MODELING OF CONFINED CONCRETE

by

Esneyder Montoya

A Thesis submitted in conformity with the requirements
for the Degree of Master of Applied Science
Graduate Department of Civil Engineering
University of Toronto

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Modeling of Confined Concrete


Graduate Department of Civil Engineering
University of Toronto

Abstract

Constitutive models for confined concrete were investigated using the nonlinear finite element program SPARCS, in which the influence of lateral pressure in tied reinforced concrete (RC) columns subjected to monotonic axial compression is formulated in terms of three-dimensional stress states. A combination of ascending and descending branches of axial stress-axial strain relationships for confined concrete proposed by various authors were evaluated. The influence of variable Poisson's ratio, compression softening of concrete, and concrete cover spalling in the response of RC columns were also analyzed. Data from columns tested by various researchers was used to establish the validity of the procedures implemented in SPARCS. In general, good agreement with the experimental results was obtained.
Acknowledgements

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## Contents

<table>
<thead>
<tr>
<th>Abstract</th>
<th>ii</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgements</td>
<td>iii</td>
</tr>
<tr>
<td>List of figures</td>
<td>xlii</td>
</tr>
<tr>
<td>List of tables</td>
<td>xxi</td>
</tr>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1. Overview of Concrete Response</td>
<td>1</td>
</tr>
<tr>
<td>1.2. Objectives</td>
<td>3</td>
</tr>
<tr>
<td>1.3. Summary</td>
<td>4</td>
</tr>
<tr>
<td>2. Literature Review</td>
<td>7</td>
</tr>
<tr>
<td>2.1. Plasticity and Fracture Energy based models</td>
<td>8</td>
</tr>
<tr>
<td>2.1.1. Liu and Foster (1998)</td>
<td>8</td>
</tr>
<tr>
<td>2.1.2. Xie; MacGregor; and Elwi (1996)</td>
<td>10</td>
</tr>
<tr>
<td>2.1.4. Chen and Mau (1989)</td>
<td>13</td>
</tr>
<tr>
<td>2.2. Linear and Nonlinear Elastic Models</td>
<td>15</td>
</tr>
<tr>
<td>2.2.1. Mau; Elwi; and Zhou (1998)</td>
<td>15</td>
</tr>
<tr>
<td>2.2.2. Barzegar and Maddipudi (1997)</td>
<td>16</td>
</tr>
</tbody>
</table>
2.2.3. Bortolotti (1994)  17
2.2.4. Selby (1990)  17
2.2.5. Vecchio (1992)  18
2.2.6. Selby and Vecchio (1993)  21
2.2.7. Abdel-Halim and Abu-Lebdeh (1989)  22

3. SPARCS  24
3.1. Program Description  24
3.2. Finite Element Library  26
   3.2.1. Hexahedron  26
   3.2.2. Pentahedra (Wedge)  27
   3.2.3. Truss Bar  28
3.3. Constitutive Models for Concrete  29
   3.3.1. Base Stress-Strain Curves for Concrete in Compression,
          Pre-Peak Behaviour  29
      3.3.1.1. Hognestad parabola  29
      3.3.1.2. Thorenfeld et al.  30
      3.3.1.3. Hoshikuma et al.  31
   3.3.2. Base Stress-Strain Curves for Concrete in Compression,
          Post-Peak Behaviour  33
      3.3.2.1. Modified Kent and Park (Scott 1982)  34
      3.3.2.2. Popovics (1973)  35
      3.3.2.3. Hoshikuma et al. (1996)  36
<table>
<thead>
<tr>
<th>Section</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.3.</td>
<td>Base Stress-Strain Curve for Concrete in Tension</td>
</tr>
<tr>
<td>3.3.3.1.</td>
<td>Vecchio (1982)</td>
</tr>
<tr>
<td>3.3.3.2.</td>
<td>Collins-Mitchell (1987)</td>
</tr>
<tr>
<td>3.3.3.3.</td>
<td>Izumo, Maekawa et. al.</td>
</tr>
<tr>
<td>3.3.4.</td>
<td>Failure Criteria for Concrete</td>
</tr>
<tr>
<td>3.3.4.1.</td>
<td>Hsieh-Ting-Chen Criterion (1979)</td>
</tr>
<tr>
<td>3.3.4.2.</td>
<td>Cracking Criteria</td>
</tr>
<tr>
<td>3.3.4.2.1.</td>
<td>Mohr-Coulomb Criterion (stress formulation)</td>
</tr>
<tr>
<td>3.3.4.2.2.</td>
<td>Mohr-Coulomb Criterion (strain formulation)</td>
</tr>
<tr>
<td>3.3.4.2.3.</td>
<td>CEB-FIP Criterion</td>
</tr>
<tr>
<td>3.3.5.</td>
<td>Confined Strength $f_p$ and Strain at Peak Stress $\varepsilon_p$</td>
</tr>
<tr>
<td>3.3.5.1.</td>
<td>Selby (1993)</td>
</tr>
<tr>
<td>3.3.5.2.</td>
<td>Vecchio (1992)</td>
</tr>
<tr>
<td>3.3.6.</td>
<td>Compression Softening</td>
</tr>
<tr>
<td>3.3.6.1.</td>
<td>Vecchio and Collins (1982)</td>
</tr>
<tr>
<td>3.3.6.2.</td>
<td>Vecchio and Collins (1986)</td>
</tr>
<tr>
<td>3.3.6.3.</td>
<td>Vecchio 1992-A</td>
</tr>
<tr>
<td>3.3.6.4.</td>
<td>Vecchio 1992-B</td>
</tr>
<tr>
<td>3.3.7.</td>
<td>Variable Poisson's Ratio</td>
</tr>
<tr>
<td>3.4.</td>
<td>Constitutive Model for Steel</td>
</tr>
<tr>
<td>3.5.</td>
<td>SPARCS Structure</td>
</tr>
<tr>
<td>4.</td>
<td>Parametric Study and Poisson's Ratio</td>
</tr>
</tbody>
</table>
4.1. Introduction

4.2. Selection of Parametric Variables
   4.2.1. Pre-Peak Base Curves for Concrete
   4.2.2. Post-Peak Base Curves for Concrete
   4.2.3. Concrete Cracking
   4.2.4. Confinement Enhancement
   4.2.5. Variable Poisson's Ratio
   4.2.6. Compression Softening

   4.3.1. Column Geometry
   4.3.2. Longitudinal Bar and Tie Setup
   4.3.3. Test Instrumentation and Procedure
   4.3.4. Selected Sheikh and Uzumeri Columns

4.4. Finite Elements Models
   4.4.1. Geometry
   4.4.2. Material Types
   4.4.3. Parameter Combinations
   4.4.4. Analysis Procedure

4.5. Analysis Results
   4.5.1. Column 2A1-1
      4.5.1.1. Cross Section and Profile Results
      4.5.1.2. Load-Deformation Curves Combination 2
      4.5.1.3. Summary of Load-Deformation Curves
4.5.2. Column 4B3-19

4.5.3. Column 2C5-17
  4.5.3.1. Cross Section Results
  4.5.3.2. Summary of Load-Deformation Curves

4.5.4. Column 4D6-24

4.6. Effects of Model Combination on the Response of the
  Selected Columns

4.6.1. Peak Load

4.6.2. Strain at Peak Load

4.6.3. Post-Peak Behavior

4.6.4. Effect of Compression Softening

4.7. Comparison of Analytical Results with a Previous Version
  of the Program

4.8. Study of Poisson's Ratio

4.8.1. Procedure to Obtain the Experimental Poisson's Ratio

4.8.2. Experimental Variable Poisson's Ratio

5. Corroboration with Experimental Studies

5.1. Introduction

5.2. Lui, Foster, and Attard Tests (1998)
  5.2.1. Column Geometry
  5.2.2. Longitudinal and Lateral Steel Arrangements
  5.2.3. Test Instrumentation and Procedure
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2.4. Selected Liu et al. Columns</td>
<td>125</td>
</tr>
<tr>
<td>5.2.5. Finite Element Models</td>
<td>126</td>
</tr>
<tr>
<td>5.2.5.1. Geometry</td>
<td>126</td>
</tr>
<tr>
<td>5.2.5.2. Material Types</td>
<td>127</td>
</tr>
<tr>
<td>5.2.5.3. Material Models</td>
<td>128</td>
</tr>
<tr>
<td>5.2.6. Analytical and Experimental Results of Liu et al. Columns</td>
<td>129</td>
</tr>
<tr>
<td>5.2.6.1. Column 2C60-10S50-15</td>
<td>129</td>
</tr>
<tr>
<td>5.2.6.2. Column 2C60-10S100-15</td>
<td>130</td>
</tr>
<tr>
<td>5.2.6.3. Column 2C60-10S150-15</td>
<td>133</td>
</tr>
<tr>
<td>5.2.6.4. Column 2C80-10S50-15</td>
<td>133</td>
</tr>
<tr>
<td>5.2.6.5. Column 2C80-6S50-15</td>
<td>135</td>
</tr>
<tr>
<td>5.3.1. Column Geometry</td>
<td>139</td>
</tr>
<tr>
<td>5.3.2. Longitudinal and Lateral Steel Arrangements</td>
<td>141</td>
</tr>
<tr>
<td>5.3.3. Test Instrumentation and Procedure</td>
<td>141</td>
</tr>
<tr>
<td>5.3.4. Selected Mander et al. Specimens</td>
<td>142</td>
</tr>
<tr>
<td>5.3.5. Finite Element Models</td>
<td>143</td>
</tr>
<tr>
<td>5.3.5.1. Geometry</td>
<td>143</td>
</tr>
<tr>
<td>5.3.6. Analytical and Experimental Results of Mander et al. Specimens</td>
<td>144</td>
</tr>
<tr>
<td>5.3.6.1. Wall 11</td>
<td>144</td>
</tr>
<tr>
<td>5.3.6.2. Scott Column</td>
<td>146</td>
</tr>
<tr>
<td>5.4. Sheikh and Uzumeri Columns</td>
<td>148</td>
</tr>
<tr>
<td>5.4.1. Column 4C6-5</td>
<td>149</td>
</tr>
<tr>
<td>Section</td>
<td>Content</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>A.1. Section Properties</td>
<td>179</td>
</tr>
<tr>
<td>A.2. Design using Canadian Standard CSA23.3-94</td>
<td>182</td>
</tr>
<tr>
<td>(Seismic Provisions Section 21)</td>
<td></td>
</tr>
<tr>
<td>A.3. Design check American Standard ACI-318-95R</td>
<td>183</td>
</tr>
<tr>
<td>(Seismic Provisions Section 21)</td>
<td></td>
</tr>
<tr>
<td>B. Sheikh and Uzumeri, and Rasvi and Saatcioglu Model Calculations</td>
<td>185</td>
</tr>
<tr>
<td>B.1. Data for Wall 11</td>
<td>185</td>
</tr>
<tr>
<td>B.2. Sheikh and Uzumeri Model</td>
<td>186</td>
</tr>
<tr>
<td>B.2.1. Parameters</td>
<td>186</td>
</tr>
<tr>
<td>B.2.2. Stress-Strain Curves</td>
<td>189</td>
</tr>
<tr>
<td>B.2.3. Axial Load Versus Axial Strain Curve</td>
<td>191</td>
</tr>
<tr>
<td>B.2.4. Parameters $\sigma_{ef}$, $\sigma_{e2}$, $\sigma_{es}$ and $k_e$ for All Columns</td>
<td>192</td>
</tr>
<tr>
<td>B.3. Rasvi and Saatcioglu Model</td>
<td>193</td>
</tr>
<tr>
<td>B.3.1. Parameters</td>
<td>193</td>
</tr>
<tr>
<td>B.3.2. Stress-Strain Curves</td>
<td>197</td>
</tr>
<tr>
<td>B.3.3. Axial Load versus Axial Strain Curve</td>
<td>198</td>
</tr>
<tr>
<td>B.3.4. Rasvi and Saatcioglu Parameters for All Columns</td>
<td>199</td>
</tr>
<tr>
<td>B.4. Axial Shortening Curves</td>
<td>199</td>
</tr>
<tr>
<td>B.4.1. Sheikh and Uzumeri Columns</td>
<td>200</td>
</tr>
<tr>
<td>B.4.2. Liu et al. Columns</td>
<td>203</td>
</tr>
<tr>
<td>B.4.3. Scott Column (Mander)</td>
<td>203</td>
</tr>
</tbody>
</table>
C. Variable Poisson's Ratio in Liu et al. Columns
### List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1.</td>
<td>Eight-noded brick (hexahedron)</td>
<td>26</td>
</tr>
<tr>
<td>3.2.</td>
<td>Deformed brick</td>
<td>27</td>
</tr>
<tr>
<td>3.3.</td>
<td>Pentahedra (Wedge)</td>
<td>28</td>
</tr>
<tr>
<td>3.4.</td>
<td>Truss Bar</td>
<td>28</td>
</tr>
<tr>
<td>3.5.</td>
<td>Hognestad Parabola</td>
<td>30</td>
</tr>
<tr>
<td>3.6.</td>
<td>Generalized Popovics Stress-strain Curve</td>
<td>31</td>
</tr>
<tr>
<td>3.7.</td>
<td>Ascending Branch of Hoshikuma et al. Model</td>
<td>33</td>
</tr>
<tr>
<td>3.8.</td>
<td>Description of Lateral Pressure $f_{lat}$</td>
<td>35</td>
</tr>
<tr>
<td>3.9.</td>
<td>Adapted Version of the Modified Kent and Park Post-Peak Model</td>
<td>35</td>
</tr>
<tr>
<td>3.10.</td>
<td>Popovics Post-Peak Relationship</td>
<td>36</td>
</tr>
<tr>
<td>3.11.</td>
<td>Hoshikuma et al. Post-Peak Curve</td>
<td>38</td>
</tr>
<tr>
<td>3.12.</td>
<td>Reinforced Concrete in Tension</td>
<td>39</td>
</tr>
<tr>
<td>3.13.</td>
<td>Mohr-Coulomb Criterion</td>
<td>41</td>
</tr>
<tr>
<td>3.15.</td>
<td>Lateral Expansion</td>
<td>47</td>
</tr>
<tr>
<td>3.16.</td>
<td>Poisson's Ratio Relationship</td>
<td>48</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.17</td>
<td>Steel Model</td>
<td>48</td>
</tr>
<tr>
<td>3.18</td>
<td>Secant Modulus Definition</td>
<td>51</td>
</tr>
<tr>
<td>3.19</td>
<td>Solution Algorithm for SPARCS (Selby and Vecchio 1993)</td>
<td>53</td>
</tr>
<tr>
<td>4.1</td>
<td>Column Dimensions</td>
<td>58</td>
</tr>
<tr>
<td>4.2</td>
<td>Sheikh-and-Uzumeri Column Sections</td>
<td>59</td>
</tr>
<tr>
<td>4.3</td>
<td>Section Detail for 2A1-1</td>
<td>62</td>
</tr>
<tr>
<td>4.4</td>
<td>Finite Element Setup</td>
<td>63</td>
</tr>
<tr>
<td>4.5</td>
<td>Material Types</td>
<td>65</td>
</tr>
<tr>
<td>4.6</td>
<td>Concrete Axial Stress for Combination 2 (2A1-1)</td>
<td>69</td>
</tr>
<tr>
<td>4.7</td>
<td>Variation of $\frac{\epsilon_{dl}}{\epsilon_p}$ Ratio for Combination 2 (2A1-1)</td>
<td>71</td>
</tr>
<tr>
<td>4.8</td>
<td>Profile Deformation, Column 2A1-1 (Combination 2)</td>
<td>72</td>
</tr>
<tr>
<td>4.9</td>
<td>Concrete Lateral Stress States, 2A1-1, Combination 2</td>
<td>73</td>
</tr>
<tr>
<td>4.10</td>
<td>Response of 2A1-1, Combination 2</td>
<td>73</td>
</tr>
<tr>
<td>4.11</td>
<td>Lateral Reinforcement Response of 2A1-1, Combination 2</td>
<td>74</td>
</tr>
<tr>
<td>4.12</td>
<td>Axial Response of 2A1-1, Combination 1</td>
<td>75</td>
</tr>
<tr>
<td>4.13</td>
<td>Axial Response of 2A1-1, Combination 3</td>
<td>75</td>
</tr>
<tr>
<td>4.14</td>
<td>Axial Response of 2A1-1, Combination 4</td>
<td>76</td>
</tr>
<tr>
<td>4.15</td>
<td>Axial Response of 2A1-1, Combination 5</td>
<td>76</td>
</tr>
<tr>
<td>4.16</td>
<td>Axial Response of 2A1-1, Combination 6</td>
<td>77</td>
</tr>
<tr>
<td>4.17</td>
<td>Axial Response of 2A1-1, All Model Combinations</td>
<td>77</td>
</tr>
<tr>
<td>4.18</td>
<td>Lateral Reinforcement Response of 2A1-1, All Combinations</td>
<td>78</td>
</tr>
<tr>
<td>4.19</td>
<td>Axial Response of 4B3-19, Combination 1</td>
<td>81</td>
</tr>
<tr>
<td>4.20</td>
<td>Axial Response of 4B3-19, Combination 2</td>
<td>82</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.21</td>
<td>Axial Response of 4B3-19, Combination. 3</td>
<td>82</td>
</tr>
<tr>
<td>4.22</td>
<td>Axial Response of 4B3-19, Combination. 4</td>
<td>83</td>
</tr>
<tr>
<td>4.23</td>
<td>Axial Response of 4B3-19, Combination. 5</td>
<td>83</td>
</tr>
<tr>
<td>4.24</td>
<td>Axial Response of 4B3-19, Combination. 6</td>
<td>84</td>
</tr>
<tr>
<td>4.25</td>
<td>Axial Response of 4B3-19, All Model Combinations</td>
<td>84</td>
</tr>
<tr>
<td>4.26</td>
<td>Confinement of 2C5-17, Combination. 2</td>
<td>86</td>
</tr>
<tr>
<td>4.27</td>
<td>Sketch of Tie Strain Distribution, Column 2C5-17</td>
<td>87</td>
</tr>
<tr>
<td>4.28</td>
<td>Axial Response of 2C5-17, Combination. 1</td>
<td>89</td>
</tr>
<tr>
<td>4.29</td>
<td>Axial Response of 2C5-17, Combination. 2</td>
<td>90</td>
</tr>
<tr>
<td>4.30</td>
<td>Axial Response of 2C5-17, Combination. 3</td>
<td>90</td>
</tr>
<tr>
<td>4.31</td>
<td>Axial Response of 2C5-17, Combination. 4</td>
<td>91</td>
</tr>
<tr>
<td>4.32</td>
<td>Axial Response of 2C5-17, Combination. 5</td>
<td>91</td>
</tr>
<tr>
<td>4.33</td>
<td>Axial Response of 2C5-17, Combination. 6</td>
<td>92</td>
</tr>
<tr>
<td>4.34</td>
<td>Axial Response of 2C5-17, Combination. 7</td>
<td>92</td>
</tr>
<tr>
<td>4.35</td>
<td>Axial Response of 2C5-17, All Model combinations</td>
<td>94</td>
</tr>
<tr>
<td>4.36</td>
<td>Lateral Reinforcement Response of 2C5-17, All combinations</td>
<td>94</td>
</tr>
<tr>
<td>4.37</td>
<td>Axial Response of 4D6-24, All Model Combinations</td>
<td>96</td>
</tr>
<tr>
<td>4.38</td>
<td>Comparisons of Strength Increase</td>
<td>100</td>
</tr>
<tr>
<td>4.40</td>
<td>Effect of Compression Softening</td>
<td>103</td>
</tr>
<tr>
<td>4.41</td>
<td>Result Comparisons for 2A1-1</td>
<td>107</td>
</tr>
<tr>
<td>4.42</td>
<td>Result Comparisons for 2C5-17</td>
<td>108</td>
</tr>
<tr>
<td>Content</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.43. Equilibrium of Forces in a Circular Section</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>4.44. Load-Strain Curve of Column 2C90-10S100-25</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>4.45. Load-Poisson's Ratio Curve of Column 2C90-10S100-25</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>4.46. Poisson's Ratio Axial Strain Curve of Column 2C90-10S100-25</td>
<td>116</td>
<td></td>
</tr>
<tr>
<td>4.47. Volumetric Strain Variation of 2C90-10S100-25</td>
<td>116</td>
<td></td>
</tr>
<tr>
<td>4.48. Load-Strain Curve of Column 2C80-6S50-15</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td>4.49. Load-Poisson's Ratio Curve Column 2C80-6S50-25</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td>4.50. Poisson's Ratio Axial Strain Curve of Column 2C80-6S50-25</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>4.51. Volumetric Strain Variation of 2C80-6S50-25</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>4.52. Lateral Expansion Variation of Series I</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>4.53. Lateral Expansion Variation of Series II</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>4.54. Lateral Expansion Variation of Series III</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>5.1. Detail of Columns Tested by Liu et al.</td>
<td>123</td>
<td></td>
</tr>
<tr>
<td>5.2. Cross Section Model for Liu Columns</td>
<td>126</td>
<td></td>
</tr>
<tr>
<td>5.3. Material Types for Liu et al. Columns</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>5.4. Axial Shortening of 2C60-10S50-15</td>
<td>131</td>
<td></td>
</tr>
<tr>
<td>5.5. Lateral Expansion of 2C60-10S50-15</td>
<td>131</td>
<td></td>
</tr>
<tr>
<td>5.6. Axial Shortening of 2C60-10S100-15</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>5.7. Lateral Expansion of 2C60-10S100-15</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td>5.8. Axial Shortening of 2C80-10S150-15</td>
<td>134</td>
<td></td>
</tr>
<tr>
<td>5.9. Lateral Expansion of 2C80-10S150-15</td>
<td>134</td>
<td></td>
</tr>
<tr>
<td>5.10. Axial Shortening of 2C80-10S50-15</td>
<td>136</td>
<td></td>
</tr>
<tr>
<td>5.11. Lateral Expansion of 2C80-10S50-15</td>
<td>136</td>
<td></td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>5.12.</td>
<td>Axial Shortening of 2C80-6S50-15</td>
<td></td>
</tr>
<tr>
<td>5.13.</td>
<td>Lateral Expansion of 2C80-6S50-15</td>
<td></td>
</tr>
<tr>
<td>5.14.</td>
<td>Mander et al. Wall</td>
<td></td>
</tr>
<tr>
<td>5.15.</td>
<td>Square Column of Scott Series</td>
<td></td>
</tr>
<tr>
<td>5.16.</td>
<td>Finite Element Model for Wall 11</td>
<td></td>
</tr>
<tr>
<td>5.17.</td>
<td>Prototype of a Hollow Bridge Pier</td>
<td></td>
</tr>
<tr>
<td>5.18.</td>
<td>Axial Shortening of Wall 11</td>
<td></td>
</tr>
<tr>
<td>5.19.</td>
<td>Axial Shortening of Scott Column</td>
<td></td>
</tr>
<tr>
<td>5.20.</td>
<td>Axial Response of 4C6-5</td>
<td></td>
</tr>
<tr>
<td>5.21.</td>
<td>Axial Response of 4B4-20</td>
<td></td>
</tr>
<tr>
<td>5.22.</td>
<td>Lateral Expansion of 4B4-20</td>
<td></td>
</tr>
<tr>
<td>5.23.</td>
<td>Axial Response of 4D3-22</td>
<td></td>
</tr>
<tr>
<td>5.24.</td>
<td>Maximum Analytical to Experimental Load Ratio (wo. / Softening)</td>
<td></td>
</tr>
<tr>
<td>5.25.</td>
<td>Maximum Analytical to Experimental Load Ratio (w. / Softening)</td>
<td></td>
</tr>
<tr>
<td>5.26.</td>
<td>Localization of Cross Sections</td>
<td></td>
</tr>
<tr>
<td>5.27.</td>
<td>Column Section and FE Mesh (CSA, ACI)</td>
<td></td>
</tr>
<tr>
<td>5.28.</td>
<td>Axial Shortening of Section in Central Zone</td>
<td></td>
</tr>
<tr>
<td>5.29.</td>
<td>Axial Shortening of Section in Plastic Hinge Zone</td>
<td></td>
</tr>
<tr>
<td>5.30.</td>
<td>Model Comparison for Column 4B3-19</td>
<td></td>
</tr>
<tr>
<td>5.31.</td>
<td>Model Comparison of Column 4D6-24</td>
<td></td>
</tr>
<tr>
<td>5.32.</td>
<td>Model Comparison of Column 2C80-6S50-15</td>
<td></td>
</tr>
<tr>
<td>5.33.</td>
<td>Model Comparison for Wall 11</td>
<td></td>
</tr>
<tr>
<td>5.34.</td>
<td>Lefas et al. Wall SW16</td>
<td></td>
</tr>
</tbody>
</table>
5.35. Finite Element Mesh for Wall SW16 166
5.36. Material Zone distribution for SW16 166
5.37. Horizontal Response of Wall SW16 168
5.38. Sketch of Horizontal Displacement of Wall SW16 168
A.1. Design Section Using CSA and ACI Codes 180
B.1. Cross Section of Wall 11 185
B.2. Strain Curves for Concrete, Wall 11, Sheikh and Uzumeri Model 190
B.3. Strain Curve for Longitudinal Steel, Wall 11 190
B.4. Load-Strain Curve of Wall 11, Sheikh and Uzumeri Model 191
B.5. Strain Curves for Concrete, Wall 11, Rasvi and Saatcioglu Model 198
B.6. Load-Strain Curve of Wall 11, Rasvi and Saatcioglu Model 198
B.7. Column 2A1-1 200
B.8. Column 4C6-5 200
B.9. Column 2C5-17 201
B.10. Column 4B3-19 201
B.11. Column 4B4-20 202
B.12. Column 4D3-22 202
B.13. Column 4D6-24 203
B.14. Column 2C60-10S50-15 203
B.15. Column 2C60-10S100-15 204
B.16. Column 2C60-150-15 204
B.17. Column 2C80-10S50-15 205
B.18. Column 2C80-6S50-15 205
B.19. Scott Column

