AN INVESTIGATION OF A SIMPLIFIED FLAT SURFACE GEOMETRY
OF THE SAILRAIL AIR BEARING SYSTEM

by

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A thesis submitted in conformity with the requirements
for the degree of Master of Applied Science
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Abstract

A novel compliant air bearing is being developed for materials handling and low speed guided transportation systems. In a typical application the load being transported is carried on a 1.2m square platform; this platform is supported on two compliant elements, or runners, which move along shallow concave guideways, or rails. Currently the runners are 1m long cylinders having an 115mm x 48mm section; the rails have a 152mm radius and are 127mm wide. Air for bearing action is introduced from supply manifolds integral with the underside of the rail through 0.52mm diameter nozzles which are spaced at 152mm intervals along the rail axis. With loads as high as 1500 kg the system can achieve effective coefficients of sliding friction as low as 0.1 percent with only modest air consumption. Although it is an externally pressurized compliant surface air bearing it has been developed on a trial-and-error basis and, as such, has controversial features. One is the construction of the runners which are currently made by winding cellulose fibre tissue onto a wooden core and then by enclosing the combination in a cover made from a 1mm thick plastic sheet. The present work is aimed at understanding of the rationale for this construction. Development of a simplified flat surface experimental model is described; its purpose is twofold: to allow simple characterization of tissue and cover properties, and to allow comparisons with the predictions of a simplified mathematical model. Results obtained to date show that the amount of tissue required to achieve air bearing action is much less than that used in practise; furthermore the system shows an unexpected ability to respond to asymmetric loading by redistribution of the air pressure at the rail-runner interface. Initial comparisons with an alternate compliant material, a cast urethane block, show marked differences in pressure distributions, with the tissue distribution being closer to the ideal of uniform footprint pressure.
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"We are an intelligent species and the use of our intelligence quite properly gives us pleasure. In this respect the brain is like a muscle. When it is in use we feel very good. Understanding is joyous"
- Carl Sagan, 1934-1997

"There are in fact two things, science and opinion; the former begets knowledge, the later ignorance"
- Hippocrates, 460-400 B.C.

"The place where optimism most flourishes is the lunatic asylum"
- Havelock Ellis, 1859-1939
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Nomenclature

Arabic

$A_T$ effective throat area of nozzle
$a_a$ acceleration of runner
$B$ half-width of runner in flat surface model
$B_R$ available bearing width of rail
$C_D$ discharge coefficient for nozzle
$C_L$ $L_{air}/L_L$
$D$ total drag between runner and rail, flexural rigidity of cover
$d_N$ diameter of nozzle
$E$ Young’s modulus of cover
$f_{ar}$ fraction of load weight $W$ supported by air pressure
$F_T$ functional dependence on $\sigma$ on $\eta$ in local spring model of tissue
$g$ acceleration due to gravity
$H$ in flat surface model, unloaded thickness of tissue
$h$ gap between runner and rail; $h=h(x,y)$
$h_b$ in flat surface model, height of rigid base above rail
$h_C$ thickness of runner cover
$h_i$ minimum gap between runner and rail; or seal height
$K_M$ $24\mu RT$
$K_s$ constant in linear spring model of tissue
$L_{air}$ restoring moment provided by air pressure about lateral runner axis
$L_E$ moment applied to cover edge in flat surface model
$L_L$ applied moment about lateral runner axis
$L_R$ length of runner
$m_f$ mass flow from one nozzle
$m_s$ mass flow from nozzles per unit length of runner
$m_D$ mass of driving weight in monorail
$m_p$ total mass supported by air bearing or runner
$n$ pressure tap number
$N_N$ number of nozzles per unit length of runner
$N_I$ in simplified runner and flat surface models, number of sheets of tissue (ply)
$p$ pressure in air bearing film
$p_A$ atmospheric pressure
$p_C$ pressure in cavity region just downstream of nozzle exit
$p_{CI}$ ideal air bearing pressure $= W/L_R B_R$
$p_T$ pressure in rail manifold, or “track pressure”
$p_T^*$ normalized track pressure
$Q_T$ volume flow at track pressure
$R$ gas constant in ideal gas law
$R_b$ radius of block in simplified runner
$Re_D$ Reynolds number of flow through nozzles
$Re_L$ Reynolds number based on a characteristic length $l$
$Re^*$ reduced Reynolds number
$R_r$  radius of rail surface  
$S_E$  slope of cover edge in flat surface model  
$T$  absolute gas temperature  
$t$  time  
$T_a$  atmospheric temperature  
$t_c$  thickness of flat surface model runner cover  
$T_T$  temperature of the air in the rail manifold  
$v$  fluid velocity component in the $y$-direction  
$V_E$  shear force applied at the cover edge in the flat surface model  
$W$  supported or load weight; usually $W = m \cdot g$  
$x$  distance along the rail  
$x_{cg}$  fractional longitudinal centre of gravity shift due to asymmetric loading $= L_L/W L_R$  
$y$  transverse distance across rail or bearing surface, measured from rail centreline  
$y_s$  lateral direction of seal height  
$y_h$  transverse distance of rail pressure tap from rail centreline  
$Y_h$  non-dimensional distance of pressure tap from rail centreline  
$z$  distance normal to rail surface  

**Greek**  
$\eta$  deflection of tissue in flat surface model  
$\theta$  angle of inclination of nozzles relative to track surface  
$\kappa$  effective local stiffness of runner at rail-runner interface  
$\lambda$  Poisson’s ratio of cover  
$\mu$  coefficient of sliding friction due to direct contact; also viscosity of air in Section ?  
$\mu_{ef}$  effective coefficient of sliding friction due to bearing action  
$\mu_s$  coefficient of static friction  
$\xi$  thickness of loaded tissue in flat surface model  
$\rho$  air density  
$\sigma$  normal surface stress at cover-tissue interface
Chapter 1

Introduction

1.1 Background of the SailRail System

The SailRail system is best described as a unique type of air bearing device, allowing large loads to be moved upon a very thin cushion of air with virtually no friction. The late Herbert E. Gladish (HEG) invented the system in 1971 while working as a design engineer for E.B. Eddy Forest Products of Ottawa, Ontario. SailRail Automated Systems (SAS) of Markham, Ontario, currently holds the patent on the technology. SAS uses the air bearing device for various automated material handling applications, mainly for use by the automotive industry. One of the original applications of the system was a warehouse stacking system depicted in Figure 1.1. A review by Sullivan (1997) describes the history and the development of the SailRail technology spanning from 1971-1996.

The most important components of the system are oval cylinders of compliant material called runners and concave guideways or rails. Figure 1.2 depicts the main components of the SailRail system. The original standard runners were made by winding cellulose fiber tissue onto a 37mm (1.5in) cardboard cylinder core, collapsing this to an oval shape by compressing it in between two rails, and then enclosing it in a 1.22mm (0.05in) thick high molecular weight polyethylene (HMWPE) cover. The standard runner was further developed into the configuration used today known as the minimum cellulose runner (MCR). The MCR was developed out of the need for less deformation of the runner when under load, which was essential to newer applications involving proximity sensors and robotics.
Figure 1.1 One of the original applications of the SailRail system, a warehouse storage system designed to store and transport goods on standard industrial pallets.
Figure 1.2  The main components of the original SailRail air bearing system
The runners have traditionally been made to be about 1m (3.28ft) in length, but depending on the application this may vary. A comparison of the standard runner and the MCR can be seen in Figure 1.3.

The runners were designed to move along the concave rails, supported on a cushion of air. Currently, the concave rail surface has a radius of 152mm (6.0in) and width of 127mm (5.0in). Air is supplied to the rail-runner interface through nozzles located on the rail surface, generating lift and creating a unique and highly effective air bearing. The nozzles are supplied from manifolds with the hollow rail interior, and are integrated into the rail surface at an angle of 20°, and to the rail axis at 45°. Initially the nozzles were located in two rows at angular distances of +5° and -15° from the rail centerline. They were placed at a longitudinal spacing of 305mm (12.0in) apart. Based on the result of UTIAS testing (Sullivan et al., 1989), the configuration was changed to have a single row of nozzles at +5° with a longitudinal spacing of 152mm (6.0in). This single row configuration is still in use by SAS for all applications and by UTIAS for all testing. Both nozzle configurations are shown in Figure 1.4.

For most applications, two runners are attached to the underside of a standard industrial pallet. Two rails are spaced apart at about 1.2m (4.0ft) parallel to each other. This allows for the pallet to support the load, and for the two runners to move along the rails, which act as both guideways and air suppliers for bearing action. Depending on the needs of the customers, SAS uses various sized material handling racks, instead of the traditional pallets. The rails must subsequently be placed at appropriate distances apart to accommodate any changes in rack size.

HEG also invented a linear induction motor (LIM) device to provide propulsion to the runners. The LIM technology has numerous applications that have become of particular interest to the Ford Motor Company. Research and development of the LIM technology has been on going at UTIAS by certain personnel through various projects.
Figure 1.3  A comparison of the cross-sections of the original standard runner (top), and the MCR (bottom), shown as actual size. The cardboard core of the standard runner is replaced by a larger wooden core in the MCR.
Figure 1.4  Nozzle layout patterns; the original standard, having 2 rows of nozzles, was invented by HEG. The new standard, having one row of nozzles, was the result of UTIAS research.
1.2 Technical Issues & Controversial Features

The SailRail system was conceived from a simple idea and evolved quickly from the initial concept to something very close to the current configuration. This current configuration is based on test results conducted by HEG as he tried to improve the material handling capabilities of the system. Through early test results the runner and rail design were developed on a trial-and-error basis. There was some evidence obtained that certain compliant materials greatly reduced the flow requirements of the system (Sullivan, 1997). However, there was no direct scientific basis for the choice of wrapped cellulose tissue or the unusual nozzle configuration. The decision was based on the fact that this configuration appeared to provide the best air bearing performance. Obviously the main question throughout the research has been trying to determine why the SailRail system works as well as it does. From the outset, it has remained unclear if these certain controversial features were in fact needed to produce a highly effective air bearing. In this respect, research conducted at UTIAS between 1971 and 1996, and comparison with the relevant technical literature has suggested that the SailRail system can best be described as an unconventional type of externally pressurized compliant surface gas-lubricated thrust bearing (Sullivan et al., 1998).

