THE EFFECT OF REFERENCE ENVIRONMENTS ON THE ACCURACY OF THE RESULTS OF AN EXERGY ANALYSIS OF AN AEROSPACE ENGINE

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science (M.A.Sc.)
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ABSTRACT

An exergy analysis is applied to a turbojet engine for a range of altitudes from 0 to 15,000 m (-50,000 ft) and for a 3,500 km flight to examine the effects of using different reference-environment models. The results of this analysis using a variable reference environment (equal to the operating environment at all times) are compared to the results obtained using two constant reference environments (0 and 15,000 m). The rational efficiency of a turbojet was observed to decrease with increasing altitude, due mainly to an increase in exhaust exergy emissions. The accuracy of exergy results was found to be dependent on the choice of reference environment, where the use of a constant reference environment can lead to errors as large as 52%. For most atmospheric applications, the use of a variable reference environment does not add great complexity to the exergy analysis while yielding the most accurate results.
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I am proud to be the first Master of Applied Science student under the joint supervision of both the University of Toronto and Ryerson Polytechnic University, and I sincerely hope that I am the first of many to enjoy the benefits of this partnership.

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NOMENCLATURE

\( a \)  
\text{speed of sound}

\( c \)  
\text{velocity relative to the fixed reference environment (}\( c = U - V \)\)

\( c_p \)  
\text{specific heat at constant pressure}

\( c_v \)  
\text{specific heat at constant volume}

\( E \)  
\text{total exergy}

\( f \)  
\text{fuel to air ratio}

\( h \)  
\text{specific enthalpy}

\( L \)  
\text{ratio of lost exergy to incoming exergy}

\( m \)  
\text{mass}

\( M \)  
\text{molar mass}

\( p \)  
\text{pressure}

\( P_s \)  
\text{shaft power}

\( P_T \)  
\text{thrust power}

\( q \)  
\text{heat transfer rate}

\( Q_R \)  
\text{heating value}

\( R \)  
\text{gas constant}

\( s \)  
\text{specific entropy}

\( t \)  
\text{time}

\( T \)  
\text{temperature}

\( u \)  
\text{local velocity}

\( U \)  
\text{flight vehicle velocity}

\( V \)  
\text{velocity relative to the propulsion unit}

\( w \)  
\text{work per unit mass}

\( x \)  
\text{mole fraction}
Greek Symbols

$\alpha$ proportion by mass of constituent in reference environment

$\beta$ proportion by mass of constituent in fuel

$\delta$ change in mass of constituent per unit of fuel burned

$\varepsilon$ specific exergy

$\gamma$ net product of constituent by mass per unit of fuel burned ($\gamma = \delta + \beta$) or the ratio of specific heats

$\lambda$ proportion by mass of constituent in post-combustion mixture

$\eta$ turbojet component efficiency

$\zeta$ specific exergy function

$\psi$ rational efficiency

Subscripts

$\infty$ reference environment

$c$ compressor

$cum$ cumulative

$i$ individual constituent

$n$ nozzle

$rel$ relative

$std$ standard

$t$ turbine

$tot$ total

$vel$ velocity

$1, 2, ...$ station within propulsion unit

Superscripts

$-$ rate per unit time

$0$ total or stagnation

$-$ working fluid values (weighted average of constituent values)
1.0 INTRODUCTION

Exergy, or availability, has been used for several decades for the analysis of ground-based power systems and processes including gas and steam turbines (Bisio, 1998; El-Masri, 1987; Facchini, 1999; Fiaschi, 1998; Gallo, 1997; Jin, 1997; Oh, 1996; Tuma, 1999), diesel engines (Fijalkowski, 1997; Rakopoulos, 1997), solar power systems (Torres-Reyes, 1998; Liu, 1995), heat pumps and exchangers (Cornelissen, 1999; Rosen, 1999), and fuels and fuel processing (de Oliveira, 1997; Stepanov, 1995). As well as engineering processes, exergy has also been applied to naturally occurring phenomena (Zalda-Aguilar, 1998), specific physical phenomena (Sahin, 1998; Saidi, 1999), and entire countries (Rosen, 1992). However, given the extensive treatment of exergy in literature (Ackeret, 1962; Ahern, 1980; Barclay, 1995; Bedringas, 1997; Czysz and Murthy, 1991; Dunbar, 1995; Jin, 1993; Kotas, 1995; Moran, 1994, 1989; Stepanov, 1998) its application to the aerospace engine has been limited, with the first such efforts being the works of Clarke and Horlock (1975) and Lewis (1976). Later works provide examples of the application of the exergy concept to various types of aerospace engines (turbojet, turbofan, scramjet) (Brilliant, 1995a, 1995b; Kresta, 1992; Malinovskii, 1984; Murthy, 1994; Murthy and Ravichandran, 1996) following the approach of the earlier efforts. However, in all these works the application of the exergy concept to the aerospace engine differs from the traditional approach for analyzing terrestrial systems in two distinct ways.

The first distinction is that the aerospace engine is typically based on an open (Brayton) cycle, where the production of thrust generally involves the ejection of exhaust gases at high temperatures and velocities. This mode of operation leads to large exergy losses with the exhaust, which differ from the exergy losses due to irreversibilities within the system. This large exhaust loss, which is typical of the aerospace engine, leads to low exergy efficiencies and has led to efforts (Riggins, 1997, 1996a, 1996b, 1996c; Riggins and McClinton, 1995) to develop a more relevant second law-based method for evaluating efficiencies based on an earlier concept described by Curran (1973). In these efforts the second law analysis approach is tailored for aerospace engines, by comparing the desired output not to the overall exergy input but to the output of the idealized version of the engine under consideration. In this manner the aerospace engine is not unreasonably "penalized" for its large exhaust exergy content.
The second major distinction between the application of exergy analysis to aerospace rather than ground-based systems relates to the selection of the reference environment, and this topic is the focus of the present thesis. An exergy analysis requires the definition of a reference environment. This reference environment is usually modeled as the ambient environment, as this is the actual environment in which the system operates and with which all exchanges of matter and energy take place. For ground-based systems this environment normally remains relatively constant in practice. For aerospace engines however, the ambient operating conditions can vary significantly during a single flight. In the previous works noted above, the exergy analyses were performed using a fixed environment, selected as a typical operating environment to which the particular engine under consideration might be exposed. This approach appears to follow that for ground-based systems, where it is sufficient in terms of analysis accuracy and realism to establish a single reference environment. However, the variations in ambient pressure and temperature over the typical operating range of any aerospace engine (from sea level to 15,000 m (~50,000 ft)) are significant and can affect the accuracy of relevant exergy analyses if ignored.

The traditional approach of a fixed reference environment is unrealistic for most aerospace applications. When one wishes to model the reference environment as the ambient operating environment the reference environment must be permitted to vary as the operating environment changes. A variable reference environment needs to be able to accommodate conditions ranging from those at sea level to the near absolute zero temperature and vacuum conditions of space (although the degree to which space conditions are experienced depends on the type of aerospace engine being considered). Thus the selection of a reference-environment model involves a trade-off. For example, the use of a fixed reference environment, arbitrarily set at some operating environment, has (i) the advantages of reduced calculation complexity and the ability to straightforwardly assess the engine over flight altitudes ranging from ground level to low Earth orbit and beyond, and (ii) the disadvantage of having a reference environment different from the environment in which the system operates.

The objective of this thesis is to assess the sensitivity of exergy efficiencies of aerospace engines to the use of different reference-environment models, to assist those applying exergy analyses to such systems in maintaining reasonable accuracy, while not making the analysis overly complex. Such knowledge should make exergy analyses more widely used in aerospace design than is presently the case. The thesis focuses on the significantly varying
operating environment encountered when applying exergy analysis to aerospace engines (as outlined in the work of Clarke and Horlock, 1975) as opposed to ground-based systems. In addressing this issue, both continually varying and constant reference environments are considered. As well, the impact these choices of reference environment have on the accuracy of relevant exergy quantities is examined from both an instantaneous and complete flight viewpoint.
2.0 THEORY

2.1 Thermodynamic Background

In order to examine the performance of the various components within a turbojet engine, it is convenient to define a stagnation state. The stagnation state defined as the state reached by a fluid as it is decelerated to rest adiabatically, reversibly, and without work being extracted can be expressed using the following form of the energy equation (Hill and Peterson, 1992),

\[ dh + u \, du = 0 \]  

which can be integrated (using the fact that the stagnation state is at rest) to

\[ h^0 = h + \frac{u^2}{2} \]  

(2)

where \( h^0 \) is the stagnation (or total) enthalpy of the fluid
\( h \) is the local enthalpy of the fluid
\( u \) is the local velocity of the fluid

If the flow is further assumed to be calorically perfect \( (h = c_p T) \), Eq. (2) can be expressed as

\[ \frac{T^0}{T} = 1 + \frac{u^2}{2c_p T} \]  

(3)

where \( T^0 \) is the stagnation (or total) temperature of the fluid
\( T \) is the local temperature of the fluid
\( c_p \) is the specific heat of the fluid at constant pressure
With the additional assumption that the fluid is thermally perfect \((a^2 = \gamma RT)\), the Mach number of the fluid can be expressed as

\[
M^2 = \frac{u^2}{a^2} = \frac{u^2}{(\gamma - 1)c_p T}
\] (4)

where \(a\) is the speed of sound

\(\gamma\) is the ratio of specific heats of the fluid \((c_p / c_v\), noting that \(c_v\) is the specific heat at constant volume).

Substituting Eq. (4) into Eq. (3) yields the following expression for the relation between the stagnation temperature and the local temperature in terms of the local Mach number and the ratio of specific heats (Hill and Peterson, 1992),

\[
\frac{T^o}{T} = 1 + \frac{\gamma - 1}{2} M^2
\] (5)

A similar relation can be derived for the relation between stagnation pressure and local pressure, assuming the flow to be isentropic (Hill and Peterson, 1992):

\[
\frac{P^o}{P} = \left(\frac{T^o}{T}\right)^{\gamma - 1} = (1 + \frac{\gamma - 1}{2} M^2)^{\frac{\gamma}{\gamma - 1}}
\] (6)
2.2 Turbojet Performance

The performance analysis of a turbojet engine is simplified by breaking the engine down into its various components and examining each under various conditions. This process results in the seven stations shown in Fig. 1. By specifying the flow velocity through the engine in terms of the Mach number, it is possible to theoretically determine the complete thermodynamic state of the flow within the engine given certain assumed performance values for each engine component. For the instantaneous exergy analyses (excluding those in the flight profile section) the performance criteria where taken from Clarke and Horlock (1975) and are shown in Table 1. In Table 1,

\[
\eta_c = \frac{h_{3\text{isentropic}} - h_2^o}{h_3^o - h_2^o} \quad \eta_t = \frac{h_4^o - h_5^o}{h_4^o - h_{5\text{isentropic}}} \quad \eta_h = \frac{h_6^o - h_7^o}{h_6^o - h_{7\text{isentropic}}} \quad (7)
\]

while \( f \) is the fuel to air ratio and \( Q_k \) is the heating value of the fuel (which in this study is methane, \( \text{CH}_4 \)).
Table 1
Summary of Specified Turbojet Operating Parameters (adapted from Clarke and Horlock, 1975).

<table>
<thead>
<tr>
<th>Section</th>
<th>Engine Component</th>
<th>Performance Criteria</th>
<th>Mach Number (at exit plane)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞</td>
<td>Freestream</td>
<td>-</td>
<td>0.80</td>
</tr>
<tr>
<td>∞ to 1</td>
<td>External Diffusion</td>
<td>$p^* / p^\infty = 1.00$</td>
<td>0.70</td>
</tr>
<tr>
<td>1 to 2</td>
<td>Internal Diffuser</td>
<td>$p^\infty_2 / p^\infty_1 = 0.995$</td>
<td>0.50</td>
</tr>
<tr>
<td>2 to 3</td>
<td>Compressor</td>
<td>$p^\infty_3 / p^\infty_1 = 6.00$</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\eta_\infty = 0.85$</td>
<td></td>
</tr>
<tr>
<td>3 to 4</td>
<td>Combustor</td>
<td>$p^\infty_4 / p^\infty_1 = 0.95$</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f = 1/40$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$Q_R = 51445 \text{ kJ/kg}$</td>
<td></td>
</tr>
<tr>
<td>4 to 5</td>
<td>Turbine</td>
<td>$\eta = 0.90$</td>
<td>0.40</td>
</tr>
<tr>
<td>5 to 6</td>
<td>Jet Pipe</td>
<td>$p^\infty_6 / p^\infty_3 = 0.98$</td>
<td>0.30</td>
</tr>
<tr>
<td>6 to 7</td>
<td>Nozzle</td>
<td>$\eta_\infty = 0.98$</td>
<td>-</td>
</tr>
</tbody>
</table>

A more modern engine is modeled for the flight profile study, with an increased compressor pressure ratio and more realistic efficiency values (see Table 2). As well, two engine operating conditions, climb and cruise, are modeled to accurately simulate the entire flight profile. Note that the aircraft uses a cruising descent and hence does not change engine operating conditions during this segment of the flight.

For the rest of the data needed to calculate engine performance (molecular composition of the ambient air, atmospheric conditions at various altitudes, thermodynamic properties of the species involved in the combustion process, etc.), see Appendix I.
Table 2  Summary of Specified Turbojet Operating Parameters for a Complete Flight.