C.1. Load-Strain Curve of 2C60-10S50-15

C.2. Load-Poisson’s Ratio Curve of 2C60-10S50-15

C.3. Poisson’s Ratio-Axial Strain Curve of 2C60-10S50-15

C.4. Volumetric-Axial Strain Curve of 2C60-10S50-15

C.5. Load-Strain Curve of 2C60-10S100-15

C.6. Load-Poisson’s Ratio Curve of 2C60-10S100-15

C.7. Poisson’s Ratio-Axial Strain Curve of 2C60-10S100-15

C.8. Volumetric-Axial Strain Curve of 2C60-10S100-15

C.9. Load-Strain Curve of 2C60-10S150-15

C.10. Load-Poisson’s Ratio Curve of 2C60-10S150-15

C.11. Poisson’s Ratio-Axial Strain Curve of 2C60-10S150-15

C.12. Volumetric-Axial Strain Curve of 2C60-10S150-15

C.13. Load-Strain Curve of 2C80-10S50-15

C.14. Load-Poisson’s Ratio Curve of 2C80-10S50-15

C.15. Poisson’s Ratio-Axial Strain Curve of 2C80-10S50-15


C.17. Load-Strain Curve of 2C80-6S100-15

C.18. Load-Poisson’s Ratio Curve of 2C80-6S100-15

C.19. Poisson’s Ratio-Axial Strain Curve of 2C80-6S100-15

C.20. Volumetric-Axial Strain Curve of 2C80-6S100-15

C.21. Load-Strain Curve of 2C90-6S100-25

C.22. Load-Poisson’s Ratio Curve of 2C90-6S100-25
C.23. Poisson's Ratio-Axial Strain Curve of 2C90-6S100-25
C.24. Volumetric-Axial Strain Curve of 2C90-6S100-25
C.25. Load-Strain Curve of 2C90-6S50-25
C.26. Load-Poisson's Ratio Curve of 2C90-6S50-25
C.27. Poisson's Ratio-Axial Strain Curve of 2C90-6S50-25
C.28. Volumetric-Axial Strain Curve of 2C90-6S100-25
C.29. Load-Strain Curve of 2C90-10S100-0
C.30. Load-Poisson's Ratio Curve of 2C90-10S100-0
C.31. Poisson's Ratio-Axial Strain Curve of 2C90-10S100-0
C.32. Volumetric-Axial Strain Curve of 2C90-10S100-0
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1.</td>
<td>Sheikh-and-Uzumeri Selected Column Properties</td>
<td>61</td>
</tr>
<tr>
<td>4.2.</td>
<td>Finite Element Model Geometry</td>
<td>64</td>
</tr>
<tr>
<td>4.3.</td>
<td>Element Material Types</td>
<td>65</td>
</tr>
<tr>
<td>4.4.</td>
<td>Parametric Combinations for SPARCS Analyses</td>
<td>66</td>
</tr>
<tr>
<td>4.5.</td>
<td>Analytical to Experimental Ratios for 2A1-1</td>
<td>78</td>
</tr>
<tr>
<td>4.6.</td>
<td>Analytical to Experimental Ratios for 4B3-19</td>
<td>85</td>
</tr>
<tr>
<td>4.7.</td>
<td>Analytical to Experimental Ratios for 2C5-17</td>
<td>93</td>
</tr>
<tr>
<td>4.8.</td>
<td>Analytical to Experimental Ratios for 4D6-24</td>
<td>96</td>
</tr>
<tr>
<td>4.9.</td>
<td>Analytical to Experimental Peak Load Ratios</td>
<td>98</td>
</tr>
<tr>
<td>4.10.</td>
<td>Strength Increase</td>
<td>99</td>
</tr>
<tr>
<td>4.11.</td>
<td>Analytical to Experimental Peak Strain Ratios</td>
<td>101</td>
</tr>
<tr>
<td>4.12.</td>
<td>Selby's and Current Models</td>
<td>105</td>
</tr>
<tr>
<td>4.14.</td>
<td>$\sigma_0$ and $V_e$ Values for Liu et al. Columns</td>
<td>113</td>
</tr>
<tr>
<td>5.1.</td>
<td>Liu et al. Columns, Material Properties</td>
<td>125</td>
</tr>
<tr>
<td>5.2.</td>
<td>Geometry of Finite Element Models</td>
<td>127</td>
</tr>
</tbody>
</table>
5.3. Liu et al. Material Types 127
5.4. Mander et al. Material Properties 142
5.5. Material Properties of Additional Sheikh and Uzumeri Columns 148
5.6. Maximum Analytical to Experimental Load Ratio 153
5.7. Short Column Sections in Seismic Zones, Model Comparisons 158
5.8. Theoretical to Experimental Maximum Load Ratios for Square and Rectangular Columns 162
5.9. Theoretical to Experimental Maximum Load Ratios for Circular Columns 163
5.10. Material Properties of Wall SW16 164
5.11. Material Zones for FE Model of SW16 165
A.1. Code Provisions for designing Compression Members 182
B.1. Wall 11 Properties 186
B.2. Parameters for Sheikh-Uzumeri Model 192
B.3. Rasvi and Saatcioglu Parameters 199
Chapter 1
Introduction

1.1 Overview of Concrete Response

Three-dimensional nonlinear finite element analysis of reinforced concrete has been investigated for almost four decades. Constitutive material models that take into account the influence of triaxial states of strains and stresses have been developed for both plain concrete and steel.

These models have been based on principles founded in theories such as plasticity, fracture mechanics and elasticity (linear and nonlinear). In classical plasticity theory, both materials (i.e., concrete and steel) behave elastically until "yielding"; then, materials behave plastically and follow associative or nonassociative flow rules. The concept of "fracture energy" is used in fracture mechanics to establish failure criteria that depend on the state of stress to which an element of concrete is subjected. Failure surfaces define the upper boundary of concrete strength. Finally, in linear elasticity, the simplest model follows a Hooke's law where stresses are directly proportional to strains, without changes in material properties. In nonlinear elasticity, concrete and steel behave elastically within small increments of load or imposed displacements; however,
material properties are changed as load increases (or decreases). Secant and
tangential stiffness matrices have been developed to account for the change in
material behaviour, and a number of models have been implemented in finite
element programs that use either formulation (Chen 1982).

Behaviour of confined concrete is different from that of unconfined
cement. Concrete can be considered confined when subjected to triaxial
compressions; the triaxial compression increases the concrete's capacity to
sustain larger compressive strengths and deformations. When a concrete
element is laterally reinforced (e.g., by ties, hoops or spirals) and subjected to
axial compression, lateral expansion of the element in the plane perpendicular to
the axial compression activates the lateral steel, which confines the element by
exerting lateral pressure. Confined concrete generally fails in a ductile manner,
whereas unconfined concrete fails in a brittle manner. As tensile strains develop
in unconfined concrete subjected to compression, concrete softens and strength
decreases. It is also known that Poisson's ratio for concrete is not constant as
load increases; it increases with axial strain increments. This phenomenon is
beneficial in activating lateral steel.

The degree of confinement is measured analytically by the increment in
compressive strength and compressive strain at peak stress with respect to the
unconfined compressive strength and strain at peak, respectively.
1.2 Objectives

The general purpose of this work was to determine the capabilities of SPARCS to model confined concrete. SPARCS is a nonlinear elastic finite element program developed at the University of Toronto (Selby 1990, Selby and Vecchio 1993) for the analysis of reinforced concrete solids. The program uses constitutive models based on the Modified Compression Field Theory (MCFT) (Vecchio and Collins 1986) and has incorporated simple finite elements such as: truss bars, 8-noded bricks and 6-noded wedges. Details of the program will be given in Chapter 3.

The specific objectives of this work can be summarized as follows:

- To extend the capabilities of SPARCS to analyze large models under imposed displacements.
- To make a parametric study on the influence of various models on confined concrete behavior; including: base stress-strain curve for concrete in compression; compression softening; cracking criteria; strength enhancement; and Poisson's ratio.
- To implement base stress-strain curves for pre-peak and post-peak behavior.
- To corroborate the analytical model for variable Poisson's ratio implemented in SPARCS with experimental results.
- To corroborate the confinement models implemented in SPARCS with columns tested under monotonic increasing axial compression.
- To examine three-dimensional states of stress in other structural elements, such concealed columns in walls, and
To comment on ACI-318-95 and CSA/CAN A23.3-94 code provisions for the design of tie setups of short columns based on results obtained from modeled sections that satisfy their requirements.

1.3 Summary

A set of ten columns tested by Liu et al. (1998) was analyzed to determine the validity of the variable Poisson’s ratio model in SPARCS (Kupfer 1969). Plots of the axial load versus Poisson’s ratio, and axial strain versus Poisson ratio obtained from the experimental results will be given in Chapter 4.

For the parametric study, a set of four columns tested by Sheikh and Uzumeri (1978) were modeled with SPARCS and analyzed to establish the sensibility of each of the models mentioned above, in the behaviour of confined concrete. During this study, the stress-strain curve for concrete proposed by Hoshikuma et al. (1996), and a tentative model for strength enhancement proposed by Vecchio (1992) were implemented in SPARCS and used in the parametric study. A thorough revision of the results of this study will also be given in Chapter 4.

Once the parametric study was completed and the influence of each model had been established, sets of columns tested by Liu et al. (1998), Mander et al. (1988), Scott et al. (1982), and three additional Sheikh and Uzumeri columns were modeled in SPARCS and compared and discussed with the experimental results. Of special interest were the comparisons of the load versus
axial strain curve, the experimental to analytical peak load ratio, the strength gain in the concrete, and the lateral expansion (i.e., tie or spiral strain history).

Two short column sections designed according to the ACI 318-95, and CSA/CAN 23.3-94 provisions were modeled with SPARCS. The results are discussed along with the analytical corroboration in Chapter 5.

The Sheikh-Uzumeri (1982) and Razvi-Saatcioglu (1999) analytical models for confined columns were used to compute the axial response of the specimens modeled with SPARCS. Comparisons of these models with the finite element solutions are given in Chapter 5 and Appendix B.

A general discussion and conclusion of the results is presented in Chapter 6, with recommendations for future work.

Due to the amount of results derived from SPARCS, an special-purpose post-processor program with three-dimensional graphical and analytical capabilities was developed for checking and plotting: geometry, deformation animation, stress and strain states in cross sections, key indicators of damage, secant moduli convergence. Some of the plots presented in this work have been taken directly from this post-processor.

In general, all the column models analyzed with SPARCS showed excellent correlation with the experimental results. The strength gain average was 3% higher than the actual tests with a standard deviation of 11%. The post-peak behavior of some of the modeled Sheikh and Uzumeri columns was significantly improved when compared with a previous version of the program (Selby and Vecchio 1993). Experimental load-axial strain curves for Liu et al.
columns were adequately simulated. The overall behavior of the Scott and Mander specimens was well captured by SPARCS.

Finally, the current modeling tools of SPARCS will help in the understanding of complex elements or structures where three-dimensional analysis is unavoidable.
Chapter 2

Literature Review

Confined concrete can be defined as concrete that is restrained laterally by reinforcement consisting of steel stirrups or spirals. This reinforcement exerts lateral passive pressure against the concrete as it expands due to the Poisson’s effect when subjected to compressive load in one direction. The major effect of the confinement is to enhance the strength and ductility of reinforced concrete.

Concrete confinement has been studied since early in the century (Richart et al. 1928), and with special interest in column behavior for the last three decades (from Kent and Park (1972) to Razvi and Saatcioglu (1998)).

Constitutive material models for concrete in triaxial stress states have been proposed and adopted for use in numerical analyses, using fracture mechanics, plasticity or nonlinear elastic analysis approaches. Stress-strain curves for confined concrete have been derived from axially loaded columns tested under different load rates.

Some of the parameters that have been studied are: unconfined concrete strength; volumetric ratio of lateral steel; longitudinal reinforcement
arrangements; tie setup; lateral steel spacing; and cover dimensions (e.g., Kent
and Park 1972, Sheikh and Uzumeri 1978, Scott et al. 1982, Mander et al. 1988,
Razvi and Saatcioglu 1992, Cusson and Paultrre 1995). The analytical stress-
strain curves were fit with great accuracy to sets of columns tested by their own
authors, but they lack general applicability.

The search for a general constitutive model that can be applied not only
to reinforced concrete columns, but to confined concrete in other structure
elements where three-dimensional behavior can be expected (i.e., concealed
columns in walls, beam-column joints), is underway.

This chapter describes briefly some of the recent published works in
modeling of confined concrete, its applicability and success, as well as a brief
discussion of each one.

2.1 Plasticity and Fracture Energy Based Models

2.1.1 Liu and Foster (1988)

The authors revised the material model proposed by Carol et al. (1992),
and calibrated the parameters of the model for high-strength concrete (HSC).

The "microplane model" of Carol et al. is based on the microplane concept
introduced by Bazant et al. (1984). At the micro level, an arbitrary plane that
passes through a point within the concrete is used to define normal and shear
strains acting on the plane. The former acts perpendicularly, and the latter acts
parallel to the plane.
The normal strain is divided into its volumetric and deviatoric components, and stress-strain relationships for the volumetric, deviatoric, and tangential stresses (i.e., shear stresses) are determined using empirical parameters. A relationship is then established between the microplane stresses and macro stress tensors using the principle of virtual work. The change in the stress tensor \(d\sigma_y\) is given by

\[
d\sigma_y = D_{\text{tan}} \cdot ds_y
\]

(2.1)

where \(D_{\text{tan}}\) is the tangential material stiffness matrix, and \(ds_y\) is the change in the strain tensor. It was assumed that corresponding stresses and strains have the same direction (i.e., shear stress and shear strain, normal stress and normal strain).

The empirical parameters needed to calibrate the model showed a wide range of variation; as pointed out by various researchers. The authors developed an axisymmetric model, using the model of Carol et al. to analyze confined and unconfined HSC, with strengths up to 100 MPa.

A set of five parameters were calibrated for different concrete strengths and different values of the \(\sigma_1/\sigma_3\) ratio, where \(\sigma_1\) is the major principal stress, and \(\sigma_3\) is the minor principal stress.

The program uses four-noded and eight-noded isoparametric elements, as well as numerical integration to compute the stiffness matrix. Tests of confined cylinders with different confining pressures tested by Dahl (1992) were
used to match the experimental data with the analytical results, showing good agreement in both pre- and post-peak behavior.

An analytical circular column was also modeled using either four- or eight-node axisymmetric elements, truss bars for the longitudinal steel, and a point-type element for the lateral confining steel. It was shown that the analyses using four or eight node elements are similar for concrete strength up to 60 MPa and differ for 90 MPa concrete. Radial expansion in the circular section seems to be concentrated in a hinge region, after the peak stress is reached. The stirrups did not yield at peak load and the model was able to represent concrete softening and cover spalling, as well as ductile or brittle post-peak behavior.

2.1.2 Xie; MacGregor; and Elwi (1996)

In recent years, various researchers have tested high strength concrete (HSC) columns, and have been investigated finite element column models of normal strength concrete (NSC).

Numerical analysis plays an important role in investigating parameters of column failure and stretches the study of column behaviour beyond experimental tests. The objective of this paper was to evaluate analytically, the behavior of four columns tested at the University of Alberta. Those analyses were carried out using a finite element model in which the material model proposed by Pramono and Willam (1989) was implemented.

The constitutive model is a fracture energy-based plasticity formulation that uses the same formulation for compression and tension in concrete and
considers strain softening for triaxial states (tensile, compressive and triaxial compressive stresses). At initial load stages, the model behaves elastically within a yield surface. As load increases, plastic flow occurs within a failure surface according to a nonassociative rule. If plastic flow continues beyond failure surface, the material will follow a softening path.

The failure criterion used was proposed by Leon (Romano 1969) and was modified to account for the triaxial stress states. Parameters needed in the material model were adjusted for HSC and were taken from data obtained from cylinders tested at the University of Alberta. Specifically, a new definition for "crack spacing" as a function of principal stress \( \sigma \), accounts for less stiffened HSC columns with moderate confinement. Finally, the program ABAQUS was used to implement the new model.

Four columns tested by Ibrahim and MacGregor (1994) were chosen to prove the analytical model. These columns had equal dimensions and steel properties, but different hook spacing and concrete strength, with the latter ranging from 59.3 to 124.8 MPa. Eccentric loading was applied, causing a zero stress condition on one of the edges of each column.

One quarter of each of the columns was modeled using the ABAQUS 20-node brick for concrete elements in which longitudinal and parallel-to-edge reinforcement were embedded (i.e., smeared out into concrete). Diamond-type internal stirrups were modeled as truss bars, and appropriate assumptions were made for displacement-controlled loading and restraints.
The results agree well with experimental values and the material model has enough capacity to describe failure modes for columns ranging from poorly confined to well confined. Poorly confined columns presented brittle behavior after peak load. Although triaxial compressive stress states developed at tie levels, local failure occurred at mid-level between stirrups due to tensile strains in the concrete core.

Cover spalling along the compressive face was typical of well-confined column failure. High moment and load capacities after cover spalling were the result of higher lateral pressures due to yielding of stirrups.

ABAQUS allows the user to input material models and failure criteria. It seems that some difficulties arose from the lack of automation in load increments within the program. Use of truss bars attached only to the ends could not adequately represent the actual confining behavior.

2.1.3 Karibanis and Kiousis (1993)

A Drucker-Prager plasticity-type model was developed to analyze the behaviour of columns confined with either ties or spirals. The model includes a nonassociative flow rule, strain hardening, and a limited tensile strength for the concrete. Steel plasticity was modeled with a Ramberg-Osgood stress-strain curve.

The elastic behaviour of concrete follows a simple Hooke's law, and in the plastic range follows a loading function which is based on a strain hardening function of plastic strains. (i.e., irrecoverable strains).
Elastoplastic stress increments were computed using a rather complicated constitutive matrix that was evaluated numerically. In this model, different confining zones between ties or spirals were defined. The least confined zone was mid-way between ties. While plastic behavior was observed in the weakest zone, other zones could experience elastic unloading during large deformations (i.e., post-peak behavior).

The model successfully predicted the behaviour of some of the columns tested by Mander et al. (1998), and Iyengar et al. (1970). It is to be noted that the model was developed only for circular columns, but can be used in columns of different size because it is not based on statistical regressions of a determined set of specimens.

2.1.4 Chen and Mau (1989)

The authors recalibrated the model proposed by Bazant and Kim (1979) with experimental data from uniaxial, biaxial and triaxial tests from other researchers. Regions of high axial compressive stress and low lateral stresses in those tests were of special importance in effectively simulating the behaviour of concrete columns.

The original model of Bazânt and Kim is an incremental stress-strain relationship of the type

\[ d \sigma = C \varepsilon \]

(2.2)
where $d\theta$ and $d\varepsilon_{\theta}$ are the incremental stress and incremental strain matrices, respectively; and the material matrix is calculated as

$$C_{\theta} = C_{\theta}^{\text{el}} - C_{\theta}^{\text{pl}} - C_{\theta}^{\text{fr}}$$

(2.3)

where the right hand side terms stand for elastic, plastic and fracture material matrices, respectively. The material matrix $C_{\theta}$ is written in terms of invariants that make it suitable for finite element analysis (FEA).

The recalibrated model was implemented in a FEA program that uses two-dimensional (2D) axisymmetric triangular ring elements for plain concrete, and elasto-plastic springs for steel spirals. A Newton-Raphson scheme was used for the load-displacement history, and the cracking of concrete was modeled using an iterative procedure that accounted for stress release (i.e., stress softening).

Circular cylinders with spiral confinement tested by Ahmad and Shah (1982) and by Mander et al. (1984) were geometrically modeled in the numerical analyses. Only slices of experimental columns were modeled due to their symmetry and the assumption that the behavior of the rest of the column is similar.

The analytical results showed very good agreement with those obtained from the tests. The numerical results proved to be an excellent method for analyzing stress distributions within the concrete core of columns. The authors compared their recalibrated model with cylinders with moderate to high volumetric ratios $\rho$ ($\rho = 1.57\% - 3.1\%$) and with a well-confined column tested.
by Mander et al. The model, however, was not compared with poorly confined columns to demonstrate its ability to predict a wide range of column behaviors.

2.2 Linear and Nonlinear Elastic Models

2.2.1 Mau; Elwi; and Zhou (1998)

An analytical study of confinement in circular columns is presented in this paper. The general solution follows a linear elastic approach, whereby the radial, tangential and axial stresses are obtained from a solution of the displacements of a cylinder subjected to a ring load (i.e., radial compression).

The writers defined an "effective confining stress" as the average of the tangential and radial stresses in the cylinder. The effective confining stress is a function of the distance from the axis of the cylinder. Several comparisons were made for different spiral pitch-to-radius ratios, and for different transverse sections along the axis of the cylinder.

From these comparisons, an "average confinement factor" was defined for sections at mid-level between spirals as a function of the spiral pitch-to-radius ratio. It was found that this factor can be used in the interpretation of nonlinear tests as was established from analyses of specimens tested by other authors.

Linear elastic analyses were performed in rectangular columns confined with welded wire fabric (WWF), where similar confinement factors were obtained.
2.2.2 Barzegar and Maddipudi (1997)

The authors developed a three-dimensional (3D) nonlinear finite element analysis (NLFEA) program for modeling of reinforced concrete structures. The concrete is modeled as an orthotropic material, with smeared cracks in nonorthogonal directions.

Concrete failure stresses are calculated using the five-parameter ultimate strength criterion proposed by Willam and Waranke (1975). The hypoelastic constitutive model of Stankowski and Gerstle (1985) was adopted with some modifications. In this model, increments of principal strains are transformed into octahedral normal and shear strain increments. The latter are multiplied by a material matrix to calculate increments in octahedral normal and shear stresses. The material matrix is a function of the tangential shear and bulk moduli, and coupling moduli.

The post-peak behaviour of concrete was also taking into account. A stress decrement is computed using a reduction factor that affects the tangent stiffness material matrix. Smeared cracks in multiple directions are based on a fracture energy model developed by De Borst and Nauta (1985). Finally, the authors also implemented models for embedded reinforcement and bond slip. To corroborate the post-peak model, the program was used to analyze one of the columns tested by Scott et al. (1982); the results showed good agreement with the axial load-axial strain experimental curve.
2.2.3 Bortolotti (1994)

The author proposed a failure criterion for concrete in terms of the tensile strength of concrete, and the internal friction angle. Based on this criterion, a constitutive law for concrete in axial tension was derived. The failure criterion was developed to be applied to axially loaded confined concrete columns.

It was shown that the confinement strength is a function of the tensile strain of the cover concrete at failure. In the case of circular columns, the cover shell is idealized as a tube subjected to internal radial tensile pressure due to expansion of the concrete core. The cover starts its tension softening behaviour after it reaches its peak tensile strain.

A formula for the minimum transverse steel ratio was deduced from the failure criterion. If the quantity of transverse steel exceeds the minimum, cover concrete could reach the ultimate tensile strain.

A set of columns tested by others was compared with the formulae deduced for peak load and peak strain. Analytical results varied from about 80% to 160% of the experimental results.

2.2.4 Selby (1990)

The author developed program SPARCS, the first University of Toronto FEA program for nonlinear elastic analysis of reinforced concrete solids. The constitutive relationships from the Modified Compression Field Theory (MCFT) were extrapolated to account for triaxial states of stresses.
Concrete was modeled as an isotropic material before cracking, and with orthotropic properties after developing cracks. Steel was smeared out in the concrete. Principal stress directions coincided with principal strains. The nonlinear elastic analyses were carried out updating the secant stiffness material matrices for both concrete and steel. An isoparametric eight-noded brick, with three degrees of freedom (DOF) per node and a closed-form stiffness matrix, was implemented. Material behaviour included: a Hoggestad parabola for stress-strain curve for concrete in compression; compression softening due to transverse tensile strains; linear elastic behaviour of uncracked concrete in tension; tension stiffening; and stress checks in cracked concrete. Steel could be oriented in any direction, and was modeled following an elastic-perfectly-plastic curve, which also included strain hardening. Perfect bond was also assumed.

First checks of the program were made comparing analytical to experimental results of beams subjected to torsion and tested by other researcher.

