Meeting in the Middle: Teacher-Facilitation of Play and Math Learning
in Contemporary Kindergarten

By

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A thesis submitted in conformity with the requirements for the degree of Doctor of
Philosophy in Developmental Psychology and Education

Department of Applied Psychology and Human Development
University of Toronto

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Abstract

Early math ability is a powerful predictor of future academic learning and opportunity (Duncan et al., 2007). To teach early math skills, many kindergarten programs world-wide endorse (Tafa, 2008), or mandate, play-based pedagogy (Ontario Ministry of Education, 2016), yet all play is not equal when it comes to math learning. Experimental evidence shows that a specific type of teacher-facilitated play, called guided play, leads to greater math learning gains than traditional approaches like direct instruction or free play (Fisher et al., 2013; Skene et al., 2022), however, classroom-based research shows that guided play is uncommon in practice (Wickstrom et al., 2019). Further work is needed to understand how teachers are successfully translating theory into practice by optimizing the middle ground of guided play to support early math learning.

This study used qualitative inquiry and thematic analysis to respond to this central challenge. It began by collecting survey data to understand kindergarten educators’ perspectives regarding play, math, and the integration of play and math. Participants who described guiding play to support math were purposefully selected for follow up interviews. Results identified three key findings. First, the majority of educators described more indirect approaches to play and
math learning in kindergarten, where children learn math through opportunities for free play without adult intervention. Conversely, only a minority of educators described more direct, or teacher-facilitated, approaches to math learning in kindergarten through various approaches to play and direct instruction. Second, educators who described these teacher-facilitated approaches also described evolving their conceptualizations of play and math learning into a singular and unified construct, that young children learn math through play. Third, this evolved perspective helped educators to then implement various approaches to guided play in practice. Implications of these findings point to a larger conversation about the need to move away from child-led approaches to early math learning and towards teacher-facilitated approaches instead. Importantly, this discussion will identify the key factors that educators leveraged to help them successfully transition towards teacher-facilitated approaches to math and will discuss what can be done to support more educators in also successfully making this transition.
Acknowledgments

I am deeply grateful for the openness, humility, generosity, and expertise of all the participating educators who opened up their classrooms, minds, and hearts to help me learn. I asked more of educators in one of the most demanding times in educational history, and they responded with such kindness. I’m honestly still in awe with how responsive they were. Thank you, educators.

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Dedication

Teaching makes me feel connected to my parents, my grandparents, and to all the educators in my family that have come before me. I feel my parents’ love, patience, and humor in the interactions I share with students and teachers. I am deeply privileged to have received such unconditional love and it’s an honour to pay a mere fraction of it forward through the art of teaching.

For Mom and Dad.

My first, and favourite, teachers.
# Table of Contents

**Abstract** .......................................................................................................................... ii

**Table of Contents** ............................................................................................................. vii

**Chapter 1: Introduction** .................................................................................................... 1

**Chapter 2: Literature Review** .......................................................................................... 9

Play in Education .................................................................................................................. 9

  - Contemporary Kindergarten Education ......................................................................... 9
  - Shifting Purpose of Play in Kindergarten ..................................................................... 10
  - Teacher Perspectives and the Implementation of Play ................................................. 13
  - Challenges Implementing Play ..................................................................................... 14
  - A Continuum of Play-Based Learning ......................................................................... 15
  - Guided Play ................................................................................................................ 17

Mathematics Knowledge for Teaching ................................................................................ 21

  - Subject Matter Knowledge .......................................................................................... 22
  - Pedagogical Content Knowledge ............................................................................... 23
  - Educators’ Relation with Mathematics ....................................................................... 24

Mathematics Education in Kindergarten .............................................................................. 25

  - Subject Matter Knowledge in Kindergarten .............................................................. 25
    - Young Children’s Mathematical Competencies ...................................................... 26
    - The Principles of Counting and Quantity ............................................................... 26
    - The Learning Trajectories ..................................................................................... 28

Pedagogical Content Knowledge in Kindergarten ............................................................. 32

  - Constructivist vs Didactic Mathematics Pedagogies ............................................... 32
### Comparing Pedagogies

- Gaps in Kindergarten Mathematics Education

### Chapter 3: Method

- Theoretical Orientation
- Philosophical Assumptions
  - Interpretive Framework
  - Orientation to the Data
  - Semantic vs Latent Analysis

### Data Collection

- Phase One
  - Participants
  - Recruitment
  - Survey
- Phase Two
  - Recruitment
  - Virtual Semi-Structured Interviews

### Data Analysis

- Theoretical Framework
- Phase One
- Phase Two
  - Step 1: Familiarization
  - Step 2: Generating Initial Codes
  - Step 3: Searching for Themes
Chapter 4: Results, Part 1. Educators’ Perspectives of Play and Math

Phase One: Traditional vs. Contemporary Perspectives of Play

The Purpose of Play in Kindergarten
How Children Learn Math
Comfort Teaching Math
Integrating Play and Math

Phase Two: Evolution and Integration of Play and Math Perspectives

Play Perspectives: Stages 1, 2, and 3

Math Perspectives: Stages 1, 2, and 3

Stage 4: Actualize

How Young Children Learn

Integrated Perspective of Play and Math
Chapter 5: Results, Part 2. Implementation: Supporting Math Along a Continuum of Guided Play .............................. 86

Approaches for Implementing Guided Play: Extending, Facilitating, and Inviting ........... 87

Extending ................................................................. 87
    Offering ............................................................ 87
    Commenting ....................................................... 88
    Joining ............................................................ 88
    Questioning ....................................................... 89

Facilitating ............................................................ 90
    Coordinating .................................................... 91
    Co-constructing ................................................ 91
    Co-directing Inquiries ........................................ 92

Inviting ................................................................. 93
    Provoking ......................................................... 93
    Enticing .......................................................... 94

An Iterative Process of Play: Planning, Guiding, and Debriefing .............................. 96

Before Play: Planning ............................................... 96
    Discussion ....................................................... 96
    Voting ........................................................... 97
    Entry Tickets ................................................... 97

During Play: Guiding ............................................... 98

After Play: Debriefing ............................................... 98
    Documenting .................................................... 98
    Celebrating ..................................................... 99
Chapter 6: Discussion ................................................................. 101

Underdeveloped Mathematics Education in Kindergarten ..................... 102
  Kindergarten as “A Separate Beast” ............................................ 102
  Limited Mathematics Knowledge for Teaching in Kindergarten .......... 107

“Whole Math” Approach is Not Sufficient ...................................... 110
  Indirect vs. Direct Approaches to Math Instruction ........................ 110
  Reflecting the Whole Language Movement .................................... 113
  Lessons Learned from the Whole Language Movement .................... 115

Teacher-Facilitation of Math Skills ................................................ 117
  Teacher-Facilitation of Play ....................................................... 117
  Balancing Pedagogical Approaches .......................................... 118
  (Dis)comfort Teaching Math .................................................... 120

Transitioning from Whole-Math to Teacher-Facilitated Math ................. 121
  Expanding Notions of Play ....................................................... 121
  Changing Relationship with Math .............................................. 122
  Strengthening Mathematics Knowledge for Teaching ..................... 124
  Strengthening Knowledge of Child Development .......................... 125
  Knowledge of Child Development Informs Pedagogical Decision-Making 126

Next Steps Towards Supporting Educators .................................... 128

Limitations ................................................................................. 132

Conclusion .................................................................................. 136

References .................................................................................. 138
Figures  ............................................................................................................................. 165
  Figure 1. Pyle and Danniel’s (2017) Continuum of Play-Based Learning .......... 165
  Figure 2. Ball et al.’s (2008) Mathematics Knowledge for Teaching Framework ..... 166
  Figure 3. Stages of Educators’ Evolution and Integration of Play and Math
  Perspectives .................................................................................................................. 167
  Figure 4. A Continuum of Guided Play to Support Early Math Learning ............ 168
Tables .................................................................................................................................. 169
  Table 1. The Principles of Counting and Quantity ...................................................... 169
  Table 2. Phase One Participant Demographic Information ......................................... 171
  Table 3. Phase Two Participant Demographic Information .......................................... 172
  Table 4. Six Steps of Thematic Analysis ....................................................................... 173
Appendices .......................................................................................................................... 176
  Appendix A. Approved Ethics Letter ........................................................................... 176
  Appendix B. Phase One Consent Letter of Information ............................................... 177
  Appendix C. Phase One Survey Questions .................................................................... 179
  Appendix D. Phase Two Consent Letter of Information .............................................. 181
  Appendix E. Phase Two Interview Questions ............................................................... 183
Chapter 1: Introduction


Throughout the fall of 2016 I heard this sentiment daily from one of my students, and all I could think was, “why?”. Jaden was six, and so much of him was unformed, growing, and malleable. How could this one piece of him already be fixed? While Jaden’s words clearly shared his perspective, his actions told a different story. Jaden loved to build. With blocks, cups, sticks, books, or anything he could find. He loved to talk about the patterns and designs on his clothing, and never missed an opportunity to highlight a “twinning” moment when his outfit matched his peers. He would also certainly be among the first to point out when something wasn’t fair. If someone had more snacks than him, if another class spent more time outside, or if we had less time to play today than the day before.

Despite his articulated stance towards math, Jaden engaged in mathematical thinking with the world around him on a daily basis, though he did not see it as such. I had a hard time understanding why that may be so, until I became the early years math lead at my school. Given my background and training with young children, my principal trusted me to lead our small team of primary teachers in strengthening our math program. Through chairing professional learning communities, I quickly learned how differently the subject of math was perceived, taught, and modelled in the early years. Many of my teaching colleagues rooted their early years math programs in worksheet-driven lessons, calendar and morning message routines, or through free exploration in play. In doing so, they were simply repeating what was familiar to them, by mirroring their own experiences learning math or by following the lead of their teaching partners. I developed many questions from this experience about how we can better understand the ways in which young children learn math and how we can better support educators in their
implementation of math education through developmentally appropriate practices. Fortunately, I had the opportunity and support to answer these questions by returning to graduate school and exploring the research literature on mathematics education in kindergarten.

The research is very clear that early math ability is a powerful predictor of future academic success and opportunity (Duncan et al., 2007; Rittle-Johnson, 2016). To teach early math skills, Ontario’s Kindergarten Program mandates play-based pedagogy [Ontario Ministry of Education (OME), 2016], yet all play is not equal when it comes to math learning. Research endorses a specific type of collaborative play where teachers infuse instruction into children’s play to guide learning towards targeted math skills (Fisher et al., 2013). This type of play is described as guided play and involves a balance between child autonomy and adult facilitation of play. Conversely, the research shows that free play, play that is led by the child with no adult involvement, is an ineffective method of supporting the development of new math skills (Alfieri et al., 2011; Fisher et al., 2013; Skene et al., 2022). In practice, however, free play continues to be a prominent type of play that educators implement, while guided play is relatively uncommon (Pyle et al., 2018a; Wickstrom et al., 2019). Possible reasons for this relate to evolving conceptualizations regarding the purpose of play in kindergarten education, including educators’ role in play, as play has historically been conceptualized as strictly free play that does not allow for adult intervention (Goouch, 2008; Pramling Samuelsson & Johansson, 2006; Pyle et al., 2017). Contemporary perspectives of play push back against these historical notions and advocate that adults should be involved in play to help support, scaffold, and extend student learning within child-centered contexts. In response to this shifting purpose of play, and adults’ role in play, many educators have expressed uncertainty in how to follow children’s lead while also directing play towards targeted academic goals (Pyle et al., 2018a). Consequently, there is a
disconnect between current recommendations from research and contemporary kindergarten classroom practice. As a result, some question the effectiveness of current pedagogies, such as play, and call for a “back-to-basics” approach to math instruction (Stokke, 2015).

Despite these calls, corroborating evidence from theoretical and experimental studies continue to demonstrate how collaborative, or guided, play is superior to traditional pedagogies like direct instruction and free play for math learning (Alfieri et al., 2011; Fisher et al., 2013; Skene et al., 2022). While this line of research suggests that children’s math ability thrives when teachers and children meet in the middle between free play and direct instruction, my research shows that guided play is uncommon in classroom practice as teacher-directed approaches to math instruction take precedence (Wickstrom et al., 2019). Compounding this problem is a limited understanding of guided play and math learning that is rooted within the realities of kindergarten classroom practice, as much of our current understanding comes from laboratory-based studies (Fisher et al., 2013; Skene et al., 2022). Further work is needed to understand how classroom teachers are successfully translating theory into practice by optimizing the middle ground of guided play to support children’s early math learning.

This dissertation responded to this need. It used qualitative inquiry to gain an in-depth understanding of how kindergarten educators were successfully implementing guided play to support children’s early math learning. This knowledge was rooted in classroom practice, as it developed a rich understanding of the lived experiences of kindergarten educators, including the successes and challenges they faced integrating play and math learning in the early years. Results from this research identified key approaches and strategies educators used to implement guided play and early math learning. These findings can now be used to support other educators towards successfully implementing guided play and early math learning in their own classrooms, which
in turn, can help to provide children with a strong foundation of math knowledge that can promote future success. To this end, the present study answered the following research questions:

1. What are kindergarten educators’ perspectives regarding the role of play in early math learning?

2. How are kindergarten educators successfully facilitating guided play to support children’s early math learning?

My ability to answer these questions, and the lens through which I viewed this study, is highly influenced by my training and experience as both a teacher and a researcher. I am privileged to have had the opportunity to gain training in the MA Child Study and Education program at the University of Toronto. This program provided me with the opportunity to understand the intimate connections between research and practice. I learned about teaching practices that are rooted in research and evidence for what best supports student learning. I also learned how to understand and respond to the needs of young learners and how to make decisions that are in the best interest of children. This program helped me to understand a larger view on the role of public education in our society, including the importance of supporting diversity, equity, and inclusion not just in our schools, but in our world more broadly. With productive struggling, I learned a lot about myself, particularly the privileges I have experienced, and continue to experience, in my life. This program pushed me to be critically reflective and taught me how to use this reflection to inform and evolve my practice as a teacher. A main gift from this training, however, was the opportunity to learn from people who love what they do. I experienced the thoughtfulness, pragmatism, and optimism of skilled educators, which I brought with me into the classrooms I shared, and continue to share, with students.
Training in this masters’ program also exposed me to a comprehensive range of research methodologies including experimental, quasi-experimental, and field-based research. Coming from a background in Psychology, which was heavily rooted in lab-based research, this exposure to different types of research was both familiar and foreign to me, in the best possible way. I had never seen, or experienced, field-based research, which completely opened up the way I viewed research and the knowledge we can gain from it. I then had the opportunity to expand and deepen my understanding of research methodology through training in my current PhD program. In particular, this program helped me to deepen my understanding of classroom-based research, including why this type of research is needed, the unique benefits it can offer, the challenges and limitations it comes with, and how to build strong relationships with practicing educators, which is essential for conducting classroom-based research. This research would simply not happen without the trust and partnership of practicing educators.

In between my masters’ and PhD program, I worked as a permanent elementary teacher in Calgary, Alberta, and taught grades one, two, and three. Throughout my entire PhD program, I also worked as an occasional teacher in Ontario. As a permanent teacher, I gained a new understanding of the role schools play within a community. I worked at schools that were in under resourced communities and played an essential role in supporting many aspects of children’s lives, including a place to provide meals, warm winter clothing, family resources, and ideally, a safe and happy place to spend the day. While I knew about this fundamental role that schools played in theory, it was very different to experience it in practice. I gained a new appreciation for ensuring children’s fundamental needs, including their social and emotional well-being, were met alongside their goals for academic learning. Academic learning is hard, but it becomes easier when children are fed, comfortable, and happy. I learned more about the
opportunities that come along with developing a strong foundation of academic skills, and importantly, the challenges that lie ahead without this strong foundation.

I currently work as an occasional teacher, and while I do not experience the same proximity to children’s learning, I have gained a new perspective that comes with being more removed from the day-to-day interactions within a school. I’ve had the opportunity to see, and work in, a large number of classrooms spanning kindergarten to grade eight, including special education and learning center classrooms. Most of my occasional teaching work, however, has been in kindergarten, which has helped me to gain a broad perspective on the many ways in which Ontario’s Kindergarten Program is being interpreted and implemented in practice. Through this work, I’ve witnessed a lot of similarities across classrooms, such as general classroom structure (e.g., first the teacher leads a whole group mini lesson, then children complete structured “tabletop” activities before transitioning to free play for most of the block) as well as a lot of differences across classrooms (e.g., how educators interpret and enact the academic objectives within the curriculum). This experience has not only helped me to feel connected to the classroom throughout graduate school, but has also helped me to frame my thinking when designing, conducting, and writing this dissertation.

My academic and professional experiences have strengthened and informed each other, which has contributed to my beliefs and practices that have influenced the choices I’ve made throughout each stage of this project. Primarily, I believe teachers play a leading, rather than a supportive, role in classroom settings. While I do believe children should have a voice in their learning and that educators can follow children’s interests, I don’t believe that means the educator has to take a back seat in decision-making. I believe it’s essential for educators to actively guide, support, and extend students’ learning. This perspective will be evident in the
leading theoretical frameworks I chose to support this study, the themes I identified through qualitative analysis, the interpretation of this study’s findings, as well as the contributions to the research literature that will be shared in the discussion.

Secondly, as an educator, I would describe my curricular stance as flexible. As a permanent teacher, I did not view the curriculum as a rigid set of rules to be followed, but rather chose to prioritize meeting my students where they were at and to move them forward in their learning. I relied on my knowledge of evidence from research when making decisions to assess and support student learning. I think this flexible curricular stance, and emphasis on the research literature, is also present in this dissertation. While Ontario’s Kindergarten Program will be the main curricular document providing context for this study, this document including its description of play and math learning objectives does not guide my interpretation of the study’s findings. Instead, the research literature will be used to guide both particular phases of the data analysis and the interpretation of the study’s findings.

Third, I do not currently believe in strictly adhering to one paradigm for conducting research. I believe a strength of my time in graduate school is that I’ve gained training, support, and experience conducting both quantitative and qualitative research, which has equipped me with tools and skills that can answer a range of research questions. I’m not interested in participating in a debate about quantitative versus qualitative methodology and would rather choose to prioritize methodological fit by answering research questions in the most appropriate way possible. This perspective will be further unpacked in Chapter 3.

Lastly, as a former classroom teacher, I understand the breadth and depth of the responsibilities that classroom teachers must balance. I think this perspective has been instrumental in my ability to work with educators in the role of a researcher and in interpreting
findings from classroom-based research. While this study is designed to address some very specific questions about play and math learning in kindergarten, I am acutely aware that this reflects a mere fraction of the responsibilities of classroom practice. While I will indulge in the details about play and math learning in kindergarten, I will also strive to share larger, overarching ideas about play and math learning in kindergarten education. My training and experience as a teacher and a researcher have strengthened and informed each other, which has also influenced the lens through which I viewed this current study.
Chapter 2: Literature Review

**Play in Education**

*Contemporary Kindergarten Education*

Kindergarten classrooms have changed considerably from Froebel’s (1967) original vision of “the garden of children”. While initially conceptualized as a program to support children’s developmental learning through playful child-led exploration, contemporary kindergarten programs have witnessed the steady introduction of academic curricular objectives that have shifted both the content and structure of kindergarten programs (Pyle, 2013; Russell, 2011; Stipek, 2004; Synodi, 2010). Such academic push down effects have been observed internationally, including countries in North America (Stipek, 2004), Western Europe (Grey & Ryan, 2016), and Australasia (Foote et al., 2004), among others. This curricular push down of academic expectations accompanied an accountability-driven era of standardized testing that emphasized proficiency with academic skills, including targeted literacy and mathematics objectives (Hargreaves & Goodson, 2006; Heydon & Wang, 2006). As such, kindergarten educators are currently faced with the task of supporting children’s mastery of these standardized academic expectations that were introduced to kindergarten curricula, in addition to the traditional developmental learning goals (Pyle, 2013; Pyle & Bigelow, 2015).

At the same time, research on play-based learning has produced growing evidence for the suitability of play to support both academic and developmental learning in young children (Pyle et al., 2017; Wallerstedt & Pramling, 2012). Accordingly, many kindergarten programs and policies world-wide endorse (Baker, 2014; Pan & Li, 2012; Synodi, 2010), and even mandate [Ontario Ministry of Education (OME), 2016], play-based learning as a leading pedagogical practice in kindergarten education. In Ontario, Canada, specifically, curricular policy currently
mandates play-based learning as the primary pedagogical practice to deliver The Kindergarten Program (OME, 2016). To this end, educators are required to abide by this pedagogical directive, despite the fact that many of them have received low, or no, levels of professional development pertaining to the study and implementation of play-based learning (Pyle & Danniels, 2017). The professional development concerning play-based learning that is available to educators varies widely in terms of how play is defined, implemented, and supported within contemporary kindergarten classrooms (Bubikova-Moan, 2019; Hunter, 2019; Schmidtke, 2020). Evidence from research promotes play, in particular teacher-facilitated play (Weisberg et al., 2013; 2016; Zosh et al., 2018), to address this academic push down effect as it positions play as a developmentally-appropriate practice that can support both academic and developmental learning within young children (Pyle et al., 2017). A remaining issue, however, is that educators face several challenges towards implementing play-based pedagogy in practice (Cooney, 2004; Hu et al., 2014), including conceptual and practical challenges regarding how to facilitate children’s play towards targeted academic goals (Pyle et al., 2018). This speaks to educators’ overarching challenge of adapting to the shifting purpose of play in contemporary kindergarten programming.

**Shifting Purpose of Play in Kindergarten**

The notion that young children learn through play has been a cornerstone of early years education for decades (Piaget, 1951). While this central tenet remains unchanged, the purpose of play in kindergarten education has evolved over the years (Pyle, 2013; Pyle et al., 2017). In keeping with Froebel’s vision of kindergarten, play was originally conceptualized as a method of supporting children’s developmental learning (Ghafouri & Wien, 2005), including their self-regulation (Bodrova & Leong, 2008), social-emotional (Tal et al., 2008), and general cognitive
growth (Hansel, 2015). To achieve this purpose, play was defined as child-led free play, where children spontaneously engaged in free exploration without adult intervention (Pramling Samuelsson & Johansson, 2006). In fact, play was considered to be ruined, or “hijacked” (Goouch, 2008, p. 95) if adults were to become involved, as play was largely viewed as the property of children that provided them with the opportunity for high levels of autonomy (Pramling Samuelsson & Johansson, 2006). Within this view of play, children’s independence and decision-making were prioritized as a means of supporting enjoyment, authenticity, and motivation for learning (Ashiabi, 2007; Sturgess, 2003). As such, adults described playing a more supportive role along the periphery of play to protect the sense of joy and autonomy that was emphasized within the notion of free play (Andrews, 2015; Wood, 2014). To this end, educators commonly described how their role was to provide large blocks of uninterrupted time for children to become deeply immersed in play (Fredriksen, 2012; Gmitrová & Gmitrov, 2004; Hansel, 2015; McNamee, 2005; Nordtomme, 2012) and also described the importance of setting up the classroom environment to support enriching and engrossing opportunities for play (Hansel, 2015; Schwartz, 2008). This involved a large emphasis on the materials that educators made available for children, ensuring that they built upon students’ interests, background knowledge, and lived experiences (Fredriksen, 2012). With this view of play, adults prioritized protecting the time and preparing the space for children to play, emphasizing the role of the classroom as “the third teacher” (Fraser & Wien, 2001; OME, 2016). After the space was thoughtfully prepared, educators would then describe stepping back to allow for children’s free engagement in play without adults’ intervention (Fredriksen, 2012), beyond simply providing materials or managing behaviours as needed. Largely, the research describes this perspective as a
traditional view of play that dichotomized the constructs of play and learning (Pyle, 2013; Pyle et al., 2017).

Contemporary views concerning the purpose of play, on the other hand, integrate the constructs of play and learning, such that children learn *through* play, and differ from traditional perspectives in regard to both the content that can be learned through play as well as adults’ role in play. While traditional views of play promote its suitability for children’s developmental learning (Ghafouri & Wien, 2005), contemporary views of play advocate for it as a means of supporting both the developmental *and* academic learning of young children (Pyle et al., 2017). This contemporary perspective of play maintains the traditional notion that play responds to the developmental needs of young children, for example that they learn best through hands on joyful activities (Hansel, 2015), but also introduces the idea that adults need to scaffold learning within these playful contexts in order to help children both deepen existing skills and build new skills (Newbury et al., 2015; Sarama & Clements, 2006). This contemporary perspective of play has largely focused on how play can be used as a method of supporting children’s academic learning and emphasizes how adults can be involved in play to help support, extend, and guide student learning towards discrete academic skills (Pyle et al., 2017). This line of research consistently points to the important role of teacher-facilitated activities to support early academic learning and argues that teachers need to take an active role during play through both creating and directing a variety of play opportunities (Kamii, 2003; Newbury et al., 2015), including intentionally intervening in free-play (Pyle et al., 2017). Advocates for this perspective argue that it’s beneficial for educators to act as co-participants in play as it allows them to observe children’s thinking and optimize learning opportunities that arise, which may otherwise go unnoticed by children (Wang & Hung, 2010). More so, this perspective emphasizes the
importance of play that is mutually-directed by both educator and child, as it takes into consideration the children’s point of view, reflects their interests, and leaves room from child-led problem solving (Korat et al., 2002), while also allowing the educator to provide appropriate scaffolding and support (Regush et al., 2002), often through questioning, discussion, introducing new concepts, and modelling academic skills (Isom, 2014; Sarama & Clements, 2006). While the research describes various perspectives of play, it also describes various implementations of play.

**Teacher Perspectives and the Implementation of Play**

Teachers’ perspectives and classroom practice are intimately connected, as teachers’ perspectives influence and shape pedagogical implementation (Foote et al., 2004; Snider & Roehl, 2007). This is particularly salient when it comes to the study of play in educational settings as previous research has shown that the way teachers perceive the purpose of play affects how they implement play in their kindergarten classrooms (Fesseha & Pyle, 2016; Pramling Samuelsson & Johansson, 2006; Pyle & Danniels, 2017; Pyle et al., 2018b). Specifically, research has found that when teachers express traditional perspectives of play that dichotomize the constructs of play and learning, they implement only free play in their classrooms for the purpose of supporting children’s social and emotional development (Pramling Samuelsson & Johansson, 2006; Pyle & Bigelow, 2015; Pyle & Danniels, 2017). Educators who aligned with more traditional perspectives supported academic learning outside of the context of play, typically though whole group lessons or withdrawal from play for one-on-one or small group direct instruction (Pyle et al., 2018b). Conversely, teachers who described a contemporary and integrated perspective of play and learning implemented a range of play-based learning in practice that supported both developmental and academic learning within the context of play.
(Pyle & Danniels, 2017; Pyle et al., 2018b; Pyle et al., 2020). As such, educators who expressed contemporary views of play actively got involved in play to help direct the play towards specific learning objectives (Pyle & Danniels, 2017; Pyle et al., 2018b; Pyle et al., 2020). While this contemporary view and implementation of play offers many new opportunities to support students’ academic learning in a developmentally sensitive way, it also comes with many challenges in practice.

**Challenges Implementing Play**

Educators face several conceptual and practical barriers towards implementing play-based learning in practice. Conceptual barriers are reflected through the shifting purpose of play in kindergarten, including the educator’s role in play (Pyle et al., 2017). Understandably, asking educators to now get involved in children’s play, to influence and purposefully “hijack” it, represents a significant point of tension in educators’ practice as the idea of getting involved in children’s play was traditionally viewed as taboo (Goouch, 2008). This shifting perspective of the purpose of play and adults’ role in play in the research literature has consequently led to inconsistent and conflicting messages that are shared with educators (Bubikova-Moan, 2019; Hunter, 2019), which contributes towards educators’ conceptual uncertainty regarding the role of play in their practice (Pyle et al., 2018a). These conceptual barriers are further compounded by practical barriers, including limited professional training and professional development about play-based learning (Howard, 2010; Pui-Wah & Stimpson, 2004), pressure from parents to use didactic instruction (Fung & Cheng, 2012), pressure for children to achieve explicit academic curricular objectives in preparation for grade one (Lynch, 2014), large educator-child ratios (Lynch, 2014), as well as limited classroom space, materials, or time to support play (Hedge & Cassidy, 2009). More so, in order to help children achieve both developmental and academic
expectations through play, as outlined in curriculum (OME, 2016), educators express uncertainty in how to follow children’s lead in play while also directing play towards explicit academic goals (Pyle et al., 2018a). In response to some of the conceptual challenges faced by educators, Pyle & Danniels (2017) developed a framework to reconceptualize the construct of play-based learning to acknowledge the shifting roles of both children and adults in play and described play as a continuum varying in levels of child-autonomy and adult facilitation.