<table>
<thead>
<tr>
<th>Section</th>
<th>Engine Component</th>
<th>Performance Criteria Climb [Cruise]</th>
<th>Mach Number (at exit plane) Climb [Cruise]</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞</td>
<td>Freestream</td>
<td>-</td>
<td>0.80 [0.80]</td>
</tr>
<tr>
<td>∞ to 1</td>
<td>External Diffusion</td>
<td>$p_1'' / p_2'' = 1.00$ [1.00]</td>
<td>0.70 [0.70]</td>
</tr>
<tr>
<td>1 to 2</td>
<td>Internal Diffuser</td>
<td>$p_1'' / p_2'' = 0.95$ [0.95]</td>
<td>0.50 [0.40]</td>
</tr>
<tr>
<td>2 to 3</td>
<td>Compressor</td>
<td>$p_1'' / p_2'' = 26$ [20]</td>
<td>0.50 [0.40]</td>
</tr>
<tr>
<td>3 to 4</td>
<td>Combustor</td>
<td>$p_1'' / p_2'' = 0.90$ [0.95]</td>
<td>0.35 [0.30]</td>
</tr>
<tr>
<td>4 to 5</td>
<td>Turbine</td>
<td>$\eta = 0.88$ [0.92]</td>
<td>0.50 [0.40]</td>
</tr>
<tr>
<td>5 to 6</td>
<td>Jet Pipe</td>
<td>$p_1'' / p_2'' = 0.98$ [0.98]</td>
<td>0.40 [0.30]</td>
</tr>
<tr>
<td>6 to 7</td>
<td>Nozzle</td>
<td>$\eta = 0.98$ [0.98]</td>
<td>-</td>
</tr>
</tbody>
</table>

2.3  Exergy

Exergy is defined as the maximum work obtainable from a system. To evaluate the exergy of a stream of matter, it is convenient to define the specific exergy function, $\zeta$. This term provides an expression for calculating the physical exergy of a stream of matter, which can be defined as the maximum work obtainable from the stream of matter by bringing it from an initial state to the reference state through processes involving thermal interaction only. For an internally reversible heat transfer process (hence the heat transfer occurs over an infinitesimal temperature gradient) the specific entropy increase can be expressed as (Kotas, 1995)

$$d\bar{s} = \frac{dq_{\text{reversible}}}{T}$$  \hspace{1cm} (8)

which can be integrated to

$$q_{\text{reversible}} = T(s_0 - s_1)$$  \hspace{1cm} (9)
where \( q_{\text{reversible}} \) is the reversible heat transfer between states 1 and 0 per unit mass

\( T \) is the temperature at which the heat transfer occurs

\( s_o \) is the specific entropy of the stream of matter at the reference state 0

\( s_i \) is the specific entropy of the stream of matter at the initial state 1

Neglecting any change in potential energy, for steady flow through a control volume where the stream of matter is brought from its initial state to the reference state, the energy equation (per unit mass) can be expressed as

\[
q|_1^0 - w|_1^0 = h_o^o - h_i^o
\]

(10)

where \( w \) is the work per unit mass extracted from the system between states 1 and 0 (positive out of the system)

\( h_o^o \) is the total specific enthalpy of the stream of matter at the reference state 0

\( h_i^o \) is the total specific enthalpy of the stream of matter at the initial state 1

Assuming the heat transfer to occur at the reference state temperature, the maximum work obtainable from the system would occur when the heat transfer occurs reversibly, hence allowing the substitution of Eq. (9) into Eq. (10), yielding

\[
w|_1^{0}_{\text{max}} = T_o(s_o - s_i) - (h_o^o - h_i^o)
\]

(11)

Rearranging Eq. (11) and expressing the reference state (or reference environment) temperature as \( T_o \) yields

\[
e_1 = (h_i^o - T_o s_i) - (h_o^o - T_o s_o) = \zeta_1 - \zeta_0 = w|_1^{0}_{\text{max}}
\]

(12)
where $\xi$ is the specific exergy function (at a given state)

$\varepsilon_i$ is the specific exergy of the stream of matter at state 1

Thus the exergy of the stream of matter can be expressed as the difference between the specific exergy functions at the initial and reference states. Further, if the stream of matter is assumed to be an ideal gas (both calorically and thermally perfect) then the specific exergy of a constituent of the stream of matter can be expressed as (using state 0 as the reference state $=0$),

$$
\varepsilon_i = c_{p_i} (T_i - T_0) - T_0 (c_{p_i} \ln \frac{T_i}{T_0} - R_i \ln \frac{P_i}{P_0}) + \frac{c_i^2}{2}
$$

where $T_i$ is the temperature of the constituent

$p_i$ is the pressure of the constituent

$c_{p_i}$ is the specific heat at constant pressure of the constituent

$R_i$ is the gas constant of the constituent

$p_0$ is the reference environment pressure

$c_i$ is the absolute velocity of the constituent with respect to a fixed reference environment

2.4 **Turbojet Exergy Balance**

For a general control volume in motion one can write an exergy balance as follows (Clarke and Horlock, 1975):

$$
(\sum_i m_i \varepsilon_i)_{incoming} \geq P_S + P_T + \sum_i q_i \left( \frac{T_i - T_0}{T_i} \right) + (\sum_i m_i \varepsilon_i)_{outgoing}
$$

where $P_S$ is shaft power extracted from the control volume

$P_T$ is thrust power extracted from the control volume

$q_i$ is the heat transfer rate across the control volume
$T_i$ is the temperature at the point of heat transfer

$\varepsilon_i$ is the specific exergy of constituent $i$ in the mixture

$m_i$ is the mass flow rate of constituent $i$ in the mixture

The difference between the left and right hand sides of Eq. (14) is equal to the irreversibility of the system. The equality in Eq. (14) applies for ideal systems; for real systems there exist irreversibilities.

The thrust power across any component within a turbojet engine, where the mass flow rate is constant across the component boundaries and the flight velocity of the engine is $U$, can be written as follows (Clarke and Horlock, 1975):

$$P_t = m U (V_{outgoing} - V_{incoming})$$  \hfill (15)

where $U$ is the flight velocity

$V$ is the flow velocity, relative to the control volume boundaries, entering and leaving the component (where the flow is parallel to the flight direction)

For a turbojet engine, the incoming exergy is provided by the fuel and as such most if not all of the exergy is chemical (a small amount of physical exergy may exist due to the difference between the conditions of the fuel storage and the reference environment). The specific exergy expression in Eq. (13) is insufficient to determine the chemical exergy of the fuel as it implicitly assumes that the substance one is considering exists in the reference environment. A different method is therefore used to determine the fuel exergy. Separating the specific fuel exergy into different terms allows a simpler calculation and better understanding of the total fuel exergy:

$$\varepsilon_{fuel} = \varepsilon_{std} + \varepsilon_{rel} + \varepsilon_{vel}$$  \hfill (16)

where $\varepsilon_{std}$ is the exergy of the fuel at a standard reference temperature and pressure

$\varepsilon_{rel}$ is the exergy of the fuel due to the difference between (a) the injection and reference environment
temperature and pressure and (b) the standard reference temperature and pressure used to find \( e_{std} \) (this value can be positive or negative)

\( e_{std} \) is the exergy of the fuel due to its kinetic energy, or velocity

Also, the terms in Eq. (16) can be expressed following the approach of Clarke and Horlock (1975):

\[
\begin{align*}
    e_{std} &= \sum_i \left( \beta_i h_{i_{std}} - \gamma_i h_{i_{std}} \right) - T_s \sum_i \left( \beta_i s_{i_{std}} - \gamma_i s_{i_{std}} \right) \\
    e_{rel} &= \sum_i \left( \beta_i \Delta h_{i_j} - \gamma_i \Delta h_{i_j} \right) - T_s \sum_i \left( \beta_i \Delta s_{i_j} - \gamma_i \Delta s_{i_j} \right) \\
    e_{vel} &= \sum_i \beta_i \frac{c_i^2}{2} 
\end{align*}
\]

where \( \beta_i \) is the number of mass units of constituent \( i \) in one mass unit of fuel

\( \gamma_i \) is the number of mass units of constituent \( i \) produced by the complete combustion of one mass unit of fuel

\( h_{i_{std}} \) and \( s_{i_{std}} \) are the enthalpy and entropy of constituent \( i \) at specified conditions (see Appendix I)

Note that \( \gamma_i \) can be positive or negative as it represents the net products of combustion. Thus for \( O_2 \) this value is -4 when using \( CH_4 \) as fuel since there are no mass units of \( O_2 \) present in the fuel itself, and during complete combustion 4 mass units of \( O_2 \) are consumed with every one mass unit of \( CH_4 \).

The parameters in the relative fuel exergy term (Eq. (18)) can be expressed using ideal gas laws (Clarke and Horlock, 1975):

\[
\begin{align*}
    \Delta h_{i_j} &= c_{p_i} (T_{i_j} - T_{std}) \\
    \Delta h_{i_e} &= c_{p_i} (T_{i_e} - T_{std})
\end{align*}
\]
\[ \Delta s_{i_3} = c_{p_i} \frac{T_{i_3}}{T_{\text{std}}} - R_i \ln \frac{P_{i_3}}{P_{\text{std}}} \]
\[ \Delta s_{i_*} = c_{p_i} \frac{T_{i_*}}{T_{\text{std}}} - R_i \ln \frac{P_{i_*}}{P_{\text{std}}} \]  

(21)

where  \( T_{i_3} \) and  \( p_{i_3} \) are the temperature and partial pressure of constituent  \( i \) as it enters the combustion process (i.e., at station 3).

\( T_{i_*} \) and  \( p_{i_*} \) are the temperature and partial pressure of constituent  \( i \) in the reference environment.

\( T_{\text{std}}, p_{\text{std}} \) are the standard temperature and pressure corresponding to those used for the tabulated data.

Since the specific exergy expression in Eq. (13) does not include terms for chemical exergy, it is useful when the chemical composition of the substance under consideration is the same in both the operating and reference environments. For the turbojet analysis this assumption is valid in all stations prior to the combustion chamber. However, beyond this point an additional term must be added to Eq. (13) to account for the chemical exergy created by the change in working-fluid chemical composition during combustion (i.e., the mole fractions of each constituent in the working fluid after combustion differ from those of the same constituent in the reference environment). Thus after combustion a modified specific exergy function is applied for the working fluid (Clarke and Hoomlock, 1975):

\[ \mathcal{e} = \bar{c}_p (T - T_{\text{ref}}) - T_{\text{ref}}(\bar{c}_p \frac{T}{T_{\text{ref}}} - R \ln \frac{P}{P_{\text{ref}}}) + \frac{c^2}{2} + T_{\text{ref}}(\sum_i \lambda_i R_i x_i \ln \frac{x_i}{x_{i*}}) \]  

(22)

where the barred specific heat and gas constant values are those that pertain to the working fluid as a whole.

\( T \) and  \( p \) are the temperature and pressure of the working fluid.

\( c \) is the absolute velocity of the working fluid.

\( \lambda_i \) is the mass fraction of constituent  \( i \) per unit of post-combustion working fluid.

\( x_i \) is the mole fraction of constituent  \( i \) in the working fluid.

\( x_{i*} \) is the mole fraction of constituent  \( i \) in the reference environment.
2.5 **Rational Efficiency**

The rational efficiency, $\psi$, is used here as a measure of merit for assessing and comparing systems and is defined as the ratio of useful or desired work obtained from the system to the total quantity of incoming exergy (Murthy, 1994; Czysz and Murthy, 1991; Clarke and Horlock, 1975). For a turbojet, where the useful work is the thrust,

$$\psi = \frac{P_T}{E_{\text{incoming}}}$$  \hspace{1cm} (23)

where the incoming exergy includes the exergy of both the fuel and the incoming air, i.e.,

$$E_{\text{incoming}} = m_{\text{fuel}} e_{\text{fuel}} + m_{\text{air}} e_{\text{air}}$$  \hspace{1cm} (24)

Since the reference environment by definition has no exergy, the freestream air exergy term in Eq. (24) is zero and the incoming exergy is solely associated with the fuel when the operating and reference environments are identical.

If the engine is considered adiabatic and no shaft power is extracted, then the sum of the thrust power produced and the total exergy lost is equal to or less than the incoming exergy (see Eq. (14)). The rational efficiency can then be written as follows:

$$\psi = 1 - \frac{\sum \text{Exergy Loss}}{m_{\text{fuel}} e_{\text{fuel}} + m_{\text{air}} e_{\text{air}}}$$  \hspace{1cm} (25)

where the total exergy loss in the numerator is the sum of the exergy losses for each engine component. The rightmost term in Eq. (25) is also referred to as the loss ratio, $L$.

To assess an engine over an entire flight using the rational efficiency, a modification must be made to the assessment variables. If considering only a brief instant in time, the rational efficiency as defined by Eq. (23) is suitable as it represents the instantaneous values of thrust power and incoming exergy flow rate. However, over an
entire flight, cumulative measures of these quantities are required. Modifying the rational efficiency to account for the cumulative effect of an entire flight can be done as follows:

$$
\psi_{\text{cum}} = \frac{\int_{0}^{t} P_{T}(t) dt}{\int_{0}^{t} E_{\text{incoming}}(t) dt}
$$

(26)

where $\psi_{\text{cum}}$ is the cumulative rational efficiency.

\(\int P_{T}(t) dt\) is the instantaneous thrust power summed over the duration of the flight (which lasts from time \(= 0\) to some final time \(= t\)).

\(\int E_{\text{incoming}}(t) dt\) is the instantaneous incoming exergy flow rate summed over the duration of the flight.

At the beginning of a flight, both the instantaneous (Eq. (23)) and cumulative (Eq. (26)) rational efficiencies are identical. However, at all times following, this is not necessarily the case (see Appendix II).

2.6 Loss Analysis

For design improvement it is often insightful to divide the total exergy loss into waste exergy emissions (e.g., exergy discarded by the turbojet with the exhaust gases) and the internal consumptions (or destructions) of exergy due to irreversibilities occurring within the engine and its components. Since the waste exergy emissions are often the single largest loss in a turbojet, it is helpful to further subdivide this loss into components identifying loss characteristics. The three main types of exergy in the waste exhaust emissions are kinetic, chemical, and physical exergy. These three components, present in Eq. (22), can be expressed individually as:

$$
\varepsilon_{\text{physical}} = c_{p}(T - T_{\infty}) - T_{\infty}(c_{p}\ln\frac{T}{T_{\infty}} - \bar{R}\ln\frac{P_{\infty}}{P})
$$

(27)
If the exhaust is assumed to contain no unburned fuel, the chemical exergy as expressed by Eq. (28) is sufficient to calculate the chemical exergy of the exhaust stream. With this assumption, the chemical exergy is due solely to the partial pressure differences between the constituents in the exhaust gas and the same constituents in the reference environment.