2.2.5 Vecchio (1992)

In 1986, Vecchio and Collins proposed The Modified Compression Field Theory (MCFT). This theory has been used to predict and to analyze the behavior of plane-stress and plane-strain elements subjected to shear and normal stresses. The original MCFT was extended to account for the effects of low tensile stresses, strength enhancement due to confinement, and variable
Poisson's ratio. Modifications to concrete material matrix derived from the MCFT, and to stress-strain curves were proposed.

In the finite element formulation of the MCFT, orthotropic materials have symmetrical material matrices. As compression in concrete increases, the secant stiffness for different values of compressive stress varies. The Poisson's ratio is expected to change as a result, thus making the material matrix for concrete unsymmetrical.

The concept of a "prestrain matrix" is introduced to account for non-stress related strains in concrete elements. Prestrains are transformed into "forces" in this formulation. The expansion effect is included in the "prestrain matrix" as

\[ \varepsilon_{e12}^o = \left[ -\nu_{12} \frac{f_{c2}}{E_{c2}} -\nu_{21} \frac{f_{c1}}{E_{c1}} \right] \]

where \( \varepsilon_{e12}^o \) is the prestrain vector in the principal directions 1 and 2, \( \nu_f \) is the Poisson's ratio in the direction \( l \) when subjected to a stress \( f_{c_l} \) in the direction \( f \), and \( E_{c_l} \) is the secant stiffness of concrete in the direction \( l \).

To avoid numerical problems in the solution of the FEA, the Poisson's ratio \( \nu_f \) was divided into an "elastic portion" \( \nu_f^e \) and a residual component \( \nu_f^r \) = \( \nu_f - \nu_f^e \). The elastic portion \( \nu_f^e \) was directly included into the material matrix, and the residual portion \( \nu_f^r \) into the prestrain matrix, keeping the symmetry and the orthotropical conditions of the material matrix.
To account for strength enhancement, a formula proposed by Kupfer (1969) for biaxial stress-states is also included in the formulation. The increment in the concrete strength is

\[ f_p = K_e f_c \]

(2.5)

where \( K_e \) is the strength enhancement factor (Kupfer), \( f_c \) is the unconfined strength of concrete and \( f_p \) is the confined strength. And the strain at peak stress \( \varepsilon_p \) (i.e., at \( f_p \)) is calculated as

\[ \varepsilon_p = K_e \varepsilon_c \]

(2.6)

where \( \varepsilon_c \) is the strain at peak unconfined stress.

In the case of triaxial stress states, the author suggested a tentative formula to compute the strength enhancement, which is a combination of the relationships proposed by Richart (1928) and Kupfer (1969).

The stress-strain curve for confined concrete is a liberal modification of the Modified Kent and Park model (Scott et al. 1982) which consists of a parabola for the ascending branch of the curve, and a straight line for the post-peak behavior.

A total of 11 panels tested by Vecchio and Collins (1986) and 13 shear walls tested by Lefas et al. (1990) were analyzed using the FEA program TRIX. The panels were subjected to combinations of biaxial and shear stresses. The set of panels chosen were those which experienced crushing of concrete, and where tensile strains were low compared to compressive strains. The shear
walls were subjected to monotonic increments of lateral load, and constant compression at the top. The walls were constructed with concealed columns at each end, thus providing an excellent test to check triaxial compressive stresses under the load conditions mentioned above.

The effect of concrete expansion was well modeled in the finite element approach, the strength enhancement was also well captured for all tests. However, the analytical results showed a stiffer response, and underestimated the deflection of the walls.

The author extended the scope of the MCFT to account for strength enhancement and variable expansion of concrete in two-dimensional structural elements.

2.2.6 Selby and Vecchio (1993)

SPARCS was further updated and improved to include two additional finite elements: six-noded isoparametric wedges (regular pentahedra), and truss bars. The effect of expansion due to Poisson's ratio was implemented using the concept of a "prestrain matrix" (Vecchio 1992).

New constitutive models were added to SPARCS. Base stress-strain curves for concrete in compression now include Popovics (1973) stress-strain curve for high strength concrete (HSC). The effect of confinement due to triaxial stress states was also implemented; a failure curve proposed by Hsieh et al. (1979) is used to compute the ultimate compressive strength. Finally, a revised version of the modified Kent and Park post-peak compressive curve (Scott et al.
1982), and the variable Poisson's ratio proposed by Kupfer et al. (1969), were added to account for ductility and variable lateral expansion in concrete.

The first author modeled six of the columns tested by Sheikh and Uzumeri (1978). Eight-noded bricks were used in two different types of meshes; one considered both core and cover concrete, and the other considered only the concrete core. Imposed loads were applied to a stiff plate at the top of each model, and a set of springs was used to model the post-peak behaviour.

Column peak strengths obtained with SPARCS agreed fairly well with the experimental results from Sheikh and Uzumeri. However, the post-peak responses were not satisfactory.

2.2.7 Abdel-Halim and Abu-Lebdeh (1989)

A set of 8 rectangular columns, two of them originally tested by Scott et al. (1982) were examined analytically. The linear elastic FEA program SAP IV was used in a step-by-step load increment approach to account for nonlinear material behaviour of reinforced concrete.

At each load stage, the finite elements (8-noded bricks) were checked to determine if they either cracked or reached a previously defined failure surface. Also, material properties (i.e., material stiffness matrices) were changed according to the current strain state. The columns were brought to failure and the analytical results compared against the experimental values, and against some of the early concrete stress-strain base curves from other researchers.
The three-dimensional stress-strain relationships and ultimate strength surface proposed by Celodin et al. (1977) were adopted in these analyses. Steel was modeled using experimental stress-strain curves with strain hardening. Good agreement with the experimental results of Scott et al. was obtained.

This is one of the first nonlinear elastic finite element analyses of confined concrete reported in the literature. Volumetric ratio of confining steel, arrangement of longitudinal steel, and the increment in compressive strength due to confinement were some of the aspects investigated.
Chapter 3

SPARCS

In this chapter, a brief description of the program SPARCS is presented. The finite elements developed for the program are described, followed by a review of each of the constitutive models for concrete and steel that have been implemented to date. Finally, the SPARCS structure and solution algorithm is described.

3.1 Program Description

Program SPARCS (i.e., Selby's Program for the Analysis of Reinforced Concrete Solids) is a nonlinear finite element program that has been developed at the University of Toronto (Selby 1990, Selby and Vecchio 1993 and 1997) for the analysis of reinforced concrete solids. The three-dimensional (3D) state of stresses in reinforced concrete solids is taken into account by extrapolating the stress-strain curves derived from the Modified Compression Field Theory (MCFT) (Vecchio and Collins 1986) to its three-dimensional formulation.

The 3D stress state is related to the 3D strain state through a constitutive material matrix, as shown below
\[
\{\sigma\} = [D]\{\varepsilon\}
\]

(3.1)

where \(\{\sigma\}\) is the stress vector, \([D]\) is the material stiffness matrix, and \(\{\varepsilon\}\) is the strain vector.

The material matrix \([D]\) is given in terms of the secant stiffness moduli, Poisson's ratio \(v\), and Shear moduli \(G\) in three directions (i.e., local, global or principal directions). Secant moduli vary at each load state as a function of the stress state. Stresses are computed according to base stress-strain curves derived for both concrete and steel subjected to either compressive or tensile strains. Material behavior relations have been adopted from the MCFT and theory of plasticity; including: strength softening due to tensile strains, strength enhancement due to confinement, variable lateral expansion, concrete cracking, and tension stiffening.

It is assumed in SPARCS that concrete behaves isotropically before cracking, and orthotropically afterwards. Cracks are assumed to be smeared within concrete, thus allowing the user to maintain the same finite element mesh during the analysis process, and not having to change it due to localized cracks. Although cracks are assumed smeared, stress checks at crack surfaces are performed to satisfy compatibility and equilibrium.

Steel can be modeled as smeared within the concrete elements, or represented as truss bars attached to solid elements. In any case, perfect bond is assumed between the two materials.
A description of the finite element library, and the material constitutive laws and failure criteria follows.

3.2 Finite Element Library

SPARCS has three finite elements in its library: an 8-noded brick (hexahedron), a 6-noded brick (pentahedra or wedge), and a truss bar.

3.2.1 Hexahedron

The 8-noded brick is shown in Fig. 3.1. It is an isoparametric element with orthogonal sides and 24 degrees of freedom (DOF), three at each node.

Figure 3.1 Eight-noded Brick (hexahedron)

The relative displacement between two adjacent nodes is assumed linear, so that edges remain straight, as shown in Fig. 3.2. Infinitesimal rotations and small deformations are assumed in the computation of the element stiffness matrix $k$:
\[ k = \int [B]^T [D][B] \, dV \]

(3.2)

where \([B]\) is the strain displacement matrix that depends on linear displacement functions, and \([D]\) is as defined above. The closed-form solution of Eq. (3.2) is obtained by direct integration, as incorporated in SPARCS by Selby (1990).

![Figure 3.2 Deformed Brick](image)

It is noted that nonorthogonal hexahedrons are not allowed in SPARCS; the program has a subroutine to check element geometry. Numbering must be counterclockwise as shown in Fig. 3.1.

### 3.2.2 Pentahedra (Wedge)

The 6-noded brick is shown in Fig. 3.3. It is also an isoparametric element with 18 degrees of freedom (d.o.f.); three at each node. It must be prismatic (i.e., maintaining the same transverse section throughout), and the bottom and top faces (face 123 and face 456 in Fig. 3.3) must be of equal area and must be located in parallel planes. Distortions are not allowed in its generation. Linear
displacements and infinitesimal deformations are also assumed. The element stiffness matrix $k$ (Eq. 3.2) is obtained from numerical gauss integration.

![Diagram of pentahedra (Wedge)](image)

**Figure 3.3 Pentahedra (Wedge)**

3.2.3 **Truss Bar**

This element has two nodes, and three d.o.f. at each end, as shown in Fig. 3.4. The element deformation is computed as the relative displacement between the two nodes divided by the length of the element. A simple direct computation of the element stiffness matrix is given in the program (Selby and Vecchio 1993).

![Diagram of truss bar](image)

**Figure 3.4 Truss Bar**
Although the element formulation is based solely on axial deformations, buckling is not taken into account when the bar is subjected to compression. Bending is also ignored in its stiffness matrix.

3.3 Constitutive Models for Concrete

This section describes the models for concrete implemented in SPARCS and used in the analysis of confined concrete. It begins with the base stress-strain curves for concrete in compression and tension, and continues with the failure criteria for concrete under triaxial state of stresses, and cracking. The suggested models for compressive strength enhancement are also reviewed. Finally, compression-softening models derived from the MCFT are presented. The section ends with a description of the variable Poisson's ratio.

3.3.1 Base Stress-Strain Curves for Concrete in Compression, Pre-peak Behaviour.

The following expressions are intended for the modeling of the ascending branch of the stress-strain curve.

3.3.1.1 Hognestad Parabola

The parabola proposed by Hognestad is a widely used stress-strain curve for the behaviour of normal strength, and is calculated as:
where $f_d$ and $\varepsilon_d$ are the compressive stress and strain in the principal $i$-direction respectively, and $f_p$ and $\varepsilon_p$ are the peak stress and strain at peak stress, respectively. Eq. (3.3) is depicted in Fig. 3.5.

$$f_d = f_p \left( 2 \frac{\varepsilon_d}{\varepsilon_p} - \left( \frac{\varepsilon_d}{\varepsilon_p} \right)^2 \right)$$

(3.3)

3.3.1.2 Thorenfeldt et al. (see Collins et al.)

This relationship is a generalized model of the base stress-strain curve proposed by Popovics (1973). The relationship represents well the stiffer ascending branch and steeper falling branch of high strength concrete (HSC). The stress-strain curve is computed as follows:

$$f_d = f_p \left( \frac{\varepsilon_d}{\varepsilon_p} \right) \left( \frac{n}{n-1+\left( \frac{\varepsilon_d}{\varepsilon_p} \right)^n} \right)$$

(3.4)

where:
\[ n = 0.80 + \frac{f_c}{17} \]

(3.5)

and

\[ k = 1.0 \text{ for the ascending branch, or for the descending branch} \]

\[ k = 0.67 + \frac{f_c}{62} \]

(3.6)

The generalized Popovics curve is graphed in Fig. 3.6. A variation in the descending slope can be seen as the concrete strength increases.

![Graph of Generalized Popovics Stress-Strain Curve](image)

**Figure 3.6 Generalized Popovics Stress-Strain Curve**

3.3.1.3 Hoshikuma et al.

Hoshikuma et al. (1996) presented a model for confinement effects on the stress-strain relation of reinforced concrete, suitable for pier design in seismic zones. The base stress-strain curve of this model was extracted and implemented in SPARCS. The ascending branch satisfies the following boundary conditions:
i) \( f_a = 0 \) at \( \varepsilon_a = 0 \)

ii) The initial stiffness \( E_a \) (tangential) is computed as the derivative of the stress with respect to the strain, at zero initial strain \( \varepsilon_a = 0 \):

\[
\frac{df_a}{d\varepsilon_a} = E_a
\]

(3.7)

iii) \( f_a = f_p \) at \( \varepsilon_a = \varepsilon_p \), and

iv) The curve is horizontal at peak stress (i.e., at \( \varepsilon_a = \varepsilon_p \))

\[
\frac{df_a}{d\varepsilon_a} = 0
\]

(3.8)

The expression for the ascending branch is given as:

\[
f_a = E_a \cdot \varepsilon_a \left( 1 - \frac{1}{n} \left( \frac{\varepsilon_a}{\varepsilon_p} \right)^{n-1} \right)
\]

(3.9)

where

\[
n = \frac{E_c}{E_c - E_{\infty}}
\]

(3.10)

and

\[
E_{\infty} = \frac{f_p}{\varepsilon_p}
\]

(3.11)

is the secant stiffness at peak stress.
Figure 3.7 shows several ascending branches obtained with the Eqs. (3.9) to (3.11). It is noted that all curves flatten at peak, thus validating the assumptions of the model.

![Figure 3.7 Ascending Branch of Hoshikuma et al. Model](image)

3.3.2 Base Stress-Strain Curves for Concrete in Compression, Post-peak Behaviour.

The post-peak behaviour of confined concrete (i.e., after peak stress) depends mainly on the triaxial stress state. In well-confined concrete, as lateral pressure increases or remains constant after the peak load has been reached, the element will be able to sustain load at large deformations, thus exhibiting ductile behaviour. On the other hand, poorly confined concrete (i.e., low levels of lateral pressure) will fail shortly after the peak load. This section describes the descending stress-strain curves used in SPARCS for confined concrete after peak stresses.
3.3.2.1 Modified Kent and Park (Scott 1982)

The current version of SPARCS contains a version loosely based on this stress-strain curve, which was introduced originally by Selby and Vecchio (1993). The compressive stress in the principal $l$-direction, $f_{al}$ is calculated as:

$$f_{al} = f_p [1.0 + Z_m (e_{al} - e_p)]$$  \hspace{1cm} (3.12)

where:

$$Z_m = \frac{0.5}{3 + 0.29 f'_c} \left[ \frac{(-e_o)}{145 f'_c - 1000} + \frac{f_{lat}^{0.9}}{170 + e_p} \right]$$  \hspace{1cm} (3.13)

and

$$f_{lat} = -I_1 + f_{al}$$  \hspace{1cm} (3.14)

$$I_1 = f_{al} + f_{a2} + f_{a3}$$  \hspace{1cm} (3.15)

Here, $I_1$ is the first stress invariant in terms of the principal concrete stresses $f_{a1}$, $f_{a2}$ and $f_{a3}$ ($f_{a3} < f_{a2} < f_{a1}$, compression negative), $e_o$ is the strain at peak unconfined stress $f'_o$. $f_{lat}$ is the assumed lateral pressure and is calculated as the algebraic sum of the principal stresses in the plane perpendicular to the principal stress being calculated (i.e., perpendicular to $f_{al}$), as shown in Fig. 3.8 for the particular case $f_{a3} = f_{a2}$.
Figure 3.8 Description of Lateral Pressure $f_{lat}$

Figure 3.9 shows the Post-peak branch of this stress-strain curve.

Figure 3.9 Adapted Version of the Modified Kent and Park Post-Peak Model

3.3.2.2 Popovics (1973)

Popovics stress-strain curve was implemented in SPARCS as another alternative for representing the post-peak behaviour of confined concrete. This version was also selected by Mander et al (1988) in the formulation of his confined concrete model. The stress-strain curve is computed as:
The relationship is schematically presented in Fig 3.10.

\[ f_a = f_p \frac{\left( \frac{e_{ul}}{e_p} \right)^n}{n - 1 + \left( \frac{e_{ul}}{e_p} \right)^n} \]

(3.16)

where \( n \) is the same as in Eq. (3.10) and the initial tangent stiffness is (in MPa.):

\[ E_o = 5000 \sqrt{f'_c} \]

(3.17)

3.3.2.3 Hoshikuma et al. (1996)

The straight line proposed by Hoshikuma et al. (1996) for the post-peak behaviour of confined concrete was implemented in SPARCS, with a slight variation in the computation of descending slope \( E_{des} \) of the curve. The stress-strain curve is given as:

Figure 3.10 Popovics Post-Peak Relationship

Modeling of Confined Concrete
\[ f_{\alpha} = f_p - E_{\alpha}(\varepsilon_{\alpha} - \varepsilon_p) \]  

(3.18)

where:

\[ E_{\alpha} = 11.2 \frac{f_p^2}{\left( \frac{f_{\text{ue}}}{2} \right)} \]  

(3.19)

Hoshikuma et al. defined an ultimate strain \( \varepsilon_{\alpha} \) as the strain corresponding to a stress equal to half the peak stress \( f_p \) in the descending branch. This value was determined based on observations of crushing of concrete and buckling of longitudinal bars in their specimens. However, this criterion was not taken into account in the SPARCS formulation. The failure criteria for concrete will be reviewed later in this section.

The original formula for the descending slope \( E_{\alpha} \) (see Fig. 3.11.) was:

\[ E_{\alpha} = 11.2 \frac{f_p^2}{(\rho_v f_{\text{ye}})} \]  

(3.20)

where \( \rho_v \) is the volumetric ratio of lateral steel in a column, and \( f_{\text{ye}} \) is the yielding stress of the lateral steel. It is noted that the denominator in the right hand side of Eq. (3.20) is replaced with the term \( f_{\text{ue}}/2 \) in Eq. (3.19)
3.3.3 Base Stress-Strain Curve for Concrete in Tension

Reinforced concrete follows a linear ascending branch up to the tensile strength $f'_t$, which is an input parameter for the program. The strain at cracking stress $\varepsilon_\sigma$ is given as:

$$\varepsilon_\sigma = \frac{f'_t}{E_s}$$  \hspace{1cm} (3.21)

where $E_s$ is the initial tangential stiffness of concrete.

As cracks are smeared out in reinforced concrete, average tensile stresses after reaching $f'_t$ can be computed due to tension stiffening (see Figure 3.12). Three tension stiffening models have been added to SPARCS:

3.3.3.1 Vecchio (1982)

Tensile stress (considered positive in SPARCS) are computed as:
where $\epsilon_d$ must be a principal tensile strain in the direction under consideration. This tension-stiffening model was derived from the Modified Compression Field Theory (MCFT) and obtained from test results of panels subjected to biaxial stresses, and extrapolated to the triaxial condition.

![Diagram of Reinforced Concrete in Tension](image)

**Figure 3.12 Reinforced Concrete in Tension**

### 3.3.3.2 Collins-Mitchell (1987)

This is a variation of the Vecchio-1982 model in which tension stiffening is computed with the relationship (see Collins and Mitchell 1997):

\[
f_d = \frac{f'_t}{1 + \sqrt{500\epsilon_d}}
\]

(3.23)
3.3.3.3 Izumo, Maekawa et. al.

After cracking, average tensile stress is kept constant (i.e., \( f_a = f'_t \)) up to a strain twice as large as \( \varepsilon_{cr} \). The stresses thereafter are given by:

\[
f_a = f'_t \left( 2 \frac{\varepsilon_{cr}}{\varepsilon_a} \right)^{0.4}
\]

(3.24)

3.3.4 Failure criteria for concrete

3.3.4.1 Hsieh-Ting-Chen Criterion (1979)

Brittle failure of concrete is expected under tensile stresses, and ductile failure can occur under triaxial compressive stresses. The failure surface proposed by Hsieh et al. (1979) (see Chen 1982) represents with good accuracy the failure of concrete in: uniaxial compression, uniaxial tension, biaxial compression, triaxial compression and/or tension. It was chosen (Selby and Vecchio 1993) to be implemented in SPARCS, and is given below in terms of stress invariants.

\[
2.0108 \frac{J_2}{f'_c} + 0.9714 \sqrt{J_2} + 9.1412 \frac{f_{el}}{f'_c} + 0.2312 \frac{I_1}{f'_c} - 1 = 0
\]

(3.25)

where \( I_1 \) is determined from Eq. (3.15), and:

\[
J_2 = \frac{1}{6} \left[ (f_{el} - f_a)^2 + (f_a - f_{ct})^2 + (f_{ct} - f_{el})^2 \right]
\]

(3.26)
A Newton method is used in the program to solve Eq.(3.25) for the compressive failure stress $f_{c2r} = f_{c3}$.

3.3.4.2 Cracking Criteria

The Mohr-Coulomb criterion and some proposed variations were also implemented in the current version of the program to account for cracking stress $f_{cr}$ under triaxial conditions. A review of this two-parameter failure criterion is given below.

3.3.4.2.1 Mohr-Coulomb Criterion (stress formulation)

If $f_{c1} > f_{c2} > f_{c3}$, the largest Mohr’s circle is defined by the extreme principal stresses $f_{c1}$, and $f_{c3}$, as shown in Fig. 3.13

![Mohr-Coulomb Criterion Diagram](image-url)

Figure 3.13 Mohr-Coulomb Criterion
The straight-line envelope curve is determined as (see Chen 1982):

\[
f_{\text{cf}} \frac{1 + \sin \phi}{2c \cos \phi} - f_{\text{er}} \frac{1 - \sin \phi}{2c \cos \phi} = 1
\]

(3.27)

where \( \phi \) is the internal friction angle of concrete (assumed 37° in SPARCS), and \( c \) is the cohesion. Defining \( f_{\text{er}} \) as:

\[
f_{\text{er}} = \frac{2c \cos \phi}{1 + \sin \phi}
\]

(3.28)

if \( f_{\text{er}} = 0 \) then \( f_{\text{er}} \) becomes \( f'_{\text{c}} \), and:

\[
f'_{\text{c}} = \frac{2c \cos \phi}{1 - \sin \phi}
\]

(3.29)

from which the cohesion \( c \) can be readily determined. The cracking stress \( f_{\text{cr}} \) is determined from Eq. (3.27) to (3.29) as:

\[
0.25 f'_{\text{c}} \leq f_{\text{cr}} = f_{\text{cf}} \left[ 1 + \frac{f_{\text{er}}}{f'_{\text{c}}} \right] \leq f'_{\text{c}}
\]

(3.30)

3.3.4.2.2 Mohr-Coulomb Criterion (strain formulation)

The terms \( f_{\text{er}} \) and \( f'_{\text{c}} \) in Eq. (3.30) are replaced by for \( \varepsilon_{\text{er}} \) and \( \varepsilon_{\text{c}} \), respectively:

\[
f_{\text{cr}} = f_{\text{cf}} \left[ 1 + \frac{\varepsilon_{\text{er}}}{\varepsilon_{\text{c}}} \right]
\]

(3.31)
3.3.4.2.3 CEB-FIP Criterion

For this case:

\[ f_{\alpha f} = 0.6 \sqrt{f'_c} \]  \hspace{1cm} (3.32)

and

\[ f_\sigma = f_{\alpha f} \left[ 1 + 0.8 \left( \frac{f_\sigma}{f'_c} \right) \right] \]  \hspace{1cm} (3.33)

3.3.5 Confined Strength \( f_p \) and Strain at Peak Stress \( \varepsilon_p \)

Once the base stress-strain curves for concrete in compression have been defined, the confined stress (i.e., peak stress) and its corresponding strain are computed following either of the following criteria.

3.3.5.1 Selby (1993)

The author uses the failure criterion proposed by Haileh et al. (1979) (Section 3.3.4.1) to define the peak stress factor \( K_\sigma \):

\[ K_\sigma = \frac{f_{\alpha f}}{f'_c} \] \hspace{1cm} (3.34)

When compression softening of concrete (i.e., strength reduction due to high tensile strains) is not taken into account, \( f_p \) and \( \varepsilon_p \) are computed in SPARCS as:

\[ f_p = K_\sigma \cdot f'_c \] \hspace{1cm} (3.35)
\[ \varepsilon_p = K_o \varepsilon_o \]  
\[(3.36)\]

### 3.3.5.2 Vecchio (1992)

A stress enhancement factor \( K_o \) is defined by the author as:

\[ K_o = \left[ 1.0 + 0.92 \frac{f_{cm}}{f'_{c}} - 0.76 \left( \frac{f_{cm}}{f'_{c}} \right)^2 \right] + 4.0 \frac{f_{cl}}{f'_{c}} \]  
\[(3.37)\]

where

\[ f_{cm} = -(f_{c2} - f_{c1}) \]  
\[(3.38)\]

and

\[ f_{cl} = -f_{c1} \]  
\[(3.39)\]

Eqs. (3.37) to (3.39) are valid if \( 0 > f_{c1} > f_{c2} > f_{c3} \), \( f_{c3} \) being the maximum compressive stress, and \( K_o \) computed in the direction of \( f_{c3} \). The expression in brackets in the right hand term of Eq. (3.37) is a relationship that approximates that of Kupfer et al. (1969) for biaxial compression, and the second term represents a slight modification of the model proposed by Richart et al. (1928) (see Vecchio 1992).

