INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

Bell & Howell Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI®
FIXED-FREQUENCY BEAM-STEERABLE LEAKY-WAVE ANTENNAS

by

Karam M. Noujeim

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Department of Electrical and Computer Engineering
University of Toronto

© Copyright by Karam Michel Noujeim 1998
The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author’s permission.

L’auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L’auteur conserve la propriété du droit d’auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-45831-8
# Table of Contents


ACKNOWLEDGMENTS ............................................................................................................ vii

Chapter 1
**INTRODUCTION**
1.1 Background .................................................................................................................... 1
1.2 Traveling-Wave Antennas .............................................................................................. 2
1.3 Survey of the Literature ................................................................................................ 3
1.4 Thesis Outline ................................................................................................................ 7

Chapter 2
**FIXED-FREQUENCY BEAM-STEERABLE LEAKY-WAVE MICROSTRIP ANTENNA ANALYSIS**
2.1 Higher-order modes in microstrip ................................................................................ 9
2.2 Leaky-wave microstrip antenna ..................................................................................... 11
2.3 The Leaky-Wave Microstrip Antenna as a Periodic Structure ..................................... 12
   2.3.1 Network description of a microstrip cell ................................................................. 15
   2.3.2 Periodic-structure analysis of a chain of two-port networks .................................. 16
2.4 Four-Port Impedance Matrix of a Microstrip Cell ......................................................... 19
   2.4.1 Four-port impedance matrix of the microstrip transmission-line pair .................. 20
   2.4.2 Four-port impedance matrix of the microstrip patch ............................................ 22
   2.4.3 Four-port impedance matrix of a microstrip cell as a cascade of three networks ...26
2.5 Power Analysis of a Periodically Loaded Leaky-Wave Microstrip Antenna ............... 28
2.6 The H-Plane Power Pattern of the Leaky-Wave Microstrip Antenna ......................... 30
   2.6.1 The leaky-wave microstrip antenna as a line source .............................................. 31
   2.6.2 The leaky-wave microstrip antenna as a nonuniformly excited linear array ..........33
2.7 Summary ....................................................................................................................... 35

Chapter 3
**FIXED-FREQUENCY BEAM-STEERABLE LEAKY-WAVE MICROSTRIP ANTENNA DESIGN**
3.1 Introduction .................................................................................................................... 36
3.2 Design Guidelines for a Periodically Loaded Leaky-Wave Microstrip ......................... 36
3.2.1 Dielectric thickness and cell dimensions .........................................................36
3.2.2 Length of a periodically loaded leaky-wave microstrip antenna ....................39
3.2.3 Input impedance ...............................................................................................39
3.2.4 Operating bandwidth .....................................................................................40
3.3 Periodically Loaded Leaky-Wave Microstrip Antenna at 6.25 GHz ....................40
  3.3.1 Coupling between adjacent cells .................................................................42
  3.3.2 Power gain, cross-polarization, and standing-wave ratio ...............................46
  3.3.3 Operating bandwidth .....................................................................................53
  3.3.4 Power analysis ................................................................................................54
  3.3.5 Periodic-structure analysis ...........................................................................55
  3.3.6 Interference measurements ..........................................................................57
3.4 Periodically Loaded Leaky-Wave Microstrip Antenna at 5.2 GHz ....................61
  3.4.1 Coupling between adjacent microstrip cells ..................................................63
  3.4.2 Power gain, cross-polarization, and standing-wave ratio ...............................65
  3.4.3 Operating bandwidth .....................................................................................71
  3.4.4 Power analysis ................................................................................................72
3.5 Summary .............................................................................................................73

Chapter 4
FIXED-FREQUENCY BEAM-STEERABLE LEAKY-WAVEGUIDE-ANTENNA ANALYSIS
4.1 Introduction .........................................................................................................74
4.2 Periodically Loaded Leaky-Waveguide Antenna .................................................74
  4.2.1 A transmission-line model for wave propagation in the x direction .............77
  4.2.2 The complex propagation constant of the periodically loaded leaky waveguide 81
4.3 The Effective-Dielectric-Constant (EDC) Technique ...........................................85
4.4 The Periodically Loaded Leaky-Waveguide Antenna as a Line Source .............86
4.5 Radiation Efficiency of a Periodically-Loaded Leaky-Waveguide Antenna ........88
4.6 Summary .............................................................................................................90

Chapter 5
FIXED-FREQUENCY BEAM-STEERABLE LEAKY-WAVEGUIDE-ANTENNA DESIGN
5.1 Introduction .........................................................................................................91
5.2 Design Guidelines for Periodically Loaded Leaky-Waveguide Antennas ..........91
Chapter 5

5.2.1 Array height over ground .................................................................91
5.2.2 Length of a periodically loaded leaky-waveguide antenna ...............94
5.2.3 Operating bandwidth and radiation efficiency ...................................94
5.3 Array of $\lambda_0$-Long Thin Strips Over a Grounded Styrofoam Slab ........95
5.4 Array of Varactor-Loaded Thin Strips Over a Grounded Teflon Slab ......101
  5.4.1 Power gain, cross-polarization, and standing-wave ratio ..................102
5.5 Array of Varactor-Loaded Thin Strips Over a Grounded Ceramic Slab ..107
  5.5.1 Power gain, cross-polarization, and standing-wave ratio ..................107
  5.5.2 Periodic-structure analysis ...........................................................113
  5.5.3 Interference measurements ...........................................................114
5.6 Side Radiation .....................................................................................118
5.7 Summary .............................................................................................122

Chapter 6

CONCLUSIONS

6.1 Introduction .......................................................................................123
  6.1.1 Periodically loaded leaky-wave microstrip antenna .........................123
  6.1.2 Periodically loaded leaky-waveguide antenna .................................124
6.2 Impact .................................................................................................125
6.3 Directions for Future Work .................................................................126

Appendix A .................................................................128

MICROSTRIP PARAMETERS

Appendix B .........................................................................................130

IMPEDANCE MATRIX OF AN N-PORT MICROSTRIP CIRCUIT

Appendix C .........................................................................................133

RESONANT RECTANGULAR MICROSTRIP PATCH

Appendix D .........................................................................................138

POWER ANALYSIS OF A PERIODICALLY LOADED
LEAKY-WAVE MICROSTRIP ANTENNA

REFERENCES ..................................................................................150
Fixed-Frequency Beam-Steerable Leaky-Wave Antennas


Karam M. Noujeim

Department of Electrical and Computer Engineering

University of Toronto

ABSTRACT

A detailed description of the fundamentals of operation of two fixed-frequency beam-steerable leaky-wave antennas developed during the course of this research is presented. This is accompanied by measurements used to validate the theories developed herein.

A study is presented of the wave propagation along the first of these antennas, a periodically loaded leaky-wave microstrip. The theoretical predictions linking the phase velocity along such an antenna to the values of the voltage-controlled capacitors placed at regular intervals along it are verified experimentally. It is found that the phase velocity along the antenna can be varied continuously by properly adjusting the DC voltage across its capacitors. This effect is used to achieve continuous fixed-frequency main-beam steering.

Analysis is also given of the wave propagation along the second antenna, a leaky waveguide made up of a set of parallel strips periodically loaded with voltage-controlled capacitors and printed on a grounded dielectric slab. It is found that the phase velocity along such an antenna, and thus the direction of its main-beam maximum can be controlled continuously at constant frequency by properly adjusting the DC voltage across its capacitors.

Finally, the effect of the dielectric constant on the scan range of fixed-frequency periodically loaded leaky-wave microstrip and leaky-waveguide antennas is addressed. Expressions for the radiation efficiency, power gain, radiated power, and power dissipated in the various parts of these antennas are derived. In addition, the response of these antennas to
harmonic and intermodulation interference is experimentally investigated, and is followed by a set of measurements that address the issue of radiation from the sides of leaky-waveguide antennas.
ACKNOWLEDGMENTS

I would like to express my gratitude to my thesis supervisor, Professor Keith G. Balmain, for his expert guidance and keen interest throughout the course of this research. His constant encouragement helped make this work an enjoyable experience.

I am grateful to Mr. Gerald Dubois for his indispensable help in facilitating numerous laboratory tasks, and Mr. Peter Kremer for his skillful etching of a wide variety of printed-circuit boards. I have greatly enjoyed the friendship and discussions with past and present graduate and post-doctoral students in the Electromagnetics Group. I would like to thank them all for making my stay here a wonderful one.

Finally, I would like to gratefully acknowledge the financial assistance provided by the Ontario Information Technology Research Centre, and by the NSERC/Bell Canada/Nortel Industrial Research Chair in Electromagnetics.
Chapter 1

INTRODUCTION

1.1 Background

Congestion in the lower bands of the radio-frequency spectrum, coupled with an increasing demand for broadband wireless communications systems and small highly directive antennas, have pushed the operating frequency of many emerging applications into the higher microwave and even into the millimeter-wave spectrum [1-2]. Examples of such applications include earth-satellite communications systems, automotive collision-avoidance radar and product-tracking systems in manufacturing plants, all of which involve communications with a moving target, and thus at least potentially a need for antennas capable of beam steering.

Driven mainly by a market need for low-cost, light-weight and easy-to-fabricate low-profile antennas, the microstrip patch antenna has become the radiating element of choice in many antenna applications, in spite of its narrow bandwidth and low power-handling capability [3]. Corporate-fed arrays of such patches have been developed along with phase shifters for beam steering, and have been integrated with microwave and millimeter-wave monolithic integrated circuits for use in a wide variety of military and civilian applications [4-5].

Despite their many attractive features, corporate-fed microstrip antenna arrays suffer from scan-blindness due to surface-wave-enhanced mutual coupling between array elements. In addition, conductor losses and spurious radiation from the feed and phase-shifter networks of such arrays result in low radiation efficiency [6].

A class of antennas that is well suited for operation at high frequencies is that of leaky-wave antennas. This is primarily due to the fact that in such antennas a single feed is used to launch a wave that travels along the antenna structure while leaking energy into free
space, thus eliminating the spurious-radiation and ohmic losses associated with the corporate feed of a microstrip antenna array, and consequently resulting in a higher radiation efficiency.

1.2 Traveling-Wave Antennas

This section presents a brief discussion of traveling-wave antennas so as to set the stage for a detailed presentation of the various antennas considered in this thesis. The traveling-wave antenna family includes leaky-wave and surface-wave antennas. A leaky-wave antenna radiates continuously along its length and, since it supports a traveling wave whose phase velocity \( v \) is greater than the velocity \( c \) of a plane wave in free space, it is sometimes referred to as a fast-wave structure [7]. The earliest example of a leaky-wave antenna is the slitted-wall rectangular waveguide [8] illustrated in Fig. 1-1. The main beam radiated by such an antenna emerges at an angle \( \theta = \sin^{-1}(c/v) \) away from broadside (perpendicular to the antenna axis).

![Figure 1-1. A slitted-wall rectangular-waveguide antenna [8]. Power leaks continuously through the narrow slit resulting in the emergence of a \( z \)-polarized main beam at an angle \( \theta = \sin^{-1}(c/v) \) away from broadside (the \( x \) direction).](image-url)
In contrast with a leaky-wave antenna, a surface-wave antenna is known as a slow-wave structure since the phase velocity of the wave traveling along it is less than \( c \). Moreover, surface-wave fields are bound to the surface so that radiation from such an antenna can take place only at discontinuities, curvatures, and nonuniformities [7]. In this thesis, surface-wave antennas will not be discussed any further.

Although the main-beam direction of a leaky-wave antenna scans well with frequency, scanning at a fixed frequency has so far been either impractical (e.g. use of liquid dielectric [9] or biased ferrite [10]), inefficient (only 50% efficiency at 40 GHz) [11], or it did not provide a large scan range (only 5°) [12].

A chronological review of previous work leading up to the state of the art in the area of fixed-frequency beam-steerable leaky-wave antennas will be presented in the following section. This will be followed by an outline of the research work reported herein.

### 1.3 Survey of the Literature

In 1978, Bahl and Bhartia [9] suggested using a periodic antenna structure immersed in a liquid-dielectric waveguiding medium whose permittivity may be controlled via a biasing electric field for the purpose of steering the main beam at constant frequency. Their theoretical study indicated that an approximate 50° scan range is possible with a moderate change in the permittivity of existing liquid dielectrics. They also reported that the beam width of such antennas remains practically constant in the aforementioned scan range.

Four years later, Horn et al. [12] showed that the main-beam direction \( \theta_m \) of a metal-grating silicon-waveguide antenna may be steered by varying the DC current through a set of distributed \( p-i-n \) diodes attached to one or both side walls of the silicon waveguide shown in Fig. 1-2. In their experiment, they were able to steer continuously the main-beam direction of the waveguide-fed antenna by 5° at a constant frequency of 63.8 GHz. They found that it was necessary to insert a low-relative-permittivity insulating layer between the waveguide side walls and the distributed \( p-i-n \) diodes in order to reduce the amount of RF power dissipated in the latter.
Figure 1-2. The constant-frequency beam-steerable dielectric-waveguide antenna designed by Horn et al. [12]. The direction of the main beam is steered in the \( H \)-plane (\( y-z \) plane) of the antenna by varying the DC current through the distributed \( p-i-n \) diodes attached to one or both side walls of the silicon waveguide.
In 1988, Maheri et al. [10] showed that the main-beam direction of the corrugated polycrystalline yttrium-iron-garnet (YIG) antenna illustrated in Fig. 1-3 may be steered by varying the intensity of the z-directed DC magnetic field across the YIG slab. In their experiment, they inserted the rectangular-waveguide-fed antenna into an electromagnet, and by varying the DC magnetic field up to 1.4 T at a frequency of 46.8 GHz, they were able to scan continuously the main-beam direction \( \theta_m \) by 41°. Within this scan range, they found that the half-power beam width varies between 3.2° and 3.6°. Extension of their work to higher frequencies currently awaits the development of low-loss ferrite materials.

![Diagram](image_url)

**Figure 1-3.** The constant-frequency beam-steerable corrugated-ferrite leaky-waveguide antenna designed by Maheri et al. [10]. A z-directed DC magnetic field across the YIG slab is used to steer the main beam in the H-plane (x-y plane) of the antenna.

In 1993, James et al. [13] obtained a scan range of 20° at a fixed frequency of 7.5 GHz by integrating 24 varactor diodes in the feed line of the two-layer microstrip array shown in Fig. 1-4. The feed line is a \( 7 \lambda_g \)-long microstrip transmission line (\( \lambda_g \) is the wavelength in the microstrip), and is shunted by reverse-biased varactor diodes at regular intervals along its length. A thin dielectric slab on which parallel strips of radiating elements spaced \( \lambda_g/2 \) apart are printed, is placed on top of the feed line. The radiating strips are arranged such that they
are transverse to the microstrip transmission line, and are electromagnetically coupled to it at their edges. Due to the presence of varactor diodes in shunt along the feed, this multi-layer structure does not lend itself to easy fabrication.

Figure 1-4. The constant-frequency beam-steerable microstrip array introduced by James et al. [13].

Recently, Vendik et al. [11] used the voltage-controlled permittivity of a ferrite film to steer the main-beam direction of the waveguide-fed leaky-wave antenna illustrated in Fig. 1-5. In the temperature range +5° to +45° C, they reported a scan range of about 75°, and an efficiency of 50% at a constant frequency of 40 GHz. They attributed the low efficiency to the
relatively lossy 3.5 μm-thick $\text{Ba}_{1-x}\text{Sr}_x\text{TiO}_3$ (BSTO) ferroelectric film, and indicated that the most difficult barrier to realizing such materials is the inability to produce ferroelectric layers with single-crystal electrical characteristics. In addition, their initial investigations showed that at frequencies below 100 GHz, the antenna efficiency may be improved by operating the antenna at superconducting temperatures.

![Diagram](image)

**Figure 1-5.** The constant-frequency beam-steerable BSTO-film leaky-waveguide antenna designed by Vendik et al. [11]. The applied static electric field between adjacent copper strips is used to alter the relative permittivity of the BSTO film, and thus the direction of the main-beam maximum in the $H$-plane (x-y plane) of the antenna.

### 1.4 Thesis Outline

A detailed description of the fundamentals of operation of two fixed-frequency beam-steerable leaky-wave antennas developed during the course of this research will be presented in the next five chapters. The first of these antennas, built from microstrip and periodically loaded with variable capacitors, will be referred to as a periodically loaded leaky-wave microstrip antenna. On the other hand, the second antenna is based on a parallel-plate waveguide with one of its plates replaced by a parallel-strip grid periodically loaded with variable capacitors, and will be referred to as a periodically loaded leaky-waveguide antenna.
In Chapter 2 of this thesis, an analysis will be given of the wave propagation along a periodically loaded leaky-wave microstrip antenna. Expressions for the radiation efficiency, power gain, radiated power, and power dissipated in the various parts of the antenna will be derived. It will be shown that the complex propagation constant along such a structure is a function of the variable capacitors loading it. This effect will be used to achieve constant-frequency main-beam steering.

In Chapter 3, measurements will be used to confirm the theoretical predictions linking the main-beam direction of a periodically loaded leaky-wave microstrip antenna to its variable-capacitor values. Its measured power patterns will be compared with those found from the theory developed in Chapter 2. In addition, the effect on the scan range of this antenna of using microstrip with different relative permittivity will be addressed, which will be followed by an experimental assessment of its response to harmonic and intermodulation interference.

Chapter 4 closely follows the outline of Chapter 2, first presenting an analysis of the wave propagation along a periodically loaded leaky-waveguide antenna, then linking the direction of the antenna’s main-beam maximum to the values of the variable capacitors loading it. Expressions for the radiated power and the power dissipated in the various parts of the antenna are also derived and used to calculate the radiation efficiency.

The theoretical predictions linking the main-beam direction of a leaky-waveguide antenna to the values of its variable capacitors will be verified experimentally in Chapter 5. In addition, the measured power patterns of this antenna will be compared with those obtained from theory for different dielectrics filling the waveguide. Chapter 5 concludes with an experimental assessment of the harmonic and intermodulation distortion of the antenna, followed by a set of measurements for assessing radiation from its sides.

Finally, Chapter 6 presents a summary of the research findings.
Chapter 2

FIXED-FREQUENCY BEAM-STEERABLE LEAKY-WAVE MICROSTRIP ANTENNA ANALYSIS

2.1 Higher-order modes in microstrip

Consider a conducting strip of width $a$ printed on a grounded dielectric slab of infinite extent as shown in Fig. 2-1-(a). In addition to the quasi-TEM mode, this structure supports an infinite number of quasi-TE$_{m0}$ modes ($m = 1, 2, ...$), commonly referred to as higher-order modes or simply as EH$_m$ modes (EH refers to the fact that both electric and magnetic components of the field are present in the direction of wave propagation) [14]. In these modes, the electric-field lines lie mainly in planes transverse to the direction of wave propagation $y$ (planes parallel to the $x$-$z$ plane), with magnetic-field components in these planes as well as in the direction of wave propagation.

The complex propagation constants of the higher-order modes may be found by applying the transverse-resonance technique [15] as was shown by Zaitsev and Fialkovskii [16], and as illustrated in Fig. 2-1-(b). For the first higher-order mode (EH$_1$), the strip width $a$ is of the order of $\lambda_g/2$, where $\lambda_g$ is the guide wavelength. In addition, this mode is symmetric about an electric wall located along the center of the conducting strip and perpendicular to it, with current flow on the strip mainly in the $x$ direction. A dominant-field diagram of the first higher-order microstrip mode is shown in Fig. 2-2.
Infinite long perfectly conducting strip

Infinite perfectly conducting ground plane

Figure 2-1. (a) A microstrip line of width $a$, thickness $h$, and relative permittivity $\varepsilon_r$. (b) A transmission-line circuit used to find the complex propagation constants of the higher-order modes supported by the microstrip. Here, $Z_a$ is the impedance presented by the left strip edge to a $z$-polarized TEM wave traveling in the negative $x$-direction between the conducting strip and the ground plane shown in (a), with $a \to +\infty$ [16].

Figure 2-2. The dominant field lines of the first higher-order mode of the microstrip shown in Fig. 2-1-(a).
2.2 Leaky-wave microstrip antenna

In 1979, Menzel [17] introduced a traveling-wave microstrip antenna based on the first higher-order mode (EH$_1$). The antenna was asymmetrically fed by means of a microstrip line as shown in Fig. 2-3, and transverse slots located along the center line of the antenna were used to suppress the fundamental mode. Using a quarter-wave transformer, the input impedance of the antenna was matched to the characteristic impedance of the microstrip feed line. The antenna radiated an $x$-polarized main beam at an angle $\theta = 37.5^\circ$ away from broadside (the $z$ direction). It exhibited an impedance bandwidth broader than that of the resonant microstrip patch, but also produced a high backlobe level.

![Side view of the antenna with dimensions and feed point](image)

**Figure 2-3.** The traveling-wave microstrip antenna introduced by Menzel [17]. Transverse slots located along the center line of the antenna are used for suppressing the fundamental mode.

Using the transverse-resonance technique [15], Oliner and Lee [18-19] later showed that the antenna introduced by Menzel [17] could have been operated as a leaky-wave antenna had it been made longer (4.85 $\lambda_0$-long instead of 2.23 $\lambda_0$, where $\lambda_0$ is the free-space wavelength at the design frequency). They also showed that the high backlobe level exhibited by
Menzel's antenna is due to the fact that 35% of the incident power is reflected at the terminated end, with the backlobe appearing at the same angle as the main beam when measured from broadside.

2.3 The Leaky-Wave Microstrip Antenna as a Periodic Structure

If the leaky-wave microstrip antenna considered by Oliner and Lee [18-19] is split into identical rectangular microstrip patches connected via short and narrow transmission line sections of width \( W \) and length \( s \) each, the structure shown in Fig. 2-4 results. The chain of microstrip cells thus formed represents a periodic structure of period \( d \) made up of identical four-port microstrip patches. Here, it will be assumed that the transmission-line pairs linking adjacent rectangular microstrip patches are responsible for most of the electromagnetic coupling between them. That this is so will be shown to be the case in Chapter 3 for the different microstrip materials used in this work. In this case, analysis of the wave propagation along such a structure requires knowledge of the four-port network parameters of a single microstrip cell only.

![Figure 2-4. The leaky-wave microstrip antenna as a periodic structure.](image-url)
If the rectangular microstrip patch of a single microstrip cell is considered in isolation, the structure shown in Fig. 2-5 results. For an electrically thin dielectric of relative permittivity \( \varepsilon_r \), the electric field below the patch is predominantly \( z \) directed. For a resonant patch length \( a \) of the order of a half a guide wavelength, the rectangular patch supports a mode whose electric-field lines are as shown in Fig. 2-5. The electric field is sinusoidal along the resonant edges (i.e. the edges along which a standing-wave pattern is evident), and is uniform along the non-resonant ones as pointed out by Carver et al. [20]. In addition, this mode is symmetric about an electric wall lying in the \( y-z \) plane.

Due to the different mode designations used in the literature, a brief overview of them will be presented, followed by the mode designation used in this chapter. The first mode designation is the one employed almost exclusively in the hollow waveguide literature. A transverse electric mode (abbreviated TE), also known as an H mode, is one in which components of the electric field lie entirely in parallel planes transverse to the direction of wave propagation (the reference direction), with magnetic-field components in these planes as well as in the reference direction. Similarly, for a transverse magnetic mode (abbreviated TM), also known as an E mode, the magnetic-field components lie entirely in parallel planes transverse to the wave-propagation direction (the reference direction), with electric-field components in the reference direction as well as in the transverse planes.

The second designation is often encountered in the literature dealing with microstrip antennas. In the case of the rectangular patch antenna shown in Fig. 2-5, the normal to the conductor (\( z \) in Fig. 2-5) rather than the direction of wave propagation is chosen as the reference for designating the “transverse” modes (such as TM modes) supported by the patch. Using this reference direction, one would designate the mode whose field lines are shown in Fig. 2-5 as the “TM\(_{10}\)” mode [20]. Such usage of “transverse” will not be employed in the present work.

