DESIGN AND MODELING OF A NEW ELECTRO HYDRAULIC ACTUATOR

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science Graduate Department of Mechanical Engineering University of Toronto

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Abstract

This thesis addresses the issues of design and mathematical model identification of an advanced hydraulic system referred to as Electro Hydraulic Actuator (EHA).

In the design part, insights into system design methodology are presented and used for the EHA prototype manufacturing. To get better knowledge of the system behavior, extensive open loop tests were performed. Test results are presented in the time and frequency domain, and it was concluded that system behavior is nonlinear.

In the modeling part, a nonlinear mathematical model of the EHA is derived. Some of the unknown model parameters had to be identified experimentally by optimization. The last step in the system mathematical modeling was open and closed loop model validation using the experimental data. Proposed nonlinear mathematical model gives good correspondence with experimental data and can be used for system studying or controller design.

Performed test show that the EHA system has high positioning accuracy (0.01mm), fairly high bandwidth (23 Hz with 20kg inertial load) and is easy to control. It shows significant benefits when compared to conventional hydraulic systems in the areas of control, efficiency and physical dimensions.
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1 Introduction

1.1 Motivation

There are two main areas of research and development in the field of robotics: development of a new control algorithms and development in the field of a robot mechanical design. As a cross-disciplinary field, robot design requires knowledge in electronics, system control and mechanical design.

Large number of papers published in the past deal with robotics control [1,2,3]. The main concern of these authors was to improve dynamic behavior of existent robot by introducing advanced control algorithms. Good control schemes can improve existent system behavior to some extent, but any further improvements can be achieved only by mechanical re-design. A very important aspect of robot mechanical design is the design of robot actuators as the prime movers. Actuators are very important for developing robot dynamics. Lately, the attention paid to actuator design has been increased, and it can be expected that any future breakthrough in robotics will be associated with a development in this area [4].
CHAPTER 1

Introduction

The focus of this thesis is on the design and testing of a novel Electro Hydraulic Actuator referred to as EHA. This project was carried out in the Robotics and Automation Laboratory, Department of Mechanical Engineering at the University of Toronto. The thesis work has two main stages:

1. EHA mechanical design
   2. EHA testing and mathematical modeling

The first stage presents a concept of a new electro-hydraulic circuit and basic engineering calculations used for its design. This circuit was designed and manufactured as a research prototype.

The second stage involves rigorous open loop circuit testing and development of a corresponding mathematical model. Since some of the model parameters such as leakage, effective bulk modulus and viscous friction were not measurable, it was necessary to perform parameter identification to determine their values. The EHA prototype setup used for testing and mathematical modeling is presented in Appendix B.

1.2 Prior Work

1.2.1 System concept

Basically, there are two types of hydraulic transmission circuits widely used in industry: valve controlled and pump controlled. Each type of circuit has its own specificity and will be discussed in the following sections.
Valve controlled circuits

The valve controlled hydraulic circuit uses a hydraulic valve as a control element while the hydraulic pump runs at constant speed (giving a constant flow rate). A typical circuit is depicted in Figure 1.1. The hydraulic valve directs the oil flow generated by a pump to the hydraulic actuator (a hydraulic motor or cylinder). This type of circuit is widely used in industry due to its high performance quality. The best performance is accomplished by use of a servovalve as a control element. A circuit usually requires a large oil reservoir exposed to the atmospheric pressure. The disadvantage of the circuit is its low efficiency which can reach values under 30% as a result of throttle losses at the valve. The valve controlled circuit can use any type of actuator, such as hydraulic motors, rotational actuators or single and double rod cylinders. However, problems with single rod cylinders (differential cylinders) arise from different piston areas (the

Figure 1.1  Typical valve controlled hydraulic circuit
usual area ratio is 2:1). This difference produces different speeds and/or forces for piston extrusion and retraction when used with symmetrical valves. Part of this problem can be solved by using a "differential valve" that has a ports size ratio of 2:1 such as Rexroth proportional valves [5]. A differential valve provides an equal piston speed for extrusion and retraction but unfortunately with different cylinder output forces for these two regimes.

From the control point of view, double-rod cylinders are very convenient. They have equal piston areas so there are no problems with speed or force asymmetry. However, in industry their application is limited because additional dead space is required for the second cylinder rod to move freely. The provision of the dead space is restrictive in applications where the effective length of the cylinder is limited by the fixtures surrounding the piston.

**Hydrostatic transmission circuits**

The second type of hydraulic transmission is achieved by a direct coupling of the hydraulic pump and the hydraulic motor. This type of circuit is called a hydrostatic transmission circuit. Here, the hydraulic pump has two tasks: to generate oil pressure and flow, and to perform as an actuator control element. There are four types of circuits as discussed in [6] and presented in Figure 1.2. All four configurations use a constant speed electric motor as a circuit prime mover.

The simplest type of hydrostatic transmission circuit is created by coupling a fixed displacement hydraulic pump to a fixed displacement hydraulic motor Figure 1.2 a). In this case the hydraulic motor speed is constant so there is no speed control. The configuration presented in Figure 1.2 b) employs a fixed displacement hydraulic pump coupled to a variable displacement hydraulic motor. By changing the motor swash plate angle it is possible to vary its volumetric displacement and to control shaft velocity.
The hydraulic circuit presented in Figure 1.2 c) is widely used in industry and contains a variable displacement pump coupled to a fixed displacement hydraulic motor. Pump flow rate is varied by changing its displacement. This type of hydrostatic transmission is well established. It usually requires an additional actuation circuit for swash plate angle control. The additional circuit presented in Figure 1.3 consists of one small cylinder (8) controlled by a proportional valve (9) integrated with the pump housing. This type of pump is referred to as a servo-pump.

The last case, Figure 1.2 d), involves variable displacement pump and motor. Theoretically, this circuit should provide an excellent opportunity to set optimally the circuit working point (from the point of circuit efficiency). Unfortunately, in practice, this type of circuit is difficult to control and is less often used [6].

Application of hydrostatic transmission circuits for linear actuation is more complex than for rotational actuation. A linear motion circuit involves a bi-directional variable displacement pump coupled to a double rod cylinder, Figure 1.3. In industry, single rod cylinders are predominantly

![Figure 1.2 Hydraulic transmission circuit types](image)
used, but their application in hydrostatic transmission creates difficulties. Most of the commercial single rod cylinders have a free piston side and rod side area ratio of 1:2. When the piston is moving, this difference requires twice the flow rate on the free piston side as on the rod side. Since all hydrostatic transmission circuits are closed circuits, input and output pump flows must be nearly equal. A large difference would lead to pump cavitation. Hence, direct application of a conventional single rod cylinder in the circuit depicted in Figure 1.3 is physically impossible.

There were attempts to solve this problem as presented in [7]. The proposed solution requires an

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Figure 1.3   Typical high power hydrostatic circuit
additional element called a hydraulic transformer, Figure 1.4. This element consists of two mechanically coupled pumps that add or deduct volume flow depending on the direction of piston movement. The hydraulic valve used in this circuit can select one of two regimes: load stroke or rapid stroke. In the load stroke regime when the valve is in position 1, the system behaves like a symmetrical cylinder with a large piston area. Hence, it has low maximal speed and high output force. In the rapid stroke regime it behaves like a symmetrical cylinder with a small piston area, giving high maximal speed but low output force. Unfortunately, the circuit is quite complex and much more expensive than the conventional hydrostatic transmission. This type of circuit also requires pump swash plate actuation and an additional pump, presented in Figure 1.3, but omitted for simplicity in Figure 1.4.

To the best of author's knowledge, to date, hydraulic transformer is the only solution to the problem of applying single rod cylinders in hydrostatic transmission circuits.

Figure 1.4  Modified hydrostatic transmission circuit
1.2.2 Mathematical modeling

Hydraulic control systems have been intensively studied in the past. Most of the research is performed for conventional valve control systems. An important foundation for hydraulic control systems in general was laid by Blackburn [8], Merritt [9] and Viersma [10].

Merritt has developed and discussed linear models and some nonlinear phenomena of electrohydraulic systems and components in general. His work, mostly based on the description of conventional valve controlled circuits, derives linear mathematical models. He presented simplified linear mathematical model of hydrostatic transmission circuits.

Viersma systematized and presented commonly used combinations of valve-cylinder systems and their dynamics. His work also expands onto the dynamics of hydraulic lines. He discusses issues, such as the influence of transmission line length and accumulator position on hydraulic systems dynamics. He contributed much to the development of conical hydrostatic bearings for the high speed frictionless hydraulic cylinders used in flight simulator setups.

Further, many of the papers published recently deal with control aspects of hydrostatic transmission circuits. A hydrostatic transmission circuit similar to the one presented in Figure 1.3 was studied by Wang Sun An et. al. [11]. They noted that derivation of the mathematical model of this system is very complicated and model parameter values are difficult to obtain. In order to design a dead beat controller they used a simplified second order mathematical model with a time delay, obtained from the system step response.

An analysis of hydrostatic transmission position control servo was performed by Burton, Edge and Burrows [12]. They derived a distributed parameter system model that includes transmission
line modeling. This approach could be used for simulation of a wave propagation effects, transmission lines effect and distributed line friction.

Katsuhiko Hattori et al. [13], developed a unidirectional hydrostatic circuit for a cooling fan actuation. The novelty of the control approach is that the fan speed is controlled by an electro-hydraulic pressure control valve, while a hydraulic pump delivers a constant flow rate. In order to study pressure fluctuations in such a system, they derived a mathematical model involving the influence of elasticity of hoses, viscous friction and load inertia. The deficiency of the method is that it produces pressure fluctuations of a low frequency and high amplitude within the circuit.

From the point of view of mathematical modeling and unknown parameter identification, a study on the variation of oil effective bulk modulus with pressure was performed by Yu Jing Hong et. al. [14]. They derived a theoretical model of bulk modulus value as a function of four unknown parameters. The derived model was significantly nonlinear, so they used optimization in order to fit the model to the experimental results, and obtain the values of unknown parameters. For the EHA model parameter identification this approach was adopted and used.

Another study on the identification of model parameters was performed by Jelali and Schwarz [15]. They introduced modified Recursive Instrumental Variables algorithm and applied it to identify nonlinear model parameters of the hydraulic system.

1.3 Contributions and Organization of the Thesis

The major contributions of the thesis are summarized as follows:
1. Based on the research and conceptual design performed in the Robotics and Automation Laboratory at the University of Toronto [16,17], a prototype of Electro Hydraulic Actuator was designed and manufactured. EHA involves a novel hydraulic transmission control concept and a new type of hydraulic cylinder - the New Linear Actuator.

2. Extensive experimental testing of the system prototype was performed and the relevant experimental results are analyzed and presented.

3. The system mathematical model was derived and experimentally validated. The mathematical model was derived based on physical relations and experimental model optimization-based parameter identification was performed. The results obtained enhance the continuation of this project, and the understanding of electro hydraulic system.

There are five chapters in the thesis. Chapter 1 provides the background and motivation for research and development of the system, and reviews the relevant literature. Chapter 2 explains the system concept and concentrates on the basic engineering calculations necessary for the system design. Chapter 3 provides a nonlinear mathematical model of the EHA facility, and simplifies it to allow for some insights into the system dynamic. Chapter 4 begins by presenting the open loop system tests, discusses the influence of electrical and hydraulic parts of the system on the overall system behavior, and concludes with a nonlinear mathematical model optimization and resulting model experimental validation. Chapter 5 presents the conclusions.
2 Mechanical Design of the Electro Hydraulic Actuator

2.1 Introduction

Two types of actuation systems are commonly used in robotics: electric motors and hydraulic actuation systems. This section will describe some of the advantages and disadvantages of each.

Electric motors are clean and do not need the same level of maintenance required by hydraulic power transmission systems. Their application is limited since they need gearing to accommodate high loads and low speed. Disadvantages of gearing include low efficiency and increased weight and backlash. Furthermore, electric motors are not suitable for hostile environments involving water or radiation.

On the other hand, hydraulic actuation generates high torque even at low speed, without the need for gearing. In hydraulic systems mechanical energy is generated by a power unit which is located apart from the actuator. Therefore, it is possible to obtain a much higher torque to actuator weight ratio than with electric motors. In hydraulic systems, torque to mass ratio is limited only by the
strength of the actuator material, and is determined by the oil supply pressure within the hydraulic circuit. The deficiencies of hydraulic actuation include:

- maintenance
- nonlinearity of the circuit
- circuit complexity
- high level of noise
- low conventional circuits power efficiency (under 30%)
- strong coupling between the actuators of the same circuit

Due to complexity and nonlinearity of hydraulic circuits, precise circuit control is quite complex and often requires advanced control techniques [18, 19]. These deficiencies have significantly affected the application of hydraulic systems. Figure 2.1 graphically compares the time constant and power to mass ratio of electric and hydraulic motors [20].

![Figure 2.1](image_url)  
**Figure 2.1** Comparison between time constants and power to mass ratio for electric and hydraulic motors
CHAPTER 2  Mechanical Design of the Electro Hydraulic Actuator

The objectives of this research are the prototyping and verification of a new actuation system, referred to as the Electro Hydraulic Actuator (EHA) which would compete with conventional hydraulic systems. EHA has a high torque to mass ratio, low friction and deadband and preserves the good characteristics of conventional hydraulic actuation. It has a much higher efficiency, more linear characteristics and reduced coupling between actuators.

2.2 System Description

The EHA operates on the principle of closed-circuit hydrostatic transmission. As mentioned in Section 1.2.1, in these systems the exhaust oil from the hydraulic cylinder (or motor) is returned directly to the pump inlet rather than to the reservoir (see Figure 2.2). Because the transmission of the fluid power is obtained by coupling a positive displacement pump to the hydraulic motor, a control valve is not required. The pump is driven by a prime mover, which is an AC electric motor. Since the system is of closed-type, there is no direct contact between the oil and air.