Again, when compression softening is not a factor in the analysis, \( f_p \) and \( \varepsilon_p \) are determined as (recalling Eqs. (2.5) and (2.6)):

\[ f_p = K_e f'_{c} \]  
\[(3.40)\]
The MCFT (Vecchio and Collins 1986) recognized the effects of tensile strains in cracked concrete. As concrete cracks, tensile stresses develop in the concrete between cracks; from zero stress at a crack location to a maximum half way between cracks. (see Figure 3.12). The effect of tensile strains is to reduce the compressive strength in the direction parallel to the cracks.

To account for this effect, several compression-softening expressions have been proposed and implemented in SPARCS.

3.3.6.1 Vecchio and Collins (1982)

The reduction factor $K_c$ ($K_c \leq 1.0$) to account for compression softening is calculated as:

$$K_c = \frac{1}{0.85 - 0.27 \frac{\varepsilon_{ct}}{\varepsilon_d}}$$

(3.42)

where $\varepsilon_{ct}$ is the tensile strain in the direction normal to the compressive strain $\varepsilon_d$. It is assumed that corresponding principal strains and stresses have the same direction in the MCFT. The peak compressive stress, and the strain at peak stress, assuming no strength enhancement (i.e., $K_c = 1.0$ or $K_c = 1.0$) are:
\[ f_p = K_d f' \]  \hspace{1cm} (3.43)

\[ \varepsilon_p = K_d \varepsilon_o \]  \hspace{1cm} (3.44)

3.3.6.2 Vecchio and Collins (1986)

\[ K_d = \frac{1}{0.80 - 0.34 \frac{\varepsilon_{dl}}{\varepsilon_o}} \]  \hspace{1cm} (3.45)

where \( \varepsilon_o \) is the strain at \( f'_p \) for a uniaxially compressed cylinder.

3.3.6.3 Vecchio 1992-A

\[ K_d = \frac{1}{1.0 - 0.35 \left[ \frac{-\varepsilon_{dl}}{\varepsilon_d} - 0.28 \right]^{0.8}} \]  \hspace{1cm} (3.46)

3.3.6.4 Vecchio 1992-B

\[ K_d = \frac{1}{1.0 - 0.27 \frac{\varepsilon_{dl}}{\varepsilon_o} - 0.37} \]  \hspace{1cm} (3.47)

In all cases the peak stress \( f_p \) and \( \varepsilon_p \) are obtained from Eq. (3.43) and (3.44).

A general sketch of the \( f_p \) versus \( \varepsilon_{df} \) relationship is shown in Fig. 3.14.

---

Modeling of Confined Concrete
3.3.7 Variable Poisson’s Ratio

Concrete subjected to compression expands laterally. (see Figure 3.15).

As compressive strains increase, the Poisson’s ratio $v$ also increases from its initial value. An expression proposed by Vecchio (1992) and slightly modified in the current version of SPARCS is presented.

$$v = v_0 \left[ 1.0 + 15 \left( \frac{2.0 \varepsilon_{ul}}{\varepsilon_o} - 1.0 \right)^2 \right] \leq 0.5$$

(3.48)

where $\varepsilon_{ul}$ is the compressive strain in $l$-direction, and $\varepsilon_o$ is the strain at peak unconfined stress $f'_u$, and $v_o$ is the initial Poisson’s ratio. The original formula (Vecchio 1992) includes $\varepsilon_p$ instead of $\varepsilon_o$ in Eq. (3.48). The Poisson’s ratio is depicted in Figure 3.16.

Figure 3.14 Compression Softening Effect

Figure 3.16 Lateral Expansion
Changes in Poisson's ratio are expected if the compressive strain \( \varepsilon_{c0} > 0.5 \varepsilon_0 \) as indicated in Fig. 3.16. This threshold value could be considered as the beginning of nonlinear behavior of normal strength concrete. The maximum value allowed in SPARCS is \( \nu_{\text{max}} = 0.5 \).

### 3.4 Constitutive Model for Steel

The monotonic response of steel was modeled with a simple elasto-plastic model with strain hardening. (Figure 3.17.) Although SPARCS has models for hysteretic response of steel in cyclic loading, they will not be presented in this work.
where $\varepsilon_y$ is the yielding strain, $\varepsilon_{sh}$ is the strain at hardening, $E_s$ is the initial stiffness, $E_{sh}$ is the strain hardening modulus, $f_y$ is the yielding stress and $f_u$ is the rupture stress. The model is valid for either compressive or tensile stresses.

### 3.5 SPARCS Structure

The material matrix of Eq.(3.1), $[D]$, comprises the material matrices for concrete and smeared steel; $[D_s]$ and $[D_o]$, respectively (in global coordinates). $[D_o]$ is the sum of the steel material matrices, calculated as:

$$ [D_o] = \sum_{i=1}^n [D_{oi}] $$

(3.49)

where $n$ is the number of steel components. Thus, the total material matrix can be written as:

$$ [D] = [D_c] + \sum_{i=1}^n [D_{oi}] $$

(3.50)

The orthotropic material stiffness matrix for concrete (Selby and Vecchio 1993) in principal directions is:

$$ [D_c'] = \begin{bmatrix}
E_{c1} & 0 & 0 & 0 & 0 & 0 \\
0 & E_{c1} & 0 & 0 & 0 & 0 \\
0 & 0 & E_{c1} & 0 & 0 & 0 \\
0 & 0 & 0 & G_{c12} & 0 & 0 \\
0 & 0 & 0 & 0 & G_{c22} & 0 \\
0 & 0 & 0 & 0 & 0 & G_{c13}
\end{bmatrix} $$

(3.51)
where $E_{il}$ is the secant modulus in the principal direction $l=1,2,3$. $G_f$ is the shear modulus given by

$$G_{ij} = \frac{E_{il}E_{lj}}{E_{il}(1+v_{ij}) + E_{lj}(1+v_{ji})}$$

(3.52)

where $v_i$ is the Poisson's ratio component of strain in the $i$-direction due to a stress in the $j$-direction.

The smeared steel material matrix in $i$-direction is given by:

$$[D_i] = \begin{bmatrix}
\rho_s \frac{f_{si}}{\epsilon_{si}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

(3.53)

where $f_{si}$ and $\epsilon_{si}$ are the stress and strain in the $i$-direction, respectively, and $\rho_s$ is the reinforcement ratio in that direction. It is noted that the secant modulus for steel is determined as:

$$E_{si} = \frac{f_{si}}{\epsilon_{si}}$$

(3.54)

Matrices $[D_i]$ and each $[D_i]$, must be transformed from principal directions to global coordinates before summing them up.

At any load stage the secant moduli depend on the stress-strain condition in the direction being analyzed. In the case of concrete (see Fig. 3.18):
\[ E_{cl} = \frac{f_{cl}}{\varepsilon_{cl}} \]

(3.55)

where \( \varepsilon_{cl} \) is the stress-related strain in that direction. Other strains due to prestress, thermal loading or lateral expansion are introduced as "prestrains" (Vecchio 1992) and converted into equivalent forces. As lateral expansion changes at each load stage according to the strain state, its related prestrains are updated at every iteration.

Figure 3.18 Secant Modulus Definition

The prestrain vector due to lateral expansion, \( \{\varepsilon_{ep}\} \), can be written as:

\[ \{\varepsilon_{ep}\} = \begin{bmatrix} \varepsilon_{cl} \\ \varepsilon_{c2} \\ \varepsilon_{c3} \end{bmatrix} \]

(3.56)

where the superscript \(^o\) indicates "prestrain".
Lateral expansion is a non-stress related strain, as it has no associated stress in the expansion direction (i.e., diagonal terms in Eq.(3.57) are null).

As mentioned in the introduction of this Chapter, SPARCS is a nonlinear elastic program that solves for displacements as in a linear elastic program. The global stiffness matrix, which is the assembly of element stiffness matrices computed with Eq.(3.2), is updated and used to solve for new displacements as load or imposed nodal displacements are incremented.

A flow chart of the SPARCS solution algorithm adapted from Selby and Vecchio (1993) is shown in Figure 3.19.

Chapter 4 shows the parametric study of the models implemented in SPARCS for the analysis of confined concrete, and the analyses made to determine the variation of Poisson's ratio in columns subjected to monotonic increasing compression.
Figure 3.19 Solution Algorithm for SPARCS (Selby and Vecchio 1993)
Chapter 4

Parametric Study and Poisson’s ratio

Parametric Study

4.1 Introduction

In order to apply the material behavior models presented in Chapter 3 and implemented in program SPARCS- to the analysis of confined concrete, it was necessary to study the influence of each model on the three-dimensional behavior of reinforced concrete when subjected to triaxial stress states.

The parameters analyzed were:

- Pre-peak compression base curves for concrete.
- Post-peak behavior of confined concrete.
- Concrete cracking.
- Confinement enhancement.
- Variable Poisson’s ratio, and
- Compression softening of concrete.
The models for each parameter were used in the analysis of a set of four columns from the experimental work of Sheikh and Uzumeri (1980).

The columns had different tie arrangements, tie spacing, and longitudinal bar cages. They failed in different manners, from brittle to ductile behavior. The set represented a broad spectrum of possible configurations of confined concrete. One limitation would be that all the columns were made of normal strength concrete, and had equal size and shape. A new set of specimens with different size and material properties was analyzed and presented in Chapter 5.

Each column was modeled in SPARCS and analyzed using combinations of the parameters mentioned above. The analytical results were compared with the experimental results, and decisions were made on the most effective models that represented the tests accurately. The selected models were then used in modeling a new set of columns, and the results presented in the next Chapter.

Later in this Chapter, two of the columns of this parametric study were compared with the results obtained by Selby and Vecchio (1993) using a previous version of SPARCS. Improvement in the modeling capabilities of the program was achieved.

4.2 Selection of Parametric Variables

4.2.1 Pre-peak Base Curves for Concrete.

Two of three base curves for pre-peak behavior of concrete were chosen for the parametric analysis: The Hoggestad parabola (Eq. 3.3) and the Hoshikuma et al. curve (Eq. 3.9). The generalized Popovics curve (Thorenfeldt
et al. Eq. 3.4) was not accounted for as it would “degenerate” into a parabola at lower concrete strength levels.

4.2.2 Post-peak Base Curves for Concrete.

All three models included in SPARCS were used in this parametric study, those were: A modified version of the Modified Kent and Park (MKP) model (Eq. 3.12), The Popovics curve (Eq.3.16), and the Hoshikuma et al. curve (Eq.3.18).

The first and third models are straight lines that depend directly on the definition of lateral pressure. The Popovics curve is indirectly related to the lateral pressure through the strength enhancement factors discussed in section 3.3.5.

4.2.3 Concrete Cracking

The “stress” formulation of the Mohr-Coulomb criterion (Eq. 3.30) was selected for the parametric study. The criterion has extensively been used to predict concrete failure in biaxial compression-tension, or uniaxial compression (e.g., test cylinders) (see Chen 1982). In columns, concrete cover fails at an earlier stage than the confined core due to the core expansion, formation of cracks, and concrete crushing.

4.2.4 Confinement Enhancement

Both enhancement factors implemented in SPARCS were used: Selby’s (Eq. 3.34), and Vecchio’s (Eq. 3.37). Both strength enhancement criteria have
different definitions of lateral pressure (see Eqs 3.14, 3.38, and 3.39). In any triaxial stress state, the lateral pressure defined by Vecchio will always be smaller than Selby's.

It should be noted that the enhancement factor \( K_e \) was also used in calculating both peak stress and peak strain. The previous version of the program had a factor \( K_e \) to compute the peak strain. (Selby and Vecchio 1993)

4.2.5 Variable Poisson's Ratio

Although SPARCS has the option of a constant Poisson's ratio \( \nu_e \), it was decided to keep it variable. Its variability was corroborated using the experimental results of Liu et al. (1998) columns, and is described at the end of this Chapter. The suggested expression for \( \nu \) in Eq.(3.48) was then used.

4.2.6 Compression Softening

The Vecchio-A model (1992) (Eq. 3.46) was selected. As this parametric study was the first attempt made to evaluate the sensibility of the tentative models for confined concrete, no preference was given to any particular softening model. If softening is not considered, the "softening factor" \( K_e \) is equal to 1.0.

4.3 Sheikh and Uzumeri Tests (1980)

This section describes the set of columns tested by Sheikh and Uzumeri (1980), and the variables studied.
4.3.1 Column Geometry

A set of 24 short columns was tested under monotonic axial compression. Each column was 305-mm square and 1960-mm high; 368-mm tapered ends confined with welded steel plates were built before casting to prevent failure away from the testing zone. Details of the column geometry are shown if Fig. 4.1.

![Diagram of Column Dimensions](image)

Figure 4.1 Column Dimensions

4.3.2 Longitudinal Bar and Tie Setup

Deformed and plain bars were used for the longitudinal and tie reinforcement, respectively. The centre-to-centre spacing between outermost ties was 267 mm in all specimens, providing a constant core area equal to 77% of the gross area, but with different cover thicknesses.
Four arrangements were selected by the authors, and shown in Fig. 4.2. Each arrangement was symmetric with respect to centre-line axes, and the number of longitudinal bars and spacing was kept constant along section edges. Longitudinal bars in a set had equal diameter. The minimum number of bars in a column was 8.

The lateral reinforcement was provided by 2, 3, or 4 closed ties or "seismic hoops" (see ACI-318-95R, CSA23.3-94); each one supported at least 4 longitudinal bars, with the exception of the internal tie in the D setup (see Fig. 4.2) that supported 8 bars. Tie diameter was also kept constant for both external and internal ties in a section. In order to prevent undesired failures, tie hooks were extended 14 tie-diameters into the concrete core.

![Diagram of four arrangements](image)

**Figure 4.2. Sheikh-and-Uzumeri Column Sections**

The spacing between ties in the test region (see Fig. 4.1) was defined in terms of the centre-to-centre vertical distance.
4.3.3 Test Instrumentation and Procedure

The 24 columns were instrumented with strain gauges to measure deformations in the longitudinal steel, ties and concrete. Each hoop in a set near the column midheight had four strain gauges. The load versus axial shortening was measured using two linear variable differential transformers (LVDT) placed at opposite sides of the column. Column tests lasted 3 to 6 hours.

The experimental variables studied were: longitudinal bar setup, tie configuration and volumetric ratio, tie spacing, and tie steel properties.

4.3.4 Selected Sheikh and Uzumeri Columns

Four of the 24 columns tested by Sheikh and Uzumeri were selected for the parametric study. Each column had one of the tie configurations showed in Fig. 4.2, and differed in volumetric ratios, tie spacing and longitudinal bar setup. Failure modes varied from brittle to ductile, thus giving a reasonable spectrum of behaviors that might be modeled with SPARCS. The purpose of the parametric study was to establish a combination of the implemented material models in the program that best represented the actual column behavior.

The material properties of the selected columns are shown in Table 4.1, where $d_b$ is the longitudinal bar diameter, $A_e$ is the total longitudinal bar cross sectional area in the column, $\rho_s$ is the longitudinal steel ratio with respect to the gross section, $f_y$ is the longitudinal steel yielding stress, $E_s$ is the initial steel stiffness, $E_{sh}$ and $\varepsilon_{sh}$ are the stiffness modulus and strain at hardening,
respectively. Also, \( d_b \) and \( A_n \) are the tie diameter and area, \( \rho_v \) is the volumetric ratio, \( f_{ty} \) is the tie yielding stress, \( s \) is the spacing between sets of ties.

### TABLE 4.1. Sheikh-and-Uzumeri Selected Column Properties

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Conf. type</th>
<th>( d_b ) [mm]</th>
<th>( A_n ) [mm²]</th>
<th>( \rho_v ) [%]</th>
<th>( f_{ty} ) [MPa]</th>
<th>( E_o ) [MPa]</th>
<th>( E_{con} ) [MPa]</th>
<th>( e_{con} ) [mm/m]</th>
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<tbody>
<tr>
<td>Column (1)</td>
<td>Conf. type</td>
<td>( d_b ) [mm]</td>
<td>( A_n ) [mm²]</td>
<td>( \rho_v ) [%]</td>
<td>( f_{ty} ) [MPa]</td>
<td>( E_o ) [MPa]</td>
<td>( E_{con} ) [MPa]</td>
<td>( e_{con} ) [mm/m]</td>
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<td>1600</td>
<td>1.72</td>
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<td>3.67</td>
<td>392</td>
<td>198400</td>
<td>6200</td>
<td>7.80</td>
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</table>

<table>
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<th>Specimen</th>
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<th>( d_b ) [mm]</th>
<th>( A_n ) [mm²]</th>
<th>( \rho_v ) [%]</th>
<th>( f_{ty} ) [MPa]</th>
<th>( E_o ) [MPa]</th>
<th>( E_{con} ) [MPa]</th>
<th>( e_{con} ) [mm/m]</th>
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<tbody>
<tr>
<td>Column (1)</td>
<td>Conf. type</td>
<td>( d_b ) [mm]</td>
<td>( A_n ) [mm²]</td>
<td>( \rho_v ) [%]</td>
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<td>( E_o ) [MPa]</td>
<td>( E_{con} ) [MPa]</td>
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<td>198500</td>
<td>38.1</td>
<td>35.9</td>
</tr>
</tbody>
</table>

* Calculated by the writer

Column (2) in Table 4.1 identifies the cross section type of Fig. 4.2, and the label in column (1) was given by the researchers. The volumetric ratio is defined as the volume of tie steel to volume of concrete ratio; \( \rho \), varied from highly confined column (2.37%) to poor-confined (0.80%). Tie spacing varied from 0.125 \( b \) to 0.33 \( b \), where \( b \) is the column size (305 mm). Finally, the average standard cylinder strength was 35 MPa. (i.e., normal strength concrete)
4.4 Finite Element Models

4.4.3 Geometry

Eight-noded hexahedrons and truss bars were used to model concrete and ties, respectively. Due to the symmetry of the columns, only one quarter was modeled. Longitudinal bars were smeared out, and truss bars were attached to two nodes of the same brick as shown in Fig. 4.3. The number of hexahedrons in a section perpendicular to the column longitudinal axis (hereafter called layer), and the number of layers for each column described in Table 4.1 varied from 13 to 17. Each layer contained 36, 49 or 64 hexahedrons. The layer depth depended on the tie spacing for each column. An example of layer setup is given in Fig. 4.4 for column 2A1-1.

![Diagram](image)

Figure 4.3 Section Detail for 2A1-1
A vertical view of the finite element configuration is presented in Fig. 4.4a. Imposed displacements were applied to the top layer, which had stronger concrete properties and smeared steel in the three global directions. The top layer was necessary to better distribute the imposed displacements through the column depth. The disturbance caused by the presence of the stiffer layer at the top (e.g., lateral restraint) was overcome with the amount of layers used in each model. An effort was made to keep hexahedron dimensions approximately equal (i.e., aspect ratio 1:1:1), and to avoid early failure of "flat" bricks due to large deformations; for this reason, a limited number of layers were used between tie setups (from 1 to 4 for the examined columns). However, this restriction did not compromise the overall column behaviour, as will be explained later.

![Figure 4.4 Finite Element Setup](image-url)
The bottom layer was restrained against axial displacements, but allowed to deform laterally. Lateral restraints along the axes of symmetry are also shown in Fig. 4.4b. Column meshes included both cover and core concrete. Table 4.2 gives the geometric properties of all the modeled columns.

### TABLE 4.2 Finite Element Model Geometry

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Geometry</th>
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<tr>
<td>Column</td>
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</tr>
<tr>
<td>4B3-10</td>
<td>17</td>
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<tr>
<td>2C5-17</td>
<td>13</td>
</tr>
<tr>
<td>4D6-24</td>
<td>14</td>
</tr>
</tbody>
</table>

#### 4.4.4 Material Types

Longitudinal reinforcement was smeared over a small area corresponding to the location of each bar. The steel ratio $\rho$ within a brick depends on the actual bar size and the brick dimensions. Figure 4.5 shows the different material types specified in SPARCS, and Table 4.3 shows the longitudinal steel ratios and concrete properties for each material type of the Sheikh-and-Uzumeri columns.
TABLE 4.3 Element Material Types

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Material Type</th>
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<th></th>
<th></th>
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<td>Type 1</td>
<td>Type 2</td>
<td>Type 3</td>
<td>All types</td>
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<td>f'_c</td>
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<td>ρ [%]</td>
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<td>[MPa]</td>
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<td>(5)</td>
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<td>——</td>
<td>30.50</td>
<td>1.86</td>
</tr>
</tbody>
</table>

The tensile strength for concrete given in column (6) of Table 4.3 was computed using:
\[ f' = 0.33 \sqrt{f'_c} \]  
\[(4.3)\]

It is widely accepted that the plain concrete column strength to standard cylinder strength ratio is about 0.85 (e.g., Razvi et al. (1996) showed results with a ratio of 0.89 for high strength concrete columns). Instead of using the unconfined concrete strength \( f'_c \) in this study, the values in column (5) of Table 4.3 were assumed in the finite element analyses.

Smeared steel and truss bars were modeled using the material properties of Table 4.1, and followed the strain-hardening model depicted in Fig. 3.17.

### 4.4.5 Parameter Combinations

From all the possible combinations of models selected in section 4.2, only those shown in Table 4.4 were chosen for running each column in SPARCS. The set of models that best represented the actual behavior of all the columns was also used in corroborating the experimental results of other specimens.

<table>
<thead>
<tr>
<th>Table 4.4 Parametric Combinations for SPARCS Analyses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7*</td>
</tr>
</tbody>
</table>

* Used only with 2CS-17 Column, MC: Mohr-Coulomb
The softening model specified in column (7) of Table 4.4 was the Vecchio-A (1992) model (Eq.(3.46)).

4.4.6 Analysis Procedure

For each column model:

- A parameter combination according to Table 4.4 was selected in SPARCS material specifications.
- The geometry and restraint conditions were checked.
- An initial imposed displacement, and a number of load stage increments were established.
- The reactions at the bottom layer were computed for each load stage, and the load versus axial strain was graphed.
- If the pairs of axial load-axial strain points were too sparse, due to a large imposed displacement, a new run with a smaller displacement was done, until a smooth curve was obtained.
- A new combination of models was chosen and SPARCS was run again with the new parameters.
- The axial load versus axial strain for each model combination was plotted and compared.

The next section shows the analysis results for the Sheikh and Uzumeri columns.
4.5 Analysis Results

Complete analysis results for column 2A1-1 are given next. A summary of the principal results for the rest of columns is presented in this section.

4.5.3 Column 2A1-1

In order to provide an inside view of the cumbersome finite element results; only detailed results obtained from combination 2 of Table 4.4 column(1) are presented. The axial load versus axial strain for the other combinations will be presented in a summary graph.

Recalling combination 2, the models studied were:

Pre-peak curve for concrete: Hognestad
Post-peak curve for concrete: Popovics
Confinement enhancement: Vecchio
Cracking criterion: Mohr-Coulomb
Poisson's ratio: Variable
Concrete Softening: Vecchio-A

4.5.1.1 Cross Section and Profile Results

The variation of axial stress in concrete for various load stages in a typical cross section is shown in Fig. 4.6. The ascending portion of the concrete curve is presented in Fig. 4.6 (a) and (b). The figures show a uniform stress distribution with small differences among elements (see color scale). There is little difference between cover and core behaviors. Fig. 4.6 (c) shows the stress...
state when the model had reached the maximum analytical load. \( (P_{\text{max}}: 3370 \text{ KN}) \). The difference between the concrete shell and the core behaviors became evident; the shell is now in the descending portion of the curve and crushing. The core elements had reached stresses almost equal to that of the standard cylinder strength. It should be noted that the plain concrete strength for the SPARCS analysis was set to 0.85 \( f'_c \) \( (f'_c = 37.5 \text{ MPa}) \), thus lateral reinforcement was activated. Fig. 4.6 (d) shows the stress state before failure, a considerable area of the concrete core could not carry more load and the column failed.