2.7 Operating and Reference Environments

The analyses presented here involve both operating and reference environments. The operating-environment temperature and pressure are those for the current altitude, as the actual performance of the turbojet is dependent on the incoming flow conditions. The thrust produced and the thermodynamic properties at points within the engine are determined using operating environment values. However, the results of the exergy analysis depend both on the performance of the engine (and hence the operating environment) and on the reference environment conditions (including the reference environment temperature and pressure). Thus changes in the reference environment leave quantities such as the thrust unaffected while causing efficiencies and losses calculated using the thrust to vary, sometimes significantly.

From a purely physical viewpoint, the most accurate choice of reference environment is one that models the instantaneous environment in which the system operates. For a turbojet engine, the use of a reference environment other than the operating environment can create the illusion of the ingested air containing exergy, and hence the possibility of producing useful work, or thrust, without the need for the exergy contained in the fuel.
3.0 INSTANTANEOUS EXERGY ANALYSIS

3.1 Fuel Exergy

An understanding of how the exergy of the fuel is affected by a changing reference environment is important as fuel is the primary source of exergy entering the turbojet and because most exergy-based efficiency measures involve fuel exergy. The accuracy of exergy analyses and engine comparisons is improved with such an understanding. The chemical exergy of the fuel entering the turbojet for three cases of reference environments is illustrated in Fig. 2. The fuel is taken to be methane (CH₄). In two cases, the reference environment corresponds to a fixed altitude (sea level or 15,000 m). In the third case, a reference environment which varies with altitude is considered.

3.1.1 Variable Reference Environment

As seen in Fig. 2, the difference between the fuel exergy at sea level and 15,000 m (~50,000 ft) is less than 0.6%. As discussed later, however, the overall engine efficiency varies by approximately 2% (see Fig. 3) when using a variable reference environment. The variation in fuel exergy causes part of the engine efficiency variation and is discussed here. Note that in most discussions after this section, the fuel exergy is treated as nearly constant in relation to other factors. The fuel exergy can be broken down into three main components (Eq. (16)): standard, relative, and velocity. The standard term utilizes standard thermodynamic data in determining enthalpy and entropy, while the relative term modifies these standard values to the appropriate environmental conditions. Since the standard exergy of the fuel (Eq. (17)) is calculated on the basis of tabulated enthalpies and entropies taken at a specified constant temperature and pressure which do not vary with the reference environment, a change in the reference pressure has no effect on any of the standard fuel enthalpies or entropies. However, since all entropy values used in the calculations are multiplied by the reference temperature to determine exergy, the lowering of the reference temperature as the altitude is increased has the effect of increasing the overall value of the standard fuel exergy term. Once the tropopause is reached (~11,000 m) and the reference temperature becomes constant the standard fuel exergy term remains unchanged.
The velocity contribution to the incoming fuel exergy is relatively small compared to the relative and standard terms, which are approximately two and three orders of magnitude larger than the velocity term, respectively. However, the velocity term is also significantly affected by the incoming fuel conditions and as such can be more dominant depending on the injection state of the fuel. The behaviour of the velocity term parallels that of the standard fuel exergy term in that it remains constant once the tropopause is reached. This behaviour is due to the fact that a constant Mach number flight profile was considered for this analysis. Since the fuel exergy due to velocity is equal to the kinetic energy of the incoming fuel (Eq. (19)) which is in turn equal to the kinetic energy of the flight vehicle in this case (as the incoming fuel injection velocity was specified as zero), the velocity component of the fuel exergy decreases with decreasing flight velocity. With a constant Mach number flight profile, as the altitude increases and the operating temperature decreases, the flight velocity decreases, thereby decreasing the fuel exergy due to velocity. However, once the tropopause is reached and the operating temperature remains constant, the flight velocity and the velocity component of the fuel exergy also remain fixed.
In assessing the behaviour of the relative fuel exergy component (Eq. (18)), it is convenient to subdivide it into enthalpy and entropy portions for each of the constituents involved in the combustion process. Since both the specified fuel injection temperature and pressure, as well as the temperature and pressure for the tabulated data, are constant (at 320 K, 1 MPa and 298 K, 0.1 MPa respectively), the corresponding property differences and ratios are also constant. For the fuel, which in this case is CH₄, Eqs. (20) and (21) show that the relative enthalpy and entropy are constant and thus independent of the reference environment. Combined with the assumption of complete combustion (hence γ = 0 in Eq. (18)) all the constituent variables in Eq. (18) are constant for CH₄ (however, although the entropy term is constant, the exergy evaluated using this entropy is not, as it is dependent on the reference environment temperature).

Therefore, when assessing the behaviour of the relative enthalpy and entropy terms, one only needs to consider the behaviour of the other species involved in the combustion process. Taking all the other species (O₂, H₂O, and CO₂) combined, several terms in Eq. (18) can be eliminated due to the fact that β = 0. Since the N₂ present in the operating environment is considered inert, all the exergy values associated with this species cancel out of the overall exergy balance and as such need not be considered.

The greater the difference between the reference environment and the standard temperatures, the larger is the combined enthalpy term for the constituents O₂, H₂O, and CO₂ (Eq. (20)). This observation is due to the fact that although the enthalpy term for the O₂ is negative, the enthalpy terms for both the H₂O and the CO₂ are positive and greater in magnitude than the negative O₂ term. Thus the lower the reference environment temperature, the greater the total relative enthalpy portion of the relative fuel exergy term.

Considering the relative entropy terms, there exists an opposition between the temperature and pressure terms. For the H₂O and CO₂ terms, a lower reference temperature results in a more positive entropy value whereas a lower reference pressure results in a more negative entropy value. The opposite behaviour is true of the entropy terms associated with the O₂. Further, the multiplying of the relative entropy terms for all the species by the reference environment temperature also affects the overall relative fuel exergy term.

However, since it has been established that the standard fuel exergy term increases as the altitude is increased (due to the decrease in reference temperature) and that the effect of the velocity term is negligible with
respect to the other two terms, it can be concluded that since the overall fuel exergy decreases the net effect of the relative exergy term (the combination of the enthalpy and entropy terms of all the species) is to decrease as the altitude is increased. Thus although the relative enthalpy terms act to increase the relative fuel exergy term, the net effect of the relative entropy terms is to decrease its value, and this decrease is greater in magnitude than the increase due to the enthalpy terms (by approximately two orders of magnitude).

From a more physical viewpoint, one would expect that the decreasing reference environment pressure with increasing altitude would lead to an increase in fuel exergy if the fuel injection pressure is higher than the reference environment pressure. However, it is also expected that any sort of increasing temperature difference would also increase the exergy of the fuel, which is not true in this case. This counter-intuitive result arises from the manner in which the overall fuel exergy is calculated. For any single substance within a mixture, chemical exergy is created by the difference in the partial pressure of the substance within the mixture and within the reference environment. However, since most fuels do not exist in a traditional reference environment (usually taken as ambient air) it is impossible to determine a partial pressure of the fuel in the reference environment. Thus a more detailed method is required for calculating the fuel exergy taking into account its absence from the reference environment. For this purpose the process of combustion is used, as for most cases (as in this case) the products of combustion are found in the reference environment and hence their chemical exergy can be calculated in the standard manner. The overall exergy of combustion (and hence the fuel) is related to the difference between the exergy of the reactants and the products.

The fact that each element involved in the combustion process (reactants and products) reacts differently to the changing reference environment when calculating their relative entropy contribution (Eq. (21)) to the relative fuel exergy gives rise to the counter-intuitive results observed. Since the effect of the decreasing reference pressure is dependent on the partial pressure and hence mole fraction of each substance involved in combustion, it is more dominant in the $O_2$ relative entropy terms as the mole fraction of this constituent is at least one order of magnitude larger than any other (neglecting $N_2$). Hence the decreasing reference pressure adds more exergy to the reactant side of the combustion process than it does to the product side. However, the effect of the decreasing reference temperature is more dominant in the $H_2O$ and $CO_2$ relative entropy terms as these are proportional to the values of
the specific heats, which are larger for these substances than for \( O_2 \). The expected increase in exergy due to a larger temperature difference is present, only this increase is larger in the products of combustion than the reactants. By adding more exergy to the products of combustion, the difference between reactants and products is reduced thereby reducing the exergy available from the combustion process and hence the fuel. Overall, the effect on the relative entropy term of the decreasing reference temperature is larger in magnitude than the effect of the decreasing reference pressure and hence the relative fuel entropy term decreases with increasing altitude as deduced earlier. Furthermore, since the decrease in the relative fuel entropy term is due to the effect of the decreasing temperature, once the tropopause is reached the effect of the only remaining changing reference parameter, the decreasing pressure, is to increase the relative fuel entropy term thereby increasing the relative fuel exergy term as well.

The behaviour of the relative entropy term dominates the behaviour of the relative fuel exergy term, since it is approximately 25 times larger than the relative enthalpy term after being multiplied by the reference environment temperature. Once in the tropopause both the standard and relative fuel exergy terms act to increase the overall fuel exergy with further increases in altitude as shown in Fig. 2 (see the variable reference environment curve).

### 3.1.2 Constant Reference Environment

In the cases in Fig. 2 where the reference environment remains constant with parameter values set to those for a specific altitude, the incoming fuel exergy remains almost unchanged as altitude changes. As was earlier established, the standard fuel exergy terms are dependent solely on the reference environment temperature and so remain fixed when the reference environment temperature is fixed. The relative fuel exergy term remains constant as it is dependent on both the reference environment temperature and pressure which are both constant when a constant reference environment is used. It is also noted that the relative fuel exergy term is dependent on the mole fraction of the products of combustion in the reference environment. But in this study the chemical composition of the reference environment remains constant with altitude (i.e., air is treated on a molar basis as 79.67% \( N_2 \), 18.77% \( O_2 \), 1.53 % \( H_2O \), and 0.03% \( CO_2 \)).
The velocity term of the fuel exergy is responsible for the very small decrease with altitude exhibited by the two constant reference environment curves in Fig. 2, below the tropopause. The small magnitude of this decrease confirms our earlier observation of the minor effect of this term on the overall fuel exergy. This small decrease is due to the fact that the kinetic energy of the flight vehicle (which is the same for any reference environment considered), and hence the fuel, is calculated using the current operating environment. Since kinetic energy is directly equal to exergy, as the altitude increases, the operating temperature decreases changing the velocity term in the fuel exergy as discussed earlier, which accounts for the small decrease shown in the curves. Once the tropopause is reached, however, the operating temperature and hence the velocity term in the fuel exergy become constant. Above 11,000 m all the terms comprising the fuel exergy are constant and hence the constant reference environment curves in Fig. 2 are constant in the tropopause.
3.2 **Rational Efficiency**

Exergy analysis helps develop a good understanding of how efficiently a system is operating relative to ideality and of the sources (locations and causes) of the major inefficiencies. The rational efficiency defined by Eq. (23) is used here to assess system performance.

3.2.1 **Variable Reference Environment**

For the case of the variable reference environment, the rational efficiency of the turbojet is seen in Fig. 3 to decrease as altitude increases, from a maximum value of approximately 16.9% at sea level (S/L) to 15.4% at 15,000 m. Since the rational efficiency depends both on the incoming fuel exergy and the thrust produced by the engine (the useful work extracted) it is useful to examine the behaviour of the thrust as the altitude is increased. The decreasing efficiency below 11,000 m might suggest that the thrust produced by the engine decreases since the analysis of the incoming fuel exergy showed that the change in this value over the given altitude range is less than 0.6% (hence nearly constant). However, the thrust actually increases with altitude, going from a value of 797 N at sea level to 833 N at 15,000 m (using an incoming air mass flow rate of 1 kg/s). This increase in thrust is due to the manner in which the thermodynamic analysis is performed.

Since the engine is specified as always expanding the exhaust gases to atmospheric pressure (which in practice means having a variable geometry nozzle), the ejected relative velocity of these gases remains fairly constant with altitude (see Tables 3 and 4). However, the incoming velocity decreases due to the decreasing operating temperature and constant flight Mach number. Thus the increase in velocity across the turbojet is greater at higher altitude, translating into a larger thrust.

The decrease in rational efficiency with increasing altitude, even though more thrust is produced, can be explained by noting that the greater thrust is being produced at the expense of higher losses. The largest contribution to the losses in the turbojet are seen in Tables 3 and 4 to be associated with the exhaust, which accounts for more than half of the total engine loss (the other major contributor being the combustor). Clearly, the increase in the exhaust loss from approximately 54% at sea level to 59% at 15,000 m is the principle reason for the decreased efficiency of the overall engine with increasing altitude. Although Tables 3 and 4 indicate that all stations except
for the nozzle actually incur lower losses at higher altitudes (note that each station in the tables is taken at the exit plane of the specified component), these increased efficiencies are more than offset by the 5% increase in the exhaust loss.

The increase in exhaust loss with increasing altitude is caused by the changing reference environment. The decreasing reference environment temperature and pressure with increasing altitude increase the exergy of the exhaust gases. As can be seen from Eq. (22), a larger temperature gradient and a more negative pressure difference (final reference environment pressure - initial exhaust pressure) increase the exergy of a given flow. In the particular case considered here, since the exhaust and reference environment pressures are always equal (as the gases are always expanded to the operating environment pressure), the pressure terms in the exergy expression are zero. As environment temperature decreases with increasing altitude the exhaust exergy increases, i.e., in going from sea level to 15,000 m the actual exhaust exergy flow rate increases by approximately 8.5% from 697 to 757 kW (in each of the Tables 3 through 7, the "Exhaust Exergy Rate Exiting Engine" is not the same as the specific total exergy $(\varepsilon_{\text{tor}})$ due to the fact that at the exhaust plane (station 7) the mass flow rate is no longer equal to the incoming air mass flow rate of 1 kg/s, but rather the incoming air mass flow rate plus the incoming fuel mass flow rate).