Until now, closed circuit hydrostatic transmission systems have used variable displacement piston pumps and a prime mover (electric motor) with a constant angular speed. Other common circuit control configurations were presented in Section 1.2.1 Figure 1.2.

EHA uses a bi-directional gear pump (P) coupled to a single-rod hydraulic cylinder. The pump is of fixed displacement type, and the hydraulic cylinder rod position is controlled by varying the speed of the electric motor. This is a new approach from the point of circuit control, since the need for additional actuation of pump swash plate is eliminated.

Another novelty of the EHA circuit is the design and application of the New Linear Actuator
CHAPTER 2  Mechanical Design of the Electro Hydraulic Actuator

(NLA)[17]. Application of a conventional single rod cylinder in a hydrostatic transmission circuits results in design and control complexity as presented in Section 1.2.1 Figure 1.4. EHA requires no additional hydraulic transformer. A picture of the EHA setup and assembly drawing of the circuit are given in Appendix B. The main EHA parts that will be described in the following sections are:

- Electric Motor with Amplifier
- Accumulator
- Hydraulic Pump
- Filters
- Inertial Load
- System Controller

2.2.1 Electric Motor and Amplifier

The prime mover of EHA is an AC electric motor which is controlled by a Digital Signal Processing (DSP) amplifier. The amplifier analog input is speed command signal from the range of ±10 Volts. The amplifier analog output used for measurements is motor speed, monitored by a resolver (±8 Volts for the motor full range speed). This signal is acquired via a National Instruments Data Acquisition Board AT-MIOH-16 installed in a PC. The board is also used for acquisition of the signals from pressure sensors (S₁) and (S₂) Figure 2.2. The link between the electric motor and hydraulic pump is obtained by a very stiff coupling (C) which minimizes the system deadband.

2.2.2 Accumulator

The minimal system pressure can be set up between 40-100 psi (2.76-6.9 bar), and is necessary to prevent both, oil cavitation and the introduction of air into the system. The EHA system
presented in Figure 2.2 can be divided into two hydraulic sub-systems: a high pressure outer circuit connected to a cylinder, and an inner circuit that employs a low pressure accumulator.

The accumulator replaces leakage losses from each line and maintains a minimal system pressure. Conceptually, it is similar to a reservoir in open type systems, serving as a source of system oil. In this application it also prevents cavitation by pressurizing the forward and return circuit lines at a preset pressure.

The check valve from the accumulator to pressure line opens when the pressure in the main line is bellow the value of

\[
\text{pressure} = (\text{accumulator preset pressure}) - (\text{check valve cracking pressure})
\]

Thus, the secondary circuit (accumulator) delivers additional oil for the main line in order to prevent pump cavitation. The check valve cracking pressure of 5 psi is quite low.

2.2.3 Hydraulic Pump

The hydraulic pump in this circuit is the main control element and the source of hydraulic power. It converts mechanical energy, received from the electric motor, to fluid pressure and flow. This
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is a bi-directional fixed displacement gear pump with three ports. Two large diameter ports are input-output ports and the third is a case drain port provided for pump leakage. The input-output ports are directly connected to cylinder ports via two steel tubes. The case drain line, as shown in Figure 2.2, connects the pump case drain to the inner circuit via the check valve. This minimizes oil loss by returning the oil leakage back to the circuit.

Based on the continuity equation, which states that for steady flow in a pipeline the mass flow rate must be the same for all cross sections of the pipe, the flow rate in both pressure lines must be equal. In the case of conventional single-rod cylinder, the flow rates are not equal since the piston areas are not equal. To obtain equal flow rates, a custom single-rod cylinder (NLA) was designed with equal pressure areas on both sides of the piston.

2.2.4 Filters

Filtering is necessary in the industrial version of the EHA to extend the life of hydraulic elements by preventing oil contamination. The pump (P) in Figure 2.2 is bi-directional so neither of the pressure lines (L₁, L₂) can be specified as forward or return flow lines. The rectifier circuits (R₁, R₂) ensure one-way flow through corresponding filter elements F₁, F₂. They are not used in the EHA prototype, since clearance within the gear pump and cylinder are quite large, meaning that only large particles of dirt would erode them. Filtering in EHA is not as critical as in conventional hydraulic circuits with servovalve, where clearances between the valve spool and body are only a few micrometers. Servovalves require filters with 6–10 μm absolute filtering for correct operation, while the EHA circuit requires only 20 μm filtering.
2.2.5 Inertial Load

For testing and studying the prototype circuit, an inertial load of 20 kg is attached to the cylinder rod and placed on linear bearings (Appendix B). The complete system is mounted on an aluminum plate, and when placed horizontally the load is purely inertial. If the system is mounted vertically, the load behaves as a gravitational and inertial load.

2.2.6 System Controller

The controller parameters are load position and velocity. Output force can be calculated by multiplying the pressure difference by the affected area:

\[
\text{force} = (\text{pressure difference across piston}) \times (\text{pressure area})
\]

where the numerical values are obtained from the pressure sensors (S₁) and (S₂), Figure 2.2. Position of the load is directly determined by a linear optical encoder. Speed can be calculated by a numerical differentiation of the position readings from the linear optical encoder (E).

The EHA controller is a Pentium-based PC with a Data Acquisition Board (12 bit DAC and ADC), Digital Input-Output board and the encoder counter board. The control software with all low-level functions is written in the C language. Controller sampling rate is 1 kHz.

2.3 Circuit Design Considerations

This section contains the circuit design and associated engineering calculations of the EHA. These
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calculations determine the required electric motor power, the accumulator charging pressure, pipe
dimensions and basic pump-actuator calculations. The usual procedure for hydraulic circuit
design involves the following steps:

1. Determine the required power output.

2. Select the actuator type based on kinematic requirements of the load (cylinder, motor, or
   vane type). Decide whether gearing should be used.

3. Select the control valve type and size (on-off valve, servovalve or proportional valve).
   Selection depends on the required type of control and system dynamics.

4. Select the pump type and size based on application (gear, vane, or piston).

5. Select the other miscellaneous components such as reservoir, piping, check and relief
   valves, accumulator and filter elements.

6. Calculate the overall system cost.

This procedure can be repeated several times with different sizes and types of components until
the best overall system is obtained. The goal is to design the best possible system in terms of cost
and performance.

A step 7 could involve system modeling in order to predict its dynamic behavior. This is a good
practice, but very often avoided (to shorten the design time). Instead, the system performance
could be tested on a prototype. This may reduce cost-effectiveness, since any subsequent upgrades
may require hardware changes.
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The above modified procedure was used for the EHA design. Since there was no a priori application, the EHA was designed around a NLA prototype, with specific constraints such as maximal positioning accuracy, small physical dimensions and high power output. Hence, the first two steps of the aforementioned design procedure were determined by the design of NLA. Because the EHA concept does not include control valve, step three was omitted. Steps four and five together with the NLA concept are described in the following sections.

2.3.1 New Linear Actuator (NLA)

Section 1.2.1 explained that the conventional single-rod cylinder is an inappropriate actuator in the hydrostatic transmission circuit. On the other hand, single-rod cylinder is convenient for robotics and other applications, due to ease of mounting. Presently, there are no commercial single-rod cylinders with equal piston areas on the commercial market. This limits the application of hydrostatic transmission circuits to hydraulic motors or double-rod cylinders.

A symmetrical linear actuator - NLA, was proposed and prototyped by the Robotics and Automation Laboratory [17]. The novelty of the design is that although the actuator is a single-rod type it has equal piston areas on both sides. This new design in actuator technology has many advantages over the conventional single-rod cylinder design. The actuator prototype is made of stainless steel, and all dynamic seals are of a lip pre-energized type made from low-friction Teflon.

The actuator has three inner volumes. V1 and V2, as shown in Figure 2.4, are working volumes and V3 is at the atmospheric pressure. The existence of the third volume made it possible to design a cylinder with equal working pressure areas $A_1 = A_2 = 5.051 \times 10^{-4} \, m^2$. 
The benefits of this design are the following:

1. A linear version of EHA must use a symmetrical actuator such as NLA. NLA is ideally suited to such a hydrostatic transmission circuit.

2. If the NLA is used in a conventional hydraulic circuit involving control valve, rod speed and output force are equal in both directions of piston movement - there is no need for compensation as explained in section 1.2.1. This also simplifies the control algorithm.

3. Greater flexibility in applications by virtue of having a third chamber, V3. The combination of three different pressure sources can be used to control the output stroke. Also, two of the volumes could be pressurized by oil and one by gas, or all three volumes by
different pressurized oil sources. This versatility means that it can be used as a differential element in a hydraulic circuit.

The disadvantage of this design is a more complicated fabrication procedure than for a conventional single-rod cylinder. The three sets of dynamic seals provide more opportunity for oil leakage than for the conventional single rod cylinder (which has only two seals). All three seals are exposed to atmospheric pressure from one side and to high pressure on the other (maximal pressure difference) yielding is the worst case in terms of leaks.

2.3.2 Pump-Hydraulic Cylinder Connection

Cylinder rod positioning accuracy is highly dependent on pump volumetric displacement. If the pump has a very small displacement per revolution it will discharge a small amount of fluid per revolution, resulting in very fine motion of the cylinder rod. A pump with smaller volumetric displacement will provide higher control resolution. On the other hand, rod speed is proportional to the pump displacement. So, there is a trade-off between maximal rod speed and positioning accuracy - a higher pump displacement would provide a higher maximum speed but a lower positioning accuracy.

The main factors for pump selection are: pump size, efficiency, working pressure and volumetric displacement. A fixed displacement gear pump (John S. Barnes G.C. series No. 04) was selected (see Appendix A). The pump has a volumetric displacement $q_p$ of 0.065 cu.in./rev (1.065 cc./rev) with a maximum angular speed of 4000 rpm. This is a high pressure pump that can produce a nominal output pressure of up to $P_{\text{max}} = 3000$ psi ($207 \cdot 10^5$ Pa) and maximum intermittent output pressure of $P_{\text{max}} = 4000$ psi ($276 \cdot 10^5$ Pa). All calculations performed here are for the ideal system, neglecting leakage flows and associated pressure drops.
At a maximum angular speed \( N \) of 4000 rpm the pump delivers a flow rate \( Q_p \) of:

\[
Q_p = q_p \cdot N = 4.26 \frac{1}{\text{min}} = 7.1 \times 10^{-5} \frac{m^3}{s}
\]

Cylinder piston area in Figure 2.4 is:

\[
A_c = [1.7502^2 - 1.4375^2] \cdot \frac{\pi}{2} = 0.78288 \text{ sq.in.} = 5.051 \times 10^{-4} m^2
\]

which gives a theoretical maximum rod speed of:

\[
\nu_{\text{max}} = \frac{Q_p}{A_c} = 1.406 \times 10^{-1} \frac{m}{s} = 14 \frac{cm}{s}
\]

The theoretical nominal output force is:

\[
T = P_{\text{max}} \cdot A_c = 207 \times 10^5 \cdot 5.051 \times 10^{-4} = 10.5 kN
\]

and maximal intermittent output force:

\[
T = P_{\text{max}} \cdot A_c = 276 \times 10^5 \cdot 5.051 \times 10^{-4} = 13.9 kN.
\]

### 2.3.3 Electric Motor Selection

I order to achieve high dynamic performance of the EHA, the electric motor and motor controller were selected according to the following characteristics: output torque, maximal speed, mechani-
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cal and electrical time constants, and precision of motion at low speed. During its operation, the
EHA circuit spends most of the time in the low speed zone. Therefore, motor low speed precision
directly affects the overall system dynamics and positioning accuracy.

Nominal power output of the hydraulic pump is:

\[ P_{pn} = Q_p \cdot P_n = 7.1 \cdot 10^{-5} \cdot 207 \cdot 10^5 = 1.5 \text{ kW} \]

and maximal output is:

\[ P_{pmax} = Q_p \cdot P_{max} = 7.1 \cdot 10^{-5} \cdot 276 \cdot 10^5 = 2 \text{ kW}. \]

Motor power requirements were calculated by assuming the worst case scenario of pump power
efficiency, \( \eta_p = 80\% \):

\[ P_{mot} = \frac{P_{pmax}}{\eta_p} = \frac{2}{0.8} = 2.5 \text{ kW}. \]

Based on this requirement the closest motor that satisfies the requirements is Mavior MA 30 (see
Appendix A). This motor is an AC brushless motor with a pancake construction which results in
very compact size and high efficiency. Motor rated continuous power output is 3184 W. It comes
with an integrated resolver which is used to provide motor velocity feedback.

Based on the hydraulic pump and electric motor diagrams (Appendix A), velocity vs. output
torque diagram is depicted in Figure 2.5. It can be seen that the electric motor has more power
than needed for the selected pump in the nominal working regime. A Mavior MA 20 motor would
have been appropriate for the selected hydraulic pump, but MA 30 was chosen to provide the option of testing the circuit with a larger hydraulic pump.

The motor controller has two main tasks: to deliver energy to the electric motor and to control its speed. It also monitors motor overheating and current overload. Hence, the Infranor SMTBD1 digital brushless servo controller was selected. It is a high output amplifier (Imax=30 A) integrated with a digital reconfigurable controller. It can be connected to a PC by a RS232 line. All necessary controller parameters can be adjusted by using Microsoft Windows™ based software, including: maximum motor speed, maximum acceleration and deceleration times, offset compensation and type of control - P, PI or PI².
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2.3.4 Accumulator charging pressure

The accumulator makes up for all volumetric oil losses, and maintains a minimal circuit pressure of $P_{\text{min1}} = 40 \text{ psi} \ (2.76 \text{ bar})$. This is important because pressure drop in the circuit could lead to oil cavitation; therefore, the minimal working pressure should be higher than $P_{\text{min1}}$. The accumulator pressure is limited to $P_{\text{min2}}=100 \text{ psi} \ (6.9 \text{ bar})$ since a pressure rise in excess of 100 psi would damage the pump seals. The accumulator is of bladder type as shown in Figure 2.3 and has a working volume of $V_{\text{acc}} = 1.5 \text{ cu.in} \ (2.458 \cdot 10^{-5} \text{ m}^3)$.