Figure 4.6 Concrete Axial Stress States for Combination 2 (2A1-1)
The axial strain to strain at peak stress ratio; \( \varepsilon_{ax} / \varepsilon_p \), for the same load stages is graphed in Fig. 4.7. Concrete behavior at the first load stage (Fig. 4.7a) was linear elastic; concrete strains were uniformly distributed and smaller than 40% of peak strains. As the load approached 90% of the peak analytical load (Fig. 4.7b), the strain-ratio distribution remained unaltered between cover and core. Nonlinearity was evident in the ascending branch, with the cover and core concrete following different paths as load increased to its peak value (Fig. 4.7c). The concrete cover then failed due to softening of concrete and supported no load afterwards. At failure (Fig. 4.7d), some portions of the concrete core reached a strain ratio of about 2.0 (i.e., a strain twice larger than peak strain), and were no longer able to carry load. It could be concluded that the column did not develop ductility.

A "wire frame" view of the vertical deformation is shown in Fig. 4.8 for the load stages mentioned above. Also shown is the average axial strain at each load stage \( \varepsilon_{ave} \). The peak strain (i.e., strain at peak load) was \(-3.0 \times 10^3\) for this model combination. The average strain at failure was about twice the peak strain. As the Popovics post-peak curve used in this analysis can "degenerate" into a parabola, it will be seen later that this was the case for this combination where the load carrying capacity declined sharply after peak load.

The last of the column-profile and cross-section graphs shows the variation of the concrete lateral strain in one of the cross sectional directions (Fig. 4.9). The behavior in the other direction was symmetric.
Figure 4.7 Variation of $\varepsilon_{cl}/\varepsilon_p$ Ratio for Combination 2 (2A1-1)

At the first load stage (Fig. 4.9a) restraint due to lateral steel was not perceptible; however, it was possible to notice the "arching" variation of lateral stresses between longitudinal supported bars. Before peak load (Fig. 4.9b), the lateral steel was not activated, but cover elements had cracked at strains of $-1.524 \text{ mm/m}$. At $P_{\text{max}}$ confinement of the core occurred to a certain degree, lateral stresses were high near longitudinal bars and decreased with the distance from the supported bars. Near failure, some elements were in lateral compression within the diagonal band shown in Fig.4.9d.
Figure 4.8 Profile Deformation, Column 2A1-1 (Combination 2)

4.5.1.2 Load-Deformation Curves for Combination 2

The axial load versus average axial strain curve for the 2A1-1 model is presented in Fig. 4.10, along with the experimental curve. In the same manner, Figure 4.11 shows the axial load versus average strain in outer tie. Comparisons among the model combinations and the experimental results will be discussed later for all columns.
Figure 4.9 Concrete Lateral Stress States, 2A1-1, Combination 2

Figure 4.10 Response of 2A1-1, Combination 2
The average tie strain at peak load was $1.4 \times 10^{-3}$ in the analysis, which was half the yield strain for this column ($\varepsilon_y = 2.7 \times 10^{-3}$, $f_y = 540$ MPa). The lateral pressure was insufficient in developing strength enhancement and ductility. It should be noted that the volumetric ratio for this column was the lowest of the set ($\rho_v = 0.80\%$), even though tie spacing was small ($0.19b$).

4.5.1.3 Summary of Load-Deformation Curves

The load versus average axial strain curves for the remaining model combinations of Table 4.4 are given in Figs. 4.12 to 4.16. Figure 4.17 and 4.18 show plots of all the combinations, and Table 4.5 shows the analytical to experimental ratios for the peak load and peak strain.
Figure 4.12 Axial Response of 2A1-1, Combination 1

Figure 4.13 Axial Response of 2A1-1, Combination 3
Figure 4.14 Axial Response of 2A1-1, Combination 4

Figure 4.15 Axial Response of 2A1-1, Combination 5
Figure 4.16 Axial Response of 2A1-1, Combination 6

Figure 4.17 Axial Response of 2A1-1, All Model Combinations
Figure 4.18 Lateral Reinforcement Response of 2A1-1, All Combinations

TABLE 4.5 Analytical to Experimental Ratios for 2A1-1

<table>
<thead>
<tr>
<th>Comb. Number</th>
<th>$P_{max}$ analytical [KN]</th>
<th>$P_{max}/P_{test}$</th>
<th>$\varepsilon_p$ analytical [mm/m]</th>
<th>$\varepsilon_p$ (anal.)/$\varepsilon_p$ test</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>3361</td>
<td>0.98</td>
<td>3.004</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>3369</td>
<td>0.99</td>
<td>2.984</td>
<td>0.93</td>
</tr>
<tr>
<td>3</td>
<td>3689</td>
<td>1.08</td>
<td>2.447</td>
<td>0.83</td>
</tr>
<tr>
<td>4</td>
<td>3875</td>
<td>1.08</td>
<td>2.435</td>
<td>0.83</td>
</tr>
<tr>
<td>5</td>
<td>3366</td>
<td>0.99</td>
<td>3.337</td>
<td>0.83</td>
</tr>
<tr>
<td>6</td>
<td>3659</td>
<td>1.07</td>
<td>2.727</td>
<td>0.76</td>
</tr>
</tbody>
</table>

$P_{max}$ (test): 3418 KN  
$\varepsilon_p$ (test): 3.6 mm/m
Although conclusions will be made later on the model combination that best represents the failure behaviors studied, a brief comment on the presented curves follows.

In terms of peak load and peak strain, the best results were obtained with combinations 1, 2 and 5; the difference among these models was the post-peak curve. Combination 5 had the Hoshikuma et al. model for post-peak behavior, the Popovics curve was used in combination 2, and the modified Kent and Park model (MKP) in combination 1. The Hoshikuma et al. model was developed for concrete piers of bridges, with low volumetric ratios (less than 0.50%) as was the case of this column.

Combinations 3, 4 and 6 overestimated the peak load and underestimated the peak strain. In this case, combinations 3 and 4 used the Selby model for confinement, and MKP and Popovics for post-peak behavior, respectively. No difference was apparent in the descending branches.

In general, the behavior of column 2A1-1 was captured with good accuracy for all model combinations of SPARCS. Although the specimen had poor confinement and showed no ductility, it was not tested to failure. Concrete cracked at longitudinal compressive strains of between -1.5 and -2.0 mm/m, and the lateral steel did not yield at peak (see Sheikh et al. 1980). The average tie strain predicted by SPARCS was smaller than the actual tie strain measured at $P_{\text{max}}$ in the test ($\epsilon_{\text{anal.}}$: $1.4 \times 10^{-3}$, $\epsilon_{\text{test}}$: $1.9 \times 10^{-3}$).
4.5.2 Column 4B3-19

The axial response of column 4B3-19 is summarized in Figs. 4.19 to 4.25, for all the model combinations. This specimen sustained the peak load over a slightly larger range of axial strain than column 2A1-1, thus exhibiting some ductility. However, due to the larger tie spacing, which was 0.33b, its capacity dropped sharply.

From the analysis, all combinations yielded somehow the same general axial response, with one peak at the beginning of the plateau of the experimental curve, followed by a descending branch, which extended approximately up to the same longitudinal strain as in the experimental curve.

The plateau in the test occurred at an axial strain of -3 x 10^-3 to -8 x 10^-3. The researchers reported that at an strain of -6.1 x 10^-3 the column reached the maximum capacity and the average tie strain was 2.3 x 10^-3. It was found in the analysis that at the analytical peak load the average tie strain was between 0.8 x 10^-3 and 1.3 x 10^-3; and that in the descending branch; at an axial strain of -6.0 x 10^-3, the average outer tie strain was between 2.3 x 10^-3 and 2.6 x 10^-3.

The cover concrete cracked at an axial strain of between -1.2 x 10^-3 and -1.8 x 10^-3, and by the time the load reached the peak load, the cover had spalled off.

A gain in axial capacity after peak was observed for model combinations 3 and 4. (Figs. 4.21, 4.22). This was more pronounced for model combination 3. After the column lost its cover, the concrete core was able to assume the load
shed by the cover and reached the initial peak load level. Both combinations used the Selby model for confinement.

Combination 6 is similar to combination 2 but without compression softening considered. The decline in the descending branch was less apparent for the former and exhibited more ductility than combination 2 curve.

Finally, Table 4.6 presents the analytical to experimental peak strain and peak load ratios for this column.
Figure 4.20 Axial Response of 4B3-19, Combination 2

Figure 4.21 Axial Response of 4B3-19, Combination 3
Figure 4.22 Axial Response of 4B3-19, Combination 4

Figure 4.23 Axial Response of 4B3-19, Combination 5
Figure 4.24 Axial Response of 4B3-19, Combination 6

Figure 4.25 Axial Response of 4B3-19, All Model Combinations
From Table 4.6 combinations 1, 2, and 5 give the best predictions for the peak load, and combinations 3, and 4 give good predictions of peak strain. It should be noted that the experimental peak strain was taken as the strain at the beginning of the plateau in the test curve.

All combinations overestimated the strength of the column, but combination 2 gave the least difference with respect to the measured capacity (43 KN). The confinement model proposed by Selby produced the highest values for lateral pressure in the core.

Although the volumetric ratio of this specimen was relatively high ($\rho_v: 1.80\%$), the tie spacing was large and the concrete core at mid-level between ties behaved unconfined after peak load had been reached. Further deterioration of the concrete core impeded the lateral steel from developing efficient pressure, and the descending post-peak response started just after the peak load without recovering capacity.
The program seems to capture the effect of tie spacing in the global shortening response of the specimen.

4.5.3 Column 2C5-17

A brief review of the cross section results is presented, followed by the axial and tie responses.

4.5.3.1 Cross Section Results

Column 2C5-17 had the highest volumetric ratio of the set ($\rho_v = 2.37\%$), and a tie spacing of 101.6 mm. (or $0.33b$). The degree of confinement at the tie level and at mid-height between ties is shown in Fig. 4.26 at the first peak load for model combination 2.

![Figure 4.26](image)

**Figure 4.26 Confinement of 2C5-17, Combination 2**

This figure shows the axial concrete strain to peak strain ratio $\varepsilon_{ct} / \varepsilon_p$. Although the color scale is different for each level, the concrete core at tie level exhibited lower ratios than that of the core midway between ties.
Strain was 1.8 x 10^-3 at peak load. The reported average peak load (see axial response of 2C5-17 graph) was 1.7 x 10^6 at the first peak load, and 2.4 x 10^6 at the second peak load. The average strain (i.e., the sum of all the tensile strains in perimeter hoop). The average strain at the center core was more stressed than the most stressed tensile bars are shown in red and the least stressed are in blue. As shown in the colored sketch of this distribution after the first peak load, for combination 2, the axial strain in the longitudinal and external hoop can also be analyzed with the help of this finite element solution. Figure 4.27 shows a comparison of the core center and the column collapsed. Provided no lateral support for the central elements, consequently, lateral pressure acting on the core finite elements near the concrete cover was smaller. The effectively confined area at midlevel was reduced even further with...
4.5.3.2 Summary of Load-Deformation Curves

The column 2C5-17 had a complex tie arrangement, with the largest number of longitudinal bars (i.e., the smallest centre to centre distance between longitudinal reinforcement). It also had the largest volumetric ratio and tie spacing. Ductile response was expected from the analyses. The axial response is depicted in Figs. 4.28 to 4.35; tie response is graphed in Fig. 4.36, and the analytical to experimental peak ratios are given in Table 4.7.

In general, the axial load-axial shortening curves presented two peaks. The first peak occurred as cover spalled off; a slight decrease in capacity then occurred due to the cover loss, followed by a gain in strength due to the activation of the lateral steel up to a second peak. After the last peak, the column model was able to carry the load without losing resistance during large deformations. Due to the large tie spacing, cross sections at midlevel between ties collapsed and the model carried no further load.

Peak values varied for each combination. The first peak was smaller than the second peak for model combinations 1, 3, and 4. On the other hand, the first peak was slightly larger for combination 2, and both peaks were equal for combination 7. Combination 5, which used the Hoshikuma et al. model for post-peak behavior only captured one peak load and descended steeply.

Initial stiffness for all models was very similar to that of the specimen, but all showed a stiffer response at first peak. Although model combinations 3 and 6 presented the highest peak load ratios (1.25), they exhibited the largest ductilities. In the case of combinations 2 and 6, which were similar except for the
compression-softening model activated in combination 2, it was apparent that some degree of concrete softening should have occurred between ties where confinement is not totally efficient. Both curves could be judged as the lower and upper limits when the effect of tensile strains is taken into account.

**Figure 4.28 Axial Response of 2C5-17, Combination 1.**
Figure 4.29 Axial Response of 2C5-17, Combination 2.

Figure 4.30 Axial Response of 2C6-17, Combination 3.
Figure 4.31 Axial Response of 2C5-17, Combination 4.

Figure 4.32 Axial Response of 2C5-17, Combination 5
Figure 4.33 Axial Response of 2C5-17, Combination 6

Figure 4.34 Axial Response of 2C5-17, Combination 7
**Figure 4.35 Axial Response of 2C5-17, All Model Combinations**

**TABLE 4.7 Analytical to Experimental Ratios for 2C5-17**

<table>
<thead>
<tr>
<th>Comb. Number</th>
<th>First Peak analytical [KN]</th>
<th>Second Peak analytical [KN]</th>
<th>$P_{max}/P_{test}$</th>
<th>$\varepsilon_p$ (first) analytical [mm/m]</th>
<th>$\varepsilon_p$ (second) analytical [mm/m]</th>
<th>$\varepsilon_p$ (anal.)/$\varepsilon_p$ test$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3528</td>
<td>3807</td>
<td>1.08</td>
<td>3.343</td>
<td>9.681</td>
<td>0.62</td>
</tr>
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<td>3622</td>
<td>3570</td>
<td>1.03</td>
<td>3.291</td>
<td>8.373</td>
<td>0.53</td>
</tr>
<tr>
<td>3</td>
<td>3820</td>
<td>4390</td>
<td>1.25</td>
<td>3.500</td>
<td>12.180</td>
<td>0.78</td>
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<tr>
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<td>3798</td>
<td>4140</td>
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<td>3.534</td>
<td>9.763</td>
<td>0.62</td>
</tr>
<tr>
<td>5</td>
<td>3625</td>
<td>——</td>
<td>1.03</td>
<td>3.267</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>6</td>
<td>——</td>
<td>4355</td>
<td>1.24</td>
<td>——</td>
<td>14.640</td>
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<tr>
<td>7</td>
<td>3854</td>
<td>3854</td>
<td>1.09</td>
<td>4.267</td>
<td>11.228</td>
<td>0.72</td>
</tr>
</tbody>
</table>

$^*$Obtained from Shahid et al. (1960)

$P_{max}$ (test): 3524 kN

$\varepsilon_p$ (test): $15.7 \text{ mm/m}$
Load versus average tie strain curves are plotted in Fig. 4.36. Model combinations 3, 4, 6, and 7 reached the second peak just after yielding of lateral steel ($\sigma_{y}: 2.4 \text{ mm/m}$), and combinations 1 and 2 reached their peak at an average tie strain between 1.8 and 2.4 mm/m. As mentioned above, the tie strain at peak for the specimen was 1.8 mm/m. It was apparent from Fig. 4.36 that a change in stiffness occurred at a tie strain of about 0.3 mm/m, which occurred at an axial strain of -1.6 mm/m. This corresponded to the initiation of cracks on the concrete cover.

![Graph of Load vs. Average Tie Strain](image)

**Figure 4.36 Lateral Reinforcement Response of 2C5-17, All Combinations**

### 4.5.4 Column 4D6-24

This column had a slightly lower volumetric ratio than 2C5-17 ($\rho_v : 2.30\%$), and a smaller tie spacing ($s : 0.13b$).
The response to monotonic compression of the specimen was ductile due to the close-spaced steel cage. There was one layer of bricks between tie setups for the finite element model, thus avoiding "flat" elements (i.e., material instability) and the aspect ratio was kept close to 1:1:1.

All model combinations but combination 6 showed two-peak curves, as could be the case for well-confined columns. The program was able to predict cover spalling (at an axial strain of about \(-1.6 \text{ mm/mm}\), the maximum peak load in a range of 96% to 109% of the actual peak load, and to a certain extent the observed ductility. The lateral steel yielded after the first peak but well before the second peak in the majority of cases. For instance, taking combination 2, the average tie strain was 1.5 \(\text{ mm/mm}\) for the first peak and 5.2 \(\text{ mm/mm}\) for the second peak. It should be noted that the experimental average tie strain was 4.5 \(\text{ mm/mm}\) at peak.

A plot with all the combinations is shown in Fig. 4.37. It is apparent that combination 6 is the best fit of the experimental curve. Recalling from Table 4.4, this was the only combination where compression softening of concrete was not taken into account. As longitudinal and lateral steel encased the concrete core very closely, triaxial compression stresses developed for large deformations, and the capacity was sustained.

Table 4.8 gives the analytical to experimental peak load and peak strain ratios for this column. Although an axial strain of \(-17.7 \text{ mm/mm}\) was reported at peak load in the experiment, a plateau stretching from about \(-3.5\) to \(-25.0 \text{ mm/mm}\) was observed, and column (7) of Table 4.8 may lack of accuracy.
4D6-24

Figure 4.37 Axial Response of 4D6-24, All Model Combinations

TABLE 4.8 Analytical to Experimental ratios for 4D6-24

<table>
<thead>
<tr>
<th>Comb. Number</th>
<th>First Peak analytical [KN] (2)</th>
<th>Second Peak analytical [KN] (3)</th>
<th>$P_{\text{max}}/P_{\text{test}}$ (4)</th>
<th>$\varepsilon_p \text{ (first)}$ analytical [mm/m] (5)</th>
<th>$\varepsilon_p \text{ (second)}$ analytical [mm/m] (6)</th>
<th>$\varepsilon_p \text{ (anal.)}/\varepsilon_p \text{ test}$ (7)</th>
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<tr>
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<td>0.99</td>
<td>3.529</td>
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<td>4651</td>
<td>0.98</td>
<td>3.516</td>
<td>11.784</td>
<td>0.66</td>
</tr>
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<td>1.03</td>
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<td>8.379</td>
<td>0.47</td>
</tr>
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</table>

$P_{\text{max (test)}}$: 4725 kN
$\varepsilon_p \text{ (test)}$: 17.7 mm/m

Modeling of Confined Concrete
An analysis of the sensibility of all the parametric variables in the model combinations is presented in the next section, and conclusions will be drawn for the model combination(s) that best fit all of the column behaviours studied.

4.6 Effects of Model Combination on the Response of the Selected Columns

Table 4.4 is reproduced in this section to facilitate the description of the parameters involved in the study of confined concrete behavior.

TABLE 4.4 Parametric Combinations for SPARCS Analyses

<table>
<thead>
<tr>
<th>Comb. Number</th>
<th>Pre-peak</th>
<th>Post-Peak</th>
<th>Confinement</th>
<th>Cracking</th>
<th>Variable Poisson's</th>
<th>Concrete Softening</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Hognestad</td>
<td>MKP</td>
<td>Vecchio</td>
<td>MC (stress)</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>2</td>
<td>Hognestad</td>
<td>Popovics</td>
<td>Vecchio</td>
<td>MC (stress)</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>3</td>
<td>Hognestad</td>
<td>MKP</td>
<td>Selby</td>
<td>MC (stress)</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>4</td>
<td>Hognestad</td>
<td>Popovics</td>
<td>Selby</td>
<td>MC (stress)</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>5</td>
<td>Hognestad</td>
<td>Hoshikuma</td>
<td>Vecchio</td>
<td>MC (stress)</td>
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<td>YES</td>
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<td>6</td>
<td>Hognestad</td>
<td>Popovics</td>
<td>Vecchio</td>
<td>MC (stress)</td>
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<td>NO</td>
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<td>7*</td>
<td>Hoshikuma</td>
<td>Popovics</td>
<td>Vecchio</td>
<td>MC (stress)</td>
<td>YES</td>
<td>YES</td>
</tr>
</tbody>
</table>

* Used only with 2CS-17 Column, MC: Mohr-Coulomb

All the combinations have the same pre-peak curve for concrete, and different post-peak curves and confinement models. Combinations 2 and 6 differ only in the concrete softening criterion. The purpose was to identify which group of models might reproduce the general behavior of confined concrete. Peak
loads, peak strains, post-peak behavior, and compression softening are to be compared in this section. As the strain data from the experiments was only available for columns 2A1-1 and 2C5-17 in this study, separate comparisons for lateral expansion will be made for these two columns against results obtained with a previous version of the program.

4.6.1 Peak Load

The results of analytical to experimental peak load ratios for each column were reorganized in Table 4.9.

<table>
<thead>
<tr>
<th>Column</th>
<th>Model Combination</th>
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<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
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<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>4B3-19</td>
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<td>1.01</td>
</tr>
<tr>
<td>2C5-17</td>
<td>1.06</td>
<td>1.03</td>
</tr>
<tr>
<td>4D6-24</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>mean</td>
<td>1.02</td>
<td>1.00</td>
</tr>
<tr>
<td>std. dev(%)</td>
<td>4.44</td>
<td>2.10</td>
</tr>
</tbody>
</table>

Combinations 1, 2, and 5 used the tentative confinement model proposed by Vecchio. The prediction of the peak load using this model agreed well with the experimental results. The confinement model proposed by Selby predicted higher peak loads (combinations 3 and 4) due to its definition of lateral pressure.
Combination 6 deserves some attention. Well-confined columns (i.e., with high volumetric ratios and small tie spacing as was the case of 4B3-19 and 4D6-24) are less susceptible to develop concrete softening in the concrete core than those not so well confined (e.g., column 2C5-17, very high tie spacing), or poorly confined (e.g., 2A1-1). As tie spacing increases, lateral pressure weakens and softening might develop within the concrete core. SPARCS cannot account for softening if it had not been set up at the beginning of the analysis.

For discussion purposes, the "strength increase" is defined as:

\[
k = \frac{P_{\text{max}}}{P_{\text{conf}}}
\]

(4.4)

where \(P_{\text{conf}}\) is the maximum unconfined load in the concrete core:

\[
P_{\text{conf}} = 0.85 f'c (A_{\text{conf}} - A_{t}) + f_y A_t
\]

(4.5)

and \(A_{\text{conf}}\) is the area of the core from centre to centre of the perimeter hoop.

These expressions were used to compute the strength increase for each combination and are presented in Table 4.10.

### TABLE 4.10. Strength Increase

<table>
<thead>
<tr>
<th>Column</th>
<th>(\rho_r)</th>
<th>Model Combination</th>
<th>Strength increase</th>
<th>(k) (exp)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(%)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2A1-1</td>
<td>0.80</td>
<td>1.20</td>
<td>1.20</td>
<td>1.31</td>
</tr>
<tr>
<td>4B3-19</td>
<td>1.80</td>
<td>1.27</td>
<td>1.27</td>
<td>1.33</td>
</tr>
<tr>
<td>4D6-24</td>
<td>2.30</td>
<td>1.37</td>
<td>1.37</td>
<td>1.52</td>
</tr>
<tr>
<td>2C5-17</td>
<td>2.37</td>
<td>1.37</td>
<td>1.30</td>
<td>1.58</td>
</tr>
</tbody>
</table>
These results are plotted in Fig 4.38 against the volumetric ratio \( \rho_r \).

![Strength Increase Graph](image)

**Figure 4.38 Comparisons of Strength Increase**

Combination 2 fitted well all the experimental peak loads. Combinations 3, 4 and 6 showed scattered strength increases for columns with high volumetric ratios.

The unconfined plain concrete strength (taken as \( 0.85f_c \)) was a good approximation of the actual value for the columns analyzed.

### 4.6.2 Strain at Peak Load

The analytical to experimental peak strain relationships are summarized in Table 4.11. For ductile behavior, the specimens sustained load without losing capacity for large deformations; thus there would be a range of axial shortening.
to which the post-peak ductile regime occurs. The analytical values taken in Table 4.11 were those at which the second peak loads occurred in ductile column behavior. The results show more scatter than the predicted peak loads, and the program underestimated these values in general.

**TABLE 4.11 Analytical to Experimental Peak Strain Ratios**

<table>
<thead>
<tr>
<th>Column</th>
<th>Model Combination</th>
<th>( \varepsilon_p ) (anal.) / ( \varepsilon_p ) (test)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>2A1-1</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>4B3-19</td>
<td>0.68</td>
<td>0.68</td>
</tr>
<tr>
<td>2C5-17</td>
<td>0.62</td>
<td>0.53</td>
</tr>
<tr>
<td>4D6-24</td>
<td>0.56</td>
<td>0.66</td>
</tr>
<tr>
<td>mean</td>
<td>0.72</td>
<td>0.73</td>
</tr>
<tr>
<td>std. dev(%)</td>
<td>15.77</td>
<td>16.83</td>
</tr>
</tbody>
</table>

For combinations 3 and 4 where the Selby's confinement model was used, the predicted peak strains were larger than those predicted for the other combinations, in three of the four columns. A better result was obtained for the poorly-confined 2A1-1 column with Vecchio's model. Although combination 3 presented the least scatter in the peak strain calculation, it overestimated column capacity well above the others.

Combinations 1 and 2 showed similar mean and standard deviation for both the peak load and the peak strain; however, combination 2 showed somewhat better ductility.
4.6.3 Post-Peak Behavior

The three post-peak curves for concrete are plotted for each column in Fig. 4.39, while keeping the other combination parameters constant.