In this thesis, the following designation is used to describe the modes of a rectangular microstrip patch. Let the resonant modes supported by the patch be denoted by \( E_{mn}^z \), where \( m \) and \( n \) are two non-negative integers used to designate the number of electric-field nulls below the patch along the \( x \) and \( y \) axes respectively. Then, the mode whose electric-field lines are shown in Fig. 2-5 is the \( E_{10}^z \) mode.
The electric-field lines of the \( E_{10}^z \) mode of a rectangular microstrip patch antenna of resonant length \( a \) and width \( b < a \) [20]. Here, the origin of the rectangular coordinate system is located at the centroid of the region between the rectangular patch and the ground plane directly below it. The electric field in the dielectric region below the patch is a sinusoidal standing wave in any plane parallel to the \( x-z \) plane, and is uniform in any plane parallel to the \( y-z \) plane.

Before proceeding with the analysis of the structure shown in Fig. 2-4, it is important to realize that the driving mechanism used to excite the first higher-order mode of the continuous microstrip antenna studied by Oliner and Lee [18-19] has the effect of exciting the \( E_{10}^z \) mode of each of the individual patches in the chain of microstrip cells studied herein. For an electrically thin rectangular microstrip patch such as that shown previously in Fig. 2-5, the electric-current density at or near the resonant frequency is directed parallel to the resonant edges (\( x \) direction) [21-22].


2.3.1 Network description of a microstrip cell

A four-port network representation of a microstrip cell is shown in Fig. 2-6 where the port currents \([I]\) and the port voltages \([V]\) of the equivalent network are related to its impedance matrix \([Z]\) by

\[
[V] = [Z][I]
\]  

(2-1)

where

\[
[V] = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}, \quad [I] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}, \quad [Z] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix}
\]  

(2-2)

For the \(E_{10}^z\) mode the electric field is zero everywhere on that portion of the \(y-z\) plane contained between the ground plane and the microstrip patch shown in Fig. 2-6, and current flow on the patch is transverse to the direction \(y\) of wave propagation on the patch array. In addition, odd electric symmetry about the \(y-z\) plane requires that the port voltages and currents on one side of this plane be related to their counterparts on the other side via

\[
V_2 = -V_1, \quad V_3 = -V_4, \quad i_2 = -i_1, \quad i_3 = -i_4
\]  

(2-3)

Consequently, substitution of eq. (2-3) into eq. (2-1) results in the following two-port description of a microstrip cell in terms of its four-port network parameters:

\[
\begin{bmatrix} V_1 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} - Z_{12} & Z_{14} - Z_{13} \\ Z_{41} - Z_{42} & Z_{44} - Z_{43} \end{bmatrix} \begin{bmatrix} i_1 \\ i_4 \end{bmatrix}
\]  

(2-4)

In this case, periodic-structure analysis of a chain of identical four-port microstrip cells has been reduced to that of two-port cells.
Figure 2-6. Top view of a unit microstrip cell and its equivalent four-port network representation in terms of \( z \) parameters. The \( y \) axis coincides with the center line of the cell.

2.3.2 Periodic-structure analysis of a chain of two-port networks

For a periodic structure made up of identical two-port networks such as shown in Fig. 2-7, the input-port voltage \( V_n \) and current \( I_n \) of cell \( n \) are related to those of cell \( n+1 \) via the transmission matrix \([ABCD]\):

\[
\begin{bmatrix}
V_n \\
I_n
\end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\
I_{n+1}
\end{bmatrix}
\]

(2-5)

In addition, as is shown in [23], the periodicity of the structure requires that the voltage and current at the input-port of cell \( n \) be related to those of cell \( n+1 \) via
where $\gamma$ is the complex propagation constant of the periodic structure, and $d$ is the period or width of a single cell. In this case, substitution of eq. (2-5) into eq. (2-6) gives

$$\begin{bmatrix} A - e^{\gamma d} & B \\ C & D - e^{\gamma d} \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(2-7)

or

$$AD - BC + e^{2\gamma d} - (A + D)e^{\gamma d} = 0$$

(2-8)

For a reciprocal network, $AD - BC = 1$, and eq. (2-8) may be solved for $\gamma$ as

$$\gamma = \alpha + j\beta = \frac{1}{d} \text{acosh} \left( \frac{A + D}{2} \right)$$

(2-9)

where

$\alpha$ is the attenuation or leakage constant of the periodic structure;

$\beta$ is the phase constant of the periodic structure.

Figure 2-7. A periodic structure of identical two-port networks described by $ABCD$ parameters.
Using eqs. (2-4)-(2-5) and (2-9), it may be shown that the complex propagation constant of the chain of four-port microstrip cells illustrated in Fig. 2-4 is given by

\[ \gamma = \alpha + j\beta = \frac{1}{d} \text{acosh} \left( \frac{Z_{11} - Z_{12} - Z_{43} + Z_{44}}{2(Z_{41} - Z_{42})} \right) \]  

(2-10)

which may be further reduced by using the symmetry property of the microstrip cell,

\[ Z_{11} = Z_{44}, \quad Z_{12} = Z_{43} \]  

(2-11)

to

\[ \gamma = \alpha + j\beta = \frac{1}{d} \text{acosh} \left( \frac{Z_{44} - Z_{43}}{Z_{41} - Z_{42}} \right) \]  

(2-12)

If the input and output ports of adjacent microstrip cells in Fig. 2-4 are connected by identical lumped loads \( Z_d \) such as shown in Fig. 2-8, it may be shown that the complex propagation constant in this case becomes

\[ \gamma = \alpha + j\beta = \frac{1}{d} \text{acosh} \left( \frac{Z_d + Z_{44} - Z_{43}}{Z_{41} - Z_{42}} \right) \]  

(2-13)

Here, lumped loads are treated as being infinitesimal.

When the periodically loaded structure of Fig. 2-8 is excited in its \( E_{10}^2 \) mode, a \( y \)-directed traveling wave is set up that results in an \( x \)-polarized main beam emerging at an angle \( \theta \) away from broadside (\( z \) direction). Based on the discussion in Section 1.2 and eq. (2-13), one may show that the main-beam-maximum direction \( \theta \) is related to the network parameters of a microstrip cell via

\[ \sin(\theta) = \frac{c}{v} = \frac{\beta}{k_0} = \text{Im} \left[ \frac{1}{k_0 d} \text{acosh} \left( \frac{Z_d + Z_{44} - Z_{43}}{Z_{41} - Z_{42}} \right) \right] \]  

(2-14)

where \( k_0 \) is the free-space wave number, and \( \beta \) is the phase constant of the periodic structure.
Figure 2-8. Periodically loaded leaky-wave microstrip antenna. Lumped loads of equal value $Z_d$ are placed at regular intervals $d$ along the direction of wave propagation $y$.

2.4 Four-Port Impedance Matrix of a Microstrip Cell

In order to determine the complex propagation constant $\gamma$ of the periodically loaded leaky-wave microstrip antenna shown in Fig. 2-8, one must first determine the impedance matrix $[Z]$ of the unloaded four-port microstrip cell. This may be accomplished by dividing the microstrip cell into three four-port sub-networks as shown in Fig. 2-9. Then, by calculating the impedance matrix of the individual sub-networks, one may determine the impedance matrix $[Z]$ of the unloaded microstrip cell. The complex propagation constant $\gamma$ of the loaded antenna may then be found from eq. (2-13).
2.4.1 Four-port impedance matrix of the microstrip transmission-line pair

For a lossy transmission line of length \( l \) and characteristic impedance \( Z_0 \), it may be shown that the two-port impedance matrix \( [z] \) relating the port voltages \( [v] \) to the port currents \( [i] \) is given by

\[
[z] = \begin{bmatrix}
Z_0 \text{coth}(\gamma_\text{vl}l) & Z_0 \text{csch}(\gamma_\text{vl}l) \\
Z_0 \text{csch}(\gamma_\text{vl}l) & Z_0 \text{coth}(\gamma_\text{vl}l)
\end{bmatrix}
\] (2-15)
where

$$\gamma_{ul} = \alpha + j\beta$$  \hspace{1cm} (2-16)$$

is the complex propagation constant of the transmission line, and $\alpha$ and $\beta$ are the attenuation and phase constants respectively.

Using eq. (2-15) and network theory, one may show that the four-port impedance matrix $[z_1] = [z_3]$ of the two identical widely separated parallel transmission lines shown in Fig. 2-10 is related to the port currents and voltages via

$$
\begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4 \\
\end{bmatrix} =
\begin{bmatrix}
Z_0\coth(\gamma_{ul}l) & 0 & 0 & Z_0\csch(\gamma_{ul}l) \\
0 & Z_0\coth(\gamma_{ul}l) & Z_0\csch(\gamma_{ul}l) & 0 \\
0 & Z_0\csch(\gamma_{ul}l) & Z_0\coth(\gamma_{ul}l) & 0 \\
Z_0\csch(\gamma_{ul}l) & 0 & 0 & Z_0\coth(\gamma_{ul}l) \\
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4 \\
\end{bmatrix}
$$  \hspace{1cm} (2-17)$$

where mutual coupling between the transmission lines has been neglected due to the wide separation between them.

Figure 2-10. A pair of identical, widely separated, parallel transmission lines.
Specializing eq. (2-17) to the \( \frac{s}{2} \)-long microstrip transmission-line pair shown in Fig. 2-9, one gets

\[
\begin{bmatrix}
  v_1 \\
  v_2 \\
  v_3 \\
  v_4
\end{bmatrix}
= \begin{bmatrix}
  Z_0 \coth(\gamma_u \frac{s}{2}) & 0 & 0 & Z_0 \text{csch}(\gamma_u \frac{s}{2}) \\
  0 & Z_0 \coth(\gamma_u \frac{s}{2}) & Z_0 \text{csch}(\gamma_u \frac{s}{2}) & 0 \\
  0 & Z_0 \text{csch}(\gamma_u \frac{s}{2}) & Z_0 \coth(\gamma_u \frac{s}{2}) & 0 \\
  Z_0 \text{csch}(\gamma_u \frac{s}{2}) & 0 & 0 & Z_0 \coth(\gamma_u \frac{s}{2})
\end{bmatrix}
\begin{bmatrix}
  i_1 \\
  i_2 \\
  i_3 \\
  i_4
\end{bmatrix}
\] (2-18)

where, for a microstrip line of width \( W \), thickness \( t \), dielectric thickness \( h \), and relative permittivity \( \varepsilon_r \), the characteristic impedance \( Z_0 \) is given in Appendix A by eq. (A-1). Here, the complex propagation constant \( \gamma_u = \alpha + j \beta \), where

\[
\beta = \omega \sqrt{\mu_0 \varepsilon_0 \varepsilon_r}
\] (2-19)

and \( \omega \), \( \mu_0 \), and \( \varepsilon_0 \) are the radian frequency, permeability, and permittivity of free space respectively. On the other hand, the attenuation constant \( \alpha \) is the sum of the conductor loss factor \( \alpha_c \), and the dielectric loss factor \( \alpha_d \) given in Appendix A by eqs. (A-5) and (A-9) respectively.

### 2.4.2 Four-port impedance matrix of the microstrip patch

If \([z_2]\) is used to denote the four-port impedance matrix of the rectangular microstrip patch shown in Fig. 2-11, then the port voltages \([v]\) and the port currents \([i]\) of the patch are related via

\[
[v] = [z_2][i]
\] (2-20)

where
Due to the symmetry of the structure,

\[
\begin{align*}
z_{11} &= z_{22} = z_{33} = z_{44} \\
z_{14} &= z_{23} \\
z_{12} &= z_{43} \\
z_{13} &= z_{24}
\end{align*}
\]  

and by reciprocity,

\[
\begin{align*}
z_{12} &= z_{21}, \quad z_{13} &= z_{31}, \quad z_{14} &= z_{41} \\
z_{23} &= z_{32}, \quad z_{24} &= z_{42} \\
z_{34} &= z_{43}
\end{align*}
\]  

Therefore, the impedance matrix \([z_2]\) of the four-port rectangular microstrip patch becomes
Using eqs. (B-1)-(B-2) of Appendix B, it may be shown that the elements of the impedance matrix \([z_2]\) are given by

\[
\begin{align*}
\mathbf{z}_{11} &= \frac{-j\kappa h}{abW^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_{0}^{W} \int_{0}^{W} \varepsilon_{om}\varepsilon_{on} \cos(k_x x) \cos(k_y y) \cos(k_x x_0) \cos(k_y y_0) dxdx_0 \frac{k^2(1 - \frac{j}{Q})}{k^2 - k_x^2 - k_y^2} \bigg|_{y = 0, y_0 = 0} \\
\mathbf{z}_{12} &= \frac{-j\kappa h}{abW^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_{0}^{W} \int_{-W}^{W} \varepsilon_{om}\varepsilon_{on} \cos(k_x x) \cos(k_y y) \cos(k_x x_0) \cos(k_y y_0) dxdx_0 \frac{k^2(1 - \frac{j}{Q})}{k^2 - k_x^2 - k_y^2} \bigg|_{y = 0, y_0 = 0} \\
\mathbf{z}_{13} &= \frac{-j\kappa h}{abW^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_{0}^{W} \int_{-W}^{W} \varepsilon_{om}\varepsilon_{on} \cos(k_x x) \cos(k_y y) \cos(k_x x_0) \cos(k_y y_0) dxdx_0 \frac{k^2(1 - \frac{j}{Q})}{k^2 - k_x^2 - k_y^2} \bigg|_{y = 0, y_0 = b} \\
\mathbf{z}_{14} &= \frac{-j\kappa h}{abW^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \varepsilon_{om}\varepsilon_{on} \cos(k_x x) \cos(k_y y) \cos(k_x x_0) \cos(k_y y_0) \bigg(\frac{k^2(1 - \frac{j}{Q})}{k^2 - k_x^2 - k_y^2}\sec\left(\frac{mn \pi b}{a}\right)\bigg) \bigg|_{y = 0, y_0 = b}
\end{align*}
\]

\[ (2-24) \]
\[ z_{14} = \frac{-j k \eta h}{a b W^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_{0}^{W} \int_{0}^{W} \epsilon_{0m} \epsilon_{0n} \cos(k_x x) \cos(k_y y) \cos(k_x x_0) \cos(k_y y_0) dx dy \frac{k^2(1 - \frac{j}{Q}) - k_x^2 - k_y^2}{k^2(1 - \frac{j}{Q}) - k_x^2 - k_y^2} \] 

\[ = \frac{-j k \eta h}{2ab} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\epsilon_{0m} \epsilon_{0n} \cos(n \pi b/a) \left[ \text{sinc}(k_x W) \right]^2}{k^2(1 - \frac{j}{Q}) - k_x^2 - k_y^2} \]  

(2-28)

where

\[ \text{sinc}(x) = \frac{\sin(x)}{x} \]  

(2-29)

and \( Q \) is the quality factor of the microstrip patch given in Appendix C by eq. (C-1). Note here that since the microstrip patch is strongly excited in its \( E_{10}^z \) mode, its impedance matrix is dominated largely by the quality factor \( Q \) of this mode and depends little on the quality-factor value of other non-resonant cavity modes [47]. For such microstrip patch, one may conveniently define the radiation resistance \( R_{rad} \), the surface-wave resistance \( R_{sw} \), the dielectric resistance \( R_d \), and the conduction resistance \( R_c \) as follows:

\[ R_d = \frac{V^2}{2P_d} \]  

(2-30)

\[ R_c = \frac{V^2}{2P_c} \]  

(2-31)

\[ R_{rad} = \frac{V^2}{2P_{rad}} \]  

(2-32)

\[ R_{sw} = \frac{V^2}{2P_{sw}} \]  

(2-33)

where

\( P_d \) is the power lost in that part of the dielectric located directly below the patch and is given in Appendix C by eq. (C-2);
$P_c$ is the power lost in the patch conductor and that part of the ground plane located directly below the patch and is given in Appendix C by eq. (C-3);

$P_{rad}$ is the power radiated into the upper hemisphere by the microstrip patch and is given in Appendix C by eq. (C-8);

$P_{sw}$ is the power lost to surface waves and is given in Appendix C by eq. (C-10);

$V$ is the voltage between any point on the edge $x=0$ or $a$ of the microstrip patch shown in Fig. 2-11, and a point directly below it on the ground plane.

2.4.3 Four-port impedance matrix of a microstrip cell as a cascade of three networks

Using eq. (C-1) of Appendix C for the quality factor $Q$ in eqs. (2-25)-(2-28), one may readily calculate the four-port impedance matrix of a rectangular microstrip patch. It was found that a choice of an upper limit $M=N=150$ on the summations of eqs. (2-25)-(2-28) is enough to achieve convergence. Here, convergence is defined as that value of $M=N$ beyond which the change between successive calculations of the real and imaginary parts of the impedance-matrix elements is less than one percent.

From the calculated four-port impedance matrices $[z_1]$, $[z_2]$, and $[z_3]$ of sub-networks 1, 2, and 3 in Fig. 2-9, the impedance matrix $[Z]$ of the microstrip cell may be found using the following procedure:

1. The impedance matrices $[z_1]$, $[z_2]$, and $[z_3]$ are converted into transmission matrices $[t_1]$, $[t_2]$, and $[t_3]$ respectively as illustrated in Fig. 2-12 and eq. (2-34);

2. The transmission matrix $[T] = [t_1][t_2][t_3]$ is converted to an impedance matrix $[Z]$ using eq. (2-35). $[Z]$ is the impedance matrix of the microstrip cell.

As was shown earlier in Subsection 2.3.2, elements of $[Z]$ are used in eq. (2-14) to find the direction of the main-beam maximum. The latter is compared in Subsection 3.3.2 with its measured counterpart and found to be in agreement to within 2.5° over a scan range of 26°, thus providing confidence in the calculated impedance matrix elements.
Figure 2.12. (a) Four-port network described by an impedance matrix.  (b) Four-port network described by a transmission matrix.

\[
\begin{align*}
t_{11} &= t_{22} = t_{33} = t_{44} = \frac{z_{12}z_{13} - z_{11}z_{14}}{2z_{13} - z_{14}} \\
t_{12} &= t_{34} = \frac{2z_{11}z_{12}z_{13} - z_{11}z_{14} - z_{12}z_{14} - z_{13}z_{14} + z_{14}^3}{2z_{13} - z_{14}} \\
t_{13} &= t_{24} = t_{31} = t_{42} = \frac{z_{11}z_{13} - z_{12}z_{14}}{2z_{13} - z_{14}} \\
t_{14} &= t_{32} = \frac{z_{11}^2z_{13} + z_{12}^2z_{13} - z_{13}^3 - 2z_{11}z_{12}z_{14} + z_{13}z_{14}^2}{2z_{13} - z_{14}} \\
t_{21} &= t_{43} = \frac{-z_{14}}{z_{13} - z_{14}^2} \\
t_{23} &= t_{41} = \frac{z_{13}}{z_{13} - z_{14}^2}
\end{align*}
\]
Power Analysis of a Periodically Loaded Leaky-Wave Microstrip Antenna

The power delivered by the voltage sources to each of the \( N \) microstrip cells of a periodically loaded microstrip antenna such as that illustrated in Fig. 2-13 is determined in Appendix D. Expressions are derived which provide the radiated power and the power lost to surface waves, both in the conductor and in the dielectric of the antenna. In addition, expressions are developed for the power dissipated in the lumped loads \( Z_d \) and in the resistive terminations \( R_L \) used to absorb some of the power remaining in the incident wave as it reaches the antenna end.

To simplify the analysis, use is made of a perfect electric plane of symmetry which reduces the chain of four-port networks into two-port ones, resulting in the equivalent network representation shown in Fig. 2-14. As is shown in Chapter 3, the microstrip materials and geometry used in this work are chosen such that coupling between adjacent microstrip patches other than that which takes place via the transmission-line sections linking them is weak. Consequently, the total power radiated and dissipated by the periodically loaded leaky-wave microstrip antenna is given, to a good approximation, by the sum of the powers radiated and dissipated by its \( N \) individual patches as may be concluded from the power expressions developed by Kraus [25] for a two-element array.
Figure 2-13. A truncated periodically loaded leaky-wave microstrip antenna made up of $N$ microstrip cells. Lumped loads of equal value $Z_d$ are placed at regular intervals $d$ along the antenna.

Figure 2-14. Equivalent network representation of the truncated periodically loaded leaky-wave microstrip antenna shown in Fig. 2-13.
The total power $P_i, i = 1...N$ dissipated in the $i^{th}$ microstrip cell is distributed among the various loss mechanisms previously outlined in Section 2.4.2. These are

1. the radiated power $P_{rad,i}$;
2. the power lost to surface waves $P_{sw,i}$;
3. the power lost in the dielectric $P_{d,i}$;
4. the power lost in the conductor $P_{c,i}$.

With $R_{rad}, R_{sw}, R_d,$ and $R_c$ as defined in eqs. (2-30)-(2-33), it may be shown that for the $i^{th}$ microstrip cell:

$$P_{rad,i} = \frac{R_{sw}R_dR_c}{R_{rad}R_dR_c + R_{rad}R_{sw}R_d + R_{sw}R_dR_c + R_{rad}R_{sw}R_c} P_i$$  \hspace{1cm} (2-36)

$$P_{sw,i} = \frac{R_{rad}R_dR_c}{R_{rad}R_dR_c + R_{rad}R_{sw}R_d + R_{sw}R_dR_c + R_{rad}R_{sw}R_c} P_i$$  \hspace{1cm} (2-37)

$$P_{d,i} = \frac{R_{rad}R_{sw}R_c}{R_{rad}R_dR_c + R_{rad}R_{sw}R_d + R_{sw}R_dR_c + R_{rad}R_{sw}R_c} P_i$$  \hspace{1cm} (2-38)

$$P_{c,i} = \frac{R_{rad}R_{sw}R_d}{R_{rad}R_dR_c + R_{rad}R_{sw}R_d + R_{sw}R_dR_c + R_{rad}R_{sw}R_c} P_i$$  \hspace{1cm} (2-39)

### 2.6 The $H$-Plane Power Pattern of the Leaky-Wave Microstrip Antenna

This section presents two techniques which will be used in Chapter 3 to determine the $H$-plane power pattern. The first of these techniques treats the antenna as a line source with current flow transverse to the direction of wave propagation, while the second considers the antenna as a nonuniformly excited, equally spaced linear array.
2.6.1 The leaky-wave microstrip antenna as a line source

Consider a line source of length $L$ and infinitesimal width $\Delta w$ positioned along the $y$ axis of a rectangular coordinate system such as shown in Fig. 2-15. For an $x$-directed traveling current wave $I_x(y)$ propagating along the line source in the positive $y$ direction, the pattern factor $f(\theta, \phi)$ is given in [24] by

$$f(\theta, \phi) = \int_0^L I_x(y) e^{jk_0 y \sin(\theta) \sin(\phi)} \, dy$$

(2-40)

$$k_0 = \frac{2\pi}{\lambda_0}$$

(2-41)

where $\lambda_0$ is the free-space wavelength, and $\theta$ and $\phi$ are the elevation and azimuth angles respectively.

