24. Tenth Student: 2/2

25. Eric: 2/2. You have 2/2. So 2/2 is the same as what? It’s the same as how much?

26. Tenth Student: One whole.

27. Eric: It’s the same as one whole. What’s another way of writing one whole or 2/2? Different way of writing it, exact same amount.


29. Eric: Sixteen over sixteen, 16/16. With those three examples, how many people see a pattern? Do you see something that might help you if I said to you, write for me another fraction that would represent one whole. How many people see a pattern here? If I said write another fraction for one whole.

30. Fifth Student: 4/4

(audio tapes, February 19, p. 25)

Once the first part of the lesson was completed and Eric established the importance of one whole and connected it to its varied fractional representations, he had students work with a partner to develop a rule or generalization for making fractions equivalent. The intention was that students would use what they had learned in the first part of the lesson to solve a series of questions that culminated in the formulation of a generalization. As Eric suggested, “Based on this whole calculation that they [the students] would do, would they see any way that maybe they could make up a little rule that would help them or somebody to make equivalent fractions all the time” (interview, February 26, p. 14).

The assignment on which students worked was designed so that the fraction sets were used to answer the questions. There were six questions in all. The questions were organized so that it was necessary to answer the first five correctly in order to solve the
sixth. The questions were as follows. In question one students had to figure out and record the value of one of each different color fraction piece. In question two they had to use the fraction pieces to determine the fractional equivalents, such as 2/2, 3/3, 4/4, of one whole, and so on. For the next two questions, students had to use different forms of one from the previous question to make fractional equivalents for 1/2 and 1/3. In the fifth question students had to discover the pattern for making fractions equivalent. This question was intended to help students figure out in question six the meaning of y/y so they could create a rule or generalization for making fractions equivalent. According to Eric question six required “taking what we talked about and going beyond” (audio tapes, February 26, p. 10). In Gardner’s (1993) terms, it was “having sufficient grasp of concepts, principles, or skills so that they can be applied to new situations or problems” (p. 21).

Once students completed the assignment and it was taken up, Eric presented his own version of the rule for making fractions equivalent. According to Eric, it was now time to concentrate on the abstract.

The way we’ve been creating equivalent fractions so far has been a lot of “hands on”. You’re taking a whole. You’re dividing it in half, coloring in a half. You’ve been taking two quarters, two fourths; you’ve been coloring them in. You’ve been looking at them. Hey, they’re the same size. Is that really a practical method of creating equivalent fractions in the real world? You know if you’re out and you’re trying to do a calculation because you’re in the store. You know if you’re trying to figure out whether one third off $25.00 is a better deal than a quarter off $30.00, things like that. But when you have to manipulate fractions in your head can you always have your picture of a fraction here when you’re messing around with it? Probably not, right. So we’ve gotta not only be able to look at things visually and see what fractions mean visually. You’ve got to be able to manipulate the numbers and the fraction activity introduced you to that. Right at the end you were asked to figure out a rule and some of you figured out that rule. Basically, it was a rule to help you numerically create equivalent fractions. I’m going to give you a piece of graph paper to copy down the following, Creating Equivalent Fractions. Here’s a statement that we already know. You were able to figure this out. I just reworded it [the rule for making fractions equivalent] so that it is hopefully worded so that it makes sense to most people. I’m going to give you the rule to help you numerically create equivalent fractions.

(audio tapes, March 1, p. 5)
As Eric led students through his version of the rule for making fractions equivalent, the following was displayed on the overhead.

**Figure 4.** Eric’s Version of the Rule for Making Fractions Equivalent

<table>
<thead>
<tr>
<th>Equivalent Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent fractions are fractions that name the same number. (They have the same value.)</td>
</tr>
<tr>
<td>$\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$, $\frac{x}{9} = \frac{18}{27}$, $\frac{x}{10} = \frac{20}{30}$, and $\frac{100}{100} = \frac{200}{300}$ are all equivalent fractions.</td>
</tr>
<tr>
<td>$\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, ... are all equivalent fractions that name the same number “1”.</td>
</tr>
<tr>
<td>To change a fraction to an equivalent fraction we multiply or divide the fraction by some form of “1”.</td>
</tr>
<tr>
<td>example: Write an equivalent fraction for:</td>
</tr>
<tr>
<td>a) $\frac{2}{6}$ b) $\frac{15}{18}$</td>
</tr>
<tr>
<td>a) $\frac{2}{6} \times \frac{3}{3} = \frac{6}{18}$, Therefore $\frac{2}{6}$ and $\frac{6}{18}$ are equivalent.</td>
</tr>
<tr>
<td>b) $\frac{15}{18} \div \frac{3}{3} = \frac{5}{6}$ Therefore $\frac{15}{18}$ and $\frac{5}{6}$ are equivalent.</td>
</tr>
<tr>
<td>$\uparrow$ This is a form of one.</td>
</tr>
</tbody>
</table>

(field notes, March 1, p. 30)

Eric concluded by saying: “If two fractions are equivalent, they’ve got exactly the same value. They’re describing the same amount or the same number. Just because they’re written differently, it doesn’t mean that the value has changed” (audio tapes, March 1, p. 6). As I suggested earlier, given the nature of this type of research, there are different interpretations that one could place on Eric’s conception of equivalent fractions. In my view his definition is conceptually more meaningful than saying that equivalent fractions are fractions that “name the same number” (Lesage et al., 1987, p. 170), or “2 fractions which are equal” (Kelly et al., 1987, p. 378).

In summary I want to point out that this episode offers a view of Eric as a pedagogue who teaches from a more conceptually oriented view in order to promote
understanding in mathematics. He provided students with "hands-on learning and effectively incorporated pictorial representations and concrete materials into his teaching. Research (Cruikshank and Sheffield, 1988; Labinowicz, 1985) suggests that the appropriate use of concrete materials in conjunction with pictorial images promotes understanding. In my experience, few teachers at the upper elementary level would have used the concrete materials in their teaching as Eric did. Neither would they have devoted the time to build visual images of the concepts. Teachers need to do more than just present information in order to teach for understanding (Alkin, 1992). Often teachers assume that students already know how to make fractions equivalent, and that it is not necessary to reteach the concept. They believe that grade seven students are at the formal operational stage in their logical thinking (Labinowicz, 1980) and do not need to work concretely.

The episode also illustrates Eric's depth of knowledge and understanding of mathematical concepts. As research (Fennema and Franke, 1992) suggests, "... what a teacher knows is one of the most important influences on what is done in classrooms, and ultimately on what students' learn" (p. 147). Not only did Eric establish the concept of one as an important anchor in mathematics, he used this principle as a basis for developing the concept of equivalency. Moreover, he demonstrated how he was able to make important connections in his teaching. In the view of Prawat (1989), teacher's content knowledge of the subject domain affects understanding of what it means to teach for understanding and their practice. Finally, the episode provides evidence of Eric's ability to challenge students to apply their knowledge of concepts and skills to new problems. In Gardner's (1993) terms, students who can do so truly understand. I move now to the second episode.