Since the beginning of the test work by HEG, two principal questions emerged (Sullivan, 1997). These two questions still form the basis for the most of the research objectives currently set by those working on the SailRail system.

(i) What is the role of the unusual structure of the runner in enabling the SailRail system obtain low coefficients of effective sliding friction with relatively low air consumption, that is, in promoting highly effective air bearing action?

(ii) What is the significance of the unusual nozzle geometry in promoting air bearing action?
HEG proposed many ideas as to why the controversial features made the system effective (Sullivan, 1997). Since the compliant SailRail runner configuration was said to outperform all other configurations, he claimed that the method of construction played a key role. He speculated that various aspects in the construction, such as the stiffness of the internal core, the tension in the tissue windings, and the bending stiffness of the cover, all contributed to the making the system highly efficient. (Sullivan, 1997). HEG also did a limited number of tests on alternative runner constructions, such as molded foam, but concluded that either they did not work, or that were not as effective. It is important to note that his work on this was never scientifically verified. The issue of alternative runner construction is one of the main driving forces behind the current research described in this report.

In relation to question (ii) it became clear that the nozzles perform a regulating orifice function, and it was discovered that the nozzle geometry could play a role in limiting air consumption. Other features of the nozzle orientation and geometry still remain controversial (Sullivan et al., 1998).

1.3 Research Strategy

At first glance the SailRail system may appear to be very simple, but in fact this appearance is illusory. To obtain an understanding of the system sufficient to provide a good basis for product improvement one has to consider the fluid mechanics of the air flow processes at the runner-rail interface, the structural aspects of the runner, the dynamics of the system, and the interaction of all three of these components. This is a difficult task that has been investigated by various personnel since the system’s invention in 1971. It is expected that the SailRail system will ultimately be described in terms of well-established physical and engineering principles. However, it’s complexity is such that one cannot expect to approach the task by formulating the equations of the continuum mechanics for the components, solving them, and then comparing the results with data.
obtained from a prototype. The availability of highly advanced digital computation methods notwithstanding, such an approach is not feasible in the near future given the substantial effort and resources required. More appropriate is the development of a mathematical model that is based on good physical insights and simplifications, which can ultimately be used for design improvement. In order to help answer the two principal questions introduced in Section 1.2, the research strategy has been to reduce the work into three distinct phases;

(i) The development of a mathematical model, including computer simulations based on varying conditions,

(ii) The verification of (i) based on detailed analytical and experimental investigations,

(iii) Design improvements suggested by (i) and (ii).

Phase (i) is still being worked on by Dr. P.A. Sullivan, Dr. C. Walsh, and Dr. Joon Chung, all of UTIAS, as part of an ongoing research collaboration over the past two years (Sullivan et al., 1998). Phase (ii) is also being worked on by several graduate level students as part of various Master’s thesis research efforts. Progress on phase (iii), is contingent on the completion of (i) and (ii).

1.4 Objectives of Current Research

1.4.1 Overall Project Objectives

The current SailRail research project objectives, as described by Sullivan et al. (1998) are:

* Development of physical understanding of the SailRail concept that will allow:
  * rational design for novel applications requiring significant changes in geometry,
  * suggest alternative methods of runner construction,
  * allow maximum flexibility in the selection of nozzle geometry and layout.
Sub-projects recently initiated to meet these main objectives are:

(i) Use of a monorail test apparatus to investigate the air bearing properties of SailRail runners,

(ii) Development and use of a runner core hardness testing instrument, and the correlation of data from this instrument with field experience,

(iii) Development of a mathematical model of the SailRail configuration based on fundamental principles of continuum and fluid mechanics,

(iv) Development and execution of experiments on configurations chosen to allow validation of the analysis of (iii),

(v) Development of apparatus and techniques to measure the electrical, thrust, and thermal properties of the integrated air bearing – LIM configuration invented by HEG, together with the measurement of these properties.

1.4.2 Objectives and Scope of Thesis

The objective of the present work was the completion of project (iv) above. Chapter 2 presents an overview of some of the pertinent research since the inception of the SailRail research project, which has lead to the development of a simplified model. Chapter 3 develops some of the rationale for the development of a flat surface model. A full description of a test apparatus used for experiments on the simplified flat surface geometry is given in Chapter 4. Chapter 5 provides a summary of the relevant test results and conclusions as taken from of an extensive test program using the flat surface apparatus. A discussion on the results and some recommendations for future work are given in Chapter 6. Since project (iv) is almost entirely experimental, the inclusion of the development of project (iii) in Appendix A allows the reader to analyze and understand the development of the theory behind the SailRail system and the simplified mathematical model.
Chapter 2

Review of Previous Research

2.1 General Classification

Previous research (Sullivan et al., 1985), suggests that the fluid mechanics of the SailRail system can be represented by three distinct phases, each occurring in a separate region within the footprint area beneath the runner. The first region encompasses the flow from the rail manifold, through the nozzles, and into the region under the runner immediately downstream. This was originally designated as the entry region but is now mainly referred to as the nozzle region. The flow processes in the nozzle region are turbulent, compressible, and usually gasdynamically choked. (Sullivan et al., 1985) The main features of the SailRail nozzle geometry are shown in Figure 2.1. As the air spreads out between the rail and the runner, in a short distance the fluid speed becomes negligible so that the fluid is effectively at rest. This static region is designated the cavity region. The cavity region consists of the pressure footprint, or in other words the main air pocket. UTIAS research has led to the conclusions that the air is essentially at rest, and that the pressure remains constant, throughout the cavity region (Sullivan et al., 1985). It is believed that most of the lift force is generated by the pressure of the air in the cavity region. The third region is identified as the narrow strip surrounding the periphery of the footprint and the cavity region, where the air escapes to atmosphere. This region is designated as the seal region. Some of the pertinent fluid and solid mechanics developed for analysis of the seal and cavity region are described in Section 2.2.
Figure 2.1 Geometry of the nozzle region.
2.2 Cavity and Seal Regions

The development of a mathematical model is based on the basic assumption that the flow in the cavity and seal regions is governed by Reynolds equation for compressible lubrication. This equation is derived assuming that the fluid is Newtonian, that it obeys the ideal gas law, and that the flow is isothermal and inertialless. Using the notation shown in Figure 2.2, with \( x \) and \( y \) being the distances along the rail and across its curved surface respectively, and with \( h(x,y) \) and \( p(x,y) \) being the gap and pressure distributions at the rail runner interface, then based on all of the above assumptions it can be shown that:

\[
\frac{\partial}{\partial x} \left[ p h \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial y} \left[ p h^3 \frac{\partial p}{\partial y} \right] = 0 \quad (2.1)
\]

For an infinitely long runner the flow between runner and rail may be assumed two-dimensional; that is, the fluid velocity in the direction parallel to the rail axis is zero. It can also be assumed that this flow is compressible but inertialless, and the gap between rail and runner is assumed to be very small relative to the pressure footprint. (Sullivan, 1997). Appendix A, which outlines the development of the mathematical model, further shows how the pressure distribution in terms of the gap distribution \( h(s) \), can be represented in the 2-D case as:

\[
p(y) = \sqrt{p_s^2 + K_M m_1 \int_y^b \frac{ds}{h(s)^3}} \quad (2.2)
\]

2.3 Measurement of the Seal Gap Height

An important parameter in the development of a mathematical model is the minimum gap between the runner and the rail that occurs in the seal region. This value has been referred to as the seal gap height, \( h_s \). Physical measurements on the profile of the cavity region suggested that the gap height decreases gradually from a maximum of 0.25mm (0.010in) near the centerline to some minimum in the seal region (HInchey et al. 1981).
Figure 2.2  Co-ordinate system used in the analysis of flow between the runner and the rail of the standard SailRail system.
However, the technique used to establish these physical measurements did not allow for the resolution of the gap profile in the seal region. Instead, the value for the seal gap height had to be obtained through an interpolation technique involving the above theory, estimated mass flow rates, and the condition that $h_s$ corresponds to a downstream constant pressure equal to atmospheric (Sullivan et al., 1985). The downstream pressure is in fact quite sensitive to the value of the $h_s$, such that a reduction of $h_s$ by 12% to 0.015 mm causes the downstream pressure to fall by 30% to 70 kPa from 101.3 kPa. Thus, optimizing the performance of the system can be done by $h_s$. All values pertaining to the gap height were obtained under the assumption that the surface roughness, of both the rail and cover, is much less than the actual gap heights (i.e. surface roughness $\ll h_s$), and therefore has a negligible effect on $h_s$ for the air bearing.

2.4 Monorail Testing

An appropriate performance index for the SailRail system is the effective coefficient of sliding friction $\mu_{eff}$, defined in the same manner as for solid contact sliding friction. A test apparatus was developed that was flexible enough to measure the effect of a variety of different test conditions on $\mu_{eff}$ (Sullivan et al., 1985). This apparatus became known as the monorail test device. Figure 2.3 is a schematic diagram of the monorail, consisting of a single rail mounted on a large I-beam, with the laid and its carriage of total mass $m_p$ being slung underneath the beam. A cable-pulley system connects the carriage to a falling weight of mass $m_D$. An optical interval timing system is used to measure the time to traverse a known distance. If acceleration is reasonably uniform, the monorail can be used to calculate $\mu_{eff}$, with $x(t)$ being the distance traveled by the carriage in time $t$ from its initial position and $g$ the acceleration due to gravity, the motion of the carriage is governed by:

$$\left(m_p + m_D\right) \frac{d^2 x}{dt^2} = \left(m_D - \mu_{eff} m_p\right) g \tag{2.3}$$
Figure 2.3 Concept of the monorail apparatus.
If the carriage travels distance $x_1$ and $x_2$ from its initial positions in times $t_1$ and $t_2$ respectively, solution of the differential equation gives:

$$
\mu_{\text{eff}} = \frac{1}{m_p} \left[ m_D - \frac{2(m_p + m_D)}{g} \left( \sqrt{x_2} - \sqrt{x_1} \right)^2 \right]
$$

(2.4)

A typical result from early testing with the monorail apparatus can be seen in Figure 2.4, where the friction data for two original standard runners is shown. The value of $\mu_{\text{eff}}$ at a normal operating track pressure of $p_r = 250$ kPa, is shown to be between 0.3% to 0.6% depending on certain test conditions, such as runner conditioning.