Figure 2-15. Line source of length $L$ and infinitesimal width $\Delta w$. A traveling wave of $x$-directed current is assumed to be propagating along the line source in the positive $y$ direction.
For an exponentially decaying traveling current wave propagating along the line source, \( I_x(y) \) may be expressed as

\[
I_x(y) = I_0 e^{-\alpha y} e^{-j\psi(y)}
\]

where \( I_0 \) is the amplitude of the current wave, \( \alpha \) is its attenuation or leakage constant, and \( \psi(y) \) is its phase. Here, current flow is taken to be transverse (x-directed) to the direction of wave propagation \( y \) as is the case for the periodically loaded leaky-wave microstrip antenna. For a traveling wave with a phase constant \( \beta \), \( \psi(y) \) may be written as

\[
\Psi(y) = k_0 y \sin(\theta_0)
\]

\[
\sin(\theta_0) = \frac{\beta}{k_0}
\]

Substitution of eqs. (2-42) and (2-43) into eq. (2-40) results in the following expression for the pattern factor

\[
f(\theta, \phi) = I_0 \int_0^L e^{-\alpha y} e^{j\beta y(\sin(\theta) \sin(\phi) - \sin(\theta_0))} dy
\]

For an x-directed infinitesimal current element lying on a grounded dielectric slab of infinite extent, eqs. (C-8) and (C-9) of Appendix C can be used to determine the element pattern \( g(\theta, \phi) \) in the \( H \)-plane \( \left( \phi = \frac{\pi}{2} \right) \):

\[
g \left( \theta, \phi = \frac{\pi}{2} \right) = \sqrt{\frac{(\cos \theta)^2}{(\varepsilon_r - (\sin \theta)^2)(\cot(\varepsilon_r k_0 h_0(\varepsilon_r - (\sin \theta)^2))^2 + (\cos \theta)^2}}
\]

Hence, the \( H \)-plane power pattern of the leaky-wave microstrip antenna may be calculated readily by direct substitution of eqs. (2-45)-(2-46) into the following expression, given in [24] as

\[
\left| F \left( \theta, \phi = \frac{\pi}{2} \right) \right|^2 = \left| g \left( \theta, \phi = \frac{\pi}{2} \right) \right|^2 \left| f \left( \theta, \phi = \frac{\pi}{2} \right) \right|^2
\]
2.6.2 The leaky-wave microstrip antenna as a nonuniformly excited linear array

For an array of \( N \) nonuniformly excited, equally spaced isotropic point sources lying along the \( y \)-axis of a rectangular coordinate system such as shown in Fig. 2-16, the array factor \( AF \) may be expressed as

\[
AF = \sum_{n=0}^{N-1} a_n e^{j\psi}
\]

(2-48)

where \( a_n \) is the amplitude coefficient of source \( n \), and \( \psi \) is the phase shift between any two adjacent point sources.

![Figure 2-16](image)

**Figure 2-16.** An array of \( N \) nonuniformly excited, equally spaced isotropic point sources lying along the \( y \) axis. The spacing between adjacent point sources is the same as the center-to-center spacing \( d \) between adjacent microstrip cells.

For large \( N \), it is convenient to view such an array as a periodic structure that supports a traveling wave having a phase velocity \( v \). In this case, \( \psi \) may be expressed as in [25] by

\[
\psi = k_0 d \sin(\theta) \sin(\phi) + k_0 \frac{c}{v} d
\]

(2-49)
where \( k_0 \) and \( c \) are the wave number and speed of light in free space respectively, and \( d \) is the spacing between any two adjacent point sources.

For an exponentially decaying amplitude distribution along the array, the amplitude coefficients \( a_n \) may be expressed as

\[
a_n = I e^{-n\alpha d} \quad n = 0 \ldots N - 1
\]

(2-50)

where \( \alpha \) is the attenuation or leakage constant, and \( I \) is an amplitude constant. Using eqs. (2-49) and (2-50), the expression for the array factor \( AF \) becomes

\[
AF = I \sum_{n=0}^{N-1} e^{jn(k_0 d \sin(\theta) \sin(\phi) + k_0 \epsilon_d)n\alpha d}
\]

(2-51)

In order to determine the \( H \)-plane power pattern of the leaky-wave microstrip antenna, the element pattern of a single microstrip patch is required. Using eqs. (C-8) and (C-16) of Appendix C, it may be shown that the \( H \)-plane element pattern of a microstrip patch such as that shown in Fig. 2-11 is given by

\[
g\left(\theta, \phi = \frac{\pi}{2}\right) = \sqrt{\frac{(\cos \theta)^2}{(\epsilon_r - (\sin \theta)^2)(\cot(hk_0\sqrt{\epsilon_r - (\sin \theta)^2})^2 + (\cos \theta)^2} \left(\sin\left(\frac{bk_0\sin \theta}{2}\right)\right)^2}
\]

(2-52)

Then, the \( H \)-plane power pattern of the leaky-wave microstrip antenna may be found from the following expression given in [24]:

\[
\left|F\left(\theta, \phi = \frac{\pi}{2}\right)\right|^2 = \left|g\left(\theta, \phi = \frac{\pi}{2}\right)\right|^2 |AF|^2
\]

(2-53)

Here, the effect of the resonant patch edges is taken into account in eq. (2-52). This contrasts with the line source treatment of Subsection 2.6.1 where the leaky-wave microstrip antenna was considered as a continuous radiating source that does not include the resonant edge effect.
2.7 Summary

In this chapter, the leaky-wave microstrip antenna considered by Oliner and Lee [18-19] has been converted into a periodic structure of identical microstrip cells made up of resonant rectangular patches connected via short transmission-line sections. Expressions for the network parameters of a microstrip cell have been derived by treating the latter as a resonant multi-port cavity with lossy walls. In addition, an expression for the complex propagation constant γ of the periodic structure has been developed. This expression depends on the network parameters of a microstrip cell, and the identical lumped loads introduced at regular intervals along the periodic structure.

Expressions for the radiated power and the power dissipated in the various parts of a truncated periodic structure have been derived based on the assumption that coupling between adjacent rectangular microstrip patches takes place mainly via the short transmission-line sections linking them. In addition, expressions for the H-plane power pattern have also been developed.
Chapter 3

FIXED-FREQUENCY BEAM-STEERABLE LEAKY-WAVE MICROSTRIP ANTENNA DESIGN

3.1 Introduction

In this chapter, examples are used to validate the theory of Chapter 2. In particular, experiments are used to validate the theoretical predictions linking the phase velocity (and thus the main-beam direction) of a periodically loaded leaky-wave microstrip antenna to the reactive loads placed along it. The effect of the dielectric on the scan range of the antenna is then examined by considering two such structures with different relative permittivities. This is followed by various measurements for assessing the response of these antennas to harmonic and intermodulation interference.

3.2 Design Guidelines for a Periodically Loaded Leaky-Wave Microstrip

This section presents the guidelines used in this chapter for the design of such an antenna. These guidelines address issues such as the choice of microstrip dielectric thickness, cell dimensions, antenna length, input impedance, and operating bandwidth.

3.2.1 Dielectric thickness and cell dimensions

For a microstrip antenna such as that shown in Fig. 3-1, the thickness of the microstrip dielectric $h$ is chosen so as to not excite higher-order surface-wave modes beyond the zero-cut-off-frequency $TM_0$ mode [26]. In addition, $h$ must be small enough to ensure that little power is dissipated in the $TM_0$ mode as was shown by Perlmutter et al. [22].
For a microstrip cell such as that depicted in Fig. 3-1, the rectangular patch length $a$ is chosen so as to make the patch resonant along its length, while a slightly smaller patch width $b$ is selected. In addition, a wide spacing $s \gg h$ between adjacent rectangular microstrip patches is chosen so that coupling between them takes place mainly via the transmission-line connections. This choice of $s$ must be traded off against the fact that in order to keep conductor losses low, the length of the transmission-line connections must be kept as short as possible. For the two microstrip antennas considered in this chapter, it was found that a value of $s$ that satisfies both of the aforementioned requirements is $s = 2.7h$.

The width $W$ of the microstrip transmission line connections is chosen such that, as the variable-capacitor value $C$ is varied between $C_{\text{min}}$ and $C_{\text{max}}$, a maximum phase-shift range $\Delta \delta$ between adjacent cells is achieved. A consequence is a wider main-beam scan range as can be deduced from eq. (2-49).

![Diagram](image)

**Figure 3-1.** Leaky-wave microstrip antenna periodically loaded at regular intervals $d$ with identical variable capacitors of value $C_{\text{min}} \leq C \leq C_{\text{max}}$. Here, the variable capacitors are considered to be small with respect to the free-space wavelength $\lambda_0$. 
To determine \( W \), consider a variable capacitor of value \( C_{\text{min}} \leq C \leq C_{\text{max}} \) embedded in a transmission line such as that shown in Fig. 3-2. By taking the derivative of the phase-shift range \( \Delta \delta \) with respect to \( Z_0 \) and setting it to zero, one may show that a maximum phase-shift range results if the characteristic impedance \( Z_0 \) of the transmission-line is

\[
Z_0 = \frac{1}{2\omega \sqrt{C_{\text{max}} C_{\text{min}}}} \tag{3-1}
\]

where \( \omega \) is the radian frequency. For a microstrip transmission line, the characteristic impedance given in eq. (3-1) may be realized by solving for the appropriate top-conductor width \( W \) in eqs. (A-1)-(A-3). For low relative permittivity microstrip, eqs. (3-1) and (A-1)-(A-3) may result in a transmission-line width which is much larger than the physical size of a variable capacitor. In this case, a value of \( W \) close to the width of the variable-capacitor leads is selected. Due to difficulties in fabrication, variable capacitors in shunt with the transmission-line sections have not been considered.

Figure 3-2. A variable capacitor of value \( C_{\text{min}} \leq C \leq C_{\text{max}} \) embedded in a transmission line of characteristic impedance \( Z_0 \). The phase-shift range \( \Delta \delta \) is maximized if the characteristic impedance of the transmission line is set to \( Z_0 = 1/(2\omega \sqrt{C_{\text{max}} C_{\text{min}}}) \).
3.2.2 Length of a periodically loaded leaky-wave microstrip antenna

The length $L$ of the antenna is chosen such that 90% or so of the input power is radiated along its length. A matched load located at the antenna end is used to absorb the power remaining in the incident traveling wave. For a given antenna leakage constant $\alpha$, and a percentage of input power to be radiated $PPR$, it has been shown in [7] that the ratio of the antenna length to the free-space wavelength $\lambda_0$ is

$$\frac{L}{\lambda_0} = \frac{k_0}{4\pi\alpha} \ln\left(1 - \frac{PPR}{100}\right)$$

(3-2)

where $k_0$ is the free-space wavenumber.

For a leaky-wave microstrip antenna periodically loaded with identical variable capacitors of value $C$ such that $C_{\text{min}} \leq C \leq C_{\text{max}}$, the leakage constant $\alpha$, and thus the antenna length $L$, depend on $C$ as may be seen from eqs. (2-13) and (3-2) respectively. In this case, the leakage constant chosen in calculating the antenna length $L$ is that which corresponds to $C_a = (C_{\text{max}} + C_{\text{min}})/2$, the middle of the capacitance range.

3.2.3 Input impedance

As is evident from the example shown in Fig. 3-3, a periodically loaded leaky-wave microstrip antenna is a two-port structure. It is fed by means of a 50 $\Omega$ 180° hybrid, and the resulting system is matched if the antenna's two input ports present a 50 $\Omega$ impedance to the output ports of the hybrid. Using Fig. 3-3 as an example, one may use the following procedure to obtain a matched system:

1. Calculate the input impedance $Z_{\text{in}}$ of the equivalent circuit shown in Fig. 3-3 using eq. D-19;
2. Transform $Z_{\text{in}}$ into a 50 $\Omega$ impedance using a microstrip matching network;
3. Use the matching network determined in (2) between the first antenna input port and the first output port of the hybrid. Use an identical matching network between the second antenna input port and the second output port of the hybrid. The resulting system is matched.
For the microstrip antennas considered in this chapter, it was found that the aforementioned matching networks are not required since a measured standing-wave ratio of less than 1.5 (without applied reverse-bias across the varactor diodes) is obtained at the input of the hybrid, indicating that the antennas are reasonably matched.

3.2.4 Operating bandwidth

The operating bandwidth of a leaky-wave microstrip antenna is not limited by its input impedance bandwidth, but rather by the fact that the main beam of such a structure scans with frequency. In this case, one may define the operating bandwidth of the antenna as that interval of the impedance bandwidth within which the power gain in a given direction varies within a given amount (3 dB is chosen here) from its maximum value.

The operating bandwidth of a leaky-wave antenna decreases as its relative permittivity \( \varepsilon_r \) is increased. This is due to the fact that the main-beam frequency-scanning rate of the antenna increases with an increase in \( \varepsilon_r \) [27].

3.3 Periodically Loaded Leaky-Wave Microstrip Antenna at 6.25 GHz

In order to demonstrate fixed-frequency beam steering in a leaky-wave microstrip antenna, the structure illustrated in Fig. 3-3 was built using 31 identical microstrip cells of width \( d = 1 \) cm each, loaded with 60 identical varactor diodes. The number of cells was so chosen to dissipate approximately 95% of the input power along the length of the antenna. This percentage of power to be dissipated is higher than the 90% suggested in Subsection 3.2.2, and is so chosen to compensate for the fact that the arbitrarily selected 50-\( \Omega \) resistive terminations located at the antenna end provide only partial absorption of the power remaining in the incident wave. Note that total absorption of the remaining incident power requires that the periodic structure be terminated in its two identical Bloch impedances [23]. For an infinite periodic structure composed of unit cells such as the one shown in the equivalent circuit in Fig. 3-3, the Bloch impedance is the impedance seen by the incident voltage and current waves at the input terminals of a unit cell [23].
The antenna is printed on a dielectric having a relative permittivity $\varepsilon_r = 3.0 \pm 0.04$, and a loss tangent $\tan\delta = 0.0013$ at 10.0 GHz. Each cell consists of a rectangular microstrip patch of length $a = 1.45$ cm and of width $b = 0.8$ cm, and 0.3-mm-wide transmission-line sections connecting each patch to the varactor diodes.

Figure 3-3. A periodically loaded leaky-wave microstrip antenna made up of 31 microstrip cells of width $d = 1$ cm each, and 60 identical varactor diodes. Each cell contains a rectangular microstrip patch of dimension $a \times b$ where $a = 1.45$ cm and $b = 0.8$ cm. The antenna is fed using a 180° hybrid and two bias tees for reverse-biasing the varactor diodes. The bias-tee and 180°-hybrid specifications are listed in Tables 3-1 and 3-2 respectively (continued on the next page).
Infinite perfectly conducting electric ground plane

Figure 3-3. Continued from the previous page.

One may recall from the previous chapter that the theory dealing with periodically loaded leaky-wave microstrip antennas is based on the assumption that coupling between adjacent rectangular microstrip patches takes place mainly via the short transmission-line sections linking them. That this is so for the structure illustrated in Fig. 3-3 will be shown to be the case in the following section, thus making this theory applicable to the current problem.

3.3.1 Coupling between adjacent cells

In order to show that the short transmission-line sections connecting two adjacent rectangular microstrip patches are responsible for most of the coupling between them, the coupling coefficient $|S_{21}|$ was calculated for the following two cases:
(1) Both electromagnetic coupling and coupling via the transmission-line sections are taken into account. This case is referred to as the fully coupled two-cell system, and is shown in Fig. 3-4;

(2) Coupling is only via the transmission-line sections (no electromagnetic coupling). This case is referred to as the partially coupled two-cell system, and is shown in Fig. 3-5.

Here, both calculations are based on the sinusoidal reaction formulation [28-29]. Conducting surfaces are modeled as quadrilaterals having zero thickness [30]. The unknown current on the quadrilaterals is expanded in a set of $N$ sinusoidal expansion functions, and the integral equation is enforced for $N$ sinusoidal electric test sources placed on the quadrilaterals [30]. The reaction integral equation is then solved using the method of moments [28-32].

In order to gain confidence in the results provided by the method of moments, the coupling coefficient of the fully coupled structure shown in Fig. 3-4 is measured using a network analyzer, and is plotted in Fig. 3-6 along with the calculated coupling coefficients of both structures in the frequency range 6-6.5 GHz. Here, the thickness of the dielectric and its relative permittivity are such that coupling between the two rectangular patches via surface waves is weak, as is shown in Subsection 3.3.4 where it is found that approximately 8% of the input power is lost to surface waves. Therefore, coupling due to surface waves is neglected when applying the method of moments, and a homogeneous dielectric of relative permittivity $\varepsilon_r = 3.0$ is assumed to fill all of space.
Figure 3-4. A fully coupled two-cell microstrip structure used for assessing the degree of coupling that exists between two identical rectangular microstrip patches connected by a pair of short transmission line sections.

In the frequency range 6-6.5 GHz, Fig. 3-6 shows that the difference between the calculated coupling coefficients of both structures is less than 0.5 dB. That is, coupling between the rectangular microstrip patches takes place mainly via the short transmission-line sections. Comparison of the measured and calculated coupling coefficients of the fully coupled structure illustrated in Fig. 3-4 shows a difference of at most 1.5 dB between the two data sets.
Figure 3-5. A partially coupled two-cell microstrip structure used for assessing the degree of coupling that exists between two identical rectangular microstrip patches connected by a pair of short transmission line sections. Here, the network parameters of a single cell are calculated using the method of moments [28-32], and are used to find those of a two-cell system. Coupling between the two patches other than that which takes place via the transmission-line pair is not included in this case.
Figure 3-6. Comparison of the calculated coupling coefficient of the structure shown in Fig. 3-4 with that shown in Fig. 3-5 in the frequency range 6-6.5 GHz. Shown also is the measured coupling coefficient of the structure illustrated in Fig. 3-4.

3.3.2 Power gain, cross-polarization, and standing-wave ratio

At a frequency $f = 6.25$ GHz, the microstrip antenna shown in Fig. 3-3 is fed using a $180^\circ$ hybrid along with two bias tees for reverse-biasing the varactor diodes. Here, use of the bias tees is of extreme importance to the survival of the RF equipment since they insure that no DC currents or voltages can reach such equipment, while simultaneously preventing RF signals from reaching the DC power supply. The bias-tee and $180^\circ$-hybrid specifications as provided by the manufacturers are listed in Tables 3-1 and 3-2 respectively.

Table 3-1 High-Voltage Bias Tee (Picosecond Pulse Labs Model 5531)

<table>
<thead>
<tr>
<th>-3 dB Bandwidth</th>
<th>AC-DC Isolation</th>
<th>Insertion Loss</th>
<th>Impedance</th>
<th>Maximum DC Voltage</th>
<th>Maximum DC Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>750 kHz - 10 GHz</td>
<td>&gt; 20 dB</td>
<td>0.3 dB</td>
<td>50 Ω</td>
<td>1.5 KV</td>
<td>20 mA</td>
</tr>
</tbody>
</table>
Using the equations of Section 2.4, the four-port impedance matrix of a single microstrip cell is determined. Then, varactor diodes, treated here as being infinitesimal, are each replaced with the equivalent circuit shown in Fig. 3-7. For the Metelics MSV-34-060-E25 abrupt varactor diode, the values of the various circuit components are given in [33] where

\[ C_p \] is the parasitic capacitance of the diode package \( = 0.07 \text{ pF} \);

\[ L_p \] is the parasitic inductance of the diode package \( = 0.4 \text{ nH} \);

\[ R_s(V) \] is the maximum bulk resistance \( = 1.3 \pm 0.13 \text{ } \Omega \);

\[ C_{j0} \] is the junction capacitance at an applied reverse-bias voltage of 0 V.

Here, \( C_{j0} = 0.98 \pm 0.098 \text{ pF} \);

\[ C_j(V) \] is the junction capacitance at an applied reverse-bias voltage \( V \) volts;

\( P_{\text{diss}} \) is the maximum peak DC power dissipation \( = 100 \text{ mW} \).

**Table 3-2** Ultra-Broadband 180° Hybrid (Loral Model 4346)

<table>
<thead>
<tr>
<th>Frequency Range</th>
<th>Coupling</th>
<th>VSWR</th>
<th>Amplitude Balance</th>
<th>Phase Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 - 18 GHz</td>
<td>3 dB</td>
<td>1.6:1</td>
<td>0.8 dB</td>
<td>12°</td>
</tr>
</tbody>
</table>

**Figure 3-7**. A voltage-dependent equivalent circuit for the Metelics MSV-34-060-E25 abrupt silicon varactor diode [33]. Here, \( Z_d(V, \omega) \) is the diode impedance as a function of the reverse-bias voltage \( 0 \leq V \leq 30 \text{ V} \), and the radian frequency \( \omega \).
For different reverse-bias voltages applied across the varactor diodes via the DC voltage source $V_{dc}$, the complex propagation constant $\gamma$ is calculated from eq. (2-13), and used to find the direction of the main-beam maximum measured away from broadside (the $z$ direction), as well as the leakage constant $\alpha$. Figure 3-8 shows the measured and calculated fixed-frequency movement of the main-beam maximum, $\theta_m$, with increasing reverse-bias voltage. Comparison of the measured results with those predicted by eq. 2-14 shows that they are in good agreement, with a maximum difference between the two data sets not exceeding $2.5^\circ$ over a measured scan range of $26^\circ$.

**Figure 3-8.** The measured and the calculated beam-maximum direction of the microstrip antenna ($\varepsilon_r = 3$) shown in Fig. 3-3 as a function of the total applied reverse-bias voltage across the varactor diodes at $f = 6.25 \text{ GHz}$. 
For values of the calculated leakage constant $\alpha$ corresponding to different applied reverse-bias voltages in the range 0-900 volts, the $H$-plane power pattern of the microstrip antenna is calculated from the line-source and array theories presented earlier in Section 2.6. The calculated power pattern is then normalized by the corresponding measured maximum power gain, and is plotted in Fig. 3-9 where the measured $H$-plane power-gain and cross-polarization patterns are also shown.

Figure 3-9. The measured power gain (absolute) and cross polarization of the microstrip antenna ($\varepsilon_r = 3$) shown in Fig. 3-3 as a function of the total reverse-bias voltage applied across the varactor diodes at 6.25 GHz. Shown also are the calculated power patterns normalized to the measured maximum power gain (continued on the next page). Here, as well as in the rest of this thesis the so-called absolute method [25] for measuring the absolute antenna gain has been used, and is based on calibrating the antenna measurement system by means of two identical horn antennas.
Figure 3-9. Continued from the previous page.
Before any comparison of the calculated results shown in Fig. 3-9 is made with the measured ones, it is important to take into consideration the assumptions made in the theory:

1. The power carried by the incident traveling wave is totally dissipated by the time the wave reaches the end of the antenna (i.e. no reflection at antenna end);
2. The theory does not model the antenna feed or predict radiation from it;
3. The diodes are assumed to be identical;
4. The microstrip antenna dielectric is assumed to be homogeneous.

Figure 3-9 shows that, as the main-beam is steered at constant frequency from $-55.5^\circ$ to $-29.5^\circ$ away from broadside, the power gain of the antenna drops from 12 to 10 dBi, while the beamwidth narrows, and the cross-polarization level remains at least 24 dB below that of the main beam. Due to the fact that the two 50-Ω resistors located at the end of the antenna act as partial absorbers, the power remaining in the incident traveling wave as it reaches the end of the antenna is partially reflected and exhibits itself sometimes as a backlobe appearing at the same angle away from broadside as the main beam. Here, total absorption of the incident power remaining at the end of the antenna requires that the Bloch impedances [23] of the periodic structure be used as terminations.

The drop in the power gain of the antenna with increasing reverse-bias voltage is accompanied by an increase in the voltage standing-wave ratio, as the measured VSWR plot shown in Fig. 3-10 indicates. That is, the input mismatch increases with increasing reverse-bias voltage, thus contributing to a decrease in the power gain of the antenna. On the other hand, the gradual narrowing of the beamwidth is due to a gradual drop in the value of the leakage constant with increasing reverse-bias voltage as Fig. 3-11 shows.

On grounds of symmetry, the theory cannot predict the measured cross-polarization patterns shown in Fig. 3-9. Here, cross polarization is most likely due to (1) asymmetry in the feed of the antenna due to amplitude and phase unbalance in the $180^\circ$ hybrid, (2) weakly excited microstrip-patch modes orthogonal to the dominant $E_{10}^z$ mode, and (3) misalignment of the microstrip antenna with respect to the rectangular horn used in the measurement.
Finally, comparison of the calculated and the measured half-power beamwidths of the patterns shown in Fig. 3-9 shows that they are in agreement to within an approximate 4°. In addition, the calculated and the measured main-beam-maximum directions agree to within 2.5° over a scan range of 26°.

Figure 3-10. The measured voltage standing-wave ratio of the microstrip antenna ($\varepsilon_r = 3$) shown in Fig. 3-3 as a function of the total applied reverse-bias voltage across the varactor diodes at 6.25 GHz. Here, the VSWR was measured at the input of the 180° hybrid.