Teaching Multiplication with Fractions

In this episode I also talk about Eric's approach to teaching fractions. It focuses on multiplication Eric taught in the latter part of the unit. I selected this particular episode because it contrasts with Eric's teaching in the one prior. As I explained earlier, Eric's
approach to teaching concepts at the beginning of the unit was different from the way he taught in the latter part. According to Eric:

I had put the time up front on making sure we had a good grounding concretely in what fractions are and the relationship between fractions, and we spent less time on other aspects. Doing a lot of concrete work on fraction equivalency would allow me to skim through it [the latter part of the unit] when I did all my other aspects of fractions.

(interview, April 19, p. 15)

Eric’s plan for multiplication was to teach only the rule because he needed to cover the curriculum. There was no longer time for concrete materials, even if they were important for building visual images. Yet, he did point out that, even if he had time, he would only use the fraction pieces for one or two more lessons. In his view, it was necessary to teach the rule for multiplying because, “when you are my age, you remember the rule for multiplying and that’s what counts” (field notes, April 23, p. 126). Once his students knew the rule, multiply top times top and bottom times bottom, all they needed to do was to apply it, that was his objective.

My goal is not that they’ll be able to think independently. It’s not that they’ll be able to problem solve in an abstract manner. It’s not that they’ll be able to internalize, synthesize, and evaluate their responses. It’s that when I give them ten questions they can get nine or better.

(audio tapes, April 23, p. 2)

He emphasized that there would be no concrete materials and no extra time for students to discover on their own. It was mostly “plug and grind”, and the goal was to achieve 87% accuracy or better. I noted earlier that, as with any colleague in conversation, Eric was colloquial when we talked. I believe what Eric meant when he said, “plug and grind” was that students would be working exclusively on questions from the textbook, rather than working on exploration and discussion of ideas.

I want to point out that often students’ misconceptions or commonly held beliefs that multiplication always makes larger and division always makes smaller conflicts with their learning of multiplication and division of fractions (Graeber, Tirosh, and Glover,
Because of this problem, teachers need to teach multiplication and division with fractions so that students can make sense of the idea why multiplication with fractions makes smaller, and division makes larger. In my experience, the use of concrete materials and/or pictorial images helps students understand why this situation occurs. In so doing, they see why the product of two fractions is generally less than its factors and the quotient greater than its dividend.

Because Eric believed that multiplication of fractions was an easier skill to learn than addition or subtraction of fractions, although not necessarily easier to understand, a rote approach to teaching made sense. In conversation with Eric, he suggested that all he really wanted his students to know about multiplication was:

I’d want you to know how to multiply them [the fractions]. I’d want you to take this one and multiply it by that one, get your answer, reduce it. It may involve changing that mixed number to an improper fraction, then multiply. Depending on how smart you were I might even take the time to point out to you that you notice that when you multiply fractions the number doesn’t necessarily get bigger. There’s a couple of kids who I would point that out to and they’d go “Huh”.
Can you do it? Great. Let’s go. That’s what I’m aiming for now.
(interview, April 23, p. 2)

Let me turn now to how these ideas worked in practice. To teach multiplication of fractions, Eric introduced the lesson by connecting new ideas to concepts he believed that students already knew. Research (Fennema, Carpenter and Peterson, 1989) suggests that teachers need to pay attention to what students already know because understanding involves linking new knowledge to existing knowledge. First, Eric talked about the similarities between multiplication with whole numbers and multiplication with fractions. Then, he focused on multiplication of fractions. He pointed out how easy the task would be as students already had learned the basics for multiplying fractions from their work with equivalent fractions. The following is Eric’s introduction to the lesson. I want to draw attention to the manner in which Eric connects the task of multiplying common fractions to what students already knew about multiplication.
1. Eric: Let's think back to whole numbers, grade two, maybe grade three. You've already learned how to add. You've learned how to subtract. What should you learn to do next?

2. First Student: Multiply.

3. Eric: Multiply. See that, fractions are just a different way to write a number. We're going to learn how to multiply. Now, how many people, if I gave you this question could get the correct answer, 6 x 5? Put your hand up if you think that you could do that and get the correct answer, 6 x 5. I don't care if you have to do it in your head or you'd have to work it out on paper, if you could get the correct answer. How about this one, 13 x 4? You could work it out or you could do it in your head, whatever, but you can tell me what 13 x 4 is. All right good. How about this, 11 x 5? 6 x 9? How many people think that they can do that? How many people know how to reduce to lowest terms? Is any of this stuff I just asked you new? Is it new or do we know this already? We've learned this already right.


5. Eric: Then you know how to multiply fractions. I'm not going to show you one things that's new. Honestly. I'm just going to show how, look, you can do this, this, and this, and put it all together. Nothing new I'm showing you here. I'm not showing you new rules to memorize. I'm not showing you new tricks. I'm just saying this is how we're going to do it. You multiply. You convert. You reduce. Can you make sure that you have the title, Multiplying Fractions, and the date as well.

And today we're going to learn how to multiply fractions.

(audio tapes, April 23, p. 7)

According to his usual style Eric displayed on the overhead projector a transparency of a written note (see Figure 5). The note, which was covered with a sheet of paper so its contents were not visible, outlined the steps for multiplying common fractions and mixed numbers.
For the purpose of this episode I focus only on multiplication of common fractions. Eric began with $1/2 \times 3/4$. At that point he moved the paper down so the expression was visible. To help students understand how to multiply common fractions, Eric suggested that they think of $1/2 \times 2/3$ as multiplying $3/4$ by $3/3$. He explained that they already knew how to multiply any fraction by a form of one, and multiplying two common fractions was the same. Research (Good, McCaslin and Reys, 1992; Haylock, 1982) suggests that students' ability to make connections is important to their learning for understanding. As Eric led students through the steps of multiplying fractions, gradually uncovering more of the transparency, he invited students to offer their input. At strategic intervals Eric stopped to
reassure students that learning multiplication of fractions was going to be easy. In Eric’s view, they already knew the necessary subskills, so it was going to be “a piece of cake”.

The following excerpt demonstrates how Eric lead his students through the procedure for multiplying common fractions. I want to draw attention to the connections he makes between students’ previously acquired knowledge and the task of multiplying fractions. Note (3, 5 and 15) how he focuses on the similarities between multiplication of common fractions and the task of creating equivalent fractions.

1. Eric: And today we’re going to learn how to multiply fractions. Now here’s another one of my famous really wordy sentences. You read and you go, “Huh”. But, as soon as you see an example, it will make total sense to you. So we have one sentence here. To multiply fractions, find the product of the numerators and place it over the product of the denominators.
   The interesting thing about this is, by the time we get to this part of the unit, when I teach multiplying fractions, I’ve already taught you how to do it. When have you already used multiplying fractions? Anybody know? I mean this year. When have I taught you this year to do this?