If isothermal conditions are assumed, then:

$$V_{\text{acc}} = \frac{\Delta V}{P_0 - P_{\text{min1}}} \frac{P_0}{P_{\text{min2}}}$$  \hspace{1cm} (1.1)

where $\Delta V = Q_L \cdot t$ is the oil volume that comes form leakage $Q_L$ during the time $t$.

Following the recommendations of [21], when the accumulator is used for leakage compensation, the accumulator charging pressure is $P_0 = 0.9 \ P_{\text{min1}} = 2.48 \text{ bar}$. From Equation 1.1 the maximum oil leakage compensation is $\Delta V = 13.273 \cdot 10^{-6} \text{ m}^3$. This implies that the accumulator can compensate leakage flows of $Q_L = 0.01327 \text{ l/min}$. This is a low value, but if the external leakages are low, it is sufficient for long-term system operation.

2.3.5 Pressure lines dimensioning

In bi-directional hydraulic transmission circuits, there are no designated supply and return lines.
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For that reason lines between the pump and cylinder should be dimensioned according to criterion that generates maximum inside pipe diameter, i.e. as a return line. This reduces the possibility of cavitation at the pump inlet port. The local pressure drop due to the filter elements is not accounted for in calculations since they were not used in the EHA prototype.

For cylinder to pump connections, steel pipes C-1010 are used with a 3/8 inch outside and a d = 0.277 inch inside diameter. They can withstand a nominal pressure of 3267 psi and maximum burst pressure of 19.600 psi. The pressure drop in the return line of the EHA circuit can be calculated using the following procedure:

The Reynolds number is:

\[ \text{Re} = \frac{K_2 Q_p}{v} = \frac{180.966 \cdot 7.1 \cdot 10^{-5}}{4 \cdot 10^{-5}} = 321 \]

where \( K_2 = \frac{4}{\pi d} \) = 180.966 and \( v = 4 \cdot 10^{-5} \frac{m^2}{s} \) is kinematic viscosity [22]. Low value of Reynolds number \( (Re < 2000) \) indicates that fluid flow is laminar.

The friction factor for laminar flow is:

\[ f = \frac{64}{\text{Re}} = 0.199 \]

The return line has a length \( l = 0.3 \) m and the pressure loss due to friction in a pipe according to Darcy's formula for a circular pipe is:
The coefficient of losses in the pipe bend (inlet T branch on the pump end, Appendix B) is $K_{bend} = 1$. The pressure loss due to bend is obtained as:

$$\Delta P_{bend} = \frac{K_{bend} Q^2}{K_1} = \frac{(7.1 \cdot 10^{-5})^2}{3.515 \cdot 10^{-12}} = 1.4 \text{ kPa}$$

The pressure loss due to the elevation difference between the pump and cylinder is:

$$P_{ele} = \rho \cdot g \cdot H = 9797 \cdot 9.81 \cdot 0.17 = 16.3 \text{ kPa}$$

where $\rho = 9797 \text{ kg/m}^3$, is oil density

$g = 9.81 \text{ m/s}^2$, is acceleration of gravity

$H = 0.17 \text{ m}$, is elevation difference.

The total pressure loss is:

$$P = \Delta P_{\text{lines}} + \Delta P_{\text{bend}} + \Delta P_{\text{ele}} = 29.9 \text{ kPa} = 4.4 \text{ psi}$$

Since the accumulator pressure is 40 psi, the total pressure loss on the path from cylinder to pump will reduce it to 35.6 psi at the pump inlet port. This is sufficient for proper operation. Small positive pressure at the pump inlet port minimizes the possibility of oil cavitation.
2.4 Summary

This chapter contains the description of the EHA circuit with preliminary engineering calculations. It provides insights into circuit components and its operation. This is the basis for further system analysis and mathematical modeling. The central component of the system is a New Linear Actuator and benefits from its use have been outlined. The NLA application is not limited to the EHA circuit and can be utilized in any conventional circuit with the same benefits.

The following chapter deals with the detailed system mathematical modeling and study of model parameters influence on the system response. The system is divided in two parts: electrical and hydraulic. Respective mathematical models is derived.
CHAPTER 3  Dynamic Modeling of the EHA

3 Dynamic Modeling of the EHA

3.1 Introduction

The design of a high performance system should be based on an in-depth knowledge of its dynamic characteristics. This in-depth knowledge of system dynamics can be obtained by mathematical modeling that provides opportunity of predicting the system’s behavior. In industrial applications the model is used for the following:

- Designing the control system.

- Meeting design objectives and market requirements.

- Ensuring data capture and design documentation for future re-use and system re-design, particularly when the influence of different parameters needs to be investigated.

Physical systems can be described by models of different levels of mathematical complexity. Given the complexity of real processes, only those characteristic of the process that are of
significance need to be modeled. For example, the model of the EHA can be obtained by taking into consideration all micro phenomena in the fluid flow (molecular theory) or, more appropriately, by assuming a one-dimensional continuous fluid flow. These two approaches produce models with significantly different levels of mathematical complexity, the choice of which is determined by the purpose of the modeling. Herein, the EHA is modeled by assuming a one-dimensional fluid flow.

The procedure for the EHA modeling is described as follows:

1. A model type is selected according to the characteristics of the system study (linear or non-linear, distributive or lumped, etc.). EHA is described by a non-linear continuous lumped model.

2. Mathematical model is derived according to physical principles. This determines the model structure.

3. Initial values of model parameters are obtained from the data provided by manufacturers of system components. Model parameters are estimated by model optimization.

4. The model is validated by comparing simulation and experimental responses.

In this section, two mathematical models of the EHA facility are derived for the following working regimes: 1) unidirectional motion; and 2) bi-directional motion. In each case the models are simplified and assumed to behave linearly within their working envelopes. These models are used for the analysis of the system dynamics.
3.2 Hydraulic circuit mathematical model

3.2.1 Simplifying assumptions

The following simplifying assumptions are made during the process of modeling and analysis of the EHA:

**Assumption 1.** The discharge flow from the accumulator is instantaneous (the EHA inner circuit - Figure 2.1) and transients associated with accumulator discharge are neglected. This implies that the inner circuit is at constant pressure $P_r$ and acts instantaneously to prevent the pressure in the outlet circuit from falling below a minimum value $P_r$. This assumption becomes invalid if the check valves are leaking or have very high time constants (slow dynamical response).

This assumption is valid if there are no external leakages, i.e. the volume of oil in the system is constant. In normal system operation when external leakage is maintained low this is justified. This condition can be ensured even with a higher leakage rates, if the experiments performed are sufficiently short. Therefore, the system pressure due to leakage would not change significantly during this period of time.

Since the influence of the inner circuit on the system dynamics is neglected, the circuit diagram simplifies to the one depicted in Figure 3.1. The operation of the check valve (C1) and (C2) is assumed to be instantaneous.
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Dynamic Modeling of the EHA

Figure 3.1 EHA modeling diagram

Figure 3.2 Schematic of the NLA with load
CHAPTER 3 Dynamic Modeling of the EHA

Assumption 2. The external leakage of NLA and the internal and external leakage of the hydraulic pump are assumed to be linear functions of the pressure difference across their corresponding leaking orifices. This is the usual model of leakage used in literature [Merit, MIT, Vierama]. The external leakage of NLA Figure 3.2, consists of a component \((\frac{C_{ec2}}{2}) \cdot P_2\) from the chamber V2 between the cylinder rod and housing, where \(C_{ec2}\) is leakage coefficient. The chamber V3 is at the atmospheric pressure, so the second leakage component \((\frac{C_{ec2}}{2}) \cdot P_2\) occurs from chamber V2 by rod and cylinder housing to chamber V3. These two components are assumed to be half of the total V2 chamber leakage, since it is not possible to accurately predict their participation in the total chamber leakage rate. External leakage from the chamber V1 occurs between the internal bush and the inner side of the cylinder rod as shown in Figure 3.2, and is equal to \(C_{ec1} \cdot P_1\).

The hydraulic pump external leakage has two components Fig 3.3, as follows: 1) leakage from

![Diagram](image-url)

**Figure 3.3** External gear pump operation
the low pressure chamber to the pump case drain; and 2) leakage from the high pressure chamber to the pump case drain. The case drain pressure is assumed to be equal to the replenishing pressure $P_r$, Figure 3.1. The external leakage of the pump is obtained as $C_{ep}P_r$, where $C_{ep}$ is the external leakage coefficient. The internal pump leakage occurs between the stationary casing and the tips of the moving gear teeth. As pressure builds up on the outlet side of the pump, the pressure difference across the gears causes leakage as shown in Figure 3.3. This leakage is referred to as "slip" and affects the pump efficiency. Further internal leakage results from fluid being trapped between gear teeth and being transferred back, Figure 3.3. The total internal pump leakage coefficient is denoted as $C_{ip}$, and total internal leakage is $C_{ip} (P_1 - P_2)$.

**Assumption 3.** The inlet/outlet port of chamber V3 is open to atmospheric environment without restriction. Therefore, the chamber pressure is assumed to remain constant during the EHA operation.

**Assumption 4.** Coupling between the electric motor shaft and pump shaft is very stiff (stiffness=$20 \times 10^3$ Nm/rad) and its twist is negligible. The angular speed of the electric motor $\dot{\theta}_p$ and the pump shafts $\dot{\theta}_p$ are, therefore, assumed to be equal.

**Assumption 5.** A commonly used model of the friction force $F_f$ is depicted in Figure 3.4 and represented by three components as follows: $F_c$ is the Coulomb friction and is assumed to be constant throughout the whole working regime. $F_s$ is the static friction and is dominant at the beginning of motion. The viscous friction $F_v$ is a function of velocity. The Coulomb and static friction forces occur in the linear bearings and at the cylinder seals and are assumed to be negligible due to the use of linear bearings and Teflon seals, which have minimal friction. Viscous friction cannot be neglected and is modeled as $F_v = B_c \dot{x}_c$. 

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Figure 3.4 Friction force components

However, even with the very low friction bearings, Coulomb and static friction forces are still present and would dominate the system response for slow piston movement. It will be shown in Chapter 4 that for low input levels (1 V) system damping is influenced by these two friction components. The EHA model optimization was performed for a mid-range input signal level of 5 Volts (50%), so that the realized piston forces are much larger than friction components. Therefore, Coulomb and static friction influence on the 5 Volts system response is not dominant and can be neglected.

Assumption 6. The hydraulic transmission lines connecting the pump and the NLA are very stiff. Their expansion due to pressure changes is considered negligible. Consequently, the dynamics effect resulting from the pipe elasticity is neglected.

Assumption 7. The pressure transmissions in the pipes are assumed instantaneous and the pressure drop across the transmission lines are neglected. Furthermore, it is assumed that both
CHAPTER 3  Dynamic Modeling of the EHA

lines are of the same length, but without equal total volumes. However, these two volumes would be equal in the case of a small piston displacement and adequate piston starting position. In general, the piston motion is significant and not always from the middle position, but in order to obtain a linearized mathematical model, volume equality will be assumed.

3.2.2 Nonlinear mathematical model

In this section, a nonlinear mathematical model of the EHA is derived. Fundamental fluid mechanics equations used for mathematical modeling are given in Appendix C. Based on these equations, components of the EHA are modeled and their models are combined to provide an overall description of the system. The overall system model will be used in Chapter 4 to identify the unknown system parameters.

Pump flow equations

From Figure 3.1 and according to assumption 2 the outlet and inlet pump flow rates can be expressed as follows [8]:

\[
Q_{1p} = D_p \dot{\theta}_p - C_{ip} (P_1 - P_2) - C_{ep} (P_1 - P_r) \tag{3.1}
\]

\[
Q_{2p} = D_p \dot{\theta}_p - C_{ip} (P_1 - P_2) + C_{ep} (P_2 - P_r) \tag{3.2}
\]

where

\[D_p = \text{pump displacement, } m^3/\text{rad}\]

\[\dot{\theta}_p = \text{pump shaft speed, } \text{rad/s}\]

\[C_{ip} = \text{internal leakage coefficient of the pump, } m^3/\text{s/Pa}\]

\[C_{ep} = \text{external leakage coefficient of the pump, } m^3/\text{s/Pa}\]

\[P_1, P_2 = \text{outlet and inlet pressures, } Pa\]
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Hydraulic line equations

The continuity equation, Equation A.9, applied to transmission lines and assuming compressibility flow, gives:

\[ ( k_e + \frac{V_1}{\beta_e} ) \frac{dP_1}{dt} = Q_{1p} - Q_{1c} \]  
\[ ( k_e + \frac{V_2}{\beta_e} ) \frac{dP_2}{dt} = Q_{2c} - Q_{2p} \]  

where \( k_e \) = elasticity coefficient of pipe, \( m^3 / Pa \)
\( \beta_e \) = effective bulk modulus of fluid, \( Pa \)
\( V_1 \) = oil volume in the forward line plus volume in pump and cylinder chambers, \( m^3 \)
\( V_2 \) = total oil volume in return line, \( m^3 \)
\( Q_{1c} \) = inlet flow rate of the cylinder, \( m^3/s \)
\( Q_{2c} \) = outlet flow rate of the cylinder, \( m^3/s \).

From Figure 3.2, the total volume of each line is obtained as:

\[ V_1 = V_{01} + A_c x_c \]  
\[ V_2 = V_{02} + A_c ( L_c - x_c ) \]  

where \( x_c \) = piston absolute position, \( m \)
\( L_c \) = total piston stroke, \( m \)
\( V_{01} \) and \( V_{02} \) = dead volumes of the hydraulic lines, \( m^3 \).