Figure 3-11. The calculated leakage constant $\alpha$ (normalized by the free-space wave number $k_0$) of the microstrip antenna ($\varepsilon_r = 3$) shown in Fig. 3-3 as a function of the total reverse-bias voltage applied across the varactor diodes at 6.25 GHz. Shown also is the calculated phase constant $\beta$ (normalized to $k_0$).
3.3.3 Operating bandwidth

In the frequency range 6.0 - 6.5 GHz, the reflection coefficient at the input of the 180° hybrid feeding the antenna shown in Fig. 3-3 was measured at three different reverse-bias voltages across the varactor diodes. The results are plotted on a Smith chart in Fig. 3-12 and show that as the frequency deviates from its center value of 6.25 GHz, the impedance mismatch at the input of the hybrid increases.

![Smith Chart](image)

**Figure 3-12.** The measured reflection coefficient at the input of the 180° hybrid feeding the microstrip antenna (ε_r = 3) shown in Fig. 3-3 in the frequency range 6.0 - 6.5 GHz, for different values of the reverse-bias voltage across the varactor diodes.

The H-plane power-gain patterns of the microstrip antenna shown in Fig. 3-3 were measured at three different frequencies and reverse-bias voltages. The results are shown in Fig. 3-13 where the movement of the main-beam direction with frequency is evident. If the operating bandwidth definition introduced in Subsection 3.2.4 is applied here, then the operating bandwidth of the leaky-wave microstrip antenna is at least 8%, with a maximum VSWR of 2.1.
Figure 3-13. The measured H-plane power gain of the microstrip antenna \((\varepsilon_r = 3)\) shown in Fig. 3-3 at three different frequencies and reverse-bias voltages.

3.3.4 Power analysis

Application of the equations presented in Sections 2.4-2.5 to the 31 cells of the microstrip antenna shown in Fig. 3-3 results in the power-distribution chart shown in Fig. 3-14, where it is assumed that the antenna is perfectly matched, and the applied reverse-bias voltage across the varactor diodes is set to 0 V.

Figure 3-14 shows that the power carried by the incident traveling wave decays exponentially along the antenna, and that the radiation efficiency in this case is approximately 63%. The remainder of the power is dissipated in the various loss mechanisms outlined in Fig. 3-14. Note here that the power lost in the \(\text{TM}_0\) surface-wave mode is less than 8% of the input power.
Figure 3-14. The distribution of power along the 31 cells of the microstrip antenna ($\varepsilon_r = 3$) shown in Fig. 3-3 at $f = 6.25$ GHz. The total reverse-bias voltage applied across the varactor diodes is set to 0 V.

3.3.5 Periodic-structure analysis

In order to gain a better insight into the theory of operation of a periodically loaded leaky-wave microstrip antenna, the Brillouin diagram of an infinite chain of capacitor-loaded microstrip cells such as that shown in the inset of Fig. 3-15 was generated based on the theory of Chapter 2. Here, all capacitors are treated as being infinitesimal devices of equal value $C$. In addition, the dimensions of the microstrip cell as well as the microstrip parameters are identical to those of the antenna illustrated in Fig. 3-3.
Figure 3-15. The Brillouin diagram of the microstrip antenna ($\varepsilon_r = 3$) shown in Fig. 3-3 when treated as a periodic structure of infinite extent. Here, the varactor diodes have been replaced with capacitors of equal value $C$.

At a constant frequency $f = 6.25$ GHz, Fig. 3-15 shows that as the capacitor value $C$ is increased from 0.5 pF to infinity, the phase velocity along the periodic structure decreases, and as a result, the direction of the main-beam maximum approaches endfire. Consequently, a change in the capacitor value $C$ may be regarded as causing a transition at constant frequency from one dispersion curve to the next. Based on Fig. 3-15, one concludes that constant-frequency steering of the main-beam toward broadside requires smaller capacitor values.
3.3.6 Interference measurements

In this section, spectrum-analyzer measurements are used to assess the response of the microstrip antenna shown in Fig. 3-3 to harmonic and intermodulation interference for different reverse-bias-voltage settings across the varactor diodes. The voltage settings are specifically chosen so as to exercise different degrees of nonlinearity in the varactor diodes.

**Harmonic distortion.** In Fig. 3-16, the microstrip antenna shown in Fig. 3-3 is operated in the receiving mode at a frequency $f = 6.5$ GHz. A transmitting horn, located 2.5 m away, is positioned such that it is aligned with $\theta_m$, the main-beam-maximum direction of the receiving microstrip antenna, so as to deliver maximum power to it at three different reverse-bias-voltage settings across the varactor diodes. In this experiment, a spectrum analyzer connected to the receiving antenna is used to monitor the amount of power delivered to its 50-$\Omega$ internal impedance. In addition, 13.2 dBm, the maximum power output available from one signal generator, is delivered to the transmitting horn.

Table 3-3 lists the power $P$ (dBm) delivered to the 50-$\Omega$ internal impedance of the spectrum analyzer at the fundamental frequency and at the first two harmonics for different voltages across the varactor diodes. For a radiated power density of 13.7 mW/m$^2$ at the receiving antenna, the received power at each of the harmonics is at least 62 dB below that received at the fundamental. In this case, the receiving antenna may be regarded as a linear system. Note here that the 180° hybrid covers the frequency range 2-18 GHz as listed in Table 3-2, while the half-power bandwidth of the bias tee extends from 750 kHz to 10 GHz only as Table 3-1 shows.
Figure 3-16. The measurement set-up used for assessing the response of the microstrip antenna \( \varepsilon_r = 3 \) illustrated in Fig. 3-3 to harmonic interference.

Table 3-3  The Measured Response of the Microstrip Antenna \( \varepsilon_r = 3 \) Shown in Fig. 3-3 to Harmonic Interference

<table>
<thead>
<tr>
<th>( V_{dc} ) (V)</th>
<th>( P ) (dBm) at 6.5 GHz</th>
<th>( P ) (dBm) at 13 GHz</th>
<th>( P ) (dBm) at 19.5 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-25.4</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
</tr>
<tr>
<td>400</td>
<td>-27.1</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
</tr>
<tr>
<td>900</td>
<td>-26.6</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
</tr>
</tbody>
</table>
Intermodulation distortion. In Fig. 3-17, the microstrip antenna shown in Fig. 3-3 is used as a receiver of two signals, the first of which is transmitted by horn 1 at a frequency \( f_1 = 6.25 \) GHz, while the second is transmitted by horn 2 at \( f_2 = 6.5 \) GHz. The transmitting horns are positioned such that they deliver maximum power to the receiving antenna at three different reverse-bias voltages. The first horn is positioned such that it is aligned with \( \theta_{m1} \), the direction of the beam maximum of the receiving antenna at \( f_1 = 6.25 \) GHz. Similarly, the second horn is positioned such that it is aligned with \( \theta_{m2} \), the direction of the beam maximum of the receiving antenna at \( f_2 = 6.5 \) GHz. A spectrum analyzer connected to the bias tees of the receiving antenna via the 180° hybrid is used to monitor the power delivered to its 50-Ω internal impedance at the various fundamental and intermodulation frequencies. The 180° hybrid is the same as that used previously, having a bandwidth of 2-18 GHz as listed in Table 3-2. On the other hand, the 3-dB bandwidth of the bias tees extends from 750 kHz to 10 GHz only as shown in Table 3-1. Here, 2.8 dBm, the maximum power available from one signal generator, is delivered to the first transmitting horn, while 13.2 dBm, the maximum available from another generator, is delivered to the second.

For different reverse-bias voltages across the varactor diodes, Table 3-4 lists the power \( P \) (dBm) delivered by the receiving microstrip antenna to the 50-Ω internal impedance of the spectrum analyzer at the fundamental and various intermodulation frequencies. For power densities of 1.1 mW/m² (at \( f_1 = 6.25 \) GHz) and 13.7 mW/m² (at \( f_2 = 6.5 \) GHz) at the receiving antenna, the power received at the intermodulation frequencies is at least 50 dB below that received at the fundamentals. This leads one to the conclusion that the power transmitted by both horns is not large enough to cause any measurable intermodulation distortion. In this case, the receiving antenna may be regarded as operating in the linear mode.
Figure 3-17. The measurement set-up used for assessing the response of the microstrip antenna (\(\varepsilon_r = 3\)) shown in Fig. 3-3 to intermodulation interference.
Table 3-4 The Measured Response of the Microstrip Antenna ($\varepsilon_r = 3$) Shown in Fig. 3-3 to Intermodulation Interference

<table>
<thead>
<tr>
<th>$V_{dc}$ (V)</th>
<th>$P$ (dBm) at $f_1$ (6.25 GHz)</th>
<th>$P$ (dBm) at $f_2$ (6.5 GHz)</th>
<th>$P$ (dBm) at $2f_2-f_1$ (250 MHz)</th>
<th>$P$ (dBm) at $2f_2-f_1$ (6.75 GHz)</th>
<th>$P$ (dBm) at $f_1+f_2$ (12.75 GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-35.77</td>
<td>-25.4</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
</tr>
<tr>
<td>400</td>
<td>-37.27</td>
<td>-27.1</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
</tr>
<tr>
<td>900</td>
<td>-39.90</td>
<td>-26.6</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
</tr>
</tbody>
</table>

3.4 Periodically Loaded Leaky-Wave Microstrip Antenna at 5.2 GHz

In order to determine the effect of the microstrip relative permittivity on the phase velocity of a leaky-wave microstrip antenna, the structure shown in Fig. 3-18 was subsequently built from microstrip having a relative permittivity $\varepsilon_r = 6.15 \pm 0.15$, and a loss tangent $\tan \delta = 0.0025$ at 10.0 GHz. The antenna consists of 31 identical microstrip cells of width $d = 1.2$ cm each, and 60 identical varactor diodes as shown in Fig. 3-18. Each cell is made up of a rectangular microstrip patch of resonant length $a = 1.20$ cm and of width $b = 1.0$ cm, and 0.3-mm-wide transmission-line sections connecting the microstrip patch to the varactor diodes. Following the design guidelines presented in Section 3.2, the number of microstrip cells is chosen such that approximately 95% of the input power is dissipated along the length of the antenna, with the remainder of the power to be partially absorbed in two 100-Ω resistors located at the antenna end farthest from the feed. As was the case for the antenna considered in Section 3.3, a PPR (percentage of power to be radiated along the length of the antenna) higher than that suggested in Subsection 3.2.2 is chosen so as to compensate for the partial power absorption provided by the 100-Ω resistors. Here, total absorption of the power remaining in the incident wave as it reaches the end of the antenna requires that the latter be terminated in its Bloch impedances [23].
Figure 3-18. A periodically loaded leaky-wave microstrip antenna made up of 31 microstrip cells of width $d = 1.2$ cm each, and 60 identical varactor diodes. Each cell contains a rectangular microstrip patch of dimension $a \times b$ where $a = 1.2$ cm and $b = 1.0$ cm. The antenna is fed using two bias tees and a $180^\circ$ hybrid having the specifications shown in Tables 3-1 and 3-2 respectively.
3.4.1 Coupling between adjacent microstrip cells

The same procedure used previously in Subsection 3.3.1 to examine the coupling between two adjacent microstrip cells is applied here to a higher relative permittivity ($\varepsilon_r=6.15$) microstrip. The coupling coefficient $|S_{21}|$ of two adjacent microstrip cells is calculated using the method of moments [28-32] described briefly in Subsection 3.3.1 for the following two cases:

1. Electromagnetic and transmission-line coupling between cells is accounted for. This case is illustrated in Fig. 3-19, and is known as the fully coupled two-cell system;

2. The two cells are coupled via the short transmission-line sections only (no electromagnetic coupling). This case is depicted in Fig. 3-20, and is referred to as the partially coupled two-cell system.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3-19.png}
\caption{Fully coupled two-cell system used to assess the degree of coupling that takes place between two rectangular microstrip patches connected via a pair of short transmission-line sections.}
\end{figure}
The calculated coupling coefficients of the structures shown in Figs. 3-19 and 3-20 are shown in Fig. 3-21, in the frequency range 5-5.5 GHz. Since less than 6% of the input power to the antenna is lost to surface waves as is shown in Subsection 3.4.4, surface-wave effects are neglected when applying the method of moments, and a homogeneous dielectric of relative permittivity \( \varepsilon_r = 6.15 \) is assumed to fill all of space.

Figure 3-20. Partially coupled two-cell system used in assessing the degree of coupling between two identical rectangular microstrip patches connected via a pair of short transmission line sections. Note here that the network parameters of the two-cell system are found from those of a single cell. Therefore, coupling between the two patches other than that which takes place via the transmission-line pair is not included.
In the frequency range 5-5.5 GHz, Fig. 3-21 shows that the difference between the calculated coupling coefficients of both structures is less than 0.5 dB. This leads one to conclude that the short transmission-line sections connecting adjacent microstrip patches are responsible for most of the coupling between them. Therefore, the equations of Chapter 2 are applicable to the antenna considered in this section.

Figure 3-21. The calculated coupling coefficient of the structure shown in Fig. 3-19 compared with that of the structure shown in Fig. 3-20 in the range 5-5.5 GHz.

3.4.2 Power gain, cross polarization, and standing-wave ratio

At a frequency \( f = 5.2 \) GHz, the microstrip antenna shown in Fig. 3-18 is fed using the 180° hybrid and the two bias tees described previously in Tables 3-1 and 3-2. The equations of Section 2.4 are then used to determine the four-port impedance matrix of a single microstrip cell. For different reverse-bias voltages applied across the varactor diodes via the DC voltage source \( V_{dc} \), the diode impedance is found and used in eq. (2-13) to calculate the complex propagation constant \( \gamma \) of the periodic structure. As a result, the leakage constant \( \alpha \) of the antenna as well as the direction of its main-beam maximum measured from broadside
are readily calculated using eqs. (2-13) and (2-14) respectively. Note here that the varactor diodes are treated as being infinitesimal, and are identical to those used in the antenna of Fig. 3-3.

Figure 3-22 shows the measured direction of the main-beam maximum, $\theta_m$, as a function of the total reverse-bias voltage applied across the varactor diodes, at a fixed frequency $f = 5.2$ GHz. Shown also in the same figure is the calculated direction of the main-beam maximum, found using eq. (2-14). Inspection of Fig. 3-22 reveals that a $60^\circ$ scan range was achieved both theoretically and experimentally, with a difference of less than $7.5^\circ$ between the measured and the calculated data sets.

![Figure 3-22](image_url)

**Figure 3-22.** The measured and the calculated main-beam-maximum direction of the microstrip antenna ($\varepsilon_r = 6.15$) shown in Fig. 3-18 as a function of the total reverse-bias voltage applied across the varactor diodes at $f = 5.2$ GHz.
Having shown the constant-frequency main-beam scanning action as a function of applied reverse-bias voltage, the measured $H$-plane power gain of the microstrip antenna shown in Fig. 3-18 was then plotted in Fig. 3-23 for different reverse-bias voltages across the varactor diodes. Shown also in the same figure are the corresponding measured cross-polarization patterns. Next, the microstrip antenna was treated as a (1) line source, and (2) nonuniformly excited linear array. In both cases, the $H$-plane power patterns were calculated from the equations of Section 2.6 and the calculated leakage constant at the different reverse-bias voltages. These patterns were then normalized to the corresponding measured maximum power-gain, and are shown in Fig. 3-23.

![Figure 3-23](image)

**Figure 3-23.** The measured power gain and cross-polarization patterns of the microstrip antenna ($\varepsilon_r = 6.15$) shown in Fig. 3-18 as a function of the total reverse-bias voltage applied across the varactor diodes at 5.2 GHz. Shown also are the calculated power patterns normalized to the measured maximum power gain (continued on the next page).
Figure 3-23. Continued from the previous page.
Figure 3-23 shows a constant-frequency gradual drop in the power gain of the antenna (from 12 dBi to -6 dBi) as the direction of the main-beam maximum approaches broadside, while the cross-polarization level remains below -10 dBi. The drop in the power gain is due to an increase in the antenna input mismatch with increasing reverse-bias voltage as the measured VSWR plot shown in Fig. 3-24 suggests. On the other hand, since on theoretical grounds the $E_{10}^z$ mode cannot contribute to the cross-polarization field, possible sources of cross polarization are

1. asymmetry in the feed mechanism (i.e. phase and amplitude unbalance in the $180^\circ$ hybrid) resulting in the generation of common-mode currents on the transmission-line sections connecting adjacent microstrip patches;

2. the presence of weakly excited higher-order modes orthogonal to the dominant $E_{10}^z$ mode;

3. the $\pm 10\%$ variation in the varactor-diode parameters as quoted by the manufacturer [33];

4. misalignment of the leaky-wave microstrip antenna with respect to the rectangular horn used in the antenna measurement set-up.

A close look at Fig. 3-23 reveals that the width of the main beam increases as the latter approaches broadside. This effect is attributed to the sharp increase in the calculated leakage constant in that region as shown in Fig. 3-25. That is, when the leakage constant becomes large, the input power into the leaky-wave microstrip antenna gets radiated by the first few patches so as to make the effective aperture short, thus resulting in a wide main beam.

Since no provisions were made to totally absorb the power remaining in the incident wave at the end of the antenna, backlobes appearing at the same angle away from broadside as the main beam are sometimes evident in the measured power patterns shown in Fig. 3-23. Here, the two 100-$\Omega$ resistors located at the antenna end act as partial absorbers, and total absorption of the remaining incident power requires that the last radiating patch be terminated in the Bloch impedances [23] of the periodic structure.
Figure 3-24. The measured voltage standing-wave ratio of the microstrip antenna ($\varepsilon_r = 6.15$) shown in Fig. 3-18 as a function of the total reverse-bias voltage applied across the varactor diodes at 5.2 GHz. Here, the VSWR was measured at the input of the $180^\circ$ hybrid.

Figure 3-25. The calculated leakage constant $\alpha$ (normalized by the free-space wave number $k_0$) of the microstrip antenna ($\varepsilon_r = 6.15$) shown in Fig. 3-18 as a function of the total reverse-bias voltage applied across the varactor diodes at 5.2 GHz. Shown also is the calculated phase constant $\beta$ (normalized to $k_0$).
3.4.3 Operating bandwidth

For three different reverse-bias voltages across the varactor diodes, the reflection coefficient of the antenna shown in Fig. 3-18 was measured at the input of the 180° hybrid in the frequency range 4.9 - 5.5 GHz. The results are plotted on a Smith chart in Fig. 3-26. These results show that an increase in the frequency is accompanied for the most part by an increase in the input mismatch. On the other hand, the measured H-plane power-gain patterns of this antenna (\(\varepsilon_r = 6.15\)) at a reverse-bias voltage of 400 V show a faster main-beam frequency-scanning rate as compared with the antenna (\(\varepsilon_r = 3\)) considered in Section 3.3. Using the operating bandwidth definition introduced in Subsection 3.2.4, one concludes that the operating bandwidth decreases with an increasing relative permittivity. For the antenna considered here, the operating bandwidth is approximately 3% at a center frequency of 5.2 GHz and a maximum VSWR of 2.1.

![Figure 3-26](image)

Figure 3-26. The measured reflection coefficient at the input of the 180° hybrid feeding the microstrip antenna (\(\varepsilon_r = 6.15\)) shown in Fig. 3-18 in the frequency range 4.9 - 5.5 GHz, for different reverse-bias voltages across the varactor diodes. Shown also are the measured H-plane power-gain patterns at a fixed reverse-bias voltage and three different frequencies.
3.4.4 Power analysis

In order to gain a clearer insight into the distribution of power along a microstrip antenna such as that illustrated in Fig. 3-18, the power-distribution chart shown in Fig. 3-27 was generated. Note that in producing this chart, use was made of the equations developed in Sections 2.4-2.5, and the following assumptions:

(1) the antenna is perfectly matched;

(2) the reverse-bias voltage across the varactor diodes is 0 V.

Figure 3-27. The power breakdown along the 31-cell periodically loaded leaky-wave microstrip antenna ($\varepsilon_r = 6.15$) shown in Fig. 3-18 at $f = 5.2$ GHz. The total reverse-bias voltage applied across the varactor diodes is set to 0 V.
Figure 3-27 clearly shows the exponential decay of the power carried by the incident traveling wave as it makes its way along the antenna. It also shows the distribution of power among the various loss mechanisms outlined in the figure's legend. Note that the radiation efficiency is approximately 43% in this case, with high dielectric losses (tanδ = 0.0025) compared with the microstrip antenna (εr = 3, tanδ = 0.0013) studied in Section 3.3. Here, less than 6% of the input power is lost in the TM0 surface-wave mode.

3.5 Summary

In this chapter, the theoretical predictions linking the main-beam direction of a leaky-wave microstrip antenna to the variable reactive loads placed periodically along it have been verified experimentally. The effect of the dielectric on the scan range of the antenna has been demonstrated through two examples having different relative permittivities. For a relative permittivity εr = 3, an approximate 26° scan range was obtained from both theory and measurements at a constant frequency f = 6.25 GHz. On the other hand, a 60° scan range was achieved both theoretically and experimentally at a constant frequency f = 5.2 GHz using a microstrip with a relative permittivity εr = 6.15.

Finally, experiments were used to assess the response of a periodically loaded leaky-wave microstrip receiving antenna with a relative permittivity εr = 3 to harmonic and intermodulation interference. In the harmonic interference test, the antenna was subjected to a power density of 13.7 mW/m² (at f = 6.5 GHz). In this case, the power received at the first two harmonic frequencies is at least 62 dB below that received at the fundamental. In the intermodulation interference case, two radiating sources providing power densities of 1.1 mW/m² (at f₁ = 6.25 GHz) and 13.7 mW/m² (at f₂ = 6.5 GHz) at the receiving antenna were used. Here, the power received at the intermodulation frequencies is at least 50 dB below that received at the fundamentals. One concludes that, for power densities below those used in the aforementioned measurements, the receiving antenna may be regarded as a linear system.
Chapter 4

FIXED-FREQUENCY BEAM-STEERABLE LEAKY-WAVEGUIDE-ANTENNA ANALYSIS

4.1 Introduction

In this chapter, an analysis is given of the wave propagation along a structure of periodically loaded infinitely long thin wires parallel to a perfectly conducting plane. It is shown that the phase velocity of waves traveling along such structure is a function of the lumped loads of equal value placed at regular intervals along each wire. This property is used in Chapter 5 to design fixed-frequency beam-steerable leaky-wave antennas.

4.2 Periodically Loaded Leaky-Waveguide Antenna

In 1959, Honey [34] introduced a leaky-wave antenna based on the TE_{10} propagation mode of a rectangular waveguide. He showed that by properly controlling the height above ground and the spacing between long closely spaced parallel wires, power patterns with low sidelobe levels can be achieved. In addition, by varying the operating frequency in the range 7-13 GHz, he was able to steer the direction of the main-beam in the H-plane of the antenna between 20° and 60° away from broadside.

A depiction of the leaky-wave antenna considered by Honey is shown in Fig. 4-1. This antenna is commonly referred to as an inductive-grid leaky waveguide [27], and consists of an array of closely spaced long wires of radius \( r \) located in free space at a height \( \lambda_0/2 < a < \lambda_0 \) above a conducting ground plane, where \( \lambda_0 \) is the free-space wavelength.
Figure 4-1. The inductive-grid leaky-waveguide antenna introduced by Honey [34] in 1959. An array of long wires of radius $r$ is placed at a height $\lambda_0/2 < a < \lambda_0$ above a conducting ground plane, with adjacent wires separated by a small spacing $s \ll \lambda_0$.

Inspection of the slitted-wall rectangular-waveguide antenna considered previously in Fig. 1-1 shows that by varying its width $a$, the directions of the component plane waves inside it change, thus resulting in a change in the direction of its main beam. The same observation applies also to the inductive-grid leaky-waveguide antenna considered here. That is, the main-beam direction of this antenna may be steered at constant frequency by varying the height of its inductive-grid wall above ground.