2. First Student: Sometimes you do it when you’re converting to lowest terms.

3. Eric: Sure. Sometimes when you’re converting to lowest terms. Not usually multiplying, but we’re applying the same rules when we’re creating equivalent fractions. Right.

4. First Student: Yeah.

5. Eric: When you multiply by a form of one. I said that to you several times when we took up the quiz. What form of one did you multiply this fraction by? So I’ve already shown you how to multiply fractions.

6. Second Student: When you go to lowest terms, you divide, so it’s exact opposite.

7. Eric: You got it. You got it. So I said, if you want to multiply fractions, find the product of the numerators. Well, product, what does that mean? What is a product? This is a math term. What is a product?

8. Third Student: The answer to the question.

9. Eric: Yeah. What type of question is the answer called a product?

10. Third Student: A fraction.
11. Eric: No. When you’re doing some type of operation the answer is called a product. What operation would you be doing?

12. Third Student: BEDMAS.

Note: BEDMAS is an acronym teachers frequently use with their students to help them remember the rules for the order of operations in mathematics. The order of the letters represents the order in which the operations are performed. For example, B, the first letter, stands for brackets. It reminds a student that all work in brackets in any question in which there are brackets and other operations must be done first, and so on.

13. Eric: Well, BEDMAS is six different things. It’s not one operation. Let’s see if somebody can help out.


15. Eric: Multiplication. The answer to a multiplication question is the product. So you find the product of the numerators and you place it over top of the denominators. Wow simple. Let’s see how many of you can solve this question based on what I just told you. There’s the first example, 1/2 x 3/4. Copy the example and try to write down the answer for me.

Remember, when you multiply, you place the product of the numerators over top of, over top of the product of the denominators. Think how you would multiply fractions if you were creating equivalent fractions. If you were creating equivalent fractions, what would you do? I haven’t told you anything new. Pretend that you’re making an equivalent fraction. Pretend just like you had 1/2 x 3/3. What would you get right here? Well, it’s not 3/3; it’s 3/4. I’m not showing you anything new. Anything we’re doing, you know how to do.

(audio tapes, April 23, p. 9)

As students worked, Eric circulated about the class observing their strategies. I noticed that at particular intervals he would return to the overhead at the front of the room and address the whole class. It was at these times that he would reassure students that they really knew how to do multiplication. Note (22-25) the emphasis on connecting what students already knew with the task of multiplying common fractions.

16. Eric: Okay, this is like adjectives, right? You’re trying to make up your own rules. Trust me. Believe me. There’s nothing new. You take the product of the numerators. Multiply the numerators together and you get the product. What’s one times three?
17. Fifth Student: Three.

18. Eric: Place it over top of the product of the denominators. Well the denominator here is two. The denominator here is four. What's the product of the denominators?

19. Sixth Student: Eight.

20. Eric: Eight. Two times four is eight. Your answer is going to be... What's it going to be?


22. Eric: 3/8. Some of you are making up, and this is just like your adjectives work. You're making up your own rules. I even stopped you, I said, look, I'm not going to teach anything new. I'm not going to show you anything different. You're making up your own rules. What would you have done if I'd said this, make 1/2 an equivalent fraction with a denominator of ten. What would you have multiplied 1/2 by to create an equivalent fraction with a denominator equal to ten?

23. Seventh Student: 5/5.

24. Eric: 5/5. Do you see how this is no different than this? 1x3 is 3. 2x4 is 8. We've been multiplying fractions for weeks. I've just never said this is multiplying fractions. I've just said, multiply this number multiplied by that one, and put it there. Don't make up your own rules. Some of you have like eight steps. You have like eight steps. It was like this long. You had things equal to 46, 858. I'm going, 'Where did this come from?' You're making up your own rules. Let's try this one. Before I go ahead, 1/2x 3/4. What word might I, or you substitute for the multiplication sign? It means multiply. Sometimes, if you hear the word, and, it means addition. What's six and four? It's ten. So, and, means addition. What word can be substituted to mean multiplication? I've got an example here because your textbook uses it quite often.

25. Seventh Student: Times.

I want to draw attention to the manner in which Eric emphasizes the importance of the word, of, to the multiplication of fractions.

26. Eric: No. Times, times is slang. We do say that many times but it is slang. Of, 1/2 of 10 is 5. Of implies multiplication. So, if you see the word of..., your textbook uses it quite a lot. About as often as you see a multiplication sign, you see the word of. So you might see something like this, 2/3 of 1/4.
Wherever you see the word of, it can be replaced with a multiplication sign. That’s what it means. It has the exact same meaning. So you try to solve the question for me. What is $\frac{2}{3}$ of $\frac{3}{4}$? Don’t forget all answers must be placed into ...

27. Eight Student: Lowest terms.

28. Eric: You’ve got it. Don’t forget. [Eric walked around the room to check their work.] All answers must be placed into lowest terms. That’s sort of an overriding rule that we have for fractions. Okay, $\frac{2}{3}$ of $\frac{1}{4}$ equals or means the same as $\frac{2}{3} \times \frac{1}{4}$. You didn’t have to write that step. I’m just showing you. It equals the same thing here. I just replaced the word of with multiply. So, when we multiply fractions, it’s the 2 multiplied by, or with what number? What number does the 2 multiply with?

29. Ninth Student: One.

30. Eric: The one. Two times one is what?

31. Tenth Student: Two.

32. Eric: The four, the other denominator. Three times four equals what?

35. Third Student: Twelve.

36. Eric: Twelve, so our answer is $\frac{2}{12}$. Ask yourself is that in lowest terms? And a question like that, you should be able to say, no, right away. They’re both even numbers. They both divide easily by two. No way, it’s in lowest terms. What is $\frac{2}{12}$ in lowest terms?

37. First Student: $\frac{1}{6}$.

38. Eric: Wow, it’s getting so simple. It’s scary, It’s scary that they actually made me teach this in grade seven, and not in grade three. I mean it’s so simple, isn’t it? Let’s take another example. My favorite example, my favorite example because, I like these examples, because they mess you up. You’re going to look at it and you’re going to go, ‘I don’t have a clue how to do it’. So copy it down, but don’t go ahead. Just copy down what you see, and we’ll see if we can figure it out. Anybody have an idea how to do it? [About six students raised their hands to acknowledge that they knew how to do it.] The first thing that you have to recognize is, this is going to be a multiplication question. The word, of, it’s multiplication, $\frac{2}{3}$ of 6. Now, the trick is going to be rewriting the six as a fraction. That’s going to be the trick. $\frac{3}{3}$, $\frac{3}{3}$ equals how much?