_A Continuum of Play-Based Learning_

Building onto decades of work about play-based learning, Pyle and Danniels (2017) expanded notions of play in the research literature by describing play as a continuum that ranges in level of child and adult participation. Overall, this continuum outlines five types of play-based learning that describe varying levels of child and adult involvement and direction over the play. From lowest to highest levels of adult involvement, these types of play include, Free Play, Inquiry Play, Collaborative Play, Playful Learning, and Learning Through Games (Figure 1). When compared to the broader literature, these five types of play can be condensed into three overarching categories of play that vary by locus of control. At opposing ends of the play continuum are _free play_ and _teacher-directed play_, respectively. As the names suggest, free play is entirely child-directed and often involves children’s spontaneous socio-dramatic or pretend play (Wallerstedt & Pramling, 2012), whereas teacher-directed play is exclusively controlled by the educator and is often demonstrated through highly structured games such as board or card games (Gmitrová et al., 2009; Kamii, 2003). Occupying the middle ground between these distinct types of play is _guided play_, in which the control over the play is shared between the child and educator through a collaborative playful experience (Fisher et al., 2013; Jensen et al., 2019; Weisberg et al., 2013).
Pyle and Danniels’ (2017) Continuum of Play-based Learning has been used as a framework of play that has been widely accepted in both literature (Mardell et al., 2019; Taylor & Boyer, 2020) and policy (British Columbia Ministry of Education, 2019; Honourable Margaret Norrie McCain, 2020) and builds onto previous work of play researchers such as Fisher et al., (2012; 2013) and Weisberg et al., (2013; 2015; 2016) that called for a need to distinguish a playful middle ground between child-led free play and teacher-led didactic instruction, which emphasizes teachers’ facilitation of play. The Play-Based Learning Continuum (Pyle & Danniels, 2017) has also been used as a theoretical framework to study the interplay between play-based learning and a multitude of learning domains including literacy (Pyle et al., 2018a; 2018b), mathematics (Wickstrom et al., 2019), self-regulation (Pyle et al., 2022), assessment (Pyle et al., 2020), and inclusion (Danniels & Pyle, 2022).

I chose this framework for this study because it not only includes a description of various approaches to guided play, which is the central concept I aim to unpack, but it also makes an important distinction between child-directedness and child-centredness, which I believe lays the groundwork for contemporary perspectives of guided play. Specifically, child-directedness refers to activities that are led by a child, whereas child-centeredness refers to a general teaching stance of honouring young children’s developmental needs, interests, and abilities (NAEYC, 2009). Pyle & Danniels (2017) describe the entire continuum as child-centered, as it emphasizes the importance of supporting children’s academic learning through developmentally appropriate strategies, such as play. The authors also explicitly state that not all types of play along the continuum are, or need to be, child-directed. They highlight the importance of teacher-facilitation within some types of play and argue that some approaches to play can contain
teacher-directed elements while still remaining child-centered. Critically, this framework espouses the belief that:

By emphasizing the need to honour children and their developmental needs (child-centeredness) rather than emphasizing the locus of control (child-directedness), classrooms can integrate both play and play-based learning, allowing teachers to provide the guidance necessary to extend children’s learning in an engaging, play-based manner. In order for play-based learning to be implemented effectively, teachers and researchers need to shift away from the perspective that the teacher’s role is “to support, not to disturb” (Pramling Samuelsson & Johansson, 2006: 48) and to avoid “hijacking” play (Gooouch, 2008, p. 95). (Pyle & Danniels, 2017, p. 13-14).

Researchers in the field of early years mathematics were also working towards descriptions of guided play before Pyle and Dannièl’s play continuum was published. Over a decade before this play continuum, research by Ginsburg (2006), Clements and Sarama (2004), and Baroody (2006) among others, discussed the role of playful learning in children’s mathematics development, while researchers like Wood, (2009), Weisberg et al., (2015), and Gasteiger et al., (2015) identified a need to distinguish between child-led and adult-led approaches to play, highly structured games, and direct-instruction. Taken together, the field of early years math research was also pointing towards identifying a continuum of playful learning that ranges in locus of control, from child-led, to guided, to teacher-led approaches to learning (Baroody et al., 2006). The convergence in perspectives, between the fields of early mathematics and play-based learning, that emphasized the need to better understand the middle ground of guided, or teacher-facilitated, approaches to play demonstrates an overarching theme in the field of early years education to better understand the ways in which educators can leverage guided play to support students’ learning.

**Guided Play**

Unlike either free play or teacher-directed play, where one party controls the playful context, guided play calls for mutual direction and collaboration within play. The concept of
guided play has been difficult to define in the literature (Weisberg et al., 2013), but has gained clarity over recent years (Jensen et al., 2019; Skene et al., 2022; Zosh et al., 2018). In essence, definitions of guided play aim to describe a balance between child-autonomy and adult-facilitation within playful contexts. Weisberg et al. (2013) were among the first research teams to highlight the potential of guided play to support young children’s learning. Through their work, guided play has been defined as play that remained child-directed, but incorporated elements of adult scaffolded learning objectives (Weisberg et al., 2013). Broadly, Weisberg et al. (2016) described two approaches to guided play. In the first approach, educators observed children in free play and encouraged children to extend their interests through questioning and commenting (Weisberg et al., 2016). In the second approach, the educator designed a playful setting, intended to target a specific learning goal, and then stepped back and allowed the child to engage in that activity with high levels of autonomy (Weisberg et al., 2013; 2016). Jensen et al. (2019) described similar approaches to play, and named them extending and initiating, respectfully. In their work, Jensen et al. (2019) described how initiating refers to activities that are created by an educator and then actively directed by the children, whereas extending refers to contexts in which children are engaged in playful activities and then adults enrich or scaffold these contexts to help students reach their intended goals. While adult-facilitation is present in more of a passive way, Weisberg et al.’s (2013; 2016) view of guided play continues to prioritize child-autonomy by continuing to describe the play as child-directed and cautions against adults “co-opting” or taking over the play (Weisberg et al., 2013).

Zosh et al. (2018) pushed the conceptualization of guided play further, highlighting higher levels of adult involvement by situating guided play in the middle between free play and direct instruction and suggesting that guided play borrows techniques from direct instruction to
help children focus on specific learning tasks, while still acknowledging the child as an active participant in their learning. This research team effectively incorporated key components of more traditional views of play that highlight the importance of the social elements involved in learning while still connecting the construct of guided play to contemporary notions of play. Specifically, Zosh et al. (2018) emphasized the importance of learning that is engaging, meaningful, socially interactive, iterative, and joyful. While these are factors that are commonly associated with traditional notions of play (Andrews, 2015; Pramling Samuelsson & Johansson, 2006), Zosh et al. (2018) make a strong argument for how they are also fundamental components of contemporary perspectives of play, including guided play, while still advocating for adults’ involvement in play to scaffold and support learning.

Within this evolved conceptualization of guided play, research is beginning to not only accept, but also promote the idea that adults can actively be involved in play to support students’ learning (Pyle & Danniels, 2017; Wickstrom et al., 2019; Zosh et al., 2018). Most recently, a study conducted by Dr. Pyle’s research team, which is currently under review, presented empirical evidence for describing guided play as a continuum that ranges in level of child or adult direction while still remaining a collaborative experience (Pyle et al., submitted). More so, this study demonstrated how teachers are not only facilitating a range of guided play opportunities to support early literacy learning, but also how teachers are beginning to transition from free and teacher-directed types of play towards more guided approaches to play as a means of supporting a greater range of early literacy skills (Pyle et al., submitted).

The potential of guided play has quickly gained interest and has been studied relative to a wide range of early learning skills. Growing evidence demonstrates the appropriateness of guided play to support children’s developmental learning, including their self-regulation
(Cavanaugh et al., 2017), inhibitory-control (Goble & Pianta, 2017), as well as creativity and problem solving (Zosh et al., 2017). Largely, guided play has been studied in relation to children’s academic learning, noting particular benefits for children’s literacy learning, including advantages for children’s vocabulary, print, and phonological awareness (Goble & Pianta, 2017; Strauss & Bipath, 2020; Toub et al., 2018) as well as their development of scientific knowledge (Sliogeris & Almeida, 2019).

There have been several studies noting the benefits of guided play for children’s mathematics learning in particular. Studies have shown that parents and children engage in more math talk, which supports greater math vocabulary and spatial language, in guided play than in either free play or direct instruction conditions (Borriello & Liben, 2018; Eason & Ramani, 2020). More so, teacher-led approaches to guided play have also been shown to lead to statistically significantly greater gains in number naming, counting, and applied problem solving in four- to six-year-old children compared to age-matched peers that were not exposed to a guided play-based math intervention program (Cohrssen & Niklas, 2019). A seminal study by Fisher et al., (2013) demonstrated that when kindergarten-aged children were exposed to guided play pedagogy they learned, and retained, statistically significantly more novel math knowledge about geometric shapes, than children who learned about the same content knowledge through either free play or direct instruction (Fisher et al., 2013). This study also demonstrated that children who were exposed to novel math knowledge through direct instruction learned statistically significantly more than children who were exposed to the same concepts through free play. In the free play condition, children did not acquire any statistically significant amount of novel math knowledge (Fisher et al., 2013). The general finding that guided play leads to greater math learning gains, when compared to either free play or direct instruction, was
corroborated by a systematic review and meta-analysis which demonstrated that guided play interventions had a statistically significant positive effect on young children’s maths skills including shape knowledge and spatial vocabulary (Skene et al., 2022).

While these studies provide a strong foundation that is crucial for our understanding of the power of guided play, many of them occurred within highly controlled laboratory settings, which differ substantially from real life classrooms, and thus educators are faced with a unique problem of how to implement guided play in order to unlock its potential to support early mathematics development in practice. Currently, there are few classroom-based studies that address the issue of how to implement guided play for early math learning within the realities of classroom practice. Studies that have examined this topic have found that while guided play is touted as best practice in the literature, educators face difficulty translating theory into classroom practice, as teacher-directed forms of math instruction take precedence (Wickstrom et al., 2019). Possible reasons for this limited translation of theory into practice may, in part, be due to challenges educators face implementing play, as described above. Another possible reason for this limited translation of knowledge may exist outside the construct of play and be rooted in our understanding of the mathematics knowledge required for teaching in kindergarten.

**Mathematics Knowledge for Teaching**

Research has long examined what knowledge educators require in order to effectively teach mathematics to students (Ball et al., 2008; Hill et al., 2005; Reid, 2013; Thames & Ball, 2010). Perhaps most prevalently and widely agreed upon is Ball et al.’s (2008) framework of Mathematics Knowledge for Teaching. This framework not only details the various domains of knowledge involved in teaching math, but also contextualizes these domains of knowledge within the many responsibilities that teaching entails. By “teaching” Ball et al.’s (2008) work
describes everything that teachers must do to support students’ math learning, including the interactive work of teaching lessons, planning for lessons, evaluating students’ work, writing and grading assignments, providing feedback to students and parents, coordinating homework, attending to issues of equity in math learning, and potentially navigating conflicting views of mathematics education among multiple stakeholders (Ball et al., 2008). In particular, this framework builds upon Schulman’s (1986) foundational work concerning pedagogical content knowledge and demonstrates connections between six domains of knowledge that are required for the effective teaching of mathematics (Figure 2). Broadly, these six domains can be conceptualized into two main camps of knowledge: Subject Matter Knowledge and Pedagogical Content Knowledge (Reid, 2013; Thames & Ball, 2010).