![Figure 3](image.png)

**Figure 3** Variation of turbojet rational efficiency at various operating altitudes using different reference environments.
Although the exergy of the exhaust is increased, this increased work potential is emitted as a waste since the turbojet has no means of extracting it. It is due to the increased exhaust exergy loss that the overall engine efficiency decreases with increasing altitude despite producing more thrust and incurring reduced losses in most engine components. Above the troposphere where the reference temperature becomes constant, the decrease in rational efficiency with further increases in altitude becomes much less pronounced (less than 0.1% between 11,000 and 15,000 m). For a constant reference environment temperature, the exhaust exergy remains constant. In the tropopause, the thrust remains constant as it varies only with inlet temperature for the thermodynamic calculation method employed here for engine performance. The very slight decrease in rational efficiency in the tropopause is caused by the small increase in incoming fuel exergy (approximately 0.1% as seen in Fig. 2).

3.2.2 Constant Reference Environment

Fixing the reference environment at the environment corresponding to 15,000 m results in a decrease in efficiency at sea level of 1.6% over the case where the actual altitude is used as the reference environment. Although the thrust produced is independent of the choice of reference environment, the engine efficiency decreases due to the behaviour of the incoming air. In most exergy analyses, the surroundings are a source of zero exergy. However, when the reference environment is different than the operating environment from which the turbojet ingests air (as is the case when a 15,000 m reference environment is used at sea level), the incoming air possesses physical exergy. Since in this case the incoming air is at a higher pressure and temperature than the reference environment, the engine appears to be receiving exergy from the incoming air flow as well as from the fuel. The exergy of the air is physical in nature, as the chemical composition of the atmosphere is not varied with altitude (also note that the incoming air possesses no kinetic exergy as the reference environment is always assumed to be at rest).

This increase in incoming exergy is responsible for the decreased engine efficiency, as the engine appears to be receiving additional exergy while still producing the same amount of thrust. At sea level, the choice of a 15,000 m reference environment creates the “illusion” of an extra 142 kW (see Table 5) of incoming exergy while the thrust produced is still 797 N (the same value as found in the variable reference environment case), thereby reducing the rational efficiency (Eq. (23)). As the operating altitude increases, the difference between the operating
and reference environments decreases, reducing the fictitious exergy in the incoming flow. At 15,000 m, the efficiencies for the variable and 15,000 m reference environments are equal, as the incoming air flow exergy is zero in both cases.

A similar phenomenon occurs when the reference environment is fixed at sea level conditions, only the effects are opposite. In this case the efficiencies for the variable and constant reference environments are equal at sea level and diverge as the altitude increases. Since the operating temperature and pressure decrease from the sea level values as the altitude is increased, a fictitious negative exergy is attributed to the incoming air flow. This negative exergy indicates that the operating environment is no longer a source of zero exergy (or even positive exergy as in the previous case) but rather that work must be done on the air flow taken from the operating environment to bring it to reference environment conditions. Thus it appears as though the exergy entering the engine in the fuel must first overcome the incoming negative exergy before it can produce thrust. But, since the thrust produced remains unchanged by the reference environment (833 N at 15,000 m), the efficiency of the turbojet increases. At an altitude of 15,000 m the use of a sea level reference environment causes the exergy of the incoming air to be -165 kW (see Table 6), and the efficiency to increase by approximately 2.2%.

The behaviour of the constant reference environment curves in Fig. 3 exhibit a distinct change at the 11,000 m isotherm. For the sea level curve this effect occurs because, although the use of a constant sea level reference environment increases efficiency with increasing altitude, the actual engine efficiency decreases with altitude up to the tropopause by a similar amount such that the two effects approximately cancel out. However, in the tropopause the actual engine efficiency remains nearly constant and thus there is no trade off between opposing tendencies, allowing the efficiency to increase due to the negative exergy of the incoming air. The change in the 15,000 m reference environment curve in Fig. 3 at the tropopause is caused by the same effect. In this case the effect of using a constant reference environment causes the engine efficiency to decrease as altitude decreases from 15,000 m to the troposphere (~11,000 m). However, in the troposphere the actual engine efficiency increases as altitude decreases, thereby canceling the tendency for the incoming-air positive exergy to decrease the engine efficiency.
Table 3 Thermodynamic Quantities at the Outlets of the Specified Engine Stations for Operating and Reference Environments of Sea Level.

<table>
<thead>
<tr>
<th>Engine Component</th>
<th>Engine Station</th>
<th>Mach Number</th>
<th>Sonic Velocity, ( V_{sa} ) [m/s]</th>
<th>Velocity, ( V ) [m/s]</th>
<th>Temperature, ( T ) [K]</th>
<th>Pressure, ( p ) [kPa]</th>
<th>Specific Energy Components [kJ/kg]</th>
<th>Shaft Power, ( P_1 ) [kW]</th>
<th>Thrust Power, ( P_T ) [kW]</th>
<th>Loss Rate, ( P_L ) [kW]</th>
<th>Rational Efficiency, ( \psi )</th>
<th>Loss Ratio, ( \zeta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Stream</td>
<td>0</td>
<td>0.80</td>
<td>296</td>
<td>223</td>
<td>287</td>
<td>100.3</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>External Diffuser</td>
<td>1</td>
<td>0.70</td>
<td>245</td>
<td>328</td>
<td>111.4</td>
<td>7.91</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Internal Diffuser</td>
<td>2</td>
<td>0.50</td>
<td>353</td>
<td>218</td>
<td>310</td>
<td>21.11</td>
<td>4.80</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Compressor</td>
<td>3</td>
<td>0.50</td>
<td>471</td>
<td>236</td>
<td>653</td>
<td>177.3</td>
<td>247.75</td>
<td>0.67</td>
<td>51489.17</td>
<td>51730.58</td>
<td>-263.1</td>
<td>10.2</td>
</tr>
<tr>
<td>Combustor</td>
<td>4</td>
<td>0.30</td>
<td>640</td>
<td>232</td>
<td>1701</td>
<td>823.0</td>
<td>1149.24</td>
<td>0.21</td>
<td>29.61</td>
<td>1178.3</td>
<td>0.00</td>
<td>6.1</td>
</tr>
<tr>
<td>Turbine</td>
<td>5</td>
<td>0.40</td>
<td>714</td>
<td>231</td>
<td>1447</td>
<td>439.8</td>
<td>872.77</td>
<td>0.70</td>
<td>26.91</td>
<td>889.89</td>
<td>0.00</td>
<td>16.1</td>
</tr>
<tr>
<td>Jet Pipe</td>
<td>6</td>
<td>0.30</td>
<td>790</td>
<td>234</td>
<td>1667</td>
<td>451.2</td>
<td>881.82</td>
<td>0.73</td>
<td>26.91</td>
<td>919.47</td>
<td>0.00</td>
<td>21.2</td>
</tr>
<tr>
<td>Nozzle</td>
<td>7</td>
<td>1.65</td>
<td>633</td>
<td>1043</td>
<td>771.0</td>
<td>968</td>
<td>358.18</td>
<td>297.21</td>
<td>26.91</td>
<td>660.31</td>
<td>0.00</td>
<td>725.6</td>
</tr>
</tbody>
</table>

Exhaust Energy Rate Exiting Engine [kW] = 697.3

Relative to the propulsion unit

Relative to the fixed reference environment

Table 4 Thermodynamic Quantities at the Outlets of the Specified Engine Stations for Operating and Reference Environments of 10,000 m.

<table>
<thead>
<tr>
<th>Engine Component</th>
<th>Engine Station</th>
<th>Mach Number</th>
<th>Sonic Velocity, ( V_{sa} ) [m/s]</th>
<th>Velocity, ( V ) [m/s]</th>
<th>Temperature, ( T ) [K]</th>
<th>Pressure, ( p ) [kPa]</th>
<th>Specific Energy Components [kJ/kg]</th>
<th>Shaft Power, ( P_1 ) [kW]</th>
<th>Thrust Power, ( P_T ) [kW]</th>
<th>Loss Rate, ( P_L ) [kW]</th>
<th>Rational Efficiency, ( \psi )</th>
<th>Loss Ratio, ( \zeta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Stream</td>
<td>0</td>
<td>0.80</td>
<td>296</td>
<td>223</td>
<td>287</td>
<td>100.3</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>External Diffuser</td>
<td>1</td>
<td>0.70</td>
<td>245</td>
<td>328</td>
<td>111.4</td>
<td>7.91</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Internal Diffuser</td>
<td>2</td>
<td>0.50</td>
<td>353</td>
<td>218</td>
<td>310</td>
<td>21.11</td>
<td>4.80</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Compressor</td>
<td>3</td>
<td>0.50</td>
<td>471</td>
<td>236</td>
<td>653</td>
<td>177.3</td>
<td>247.75</td>
<td>0.67</td>
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<td>51730.58</td>
<td>-263.1</td>
<td>10.2</td>
</tr>
<tr>
<td>Combustor</td>
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<td>0.30</td>
<td>640</td>
<td>232</td>
<td>1701</td>
<td>823.0</td>
<td>1149.24</td>
<td>0.21</td>
<td>29.61</td>
<td>1178.3</td>
<td>0.00</td>
<td>6.1</td>
</tr>
<tr>
<td>Turbine</td>
<td>5</td>
<td>0.40</td>
<td>714</td>
<td>231</td>
<td>1447</td>
<td>439.8</td>
<td>872.77</td>
<td>0.70</td>
<td>26.91</td>
<td>889.89</td>
<td>0.00</td>
<td>16.1</td>
</tr>
<tr>
<td>Jet Pipe</td>
<td>6</td>
<td>0.30</td>
<td>790</td>
<td>234</td>
<td>1667</td>
<td>451.2</td>
<td>881.82</td>
<td>0.73</td>
<td>26.91</td>
<td>919.47</td>
<td>0.00</td>
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</tr>
<tr>
<td>Nozzle</td>
<td>7</td>
<td>1.65</td>
<td>633</td>
<td>1043</td>
<td>771.0</td>
<td>968</td>
<td>358.18</td>
<td>297.21</td>
<td>26.91</td>
<td>660.31</td>
<td>0.00</td>
<td>725.6</td>
</tr>
</tbody>
</table>

Exhaust Energy Rate Exiting Engine [kW] = 757.2

Relative to the propulsion unit

Relative to the fixed reference environment

\( \text{Thrust [kN]} \times 707.1 = \text{Total [kW]} \times 0.1657 \div 0.5314 \)
Table 8  Thermodynamic Quantities at the Outlets of the Specified Engine Stations at an Operating Environment of Sea Level and a Reference Environment of 15,000 m.

<table>
<thead>
<tr>
<th>Engine Component</th>
<th>Engine Station</th>
<th>Mach Number</th>
<th>Sonic Velocity, (a) [m/s]</th>
<th>Velocity(^a), (V) [m/s]</th>
<th>Temperature, (T) [K]</th>
<th>Pressure, (p) [kPa]</th>
<th>Specific Energy Components [kJ/kg]</th>
<th>Shaft Power, (P_s) [kW]</th>
<th>Thrust Power, (P_t) [kW]</th>
<th>Loss Rate, (\psi)</th>
<th>Rational Efficiency, (\eta)</th>
<th>Loss Ratio, (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freestream</td>
<td>0</td>
<td>0.80</td>
<td>340</td>
<td>272</td>
<td>101.3</td>
<td>0.0</td>
<td>142.21 0.00 0.00 142.21 0.0 0.0 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>External Diffuser</td>
<td>1</td>
<td>0.70</td>
<td>345</td>
<td>241</td>
<td>95.9</td>
<td>0.0</td>
<td>150.12 0.47 0.00 150.12 0.0 0.0 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internal Diffuser</td>
<td>2</td>
<td>0.50</td>
<td>353</td>
<td>179</td>
<td>91.6</td>
<td>0.0</td>
<td>163.79 4.80 0.00 163.79 0.0 0.0 0.0000 0.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressor</td>
<td>3</td>
<td>0.60</td>
<td>471</td>
<td>236</td>
<td>86.2</td>
<td>0.0</td>
<td>1705.18 5100 0.00 1705.18 0.0 0.0 0.0000 0.0000</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Combustor</td>
<td>4</td>
<td>0.30</td>
<td>840</td>
<td>252</td>
<td>20.3</td>
<td>0.0</td>
<td>20.24 0.00 0.00 20.24 0.0 0.0 0.0000 0.0000</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turbine</td>
<td>5</td>
<td>0.40</td>
<td>774</td>
<td>310</td>
<td>37.6</td>
<td>1447</td>
<td>432.9 0.70 0.00 432.9 0.0 0.0 0.0000 0.0000</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jet Pipe</td>
<td>6</td>
<td>0.30</td>
<td>780</td>
<td>234</td>
<td>24.3</td>
<td>1487</td>
<td>481.2 0.73 0.00 481.2 0.0 0.0 0.0000 0.0000</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Nozzle</td>
<td>7</td>
<td>1.50</td>
<td>633</td>
<td>1543</td>
<td>-771.0</td>
<td>988</td>
<td>101.3 1.01 0.00 101.3 0.0 0.0 0.0000 0.0000</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Relative to the propulsion unit

* Relative to the fixed reference environment

Exhaust Energy Rate Exiting Engine [kW] = 936.9

| Thrust [N] | 797.1 | Total | 0.1875 0.8475 |

Table 9  Thermodynamic Quantities at the Outlets of the Specified Engine Stations at an Operating Environment of 15,000 m and a Reference Environment of Sea Level.