If lumped reactive loads of equal value $Z_d$ are introduced at regular intervals $p$ along each wire of the inductive grid, the resulting system may be viewed as a reactive wall whose reactance controls the phase of the internal component plane waves impinging on it. A change in the value of this reactance has the effect of changing the effective height of the unloaded grid wall above ground. Consequently, one expects the direction of the main beam to change at constant frequency.

If in addition to the lumped loads $Z_d$, a homogeneous dielectric medium of relative permittivity $\varepsilon_r$ is introduced between the wire array and the ground plane, the structure shown in Fig. 4-2 results, where the spacing $s$ between adjacent wires and the loading interval $p$ are taken to be small in comparison with the wavelength in the dielectric medium, $\lambda_d$. This structure may be regarded as a guiding system which supports transverse-electric (TE) waves prop-
agating in the $z$ direction, and will be studied using a technique similar to that used by Honey [34] for analyzing the inductive-grid leaky-waveguide antenna. This technique is based on modeling wave propagation in the $x$ direction by an equivalent transmission-line circuit to which the transverse resonance method [15] is applied and used in conjunction with the wave equation to determine the complex propagation constant of the TE waves traveling along the antenna in the $z$ direction.

![Diagram](image)

**Figure 4-2.** The periodically loaded leaky-waveguide antenna studied in this chapter. Lumped loads of equal value $Z_d$ are placed at regular intervals $p$ along each wire. A homogeneous dielectric medium of relative permittivity $\varepsilon_r$ is assumed to exist between the infinite array of infinitely long, closely spaced wires and the perfectly conducting ground plane. Here, the loading interval $p$ is taken to be small in comparison with the wavelength in the dielectric medium, $\lambda_d$. 
4.2.1 A transmission-line model for wave propagation in the x direction

Wait [35] considered the problem of reflection of a uniform plane wave obliquely incident on, and polarized parallel to a periodic structure of infinitely long thin wires parallel to a plane interface between two homogeneous dielectrics. He showed that the space on either side of the interface can be represented by a transmission line, while the array of wires itself can be represented by a shunt impedance across one of these lines.

A depiction of the problem is given in Fig. 4-3 where the wire array is located at a distance \( a \) away from the plane interface between two dielectrics of relative permittivities \( \varepsilon \) and \( \varepsilon_d \). The distance \( s \) between adjacent wires is assumed to be large in comparison with the wire radius \( r \). A uniform plane wave incident on the wire array at an angle \( \theta \) has its electric field polarized parallel to the wires. An equivalent circuit for this structure, determined by Wait [35], is shown in Fig. 4-4 and consists of two semi-infinite transmission lines of characteristic impedance

\[
Z_c = \eta \sec(\theta)
\]

and

\[
Z_{cd} = \eta_d \sec(\theta_d)
\]

where

\[\eta\] is the intrinsic impedance of the dielectric medium of relative permittivity \( \varepsilon \);

\[\eta_d\] is the intrinsic impedance of the dielectric medium of relative permittivity \( \varepsilon_d \).

and a shunt impedance \( Z_g \) representing the array of wires.
Figure 4-3. A uniform plane wave incident on an array of infinitely long thin wires parallel to a plane interface between two homogeneous dielectrics. The electric field is polarized parallel to the wires.

Figure 4-4. An equivalent transmission-line model for the system shown in Fig. 4-3 as determined by Wait [35].
For perfectly conducting wires, the impedance of the wire array is given in [35] by

\[ Z_g = j\frac{s}{\lambda} \eta \left[ \ln \left( \frac{s}{2\pi r} \right) + \Delta \right] \]  \hspace{1cm} (4-3)

where

- \( s \) is the distance between adjacent wires;
- \( \lambda \) is the wavelength in the medium containing the wire array;
- \( r \) is the wire radius;
- \( \Delta \) is a correction factor which depends on \( \sigma, s, \) and the angle of incidence \( \theta \).

For \( a, s \ll \lambda \), the correction factor may be neglected, and the impedance of the wire array may be approximated as

\[ Z_g = j\frac{s}{\lambda} \eta \ln \left( \frac{s}{2\pi r} \right) \] \hspace{1cm} (4-4)

On the other hand, for wires having a finite conductivity \( \sigma \), it has been shown by Wait [35] that eq. (4-4) takes the form

\[ Z_g = j\frac{s}{\lambda} \eta \ln \left( \frac{s}{2\pi r} \right) + (1 + j) \frac{s}{2\pi r} \eta \frac{\mu \omega}{2\sigma} \] \hspace{1cm} (4-5)

where \( \omega \) is the radian frequency.

If lumped loads of equal value \( Z_d \) are introduced at regular intervals \( p \) along each wire in the periodic structure of Fig. 4-3, the impedance of the wire array \( Z_g \) must be modified to incorporate the effect of these loads. For a small interval \( p \) between adjacent loads, and a small spacing \( s \) between adjacent wires (i.e. \( p, s \ll \lambda \)), the lumped loads contribute an amount

\[ Z_1 = \frac{Z_d}{p} s \] \hspace{1cm} (4-6)
to the impedance $Z_g$ of the array of unloaded wires. In this case, the impedance of the periodically loaded wire array becomes

$$Z_{gl} = j \frac{s}{\lambda} \ln \left( \frac{s}{2\pi r} \right) + \frac{Z_d}{p} s$$

(4-7)

and the corresponding equivalent circuit is shown in Fig. 4-5.

![Equivalent Circuit Diagram](image)

**Figure 4-5.** An equivalent transmission-line model for the system shown in Fig. 4-3 with lumped loads of equal value $Z_d$ placed at regular intervals $p$ along each wire.

The transmission-line model shown in Fig. 4-5 may be readily used to obtain an equivalent circuit for the wave propagation along the $x$ direction in the periodically loaded leaky-waveguide antenna shown in Fig. 4-2. This equivalent circuit is shown in Fig. 4-6 where

- $Z_c$ is the characteristic impedance of the transmission line representing the free-space medium located above the wire array;
- $Z_{cd}$ is the characteristic impedance of the transmission line representing the homogeneous dielectric medium located between the wire array and the ground plane;
- $Z_{gl}$ is the impedance of the periodically loaded wire array as given in eq. (4-7);
- $\eta$ is the intrinsic impedance of free space;
\( \eta_d \) is the intrinsic impedance of the homogeneous dielectric medium located between the wire array and the ground plane;

\( \theta_d \) is the angle of plane-wave incidence or reflection at the dielectric-free-space interface, measured from the normal to the wire array;

\( \theta \) is the angle of plane-wave transmission through the dielectric-free-space interface, measured from the normal to the wire array.

\[ Z_c = \eta \sec(\theta) \]
\[ Z_{gl} = Z_g + Z_1 \]
\[ Z_{cd} = \eta_d \sec(\theta_d) \]

**Figure 4-6.** A transmission-line model for the wave propagation along the \( x \) direction of the periodically loaded leaky-waveguide antenna shown in Fig. 4-2.

### 4.2.2 The complex propagation constant of the periodically loaded leaky waveguide

In order to determine the propagation constants of the TE waves that are supported by the leaky-waveguide antenna shown in Fig. 4-2, the transverse-resonance technique [15] is applied to the transmission-line model shown in Fig. 4-6. This technique ensures that the boundary conditions are satisfied in any plane parallel to the wire array by requiring that the
sum of either the impedances or admittances seen when looking on opposite sides of this plane be equal to zero. With the plane of the wire array chosen as a reference, this technique results in the following equation:

$$\frac{Z_{cd}}{Z_{gl}} + \frac{Z_{cd}}{Z_c} + \coth(\gamma_{xd}a) = 0$$

(4-8)

where $\gamma_{xd} = \alpha_{xd} + j\beta_{xd}$ is the $x$ component of the complex propagation constant, and $\alpha_{xd}$ and $\beta_{xd}$ are the corresponding attenuation and phase constants respectively. Then, by substituting eqs. (4-1)-(4-2), (4-7), and the identity

$$\eta_d = \frac{\eta}{\lambda_d}$$

(4-9)

into eq. (4-8), one may show that

$$0 = \frac{j2\pi \cos(\theta_d)}{\lambda_d} \left[ \frac{s}{2\pi a} \ln\left(\frac{s}{2\pi r}\right) - j\frac{s}{2\pi a} \right] + \frac{1.0}{\frac{Z_{cd}}{Z_c} + \coth(\gamma_{xd}a)}$$

$$= \frac{(\gamma_{xd}a)s}{2\pi a} \left[ \ln\left(\frac{s}{2\pi r}\right) - j\frac{Z_{norm}}{p} \right] + \frac{1.0}{\frac{Z_{cd}}{Z_c} + \coth(\gamma_{xd}a)}$$

(4-10)

where $\lambda$ and $\lambda_d$ are the wavelengths in free space and in the homogeneous dielectric medium respectively, and $Z_{norm}$ is defined as

$$Z_{norm} = \frac{\lambda_d}{\eta_d}$$

(4-11)

In order to solve for $\gamma_{xd}$, the complex propagation constant along the antenna of Fig. 4-2, use is made of the wave equation which requires that

$$\gamma_{xd}^2 + \gamma_{yd}^2 + \gamma_{zd}^2 + k_d^2 = 0$$

(4-12)
where

\[ k_d^2 = \omega^2 \mu_0 \varepsilon_0 \varepsilon_r \]  \hspace{1cm} (4-13)

\[ \lambda_d = \text{Im}(k_d) \]  \hspace{1cm} (4-14)

and

\[ \gamma_{yd} = \alpha_{yd} + j\beta_{yd} \] is the complex propagation constant in the y direction;

\[ \gamma_{zd} = \alpha_{zd} + j\beta_{zd} \] is the complex propagation constant in the z direction;

\[ \omega = 2\pi f \] is the radian frequency;

\[ f \] is the operating frequency.

With no variations in the y direction, \( \gamma_{yd} = 0 \), and eq. (4-12) reduces to

\[ \gamma_{zd}^2 + k_d^2 = 0 \]  \hspace{1cm} (4-15)

or, alternatively,

\[ \gamma_{zd} = \sqrt{-k_d^2 - \gamma_{xd}^2} = \alpha_{zd} + j\beta_{zd} \]  \hspace{1cm} (4-16)

so that use of the identity

\[ \sin(\theta_d) = \frac{\lambda_d}{\lambda} \sin(\theta) = \frac{\lambda_d}{\lambda_r} = \frac{\lambda_d}{2\pi \beta_{zd}} \]  \hspace{1cm} (4-17)

in eqs. (4-1) and (4-2) results in the following expression for the impedance ratio \( \frac{Z_{cd}}{Z_c} \):

\[
\frac{Z_{cd}}{Z_c} = \frac{\eta_d}{\eta} \frac{\cos\left(\arcsin\left(\frac{\lambda_d \beta_{zd}}{2\pi}\right)\right)}{\cos\left(\arcsin\left(\frac{\lambda_d}{2\pi}\right)\right)} = \frac{\eta_d}{\eta} \frac{\cos\left(\arcsin\left(\frac{\lambda \text{Im}\left(\sqrt{-k_d^2 - \gamma_{xd}^2}\right)}{2\pi}\right)\right)}{\cos\left(\arcsin\left(\frac{\lambda_r}{2\pi}\right)\right)} \]  \hspace{1cm} (4-18)
where $\beta_{zd}$ and $\lambda_g$ are the phase constant and dielectric-guide wavelength along the $z$ direction respectively. In this case, eq. (4-10) becomes

\[
\frac{(\gamma_{zd}a)s}{2\pi a} \left[ \ln \left( \frac{s}{2\pi r} \right) - j \frac{Z_{norm}}{p} \right] + \frac{1.0}{\eta_d \eta \cos \left( \frac{\lambda_{zd} \text{Im}(\sqrt{\frac{k_d^2 - \gamma_{zd}^2}{2\pi}})}{2\pi} \right)} + \coth (\gamma_{zd}a) = 0 \tag{4-19}
\]

which can be solved for the unknown variable $\gamma_{zd}$ using a complex secant technique [36]. As a result, the complex propagation constant along the periodically loaded leaky-waveguide antenna of Fig. 4-2 can be determined using eq. (4-16), and used to find $\theta_m$, the direction of the $y$-polarized main beam measured from broadside, using

\[
\theta_m = \sin \left( \frac{\lambda}{\lambda_g} \right) = \sin \left( \frac{\lambda}{2\pi} \beta_{zd} \right) \tag{4-20}
\]

and the leakage constant $\alpha_{zd} = \text{Re}(\gamma_{zd})$. Since eq. (4-19) is a function of the lumped loads $Z_d$ and the relative permittivity $\varepsilon_r$, its solution depends on the values of these variables. Consequently, the direction of the main-beam maximum, $\theta_m$, and the leakage constant, $\alpha_{zd}$, are functions of the lumped loads and the relative permittivity. The extent of this dependence will be addressed in Chapter 5.

Let the lumped loads of the wire array be replaced with short circuits (i.e. $Z_d = 0$), and the dielectric medium contained between the wire array and the ground plane be taken as free space (i.e. $\varepsilon_d = \varepsilon_0$). Then, the periodically loaded leaky-waveguide antenna shown in Fig. 4-2 becomes identical to the inductive-grid leaky waveguide considered by Honey [34] for which eq. (4-19) for the complex propagation constant $\gamma_{zd}$ reduces to

\[
(\gamma_{zd}a) \left[ \frac{s}{2\pi a} \ln \left( \frac{s}{2\pi r} \right) \right] + \frac{1.0}{1.0 + \coth (\gamma_{zd}a)} = 0 \tag{4-21}
\]
4.3 The Effective-Dielectric-Constant (EDC) Technique

The analysis technique presented in Section 4.2 assumed that the leaky waveguide is infinitely wide, and as such is not practical. In particular, one expects the phase constant $\beta_{zd}$ along a leaky-waveguide antenna of infinite width to be different from that of finite width due to the fact that the latter no longer supports pure TE modes, but rather hybrid modes involving TE-TM coupling as shown by Schwering and Oliner [4]. On the other hand, for a leaky-waveguide antenna having a large width-to-height ratio, the leakage constant $\alpha_{zd}$ is, to a good approximation, the same as that of an infinitely wide antenna [4].

A simple and accurate technique that has been successfully used to calculate the phase constant of metal and dielectric grating antennas of finite width is the effective-dielectric-constant technique [4] illustrated in Fig. 4-7. In this technique, the finite-width dielectric slab of a leaky-waveguide antenna is replaced with one of infinite extent, having an effective dielectric constant $\varepsilon_{\text{eff}}$ that depends on the width-to-height ratio of the finite-width dielectric slab. Consequently, the finite-width leaky waveguide is transformed into an antenna of infinite width whose phase constant $\beta_{zd}$ may be determined using the technique previously developed in Section 4.2.

Using the effective-dielectric-constant technique [4], one may show that when the leaky-waveguide-antenna width $w > \lambda_0/(\sqrt{\varepsilon_{\text{eff}} - 1})$, where $\lambda_0$ is the free-space wavelength, the effective dielectric constant $\varepsilon_{\text{eff}}$ can be calculated from the following equation:

$$\frac{\sqrt{\varepsilon_{\text{r}} - \varepsilon_{\text{eff}}}}{\varepsilon_{\text{eff}} - 1} \tan(k_0a \sqrt{\varepsilon_{\text{r}} - \varepsilon_{\text{eff}}}) = 1.0$$ (4-22)

where $k_0$ is the free-space wave number. On the other hand, if $w \leq \lambda_0/(\sqrt{\varepsilon_{\text{eff}} - 1})$, the effective dielectric constant can be calculated from

$$\frac{\sqrt{\varepsilon_{\text{r}} - \varepsilon_{\text{eff}}}}{\varepsilon_{\text{eff}} - 1} \tan(k_0a \sqrt{\varepsilon_{\text{r}} - \varepsilon_{\text{eff}}}) = \varepsilon_{\text{r}}$$ (4-23)
4.4 The Periodically Loaded Leaky-Waveguide Antenna as a Line Source

In order to determine the $H$-plane power pattern of a leaky waveguide, the latter is treated as a line source of infinitesimal width $\Delta w$ and of length $L$, positioned along the $z$ axis of a rectangular coordinate system such as shown in Fig. 4-8. For a wave of $y$-oriented current $I_y(z)$ traveling along the line source, the pattern factor $f(\theta, \phi)$ is given in [7] by

$$f(\theta, \phi) = \int_0^L I_y(z)e^{jkz\sin(\theta)} dz$$  \hspace{1cm} (4-24)
\[ k_0 = \frac{2\pi}{\lambda_0} \]  

(4-25)

where \( \lambda_0 \) is the free-space wavelength, and \( \theta \) and \( \phi \) are the elevation and azimuth angles respectively.

---

![Diagram](image)

**Figure 4-8.** Line source of length \( L \) and infinitesimal width \( \Delta w \). A wave of \( y \)-oriented current is assumed to be traveling along the line source in the positive \( z \) direction.

For an exponentially decaying current wave traveling along the length of the antenna, \( I_y(z) \) may be expressed as

\[ I_y(z) = I_0 e^{-\alpha z} e^{-j\psi(z)} \]  

(4-26)

where \( I_0 \) is the current-wave amplitude, \( \alpha \) is its attenuation or leakage constant, and \( \psi(z) \) is its phase. For a traveling wave with a phase constant \( \beta \), \( \psi(z) \) may be written as

\[ \psi(z) = k_0 z \sin(\theta_0) \]  

(4-27)
where
\[
\sin(\theta_0) = \frac{\beta}{k_0} \tag{4-28}
\]

Substitution of eqs. (4-26) and (4-27) into eq. (4-24) results in the following expression for the pattern factor:
\[
f(\theta, \phi) = I_0 \int_0^L e^{-\alpha z} e^{j k_0 z (\sin(\theta) - \sin(\theta_0))} dz \tag{4-29}
\]

For a \(y\)-directed infinitesmal current element lying on a grounded dielectric slab of infinite extent, eqs. (C-8) and (C-9) can be used to determine the current element pattern \(g(\theta, \phi)\) in the \(H\)-plane \(\left(\phi = \frac{\pi}{2}\right)\)
\[
g\left(\theta, \phi = \frac{\pi}{2}\right) = \sqrt{\frac{(\cos \theta)^2}{\left(\varepsilon_r - (\sin \theta)^2\right)\left(\cot(h k_0 \sqrt{\varepsilon_r - (\sin \theta)^2})\right)^2 + (\cos \theta)^2}} \tag{4-30}
\]

Hence, the \(H\)-plane power pattern of the leaky-waveguide antenna may be calculated by substituting eqs. (4-29)-(4-30) into eq. (4-31), given in [24] by
\[
\left|F\left(\theta, \phi = \frac{\pi}{2}\right)\right|^2 = \left|g\left(\theta, \phi = \frac{\pi}{2}\right)\right|^2 \left|f\left(\theta, \phi = \frac{\pi}{2}\right)\right|^2 \tag{4-31}
\]

### 4.5 Radiation Efficiency of a Periodically-Loaded Leaky-Waveguide Antenna

For such an antenna of length \(L\), it has been shown in [7] that the radiation efficiency is
\[
\varepsilon_{\text{rad}} = \frac{100\alpha_r}{\alpha_r + \alpha_c + \alpha_d + \alpha_1 + \frac{(\alpha_r + \alpha_c + \alpha_d + \alpha_1) P(L)}{(1 - e^{-2L(\alpha_r + \alpha_c + \alpha_d + \alpha_1) P(0)}} \tag{4-32}
\]

where
\(P(0)\) is the power delivered to the antenna;

\(P(L)\) is the power remaining at a distance \(L\) along the antenna;

\(\alpha_r\) is the attenuation or leakage constant due to radiation;

\(\alpha_c\) is the attenuation constant due to conduction loss;

\(\alpha_d\) is the attenuation constant due to dielectric loss;

\(\alpha_l\) is the attenuation constant due to loss in the lumped loads.

and \(\alpha_r = \alpha_{zd}\).

If the length \(L\) of the antenna is chosen such that 10\% of the power delivered to it is absorbed in a matched resistive load at the antenna end, the radiation efficiency becomes

\[
e_{rad} = \frac{\frac{100\alpha_r}{\alpha_r + \alpha_c + \alpha_d + \alpha_l + \frac{0.1(\alpha_r + \alpha_c + \alpha_d + \alpha_l)}{1 - e^{-2L(\alpha_r + \alpha_c + \alpha_d + \alpha_l)}}}}{(4-33)}
\]

Calculation of the radiation efficiency using eq. (4-33) is performed in three steps. First, the leakage constant \(\alpha_r = \alpha_{zd}\) is found by setting the losses in the lumped loads, the conductors, and the dielectric to zero, and by solving eqs. (4-19) and (4-16). Then, eqs. (4-19) and (4-16) are solved for the attenuation sum \(\alpha_r + \alpha_c + \alpha_d + \alpha_l\) while the losses in the lumped loads, the conductors, and the dielectric are in effect. Finally, the radiation efficiency of the antenna is found from eq. (4-33).

In a similar fashion, one may also calculate \(e_l\), the percentage of the input power lost in the lumped loads, \(e_c\), the percentage of the input power lost in the conductor, and \(e_d\), the percentage of the input power lost in the dielectric using eqs. (4-34) through (4-36).

\[
e_l = \frac{\frac{100\alpha_l}{\alpha_r + \alpha_c + \alpha_d + \alpha_l + \frac{0.1(\alpha_r + \alpha_c + \alpha_d + \alpha_l)}{1 - e^{-2L(\alpha_r + \alpha_c + \alpha_d + \alpha_l)}}}}{(4-34)}
\]
\[ e_c = \frac{100\alpha_c}{\alpha_r + \alpha_c + \alpha_d + \alpha_l + \frac{0.1(\alpha_r + \alpha_c + \alpha_d + \alpha_l)}{1 - e^{-2L(\alpha_r + \alpha_c + \alpha_d + \alpha_l)}}} \]  
(4-35)

\[ e_d = \frac{100\alpha_d}{\alpha_r + \alpha_c + \alpha_d + \alpha_l + \frac{0.1(\alpha_r + \alpha_c + \alpha_d + \alpha_l)}{1 - e^{-2L(\alpha_r + \alpha_c + \alpha_d + \alpha_l)}}} \]  
(4-36)

4.6 Summary

In this chapter, a study was presented of the wave propagation along a periodically loaded leaky-waveguide antenna. Equations that incorporate the effect of the lumped loads and the dielectric medium have been developed whose solutions provide the propagation and leakage constants of the antenna. Using the effective-dielectric-constant technique [4], the analysis procedure developed for leaky-waveguide antennas of infinite width has been extended to deal with finite-width antennas.

An expression for the \(H\)-plane power pattern of the antenna has been derived by treating the latter as a line source with current flow transverse to the direction of wave propagation. In addition, expressions for the radiation efficiency and fractions of the input power dissipated in the various parts of the antenna have been developed.
Chapter 5

FIXED-FREQUENCY BEAM-STEERABLE
LEAKY-WAVEGUIDE-ANTENNA DESIGN

5.1 Introduction

In this chapter, various periodically loaded leaky-waveguide antennas are built and used to validate the theory of Chapter 4. Specifically, experiments are used to verify the theoretical predictions linking the direction of the main-beam maximum of a leaky waveguide to the reactive loads placed periodically along its radiating strips.

Three periodically loaded leaky waveguides with different relative permittivities are used to investigate the effect of the dielectric on the scan range of such antennas. In addition, the issue of radiation from the sides of these antennas is examined experimentally, followed by various measurements for assessing their response to harmonic and intermodulation interference.

5.2 Design Guidelines for Periodically Loaded Leaky-Waveguide Antennas

This section presents the various guidelines used in this chapter for the design of such antennas. These guidelines are based on the theory of Chapter 4, and deal mainly with such issues as array height over ground and leaky-waveguide length.

5.2.1 Array height over ground

By applying eqs. (4-16) and (4-19)-(4-20) to the periodic structure shown in Fig. 5-1 while varying the height of the wire array over ground, and the values of the identical variable capacitors loading it, one may generate the set of constant-phase-velocity curves shown in
Fig. 5-2. Here, \( C_c \) is defined as

\[
C_c = \frac{2\pi a}{s \left( \ln \left( \frac{s}{2\pi r} \right) + \frac{\lambda_0}{\omega \eta p C} \right)}
\]  

(5-1)

where \( \omega \) and \( \eta \) are the radian frequency and the intrinsic impedance of free space respectively.