**Subject Matter Knowledge**

The first category of Ball et al.’s (2008) Mathematical Knowledge for Teaching is Subject Matter Knowledge, which is composed of three specific domains: *Common Content Knowledge*; general mathematical skill that can be used in any context, not specific to teaching, *Specialized Content Knowledge*, mathematical skill that is unique to teaching, and *Horizon Knowledge*, an awareness of how mathematical concepts are related and span throughout a curriculum (Ball et al., 2008; Hill & Ball, 2009). The domains of Common Content Knowledge and Specialized Content Knowledge are particularly relevant to this study, as they most closely relate to the direct teaching of discrete mathematical skills. Let’s consider these concepts in greater depth through an example. Consider the following subtraction problem:

```
307
- 168
---
261
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Common Content Knowledge is the skill required to solve this subtraction problem. It is also the knowledge that tells us the answer to this problem is incorrect. It does not require any specialized knowledge or training and this Common Content knowledge can be applied in any setting (Thames & Ball, 2010). For example, several professions like nursing, engineering, and accounting require the ability to execute mathematical operations and solve mathematical problems, necessitating the use of Common Content Knowledge. Specialized Content Knowledge, on the other hand, is the skill required to understand why the answer to the subtraction problem is incorrect. It is the specialized knowledge required for teaching and supporting the learning of mathematics (Hill et al., 2005). For example, Specialized Content Knowledge requires an ability to analyze students’ errors with flexibility and fluency to understand, and correct, students’ mathematical misconceptions. In the above example, it’s the understanding that the student has subtracted the smaller digit from the larger digit in each column (e.g., 3-1=2, 6-0=6, 8-7=1, therefore 261) (Ball et al., 2008). Ball et al. (2008) described, “This work involves an uncanny kind of unpacking of mathematics that is not needed— or even desirable—in settings other than teaching” (p. 400).

**Pedagogical Content Knowledge**

Just like Subject Matter Knowledge, Pedagogical Content Knowledge can also be decomposed into three other domains; knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum (Ball et al., 2008; Reid, 2013). Each of these domains reflect blending an understanding of mathematics with various aspects of teaching (Thames & Ball, 2010). Knowledge of content and students refers to a teacher’s ability to anticipate what students are likely to think, what they may find confusing, and what might be of interest or relevance to them when delivering math lessons. It also involves an ability to
anticipate how students may respond to a mathematical task or assignment and to hear and make sense of children’s emerging mathematical thinking (Ball et al., 2008). Communicating and interpreting children’s mathematical thinking may pose additional challenges when working with younger learners, as their communication skills in general are still developing. This requires an ability to be sensitive to the multiple ways in which young children communicate meaning, for example through gestures, facial expressions, and drawings (Einrsdottir et al., 2009; Gordon & Ramani, 2021; Wickstrom & Pelletier, 2019). Knowledge of content and teaching reflects the ability to design, sequence, and deliver mathematical content. For example, which mathematical concepts to first expose children to, what examples will best demonstrate these concepts and which examples and activities will push their thinking further. This requires an understanding of the interaction between specific mathematical concepts as well as pedagogical issues that influence student learning (Ball et al., 2008). Lastly, knowledge of content and curriculum connects a teachers’ understanding of mathematics with their knowledge of the curriculum. Ball et al., (2008) provisionally placed this domain within Schulman’s’ category of Pedagogical Content Knowledge, though suggested it may permeate across other domains, or be a separate category entirely. For the purpose of this study, content knowledge and curriculum will remain within the overarching category of pedagogical content knowledge, as originally described by Ball et al. (2008).

**Educators’ Relationship with Mathematics**

Research shows how educators’ capacity to teach math, including their Mathematics Knowledge for Teaching, is influenced by their experiences and overall relationship with math (Ashcraft, 2002). For example, teachers’ past experiences learning math as a student can be associated with fear, anxiety, and negative attitudes towards math as a subject (Brady & Bowd,
Such negative associations with mathematics can lead some educators to shy away from, or avoid, the subject in practice (Ambrose, 2004). Some research suggests how high levels of math anxiety and avoidance are common in elementary educators, particularly early years educators (Malinsky et al., 2006), as there is no, or limited, mathematics course requirements to become a certified elementary teacher (Malzahn, 2002). As such, elementary teachers typically have lower levels of mathematics content knowledge, which can often be the result of high levels of anxiety and avoidance learning math as a student (Ganley et al., 2019). Thus, we can commonly see a viscous cycle of math anxiety as a student, avoidance learning the subject, resulting in lower content knowledge, which can lead to math anxiety and avoidance as an educator (Ganley et al., 2019).

Fortunately, research also shows how educators’ relationship with math can improve, and so too can their approaches to math instruction. Pre-service teacher education programs that not only provide students with the opportunity to reflect upon their relationship with math, but also actively support pre-service educators in building confidence with the subject are key to helping improve educators’ relation and approaches to mathematics (Reid, 2013; Samuel & Warner, 2019). More so, research continues to point to the important role a growth mindset plays in improving an educators’ relationship with math (Peterman & Ewing, 2019), such that believing that one’s mathematics skills, attitudes, and abilities are malleable and can improve with practice (Dweck 2006; 2015).

**Mathematics Education in Kindergarten**

*Mathematics Subject Matter Knowledge in Kindergarten*

The study of mathematics has historically focused on older elementary and high school grades, in part due to the volume and complexity of the subject matter and curricular reform
pertaining to these grades (Mueller, 2019), but also in part due to a historical focus on children’s developmental learning in kindergarten that was prioritized over young children’s academic learning (Saracho & Spodek, 1998). Over the past few decades, however, there has been growing interest in early years mathematics education, including Mathematics Knowledge for Teaching, as growing evidence demonstrates how early mathematical competency is a powerful predictor of future academic success and opportunity (Duncan et al., 2007; Rittle-Johnson et al., 2016).

Mathematics Subject Matter Knowledge in kindergarten has largely focused on number sense and spatial reasoning, which are the cornerstones of early mathematics education (Sarama & Clements, 2009). Number sense refers to the meaning of, and relationship between, numbers while spatial reasoning is the ability to represent, manipulate, and communicate about shapes and spatial relations (National Council of Teachers of Mathematics, 2000; National Research Council, 2006). Through the cumulative efforts of many research teams over the past decades, we’ve come to develop an initial understanding of what number sense and spatial reasoning skills are and what they may look like in kindergarten classrooms (Clements & Sarama, 2014; 2022; Sarama & Clements, 2009). The research has historically focused on the concepts and skills involved in children’s early number sense (Baroody, 2006; Baroody et al., 2006; Clements, 1984; 1999) and has somewhat recently just begun to discover, or reconceptualize, spatial reasoning as a foundational mathematical skill (Davis et al., 2015; Hawes et al., 2017; Mix & Cheng, 2012; Moss et al., 2016). While this emerging line of research is a promising start, there remains much more to learn about spatial reasoning in the early years.

**Young Children’s Mathematical Competencies.** Understanding young children’s mathematical skills began by revisiting, and correcting, misconceptions researchers and practitioners previously held about young children’s competency and capacity to learn
mathematics. Unfortunately, it was commonly believed that young children were simply incapable or had little capacity for mathematical reasoning, as the subject matter was considered too complex for their developing cognitive skills (Piaget, 1960; Piaget & Szmwinak, 1952; Thorndike, 1922). Overall shifts in the literature of child development that viewed children as capable and competent beings (Gelman & Gallistel, 1978; Perry & Dockett, 2002), coupled with the rise of constructivist approaches to teaching that endorsed children as active agents in learning (Vygotsky, 1978), lead to changing perspectives of young children’s mathematical abilities (Baroody et al., 2006; Gelman & Gallistel, 1978; Ginsburg, 1977; 2006).

Herbert Ginsburg (1977; 1999; 2006) is well known as a leading researcher who advocated for this shift in perspective by demonstrating how young children arrived at school with an innate ability to reason and think mathematically. He described how young children begin to build their mathematical competencies through their daily exposures and interactions with their environment, a phenomenon he described as “everyday math”. He demonstrated how children’s everyday experiences helped them to develop an understanding of more or less, bigger and smaller, and to begin to name and identify number (Ginsburg, 2006). The view that young children are capable of learning mathematics, and that they bring many meaningful mathematical experiences with them to school, is commonly held in both contemporary theory and practice (Clements & Sarama, 2014).

**The Principles of Counting and Quantity.** Alongside this shift of acknowledging young children’s mathematical abilities, researchers also began to build a strong understanding of the general principles of counting and quantity. These are overarching rules that govern our understanding of number (Gelman & Gallistel, 1986). The principles of counting and quantity are the building blocks of number, much in the same way phonemic awareness skills are the
build blocks of language. Beginning with the work of Piaget and Szmwinak (1952), which was further refined by Kamii (1985), Gelman and Gallistel (1986), and Ginsburg (1998; 2006), research has identified 10 principles of counting, which are outlined and defined in Table 1. The first five principles are the most pertinent to mathematics education in kindergarten as they are the most foundational to developing an understanding of number (Clements & Sarama, 2009; Sarama & Clements, 2014). If children fail to grasp an understanding of stable order, one-to-one-correspondence, cardinality, abstraction, and order irrelevance, they will struggle to become proficient mathematical thinkers (Clements & Sarama, 2009).

**The Learning Trajectories.** Over the past several decades, many research teams have described how children acquire mathematical skills along a developmental progression, commonly referred to as a learning trajectory (Clements & Sarama, 2004; Confrey et al., 2014; Simon, 1995). These research teams, however, often describe different conceptualizations of learning trajectories and how they relate to mathematical thinking, teaching, learning, and curricula (Clements & Sarama, 2004). For example, some teams emphasize instructional sequencing (Clements & Sarama, 2004; 2009; 2014; Gravemeijer, 1999; Simon, 1995), or the ability to connect a mathematical goal with activities to support children’s mathematical thinking and learning towards this goal, while other teams may emphasize the psychological developmental progression of the mathematical skills themselves (Battista, 2004; Carpenter & Moser, 1984; Griffin & Case, 1997). Other research teams also consider the dynamic nature of learning trajectories, how our understanding of them can change over time, and how they may be influenced by forms of instruction and pedagogy, socio-cultural factors, and tools. Importantly, this critical interpretation of learning trajectories also considers how our research evidence of
learning trajectories may influence curricular standards and expectations within a variety of countries (Confrey et al., 2014).

For this dissertation, I chose to view children’s mathematical skills from a learning trajectories perspective that emphasizes the connection between instruction and children’s active engagement in learning activities (Simon, 1995; Clements & Sarama, 2004), as the notion that children develop mathematical abilities along developmental trajectory that connects guided instruction with meaningful opportunities for learning aligns with constructivist notions of learning that are present in this dissertation. A constructivist view, that children are active participants in building knowledge under the guidance of a more knowledgeable other, is an overarching perspective on teaching and learning that runs throughout this dissertation and aligns with other fundamental frameworks used in this study, such as Pyle & Danniels’ (2017) continuum of play-based learning. More so, a learning trajectory view of math skills demonstrates strong horizon knowledge of the progression of mathematical skills, as they describe in detail the specific mathematical milestones children must meet as they acquire overarching mathematical skills. This strong horizon knowledge helps to break down the complexities of acquiring overarching mathematical skills, such as learning to count, into discrete subskills, such as gaining an understanding of stable order, one-to-one correspondence, and cardinality. In doing so, a learning trajectories perspective that emphasis the role of instructional sequencing also aligns with Ball et al.’s (2008) framework of Mathematics Knowledge for Teaching, which is another fundamental theoretical framework used in this study.

While several different research teams describe a learning trajectory approach to mathematical skills, I specifically chose Clements and Sarama’s interpretation of learning trajectories for three reasons. First, Clements and Sarama’s work has a distinguishable focus on
mathematical learning in the early years. More so, their work commonly discusses the role of play in children’s mathematical learning. While their discussions of play are often broad in scope, they continue to emphasize the role of what they call “purposeful play”, or play that is intentionally planned and facilitated by an educator, in supporting early math learning (Clements & Sarama, 2014; 2022). Given that this present study is focused on mathematics education within play-based kindergarten, and in particular teacher-facilitated approaches to play, their approach to mathematical learning trajectories was a particularly good fit. Second, Clements and Sarama’s work is highly cited within the research literature, particularly when it comes to studies of early years mathematics teaching and learning (Blanton et al., 2015; Fisher et al., 2012; Papadakis et al., 2017) which demonstrates a strong acceptance of their work within the research community. Third, Clements and Sarama’s work is also commonly cited, and used, within communities of professional practice. Their research is explicitly cited within Ontario’s Kindergarten Program (2016) as well as Ontario’s professional development resources (e.g., Capacity Building Series, 2011). More so, their work is commonly cited within Ontario school board-approved professional development series, including my own school board where I work (e.g., Simcoe County District School Board), as well as professional workshops for practicing educators that are organized by Ontario Universities (e.g., Math Knowledge Network, 2022). I believed this strong overlap between research and practitioner communities provides strong consistency in knowledge that was well-suited for the classroom-based research I conducted.

Douglas Clements and Julie Sarama (2014) developed one of the most comprehensive, systematic, and evidenced-based frameworks for understanding the development of children’s mathematical skills. Through a rigorous review of the existing literature, and through the collection and analysis of novel data, Clements and Sarama constructed an irreplaceable
framework, named The Learning Trajectories, that delineates the progression of 20 mathematical skills (Clements & Sarama, 2014; 2022; Sarama & Clements, 2009). For a detailed description of these 20 math skills, see Clements & Sarama (2014; 2022). This research team has also translated their findings into numerous educator resources, including several published books and a free website, to support the implementation of their mathematics Learning Trajectories in practice. Their Learning Trajectories framework will be the leading theoretical framework used in this study to conceptualize the foundational mathematical skills in kindergarten (Clements & Sarama, 2014; 2022; Sarama & Clements, 2009).

Rooted in developmental psychology theory and research, Clements and Sarama (2014) established Learning Trajectories that denote the development of children’s number sense and spatial reasoning abilities throughout the early years of schooling. Illustrated by these Learning Trajectories, young children acquire number sense through a gradual progression in their ability to count verbally, count objects, compare, order, and estimate (Baroody et al., 2006; Clements, 1984; Clements, 1999; Clements & Sarama, 2014) as well as the development of arithmetic abilities, including early counting, addition, and subtraction strategies (Clements & Sarama, 2014; Huttenlocher et al., 1994; Wu, 2007). While children may be separately exposed to these aspects of early numerical knowledge, these skills gradually connect throughout the preschool years (Linnell & Fluck, 2001) and are interrelated as the development of one skill supports and motivates the growth of the others (Clements & Sarama, 2014).

Children’s spatial reasoning ability also develops along a trajectory and centers around two main skills: spatial orientation - understanding and navigating relationships between different positions in space - and spatial visualization - the ability to generate and manipulate two and three-dimensional objects (i.e., visually moving, matching and combining objects) (Clements...
& Sarama, 2014; Harris, 1981; McGee, 1979). The study of children’s spatial reasoning is an evolving field, with more recent research helping to expand our conceptualization of spatial reasoning to consider children’s actions including, but not limited to, perspective taking, gesturing, map-making, diagramming, sliding, rotating, and reflecting (Davis et al., 2015; Moss et al., 2016). Accordingly, the concepts of number sense and spatial reasoning are intended to be taught, through play, in Ontario’s Kindergarten Program (OME, 2016).

**Pedagogical Content Knowledge in Kindergarten**

**Constructivist vs. Didactic Mathematics Pedagogies.** Math pedagogy has been a long-standing topic of debate (Mueller, 2019), which has largely been situated within older grades (Alfieri et al., 2011), though academic push down effects have begun to apply this debate within kindergarten settings as well (Schmidtke, 2020). In general, one side of this debate argues for constructivist approaches to learning, such as play-based learning, while the other advocates for more didactic approaches to teaching mathematics.

Constructivist learning reflects the overarching idea that children take an active role in their learning as they build their knowledge through interactions with their environment, often under the guidance of a more knowledgeable other (Vygotsky, 1978). Constructivist learning is deeply rooted in the social world of children, including the socializing experiences of their interactions with others and their environment, which involves a high degree of language comprehension (Liu & Matthews, 2005). Conceptualizations of play, including both traditional and contemporary, align with Vygotsky’s notion of constructivist learning as they endorse the perspective that children have a high degree of agency in their learning as they actively construct their knowledge through their interactions with others in play (Pyle et al., 2017; Weisberg et al., 2013; 2016). Ontario’s Kindergarten Program endorses a constructivist view of learning through
play and reflects a modern perspective that “play and academic work are not distinct categories for young children” and espouses a theory of change that children learn through play, “especially in the areas of problem solving, language acquisition, literacy, and mathematics” (OME, 2016, p. 18).

Advocates of constructivist learning endorse its many benefits, which largely center around the agency of the learner, who is viewed as active rather than passive (Ackermann, 2001). Those who endorse constructivist approaches to learning argue that students learn best by solving authentic problems that have relevance and meaning to them (Hmelo-Silver et al., 2007). Often these problems, or contexts for learning, reflect students’ interests and curiosities and build off what students may already know (Alanazi, 2019). Importantly, advocates for constructivist approaches to learning emphasize the necessity of scaffolding and guidance by an educator as children are actively building their knowledge (Alanazi, 2019). Scaffolding helps to reduce students’ cognitive load, provide expert guidance, and support students to develop structured ways of thinking while still providing room for child-autonomy and choice in learning (Hmelo-Silver et al., 2007).

While constructivist learning has been a widely promoted and accepted stance in practice and literature (Ackermann, 2001; Alanazi, 2019; Hmelo-Silver et al., 2007), it is not without limitations. Primarily, those that are skeptical of the benefits of constructivist learning argue that constructivist approaches to teaching can lead to minimal guidance for students, which can be ineffective for student learning and can leave students feeling lost and frustrated (Kirschner et al., 2006). The heavy demands that constructivist learning makes on students’ oral language comprehension pose a significant challenge for students who may still be developing their language skills (Liu & Matthews, 2015), including kindergarten-aged children.
Despite the constructivist views on learning that are endorsed in Ontario’s Kindergarten Program (OME, 2016), the literature also describes traditional math strategies, such as direct instruction, as a competing math pedagogy. Traditionally, mathematics pedagogy focused on procedural learning through the memorization of basic facts, explicit teacher instruction, algorithmic learning, and pencil-and-paper practice (Stokke, 2015). This approach has been referred to as “traditional math”, which commonly involves direct or explicit instruction (Engelmann, 1981). Advocates of traditional math pedagogy argue that children require this systematic and intentional approach to teaching in order to develop their mathematical knowledge and explicit instruction is said to achieve this, as it entails clear teacher modeling of math concepts and guided practice opportunities (Doabler & Fien, 2013; Klahr, 2009; Mayer, 2004). More so, advocates of this pedagogical approach promote its suitability for supporting specific populations of children, such as those with math learning difficulties (Doabler & Fein, 2013).

A main critique of this didactic approach to learning is that it prioritizes procedural learning over conceptual understanding (Ansari, 2016; Kamii & Dominick, 1997), which can create large gaps in students’ conceptual knowledge (Ansari, 2016). While students who are taught with this approach may be good at reciting mathematical facts, their understanding of the underlying mathematics can be limited and therefore they may struggle to apply their conceptual understanding when problem solving (Ansari, 2016). Such limited conceptual understanding can often lead to underdeveloped problem-solving strategies, as children who learn mathematics through direct instruction often use a single approach to mathematical problem-solving by mirroring the method modeled by their educator (Bonawitz et al., 2011).
Comparing Pedagogies. Previous research has sought to understand which of these constructivist and didactic pedagogical approaches are the most beneficial to children’s learning. Alfieri et al. (2011) began this inquiry by conducting a meta-analysis which compared the impact of unassisted discovery-learning, enhance discovery-learning, and direct instruction teaching approaches on students’ mathematics learning. While the terminology differs slightly, unassisted discovery learning aligns closely to free play, as both are child-directed learning approaches that lack educator support, while enhanced discovery-learning and guided play are similar as both involve collaborative learning between student and educator. Results of this meta-analysis demonstrated that direct instruction was a more effective method than unassisted discovery-learning, but that enhanced discovery-learning was a superior approach than both unassisted discovery-learning and direct instruction (Alfieri et al., 2011). Findings from Fisher et al. (2013) corroborated evidence from Alfieri et al. (2011) and demonstrated that kindergarten-aged children learned and retained statistically significantly more geometric shape knowledge when they were taught through guided play than when taught through direct instruction or free play. While the research continues to develop a stronger understanding of how to teach and support students’ mathematical knowledge, several gaps in how to apply this knowledge to kindergarten education remain evident.

Gaps in Kindergarten Mathematics Education

While our knowledge of mathematics education in kindergarten has come a long way over the last few decades, we have much further to go. Broadly, this literature review has outlined three leading frameworks that, when viewed alongside each other, begin to paint a clearer picture of mathematics education in kindergarten. This includes Ball et al.’s (2008) framework for the Mathematics Knowledge for Teaching, which is composed of both Subject
Matter Knowledge and Pedagogical Content Knowledge. In kindergarten, Clements and Sarama’s (2009) mathematics Learning Trajectories presents one of the most thoroughly available understandings of what constitutes Mathematics Subject Matter Knowledge at the kindergarten level. Lastly, Pyle and Danniels (2017) Continuum of Play-Based Learning presents a contemporary conceptualization of play-based learning that helps to unpack the Pedagogical Content Knowledge that is required to deliver Mathematics Subject Knowledge to young learners in a developmentally sensitive way. While these frameworks have been studied, and widely agreed upon as separate entities, little work has addressed the interplay among them. Identifying the disconnections and considering the connections across these leading theoretical frameworks may help to address some prevailing gaps in the literature regarding play, math, and most relevant to the present study, the integration of guided play and math.

Primarily, very little research has addressed the connection between the Subject Matter Knowledge and Pedagogical Content Knowledge of Ball et al.’s (2008) framework of Mathematics Knowledge of Teaching, particularly with respect to mathematics education in kindergarten. A possible opportunity to address this gap is to apply Clements and Sarama’s (2014) mathematics Learning Trajectories to Ball et al.’s (2008) framework. This would help to provide greater clarity as to what the math skills in kindergarten are, including common student misconceptions and errors (i.e., Subject Matter Knowledge), as well as how to anticipate, plan for, and address these misconceptions and errors in practice (i.e., Pedagogical Content Knowledge).

A remaining issue, however, is that the Learning Trajectories have largely focused on the Subject Matter Knowledge aspects of kindergarten math education and less on the Pedagogical Content Knowledge components. Consequently, the literature regarding the pedagogical aspects
of supporting this early mathematical knowledge in practice has somewhat lagged behind the development of Mathematics Subject Matter Knowledge. When the pedagogical pieces are studied, however, they are often discussed so broadly that it lacks support for educator reproducibility (Alfieri et al., 2011), or in such minutia that it may not accurately reflect the reality of educators’ classroom practice (Fisher et al., 2013), or be lost within a debate about what math pedagogy should entail (Ansari, 2016; Stokke, 2015), or may fail to account for the developmental needs of kindergarten-age children (Englemanm, 1981).

Pyle and Danniels’ (2017) Continuum of Play-Based Learning, however, directly responds to this limitation, as this framework addresses the pedagogical pieces that are needed to support Mathematics Content Knowledge in kindergarten in a developmentally sensitive way. Although, a remaining gap with this framework is that all types of play along this continuum are not understood equally, as we are lacking in our knowledge of guided play, despite the fact that research continues to suggest this as the most beneficial type of play for early math learning (Fisher et al., 2013; Skene et al., 2020). For example, the research has thoroughly reported upon how math learning can naturally emerge from children’s free play without adult intervention (Piaget, 1951), yet research continues to show this to be an ineffective method for acquiring novel math knowledge (Alfieri, 2011; Fisher et al., 2013). Conversely, research continues to strengthen its understanding of teacher-directed play (Gmitrová et al., 2009) and demonstrates how math learning can occur through highly structured and teacher-led approaches to play, such as through games (Kamii, 2003; Pyle et al., 2017). What the research is missing, however, is an in-depth understanding of what lies in the middle – guided play – despite growing evidence that this type of play can be considered the “sweet spot” for early mathematics learning (Fisher et al., 2013; Skene et al., 2020). Furthermore, while endorsement for guided play is rooted in studies of
experimental design, there are growing calls to better understand how educators can successfully implement guided play within the realities of classroom practice (Wickstrom et al., 2019). The issue of implementing guided play to support math learning is compounded in practice, however, as educators hold divergent perspectives of play that either dichotomize or integrate the constructs of play and academic learning (Fesseha & Pyle, 2016; Pyle et al., 2017), which is further challenged by uncertainty in how to follow children’s lead in play while also directing play towards targeted curricular objectives (Pyle et al., 2018a). The present study responds to this central challenge and aims to understand the ways in which kindergarten educators are successfully collaborating with children in play to support their early mathematics development. To this end, the present study will aim to answer the following research questions:

3. What are kindergarten educators’ perspectives regarding the role of play in early math learning?

4. How are kindergarten educators successfully facilitating guided play to support children’s early math learning?
Chapter 3: Method

Theoretical Orientation

Much of our existing knowledge about the role of guided play to support early math development comes from experimental studies (Fisher et al., 2013; Skene et al., 2022). These studies serve an irreplaceable function in the literature as they provide the foundation for our understanding of how to support young learners with mathematics. What’s missing from the literature, however, is an understanding of the implementation of guided play to support math that is contextualized within the realities of classroom practice. It is not surprising to highlight that a heavily controlled laboratory setting differs substantially from a real-life classroom. As such, gaining an understanding of the real-world applications of guided play and mathematics will provide a deeper understanding that is more closely rooted in classroom practice. More so, while the research has a strong overall understanding of the intimate connection between educators’ perspectives and their practice (Pajares, 1992; Snider & Roehl, 2007) it has a weaker understanding of how this influential relationship relates to their specific perspectives of mathematics and play-based learning in kindergarten. Gaining insight into educators’ perspectives will help to strengthen our understanding of their implementation of guided play and mathematics within their real-life classrooms.

To this end, the present study used qualitative methodology to gain a deeper understanding of educators’ perspectives and implementation of guided play to support mathematics in kindergarten. Qualitative methodology is best suited for this study as it aligns with interpretive frameworks that respond to real-world problems (Creswell & Poth, 2018), it offers data collection methods that gain a deeper understanding of the lived experiences, perspectives, and stories of others (Patton, 2014), and it provides data analysis techniques that
can identify both semantic and latent findings within a data set through deductive and inductive analysis (Braun & Clarke, 2013). The flexibility to use deductive and inductive coding, both independently and in partnership, provides the opportunity to answer a range of research questions that can include both specific and targeted questions as well as more open-ended and exploratory questions (Braun & Clarke, 2006). In the context of the present study, this flexibility will help me to answer more targeted questions, such as the specific strategies educators are using to implement guided play and math, as well as more open-ended questions pertaining to educators’ perspectives of play and math more broadly. Taken together, this qualitative methodology is well-suited for gaining a deeper understanding of educators’ perspectives and practices for implementing guided play to support early math learning in kindergarten.

**Philosophical Assumptions**

*Interpretative Framework*

This study aligns with a pragmatism interpretive framework as it focuses on the outcomes of research in response to a real-world problem (Creswell & Poth, 2018; Patton, 2014). The present study begins and ends with an emphasis on “what works” (Patton, 2014). It begins by seeking to understand how educators are successfully integrating math and play pedagogies and ends with comprehensive teaching strategies that other educators could use to support math learning in kindergarten. I was nudged (and sometimes dramatically pushed) towards this stance in response to the real-life context in which this study took place. The global Coronavirus pandemic occurred throughout the planning, data collection, and writing of this study, which necessitated the restructuring of several key elements. While the research purpose and questions remained unchanged, I had to adapt my original plans for collecting data within naturalistic classroom environments as schools were closed to both in-person learning and nonessential
visitors. I responded by accepting the reality of the situation and shifting my data collection to virtual methods, which had some unanticipated benefits that I will discuss below. I believe this pivot was possible by an unrelenting effort to focus on “what works”.

Pragmatism reflects an ontological view that “reality is what is useful, practical and works” (Creswell & Poth, 2013, p. 35) and an epistemological view that “reality is known through using many tools of research that reflect both deductive (objective) evidence and inductive (subjective) evidence” (Creswell & Poth, 2013, p. 35). Pragmatism does not commit to any one system of philosophical assumptions, but rather affords the researcher freedom to choose methods, techniques, and procedures that best fit their research needs and purposes (Cherryholmes, 1992). As such, a key benefit of pragmatism as an interpretive framework for this study is that it supports both inductive and deductive approaches to analysis (Patton, 2014), which will be used to identify both latent themes relating to educators’ perspectives and experiences integrating play and math in kindergarten as well as semantic themes outlining specific approaches and strategies educators used to implement collaborative play to support math.

**Orientation to the Data**

This study has an experiential orientation towards the data and seeks to understand the meanings, views, perspectives, experiences, and practices expressed by the participants in this study (Braun & Clarke, 2013; Bryne, 2022). I acknowledge that while the thoughts, feelings, and perspectives expressed by the participants may be subjective, I accept the meaning and meaningfulness of the experiences that are described by the participants (Braun & Clarke, 2014). In other words, I believe that the thoughts, feelings, and experiences of integrating play and math in kindergarten that are shared by the participants in this study are a reflection of their own
personally held beliefs (Bryne, 2022) and the interpretation of their experiences will be prioritized, accepted, and reported upon (Braun & Clarke, 2013). Conversely, in this study, I did not seek a critical orientation to the data, to take an interrogative stance towards the experiences participants shared as a means of exploring a culturally constructed phenomenon (Bryne, 2022). Instead, I wanted to understand kindergarten educators’ perspectives and practices towards integrating play and math pedagogy through collaborative approaches to play, and thus maintaining an experiential orientation to the data was best suited for addressing my research objectives (Braun & Clarke, 2006; 2013; 2014; Bryne, 2022).

**Semantic vs. Latent Analysis**

Proponents of thematic analysis highlight how it’s important for researchers to indicate the level of their analysis as semantic, a literal, surface-level interpretation or synopses of a data set or latent, an interpretation of the deeper ideas, patterns, and themes, that go beyond what is literally presented in a data set (Braun & Clarke, 2006; 2013; Bryne, 2022). This present study will use both semantic and latent analysis, that will be shared through three different phases of analyzing the data set.

The first phase of analysis will use inductive, or data-driven, coding that will be focused at the semantic level to present a summary of the survey responses that describe educators’ general perspectives of play and mathematics in kindergarten. The second phase of analysis, which is the main analysis in this study, will use inductive coding to generate latent themes that will identify underlying ideas that shape the surface level of educators’ interview responses (Braun & Clarke, 2006; 2013; Bryne, 2022). Latent analysis involves my interpretation of the data, as I play an active role in identifying and interpreting the codes and themes that can best answer my research question. My interpretation of this data is not without influence. My
professional experience, training, beliefs, and biases will influence how I identify latent themes within this study and I think it is important to keep that in mind when analyzing and reporting upon the data. The third, and final, phase of analysis will focus on the semantic level, to provide a summary of educators’ approaches and strategies for implementing guided play to support math. It will begin by using deductive, or theory-driven, coding to isolate specific descriptions of guided play. These descriptions of guided play will then be further unpacked, through inductive coding, to gain a deeper understanding of how educators are implementing guided play to support math learning in kindergarten. This dance between inductive and deductive coding is a leading reason why this study aligns with a pragmatic interpretative framework, as utilizing both of these approaches to thematic analysis is well suited for answering my research questions (Creswell & Poth, 2018; Patton, 2014).

Data Collection

To answer this study’s research questions (i.e., What are kindergarten educators’ perspectives regarding the role of play in early math learning? How are kindergarten educators facilitating guided play to support children’s early math learning?) data were collected in two phases, with results of Phase One informing Phase Two. The first phase used an online survey to collect educators’ perspectives on play, mathematics, and approaches towards integrating play and math in kindergarten. Educators who described collaborative approaches to integrating play and math were invited for follow up interviews, or Phase Two, to understand their perspectives and practices in greater depth.

Phase One

Participants. One hundred and one participants took part in Phase One of the study.
While 101 participants responded to the online survey, 55 participants completed the survey in full. These 55 participants were included in the analysis for Phase One. The other 46 participants were not included as they did not provide answers to the open-ended questions, started but did not complete the survey, and in one case, was clearly a nonsensical response likely produced by a Twitter bot. It was necessary that participants answer the open-ended questions, as these questions were used to identify their perspectives towards play and math and to recruit participants for Phase Two. Participants included current and former Early Childhood Educators (ECEs) and kindergarten teachers. Inclusion criteria required that all participants were certified ECEs or teachers, and either currently or previously held a permanent or Long-Term Occasional contract in a kindergarten classroom. Educators from public, private, and independent schools from around the globe were invited to participate in the online survey. Demographic information for the 55 participants from Phase One can be found in Table 2. This study received approval from the University of Toronto’s ethical review board and all participants gave informed consent prior to participating in the study (Appendix A).