<table>
<thead>
<tr>
<th>Engine Component</th>
<th>Engine Station</th>
<th>Mach Number</th>
<th>Sonic Velocity, (a) [m/s]</th>
<th>Velocity(^a), (V) [m/s]</th>
<th>Temperature, (T) [K]</th>
<th>Pressure, (p) [kPa]</th>
<th>Specific Energy Components [kJ/kg]</th>
<th>Shaft Power, (P_s) [kW]</th>
<th>Thrust Power, (P_t) [kW]</th>
<th>Loss Rate, (\psi)</th>
<th>Rational Efficiency, (\eta)</th>
<th>Loss Ratio, (L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freestream</td>
<td>0</td>
<td>0.80</td>
<td>295</td>
<td>236</td>
<td>0.0</td>
<td>127.0</td>
<td>-188.45 0.00 0.00 -188.45 0.0 0.0 0.0056 0.0000</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>External Diffuser</td>
<td>1</td>
<td>0.70</td>
<td>299</td>
<td>209</td>
<td>29.7</td>
<td>223</td>
<td>-198.59 0.35 0.00 -198.59 0.0 0.0 0.0056 0.0000</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internal Diffuser</td>
<td>2</td>
<td>0.60</td>
<td>308</td>
<td>153</td>
<td>63.1</td>
<td>233</td>
<td>-149.54 3.48 0.00 -149.54 0.0 0.0 0.0056 0.0000</td>
<td>0.0</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compressor</td>
<td>3</td>
<td>0.60</td>
<td>309</td>
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<td>416</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Combustor</td>
<td>4</td>
<td>0.30</td>
<td>809</td>
<td>241</td>
<td>5.4</td>
<td>1263</td>
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<td>0.0</td>
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<td></td>
</tr>
<tr>
<td>Turbine</td>
<td>5</td>
<td>0.40</td>
<td>753</td>
<td>301</td>
<td>68.1</td>
<td>1368</td>
<td>632.61 2.12 0.00 632.61 0.0 0.0 0.0129 0.0003</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jet Pipe</td>
<td>6</td>
<td>0.30</td>
<td>783</td>
<td>247</td>
<td>8.8</td>
<td>1387</td>
<td>850.98 0.04 0.00 850.98 0.0 0.0 0.0129 0.0003</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nozzle</td>
<td>7</td>
<td>1.72</td>
<td>908</td>
<td>1043</td>
<td>49.8</td>
<td>847</td>
<td>116.87 323.38 0.00 116.87 0.0 0.0 0.1759 0.0172</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Relative to the propulsion unit

* Relative to the fixed reference environment

Exhaust Energy Rate Exiting Engine [kW] = 478.3

| Thrust [N] | 832.6 | Total | 0.1713 0.8287 |
3.3 **Loss Analysis**

Exergy analysis provides a tool for evaluating not only efficiency, but also the causes and locations of losses. A proper identification of system inefficiencies aids in performance improvement efforts.

3.3.1 **Variable Reference Environment**

As pointed out in the previous section, the exhaust loss is the major contributor to the inefficiency of the turbojet. Most efforts to increase the thrust of a turbojet also increase the exhaust loss (e.g., increasing the exhaust gas temperature and/or velocity to produce more thrust). However, this emission loss is different than losses due to internal irreversibilities (e.g., friction, pressure loss, mixing) which exist in any real system. The external loss incurred through the ejection of the exhaust gases is somewhat retrievable.

The division between internal and external losses when the operating environment is used as the reference environment is shown in Fig. 4 at both sea level and 15,000 m. At sea level the rational efficiency is approximately 17% and hence the exergy loss is 83%. Of this loss, 65% is external in that exergy is ejected with the exhaust gases. The remaining 35% is internal losses due to irreversible processes (mixing, combustion, friction, etc.).

![Breakdown of Total Loss at S/L (S/L Reference Environment)](image1)

![Breakdown of Total Loss at 15,000 m (15,000 m Reference Environment)](image2)

**Figure 4** Breakdown of overall engine exergy losses into external and internal components using a variable reference environment at (a) sea level and (b) 15,000 m.
At 15,000 m it can be seen in Fig. 4 that the internal losses decrease from 35% to 30% while the external losses increase from 65% to 70% as altitude is increased. Thus the engine reduces the percentage of the total loss due to irreversibilities despite decreasing overall efficiency with increasing altitude. This observation supports the trend seen in Tables 3 and 4, where all the engine components except the nozzle and the exhaust have lower individual losses at higher altitudes. The increase in the external loss percentage is due to the fact that as the operating environment increases altitude, the temperature and pressure both decrease. Since in this case the reference environment is the same as the operating environment, the lower reference environment temperature creates more exergy in the exhaust gas emissions (the pressure term in Eq. (22) is zero as the exhaust gas pressure is equal to the reference environment pressure). Note that at 15,000 m the total exergy loss increases to approximately 85% of the total incoming exergy (as the rational efficiency decreases to approximately 15%).

From a practical viewpoint these results are important as they provide a clearer understanding of the behaviour of the engine. The rational efficiency indicates that the engine becomes less efficient with increasing altitude. Further, these results show that internal losses, or irreversibilities, are reduced at higher altitudes. The external loss due to the exhaust loss emission is responsible for the decreased engine efficiency at increased altitudes, not an increase in the irreversible loss mechanisms (friction, mixing, etc.) traditionally associated with decreased efficiencies.

3.3.2 Constant Reference Environment

Following Fig. 4, a loss breakdown is shown in Figs. 5a and 5b, but with the reference environment held constant at 15,000 m and sea level respectively. It is evident that at sea level the use of a 15,000 m reference environment leads to a false increase (from 65% to 78%) of the exhaust emission losses for the turbojet as a fraction of the total loss. This is due to the fact that using a higher altitude reference environment than the operating environment increases the exhaust gas exergy due to the decreased reference environment temperature and pressure (see Eq. (22)). The increased exergy of the exhaust gases increases the external percentage of the total loss and thus decreases the internal percentage.
Figure 5  Breakdown of overall engine exergy losses into external and internal components at (a) sea level and (b) 15,000 m using constant reference environments.

This increase in the external loss percentage is to be expected given the fictitious extra exergy entering the turbojet (described in the rational efficiency section, 3.2). Since this excess exergy does not really exist, it cannot be converted into thrust and hence must be considered a loss. However, this loss is independent of any of the irreversible processes present within the engine, as its magnitude is established before the incoming flow enters the engine. Thus the fictitious exergy must be ejected as an external loss thereby increasing the external percentage of the total loss.

The opposite trend is evident when a constant sea level reference environment is used at an operating environment of 15,000 m. By comparing Figs. 4b and 5b it can be seen that in this case the external losses decrease from 70% to 52% of the total loss. The mechanisms responsible for this shift are similar to those for the constant 15,000 m reference environment case only opposite in effect. The increased pressure and temperature of the sea level reference environment over the 15,000 m operating environment reduce the exergy of the exhaust gases and hence decrease the percentage of external exergy loss as a fraction of the total loss.

In this case, the incoming air contains negative exergy (which indicates that work must be done on the operating environment air to bring it to reference environment conditions) but the properties of this exergy remain the same. It is still a fictitious quantity and thus must still count as a loss (as it cannot be converted into thrust), and as before, this loss is independent of any internal irreversibilities within the engine. However, the negative value of this quantity reduces the magnitude of the external loss and hence gives rise to a decrease in the external loss.
portion of the total loss, as opposed to the increase seen when using a constant 15,000 m reference environment.

Although the choice of reference environment has only a minor effect on the exergy based rational efficiency (less than 2.5%), the total loss breakdown reveals further information. One of the main practical benefits of exergy analysis is that it permits losses to be better defined and characterized, thereby allowing designers to better direct efforts to increase efficiency to the areas that have the most potential for, or need of, improvement. Here, the impact of the choice of reference environments is significant, as variations of approximately 18% are seen in some important parameters. Such discrepancies not only affect the accuracy of analysis calculations but also obscure general trends. In going from sea level to 15,000 m, for example, the internal losses decrease by approximately 5% (see Fig. 4) but when using a constant reference environment the opposite trend appears (as seen in Figs. 4a and 5b or Figs. 5a and 4b).
3.4 Exhaust Loss Analysis

The loss analysis in section 3.3 identified exhaust emission as the single largest contributor to the overall exergy loss of the engine. As such, this area presents the greatest possibility for increased efficiency. However, in order to consider proper recovery of the exergy from the exhaust, it is necessary to analyze the exergy of the flow to determine both the potential size of any gains and the nature of the exergy to be recovered. By doing this, the areas of greatest potential loss reduction can be readily identified.

3.4.1 Variable Reference Environment

Since the exergy of the fuel is purely chemical in nature (the velocity exergy component being negligible due to the specified input conditions), the exergy in the exhaust might be expected to contain much chemical exergy. However this is not the case as seen in Fig. 6. At both sea level and 15,000 m the chemical exergy in the exhaust stream is only 4% and 3%, respectively, of the total exhaust exergy. This small contribution arises from the fact that the exhaust stream is non-combustible (in this analysis complete combustion is assumed). Thus the only chemical exergy present in the exhaust is due to the difference in mole fractions of the exhaust gases leaving the turbojet and the same constituents present in the reference environment. The small decrease observed in chemical exergy as the altitude increases is due to the fact that the temperature decreases with altitude and the specific chemical exergy expression Eq. (28) is dependent on the reference environment temperature.

Physical exergy makes up most of the exhaust exergy (52% at sea level and 53% at 15,000 m). Physical exergy is treated here as the exergy obtained by reversibly bringing a flow to thermal and mechanical equilibrium with the reference environment. In this case, the physical exergy is strictly thermal since the exhaust gases are expanded to the operating environment pressure. However, had the analysis been performed with a fixed geometry nozzle (hence a constant exhaust pressure), a portion of the physical exergy contained in the exhaust would be due to the exiting pressure being different from the reference environment pressure (Eq. (27)). The second largest component of the exhaust exergy is the kinetic exergy of the expelled gases (44% of the total exhaust loss at both altitudes). Thus the two factors (i.e., the high temperature and velocity of the expelled gases) that cause the exhaust to contribute greatly to overall engine loss are the same factors that allow the engine to produce thrust. Fig. 6 also
indicates that although the total exhaust loss varies with altitude, from 697 kW at sea level to 757 kW at 15,000 m (see the Exhaust Exergy Rate Exiting Engine in Tables 3 and 4), the composition of this loss remains fairly constant with altitude.

Figure 6 Breakdown of exhaust gas exergy emission into kinetic, physical, and chemical components using a variable reference environment at (a) sea level and (b) 15,000 m.

3.4.2 Constant Reference Environment

The errors introduced by using a constant reference environment are more pronounced when evaluating the exhaust loss breakdown than when evaluating the rational efficiency (as is the case for the breakdown of the total loss as well). Considering the case where the reference environment is held constant at 15,000 m while operating at sea level (Fig. 7a), 65% of the exhaust loss is physical in nature. This value is 13% greater than when the operating environment is used as the reference environment (Fig. 6a). The increase is due to two factors: the increase in the thermal portion of the physical exergy as well as an additional pressure related component. Since the reference environment temperature is lower in this case than the operating environment temperature, there exists a larger temperature difference between the exhaust temperature (which is the same for both Figs. 6a and 7a as the operating environment is the same in both cases) and the reference environment temperature. In addition there is a difference between the exhaust pressure and the reference environment pressure as the analysis specifies expansion
to the operating environment pressure, which in this case is at sea level conditions. Since the pressure at sea level is higher than that at 15,000 m, a fictitious positive physical exergy component is introduced in the exhaust.

Since the kinetic exergy component of the exhaust is solely dependent on the velocity of the outgoing gases (which is dependent on the operating environment), the choice of reference environment has no effect on the magnitude of this component of the exergy loss. However, as seen in Figs. 7a and 6a it appears as though the kinetic contribution to the overall exhaust loss has been reduced to 33% from 44%. This decrease is attributable to the increase in the total amount of exergy being ejected from the engine due to the increase in physical exergy, from 697 kW to 937 kW (see Tables 3 and 5), while the kinetic exergy of the gases remains fixed.

This effect is also partially responsible for the decrease in the percentage of chemical exergy in the exhaust. However, in this case the actual magnitude of the chemical exergy also decreases due to the lower reference environment temperature. It should be noted, however, that in both the variable and constant reference environment cases the percentage of exhaust loss due to chemical exergy is relatively small (approximately 4%).

![Breakdown of Exhaust Loss at S/L (15,000 m Reference Environment)](a)

![Breakdown of Exhaust Loss at 15,000 m (S/L Reference Environment)](b)

**Figure 7** Breakdown of exhaust gas exergy emission into kinetic, physical, and chemical components at (a) sea level and (b) 15,000 m using constant reference environments.

In the case where the reference environment is held constant at sea level while operating at 15,000 m, the physical exergy component of the exhaust decreases to 25% from 53% (Figs. 6b and 7b). This difference is due to
the same two mechanisms as in the constant 15,000 m reference environment case, only the effects are reversed. The increased reference environment temperature decreases the temperature gradient between the exhaust gases and the reference environment, thereby decreasing the thermal component of the physical exergy. Also, a negative physical exergy exists due to the fact that the exhaust pressure is lower than the reference environment pressure. That is, since the exhaust gases are expanded to the operating environment pressure (which is at 15,000 m in this case), work must be done on the gases to bring them to the reference environment pressure. As seen in the loss analysis (section 3.3), this negative exergy is fictitious and due solely to the fact that the reference environment is not identical to the operating environment. However, since the thermal component of the physical exergy is larger than this fictitious negative component, the overall physical exergy component remains positive (although this does not have to be the case as illustrated in the cumulative exhaust loss analysis (section 4.4)).

This drastic reduction in physical exergy reduces the total amount of exergy exiting the engine, from 757 kW to 478 kW (see Tables 4 and 6), and hence the percentage of the total exhaust loss due to kinetic exergy increases to 69% from 44%. This increase is again due solely to the fact that the overall exhaust exergy changes due to the physical exergy component’s dependence on the choice of reference environment, as the magnitude of the kinetic component is dependent on the operating environment only. The chemical exergy increases slightly from the variable reference environment case due to the increase in reference environment temperature. This effect, combined with the overall decrease in exhaust exergy, gives rise to the increase in the percentage of chemical exergy to 6% as shown in Fig. 7b.