(audio tapes, April 23, p. 10)
In summarizing I want to point out that the predominante approach evident in this episode, unlike the previous one, is teaching by rote. As I explained earlier, Eric’s plan for multiplication was to teach only the rule because he needed to cover the curriculum. He no longer had time for concrete materials, even if they were important for building visual images. Yet, in my view, he did incorporate strategies to promote conceptual understanding in students’ learning. First he linked new learning to students’ previous knowledge as he connected multiplication of fractions to multiplication of whole numbers. Then, he showed students how multiplying common fractions was the same as making fractions equivalent. As I noted earlier, the ability to make connections is important for students’ learning for understanding (Good, McCaslin and Reys, 1992; Haylock, 1982). Moreover, students who view mathematics as a connected body of knowledge generally have a better understanding of the discipline. At the same time that the episode illustrates that building students’ understanding of mathematics was important to Eric, it also reveals that covering the curriculum was more pressing. In the next chapter, I will talk about factors within the context of teaching work, such as covering the curriculum, that mitigate against teaching for understanding.

The previous two episodes provide evidence that Eric’s teaching of mathematics was a contrast in perspectives. At the same time that teaching rules and telling were important to his practice in mathematics, Eric provided opportunities for students to work through ideas on their own so “they got a good understanding of fractions, what they are, so that they actually know them” (interview February 16, p. 9). To complete the portrait of Eric’s life in the classroom I now focus on the last teaching episode. The following describes Eric’s approach to teaching mixed numbers and improper fractions. I have selected this episode because it further illustrates Eric’s deep understanding of mathematical concepts, and his ability to present ideas in meaningful ways to his students, factors that affect teaching for understanding (Prawat, 1989).
Visuализация Мешанных Чисел и Неправильных Фракций

Очень часто я наблюдал, как учителя рассказывают ученикам, что для перевода смешанного числа в неправильную дробь, умножают целое число на знаменатель и прибавляют числитель. Они объясняют обратную процедуру, изменяя неправильную дробь в смешанное число, как делят числитель на знаменатель, затем записывают целое число и дробь. Этот метод акцентирует, как, но не почему. По мнению Национального Совета Учителей Математики (НСУМ, 1989; НСУМ, 1991), чтобы обучить пониманию, учителям необходимо предоставлять ученикам возможности исследовать концепции и идеи, и создавать знание в ходе активного участия в дисциплине. Этот вид обучения акцентирует "сделать" вместо "понимать". Вместо того чтобы представить ученикам шаги для перевода смешанных чисел в неправильные дроби и обратную процедуру, Эрик разработал задание в котором ученики использовали конкретные и пикториальные изображения, чтобы обнаружить связь и тесную связь между смешанными числами и неправильными дробями.

Задание было спроектировано так, чтобы студенты работали на своих собственные использованием фракций частей, чтобы узнать о смешанных числах и неправильных дробях. было три части. с целью построения на основании студентов' предыдущее обучение, первая часть включала ряд вопросов, которые сосредотачивались на различных способах, которыми можно было бы представлять целое число в дробной форме. Это задание устанавливало основу для развития концепций в частях, которые следовали. Вторая часть сосредоточилась на смешанных числах. Первые студенты использовали фракционные части, чтобы создать визуальные изображения смешанных чисел. Эти изображения также показывали им смешанные эквиваленты неправильных дробей. Чтобы применить то, что они узнали о неправильных дробях, студенты затем ответили на вопросы по сложению смешанных чисел. Третья часть сосредоточилась на неправильных дробях. После этого вновь использовались части фракций, чтобы иллюстрировать дроби. Однако, в этом случае произошла обратная связь и изображения неправильных дробей также показывали смешанные эквиваленты. Продолжение к этому
section of the assignment consisted of a series of questions in which students created pictorial representations of different mixed numbers and improper fractions and recorded both forms of the number for each representation.

Using the fraction pieces, Eric introduced the lesson with a brief review of the different ways for representing one whole using the fraction pieces. Once he was certain that students remembered the different fractional forms for one whole, he moved to the focus of the lesson. First he pointed out that mixed numbers were part whole number and part fraction. Then he talked about the different ways, concretely, pictorially, or abstractly, that a mixed number could be represented. The following is an excerpt from the data in which Eric introduced the concept of mixed numbers. I have included it because it reveals how Eric incorporated the use of the concrete and pictorial to develop the concept of a mixed number. As noted, research (Cruikshank and Sheffield, 1988; Labinowicz, 1985) suggests that the appropriate use of concrete materials in conjunction with pictorial images promotes understanding. The dialogue begins with Eric’s definition of the concept of a mixed number.

1. Eric: It’s a number that has both whole numbers and fractions. It doesn’t just have to have one. It could have the whole number 64 in it. It has a whole number and it has a fraction. And together, you put them together and what you call that is a mixed number. It’s not just a fractional number. It’s not just a whole number. It’s a mixed number. It has a whole number part, and I has a fraction as part of it. Now if I gave you the mixed number 1 1/3. The number might be represented by a picture something like this (see Figure 6). It doesn’t have to be. Try to see what you can do, either looking at this picture or messing around with your fraction set. What pieces, what colored pieces did I put together to make that representation? What colored pieces out of the fraction set would you have to put together to make this illustration? If I said, make it on your table right now, what colored pieces would you have to put together?

2. First Student: Blue. [Note: the blue pieces of the fraction set each represent 1/3 of the whole.]

3. Eric: Blue ones. You would have to use thirds. How many of those pieces would you need in order to make the
representation right there that I’ve shown you? How many of those pieces will you need to make this?


5. Eric: You need four. Can you do that at your table right now?


7. Eric: No, you can’t. You’ve got only three blue pieces. If I said get with another pair [of students], could you guys then make this?

8. Third Student: Yes.

In the comment that follows note how Eric draws students’ attention to the connection between one whole and its fractional equivalent as he develops the concept of a mixed number.

9. Eric: Sure you could. You have to, each of you, kick in two blue pieces so that you have in total four blue pieces. The three blue pieces right here, and I’m pointing at this picture so you should see where I’m pointing, the three blue pieces together right here represent one whole. That’s one whole and this is one third. We read a mixed number by reading the whole number first, one and one third. Now up here I gave you a couple of different ways to represent fractions. Down here, can you take your pencil or a pencil crayon, and draw for me some other representation, some other illustration that might represent the same mixed number, 1 1/3? Can you draw some illustration for me down here in the space at the bottom of the page that would represent to you, or to me, 1 1/3? I gave you one way that it might look. Can you come up with some other way that it might look? Just a rough sketch, right. We don’t have to be artists.

At this point Eric had the students illustrate their version of the mixed number 1 1/3. As he circulated about the room observing students’ representations, Eric continued to focus on the relationship of one whole with its fractional equivalent.