**Recruitment.** Participant recruitment for Phase One included three approaches, with the intention of casting a broad net to gather as many diverse perspectives on play and math as possible. The first recruitment approached involved posting a link to the online survey on various social media platforms. Specifically, I posted the link on my professional Twitter account as well as the Play Learning Lab’s Twitter account. From here, the link to the survey was retweeted several times by accounts representing professional organizations (e.g., OISE, individual schools) as well as personal accounts from classroom educators, graduate students, and professors at various universities. The second recruitment approach involved emailing my personal and professional contacts, gathered through my experience working as a permanent and
occasional teacher, to invite educators to participate in the online survey. Lastly, the link to the survey was shared with educators at a Canadian conference about early years math education, with the approval of the conference organizers. I presented at this conference but had no financial incentive or reimbursement for my participation. In fact, I was retroactively offered a stipend for my participation in the conference, which I declined, as I stated in my approved ethics protocol that I would not be receiving any financial incentive or reimbursement for presenting at this conference. In all three approaches, a snowballing technique was used, as all participants were invited to share the link with their personal and professional contacts (Appendix B).

Survey. The online survey consisted of 17 questions; 11 closed-ended questions and six open-ended questions (Appendix C). The closed-ended questions gathered participants’ demographic information, focusing on their teaching experience, education, and training. Gathering this demographic information was important because it not only allowed me to gain a deeper understanding of participants’ backgrounds, but also allowed me to explore if there were any relations between educator training and their perceptions of play and math. Previous research has shown that while play-based learning is mandated in many jurisdictions, like Ontario, teachers have received minimal levels of training relating to play-based pedagogy (Pyle & Danniels, 2017) and consequently describe several challenges relating to the implementation of play to support student learning (Fesseha & Pyle, 2016; Pyle & Danniels, 2017). I wanted to know if teachers who had received more play-training, and where they have received their play training (i.e., school board professional development, teacher training program, additional qualifications), related to their possible conceptualizations and implementation of collaborative approaches to play. Similarly, I wanted to know if educators’ education level in mathematics
(i.e., highest course level completed) and their training in how to teach math related to their conceptualizations and approaches to mathematics instruction in the early years. Research has demonstrated connections between educators’ mathematics content knowledge and their experiences teaching math to students, with stronger levels of both mathematics content knowledge and pedagogical content knowledge relating to more successful experiences teaching mathematics (Thames & Ball, 2010). A concern that has been noted in the research is that many elementary teachers (K - Grade 6) often have lower levels of mathematics content knowledge as there is no requirement to complete coursework relating to mathematics content knowledge as part of their teaching qualifications [Ontario College of Teachers (OCT), 2022]. For comparison, to be qualified as a high school math teacher, teachers must have a minimum of three undergraduate or graduate courses in mathematics (OCT, 2022; Reid & Reid, 2017). As there is no university level math requirement to become an elementary teacher, many elementary educators’ exposure to learning mathematics content knowledge may have stopped in high school, around Grade 10 or 11, after which math courses were no longer compulsory (Reid & Reid, 2017).

The open-ended questions sought to gather participants’ perspectives on the role of play in kindergarten, their approaches to implementing play in educational settings, their approaches to teaching mathematics in kindergarten, as well as possible challenges they may face while supporting children’s mathematics development. It was important to ask questions about educator’s perspectives, not only because teacher perspectives deeply influence their practice (Snider & Roehl, 2007), but also because research has demonstrated that educators hold onto very different perspectives regarding the purpose of play in kindergarten, which leads to categorically different implementations of play to support student learning (Fesseha & Pyle,
2016; Pyle et al., 2018b). Specifically, educators who align with more traditional perspectives of play implement mainly free play in their classroom to support children’s developmental learning and deliver academic instruction outside the context of play through more didactic methods of instruction. Conversely, educators who express a more modern view of play implement a continuum of play-based learning to support both developmental and academic learning within contexts of play (Fesseha & Pyle, 2016; Pyle & Danniels, 2017; Pyle et al., 2018b). Accordingly, asking these open-ended questions was essential for recruiting for Phase Two, as it allowed me to isolate educators who expressed more modern perspectives of play, including collaborative approaches to play, and invite them for follow-up interviews to learn more about their perspectives and practices. I limited the open-ended questions at six, to help avoid attrition, as similar surveys conducted in Dr. Pyle’s lab have found that participants’ responses to open-ended questions tend to taper off after five or six questions (Fesseha & Pyle, 2016; Danniels & Pyle, 2021). All survey questions for Phase One can be found in Appendix C.

All participants gave informed consent to participate the study. The first page of the online survey was a letter of informed consent (Appendix B). At the end of this letter, the participants were explicitly asked to click a button that either said, “Yes, I consent to participate in this study” or “No, I do not consent to participate in this study”. It was mandatory that participants clicked one, but not both, of these buttons in order to continue to the survey. If they clicked the “yes” button, they proceeded to the survey questions and if they clicked the “no” button the survey would end, and they could not proceed to the questions. On the second page of the survey, participants were required to complete an educator declaration to confirm that they are in fact a certified teacher or early childhood educator. Like the consent page, this page was also mandatory. If the participants clicked “yes”, they would proceed onto the survey questions
and if they clicked “no” the survey would end, and they would not be permitted to proceed to the questions. Besides these first two questions, consent and educator declaration, all other questions in the survey were optional and the participants were informed that they could skip any question they did not wish to answer. At the end of the survey, participants were invited to provide their email address to indicate their interest in participating in a follow-up Zoom interview.

The survey responses of participants that provided their email address were kept confidential. Responses of participants that did not provide an email address were anonymous, as no other personally identifiable information was collected through the online survey. All data were stored in password-protected, invite-only folders on OISE's SharePoint platform. This is an established method of securely storing data that is used in Dr. Pyle's lab and was approved by the University of Toronto’s ethical review board. Consent and personally identifiable information were stored in a separate password-protected, invite-only, folder on SharePoint, which was stored separately from the survey responses.

Phase Two

Recruitment. Participants for Phase Two were purposefully selected based upon their responses to the open-ended questions from Phase One. Specifically, all participants who described academic learning, among other types of learning, as a purpose of play, collaborative approaches to play, and a willingness to participate in follow-up interviews were invited to participate in Phase Two. It was imperative that educators explicitly state the belief that academic learning can occur in play, as research shows that not all educators share this perspective regarding the purpose of play (Pyle et al., 2017). Specifically, previous research has shown two general play profiles; educators who believe the purpose of play is to support developmental learning and that academic learning occurs outside the context of play, and
educators who describe the purpose of play to support both developmental and academic learning (Fesseha & Pyle, 2016; Pyle & Danniels, 2017; Pyle et al., 2018b). Since the objective of the current study was to understand how educators are integrating play and math learning in kindergarten, it was essential that participants expressed the belief that children can learn math through play. Similarly, since a secondary objective of this study was to understand educators’ approaches for implementing collaborative play, it was necessary that participants described actually collaborating or working with children in the context of play. All participants from Phase One that described academic learning as one of the purposes of play as well as approaches to collaborating with children in play, and those who expressed interest in participating in a follow-up interview were invited to participate in Phase Two of data collection. The first 10 participants to respond were interviewed and gave informed consent prior to participating in Phase Two (Appendix D). Demographic information for participants in Phase Two can be found in Table 3.

Virtual Semi-Structured Interviews. Follow-up interviews occurred virtually on Zoom and followed a semi-structured interview format (Leavy, 2014). A semi-structured interview format facilitates cross-case comparison, as all participants answer the same questions, while also allowing flexibility to explore emergent ideas (Patton, 2014). All participants were asked the same six questions (Appendix E) about their approaches to math, play, and collaborative play in kindergarten. I also embedded follow-up questions within these six questions to further probe at educators’ use of math manipulatives to support learning as well as their mathematics content knowledge. After responding to these six core questions, I asked participants 2-4 additional questions that focused on educators’ approaches to planning for math and play and where they get their ideas for supporting math development in play (Appendix E). When conducting the
semi-structured interviews, I endeavoured to take a stance of empathetic neutrality (Patton, 2014), to be supportive and encouraging, yet reserving my own reactions and opinions as to not influence the participants’ responses. I also planned prompts and redirection phrases to help refocus the interview when necessary (e.g., can you tell me more about your approaches to play?).

While pivoting to virtual interviews was a response to the Coronavirus pandemic, it provided several unanticipated advantages. Primarily, it allowed for greater accessibility. It removed barriers due to geographic location, as I was able to interview educators from across Canada (i.e., British Columbia, Alberta, Ontario) which otherwise would not have been possible. This provided a unique opportunity to learn about educators’ approaches to collaborative play within schoolboards that have different kindergarten curricula, policy, and mandates. Virtual interviews also increased accessibility as they provided greater flexibility for participants’ schedules. Participants expressed how virtual interviews were more conducive to their professional and personal schedules. While some educators scheduled interviews at convenient times within their workday (e.g., before/after school, prep, lunch breaks), most educators chose to schedule the virtual interview during their personal time over the unprecedented “April Break” of 2021. They expressed how easy it was to complete the interview from the comfort of their home and appreciated the flexibility to schedule it at a time that worked best for them. In many cases, participants even expressed how the virtual interview provided them with the opportunity to participate in research while still balancing their child-care responsibilities. Given that the field of education, and most notably elementary education, is predominately staffed by women (Hoffman & Carey, 2017), who are also disproportionately responsible for child-care (Leclerc,
2020), broadening our use of data collection methods that allow for greater accessibility for participation may be a promising future direction for educational research.

Secondly, virtual interviews provided the opportunity to tap into an educator’s virtual world. Regardless of whether an educator’s classroom exists in a virtual or bricks-and-mortar space, a large part of their job, resources, communication, organization, and practice involves technology. Every participant shared an idea, lesson, resource, photo, video, or an artifact from their practice that “lived” on their computer. The screen-share option on Zoom allowed educators to visually share these key components of their practice with me.

Third, this screen-share option on Zoom holds great potential for virtually collecting visual data. Due to my ethics protocol, I was not able to capitalize on this possibility, as I was ethically bound to destroy the video component of my Zoom recordings and only keep the audio portion. However, having conducted these virtual interviews, I now see the benefit of using the screen-share function to collect visual data. Participants shared Google drives, PowerPoint slides, assessment forms, lesson plans, long-range plans, photos, Pinterest accounts, Instagram pages and other online media to help demonstrate their approaches to integrating math and play. Many of them even used Zoom’s annotation feature to draw my attention to specific features within their artifacts from practice. Regardless of the lost visual data, the screen share feature on Zoom drew on methodology from photo and video-elicitation interviewing techniques (Alonzo & Kim, 2016) to help elicit participant responses that were connected to their classroom practices.

Virtual interviews were not without their drawbacks. Most noticeably, building rapport in a virtual space can come with unique challenges. With in-person interviews, you can have a more natural “soft start” with participants. There’s a longer transition period as you meet them in the school, walk to their classroom, and set up your interviewing materials, which often provides
a small window of time to chat and break the ice. With virtual interviews, there is no organic transition period, as participants “pop” into an interview setting. I found it incredibly important to integrate a transition period to help build rapport and make participants feel comfortable before proceeding to the interview questions. Techniques for supporting this transition period involved simple small talk (e.g., asking how their day was going or how their “April Break” was going), thanking them for joining me to demonstrate gratitude, and using humor to help lighten the atmosphere. Often though, we just engaged in “teacher-talk” (e.g., chatting about school communities, complaining about covid regulations, swapping funny teaching stories) which I found to be a highly effective way of breaking the ice and building rapport. I believe this helped participants to relate to me as an educator and helped to reduce a potential perceived power differential between participant and researcher. Importantly, I only started recording after this transition period and after rapport had been established. I would let the participants know when I would start and stop the recording to keep them informed and maintain transparency throughout the data collection process.

Due to the need to build in a transition period for rapport building, the interviews took more time than I had anticipated. Interviews that take too long can lead to participant exhaustion and attrition, and importantly, do not respect the time commitments of participants (Adams, 2015). In the case of my study, lengthy interviews would extend beyond what was outlined in the letter of consent and what the participants had agreed to prior to participating in the study. I was incredibly mindful of the time and ensured that I prioritized my six required interview questions, with appropriate follow-up questions as necessary, within the one hour that participants had agreed to. The interview extended beyond one hour with two participants in particular. In both interviews I asked each participant if they would like to end the interview, or if they had other
scheduling commitments they had to attend to as we were going to exceed the one hour. In both cases, each participant expressed how they wanted to continue with the interview, and that they had no other scheduling commitments. One participant even shared a 12-page Google document about her approaches to play in kindergarten and she was excited, and determined, to show me the entire document.

Lastly, a significant limitation about conducting virtual interviews with educators is that it can remove them from their physical classroom. While it is possible to conduct virtual interviews with educators from their classroom, many educators chose to conduct them at home, which comes with the benefits noted above. When conducting interviews with educators in their classroom, they are able to access artifacts from practice to support their answers. For example, they can reference student work within the classroom, share lesson plans, assessment folders, and physically show the researcher various materials and activities that they use to support play-based learning within their classroom (Pyle et al., 2020). Another key benefit about conducting interviews within an educator’s classroom is that it is in their space, which can help them to feel more comfortable throughout the interview process.

An approach I used to address this limitation was by inviting educators to bring artifacts from their practice to our virtual interviews. When scheduling the interview, I would invite them to bring anything from their classroom they felt would help demonstrate their approaches to play and math in kindergarten. I made sure to clearly articulate that the participants were not required to create any artifacts for this interview, but rather bring items that they routinely used in their practice and were readily available. Every participant brought something to share, including both physical and virtual artifacts. The physical artifacts included their favourite teaching resource books (e.g., *Good Questions*, by Marian Smalls, *Messy Maths* by Juliet Robertson, *What to Look
For by Alex Lawson) as well as some of their favourite tools for teaching math (e.g., dot plates, board games, storybooks). Educators mostly shared virtual artifacts including Google drives with their lesson plans and assessment folders, PowerPoint slides documenting their approaches to play, assessment forms, lesson plans, long-range plans, photos of children engaged in play, as well as their favourite Pinterest pages, Instagram accounts, and educator blogs.

All semi-structured interviews were recorded on Zoom. The recording is automatically split into an audio and video file upon saving. The video file was then deleted and purged, while the audio file was stored on a password-protected, invite-only folder on SharePoint. All interviews were transcribed verbatim and stored on the same SharePoint folder. All consent forms, and identifiable information, were stored in a separate password-protected, invite only, SharePoint folder to protect participant’s privacy.

Data Analysis

Theoretical Framework

The data analysis aligns with a theoretical framework that describes play-based learning as a continuum, ranging in level of child and adult direction (Pyle & Danniels, 2017; Zosh et al., 2018). This conceptualization of play-based learning has been widely agreed upon in the literature (Mardell et al., 2019; Taylor & Boyer, 2020; Zosh et al., 2018) and has also been adopted into some curricular and policy documents (British Columbia Ministry of Education, 2019). I chose to use this theoretical lens because it includes a description of guided play, play that is mutually directed by children and adults, which is a fundamental concept I aim to unpack in this study. To further support this theoretical framework, I also integrated work from Weisberg et al. (2013; 2016) Fisher et al. (2013) and Jensen et al. (2019) as they have published extensively on the construct of guided play, which builds onto the theoretical framework of play.
as a continuum (Pyle & Danniels, 2017; Zosh et al., 2018). Importantly, this theoretical framework has been widely used as an analytical lens for studying play-based learning, including studies relating to literacy development, (Pyle et al., 2018a; Pyle et al., 2018b), assessment (Pyle et al., 2020), inclusion (Danniels & Pyle, 2021), as well as mathematics (Wickstrom et al., 2019).

Three phases to analysis were used to answer the study’s research questions (1) *What are kindergarten educators’ perspectives regarding the role of play in early math learning?* 2) *How are kindergarten educators successfully facilitating guided play to support children’s early math learning?*). The first two phases of analysis answered the first research question, while the third phase of analysis answered the second research question. The results from each phase of analysis informed the next.

**Phase One**

The first phase of analysis used the survey data collected from Phase One of data collection. Descriptive statistics for the demographic information gathered from the closed-ended survey questions were generated using Excel and are displayed in Table 2. The six open-ended survey questions, pertaining to educators’ perspectives on play and math, were inductively coded, line-by-line, through a method of constant comparison (Patton, 2014). This inductive analysis remained at the semantic level and aimed to summarize and report upon educators’ literal, or surface level, descriptions of their perspectives relating to play and mathematics in kindergarten.

I read through all of the participants’ responses, giving equal attention to each line of text, and open coded (Patton, 2014) participants’ responses to the six open-ended questions. I used open, or data-driven, coding because I wanted to gain a broad understanding of educators’ general perspectives towards play and mathematics in kindergarten. More so, there is no existing
theoretical framework pertaining to educators’ perspectives of early math practices that could guide a more deductive approach to coding. This stage of open coding produced 110 codes, which were further refined into seven categories reflecting similar ideas or themes. The seven categories included: math comfort, math affect, learn math, purpose of play, adult role in play, integrate play and math, and challenges. For example, the codes, developmental/social learning, creativity and imagination, enjoyment, authentic learning, build friendships, self-regulation, release energy, general learning, social and academic learning, apply learning from direct instruction, problem-solving, understand the world, and hands on learning were collapsed into a category called purpose of play. Next, these seven categories were grouped together based upon overarching themes. For example, the categories purpose of play and adult role in play were grouped together to form the overall theme of the purpose of play in kindergarten. The final four themes included the purpose of play in kindergarten, how children learn math, comfort teaching math, integrating play and math. Throughout the process of generating open codes, which were collapsed into categories, and then further organized into themes, it became apparent that some of the perspectives that educators were sharing aligned with existing research. This was particularly true regarding educators’ perspectives of the purpose of play, which aligned with either traditional or contemporary notions of play-based learning (Fesseha & Pyle, 2016; Pyle et al., 2017).

Phase Two

Data from the semi-structured interviews were used for the second phase of analysis. To analyze the interview data, an in-depth inductive thematic analysis was conducted, following Braun & Clarke’s (2006) six steps of thematic analysis. A summary of these six steps, along with data extracts and examples of coding, are presented in Table 4. The greater quantity of data
resulting from the semi-structured interviews provided the opportunity to conduct a more in-depth inductive analysis than in phase one.

**Step 1: Familiarization.** Analysis began by immersing myself in the data through familiarization to understand the depth and breadth of participants’ responses to my interview questions. Familiarization began when collecting the interview data, as I conducted all the interviews myself, which allowed me to gather initial thoughts and impressions of the educators’ responses throughout the interview process (Braun & Clarke, 2006). Familiarization progressed as I transcribed the audio recordings of the interviews, which was completed orthographically (Braun & Clarke, 2013), indicating inflections, breaks, and pauses of both myself and the participants. Listening and re-listening to the interviews throughout the act of transcription provided an opportunity to reflect upon each participant’s interview (Bird, 2005; Riessman, 1993). After the interviews were transcribed, I re-listened to each interview while proof-reading the transcript to not only ensure the accuracy of every word that was spoken, but also to ensure that punctuation, pauses, and fluency were reflected within the transcript (Poland, 2002).

Lastly, I re-read each interview transcript before beginning any coding. I kept a journal throughout the familiarization process, where I recorded my thoughts, impressions, and questions. I added to this journal after every interview, throughout the transcription process, and throughout the final read-through of each transcript before progressing towards initial coding. This journal provided a centralized location to record all my notes, which I frequently referenced throughout the analysis. I continued to journal throughout all six steps of the thematic analysis as Braun & Clarke (2013) recommend this practice to support the analytical process. I am very grateful for their suggestion, as journaling proved to be extremely helpful as it helped to reduce
the cognitive load while coding, which helped me approach coding with a clearer and more focused mind, which then helped me to piece together the overall narrative of the analysis.

**Step 2: Generating Initial Codes.** After I had familiarized myself with the data, I began to generate initial codes. Codes condense key ideas from raw data into more manageable and short word segments that can be analyzed in a meaningful way (Boyatzis, 1998). The process of coding is an essential step in the analysis, as it helps to organize the raw data into meaningful groups (Tuckett, 2005), which are then organized into categories and then further refined into themes, which will be discussed in the next three steps. In the present study, I inductively coded the interview text using line-by-line coding (Patton, 2014), which followed a more data-driven analysis (Braun & Clarke, 2006). I chose this inducive, or data-driven, approach to generating codes as I considered it to be the most appropriate methodological fit (Patton, 2014) for answering my first research question; *What are kindergarten educators’ perspectives regarding the role of play in early math learning?* I determined methodological fit, or appropriateness (Patton, 2014), based upon two factors: the nature of the research question and the existing body of research. The first research question is more exploratory in nature, as it inquires about the possible perspectives that educators may have of play and math learning in the early years. It’s seeking to explore, rather than confirm, a phenomenon. As such, it would be more appropriate to answer this question using an inductive, or data-driven, coding technique, to more accurately capture educators’ perspectives on play and mathematics without any preconceived notions, expectations, or frameworks for interpreting their responses (Braun & Clarke, 2006). Second, this open-ended question is born from the existing literature, as there currently is no comprehensive, or widely agreed-upon, theoretical framework regarding educators’ perspectives on play and math learning in kindergarten that could support a more deductive, or theory-driven,
approach to coding. Using a method of constant comparison (Patton, 2014), I systemically worked through all 10 interview transcripts, giving equal attention to each line of text, and coded for as many potential themes/patterns as possible, to be further revised and refined in future steps of the thematic analysis. I chose to complete this initial coding using NVivo software for the purposes of data organization, coding context, and data retrieval. At the end of this phase, I had generated an initial list of 273 codes. Samples of these codes can be seen in Table 4.

**Step 3: Searching for Themes.** In Step 3, I began to refine the long list of initial codes into broader themes or patterns, which may sometimes be referred to as categories at this stage (Patton, 2014). While codes are condensed segments of raw data, themes are broader representations of overarching patterns among the data set (Boyatzis, 1998). Accordingly, this stage of the analysis began to shift from a narrow focus on codes to a broader focus on themes. To begin identifying themes, I refined, collapsed, and sorted codes into relevant groups, which began to form categories or candidate themes. To facilitate this reorganization process, I wrote many of the codes, identified in Step 2, on sticky notes and physically moved them around as I sorted. It is common for researchers to construct mind maps, tables, or other forms of visual representations of their data throughout this stage of the thematic analysis (Braun & Clarke, 2006). Throughout this process, some initial codes became a leading theme, while other codes were collapsed together and became sub themes. I had several codes that did not appear to fit anywhere, yet at this stage I did not remove any codes but rather housed them under a temporary theme titled “lost and found” as it was possible that they could be sorted into another overarching theme as the analysis progressed. I continued to record thoughts, ideas, and questions in a journal throughout this stage. After I was satisfied with the candidate themes and sub themes, I transferred the mind map I constructed with sticky notes into NVivo. I did this by replicating the
exact same organizational structure of the physical sticky notes (i.e., collapsing and grouping
codes), into corresponding electronic grouping of codes on NVivo. Further reviewing, defining,
and naming of themes continued on NVivo. At the end of this stage, the 273 codes from Step 2,
were grouped into 26 candidate themes or categories. An example of these candidate themes can
be found in Table 4.

**Step 4: Reviewing Themes.** In Step 4, I began to refine the candidate themes I identified
in Step 3. I used Patton’s (2014) dual criteria for judging categories, including internal
homogeneity and external heterogeneity, to facilitate this process. In other words, I used the
following questions to guide this phase of analysis: is each theme internally consistent? Are the
themes distinct from each other? Throughout the refinement process, it became evident that some
candidate themes were not viable, as they did not have enough data to support them, while other
themes were broken apart and separated or collapsed together as themes and subthemes.

As described by Braun and Clarke (2006), this step involved two levels of reviewing and
refining themes. In the first level, I reviewed the themes with respect to the coded data extracts.
Accordingly, I read all the coded extracts from the interview transcripts for each candidate theme
and considered whether they formed a coherent pattern within each candidate theme, and that the
candidate themes were distinct from each other. At this level of Step 4, I had refined and
shortened the list of 26 candidate themes, into 9 candidate themes, which are presented in Table
4.

While the first level of review and refinement focused on coded data extracts, the second
level of refinement focused on reviewing the themes in relation to the entire data set. This
involved re-reading all the interview transcripts to ascertain if the themes reflected the data set as
a whole and to code for any additional data within the themes that I may have missed during the
initial coding process (Braun & Clarke, 2006). At this stage of the analysis, it became apparent to me that the results reflected an overall story about educators’ evolution in their professional practice and perspectives. I generated four overarching themes that reflected four general stages of the evolution of educators’ perspectives of play and math in kindergarten (Table 4). During Step 4, I thought the teachers progressed through each phase having already integrated the constructs of play and math. This interpretation will be further refined in Step 5.

**Step 5: Defining and Naming Themes.** In Stage 5, I “defined and refined” (Braun & Clarke, 2006, p. 92) my themes by identifying the “essence” of what each theme represented as well as how the themes fit together to form an overall narrative of the results. I did this by returning to the collated data extracts for each theme to review if the codes were internally consistent and formed a coherent narrative that could highlight the unique contribution and main ideas of each theme. A key factor that facilitated this step of analysis was reflecting upon whether I was able to identify latent themes throughout the data set, or if I was simply summarizing and reporting upon semantic codes. A guiding question, presented by Braun & Clarke (2006) helped me with this phase; do my themes simply reflect summaries of the interview questions? Or do they go beyond the main ideas in the interview questions to identify deeper patterns, themes, and meanings from the participants’ responses? This phase of the thematic analysis made three fundamental shifts in my interpretation of the findings.

First, I originally thought that teachers progressed through each stage of their evolution in practice by already viewing play and math as one construct. Upon revisiting the data, it became apparent to me that teachers viewed play and math as separate constructs throughout Stages 1-3 of their evolution, and it wasn’t until Stage 4 that they actually began to merge these constructs together. Second, while Step 4 of the analysis helped me to identify four distinct phases of
evolution in educators’ professional development, Step 5 helped me to refine how these four phases fit together to form an overall narrative of professional growth. Specifically, in Step 5, I was able to identify where educators’ evolution in practice began and what motivated the connections between their phases of professional development. Lastly, throughout Step 5, I let go of an interpretation of these phases of professional development being a cyclical and iterative process. I accepted that I did not have enough data within a candidate theme to support this interpretation. Instead, I accepted that the data, as collected, more closely represented a linear progression between each stage of professional development. Further data collection, by asking educators questions about their next steps for learning would help to further unpack the cyclical, or linear, relation between each phase in their evolutionary journey.

**Step 6: Producing the Report.** Lastly, the results from this thematic analysis were written-up and formally presented in Chapter 4. Illustrative excerpts from the data will be shared, along with visual representations of the main findings to help support my interpretation of the results from the in-depth thematic analysis (Braun & Clarke, 2006).

While the final report from this thematic analysis will present themes pertaining to educator’s perspectives on math and play, several themes relating to educators’ implementation of guided play emerged throughout the thematic analysis. These themes regarding implementation did not fit into the overall narrative about educators’ evolution in their perspectives about play and math, yet they inspired further analysis to gain a deeper understanding of educators’ implementation of guided play to support early math learning.

**Phase Three**

The third phase of the data analysis dove deeper into the codes that were identified in relation to educators’ implementation of play, though were not included in the final report of the
thematic analysis presented in Phase Two of the data analysis. Specifically, codes, and the associated data extracts, that referenced educators’ involvement in play (e.g., collaborative play, mutually-directed play, learning invitations) were compiled into an overarching category called “guided play”. Next, I re-read through the entire data set and deductively coded for any description or example of guided play and assigned it under the same overarching code of guided play. This part of the analysis was theory-driven, as I used a widely agreed upon definition of guided play, play that is to some degree mutually led by child and educator (Pyle & Danniels, 2017; Weisberg et al., 2016; Zosh et al., 2018), to guide the coding at this stage of the analysis.

After all instances of guided play had been identified and isolated under the umbrella code of “guided play”, I then read through all the collated data extracts to broadly group each example of guided play based upon the level of child and adult involvement. This grouping aligns with theoretical definitions of guided play that currently exist in the literature, as guided play has been described as a continuum (Pyle et al., submitted) that can be child-led, mutually led, or educator-led (Pyle et al., 2017). Additionally, grouping based upon level of child and adult control, or direction, over the play has been an accepted method for analyzing and presenting qualitative data in previous research (Pyle et al., 2020; Wickstrom et al., 2019). At this stage, the analysis continued to be theory-driven, as the grouping of data extracts was determined based upon an existing, and agreed upon, theoretical framework of play and guided play in particular (Jensen et al., 2019; Pyle et al., 2017; Weisberg et al., 2016). This stage of the analysis generated three overall approaches to guiding play; extending, facilitating, and inviting children to play, which reflected play that was child-led, mutually led, or educator-led, respectively.
The next stage of analysis then inductively (Patton, 2014) coded the data extracts within these three approaches to guided play (i.e., extending, facilitating, inviting), with the aim of identifying specific strategies educators used to implement these three general approaches. While there was a guiding question, or focus, to this inductive stage of analysis, the specific strategies that emerged were organic to the data set. This stage of analysis could be considered more data-driven, as opposed to theory-driven, as there is currently no theoretical framework or definition available in the literature to deductively code for specific strategies that educators are using to implement guided play, hence, the purpose of this study’s second research question (i.e., how are kindergarten educators facilitating guided play to support children’s early math learning?), and this corresponding phase of data analysis.
Chapter 4 - Results, Part 1

Educators’ Perspectives of Play and Math

Three overarching findings resulted from the three-phase data analysis. The first two findings pertain to educators’ perspectives of play and mathematics and will be presented together in Chapter 4. The third finding will report upon educators’ implementation of play and mathematics and will be reported separately in Chapter 5. In this current chapter, results from the Phase One survey data will be presented first and will reflect a broad scope of educators’ perspectives regarding the integration of play and early mathematics. Overall, results from Phase One will reveal that educators perceived divergent purposes of play in their kindergarten classrooms that aligned with either traditional or contemporary notions of play-based learning. These perspectives either dichotomized or integrated the constructs of play and academic learning. Next, results from the Phase Two interview data will be shared, to gain a deeper understanding of the educators who described more modern notions of play to unpack how they were able to conceptually integrate notions of play and academic learning in kindergarten. Results from Phase Two will demonstrate both an evolutionary journey in educators’ perspectives as well as an integration of the constructs of play and early mathematics learning. Findings from Phase One and Phase Two confirmed consistency in educators’ perspectives towards collaborating with children in play to support early math learning, and points to connections between educators’ evolution in their professional practice and their contemporary notions of play in kindergarten.
Phase One: Traditional vs. Contemporary Perspectives of Play

The Purpose of Play in Kindergarten

When asked, “what is the purpose of play in kindergarten”, the vast majority of educators (n = 45, 82%) described how the purpose of play was to solely foster children’s developmental learning. Specifically, they described play as a way of supporting children’s “social skills and interaction, self-regulation skills, problem solving skills” (Participant 86). These educators emphasized the importance of joy in children’s learning, describing that “the purpose of play in kindergarten is for enjoyment” (Participant 23) and explained that “children need to enjoy their kindergarten experience and play-based learning is a fun way to learn” (Participant 94). While prioritizing joy in learning is important, the perspectives of these teachers aligned with more traditional notions of play as they do not yet describe academic learning, or the potential for academic learning, in contexts of play.

A minority of educators (n = 10, 18%), however, described how academic learning can be integrated into play. This minority of educators aligned with more contemporary notions of play and described a comprehensive purpose of play to support both developmental and academic learning in kindergarten. They acknowledged the benefits of play for children’s social interaction and self-regulation while also highlighting its suitability for supporting academic learning in a developmentally appropriate way. Participant 101 explained:

*Play in kindergarten is for both academic and developmental learning. With play, students make connections and learn "how to learn" by asking questions, forming hypothesis, performing experiments, and making observations. Furthermore, they learn how to interact more successfully with others while regulating their own needs.*

These participants also described how teachers can infuse instruction into play to support both social and academic learning:
In my professional opinion, the purpose of play in kindergarten is to help students develop academically and socially. Play allows students to develop many important skills (cooperation, collaboration, leadership, communication). It also provides a great avenue for injecting instruction/teaching in an engaging atmosphere. (Participant 73)

This separation between traditional and contemporary perspectives of play has been reported in the literature (Fesseha & Pyle, 2016; Pyle et al., 2017) yet the distribution of perspectives was more balanced, with 41% aligning with traditional perspectives and 59% aligning with contemporary notions of play (Fesseha & Pyle, 2016). A similarly balanced distribution of perspectives was also reported when kindergarten teachers were specifically asked about their perspectives regarding play and literacy, with 42% describing a traditional perspective that dichotomized play and academic learning and 58% that described an integrated perspective of play and academic learning (Pyle et al., 2018b). Why, then, is there a larger disparity in the proportion of teachers that described traditional (82%) versus contemporary (18%) notions of play, in relation to children’s math learning? To better understand this disproportionate separation between dichotomized and integrated perspectives of play and academic learning, we must gain a deeper understanding of these educators’ perspectives regarding early mathematics learning and instruction.

**How Children Learn Math**

When asked how children best learn math, educators’ responses reflected one of two distinct perspectives: hands-on learning or balanced instruction. A large majority of participants (n = 39, 70%) expressed the belief that young children learn math best through hands-on, authentic learning that reflects their real-life experiences. They explained that “hands on learning has been the most productive way for students to learn” (Participant 69) and described how children learn best “when math is intertwined with their centers and throughout their daily routine especially with hands on activities like cooking, baking, building, and drawing”
(Participant 84). These educators broadly mentioned the use of play in children’s math learning by describing it as a mechanism to connect to, or act out, these hands-on learning experiences and that children learn math through “hands on, in authentic meaningful ways, in play” (Participant 37). While these teachers described elements of play, they are not yet describing comprehensive notions of play in relation to children’s math learning. In particular, they described a singular, yet broad approach to free play, that promotes incidental math learning through children’s self-directed and spontaneous exploration. For example, Educator 33 described the importance of children’s free exploration with various materials to support mathematical thinking, “I think that many students learn math through hands on play experiences. Exploring with different materials, building, and creating. In this play is where so much of their mathematical understanding is.” With this perspective, educators did not articulate how, or if, adults could be involved to support, guide, extend, or even interact with children in play to support math learning.

Conversely, approximately one third of participants (n = 16, 30%) explained how children learn math best through balanced instruction, including both a continuum of play-based learning as well as direct instruction. They articulated the belief that children learn math “by a combination of free exploration, guided exploration, teacher-led exploration, explicit instruction, demonstrations, explaining their thoughts and ideas to others” (Participant 71) and that “there has to be a balance between direct instruction and group play with teacher guidance” (Participant 21). It is perhaps important to highlight here that more educators described how children acquire academic learning through play when asked, “how do children best learn math” (i.e., 30%) than when directly asked, “what is the purpose of play” (i.e., only 18% described play for academic learning). This may be reflective of a larger issue surrounding a disagreement on
the definition of play that has been heavily reported upon in the literature (Ghafouri & Wien, 2005; Pramling Samuelsson & Johansson, 2006; Pyle et al., 2017; Tal et al., 2008) which can contribute to conceptual barriers regarding the construct of play in practice, which results from the interview data will describe in greater depth below. When asking about play and academic learning in a more indirect way (e.g., how do children best learn math?) educators may be less constrained by the baggage surrounding the construct of play, and may be freer to talk about their views on learning more broadly, than when they are specifically asked about the functionality and purpose of play directly.

While these teachers described the importance of hands-on learning for young children, they also articulated the need for teacher involvement to help students achieve specific learning objectives, as explained by Participant 91:

I think children learn most through hands-on exploration and application of the concepts, through play, real-world application, and manipulatives. I think they also benefit from small group instruction and activities to target certain math trajectories.

This emphasis on adults’ role in children’s math learning is a distinguishing feature of this perspective. In doing so, teachers who described this balanced perspective of math learning articulated both varied approaches to math instruction, as well as more comprehensive notions of play that highlight the intentional involvement of adults in play to support academic learning. While it is promising that approximately one third of educators are describing balanced and targeted approaches to math instruction, it remains unclear why the majority of educators are describing more indirect approaches to mathematics through child-directed free play. To further understand this divide, we can begin to understand educators’ experience and comfort teaching mathematics.

Comfort Teaching Math
Why might some educators not be seeing the potential of supporting math learning through play? Previous research has hypothesized that some educators may experience discomfort teaching math, are math anxious, and that such negative affect towards math may lead to avoidance of the subject (Ambrose, 2004). This appears to not be the case in this study. In fact, all but one participant expressed comfort and confidence teaching math in kindergarten. Participant 96 illustrated, “I feel very confident teaching math to students - it is my favourite subject to teach! I love expanding their idea of what math is as they explore the world around them!” Instead of math avoidance, results of this study begin to point towards tension integrating the constructs of play and math learning.

**Integrating Play and Math**

Teachers’ responses to the question, “How might play be used to support children's math learning in kindergarten?” provided greater insight into the pedagogical discrepancies between traditional vs contemporary notions of play as well as the hands-on vs balanced instruction approaches to mathematics. While approximately one third (n = 16, 30%) of teachers had positive views of play, they did not actually answer the question of how they may integrate play and math learning. They provided general responses like, “it's everything” (Participant 67), or “math is everywhere” (Participant 53). The lack of clarity in their responses reflects the absent description of academic learning in their beliefs about the purpose of play. Quite simply, if teachers don’t describe the purpose of play for academics, then how would they possibly integrate play and math learning? This finding aligns with research that demonstrates how teachers’ practices are influenced by their perspectives, and how teachers that view play for the purpose of social learning do not implement it in their classrooms to support students’ academic learning (Fesseha & Pyle, 2016).
Nearly half \((n = 26, 47\%)\) of participants clearly described how free play can be used as an opportunity for children to explore and interact with mathematical ideas that are innate to their environment. For example, Participant 95 explained:

_I feel that young children often naturally use mathematics in their play. They love to line up their toys, sort materials, and decompose large groups. Students are naturally curious about exploring their worlds through a math lens as they count things in their environment or point out the shapes around them._

In their responses, these educators did not include any description of how adults may join, influence, or guide the play to support math learning. Again, aligning with more traditional views regarding adult’s role in play.

Just under one quarter \((n = 13, 23\%)\) of teachers, however, clearly articulated how educators are involved in learning math through various opportunities for play. Participant 79 illustrated:

_Children are learning math in all areas of the classroom, from each other as well as the staff. Money in the drama centre, shapes/figures, fractions, positional language in the block centre, quantities, representation, geometry, measurement in the manipulatives area, etc._

In doing so, these educators are describing how “various types of play-based learning models can be utilized to engage students in exploring math concepts and topics” (Participant 4), including free play, guided play, and teacher-directed play. These educators were then invited to complete an in-depth follow up interview, to better understand how they were successfully integrating contemporary constructs of play and math learning in kindergarten.

**Phase Two: Evolution and Integration of Play and Math Perspectives**

The results from Phase Two of the data analysis revealed a process of educators’ evolution, and integration, in their perspectives of play and math learning in kindergarten education. Four themes emerged from Braun and Clarke’s (2006) Thematic Analysis: Reflect,
Identify, Learn, and Actualize. These four themes will be presented as stages along educators’ evolutionary journey in their perspectives of play and math. Stages 1, 2, and 3 for play and math perspectives will first be shared and represent stages when educators viewed play and math as separate constructs. Stage 4 will then be shared, where educators were able to integrate their perspectives of play and math learning into a singular construct. Throughout this evolutionary journey, educators engaged in parallel paths of professional development to learn about modern notions of play and approaches to math instruction in the early years. Throughout their learning, however, an overlapping emphasis on child-centeredness helped teachers to bridge their conceptualizations of play and math instruction by focusing on what young children need in order to learn. In doing so, the boundaries between play-based learning and math instruction blended together and gave rise to an integrated and unified perspective of how children can learn math through teacher-facilitated play.

**Figure 3. Stages of Educators’ Evolution and Integration of Play and Math Perspectives**

![Diagram showing stages of evolution and integration of play and math perspectives]

**Play Perspectives: Stages 1, 2, and 3**

**Stage 1: Reflect.** Educators’ evolving perspectives of play began by reflecting upon and questioning their current approaches to play-based learning. They commonly reported reflecting upon their role in play, the extent to which they should get involved in children’s play, and balancing child vs teacher-led opportunities for learning. Teacher 8 described reflecting upon these concerns:
I found it so interesting because when I started back in 2010, we were always trying to find the sweet spot. What’s the area that works? Are we allowing for too much free? Are we doing too much guided? Are we too instructional? Are we interrupting their play? So it’s always this questioning of ourselves and the DECE, that we weren’t doing enough.

This was particularly salient for more experienced kindergarten teachers in Ontario that witnessed the introduction of the Full-Day Kindergarten program in 2010, as described by Teacher 8, which also included a provincial mandate to use play-based learning to support children’s developmental and academic learning. Teacher 7 described reflecting upon and questioning her approaches to play-based learning in response to this curricular and policy shift:

*My first teaching job was a kindergarten class that was every other day. It wasn’t this new program document. I was pretty traditional back then...So, it was very much, I teach you a lesson and you do some worksheet. That kind of approach. Then they played. Once I moved into this new program, I was told that the play was more free play. That they get an hour of free play in the morning and an hour of free play in the afternoon. And really, that was their time, I don’t interfere. I just provide materials, if they need it. I’m managing the room. It took me a few months to kind of go, “This seems really, um, not an effective use of time. Could we be doing more during the play?”*

By reflecting on their practice, teachers identified a need to “be doing more during the play” and began to question how they could adapt their approaches to play-based learning to achieve this goal. This reflection also allowed educators to begin to identify and work through conceptual barriers towards implementing play-based learning in their kindergarten classrooms.

**Stage 2: Identify.** After identifying a need to adapt their approaches to play-based learning, teachers described how the implementation of play could be limited by competing conceptual barriers imposed from many sources. These conceptual barriers apply not only to play, but also to teaching and learning in general. Teacher 2 summarized the challenges integrating educators’ diverse experiences and philosophies within a school community:

*It is hard when you’re in public education, you get hired to a school and not everyone comes with the same experiences, not everyone comes to the same risk level that they're willing to take in their classrooms and a lot of people come with their own ideas of what school should look like and sometimes they don't all mesh together.*
These differing philosophies about teaching and learning are further reflected in general beliefs about play-based learning. In particular, the educators shared how they’re often faced with skepticism regarding play as an effective form of learning and described, “That’s a scare with play-based learning. That it’s like, ‘oh, they are never going to do math, they are just going to play all day’” (Teacher 5). The teachers continued to describe how this skepticism is often shared by their students’ families and how they feel a need to defend their use of play through sharing research and evidence-based resources. Teacher 2 described:

It's hard to explain to parents, I found. Just letting them get their head wrapped around, “what do you mean my kids plays all day.” ... I think that you owe parents sometimes too, if they’re questioning why are you doing this? Well, here you go, here’s why I'm making these choices as the teacher. I'm just not dreaming this up out of my own thoughts and opinions, this is research-based and it's best practice.

Conceptual barriers to play were also present among professionals within a school community. Specifically, teachers described how many of their colleagues shared different beliefs and uncertainties regarding the integration of play and academic learning. They described a delicate balancing act when trying to navigate the conceptual barriers that are shared by multiple professionals within their school. Perhaps most significantly, were differing views regarding the integration of play and academic learning between teaching partners within a kindergarten class, as described by Teacher 3:

You’re also collaborating with your partner. So, what do they think is important for math? And it is very different between the OCT and the ECE. Especially being a new teacher, I came out with all this learning that I think is fresh and here’s all the best practices, but you also kind of have to balance with what they’ve maybe done and what they view as important or helpful for mathematical learning. And sometimes having to step back from ideas that you’re really wanting to put out. So, that professional relationship can influence your math in the classroom.

Such differing views are also present across kindergarten classrooms, as different kindergarten teachers interpret and implement play-based learning differently. Teacher 2 described:
I want to say it [approach to play] definitely evolved the more I was teaching. My first year I would say, I was just kind of following a lovely partner teacher but a very traditional teacher at the same time, who really focussed on just keeping the kids managed vs. allowing the learning to become what it could potentially be. So then when I moved out and then I was all of a sudden the only kindergarten teacher at a school and I had no one else to look at, I was like, “oh, this is where, it really kind of turns on.”

These differences continued to persist between kindergarten and older primary grade educators, as many kindergarten educators endorsed the perspective that children can learn academically through play yet described how this perspective is often not supported in the older primary grades, as described by Teacher 1:

We’re not as prepared to have them play [in older grades] … you know in grade one, you have to put something else on the calendar for the principal look at it and go, “yes, they are using that time”… They need it. You don’t leave kindergarten, go through two summer months, and change the way you think.

Yet despite these differing conceptual barriers to play, teachers in this study continued to identify the importance of play for young children’s academic learning, as described by Teacher 5:

Play matters. And math will happen in play. It’s going to happen at different times. That stress is something I feel like a lot of us, especially the OCTs get stressed about is the Grade 1 preparedness rather than “they’re in JK, like they have two full years before they’re going to grade one this is fine”. A lot is going to happen. With the balance of, yes, I do want them to be prepared. Early mathematics matters because a strong foundation now does provide a better insight for them later. So, finding that balance of a strong foundation and not stressing.

This belief about the importance of play for young children’s learning, coupled with their reflection and questioning of their current play practices, inspired these educators to begin to approach play differently in their classrooms. Specifically, these teachers described a need to begin to get involved in play, to direct it towards intentional math goals. They began to get involved in play through introducing and directing playful games to support math learning. Teacher 6 illustrated:

I had a lot of open play. And while I still have the open play, and I love the open play, we’ve kind of now made a small shift in making the play a little bit more directed. And I
would say that kind of goes around the games and the things that we’re setting out as provocations.

While facilitating games is a great start, these educators began to update their approach to play by flipping from one extreme (entirely child-directed free play) to another extreme (entirely teacher-directed play) and were not yet meeting children in the middle between these approaches to play. They acknowledged this limitation and discussed how learning to find a balance between these opposing approaches to play was initially a challenge for them. Teacher 8 explained:

*I would say that’s our biggest challenge is helping educators see, I know you know how to do learning through games, and I know you know free play. It’s those middle ones that we need to spend more time unpack and practicing in our classrooms.*

By reflecting upon their practice, educators identified a need to improve their approaches to play, conceptual barriers towards implementing play, and a desire to get involved in children’s play. They began to experiment with new approaches to directing play by introducing and facilitating playful games to target academic skills. However, they continued to describe a need to become more involved and to engage more deeply with children in play, so they decided to seek out professional development to enhance their approaches to play-based learning in kindergarten.

**Stage 3: Learn.** After identifying a need and experimenting with new approaches to play, these teachers then sought out professional development to learn more about modern notions of play. In particular, teachers described learning about contemporary conceptualizations of play that advocate for adult involvement to support academic learning. Teacher 1 explained how they learned to make minor modifications to free play so that they could enter into the play to influence and direct the play towards academic goals.

*You’ve got lots of materials? Oh good, I’m just going to jump in with you. Not recognizing the cardinal number yet? Well, that’s okay, we’re just going to keep practicing. We can keep counting wherever you are, I’m going to jump in and we are going to do it again and I’m jumping out. It doesn’t have to be laborious and pounded on. It has to be a little bit fun, it belongs in our play.*
Accepting that adults can be involved in children’s play represented a significant shift in these educators’ perceptions of play. Through this shift, educators described learning about play as a continuum, ranging in level of adult and child involvement. They also learned how academic skills, including mathematics, can be integrated along this continuum. Beginning with the most child-directed approach to play, educators described how free play, or child-led play, supported math development. Teacher 10 explained, “when it’s more on the free play side of the continuum – that’s when we introduce those materials and put them out at free play, to see how they can naturally extend onto it.” While not involved in the play directly, educators used free play as an opportunity for children to explore math-rich materials. Teachers also described more collaborative approaches to play, where they would engage with children in the context of play by helping to explore their interests and facilitate their inquiries. Teacher 3 illustrated:

*Then you can have in the middle, more inquiry, where you have something that you might be exploring with the students. So, we're doing an inquiry last year on structures and with that some of the students will just naturally be asking questions and wanting to dive into different areas with the inquiry. Some of it might be things that you bring in, questions you ask, or prompts that you make, or materials that you bring in to address a question or something that they were wondering about."

Similarly, teachers described getting involved in play, but with higher levels of teacher-direction. This approach to play is commonly referred to as teacher-directed play and often involves highly structured activities with very specific objectives. Teacher 3 continued:

*All the way to more teacher-driven, which, I would say could be working with a small group and having them work with manipulatives or different learning through games. So, having students working and playing like a game of Bingo and that’s very teacher-led, but there's a game element to it. It's playful and so they're still engaging in play but it’s kind of very prescribed, has rules and kind of steps for them to follow, to get at a specific learning."

Educators’ professional development helped to expand their conceptualizations of play. This expanded view of play helped educators to understand that they can get involved in play,
and that the extent of their involvement can range. In doing so, these educators embraced modern notions of play that supports the inclusion of early mathematics along a continuum of play-based learning.

**Math Perspectives: Stages 1, 2, and 3**

**Stage 1: Reflect.** Paralleling their reflection on their play practices, teachers simultaneously reflected upon their approaches to math instruction in kindergarten. At this point, teachers’ approaches to math instruction were perceived separately from their approaches to play. Overall, teachers’ reflections about math pedagogy were focused on their general approach to mathematics education in the early years. Specifically, many educators described following ideas outlined in curricular documents, but without really understanding a deeper meaning behind what they were doing. Teacher 7 articulated this phenomenon:

> We applied for a grant through our union. You would do a project, and you would get release time. So, I applied, and I wanted to look at math because I knew, “I don’t really know why I’m doing the things that I’m doing.” Like, I’m doing them, I’m looking at the program document. I think this is right, but I really don’t know.”

Just like with their reflections about play, reflecting upon their approaches to math instruction helped educators to identify a need; to deepen their understanding of mathematics education in the early years. Similarly, identifying this need also help teachers begin to unearth barriers they faced towards understanding, and implementing, approaches to mathematics education in kindergarten.

**Stage 2: Identify.** While teachers described conceptual barriers associated with play, they also described emotional barriers in their experience and relationship with mathematics. All, but one, teacher described a deeply negative experience learning math as a student across elementary school, high school, and/or throughout their teacher training program. Teacher 2
shared, “I hated math going through high school. As soon as I didn't have to take it, I was, like, “I'm done, that's totally fine.”

Regrettably, these kindergarten teachers shared how incorrect and harmful stereotypes about gender and math were shared and reinforced throughout their schooling. Teacher 7 illustrated how the oppression of this stereotyped perspective remained for years.

I can hear this teacher in Gr. 2 saying, “well, you’re not really good at this. It could be because you’re a girl. Girls generally aren’t”. So, it’s interesting because that kind of stays with you. Because math was never, I never felt like it was a strong suit.

Accordingly, these negative and formative experiences with mathematics came with a lot of emotional baggage for teachers to work through, as described by Teacher 4:

I’ll just say, my experience with math was awful. I always wanted “advanced math for dummies” …I wanted to understand it, but they were going way too fast. Even in teacher’s college, I was like, “oh, I'm just not really good at math”. Which is not true.”. So, I came with a lot of baggage.

Despite their dislike of math as a student, educators in this study described a love for teaching the subject now. Teacher 4 continued to describe, “I love math now. I’m our primary math lead, I love math.” Similarly, Teacher 2 continued to describe a love of math as a teacher, despite her hatred of the subject in high school.

My first job when I came out as a pre-service teacher, I got a job, as a math interventionist. My parents were like, “are you kidding? What are you doing?” But I'm like "I'm a certified teacher, I'm supposed to be able to do this" and that job really made me realize, oh, I was not great at math [as a student] but I really enjoyed teaching math.

This new love of math was also coupled with educators’ desire to help their students enjoy math, explaining “I’ve got to make sure you guys think this is the best” (Teacher 7) to ensure their students didn’t dislike the subject in the same way they did. Accordingly, educators in this study were able to use their negative experience with math as a student to create a positive experience for their current students and in their current role as a teacher.
A driving force behind teachers’ ability to approach mathematics with a new perspective and attitude was a growth mindset. Teacher 5 articulated:

“I would say the idea of changing your words... I guess I would say, “yet”. The power of yet, like, ‘Oh, I can’t do that yet”, rather than I can’t do that. So, it implies this idea that when you practice, you get better, which I know to be true.

This growth mindset was coupled with a sincere desire and commitment to improve their relationship and affect towards math, as described by Teacher 4:

At the end of the year, I look and say, what really worked? And what can I do better? What needs to change. And the thing that really for me, maybe 10 years ago, was I really needed to change my math program. I needed to make me be excited, because if I am, they will be.

Taken together, the growth mindset and commitment to improve their math program helped educators to reconceptualize their relationship with mathematics. They were able to override their previously negative experience with mathematics, with more positive attitudes and approaches. Teacher 2 explained:

“I think the thing I realized was that I didn't have to teach math the way that I was taught math... So, I think when I finally shifted my practice and thinking in my approach to it, I was like, ‘no, this doesn't have to be difficult.”

Following this shift in perspective, these educators described a pressing need to learn more about mathematics in early years education. Instead of avoiding math, they wanted to conquer it, and they accomplished this by seeking out professional learning.

**Stage 3: Learn.** To combat their negative experience with math, these educators sought out professional learning on mathematics in early years education. Specifically, they learned about mathematical content knowledge, as well as instructional strategies to support mathematics learning in kindergarten.

Many teachers described a need to acquire information about early mathematics because it was learning they were not exposed to during their teacher-training program, as described by
Teacher 7, “I’d argue I didn’t learn this in teachers’ college, none of this was shared. We learned a lot of pedagogical practices, but not content”. They were determined to fill in the gaps in their knowledge and so they began to connect with research, and researchers, to learn more about mathematical content knowledge. In particular, Teacher 7 described leading a group of kindergarten educators in London Ontario to meet with a mathematics researcher at their local university to learn about concepts relating to counting and quantity.

*It might have been Daniel Ansari, because we live in London, and we read something he wrote. And we we’re like, “well this guy is at Western”. So, we called him, and said, “we’re just a just a bunch of educators in the dark. We need you to kind of shine some light on what we’re doing in the early years.” So, we met with him and he talked specifically about quantity. And how that’s a really big piece for early learning. We started with Daniel and we started looking at – we didn’t even know counting principles. We were kind of doing it, but we didn’t know what they were called. That was for us, the tipping point… I’m so grateful for that because it changed how we taught. Because prior to that we were just looking at the document to kind of tell us what to do, but we didn’t know why we were doing it. What was the reason?*

In addition to learning about the principles of counting and quantity, Teacher 7 continued to describe how they learned about trajectories of math learning and development, “So, they handed us Alice Lawson’s *What to look for* and showed us her continuum. And that was a huge light bulb. Oh my gosh! You mean kids learn on a trajectory? Like, what?” Throughout their professional learning on mathematics content knowledge, these educators described learning from evidence-based resources to connect their practice with research.

Similarly, they described learning how to systematically teach children math skills along these math trajectories by differentiating and scaffolding student learning. In particular, Teacher 3 referred to The Mathematics Learning Trajectories by Douglas Clements and Julie Sarama (2009):

*Sometimes I might have kind of a goal for a student, but it would be on an individual basis. So, what this also makes me think of is the learning trajectories. Have you heard of Clements and Sarama?... With that if you've done some diagnostics, to know where*
students are at with counting principles or whatever it might be, that you know here's the next steps, because it can kind of blend, developmentally, of where the next kind of milestone or skill for them to develop is.

Their evidence-based professional learning also included developing instructional strategies to inspire and support mathematical conversations. Guided by the work of Marian Small, these educators all described using math talks, number talks, and/or dot talks to support children’s thinking and to help them verbalize their mathematical knowledge. Many educators, like Teacher 8, described this instructional technique as a very eye-opening and transformative experience.

*I also like, in the classroom, doing math talks. I think that was my “ah-ha” moment in teaching kindergarten. Something that I never did in any other grade was just having a conversation about how math lives. Whether it’s in a picture, whether it’s in a problem... So, when we were looking at dotted plates that we projected onto the screen, half the kids share, “ok, [child’s name], how do you see the number?” And so, someone would share something and someone else would share a different way. And then they would have conversations about how they saw it differently. And seeing their lights go off, that was also a very enlightening moment for me.*

Throughout their professional learning about math in the early years, these educators connected with evidence-based resources to expand their mathematical content knowledge and instructional skill set for supporting math learning in kindergarten. While educators were learning about mathematics in the early years and modern approaches to play, they were exposed to overlapping learning that helped them to integrate their perspectives on play and math development in kindergarten.

*Stage 4: Actualize*

**How Young Children Learn.** While these teachers embarked on two distinct paths of professional development to learn about play and math, separately, there was overlapping learning that bridged these distinct fields of study; an emphasis on what young children need in order to learn. This shared emphasis on child-centered pedagogies helped teachers to blend the
boundaries between their professional learning of play and math education and allowed them to integrate these formerly separate notions into a singular and unified construct.

Primarily, these teachers described how young children learn best when they feel safe and secure. This perspective reflects Blatz’ (1967) security theory, which describes how emotional security can be foundational to learning, as described by Teacher 5:

“So even if it’s not the targeted skill we necessarily wanted you to get, there are other things going on. And you felt safe. And you felt that you were building friendships, so that we kind of had that safe security place that is school.”

Teachers described how this emotional security can help to bolster children’s learning. That it can be easier for children to engage in learning when they feel comfortable, safe, and secure. With this perspective, educators described the importance of school being a positive and safe place for children.

Similarly, the teachers also described the essential role that relationships, and the building of relationships, plays in supporting children’s learning. Teacher 2 articulated how the relational aspects of learning are central to instruction.

“I think just really listening to the experiences that they’re having, as much as a lot of people say oh that’s relationship-building, it's not separate from math though. It's not separate from literacy. Relationship is how you get to know your students’ story well enough to be able to drive your instruction.

After emotional security and strong relationships are established, the teachers described how young children learn best when learning is relatable, authentic, and meaningful to them. Teacher 8 described trying to integrate math into as many places as possible, to make math come to life in an authentic way.

“I always try to bring in math with other things that we were learning, so they could see that it’s just everywhere. I didn’t want it to be like math only lived during math time. It lived in our everyday experiences, whether we were going for walks, whether we were outdoor playing, or whether we were inside doing our learning groups.”
Lastly, teachers articulated the need for young children to experience joy in their learning. That joy and positive associations with school and learning will help to bolster their development, both socially and academically. Teacher 5 clearly articulated how the need for joy and positivity at school is a foundational aspect of learning. That it is required for children to tackle academic learning and that play is a vehicle for connecting joy and learning.

*I feel like with math in play, coming back to the growth mindset, I want them to have fun. I want their connection to school to be like, “Oh, this is fun. I’m excited to go back tomorrow.” Because that positive attitude and positive association will [help them] to get there. They’re going to be able to count to 10 eventually. That's going to happen. But if they don’t like school and don’t feel like they’re having fun at school or if school is a place where they don’t do well, then what's that going to lead them to in the long run?*

These teachers all clearly articulated the importance of the social aspects of learning, and in doing so, this stage of Actualization mirrored what educators said in their responses to the Phase One survey, that the purpose of play is to support children’s social and academic learning, thereby demonstrating consistency and connection between their responses to both phases of this study. Consistency between educators’ survey and interview responses suggest that the evolutionary process described in Phase Two of the results, culminating in the final stage of actualization, may begin to explain why educators who have undergone this evolution in perspectives describe contemporary notions of play to support children’s social and academic learning, while those who may not have undergone this evolution in perspectives hold onto traditional views of play to support children’s social learning alone.

**Integrated Perspective.** Throughout their professional development on play and math, educators were exposed to a common theme that focused on children’s needs that are essential for learning. This shared emphasis on child-centered approaches to teaching and learning helped to bridge the constructs of play and math learning and merged these formerly separate notions into an integrated perspective by helping educators to understand how play can be a
developmentally sensitive way to address the social aspects of children’s learning, but also how educators can become involved in play to guided it towards academic learning goals. This integrated perspective brought educators to a place where they could then implement guided play in practice to support children’s math learning, while still remaining sensitive to the needs of young learners. Teacher 10 describes providing children with opportunities to initiate play that are interesting and meaningful to them, but then joins their play to help direct it towards mathematical goals.

*I usually let them set the foundation and then see what they want to do. And that’s when I start building around [what they’re doing]. And then once they’re already engaged, I make the decision [to enter play] by just knowing what they are capable of and seeing where the math is and seeing where I can extend it.*

Teachers’ implementation of guided play will be further unpacked in Chapter 5 by demonstrating the specific approaches and strategies they used to facilitate play to support math learning. While the implementation of guided play will be the focus of the next chapter, it is important to understand how educators got to this place in their practice, which was driven by their evolution, and integration, of their play and math perspectives.

Overall, these educators initially perceived play and math learning as separate constructs, which they were able to integrate into a singular notion through an evolution in their practice. The process of integrating these constructs consisted of four stages: reflecting upon their practice to recognize a need to improve their approaches to play and math, identifying barriers towards implementing play and math, learning about modern notions of play and mathematics instruction in the early years to overcome these barriers, and actualizing this learning through integrating their perspectives of play and math into a singular construct, that children can learn math through teacher-facilitated play.
Chapter 5 - Results, Part 2

Implementation: Supporting Math Along a Continuum of Guided Play

The evolution and integration of play and math perspectives, as described in Chapter 4, allowed educators to support children’s early math learning through collaborative approaches to play. Through this evolutionary journey, teachers gained a deep understanding of both play and math pedagogy, which was connected by a shared emphasis on child-centeredness. Teachers were able to simultaneously hold, integrate, and apply their understanding of these two fields of pedagogical knowledge, which underpinned their facilitation of collaborative play. In doing so, educators were able to not only identify the math learning that was occurring in children’s play, but also to identify the potential for math learning in play.

There’s something to say for really understanding the math curriculum. And the big ideas behind it. I felt like once I really dove into actually reading the research behind the curriculum and really understanding the big ideas, then I could see it everywhere in play. But if you’re just focusing on like one content expectation, and it’s a shallow understanding of that content expectation, you don’t see it in play or you don’t see how play can lead to it. (Teacher 10)

This chapter will unpack how educators were able to apply their understanding of play and math, to “see it everywhere in play” and to “see how play can lead to it”. By comparing and combining codes across all participants, teachers described three approaches to collaborating with children through guided play, which ranged from lower to higher levels of adult-facilitation. These three approaches were also situated within an overarching and iterative process of play, where educators were involved before, during, and after periods of play to guide student learning. The three approaches to facilitating guided play, each with specific strategies for implementation, will first be shared to demonstrate how educators can support early mathematics learning along a continuum of guided play. Next, these three approaches will be contextualized within a larger and iterative process of play, to demonstrate how educators integrated math