In both cases considered, it is important to note that not only does the choice of a constant reference environment skew the accuracy of the analysis by as much as 28%, but it also indicates false general trends. In either case a constant reference environment indicates that the composition of the exhaust loss varies with altitude, thus suggesting the possibility of minimizing a given type of loss through a carefully chosen cruising altitude. In going from Fig. 6a to 7b, the use of a constant sea level reference environment indicates that the physical exergy component of the exhaust loss decreases with increasing altitude while the kinetic component increases (the same trend is predicted using a constant 15,000 m reference environment, see Figs. 6a and 7b). But in actuality the exhaust exergy composition is approximately constant with altitude despite the actual magnitude of the exhaust loss varying.
4.0 FLIGHT PROFILE EXERGY ANALYSIS

4.1 Flight Profile Description

To examine the effects of different reference environment models on the accuracy of an exergy analysis applied to an entire flight profile, the characteristics of the flight must first be established. For the present analysis, a cruising altitude of 15,000 m (-50,000 ft) is used over a ground distance of approximately 3,500 km (approximately the distance between Toronto and Vancouver) with both the departure and destination aerodromes assumed to be at sea level. To reach the cruising altitude the aircraft uses a constant rate of climb of 3,000 m/min (-10,000 ft/min) which results in a time of 5 min to reach cruise altitude. The descent portion of the flight is accomplished using a constant descent angle of 10 degrees under cruise power conditions. The engine operating parameters in climb are different from those in cruise (see Table 2 for details) but because a cruising descent is used, the engine operating parameters in both cruise and descent are identical. The total flight time is approximately 4 hrs.

4.2 Cumulative Rational Efficiency

As a measure of merit for the overall efficiency of the engine during the flight, the cumulative rational efficiency as defined by Eq. (26) is used.

4.2.1 Variable Reference Environment

The variable reference environment curve in Fig. 8 shows the cumulative rational efficiency of the turbojet decreasing rapidly at the beginning of the flight and then leveling off asymptotically. At a distance of 0 km (and hence at an altitude of sea level) the cumulative rational efficiency is 22.27% which is identical to the instantaneous rational efficiency (Eq. (23)) value at this point (see Fig. 9). However, as the aircraft climbs and then establishes itself at the cruise altitude, the cumulative rational efficiency decreases to a value of 20.04% at a distance of 3,445 km. This yields a maximum variation of 2.23% over the entire flight. The instantaneous rational efficiency values also vary by approximately the same amount, with the value decreasing from that at sea level to 20.57% at the end of the climb segment (a distance of 73 km) and dropping further to a value of 20.02% as the engine operating
parameters are modified for the cruise condition, thus yielding a maximum variation of 2.25%.

The fact that the cumulative rational efficiency at 3,445 km (20.04%) is almost identical to the instantaneous rational efficiency during cruise (20.02%) is to be expected given the length of the flight. Since the aircraft spends the majority of its operating time under cruising conditions which are constant, any variations caused by the climbing and descending portions of the flight are overwhelmed by the significantly larger cruise segment. This fact is evident from the shape of the cumulative rational efficiency curve. At the beginning of the flight where the aircraft has spent no time cruising, the climb conditions dominate the behaviour of the cumulative curve. Thus the rapidly decreasing instantaneous rational efficiency during the climb portion of the flight (Fig. 9, for ground distances from 0 to 73 km) dominates the behaviour of the cumulative curve in this region (Fig. 8, for ground distances from 0 to 73 km). At the end of the flight the instantaneous rational efficiency increases due to the descent in the same manner that it decreased during the climb segment. However even given this rapid increase, the effect of the instantaneous rational efficiency is much less pronounced on the cumulative curve as only a very small increase in the cumulative rational efficiency is seen in Fig. 8 starting at 3,445 km.

Figure 8 Variation of turbojet cumulative rational efficiency over a flight range of 3,500 km at a cruising altitude of 15,000 m using various reference environments.
Thus the more time is spent under cruising conditions, the more the cumulative efficiency results tend to reflect the instantaneous results during cruise (which are constant).

The stabilizing and averaging nature of cumulative results causes the sudden variations in instantaneous efficiencies to be much less visible in the cumulative results. Specifically, the cumulative efficiencies somewhat mask (i) the sharp decrease in instantaneous efficiency during the climb segment of the flight, and (ii) the small instantaneous-efficiency plateau seen as the engine enters the tropopause under climb conditions (past this plateau, the engine switches operating parameters from climb to cruise settings, thus creating the discontinuous (vertical) change in the instantaneous rational efficiency). This stabilizing and averaging effect is even more noticeable during the descent portion of the flight, as previously mentioned, as only a small increase in the cumulative rational efficiency is observed despite the relatively large increase in the instantaneous efficiencies.

Note that although the aircraft starts to descend at a distance of 3,425 km, the instantaneous rational efficiency changes very little at this point. It is not until the troposphere is reached at a distance of 3,445 km that the instantaneous efficiency starts to increase rapidly.

Figure 9 Variation of turbojet instantaneous rational efficiency over a flight range of 3,500 km at a cruising altitude of 15,000 m using various reference environments.
In Fig. 9, the 15,000 m and variable reference environment curves are identical during cruise, but the 15,000 m reference environment curve starts to decrease at the start of the descent whereas the variable reference environment curve does not increase dramatically until a small distance later where the aircraft re-enters the troposphere. This delay in increasing instantaneous efficiency on the variable reference environment curve is due to the fact that in the tropopause the instantaneous efficiency is nearly constant (see Fig. 3) and as such no change is visible.

4.2.2 Constant Reference Environment

The use of a constant sea level reference environment to evaluate the cumulative rational efficiency produces errors in both numerical accuracy and predicted trends. At an operating altitude of sea level (for a distance traveled of 0 km), the variable and sea level curves in Fig. 8 are identical at a value of 22.27%. However, whereas the variable reference environment curve indicates that the engine efficiency decreases as the flight progresses, the sea level reference environment curve shows the opposite trend, with the curve reaching a maximum value of 23.71% at a ground distance of 3,425 km, a variation of 1.44%. The cumulative sea level curve starts to decrease at the start of the descent due to the marked change in instantaneous rational efficiency shown in Fig. 9 at the start of the descent. This behaviour is in contrast to that for the variable cumulative rational efficiency curve which reaches a minimum at the point the aircraft descends into the troposphere, at a ground distance of 3,445 km.

The cumulative sea level reference environment curve tends asymptotically towards the instantaneous sea level reference environment value during cruise (23.72%) and, as shown by the value of the cumulative rational efficiency at 3,425 km, this value is nearly reached. The maximum error (the maximum difference between the variable and sea level reference environment cumulative rational efficiencies) occurs at the start of the descent portion of the flight and is equal to 3.67%. This result is different from the instantaneous results, where the maximum error occurs at the end of the climb segment (73 km) while the engine is still operating under climb conditions. In this case, the use of a sea level reference environment predicts an instantaneous rational efficiency of 24.75%, which, when compared to the value predicted for the variable reference environment curve of 20.57%, yields a maximum error of 4.18%. However comparing the instantaneous results during cruise, the error between
using a variable and sea level reference environment is 3.70%. This is the asymptotic limit, i.e., the maximum value which is approached but never reached of the error between the cumulative curves as the flight distance is increased.

The reasons for the increasing cumulative rational efficiency when using a constant sea level reference environment while cruising at an altitude of 15,000 m are the same as those outlined in the instantaneous rational efficiency discussion (section 3.2). The use of this reference environment creates the “illusion” of negative exergy entering the engine with the airflow at all altitudes above sea level. As the flight time increases (which requires the aircraft to increase altitude to the cruising height), the quantity of this negative exergy increases, causing the cumulative rational efficiency to increase. Since the entire flight is spent at altitudes above sea level, the engine continues to “ingest” negative exergy, resulting in a total accumulation of approximately -2.40 GJ. This fictitious exergy is significant in quantity, representing approximately 15.72% of the total exergy input through the fuel of 15.27 GJ. (Note: the exergy input with the fuel evaluated using a variable reference environment is approximately 15.19 GJ.)

The use of a constant 15,000 m reference environment produces a cumulative rational efficiency curve with a similar shape as the constant sea level reference environment curve, but displaced negatively on the efficiency axis. This result is to be expected, as the use of a 15,000 m reference environment at an operating altitude of sea level creates the “illusion” of positive exergy in the incoming airflow. This added exergy decreases the rational efficiency compared to the case for a variable reference environment, yielding a value of 19.42% at sea level (for both the instantaneous and cumulative values). However, as the flight time increases during climb, the reference and operating environments approach and eventually meet at the cruising altitude, thus eliminating the fictitious positive exergy in the incoming airflow. In this case, the total accumulation of fictitious exergy is approximately 0.05 GJ compared to the cumulative exergy input through the fuel of approximately 15.19 GJ (a difference of three orders of magnitude). Thus the fictitious exergy represents a much smaller percentage of the total actual exergy input, approximately 0.32%.

The largest difference between the 15,000 m and variable reference environment cumulative rational efficiencies (Fig. 8) occurs at sea level and is equal to 2.85% (since this is at the beginning of the flight, it is also the largest difference in the instantaneous values). The cumulative 15,000 m reference environment curve increases with altitude (again predicting the opposite trend from the variable reference environment case) to a value of 20.01%
at 3,425 km which is very near the asymptotic value of 20.02% (the instantaneous cruise value using a 15,000 m reference environment). Thus the predicted variation in the cumulative rational efficiency over the entire flight is 0.59%.

The results indicate two main advantages in using the cumulative, rather than the instantaneous, rational efficiency to evaluate engine performance over an entire flight:

i. The first is that the sharp changes and irregularities seen in the instantaneous efficiencies (Fig. 9) are put into better perspective in terms of their impact on engine efficiency over an entire flight. For example, the peak instantaneous rational efficiency of 24.75% at 73 km under climb conditions when using a constant sea level reference environment is not observed in the cumulative results because it occurs for such a short duration.

ii. The second advantage is that the cumulative results demonstrate more clearly the advantage of using a constant reference environment equivalent to the cruising altitude conditions. From the instantaneous viewpoint alone, both the sea level and 15,000 m reference environments produce somewhat similar maximum errors (approximately 4% and 3% respectively). From these results alone, one might expect that either choice of constant reference environment would produce similar errors. However, since the majority of the flight is conducted at a cruise altitude of 15,000 m, the sea level reference environment maintains its error as distance traveled increases, while the 15,000 m reference environment reduces it. This error reduction is clearly shown on the cumulative results (Fig. 8) as the 15,000 m and variable reference environment curves converge and almost intersect. Thus for the 15,000 m case the error is reduced by orders of magnitude as distance traveled increases whereas for the sea level case, the curve diverges from the variable curve and asymptotically approaches a 3.70% error. Furthermore, the dependance of the amount of error reduction or propagation on distance is also shown in the cumulative results. At a distance of approximately 1,000 km, the cumulative efficiency for the 15,000 m reference environment no longer exhibits most of the error produced during the beginning portion of the flight through the use of this choice of reference environment, while at the same distance the sea level reference environment curve has attained most of its maximum error. The instantaneous results show no correlation between accumulated error and distance flown.
4.3 Cumulative Total Loss Analysis

This section examines the contribution of the exhaust gas emission to the cumulative exergy loss, in a similar manner as done for the total loss breakdown in section 3.3. In this case however, the percentage of exergy contained and accumulated in the exhaust over an entire flight is expressed as a percentage of the total cumulative incoming exergy, as opposed to a percentage of the total exergy loss (which is used in section 3.3). Since the exhaust has already been identified as the major contributor to engine losses, it is believed that it is more informative to know the exhaust emission exergy as a percentage of the cumulative exergy input as opposed to the cumulative total loss (which is itself a percentage of the cumulative exergy input).

4.3.1 Variable Reference Environment

The variable reference environment curve in Fig. 10 shows that the cumulative exhaust exergy percentage increases at the beginning of the flight, and then levels off asymptotically to a constant value. The rapid increase in exhaust exergy percentage between distances of 0 and 73 km is due to the increasing altitude during this phase of flight, when the reference environment pressure and temperature decrease and the exergy of the exhaust gases correspondingly increase. At sea level (a distance of 0 km) 50.29% of the cumulative exergy input is lost through the exhaust while at 3,445 km this value increases to 56.41%, a variation of 6.12%. As with the cumulative rational efficiency results, the cumulative exhaust loss curve asymptotically approaches the instantaneous exhaust loss value during cruise of 56.47% as the flight distance is increased. There is a small decrease beyond a distance of 3,445 km in the instantaneous percentage of the input exergy contained in the exhaust, because the descent occurs. But due to the much greater time spent at cruising conditions, the short duration of this phase of flight has little impact on the cumulative results.

4.3.2 Constant Reference Environment

With a constant sea level reference environment, the cumulative exhaust exergy percentage decreases as the flight progresses, going from a value of 50.29% at a ground distance of 0 km (sea level) to a value of 37.87% at 3,425 km, a variation of 12.42% (as expected, the latter percentage is near the instantaneous exhaust exergy
percentage during cruise of 37.76%). This trend is opposite to that exhibited by the variable reference environment curve in Fig. 10 where exhaust emissions contain increasing exergy as the flight progresses.

Since the decrease in exhaust exergy becomes greater as the difference between the operating and reference environment pressures increases (due to the fact that the exhaust gases are expanded to operating environment pressure), the constant sea level curve in Fig. 10 decreases as the aircraft climbs (between 0 and 73 km). Also, although the exhaust gases are at a higher temperature than the reference environment temperature at sea level, the thermal portion of the exhaust exergy is still decreased when compared to the variable reference environment case. This decrease occurs because the sea level temperature is higher than the temperature at 15,000 m, thus decreasing the apparent thermal difference.

![Figure 10](image)

**Figure 10** Variation of turbojet cumulative exhaust emission exergy over a flight range of 3,500 km at a cruising altitude of 15,000 m using various reference environments.

At the start of the flight the values of the exhaust exergy as a percentage of incoming exergy for both the sea level and variable reference environments are the same. However, as the flight progresses, the sea level curve diverges from the variable curve and reaches a maximum error at 3,425 km of 18.54%. This is in contrast to the
maximum error of 3.67% observed in the cumulative rational efficiency values using a constant sea level reference environment (which also occurs at the same point in the flight). Although both the cumulative rational efficiency and exhaust exergy behave qualitatively in the same manner when a constant sea level reference environment is used, the cumulative exhaust exergy exhibits both a larger variation in values during a single flight and a larger error when compared to the values obtained using a variable reference environment.