Early test results showed the ability of the standard runners to improve in performance, or condition, by reducing $\mu_{\text{eff}}$ over an extended period of time. This provided a major difficulty in the early prototype testing (Sullivan, 1997) by contributing to the non-repeatability of test results. The monorail apparatus proved to be an effective tool for measuring and controlling the impact of the conditioning effect on the system. Figure 2.4 also shows the effect of runner conditioning on the coefficient of sliding friction, as shown by a 2 hour conditioning study. In some cases, this conditioning effect has been shown to last as long as several days. Preconditioning of the runners became a prerequisite to all tests using the monorail. The conditioning effect was eventually traced to viscoelastic properties of the tissue in the runner. Evidence obtained through structural testing showed that the tissue is both hysteretic and highly nonlinear, suggesting that it could creep and gradually conform to the shape of the rail, further explaining the conditioning effect. (Sullivan et. al, 1985).
Figure 2.4 Friction data for original standard runners subjected to two hours of conditioning on the monorail apparatus.
It has also been suggested that $\mu_{eff}$ is almost entirely due to sliding friction from runner-rail contact in the seal region (Sullivan, 1997), implying that the performance of the system is highly dependent on $h_t$. Based on this relationship, we can state that for a given mass flow rate $m_f$ and applied load $W$, the system can be optimized by determining the configuration with the maximum $h_t$.

2.5 The Simplified Runner

Throughout some of the early research, HEG had frequently suggested that both basic tissue properties and the tension used to wind the tissue onto the core played a significant role in runner performance (Sullivan, 1997). As a first step in empirically determining the relative importance of all the various factors, it was proposed that some investigation should be done on the configuration depicted in Figure 2.5 which eventually became known as the simplified runner (Sullivan et al., 1985). It consists of a rigid block have one surface machined to a radius $R_b$ less than the radius of the rail $R$, a specified number of layers of tissue $N_T$, and a sheet of cover material placed between the rail and the tissue.

The monorail test apparatus was used extensively to investigate the performance of the simplified runner. Several key results came out of these tests, the first being that $\mu_{eff}$ was close to that for the standard runner. Figure 2.6 shows the results from testing that shows a comparison of the monorail transit times for the standard runner and the simplified runner configuration. Another key result was the fact that only a small number of layers of tissue were needed to establish an effective air bearing with the simplified runner. Figure 2.7 shows the effect of $N_T$ on the effective coefficient of sliding friction $\mu_{eff}$. To establish further comparison with the standard runner, the conditioning effect described in Section 2.4 was also investigated for the simplified runner.
Figure 2.5 The simplified runner geometry.
Low Profile Track, 152 mm
Cover Thickness: .86 mm
% Driving Weight: 1.1%

Figure 2.6 Comparison of transit times for the simplified runner configuration and the standard runner (shoe).
Figure 2.7 The effect of the amount of tissue layers $N_T$ on performance the simplified runner.
As a general statement on the results, it was suggested that the simplified runner possessed the ability to condition quicker than the standard rail, possibly due to the fact that the tissue did not have to conform to the rail nearly as much as the standard rail would have to (Sullivan et al., 1998). Figure 2.8 shows the results from a series of test showing the conditioning effect for the simplified runner.

One final significant result from the simplified runner testing was the fact that the runner possessed the ability to produce longitudinal restoring moments to offset load conditions. However, since no quantification of this ability to restore applied moments could be ascertained from the previous research efforts, it is given as a generalized statement only. The results of all aspects of testing on the simplified runner provide encouraging results for alternative runner construction and optimization, as well as for the formulation of a mathematical model.

2.6 The Flat Surface Geometry Model

The initial aim of the investigation (Sullivan et al., 1998) was to develop a detailed model of the simplified runner, but this geometry confounds compliant air bearing and geometrical effects, such as the curvature of the rail. It was decided that it would be possible to simplify the problem by investigating a flat surface model of the air bearing system, depicted in Figure 2.9. The notation used in the analysis of this flat geometry is shown in Figure 2.10. The runner consists of a rectangular block of tissue or other compliant material having unloaded thickness $H$ and width $2B$ mounted on top of a cover in the form of a thin plate of thickness $t_c$. An added advantage of this configuration is that modelling of the behaviour of the tissue can be based on direct measurement of the structural properties. Furthermore, it facilitates investigation of such issues as the adequacy of proposed tissue models, as well as certain justifications for the simplified geometry model. Research efforts in the determination of a mathematical model for the SailRail system are currently all focussed on this flat surface geometry.
Figure 2.8 Results of repeatability and conditioning tests on the simplified runner with $N_T = 80$. 
Figure 2.9 Simplified flat surface model of the air bearing system used for both experiment and theoretical analysis.
Figure 2.10 Notation used in the analysis of the flat surface model.
Chapter 3

Theory of the Flat Surface Model

3.1 Introduction

In order to get a complete understanding of the SailRail system, it became necessary to develop a mathematical model of the air bearing system, in particular one based on the flat surface geometry model outlined in Section 2.6. The beginning of the mathematical model was introduced through research conducted at UTIAS starting in 1981 (Hinchey et al. 1981). Over the years the model has been both updated and added to as new theories, models, and analytical techniques have developed. The results of many validation and exploratory experiments have also aided the development of the model.

The research and experiments described in Chapters 4 and 5 of this report have arisen from a need for a better understanding of the basic principles involved in the SailRail system. A few main factors are responsible for the direction of the current research. The first is that for some time there has been a need expressed by both UTIAS personnel and SailRail personnel for finding alternative methods of constructing the runner which simplify its construction, which make its properties more predictable, and which make it more robust in operation (Sullivan, 1997). The second factor comes from test results shown in Section 2.6, which concluded that only a thin layer of un-tensioned tissue is required to obtain effective bearing action. A third factor to consider is that although some previous and current work has suggested a number of theories concerning both the fluid mechanics and the structural properties of the runner, none of these theories has yet to be strongly validated.
Taking these factors together, it was deemed counter-productive to attempt modeling of the SailRail. Instead, it was decided that the effort should be focused on the flat surface geometry model outlined in Section 2.8, which could provide strong validation of the theories referred to above. It is believed that this geometrical configuration will guide both the selection of a simplified runner configuration and development of a mathematical model that can be used as a direct aid to design.

For the present project, the mathematical model chosen was an extension of that previous developed and was adapted to the flat surface geometry (Sullivan, 1997 and Sullivan et al., 1985). The original model, as given in these references, assumed that the fluid mechanical processes and structural aspects could be classified as described in Chapter 2. This means the flow in the nozzle region can be described by an inviscid compressible orifice flow law together with an empirically determined discharge coefficient, and that the flow in both cavity and seal regions could be described by Equation 2.1. Previous research outlines the assumption that the main effect of runner compliancy was to produce a local spring behaviour at the rail-runner interface (Sullivan et al. 1985). Described in Appendix A is the progress made to date by Dr. P.A. Sullivan and Dr. C. Walsh in developing a comprehensive mathematical model for the flat surface geometry (Sullivan et al., 1998).
Chapter 4

Simplified Flat Surface Testing Apparatus

4.1 Design and Construction

The major elements of the test apparatus designed to verify the theory of Sullivan et al. (1985) are depicted in Figures 4.1 and 4.2. Two 457mm × 152mm (18in × 6in) rectangular plates, form the most important components of the system. Between these two plates, compliant material is placed. This is usually in the form of a HMWPE cover and flat layers of cellulose tissue. This represents an unwound version of the standard runner. The lower plate contains a single row of 1.02mm (0.040in) diameter, 2.54mm (0.10in) long radial nozzles along the centerline of the plate. These nozzles are spaced at 95.00mm (3.75in) apart. This lower plate is supported by vertical spacer bars and a third rectangular base plate.

A cylindrical air supply manifold supplies the flow requirements of the nozzles. The flow is delivered from the supply manifold to the nozzles by flexible hosing with an inner diameter of 9.53mm (0.375in), through 7.93mm (0.313in) nozzle plenums. The system was designed this way for a specific reason. Due to the large diameter of the nozzle plenums, the pressure at the nozzle entrance is assumed to be equal to the pressure measured in the manifold. In other words, the pressure measured in the manifold is taken to be equivalent to the SailRail track pressure $p_T$, for the flat geometry apparatus. The nozzle layout and configuration was chosen to ensure that the flow could accurately be described by the orifice model outlined in Appendix A. Longitudinal and lateral pressure taps are also located in the lower plate. These pressure taps were drilled at various locations along both centerlines of the plate surface, in order to try and give an accurate profile of the pressure in the footprint area.
Main components of the apparatus designed and built to verify the simplified flat surface geometry model. Drawing includes the load carriage, and the vertical alignment mechanism (drawing not to scale).
Figure 4.2  Drawing of the base of the apparatus, containing the nozzles, pressure taps, and flat rail surface used to generate the air bearing (drawing not to scale).
The design of the apparatus is such that the lower plate can be replaced with a plate having a different nozzle configuration or a different layout of pressure taps to obtain other profiles. Figure 4.4 shows the current pressure tap and nozzle layout installed in the model, as well as a representative diagram of the nozzle geometry and air supply system. Figure 4.5 shows a photograph of the apparatus as set up for laboratory experiments.