learning throughout an ongoing and comprehensive approach to play in kindergarten. The following diagram (Figure 4) represents the findings from Phase Three of the analysis, which depict a continuum of guided play educators used to support math learning, and was developed based on the results of the participants’ interviews.

**Figure 4. A Continuum of Guided Play to Support Early Math Learning**

![Figure 4. A Continuum of Guided Play to Support Early Math Learning](image)

**Approaches for Implementing Guided Play: Extending, Facilitating, and Inviting**

**Extending**

The most common approach to collaborating with children in play was through extending children’s play and involved the highest level of child-autonomy. In this approach, children created a play scenario, such as playing “restaurant” in the drama center, and the educator joined into this child-generated play context to identify and extend math learning. Teachers discussed four common strategies for extending play to infuse and support math learning: offering, commenting, joining, and questioning.

**Offering.** By offering, the educator observed children in play, identified a natural extension, and then shared, or offered, this extension with the children. It often involved offering
new materials, such as a cash register, toy money, or writing materials, to help the children connect previously learned skills to their play, as well as offering new ideas or suggestions for how the play could evolve. Teacher 3 explained her rationale behind using offering as a strategy for extending child-led play, “Just thinking about the materials they're using. Do you have something else that you could offer them or something that you already talked about that can tie in and make a connection to the thing that they’re doing?”. Offering involved lower levels of adult facilitation while maintaining higher levels of child autonomy, as the children decided how they will accept the offer, if at all.

Commenting. Educators used commenting to identify, label, and make children’s math learning explicitly known during play. Within Ontario’s Kindergarten Program (2016) this strategy is referred to as “noticing and naming” the learning. Instead of asking children to explain or identify the math learning within their play, the educators highlighted the learning for them, drawing their attention to key math concepts within their play that may have gone unnoticed. Educators used this technique to reinforce learned math skills and to introduce new mathematical concepts. Teacher 3 illustrated how she used commenting to draw students’ attention to spatial reasoning and measurement skills within children’s block play.

I think commenting too. So just how you’re phrasing it. So, I’ve been trying to think more about commenting on things and not just questioning that needs them to respond, that will interrupt their flow of play... So, maybe they have built something in the blocks area saying, “oh, this looks very tall” or “it looks like you've added this on top.” Just kind of maybe describing it to them. More describing the play.

By commenting on children’s play, educators drew children’s attention to specific attributes of child-led play to highlight, reinforce, and introduce mathematical ideas.

Joining. Educators also extended child-initiated play by joining children in play as a participant. Often, educators would join children in play to help direct it towards particular math
learning. For example, Teacher 3 described joining play as a customer at a pizza store, to lead and support a conversation around counting and quantity.

*Joining the play, you can either be prompting and asking questions, or joining as a play participant. So that might look like, you know, you're in the dramatic center and you're going to take on the role as a customer and you can kind of start to prompt the students, by saying, “Ok, I’d like to purchase a pizza”. And then kind of having that dialogue to see where they take it from there so that you can insert those learning opportunities. Or, “oh, how much does it cost” and that it's more natural within their play.*

Through joining the play as a participant, educators were able to influence the direction of play towards mathematical learning goals, while still following children’s interests within play.

**Questioning.** The most common strategy to extend child-initiated play was through questioning. Educators described a variety of uses for questioning children in, and about, their play. Most commonly, teachers used questioning to understand what children were doing in play and to support their verbal reasoning and communication skills. Teacher 1 described using questioning frequently when children were playing with loose parts, to better help identify the math learning in their open-ended play, “I’d have her tell me, ‘Well, what was your sorting rule? How do you know this one belongs here?’ She could explain what she had done”. Through questioning, Teacher 1 was able to understand what the child was doing in play as well as identify and communicate a data management skill of sorting.

Questioning was also used to support children in explaining their mathematical thinking and understanding. Teacher 6 described a process of supporting children in developing their communication skills to explain their mathematical reasoning.

*The other thing we do, is we question... we’ll ask them, “so how did you come to that answer?” And initially, their answers were not the greatest. A lot of them have trouble explaining how they come to that answer. But overtime they get better at answering that kind of question.*
Helping children to explain their thinking in play allowed teachers to gain a deeper understanding of their students’ math knowledge. Similarly, teachers also used questioning to challenge and push children’s mathematical thinking further within the context of play. Often, after children would explain their math thinking, teachers would test their knowledge by posing a challenge to encourage students to think deeply about a math concept. For example, Teacher 3 described posing hypothetical questions to students about limiting materials while they were building structures:

*And then I think from there, depending on the kids, you could ask kind of those more provoking questions, or you could push them a little bit more. Have you considered how you could set some parameters around, for example, what if I took this piece away from you? Now what are you going to do? Again, back to kind of that problem-based solving.*

Extending children’s’ play, using a variety of strategies like offering, commenting, joining, and questioning allowed educators to collaborate with children in the context of play to support their math learning. This approach to guided play had the highest level of child-autonomy, as the play context was initiated by the children, and the lowest levels of adult-facilitation, as the educator would make minor modifications to the play context that was created and directed by the students.

**Facilitating**

The second approach for supporting math along a continuum of guided play was through facilitating children’s play. Here, educators worked alongside children in play to help them ask and answer questions, to explore new ideas and skills, and to help them accomplish their goals in play. While facilitating may sound similar to extending, it differs by higher levels of adult-facilitation. By co-piloting play with students, the educator ensures that they pick up specific learning objectives throughout their play journey. Facilitating children’s play is the difference between what a child can already do on their own and what they are unable to do, it’s what they
can do with help. It is the most direct application of Vygotsky’s (1978) Zone of Proximal Development. Teachers discussed three strategies for facilitating guided play: coordinating, co-constructing, and co-directing inquires.

**Coordinating.** When coordinating play, teachers met with children within their play and helped them organize their thoughts, reduce their cognitive load, and refine their ideas to help accomplish their goals for play. Often, the educator joined children in play to help them breakdown their objectives into more feasible goals that still reflected their interests and intentions. As the teacher is helping children to coordinate their play, they are also guiding the play towards math objectives, or inserting math goals into what the children have already created. For example, Teacher 7 described helping children to plan for a Halloween party by helping them to organize their efforts and by inserting, and supporting, numeracy skills required to plan and execute the party.

> So, it’s Halloween and there are some parameters around celebrating Halloween. So, I ask the kids, how do you guys want to do this? Here’s the criteria... they organized, they told me, “Well, we want to have a party and we want these things: cookies, drinks, scary punch. And so, we split the kids into groups and said, who wants to take on food? Who wants to be in charge of decorating? And, you know, they wanted something fun, some scary stuff. Like the box with the spaghetti that feels like worms... So, that’s what we worked on for most of October. Was, they wanted to create this crazy room, and that’s when we started to see, but you need literacy for that, you need numeracy for that, based on, “well we need to know how many kids are in our class?” I don’t know? How many kids are in our class? How could you find out?

By helping to coordinate children’s goals for play, teachers facilitated math learning within collaboratively directed play.

**Co-constructing.** Teachers also facilitated guided play by co-constructing with children. While facilitating, the teacher joined children in play to help them build structures, either complex or simple, to help the children accomplish tasks, and to help draw their attention to key features to support their spatial thinking and geometric language. Building, with any material, is
a very common activity in kindergarten classrooms, and joining the play to build, or co-
construct, with the children represents a significant shift from letting children build in free play
alone. By building alongside the students, teachers helped to push children’s thinking and
mathematical reasoning further, to directly support their spatial reasoning skills, and to help
name and verbalize spatial language. Building with children also engaged them more deeply in
the learning process and helped to extend and enrich the play than if either the child or teacher
had built the structures independently. Teacher 5 described how children’s learning was
enhanced when she helped her students to build a rocket ship out of an old refrigerator box.

*It feels more meaningful for them, and I feel like it lasts longer. I kind of wonder if I had
painted it on the weekend and it just looks like a rocket ship and they never saw the
before, would it have lasted as long? If it would have been as fun to play with? In both
ways, it’s like, “Okay, it would be kind of cool if they just saw a rocket ship come to
class.” But I feel like they really did enjoy the process of making something.*

**Co-directing Inquires.** Lastly, teachers described facilitating guided play by co-directing
inquires. With this strategy, teachers helped to support the inquiry process, often initiated by
students, to deepen their learning and interest in a topic. Throughout the inquiry process, the
teacher would intentionally direct the learning, questions, and activities towards mathematical
goals. Teacher 5 described facilitating a student inquiry on pets and animals, by supporting
children to turn their drama center into a veterinary clinic, which they used to learn about
different animals, how x-rays work, and how to care for their pets. Throughout the inquiry,
Teacher 5 initiated, and supported several mathematical conversations to get the students
thinking and communicating about their mathematical reasoning.

*And then asking them, in terms of building on that inquiry, what should we add next?
Because they have really good ideas, like “let’s have a chart! Or “let’s have x-rays in
here!” Oh ya, absolutely. Let’s have animal x-rays in our room, that’s such a cool thing
to explore. And then getting some good questions. [For example] someone brought in a
dog and [it] was 4 years old and it was a big German shepherd and one of our students is
like, “my dog’s 12 years old and it’s really small” and you’re like, “oh my, so age doesn’t
Facilitating guided play using strategies like coordinating, co-constructing, and co-directing inquires, allowed educators to follow children’s interests in play and to help children accomplish their goals in play, while also directing the play towards mathematical learning goals. Facilitating involved similar levels of child and adult input, or direction, over the play as both student and teacher had similar control over the direction and outcome of the play.

**Inviting**

The last approach teachers used to guide play was by inviting children to play. In this approach, educators created a play environment, game, or activity, and then invited the children to join them in this playful creation. Often, this playful context had a very specific goal, for example practicing counting, exploring measurement, or learning math vocabulary, that was determined by the educator. It is important to distinguish inviting, an approach to guided play, from teacher-directed play. While they both have higher levels of teacher-direction, inviting still leaves room for some level of child autonomy within the playful context that was educator-created, whereas teacher-directed play is controlled entirely by the educator and the child has relatively little say in the direction or outcome of play. Inviting could be conceptualized as the inverse of extending. While extending is child-created and educator-extended, inviting is educator-created and child-extended. Teachers in this study discussed two strategies for inviting children to guided play: provoking and enticing.

**Provoking.** Provoking was the most common strategy for inviting children to guided play. With this strategy, teachers would set out activities, signs, or clues that would serve as a provocation or inspiration for play. Sometimes these provocations would be more closed-ended, for example, “matching the number and the symbol to the items” (Teacher 7) and sometimes
they would be more open-ended, for example leaving out “10 frames and whole bunch of loose parts” (Teacher 8). Teacher 3 explained various approaches she used for setting up provocations in her classroom:

*I’m setting up - we call them in our board, learning invitations - which are essentially provocations. So, setting up different things in the classroom. Sometimes based on where students needed more practice. Sometimes just kind of if we had a focus in mind of, you know we are kind of working a little bit more on shapes right now or geometry, so gearing it towards that. So, setting up materials sometimes with a question that was very specific and very closed. Sometimes just leaving materials with the opportunity for students to take it in their own direction but in ways that there could be mathematical elements.*

While Teacher 3 would set up small tabletop activities as provocations, Teacher 8 described setting up and playing games with students as an approach to provocations:

*So we would put our invitations for learning on where we would take the learning. For example, when we were looking at, numbers we would put out games. So, we had an idea of where these games were going. And we had a specific outcome that we wanted. But then, when students went there and they were playing with the material that was provided for them, but it was going in a different direction, we weren’t stuck in our ways in being like “no, it has to be done this way.” We were like, “ok, well what are your thoughts? How do you think this should go?” … I thought at the same time, “ok, it’s ok if it doesn’t go in this direction. What learning can I derive from what they’re doing and how can I bring in the curriculum to what they’re doing?”*

With provocations, sometimes the educators would set up the activity and walk away, like Teacher 3, and sometimes they would sit with the children and use the provocation to play with them, like Teacher 8. Teachers described using provocations frequently in their classrooms as an everyday approach to facilitating children’s play.

**Enticing.** The second approach to inviting children to guided play was by enticing them to engage in an educator-created play context. With enticing, the educator created a highly engaging, exciting, and novel play center or context to lure the children into play. While it is very similar to provocations, provocations were used as everyday classroom activities, while enticing was reserved for special events and activities, and involved the creation of a much more
complex and enriching play scenario. For example, Teacher 9 described building an ice cream shop in their drama center, complete with toy ice cream, money, and menus, to introduce the concept of money and deepen children’s counting skills while also leveraging their interest in ice cream to entice them into play.

They were really into ice cream. So, we set up an ice cream shop in the drama center. And certain ice creams cost so much money. We had the play money in there, to help familiarize them with money. But instead of being, like 25 cents, it was 5 coins. So, they’d have to count out 5 coins. So, getting their counting practice and some kids would be able to like, “oh, 25 cents that’s a quarter. I’ll find the quarter”. So, right there, there’s a variety of ways to incorporate the math.

Teacher 4 had a collaborative approach to enticing. During a morning meeting, she showed her students a picture of a giant two metre cow that lived in Australia. The children were really interested in this cow and had many questions about why it was so big. When the students were in gym, the teacher drew and cut out a scaled diagram of the cow and left it out, along with some rulers and pencils, for the children to explore. This led to an ongoing investigation of measurement and scale, as described by Teacher 4:

So there’s a giant cow in Australia...he’s two metres tall... So, I showed them a picture and said, “hey, there’s a cow and it’s really big.” And they went, “really big?” and I went “ya”. So when they went to gym, I drew it and I put it out. And I just left it on our front bulletin board and I made a ruler.... And then, we made a flamingo... and this is an ostrich..., someone drew a woodpecker, and there’s a butterfly over here...I realized that it just spoke about things that’s really hard to explain to people and that you could teach a lot of math... in such a cool way... because there’s a number line and measurement, like, how tall is two metres? How tall is 200cm? And then you could take that and start putting them around the room and go, “which is the tallest?” .... What were you surprised about? What was I surprised about? I didn’t realize how big things were and I didn’t understand the concept of size.... they suddenly understood measurement in a visual way. But they got into it. And there’s so many kids that just stood there and they wanted more of them. So, we just added. We put more of them. There was a t-rex, we made a streetcar at one point. But they were asking these questions, and I was like, “I don’t know... let’s figure it out!”
Teacher 4 capitalized on children’s interests in animals and their questions about size to entice them into an inquiry on scale and measurement, by strategically placing out novel and exciting materials to engage children into guided play.

Inviting children to guided play used provoking and enticing as key strategies to lure children into play. This approach to guiding play had the highest levels of teacher-facilitation, while still leaving room for child autonomy and choice. With inviting, the educators created a playful context and then invited the children to engage and extend the learning within this playful activity.

**An Iterative Process of Play: Planning, Guiding, and Debriefing**

Play is commonly described as an iterative process of wondering, discovering, hypothesizing, and revising theories that underpin learning (Zosh et al., 2018). Teachers in this study described three approaches towards intentionally facilitating an iterative process of play to support student learning; planning, guiding, and debriefing, which occur before, during, and after play, respectfully. Taken together, these three approaches helped to support children through a systematic process of learning through play and align with both historical and contemporary descriptions of play as an iterative process of learning for young children (Piaget, 1962; Zosh et al., 2018).

**Before Play: Planning**

The iterative process of play began with educators helping children to plan for and consider their intentions for play. Planning involved several strategies including, class discussions, voting, and entry tickets.

**Discussion.** The most common strategy for planning involved the facilitation of class discussions. As a class, the educator facilitated a discussion about students’ interests, inquiries,
and questions, and would help to connect their wonderings to different play opportunities within
the classroom. Teacher 2 explained, “We always kind of started with a wondering. So, what are
you wondering about in your play? Is there a question you have that you’re wanting to solve?”

**Voting.** Following these class discussions, the teacher would then commonly facilitate a
class vote to determine the purpose of various classroom spaces. Through this process, the
educator would further strengthen the connections between children’s interest and opportunities
for enriching play experiences within the classroom. For example, Teacher 1 described
facilitating a class vote to decide what the drama center should transform into:

> Well, at circle time we would determine what should the drama area become next... And
> then once they voted and knew what it was going to become, I would ask them, “what
> materials are we going to require in order to make this happen?” They would come up
> with a list and they could bring it in from home, or I could bring it in, or whatever we
> had. And we would establish it together, the cafe, it was the Bird’s Creek Café.

**Entry Tickets.** Teachers also took a more direct role in helping children prepare for play
by asking them to complete entry tickets before joining open-ended centres. Often, this was done
to help children expand the possible ways they may engage with a centre. For example, Teacher
10 described asking children to draw a blueprint of a structure before entering the block centre,
to help children think critically about what they were going to build and to help support them in
their planning process:

> If they want to go to a certain center in the classroom – let’s say the building center –
during play, we did a lot of “well, ok, let’s draw a blueprint, first”. And they all have
blueprint books, that I make for them at the start of the year. And their blueprints get
more and more detailed as the year goes on. So, at the start, they are just kind of circles,
but then they really start to understand the classroom, and then it’s a lot of, “well, how
many [blocks] do you need for that?”. Like, how many big blocks should we get out? So,
they end up labeling them, and that’s a lot of their literacy skills as well.

Planning for play is an important part of the evolving and iterative process of play. Here,
teachers took an active role in helping children determine, organize, and prepare for their
learning and exploration in play. This planning was then actualized through educators’ involvement in guiding children’s play.

**During Play: Guiding**

While the approaches to guiding play (i.e., extending, facilitating, inviting) have been described in detail above, it is important to reiterate the role of guiding children’s play within the larger, iterative, process of play itself. The examples previously discussed illustrate the crucial ways in which educators guide play towards academic goals, integrate opportunities for new learning, and help children make connections and apply their learning in play. In doing so, the educators helped children generate questions, form hypotheses, test, and revise learning through play. Educator involvement in play, to guide, support, and extend learning, is an essential component that makes this iterative, authentic, and meaningful approach to student learning possible.

**After Play: Debriefing**

After play occurred, educators would often gather the class to debrief their play and provide an opportunity to review, reflect, and consolidate their learning. Teachers discussed using four strategies to help children debrief their learning following play: sharing documentation of play, celebrating the learning, exit tickets, and planning for future periods of play.

**Documenting.** The most common strategy was sharing documentation of play, as the teacher facilitated a whole class discussion to share any documentation (e.g., photos, videos) that was recorded while children were playing. Teacher 2 described sharing her documentation of play to help students reflect upon what they learned in play:
At the very end of that round of play we would come back and be like "So, were your questions answered? Did you achieve what you wanted to? We would go to the documentation that I took and be able to have a chance to explain it to our peers."

Teacher 2 used this debriefing period to share her documentation of play and to support students in sharing their accomplishments from play. She also used this opportunity to “use the kids as experts” and invite them to share and explain what they had done in play. She described using gallery walks as an effective technique for eliciting children’s reflections, explanations, and questions from a period of play. Through these discussions, children and teachers were able to celebrate the learning and accomplishments that children made in play.

Celebrating. Celebrating students’ learning in play also inspired children to share their ideas, and approaches to play to support the learning of other students. Teachers commonly facilitated this student-to-student sharing, by suggesting students use developmentally appropriate exit tickets as part of their centers. For example, Teacher 4 described guiding a student to create a blueprint of their structure, as a method of supporting and encouraging other students to engage in similarly complex building: “I love what you’re doing, you should make it on paper. Make a blueprint and then other people can make it.” The use of exit tickets allowed students to both record and share their learning in play.

Planning. Sharing and celebrating learning, supported through strategies like exit tickets and gallery walks, helped classes plan for future periods of play. Through debriefing, both the educator and children were able to reflect upon and consider next steps for learning. For the educator, this debrief allowed them to consider and plan for the next learning goal for their students, to “push their learning somewhere, either developmentally or academically” (Teacher 3). For children, planning for the next period of play allowed them to identify strategies that worked well and possible ways to engage in future play, for example, by using a blueprint to
build an intricate structure, as described above by Teacher 4. Debriefing play by planning for future play is a key step that educators used to support an ongoing and iterative process of play within their classrooms.

In conclusion, teachers integrated and applied their pedagogical knowledge of play and mathematics through collaborative approaches to play. By varying levels of adult-facilitation and child-autonomy, educators enacted three approaches towards implementing a continuum of guided play to support early math learning: extending, facilitating, and inviting. These three approaches to guided play were also situated within an overarching and iterative process of play, which was facilitated by educators’ involvement before, during, and after play to consolidate, connect, and deepen student learning.
Chapter 6: Discussion

This study sought to understand the ways in which kindergarten educators were successfully collaborating with children in play to support their early math learning. Educators’ successes were shared in Chapters 4 and 5, which outlined their perspectives and implementation of play and math. Broadly, the majority of educators’ perspectives aligned with a traditional conceptualization of play that is child-led with no adult involvement, while only a minority of educators articulated contemporary perspectives that described how educators can support math learning along a continuum of play. Educators who articulated these contemporary perspectives of play and math learning were invited for follow-up interviews to learn about their practices in greater depth. Throughout these interviews, educators depicted an evolution in their practice that helped them to not only shift their perspectives of play and improve their relationship with math, but also to integrate the constructs of play and math learning through developing an understanding of collaborative approaches to play. In doing so, these educators described three overall approaches, each with specific strategies for implementation, that they used to guide children’s play to support math learning. They also described how collaborating with children in play was part of an ongoing and iterative process, and that the educator played a key role in supporting learning before, during, and after periods of play.

The following discussion will outline an overall argument for why, and how, we can improve the approaches to mathematics teaching and learning that are currently occurring in kindergarten classrooms. Reasons to improve these prevailing math approaches are rooted in underdeveloped Mathematics Knowledge for Teaching (Ball et al., 2008) in kindergarten as well as outdated perspectives of play and math learning that lags behind knowledge gained in other areas of academic learning in the early years. Vestigial traits from the whole language movement
continue to influence educators’ perceptions, and implementation, of academic learning in kindergarten and contribute towards incidental approaches to math learning that reflect low levels of educator involvement (Sarama & Clements, 2009; Weaver et al., 1990). Such indirect approaches to early math learning are insufficient, and I argue for movement away from a stance of “whole-math” and towards the action of teacher-facilitation of math learning instead. Despite an emphasis on whole-math perspectives, there are teachers who have successfully transitioned towards teacher-facilitation of math learning within their kindergarten classrooms. This discussion will take a deep dive into how these educators were able to execute this successful transition and outline key strategies that may support other educators in their practice. Critically, I will also discuss the immediate and future next steps that can be taken to support educators through this evolution in practice.

Underdeveloped Mathematics Education in Kindergarten

Kindergarten as “A Separate Beast”

In practice, kindergarten is conceptualized and implemented as a separate program that is distinct from the elementary grades. As such, the math program in kindergarten is separate from the math program in the elementary grades, and so too is the Mathematics Knowledge for Teaching (Ball et al., 2008) in kindergarten. Teacher 7 effectively described this issue, “It’s not known in kindergarten [Mathematics Knowledge for Teaching]. Grade 1-8, maybe. But certainly, kindergarten is treated like this separate beast from the rest of the grades.” In Ontario, this “separate beast” is evidenced by an entirely different curriculum, named “The Kindergarten Program”, which includes a teaching structure and mandated pedagogical approach that is distinct from the elementary grades. Specifically, in Ontario, The Kindergarten Program (OME, 2016) is implemented by two educational professionals, a certified teacher and an Early
Childhood Educator, collectively referred to as “the teaching team”, who are mandated to use play as the primary pedagogy for achieving curricular objectives. Despite this mandate to support children’s learning through play, teachers receive very little, if any, training in how to implement play, particularly in relation to the integration of play and academic learning (Pyle & Danniels, 2017). Conversely, the elementary mathematics curriculum is delivered by only one certified teacher and does not mandate a play-based pedagogy. Instead, the elementary mathematics curriculum requires teachers to embed process standards within their math instruction. These process standards reflect overall mathematical processes (e.g., problem solving, reasoning and proving, reflecting, connecting, communicating, representing, selecting tools and strategies) that students must engage with across all strands of the mathematics curriculum (e.g., social-emotional learning, number, algebra, data, spatial sense, and financial literacy). While perhaps less explicit than the kindergarten curriculum, the elementary mathematics curriculum also requires educators to engage in specific pedagogical practices.

The structure and content of both the kindergarten and elementary curriculums differ drastically. The kindergarten program is divided into four overall curricular areas: Belonging and Contributing, Self-Regulation and Well-Being, Demonstrating Literacy and Mathematics Behaviours, and Problem-Solving and Innovating (OME, 2016). Conversely, in Ontario, the elementary curriculum is organized by subject area, rather than grade, which outlines the progression of subject-specific skills and objectives throughout grades 1-8, exemplifying strong horizon knowledge of each subject to be taught at school (Hill & Ball, 2009; OME, 2020). The elementary math curriculum is not without limitation, however, most notable of which is that this new curriculum was introduced during the uncertainty of a global pandemic with little to no educator support for implementation (Wong, 2020). The timing of this new curriculum was
problematic, as schools faced monumental challenges, such as closures to in person learning, transitions to online learning, and general uncertainly about the safety and well-being of their staff and students. As such, while this new curriculum offers many new insights, including an emphasis on coding and financial literacy, it still requires time and support for educators to adapt, further dividing a disconnect between the kindergarten program and the elementary mathematics curriculum.

When looking at the mathematics component of the kindergarten program specifically, it is evident that it has been clustered with literacy, and very few connections between these mathematical objectives and those of the future elementary grades are made explicit, as is the case with the elementary program. This demonstrates poor communication of mathematical horizon knowledge that connects kindergarten with the future elementary mathematics curriculum. More so, mathematical objectives of the kindergarten curriculum focus on more basic skills, like counting (OME, 2016), whereas the elementary curriculum begins to focus on higher-order math skills, such as problem-solving (OME, 2020), which reasonably reflects the developmental progression of the mathematical skills and abilities (Clements & Sarama, 2009). However, without strong horizon knowledge between the kindergarten and elementary curriculums, it is challenging for educators to make explicit connections between the basic skills present in the math section of the kindergarten program and the more complex mathematical problem-solving skills required in older elementary grades. Strengthening the horizon knowledge between the kindergarten and elementary mathematics curriculum is an important next step.

Perhaps most importantly, kindergarten students have different developmental needs than elementary students, with the biggest differences seen between junior kindergarten students (age 3 or 4) and grade six students (age 10 or 11). For example, kindergarten students are still
developing their communication skills, including verbal and non-verbal forms of communicating (Justice & Pullen, 2003; Wu, 2009). This can influence how young children both receive instruction and how they provide feedback to educators. For example, young children communicate meaning, including mathematical understanding, through multiple modalities such as gestures, facial expressions, engagement/disengagement, as well as through more tactile means, such as drawings, buildings, or through representations in play (Einrsdottir et al., 2009; Gordon & Ramani, 2021; Hansel, 2015; Pelletier, 2016; Vygotsky, 1978). More so, as kindergarten is the entry point into formal schooling, many kindergarten students are still learning the foundational skills of how to behave while at school, including basic skills like following classroom routines, developing independence, sharing adult attention, and navigating social relationships with peers. They are also developing foundational fine-motor skills like how to hold a pencil, book, or how to gently play with classroom materials (Grissmer et al., 2010). As such, support for these important foundational developmental skills has to be sensitively and adequately balanced alongside support for children’s academic learning, including mathematics (Pyle, 2013). In response, Ontario introduced the play-based learning mandate within their kindergarten program as a method for addressing the developmental and academic needs of young learners (OME, 2016), however, conflicting views of regarding the purpose of play have posed barriers towards the integration of play and academic learning in classroom practice (Pyle et al., 2018a; 2018b).

In the resources provided to educators, however, kindergarten is frequently grouped in with elementary mathematics, despite the fundamental differences between these two programs. These resources can span kindergarten to grade six within one text (Lawson, 2016; Newton, 2016; Small, 2013; Van de Walle et al., 2011), and even the resources designated for the early
years cluster kindergarten in with the primary grades (Gr. 1-3) (Hughes, 2019; Moss et al., 2016, Small, 2020). While encapsulating kindergarten into resources for the primary grades can demonstrate strong horizon knowledge, there are also some overarching concerns. Primarily, as just discussed, the developmental needs of kindergarten and elementary children differ. The largest differences are obviously seen between kindergarten and grade six students, but I also argue that there are meaningful differences in the developmental needs of kindergarten and primary students (Grades 1-3). We cannot simply conceptualize a four-year-old as half an eight-year-old and therefore we cannot just present kindergarten resources as a simplified version of grade three resources. Students’ needs and context differ. Second, kindergarten education has a different pedagogical mandate, which requires different resources and educator supports that specifically respond to this mandate. To address the developmental needs of young children alongside their academic learning, teachers are required to use play to support kindergarten children’s mathematics development, so we need to also provide educators with evidenced-based resources to integrate play and math learning. Third, the mathematical curricular objectives in kindergarten focus on more basic mathematical skills, such as counting, whereas older elementary grades also include a focus on higher order skills like problem-solving (OME, 2016; 2020). When kindergarten is encapsulated within elementary mathematics resources, emphasis is often placed on more complex concepts that are taught in the older elementary grades and pulls focus from the more basic skills that are required in kindergarten. Consequently, the Specialized Content Knowledge (Ball et al., 2008) required to support these basic skills, such as an understanding of children’s common misconceptions about counting and quantity, is often overlooked or underrepresented in the resources available to educators. While both basic and higher-order skills are important, we still need to take the time to thoroughly develop a strong
foundation of the basic skills that will help the development of higher-order mathematical skills and thinking (Clements & Sarama, 2009). Having evidenced-based resources that are dedicated to kindergarten, specifically, would help to build these strong, and essential, foundational math skills.

The lack of evidence-based resources that are specific to kindergarten is further evidenced by the resources shared by participants in this study. For example, even though Phase Two educators shared high quality, evidence-based resources to support math learning (e.g., Clements and Sarama’s Mathematics Learning Trajectories website, What to Look For by Alex Lawson, and Good Questions, by Marian Smalls), all of these resources cluster kindergarten in with the elementary grades. No educator in this study shared a published, edited, or externally reviewed resource about play-based learning or about integrating play-based learning and math. Instead, much of the information they shared about play was piecemealed together from their school board’s professional development, Additional Qualification courses, or from reading general information about play-based learning on the internet. When it came to finding discrete play-based activities for supporting math learning, all the educators told me how they either found ideas on the internet (Twitter, Instagram, Pinterest, Educator Blogs, Teachers-Pay-Teachers) or from other kindergarten teachers at their school. The lack of systematic and evidenced-based resources for math, play, and the integration of math into play available for educators is a concern. This is an evident next step towards supporting educators in their practice, which will be discussed in greater detail later in this discussion.

**Limited Mathematics Knowledge for Teaching in Kindergarten**

Without systematic and evidence-based resources to support the integration of play and math, a resource void is created, which can be filled with amateur or non-substantiated claims