By applying assumption 6, Equations 3.3 and 3.4 simplify to:
\[ \frac{V_1}{\beta_e} \frac{dP_1}{dt} = Q_{1p} - Q_{1c} \quad (3.7) \]
\[ \frac{V_2}{\beta_e} \frac{dP_2}{dt} = Q_{2c} - Q_{2p}. \quad (3.8) \]

**The NLA flow equations**

The flow equations associated with the NLA, Figure 3.2 are the following:

\[ Q_{1c} - C_{e1} P_1 = \frac{dV_1}{dt} \quad (3.9) \]
\[ - Q_{2c} - C_{e2} P_2 = \frac{dV_2}{dt} \quad (3.10) \]

where \( C_{e1} \) and \( C_{e2} \) are external leakage coefficients from the cylinder's chambers, \( m^3/s/Pa \) and \( V_1, V_2 \) are given by Equations 3.5 and 3.6.

**Cylinder force equilibrium equation**

From Figure 3.2 the force equilibrium equation is as follows:

\[ A_c (P_1 - P_2) = M \ddot{x_c} + K x_c + F_L + F_f \quad (3.11) \]

where \( A_c = \) piston pressure area, \( m^3 \)
\( P_1 \) and \( P_2 \) = pressures in the inlet and outlet cylinder chamber, \( Pa \)
\( M = \) load and piston mass, \( kg \)
\( K = \) spring constant, \( N/m \)
\( F_L = \) external force acting on load mass, \( N \)
\( F_f = \) total friction force occurring in linear bearings and at cylinder seals, \( N \).
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Assuming that there are no external forces acting on the cylinder, and following assumption 5, the Equation 3.11 is simplified to:

\[ A_c (P_1 - P_2) = M \dot{x}_c + B_c \ddot{x}_c \]  

(3.12)

where \( B_c \) = viscous friction coefficient, \( N/m/s \).

**Mathematical model of hydraulic sub-system**

Combining Equations 3.1, 3.2, 3.5, 3.6, 3.7, 3.8, 3.9 and 3.10, the following is obtained:

\[
\frac{V_{01} + A_c x_c}{\beta_e} \frac{dP_1}{dt} = D_p \dot{\theta}_p - C_{ip} (P_1 - P_2) - C_{ep} (P_1 - P_r) - C_{ec1} P_1 - A_c \dot{x}_c 
\]

(3.13)

\[
\frac{V_{02} + A_c (L_c - x_c)}{\beta_e} \frac{dP_2}{dt} = -D_p \dot{\theta}_p + C_{ip} (P_1 - P_2) - C_{ep} (P_2 - P_r) - C_{ec2} P_2 + A_c \dot{x}_c 
\]

(3.14)

Equations 3.12 through 3.14 represent the mathematical model of the hydraulic part of the EHA given the assumptions listed in Section 3.1. The derived model is nonlinear, and closed-form solutions to these differential equations are not possible. Model nonlinearity results from the terms on the left side of Equations 3.13 and 3.14, where piston position \( x_c \) is multiplied by the pressure derivative. Practical application of this model is conditioned by exact knowledge of values of relevant system parameters. In this case, the unknown values of effective bulk modulus (\( \beta_e \)), leakage coefficients (\( C_{ec1}, C_{ec2} \)) and viscous friction coefficient (\( B_c \)) need to be determined.

In order to get a qualitative picture of the EHA system dynamic behavior, Equations 3.12 through 3.14, are linearized and analyzed in the following section.
3.2.3 Analysis of the mathematical model

The use of the accumulator establishes a minimum line pressure $P_r$ as described in Section 2.2.2. Pressures $P_1$ and $P_2$ cannot fall below this value. The operational envelope of EHA can be divided into two distinct modes as described below:

- **Mode 1.** Applies for conditions when both pressures $P_1$ and $P_2$ vary simultaneously. This occurs when pump switches the flow direction and the pressure in the return line starts to increase from its constant value of $P_r$, while the pressure in the forward line starts to drop to $P_r$. This is a general mode of operation.

- **Mode 2.** Occurs when the system is running in one direction and when all transients resulting from any changes in direction have vanished. In this operation, the pressure in the return line is equal to the replenishing pressure $P_r$, while the pressure in the forward line varies according to pump action. The forward line is the control line, while the return line is passive.

Two linear models can be derived from Equations 3.12 to 3.14. These models will not be as accurate as a non-linear model, but can provide insight into the system dynamic behavior. To obtain a linear model of the system it is necessary to introduce additional assumptions:

**Assumption 8.** It is convenient to assume such piston position $x_c$ that gives an equal transmission lines volume:

$$V_1 = V_2 = V_r$$
where \( V_t \) is total volume and includes the cylinder and pump chamber with the connecting line. Further, it is assumed that changes of piston position \( x_c \) from the above mentioned initial position are small. Therefore, the volumetric term on the left-side of Equations 3.13 and 3.14 can be assumed to be constant.

**Assumption 9.** Value of the replenishing pressure \( P_r \) can be neglected when compared to the actual line pressure. Replenishing pressure has a significantly lower value than the circuit working pressure and can be neglected. This assumption will be used for the derivation of the linearized model for Mode 2 operation.

**Linear model for Mode 1**

In this mode, pressures \( P_1 \) and \( P_2 \) are both varying. Equation 3.14 may be subtracted from 3.13 to give:

\[
\frac{V_t}{\beta_e} \left( \frac{dP_1}{dt} - \frac{dP_2}{dt} \right) = 2D_p \dot{\theta}_p - 2 C_{ip} (P_1 - P_2) - C_{ep} (P_1 - P_2) - C_{ec1} P_1 + C_{ec2} P_2 - 2 A_c \dot{x}_c 
\]

(3.15)

The leakage coefficient for the left and right cylinder chambers are assumed equal such that:

\[ C_{ec1} = C_{ec2} = C_c \]

Applying a Laplace transformation to Equation 3.15, yields:

\[
\frac{V_t}{\beta_e} (s P_1(s) - s P_2(s)) = 2D_p \hat{\theta}_p (s) - 2 C_{ip} (P_1(s) - P_2(s)) - C_{ep} (P_1(s) - P_2(s)) - 
\]
Grouping the terms containing $P_1(s) - P_2(s)$ and combining Equation 3.12 with Equation 3.16, the following is obtained:

$$\frac{V_t}{\beta_e} s + 2 C_{ip} + C_{ep} + C_c \left( \frac{1}{A_c} (M_e s^2 + B_c x_c) X_c(s) = 2 D_p \dot{\theta}_p(s) - 2 A_c s X_c(s) \right)$$

The transfer function of the system is obtained by rearranging Equation 3.17:

$$\frac{X_c(s)}{\dot{\theta}_p(s)} = \frac{2 D_p}{A_c} \frac{M V_t}{2 \beta_e A_c^2} s^3 + \frac{B_c V_t}{2 \beta_e A_c^2} \frac{M C_t}{2 A_c^2} s^2 + \left( 1 + \frac{B_c C_t}{2 A_c^2} \right) s$$

where $C_t = 2 C_{ip} + C_{ep} + C_c$ is the total leakage coefficient.

**Linear model for Mode 2**

In Mode 2, the pressure in the return line is constant and equal to the replenishing pressure $P_r$. By substituting $P_2$ with $P_r$ in Equations 3.12 to 3.14, the continuity and force equilibrium equations are obtained as:

$$D_p \dot{\theta}_p - C_{ip} (P_1 - P_r) - C_{ep} (P_1 - P_r) - C_{et} P_1 - A_c \dot{x}_c = \frac{V_t}{\beta_e} \frac{d P_1}{d t}$$

$$A_c (P_1 - P_r) = M \ddot{x}_c + B_c \dot{x}_c$$

Rearranging Equation 3.19 gives:
Dynamic Modeling of the EHA

\[ D_p \dot{\theta}_p + C_p r P_r - (C_p + C_{e1})P_1 - A_c x_c = \frac{V_t}{\beta_e} \frac{dP_1}{dt} \]  
(3.1)

where \( C_{p} = C_{ip} + C_{ep} \) is the total pump leakage coefficient.

Applying the Laplace transformation on Equations 3.20 and 3.21 yields:

\[ D_p \dot{\theta}_p(s) + C_p r P_r = \left( \frac{V_t}{\beta_e} s + C_t \right) P_1(s) + A_c s X_c(s) \]  
(3.22)

\[ A_c (P_1(s) - P_r) = (M s^2 + B_c s^2) X_c(s) \]  
(3.23)

where \( C_t = C_p + C_{e1} \) is the total leakage coefficient.

Combining Equations 3.22 and 3.23 and assuming that \( P_r \) is negligible (assumption 9), the system transfer function is obtained as:

\[ \frac{X_c(s)}{\dot{\theta}_p(s)} = \frac{D_p}{A_c} \frac{\frac{M V_t}{\beta_e A_c^2} s^3 + \left( \frac{B_c V_t}{\beta_e A_c^2} + \frac{M C_t}{A_c^2} \right) s^2 + \left( 1 + \frac{B_c C_t}{A_c^2} \right) s}{\frac{B_c C_t}{A_c^2}} \]  
(3.24)

This transfer function describes a hydraulic system moving in one direction when all transients from changes in direction are neglected. The term \( \frac{B_c C_t}{A_c^2} \) is significantly less than 1 and can be neglected.
CHAPTER 3  
Dynamic Modeling of the EHA

Analysis of transfer functions

The transfer functions Equations 3.18 and 3.24 can be generalized as:

\[
\frac{X_c(s)}{\dot{\theta}_p(s)} = \frac{D_p}{A_c s} \frac{\omega_h}{\omega_h^2 s^2 + \frac{2 \zeta_h}{\omega_h} s + 1}
\]  
(3.25)

where \( \omega_h = \) hydraulic undamped natural frequency, rad/s  
\( \zeta_h = \) hydraulic damping ratio, dimensionless

The formulas for calculating the values of \( \omega_h \) and \( \zeta_h \) for the two modes of operation are given in Table 3.1. The undamped natural frequency of the system operating in Mode 1 (Equation 3.18) is \( \sqrt{2} \) times higher than that of Mode 2 (Equation 3.24). Because of the compressibility, oil trapped between the pump and cylinder piston behaves as an oil spring. The system model describing a system operating in Mode 1 includes both transmission lines while the model for Mode 2 only one. Therefore, the natural frequency of the Mode 1 model is \( \sqrt{2} \) higher than the Mode 2 model. In both cases, increase of inertial mass \( M \) and total volume \( V_t \) lowers the system's natural frequency. The influence of the effective bulk modulus \( \beta_e \) and piston area \( A_c \) is opposite. Therefore, the designer should attempt to keep the transmission lines as short as possible and try to achieve highest bulk modulus by bleeding the air out of system.

The influence of the system parameters on the damping ratio \( \zeta_h \) for the two modes of operation is compared in Table 3.1. It can be concluded that the most effective way to increase the system damping is by increasing the values of the leakage coefficient \( C_t \) and the viscous friction \( B_e \). Therefore, there is a trade-off between system damping and system efficiency.
CHAPTER 3  Dynamic Modeling of the EHA

From Equations 3.18 and 3.24, the system gain is proportional to the ratio of pump displacement over piston area $D_p/A_c$. Therefore, higher system maximal velocity can be obtained by a higher pump displacement. It should be noted that for single-rod asymmetrical cylinders which have different areas on each side of the piston, the system gain differs according to stroke direction. This introduces additional complexities in the control of conventional circuits.

Table 3.1  Transfer functions parameters for Mode 1 and Mode 2

<table>
<thead>
<tr>
<th></th>
<th>Equation 3.18</th>
<th>Equation 3.24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_h$</td>
<td>$\sqrt{\frac{2 \beta_\epsilon A_c^2}{M V_t}}$</td>
<td>$\sqrt{\frac{\beta_\epsilon A_c^2}{M V_t}}$</td>
</tr>
<tr>
<td>$\zeta_h$</td>
<td>$\frac{C_l}{2 \sqrt{2 A_c}} \sqrt{\frac{\beta_\epsilon M}{V_t}} + \frac{B_c}{2 \sqrt{2 A_c}} \sqrt{\frac{V_t}{\beta_\epsilon M}}$</td>
<td>$\frac{C_l}{2 A_c} \sqrt{\frac{\beta_\epsilon M}{V_t}} + \frac{B_c}{2 A_c} \sqrt{\frac{V_t}{\beta_\epsilon M}}$</td>
</tr>
<tr>
<td>$C_l$</td>
<td>$2 C_{ip} + C_{ep} + C_{ec1}$</td>
<td>$C_{ip} + C_{ep} + C_{ec1}$</td>
</tr>
</tbody>
</table>

3.2.4 EHA versus conventional hydraulic systems

From [9] the transfer function of a valve controlled cylinder can be expressed as:

$$\frac{X_p(s)}{X_v(s)} = \frac{K_q}{A_c} \frac{\omega_h}{s \left( \frac{s^2}{\omega_h^2} + \frac{2 \zeta_h}{\omega_h} + 1 \right)}$$  \hspace{1cm} (3.26)

where $X_p$ and $X_v$ are the piston and valve spool position

$K_q(x_v, P_L)$ is the valve flow gain, $m^3/s/m$
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Dynamic Modeling of the EHA

\[ \omega_n = \sqrt{\frac{4 \beta_e A_e^2}{M V_t}} \] is the natural frequency, rad/s

\[ \zeta_h = \frac{K_{ce} A_c}{4} \left( \frac{\beta_e M}{V_t} \right) + \frac{B_c}{4 A_c} \sqrt{\frac{V_t}{\beta_e M}} \] is the damping ratio, dimensionless

\[ K_{ce} = K_c + C_{tp} + \frac{C_{ep}}{2} \] is the total flow-pressure coefficient, \( m^3/s/Pa \)

\[ K_c(x_c, P_L) \] is valve flow-pressure coefficient, \( m^3/s/Pa \)

\[ P_L \] is the load pressure, Pa.