4.2 Data Collection

Each of the 10 longitudinal and 16 lateral pressure taps shown in Figure 4.4 are connected to a Scani-valve pressure transducer by flexible Tygon tubing. The Scani-valve is capable of rotating through numerous pressure inputs that can be read by the same transducer. The actual transducer is a piezo-resistive type with an experimentally determined calibration constant of 0.0337646 mV/psi. The calibration of the transducer was achieved by applying a known pressure, as measured by a calibrated Heise® pressure gauge accurate to ±2 mm of Hg, to the transducer and measuring the voltage output with an oscilloscope. This constant was re-confirmed on a number of occasions to ensure the system was operating properly. The Scani-valve is connected to a signal conditioner that boosts the received signal by a gain of 10. The signal then travels into a Soft500 series digital acquisition board, set to sample at a rate of 0.1kHz maximum. The input data is interpreted and saved by a PC running a Basic program written by the author specifically for the apparatus. This program is listed in Appendix B. An integration program recovery.c, listed in Appendix C, is used in the analysis of the simplified flat surface model test results, providing a tool for integrating the pressure distributions for vertical force and moment.
Figure 4.3  Layout of the nozzles and pressure taps currently installed in the lower plate of the apparatus (upper diagram), and a representative depiction of the air supply system, including the nozzle geometry (lower diagram).
Figure 4.4  Photographs of the flat surface apparatus as currently set-up for laboratory experiments. The air supply, and vertical alignment mechanisms can be seen in the foreground.
Chapter 5

Experimental Results

5.1 Introduction

The results of a comprehensive experimental test program are described within this section of the report. The simplified flat surface geometry test apparatus, described in Chapter 4, was the primary testing device used in most of the experiments. Numerous exploratory tests using this apparatus were conducted to gain a better understanding of the physics that were involved in the system. The test program was designed such that certain results could ensure that the test apparatus was both predictable and repeatable under varying test conditions.

Variables such as tissue conditioning and model stability were thoroughly investigated in the initial portions of the test program. Considerable effort was made to ensure that the apparatus was in fact capable of accurately producing the desired results. Summaries of the significant results found from the exploratory test program are presented below in Sections 5.2 and 5.3. Section 5.4 explains the progress made in developing experiments actually designed to aid in the verification of the mathematical model.

5.2 Initial Experimental Results

In order to function as an air bearing a device such as the flat surface model must be statically stable. This implies that the apparatus must possess the ability to generate lateral and longitudinal restoring moments sufficient to compensate for both loading asymmetries and other external disturbances. The ability to generate these restoring moments has been well documented for the original SailRail system in several publications (Sullivan et al., 1985). Having no knowledge of the stability of the test apparatus it was
decided that the stability problem should be bypassed by using fixed displacement experiments. A Tinius-Olsen structural test machine was used to establish this fixed displacement criterion. The machine operates by having a load plate capable of height adjustment that can be lowered down on to a test sample sitting on a fixed base. The machine is capable of producing very high loads that can be retrieved as output from an accompanying digital acquisition system.

A test tissue sample was obtained by cutting a section of a standard SailRail runner to the dimension of the apparatus (457mm by 152mm). The same was done for the HMWPE plastic cover. This original tissue sample had a tissue count of $N_T = 250$, which corresponds with an unloaded thickness of about 0.5in, and was used for most of the original testing. Stress relaxation and compression tests were conducted on the test sample using the Tinius-Olsen machine. Results of these tests can be seen in Figure 5.1. The purpose of this was to determine if the same viscoelastic effects existed in an unwound sample of the cellulose tissues. Both the hysteresis in the loading curve and the stress relaxation properties suggest that some non-linearity exist within the tissue. Further confirmation of the tissue conditioning effect was the results from early testing that showed the measured load $W$ changed continuously with time while at a fixed displacement. Since the apparatus was designed to measure a true pressure gradient beneath the flat surface runner sample while at a constant load $W$, the decision was made to abandon the use of the Tinius-Olsen machine.

During these initial test we discovered that the flat surface model could sustain substantial longitudinal pressure gradients. Tests using loads that were offset in the longitudinal direction were set up by inserting shims in one end of the test apparatus. The results of these tests gave us our first indication that the apparatus did in fact have the ability to produce restoring moments.
Figure 5.1 Stress relaxation test for planar tissue model put in the Tinius Olsen machine, and the load compression curve for the same tissue sample showing a hysteresis effect.
A decision was made that the apparatus should be constrained in some type of to allow for a known fixed load $W$ to be applied while controlling the lateral stability. In order to accomplish this an alignment mechanism was designed and built in the ACV lab at UTIAS. This alignment device is depicted in Figure 5.2. The upper plate (1) supports an I-beam, on top of which is mounted a load platform (5). The motion of the upper plate is constrained by two assemblies (6) attached to the I-beam through extension brackets. These assemblies consist of linear ball bushings (7) allowing vertical motion along two vertical shafts (not shown), together with ball bearing mounted shafts which permit rotation about a horizontal axis located above the middle of the plate (2). A photograph of the test apparatus and alignment device is shown in Figure 5.3. However, experiments subsequently showed that the moment restoring capabilities of the apparatus are such that the alignment mechanism was unnecessary and hence discontinued. The result of the ability of the model to produce lateral-restoring moments also justified the elimination of the lateral constraint.

5.3 Exploratory Test Results

5.3.1 Effect of Tissue Conditioning

Using the alignment mechanism, some exploratory tests were conducted to identify if a significant tissue conditioning effect existed for the flat surface geometry model. One of the test results that examined the tissue conditioning effect is shown in Figure 5.4. Over the 24-hour period that the tissue sample remained under load, only minimal changes in both the lateral and longitudinal centerline pressure distributions were observed. This supports the hypothesis of researchers (Sullivan et al., 1998), that the conditioning effect of both the original standard and MCR runners is a result of the runner geometry not matching the rail geometry, as opposed to the properties of the tissue itself. One might say that with the rail-runner system, the tissue takes time to flow into conformity to produce the desired air bearing. The unwound, untensioned tissue in the flat surface geometry experiments does not need to flow, because it matches the geometry of the flat surface model.
Figure 5.2  CAD representation of the main components of the alignment mechanism used in conjunction with the simplified flat surface geometry model
Figure 5.3  Photograph of the entire test apparatus including the alignment mechanism, as set-up in the ACV laboratory at UTIAS
Figure 5.4 Results showing the tissue conditioning effects on the centreline pressure distribution for the flat surface model. The upper and lower graphs represent the lateral and longitudinal pressure profiles respectively. Test conditions; $H = 12.7$ mm (0.50 in), $B_R = 127$ mm (5.0 in), $t_C = 1.01$ mm (0.040in), $p_T = 413$ kPa (60 psi), $W = 199.6$ kg (439.8 lb).
This was a fundamental result of the experiments to date, and was also the first step in verifying the repeatability of the test apparatus. With the knowledge that no significant tissue conditioning effects existed within the flat surface geometry sample, experiments could be conducted without the need to prepare the samples by pre-conditioning them while under load for a set time period. The elimination of tissue conditioning as a variable in the system also allowed the research effort to focus on ensuring that other non-linearities in the system did not affect the test results.

As described in Section 5.2, a hysteresis exists within the load-compression curve of the cellulose tissue, as well as some stress relaxation effects after the initial loading period. The test program was designed around eliminating these effects as potential sources of variation in the results. All tests were conducted on what was termed "load increase" conditions. This means that tests were never conducted on the apparatus by removing weight, unless the entire load was completely removed and then reloaded after an appropriate amount of time. This ensured that the test results all fell on the upper portion of the load-compression curve. The tests were also delayed at least ten minutes after loading to ensure that the stress relaxation effect during testing would be minimal. Figure 5.4 also shows that a slight non-uniformity across the centerline of the lateral pressure distribution exists. This effect was found to be caused by complications within the alignment mechanism that eventually lead the research to an unconstrained system to confirm lateral and longitudinal stability.

5.3.2 Model Stability

To verify the stability of the flat surface geometry model, a number of tests were conducted with offset loads in both the longitudinal and lateral directions. Figure 5.5 shows the results of tests performed with three different offset loads applied in the lateral direction. The offsets were applied by shifting the weights placed in the load platform, (5) in Figure 5.2, from a center position to two different off-center positions, resulting in an overall shift in the center of gravity (c.g). The resulting pressure distributions indicated
Figure 5.5  Three lateral profiles tested under a offset load condition in the lateral direction; equivalent to a center of gravity shift of ±2.39%.
that the fraction of the load supported by the air bearing remained comparable for all three
tests. These results also allowed for a conclusion to be made that a slight asymmetry
exists in the model, even after the elimination of the alignment mechanism as a source of
variance. It has been suggested that the model's lateral weight distribution is the reason
for this slight asymmetry. To confirm the trend, the test was repeated with comparable
results. The data clearly suggests that the model has a lateral weight asymmetry
corresponding to about 2.4 percent of its width $B_R$, or about 3.66 mm. (0.14 in.),
indicating that appropriate trimming is required as a prelude to testing.

As noted in Section 5.2, the SailRail air bearing can sustain longitudinal pressure
gradients capable of providing most of the moment required for longitudinal stability, the
above tests are the first to suggest that significant lateral pressure gradients can also be
sustained. Since transverse pressure profiles were obtained only at the model centre, an
accurate determination of the extent to which the pressure profiles supply the restoring
moment is not possible. However, the uniformity of the longitudinal pressure profiles in
all test results suggests that an approximate estimate can be obtained from the lateral
profiles by assuming that they act over the entire bearing length. These estimates,
obtained by numerical integration of the profiles in Figure 5.5, are shown in Table 5.1,
which gives the applied moment as a c.g. shift, the pressure lift as a percentage of $W$, and
the pressure moment as a c.g. shift. The result of the pressure lift exceeding the applied
load $W$ is explained by the fact that the calculation does not allow for the fall-off in the
pressure at the model ends. We concluded that the pressure variations under a laterally
offset load are capable of supplying most of the lateral restoring moment.