![Graphs showing post-peak behavior for different columns.](image)

**Figure 4.39** Effect of Post-Peak Curves (1: Modified Kent & Park, 2: Popovics, 5: Hoshikuma et al.)

No difference was apparent in column 2A1-1 in the post-peak regime. However, the Hoshikuma et al. model showed a steep descending branch for the remaining three columns. The Modified Kent and Park and the Popovics curves...
sustained load similarly, but the Popovics curve produced larger deformations. Thus, it traced the experimental curves with slightly better accuracy for highly confined columns.

4.6.4 **Effect of Compression Softening**

Figure 4.40 shows combinations 2 and 6, which incorporated the same models, but the former considered concrete softening due to tensile strains.

![Graphs showing effect of compression softening](image)

*Figure 4.40 Effect of Compression Softening*
Influence on the ductility was noted if compression softening was not considered in the analysis. As the concrete strength did not weaken due to tensile strains, lateral expansion increased quickly at early load stages, resulting in early activation of lateral steel. By the time the column had reached its peak, the confining pressure had eventually reached its maximum value, and ties had undergone deformations beyond yielding.

The two combinations could be considered as bounds for the axial shortening curves of the analyzed columns. They were used for corroborating the confinement behavior of the specimens presented in Chapter 5.

4.7 Comparison of Analytical Results with a Previous Version of the Program

Selby (1993) reported the analysis of six of the Sheikh-Uzumeri columns using SPARCS. Two of these columns were also analyzed in this study, 2A1-1 and 2C5-17. In Selby’s early attempt to model strength enhancement, program SPARCS had limited options in both constitutive modeling and analysis capabilities. Some of these limitations have been overcome through recent improvements. Differences in modeling and analytical results are discussed in this section.

Table 4.12 shows a comparison of the geometry, steel and material models used. Although a refiner mesh was used in the current study, the number of layers between ties was smaller to keep an average aspect ratio of 1:1:1 (i.e., width: height: depth) and avoid early crushing of “flat” brick layers. With the
state-of-the-art computer processors and some modifications to vector dimensions in the SPARCS solver, it was possible to increment the number of elements substantially, and to solve considerably larger models in a fraction of the previous running times (that lasted up to one week!)

Some analysis assumptions remained the same. Perfect bond was assumed between steel bars and concrete, truss bars were used to model the ties, and buckling of longitudinal bars was not considered.

### TABLE 4.12 Selby’s and Current Models

<table>
<thead>
<tr>
<th>Properties</th>
<th>Selby 2A1-1</th>
<th>2C5-17</th>
<th>SPARCS90 2A1-1</th>
<th>2C5-17</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bricks in cross section</td>
<td>36</td>
<td>36</td>
<td>49</td>
<td>64</td>
</tr>
<tr>
<td>Full mesh</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spalled mesh</td>
<td>36</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layers per tie spacing</td>
<td>6</td>
<td>6</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Number of Layers</td>
<td>10</td>
<td>10</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>1:1:0.38</td>
<td>1:1:0.67</td>
<td>1:1:0.80</td>
<td>1:1:1.33</td>
</tr>
<tr>
<td>Total number of Bricks</td>
<td>360</td>
<td>360</td>
<td>637</td>
<td>832</td>
</tr>
<tr>
<td><strong>Steel</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal Bars</td>
<td>Truss Bars</td>
<td></td>
<td>Embedded</td>
<td></td>
</tr>
<tr>
<td>Ties</td>
<td>Truss Bars</td>
<td></td>
<td>Truss Bars</td>
<td></td>
</tr>
<tr>
<td><strong>Analysis Method</strong></td>
<td>Imposed Forces</td>
<td></td>
<td>Imposed Displacements</td>
<td></td>
</tr>
<tr>
<td><strong>Material Models</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-peak</td>
<td>Hagemsted</td>
<td></td>
<td>Hagemsted</td>
<td></td>
</tr>
<tr>
<td>Post-Peak</td>
<td>MGP</td>
<td></td>
<td>Pappovics</td>
<td></td>
</tr>
<tr>
<td>Strength Enhancement</td>
<td>Selby</td>
<td></td>
<td>Vecchio</td>
<td></td>
</tr>
<tr>
<td>Lateral Expansion</td>
<td>Kupfer</td>
<td></td>
<td>Kupfer w. modification</td>
<td></td>
</tr>
</tbody>
</table>

Post-peak modeling with the previous version was done using imposed forces at the top of the column model. In order to transmit forces after peak
loads, it was necessary to add very stiff springs at the four corners of the model, the difference between the applied load and the load taken by the springs was the load the column supported. It was also necessary to model a spalled mesh (i.e., a model without concrete cover) once peak loads had been reached. The current version of the program overcomes this difficulty, and allows the user to specify imposed displacements.

Figures 4.41 and 4.42 show the axial shortening and average tie strain for 2A1-1 and 2C5-17, respectively. Tie strains followed the same load path in the initial loading phase, showing a stiffer response than the specimens. No difference in the pre-peak response was noted. However, stretching of ties occurred to some extent in the post-peak branch for both columns in the current analyses.

The use of spalled meshes was not successful in capturing the post-peak response of 4 of the 6 columns analyzed by Selby. As the author commented, premature crushing of the concrete core contributed to the lack of ductility. This could be attributed in part to the aspect ratio of the finite elements used.

The model implemented in SPARSC for variable Poisson's ratio was compared with ten of the columns tested by Liu et al (1998). The next section presents the results of these comparisons.
Figure 4.41 Result Comparisons for 2A1-1
Figure 4.42 Result Comparisons for 2C5-17
4.8 Study of Poisson's Ratio

Ten 250-mm circular columns tested by Liu et al. (1998) were chosen to investigate the variation of the Poisson's ratio with the increase in axial compression. Column properties are shown in Table 4.13; detailed description of these columns will be given in Chapter 5. These columns had spirals or hoops as lateral steel, cylinder strengths for concrete ranging from 60 to 90 MPa, volumetric ratios from 1.2 to 6.0%, and concrete covers of 0, 15 and 25 mm.

The objective of this study was to corroborate, by direct observation of the plots given herein, that lateral expansion varies as axial load increases, and to determine if the tentative expansion model proposed by Vecchio (see Vecchio 1992) reasonably reproduces this phenomenon.

<table>
<thead>
<tr>
<th>Specimen Name</th>
<th>Tie Steel</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>type</td>
<td>$d_h$ [mm]</td>
</tr>
<tr>
<td>2C60-10S50-15</td>
<td>hoops</td>
<td>10.0</td>
</tr>
<tr>
<td>2C60-10S100-15</td>
<td>hoops</td>
<td>10.0</td>
</tr>
<tr>
<td>2C60-10S150-15</td>
<td>hoops</td>
<td>10.0</td>
</tr>
<tr>
<td>2C80-10S50-15</td>
<td>spiral</td>
<td>10.3</td>
</tr>
<tr>
<td>2C80-6S50-15</td>
<td>spiral</td>
<td>7.1</td>
</tr>
<tr>
<td>2C80-6S100-15</td>
<td>spiral</td>
<td>7.1</td>
</tr>
<tr>
<td>2C90-10S100-25</td>
<td>spiral</td>
<td>9.2</td>
</tr>
<tr>
<td>2C90-6S50-25</td>
<td>spiral</td>
<td>3.7</td>
</tr>
<tr>
<td>2C90-6S100-25</td>
<td>spiral</td>
<td>3.7</td>
</tr>
<tr>
<td>2C90-10S100-0</td>
<td>spiral</td>
<td>9.2</td>
</tr>
</tbody>
</table>

TABLE 4.13 Liu et al. Column Properties for the Study of Poisson's ratio

* Calculated by the writer
4.8.1 Procedure to Obtain the Experimental Poisson's ratio

The Poisson's ratio variation was computed using the procedure described below:

- From the experimental axial load versus lateral steel strain curve, a load stage was selected at which the Poisson's ratio $v$ was to be computed (i.e., tie strain $\varepsilon_t$).

- The lateral pressure $f_{lat}$ on the concrete core was determined from the equilibrium of the section as shown in Figure 4.43.

$$f_{lat} = \left( \frac{2A_v f_s}{d_v s} \right)$$

(4.6)

where $f_s$ is the actual tie stress, $s$ is the hoop spacing (or spiral pitch), $d_v$ is the centre-to-centre tie diameter, $A_v$ is the area of one hoop leg, and $d$ is the column diameter.
Equation 4.6 can also be written as:

\[ f_{\text{lat}} = \frac{1}{2} \rho_s f_s \]  

(4.7)

where

\[ \rho_s = \frac{4A_s}{d_s s} \]  

(4.8)

\( \rho_s \) is the volumetric lateral steel ratio in the concrete core. It should be noted that \( f_{\text{lat}} \) was assumed uniformly distributed along the tie spacing; however, this was not the case for confined columns.

- As the hoop spacing varied from 50 to 150 mm for the Liu et al. columns, the lateral pressure on the concrete core was no longer considered uniformly distributed. Thus, in order to obtain an equivalent pressure (i.e., uniform lateral pressure), a factor to reduce \( f_{\text{lat}} \) was applied. The confinement effectiveness coefficient \( k_s \) proposed by Mander et al. (1998) was used in this study. The factor \( k_s \) is computed for circular hoops as:

\[ k_s = \frac{\left(1 - \frac{s'}{2d_s}\right)^2}{1 - \rho_{\infty}} \]  

(4.9)

and for circular spirals as:

\[ k_s = \frac{\left(1 - \frac{s'}{2d_s}\right)}{1 - \rho_{\infty}} \]  

(4.10)
where \( s' \) is the clear spacing between hoops (or clear spiral pitch), and \( \rho_{cs} \) is the longitudinal steel ratio in the concrete core.

- The lateral pressure on the concrete core was then computed as:

\[
f_{\text{int}} = \frac{1}{2} k_c \cdot \rho_c \cdot f_c
\]

(4.11)

- As the lateral strain in concrete was expected to be small, the concrete behavior in the direction perpendicular to the applied load could be considered linear elastic. It was assumed that if the lateral stress \( f_{\text{int}} \) was smaller than 40% of \( f'_c \) (cylinder strength for concrete), the strain in the concrete \( \varepsilon_c \) due to confinement could be computed as:

\[
\varepsilon_c = -\frac{f_{\text{int}}}{E_c}
\]

(4.12)

where \( E_c \) is the stiffness modulus for the concrete (shown in Table 4.13).

- The Poisson's ratio \( \nu \) is given as:

\[
\nu = \frac{(\varepsilon_h + \varepsilon_v)}{\varepsilon_{av}}
\]

(4.13)

where \( \varepsilon_h \) is the tie strain, and \( \varepsilon_{av} \) is the average axial strain in the column, obtained from the experimental results.

- The volumetric strain \( \varepsilon_v \) was computed as:

\[
\varepsilon_v = \varepsilon_{av} + 2(\varepsilon_h + \varepsilon_v)
\]

(4.14)
The procedure was repeated for various load stages until the axial shortening and tie strain curves for each column were completely determined.

Axial load-Poisson's ratio curves were normalized with respect to the initial Poisson’s ratio \( \nu_0 \) and the strain at peak cylinder stress \( \varepsilon_0 \). As the columns were made of high strength concrete, the values for \( \varepsilon_0 \) were obtained as follows:

\[
\varepsilon_0 = \frac{f'_e}{E_c} \frac{n}{n-1}
\]

(4.15)

\[
n = 0.8 + \frac{f'_e}{17}
\]

(4.16)

where the stiffness modulus \( E_c \) was obtained from the experimental results. Table 4.14 shows these initial values for all the columns.

### TABLE 4.14 \( \varepsilon_0 \) and \( \nu_0 \) values for Liu et al. Columns

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Series</th>
<th>( \varepsilon_0 ) [mm/m]</th>
<th>( \nu_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2C80-10S50-15</td>
<td>I</td>
<td>1.858</td>
<td>0.25</td>
</tr>
<tr>
<td>2C80-10S100-15</td>
<td>I</td>
<td>1.858</td>
<td>0.33</td>
</tr>
<tr>
<td>2C80-10S150-15</td>
<td>I</td>
<td>1.858</td>
<td>0.22</td>
</tr>
<tr>
<td>2C80-10S50-15</td>
<td>II</td>
<td>2.154</td>
<td>0.04</td>
</tr>
<tr>
<td>2C80-6S50-15</td>
<td>II</td>
<td>2.154</td>
<td>0.04</td>
</tr>
<tr>
<td>2C80-6S100-15</td>
<td>II</td>
<td>2.154</td>
<td>0.22</td>
</tr>
<tr>
<td>2C90-10S100-25</td>
<td>III</td>
<td>2.245</td>
<td>0.26</td>
</tr>
<tr>
<td>2C90-6S50-25</td>
<td>III</td>
<td>2.245</td>
<td>0.14</td>
</tr>
<tr>
<td>2C90-6S100-25</td>
<td>III</td>
<td>2.245</td>
<td>0.12</td>
</tr>
<tr>
<td>2C90-10S100-0</td>
<td>III</td>
<td>2.245</td>
<td>0.22</td>
</tr>
</tbody>
</table>
These normalized plots were compared with the variable Poisson's ratio model (see Figure 3.16):

\[ v = v_o \left[ 1.0 + 1.5 \left( \frac{\varepsilon - 1.0}{\varepsilon_o} \right)^2 \right] \leq 0.5 \]

(Eq. 3.48)

It should be noted that two assumptions were made: concrete behavior was considered elastic up to a stress of 0.4f', in the direction of expansion, which was deemed reasonable; and the use of a factor to obtain an equivalent lateral pressure. Although various factors have been proposed in the literature (e.g., Razvi and Saatcioglu (1999)) it was not an objective here to investigate them, but to obtain a value for the lateral pressure that reflected the actual confinement behavior.

4.8.2 Experimental Variable Poisson's Ratio

Axial load-axial strain, axial load-tie strain, axial load-Poisson's ratio, axial strain-Poisson's ratio curves, and the variation of the volumetric ratio are presented in Figs. 4.44 to 4.51 for two of the Liu et al. Columns. The results for the remaining columns are shown in Appendix C.
Figure 4.44 Load-Strain Curve of Column 2C90-10S100-25

Figure 4.45 Load-Poisson's Ratio Curve of Column 2C90-10S100-25
Figure 4.46 Poisson’s Ratio Axial Strain Curve of Column 2C90-10S100-25

Figure 4.47 Volumetric Strain Variation of 2C90-10S100-25
Figure 4.48 Load-Strain Curve of Column 2C80-6S50-15

Figure 4.49 Load-Poisson’s Ratio Curve of Column 2C80-6S50-25
Figure 4.50 Poisson's Ratio Axial Strain Curve of Column 2C80-6S50-25

Figure 4.51 Volumetric Strain Variation of 2C80-6S50-25
Figures 4.52 to 4.54 show plots of the variation of Poisson's ratio with the axial strain for each of the column series described in Table 4.14. Also shown is the analytical expression for the lateral expansion model of Eq. 3.48.

**2C80-Series I**

![Graph showing variation of Poisson's ratio with axial strain for 2C80-Series I](image)

**Figure 4.52 Lateral Expansion Variation of Series I**

**2C80-Series II**

![Graph showing variation of Poisson's ratio with axial strain for 2C80-Series II](image)

**Figure 4.63 Lateral Expansion Variation of Series II**
The following can be concluded from the observed behavior of these columns:

- The increase in the Poisson’s ratio began at about 0.5 to 0.6 $P_{\text{max}}$, where $P_{\text{max}}$ is the maximum load sustained by the column.
- Lateral steel strains are practically negligible for axial compressive loads smaller than 0.5 to 0.6 $P_{\text{max}}$.
- It is apparent that as the Poisson’s ratio began to change, the volumetric strain $\varepsilon_v$ increased (i.e., the column section changed from being contracted to being expanded).
- The Poisson’s ratio increase from the initial value $\nu_0$ began at axial strains at about 0.2 to 0.6 $\varepsilon_0$. 

![Figure 4.54 Lateral Expansion Variation of Series III](image-url)
In general, the analytical curve traced the actual variation in lateral expansion as load increased.

It should be noted that this analysis was limited to circular high-strength columns. Further analysis of the actual variation of lateral expansion in other sections and concrete types may be needed to validate the proposed formula for the Poisson's ratio variation implemented in program SPARCS.

It seemed that Equation 3.48 was adequate in modeling the Poisson's effect on confinement, and it was decided to keep $\nu$ variable during all of the analyses made with SPARCS for all the specimens studied.
Chapter 5
Corroboration with Experimental Studies

5.1 Introduction

In order to provide a closer view of the capabilities of the models implemented in the program SPARCS, a series of specimens tested by other researchers were modeled and analyzed. This series included five high strength concrete (HSC) circular columns tested by Liu et al. (1998), two normal strength concrete (NSC) specimens tested by Mander et al. (1984), three additional Sheikh and Uzumeri columns, and a shear wall tested by Lefas et al. (1990).

The objective was to corroborate the effectiveness of the confinement model through comparisons of analytical and experimental axial and lateral deformation curves. A description of the specimens and testing procedures are presented, followed by the finite element model and analysis results.
5.2 Liu, Foster, and Attard Tests (1998)

Circular HSC columns with hoops or spirals were tested and modeled using the "microplane model" reported by the authors.

5.2.1 Column Geometry

A set of 18 short HSC circular columns divided into 3 series (i.e., I, II and III), were subjected to concentric monotonic axial compression. The columns had a 250-mm diameter and were 1600-mm high. Top and bottom ends were haunched to prevent failure. Each haunch was 400 x 400 x 300 mm. The middle third of each column was gauged with strain gauges. Details of the column geometry and longitudinal and lateral steel distribution for all the series are shown in Fig. 5.1.

Figure 5.1 Detail of Columns Tested by Liu et al.
5.2.2 Longitudinal and Lateral Steel Arrangements

Hot-rolled bars were used for the longitudinal reinforcement, and deformed bars for the circular hoops and spiral ties. The circular hoops were welded at the ends.

Hoop spacing or spiral pitch was 50, 100, or 150 mm, and concrete cover varied from 0 mm to 25 mm (see Fig. 5.1c). Eight 12-mm bars were used for the longitudinal steel in all specimens, and 6-mm or 10-mm bars were used for hoop or spirals. Since the cover thickness varied, the centre-to-centre diameter of the hoops or spirals was not constant.

5.2.3 Test Instrumentation and Procedure

Strain gauges were mounted on both lateral steel and longitudinal steel at the midheight of the testing zone (600 mm). For series I, three gauges at 120° were used in each of two adjacent hoops, and one gauge in each of two opposite longitudinal bars. For series II and III, four strain gauges were used along one pitch on the central spiral, and four gauges on every second longitudinal bar.

LVDT's spanning 400 mm were installed to measure the vertical displacement of each column. Load cells were used to measure the applied load.

The specimens were casted vertically and tested horizontally at a slow rate. Some difficulties in keeping the load concentric was reported by the
authors; large eccentricities were recorded during the testing of some of the elements.

The experimental variables studied included spacing and volumetric ratio of spirals, concrete cover and concrete strength.

5.2.4 Selected Liu et al. Columns

Five of the 18 columns were modeled in SPARCS and compared with the experimental results. In this selection, the concrete cover was kept constant (15 mm). Three of the columns had a nominal concrete strength of 60 MPa, corresponding to Series I (see Fig. 5.1a), and two were of 80 MPa, corresponding to Series II (Fig. 5.1b). The variables accounted for in the modeling were the hoop or spiral spacing, the concrete strength, and the effect of lateral steel yielding stress.

The material properties of the selected columns are shown in Table 5.1.

**TABLE 5.1 Liu et al. Columns, Material Properties**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Series</th>
<th>Tie Steel</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name (1)</td>
<td></td>
<td>(2) type</td>
<td>(3) (d_r ) [mm]</td>
</tr>
<tr>
<td>2C60-10S50-15</td>
<td>I</td>
<td>hoops</td>
<td>10.0</td>
</tr>
<tr>
<td>2C60-10S100-15</td>
<td>I</td>
<td>hoops</td>
<td>10.0</td>
</tr>
<tr>
<td>2C60-10S150-15</td>
<td>I</td>
<td>hoops</td>
<td>10.0</td>
</tr>
<tr>
<td>2C60-10S50-15</td>
<td>II</td>
<td>spiral</td>
<td>10.3</td>
</tr>
<tr>
<td>2C60-6S50-15</td>
<td>II</td>
<td>spiral</td>
<td>7.1</td>
</tr>
<tr>
<td>Longitudinal Bars</td>
<td></td>
<td></td>
<td>(d_0 ) [mm]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12.1</td>
</tr>
</tbody>
</table>

*Calculated by the writer*
The volumetric ratio $\rho_v$ varied from 2.0% to 6.4%, which could be considered high values. However, due to the brittle nature of HSC, large amounts of lateral steel was needed for these columns to undergo ductile behavior. Tie spacing varied from 20% to 60% of the column diameter.

5.2.5 Finite Element Models

5.2.5.1 Geometry

As with the columns in Chapter 4, only one-quarter of the column cross section was modeled due to symmetry. It should be noted that SPARCS has no axisymmetric elements in its library to model bodies of revolution (e.g., circular columns). Instead, 6-noded wedges and truss bars were used for concrete, hoops and spirals, respectively. Longitudinal bars were smeared out, and truss bars were attached to adjacent wedge nodes as shown in Fig. 5.2. A summary of the geometry for each column is given in Table 5.2. The vertical profile of the finite element model for column 2C60-10S50-15 was similar to the Sheikh and Uzumeri columns.

![Figure 5.2 Cross Section Model for Liu Columns](image)
There were no differences in the geometry of all models, but the number of truss bars used and the mechanical properties of the materials varied.

**TABLE 5.2 Geometry of Finite Element Models**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Layers per s</td>
</tr>
<tr>
<td>----------</td>
<td>--------------</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>2C60-10S50-15</td>
<td>1</td>
</tr>
<tr>
<td>2C60-10S100-15</td>
<td>2</td>
</tr>
<tr>
<td>2C60-10S150-15</td>
<td>3</td>
</tr>
<tr>
<td>2C80-10S50-15</td>
<td>1</td>
</tr>
<tr>
<td>2C80-6S50-15</td>
<td>1</td>
</tr>
</tbody>
</table>

**5.2.5.2 Material Types**

The longitudinal reinforcement was modeled as smeared within the concrete. The steel ratios \( \rho \) used in the models are shown in Table 5.3 along with the plain concrete strength and the tensile strength for concrete. A sketch of the material distribution in the cross section of the circular columns is shown in Fig. 5.3.

**TABLE 5.3 Liu et al. Material Types**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Material Types</th>
<th>Long. Steel</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series</td>
<td>Type 1 ( \rho )</td>
<td>Type 2 ( \rho )</td>
<td>Type 3 ( \rho )</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>I</td>
<td>---</td>
<td>12.22</td>
<td>11.66</td>
</tr>
<tr>
<td>II</td>
<td>---</td>
<td>12.22</td>
<td>11.66</td>
</tr>
</tbody>
</table>
The tensile strength for concrete (see column (6) of Table 5.3) was computed using:

\[ f'_t = 0.45 \sqrt{f'_c} \]  

(5.1)

5.2.5.3 Material Models

For the corroboration study, the material behavior models that best fit the experimental data in Chapter 4 were used. Those models were:

- **Pre-peak curve for concrete:** Hognestad Parabola
- **Post-peak curve:** Popovics.
- **Confinement model:** Vecchio
- **Cracking criterion:** Mohr-Coulomb (stress formulation)
- **Compression softening:** Vecchio-92A (if considered)

The strain-hardening model for steel was also used in all subsequent analyses. Although the Popovics equation for HSC could also be used in the
pre-peak regime, it was found that the simple parabola equation provided good approximations of the general axial-shortening behavior.

5.2.6 Analytical and Experimental Results of Liu et al. Columns.

These column models were also subjected to imposed displacements at the top stiff layer. The results are divided into axial strain and tie strain curves; the effect of lateral expansion is discussed along with the comparisons with the experimental data. A summary of the peak loads and respective strains is presented later in this section.

The FE model behavior was compared with the experimental result for each column. Values given in parentheses in the following section are the axial compressive strain in $\text{mm/m}$ at which an event occurred.

5.2.6.1 Column 2C60-10S50-16

The failure sequence in the analytical model was as follows: cracking of cover shell (-1.624); yielding of longitudinal steel (-2.170); first peak load of 2750KN and cover spalling (-2.811); yielding of hoops (-4.611); and second peak load of 2833 KN (-8.187). At failure the column capacity was reduced to less than 50% of the maximum load achieved. Ductility of the model was half that of the specimen. The expansion effect was captured reasonable well (see Figure 5.5).

The column specimen showed similar behavior (see Liu et al. 1998) with slight differences in the axial strain. The longitudinal bars yielded before the first
peak. As the first peak of 2860 KN was reached, the cover spalled off at an average axial strain of -3.0 \( \text{mm/m} \). The strain hardening of ties began immediately after the yield strain, and the lateral stress increased after the first peak until a second peak of 2970 KN was reached. The column was heavily confined (6.0%) and failed due to tie weld fracture. Slight buckling of longitudinal bars was apparent. The SPARCS and experimental curves are plotted in Figure 5.4.