Because Equations 3.18, 3.24 and 3.25 are identical in form, the response of a valve controlled piston and the EHA are similar in nature. The difference is in the system parameters, such as the natural frequency and damping ratio. The natural frequency of a valve controlled cylinder is at least \( \sqrt{2} \) times higher than that of EHA (from Table 3.1). Furthermore, since the internal chamber volume of a valve is much smaller than that of a pump, this difference in natural frequency favors a valve-cylinder combination additionaly.

A significant benefit of the EHA is that the pump-cylinder leakage coefficients and system gain are nearly constant for all operating regimes, i.e. \( \frac{D_p}{A_c} = const. \) (Equation 3.25) and \( C_l = const. \)

Table 3.1. The gain \( K_q \) and the leakage characteristic \( K_c \) in the valve-piston combination are functions of the spool position \( x \), and load pressure \( P_L \). They change significantly with the system operating point [9]. This results in changes in stability margins of conventional hydraulic systems. The conclusion is that the EHA is more linear and more stable than conventional hydraulic systems, but has a slower dynamic response.
3.3 Electric motor transfer function

Dynamic model of electric motors has been extensively studied in the literature [23, 24]. Electric motors are usually described by first or second order linear transfer functions, depending on the type of motor and its controller. In this application, an AC electric motor is used and its mathematical model is presented as a first order linear transfer function, [23]. This is the simplest representation for the motor and involves only the mechanical time constant $T_m$ as an unknown parameter.

The velocity of the motor is controlled by a proportional-integral (PI) controller as depicted in Figure 3.5. The combined transfer function for the motor and its controller is obtained as:

![Electric motor control circuit diagram](image)

**Figure 3.5** Electric motor control circuit diagram
\[
\dot{\theta}_m(s) = \frac{K_p}{K_I} \frac{(s \frac{K_p}{K_I} + 1)\cdot U(s)}{T_m s^2 + \frac{1+K_p}{K_I} s + 1}
\] (3.27)

or,

\[
\dot{\theta}_m(s) = \frac{(T_I s + 1)}{T_m s^2 + T_m 1 + 1}
\] (3.28)

where \( T_m = \frac{T_m}{K_I}, T_m 1 = \frac{1+K_p}{K_I} \text{ and } T_I = \frac{K_p}{K_I} \).

The overall electrical sub-system is described by a second order linear transfer function with parameters \( T_I, T_m 1 \) and \( T_m 2 \). This is simplified motor-controller representation and it reflects dominant system behavior. Real controller electronics is much more complex, so model exact parameters values could not be determined without model identification.

### 3.4 Summary

The mathematical model of the EHA was divided into hydraulic and electrical sub-systems models. Transfer functions for each of the subsystems are presented in this chapter. First the hydraulic system nonlinear mathematical model is derived. By dividing the system operation into two working modes a linearized mathematical model of the hydraulic subsystem is derived. The influence of system parameters on the dynamic behavior is discussed.

Some of the parameters of EHA model can be determined by reference to data supplied by parts.
manufacturers. The values of the parameters that could not be determined from the supplier data, were identified experimentally as discussed in Chapter 4.
4 Experimental Determination of Unknown Parameters

4.1 Introduction

The area of engineering analysis devoted to the derivation of a relevant system description from observed data is called system identification. There are numerous system identification methods and various obtained model classifications depending upon the purpose of the analysis [25, 26]. For instance, white-box modeling can be performed if it is possible to describe a system by means of physical laws. This type of modeling assumes complete knowledge of the process. Since complete knowledge of a system is rarely achievable, other modeling methodologies include:

- Black-box modeling. The process is too complex and cannot be described by a few idealized physical relations. In this case, a general model structure can be used, in which unknown parameters are estimated.

- Gray-box modeling. Physical laws are used to describe a system, but not all model parameters are known. This case also requires estimation of a model parameters.
CHAPTER 4  Experimental Determination of Unknown Parameters

Mathematical models can be parametric or nonparametric. Models defined by a finite number of real parameters are known as parametric models. Power spectrum, frequency response and empirical transfer functions (ETFE) are known as nonparametric models.

The identification of mathematical models for single-input single-output linear systems is well developed [25,26]. However, it is difficult to apply linear systems identification methods to nonlinear systems [25]. One of the possible procedures for nonlinear system parameter identification is model optimization. There are two basic approaches using model optimization:

1. Optimization of the model (model parameters) such that the output of the model and real system coincides as closely as possible for the same input signal (error minimization via optimization).

2. Optimization of the controller parameters such that the closed-loop system has the desired behavior. The criteria for optimization can be lowest energy consumption, highest speed, etc. This is the task of designing the optimal controller.

In the previous chapters, by virtue of the physical laws, a mathematical description of the EHA plant was derived. The system is described by a nonlinear model and its structure is established, but not all model parameters are known. Hence, the EHA plant is presented by a gray-box model which requires unknown parameter estimation.

The estimation of the EHA model is based on the optimization problem 1, for which some basic theoretical background will be presented. In the following sections, the system empirical transfer function will be estimated by a nonparametric identification technique, followed by model parameter identification.
4.2 Definition of optimization problems

Generally, any optimization problem can be expressed in mathematical terms as:

\[
\text{minimize a function } \quad F(x), \quad x \in \mathbb{R}^n
\]

subject to constraints \( h(x) \leq 0. \)

A properly defined optimization problem has three characteristics:

- \( F(x) \) is known as a cost function
- \( x \) represents a set of variables to be used to optimize \( F(x) \)
- Bounds in the parameters space \( h(x) \) that bounds the solution inside the allowed region

The selection of a cost function depends on the type of optimization problem and the desired goal. One form of the function is the sum of absolute error values:

\[
F(x) = \sum_{k=1}^{N} |e_k(x)| \quad (4.2)
\]

where \( e_k(x) \) is error between the goal function and model.
CHAPTER 4  Experimental Determination of Unknown Parameters

Most often used criterion is the Least Squares, which can be expressed as:

\[ F(x) = \sum_{k=1}^{N} \varepsilon_k^2(x) \]  \hspace{1cm} (4.3)

For any optimization problem it is important to ensure that the located minimum is the optimal solution of the problem within the region of constraints. When the optimization problem is not subject to constraints, the solution would be a global minimum. In Figure 4.1 three typical cost function shapes are presented: convex, unimodal and nonunimodal functions. Convex and unimodal functions, Figure 4.1 a) and b), have a unique solution to the optimization problem. The solution for a nonunimodal function depends on the initial conditions and is not unique. In general, analytical description of the cost function \( F(x) \) is not known. Therefore, once a minimum has been detected, one can not claim that the solution is optimal, Figure 4.1c). This is a deficiency of the optimization methods.
CHAPTER 4 Experimental Determination of Unknown Parameters

4.2.1 Optimization techniques

Numerous techniques are presented in the literature [27,28] for solving the optimization problem, Equation 4.1. They can be divided as presented in [27]: Descent methods, Newton methods, Quasi-Newton methods and Conjugate gradient methods. Many highly sophisticated methods were also developed and implemented in computer software. Unfortunately, these methods tend to be so complex that the authors of [27] suggest that user should not start from scratch and develop his/her own method or software. Instead, the user should have a good understanding of the method and use selected routines from a high quality mathematical software library. The description of these methods is out of the scope of this thesis. It can be found in the formerly mentioned literature.

For solving the parameter optimization problem of the EHA, MatLab software and Simulink (products of MathWorks Inc.) were used. The function constr from the MatLab Optimization Toolbox provides a solution to the optimization problem with constraints, Equation 4.1. In addition to cost function, it is possible to define lower and upper bounds on parameter vector $x$ as part of the constr function input argument. The function general input form is:

$$x = \text{constr}(\text{'fun'}, x_0, \text{options}, vlb, vub)$$

where $fun$ = a cost function that is optimized,

$x_0$ = set of starting values,

$options$ = set of optimization parameters (min. and max. step, error, etc.)

$vlb$ and $vub$ = set of lower and upper bounds on $x$, respectively.

After completing the optimization successfully, constr returns a vector $x$ that represents a set of optimal parameter values for given initial values and constraints. The function solves a general
constrained non-linear optimization problem and is based on solving a Quadratic Programming (QP) subproblem [29]. The function to be minimized and the constraints must be continuous. The solution obtained may be only a local minimum. Details about the `constr` function and its additional options can be found in the Optimization Toolbox User Guide [29]. In conjunction with MatLab, Simulink software was used for dynamic simulation of the EHA. The integration algorithm used for simulation was based the 4th order Runge-Kutta method.

4.3 EHA open loop response

An open loop EHA transfer function was obtained by spectrum estimation based on the Fourier transform. This method is based on the discrete Fourier transform of input $u(t)$ and output $y(t)$ data. The transfer function estimate can be obtained according to definition of a transfer function as the ratio between the discrete Fourier transforms of outputs and inputs:

$$G(e^{i\omega k}) = \frac{Y(\omega)}{U(\omega)} \quad \omega_k = \frac{2 \pi k}{N}, \quad k = 1, ..., N$$

This estimate is defined only for discrete frequency points $\omega_k$. The experimental transfer function can be improved by a smoothing and averaging technique. The most common way of smoothing is by introduction of window functions. Application of window functions improves the estimated transfer function by reducing the spectral leakage associated with the finite observation interval [25].

To estimate the EHA transfer function, the MatLab function `spectrum` was used. It estimates the power spectral density of the input and output data and the coherence function between the
input and output. Smoothing of the data is performed by a selected frequency window (can be Hanning, Bartlett, Hamming [25]) and averaging by specifying the number of averaging data intervals (must be a power of two). Further, it provides 95% confidence intervals for input and output data spectral density and calculates the coherence spectrum. The coherence $\Gamma^2_{uy}$, expresses the degree of correlation between the input $u$ an output $y$ in the frequency domain:

$$\Gamma^2_{uy} = \frac{|S_{uy}(i\omega)|^2}{S_{uu}(i\omega) S_{yy}(i\omega)} \quad (4.5)$$

where $S_{uy}$ and $S_{uu}$ ($S_{yy}$) denote power cross and auto spectra respectively.

Testing of the EHA plant was performed while the system was in its horizontal configuration (load inertial) and with accumulator pressure of 5 bar. The test bed configuration was presented in Figure 3.1 and does not contain rectifier circuits with a filter element. The NLA volume $V_3$ was opened to atmospheric pressure.

The spectrum of the input signal must be "rich" enough to excite all required parameters of the system such that their values are reflected in the system output. Therefore, a sinusoidal input signal with an increasing frequency was selected as depicted in Figure 4.2 a). The amplitude of this signal was 3 V and its frequency varied from 0.3 to 1 Hz in increments of 0.1 Hz, and from 1 to 80 Hz with increment of 1 Hz. Higher input frequencies were not used since the electric motor has a protection circuit which switches the motor off for frequencies in excess of 80 Hz. The sampling rate was 1 kHz. Data from Figure 4.2 is normalized and 512 point FFT transformation is performed on both input and output data. For data smoothing in the frequency domain the Hanning window is applied. Resulting estimated amplitude and phase plots are presented in Figure 4.3. and Figure 4.4 respectively.
Figures 4.2 EHA time domain response

As it was stressed, the coherence spectrum is very useful for measuring the dependence between two signals. A value of coherence close to 1 indicates in what frequency ranges there is a good approximation with a linear model. Low values of coherence spectrum indicate that nonlinear relation between input and output or that disturbance is high in that range. Coherence spectrum for the EHA response, Figure 4.2, is depicted in Figure 4.5. It can be concluded that for frequencies of up to 30 Hz there is a linear relation between input and output signals. From the time response diagram, Figure 4.2, it can be found out that for frequencies larger than ~30 Hz the piston is not responding to input signal. The reason for this might be the analogue filter on the motor controller input which protects the motor from input signal noise. The coherence spectrum plot, Figure 4.5
Figure 4.3 EHA amplitude diagram for 3 V input

Figure 4.4 EHA estimated phase diagram
Figure 4.5 Coherence for 3 V sine input

also shows that amplitude and phase plots for frequencies larger than 30 Hz are not significant.

By applying the described procedure, amplitude and phase plots for different input signal levels were obtained, Figures 4.6 through 4.17. Since the maximum input signal level for the EHA is 10V, an amplitude of 1 V represents 10% of maximal input. The test input signal used to determine position and velocity Bode diagrams is a variable frequency sinusoidal signal, Figure 4.2 a). Each test was performed with different amplitude levels (1, 2, 3, 5, 7, and 10 Volts). In the case of position response Bode diagrams signal frequency changes in increments of 1 Hz in the interval of 1- 80 Hz for all amplitude levels. Input signal data is summarized in Table 4.1.

Velocity diagrams present a more "natural" system transfer function as the relation between same physical quantities: input signal (desired speed) and output - piston speed. Piston velocity was calculated by numerical differentiation of the recorded position data. This was performed by
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TABLE 4.1  Summarized open loop test signal data

<table>
<thead>
<tr>
<th>Position diagrams</th>
<th>Figure 4.6</th>
<th>Figure 4.7</th>
<th>Figure 4.8</th>
<th>Figure 4.9</th>
<th>Figure 4.10</th>
<th>Figure 4.11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input signal frequency range for position diagrams</td>
<td>1 - 80 Hz</td>
<td>1 - 80 Hz</td>
<td>1 - 80 Hz</td>
<td>1 - 80 Hz</td>
<td>1 - 80 Hz</td>
<td>1 - 80 Hz</td>
</tr>
<tr>
<td>Velocity diagrams</td>
<td>Figure 4.12</td>
<td>Figure 4.13</td>
<td>Figure 4.14</td>
<td>Figure 4.15</td>
<td>Figure 4.16</td>
<td>Figure 4.17</td>
</tr>
<tr>
<td>Input signal frequency range for velocity diagrams</td>
<td>0.1 - 80 Hz</td>
<td>0.2 - 80 Hz</td>
<td>0.3 - 80 Hz</td>
<td>0.5 - 80 Hz</td>
<td>0.7 - 80 Hz</td>
<td>1 - 80 Hz</td>
</tr>
<tr>
<td>Signal Amplitude</td>
<td>1 V</td>
<td>2 V</td>
<td>3 V</td>
<td>5 V</td>
<td>7 V</td>
<td>10 V</td>
</tr>
</tbody>
</table>

calculating the difference between two successive piston positions (sampling instants) and dividing it by the sampling interval length (1ms). Since, the resulting velocity signal was noisy, it was filtered by 5th order Butterworth digital filter with the cut-off frequency of 160 Hz. For the filter design, the MatLab function `butter` was used. Data filtering was performed with `filtfilt` function that introduces a zero phase shift in the filtered data. Hence, the resulting filtered velocity diagrams do not have phase distortion. The above described procedure (for position frequency response) was performed to determine velocity response amplitude and phase diagrams.