<table>
<thead>
<tr>
<th>Applied c.g. shift, % of $B_R$</th>
<th>0</th>
<th>2.86</th>
<th>5.73</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure lift, % of $W$</td>
<td>103.7</td>
<td>105.5</td>
<td>106.4</td>
</tr>
<tr>
<td>Pressure c.g. shift, % of $B_R$</td>
<td>0</td>
<td>2.93</td>
<td>6.44</td>
</tr>
</tbody>
</table>

Table 5.1 Pressure lift and moment response characteristics of data in Figure 5.5; the
c.g. shifts are relative to that for the symmetric profile.
Figure 5.6 shows how the system responds to an increasing offset load in the longitudinal direction. For these tests $W$ was not held constant, instead weights were added at one end of the load platform. The model responds by substantially increasing the pressure at that end, with the longitudinal distribution being approximately linear. The determination of the extent to which the air bearing response moment is due to the air pressure distribution requires measurement of transverse pressure profiles at stations other than the model center. In the absence of this data, a simplified approach was used. If one assumes a uniform lateral and linear longitudinal pressure distribution accounts entirely for an applied $W$ and moment $L_L$, one can calculate this distribution. With $x$ being the distance from one end of the sample, $B_R$ being the sample width, and denoting this pressure as $p_{lin}$, its distribution is:

$$p_{lin}(x) = \left\{ \frac{W}{L_R B_R} - \frac{6L_L}{B_R L_R^2} \right\} + x \left[ \frac{12L_L}{B_R L_R^2} \right]$$ \hspace{1cm} (5.1)

Figure 5.7 gives the profiles for the test results shown in Figure 5.6 normalized by the distribution of Equation 5.1. Except for the values at the left hand end near $x = 0$, the pressure are closely correlated to this distribution. These normalized profiles suggest that the flat surface geometry test sample is capable of supplying virtually the entire longitudinal restoring moments, even when subjected to c.g. shifts as high as a 16.7%.

### 5.3.3 Choked Flow Condition

In order to achieve a true comparison to the mathematical model, the mass flow $m_f$ through the nozzle must be known. This is in part due to the model itself, which has been developed with the understanding that the mass flow through the nozzles is an important control characteristic in optimizing the effectiveness of the overall system. Measurement of this critical parameter has not yet been achieved for the flat surface geometry model, however recent research efforts focused on this are promising.
Figure 5.6  A series of longitudinal pressure profiles obtained during testing with offset loads in the lateral direction. Test conditions; $B_R = 127$ mm (5.00 in), $N_T = 130$, $p_T = 515$ kPa (60 psig).
Figure 5.7  Longitudinal pressure profiles of Figure 5.6 normalized by the linear distribution of equation 5.1
Without knowing the mass flow through the nozzles, some type of control mechanism is required in order to determine the true effect that the changing test variables such as load \(W\), tissue ply \(N_T\), etc., have on the flat surface geometry system. It was suggested that a necessary requirement for comparison of the test results is having a gasdynamic choking condition through the rail nozzles.

To determine whether or not the test parameters were sufficient enough to establish a choked flow condition, a series of tests were conducted. These tests were all conducted at track pressure, \(p_T = 515\text{kPa} \ (60 \text{ psig})\), a pressure that was assumed to be well above the level required to ensure gasdynamic choking. Figure 5.8 shows the effect of changing the applied load \(W\) has on the pressure profiles for the choked flow nozzle condition. The absence of a seal region is explained by the fact this series of tests were conducted using a tissue width \(B_r\) of 152mm (6.0in) as opposed to 127mm (5.0 in) for some of the previous tests. Figure 5.9 is a re-plot of the data from Figure 5.8 in which the ideal air bearing pressure \(p_{ci} = W / B_rL_r\) has been used to non-dimensionalize the results. The results show that the measured pressures are close to the ideal value, with a slight tendency to decrease as \(W\) increases. Thus we may infer that, as for the SailRail runner, this configuration supports almost the entire load by air pressure. Figure 5.10 shows the effect of changing the track source pressure \(p_T\) from 10 psi to 80 psi. The absence of any significant changes in the pressure profiles with increasing source pressure further verifies that the choked flow criterion has been sufficiently met. The uniformity of the longitudinal profiles suggests that the system is two-dimensional. This result confirms the usefulness of the rail-runner model developed in Appendix A as an analysis tool for experimental results.
Figure 5.8  Lateral and longitudinal pressure profiles for changing load $W$. Test conditions; $H = 12.7$ mm (0.50 in), $B_R = 152$ mm (6.0 in), $t_c = 1.01$ mm (0.040 in), $p_T = 413$ kPa (60 psi).
Figure 5.9  The results shown in Figure 5.8 non-dimensionalized by the ideal air bearing pressure $p_{CI}$. 
Figure 5.10  Lateral and longitudinal pressure profiles for changing track pressure $p_T$.
Test conditions; $H = 12.7$ mm (0.50 in), $B_R = 127$ mm (5.0 in), $t_C = 1.01$
mm (0.040 in), $W = 119.5$ kg (440 lb).
5.3.4 Effect of Tissue Thickness

The use of tissue as the compliant air bearing material has been controversial since the inception of the SailRail concept. Figure 5.11 gives an illustration of the role played by the tissue in generating bearing action and in providing tolerance to irregularities. The pressure distributions for three different cases are shown: (i), the test configuration; (ii), with the tissue removed; (iii), with both tissue and cover removed. Thus case (iii) is a rigid-surface air bearing. The theoretical pressure distribution for the rigid surface air bearing is also shown. Considering first the lateral distributions, the pressure profiles for the cases (ii) and (iii) are similar, displaying a region of rapid pressure decay much nearer the model centreline than for case (i). Consequently, for case (i) the weight $W$ supported by air pressure, which is proportional to the area under the pressure profiles, is much greater than for the other two cases. Note especially that this increase is obtained with a layer of tissue, which has an unloaded thickness $H$ of only 6.4 mm. (0.25 in.). Considering now the longitudinal profiles, removal of the tissue causes the pressure at the ends to increase substantially, with the increase at the right-hand end being particularly marked. This behaviour was eventually traced to a 0.13 mm. (0.005 in.) warp in the model’s lower plate occurring, that has since been removed.

Further insight as to the effect that the amount of tissue has in developing an effective air bearing was needed. Shown in Figure 5.12 is the test results obtained by varying the number of sheets of tissue in the sample $N_T$. The results clearly show that as $N_T$ is increased, the pressure distribution supports a large fraction of the applied load, however the increase is not overly substantial. These results further confirm the hypothesis, as outlined in Section 2.5 for the simplified runner case, that only a thin layer of tissue is required to obtain an effective air bearing. Figure 5.13 shows the effect of varying $N_T$ on the model’s ability to support the applied load with an air bearing condition. Under the same test conditions the metal-to-metal and cover-only cases resulted in a load recovery of only 48.6% and 54.5% respectively.
Figure 5.11 Lateral and longitudinal profiles for $N_T = 10$ compared to the rigid, theoretical rigid and cover-only cases. Test conditions; $B_R = 127$ mm (5.0 in), $W = 6.63$ kN (306.8 lb), $p_T = 10$ psig (170 kPa).
Figure 5.12 Lateral and longitudinal pressure distributions for four different values of \(N_T\). Test conditions; \(B_R = 127 \text{ mm (5.0 in)}, W = 6.63 \text{ kN (306.8 lb)}, p_r = 10 \text{ psig (170 kPa)}.\)
Figure 5.13 Load recovery of the data from Figure 5.14.
5.3.5 Air Cushion Comparison

In order to provide more validation on the accuracy and precision the flat surface geometry test apparatus, the decision was made to conduct experiments with known results. An air cushion simulation was the logical choice given the extensive amount of work that had been done previously at UTIAS on the air cushion subject. A specialized aluminum plate was designed and developed to effectively simulate a basic air cushion design. This aluminum plate was 6.0 in (152 mm) by 18.0 in (457 mm) and was attached to the bottom plate of the load plate. A specially machine slot, housing a fitted O-ring, extends around the inner perimeter (0.5 in from the edge) of the bottom face of the aluminum plate. The O-ring acts as a seal creating an air cushion pocket of 5.0 in (127 mm) by 17 in (432 mm).

Under basic air cushion principles, the pressure within the air pocket should be completely uniform; as opposed to the pressure distribution typically seen with the SailRail system. Figure 6.16 shows the results a test conducted using the flat surface geometry test apparatus with the aluminum air cushion plate acting as the effective lift device. As predicted, the pressure within the air pocket is completely uniform. This was another positive step in establishing the validity of the test apparatus. The fraction of the load supported by air pressure by the air cushion plate was 83.2% for this test, compared to values upward of 98.9% using the tissue and HMWPE combination (under high values of $N_T$). This result remains unexplained, since one would expect the load recovery of the air cushion plate to be close to 100%. A possible explanation is that the air cushion plate would actually require a much higher airflow to achieve lift-off than the device than the device can supply. All subsequent tests, using a varying load $W$, yielded comparable results. This result further emphasized the value of a compliant material, such as the cellulose tissue, for the formation of a highly effective air bearing.
Figure 5.14  Longitudinal and lateral pressure distribution of the aluminum air cushion plate. Test conditions; $B_R = 127$ mm (5.0 in), $W = 119.5$ kg (440 lb), $p_T = 413$ kPa (60 psi).
5.3.6 Urethane Test Results

It has been noted by several involved with the SailRail system that it is highly desirable to replace the cellulose tissue as the primary air bearing compliant material with an alternative material that would reduce cost, increase durability, and simplify construction. A cast elastomer block is believed to be the most promising choice. Hence as part of this test program it was decided that a number of suitable elastomers would be tested in the flat surface geometry apparatus to determine their effectiveness at creating a comparable air bearing to that of the tissue.