10. Eric: Good. Good. That’s good. Okay. Okay. Good. Good. That’s good. So remember, just like up here, you can draw some shape to represent one whole. It doesn’t have to be a circle. It doesn’t have to be a square. It’s one whole, and then you’ve got one-third. You’ve got a second whole, and all you’re concerned with is one-third of it. One-third, that’s all you need. How many thirds in total do you need to make 1 1/3? How many thirds in total would you need to make 1 1/3, to make that illustration?
11. Fifth Student: One-third.
12. Eric: How many in total? How many thirds? Each piece here is one-third. How many of those pieces are you going to need?
13. First Student: One.
14. Eric: One, to make 1 1/3?
15. First Student: Four.
16. Eric: Four. You've got it right. You're going to need four of those pieces, if you want to make that mixed number, 1 1/3. How many thirds does that make altogether?
17. Sixth Student: Four.
18. Eric: Is four thirds the same as one whole and one third? Sure it is.

(audiotapes, March 25, p. 12)

Figure 6. The Mixed Number 1 1/3

Eric repeated the process with another number before he set the students to work on their own. When this part of the assignment was completed and checked, Eric moved to the last section and briefly talked about improper fractions. With the use of pictorials Eric demonstrated how to show the improper fraction 5/3. First he shaded all three parts of the first circle that was illustrated on the overhead and wrote both one whole and 3/3 under it (see Figure 7).
Then, he shaded two of the three parts of the second circle and wrote the following two number phrases, $3/3 + 2/3$ and $1 + 2/3$, below both circles (see Figure 8). At this point Eric briefly explained that even though these phrases looked different, they both represented the same idea. He followed by writing the mixed number $1 2/3$ noting that it was yet another way of showing the concept that was illustrated. After several practice examples, students completed the last section of the assignment.
Eric's development of mixed numbers and improper fractions in this episode demonstrates his depth of knowledge and deep understanding of mathematical concepts, factors that affect teachers' understanding of teaching for understanding (Prawat, 1989). He designed an assignment that offered students an alternative to memorizing a rule and provided them with a view of mixed numbers and improper fractions that concentrated on meaning and understanding. In so doing, students were able to construct new learning on the basis of their previous knowledge. In my view this approach attends more to meaning and understanding than it does to rote. As I noted earlier, The National Council of Teachers of Mathematics (NCTM, 1989; NCTM, 1991) recommends that teachers need to provide students with opportunities to explore concepts and ideas, and to create knowledge in the course of active engagement with the discipline in order to teach for understanding.

In summarizing the chapter I want to point out that in the three episodes I have presented, I have portrayed different aspects of Eric's understanding of what it means to teach for understanding in the context of mathematics. I have also considered the
implications for his practice. In so doing I have provided evidence of Eric’s beliefs and views about the teaching and learning of mathematics, his content knowledge, and his perspective of understanding, themes which are tied to questions one. I have also focused on the manner in which Eric promoted understanding in his teaching, his teaching style and how he regarded himself as a teacher, themes which fit with question two. Whether Eric taught by telling, or by providing students with experiential learning, Eric demonstrated a deep understanding of mathematical concepts and the knowledge that students bring to their learning. Although Eric believed in the importance of developing conceptual understanding in students’ learning of mathematics, he also pointed out that, in the best of all worlds, he would still teach rules and generalizations.

Both in this chapter and the previous one, I have developed a portrait of Melany’s and Eric’s life in the classroom as I addressed the first two questions that guide this study.

1) What does it mean to teach for understanding in the context of the mathematics classroom?

2) What are the implications of teachers’ perceptions of what it means to teaching for understanding?

In the seventh chapter of this study, which follows, I address the last question.

3) What is the nature of the context in which teachers create their practice in mathematics?

I also present my conclusions and reflections on this inquiry.
Footnotes

1. A fraction set is a commercially made set of different size plastic pieces that have been cut to represent fractions such as halves, quarters, thirds, eighths, etc. The whole is the template for the other fractional pieces. The shape of the pieces in a set may be rectangular, or they may be circular. The sets are cut so that the pieces represent different fractions and are proportionate in size to the whole and to each other. Different fractions are distinguished by color. For example, one whole is black. The two pieces that each represent one half are orange. The three pieces that each represent one third are green. Quarters are blue, etc.
Chapter 7   Ending

In this final chapter of the study my purpose is twofold. First I address the third question that guides this study:

What is the nature of the context in which teachers create their practice in mathematics?

In so doing I talk about the effects of context as they relate to Melany’s and Eric’s practice. Then I talk about what I have found as a result of my research on teaching for understanding in the context of mathematics. I also consider the realities of teaching for understanding in today’s educational climate. A discussion which focuses on the strengths and limitations of the study, and plans for future research follows. The chapter concludes with reflections on personal learning.

The Context in which Teachers Create their Practice

As I noted in Chapter Two, context has the capacity to significantly shape practice. According to Talbert and McLaughlin (1993), factors ranging from students in a classroom, subject matter, curriculum, colleagues, school administrators and school organization to broader environments that extend beyond school systems such as parents, the community, and institutions such as government determine whether a teacher is likely to embrace a vision of teaching for understanding. From an alternate perspective Gardner (1993) suggests that obstacles such as test-text phenomenon, the correct-answer phenomenon, pressure for coverage, cognitive Freudianism, institutional constraints, and disciplinary constraints stand between schools and educating for understanding. In the next section I focus on factors of context that affected Melany’s and Eric’s practice. In so doing I talk about the following: pressure for coverage, the curriculum, bureaucracy, textbooks and teachers’ resources, and students.
Pressure for Coverage

In Gardner's (1993) view the greatest enemy of understanding is pressure for coverage. Teachers are exceedingly busy and often do not have sufficient time to fulfill their curriculum responsibilities (Good, McCaslin and Reys, 1992). In the four month period that I observed in each of Melany’s and Eric’s classroom, Eric in particular talked about not having enough time to cover the curriculum. Frequently our conversations would gravitate to his concerns about not being able to achieve this goal. He would tell me as he pointed to the year’s overview on the wall beside his desk, “I have a curriculum to cover and I have to stick to it” (field notes, March 4, p. 49). There were several instances during the fractions unit, as the following examples suggest, when pressure to cover the curriculum affected Eric’s teaching. When Eric devoted additional time at the beginning of the fractions unit to develop the concept of fractions and the principle of equivalent fractions, he covered the latter half and teach only the rules for the operations. During the subtraction part, Eric confessed that he had no choice but to move on, even though some students were experiencing difficulty. As he suggested, he had to “cycle” through it because there was a schedule to keep. His need to cover the curriculum also stood in the way of providing students with the opportunity to work at their own pace in mathematics. I recall the occasion when one of his better mathematics students approached Eric to ask if he would allow those “who really know their stuff” to work ahead, his response was that it was not possible. According to Eric, there was not sufficient time to individualize mathematics and allow students to work at their own pace. Yet, pressure for coverage was not an issue for Melany. There were few instances while I observed in her classroom when she talked about the need to cover the curriculum. According to Melany, it was more important to focus on investigations that connected with students’ reality than to worry about how much time she had to cover the curriculum.