For the constant 15,000 m reference environment in Fig. 10, the cumulative exhaust exergy percentage decreases from 63.85% at the beginning of the flight (a distance traveled of 0 km) to a value of 56.54% (which is similar to the instantaneous exhaust exergy percentage in cruise of 56.47%) at a distance of 3,425 km, a variation of 7.31%. As with the use of a constant sea level reference environment, this choice of reference environment leads to the cumulative exhaust exergy decreasing with increasing flight distance, a trend opposite to that observed for the variable reference environment curve. In this case, however, the reference environment temperature and pressure are lower than the operating environment values. So there is a negative pressure difference at sea level which tends to increase the value of the exhaust exergy. This effect is responsible for the apparent increased exhaust exergy percentage seen at sea level for the 15,000 m reference environment curve. As the flight distance increases, the difference between the operating and reference environments decreases (a trend opposite to that for the constant sea level case), reducing the effect of the fictitious pressure difference and hence causing the exhaust exergy percentage to decrease and to approach the variable reference environment values. Note also that the increase in the temperature difference between exhaust temperature and reference environment temperature created by the use of a 15,000 m reference environment at sea level as compared to the temperature difference present when using a variable reference environment is reduced with increasing altitude and flight distance. Thus the maximum error between the 15,000 m and variable reference environment values of exhaust exergy percentage is 13.56% and occurs at the beginning of the flight, as opposed to the constant sea level curve which has a maximum error at the end of the cruise segment of the flight.

The cumulative exhaust exergy percentage curves in Fig. 10 show more clearly the errors in trends than the instantaneous results (as in Figs. 4a and 5b or Figs. 5a and 4b). Noting that the first 73 km of the flight represents the climb segment, Fig. 10 clearly shows that the use of a constant reference environment leads to the
erroneous finding that exhaust exergy decreases with increasing altitude. While this result is discernable from the instantaneous results, it is not as obvious. Further, as with the cumulative rational efficiency results, the cumulative exhaust exergy results show the advantage of using a constant reference environment with conditions equal to those at the cruising altitude, as this selection results in a decreasing error, as opposed to an increasing error for a constant sea level reference environment, as ground distance is increased. Although both choices of reference environment can lead to maximum errors of similar magnitude (14-19%), the choice of constant reference environment significantly affects the errors in the cumulative results as the flight progresses. Both the cumulative rational efficiency and cumulative exhaust exergy percentage indicate that, with a sufficiently large cruising distance, the results for a constant 15,000 m reference environment are almost identical to those for the variable reference environment.

One significant difference between the cumulative exhaust exergy percentage and the cumulative rational efficiency results involves the magnitudes of the potential errors caused by the use of a constant reference environment. As found for the instantaneous results, the choice of reference environment has a greater impact (one order of magnitude) on the numerical accuracy of the cumulative exhaust emission exergy percentage than the cumulative rational efficiency. As well, the predicted trends using a constant reference environment are erroneous, being approximately opposite in form to the trends for the variable reference environment case, with the magnitude of the predicted variation in error by as much as 6.30% for the cumulative exhaust exergy percentage.
4.4 Cumulative Exhaust Loss Analysis

It is useful to know how the breakdown of the exhaust loss changes over an entire flight to ensure that efforts to recover any losses are effective and worthwhile over the entire flight, not just over a single flight segment. Here, variations of each of the three major components of the cumulative exhaust loss (physical, kinetic, and chemical) are illustrated separately for various reference environments over the 3,500 km flight used in the previous sections.

4.4.1 Variable Reference Environment

For a variable reference environment, each of the components of the exhaust loss varies only slightly over the flight, as illustrated by the nearly constant variable reference environment curves in each of Figs. 11, 12, and 13 (noting that the cumulative chemical exergy curve appears to exhibit a larger variation due to the enlarged vertical scale). In all cases the cumulative exhaust exergy breakdown remains nearly constant over the flight, comprised approximately of 34% physical, 63% kinetic, and 3% chemical exergy.

Figure 11 Variation of the physical exergy component of the cumulative exhaust loss over a flight range of 3,500 km at a cruising altitude of 15,000 m using various reference environments.
Figure 12: Variation of the kinetic energy component of the cumulative exhaust loss over a ground distance.

Figure 13: Variation of the chemical energy component of the cumulative exhaust loss over a ground distance.
Figure 11 shows the physical-exergy percentage contribution to the cumulative exhaust loss, which decreases slightly from a value of 35.86% at sea level to a value of 34.16% at a ground distance of 3,500 km; a variation of 1.70%. The kinetic exergy component of the cumulative exhaust loss behaves in the opposite fashion of the physical exergy component with a value of 60.39% at the start of the flight which increases slightly to a value of 63.21% at a ground distance of 3,500 km, a variation of 2.82% (Fig. 12). In both of these cases, the effect of changing altitude is sufficiently small (as evidenced by the beginning portions of the curves which represent the climb segment of the flight) that the final cruise descent has no perceptible influence on the behaviour of the curves. Thus the final values cited in this discussion are those occurring at the end of the flight, at a ground distance of approximately 3,500 km.

Figure 13 shows the cumulative chemical exergy decreasing from 3.75% of the cumulative exhaust loss at a distance of 0 km to 2.63% at a distance of approximately 3,500 km. Due to the small magnitude of the chemical exergy with any choice of reference environment, the variation of 1.12% shown in Fig. 13 appears much greater than the variations experienced by the other exhaust components, even though it experiences the smallest change of the exhaust components over the flight.

4.4.2 Constant Reference Environment

The use of a constant sea level reference environment has an impact on the accuracy of the cumulative exhaust exergy breakdown which is large, even greater in fact than its impact on the cumulative total loss breakdown (which involved errors as large as 19%). At the beginning of the flight, for both the variable and constant sea level reference environments, the physical exergy component value is 35.86% (see Fig. 11). However instead of remaining nearly constant with distance, the sea level reference environment curve decreases sharply to a value of -17.53% at a ground distance of 3,425 km (the start of the descent). This is a variation of 53.39%, compared to the variation of 1.70% for the variable reference environment curve. Also, as noted in previous sections, the cumulative curve approaches the instantaneous cruise value, which in this case is -18.25%.

Thus the cumulative physical exergy component of the exhaust is so under-predicted for the sea level reference environment that the exhaust contains not only no physical exergy, but instead a physical exergy deficit. This negative numerical value is due to the positive pressure difference created when using the sea level reference
environment pressure where the operating environment pressure is equal to that for the cruising altitude of 15,000 m. This fictitious pressure difference creates the illusion of negative physical exergy existing in the exhaust during all phases of the flight, given that only at two instants during takeoff and landing is the aircraft at an operating environment of sea level. Thus although the temperature difference creates positive physical exergy, the much greater difference between the exhaust gas pressure and the reference environment pressure during cruise overwhelms this positive exergy and thus creates the illusion that the exhaust contains negative physical exergy. At altitudes below approximately 11,000 m, the exhaust still contains positive exergy from an instantaneous viewpoint, as the positive thermal portion of the exergy is greater than the negative pressure portion. However, the positive physical exergy accumulated during the climb segment of the flight under 11,000 m is quickly diminished by the negative physical exergy accumulated during flight above 11,000 m (i.e. cruise).

Although overall the exhaust still has positive exergy, this negative physical component creates the illusion that less exergy is exiting the engine through the exhaust. This result is consistent with results for the cumulative loss analysis (section 4.3, Fig. 10) in that the use of a constant sea level reference environment decreases the percentage of the cumulative exhaust loss. Over the entire flight, in fact, for a constant sea level reference environment, approximately 4900 MJ of exergy is ejected with the exhaust over the flight range of 3,500 km. This value is approximately 43% less than the actual value of approximately 8565 MJ for a variable reference environment. This decrease in cumulative exhaust exergy is due entirely to the negative physical exergy shown in Fig. 11 for the sea level reference environment.

This negative physical exergy leads to marked increases in the kinetic component of the cumulative exhaust exergy for a constant sea level reference environment. Since, as discussed in previous sections, the kinetic exergy of the exhaust is independent of the choice of reference environment, the three curves in Fig. 12 might be expected to be identical. However, even though the total kinetic exergy ejected with the exhaust over the flight distance of 3,500 km is constant at approximately 5400 MJ, this quantity represents a larger percentage of the cumulative exhaust exergy given the behaviour of the physical exergy component. At the beginning of the flight, the values for the sea level and variable reference environment curves are identical at 60.39%, but the former increases with distance to a maximum value of 111.39% at 3,425 km. This variation of 48.18% is much greater than the maximum
variation exhibited by the variable reference environment curve of 2.82%. In fact over the majority of the flight, the sea level reference environment curve in Fig. 12 indicates that the kinetic component of the cumulative exhaust exergy is greater than (i.e., over 100%) the cumulative exhaust exergy itself. This finding can also be seen by comparing the cumulative exhaust exergy ejected over the entire flight of 4900 MJ (when using a constant sea level reference environment) to the cumulative kinetic exergy ejected through the exhaust of 5414 MJ (when using any choice of reference environment).

Chemical exergy comprises a small percentage of the cumulative exhaust exergy, varying from 3.75% at sea level to a value of 6.15% at the start of the descent (3,425 km) when using a constant sea level reference environment. This increase is due to the decrease in overall cumulative exhaust exergy for a sea level reference environment (as is the case for the kinetic exergy component), although in this case the magnitude of the chemical exergy component is also increased slightly compared to the variable reference environment case due to its dependance on the reference environment temperature (Eq. (28)) which remains at the sea level value.

When using a constant sea level reference environment, the maximum error in the chemical exergy component of 3.52% (evaluated as the greatest difference in values between the variable and sea level reference environment curves in Fig. 13) occurs at the start of the descent, at a distance of 3,425 km. This result is consistent with those for the other flight-profile results. For the physical exergy component the maximum error is approximately 51.69% whereas for the kinetic exergy component it is 48.18% (see Figs. 11 and 12 respectively). These values are similar to the maximum variations experienced over the flight, mainly because with the variable reference environment a nearly constant exhaust exergy breakdown is observed at all altitudes (and hence distances).

The use of a constant 15,000 m reference environment leads to erroneous trends similar to those for the constant sea level reference environment. The major difference is that instead of starting at values equal to those obtained using a variable reference environment, the curves for the 15,000 m reference environment start at the maximum error and become more accurate as the flight progresses, approaching the variable reference environment curves asymptotically. Thus in Fig. 11 the value of the physical component of the cumulative exhaust exergy is initially 56.58% at sea level and decreases to a minimum of 34.41% at a distance of 3,425 km, a variation of 22.17%. At sea level, the maximum error is 20.72%, but by a distance of approximately 200 km this error has been
reduced by one order of magnitude and over the flight distance of approximately 3,500 km the error is almost completely eliminated. This decrease in the physical component of the cumulative exhaust exergy is due to the reduction of the error induced through the use of a higher altitude reference environment than operating environment at the start of the flight. Since the error is at a maximum when the aircraft is at sea level, the fictitious positive pressure-related physical exergy is the largest at this point. At all times during the climb segment, however, this fictitious exergy is continually reduced, becoming zero as the cruise altitude is reached. Thus, this choice of reference environment produces error only during the climb and descent segments of the flight. Given the short span of time spent under these conditions relative to the entire flight time, the cumulative results for the 15,000 m reference environment exhibit much less error than the results using a constant sea level reference environment.

The cumulative exhaust exergy ejected over the flight using a constant 15,000 m reference environment is approximately 8633 MJ, which is only 0.80% greater than the value for a variable reference environment (approximately 8565 MJ). This small difference is attributable to the minimal impact that the climb and descent segments have on the cumulative results. The maximum errors obtained when using a constant 15,000 m reference environment are large because this error occurs at the beginning of the flight, thus the heavily weighted effect of the cruise segment is absent.

Corresponding to the increased physical component of the exergy of the exhaust at sea level is a decreased kinetic component (see the 15,000 reference environment curve in Fig. 12). As mentioned earlier, the magnitude of the kinetic component of the cumulative exhaust exergy is independent of the choice of reference environment. However, the increase in the physical component increases the overall cumulative exergy, with the effect of decreasing the percentage of the exhaust exergy in kinetic form. Thus the 15,000 m reference environment curve has a value of 41.48% at a distance of 0 km (sea level) and increases to a maximum of 62.97% at a distance of 3,425 km, a variation of 21.49%. The maximum error of 18.91% occurs at sea level and is continually reduced with increasing ground distance until the descent segment of the flight is reached (at which point there is a very small increase in error). The chemical component of the cumulative exhaust exergy has an initial value of 1.93% which increases to 2.61% at 3,425 km, a variation of 0.68%. This increase is again mainly due to the fact that the fictitious pressure-related physical component of the exhaust exergy is reduced as the aircraft climbs (and in small part is due
to the dependence of the chemical exergy component on the reference environment temperature). The maximum error in the chemical exergy component occurs at sea level at the start of the flight and is equal to 1.82%.

The choice of reference environment clearly has the greatest impact on the results presented in this section. The use of a constant sea level reference environment leads to maximum errors (defined as the largest difference between constant and variable reference environment curves) as large as 52%, 2.5 times as large as the maximum error produced in the cumulative total loss analysis (19%) and 13 times as large as the maximum error produced in the cumulative rational efficiency analysis (4%). As with the cumulative total loss analysis, the choice of any constant reference environment leads to the prediction of false trends. In the case considered here, the actual exhaust loss breakdown is approximately constant during the flight (which is shown much more clearly in Figs. 11, 12, and 13 than in Figs. 6a and 7b or Figs. 7a and 6b), with variations limited to approximately 1-3% for the three exhaust components over distances of 0 to 3,500 km. The use of a constant sea level reference environment, however, predicts variations as large as 53% between component percentage values at 0 and 3,500 km. The use of a constant reference environment equal to the cruising altitude can reduce the errors by approximately half, with maximum errors of approximately 21% (as opposed to 52%) and variations over the flight of approximately 22% (as opposed to 53%). As well, the use of this choice of reference environment tends to mitigate the error produced, with the error decreasing as the length of the cruising segment of the flight is increased in duration.
5.0 CONCLUDING REMARKS

5.1 Summary of Findings

The results of the exergy analysis performed here on a turbojet engine indicate that the exhaust emissions contain the majority of the exergy loss. This result is consistent with the results reported in previous works (Brilliant, 1995; Clarke and Horlock, 1975; Kresta, 1992; Lewis, 1976; Malinovskii, 1984). The overall rational efficiency of the engine is shown to decrease by approximately 2% from sea level to the tropopause (~11,000 m), mainly due to the increase in the exergy loss associated with the exhaust emissions. For a typical modern engine the overall rational efficiency is approximately 20%, which is better than the older engine used in Clarke and Horlock (1975) which has a rational efficiency of approximately 16%. Above the tropopause the rational efficiency remains nearly constant, exhibiting only a slight decrease with altitude (less than 0.1%). When a constant reference environment is used, the rational efficiency remains nearly constant as altitude increases up to the tropopause and increases with altitude above the tropopause. The maximum errors produced through the use of a constant reference environment on the accuracy of the rational efficiency of the engine (both instantaneous and cumulative) are approximately 2-3%.