A castable urethane elastomer named Conathane® TU-601 was obtained from a company named Conap®. A mould was created in the dimensions of the original tissue sample, and a 0.5 in (12.7 mm) thick urethane sample was created from this material for test purposes. Some of the quoted properties of the cured product are: tensile strength 850 psi, ultimate elongation 600%, specific gravity 1.40, and Shore hardness 40A. However, comparison figures for the cellulose tissue were not available during the research period. Figure 5.15 show the initial test results of a conditioning experiment that clearly show that no conditioning effects exist within the urethane sample after being under load $W$ for a 24 hour period. One oddity of the test results was the erratic longitudinal pressure profile. This non-uniformity caused difficulties within this portion of the test program. The source of the variation has yet to be determined, but it is believed that flaws in the actual mixing and casting procedures of the urethane product may have produced a non-linear test sample.

Figure 5.16 shows a comparison between pressure profile of two tissue samples with different values of $N_f$ and the cast urethane sample.
Figure 5.15  Results showing the conditioning effects on the centerline pressure distribution for the flat surface model using the Conathane® CU-601 sample. The upper and lower graphs represent the lateral and longitudinal pressure profiles respectively. Test conditions; $H = 12.7$ mm (0.50 in), $B_R = 127$ mm (5.0 in), $t_C = 1.01$ mm (0.040in), $p_R = 413$ kPa (60 psi), $W = 79.10$ kg (174.39 lb).
Figure 5.16  Longitudinal and lateral pressure distribution of three test sample. Results compare the urethane sample to two different tissue samples. Test conditions; $B_R=127 \text{ mm} \ (5.0 \text{ in}), W = 119.5 \text{ kg} \ (440 \text{ lb}), \rho_T = 413 \text{ kPa} \ (60 \text{ psi})$. 
The results show that the urethane has a very high peak in center of the lateral profile as compared to the two tissue sample, but the urethane does not seem to possess the ability to extend it’s effective sealing region to the outer edge of the apparatus as does the tissue samples. The fraction of the load supported by air pressure for the urethane sample was 94.9% compared to 92.1% for the second tissue sample with $N_T = 130$. At this time, to conclude that the urethane produces a better air bearing would be erroneous. The ability of the tissue sample to extend the pressure profile to the outer edge of the air bearing surface may have a significant effect in reducing the effective coefficient of sliding friction due to air bearing action $\mu_{eff}$. The ability of the urethane sample to produce high peaks in the center of the apparatus has the benefit of generating large amounts of vertical lift within the center of the air bearing surface. The true trade off between these two benefits requires exact determination of the fraction of the load carried by air pressure, either by more detailed pressure profiles or by establishing comparison values of $\mu_{eff}$ for each of the two types of test material. This measure will give an accurate indication of the more effective material. A recommendation for further testing, based on improving the ability to provide a proper experimental validation of the math model, would be to determining all of the physical properties of the urethane, especially Poisson’s Ratio $\lambda$.

As described in Section 5.3.2, the discovery of the ability of the flat surface geometry model to generate restoring moments was a significant break through. One essential requirement for any replacement of the cellulose tissue, will be that it possess this restoring ability. Figure 5.17 shows the test results for three different test samples subjected to an offset longitudinal load. As was seen from the earlier test results, the two tissue samples clearly show the ability to generate restoring moments to compensate for the offset load. The urethane test sample also shows an upward trend, which might suggest that it also possesses the ability to generate restoring moments. The difficulty around jumping to this conclusion is the fact that, as was previously alluded to, the longitudinal pressure profile is highly erratic.
Figure 5.17 Longitudinal pressure distributions of three test samples subjected to a longitudinal load offset. Results compare the urethane sample to two different tissue samples. Test conditions; $B_R = 127$ mm (5.0 in), $p_T = 413$ kPa (60 psi).
This complicates the use of the integration program recovery.c, listed in Appendix C, as an analysis tool. Therefore, the quantitative measure of how much of the offset load was recovered by a restoring moment cannot be obtained. Until this source of variation can be eliminated, no conclusions can be drawn.

5.4 Experimental Validation of the Mathematical Model

As detailed in Section 5.3.3, the requirement of a choked flow condition through the nozzles is necessary for comparison of test results from the model. Even though the choked flow condition was experimentally confirmed, one important measurable, mainly the mass flow through the nozzles $m_f$, remained undetermined throughout the research. An entire research effort now exists at UTIAS with developing an experimental method to accurately determine the mass flow. It is essential that the mass flow value be determined before any meaningful comparison with the math model is attempted.

The test results shown throughout Section 5.3 all lend some portion to the final validation of the math model. By identifying the effect of tissue ply, model stability, urethane as a test sample, etc. it allows simulations to be run under comparable conditions. Unfortunately the recently developed simulation programs are quite simplistic in terms of their flexibility and are somewhat constrained (Walsh, 1998). They do not possess the ability to accurately simulate the test conditions being used with the flat surface geometry test apparatus. Without this ability, no quantitative comparisons could be completed under the current scope of work of this test program. The exploratory test results have laid the groundwork for future work to be completed, including the full validation of the mathematical model.
Chapter 6

Conclusions

6.1 General Discussion of Results

Although the work completed to date on the flat surface model is preliminary in nature, four important points can nevertheless be made. First, the rapid conditioning of this model strongly supports a suggestion made as a result of tests on the simplified runner: given that tissue has creep properties, it is very important to ensure that the geometry of the runner matches that of the rail, so that the amount of time required for the runner to conform is minimized. Furthermore, it appears that existing runner designs are poorly matched in this respect. The second point is that only a very thin layer of tissue is required to establish air bearing action; this supports the idea that the bulk of the runner in existing designs may simply allowing tolerance to irregularities in rail geometry and alignment. The third point is related to this: the system has a strong capacity to respond to asymmetric loading by redistribution of pressure as opposed to by increased rail-runner contact. Finally, exploratory experiments with an alternative compliant material, a cast urethane, suggest that tissue may be providing a pressure distribution which is closer to the ideal of uniform support pressure.

The two latter results are only preliminary in nature, but they underscore the need for continued development of mathematical models which can examine the effect of such material properties as the nonlinear stiffness characteristics of the tissue and the role of Poisson's ratio.
6.2 Recommendations for Future Work

Based on the results of this test program I propose the following recommendations for future work on the simplified flat surface model and related projects:

i. Experiments be conducted to establish an accurate measure of the mass flow through the nozzles \( m_f \). This is a critical parameter of the system that needs to be determined before any comparison of the simplified mathematical model can be attempted.

ii. Pressure distributions obtained by the flat surface geometry test apparatus need to be more detailed in order for more accurate integration procedures to be used. This would give a better indication of load recovery, restoring moments, etc. A suggestion for this is either the adaptation of the current apparatus to include more pressure measurements, or the construction of an entirely new device with increased measurement capacity.

iii. More in depth testing needs to be conducted on the use of elastomers with the SailRail system. The relevant physical properties for both the elastomers and the tissue should be determined by a physical-testing laboratory. Detailed comparisons studies can than be conducted; both experimentally and by using the recently developed simulation routines that are based on the mathematical model.

iv. Comparison of \( \mu_{\text{eff}} \) in the flat surface geometry configuration for a variety of test conditions and compliant materials. This would enable a true performance measure to be obtained that quantifies optimal conditions for the formation on the air bearing.
Chapter 7

References


Appendix A

Simplified Flat Surface Model

A.1 Concept, Geometry, and Notation

The system is assumed to be static, so viscoelastic effects such as runner conditioning are excluded. The rail and runner are assumed to be infinitely long with uniform properties along their lengths. This restricts the problem to one of two dimensions at most, and corresponds most closely to the continuous support configuration of the original system. The runner is regarded as a perfectly elastic free body supported above the rail by a thin film of air. Therefore, information on both the runner deformation and its vertical displacement relative to the rail are required.

Figure 2.10 gives the notation used in the analysis. With z now being the vertical direction, the rail is a flat surface in the horizontal or (x,y) plane. The runner consists of a rectangular block of tissue or other elastic material having unloaded thickness $H$ and width $2B$ mounted on top of a cover in the form of a thin flat plate of thickness $t_c$. With $h(y)$ being the gap distribution between rail and runner, and $\xi(y)$ the local thickness of the tissue when deformed by the load $W$ and the displacement of the cover due to air flow, the base supporting $W$ is at height $h_b$ above the rail. Thus

$$h_b = h(y) + t_c + \xi(y)$$  \hspace{1cm} (A.1)

If the tissue deflection under load is $\eta(y)$, then $\xi = H - \eta$, so that the geometric compatibility condition, equation A.1 can be written as

$$h_b + \eta(y) - h(y) = H + t_c$$  \hspace{1cm} (A.2)
The tissue component is simplified to a local spring effect, that can be nonlinear. The pressure between rail and runner in the cavity and seal regions is defined by the compressible Reynolds equation for inertialess flow in thin films which, for an infinitely long runner, is Equation 2.1 with the \( \partial / \partial x \) terms set to zero. As discussed below, this is augmented by the possibility of flow separation in the seal region due to fluid inertia.

Finally, consider the nozzle region; both the review of Sullivan (1997), and tests conducted at UTIAS show that radial nozzles are as effective as inclined nozzles, implying that the latter have no significant directivity effect. It follows that, except for their regulation of the flow into the cavity region, the geometry of the nozzles has no impact on the performance of the SailRail air bearing. Consequently, the flow in the nozzle region is modeled as an orifice flow through a slot located at \( y = 0 \). The slot width is chosen so that the mass flow rate per unit length of runner is the same as that delivered by the discretely placed nozzles.