Diagrams in Figure 4.6 through 4.17 represent the overall system (electrical and hydraulic part) dynamics in the frequency domain. The diagrams support the following observations:

- System natural frequency $\omega_n$ varies in the range of 15 - 28 Hz depending on the input signal level.
Figure 4.6  Position frequency response for 1 V input

Figure 4.7  Position frequency response for 2 V input
Figure 4.8  Position frequency response for 3 V input

Figure 4.9  Position frequency response for 5 V input
Figure 4.10  Position frequency response for 7 V input

Figure 4.11  Position frequency response for 10 V input
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Figure 4.12  Velocity frequency response for 1 V input

Figure 4.13  Velocity frequency response for 2 V input
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Figure 4.14  Velocity frequency response for 3 V input

Figure 4.15  Velocity frequency response for 5 V input
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Figure 4.16  Velocity frequency response for 7 V input

Figure 4.17  Velocity frequency response for 10 V input
• From the peak of the magnitude plot, it can be observed that the system damping is not constant and depends on the input signal level. It decreases while the input level increases up to some point, after which it increases again.

• For the position response diagrams, Figures 4.6 through 4.11, the amplitude plots have a slope of -20 db/dec up to the resonant peak. For frequencies larger than natural frequency (≈30 Hz), the slope is much higher -100 db/dec. Therefore, it can be concluded that for lower frequencies system amplitude diagram resembles to one of the linear system.

Further, it is useful to estimate the values of the system undamped natural frequency $\omega_n$ and the damping ratio $\zeta_n$ for different input signal levels. This can be done by optimizing the linear model to the actual experimental data. Velocity frequency response diagrams, Figures 4.12 through 4.17, will be used for this analysis. These diagrams show a resemblance to second order system response (with the exception of a large amplitude slope at high frequencies). Hence, we can assume the overall system transfer function as a second order system of the form:

$$\frac{\dot{X}_c(s)}{U(s)} = \frac{K}{\frac{s^2}{\omega_n^2} + \frac{2\zeta_n}{\omega_n} s + 1} \quad (4.6)$$

where $\dot{X}_c(s)$ and $U(s)$ are the Laplace transformations of cylinder speed and input voltage while $\omega_n$ and $\zeta_n$ are the system natural frequency and damping ratio, respectively. This is a crude approximation but can give a range of values of natural frequency and system damping.

Equation 4.6 was optimized in the time domain by use of MatLab and Simulink. Figure 4.18
Figure 4.18 5 Volt input signal with experimental and simulated speed response
CHAPTER 4  

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presents experimental and simulated velocity response based on the transfer function, Equation 4.6. Simulink was used to simulate the transfer function response in the time domain, and MatLab `constr` function for optimization of the model parameters $\omega_n$ and $\zeta_n$. The gain $K$ was calculated as the ratio of input and output signal amplitudes. The cost function used in optimization is the sum of the squares of error values (difference between experimental and simulated response), Equation 4.3. For model optimization (Equation 4.6), the same experimental data was used as for the velocity frequency response, Table 4.1 for all input signal levels. The results of optimization are presented in Table 4.2 and Figures 4.19 through 4.24. The detailed procedure for performed optimization is explained in Section 4.4.2.

<table>
<thead>
<tr>
<th>TABLE 4.2</th>
<th>Identified linear model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal Voltage</td>
<td>$K \left( \frac{m}{V_s} \right)$</td>
</tr>
<tr>
<td>1</td>
<td>$1.619 \times 10^{-2}$</td>
</tr>
<tr>
<td>2</td>
<td>$1.409 \times 10^{-2}$</td>
</tr>
<tr>
<td>3</td>
<td>$1.468 \times 10^{-2}$</td>
</tr>
<tr>
<td>5</td>
<td>$1.420 \times 10^{-2}$</td>
</tr>
<tr>
<td>7</td>
<td>$1.405 \times 10^{-2}$</td>
</tr>
<tr>
<td>10</td>
<td>$1.309 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Figure 4.19  1V identified and experimental frequency response

Figure 4.20  2V identified and experimental frequency response
Figure 4.21  3V identified and experimental frequency response

Figure 4.22  5V identified and experimental frequency response
CHAPTER 4  Experimental Determination of Unknown Parameters

Figure 4.23  7V identified and experimental frequency response

Figure 4.24  10V identified and experimental frequency response
If we approximate the real (nonlinear) system by a second order model, the equivalent values of damping factor and natural frequency for different input signal levels have the values presented in Table 4.2.

The amplitude and phase plots of the overall system linear model and of the system based on the experimental data are presented in Figures 4.19 through 4.24. Table 4.2 shows that system gain $K$ is nearly constant through the whole range of input signal level (10-100%). This is not the case for $\omega_n$ and $\zeta_n$. The damping ratio of EHA is high for a small input signal level (0.8198) and small for mid-range input levels (from 0.1166 to 0.1872). The results obtained for 7 and 10 Volt sinusoidal input show an increase in the system damping, reaching a value of 0.58 in the latter case.

In order to get an insight into these phenomena it is useful to separate the electrical and hydraulic part of the EHA and study how they contribute to the overall system response. For this purpose the experimental data presented in Table 4.1 for the system velocity response was used. The electrical subsystem input is a voltage signal (desired speed) and the output is the electric motor speed. Input to the hydraulic part is hydraulic pump angular velocity, and the output is piston velocity. For all calculations, velocity signals were filtered by a 5th order digital Butterworth filter, with a cut-off frequency of 160 Hz. Amplitude plots of each part of the system were obtained by the procedure explained earlier and for each input signal they were plotted on separate diagrams, Figure 4.25 through 4.30. The amplitude response of the overall system is presented by a solid line, the hydraulic part by a dotted line and the electrical part by a dash-dotted line.

The amplitude diagrams Figure 4.25 trough 4.30 show that overall system behavior is influenced by both hydraulic and electrical part dynamics. It can be seen that natural frequency of the electrical part of the circuit decreases while the input signal level increases. In the case of the
Figure 4.25 Amplitude diagrams for 1 V input

Figure 4.26 Amplitude diagrams for 2 V input
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Figure 4.27  Amplitude diagrams for 3 V input

Figure 4.28  Amplitude diagrams for 5 V input
Figure 4.29  Amplitude diagrams for 7 V input

Figure 4.30  Amplitude diagrams for 10 V input
1V signal input, the amplitude plot of the electrical part is nearly flat, at 0 dB level, indicating that the overall system damping is the result of hydraulic circuit damping, Figure 4.25. On the other hand, for higher levels of input (2 - 10 V), hydraulic circuit damping has a nearly constant (low) value, while the natural frequency of the electrical part decreases. Therefore, in this range, the resulting overall system response is influenced by both, electrical and hydraulic parts. For the maximal system input of 10 V, the amplitude peak due to low hydraulic damping is being reduced by approximately -10 dB, as a result of the attenuation due to the electrical part of the circuit.

The following conclusions can be stated:

- Both the electrical and hydraulic parts of the circuit show nonlinear behavior, represented by variable damping and/or natural frequency.

- The hydraulic part has a nearly constant magnitude characteristic for higher input levels (2 - 10 Volts). For low input signal level of 1 V, the hydraulic part damping is quite high. An explanation for this might be that for this input level Coulomb and static friction force become dominant in the system response, resulting in an increased hydraulic system damping. Similar observations were reported by Merrit [9].

- The electric part of the circuit influences the system response for higher input levels (7 and 10 Volts), Figures 4.29 and 4.30. This phenomenon might come from motor saturation and results as lower overall system natural frequency.

Hence, for accurate system modeling, for a low level input signal (of 1 V and lower), it would be necessary to consider one of the friction models (see Section 3.2.1, assumption 5). This would increase the number of unknown model parameters. Herein, Coulomb and static friction forces
are neglected by assumption 5 in order to simplify the system model and reduce the number of optimization parameters. After model optimization, the Coulomb and static friction effects will be partially included in the selected optimization parameter values.

To avoid the effects of nonlinear phenomena due to friction and motor saturation, low level and high level input signals should be avoided as model optimization test signals. Instead, mid-range signals should be used, so the system identification was performed for a 5 V sinusoidal input.

From these observations it can be concluded that the real system has nonlinear behavior, and should be treated accordingly. However, the variation of system parameters is not too large, so for input signal range of 2-7V, a linear analysis is acceptable (but with less accurate results).

4.4 EHA mathematical model optimization

4.4.1 Mathematical model parameters

The nonlinear mathematical model of the hydraulic subsystem, as presented in Chapter 3, was given by:

\[
\frac{V_{01} + A_c x_c}{\beta_e} \frac{dP_1}{dt} = D_p \dot{\theta}_p - C_{ip} (P_1 - P_2) - C_{ep} (P_1 - P_r) - C_{ec1} P_1 - A_c \dot{x}_c
\]  

(3.10)

\[
\frac{V_{02} + A_c (L_c - x_c)}{\beta_e} \frac{dP_2}{dt} = -D_p \dot{\theta}_p + C_{ip} (P_1 - P_2) - C_{ep} (P_2 - P_r) - C_{ec2} P_2 + A_c \dot{x}_c
\]  

(3.11)

\[
A_c (P_1 - P_2) = M \ddot{x}_c + B_c \dot{x}_c
\]  

(3.12)
The electric motor - controller assumed transfer function was:

\[
\frac{\theta_m(s)}{U(s)} = \frac{(T_1 s + 1)}{T_m s^2 + T_m s + 1}
\]  \hspace{1cm} (3.25)

Some of the parameters figuring in Equation 3.10 through 3.12 were obtained from the manufacturer specifications or from the design data. However, viscous friction coefficient \( B_c \), cylinder leakage coefficients \( C_{ec1}, C_{ec2} \) and effective bulk modules \( \beta_e \) could not be calculated. The same stands for the electric motor parameters \( T_1, T_m1 \) and \( T_m2 \). Therefore, unknown parameters values should be determined by the numerical optimization of the mathematical model. In general, it is possible to measure a cylinder leakage by applying a constant pressure source to one cylinder chamber and measure the leakage over a long period of time. However, necessary equipment for such test was not available, so the leakage coefficients were estimated from the experimental data.

Mathematical model parameter values for electric motor transfer function are given in Table 4.3 and for the hydraulic sub-system model in Table 4.4. For the model unknown parameters, such as the leakage coefficients or effective bulk modules, expected ranges are specified. These values

**TABLE 4.3 Electric motor - controller parameters**

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value (range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain</td>
<td>( K_m )</td>
<td>( 40 - 42 \text{ rad/V s} )</td>
</tr>
<tr>
<td>Integral time constant</td>
<td>( T_1 )</td>
<td>( 5 \cdot 10^{-4} - 1 \cdot 10^{-2} )</td>
</tr>
<tr>
<td>Natural frequency</td>
<td>( \omega_m = \sqrt{1/T_m} )</td>
<td>( 10 - 30 \text{ Hz} )</td>
</tr>
<tr>
<td>Damping factor</td>
<td>( \zeta_m = T_m2 \omega_m / 2 )</td>
<td>( 0.6 - 0.8 )</td>
</tr>
</tbody>
</table>
TABLE 4.4 Hydraulic circuit parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value (range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piston area</td>
<td>$A_c$</td>
<td>$5.051 \times 10^{-4}$ m$^2$</td>
</tr>
<tr>
<td>Pump displacement</td>
<td>$D_p$</td>
<td>$1.6925 \times 10^{-7}$ m$^3$/rad</td>
</tr>
<tr>
<td>Internal pump leakage coef.</td>
<td>$C_{ip}$</td>
<td>$2 \times 10^{-13}$ m$^3$/s $/Pa$</td>
</tr>
<tr>
<td>External pump leakage coef.</td>
<td>$C_{ep}$</td>
<td>$2 \times 10^{-13}$ m$^3$/s $/Pa$</td>
</tr>
<tr>
<td>Replenishing pressure</td>
<td>$P_r$</td>
<td>$5 \times 10^5$ Pa</td>
</tr>
<tr>
<td>Dead volume 1</td>
<td>$V_{01}$</td>
<td>$3.9252 \times 10^{-5}$ m$^3$</td>
</tr>
<tr>
<td>Dead volume 2</td>
<td>$V_{02}$</td>
<td>$3.4047 \times 10^{-5}$ m$^3$</td>
</tr>
<tr>
<td>Piston stroke</td>
<td>$L_c$</td>
<td>$126 \times 10^{-3}$ m</td>
</tr>
<tr>
<td>Inertial load and piston mass</td>
<td>$M$</td>
<td>$22.5$ kg</td>
</tr>
<tr>
<td>Effective bulk modules</td>
<td>$\beta_e$</td>
<td>$0.6 \times 10^8 - 12 \times 10^8$ Pa</td>
</tr>
<tr>
<td>Cylinder leakage coef.</td>
<td>$C_{ec1}$</td>
<td>$1 \times 10^{-13} - 1 \times 10^{-11}$ m$^3$/Pa s</td>
</tr>
<tr>
<td>Cylinder leakage coef.</td>
<td>$C_{ec2}$</td>
<td>$1 \times 10^{-13} - 1 \times 10^{-11}$ m$^3$/Pa s</td>
</tr>
<tr>
<td>Viscous friction coef.</td>
<td>$B_c$</td>
<td>$1 \times 10^4 - 50 \times 10^4$ Ns/m</td>
</tr>
</tbody>
</table>

were obtained from literature reports [12,14,30] combined with engineering experience. These ranges will be used as constraints for the mathematical model optimization.