The challenge of teaching for understanding while attempting to cover mandated curriculum expectations became a reality for me when Eric let me teach a unit on integers to
the class. To reduce some of the pressure that Eric was experiencing and allow him to attend to his many classroom and extracurricular responsibilities, I offered my help. I suggested that he let me teach a unit on integers as a way to “pay back” (Glesne and Peshkin, 1992) for letting me conduct research in his classroom. Also, I thought that this offer would provide me with an opportunity to find out what it might be like to try to teach for understanding while attempting to adhere to the tight schedule of the curriculum. As I saw it, “the curriculum was structured so that it was really hard to involve students in the bigger picture [of mathematics], because you really feel obligated to stick to the expectations” (field notes, June 6, p. 239). During the two weeks that I taught the unit on integers, Eric regularly reminded me that along with teaching the concepts “for understanding”, I still had to “stick” to the curriculum guidelines. Daily, he would tell me that I had only two weeks and no longer. It was all the time that he could spare. When I failed to meet the deadline, Eric, tongue-in-cheek, pointed out that this was the reality of classroom life.

The Curriculum

According to Anderson and Roth (1989) curricular demands often mitigate against the development of classroom environments that contribute to instruction which promotes understanding. Careful examination of The Ontario Curriculum Grades 1-8 Mathematics (Ministry of Education and Training, 1997) that Melany and Eric were expected to follow, provides support for these views. For example, the introduction to the document tends to concentrate more on rigor and challenge and less on promoting a view of mathematics as invention and the creation of knowledge. It states that:

*The Ontario Curriculum, Grades 1-8: Mathematics, 1997* has been developed to provide a rigorous and challenging curriculum for students in Grades 1-8. The required knowledge and skills for each grade set high standards ...
New Features of the Mathematics Curriculum

The mathematics curriculum set out in this document is significantly more rigorous and demanding than previous curricula. This curriculum includes a broader range of knowledge and skills and introduces many skills" (Ministry of Education and Training, 1997, p. 3).

The document also presents mathematics as an extensive list of ideas to be covered, rather than a holistic body of connected knowledge. Firstly, the curriculum is divided into five separate strands. These are: Number Sense and Numeration, Measurement, Geometry and Spatial Sense, Patterning and Algebra, and Data Management and Probability. Secondly, each strand is subdivided into a series of concepts and skills in the form of expectations. There are a minimum of twenty expectations for each grade level within each of the five strands. Consequently, teachers must cover approximately one hundred expectations within a school year. They are also expected to teach concepts and skills from each of the five different strands monthly. Requirements such as these tend to inhibit teachers' flexibility, particularly those who wish to teach for understanding. Furthermore, students are left with the impression that mathematics is a series of expectations.

Eric saw the curriculum as "loaded" and felt obliged to follow it, "otherwise you end up teaching them [students] things you have no business teaching them right now and not teaching them things they should know at this level" (interview, March 25, p. 9). He pointed out that if he sent students to grade eight and grade nine without teaching topics such as fraction operations “because they had only worked on equivalent fractions, and that’s what they [students] would say to the teacher [the following year]” (field notes, March 4, p.50), then he was not fulfilling his responsibility.

Melany, on the other hand, saw the expectations as a list of concepts to be covered. In her view it was not possible to follow the expectations and still teach for understanding. For Melany, it was more important to individualize learning and provide students with experiences that connected to their reality, rather than teach to the curriculum. The following provides evidence.
Well, I'll tell you I don't use it except I know [I should]. Do I stick to it? Some times, I get really hyper, and think that I better do this. Then, I'll look at what the kids can do and I think I can only go with where they are ready to go. I mean this is ridiculous pounding into their heads. I'm just going to set up all kinds of problems for myself and them. So then I just put it on the shelf. (interview, September 25, p. 8)

**Bureaucracy**

In many instances, as the previous discussion on the curriculum demonstrates, school boards and ministries of education may stand in the way of teaching for understanding. Yet there are those who see it differently. Eric saw the school board as supporting teaching mathematics for understanding. When I first observed Eric teaching fractions, he explained that his approach to teaching the beginning concepts of the fractions unit was congruent with the board's philosophy for teaching for understanding in mathematics. According to Eric he “would pay lip service to what they [the board] wanted done [in the other areas] but not in math” (field notes, April 1, p. 88).

Melany, on the other hand, was not as positive about the support that the board provided for teachers in mathematics. In her view, if teachers were expected to teach mathematics according to guidelines recommended by The National Council of Teachers of Mathematics (NCTM, 1989, 1991), then the board needed to provide teachers with appropriate professional development. She pointed out that during the term there had only been one inservice session for teachers in her school to introduce the new mathematics curriculum. “So they came and they gave you this [the new curriculum] and that’s it. And they said ‘follow it’ and there’s no follow up” (interview, October 23, p. 17). According to Melany, the school administration also failed to provide adequate support for mathematics. There was only one half hour lunch meeting, and it was “basically passing out papers” to insert into their curriculum binders. She pointed out that in language arts it was different. Already, the school board had given her two full release days to attend meetings that covered new initiatives, and it was only the beginning of the school year.
Textbooks and Teacher Resource Books

I want to talk briefly about the role of textbooks and teachers’ resources in teaching for understanding. They may either stand in the way of teaching for understanding or help to promote it. According to Melany, unless she had a “good unit” to teach, she would rather not teach that particular area of the curriculum. She explained that her motive for not teaching three-dimensional geometry and volume was that she did not have the appropriate resources. As I noted in an earlier chapter, according to Melany Marilyn Burns’ resources were good units because the investigations connected with students’ interests and they promoted meaningful learning. However, not all Burns’ materials fit with every area of the curriculum that Melany was expected to teach. In her view, there were no other suitable options. Her infrequent use of the textbook was based on a belief that textbook learning was not meaningful. As I noted in Chapter Five Melany resorted to using the textbook only when she needed time for herself, when she was at a loss for ideas, or when a supply teacher was filling in for her.

Eric used the textbook more often. Although he developed his own teaching materials, his rationale for using the textbook was that it was equally as important for students to know how to do textbook work as it was to learn for understanding. However, the textbook that Eric used attended less to meaning than to algorithmic learning. The series is no longer approved by the Ministry of Education and Training because its philosophy does not fit with the currently approved approach to teaching mathematics for understanding. Although the board had provided Eric’s school with money to purchase new textbooks, Eric explained that the school was still in the process of deciding which books to buy.
Students

According to Talbert and McLaughlin (1993), "Students are the most salient and powerful context of teaching. Their needs, as teachers perceive them, and the constraints and opportunities that they provide for instructional choices shape teachers' goals, and conceptions of practice" (p. 186). Anderson and Roth (1989) concur that students, as viewed by their teachers, affect teachers' practice and their goals about learning. However, students' views of what counts as knowledge are often determined by the nature of instruction. The National Council of Teachers of Mathematics (1991) points out that often students are exposed to a style of teaching in which teachers describe and explain information, rather than teaching which facilitates the construction of knowledge.