**A.2 Fluid Mechanics of Cavity and Seal Regions**

The version of Equation 2.1 used below may be simply derived as follows. For an infinitely long runner the flow between runner and rail may be assumed two-dimensional; that is, the fluid velocity in the direction parallel to the rail axis is zero. If, in addition, the flow is assumed compressible but inertialess and the gap between rail and runner is assumed to be very small relative to the pressure footprint width, the equations of motion for a particle of fluid such as air reduce to a very simple form. With \( p \) being the local static pressure, and \( v(y, z) \) being the velocity component in the \( y \)-direction, they are

\[
\begin{align*}
\frac{\partial p}{\partial z} &= 0; \\
\frac{\partial p}{\partial y} &= \mu(T) \frac{\partial^2 v}{\partial z^2}
\end{align*}
\]  

(A.3)
Here $\mu$ is the viscosity which, for gases is a function of temperature $T$ alone. Following the usual gas bearing practice we assume that, because the gap is very small, the gas temperature in the cavity and seal regions is constant and equal to the temperature of the surrounding bearing hardware. The latter is taken to be the local atmospheric temperature $T_a$. Then equation A.3 can be immediately integrated; subject to the usual no-slip conditions at $z = 0$ and $z = h$ it gives

$$v(y, z) = -\frac{1}{2\mu} \frac{\partial p}{\partial y} \left( z(h - z) \right)$$  \hspace{1cm} (A.4)

Conservation of mass is used in its integral form to impose a relationship between the local gap $h(z)$ and the velocity profile given by equation A.4:

$$\int_{y=0}^{(y)} \rho v(y, z) dz = m_s$$  \hspace{1cm} (A.5)

where $m_s$ is the mass flow per unit length of runner; $m_s$ is independent of $y$. With $T$ being the absolute ideal gas temperature, we assume that the bearing air obeys the ideal gas law $p = RT$ where $R$ is the gas constant. Using this law, and substituting of Equation A.2 into A.3, followed by integration gives, with $K_M = 24\mu RT$,

$$p \frac{dp}{dy} = -\frac{K_M m_s}{2h^3}$$  \hspace{1cm} (A.6)

which is the one-dimensional equivalent of Equation 2.1. Equation A.6 can be further integrated to give the pressure distribution in terms of the gap distribution; if $P_a$ is the atmospheric pressure, then
According to one-dimensional isentropic flow theory, the unchoked mass flow $m_f$ through a nozzle having an upstream or reservoir pressure $p_r$ and temperature $T_r$, a throat of cross-sectional area $A_T$, and a downstream or exit pressure $p_C$ is given by

$$p(y) = \sqrt{p^2 + K_m m_1 \int_{s_0}^{s} \frac{ds}{h(s)^{1/3}}} \quad (A.7)$$

### A.3 Nozzle Region Model

According to one-dimensional isentropic flow theory, the unchoked mass flow $m_f$ through a nozzle having an upstream or reservoir pressure $p_r$ and temperature $T_r$, a throat of cross-sectional area $A_T$, and a downstream or exit pressure $p_C$ is given by

$$m_f = \sqrt[3]{\frac{2\gamma}{(\gamma - 1)\sqrt{RT_r}}} \left( \frac{p_C A_T}{p_r} \right)^{1/3} \sqrt{1 - \left( \frac{p_C}{p_r} \right)^{\gamma-1}} \quad (A.8)$$

In this equation $\gamma$ is the ratio of specific heat at constant pressure to that at constant volume; $\gamma = 1.4$ for air. If $p_C$ falls below a critical value $p_{cr}$, given by

$$\frac{p_{cr}}{p_r} = \left( \frac{2}{\gamma + 1} \right)^{\gamma \gamma - 1} \quad (A.9)$$

then choking occurs and $m_f$ becomes independent of $p_C$, and is given by

$$m_f = \sqrt{\frac{2\gamma}{\gamma + 1}} \left( \frac{2}{\gamma + 1} \right)^{\gamma \gamma - 1} \left( \frac{p_T A_T}{\sqrt{RT_r}} \right) \quad (A.10)$$

In the following analysis we assume that $A_T = \pi d_N^2/4$, $T_r = T_a$, so that, with $N_N$ being the number of nozzles per unit length of runner, and with $C_D$ being a discharge coefficient accounting for vena-contracta effects and fluid losses, the actual mass flow $m_s$ per unit length of runner is given by
\[ m_i = C_D N_N m_f \]  

(A.11)

From dimensional analysis considerations, referring to the nozzle geometry depicted in Figure 2.1, we expect

\[ C_D = C_D \{ \theta, (h_C/d_N), (l_n/d_N), (p_C/p_T), Re_D \} \]  

(A.12)

In this equation, \( h_C \) is the value of \( h \) over the nozzles and \( Re_D \) is a suitably defined Reynolds number. In terms of the notation of Figure 2.10, \( h_C \) is equivalent to \( h(y = 0) \). For accurate calculations, \( C_D \) must be determined experimentally; however, we are not aware of the availability of any data directly relevant to the configuration of Figure 2.11, even for the radial nozzle case (6 = 90 deg.). However, past experience suggests that, for turbulent flow \( C_D \) should be very weakly dependent of \( Re_D \) and only moderately dependent on \( p_C/p_T \); it should be strongly dependent on the other factors. In the absence of suitable data, based on results for other geometry's in the literature, we provisionally assume \( C_D = 0.78 \).

**A.4 Runner Model**

The cover is modelled as a thin flat plate which undergoes a deflection equal to that of the tissue \( \eta \) and caused by a distributed load \( w_S \) per unit area; \( w_S \) is the difference between the fluid pressure \( p \) acting upwards and a normal stress \( \sigma \) imposed by the tissue acting downwards. If the cover curvature and slope remain small when deflected and if, consistent with the assumptions made above, \( \eta \) is independent of \( x \), then the equation governing the cover shape becomes equivalent to that for a beam, namely
\[ D \frac{d^4 \eta}{dy^4} = w_i(y) = \{ p(y) - P_s \} - \sigma(y) \quad \text{(A.13)} \]

In this equation \( D \) is the flexural rigidity or bending stiffness of the cover; with \( E \) being Young’s modulus and \( \lambda \) Poisson’s ratio, \( D = E(t_c)^3/(12 [1 - \lambda^2]) \).

The plate loading \( w_s \) is an unknown function of \( y \) which is determined as part of the solution by means of Equation 2.1 for \( p \), and by a suitable elastic model of the tissue layer giving \( \sigma \). In the general \( \sigma \) case is determined by the equations of two-dimensional elasticity but, in this first analysis, as noted above, we use a local spring model in which \( \sigma \) depends only on the tissue deflection \( \eta \) through a specified functional form:

\[ \sigma(y) = F_i(\eta(y)) \quad \text{(A.14a)} \]

The form of \( F_i \) used in the first calculations is a linear spring:

\[ \sigma = K_i \eta \quad \text{(A.14b)} \]

The formulation of the runner model is completed by imposing appropriate boundary conditions; since it is fourth order, four are required. These are usually taken to be suitable combinations of displacements \( \eta \), slopes \( d\eta/dy \), applied moments \( L \) and shear forces \( V \) at specified points. According to beam theory, for small slopes

\[ L = -D \frac{d^2 \eta}{dy^2}, V = -\frac{dL}{dy} = -D \frac{d^3 \eta}{dy^3} \quad \text{(A.15)} \]
In the present problem, symmetry about the model center \( y = 0 \) requires zero slope and shear at this point, or

\[
\frac{d\eta}{dy}\bigg|_{y=0} = 0; \quad \frac{d^3\eta}{dy^3}\bigg|_{y=0} = 0
\]  

(A.16)

The remaining two boundary conditions are to be applied at the runner edge, corresponding here to \( y = B \). If the edges are not in any way externally constrained, the appropriate conditions are zero moment and shear, leading to

\[
\frac{d^2\eta}{dy^2}\bigg|_{y=B} = S_E; \quad \frac{d^3\eta}{dy^3}\bigg|_{y=B} = 0
\]  

(A.17a)

However, simulation of this runner configuration requires imposition of conditions reflecting the facts that assembly of the runner involves wrapping the cover around the log, and that the runner is compressed when loaded. None of the standard beam edge conditions - free, pinned or clamped - apply. Furthermore, the results show that application of bending moments to an unwrapped cover edge can significantly alter bearing performance. Thus determination of appropriate edge conditions is an important part of the present work.

The choice is constrained by the fact that, imposing a displacement is physically equivalent to imposing a shear, and a slope is equivalent to a moment. Hence one cannot impose both \( \eta \) and \( \frac{d^3\eta}{dy^3} \), or both \( \frac{d\eta}{dy} \) and \( \frac{d^2\eta}{dy^2} \). Within these constraints, the cover wrapping process may be viewed as imposing a moment \( L_E \) and a shear force \( V_E \), so that
\[ \frac{d^2\eta}{dy^2} \Big|_{y=\delta} = \frac{L_E}{D} = \frac{d^3\eta}{dy^3} \Big|_{y=\delta} = \frac{V_E}{D} = \overline{V_E} \quad (A.17b) \]

However, for reasons discussed below, we impose a slope \( S_E \) in place of \( L_E \):

\[ \frac{dh}{dy} \Big|_{y=\delta} = S_E; \quad \frac{d^3h}{dy^3} \Big|_{y=\delta} = \frac{V_E}{D} = \overline{V_E} \quad (A.17c) \]

with a view to determining \( L_E \) as part of an iterative process.