and materials. We saw this happen in the current study, as teachers turned to uncited and uncredited resources on the internet (Pinterest, Instagram, Blogs, etc.) to guide their practice. I think that these teachers are doing the best that they can with what’s available, but what is available is a messy field for educators to navigate as there is inconsistent and conflicting messaging regarding both play and math pedagogies. This has a deep impact on educator’s Pedagogical Content Knowledge (Ball et al., 2008) required for teaching math in kindergarten. For instance, despite the fact that play is mandated in Ontario, educators have limited access to professional development regarding play as pedagogy, and the professional development that is available can vary widely in terms of how play is defined, implemented, and supported in practice (Bubikova-Moan, 2019; Hunter, 2019; Schmidtke, 2020). This is reflective of traditional and contemporary perspectives of play described in both research and practice (Fesseha & Pyle, 2016; Pyle et al., 2017; Pyle et al., 2018b). Compounding the confusion around play as pedagogy is a simultaneous debate about math pedagogies, as some advocate for more constructivist approaches to math learning, like play, (Fisher et al., 2013), while others argue for more didactic approaches to math instruction, such as direct instruction (Stokke, 2015). Conflicting, and inconsistent information about Pedagogical Content Knowledge for math in kindergarten is a contributing factor to our underdeveloped, or limited, understanding of mathematics education in kindergarten.

A second contributing factor to our underdeveloped knowledge of mathematics education in kindergarten is a limited understanding of Subject Matter Knowledge (Ball et al., 2008) at the kindergarten level (Figure 2). It is important, here, to differentiate between kindergarten educators’ Common Content Knowledge (CCK) (i.e., general knowledge of mathematics, not specific to teaching) and Specialized Content Knowledge (knowledge and skills for teaching
Ball et al. (2008). Without question, I believe that kindergarten educators have the Common Content Knowledge required to understand kindergarten-level mathematics. Instead, what I argue is that their Specialized Content Knowledge of how to deliver math instruction is underdeveloped. While Clements and Sarama’s (2009; 2022) Learning Trajectories do a remarkable job explaining and breaking down key math skills in kindergarten, there could be stronger connections made that outline the Specialized Content Knowledge teachers need in order to deliver instruction. Specifically, while the Learning Trajectories make some mention of common mathematical misconceptions and errors children make, this work could go a lot deeper. This underdeveloped knowledge is further evidenced by findings in the present study, as educators explicitly described, “I don’t really know why I’m doing the things that I’m doing... I’d argue I didn’t learn this [Specialized Content Knowledge] in teachers’ college, none of this was shared. We learned a lot of pedagogical practices, but not content” (Teacher 7). Educators in Phase Two of this study were able to identify this gap in their knowledge and sought out professional learning to improve their Specialized Content Knowledge for teaching mathematics in kindergarten. Yet despite their initiative, I argue that their learning was still in some ways underdeveloped, because our knowledge of this topic is also underdeveloped, and therefore so is the support we can offer educators.

Ball et al.’s (2008) framework discusses how gaining a deeper understanding of the mathematic misconceptions, errors, and unsuccessful strategies students use would help to build the strong, flexible, and insightful Specialized Content Knowledge that is not only required to teach math, but also to understand how children interact with mathematical ideas. This Specialized Content Knowledge is also connected to teachers’ Pedagogical Content Knowledge, specifically Knowledge of Content and Students (i.e., anticipate how students may respond to
math activities/lessons by identifying what students might find challenging, interesting, or of relevance) as well as Knowledge of Content and Teaching (i.e., how to design, sequence, and deliver mathematical content). This idea, however, points to a third contributing factor towards our underdeveloped knowledge of math education in kindergarten, which is that there is a limited understanding that connects the two halves of Ball et al.’s (2008) framework of Mathematics Knowledge for Teaching.

Without a strong understanding of the connection between Subject Matter Knowledge and Pedagogical Content Knowledge, in kindergarten mathematics education, it can be difficult for kindergarten educators to not only anticipate which mathematics concepts may pose the biggest challenges for their students, but also how to respond to these challenges. Ball et al.’s (2008) work also discusses the importance of teachers being able to quickly and fluently respond to common mathematics misconceptions that children display when engaged in mathematical activities. This fluidity and flexibility that is a result of strong understanding of the connection between multiple components of Ball et al.’s (2008) framework, is particularly relevant in the implementation of guided play, as educators need to respond “in the moment” or “on-the-fly” as they interact with children in play to support learning and correct misbeliefs about mathematical concepts. While adults’ active and flexible responsivity to children’s math learning in play is an optimal goal, a prevailing problem in this present study is that there’s a hesitancy for educators to become involved in play to directly support math learning, and to a larger extent, a hesitancy to directly teach discrete math skills in general.

“Whole Math” Approach is Not Sufficient

*Indirect vs. Direct Approaches to Math Instruction*
Results from Phase One survey data are beginning to point towards different perspectives and approaches to math instruction in kindergarten that reflect different purposes of play and offer children different levels of educator support. Specifically, 82% of participants described the purpose of play in kindergarten for supporting children’s social learning, while only 18% described how play can be used to support both social and academic development. Of these 82% of participants who are not describing academic learning, or the potential for academic learning within play, the vast majority also articulated the belief that young children learn math best through “through hands on play experiences” (Participant 33). These educators are not yet describing any level of teacher involvement, either through various approaches to play or through direct instruction, when supporting students’ math learning. Taken together, this perspective of play and approach to math learning reflects a more indirect teaching approach as children are left to spontaneously engage in mathematical learning through child-led play without adult intervention. Consequently, the math learning that children engage in is more of an incidental by-product of their free exploration, rather than a discretely targeted goal for their learning. This child-led approach to play without adult intervention has been well documented in the literature (Fleer, 2001; Wood & Attfield, 2005), often in relation to children’s literacy learning (Elvin et al., 2007; Sharp et al., 2012; Van Oers & Duijkers, 2013; Wohlwend, 2008), and emphasizes how educators play a fundamental role in setting up a child-centered environment, providing materials, and protecting large blocks of uninterrupted time for children to engage deeply in free play (Korat et al., 2002). This perspective is not without benefit, however, as this child-driven approach to learning reflects students’ interests and has been proposed to bolster engagement, inquiry, and joy in children’s learning (Foote et al., 2004). However, this approach remains problematic in practice because research continues to
demonstrate how free play, or unassisted discovery, is a less effective math pedagogy (Alfieri et al., 2011), as students do not display any statistically significant change in their math knowledge when they learn about novel math concepts through free play (Fisher et al., 2013).

Conversely, the 18% of participants that described the purpose of play for social and academic learning also described how children learn math best through a balance of direct instruction and various approaches to play. Taken together, these educators described more direct approaches to teaching math in kindergarten as they explicitly articulated their involvement in students’ math learning through collaboration with children in play, by planning and implementing math games, and by facilitating children’s math learning through direct instruction. Their emphasis on the educators’ role in planning, implementing, and directly facilitating discrete math skills, both within and outside contexts of play, represents a categorically different perspective and approach to facilitating children’s mathematics learning in kindergarten.

Research demonstrates that approaches to math instruction, like those described by 18% of educators in Phase 1, have distinct benefits for children’s early math learning (Skene et al., 2022). Specifically, this line of research shows that when children are exposed to mathematical concepts through educator facilitated approaches to play, they acquire statistically significantly greater math learning gains than when they are exposed to math concepts through either direct instruction or free play (Alfieri et al., 2011; Fisher et al., 2013; Skene et al., 2022). More so, this line of research also demonstrates that children will learn more novel math knowledge when they learn about math concepts through direction instruction, than compared to free play (Alfieri et al., 2011; Fisher et al., 2013). This finding suggests that direct instruction has a place in kindergarten classroom practice, as it leads to student learning gains, but perhaps it should not be
the only, or main, pedagogical approach, as teacher-facilitated play leads to greater learning
gains (Alfieri et al., 2011; Fisher et al., 2013). Taken together, supporting children’s math
learning through a combination of direct instruction and varied approaches to play-based
learning, as described by 18% of educators in Phase One, aligns with contemporary research
recommendations for supporting math learning in kindergarten.

**Reflecting the Whole Language Movement**

The finding that 82% of educators described indirect approaches while only 18%
described direct approaches to math instruction begs the question as to why this is such a vastly
different representation of perspectives from previous research regarding play and academic
learning. In studies of play and literacy, for instance, we see a more even divide, of 42%
describing more indirect approaches, and 58% describing more direct approaches towards
integrating play and literacy (Pyle et al., 2018b). Pyle et al.’s (2018b) study was also conducted
in Ontario and took place with a similar population to that of the present study, given that the
majority of teachers in this study were also from Ontario and therefore were influenced by the
same curricular programs, policies, and supports. One broad interpretation of this finding is that
the math research, and consequently educator perspectives and practice, lags behind literacy
research, perspectives, and practices. As previously discussed, the underdeveloped study of
mathematics education in kindergarten is a contributing factor to this lag, as literacy research and
practice has historically been prioritized in kindergarten classroom practice (Stephan et al.,
2015). The overwhelming perspective of indirect approaches to mathematics learning may be
shocking when compared alongside current literacy and play research, but it is not a new
phenomenon. We’ve seen this stance play out in educational theory and practice before, most
notably through the whole language movement [Ontario Human Rights Commission (OHRC), 2022; Weaver et al., 1990].

In essence, the whole language movement argued that children would acquire literacy skills if they were submersed in a literacy-rich environment that posed many opportunities to engage in deeply enriching literacy materials and activities that reflected child interest, choice, and agency (OHRC, 2022; Weaver et al., 1990). More so, the whole language movement, or debate, is perhaps best known for prioritizing higher order literacy skills like, reading and comprehension, over more basic skills like phonemic awareness and phonics instruction (Fry, 1993; McKenna et al., 1994). A common critique of this movement is that there was a lack of support, or direct teaching, of the foundational skills like phonics that are required to support higher order skills like comprehension, and consequently many children struggled to become proficient readers (OHRC, 2022). Advocates for whole language, on the other hand, emphasised child-choice and agency in literacy learning and expressed the belief that educators play a more supportive role in intentionally setting up a literacy-rich environment that provided ample opportunities for children to engage in deep and meaningful literacy experiences, though they often did not get directly involved in the explicit instruction of discrete and basic literacy skills (Goodman, 1989).

The parallels between the whole language movement and the current prevailing perspectives on play and math learning in kindergarten are evident. Primarily, results from Phase One demonstrated that the vast majority of educators shared a more indirect approach to math instruction by prioritizing enriching, child-led, and hands-on activities that they believed children would spontaneously engage in if they set up enriching and meaningful environments and provide time and space for uninterrupted play. Though not explicitly discussed by educators in
this study, the research also highlights a similar pedagogical debate in the literature about prioritizing higher order mathematical skills, like problem-solving, over more basic or procedural, skills like counting and operations (Ansari, 2016; Stokke, 2015). While this debate is often situated in older grades (Alfieri et al., 2011), there is an important connection between the current debate regarding mathematics education and the ongoing debate regarding literacy education. If we want to fast-forward through this current period of “whole math” then we can turn to the recent Right to Read Report (2022) commissioned by the Ontario Human Rights Commission to see how the Whole Language debate played out in practice.

**Lessons Learned from the Whole Language Movement**

Recently, the Ontario Human Rights Commission initiated a public inquiry into human rights issues affecting students’ literacy acquisition, with a particular emphasis on students with reading disabilities. This public inquiry reviewed the current literacy practices and curriculum in Ontario’s public education system alongside the science of reading literature. In general, they found the current literacy practices to be inequitable, such that Ontario’s literacy programs and policies are failing to support all students in becoming proficient readers, and in doing so are denying many Ontario students a basic human right. They lay out a strong legal argument for this assertion by demonstrating how the Supreme Court of Canada recognizes that “learning to read is not a privilege, but a basic and essential human right” (OHRC, 2022, p. 2) and that Ontario’s literacy curriculums, practices, professional development, and pre-service teacher training programs largely reflect out of date practices that contemporary science of reading research does not endorse as effective pedagogy (OHRC, 2022). In response, they outlined key issues and recommendations to improve the literacy practices in Ontario’s public school system. Broadly, these issues and recommendations were organized around five leading themes:
Curriculum and Instruction, Early Screening, Reading Interventions, Accommodations, Professional Assessments, and Systemic Issues (OHRC, 2022). While all of these issues also pertain to the study and practice of mathematics education, the theme of Curriculum and Instruction most directly and urgently corresponds to the findings outlined in this current study.

Within the theme of Curriculum and Instruction, the Right to Read report makes several explicit recommendations. Most relevant to the current study is the recommendation that teachers must use “mandatory explicit, systematic and direct instruction in foundational word reading skills” (OHRC, 2022, p. 22) to support discrete basic literacy skills, and that professional training and development for both pre-service (i.e., teacher training programs) and in-service teachers (e.g., Additional Qualification courses) revise their content to align with contemporary science of reading research. Outdated, and ineffective, approaches to literacy instruction, such as whole language, are commonly still present in many school boards, professional development, and training, and the OHRC (2022) report calls for a need to replace these antiquated approaches with evidenced-base techniques that highlight the fundamental role the educator plays in systematically and explicitly supporting foundational literacy skills. Explicit connections can be made from these “foundational word reading skills” involved in literacy development, such as phonics, and foundational mathematical skills, such as an understanding of the principles of counting. More so, this report is making it explicitly clear that educators are required to support the development of these foundational skills through systematic instruction. While these recommendations are evidenced-based and important, there has been push back, as some continue to highlight that contextualizing foundational skills within print-rich and engaging literacy environments is still relevant and important for students’ literacy development (Cummins, 2022).
What I argue here is that the same recommendations from this report also apply to the field of mathematics. That it is necessary for children to build the foundational math skills that will contribute towards deep mathematical understanding and proficiency (Duncan et al., 2007; Rittle-Johnson et al., 2016) and that teachers play an essential role in actively and systematically supporting the development of these foundational math skills (Clements & Sarama, 2014). What I further argue is that when it comes to teaching these foundational math skills in kindergarten, we have to do a better job of supporting educators in using developmentally appropriate practices, which the research is telling us are rooted in play (Weisberg et al., 2013) and specifically in teacher-facilitated play (Fisher et al., 2013; Skene et al., 2022; Weisberg et al., 2016).

Teacher-Facilitation of Math Skills

Teacher-Facilitation of Play

Research continues to point to the importance of teacher-facilitation of math skills, particularly teacher-facilitation of play to support young children’s academic learning (Roskos & Christie, 2009; Weisberg et al., 2013; 2016; Zosh et al., 2018). This has been studied in relation to children’s literacy learning (Cavanaugh et al. 2017; Goble & Pianta, 2017; Pyle et al., 2022, submitted), science learning (Sliogeris & Almeida, 2021), and mathematics (Wickstrom et al., 2019). Across many academic domains, research corroborates the assertion that educators should facilitate play to guide and extend student learning (Skene et al., 2022). When it comes to children’s math learning in particular, the research is showing us that teacher-facilitation of play, or guided play, is considered to be the optimal approach to supporting children’s acquisition of and ability to retain new mathematical information (Fisher et al., 2013). In practice, however, educators have expressed uncertainty in how to facilitate children’s play so that they may follow
Findings from Phase Two of this study directly respond to this central challenge. In particular, the approaches and strategies, identified in Chapter 5, that educators used to support math learning along a continuum of guided play begin to describe a framework for facilitating play to support young children’s math development. A key component of this developing framework is that it helps to connect the two components of Ball et al.’s (2008) theory of Mathematics Knowledge for Teaching, which include Subject Matter Knowledge and Pedagogical Content Knowledge. Having both a deep understanding of mathematics content knowledge and contemporary views of play pedagogy allowed educators in Phase Two to see the math learning in play as well as the potential for math learning in play. Teacher 10 beautifully summarized this point by sharing,

“I felt like once I really dove into actually reading the research behind the curriculum and really understanding the big ideas, then I could see it [math] everywhere in play. But if you’re just focusing on like one content expectation, and it’s a shallow understanding of that content expectation, you don’t see it in play or you don’t see how play can lead to it.”

Educators need to know the math content knowledge, particularly Specialized Content Knowledge (Ball et al., 2008), but they also need to know how to deliver that math knowledge through developmentally appropriate means (Weisberg et al., 2013; 2016). By facilitating children’s math learning through guided play, teachers are able to merge these two necessary skills.

**Balancing Pedagogical Approaches**

While the research continues to point towards teacher-facilitated play as an optimal method for supporting young children’s learning (Skene et al., 2022), competing bodies of
research also advocate for different pedagogical approaches, such as direct instruction (Klahr, 2009; Mayer, 2004; Stokke, 2015). Rather than perpetuate the debate between constructivist and didactic approaches, I argue that there is room for both within a kindergarten classroom and that teacher-facilitation of math skills can occur through a variety of forms. The research does show us that direct instruction has a time and place in a classroom, for example, it can be a very effective pedagogy for students who have difficulties learning mathematics (Doabler & Fien, 2013). While I advocate for balance between these approaches, I do not mean that there needs to be equal prevalence of these pedagogies in classroom practice. The research is explicitly showing us that while direct instruction can be beneficial to student learning, guided play approaches result in greater math learning gains and knowledge retention (Fisher et al., 2013; Skene et al., 2022). Given these learning gains, and the noted developmental benefits of teacher-facilitated play-based learning (Jensen et al., 2019; Zosh et al., 2018), prioritizing collaboratively playful approaches to math learning in kindergarten poses an advantage. These guided play approaches to math learning can be further supported by direct instruction. Further research could explore what combinations or proportions of these pedagogical approaches may best support students’ math learning and could further support educators’ pedagogical decision-making by deepening our knowledge of mathematics education and play-based learning in kindergarten. With further research, we can also begin to unpack the ways in which teacher-facilitated play and direct instruction can be integrated or coexist within kindergarten classrooms.

While I argue for balance between approaches to teacher-facilitation of math, I also argue for greater balance among the math skills we teach kindergarten children. A main lesson learned from the Whole Language Movement was that children need both foundational skills and higher
order skills to develop literacy proficiency (OHRC, 2022). The same can be said for the field of mathematics. We need to support the development of both basic skills, like the principles of counting, and higher-order skills like mathematical problem-solving (Clements & Sarama, 2014; Sarama & Clements, 2009). Fundamentally, however, educators need to be involved in directly facilitating, supporting, and extending both basic and higher order math skills, beginning in kindergarten.

(Dis)comfort Teaching Math

The finding that some teachers are not facilitating math learning directly (i.e., 82% of Phase One participants), may be a reflection of educators’ comfort teaching math. Research has documented how elementary teachers, particularly in the early years, have described discomfort teaching math, have expressed anxiety relating to the subject, and can therefore have a tendency to avoid, or shy away from, the direct teaching of mathematics (Ambrose, 2004; Ashcraft, 2002; Gresham, 2008).

In the present study, all the educators described high levels of comfort when teaching math in kindergarten. While I believe their description of their experience to be true, I would argue that these educators are not actually directly teaching math skills, as the vast majority engage in more indirect approaches to math learning that do not reflect adult involvement in the direct teaching of math skills. What they may be comfortable with, however, is setting the foundation for math learning. They may be comfortable preparing an enriching space to engage in math learning and they may be comfortable providing the time, space, and materials to let math learning naturally emerge from play. But they are not describing comfort with the direct teaching of math skills, in either the context of play or direct instruction. For the majority of teachers, setting up the foundation for math learning is their comfort zone for teaching math,
rather the direct teaching of math skills. Consequently, we need to help push educators out of this comfort zone of “whole-math” and towards teacher-facilitated approaches instead. Crucially, though, we also need to provide educators with support through this transition. While the majority of teachers may currently practice within the comfort zone of “whole-math”, there are teachers who described shifting away from this perspective towards more teacher-facilitated approaches to math learning. Unpacking how these teachers were able to successfully make this shift will help us to understand the key factors that supported them in their practice.

**Transitioning from Whole-Math to Teacher-Facilitated Math**

**Expanding Notions of Play**

A key factor in transitioning from whole-math to teacher-facilitated approaches to math included expanding notions of play-based learning in kindergarten. All the educators in Phase Two described expanding their notions of play by engaging in professional development. In doing so, this expanded view of play helped to clarify the conflicting views of play and Pedagogical Content Knowledge (Ball et al., 2008) educators initially described. Expanding notions of play helped to provide clarity on what play is, what it can look like, what its role in a classroom may be, and importantly, what an educator’s role in play can involve. While all educators may not have explicitly described learning about play as a continuum, many did, and they all described learning about various approaches to play that involved different levels of adult involvement or facilitation. Importantly, this expanded notion of play still maintained the benefits of play for children’s social learning, including the perspective that play needs to be engaging, meaningful, socially interactive, iterative, and joyful (Zosh et al., 2018), but also introduced the idea adults can guide play to support children’s developmental and academic learning (Pyle et al., 2017). This expanded view of play not only helped educators to address
common misconceptions about play, but also provided them with multiple access points into play to support student learning.

Conceptualizing play as a continuum that ranges in levels of child and adult involvement allowed educators to access play in a way that aligned most closely with their beliefs and comfort-level (Pyle & Danniels, 2017; Wickstrom et al., 2019). For example, educators that hold more traditional perspectives of play may be more comfortable entering play from the child-led side of the continuum, by gently joining or influencing the play, before gradually entering children’s play to guide, support, and extend learning. Conversely, educators who may be uncomfortable with the high degree of unpredictability that accompanies free play may feel more confident facilitating play from the teacher-directed side of the continuum, as it would allow them to plan for and target specific learning goals. From here, educators could gradually release some control over the play to open up more collaborative or child-led opportunities for play. For many educators, accessing multiple entry points into play, and moving along the play continuum, can involve minor modifications to their current practice (Wickstrom et al., 2019). For example, providing children with choice in not only what, but also how they engage in play as well as posing open-ended questions can help both educators and children to share control over the direction and outcome of play.

Changing Relationship with Math

A second key factor educators used to shift from whole-math to teacher-facilitated approaches to math involved improving their relationship with math. To facilitate this shift, the research commonly points to the importance of embodying a growth mindset (Dweck, 2015; Peterman & Ewing, 2019; Samuel & Warner, 2019). A growth mindset relates to the underlying beliefs people hold about learning and intelligence. Someone with a growth mindset believes that
their learning and intelligence can improve with practice and effort, while someone with a fixed mindset may believe that their learning and intelligence are pre-determined or static and that it is not influenced by their efforts (Dweck, 2006). For example, a common perspective of a fixed mathematic mindset is when someone believes they are simply “not a math person” (Dweck, 2015). Every teacher in Phase Two demonstrated a growth mindset towards developing mathematical skills and towards improving their relationship with math. For example, Teacher 5 emphasized “the power of yet”; the idea that although she may not be able to currently do something, with practice, she will be able to achieve that skill. This growth mindset was fundamental to improving educators’ perspectives on math, as it helped educators to separate their negative experience learning math as a student from their positive experience as an educator.

The identification and description of the role of growth mindset was the result of an emergent approach to coding the interview data. I interpreted the data based upon what educators were telling me, and they explicitly used the term “growth mindset”, which is also an idea commonly endorsed in Ontario curricula and policy (OME, 2016; 2020). Deeper interpretation of the educators’ descriptions of growth mindset could connect to the overarching role of teachers’ self-efficacy in their ability to strengthen their mathematics teaching practice to meet the needs of their students (Klassen & Tze, 2014; Perera & John, 2020). This overarching notion of self-efficacy could relate to a teachers’ ability to teach math as well as their ability to implement, and adapt, their approaches to play-based learning. Since I did not conceptualize this project to address the role of growth mindset, or self-efficacy, I did not plan interview questions, or ask follow up questions, that probed at teachers’ self-efficacy further. A such, I did not feel as though I had enough data to support a rich interpretation of the role of teachers’ self-efficacy.
towards improving their relationship with math, though I believe this presents a promising
direction for future research.

One way they helped to build a positive experience as an educator was by gaining a
deeper understanding of the mathematics knowledge required for teaching

**Strengthening Mathematics Knowledge for Teaching**

Research demonstrates that strengthening educators’ Mathematics Knowledge for Teaching, including their Subject Matter Knowledge and Pedagogical Content Knowledge, is a leading factor towards strengthening educators’ expertise in teaching mathematics (Ball et al., 2008; Thames & Ball, 2010). In turn, the research also demonstrates that teachers’ enhanced knowledge for teaching mathematics leads to statistically significant growth in students’ math achievement (Hill et al., 2005). Findings from the present study are in keeping with the existing research, as educators in Phase Two intentionally sought out new knowledge pertaining to both their Subject Matter Knowledge and their Pedagogical Content Knowledge for teaching mathematics in kindergarten. Their professional learning began by identifying a need, in both their practice of mathematics and play, which they then addressed through separate paths of professional development to strengthen their Subject Matter Knowledge and Pedagogical Content Knowledge, respectively. In particular, educators in Phase Two focused on developing their Specialized Content Knowledge for mathematics in kindergarten. They connected with research, and researchers, to better understand what foundational mathematical skills are required in the early years. For example, Teacher 7 described learning from Dr. Daniel Ansari about the principles of counting and how they are fundamental for building a strong understanding of quantity.

*So, we met with him and he talked specifically about quantity. And how that’s a really big piece for early learning. We started with Daniel and we started looking at — we didn’t*
even know counting principles. We were kind of doing it, but we didn’t know what they were called. That was for us, the tipping point... I’m so grateful for that because it changed how we taught.

While learning about Subject Matter Knowledge, educators simultaneously learned about Pedagogical Content Knowledge by unpacking and expanding their notions of play in kindergarten. By reconceptualizing play as a continuum, educators described realizing how they could become involved in children’s play to help guide, support, and extend play towards academic goals, including the specific mathematic skills they were concurrently learning about. The realization, and acceptance, that adults can become involved in children’s play represented a significant shift in practice as these educators now described actively supporting children’s academic learning within the context of play. In doing so, these educators are now aligning with contemporary perspectives and purposes of play that advocate for both the utility of play to support academic learning, as well as the involvement of adults in play to scaffold learning (Pyle et al., 2017; Weisberg et al., 2013; 2016; Zosh et al., 2018). Throughout these parallel paths of professional development that targeted teachers’ mathematical knowledge required for teaching, educators also described engaging in shared learning that helped to bridge these paths of professional development, which included a shared focus on what young children need in order to learn.

**Strengthening Knowledge of Child Development**

While broadening conceptions of play, embodying a growth mindset, and strengthening Subject Matter Knowledge and Pedagogical Content Knowledge are all factors that the research anticipates, this study also discovered a novel factor that helped educators shift their practice, deepening their knowledge of child development. Throughout their evolutionary journey, educators engaged in parallel paths of professional development that focused on play-based
learning and mathematics, separately. Throughout this learning, however, educators identified commonalities that helped to bridge these separate paths of professional development. They were able to unpack the unique needs of young children and developed an understanding of what young children need in order to learn (Daniels & Shumow, 2003). Educators in Phase Two described a fundamental need to understand, and support, children’s social and emotional well-being alongside their academic learning. They described Blatz’ (1967) security theory, by expressing the belief that children learn best when they feel safe and secure. They also described a multitude of ways they nurtured this sense of security by “tuning-in” to children’s emotional state, expressing empathy, supporting activities that reflect children’s interests and lived experiences, and prioritizing joy in children’s learning (Zosh et al., 2018). At the same time, these educators were embodying, describing, and in some cases quoting, the work of classic developmental psychologists like Piaget and Vygotsky. They all described scaffolding children’s learning through supporting their zone of proximal development and clearly articulated the belief that young children are capable and competent learners (Vygotsky, 1978). Overall, they described a deep understanding of constructivist notions of learning that placed children, and their needs, at the center of their practice. This knowledge of child development and focus on child-centeredness, to make decisions that are in the best interest of children, was a leading factor that informed their pedagogical decision-making (Daniels & Shumow, 2003; McDevitt & Ormrod, 2020).

**Knowledge of Child Development Informs Pedagogical Decision-Making**

Educators’ knowledge and understanding of child development was at the forefront of their practice and was used to inform their pedagogical decision-making. Much of the research regarding pedagogical decision-making explores how educators integrate information about their
students, subject matter, lessons, and classroom environments in order to make judgments and decisions regarding student learning (Hedges, 2012). There are a variety of factors, commonly referred to as educators’ funds of knowledge, that influence and shape the pedagogical decisions that educators make (Hedges, 2012; Karabon, 2019). Among many, these factors can include knowledge of children’s cultural experiences (Karabon, 2019), early childhood education (Hedges, 2012), as well as curriculum and pedagogical approaches (Schulman, 1986), and construct a lens through which educators make choices regarding instruction, content, and pedagogy (Gess-Newsome, 2015). Findings from the current study demonstrate how this lens of pedagogical decision-making begins with educators’ understanding and appreciation of children’s development. For example, educators in Phase Two demonstrated a deep understanding of the developmental needs of kindergarten-aged students, the unique challenges of supporting young learners, and the importance of fostering social development alongside academic learning.

Overall, the educators in Phase Two articulated how young children learn best when they are active participants in their learning, often through concrete or tactile activities. Perhaps more importantly, these educators also prioritized the crucial role that educators play in scaffolding and extending student learning within these child-centered contexts. In particular, they emphasized the role of teacher-facilitated play as a means of meeting children where they’re at, while still pushing their learning forward. This represents a significant shift in perspectives from previous literature (Fleer, 2001; Korat et al., 2002; Wood & Attfield, 2005), as the educators in Phase Two of this study were clearly advocating for adult involvement in play and described it as a developmentally appropriate practice to target and support specific learning objectives (Pyle et al., 2017). In doing so, these educators are demonstrating an important response to Pyle &
Danniels’ (2017) call that, “in order for play-based learning to be implemented effectively, teachers and researchers need shift away from the perspective that the teacher’s role is ‘to support, not to disturb’ (Pramling Samuelsson & Johansson, 2006: 48) and to avoid ‘hijacking play (Goouch, 2008, p. 95)” (Pyle & Danniels, p. 287) as they are able to make an important distinction between play-based pedagogy that is child-centered vs child-directed. They are prioritizing pedagogical choices that are child-centred, that honour children and their developmental needs, rather than emphasizes pedagogical locus of control, or the idea that play needs to be directed by children in order for it to benefit their learning (Pyle & Danniels, 2017).

Educators in Phase Two of this study used their knowledge of child development as a starting point to inform the decisions they made regarding when and how to become involved in play to support children’s math development. For example, Teacher 10 illustrated, “I think I make the decision [to enter play] by just knowing what they are capable of and seeing where the math is and seeing where I can extend it.” While educators have been using their knowledge of child development to support student learning for decades (Daniels & Shumow, 2003; McDevitt & Ormrod, 2020; Piaget, 1952; Vygotsky, 1978), this study offers new insight such that educators are using this knowledge of child development to also collaborate with children in play to support their mathematics learning.

**Next Steps Towards Supporting Educators**

As evidenced by findings from Phase Two of this study, many educators were able to successfully transition out of their comfort zone of “whole-math” and towards new territory of teacher-facilitated approaches to math in kindergarten. I argue that research needs to further advance to help more educators make this successful transition. Specifically, I call for three
explicit next steps to help more educators move beyond indirect approaches to math instruction and towards more direct math strategies.

First, further research must be conducted to gain a deeper understanding of the Subject Matter Knowledge of mathematics at the kindergarten level. Not kindergarten and the elementary grades, inclusive, but rather kindergarten specifically. In particular, additional research must be done to unpack the Specialized Content Knowledge required to teach math in kindergarten. This would involve gaining a deeper understanding of the key ideas and skills relating to both number sense and spatial reasoning in kindergarten by unpacking exactly what these skills involve and what they can look like in early years classrooms. More so, research needs to dive deeper into understanding children’s perspectives relating to math learning, including the common misconceptions they may hold about key number sense and spatial reasoning ideas, and how educators can help to correct these misconceptions. Gaining this foundational knowledge will help to build stronger connections between mathematics in kindergarten and the later elementary grades and will help to strengthen horizon knowledge available to kindergarten educators (Ball et al., 2008; Hill & Ball, 2009).

This line of research could look very similar to what Alex Lawson did when examining how children develop an understanding of addition, subtraction, multiplication, and division through her creation of a developmental trajectory of these key mathematical skills (Lawson, 2016). Throughout this work, Lawson (2016) studied young children’s development of mathematical skills over several years to gain a deeper understanding of elementary mathematics, student misconceptions, and how educators can support children along their own development of mathematical learning. While this line of research studied kindergarten and the
elementary grades, inclusive, I propose a follow up line of research that builds upon Lawson’s (2016) work by taking a deeper dive into the foundational mathematics skills of kindergarten.