### 4.4.2 Electric motor model optimization

Electric motor-controller transfer function, Equation 3.25 has three unknown parameters: $T_I$, $T_{m1}$ and $T_{m2}$. For simulation, an additional gain $K_m$ is introduced as the system gain resulting
from a relation between physical quantities: system input voltage and motor velocity. Hence, its dimension is rad/Vs, Table 4.3. The electric motor mathematical model is represented in Simulink by the flow chart, Figure 4.31. Optimization is performed for a 5 Volt increasing frequency sinusoidal signal presented in Figure 4.32 (motor velocity signal is not filtered) in the time domain. The input is the voltage signal that was experimentally used for system testing. The first step in model optimization was the determination of a gain $K_m$ by comparing the unit gain transfer function simulated output with experimental system response. Electric motor response is not symmetrical, i.e. there is a slight difference between motor output speed for a ±5 V input signal (= 40 rev/min more for -5 V), Figure 4.32. This may come from electric motor-controller asymmetry or asymmetry resulting from the resolver used for velocity measurement. The obtained value of model gain calculated at +5 Volts input is $K_m = 40.55$ rad/Vs.

Further, the MatLab constr function was used for model optimization of three unknown parameters $T_{m1}$, $T_{m2}$ and $T_I$. The procedure flow for model optimization is presented in Figure 4.33. The optimization begins with parameters initial value $x_0$, for which model simulation is
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Figure 4.32  Test signal for electric motor-controller model optimization

Figure 4.33  Optimization procedure flow chart
performed in Simulink. Squared sum of differences between simulated and measured motor velocity is the criterion of model optimization (cost function). At each iteration the Matlab `constr` function determines new parameter values based on the cost function, and transfers them to the Simulink model for a new simulation. The procedure is interrupted when the cost function and parameter values change are within a tolerance of $1 \cdot 10^{-4}$.

In Table 4.5 optimization input values and identified parameter values are presented. For convenience, parameter values are given in vector form as a $1 \times 3$ vector $x = [T_l, T_{m1}, T_{m2}]$. Initial values are selected such that system natural frequency is 20 Hz and damping is 0.7. Parameter upper and lower bounds were calculated based on parameter limits from Table 4.3.

**Table 4.5  Electric system optimization input and output parameter values**

<table>
<thead>
<tr>
<th>Name</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter starting values</td>
<td>$x_0$</td>
<td>$[1.1 \cdot 10^{-3}, 6.3 \cdot 10^{-5}, 1.1 \cdot 10^{-2}]$</td>
</tr>
<tr>
<td>Lower bound on parameter values</td>
<td>$v_{lb}$</td>
<td>$[5 \cdot 10^{-4}, 2.8 \cdot 10^{-5}, 6.37 \cdot 10^{-3}]$</td>
</tr>
<tr>
<td>Upper bound on parameter values</td>
<td>$v_{ub}$</td>
<td>$[1 \cdot 10^{-2}, 25.33 \cdot 10^{-5}, 25.46 \cdot 10^{-3}]$</td>
</tr>
<tr>
<td>Identified parameter values</td>
<td>$x$</td>
<td>$[6.8525 \cdot 10^{-3}, 5.7803 \cdot 10^{-5}, 1.0162 \cdot 10^{-2}]$</td>
</tr>
<tr>
<td>Value of cost function at $x$</td>
<td>$F(x)$</td>
<td>$2.2249 \cdot 10^6$</td>
</tr>
</tbody>
</table>
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Figure 4.34  Optimized model and experimental response

Figure 4.35  Error between simulated and experimental response
Since the measured output velocity signal Figure 4.32 was noisy it was filtered by Butterworth 5th order filter at 160 Hz, as explained earlier. For the simulation, a 5 Volt sine signal was used. Obtained values of parameters $T_m 1$ and $T_m 2$, Table 4.5, correspond to a second order linear system with natural frequency of $\omega_m = 21$ Hz and damping ratio $\zeta = 0.68$. Model simulation and experimental system response are presented in Figure 4.34 and their difference is presented in Figure 4.35. Solid line represents simulated and dotted line experimental response. Figure 4.35 shows good correspondence between the experiment and the simulation. Hence, electric motor-controller transfer function can be represented by:

$$G_m(s) = \frac{2.779 \times 10^{-1} s + 40.55}{5.7803 \times 10^{-5} s^2 + 1.0162 \times 10^{-2} s + 1}$$

### 4.4.3 EHA model optimization

The EHA model was optimized in much the same manner as the electric motor model. A mathematical model of hydraulic system given by Equations 3.10 through 3.12 was transferred to Simulink model, depicted in Figure 4.36. The previously determined linear mathematical model of motor-controller system was added to this diagram.

Figure 4.37 presents the Simulink model of piston force equilibrium, Equation 3.12. The model can be easily expanded for a more complex load case by adding a friction model, external force and spring type load. This would enable to study the system under different loading conditions.

For optimization, model unknown parameters are viscous friction coefficient $B_c$, cylinder leakage coefficients ($C_{ec1}$, $C_{ec2}$) and effective bulk modules $\beta_e$. Hydraulic model parameter values are selected from Table 4.4. In order to improve numerical stability during simulation, all parameters
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Figure 4.36  Hydraulic system Simulink model

Figure 4.37  Structure of EHA Load block from Figure 4.36
were scaled so that the system output is expressed in mm and pressure in mPa \((1 \times 10^{-3} \text{ Pa})\). Hence, the parameter vector \(x\), Table 4.6, is 1x4 vector with elements: \(C_{ec1}, C_{ec2} (\text{mm}^3/\text{mPa s}), B_c (\text{Ns/mm})\) and \(\beta_e (\text{mPa})\) respectively. The starting values, upper and lower bonds on parameter values and resulting optimized parameter values are given in Table 4.6. Model optimization was performed with the same experimental data depicted in Figure 4.32, but in this case the relevant system output was piston position, Figure 4.38. Values obtained from optimization are presented in Table 4.6.

<table>
<thead>
<tr>
<th>Name</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter starting values</td>
<td>(x_0)</td>
<td>([7, 15, 500, 3 \times 10^4])</td>
</tr>
<tr>
<td>Lower bound on parameter values</td>
<td>(v_{lb})</td>
<td>([1, 1, 200, 1 \times 10^4])</td>
</tr>
<tr>
<td>Upper bound on parameter values</td>
<td>(v_{ub})</td>
<td>([30, 60, 1000, 6 \times 10^5])</td>
</tr>
<tr>
<td>Identified parameter values</td>
<td>(x)</td>
<td>([8.521, 17.711, 445.8, 3.5 \times 10^4])</td>
</tr>
<tr>
<td>Value of cost function at (x)</td>
<td>(F(x))</td>
<td>(2.571 \times 10^{-3})</td>
</tr>
</tbody>
</table>

Optimized model response (solid line) and experimental response (dotted line) are depicted in Figure 4.39. Model error is defined as the difference between experimental measurement and simulated response, depicted in Figure 4.40. Agreement between the model and the experiment is quite good. For lower frequencies, the error is less than 1 mm, and for higher frequencies the maximum error is around 2 mm. Parameter values from Table 4.6 converted to metric SI units are given in Table 4.7.
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Figure 4.38  EHA test sine response used for hydraulic parameters identification

Table 4.7  Identified hydraulic system parameter values

<table>
<thead>
<tr>
<th>Name</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left chamber leakage</td>
<td>$C_{el1}$</td>
<td>$8.521 \times 10^{-12} \frac{m^3}{s \cdot Pa}$</td>
</tr>
<tr>
<td>Right chamber leakage</td>
<td>$C_{el2}$</td>
<td>$17.711 \times 10^{-12} \frac{m^3}{s \cdot Pa}$</td>
</tr>
<tr>
<td>Viscous friction</td>
<td>$B_c$</td>
<td>$44.58 \times 10^4 \frac{Ns}{m}$</td>
</tr>
<tr>
<td>Effective bulk modulus</td>
<td>$\beta_{el}$</td>
<td>$3.5 \times 10^7 Pa$</td>
</tr>
</tbody>
</table>
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**Figure 4.39**  Experimental and simulated EHA sine response

**Figure 4.40**  Error between simulated and measured response
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The obtained values reflect the expectation that the leakage coefficient for the right cylinder chamber is about twice as the one for left. This was expected, since both chambers were manufactured by the same method, but the right chamber has two leaking zones (two seals) while the left chamber only one. Both leakage coefficients are quite large, as a consequence of a high external leakage rate. This was also observed during the experiments, since the cylinder housing was getting oily.

Obtained value for effective bulk modulus is quite low - $3.5\cdot10^7$ Pa due to presence of air in the system oil. This amount of air was unavoidable even after air bleeding, because the air was penetrating the system through the cylinder seals during operation. Effectively, this problem is a result of a poor manufacturing and can be resolved by a more precise fabrication of the cylinder barrel and piston.

External oil leakage is unacceptable in this system where the amount of system oil is small. Hence, any external oil losses would result in a drop of the system starting pressure $P_r$, which can lead to pump cavitation. On the other hand, leakage has a positive effect on the system response, because large leakage contributes to higher system damping, Table 3.1. These are contradictory requests in the general case. In the case of EHA, any external leakage should be minimized so that the system can operate for a longer period of time without refilling. If a higher system damping ratio is needed it can be produced "artificially" by generating internal leakage. This can be done by connecting the left and right transmission lines through a small orifice or valve. This is a compromise between system efficiency and system damping. As leakage increases, damping increases, but system efficiency decreases.
4.5 EHA mathematical model validation

The EHA model validation was performed by comparing the model time domain response with the experimental system response for different types and amplitudes of input signal. Validation was performed in three groups of experiments:

1. Open loop tests with variable frequency sinusoidal input
2. Open loop tests with pulse series input
3. Closed loop step response

All experiments were performed with \( P_r = 5 \cdot 10^5 \) Pa accumulator pressure and piston starting position at \( x_c = 40 \) mm from the left end. Maximal experiment duration was 32.5 s, selected so that the system minimal pressure would not change due to leakage.

4.5.1 Open loop model validation

Summarized input signal data used for these experiments is presented in Tables 4.8 and 4.9. The first batch of tests, Table 4.8, are sine tests similar to one used for model optimization. Mathematical model error was calculated as a difference between experimental and simulated response and expressed in mm. Second batch of experiments was performed with pulse series as the system input, Table 4.9. Experimental responses are represented by dotted lines and simulations by solid lines.
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Table 4.8  
Summarized sine signal test data used for model validation

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>4.41 &amp; 4.42</th>
<th>4.43 &amp; 4.44</th>
<th>4.45 &amp; 4.46</th>
<th>4.47 &amp; 4.48</th>
<th>4.49 &amp; 4.50</th>
<th>4.51 &amp; 4.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal Frequency</td>
<td>0.1-80 Hz</td>
<td>0.2-80 Hz</td>
<td>0.3-80 Hz</td>
<td>0.5-80 Hz</td>
<td>0.7-80 Hz</td>
<td>1-80 Hz</td>
</tr>
<tr>
<td>Signal Amplitude</td>
<td>1 V</td>
<td>2 V</td>
<td>3 V</td>
<td>5 V</td>
<td>7 V</td>
<td>10 V</td>
</tr>
</tbody>
</table>

Table 4.9  
Summarized pulse signal test data used for model validation

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>4.53 &amp; 4.54</th>
<th>4.55 &amp; 4.56</th>
<th>4.57 &amp; 4.58</th>
<th>4.59 &amp; 4.60</th>
<th>4.61 &amp; 4.62</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse Length</td>
<td>2 s</td>
<td>0.5 s</td>
<td>0.5 s</td>
<td>0.3 s</td>
<td>0.2 s</td>
</tr>
<tr>
<td>Signal Amplitude</td>
<td>1 V</td>
<td>3 V</td>
<td>5 V</td>
<td>7 V</td>
<td>9 V</td>
</tr>
</tbody>
</table>

It can be seen from Figures 4.41 through 4.62 that simulated and experimental responses are quite close. For a low frequency sine EHA response, the fit between model and real system is very good for all input signal levels. Larger differences occur in the higher frequencies (20–30 Hz).

Response drift is quite large for input signal levels less than 5 V in both cases (sine and pulse responses), Figure 4.41, 4.43, 4.53 and 4.55. Earlier, in Section 4.4.2, it was suspected, from the electric motor speed measurements, that the electric motor is not ideally symmetrical. However, it is not possible to determine if overall system asymmetry (drift) results from the difference in left and right piston side leakage or from electric motor asymmetry. It is most probable that this drift comes from a combination of both. For the electric motor excited by a 5 V sine (see section 4.4.2), the motor velocity is 40 rev/min higher for maximal minus input signal than for plus input. Hence, the piston should drift more in the negative direction. On the other hand, the leakage from
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Figure 4.41  Simulated and experimental response for 1 V sine

Figure 4.42  Model input and error - 1 V sine
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Simulated and experimental response

Figure 4.43  Simulated and experimental response for 2 V sine

Model error

Figure 4.44  Model input and error - 2 V sine
Figure 4.45  Simulated and experimental response for 3 V sine

Figure 4.46  Model input and error - 3 V sine
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Figure 4.47  Simulated and experimental response for 5 V sine

Figure 4.48  Model input and error - 5 V sine
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Figure 4.49  Simulated and experimental response for 7 V sine

Figure 4.50  Model input and error - 7 V sine
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**Figure 4.51** Simulated and experimental response for 10 V sine

**Figure 4.52** Model input and error - 10 V sine
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**Figure 4.53** Simulated and experimental response for 1 V pulse

**Figure 4.54** Model input and error - 1 V pulse
Figure 4.55  Simulated and experimental response for 3 V pulse

Figure 4.56  Model input and error - 3 V pulse
CHAPTER 4  Experimental Determination of Unknown Parameters

**Figure 4.57**  Simulated and experimental response for 5 V pulse

**Figure 4.58**  Model input and error - 5 V pulse
**Figure 4.59** Simulated and experimental response for 7 V pulse

**Figure 4.60** Model input and error - 7 V pulse
CHAPTER 4  Experimental Determination of Unknown Parameters

Figure 4.61  Simulated and experimental response for 9 V pulse

Figure 4.62  Model input and error - 9 V pulse
piston chamber two is much higher than from chamber one. This should produce positive piston position drift. However, as the electric motor is described by a linear transfer function, motor asymmetry is not accounted in the overall EHA model. Therefore, during the optimization, both sources of the piston drift are accounted as a difference of the leakage coefficients values ($C_{e1}$ and $C_{e2}$). As a results, model response agrees well with the experimental response for 5 V input, but there is a position drift for other input levels. Despite this, the following section will show, that the proposed EHA model describes the real system dynamic very accurately.