According to Melany, students presented a roadblock for the changes she was implementing in her mathematics program. In the following excerpt from the data she talks about the resistance she encountered when she attempted to implement writing as a tool for reflection in mathematics.

... part of the reason that it's hard is because the kids aren't conditioned to deal with math in that way. They haven't been taught in that way so it takes them a while to realize that they're going to write.

(interview, December 16, p. 14)

Melany compared this problem to a similar one that she had previously experienced when she was attempting to change the way that she taught language arts:

Children have to go through this because I'm going through a learning process here so it's not going to go tickety-boo because I remember that with the writing process. I remember that there's this sort of cacophony because kids aren't being told what to do, when to do it, and how to do it.

(interview, October 9, p. 2)

On one occasion when Melany assigned students exercises from the textbook they jumped into their work without a moment's hesitation after receiving their instructions. As
they worked studiously at their desks, bent over their with pencil in hand, Melany chuckled and commented with a grin on her face:

Look how enthusiastic they are when they are assigned work from the textbook. Why is that? It's because they are comfortable and they don't need to think. They're like robots and they don't need to think and this is what school's all about. It's hard to make kids think.

(field notes, November 28, p. 227)

As Melany pointed out, many students are happiest when they are doing textbook work rather than having to figure things out for themselves.

The following incident, which occurred while I taught the unit on integers, provides further evidence of how student pressure can force a teacher to remain committed to a style of teaching as telling. Of the eight lessons that I taught, one lesson in particular stands out. While teaching the lesson, I sensed students' reluctance to participate. I wondered if they were confused by my teaching, or they were not sufficiently comfortable with my teaching style. When I attempted to elicit reasons for their behavior, the responses I received revealed how students might sabotage a teacher who wishes to teach for understanding. The following are samples of their comments. Some students said that my questions were “too long”. Further probing revealed that words I used such as magnitude, counting numbers, and axis were terms that they had not heard before. In my view, these are terms with which students in grade seven should be familiar. Another student explained that my questions were “not easy”. He, as others, explained that they could only respond to my “short-answer questions”. I was also told that I posed questions for which the answers were not obvious. Further inquiry revealed that, “He [Eric] gives an explanation first, and then he asks the question so it is easier to understand” (field notes, May 31, p. 233). At the end of the two week period I asked students for additional feedback about my teaching. The following is an example of their responses. “I spent too much time, and I should have taught more like their teacher” (field notes, June 10, p. 246). The most memorable comment came from a student who privately told me, “Maybe your teaching is a little rusty
because you haven’t taught for a long time and probably need lessons from our teacher” (field notes, June 10, p. 246).

In the previous sections I have talked about the effect of context as it related to Melany and Eric. It is evident that factors within the multi-layered context in which teachers work have the potential to affect their thinking and their practice and to stand in the way of teaching for understanding. I move now to the second part of this chapter in which I talk about what I have found as a result of conducting research on teachers’ understanding of teaching for understanding in the context of mathematics.

Reflections on the Study

The purpose of the study was to inquire into teachers’ understanding of teaching for understanding in mathematics. In so doing I looked at two elementary teachers’ beliefs and views and observed their practice so that I might come to understand what they understood about teaching for understanding in mathematics. The study demonstrated the problematic nature of teaching for understanding and the challenges it presents for teaching in this way. Firstly, it showed that teachers’ views of teaching for understanding may vary and may not necessarily be congruent with those of the literature, as the research (Cohen and Ball, 1990a; Gardner, 1993; Prawat, 1989) suggests.

Melany’s perspective on teaching for understanding focused on the responsibility of the teacher. She believed that it was a teacher’s duty to motivate students to think, to make generalizations and to provide them with the opportunity to talk about their thinking—what they were thinking and why. According to Melany, teaching for understanding was also “giving kids experiences where they can start coming up with their own [ideas], expressing and clarifying their own understanding of concepts” (interview, November 20, p. 13). On the other hand, Eric’s view of teaching for understanding was tied to his idea of nontraditional teaching. According to Eric, teaching in this way provided for learning which was based in experience, and it was a teacher’s responsibility to provide students with
different experiences. He described these experiences as "hands-on" learning, providing students with in-depth activities in which they had to figure things out on their own, and learning skills for dealing with the unexpected in life.

Secondly, the study revealed, as Eisenhardt et al. (1993) suggest, that teachers may not always have well formed images of what it means to teach for understanding. At the same time that Eric believed that he taught for understanding, he also doubted himself. In a conversation I had with him, initially he stated, "I don't not teach for understanding in math" (interview, May 2, p. 6). Yet, later in that same conversation, he explained.

In math, I don't know if teaching for understanding is the way I'd like to describe it because I'm not so sure I understand myself exactly what that means. But I like to think that I'm teaching the kids to be critical and to think in my other subject areas, most definitely. Math, yes, in the extent that like look at this, does this make sense to you? Come on, think about it. Is this really a logical answer? Is this really a logical way to solve this problem?

(interview, May 2, p. 6)

Thirdly, the study provides evidence that it is not an easy task for teachers to define understanding, the very concept which is at the heart of what it means to teach for understanding (Perkins, 1993). Eric, who could easily talk about his ideas, found it difficult to find words to express what understanding meant.

Like I don't know necessarily what understanding is. I'm sure I'll see a kid that can manipulate it [new concepts or ideas] and work around it and wheel around and they can apply different strategies to it and sure I'd say, yes, that kid understands. But to define what understanding means, it beats me.

(interview, March 28, p. 3)

At one point he did suggest that understanding was more of a "gut feeling - something that I know inside of me" (interview, March 28, p. 1). It was "something" he knew when he looked at students' written work or when he talked to them. Although Melany skirted around my attempts to engage her in conversation about her views on understanding, she did point out later in the study that understanding was not "something" she could easily talk about in the abstract. For her understanding was specific to a particular content area or
concept, rather than "a generic concept that was inclusive to all disciplines" (field notes, October 7, p. 66).

Fourthly, factors within the context that teachers work add to the challenge of teaching for understanding. As I suggested earlier, for Melany it was finding "good ideas" in the style of Marilyn Burns rather than "knowing" mathematics. It was also the demands that teaching for understanding put on her time and energy. I recall her telling me how periodically she needed to revert to textbook teaching because it "gave her a break". For Eric it was having sufficient time and covering the curriculum.

In the previous paragraphs I have talked briefly about the problematic nature of teaching for understanding as it relates to this study, and the challenges it presents for teaching in this way. I want to shift now and focus on what I have found about teaching for understanding that goes beyond cases in the literature that are closely related to the present study.