A.5 Bearing Load

The final condition we require is provided by an equation relating the above dynamics to the load \( W \) per unit length being supported by the bearing; since the runner is modelled as a free body supported only by the air pressure in the gap, we have

\[ W = 2 \int_{\delta}^{\theta} p(y)dy \quad (A.18) \]

A.6 Performance Criterion of Simulations

In the experiments, the performance index is the effective coefficient of sliding friction \( \mu_{\text{eff}} \), a quantity that cannot be determined by the present model. Order-of-magnitude calculations such as that given below, together with analysis of monorail results suggest that \( \mu_{\text{eff}} \) is almost entirely due to sliding friction arising from runner-rail contact. One would expect, in turn, that this contact is confined mostly to the seal region, and that it is associated with surface irregularities in the rail and runner. Nevertheless, as calculations show, one can estimate for the seal region a minimum gap \( h_S \) corresponding to smooth rail and runner; typically \( h_S \) is thought to be about 0.017 mm.
We therefore provisionally assume that $\mu_{\text{eff}}$ is correlated with $h_S$ and that, for a given mass flow rate $m_f$ and load $W$, the configuration with the largest value of $h_S$ will perform best.

For a typical load of 500 kg (1100 lb.) per runner operating at $\mu_{\text{eff}} = 0.5$ percent the drag force is about 245 N (55 lb.). An estimate of the fraction of this due to shearing of the air film can be obtained by assuming a pressure footprint size, a typical runner velocity $U$ and an average runner-rail gap $h_m$. From Ref. 2, a typical $h_m$ is about 0.15 mm (0.006 in.); since for air at 20 deg. C., the dynamic viscosity $\mu_s$ is about 1.8 N.s/m$^2$, a typical shearing stress $\tau_{\text{air}}$ may be computed as $\tau_{\text{air}} = \mu_s U/h_m = 12$ N/m$^2$ (0.00174 psi). For a 1.0 m (3.28 ft.) long runner with a pressure footprint width of 100 mm (3.94 in.), this corresponds to a shear force of 1.2 N (0.270 lb.), or about half of one percent of the drag force.
Appendix B

Computer Code of “Planar.bas”

B.1 Introduction

“Planar.bas” was written out of the necessity to obtain large amounts of data from the planar geometry apparatus as quickly as possible. The program is written to operate with the S500 Digital Acquisition System currently set up in the ACV laboratory at UTIAS. The program has several features to ensure that the data being recorded is accurate. The main feature to accomplish this is a time weighted average recording system. The pressure within the Scani-valve must first be allowed to stabilize before any readings are taken. This is because the Tygon pressure tubes are relatively small, and the Scani-valve must rotate through the taps causing a pressure change-over. The incoming data samples are then recorded for a set period of time after which a weighted average is determined and saved as the final result for that particular pressure tap. This program automatically saves all the final data into a user specified file in a format compatible with the program “recovery.c” listed in Appendix D. Further instructions on the use and operation of the digital acquisition system and the “planar.bas” program can be provided upon request.

B.2 Code Listing

0 REM Planar.bas – Digital Acquisition Program for use with S500
10 REM Last edit: January 20, 1998
20 REM Author: Bryan Townsend
98 CLS
99 INPUT “Enter output file prefix:”, PREFIX$
100 REM Constants and initial values
110 NUMTAPs = 28:
130 CALL INIT
140 TOTALPR%=0: WAITLOOP=8; TOTLOOP=15; AVGPR=0:
150 TEMPPR%=0; PR#=0; SUFFIX$ = ”.DAT"
160 CALL IONAME’(”CHAN4”, 1,7,12,10)
170 CALIB# = 0.0033764555119#: REM:mV/psi
180 PSI2KPA# = 6.89476095#: REM: Conversion Multiplier
190 GAIN=10: REM: Signal Gain
200 INPUT "Enter test sequence #", POSIT$
210 POSITXN=VAL(POSIT$)
220 FILE$ = PREFIX$ + POSITSS$ + SUFFIX$
250 PRITN "Home Scani-valve and press Enter when ready"
260 INPUT DUMMY$
270 IF DUMMY$<> "correct GOTO 290"
280 CLOSE#1 : GOTO 100
290 OPEN FILE$ FOR OUTPUT AS 1
295 PRINT #1, DATES;" "TIMES;"
300 KEY (1) ON:CLS
400 GOTO 430
430 BEEP:BEEP
440 TOTALPR#*=0!: NEXT TAP
487 FOR N=1 to 700: NEXT N:BEEP:BEEP
490 CLOSE #1: GOTO 100
600 TOTAL PR#*=TOTALPR#+PR#
700 CLOSE #1: END
720 RETURN
Appendix C

Computer Code of “Recovery.c”

C.1 Introduction

“Recovery.c” was written by the author to aid in the analysis of the planar geometry model. Since numerous tests were conducted it became necessary to have a device capable of taking the raw data (ie. pressure distribution) produced and converting it meaningful data. The code was adapted from a similar program written for the monorail apparatus.

The program integrates the pressure distribution under a flat sample of compliant cellulose tissue in the simplified flat surface apparatus, to determine the total vertical force produced, the restoring moment produced and the relative errors. To accomplish this a double integral is solved numerically by averaging the pressure at each incremental area determined by the spacing of the pressure taps. The vertical force and moment are calculated by adding up the incremental vertical forces and moments that arise from each incremental area. The error in the pressure measurements was largely controlled by the accuracy of the A/D conversion. Using a pressure transducer that was well calibrated, the digitalization accuracy becomes $20/2^{12} = 0.005[V]$ with an output voltage range of $\pm 10[V]$. Including calibration and instrument gain, the resulting error is $\pm 1[kPa]$. Due to accurate machining, the location of each of the pressure taps was extremely accurate to $0.0001[m]$. The resulting errors were calculated at each step in the integration process. Since the vertical force and moment calculations are simply the sum of the terms, where each term is the results of a product, the resulting error can be easily determined.

By comparing this to the applied force and moment, it becomes possible to calculate the percent recovery for each configuration and test. Since the system is static,
this is the only real measure of performance available. The usage of the program is as follows: "c:\recovery.exe sample.dat", where the "sample.dat" file is obtained from the digital acquisition system, or in more accurately from the program "planar.bas" listed in Appendix C.

C.2 Code Listing

/* recovery.c - Integrates pressure distribution for vert. force and moment.
   for analysis of simplified flat surface model.
   Author: Bryan Townsend
   Date: February 22, 1997
   
   NOTE: This program is an adaptation of integral.c written by J. Lassaline
   in June, 1995. The structure and formulation are similar but changes have
   been made to adapt the program for use with the simplified flat model.
   
   Integrates the pressure distribution to determine the total vertical force
   and the restoring moment. Errors are calculated based on pre-defined errors.
   Errors calculated based upon:
   P=A*B; dP/P=dA/A+dB/B; P=A+B: dP=dA+dB;
   
   Formulas used:
   vert. force = SUM (d vert. force)
   (d vert. force) = dp*pressure*area
   moment = SUM (d moment)
   (d moment) = (d vert. force)*(moment arm)
   */
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

/* ARRAYSIZEx should be 1+(num of taps) */
#define ARRAYSIZE ??

int main(int argc, char *argv[])
{
    const float ts=0.0025 /* tap spacing */
    const char KEYWORD[]="Static:"
    const unsigned NUMTAP=ARRAYSIZE-1:
    
    float ddp=0.01;
    float dds=0.0001; /* default errors */
    float ddl=0.0001;
    
    FILE * infile;
    double vforce=0,moment=0,dv,dmom,dp,dl,vferr=0,
    momerr=0,dvfer,dmomerr;
float plist1[ARRAYSIZE], plist2[ARRAYSIZE], 
dummy, adjust, pstatstart, pstatend, momcenter, 
momarm;
char *token = malloc(80*sizeof(char));
int i, firstpass = 1;

/* Usage check */
if ( (!((argc==2) || (argc==5))) )
  {
    printf("Usage: %s datafile [error error]\n \n argv[0]);
    printf(" [] - optional \n");
    printf(" error - error [kPa] in pressure \n");
    printf(" error - error [m] in transverse measures \n");
    printf(" error - error [m] in longitudinal measures \n");
    return;
  }
if (argc==5)
  {
    ddp = atof(argv[2]);
    dds = atof(argv[3]); /* replaces default errors */
    ddl = atof(argv[4]);
  }
/* Open File */
if ( ((ifstream=fopen(argv[1], \"r\")) == NULL) )
  {
    printf("File %s not found.\n", argv[1]);
    return;
  }
/* Get moment center */
printf("Moment center (distance from free edge in [mm]): \");
scanf("\%f", &momcenter);
do
  do
    {
      fscanf(infile, \"%s\", token); /* read file ...*/
      if (feof(infile)) /* end of file */
        {
          printf("Total vertical force: %f +/- %f [kN] \n\n\n vforce,vferr); 
          printf("Total moment: %f +/- %f [kNm] (tve=ccw)\n\n\n\nmoment,momerr);
          return;
        }
    } while (strcmp(token, KEYWORD) != 0); /* ... ignoring everything until */
for (i=0;i<NUMTAPS;i++)
  fscanf(infile, \"%d %f\", &dummy, &plist2[i]); /* pressure values */
  for (i=0;i<NUMTAPS;i++)
  fscanf(infile, \"%s %f\", token, &plist2[i]); /* end static pressure */
if (!firstpass)
  { 
    dp = (plist2[i]+plist2[i-1]+plist1[i]+plist1[i-1])/4; /* Avg. press. */
dvd=dp*ds*dl; /* Incr. vert. force */
dmom=dvf*momarm; /* Inc. moment */
vforce=vforce+dvf; /* Add to total */
moment=moment+dmom;

if(dp!=0.0) /* Avg. press. can be 0 */
{

dvfrerr=ddp*dl*ds+dd*dp*dl+ddl*dp*ds /* Calc. errors */
dmomerr=dvfr*momarm+ddl*dvf;
}

vfrerr=vfrerr+dvfrerr; /* Add to total */
momerr=momerr+dmomerr;
}

firstpass=0;

for (i=0;i<ARRAYSIZE;i++) /* Move current tap press list */
{plist1[i]=plist2[i]; /* to Prev. tap press list */
}
while (feof(infile)!=0);
close(infile);
free(token);
return;
}