For both a constant and variable reference environment, the fuel exergy remains nearly constant at all altitudes up to 15,000 m, with the variable reference environment case showing the largest variation (less than 0.6%).

The internal exergy losses (those attributable to irreversible processes within the engine) decrease with increasing altitude. At sea level the internal exergy losses comprise approximately 35% of the total exergy loss, and this value decreases to 30% at 15,000 m using the older engine operating parameters found in Clarke and Horlock (1975) (hence the exhaust exergy loss increases from 65% to 70%). Over an entire flight, the exhaust exergy loss comprises between 50-55% of the cumulative incoming exergy when using typical modern engine operating parameters, this value increasing with increasing altitude (i.e., 50% at sea level, 55% at 15,000 m, a trend which is consistent with the instantaneous results). The use of a constant reference environment predicts an opposite trend with the exhaust exergy loss decreasing with increasing altitude and hence the internal exergy losses increasing, leading to errors as large as 19% over a flight distance of approximately 3,500 km. The use of a fixed
reference environment with parameters corresponding to low-altitude conditions (sea level) leads to an under
prediction of the exhaust exergy loss at all altitudes except the reference environment altitude, whereas this value
is over-predicted if the reference environment parameters are fixed at conditions for a higher altitude (15,000 m).

Although the magnitude of the exhaust loss varies with altitude, the composition of this loss remains
nearly constant when a variable reference environment is used. However, the use of a constant reference
environment causes the breakdown of the exhaust loss to be dependent on altitude. A reference environment
different than the operating environment (whether it takes on parameter values for higher or lower altitudes) causes
the physical component of the exhaust loss to decrease with increasing altitude and the kinetic contribution to
increase (the chemical component being negligible at approximately 2-6% in all cases considered). The differences
between the indicated and actual (as found using the operating environment as the reference environment, i.e., the
variable reference environment case) component contributions can be as large as 28% for the older engine
considered and 52% for a more typical modern engine when using a constant reference environment. This error is
also dependent on the choice of constant reference environment. The use of a constant reference environment with
conditions equivalent to those for the cruising altitude can reduce this maximum error by more than half and produce
cumulative results nearly identical to the variable reference environment values over a sufficiently large flight
distance.

5.2 Conclusions

The choice of reference environment was shown to have a negligible impact on the analysis of the incoming
fuel exergy, which is significant as the fuel represents the only source of exergy input into the engine. However,
it was also noted that depending on the fuel storage conditions, this conclusion could change.

Overall, the use of a constant reference environment yields rational efficiencies that are reasonably accurate
compared to those found using the actual operating environment as the reference environment. However, the
behaviour of the rational efficiency as altitude is varied is dependent on the reference environment, with a constant
reference environment predicting false trends. Although the accuracy in the magnitude of the rational efficiency
is still probably within acceptable engineering limits depending on the application, given the discrepancy in
predicted trends, the potential for larger errors exists as the difference between the operating and reference environments increases.

The choice of reference environment has a greater impact on the accuracy of results relating to the locations and causes of exergy losses. In the analyses performed herein, the maximum errors obtained in the exhaust loss evaluation (as a percentage of the total exergy input) through the use of a constant reference environment can be in excess of 4.5 times larger than the maximum errors produced in the rational efficiency results. Thus the predicted breakdown of the total loss into internal and external components can be significantly affected by the reference environment. Since this type of loss analysis is significant in determining where the key inefficiencies are located, errors on the order of 19% can greatly reduce the benefits of this type of analysis. A 15,000 m (~50,000 ft) difference in altitude between the operating and reference environment can lead to the erroneous conclusion that the exhaust emission exergy loss is not the major contributor to the overall inefficiency of the turbojet engine, indicating instead that the irreversible losses are equivalent in magnitude. As well, the use of a constant reference environment is unsuitable for predicting the behaviour of the exhaust loss during a complete flight, as this choice of reference environment indicates that the exhaust loss decreases, instead of increasing, with increasing altitude.

The choice of reference environment has the greatest impact on the breakdown of the exhaust exergy loss. Here, errors approximately 13 times greater than those for the rational efficiency results are observed when using a constant reference environment. With maximum errors of approximately 52% (using typical modern engine operating parameters), the use of a constant reference environment is likely inadequate for any design-improvement efforts. Although in all cases the chemical exergy component of the exhaust loss was properly identified as being relatively small, both the physical and kinetic components were found to be sensitive to the choice of reference environment, which is not actually the case. As shown using a variable reference environment, the composition of the exhaust loss is independent of altitude, with both the physical and kinetic exergy components contributing nearly equally (depending on the engine operating parameters) to the overall exhaust exergy loss.

From an instantaneous viewpoint, the errors produced through the use of a constant reference environment are approximately equal, whether the reference environment conditions correspond to high- or low-altitude conditions. However, over an entire flight, the choice of a constant reference environment corresponding to the
cruising altitude can significantly reduce the maximum cumulative errors produced, and mitigate the impact of any errors accumulated during the climbing and descending segments of the flight. The greater the flight distance, the more a constant reference environment corresponding to the cruising altitude will produce results approaching those obtained using a variable reference environment. This error-reduction effect is the most pronounced in the exhaust loss analysis, for which the errors are the largest when using a constant reference environment. However, the error-reduction effect is dependent on the length of the cruise segment of the flight and would be less noticeable for shorter flights, where the climb and descent segments represent a significant portion of the overall flight.

For analysis and design work, the use of a constant reference environment appears to be unsuitable in many instances for accurately guiding improvement efforts, as the locations of the greatest losses and the causes of these losses are not properly characterized.

5.3 Recommendations

Care should be exercised when using a constant reference environment for both instantaneous assessments of engine performance and cumulative engine assessments over entire flights, given the inaccuracies that can ensue. However, the use of a constant reference environment may prove suitable under conditions where the operating environment is difficult to define as a reference environment. For example, for space applications the lack of atmosphere renders the method used herein to evaluate the fuel exergy void, as the products of combustion are not present in a vacuum. In cases like this a constant reference environment corresponding to high altitude conditions may prove more suitable than a variable reference environment.

For most atmospheric aircraft applications, the use of a variable reference environment (equal to the operating environment at all times) does not add great complexity to the exergy calculations and yields accurate results that can be used to evaluate engine performance under any combination of operating conditions. The universality of the exergy efficiency parameter allows the comparison of various engine types (turboprop, turbofan, turbojet, scramjet) using a single term. It also allows components within an engine to be compared using a single term without resort to different efficiency parameters for different components (i.e., compressor efficiency, combustor efficiency, etc.). Given the clarity and consistency with which an exergy analysis has been shown to
describe the operation of a turbojet engine, it is hoped that its use will be further employed in the aerospace propulsion community to engines under current development.

Several recommendations for future work are also merited. First, as the present work is limited to altitudes below 15,000 m (~50,000 ft), the research in this area should be extended for higher altitudes (low Earth orbit and beyond) where even more severe changes are observed in the operating environment. Space as a reference environment provides a unique challenge in applying an exergy analysis, but one that should be overcome if exergy analyses are to aid in the development of future propulsive devices. As well, given the large amount of exergy contained in the exhaust emissions of the turbojet, research into methods of utilizing this exergy should be undertaken, as such efforts could yield improvements in engine efficiencies.
REFERENCES


Energy Conversion and Management, Vol. 36, March, pp. 149-159.


APPENDIX 1: Calculation Parameters

The variation in both atmospheric temperature and pressure as altitude is increased from sea level to 15,000 m and beyond is shown in Fig. 14 (with overlaying data points from McCormick, 1995). The sharp change in the variation in the temperature curve occurs at the start of the tropopause at approximately 11,000 m, above which the temperature remains constant until the stratosphere is reached (not shown on the figure). The two altitudes used most often in this report are sea level and 15,000 m which have temperatures of 288 K and 217 K and pressures of 101 kPa and 12 kPa, respectively.

The chemical composition of the reference environment, i.e. the proportion by mass or mole of each constituent in the atmosphere, is shown in Table 7. Table 8 shows the values of the various parameters used to evaluate the chemical exergy of both the fuel and the post-combustion mixture. Note that since the fuel is methane,
complete combustion occurs according to the formula

\[
CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O
\]  

(31)

Table 7  
Assumed Atmospheric Composition Used in Analysis.*

<table>
<thead>
<tr>
<th>Constituent</th>
<th>( M ) [kg/kmol]</th>
<th>( \alpha ) [kg/kg (_{\text{air}})]</th>
<th>( x ) [mol/mol (_{\text{air}})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>N(_2)</td>
<td>28</td>
<td>0.78000</td>
<td>0.79674</td>
</tr>
<tr>
<td>O(_2)</td>
<td>32</td>
<td>0.21000</td>
<td>0.18769</td>
</tr>
<tr>
<td>CO(_2)</td>
<td>44</td>
<td>0.00035</td>
<td>0.00023</td>
</tr>
<tr>
<td>H(_2)O</td>
<td>18</td>
<td>0.00965</td>
<td>0.01533</td>
</tr>
</tbody>
</table>

*adapted from Clark and Horlock (1975)

Table 8  
Combustion Parameters.*

<table>
<thead>
<tr>
<th>Constituent</th>
<th>( \beta ) [kg/kg (_{\text{fuel}})]</th>
<th>( \delta ) [kg/kg (_{\text{fuel,based}})]</th>
<th>( \gamma ) [kg/kg (_{\text{fuel,based}})]</th>
<th>( \lambda ) [kg/kg (_{\text{fuel,based}})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>N(_2)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.76471</td>
</tr>
<tr>
<td>O(_2)</td>
<td>0.0</td>
<td>-4.0</td>
<td>-4.0</td>
<td>0.12745</td>
</tr>
<tr>
<td>CO(_2)</td>
<td>0.0</td>
<td>2.75</td>
<td>2.75</td>
<td>0.05426</td>
</tr>
<tr>
<td>H(_2)O</td>
<td>0.0</td>
<td>2.25</td>
<td>2.25</td>
<td>0.05358</td>
</tr>
<tr>
<td>CH(_4)</td>
<td>1.0</td>
<td>-1.0</td>
<td>0.0</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

*adapted from Clarke and Horlock (1975)

The last set of data required to calculate the chemical exergy of the fuel are the standard thermodynamic quantities in Eqs. (17), (20), and (21) for each constituent involved in the combustion process. These values are shown in Table 9 (based on data in Clarke and Horlock, 1975) and represent the values at a temperature of 298 K and a pressure of 100 kPa.
Table 9  Standard Thermodynamic Properties of Constituents Involved in Combustion.*

<table>
<thead>
<tr>
<th>Constituent</th>
<th>$R$ [J/kg °K]</th>
<th>$c_p$ [J/kg °K]</th>
<th>$h_{st}$ [kJ/kg]</th>
<th>$s_{st}$ [kJ/kg °K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_2$</td>
<td>296.94</td>
<td>1043.7</td>
<td>0</td>
<td>6.845</td>
</tr>
<tr>
<td>$O_2$</td>
<td>259.82</td>
<td>932.9</td>
<td>0</td>
<td>6.415</td>
</tr>
<tr>
<td>$CO_2$</td>
<td>188.96</td>
<td>898.3</td>
<td>-8949</td>
<td>4.860</td>
</tr>
<tr>
<td>$H_2O$</td>
<td>461.91</td>
<td>1898.0</td>
<td>-13444</td>
<td>10.490</td>
</tr>
<tr>
<td>$CH_4$</td>
<td>519.65</td>
<td>2415.3</td>
<td>-5681</td>
<td>11.605</td>
</tr>
</tbody>
</table>

*adapted from Clark and Horlock (1975)
APPENDIX II: Mathematical Description of Cumulative Efficiency

The form of the cumulative rational efficiency curve can be determined by considering the following discussion. Taking the instantaneous thrust power and instantaneous incoming exergy flow rate to vary linearly with time, \( t \), one can represent these curves with the following equations:

\[
PT = At + b \quad E_{\text{incoming}} = Gt + h
\]

(32)

which when integrated over time yields

\[
\int PT(t) \, dt = at^2 + bt + c \quad \int E_{\text{incoming}}(t) \, dt = gt^2 + ht + k
\]

(33)

where \( a = A/2 \), \( g = G/2 \), and \( c \) and \( k \) are constants depending on the limits of integration (which represent the length of the flight in the appropriate units of time).

The analysis so far includes all the possible scenarios encountered during a flight, as the thrust power can increase (climb), decrease (descent), or stay constant (cruise) using Eq. (32). Thus to calculate the cumulative rational efficiency at any point along the flight profile requires evaluating the ratio of \( \int PT(t) \, dt \) to \( \int E_{\text{incoming}}(t) \, dt \) (see Eq. (26)) which yields the following expression:

\[
\Psi_{\text{cum}} = \frac{\int PT \, dt}{\int E_{\text{incoming}} \, dt} = \frac{a + \frac{b}{t} + \frac{c}{t^2}}{g + \frac{h}{t} + \frac{k}{t^2}}
\]

(34)
Thus as $t$ increases (which in turn means the flight distance increases for a given cruising speed) the higher order terms become less significant. In the limit as $t$ (or cruise distance) approaches infinity,

$$\psi_{cum} \to \frac{a}{g}$$

(35)