As I noted in Chapter Two, case studies developed within The California Study of Elementary Mathematics (Peterson, 1990a) provide an important context for situating this study. These studies (Peterson, 1990a) are closely related to this study as they too inquired into teachers' perspectives on teaching mathematics for understanding. However, their orientation varied in that they looked at the effects of policy change on teachers' practice. Also, researchers spent considerably less time observing in teachers' classrooms. Nevertheless, the present study corroborates findings of these case studies (Peterson, 1990a). The two teachers in this study also varied in their perceptions of what it means to teach for understanding in mathematics and their views did not necessarily fit with researchers' and those of curriculum developers. As with the previous studies (Peterson, 1990a) and others in the literature (Eisenhardt et al., 1993; Roulet, 1998) that focus on teaching for understanding, the present study also provides evidence of the complex interrelationship between teachers' beliefs and views and their practice. However, this study goes beyond those in the literature (Eisenhardt et al., 1993; Peterson, 1990a; Roulet, 1998)
in that it demonstrates that language arts provided an important referent for understanding these teachers’ understanding of teaching for understanding in mathematics. It would be surprising if this situation was not the pattern for studies carried out in a similar manner with other teachers. In the section that follows I want to talk briefly about the realities of teaching for understanding in mathematics in today’s educational climate.

Gardner (1993) points out that, “Teaching for understanding is a compelling notion— but it isn’t necessarily easy, as researchers at Harvard University’s Project Zero are finding out” (p. 22). Developing units of study for a project whose goals were to develop understanding took time. In addition, there were institutional and pedagogical constraints with which to deal. Teachers and students had to adjust their thinking. The California Study of Elementary Mathematics (Cohen and Ball, 1990a, 1990b) revealed that forces such as teachers’ attitudes, insufficient professional development, vagueness of policy reform and other factors within the multi-layered context of the educational landscape presented challenges for teaching for understanding. Despite efforts of The California State Department of Education to support elementary teachers with texts, curriculum, and professional development to encourage them to change their practice to instruction that promoted understanding, resistance was encountered. Wide scale implementation was not achieved. In fact the policy has been changed. Roulet’s (1998) study on teaching for understanding points out that a majority of high school teachers are unwilling to endorse instructional strategies recommended by The Professional Standards for Teaching Mathematics (NCTM, 1991). His study revealed that a teacher “with a more mixed subject conception” (Roulet, 1998, p. iii) who was in the process of transition and reconstruction, turned to traditional transmissive modes of instruction when confronted with the forces of opposition. As I noted in Chapter One, the views of interest groups who favor a more traditional, back-to-basics approach for dealing with students’ poor performance in mathematics are inconsistent with the views of The National Council of Teachers of Mathematics, which advocates teaching for understanding (Prawat, Remillard, Putnam,
Heaton, 1992). I move now to the final part of this chapter in which I address the strengths and weaknesses of the study, plans for future research, and conclude with some personal reflections.

Because of the paucity in the literature of comprehensive studies that focus exclusively on teaching for understanding, as I noted in Chapter One, the study, as one small piece of the research on understanding, adds two more comprehensive cases on teaching for understanding in the context of mathematics to the body of knowledge regarding cognitions and context related to teaching for understanding. It also helps fill the need for more “schools-based” research in mathematics (Goldin, 1993) and “feed the development of increasingly useful theories of mathematics teaching and learning” (Hiebert and Carpenter, 1992). The study also provides readers with further insight into contexts that significantly shape teachers’ practice, and factors within those contexts that affect teachers’ understanding of teaching for understanding.

The study’s base in teacher practice will appeal not only to researchers in the field of education, but also to practitioners who wish to understand what it means to teach for understanding in the context of mathematics. Each of the cases or bounded systems represents an “instance drawn from a class” (Adelman, Jenkins and Kemmis, 1983, p. 3) and provides the detail necessary to describe the “meaning perspectives of the particular actors in the particular events” (Erickson, 1986, p. 121). In so doing, the study will permit readers to connect personal experience to that of the participants.

Having focused in the previous paragraphs on the strengths of the study, I want to talk briefly about its limitations. Given the nature of the problem I was addressing, the appropriateness of the research methodology fit with the multidimensional nature of this phenomenon. I was able to observe two teachers in the “natural” (Hammersley and Atkinson, 1992) setting of their classrooms so that I might learn how they construct the world about them. I was also able to gain insight into their beliefs and views about mathematics and teaching for understanding. However, as I contemplate what I might have
done differently in this study, I suggest that with the luxury of time I might have inquired into more than two teachers’ practice, possibly those who teach at different grade levels so that I might add to the portraits of teaching for understanding that this study offers. I might also have included students’ perspectives. The learner’s perspective might have provided a more complete picture of what it means to teach for understanding in mathematics. Inquiry into students’ perspectives on understanding, a topic for future exploration and research, is discussed in the next section.

Although I was able to gather sufficiently rich data as I carried out my research, there were times when I found it extremely challenging in Melany’s open plan classroom to capture quality data in my audio taping. Even though videotaping tends to be more intrusive, I would consider its use if presented with a similar situation in the future. Lastly, as a new researcher who is in the process of developing and refining my interviewing and data collection techniques, I hope to become more proficient with practice.

From my view this study is the “payoff of my research” (Glesne and Peshkin, 1992). It is also the culmination of my extensive experience as a teacher educator and my continuous effort as a life long learner. The study marks the beginning of future research endeavors with teachers and students in the field. Some of the questions that I intend to explore that have evolved as a result of my work on this study are:

a) What is it that elementary students understand about what it means to understand in mathematics?

My earlier comments regarding limitations of the study were the basis for this question. It is my sense that investigating students’ understanding would provide a more complete picture of the phenomenon that this study investigated. It would also offer increased insight into students’ views on what it means to understand in mathematics.

Although standardized testing was not a concern for Melany and Eric during the time that I observed in their classrooms, it is an issue closely related to teaching for understanding that I wish to explore. In many instances standardized testing stands in the
way of teaching for understanding. Teachers are willing to alter their practice and the nature of instruction to conform to the content of tests (Talbert and McLaughlin, 1993). As a consultant working with teachers in classrooms, I have often observed this problem. For these reasons, I suggest the following question for future investigation.

b) What is the nature of the impact of Ministry testing on teaching for understanding?

The next question relates to my work with preservice teachers. The majority of preservice teachers I have taught in the past decade have been schooled on a diet of teaching in which telling and rote are common. As a result of their “apprenticeship of learning” (Lortie, 1975), many are comfortable with and believe that a traditional approach is the only way to teach mathematics. Frequently I encounter resistance when I introduce a style of teaching in class that is different. For this reason I wish to investigate the following.

c) What is the effect of preservice teachers’ beliefs about mathematics on their understanding and willingness to teach for understanding in mathematics?

As I reflect on my learning I suggest that this research endeavor has opened my eyes to the complexities of teaching for understanding in mathematics. It has also helped me to come to understand and better appreciate the challenges with which teachers, who wish to teach in this way, are faced. Moreover, I have become aware that, as a researcher, one cannot use broad brush strokes to judge a teacher’s practice. What might initially appear to be a style of teaching that does not promote understanding may on further investigation and reflection do so.
References