Figure 5-1. Leaky-waveguide antenna loaded with variable capacitors of equal value \( C \) placed at regular intervals \( p \) along each wire. Adjacent wires are separated by a distance \( s \), and are located at a height \( a \) above a perfectly conducting ground plane. Here, the array and the wires are taken to be infinite in extent, and are located in free space.
For a given height of the wire array above ground, Fig. 5-2 shows that as the capacitor value C is increased, the phase velocity along the array decreases, and the direction of the beam maximum approaches endfire. Figure 5-2 also shows that in order to maximize the scan range of the periodically loaded leaky-waveguide at a fixed frequency $f$, the height of the array above ground must approach $\lambda_0/2$. The latter observation is key to designing wide-scan-range fixed-frequency leaky waveguides, and must be traded off against the fact that the broadside $H$-plane power gain of a wire located at a distance $\lambda_0/2$ above and parallel to a ground plane is zero.
5.2.2 Length of a periodically loaded leaky-waveguide antenna

For such a structure, the length $L$ of the wire array is chosen so as to radiate 90\% or so of the input power along the antenna, with the remaining power to be absorbed in a matched load located at the antenna end farthest from the feed. Given the leakage constant $\alpha$ along the antenna, and the percentage of input power to be radiated $PPR$, it has been shown by Walter [7] that the ratio of the antenna length $L$ to the free-space wavelength $\lambda_0$ is

$$\frac{L}{\lambda_0} = -\frac{k_0}{4\pi\alpha} \ln \left(1 - \frac{PPR}{100}\right) \quad (5-2)$$

where $k_0$ is the free-space wavenumber.

The leakage constant $\alpha$ and the length $L$ of a periodically loaded leaky waveguide depend on the values of the reactive loads placed along the antenna as may be seen from eqs. (4-16), (4-19) and (5-2). For a leaky-waveguide antenna periodically loaded with identical variable capacitors of value $C_{\text{min}} \leq C \leq C_{\text{max}}$, the average capacitance value $C_a = (C_{\text{max}} + C_{\text{min}})/2$ is used in determining the antenna length $L$ for a given percentage of input power to be radiated $PPR$.

5.2.3 Operating bandwidth and radiation efficiency

The operating bandwidth of a leaky waveguide may be defined as in Section 3.2.4 (i.e. that interval of the impedance bandwidth within which the power gain is within 3 dB from its maximum value), and decreases with an increase in the relative permittivity of the dielectric used in the antenna. On the other hand, due to the fact that the antennas considered in this chapter do radiate from their sides as will be shown in Section 5.6, the radiation efficiency of these antennas will not be calculated. This is because the expression for the radiation efficiency given in eq. (4-33) does not account for the power lost in radiation from the sides of these antennas.
5.3 Array of $\lambda_0$-Long Thin Strips Over a Grounded Styrofoam Slab

In order to verify the dependence of the main-beam direction on periodic reactive loading, the transversely truncated array of $\lambda_0$-long thin wires shown in Fig. 5-3 was designed. A half-wave center-fed dipole located midway between the array and the ground plane provides a feed for this structure. A parasitic dipole of radius and length slightly larger than those of the half-wave dipole is used as a reflector. At regular intervals $p = \lambda_0/5$ along each $\lambda_0$-long wire of the array, capacitors of equal value $C$ are introduced. In addition, wires of identical length and radius are used to connect adjacent $\lambda_0$-long wire ends so as to form a ladder-like structure. The purpose of these wires will be to supply DC bias to a set of voltage-controlled capacitors placed along each wire of the array. The truncated periodic structure is illustrated in Fig. 5-3, where the wires and the ground plane are assumed to be perfectly conducting, and are located in free space.

Two versions of the structure shown in Fig. 5-3 were built on a $57\times42.5$ cm$^2$ aluminum ground plane. Copper strips of 17-μm thickness etched on a printed-circuit board with a dielectric of relative permittivity $\varepsilon_r = 3.0$ and thickness $t = 0.75$ mm (Rogers RO3003) were used in place of the $\lambda_0$-long thin wires and the bias-line wires. The strip width was chosen such that it is equal to four times the radius of the wire it is replacing. In the first version, lumped capacitors were not used ($C = \infty$), and the copper strips and bias lines were etched on one side of the printed-circuit board as shown in Fig. 5-4, while copper on the other side of the board was completely removed. In the second version, identical copper plates on both sides of the printed-circuit board were used to make capacitors of value $C = 0.1$ pF at regular intervals $p = \lambda_0/5$ along each strip as shown in Fig. 5-5. In both versions, styrofoam material ($\varepsilon_r = 1.0$) was used to support the printed-circuit boards over the aluminum ground plane, and the compensated balun [37] shown in Fig. 5-3 was used to center-feed the $\lambda_0/2$-long dipole.
Figure 5-3. Truncated periodic structure of $\lambda_0$-long thin wires parallel to a perfectly conducting ground. Capacitors of equal value $C$ are placed at regular intervals $p = \lambda_0/5$ along each $\lambda_0$-long wire. Here, the compensated balun [37] shown is used to feed the $\lambda_0/2$ dipole.
Figure 5-4. Version I of the array with copper strips lying on the same side of the printed-circuit board. Copper on the other side of the board is removed by etching.

Figure 5-5. Version II of the array with lumped capacitors of value $C = 0.1$ pF at regular $p = \lambda_0/5$ intervals.
The $H$-plane power-gain (dB) patterns of both versions were measured at 4.5 and at 5.0 GHz, and are compared with the corresponding thin-wire method-of-moments [28-29, 31-32] results in Figs. 5-6 and 5-7. Here, the method of moments is used to solve the reaction integral equation by expanding the unknown current on the wires in a set of $N$ piecewise sinusoidal expansion functions tested with $N$ piecewise sinusoidal electric sources [28-29, 31-32]. In addition, the ground plane located below the array of wires is treated as infinite in extent and perfectly conducting, while the wire conductivity is taken to be that of copper. Figures 5-6 and 5-7 show that the introduction of the 0.1 pF capacitors causes a shift in the direction of the main-beam maximum toward broadside. This shift is more pronounced at 4.5 GHz as predicted in Fig. 5-2, since at this frequency the radiating strips are electrically closer to the ground plane than they are at 5 GHz.

At both frequencies, the shift in the main-beam direction is accompanied by an increase in the power gain due to the fact that the $H$-plane power gain of a current element located parallel to a ground plane, at approximately $3\lambda_0/4$ away from it reaches a maximum near broadside and gradually decays away from it. In addition, comparison of Figs. 5-6 and 5-7 shows the main feature of a leaky-wave antenna, that of changing the direction of the main-beam maximum with frequency. As the frequency is increased, the direction of the main beam moves closer to endfire. Due to the fact that no provisions were made for absorbing the power remaining in the incident traveling wave as it reached the end of the antenna, the power patterns shown in Figs. 5-6 and 5-7 exhibit high backlobes appearing at approximately the same angle away from broadside as the main beam. Comparison of the measured power-gain patterns with those calculated using the thin-wire method of moments [28-29, 31-32] shows that the latter predicts measured pattern features such as half-power beamwidth and main-beam direction to within an approximate $\pm 3^\circ$. Finally, the difference between the measured and the calculated lobe levels is most likely due to the fact that the $57 \times 42.5 \text{ cm}^2$ aluminum ground plane was treated as infinite in extent in the method-of-moments calculations. That is, the latter did not account for knife-edge diffraction from the edges of the aluminum ground plane.
Figure 5-6. The calculated (thin-wire method of moments [28-29, 31-32]) and measured H-plane power-gain patterns of the leaky-waveguide antenna ($\varepsilon_r = 1$) shown in Fig. 5-3 at a frequency $f = 5.0$ GHz. In the method-of-moments calculations, all wires were divided into 5 segments per wavelength, with the exception of the $\lambda_d/2$ center-fed dipole which was divided into 6 segments to allow a voltage generator to be placed at its center. In addition, the reflector wire was divided into 6 segments also.
Figure 5-7. The calculated (thin-wire method of moments [28-29, 31-32]) and measured $H$-plane power-gain patterns of the leaky-waveguide antenna ($\varepsilon_r = 1$) shown in Fig. 5-3 at a frequency $f = 4.5$ GHz. Here, the wire divisions described in Fig. 5-6 were left unchanged, and were used in the method-of-moments calculations.
5.4 Array of Varactor-Loaded Thin Strips Over a Grounded Teflon Slab

Having shown the main-beam scanning feature of a periodically loaded leaky waveguide using fixed capacitors, a new version incorporating abrupt 30-volt Metelics MSV-34-060-E25 silicon varactor diodes was subsequently built on a grounded teflon slab of thickness \( d_1 = 2.015 \) cm and relative permittivity \( \varepsilon_r = 2.1 \) as illustrated in Fig. 5-8. The varactor-diode parameters as specified by the manufacturer [33] are listed in Subsection 3.3.2. One-mm-wide copper strips of 17-\( \mu \)m thickness etched on a printed-circuit board with a relative permittivity \( \varepsilon_r = 3.0 \) and thickness \( d_2 = 0.75 \) mm (Rogers RO3003) were loaded with identical varactor diodes placed at regular intervals \( p = 1 \) cm along each. In order to impede the flow of RF signals between adjacent strip ends while providing a DC path for reverse-biasing the varactor diodes, planar inductors were used to connect such ends as shown in Fig. 5-8. These inductors are connected at the antenna side farthest from the feed region to a low-current variable-voltage DC power supply located outside the anechoic chamber. A half-wave center-fed dipole located midway between the array of loaded strips and a 57\( \times \)42.5 cm\(^2\) aluminum ground plane is used to feed this structure by means of the compensated balun [37] shown in Fig. 5-3. The half-wave dipole is backed by a flat copper sheet located \( \lambda_0/4 \) away so as to act as a reflector.

![Figure 5-8. Varactor-loaded array of parallel strips placed on a grounded teflon slab (\( \varepsilon_r = 2.1 \)). A reverse-bias voltage \( V_{dc} \) applied across the varactors is used to change the phase velocity along the array for the purpose of scanning the main beam. The diodes used here are packaged 30-volt Metelics MSV-34-060-E25 abrupt varactor diodes (continued on the next page).](image)
5.4.1 Power gain, cross-polarization, and standing-wave ratio

For different reverse-bias voltages applied across the varactor rows, the leakage constant along the leaky waveguide is found from the solution to eqs. (4-16) and (4-19). The effective-dielectric-constant technique introduced earlier in Section 4.3 is then used along with eqs. (4-19) and (4-20) to find the phase constant of this structure, and thus the direction of its main-beam maximum measured away from broadside (the x direction). Due to the fact that the relative permittivity of the thin printed-circuit board is different from that of the thick teflon slab, the volume-average relative permittivity $\varepsilon_a$ was used in all calculations. It is defined as

$$\varepsilon_a = \frac{\varepsilon_{r1}d_1 + \varepsilon_{r2}d_2}{d_1 + d_2} \quad (5-3)$$
Figure 5-9 shows the calculated direction of the main-beam maximum, $\theta_m$, for different reverse-bias voltages in the range 0-150 volts, and a fixed frequency $f = 5.3$ GHz. Shown also in the same figure is the measured main-beam direction as a function of reverse-bias voltage. Comparison of the calculated results with those obtained from measurements shows a difference between the two of less than $2^\circ$ over a measured scan range of 10.5$^\circ$.

From the calculated value of the leakage constant and eq. (4-31), the $H$-plane power pattern of the periodically loaded leaky-waveguide antenna is found for different reverse-bias voltages in the range 0-150 volts. Here, the leaky waveguide is treated as a line source with current flow transverse to the direction of wave propagation as described in Section 4.4. The calculated $H$-plane power pattern, normalized to the corresponding measured maximum power gain is then plotted in Fig. 5-10 where the measured power-gain and cross polarization patterns are also shown.

![Figure 5-9](image)

**Figure 5-9.** The measured and the calculated main-beam-maximum direction of the periodically loaded leaky-waveguide antenna ($e_r = 2.1$) shown in Fig. 5-8 for different reverse-bias voltages applied across the varactor rows at $f = 5.3$ GHz.
Figure 5-10. The measured H-plane power-gain patterns of the periodically loaded leaky waveguide ($\varepsilon_r = 2.1$) shown in Fig. 5-8 for different reverse-bias voltages applied across the varactor rows at $f = 5.3$ GHz. Shown also are the measured cross-polarization patterns, and the calculated power patterns normalized to the corresponding measured power-gain maxima (continued on next page).
As was the case for the microstrip antennas considered in Chapter 3, this paragraph highlights the following assumptions made in the theory used herein:

1. the antenna is terminated in a matched load;
2. the varactor diodes are identical;
3. the antenna dielectric is homogeneous.

as well as its underlying limitations of (a) not predicting radiation from the sides of the antenna, and (b) not modeling the antenna feed or predicting radiation from it.

Figure 5-10 shows the fixed-frequency movement of the main-beam maximum of the leaky waveguide shown in Fig. 5-8 as a function of the reverse-bias voltage applied across its varactor rows. As this voltage is increased from 0 to 150 volts, the main-beam maximum moves from 48° to 37.5° away from broadside, while the cross-polarization level remains at least 24 dB below the main beam. This movement of the main-beam maximum is accompanied by a slight drop in the power gain, and by an increase in the measured voltage standing-wave ratio as Fig. 5-11 shows. The former effect may be explained by the fact that the $H$-plane power gain of an electric current element located parallel to a ground plane, at approximately a half dielectric-wavelength away from it reaches a maximum in the vicinity of $\theta = 45^\circ$ away from broadside, and decreases as $\theta$ reaches 0°. Due to the fact that a matching load was not provided for absorbing the power remaining at the end of the antenna, the power
patterns shown in Fig. 5-10 exhibit moderate-level backlobes appearing at approximately the same angle away from broadside as the main beam. On the other hand, the high lobe level in the vicinity of the backlobe is most likely due to radiation from the feed region as Fig. 5-12 suggests. As the reflector height of the leaky waveguide shown in Fig. 5-8 is increased from 2.085 cm to 4.5 cm, the power gain in the vicinity of the backlobe decreases.

Finally, Fig. 5-10 shows that the measured and the calculated half-power beamwidths agree to within an approximate ±2°. In addition, comparison of the calculated and the measured direction of the main-beam maximum, θ_m, shows that the theory predicts measured results to within ±2° over a measured scan range of 10.5°.

Figure 5-11. The measured voltage standing-wave ratio of the periodically loaded leaky-waveguide antenna (ε_r = 2.1) shown in Fig. 5-8 as a function of the reverse-bias voltage applied across the varactor rows at f = 5.3 GHz.

Figure 5-12. The measured H-plane power gain of the leaky waveguide shown in Fig. 5-8 for two cases. In the first case, the reflector height is the same as that used before, and is equal to the thickness of the dielectric (2.085 cm). In the second case, a 4.5-cm-high reflector is used. Note here that in both cases, the varactor diodes have been replaced with short circuits of the same width as the radiating strips.
5.5 Array of Varactor-Loaded Thin Strips Over a Grounded Ceramic Slab

In order to verify the dependence of the scan range on the relative permittivity of a periodically loaded leaky-waveguide antenna, the teflon slab used in Fig. 5-8 was replaced by a ceramic slab of the same dimensions, having a relative permittivity $\varepsilon_{r1} = 3.5$. The resulting structure, illustrated in Fig. 5-13, is operated at a fixed frequency $f = 4.1$ GHz so as to maximize its scan range based on the design guidelines presented in Section 5.2. It is worth mentioning at this point that although the half-wave center-fed dipole used to feed this structure was impedance matched to a compensated balun [37] in free space, no attempt was made to match it in the waveguide environment (i.e. in the presence of the grounded dielectric slab, parallel-strip grid, and reflector).

5.5.1 Power gain, cross-polarization, and standing-wave ratio

Figure 5-14 shows the measured and the calculated fixed-frequency movement of the main-beam maximum, $\theta_m$, as the reverse-bias voltage applied across the varactor rows is increased from 0 to 150 volts. Here, calculations are based on the fixed-frequency solution of eqs. (4-19) and (4-20) for different reverse-bias voltages. Comparison of the measured results shown in Fig. 5-14 with those predicted by the theory shows that the latter predicts the measured main-beam direction to within $2^\circ$ over a measured scan range of $48^\circ$.

By treating the leaky waveguide shown in Fig. 5-13 as a line source with current flow transverse to the direction of wave propagation, one may calculate its $H$-plane power pattern by using the leakage constant of this antenna in eq. (4-31). Here, the leakage constant is found from eq. (4-16) and the calculation results described in the previous paragraph, for different reverse bias voltages in the range 0-150 volts. The calculated power patterns, each normalized to the corresponding measured maximum power gain, are then plotted along with their measured counterparts in Fig. 5-15, where the measured cross-polarization patterns are also shown.
Figure 5-13. Varactor-loaded array of parallel strips placed on a grounded ceramic slab ($\varepsilon_r = 3.5$). A reverse-bias voltage $V_{dc}$ applied across the varactor diodes is used to change the phase velocity along the array for the purpose of scanning the main beam. The diodes used here are packaged 30-volt Metelics MSV-34-060-E25 abrupt varactor diodes.
Figure 5-14. The measured and the calculated main-beam-maximum direction of the leaky-waveguide antenna ($\varepsilon_r = 3.5$) shown in Fig. 5-13 for different reverse-bias voltages applied across the varactor rows at $f = 4.1$ GHz.

Figure 5-15. The measured $H$-plane power-gain and cross-polarization patterns of the leaky-waveguide antenna ($\varepsilon_r = 3.5$) shown in Fig. 5-13 for different reverse-bias voltages applied across the varactor rows at $f = 4.1$ GHz. Shown also are the calculated power patterns normalized to the corresponding measured power-gain maxima (continued on the next page).
Figure 5-15. Continued from the previous page.
Figure 5-15 shows that as the reverse-bias voltage applied across the varactor rows is increased from 0 to 150 V, the main-beam maximum of the periodically loaded leaky waveguide moves at a constant frequency from 52.5° to 4.5° away from broadside, while the cross-polarization level remains at least 20 dB below that of the main beam. The movement of the main-beam direction toward broadside is accompanied by a gradual drop in the power gain, a broadening of the main beam, and an increase in the voltage standing-wave ratio as Fig. 5-16 shows.

The drop in the power gain of the antenna is due to the fact that its array of radiating strips is located at an approximate height of a half dielectric-wavelength above the ground plane. In this case, the H-plane power gain of an electric current element located parallel to a ground plane, at approximately a half dielectric-wavelength away from it, reaches a maximum in the vicinity of $\theta = 45^\circ$ away from broadside, and gradually decreases as $\theta$ reaches $0^\circ$. On the other hand, the increase in the width of the main beam near broadside is attributed to the sharp increase in the leakage constant in that region. When the leakage constant becomes large, the input power into the leaky waveguide gets radiated by the first few strips so as to make the effective aperture short, thus resulting in a wide main beam. Note that since the calculated leakage constant shown in Fig. 5-17 did not predict correctly the voltage values at which the leakage constant increases sharply, the last two power-pattern plots shown in Fig. 5-15 show a pronounced difference between the measured H-plane power gain and the calculated power patterns. The cause for the error in the calculated leakage constant near broadside has not yet been determined.

Since no attempt was made to terminate the antenna in a matched load so as to absorb the remaining incident power, some of the measured power-gain patterns shown in Fig. 5-15 exhibit moderate-level backlobes. On the other hand, the high lobe level in the vicinity of the backlobe region is most likely due to radiation from the feed structure as was argued earlier in Subsection 5.4.1 for reflectors of different height.
Figure 5-16. The measured voltage standing-wave ratio of the leaky-waveguide antenna ($\varepsilon_r = 3.5$) shown in Fig. 5-13 as a function of the reverse-bias voltage applied across the varactor rows at $f = 4.1$ GHz.

Figure 5-17. The calculated leakage constant $\alpha_{zd}$ (normalized to the free-space wave number $k_0$) of the leaky-waveguide antenna ($\varepsilon_r = 3.5$) shown in Fig. 5-13 as a function of the reverse-bias voltage applied across the varactor rows at $f = 4.1$ GHz.
5.5.2 Periodic-structure analysis

Consider the leaky-waveguide antenna shown in Fig. 5-13, treated here as a structure of infinite length and width, and assume that the varactor diodes have all been replaced with capacitors of equal value $C$. Then, by solving eqs. (4-16) and (4-19) for different capacitor values, one may generate the Brillouin diagram for this structure as depicted in Fig. 5-18 where the dispersion curve of the TE$_{10}$ mode of a parallel-plate waveguide has also been shown for comparison.

![Brillouin Diagram of Periodically Loaded Leaky-Waveguide Antenna](image)

**Figure 5-18.** The Brillouin diagram of the leaky-waveguide antenna ($\varepsilon_r = 3.5$) shown in Fig. 5-13. Here, the antenna is treated as a structure of infinite width and length, with varactor diodes replaced with capacitors of equal value $C$. Shown also is the dispersion curve of the TE$_{10}$ mode of a parallel-plate waveguide.
Figure 5-18 shows that a constant-frequency increase in the capacitor value $C$ from 0.5 pF to infinity causes the phase velocity along the capacitor-loaded leaky waveguide to decrease, and as a result, the direction of the main-beam maximum approaches endfire. Therefore, one may view constant-frequency main-beam steering in a periodically loaded leaky waveguide as a consequence of transitions from one dispersion curve to the next, induced by a constant-frequency change in the capacitor value $C$. Here, constant-frequency steering of the main-beam maximum close to broadside requires smaller capacitor values as suggested by Fig. 5-18.

5.5.3 Interference measurements

In this section, spectrum-analyzer measurements similar to those carried out in Subsection 3.3.6 will be used to assess the response of the leaky-waveguide antenna shown in Fig. 5-13 to harmonic and intermodulation interference. In these measurements, different degrees of nonlinearity in the varactor diodes are exercised by properly setting the reverse-bias-voltage across the varactor rows.

**Harmonic distortion.** Consider the structure shown in Fig. 5-19 where the periodically loaded leaky-waveguide antenna of Fig. 5-13 is used as a receiver of a 4.1 GHz signal originating in a rectangular horn antenna located 5 m away. Here, 13.2 dBm, the maximum power available from one signal generator is delivered to the transmitting horn, which provides a power density of 3.6 mW/m² at the receiving antenna. In order to deliver maximum power to the leaky waveguide, the horn antenna is positioned such that it is aligned with $\theta_m$, the direction of the main-beam maximum of the leaky waveguide.

For two different reverse-bias-voltage settings across the varactor rows, Table 5-1 lists the power $P$ (dBm) delivered by the leaky waveguide to the 50-Ω internal impedance of the spectrum analyzer at the fundamental frequency and at the first two harmonics. Note here that the compensated balun [37] used to feed the leaky-waveguide antenna is designed to operate in the vicinity of 4.1 GHz. Based on the results listed in Table 5-1, one may view the leaky-waveguide antenna in this case as a linear system.
Figure 5-19. The measurement set-up used for assessing the response of the leaky-waveguide antenna ($\varepsilon_r = 3.5$) shown in Fig. 5-13 to harmonic interference.
Table 5-1  The Measured Response of the Leaky-Waveguide Antenna Shown in Fig. 5-13 to Harmonic Interference

<table>
<thead>
<tr>
<th>$V_{dc}$ (V)</th>
<th>$P$ (dBm) at 4.1 GHz</th>
<th>$P$ (dBm) at 8.2 GHz</th>
<th>$P$ (dBm) at 12.3 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>≤ -30.33</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
</tr>
<tr>
<td>50</td>
<td>&lt; -30.33</td>
<td>&lt; -90</td>
<td>&lt; -90</td>
</tr>
</tbody>
</table>

*Intermodulation distortion.*  If an additional transmitting rectangular horn operating at a frequency $f_1 = 4.0$ GHz is added to the structure shown in Fig. 5-19, the structure illustrated in Fig. 5-20 results. Here, 13.56 dBm, the maximum power available from one signal generator, is delivered to one of the transmitting horns, while 3.0 dBm, the maximum available from another generator, is delivered to the other horn. The two rectangular horns are positioned such that they deliver maximum power to the leaky-waveguide antenna. The first horn is aligned with $\theta_{m1}$, the direction of the main-beam maximum of the leaky waveguide at $f_1 = 4.0$ GHz, and provides a power density of 0.3 mW/m$^2$ at the waveguide. On the other hand, the second horn is aligned with $\theta_{m2}$, the direction of the main-beam maximum of the leaky waveguide at $f_2 = 4.1$ GHz, and provides a power density of 3.6 mW/m$^2$ at the waveguide also.