5.2.6.2 Column 2C60-10S100-15

The model reached a maximum load of 2830 KN at an axial strain of 2.831 \( \text{mm/m} \), which was larger than that of the experiment (2460 KN at about -1.5 \( \text{mm/m} \)). Due to this circumstance, the longitudinal reinforcement yielded before the peak load in the model. As can be seen in Figure 5.6, the initial stiffness of the model was smaller than the recorded during testing. However, for both the model and the specimen, yielding of hoops did not occur at peak.

During the testing of the column, a large eccentricity was detected as the load was applied. The column did not reach the "squash load" (for this case, calculated to be 2850 KN), which could be considered as the minimum capacity for concentric loaded column. The partial spalling at peak load observed in the experiment could also be proof of the accidental eccentricity, and could explain the difference in the axial shortening curves.

The hoop strain is shown in Figure 5.7. The FE model failed before yielding of the lateral steel.
Figure 5.4 Axial Shortening of 2C60-10S50-15

Figure 5.5 Lateral Expansion of 2C60-10S50-15
2C60-10S100-15

![Graph showing load vs. axial strain for 2C60-10S100-15 with different lines representing test and SPARCS models with and without softening.]

Figure 5.6 Axial Shortening of 2C60-10S100-15

2C60-10S100-15

![Graph showing load vs. average hoop strain for 2C60-10S100-15 with different lines representing test and SPARCS models with and without softening.]

Figure 5.7 Lateral Expansion of 2C60-10S100-15
5.2.6.3 Column 2C60-10S150-15

The model traced with good accuracy the ascending branch of the axial strain curve. Cracking of the shell began at an axial strain of \(-1.623 \text{ mm/m}\); at this load stage, the lateral steel strain was practically negligible (0.3 \text{ mm/m}). The peak load (2860 KN, compression softening not considered) was reached at an axial strain of \(-2.813 \text{ mm/m}\), after the longitudinal bars yielded. At peak load the cover spalled off. Due to the large spacing between hoops, the triaxial compression in the concrete was hardly developed and the column model failed in a brittle manner.

For the specimen, similar failure conditions prevailed as for specimen 2C60-10S100-15. A large eccentricity was recorded (9 mm) in the strain gauges installed in the longitudinal reinforcement. However, the shape of the axial shortening curve suggested that the actual behavior could have been similar to that described for the model. The analytical peak load was the minimum expected strength (i.e., the squash load). Axial shortening and tie strain graphs are plotted in Figures 5.8 and 5.9, respectively.

5.2.6.4 Column 2C80-10S50-15

The model without compression softening effect matched very well the experimental curve for axial shortening. In the model with compression softening, the post-peak regime was "parallel" to that of the experimental one, but the tracing was at a lower load.
Figure 5.8 Axial Shortening of 2C60-10S150-15

Figure 5.9 Lateral Expansion of 2C60-10S150-15
For the analytical model, the ascending branch closely matched the experimental one until 70% of the first peak, even though the Hognestad parabola was used in the modeling. First cracks in the cover appeared at about -2.0 \( \text{mm/m} \). The first peak load of 3590 KN occurred at an axial strain of -3.242 \( \text{mm/m} \); at this point, the longitudinal bars had already yielded. The tie steel strain was low (1.4 \( \text{mm/m} \)). As lateral pressure increased after the first peak, a second higher peak of 3850 KN was obtained in the model. The tie steel yielded before the second peak was reached (see Fig. 5.10).

The test specimen reached the first peak (3520 KN) at about -2.4 \( \text{mm/m} \). The vertical reinforcement yielded just before this peak, and significant cover spalling was observed. The spiral stresses increased after the first peak, until a second peak (3880 KN) was recorded. The column failed after the second peak due to fracture of the spiral reinforcement in the gauged region.

5.2.6.5  Column 2C80-6S50-15

The difference with the previous column was the diameter of the spiral (i.e., different volumetric ratio). The model (without softening effect) cracked initially at -2.0 \( \text{mm/m} \) axial strain; at this load stage the average spiral strain was only 0.8 \( \text{mm/m} \). The load-deformation response peaked (3700 KN) at -3.6 \( \text{mm/m} \), and the tie strain at this load stage was 1.6 \( \text{mm/m} \).
Figure 5.10 Axial Shortening of 2C80-10S50-15

Figure 5.11 Lateral Expansion of 2C80-10S50-15
In the analytical model, the column crushed at about -6.4 mm/m. The model with compression softening followed the ascending branch but the cover spalled off at a lower axial strain. Due to the concrete softening, the post-peak branch obtained in the finite element analysis was lower than the actual behavior. No considerable cracking within the core would have been expected due, in part, to the tight lateral reinforcement.

The failure sequence for the specimen was: yielding of longitudinal bars; peak load of 3850 KN reached at about -3.0 mm/m with cover spalling; descending post-peak branch up to -6.0 mm/m; and failure due to fracture of the spiral reinforcement in seven locations followed by longitudinal bar buckling. The actual hoop strain plot apparently did not reflect the reported spiral fracture. Plots of axial shortening and tie strain are graphed in Figs. 5.12 and 5.13.
Figure 5.12 Axial Shortening of 2C80-6S50-15

Figure 5.13 Lateral Expansion of 2C80-6S50-15
5.3 Mander et al. Specimens (1984)

Mander tested a series of circular, square, and rectangular columns. The main objective of this investigation was to evaluate column behavior under high axial strain rates, thus simulating the response of bridge columns under seismic load. A stress-strain model for the concrete that considered the effect of confinement and the strain rate was also developed. The ultimate strain was based on the rupture of the first hoop when a column was subjected to either monotonic or dynamic loading.

5.3.1 Column Geometry

A set of 17 short circular columns, 5 square columns, and 16 walls divided into two batches were subjected to concentric monotonic or dynamic load at both low and high strain rates. The square column series was prepared for testing by Scott (1982) to establish the influence of age in testing concrete. Near-full-size column dimensions were chosen to avoid scale effects. Scott's columns were 450-mm square and 1200-mm high, and the walls were 700 x 150 mm rectangular (scaled to about 0.5:1 ratio of the actual dimensions of the hollow sections, which were 1900 x 300) and 1200-mm high. In all cases, tie or spiral spacing was reduced at both ends from 80% to 50% of the spacing in the test region. Details of the column geometry, longitudinal and lateral steel distribution for the walls and square specimens are shown Figs.5.14 and 5.15.
Wall Setup

Figure 5.14 Mander et al. Wall 11

Scott Column Tested by Mander

Figure 5.15 Square Column of Scott Series
5.3.2 Longitudinal and Lateral Steel Arrangements

Wall 11 had 50-mm hoop spacing. In order to avoid sudden loss of capacity during testing, the concrete cover was kept to a minimum of 25 mm.

The Scott column shown in Fig. 5.15 had 12 longitudinal deformed bars of 20 mm and Grade 380, lateral steel was plain round bars of 10 mm and Grade 275, with a cover to the hoops of 20 mm. The tie setup was similar to the “Type D” arrangement of the Sheikh and Uzumeri columns (see Chapter 4).

5.3.3 Test Instrumentation and Procedure

For the circular columns, four linear potentiometers were installed in pairs on the East-West and North-South faces of the column, spanning over a length equal to the core diameter (450 mm). These potentiometers were used to measure the axial displacement in the column. It was assumed that longitudinal steel and concrete were perfectly bonded, thus strain gauges were not mounted on the longitudinal bars. The lateral steel strains were measured using electrical strain gauges.

For the walls, linear potentiometers were installed in pairs symmetrically placed along the 700-mm side of the wall, and spanning 400 mm. Eight strain gauges, four on one of the internal hoops and the other four on the opposite external hoop, were used to measure the lateral expansion.

For the Scott column, as for the circular column, four potentiometers were installed; one at each face of the column, spanning 400 mm. The author did not report strain gauges on the tie steel for this column.
In all cases, a DARTEC testing machine was used. The load and the overall displacement between the machine head and bottom plate were also recorded.

Wall 11 was tested at a high strain rate (0.013/s), which was a longitudinal displacement of about 20 mm/s. Cameras were mounted and synchronized with the machine operation. Each single test lasted less than 5 seconds.

5.3.4 Selected Mander et al. Specimens

The specimens selected for modeling with SPARCS are shown in Figs. 5.14 and 5.15. It should be noted that the general dimensions of these specimens were larger than the previous analyzed columns. (i.e., Liu et al. and Sheikh and Uzumeri columns). The specimens were made of normal strength concrete (25 and 41 MPa).

The purpose was to establish the capacities of the program to model the confinement effect on different specimen geometry and steel arrangements. Material properties for the specimens are given in Table 5.4.

<table>
<thead>
<tr>
<th>Label</th>
<th>Tie Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_h$ [mm]</td>
</tr>
<tr>
<td>Wall 11</td>
<td>hoops</td>
</tr>
<tr>
<td>Scott</td>
<td>hoops</td>
</tr>
</tbody>
</table>
5.3.5 Finite Element Models

5.3.5.1 Geometry

The same pattern used in modeling Sheikh and Uzumeri columns was implemented for the Mander et al. specimens. The wall was modeled using 8-noded bricks for concrete elements, the longitudinal steel was smeared into the concrete bricks, and the hoop sets were modeled with truss bars as shown in Figure 5.16 (dimensions in mm).

![Wall mesh diagram](image-url)  
*Figure 5.16 Finite Element Model for Wall 11.*
The material behavior models used for Liu et al. columns were used in modeling Mander's specimens. Eq. 4.4 was used in calculating the tensile strength for concrete, $f'_t$.

5.3.6 Analytical and Experimental Results of Mander et al. Specimens.

5.3.6.1 Wall 11

This wall was a scale model (0.5:1 ratio) of a wall in a ductile hollow section of a prototype bridge pier, as shown in Figure 5.17.

![Figure 5.17 Prototype of a Hollow Bridge Pier](image)

It was tested in monotonic compression but under high strain rates, thus simulating the dynamic effect that could be produced by an earthquake. The specimen was restrained against out-of-plane buckling during the testing. The finite element model was restrained as for the square sections of Chapter 4, but
any possible restraining effect due to the perpendicular walls joining at both ends was not taken into account, neither for the specimen nor for the SPARCS model.

The analytical and experimental axial shortening are plotted in Figure 5.18. As the wall ties were closely spaced and the volumetric steel ratio was high ($\rho_v = 2.33\%$), little cracking in the concrete core would have been expected. Thus, compression softening was not considered in this analysis.

![Wall 11](image)

**Figure 5.18 Axial Shortening of Wall 11**

The sequence of events in the analytical model was: concrete cracks in the cover at an axial compressive strain of about $-1.8 \ \text{mm/m}$; yielding of longitudinal bars; maximum axial load reached at $-3.6 \ \text{mm/m}$ with cover spalling.
Finally, significant loss of capacity occurred, but the ductility was kept comparable to that of the specimen up to a strain of \(-15 \text{ mm/m}\).

Mander reported (1984) vertical cracks developing at a strain of about \(-2 \text{ mm/m}\), large pieces of concrete cover spalling off, considerable strength gain due to the tie arrangement, and a steep post-peak descending branch. This might be attributed to the large concrete cover to gross area ratio (43%)

SPARCS captured well the overall behavior of the wall; however, some difference can be noted in the post-peak regime. The finite element model traced the actual descending part of the curve but at lower loads. The tie yield strain in the analytical model occurred after the peak load, whereas the test specimen ties yielded practically at the onset of the post-peak behavior.

5.3.6.2 Scott Column

Scott (1982) prepared for testing five square columns. Mander (1984) then tested the specimens at high strain rates in order to establish the influence of age in concrete. One of those specimens (specimen 16, see Mander 1984) shown in Fig. 5.15, was tested at an age of 942 days and at a strain rate of 0.0167/s. Mander found a strength gain of the specimen in the post-peak region. In order to avoid any assumption regarding shrinkage and creep of concrete, the finite element model was compared with the original column tested by Scott at an age of 67 days and at the same strain rate (specimen 13, Mander 1984).

The axial shortening curves are given in Figure 5.19. The analytical ascending branch separated from the experimental at about 50% of the peak
load. Although the analytical model reached concrete core stresses of 37 MPa (1.75 times the plain concrete strength, $0.85f'_c$), the peak load was only 81% of the actual maximum. Dynamic effects of the imposed load in the test were apparently more pronounced in this column than in wall 11. The cover began cracking at about -1.5 $\text{mm/m}$, and spalled off at the peak load. The longitudinal bars yielded before the maximum load was reached; the ties yielded after the peak load, and the model sustained the load without losing capacity until the concrete core crushed.

![Scott Column](image)

**Figure 5.19 Axial Shortening of Scott Column**
5.4 Sheikh and Uzumeri Columns

Three additional columns were randomly selected from the 24 tested by the researchers in order to confirm the suitability of the material behavior models that had been chosen in the parametric study. The column descriptions, finite element models, test setup and instrumentation were described in Chapter 4; hence, only the results of these columns are presented in this section. The material properties are shown in Table 5. Each column was analyzed two times, one considering concrete softening and the other without.

### TABLE 5.5 Material Properties of Additional Sheikh and Uzumeri Columns

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Conf. type</th>
<th>(d_0) [mm]</th>
<th>(A_s) [mm²]</th>
<th>(\rho_s) [%]</th>
<th>(f_y) [MPa]</th>
<th>(E_s) [MPa]</th>
<th>(E_{eh}) [MPa]</th>
<th>(e_{eh}) [mm/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4C6-5</td>
<td>C</td>
<td>15.675</td>
<td>3200</td>
<td>3.44</td>
<td>367</td>
<td>200000</td>
<td>9220</td>
<td>7.70</td>
</tr>
<tr>
<td>4B4-20</td>
<td>B</td>
<td>19.050</td>
<td>3406</td>
<td>3.67</td>
<td>392</td>
<td>198400</td>
<td>6200</td>
<td>7.80</td>
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<td>4D3-22</td>
<td>D</td>
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<td>3406</td>
<td>3.67</td>
<td>392</td>
<td>198400</td>
<td>6200</td>
<td>7.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Specimen</th>
<th>(\phi_h) [mm]</th>
<th>(A_b) [mm²]</th>
<th>(\rho_r) [%]</th>
<th>(f_{sh}) [MPa]</th>
<th>(E_c) [MPa]</th>
<th>(s) [mm]</th>
<th>(f_c) [MPa]</th>
<th>(e_c) [mm/m]</th>
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</thead>
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<tr>
<td>4C6-5</td>
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<td>17.8</td>
<td>2.27</td>
<td>540</td>
<td>200000</td>
<td>38.1</td>
<td>35.0</td>
<td>2.2</td>
</tr>
<tr>
<td>4B4-20</td>
<td>4.76</td>
<td>17.8</td>
<td>1.70</td>
<td>480</td>
<td>199500</td>
<td>38.1</td>
<td>34.7</td>
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</tr>
<tr>
<td>4D3-22</td>
<td>7.94</td>
<td>49.5</td>
<td>1.60</td>
<td>480</td>
<td>200000</td>
<td>82.5</td>
<td>35.5</td>
<td>2.2</td>
</tr>
</tbody>
</table>
5.4.1 Column 4C6-5

This column had a large volumetric ratio and a small tie spacing, combined with a tight cage and a large number of longitudinal bars. The axial response is plotted in Figure 5.20.

![Figure 5.20 Axial Response of 4C6-5](image)

Both SPARCS analyses followed the experimental curve with good accuracy. When softening was considered, the trace of the post-peak behavior closely followed the experimental curve. Two peaks, one at the spalling of the concrete cover and one after the ties yielded, were apparent in the analysis with concrete softening. The second peak (4640 kN) was slightly larger than the first peak (4450 kN) indicating some gain of capacity after loss of cover. Yielding of
longitudinal bars and initial cracking of the cover occurred at the same load stage at about -2.0 \( \text{mm/} \).n.

The model behavior was equal for both analyses up to the first peak of the softened curve; bifurcation of the post-peak regime was apparent afterwards.

5.4.2 Column 4B4-20

This specimen had a smaller volumetric ratio than the previous column, with an equal tie spacing but different tie arrangement. In this case, the analytical response without concrete softening traced the specimen ductile response very well (see Figures 5.21 and 5.22). Cover cracking started at about -1.9 \( \text{mm/} \); the longitudinal bars yielded before the peak load, and the tie steel yielded after the onset of the post-peak branch.

It was therefore apparent that the tie configuration influenced the overall response of the finite element model. Comparing this column to 4C6-5, both had similar properties for the concrete and longitudinal steel. Although there were differences in the amount of longitudinal steel and volumetric ratio, the main difference was the tie arrangement. The Type C arrangement (see description in Chapter 4) produced larger lateral stresses and larger strength gain than did Type B.
Figure 5.21 Axial Response of 4B4-20

Figure 6.22 Lateral Expansion of 4B4-20
5.4.3 Column 4D3-22

The tie spacing of this specimen was the largest among the three columns. Once the maximum load was reached, the specimen sustained no load at large strains; instead, the capacity dropped after the peak. As for the previous models, this column was analyzed with and without softening. Due to the large spacing between ties, cracking within the concrete core would have been expected leading to some softening of concrete. This behavior was captured by SPARCS as shown in Figure 5.23.

The failure sequence for the softened response was as follows: yielding of the longitudinal bars and initiation of vertical cracks in the cover. First peak load (4330 kN) and cover spalling; second slightly smaller peak load (4300 kN); and decrease of capacity until failure due to concrete crushing.

![4D3-22](image)

**Figure 5.23 Axial Response of 4D3-22**
5.5 General Results

The maximum loads obtained from the tests are compared to the analytical peak loads obtained from the SPARCS analyses in Table 5.6. The analytical values were found to be between 95% to 104% of the experimental values, with a standard deviation of 11%. The Scott column (tested by Mander) was not accounted for in the computation.

A graphical representation of Table 5.6 is shown in Figures 5.24 and 5.25. There was no significant difference in the computed values with respect to the experimental ones. Thus, it is apparent that the tentative model for strength enhancement proposed by Vecchio (1992) can be used in the analysis of confined concrete regardless of the element shape (e.g., circular, rectangular, and square columns), and concrete type (e.g., normal or high strength concrete).

### TABLE 5.6 Maximum Analytical to Experimental Load Ratio

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Column</th>
<th>( P_{\text{max}} ) (test)</th>
<th>( P_{\text{max}} ) (SPARCS) (w./sof)</th>
<th>( P_{\text{max}} ) (SPARCS) (wo./sof)</th>
<th>( P(\text{SPARCS})/P_{\text{max}} )</th>
</tr>
</thead>
<tbody>
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<td>Sheikh and Uzumeri</td>
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<td>3370</td>
<td>3660</td>
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<td>4636</td>
<td>5102</td>
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</tr>
<tr>
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<td>4B3-19</td>
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<tr>
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<tr>
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</tr>
<tr>
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<td>4D6-24</td>
<td>4725</td>
<td>4850</td>
<td>4858</td>
<td>0.98</td>
</tr>
<tr>
<td>Liu et al.</td>
<td>2C60-10S50-15</td>
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</tr>
<tr>
<td></td>
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<td>2460</td>
<td>2572</td>
<td>2831</td>
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</tr>
<tr>
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<td>2585</td>
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</tr>
<tr>
<td></td>
<td>2C60-10S50-15</td>
<td>3880</td>
<td>3324</td>
<td>3850</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>2C60-6S50-15</td>
<td>3850</td>
<td>3331</td>
<td>3698</td>
<td>0.87</td>
</tr>
<tr>
<td>Mander et al.</td>
<td>Wall 11</td>
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<td>4250</td>
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</tr>
<tr>
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<td>Scott</td>
<td>8600</td>
<td>6125</td>
<td>6970</td>
<td>0.71</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.96</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.11</td>
</tr>
</tbody>
</table>
Figure 5.24 Maximum Analytical to Experimental Load Ratio (wo./Softening)

Figure 5.25 Maximum Analytical to Experimental Load Ratio (w./Softening)
5.6 Analysis of Short Column Sections Designed according to CSA23.3-94 and ACI-318-95 Code Provisions

Two cross sections of a short square column were designed according to the Canadian and American design provisions for concrete (i.e., CSA23.3-94 and ACI-318-95, respectively). Specifically, they were designed for the minimum requirements for lateral steel in seismic zones. The sections were assumed to be one in the plastic hinge zone (i.e., near the top and bottom of the column) and one in the central region of the column away from the plastic zone (see Figure 5.26).

![Diagram of column sections]

Figure 5.26 Localization of Cross Sections

The column was assumed to be constructed of normal strength concrete of 30 MPa, with 400 MPa steel used for both the longitudinal and lateral steel. Eight longitudinal bars were supported by the outermost hoop and two cross ties, as shown in Figure 5.27a. In the SPARCS analyses, both sections were
subjected to concentric compression. As this is not the case for seismic actions, a close look at the expected ductility and strength enhancement due to confinement could be seen from these analyses. The finite element mesh used in the SPARCS is plotted in Figure 5.27b. Detailed design calculations and a summary of the code provisions for the design of short columns are given in Appendix A.

The axial load versus axial shortening curves for both sections are graphed in Figures 5.28 and 5.29, along with the curves predicted by the Sheikh-Uzumeri and Razvi-Saatcioglu models (which are explained in Appendix B). In the SPARCS analyses, the material models chosen in Chapter 4 were used with and without compression softening effect.

![Figure 5.27 Column Section and FE Mesh (CSA, ACI)](image-url)
**Figure 5.28 Axial Shortening of Section in Central Zone**

**Figure 5.29 Axial Shortening of Section in Plastic Hinge Zone**
For the SPARCS analysis, it is apparent that after the peak load had been reached, the column sustained its load carrying capacity for up to about twice the peak strain. From this point onwards, a sudden loss of capacity occurred. The Sheikh-Uzumeri and Razvi-Saatcioglu models followed parallel post-peak behaviors but at different load capacities, the former being more conservative. However, it should be noted that the finite element solution without compression softening lay between the two confinement models.

A comparison of the maximum load and the strain at peak load for both sections is given in Table 5.7. Also shown is the strain at 85% of the peak load in the post-peak region, $\varepsilon_{85}$; and the peak strain to $\varepsilon_{85}$ strain ratio. Although the column in the potential hinge zone apparently achieved some ductility, the axial column capacity is reduced between 5 and 17% (depending on the model considered) due to the higher tie spacing in the central zone, and the ductility dropped more drastically. This drawback could be of some importance in columns with high axial load and low moments caused by lateral loads (e.g., ground motions), such as internal columns in buildings.

TABLE 5.7 Short Column Sections in Seismic Zones, Model Comparisons

<table>
<thead>
<tr>
<th>Model</th>
<th>Hinge Region</th>
<th>Central Region</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{\text{max}}$</td>
<td>$\varepsilon_{\text{max}}$</td>
</tr>
<tr>
<td>Sheikh-Uz.</td>
<td>8106</td>
<td>2.19</td>
</tr>
<tr>
<td>Razvi-Saat.</td>
<td>9215</td>
<td>3.80</td>
</tr>
<tr>
<td>SPARCS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>w/soft</td>
<td>7800</td>
<td>7.60</td>
</tr>
<tr>
<td>w/soft</td>
<td>7020</td>
<td>5.70</td>
</tr>
</tbody>
</table>

* $\varepsilon_{85}$ is the calculated strain at 85% of the peak load in the post-peak region.
Comparisons with Sheikh-Uzumeri and Razvi-Saatcioglu Confinement Models

The axial shortening results obtained for the set of 14 specimens analyzed with SPARCS are compared with the analytical models for confined concrete columns subjected to concentric compression developed by Sheikh et al. (1982, 1992), and Razvi and Saatcioglu (1999). Description of both models and an example of how to compute the axial load versus axial strain using them are presented in Appendix B. Also shown in Appendix B is the complete set of axial shortening graphs for all of the specimens analyzed in this work.

The Sheikh and Uzumeri model was originally developed for columns of normal strength concrete; the model was modified to allow its use with any concrete strength. The Razvi and Saatcioglu model was intended for general application; it can be applied to rectangular, square or circular sections, and concrete may be normal or high strength.

It should be noted that the material behavior relationships used in the finite element analyses were (see Chapter 3): the Hogestad parabola for the pre-peak response; the Popovics curve for post-peak regime; the Vecchio's model for strength enhancement; the modified Kupfer model for lateral expansion (i.e., Poisson's ratio); and the Mohr-Coulomb criterion for concrete cracking.

The results for some of the columns are plotted in Figs. 5.30 to 5.33. Each graph includes the experimental curve, the FE curves from SPARCS (with and without softening), and the analytical models.
Figure 5.30 Model Comparison for Column 4B3-19

Figure 5.31 Model Comparison for Column 4D6-24
Figure 6.32 Model Comparison for Column 2C80-6S50-15

Figure 5.33 Model Comparison for Wall 11
The analytical models predict the compressive behavior but do not give an ultimate axial strain. The post-peak response of the Sheikh-Uzumeri and Razvi-Saatcioglu models were cut off in the plots, as they continue beyond a residual stress, which is $0.3f_{\text{c}}^\text{u}$ and $0.2f_{\text{c}}^\text{u}$ respectively.