4.5.2 Closed loop model validation

Closed loop experiments were performed by closing the unity feedback position loop on the system. Now, the system input is desired piston position. The respective EHA mathematical model used for simulations is depicted in Figure 4.63. A saturation element is used for limiting the electric motor input voltage in the range of 2.5 to 10 Volts. For all tests, step input of 10 mm was used, accumulator pressure was $P_r = 5 \cdot 10^5$ Pa and piston starting position was $x_c = 40$ mm. Experiment duration was limited to 2 s. The results of simulation and experimental testing are presented in Figures 4.64, 4.66, 4.68 and 4.70. Simulation response is depicted by a solid line and experimental by a dotted line.

The results of simulations show a very good agreement with obtained experimental results. For the control signal limited at 2.5 V, simulated response nearly coincides with experimental response, Figure 4.64. The highest EHA model error is for 10 V limited signal as shown in Figure 4.70, where the mathematical model is slightly underdamped when compared to the experimental data. Still, it shows a very good correspondence to experimental results.
Figure 4.63  Closed loop EHA simulation model
CHAPTER 4  Experimental Determination of Unknown Parameters

Figure 4.64  Simulated and experimental closed loop response

Figure 4.65  EHA model error for 2.5 V control signal limit
CHAPTER 4
Experimental Determination of Unknown Parameters

Closed loop step response with limit of 5 V

Figure 4.66  Simulated and experimental closed loop response

EHA closed loop model error

Figure 4.67  EHA model error for 5 V control signal limit
CHAPTER 4  Experimental Determination of Unknown Parameters

Figure 4.68  Simulated and experimental closed loop response

Figure 4.69  EHA model error for 7 V control signal limit
CHAPTER 4  Experimental Determination of Unknown Parameters

Figure 4.70  Simulated and experimental closed loop response

Figure 4.71  EHA model error for 10 V control signal limit
CHAPTER 4  Experimental Determination of Unknown Parameters

Therefore, it can be concluded that the analytical model developed for the EHA system is valid. Identified values of the model unknown parameters produce a very good agreement with the experimental results. However, it cannot be stated that these values are the exact physical parameter values. Two main reasons for this are:

1. The optimization procedure cannot guarantee a unique solution to the optimization problem even with a very narrow parameter range.

2. Phenomena such as Coulomb and static friction, measurement noise and motor asymmetry were neglected with the assumptions defined a priori. However, they exist in a physical system, so their influence was accounted by optimization in the values of the identified model parameters $C_{ec1}$, $C_{ec2}$, $B_c$ and $\beta_e$.

The increase in the values of both leakage and viscous friction coefficient increase the system damping. Consequently, several different parameter value combinations might produce similar model responses. It would be impossible to tell which one is the true value. Therefore, a good knowledge about the expected parameter values (intervals) is crucial for the identification of their values.

The best example for the second reason of parameter value uncertainty is neglecting the electric motor asymmetry (by describing it with a linear transfer function). Hence, the complete system drift was modeled by the difference of the cylinder leakage coefficients $C_{ec1}$ and $C_{ec2}$.

Although there is uncertainty about parameter values, the mathematical model obtained can be used for control algorithm development or for investigation of the design parameters influence on the system behavior. It provides a very good tool for system study. The model can be easily
CHAPTER 4  Experimental Determination of Unknown Parameters

expanded to account for external force, spring type load and friction. Further, the model quality can be enhanced by determining the real values of leakage coefficients $C_{ec1}$ and $C_{ec2}$. This could be done with adequate equipment as described in Section 4.4.1. In that case, only two unknown parameter values would be left for model optimization, thus reducing parameter value uncertainty.

4.6 Summary

This chapter covers the testing of the EHA system and identification of the model unknown parameters. The first part of the chapter gives the basic theoretical background for model optimization. Next, the major part of the chapter deals with detailed open loop system testing and discusses phenomena observed in the test data. It was concluded that generally the system is nonlinear and its dynamic behavior is influenced by both the electrical and hydraulic sub-systems. The electric part of the system has a larger influence on system dynamic at high levels of system input, while the hydraulic part is dominant for the lower level.

Based on the mathematical model derived in Chapter 3, a simulation model was built in Simulink and optimized for a 5 Volt variable frequency sine signal. Model validation was performed for open and closed loop cases, and it was shown that the derived model represents the system dynamic very accurately. Some recommendations for model enhancement were given.
5 Conclusions

This thesis addresses the issue of design and mathematical modeling of a new hydraulic system referred to as the Electro Hydraulic Actuator (EHA). The foundation of this research includes conventional valve controlled and hydrostatic transmission circuits. The objective was to design a hydraulic system that would preserve good features of a conventional hydraulic systems and provide a significant improvements. Major novelties of the EHA circuit design are:

1. Control of the output actuator position or speed is obtained by controlling the electric motor velocity. Until now, position or velocity was controlled by a servovalve or by varying a pump and/or hydraulic motor displacement. Electric motor control eliminates the need for an additional hydraulic circuit for pump swash plate actuation.

2. The design and application of a specially developed single rod cylinder with equal piston areas, referred to as a New Linear Actuator (NLA). Conventional single rod cylinders are predominantly used in practice. However, they are rarely used in hydrostatic transmission circuits because their application produces an additional circuit complexity. Use of the NLA is not limited to the EHA only. It also provides benefits when used in a conventional hydraulic circuits by giving equal speed and force in both piston stroke directions.
Open loop EHA experiments gave some insights into the circuit behavior. It was shown that the circuit has nonlinear behavior, manifesting as variable circuit natural frequency and damping coefficient at different circuit operating points. However, variation of the EHA circuit parameters is much smaller than for a corresponding valve controlled circuit, facilitating the circuit control. Further, it was shown that both the electrical and the hydraulic parts of the circuit play an important role in the overall circuit dynamics. For lower values of input signal (1-2V), the hydraulic part is dominant, whereas for the higher input levels, electrical part becomes dominant. Experimentally determined system bandwith is approximately 23 Hz for 50% system input, with inertial load of 20 kg.

The nonlinear mathematical model of the EHA system was identified by model optimization. The high values of NLA leakage coefficients reflect poor cylinder manufacturing, observed as external leakage during system testing. Model validation was performed in open and closed loop using experimental responses and it was proved that the established model describes the system behavior accurately. Although there is uncertainty about the obtained parameter values, the proposed model can be used for system studies or controller design. The closed loop experiments, with a simple proportional controller, show that the system response is extremely accurate. EHA positioning accuracy equals the optical encoder resolution (0.01mm). In regard to future issues, the proposed mathematical model can be easily expanded to include Coulomb and static friction, spring type load and an external force. This would enable studies under more complex working conditions.

To sum up, the EHA system has high positioning accuracy (0.01mm), fairly high bandwith (23 Hz), high force output (=10.5 kN)), and is easy to control. It shows significant benefits when compared to conventional hydraulic systems in the areas of control, efficiency (inherited form hydraulic transmission) and physical dimensions.
References


References

[9] H. E. Merritt, **Hydraulic Control Systems**


References


Appendix A
Electrical Motor and Hydraulic Pump Data

<table>
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<tr>
<th>Motor Parameters</th>
<th>Units</th>
<th>MA 3A</th>
<th>MA 6A</th>
<th>MA 10A</th>
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Figure A.1 Mavior MA Series Electric Motors Specifications
Appendix A

Electrical Motor and Hydraulic Pump Data

MA30A with SMTBS-220/45 Controller

\[ T_s = 117 \text{ lb.in.} \]
\[ T_{ps} = 322 \text{ lb.in.} \]
\[ T_r = 90 \text{ lb.in.} \]
\[ T_{pr} = 185 \text{ lb.in.} \]
\[ P_r = 3184 \text{ watts} \]

Figure A.2 MA30 performance diagram

Figure A.3 G.C. 04 hydraulic pump performance curves

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# Appendix B

## EHA setup assembly drawing

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Figure B.1  EHA assembly drawing
Figure B.2  Electro Hydraulic Actuator setup
Appendix C

Fluid mechanics fundamental equations

Analytical description of fluid flow is based on the motion of infinitesimally small fluid cube. This fluid volume is described in Cartesian coordinates using position \((x, y, z)\), pressure, temperature, density and viscosity. It follows that, in general, seven independent equations are needed to describe fluid flow. The first three equations are *Navier-Stokes* (NS) equations:

\[
\begin{align*}
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) &= \rho X - \frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) &= \rho Y - \frac{\partial P}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\
\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) &= \rho Z - \frac{\partial P}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)
\end{align*}
\]

(A.1) (A.2) (A.3)

where \( \rho \) = fluid mass density,
\( u, v, w \) = fluid velocity components,
\( X, Y, Z \) = body forces per unit volume,
\( P \) = pressure,
\( \mu \) = absolute viscosity.
These three equations are simplified NS equations since fluid mass density and viscosity is assumed to be constant. The left side terms in these equations result from fluid inertia, while the three last terms on right side result from fluid viscosity. In relation to this we can distinguish two types of fluid flow: 1) laminar, where viscous (friction force) is dominant and 2) turbulent, when inertial forces are dominant. For prediction of a flow type, the Reynolds number $Re$ is used. A large Reynolds number indicates that inertial forces are dominant, and that flow is turbulent. A small $Re$ indicates laminar flow. For laminar flow Reynolds number is $Re < 2000$ and for turbulent $Re > 4000$. The range $2000 < Re < 4000$ is the transition range where both, laminar and turbulent flow regimes can occur. The next equation used for fluid flow description is the law of mass conservation (continuity equation):

$$\sum W_{in} - \sum W_{out} = g \frac{dm}{dt} = g \frac{d(pV_0)}{dt} \quad (A.4)$$

where $W_{in}, W_{out}$ = mass flow rates, and $V_0$ = control fluid volume.

The law of energy conservation (the first law of thermodynamics) can be expressed as:

$$\frac{dQ_h}{dt} - \frac{dW_e}{dt} + \sum W_{in} h_{0in} - \sum W_{out} h_{0out} = \frac{dE}{dt} \quad (A.5)$$

where $\frac{dQ_h}{dt} =$ heat rate transferred to fluid volume,

$W_e =$ external work that fluid is using,

$h_{0in}, h_{0out} =$ input and output energy per unit mass, and

$E =$ total energy of fluid inside observed volume.
Appendix C  

**Fluid Mechanics Fundamental Equations**

The equation of state for liquid can be presented as [Merrit]:

\[
\rho = \rho_0 \left[ 1 + \frac{1}{\beta} (P - P_0) - \alpha (T - T_0) \right] \tag{A.6}
\]

where \(\rho_0\), \(P_0\) and \(T_0\) are initial values of mass density, pressure and temperature respectively while \(\beta\) is fluid bulk modulus. The last equation describes fluid viscosity as a function of pressure and temperature. For liquid, the following can be used [9]:

\[
\mu = \mu_0 e^{-\lambda (T - T_0)} \tag{A.7}
\]

where \(\mu_0\) is absolute viscosity at \(T_0\),

\[\lambda = \text{a constant depending on the liquid, and}\]

\[T = \text{temperature}.
\]

The influence of temperature on fluid density and viscosity can be neglected. The need for energy equation is eliminated. Further, viscosity can be assumed to be constant, and mass density from Equation A.6 can be expressed as:

\[
\rho = \rho_0 + \frac{\rho_0}{\beta} P \tag{A.8}
\]

where \(\rho_0\) and \(\beta\) are mass density and bulk modulus at zero pressure. The weight flow rate can be expressed as \(W = g \rho \ Q\) where \(Q\) is volumetric flow rate. Combining this equation with Equations A.8 and A.4, yields the following continuity equation:

\[
\sum Q_{in} - \sum Q_{out} = \frac{dV_0}{dt} + \frac{V_0}{\beta} \frac{dP}{dt} \tag{A.9}
\]
 Appendix C  Fluid Mechanics Fundamental Equations

The second term on the right equation side is called *compressibility flow* and is a result of pressure changes. This equation, together with equations describing turbulent and laminar flow through orifices, are essential for fluid control systems mathematical modeling. Turbulent flow through orifices can be described by [9]:

\[ Q = C_d A_0 \sqrt{\frac{2}{\rho} (P_1 - P_2)} \]  \hspace{1cm} (A.10)

and laminar flow as:

\[ Q = C_l (P_1 - P_2) \]  \hspace{1cm} (A.11)

where \( P_1 - P_2 \) = pressure drop caused by flow through orifice

\( C_d \) = discharge coefficient depending on orifice geometry, and

\( C_l \) = leakage coefficient depending on orifice geometry and fluid viscosity.

Equation A.10 is used for modeling of oil flow through servo valve orifices, while Equation A.11 is used as a model of *leakage flow*. 