For two different reverse-bias-voltage settings across the varactor rows, Table 5-2 lists the power $P$ (dBm) delivered by the leaky-waveguide antenna to the 50-Ω internal impedance of the spectrum analyzer. Here, the received power is measured at the fundamental and at the various intermodulation frequencies. These measurements show that the power received at the two fundamentals is at least 28 dB higher than that received at the intermodulation frequencies. In this case, one concludes that the power radiated by both horns is not large enough to produce any measurable intermodulation distortion, and the receiving antenna may be considered to be operating in the linear mode. Note that the compensated balun [37] used in the feeding structure of the leaky-waveguide antenna is designed for operation about a frequency of 4.1 GHz.
Figure 5-20. The measurement set-up used for assessing the response of the leaky-waveguide antenna ($\varepsilon_r = 3.5$) shown in Fig. 5-13 to intermodulation interference.
Table 5-2  The Measured Response of the Leaky-Waveguide Antenna Shown in Fig. 5-13 to Intermodulation Interference

<table>
<thead>
<tr>
<th>$V_{dc}$ (V)</th>
<th>$P$ (dBm) at $f_1$ (4.0 GHz)</th>
<th>$P$ (dBm) at $f_2$ (4.1 GHz)</th>
<th>$P$ (dBm) at $f_2-f_1$ (100 MHz)</th>
<th>$P$ (dBm) at $2f_2-f_1$ (4.2 GHz)</th>
<th>$P$ (dBm) at $f_1+f_2$ (8.1 GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-29.5</td>
<td>-40.33</td>
<td>$&lt; -90$</td>
<td>$&lt; -90$</td>
<td>$&lt; -90$</td>
</tr>
</tbody>
</table>

5.6 Side Radiation

Due to the fact that a grounded dielectric slab of finite width does not support pure TE or TM modes, but rather hybrid modes involving TE-TM coupling [4], one might expect a leaky-waveguide antenna of finite width $w$ to radiate along its sides. That this is so will be shown to be the case using a series of measurements performed on the receiving leaky-waveguide structure shown in Fig. 5-21, where a fixed rectangular-horn antenna used as a transmitter is positioned such that its $H$ or $E$ plane is coincident with the $x = \frac{a}{2}$ plane of the leaky waveguide. Here, the leaky waveguide is rotated about its $x$ axis, and the angle of reference, $\psi = 0^\circ$, corresponds to the case where the terminated end faces directly the horn aperture.

At 5.3 GHz, Fig. 5-22-(a) shows that in the absence of the radiating strips and the teflon slab, the measured $x$-polarized power gain of the dipole-reflector-ground system is below $-10$ dBi, and is symmetric about the $x$-$z$ plane as expected. Upon introduction of the teflon slab, symmetry about the $x$-$z$ plane is maintained while a sharp increase (10 dB) in the $x$-polarized power gain takes place, with the power-gain peaking in the vicinity of $\pm 45^\circ$. The latter effect suggests that sides 1 and 2 of the teflon slab radiate continuously along their lengths, and that the slab itself acts as a TE-to-TM-mode transformer and vice versa. That is, the teflon slab is the scene of TE-TM mode coupling.
Figure 5-21. Measurement setup used to detect radiation from the sides of a leaky-waveguide antenna ($\varepsilon_r = 2.1$) at 5.3 GHz. Here, the leaky waveguide is identical to that shown in Fig. 5-8, with the exception of the printed-circuit strips which have been replaced with those shown in Fig. 5-4.
Figure 5-22. Measured power-gain patterns for assessing radiation from the sides of the leaky-waveguide antenna shown in Fig. 5-21 at a constant frequency $f = 5.3$ GHz. Here, the $H$-plane of the rectangular horn is parallel to the ground plane of the leaky-waveguide antenna.

If the grid of radiating strips is introduced, symmetry about the $x$-$z$ plane is still maintained, while the $x$-polarized power gain pattern shown in Fig. 5-22-(a) becomes more directive, as Fig. 5-22-(b) shows. In addition, due to reflection of the incident wave at the end of the dielectric slab, backlobes emerge at approximately the same angle as the main beam on both sides of the $x$-$z$ plane as shown in Fig. 5-22-(b). Initially, it was thought that if a thin layer (0.78-mm thick) of an available Emerson and Cuming Eccosorb-VF [38] absorber is applied to the sides of the dielectric, side radiation would dampen. However, it was found that
this is not the case as Figs. 5-22-(b) and 5-22-(c) show. This is most likely due to the fact that at 5.3 GHz, the conduction and displacement currents in the absorbing material (Eccosorb VF) are approximately the same (i.e. $\sigma = \omega \varepsilon_0 \varepsilon_r = 10 \text{ S/m}, \varepsilon_r = 34$) [39]. That is, unlike the case where the absorber was absent, wave reflections at the sides of the dielectric are now accompanied by phase changes (due to the conductive part of the Eccosorb VF material) which influence the $x$-polarized power gain.

If the leaky waveguide shown in Fig. 5-21 is tilted by $45^\circ$ about its axis located in the horizontal plane toward the horn antenna, the power-gain patterns shown in Fig. 5-23 result for two different horn polarizations. When the $H$-plane of the horn is perpendicular to the horizontal plane, Fig. 5-23-(a) shows a drop in the power gain of the tilted leaky waveguide due to the presence of the absorber. On the other hand, when the $H$-plane of the horn antenna is parallel to the horizontal plane, the effect of the absorber on the power gain is minimal as Fig. 5-23-(b) shows.

![Figure 5-23. Measured power-gain patterns of the leaky waveguide shown in Fig. 5-21 when tilted at 45° about its axis, for two different polarizations of the horn antenna.](image-url)
5.7 Summary

In this chapter, various leaky-waveguide antennas were built and used to confirm the theoretical predictions linking the main-beam direction of a leaky waveguide to the reactive loads placed at regular intervals along it. The dependence of the scan range on the height of an array over ground has also been verified both theoretically and experimentally.

The effect of the relative permittivity on the scan range has been investigated through example structures having different relative permittivities. For a leaky waveguide with a relative permittivity \( \varepsilon_r = 2.1 \), a 10.5° scan range was obtained both theoretically and experimentally at a constant frequency \( f = 5.3 \text{ GHz} \). By simply replacing the teflon slab with a ceramic slab (\( \varepsilon_r = 3.5 \)) of the same dimensions, a 48° scan range was achieved at a constant frequency \( f = 4.1 \text{ GHz} \).

The response of a receiving varactor-loaded leaky waveguide (\( \varepsilon_r = 3.5 \)) to radiated harmonic and intermodulation interference was investigated experimentally. In the presence of a transmitting horn providing a power density of 3.6 mW/m² (at \( f = 4.1 \text{ GHz} \)) at the receiving antenna, the received power at the harmonic frequencies is at least 59 dB below that received at the fundamental. On the other hand, in the presence of two transmitting horns providing power densities of 0.3 mW/m² (at \( f_1 = 4.0 \text{ GHz} \)) and 3.6 mW/m² (at \( f_2 = 4.1 \text{ GHz} \)) at the receiving waveguide, the power received at the intermodulation frequencies is at least 28 dB below that received at the fundamentals. Consequently, for power-density levels below those specified here, the leaky waveguide may be considered as a linear system.

Finally, the issue of radiation from the sides of a leaky-waveguide antenna having a relative permittivity \( \varepsilon_r = 2.1 \) was addressed experimentally. Using a series of far-field measurements, it was shown that the leaky-waveguide dielectric is the scene of TE-TM mode coupling, resulting in side radiation that is at least 11 dB below the \( H \)-plane power-gain maximum of the leaky waveguide.
Chapter 6

CONCLUSIONS

6.1 Introduction

In this thesis, two fixed-frequency beam-steerable traveling-wave antennas were introduced. The first of these antennas is a series-fed array of varactor-loaded rectangular microstrip patches, and is referred to as a periodically loaded leaky-wave microstrip antenna. On the other hand, the second antenna is based on an array of varactor-loaded parallel strips printed on a grounded dielectric slab, and is referred to as a periodically loaded leaky waveguide. In both antennas, main-beam steering is based on the basic principle of traveling-wave phase control by means of voltage-controlled variable capacitors (varactors) integrated into the antenna structure.

6.1.1 Periodically loaded leaky-wave microstrip antenna

In Chapters 2 and 3 of this thesis, a study was presented of the wave propagation along a periodically loaded leaky-wave microstrip antenna. The theoretical predictions linking the phase velocity along such an antenna to the values of the voltage-controlled capacitors placed at regular intervals along it were confirmed experimentally. It was found that the phase velocity increases with decreasing capacitor value, causing the direction of the main-beam maximum to shift toward broadside at constant frequency. Similarly, increasing the capacitor value was found to cause the phase velocity along the antenna to decrease, resulting in a constant-frequency shift of the main-beam direction toward endfire.

Examples were used to examine the effect of the relative permittivity on the scan range of the antenna. For a microstrip relative permittivity of 3, a 26° scan range was calculated and obtained from measurements at a frequency $f = 6.25$ GHz. On the other hand, using a microstrip relative permittivity of 6.15, a 60° scan range was achieved both theoretically and experimentally at a frequency $f = 5.2$ GHz.
By treating a leaky-wave microstrip antenna as a line source with current flow transverse to the direction of wave propagation, an expression for the $H$-plane power pattern was derived. A similar expression was also derived by viewing the antenna as a nonuniformly excited, equally spaced linear array. The aforementioned expressions were then used for comparison with the measured power-gain patterns.

Finally, expressions for the radiated power and the power dissipated in the various parts of a periodically loaded leaky-wave microstrip antenna were derived and used to calculate the radiation efficiency of two prototype antennas. In addition, various measurements involving radiation from nearby antennas were used to assess the response of a receiving microstrip antenna ($\varepsilon_r = 3$) to harmonic and intermodulation interference. It was found that the power levels of the interfering signals used in these measurements were not high enough to push the antenna under test into a nonlinear mode of operation. To elaborate, in the harmonic interference case, a power density of 13.7 mW/m$^2$ (at 6.5 GHz) was used at the receiving antenna. On the other hand, power densities of 1.1 mW/m$^2$ (at 6.25 GHz) and 13.7 mW/m$^2$ (at 6.5 GHz) were used in the intermodulation interference case. Although the power-density levels required to drive the receiving microstrip antenna on the verge of nonlinear operation have not been measured, such an antenna may be safely used as a linear system in applications not exceeding the power densities used in the interference measurements.

6.1.2 Periodically loaded leaky-waveguide antenna

In Chapters 4 and 5, an analysis was given of the wave propagation along a periodically loaded leaky waveguide based on the transverse-resonance [15] and effective-dielectric-constant [4] techniques. Using measurements and theory, it was found that the phase velocity along such an antenna depends on the value of the variable capacitors placed at regular intervals along each of its radiating strips. An increase in the capacitor value was found to decrease the phase velocity along the antenna, resulting in a shift of the main-beam maximum toward endfire. Similarly, a decrease in the capacitor value is accompanied by an increase in the phase velocity, and causes the main-beam maximum to shift toward broadside.
In order to illustrate the effect of the relative permittivity on the scan range, two leaky-waveguide versions were built. The first, having a relative permittivity $\varepsilon_r = 2.1$, exhibited a measured $10.5^\circ$ scan range ($9.5^\circ$ calculated) at a frequency $f = 5.3$ GHz. On the other hand, a $48^\circ$ scan range was achieved both theoretically and experimentally in the second version using a relative permittivity $\varepsilon_r = 3.5$ and a fixed frequency $f = 4.1$ GHz.

By considering a leaky waveguide as a line source with current flow transverse to the direction of wave propagation, an expression for the $H$-plane power pattern was derived and used for comparison with the measured power-gain patterns. In addition, expressions giving the fraction of the input power radiated and dissipated in the various parts of the antenna were developed.

The response of a receiving periodically loaded leaky-waveguide antenna ($\varepsilon_r = 3.5$) to radiated harmonic and intermodulation interference was assessed experimentally. In the harmonic interference case, a radiating source providing a power density of $3.6$ mW/m$^2$ (at 4.1 GHz) at the receiving antenna was used. Another interference source providing $0.3$ mW/m$^2$ (at 4.0 GHz) was added in the intermodulation interference case. It was found that the power levels of the interfering signals used in these measurements were not high enough to drive the leaky waveguide into a nonlinear mode of operation. The leaky waveguides considered here may be safely used as linear systems in receiver applications not exceeding the aforementioned power densities. Finally, it was shown experimentally that such antennas support hybrid modes that result in side radiation that is at least $11$ dB below their $H$-plane power-gain maximum.

6.2 Impact

The fixed-frequency beam-steerable leaky-wave antennas introduced in this thesis provide a simple, radiation-efficient, and low-cost alternative to corporate-fed microstrip phased arrays. That this is so is a direct consequence of the absence of the corporate-feed and phase-shifter networks in such antennas. To elaborate, these antennas are
(1) Simple. They require a single feed to launch the traveling wave, and a single DC port to steer continuously the main beam;

(2) Radiation efficient. One expects these antennas not to suffer from the conductor losses and spurious radiation associated with microstrip corporate feeds and phase-shifter networks. This makes them potential candidates for operation at millimeter-wave frequencies;

(3) Frequency hopping. Since these antennas scan also with frequency in a given frequency range, their main beam may be steered at any constant frequency in that range;

(4) Low cost. One expects the simplicity of these antennas to translate into a lower fabrication cost compared with microstrip phased arrays.

A major disadvantage of periodically loaded leaky-wave antennas is their low power-handling capability. This is due to the fact that currently available varactor diodes cannot handle large amounts of power (e.g. a maximum of 100 mW peak DC power dissipation for the MSV-34-060-E25 varactors [33] used in this work). Therefore, while these antennas could be used in low-power transmitters, their main use would be in receivers. Potential applications lie in the area of communications with moving objects such as in automotive radar, satellite communications, and target tracking.

6.3 Directions for Future Work

Possible extensions of the research presented in this thesis include the following:

(1) A study of the effect of varactor diode failures on the antenna performance (i.e. main-beam direction, power gain, beamwidth, input impedance, etc.). In particular, one could address the dependence of antenna performance degradation on diode failure location;
(2) Scaling of the periodically loaded antennas to millimeter-wave frequencies. The research reported in this thesis was concerned mainly with proving first principles (e.g. fixed-frequency beam-steering of leaky-wave antennas at low cost), and did not address the performance of these antennas at millimeter-wave frequencies. Any plans to do so must ensure that the microstrip thickness is such that little power is lost in the $TM_0$ surface-wave mode, and that higher-order surface-wave modes are cut off. In the case of the leaky-waveguide antenna, one must minimize radiation from the feed region through the use of an appropriate feed (i.e. waveguide launcher);

(3) Beam steering in two dimensions. One possibility is to use a varactor-loaded transmission line to feed an array of periodically loaded leaky-wave antennas;

(4) Pattern synthesis. For a given reverse-bias voltage, the value of the leakage constant along a fixed-frequency beam-steerable leaky-wave antenna remains unchanged, and the power distribution along such structure decays exponentially. One possible area of future work is to control the value of the leakage constant along the antenna so as to synthesize a given radiation pattern.
Appendix A

MICROSTRIP PARAMETERS

A.1 Characteristic impedance

For a microstrip transmission line of width \( W \), thickness \( t \), dielectric thickness \( h \), and relative permittivity \( \varepsilon_r \), the characteristic impedance \( Z_0 \) is given in [40-41] by

\[
Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln \left( \frac{8h}{W} + 0.25 \frac{W_e}{h} \right) \quad \frac{W_e}{h} \leq 1
\]

\[
= \frac{120\pi}{\sqrt{\varepsilon_r} \left( \frac{W_e}{h} + 1.393 + 0.667 \ln \left( \frac{W_e}{h} + 1.444 \right) \right)} \quad \frac{W_e}{h} > 1
\]

\[
\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2 \sqrt{1 + \frac{12h}{W_e}}} + 0.04 \left( 1 - \frac{W_e}{h} \right)^2 \quad \frac{W_e}{h} \leq 1
\]

\[
= \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2 \sqrt{1 + \frac{12h}{W_e}}} \quad \frac{W_e}{h} > 1
\]

\[
W_e = W + \frac{t}{\pi} \left( 1 + \ln \left( \frac{2h}{t} \right) \right) \quad \frac{W}{h} > \frac{1}{2\pi}, \quad t \leq h, \quad t \leq \frac{W}{2}
\]

\[
= W + \frac{t}{\pi} \left( 1 + \ln \left( \frac{4\pi W}{t} \right) \right) \quad \frac{W}{h} \leq \frac{1}{2\pi}, \quad t \leq h, \quad t \leq \frac{W}{2}
\]

and is based on a quasi-static formulation. Here, the complex propagation constant \( \gamma_0 = \alpha + j\beta \), where

\[
\beta = \omega \sqrt{\mu_0 \varepsilon_0 \varepsilon_e}
\]

and \( \omega, \mu_0, \) and \( \varepsilon_0 \) are the radian frequency, permeability, and permittivity of free space respectively.
A.2 Conductor and dielectric attenuation factors

The attenuation constant $\alpha$ of a microstrip transmission line is the sum of the conductor loss factor $\alpha_c$, and the dielectric loss factor $\alpha_d$. For a conductor of thickness $t$ and conductivity $\sigma$, $\alpha_c$ is given in [42] by

$$\alpha_c = \frac{R_s}{2\pi Z_0 h} P \left(1 + \frac{h}{W_e} + \frac{h}{\pi W_e} \ln \left(\frac{4\pi W_e}{t} + \frac{t}{W_e}\right)\right)$$

$$\frac{W_e}{h} \leq \frac{1}{2\pi}$$  \hspace{1cm} (A-5)

$$= \frac{R_s}{2\pi Z_0 h} P \cdot Q$$

$$\frac{1}{2\pi} < \frac{W_e}{h} \leq 2$$

$$= \frac{R_s}{Z_0 h} \frac{Q}{\left(\frac{W_e}{h} + \frac{2\pi}{\ln \left(\frac{2\pi e W_e}{2h + 0.94}\right)}\right)^2} \left(\frac{W_e}{h} + \frac{W_e}{\pi h \left(\frac{W_e}{2h + 0.94}\right)}\right)$$

$$\frac{W_e}{h} \geq 2$$

where

$$R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}}$$  \hspace{1cm} (A-6)

$$P = 1 - \left(\frac{W_e}{4h}\right)^2$$  \hspace{1cm} (A-7)

$$Q = 1 + \frac{h}{W_e} + \frac{h}{\pi W_e} \left(t \ln \left(\frac{2h}{t}\right) - \frac{t}{h}\right)$$  \hspace{1cm} (A-8)

and the dielectric loss factor $\alpha_d$ is given in [43] by

$$\alpha_d = \frac{\pi \varepsilon_r \varepsilon_e - 1}{\sqrt{\varepsilon_e \varepsilon_r - 1}} \frac{\tan \delta}{\lambda_0}$$  \hspace{1cm} (A-9)

where $\lambda_0$ is the wavelength in free space, and $\tan(\delta)$ is the loss tangent of the dielectric.
Appendix B

IMPEDANCE MATRIX OF AN N-PORT MICROSTRIP CIRCUIT

B.1 Impedance matrix elements

For an arbitrary N-port microstrip circuit such as that shown in Fig. B-1, it may be shown that the port voltages \([v]\) are related to the port currents \([i]\) via the impedance matrix \([z]\) whose elements are given by Okoshi et al. [44] as

\[
z_{ij} = \frac{1}{W_i W_j} \iint G(l_i, l_j) dl_i dl_j \quad i, j = 1 \ldots N
\]  

(B-1)

![Diagram of an N-port microstrip circuit](image)

Figure B-1. An arbitrary N-port microstrip circuit.

where

\(G(l_i, l_j)\) is the Green's function. It is the voltage response between a point \((x_i, y_i)\) on the top conductor and a point on the ground plane directly below it, due to a unit \(z\)-directed filamentary electric current injected at \((x_j, y_j)\) between the top and bottom conductors;
$W_i$ is the width of port $i$;

$W_j$ is the width of port $j$;

$l_i$ is the distance along the periphery of the $i^{th}$ microstrip port;

$l_j$ is the distance along the periphery of the $j^{th}$ microstrip port.

**B.1.1 Green's function of a rectangular microstrip patch**

It has been shown by Morse et al. [45] and by Lo et al. [46-47] that the Green's function of a rectangular microstrip patch such as that shown in Fig. B-2 is

$$G(x, y, x_0, y_0) = \frac{-j\eta h}{ab} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\varepsilon_{0m}\varepsilon_{0n}\cos(k_x x_0)\cos(k_y y_0)}{k^2 \left( 1 - \frac{j}{Q} \right) - k_x^2 - k_y^2}$$

$$\eta = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$k = \omega \sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}$$

$$k_x = \frac{m\pi}{a}$$

$$k_y = \frac{n\pi}{b}$$

$$\varepsilon_{0m} = 1 \text{ if } m = 0, 2 \text{ if } m > 0$$

where

$\varepsilon_0$ is the permittivity of free space;

$\mu_0$ is the permeability of free space;

$\varepsilon_r$ is the relative permittivity of the microstrip material;
is the relative permeability of the microstrip dielectric material;

\( Q \) is the quality factor of the lossy cavity resonator formed by the patch;

\( a \) is the length of the rectangular microstrip patch;

\( b \) is the width of the rectangular microstrip patch;

\( h \) is the thickness of the dielectric.

**Figure B-2.** A rectangular microstrip patch of length \( a \) and width \( b \). The origin of the rectangular coordinate system is chosen to coincide with the lower left corner of the patch.
Appendix C

RESONANT RECTANGULAR MICROSTRIP PATCH

C.1 Quality factor

For a resonant rectangular microstrip patch antenna such as that shown in Fig. C-1, the quality factor $Q$ is defined in [46] as

$$Q = \frac{P_{\text{rad}} + P_{\text{sw}} + P_{c} + P_{d}}{2\omega W_{e}}$$  \hspace{1cm} (C-1)

Figure C-1. A rectangular microstrip patch of resonant length $a$ and width $b$. The origin of the rectangular coordinate system is chosen to coincide with the center of the patch.

where

$P_{\text{rad}}$ is the power radiated into the upper hemisphere by the microstrip patch;

$P_{\text{sw}}$ is the power lost to surface waves;

$P_{c}$ is the power lost in the patch conductor and that part of the ground plane located directly below the patch;
\( P_d \) is the power lost in that part of the dielectric located directly below the patch;

\( W_e \) is the electric energy stored at resonance in the \( axb \times h \) cavity, where \( a \) and \( b \) are the rectangular patch dimensions, and \( h \) is the thickness of the dielectric;

\( \omega \) is the radian frequency.