Second, research needs to continue to further develop approaches and strategies for supporting math development along a continuum of guided play. Not only would this line of research respond to current calls for expanded theoretical notions of guided play (Jensen et al., 2019; Weisberg et al., 2013; 2016; Zosh et al., 2018), but it would also directly respond to the play-based learning pedagogical mandate Ontario educators currently face in practice (OME, 2016). While Chapter 5 of this dissertation lays a strong foundation for this work, conducting additional research would help to further strengthen this framework for supporting children’s math learning through collaborative play. Specifically, this line of work could involve collecting data from more educators as well as conducting classroom observations to gain a clearer understanding of what educators are successfully doing in practice to integrate math learning in guided play. Classroom based research, including the collection of classroom observations, offers crucial insights into the ways in which educators are successfully translating theory into practice (Wragg, 2011). There has been an ongoing need to develop strong connections between researchers and practicing educators (Doerr & Tinto, 2000) and current trends in research continue to demonstrate how rich and practical knowledge can be gathered, that reflects the realities of classroom practice, when we leverage classroom-based approaches to research (Bostic et al., 2021; Pyle et al., 2020).

Third, resources need to be created and shared with educators that integrate this deepened understanding of Specialized Content Knowledge for mathematics in kindergarten and enhanced approaches and strategies for implementing various approaches to guided play that range in level of child and adult collaboration. While evidence-based resources currently exist that can support
educators’ integration of play and math learning in kindergarten (Moss et al., 2016; Trent Math Education Research Collaborative, 2023; The Robertson Program for Inquiry-Based Teaching in Mathematics and Science, 2023), additional resources can be created to further strengthen the support provided to educators. Specifically, additional resources that develop a comprehensive, systematic, and sequential approach to integrating critical foundational math skills with a variety of play-based approaches are needed and would provide further support to kindergarten educators and their students. In the development of these resources, it would be essential to keep in mind the realities of classroom practice educators must navigate, including structural constraints like class size, resource availability, and timetable scheduling, alongside conceptual challenges such as educators’ various conceptualizations of play as well as educators’ comfort, knowledge, and experience teaching math. Importantly, the development of resources must be mindful of a push and pull between specialized knowledge and resources that are exclusive to kindergarten education, while still keeping an eye to the horizon knowledge required for mathematical learning in future grades. Gaining a deepened understanding of specialized knowledge for kindergarten alongside horizon knowledge of mathematics in the elementary grades are not contradictory, but rather, complimentary goals, as strengthening knowledge of what kindergarten math skills are critical to future learning as well as how to best support these critical foundational skills will help to build stronger connections between early years and elementary education. An initial step that can immediately be taken is to translate the findings from Chapter 5 of this dissertation into an interim resource guide for educators, which can be further strengthened with future research as noted above. Findings from this chapter are among the first, to my knowledge, to provide evidenced-based approaches, strategies, and examples of supporting math development through a range of guided play activities.
Ultimately, to achieve the three next steps outlined above, follow up studies could be conducted to gain a deeper understanding of the Mathematics Knowledge for Teaching in kindergarten, with particular emphasis on connecting components of Subject Matter Knowledge and Pedagogical Content Knowledge through collaborative approaches to play. This future work could use a combination of educator interviews, classroom observations, student interviews, and student math tasks/measures to gain a deeper understanding of the mathematics knowledge required for teaching within a kindergarten classroom. Importantly, a continued partnership between researchers and practicing educators is crucial in order to facilitate these next steps.

Limitations

While this study offers new insights into both theory and practice, it is not without limitation. To begin, the way in which participants were recruited for this study may pose a limit upon the data that were collected. Specifically, participants were recruited through a link to an online survey. Due to the anonymous nature of online surveys, it is impossible to verify whether participants who responded to the survey were in fact practicing educators. To address this potential limitation, participants were asked to complete an educator declaration, to indicate their certification and experience as an educator. More so, as participants were asked detailed questions about their classroom practice it became evident if respondents were not practicing educators as their responses failed to answer the survey questions, or if they were Twitter bots that produced nonsensical responses. These types of responses were deleted from the data set, as described in Chapter 3.

More specifically, a large number of participants were likely recruited from a conference on mathematics education in early years education, where I presented and promoted participation in my study. As some participants in my study also attended my presentation, it is possible that
my talk, where I discussed the importance of guided play in math development, may have influenced their responses to both Phase One and Phase Two of this study. Participant, or response bias, has been described as a common limitation with interview data in that participants can sometimes feel pressure to “give the right answer” or tell the researcher “what they want to hear” (Creswell & Poth, 2018; Patton, 2014). This can be particularly relevant in studies of professional practice, as participants may feel pressure to achieve a certain professional standard, or maintain a particular professional image, and unfortunately in some cases may fear professional repercussions for unsatisfactory responses (Patton, 2014). In the current study, participants who attended my talk may have felt pressure to “give the right answer” as I had already made my position clear regarding the importance of collaborative approaches to play and math learning in kindergarten. The purpose of this study, however, was to better understand how educators were successfully integrating approaches to play and math in kindergarten, so even if my presentation influenced participants’ responses, it is possible it helped to frame their responses in a way that actually helped to answer the research question. More so, the in-depth interview was extensive, and it would take a great deal of commitment for a participant to fake their way through many questions, and follow-up questions, in order to “tell me what I wanted to hear”. Lastly, and perhaps most convincingly, 82% of responses to the Phase One data did not tell me what I wanted to hear. The majority of educators described child-led approaches to play that did not reflect any level of educator involvement. Only a minority of educators described approaches to play that aligned with my second research question.

Second, this study did not collect observations from educators’ classroom practice, and therefore was not able to capture the richness that can be documented through video recordings, photos, and field notes. Collecting classroom observations could have helped to paint a clearer
picture of the approaches and strategies that educators used to integrate play and math learning. Regardless, the semi-structured interviews were quite in-depth, as participants were asked many times to share different examples from their practice. In particular, participants were also invited to bring artifacts from their practice to the interview, as a means of generating a connection to their practice. This technique borrowed from the literature of photo-elicitation interviewing strategies as a way of helping educators connect their interview responses to particular classroom practices (Alonzo & Kim, 2016). More so, extensive research points to the strong connections between educators’ perspectives and their practices, in that the decisions educators make regarding how they teach and support their students is accurately reflective of their presently held perspectives of teaching and learning (Snider & Roehl, 2007). The connections between educators’ perspectives and their practice of play are particularly well-documented, as previous research has repeatedly demonstrated how teachers who hold traditional perspectives of play implement only free play in their classrooms, while educators who describe contemporary notions of play enact a continuum of child and educator-led approaches to play-based learning (Fesseha & Pyle, 2016; Pyle et al., 2017).

Third this study failed to capture the perspectives of children regarding their experiences learning math through play. Given that guided play involves the collaboration of both educators and children, this study may feel a bit one-sided in that it only documented the educators’ perspectives and role towards integrating play and math. While understanding children’s perspectives was not a stated goal of the present study, gaining this knowledge may have helped to gain a deeper understanding of the various approaches and strategies for collaborating with children in play to support math development. Regardless, gaining a deeper understanding of
children’s experiences with play and math in kindergarten could pose a promising direction for future research.

Fourth, I think it is important to acknowledge the sample size of this study, particularly in Phase Two. I do not wish to label this as a limitation, but rather discuss it as a reality of the qualitative research I conducted. I conducted in-depth interviews with 10 educators who were successfully collaborating with children in play to gain a deeper understanding of the approaches and strategies they used to integrate guided play and math learning in their kindergarten classrooms. With a sample size of 10 participants, it would be inappropriate to make generalizations about their practices to a larger population of educators and it would also be inappropriate to make assertions about the effectiveness of guided play compared to other pedagogical approaches. While answers to these types of questions would certainly contribute to the literature, that was not the purpose of this study, but do pose a promising direction for future inquiry. What this study intended, and accomplished, was to gain a deeper understanding of the lived experiences of kindergarten educators who are finding success in their practice integrating collaborative play and math learning. By conducting a qualitative inquiry, I was able to learn “what works” in their classrooms and paint a rich picture of their evolving relationship with play and math as well as the approaches and strategies they used to successfully collaborate with children in play to support early math learning. These key findings can now be shared with other educators to support them in their practice. By no means do the approaches and strategies for guiding play identified in this study represent an exhaustive list. They are a starting point, as there is certainly still much more to learn.

Lastly, an overarching limitation of this study is the general climate of disagreement that the fields of both play and math are situated within, which becomes even more contentious when
we look at the intersection of these fields. As discussed, the study of play has a long history with deep divides regarding the definition and role of play in children’s learning, just as math education has distinct camps regarding pedagogy and instruction. Math education even has disagreements defining “foundational” math skills, with different research teams emphasising different concepts, which often focus more on numeracy skills while disregarding, or underrepresenting foundational spatial reasoning skills (Baroody et al., 2006; Clements & Sarama, 2014; Gelman & Gallistel, 1986; Ginsburg, 2006). This conflicting and inconsistent information permeates these fields of knowledge, which leaves educators with a messy landscape to navigate in practice.

The political climate in which this study took place adds additional obstacles for Ontario teachers to navigate, specifically with respect to mathematics education and curriculum. In particular, the current government introduced a new elementary math curriculum (Gr. 1-8), in the middle of the unprecedented challenges imposed by the Coronavirus pandemic, with little to no support for implementation (Wong, 2020). While this new curriculum does not apply to kindergarten, directly, it does influence the pressures kindergarten teachers face to prepare children for academic learning in grade one without adding any clarity as to what foundational skills in kindergarten can best prepare for this transition. More so, the rollout of this curriculum demonstrated insensitivity to the needs of teachers, including both the challenges they faced teaching during a pandemic as well as the challenges they experience teaching the subject of mathematics, adding further stress to an already contentious field (Wong, 2020). Helping educators to navigate this messy landscape, by beginning to add clarity to the field of early years math education is a clear next step towards supporting educators in practice.

Conclusion
In conclusion, this study sought to understand how kindergarten educators were successfully implementing guided play in practice to support students’ early math learning. Through qualitative inquiry, results of this study found that a large majority of participants engaged in more indirect approaches to math learning that did not reflect high levels of adult involvement, whereas a minority of participants described more intentional approaches to math learning through facilitating a range of play-based learning alongside direct instruction. This minority of participating teachers described an evolution in their practice of both play and math pedagogy, which were integrated through collaborative approaches to play. This evolution in their perspectives led educators towards implementing a range of collaborative approaches to play and math in their classrooms. These educators were successfully able to shift from a more hands-off or “whole math” approach to a more hands-on, or teacher-facilitation of math learning by expanding their notions of play, improving their relationship with math, strengthening their mathematics knowledge for teaching, strengthening their knowledge of child development, which they used to inform their pedagogical decision-making. This study calls for a need to support more educators in successfully transitioning away from these more indirect approaches towards more teacher-facilitated approaches to math instruction in kindergarten. Next steps towards supporting more educators involves deepening our understanding of Subject Matter Knowledge in kindergarten, continuing to develop a framework for the approaches and strategies for implementing guided play, and creating and sharing systematic and evidence-based resources with educators that integrate this deepened knowledge that is required for teaching mathematics in kindergarten. These next steps would help to strengthen the mathematics education in kindergarten, which in turn, would help to provide more children with a strong foundation of math knowledge that will help to set them up for future success and opportunity.
References


British Columbia Ministry of Education. (2019). *Play Today*. Province of BC.


Clements, D. H. (1984). Training effects on the development and generalization of Piagetian logical operations and knowledge of number. *Journal of Educational Psychology*, 76,


Ganley, C. M., Schoen, R. C., LaVenia, M., & Tazaz, A. M. (2019). The construct validation of

doi:10.1177/2332858419839702


doi:10.1080/02568540509595071


in Childhood Education, 23(3), 367–381. doi: 10.1080/02568540909594667


Honourable Margaret Norrie McCain. (2020). Early years study 4: Thriving kids, thriving society. Margaret and Wallace McCain Family Foundation Inc.

Howard, J. (2010). Early years practitioners’ perceptions of play: An exploration of theoretical understanding, planning and involvement, confidence and barriers to practice.


Klahr, D. (2009). “To everything there is a season, and a time to every purpose under the heavens”: What about direct instruction? In S. Tobias & T. M. Duffy (Eds.), *Constructivist theory applied to instruction: Success or failure?* (pp. 291–310). Taylor & Francis.


NAEYC. (2009). *Developmentally appropriate practice in early childhood programs serving children from birth through age 8. A position statement of the National Association for*


Russell, J. L. (2011). From child’s garden to academic press: The role of shifting institutional


http://works.bepress.com/genan_anderson/18/


*Psychology in the Schools, 44*(8), 873–886. doi:10.1002/pits.20272


doi: 10.1016/j.ecresq.2004.10.010

https://www.cdhowe.org/sites/default/files/attachments/research_papers/mixed/commentary_427.pdf


doi:10.1046/j.1440-1630.2003.00362.x


doi:10.1080/09669760.2010.521299


doi:10.1111/mbe.12015

doi:10.1177/0031721715583955


**Figure 1.** Pyle and Danniels’ (2017) Continuum of Play-Based Learning

**Note:** Pyle & Danniels’ (2017) continuum of play-based learning presents the construct of play as a continuum, ranging in level of child and adult control or facilitation of the play. While the continuum outlines five types of play, they are commonly condensed into three broader types of play: *free play*, play that is entirely directed by a child, *guided play*, play that is mutually directed by child and educator, and *teacher-directed play*, play that is entirely directed by the educator.
Figure 2. Ball et al.’s, (2008) Mathematics Knowledge for Teaching Framework

Note: Ball et al.,’s (2008) Mathematics Knowledge for teaching framework outlines six domains of knowledge teachers need to teach mathematics. These domains are grouped into two overarching categories, Subject Matter Knowledge; the knowledge of mathematics as an academic subject, and Pedagogical Content Knowledge; knowledge of how to deliver mathematics instruction.

Subject Matter Knowledge is broken down into Common Content Knowledge; general mathematical skill that can be used in any context, not specific to teaching, Specialized Content Knowledge, mathematical skill that is unique to teaching, and Horizon Knowledge, an awareness of how mathematical concepts are related and span throughout a curriculum.

Pedagogical Content Knowledge is also broken down into Knowledge of Content and Students; an ability to anticipate what students are likely to think, what they may find confusing, and what might be of interest or relevance to them when delivering math lessons, Knowledge of Content and Teaching; the ability to design, sequence, and deliver mathematical content, and Knowledge of Content and Curriculum; a teachers’ ability to connect their understanding of mathematics with their knowledge of the curriculum.
Figure 3. Stages of Educators’ Evolution and Integration of Play and Math Perspectives
Figure 4. A Continuum of Guided Play to Support Early Math Learning
Table 1. The Principles of Counting and Quantity

<table>
<thead>
<tr>
<th>Principle</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stable Order</td>
<td>There is a consistent set of counting words, that are said in a consistent order, and never changes.</td>
<td>We count by saying: 1, 2, 3, 4, 5... etc.</td>
</tr>
<tr>
<td>One-to-One</td>
<td>There is one and only one number said for each object in a set.</td>
<td>![1 2 3 4 5]</td>
</tr>
<tr>
<td>Correspondence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cardinality</td>
<td>The last number spoken represents the quantity for the entire set.</td>
<td>![5 circles represented as 5]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>There are 5 circles in this set. 5 represents the quantity for the entire set.</td>
</tr>
<tr>
<td>Abstraction</td>
<td>It does not matter what you count, the process for counting remains the same, whether objects are tangible or not.</td>
<td>We can count the 5 circles above (physically present). We can also count the number of planets in our solar system, even though they are not physically present/tangible.</td>
</tr>
<tr>
<td>Order Irrelevance</td>
<td>It does not matter in which order you count; the number in the set does not change.</td>
<td>We can count the circles from left to right. ![5 circles represented as 5]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Or from right to left.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The value in the set (5) doesn’t change.</td>
</tr>
<tr>
<td>Conservation</td>
<td>The count for a set of objects remains the same, no matter if the objects are spread out or close together.</td>
<td>![1 2 3 4 5]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>![1 2 3 4 5]</td>
</tr>
<tr>
<td>Subitizing</td>
<td>The ability to “see” a small amount of objects and instantly know how</td>
<td></td>
</tr>
<tr>
<td><strong>Hierarchical Inclusion</strong></td>
<td>All numbers preceding a given number are systematically included in the value of that number.</td>
<td></td>
</tr>
<tr>
<td>---------------------------</td>
<td>------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>Movement is Magnitude</strong></td>
<td>As you move up the counting sequence (or forwards) the quantity increases, and as you move down (backwards) the quantity decreases</td>
<td></td>
</tr>
<tr>
<td><strong>Unitizing</strong></td>
<td>The ability to count a large group of items by decomposing them into smaller, equal groups and then counting those.</td>
<td></td>
</tr>
</tbody>
</table>

There are 6 dots on this die. You can know that by looking and instantly “seeing” 6.

The numbers 1, 2, 3, and 4 are included in the number 5

10 can be unitized into 5 groups of two. And then the 5 units of two can be counted to make 10. (i.e., 2, 4, 6, 8, 10)

### Table 2. Phase One Participant Demographic Information

<table>
<thead>
<tr>
<th>Total Number of Participants</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>Canada</td>
</tr>
<tr>
<td></td>
<td>88% Ontario</td>
</tr>
<tr>
<td></td>
<td>4% Newfoundland</td>
</tr>
<tr>
<td></td>
<td>2% British Columbia</td>
</tr>
<tr>
<td></td>
<td>2% Alberta</td>
</tr>
<tr>
<td></td>
<td>2% New Brunswick</td>
</tr>
<tr>
<td></td>
<td>2% Prince Edward Island</td>
</tr>
<tr>
<td>Years of Teaching Experience</td>
<td>Min: 1 year</td>
</tr>
<tr>
<td></td>
<td>Max: 40 years</td>
</tr>
<tr>
<td></td>
<td>Mean: 14.10 years</td>
</tr>
<tr>
<td>Years Teaching Kindergarten</td>
<td>Min: 1 year</td>
</tr>
<tr>
<td></td>
<td>Max: 24 years</td>
</tr>
<tr>
<td></td>
<td>Mean: 7.80 years</td>
</tr>
<tr>
<td>Highest Level of Education</td>
<td>College: 14% of participants</td>
</tr>
<tr>
<td></td>
<td>Undergraduate: 62% of participants</td>
</tr>
<tr>
<td></td>
<td>Graduate: 24%</td>
</tr>
</tbody>
</table>
### Table 3. Phase 2 Participant Demographic Information

<table>
<thead>
<tr>
<th>Location</th>
<th>Teacher 1</th>
<th>Teacher 2</th>
<th>Teacher 3</th>
<th>Teacher 4</th>
<th>Teacher 5</th>
<th>Teacher 6</th>
<th>Teacher 7</th>
<th>Teacher 8</th>
<th>Teacher 9</th>
<th>Teacher 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ontario</td>
<td>Alberta</td>
<td>Ontario</td>
<td>Ontario</td>
<td>Ontario</td>
<td>Ontario</td>
<td>Ontario</td>
<td>Ontario</td>
<td>Ontario</td>
<td>Ontario</td>
<td>BC</td>
</tr>
<tr>
<td>Current Role</td>
<td>K</td>
<td>K Consult.</td>
<td>Primary LTO</td>
<td>K</td>
<td>Virtual French K</td>
<td>K</td>
<td>K Consult.</td>
<td>K resource teacher</td>
<td>K</td>
<td></td>
</tr>
<tr>
<td>Previous Roles</td>
<td>ECE, SK/1, 1, 2</td>
<td>FDK, HDK, GR½, SERT</td>
<td>Outdoor Ed, Tutor, Occasional Teacher, FDK, Summer School</td>
<td>Junior LD, Intermediate LD, HSP, K Diagnostic, Primary Behaviour, HDK, FDK</td>
<td>French K</td>
<td>1, 2, 3, 1/2/3, K K Consult., K AQ, K/Early Learning Course at Brock</td>
<td>K, ½, 5</td>
<td>Camp Counsellor, ECE, Teacher</td>
<td>Grades 1-8 Homeroom, Planning Time, Teacher-Librarian, Spec ed</td>
<td>Grade 7 Math Teacher</td>
</tr>
<tr>
<td>Years in Ed</td>
<td>40</td>
<td>8</td>
<td>2</td>
<td>21</td>
<td>3</td>
<td>21</td>
<td>15</td>
<td>13</td>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>Years in K</td>
<td>28</td>
<td>6</td>
<td>1</td>
<td>13</td>
<td>3</td>
<td>12</td>
<td>4</td>
<td>8</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Certifications</td>
<td>ECE OCT</td>
<td>OCT</td>
<td>ECE OCT</td>
<td>OCT</td>
<td>ECE OCT</td>
<td>OCT</td>
<td>OCT</td>
<td>ECE OCT</td>
<td>OCT</td>
<td>OCT</td>
</tr>
<tr>
<td>Education</td>
<td>Bachelor</td>
<td>Masters</td>
<td>Masters</td>
<td>Bachelor</td>
<td>Masters</td>
<td>Masters</td>
<td>Masters</td>
<td>Bachelor</td>
<td>Bachelor</td>
<td>Masters</td>
</tr>
<tr>
<td>Math Content Knowledge</td>
<td>Undergrad (stats)</td>
<td>Masters (Research in math)</td>
<td>Masters (stats)</td>
<td>Highschool Undergrad (stats)</td>
<td>Highschool Masters</td>
<td>Masters Undergrad</td>
<td>Undergrad</td>
<td>Undergrad Masters</td>
<td>Undergrad Masters</td>
<td>Undergrad Masters</td>
</tr>
<tr>
<td>Play Training</td>
<td>Yes (ECE, PD, AQs)</td>
<td>Yes (OCT, PD, AQs)</td>
<td>Yes (ECE, OCT, PD, AQ)</td>
<td>No</td>
<td>Yes (ECE, AQ)</td>
<td>Yes (PD)</td>
<td>No</td>
<td>Yes (ECE)</td>
<td>Yes (AQ)</td>
<td>Yes (AQ)</td>
</tr>
<tr>
<td>Phase</td>
<td>Description of Process</td>
<td>Example from Study</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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</tr>
</tbody>
</table>
| 1. Familiarization  | Conducted interviews, transcribed data, read, and re-read transcripts, journaled ideas, impressions, and questions. | Excerpts from Journal:  
Teacher 1  
- Gets involved in play. Discusses a more child-directed approach to guided play  
- Discusses a range of instructional approaches for math – describes a need for direct instruction, but also describes a continuum of play-based approaches (free, guided, and TDP).  
- Very positive about learning – emphasizes that it should be fun and enjoyable for children  
Teacher 7  
- Bad experience learning math as a student, enjoys teaching math now.  
- Realized a new way of teaching math – moved beyond their own personal experience. I think there’s something here with the teachers being able to untangle their experiences from how they teach math – moving beyond how they were taught (repeating the familiar) and embracing new pedagogical approaches  
- Teacher math content knowledge is crucial to become involved in play and extend towards math goals  
- This teacher discusses a lot of professional growth, both in her implementation of play and understanding of math content knowledge. |
| 2. Generate Initial Codes | Open coded the entire data set, generated list of initial codes, collated data excerpts relevant to each code. | Generated 273 initial codes.  
Example of Codes:  
artifacts from practice, assessment, children’s interests, collaborative play, challenges, dot talks, ECE OCT relation, embedded math, free play, games, hands-on learning, joy/fun in learning, manipulatives, play practice evolved, professional development, math as a student, math as a teacher, math attitude, math content knowledge, math resources, outdoor play, planning for play, play in older grades, reflective practice, relationship building, teacher, training program, virtual learning,  

3. Searching for Themes
Collated similar codes into categories or candidate themes. Generated thematic map.
Grouped initial codes into 26 candidate themes/categories.
Categories:
- adult role in play, artifacts from practice, collaborating in play, covid, curriculum, educator stance, examples of play, experience with math, implementing math, implementing play, integrating play and math, manipulatives, materials/classroom environment, math content knowledge, math instruction, math skills, online learning, outdoor play, perspective of play, planning, planning for math, planning for play, play in older grades, professional development, resources, types of play

4. Reviewing Themes
Level 1: Checked if candidate themes worked in relation to the coded data extracts.
Collapsed candidate themes/categories together, broke some apart into separate themes, let some go because there wasn’t enough data to support a theme. Generated 9 revised candidate themes/categories.
Revised candidate themes:
- collaborating in play, educator experience and stance, educator resources, manipulatives, math instruction, mathematics content knowledge, planning, play implementation, play perspectives

Level 2: Checked if candidate themes worked in relation to the entire data set.
Highly iterative stage. Interpretation of codes moved from semantic to latent. It became apparent that the overall story was about educators’ evolution in perspectives and practice. Throughout this iterative process, I began to collapse more themes together and leave some out as they didn’t fit into the overall narrative of professional evolution. I settled upon four themes that fit together to tell a story of evolution of play and math perspectives.

4 stages of evolution:
1. Compartmentalize: Separated experience as a student from teaching practice
2. Learn: Sought out professional knowledge
3. Apply: Play brought this professional learning to life
4. Personalize: Flexibility to adapt professional learning to their classroom

At this phase, I thought teachers progressed through each phase, having an integrated view of play and math. This was further refined in step 5.
### 5. Defining and Naming Themes

Continued to refine the specifics of each theme alongside the overall story the analysis tells. Generated clear description and name for each theme.

In this step, I made three fundamental revisions to the themes.

First, I realized that the teachers were actually initially describing perspectives of play and math as separate constructs and it wasn’t until later in their evolution that they actually began to integrate these two constructs into a singular and unified notion. I revised the names and definitions of each theme/phase in evolution.

**Revised stages of evolution:**

1. **Reflect** (upon practice, recognize a need to improve approaches to play and math)
2. **Identify** (barriers towards implementing play and math)
3. **Learn** (about modern notions of play and math instruction in the early years to overcome these barriers)
4. **Actualize** (this learning through collaborative approaches to play)

Second, I began to identify and describe the connections between each phase.

Third, I accepted a linear relation between each theme. I thought there may be a cyclical relation, but I had to accept that I did not have enough data to support this interpretation.

### 6. Producing the Report

Selected extract examples that related back to the analysis, the research question and the literature. Produced report of the analysis.

The report is presented in Chapters 4 and 5.
Appendix A: Approved Ethics Letter

Dear Ms Hanna Wickstrom:

Re: Your research protocol application entitled, “Kindergarten Educator Perspectives on Mathematics and Play-Based Learning”

The Social Sciences, Humanities & Education REB has conducted a Delegated review of your application and has granted approval to the attached protocol for the period 2021-02-24 to 2022-02-23.

Please note that this approval only applies to the use of human participants. Other approvals may be needed.

Please be reminded of the following points:

• An Amendment must be submitted to the REB for any proposed changes to the approved protocol. The amended protocol must be reviewed and approved by the REB prior to implementation of the changes.

• An annual Renewal must be submitted for ongoing research. Renewals should be submitted between 15 and 30 days prior to the current expiry date.

• A Protocol Deviation Report (PDR) should be submitted when there is any departure from the REB-approved ethics review application form that has occurred without prior approval from the REB (e.g., changes to the study procedures, consent process, data protection measures). The submission of this form does not necessarily indicate wrong-doing; however follow-up procedures may be required.

• An Adverse Events Report (AER) must be submitted when adverse or unanticipated events occur to participants in the course of the research process.

• A Protocol Completion Report (PCR) is required when research using the protocol has been completed. For ongoing research, a PCR on the protocol will be required after 7 years, (Original and 6 Renewals). A continuation of work beyond 7 years will require the creation of a new protocol.

• If your research is funded by a third party, please contact the assigned Research Funding Officer in Research Services to ensure that your funds are released.

Best wishes for the successful completion of your research.
Appendix B: Phase One Consent Letter of Information

Dr. Eric Jackman Institute of Child Study
UNIVERSITY OF TORONTO

Consent Letter

You are invited to participate in a research study conducted by Hanna Wickstrom, a graduate student at the University of Toronto. This research is being conducted under the supervision of Dr. Angela Pyle, a faculty member of the University of Toronto. The purpose of this research is to examine current and former kindergarten educators’ perspectives regarding play and mathematics development in kindergarten.

If you decide to participate, you will be asked to complete a 10-minute online survey. This study uses an online survey hosting program (REDCap), which is maintained by the University of Toronto, Ontario Institute for Studies in Education. Survey questions focus on your teaching experience, education and training, your views on the role of play in kindergarten, and your views on teaching mathematics in kindergarten. At the end of the survey, you will be invited to share your email address if you wish to be contacted for a follow up virtual interview. If you provide your email address, your survey responses will be de-identified and confidential. If you do not provide your email address, your survey responses will be anonymous as no other personally identifiable information will be collected.

Participation in this study is completely voluntary. You may decline to answer any questions and can withdraw your participation at any time without consequence. To withdraw, simply close your web browser at any time. If you decide to withdraw participation after providing your email address, your survey responses will be retrieved and deleted. If you do not provide your email address than it will not be possible to retrieve and delete your data once the survey responses have been submitted.

Your responses will only be accessible to myself and Dr. Pyle. However, for quality assurance purposes the data may be accessed by a member of the University of Toronto’s Human Research Ethics Program (HREP). The HREP members are bound to exercise the same level of confidentiality as the researcher.

The data collected from this study will be de-identified and stored on a secured and password-protected online platform, maintained by the University of Toronto. The data will be electronically archived after completion of the study, maintained for a minimum of five years, and then deleted and purged. The results of this study may be shared in publications, academic conference presentations, and educational presentations for pre-service and in-service teachers. No personally identifiable information will ever be shared in the results.
If you have any questions about the study, please contact Hanna Wickstrom at hanna.wickstrom@mail.utoronto.ca or 416-702-7208, or contact Dr. Angela Pyle at angela.pyle@utoronto.ca.

This study has been approved by the Research Ethics Boards of the University of Toronto. If you have any questions related to your rights as a participant in this study, please contact the Ethics Review Office at the University of Toronto at 416-946-3272 or, ethics.review@utoronto.ca.

Thank you for considering participation in this study.

Sincerely,
Hanna Wickstrom
PhD Candidate, University of Toronto, OISE

I have read the consent letter and consent to participate in the current study.

Yes (Button)

No (Button)
Appendix C: Phase One Survey Questions

1. Where do you work as a kindergarten educator? Either currently or previously. (Example: Ontario, Canada)
2. What is your current teaching assignment/job?
3. What teaching job(s) have you completed in the past? Please include all jobs that pertain to the field of education.
4. Including the current year, how many years have you worked as an educator?
5. Including the current year, how many years have you worked in kindergarten?
6. What are your certifications? Check all that apply.
   An Early Childhood Educator (ECE)
   A Teacher
   Other ____________
7. What is your highest level of education? Either completed or in progress.
   College Diploma
   Bachelor’s Degree
   Master’s Degree
   Doctoral Degree
   Other ____________
8. What is your highest level of educational coursework pertaining to mathematics content knowledge? Either completed or in progress.
   High school Coursework
   College Coursework
   Undergraduate Coursework
   Master’s Coursework
   Doctoral Coursework
   Other ____________
9. Where have you taken educational coursework pertaining to the instruction of mathematics. Either completed or in progress. Check all that apply.
   College Coursework
   Undergraduate Coursework
   Master’s Coursework
   Doctoral Coursework
   Educator Training Program
   Professional Development and Workshops
   Additional Qualifications
   Other: ____________________________
10. Have you received formal training regarding play in kindergarten?
    Yes
    No
11. If yes, please specify where you received this training. Check all that apply.
    ECE Training Program
    Teacher training program
    Professional Development
    Additional Qualification
    Other: __________________________________________
12. How would you describe your comfort level when teaching math to students?
13. In your professional opinion, how do children best learn math in kindergarten?
14. In your professional opinion, what is the purpose of play in kindergarten?
15. As an educator, what is your role in play-based learning?
16. How might play be used to support children’s math learning in kindergarten?
17. What challenges, if any, might you face implementing play to support children’s math learning in kindergarten?
18. If you’re interested in participating in a follow-up Zoom interview, please provide your email address. ________________________________
Appendix D: Phase Two Consent Letter of Information

Consent Letter

You are invited to participate in a research study conducted by Hanna Wickstrom, a graduate student at the University of Toronto. This research is being conducted under the supervision of Dr. Angela Pyle, a faculty member of the University of Toronto. The purpose of this research is to examine current and former kindergarten educators’ perspectives regarding play and mathematics development in kindergarten.

If you decide to participate, you will be asked to complete a virtual interview, hosted over Zoom. This interview will last approximately one hour in length and will include questions that focus on your approaches to mathematics instruction as well as your approaches to play-based learning in your kindergarten classroom. You will be invited, but not required, to bring artifacts from your classroom, such as examples of math activities or formal/informal plans/notes, to help explain your approaches to mathematics instruction. These artifacts can include anything that is readily available and should not require any preparation prior to the interview. This interview will be recorded over Zoom. Audio and video data will be recorded during the interview and Zoom will generate audio and video files of this recording. Only the audio file of the interview will be saved. The video file of the interview will be deleted and purged. The audio file will be transcribed verbatim and no personally identifiable information will be recorded in the verbatim transcript.

Participation in this study is completely voluntary. You may decline to answer any questions and can withdraw your participation at any time without consequence. To withdraw, you may simply tell me that you no longer wish to continue with the interview. I will then delete and purge your interview data from the study.

Your responses will only be accessible to myself and Dr. Pyle. However, for quality assurance purposes the data may be accessed by a member of the University of Toronto’s Human Research Ethics Program (HREP). The HREP members are bound to exercise the same level of confidentiality as the researcher.

The data collected from this study will be de-identified and stored on a secured and password-protected online platform, maintained by the University of Toronto. The data will be electronically archived after completion of the study, maintained for a minimum of five years, and then deleted and purged. The results of this study may be shared in publications, academic conference presentations, and educational presentations for pre-service and in-service teachers. No personally identifiable information will ever be shared in the results.
If you have any questions about the study, please contact Hanna Wickstrom at hanna.wickstrom@mail.utoronto.ca or 416-702-7208, or contact Dr. Angela Pyle at angela.pyle@utoronto.ca.

This study has been approved by the Research Ethics Boards of the University of Toronto. If you have any questions related to your rights as a participant in this study, please contact the Ethics Review Office at the University of Toronto at 416-946-3272 or, ethics.review@utoronto.ca.

Thank you for considering participation in this study.

Sincerely,
Hanna Wickstrom
PhD Candidate, University of Toronto, OISE

I have read the consent letter and consent to participate in the current study.

Participant Name (Please Print): ________________________________

Participant Signature: ________________________________

Date: ________________________________
Appendix E: Phase Two Interview Questions

These questions were asked to all interview participants.

1. Can you tell me about your experience learning math as a student?
2. Can you describe approaches to math instruction in your kindergarten classroom?
3. Can you describe approaches to play-based learning in your kindergarten classroom?
4. Can you describe how you might collaborate with children in the context of play?
5. Can you describe an example of how you collaborate with children in play to support their math learning?
6. Is there anything else you’d like to share with me about play and children’s math learning?

Example Follow-Up Questions

1. Can you walk me through the artifacts/examples of math activities you brought to our interview?
2. Can you walk me through some of your approaches towards planning for math in the context of play?
3. Can you tell me about where you get your ideas for supporting math development in play?