### C.2 Conductor and dielectric loss

For the \( E_{10}^z \) mode of operation (i.e. \( a = \lambda_g / 2 \), where \( \lambda_g \) is the guide wavelength), it has been shown by Lo et al. [47] that the power lost in that portion of the dielectric located below the patch is

\[
P_d = 2\omega \tan(\delta) W_e \tag{C-2}
\]

where \( \tan(\delta) \) is the loss tangent of the dielectric. It has also been shown in [47] that the power lost in the conductor is

\[
P_c = \frac{2\omega \Delta W_e}{h} \tag{C-3}
\]

where, for a conductor with conductivity \( \sigma \), the skin depth \( \Delta \) is given by

\[
\Delta = \frac{2}{\sqrt{\omega \mu_0 \sigma}} \tag{C-4}
\]

### C.3 Stored electric energy

In order to determine the stored electric energy in the resonant rectangular microstrip patch, use is made of the electric field distribution in the cavity, and Poynting's power theorem [48]. For the \( E_{10}^z \) mode, the electric field distribution is given in [46] by

\[
E_{10}^z = C_{10} \cos \left( \frac{\pi x}{a} \right) \tag{C-5}
\]
where \( C_{10} \) is an amplitude constant. It is convenient to set \( C_{10} \) to

\[
C_{10} = \frac{V}{h}
\]

where \( V \) is the voltage between any point on the edge \( x=-a/2 \) or \( a/2 \) of the microstrip patch, and a point directly below it on the ground plane. Then, from eqs. (C-5)-(C-6) and Poynting's power theorem [48], one may show that the stored electric energy at resonance for the \( E_{10}^z \) mode is

\[
W_e = \frac{\varepsilon_0\varepsilon_r}{2} \int_0^a \int_0^{2\pi} \left| E_{10}^z \right|^2 dz dy dx = \frac{ab\varepsilon_0\varepsilon_r V^2}{4h}
\]

(C-7)

C.4 Radiated power and power lost to surface waves

The power radiated by the rectangular microstrip patch and the power lost to surface waves will be determined using a technique known as the electric surface-current model [22]. In this technique, the Green's function \( G \) of an infinitesimal electric-current element lying on a grounded dielectric slab of infinite extent is derived, and is used in conjunction with an assumed current density distribution \( J \) on the rectangular patch to find the radiated power, and the power lost to surface waves. The details of this technique are given in Perlmutter et al. [22], and only the results relevant to this work will be stated here.

Assuming the \( E_{10}^z \) mode of operation, it has been shown by Perlmutter et al. [22] that the expression for the power radiated into the upper hemisphere by a rectangular microstrip patch such as that shown in Fig. C-1 is

\[
P_{rad} = \frac{15k_0^2}{8\pi} \int_0^\pi \int_0^{2\pi} \left( \frac{\left| J_x \sin \phi \right|^2 \left( \cos \phi \right)^2}{\left( \varepsilon_r - (\sin \theta)^2 \right)^2 \left( \cot (hk_0\sqrt{\varepsilon_r - (\sin \theta)^2}) \right)^2 + \left( \cos \phi \right)^2} \right) \sin \theta d\theta d\phi
\]

\[
+ \frac{\left| J_x \cos \phi \right|^2 \left( \varepsilon_r - (\sin \theta)^2 \right) \left( \cos \phi \right)^2}{\left( \varepsilon_r - (\sin \theta)^2 \right) + \varepsilon_r^2 \left( \cos \phi \right)^2 \left( \cot (hk_0\sqrt{\varepsilon_r - (\sin \theta)^2}) \right)^2} \right) \sin \theta d\theta d\phi
\]

\]

(C-8)
where

θ is the elevation angle in a spherical coordinate system;

φ is the azimuth angle in a spherical coordinate system;

$k_0$ is the wave number in free space;

$\varepsilon_r$ is the relative permittivity of the microstrip material;

$\tilde{J}_x$ is the Fourier transform of the $x$-component of the electric current-density distribution on the rectangular patch.

It is worth noting here that eq. (C-8) is of the form

$$P_{rad} = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} f^2(\theta, \phi) \sin \theta d\theta d\phi$$

\hspace{1cm} (C-9)

where $f^2(\theta, \phi)$ is the power pattern.

The microstrip materials chosen for this work are electrically thin at the operating frequencies used, and support the lowest-order surface-wave mode (TM0) only. It has been shown by Perlmutter et al. [22] that $P_{sw}$, the power lost in this mode, is

$$P_{sw} = 15k_0^2 A_S \int_0^{2\pi} |\tilde{J}_x(\theta, \phi)|^2 (\cos \phi)^2 d\phi$$

\hspace{1cm} (C-10)

$$A_S = \frac{\varepsilon_r(x_p^2 - 1)x_p}{\varepsilon_r x_p \left( \frac{1}{\sqrt{x_p^2 - 1}} - \sqrt{x_p^2 - 1} \right) + x_p k_0 h \left( 1 + \frac{(x_p^2 - 1)\varepsilon_r^2}{\varepsilon_r - x_p^2} \right)}$$

\hspace{1cm} (C-11)

$$\theta_p = \text{asin}(x_p)$$

\hspace{1cm} (C-12)

where $x_p$ is the TM0 surface-wave pole determined by solving the nonlinear equation

$$\varepsilon_r \sqrt{x_p^2 - 1} - \sqrt{\varepsilon_r - x_p^2} \tan(hk_0\sqrt{\varepsilon_r - x_p^2}) = 0$$

\hspace{1cm} (C-13)
For the $E_{10}^{z}$ mode of operation, the assumed $x$-directed electric-current-density distribution is

$$
\bar{J}_x(x, y) = \frac{V}{Z_0 b} \sin \left( \beta \left( \frac{a}{2} + x \right) \right) \hat{x} \quad \frac{-a}{2} \leq x \leq 0
$$

$$
\frac{V}{Z_0 b} \sin \left( \beta \left( \frac{a}{2} - x \right) \right) \hat{x} \quad 0 \leq x \leq \frac{a}{2}
$$

where $V$ is the voltage between any point along one of the patch edges of width $b$ and a point on the ground plane directly below it, and $Z_0$ and $\beta$ are as defined in eqs. (A-1) and (A-4) with $W$ replaced by $b$. In this case, application of the two-dimensional Fourier transform

$$
\tilde{J}(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} J(x, y) e^{-j k_x x} e^{-j k_y y} \, dx \, dy
$$

(C-15)

to the current-density distribution defined in eq. (C-14) gives

$$
\tilde{J}_x(k_x, k_y) = \frac{4 V \beta \left( \cos \left( \frac{a k_x}{2} \right) - \cos \left( \frac{a \beta}{2} \right) \right) \sin \left( \frac{b k_y}{2} \right)}{b k_y Z_0 (\beta^2 - k_x^2)}
$$

(C-16)

where

$$
k_x = k_0 \sin \theta \cos \phi
$$

(C-17)

$$
k_y = k_0 \cos \theta \cos \phi
$$

(C-18)

Hence, the radiated power $P_{\text{rad}}$ and the power lost to surface waves $P_{\text{sw}}$ may be readily determined using eqs. (C-8) and (C-10), and may be subsequently used to calculate the quality factor $Q$ of the rectangular microstrip patch.
Appendix D

POWER ANALYSIS OF A PERIODICALLY LOADED LEAKY-WAVE MICROSTRIP ANTENNA

D.1 Introduction

Before proceeding with the derivation of the various expressions for the power dissipated in each of the $N$ cells of the periodically loaded leaky-wave microstrip antenna shown in Fig. D-1, a power analysis is given first for a loaded two-port network. This will serve as a basis for carrying out the power analysis of the aforementioned antenna.

Figure D-1. Network representation of a truncated periodically loaded leaky-wave microstrip antenna made up of $N$ microstrip cells. Lumped loads of equal value $Z_d$ are placed at regular intervals $d$ along the antenna.
D.1.1 Power analysis of a two-port network

For a two-port network such as that shown in Fig. D-2, it may be shown that the real power available from the voltage source $V_S$ with a complex source impedance $Z_S$ is

$$P_{\text{AVS}} = \frac{|V_S|^2}{8\text{Re}[Z_S]}$$  \hspace{1cm} \text{(D-1)}$$

and does not depend on the $S$-parameters of the two-port network or the complex load $Z_L$. If $Z_{\text{IN}}$ is used to denote the input impedance of the loaded two-port network, then the input reflection coefficient $\Gamma_{\text{IN}}$ is

$$\Gamma_{\text{IN}} = \frac{Z_{\text{IN}} - Z_S}{Z_{\text{IN}} + Z_S}$$
Figure D-2. Two-port network loaded with an impedance $Z_L$, and excited by a voltage source $V_S$ with a source impedance $Z_S$. The reference impedance for the $S$ parameters of the two-port network is taken to be $Z_0$.

and the reflection coefficients looking into the load, $\Gamma_L$, and into the source, $\Gamma_S$, are

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$  \hspace{1cm} (D-2)$$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$  \hspace{1cm} (D-3)$$

where $Z_0$ is the reference impedance for the $S$ parameters.

Let $P_{IN}$ and $P_L$ denote the input power into the loaded network, and the power dissipated in the load respectively. It has been shown in [49] that

$$P_{IN} = P_{AVS} \frac{(1 - |\Gamma_S|^2)(1 - |\Gamma_{IN}|^2)}{|1 - \Gamma_S \Gamma_{IN}|^2}$$  \hspace{1cm} (D-4)$$

$$P_L = P_{AVS} \frac{(1 - |\Gamma_S|^2)|S_{21}|^2(1 - |\Gamma_L|^2)}{|1 - \Gamma_S \Gamma_{IN}|^2|1 - S_{22} \Gamma_L|^2}$$  \hspace{1cm} (D-5)$$
If the source impedance $Z_S = Z_0$, $\Gamma_S = 0$, so that eqs. (D-4) and (D-5) reduce to

$$P_{IN} = P_{AVS}(1 - |\Gamma_{IN}|^2)$$  \hspace{1cm} (D-6)

$$P_L = P_{AVS} \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - S_{22} \Gamma_L|^2}$$  \hspace{1cm} (D-7)

At this stage, it is necessary to introduce a number of notations that will prove to be useful in subsequent sections. The first of these, $[ABCD]_i$, will be used to denote the transmission matrix of the $i^{th}$ microstrip cell shown in the equivalent network representation of Fig. D-1. When expressed in terms of the impedance-matrix elements of a microstrip cell, $[ABCD]_i$ may be written as

$$[ABCD]_i = \begin{bmatrix}
Z_{11} - Z_{12} & (Z_{11} - Z_{12})(Z_{44} - Z_{43}) & - (Z_{14} - Z_{13})(Z_{41} - Z_{42}) \\
Z_{41} - Z_{42} & Z_{41} - Z_{42} \\
1 & Z_{44} - Z_{43} & Z_{44} - Z_{42}
\end{bmatrix}, \ i = 1 \ldots N$$  \hspace{1cm} (D-8)

The second notation, $[ABCD]_{d_i}$, is used to denote the transmission matrix of the $i^{th}$ load $Z_d$ in the equivalent network representation of Fig. D-1 where

$$[ABCD]_{d_i} = \begin{bmatrix}
1 & Z_d \\
0 & 1
\end{bmatrix}$$  \hspace{1cm} (D-9)

Finally, the notation $[A_iB_iC_iD_i]$ will be used to refer to the transmission matrix

$$[A_iB_iC_iD_i] = \begin{bmatrix}
A_i & B_i \\
C_i & D_i
\end{bmatrix} \text{ where } i \text{ is any positive integer.}$$
D.1.2 Power dissipated in the resistive terminations

Using eq. (D-7) and the equivalent network in Fig. D-1, it may be shown that the power dissipated in the two resistive terminations $R_L$ is given by

$$P_L = \frac{|V_s|^2|S_{21}|^2(1-|\Gamma_L|^2)}{4Z_0} \frac{1}{|1 - S_{22}\Gamma_L|^2}$$  \hspace{1cm} (D-10)

where

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$  \hspace{1cm} (D-11)

$$S_{21} = \frac{2}{A_1 + B_1Y_0 + C_1Z_0 + D_1}$$  \hspace{1cm} (D-12)

$$S_{22} = \frac{-A_1 + B_1Y_0 - C_1Z_0 + D_1}{A_1 + B_1Y_0 + C_1Z_0 + D_1}$$  \hspace{1cm} (D-13)

$$[A_1B_1C_1D_1] = [ABCD]^1[ABCD]^d_1[ABCD]^d_2[ABCD]^d_{n-1}[ABCD]^d_N$$  \hspace{1cm} (D-14)

$$Y_0 = \frac{1}{Z_0}$$  \hspace{1cm} (D-15)

D.1.3 Power dissipated in the first microstrip cell

Using eqs. (D-6)-(D-7), and the equivalent network of Fig. D-1, one may show that the power dissipated in the first microstrip cell is given by

$$P_1 = 2(P_{IN} - P_L)$$  \hspace{1cm} (D-16)

where $P_{IN}$ is half the power dissipated in all microstrip cells, lumped loads $Z_0$, and resistive terminations $R_L$, and is given by

$$P_{IN} = \frac{|V_s|^2}{8Z_0}(1 - |\Gamma_{IN}|^2)$$  \hspace{1cm} (D-17)
\[
\Gamma_{IN} = \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0} 
\]

(D-18)

\[
Z_{IN} = \xi''_{11} - \frac{\xi''_{12} \xi''_{21}}{\xi''_{22} + R_L} 
\]

(D-19)

\[
[\xi''] = \begin{bmatrix}
A_2 & 1 \\
C_2 & C_2 \\
1 & D_2 \\
C_2 & C_2
\end{bmatrix} 
\]

(D-20)

\[
[A_2 B_2 C_2 D_2] = [ABCD]_1 [ABCD]_d \ldots [ABCD]_{d_{N-1}} [ABCD]_N 
\]

(D-21)

and \( P_{L_1} \) is half the power dissipated in the remaining parts of the antenna past the first cell, where

\[
P_{L_1} = \frac{|V_S|^2 |S_{21}|^2 (1 - |\Gamma_{L_1}|^2)}{8Z_0 \left| 1 - S_{22} \Gamma_{L_1} \right|^2} 
\]

(D-22)

\[
\Gamma_{L_1} = \frac{Z_{L_1} - Z_0}{Z_{L_1} + Z_0} 
\]

(D-23)

\[
Z_{L_1} = \xi'_{11} - \frac{\xi'_{12} \xi'_{21}}{\xi'_{22} + R_L} 
\]

(D-24)

\[
[\xi'] = \begin{bmatrix}
A_1 & 1 \\
C_1 & C_1 \\
1 & D_1 \\
C_1 & C_1
\end{bmatrix} 
\]

(D-25)

\[
[A_1 B_1 C_1 D_1] = [ABCD]_d_1 [ABCD]_2 \ldots [ABCD]_{d_{N-1}} [ABCD]_N 
\]

(D-26)
\[ S_{21} = \frac{2}{A + BY_0 + CZ_0 + D} \]  
(D-27)

\[ S_{22} = \frac{-A + BY_0 - CZ_0 + D}{A + BY_0 + CZ_0 + D} \]  
(D-28)

\[ [ABCD] = [ABCD]_1 \]  
(D-29)

\[ Y_0 = \frac{1}{Z_0} \]  
(D-30)

D.1.4 Power dissipated in the last microstrip cell

Using eq. (D-7) and the equivalent network in Fig. D-1, one may show that the power dissipated in the last microstrip cell is given by

\[ P_N = 2(P_{L_{N-1}} - P_L) \]  
(D-31)

where \( P_{L_{N-1}} \) is half the power dissipated in the rest of the antenna past the \((N-1)\)th lumped load \( Z_d \), and is given by

\[ P_{L_{N-1}} = \frac{|V_S|^2 |S_{21}|^2 (1 - |\Gamma_{L_{N-1}}|^2)}{8Z_0 \left| 1 - S_{22} \Gamma_{L_{N-1}} \right|^2} \]  
(D-32)

\[ \Gamma_{L_{N-1}} = \frac{Z_{L_{N-1}} - Z_0}{Z_{L_{N-1}} + Z_0} \]  
(D-33)

\[ Z_{L_{N-1}} = \xi_{11} - \frac{\xi_{12} \xi_{21}}{\xi_{22} + R_L} \]  
(D-34)
\[ [\xi] = \begin{bmatrix} \frac{A_2}{C_2} & \frac{1}{C_2} \\ \frac{1}{C_2} & \frac{D_2}{C_2} \end{bmatrix} \] (D-35)

\[ [A_2B_2C_2D_2] = [ABCD]_N \] (D-36)

\[ S_{21} = \frac{2}{A_1 + B_1 Y_0 + C_1 Z_0 + D_1} \] (D-37)

\[ S_{22} = \frac{-A_1 + B_1 Y_0 - C_1 Z_0 + D_1}{A_1 + B_1 Y_0 + C_1 Z_0 + D_1} \] (D-38)

\[ [A_1B_1C_1D_1] = [ABCD]_1[ABCD]_d\ldots[ABCD]_{N-1}[ABCD]_{d_{N-1}} \] (D-39)

\[ Y_0 = \frac{1}{Z_0} \] (D-40)

and \( P_L \) is half the power dissipated in the resistive terminations, as given by eq. (D-10).

**D.1.5 Power dissipated in the \( i^{th} \) microstrip cell**

Using eq. (D-7) and the equivalent network in Fig. D-1, one may show that the power dissipated in the \( i^{th} \) microstrip cell is

\[ P_i = 2(P_{L_{i-1}} - P_{L_i}), \quad 1 < i < N \] (D-41)

where \( P_{L_{i-1}} \) is half the power dissipated in the rest of the antenna past the \((i-1)^{th}\) lumped load \( Z_d \), and is given by

\[ P_{L_{i-1}} = \frac{|V_s|^2}{8Z_0} \frac{|S_{21}|^2}{[1 - \Gamma_{L_{i-1}}]^2} \frac{1}{[1 - S_{22} \Gamma_{L_{i-1}}]^2} \] (D-42)
\[ \Gamma_{L_{i-1}} = \frac{Z_{L_{i-1}} - Z_0}{Z_{L_{i-1}} + Z_0} \]  
(D-43)

\[ Z_{L_{i-1}} = \xi_{11} - \frac{\xi_{12} \xi_{21}}{\xi_{22} + R_L} \]  
(D-44)

\[ [\xi'] = \begin{bmatrix} \frac{A_2}{C_2} & 1 \\ \frac{1}{C_2} & \frac{D_2}{C_2} \\ \frac{1}{C_2} & \frac{D_2}{C_2} \end{bmatrix} \]  
(D-45)

\[ [A_2 B_2 C_2 D_2] = [ABCD]_i[ABCD]_{d_i} \ldots [ABCD]_N \]  
(D-46)

\[ S'_{21} = \frac{2}{A_1 + B_1 Y_0 + C_1 Z_0 + D_1} \]  
(D-47)

\[ S'_{22} = \frac{-A_1 + B_1 Y_0 - C_1 Z_0 + D_1}{A_1 + B_1 Y_0 + C_1 Z_0 + D_1} \]  
(D-48)

\[ [A_1 B_1 C_1 D_1] = [ABCD]_1[ABCD]_{d_i} \ldots [ABCD]_{i-1}[ABCD]_{d_{i-1}} \]  
(D-49)

\[ Y_0 = \frac{1}{Z_0} \]  
(D-50)

and \( P_{L_i} \) is half the power dissipated in the antenna past the \( i^{th} \) microstrip cell, and is given by

\[ P_{L_i} = \frac{|V_S|^2 |S''_{21}|^2 (1 - \left| \Gamma_{L_i} \right|^2)}{8Z_0 \left| 1 - S''_{22} \Gamma_{L_i} \right|^2} \]  
(D-51)

\[ \Gamma_{L_i} = \frac{Z_{L_i} - Z_0}{Z_{L_i} + Z_0} \]  
(D-52)
Z_L_i = \xi_{11}'' - \frac{\xi_{12}'' \xi_{21}''}{\xi_{22}'' + R_L} \tag{D-53}

[\xi''] = \begin{bmatrix} A_4 & \frac{1}{C_4} \\ \frac{1}{C_4} & D_4 \end{bmatrix} \tag{D-54}

[A_4B_4C_4D_4] = [ABCD]_d[ABCD]_{i+1}...[ABCD]_N \tag{D-55}

S_{21}'' = \frac{2}{A_3 + B_3Y_0 + C_3Z_0 + D_3} \tag{D-56}

S_{22}'' = \frac{-A_3 + B_3Y_0 - C_3Z_0 + D_3}{A_3 + B_3Y_0 + C_3Z_0 + D_3} \tag{D-57}

[A_3B_3C_3D_3] = [A_1B_1C_1D_1][ABCD]_i \tag{D-58}

Y_0 = \frac{1}{Z_0} \tag{D-59}

D.1.6 Power dissipated in the i^{th} lumped load Z_d

Using eq. (D-7) and the equivalent network in Fig. D-1, one may show that the power dissipated in the i^{th} lumped load Z_d is given by

\[ P_{d_i} = (P_{L_i} - P_{L_d}) \quad 1 \leq i \leq N - 1 \tag{D-60} \]

where \( P_{L_i} \) is half the power dissipated in the rest of the antenna past the i^{th} microstrip cell, and is given by

\[ P_{L_{i-1}} = \frac{|V_s|^2|S_{21}'|^2(1 - |\Gamma_{L_{i-1}}|^2)}{8Z_0 \left| 1 - S_{22}'\Gamma_{L_{i-1}} \right|^2} \tag{D-61} \]
\[ \Gamma_{L_i} = \frac{Z_{L_i} - Z_0}{Z_{L_i} + Z_0} \]  
\( (D-62) \)

\[ Z_{L_i} = \xi_{11}' - \frac{\xi_{12}' \xi_{21}'}{\xi_{22}' + R_L} \]  
\( (D-63) \)

\[ [\xi'] = \begin{bmatrix} A_2 & \frac{1}{C_2} \\ C_2 & \frac{1}{A_2} \\ \frac{1}{C_2} & \frac{D_2}{C_2} \end{bmatrix} \]  
\( (D-64) \)

\[ [A_2 B_2 C_2 D_2] = [ABCD]_d_i [ABCD]_{i+1} \ldots [ABCD]_N \]  
\( (D-65) \)

\[ S_{21}' = \frac{2}{A_1 + B_1 Y_0 + C_1 Z_0 + D_1} \]  
\( (D-66) \)

\[ S_{22}' = \frac{-A_1 + B_1 Y_0 - C_1 Z_0 + D_1}{A_1 + B_1 Y_0 + C_1 Z_0 + D_1} \]  
\( (D-67) \)

\[ [A_1 B_1 C_1 D_1] = [ABCD]_i [ABCD]_{d_i} \ldots [ABCD]_i \]  
\( (D-68) \)

\[ Y_0 = \frac{1}{Z_0} \]  
\( (D-69) \)

and \( P_{Ld_i} \) is half the power dissipated in the antenna past the \( i \)th lumped load \( Z_{d_i} \) and is given by

\[ P_{Ld_i} = \frac{|V_S|^2 |S_{21}''|^2 (1 - |\Gamma_{Ld_i}|^2)}{8Z_0 \left| 1 - S_{22}'' \Gamma_{Ld_i} \right|^2} \]  
\( (D-70) \)

\[ \Gamma_{Ld_i} = \frac{Z_{Ld_i} - Z_0}{Z_{Ld_i} + Z_0} \]  
\( (D-71) \)
\[Z_{Ld_t} = \xi_{11}'' - \frac{\xi_{12}'' \xi_{21}''}{\xi_{22}'' + R_L}\] (D-72)

\[
[\xi''] = \begin{bmatrix}
\frac{A_4}{C_4} & 1 \\
\frac{1}{C_4} & \frac{C_4}{C_4} \\
\frac{1}{C_4} & \frac{D_4}{C_4}
\end{bmatrix}
\] (D-73)

\[
[A_4B_4C_4D_4] = [ABCD]_{i+1}[ABCD]_{d,i+1} \ldots [ABCD]_N
\] (D-74)

\[
S_{21}'' = \frac{2}{A_3 + B_3Y_0 + C_3Z_0 + D_3}
\] (D-75)

\[
S_{22}'' = \frac{-A_3 + B_3Y_0 - C_3Z_0 + D_3}{A_3 + B_3Y_0 + C_3Z_0 + D_3}
\] (D-76)

\[
[A_3B_3C_3D_3] = [A_1B_1C_1D_1][ABCD]_{d_i}
\] (D-77)

\[
Y_0 = \frac{1}{Z_0}
\] (D-78)
REFERENCES