The finite element modeling followed the pre-peak and post-peak branches in a manner similar to these well-known analytical models. Tables 5.8 and 5.9 give the theoretical to experimental ratios for the maximum load for all the models. SPARCS presented an average ratio between of 95% to 103% of the actual peak load for rectangular columns, and between 97% to 109% for the circular columns. All models had similar standard deviations. This proves the efficiency of SPARCS to trace the actual response of columns in concentric compression.

**TABLE 5.8 Theoretical to Experimental Maximum Load Ratios for Square and Rectangular Columns**

<table>
<thead>
<tr>
<th>Column</th>
<th>$P_{\text{max}}$ (test) [kN]</th>
<th>$P_{\text{max}}$ (SPARCS) [kN] w/soft</th>
<th>$P_{\text{max}}$ (SPARCS) [kN] w/o soft</th>
<th>Sheikh-Uzumeri [kN]</th>
<th>Razvi-Saat. [kN]</th>
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</thead>
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<tr>
<td>2A1-1</td>
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<tr>
<td>4C8-5</td>
<td>4710</td>
<td>4695</td>
<td>5102</td>
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<tr>
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<td>3622</td>
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<td>4B3-19</td>
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<td>4277</td>
<td>1.01</td>
<td>1.04</td>
</tr>
<tr>
<td>4B4-20</td>
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<td>4233</td>
<td>4379</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
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<td>4464</td>
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</tr>
<tr>
<td>4D6-24</td>
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<td>4857</td>
<td>0.85</td>
<td>0.97</td>
</tr>
<tr>
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<td>6125</td>
<td>8970</td>
<td>0.71</td>
<td>0.81</td>
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</table>

Average 0.96 1.03 1.04 1.08
Std Deviation 0.10 0.11 0.10 0.11
TABLE 5.9 Theoretical to Experimental Maximum Load Ratios for Circular Columns

<table>
<thead>
<tr>
<th>Column</th>
<th>$P_{test}$</th>
<th>$P_{max}$ (w./soft)</th>
<th>$P_{max}$ (wo./soft)</th>
<th>Razvi-Saatcioglu</th>
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</thead>
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<tr>
<td></td>
<td>(test) (kN)</td>
<td>(kN) (3)</td>
<td>(kN) (4)</td>
<td>(kN) (7)</td>
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<td>2531</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>1.15</td>
<td></td>
<td>3034</td>
</tr>
<tr>
<td>2C80-10S150-15</td>
<td>2300</td>
<td>2585</td>
<td>2859</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.24</td>
<td></td>
<td>2054</td>
</tr>
<tr>
<td>2C80-10S50-15</td>
<td>3880</td>
<td>3324</td>
<td>3850</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.99</td>
<td></td>
<td>4159</td>
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<tr>
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<td>3331</td>
<td>3698</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.96</td>
<td></td>
<td>4038</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>0.97</td>
<td>1.09</td>
<td>1.16</td>
</tr>
<tr>
<td>Std Deviation</td>
<td></td>
<td>0.13</td>
<td>0.13</td>
<td>0.12</td>
</tr>
</tbody>
</table>

It should be noted from Table 5.8 column (7), in the Sheikh and Uzumeri model; that the maximum analytical load for their columns was slightly different from the experimental ones. In some of the experiments, the peak load occurred when the contribution of the concrete cover vanished due to spalling, which was not the case when using the analytical model.
5.8 Finite Element Analysis of a Shear Wall

One of the shear walls tested by Lefas et al. (1990) (see Vecchio 1992) was chosen to examine the effect of confinement on concealed columns. This wall was analyzed by Vecchio (1992) using the two-dimensional finite element program TRIX, and by Selby and Vecchio (1993) using a previous version of SPARCS.

The wall (SW16, see Vecchio 1992) had a height-to-width ratio of 1.0; it was cast integrally with a top spreader beam and a heavily reinforced beam at its base. Properties for concrete and steel for this wall are given in Table 5.10 and plotted in Figure 5.34.

<table>
<thead>
<tr>
<th>Steel</th>
<th>$d_c$ [mm]</th>
<th>$A_s$ [mm$^2$]</th>
<th>$f_y$ [MPa]</th>
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<tr>
<td>Vertical</td>
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<tr>
<td>Horizontal</td>
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<td>30.7</td>
<td>520</td>
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<tr>
<td>Stirrups</td>
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<td>420</td>
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</table>

TABLE 5.10 Material Properties of Wall SW16

The wall was subjected to constant axial load and monotonically increased lateral load, with both loads applied at the top spreader beam. The compressive axial load was 460 kN. Lefas reported triaxial stress conditions at the bottom of the concealed column subjected to compression.

The finite element model for wall SW16 is shown in Figure 5.35. A total of 285 hexahedrons were used to model reinforced concrete; vertical, horizontal,
and tie steel were smeared within the concrete bricks. Table 5.11 shows the material zones for the finite element model.

![Diagram](image)

**Figure 5.34 Lefas et al. Wall SW16**

**TABLE 5.11 Material Zones for FE Model of SW16**

<table>
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<tr>
<th>Zone</th>
<th>Zone Number (1)</th>
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<th>$\rho_f$ (3)</th>
<th>$\rho_r$ (4)</th>
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<td>1</td>
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<td>0.000</td>
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<td>1.200</td>
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<td>3</td>
<td>3</td>
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</tr>
<tr>
<td>4</td>
<td>4</td>
<td>1.875</td>
<td>0.140</td>
<td>1.046</td>
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</table>

The material zone distribution for the FE mesh is shown in Figure 5.36.
Figure 5.35 Finite Element Mesh for Wall SW16

Figure 5.36 Material Zone Distribution for SW16
The vertical load of 460 kN was applied at the top spreader beam as joint forces, and the lateral load was applied as imposed displacements at the leftmost nodes of the top beam as shown in Fig 5.35. The analytical model considered compression softening, the confinement model proposed by Vecchio, and the variable lateral expansion.

The material properties for the finite element analysis were: plain concrete strength $0.85f'_c$ (37.4 MPa), the elastic modulus 33150 MPa, and the initial Poisson's ratio $v = 0.15$. Also, for the steel: the stiffness modulus was 210000 MPa, the strain and stiffness modulus at hardening were assumed as $2.5 \times 10^5$ and 10000 MPa, respectively.

The experimental and analytical shear-lateral displacement curves are shown in Figure 5.37, and a plot of the deformed wall near failure is shown in Figure 5.38 (horizontal load applied from left to right). The analytical response was stiffer than the experimental, but the analytical shear strength was 351 kN, which was only 1% smaller than the actual (the wall shear resistance was 355 kN). SPARCS underestimated the lateral displacement. However, the analytical curve traced well the overall response of the wall. Flexural and flexural shear cracks developed at the top beam and the concealed column on the tension side. Shear cracks spreaded out into the wall web and the bottom of the concealed column on the compression side was subjected to triaxial compressive stresses reaching a maximum principal compressive stress of 43 MPa (1.15 times $0.85f'_c$).
Wall SW16

Figure 6.37 Horizontal Response of Wall SW16

Max. Disp: 4.5mm

Figure 6.38 Sketch of Horizontal Displacement of Wall SW16
Chapter 6

Conclusion

6.1 Conclusions

6.1.1 Summary

The capabilities of finite element program SPARCS used in the modeling of confined concrete were examined. Constitutive material behavior models for strength enhancement, lateral expansion, concrete softening, and post-peak ductility were combined in a parametric study of four columns subjected to monotonic concentric compression. The columns represented different steel configurations and failure modes. The combination of pre- and post-peak compression curves for concrete, strength enhancement and variable Poisson’s ratio models that best fit the experimental results was chosen from the set of parameters analyzed. The current version of the finite element program was used to compare two of the four columns mentioned above with the analytical results obtained from a previous version. Significant improvement in the ductile behavior of the models was noted.
The Poisson's ratio was shown to be variable in the test data from ten circular high-strength columns. The variable lateral expansion model of SPARCS accurately traced the behavior of the circular columns.

The selected model combination from the parametric study was used to corroborate experimental results of another set of specimens tested by various researchers. The first attempt to model the confinement behavior of high-strength circular columns was made in this project. Elements of different size, steel arrangement, and concrete type were modeled and analyzed using SPARCS. The model combination accurately predicted the strength and post-peak behavior of the specimens. It was noted that compression softening of concrete influenced the axial shortening response of the columns.

The finite element response of the columns was compared with the analytical stress-strain models for confinement proposed by Sheikh and Uzumeri and Rasvi and Saatcioglu. It was found that the capacity and post-peak behavior predicted using SPARCS compared reasonably well.

Two sections of a theoretical column were designed according to the minimum requirements for seismic zones of the Canadian CSA/CAN A23.3-94 and American ACI-318-95R codes. The sections were located at the potential plastic hinge and at the middle of the column, respectively. The SPARCS analyses showed less ductility and smaller strength in the central zone than in the potential hinge zones, for the case of axial concentric compression.

A squat shear wall with concealed columns was subjected to constant axial compression and monotonically increasing lateral load was modeled using
the material models chosen in the parametric study. It was found that triaxial compression stresses did occurred at the base of one of the concealed columns. The strength was predicted with good accuracy; however the analytical response was somewhat stiff.

6.1.2 Specific Conclusions

- The pre-peak axial response was not significantly affected by the chosen base curve; the Hognestad parabola fit the actual response with sufficient accuracy.
- The Popovics curve reproduced well the post-peak behavior of axially loaded reinforced concrete specimens.
- The strength enhancement model proposed by Vecchio predicted the element capacity within a mean of 3% and standard deviation of 11% for all the specimens analyzed in this study.
- The variable Poisson’s ratio model effectively simulated the increased lateral pressure on the axially loaded columns.
- The current version of the program reasonably modeled cover spalling and tie strain variation of the column models.
- The use of imposed displacements in SPARCS eliminated the need for stiff springs to analytically reproduce the post-peak regime of reinforced concrete subjected to compression, a significant improvement over the previous version.
• The plain concrete strength of all the columns was taken as 85% of the standard cylinder strength for concrete. This value was in accordance to the observed capacity of the plain concrete columns that have been tested.

6.2 Improvements, Limitations and Recommendations for Future Work

6.2.1 Improvements in SPARCS

The SPARCS structure was enhanced in the following aspects:
• In the solver, that handles large models in any global direction.
• In the geometry checks of bricks and wedges.
• A more efficient method of numbering elements was suggested.
• In the calculation of the peak stress $f_p$ and strain at peak stress $\varepsilon_p$.
• In the variable Poisson's ratio.

Improvements in the formulation are required as follows:
• In the post-peak stress-strain ductility relation.
• In the transition between the unconfined and confined elements
• In the softened-unsoftened transition.

6.2.2 Limitations of the Analysis

Although an effort was made to model the confinement effect through a wide spectrum of alternatives (e.g., concrete types, steel arrangements, size, failure modes), some limitations are stated:
• Models with a large number of finite elements might be time consuming.
• The lateral pressure as defined in Vecchio and Selby enhancement models might overestimate the compressive capacity of triaxially stressed members other than columns since the use of $0.85f_c$ for plain concrete strength is common only for columns.

• The analytical responses have an upper bound if softening of concrete is not included in the program options. If softening is included, a lower bound could occur as both ductility and strength decrease.

• Buckling of reinforcement in compression has not yet been implemented.

6.2.3 Recommendations for Future Work

The following recommendations are made:

• In order to model bodies of revolution in SPARCS, such as circular columns, an axisymmetric element would facilitate the mesh generation.

• The confinement model should be tested in specimens subjected to moment and axial load (i.e., eccentric loads).

• Improvement in the Poisson's ratio to model conditions beyond the 0.5 limit.

• Enhancement of the post-peak model for confined concrete in terms of triaxial stresses, to model ductility and failure conditions.

• Improvements to the tension behavior model for concrete to better model highly confined specimens with practically no cracking of the core.

• Improvements to the definition of lateral pressure in the confinement models to consider uneven triaxial compressive stress states, such as at the border elements of a concrete core near peripheral stirrups in a column.
References


Appendix A

Design of Short Column Sections According to CSA23.3-94 and ACI-318-95R

Table A.1 shows a summary of the provisions for the design of spirals and ties in short columns according to the Canadian and American code provisions. The table covers the requirements for seismic and "non-seismic" zones, and notes are included in italics where there are differences between the codes.

A.1 Section Properties

The section is shown again in Figure A.1.

- Geometry

Square column, with B = H = 490 mm, Tie diameter $d_t = 11.3$ mm, Longitudinal Bar diameter $d_b = 19.5$ mm.

- Materials

Unconfined concrete strength $f_c = 30$ MPa, Yielding stress of steel $f_y = 400$ MPa
<table>
<thead>
<tr>
<th>Article</th>
<th>Condition</th>
<th>Minimum diameter</th>
<th>Clear spacing between bars</th>
<th>Medium bar</th>
<th>Minimum reinforcement</th>
<th>Staggered and unspreaded longitudinal bars</th>
<th>Minimum spacing (minimum in mm)</th>
<th>Minimum number of longitudinal bars</th>
<th>Minimum yield stress (ksi)</th>
<th>Minimum ultimate stress (ksi)</th>
<th>Longitudinal reinforcement</th>
<th>Effective length factor (L/fe)</th>
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<td>12.7 mm (1/2&quot;)</td>
<td>14.5 mm (9/32&quot;)</td>
<td>14.5 mm (9/32&quot;)</td>
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<td>12.7 mm (1/2&quot;)</td>
<td>14.5 mm (9/32&quot;)</td>
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<td>21.4.2.1</td>
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<td>21.4.2.3</td>
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<td>21.4.4.3</td>
<td>Strength of column or beam</td>
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<td>Strength of column or beam</td>
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</tr>
</tbody>
</table>

**Description:**
- Longitudinal reinforcement
- Transverse reinforcement
- Minimum spacing between core lines

**Notes:**
- Values are in accordance with the Code for steel structures.
- Values are in accordance with ACI 318-99 and ASHRAE.
- Values are in accordance with the Code for steel structures.
- Values are in accordance with the Code for steel structures.
A.2 Design using Canadian Standard CSA23.3-94 (Seismic Provisions Section 21)

- [21.4.3.1] Longitudinal steel ratio $\rho$:

  $$0.01 \leq \rho \leq 0.06$$

  Thus,

  $$\rho = \frac{8 \times (300 \text{mm}^2)}{490^2} = 1.0\%$$

- [21.4.4.2] Minimum lateral steel area $A_{sh}$:

  $$A_{sh} \geq 0.3 sh \frac{f'c}{f_{yh}} \left( \frac{A_T}{A_{sh}} - 1 \right)$$

  or,

  $$A_{sh} \geq 0.3 sh \frac{f'c}{f_{yh}}$$
where \( h_o = 410 \text{ mm} \) (outer-to-outer external hoop distance), \( A_{oh} = h_o^2 \), \( A_g \) is the gross area section = 240100 mm\(^2\), and \( s \) is the tie spacing. Thus, for this section the former equation governs, \( A_{oh} \geq 3.951s \). As the section has 3 legs in each direction, \( A_{oh} \geq 3(100 \text{ mm}^2) = 300 \text{ mm}^2 \), and

\[
s \leq \frac{A_{oh}}{3.951} = 75 \text{mm}
\]

- [21.4.4.3] Minimum tie spacing in potential plastic hinge zones

\[
s \leq \begin{cases} \frac{1}{4}(B = 490 \text{mm}) \\ 100 \text{mm} \\ 6d_b \\ 312 \text{mm}[7.6.5.2] \end{cases}
\]

Since the calculated spacing is smaller than the values from this clause, \( s = 75 \text{mm} \)

- [21.4.4.6] Tie spacing in zones away from potential plastic hinges:

\[
s \leq \frac{6d_b}{150 \text{mm}}
\]

Thus, \( s = 6(19.5 \text{mm}) = 117 \text{ mm} \), \( s \equiv 115 \text{ mm} \)

A.3 Design check American Standard ACI-318-95R (Seismic Provisions Section 21)

- [21.4.1.1] B, \( H \geq 12^\circ \) (305 mm) o.k.
- [21.4.1.2] \( B/H = 1.0 \geq 0.4 \) o.k.
- [21.4.3.1] Longitudinal steel ratio \( \rho = 1\% \) o.k.
- [21.4.4.1] \( s = 75 \text{ mm} \) o.k.
• [21.4.4.2] Minimum spacing in plastic hinge zones

\[ s \leq \frac{1}{4}(490) = 122.5\text{mm} \]
\[ \text{100mm} \]

Thus, \( s = 75 \text{ mm} \) o.k.

• [21.4.4.3] Maximum distance between supported bars = 165 mm < 355 mm

o.k.

• [21.4.4.6] Tie spacing outside hinge region

\[ s \leq \frac{6d}{150\text{mm}(6")} \]

Thus \( s = 115 \text{ mm} \) as for the CSA23.3-94 design.
Appendix B

Sheikh and Uzumeri, and Razvi and Saatcioglu Model Calculations

The axial load versus axial strain curve of an element subjected to axial concentric compression using the two models is given in this appendix. Computations for all the specimens analyzed in this project followed the same procedure. Wall 11 (tested by Mander) was chosen as example.

B.1 Wall 11 Data

The section of this wall is reproduced in Figure B.1 and its properties are given in Table B.1.

![Wall 11 Diagram]

Figure B.1 Cross Section of Wall 11
### TABLE B.1 Wall 11 Properties

<table>
<thead>
<tr>
<th>Label</th>
<th>Tie Steel</th>
<th></th>
<th></th>
<th></th>
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<tr>
<td></td>
<td>type</td>
<td>$d_n$ [mm]</td>
<td>$A_n$ [mm$^2$]</td>
<td>$\rho_t$ [%]</td>
<td>$f_{yh}$ [MPa]</td>
<td>$E_n$ [MPa]</td>
<td>$E_{sh}$ [mm/m]</td>
<td>$\epsilon_{sh}$</td>
<td>$s$ [mm]</td>
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<tr>
<td>Wall 11</td>
<td>hoops</td>
<td>6.0</td>
<td>28.3</td>
<td>2.33</td>
<td>310</td>
<td>198000</td>
<td>3200</td>
<td>22.00</td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

| Label | Longitudinal Steel | Concrete | | | | | | | | | |
|-------|---------------------|-----------|---|---|---|---|---|---|---|---|
|       | $d_n$ [mm] | $A_n$ [mm$^2$] | $\rho_t$ [%] | $f_y$ [MPa] | $E_n$ [MPa] | $E_{sh}$ [mm/m] | $\epsilon_{sh}$ | $f_s$ [MPa] | $E_s$ [MPa] |
| Wall 11 | 12.0          | 3217      | 3.06          | 290          | 191000      | 3900          | 24.00          | 41            | 31000     |

**B.2 Sheikh and Uzumeri Model**

The "analytical model for concrete confinement in tied columns" was reported by the authors in 1982, and modified by Sheikh and Yeh (1992). A detailed description can be found elsewhere (Sheikh et al. 1982, 1992). The model is applicable to columns of normal concrete strength. Notation follows that of the papers. The model was applied to wall 11 in the following manner:

#### B.2.1 Parameters

- **Geometry:**
  - Gross Area of the section: $700 \times 500$ mm $= 105000$ mm$^2$
  - Centre-to-centre distance of outermost tie: $B = 944$ mm, $H = 94$ mm.
  - Tie spacing: $s = 50$ mm
  - Cover to centre line of tie: $25$ mm $+ d_n/2 = 25$ mm $+ 3$ mm $= 28$ mm
Volumetric ratio $\rho_s = 0.0233$

Area of confine concrete $A_{ce} = B \times H = 944 \times 94 = 60536 \text{ mm}^2$

Strain at peak unconfined stress $\varepsilon_{ce} = 2.56 \text{ mm/m}$

- Effective confined core area to core area ratio at tie level $\lambda$

$$\lambda = 1 - \frac{\sum C_i^2}{\alpha A_{ce}}$$

where $C_i$ is the centre-to-centre distance between longitudinal bars, for this case: $C_1 = 66 \text{ mm (long direction)}, C_2 = 88 \text{ mm (short direction)}$

$\alpha = 5.5$ (statistically determined factor)

$n = \text{number of arcs covered between longitudinal bars} = 16$

Thus,

$$\lambda = 0.65$$

- Area of effectively confined concrete $A_{ce}$ (midlevel between ties):

$$A_{ce} = \lambda (B - 0.5s \tan \theta)(H - 0.5s \tan \theta)$$

where $\theta$ is the arc angle, taken as $45^\circ$. Thus,

$$A_{ce} = 42711 \text{ mm}^2$$

- Ratio of area effectively confined at critical section to the area of the core $\lambda^*$:

$$\lambda^* = \frac{A_{ce}}{A_{ce}}$$

(B.3)
Thus, $\lambda^* = 0.70$.

- Capacity of unconfined concrete core:

$$P_{\infty} = 0.85 f'_c (A_w - A_s)$$  \hspace{1cm} (B.4)

where $f'_c = 41$ MPa, thus $P_{\infty} = 1998$ kN.

- Gain in concrete strength $K_c$:

$$K_c = 1.0 + \frac{1}{P_{\infty}} \lambda \left( 1 - \frac{0.5 s}{B} \tan \theta \right) \left( 1 - \frac{0.5 s}{H} \tan \theta \right) B H \beta \left( \rho_c f'_c \right)^{\gamma}$$  \hspace{1cm} (B.5)

where $\beta = 0.0071$, $\gamma = 0.5$ are statistic parameters given, and $f'_c$ is the tie stress, considered as the yielding stress. Thus for this example:

$$K_c = 1.28$$

- Determination of minimum strain corresponding to maximum concrete stress

$$\varepsilon_{n1} = 2.2 K_c \left( \frac{C}{m} \right)$$  \hspace{1cm} (B.6)

Here, $\varepsilon_{n1} = 2.816 \text{ mm/m}$

- Determination of maximum strain corresponding to maximum concrete stress

$$\frac{\varepsilon_{n2}}{\varepsilon_{\infty}} = 1 + \frac{248}{C} \left[ 1 - 5.0 \left( \frac{s}{B} \right)^2 \right] \rho_c f'_c \sqrt{f'_c}$$  \hspace{1cm} (B.7)

where $C$ is the spacing between longitudinal bars (averaged out in this calculation to 85.3 mm). Thus $\varepsilon_{n2} = 11.5 \text{ mm/m}$

- Determination of strain at 85% of the maximum concrete stress in the descending branch (post-peak behavior)

---

Modeling of Confined Concrete
The least value for $\varepsilon_{\text{ess}}$ was obtained in the short direction of the section (i.e., along the 150-mm side), thus $\varepsilon_{\text{ess}} = 16.7 \text{ mm/m}$

- Maximum stress in concrete $f_{\text{uc}}$:

$$f_{\text{uc}} = (0.85f'_c)K_s$$  \hspace{1cm} (B.9)

Thus, $f_{\text{uc}} = 44.6 \text{ MPa}$

B.2.2 Stress-Strain Curves

- Confined Concrete

  For the concrete core, the stress-strain curve begins with a parabola until the maximum confined stress; $f_{\text{uc}}$, is reached at the strain $\varepsilon_{\text{ef}}$, followed by a plateau up to the strain $\varepsilon_{\text{e2}}$, and descending with a slope determined by the strain $\varepsilon_{\text{ess}}$ at 0.85$f_{\text{uc}}$. The curve continues from this point until a stress of 0.30$f_{\text{uc}}$ has been reached. The model suggests a horizontal line from the latter point.

- Unconfined Concrete

  Sheikh and Uzumeri measured strains in the concrete cover up to about 3 to 3.5 mm/m in their columns (see Sheikh et al. 1980). It was suggested that the unconfined concrete curve used in the calculations of the total load versus axial strain were a parabola with a maximum stress of 0.85$f_c$ and with a linear descending branch up to a maximum strain between the values mentioned.
above and a stress of \(0.425f'_c\). In this work, the descending branch was calculated until a strain equal to \(1.5\varepsilon_{	ext{cr}}\), about \(3 \text{ mm/m}\) for normal strain concrete.

The stress-strain curve for both confined and unconfined concrete is graphed in Figure B.2. Also shown in Figure B.3 is the stress-strain curve for the longitudinal steel of the wall.

![Wall 11 Stress-Strain Curve](image)

**Figure B.2 Strain Curves for Concrete, Wall 11, Sheikh and Uzumeri Model**

![Wall 11 Longitudinal Steel Curve](image)

**Figure B.3 Strain Curve for Long. Steel, Wall 11**
B.2.3 Axial Load versus Axial Strain Curve

The axial load is computed using the following formula:

\[ P = f_{\infty} (A_{\infty} - A_z) + f_c (A_z - A_{\infty}) + f_s A_s \]

(B.10)

where the first term on the right hand side refers to the axial load carried by the concrete core \((f_{\infty} \text{ is the concrete core stress at a given axial strain})\), the second term is the axial load carried by the concrete cover, and the last term is the load in the longitudinal steel. Figure B.4 shows the Load-Strain curve for Wall 11. The contributions to the total load of each of the concrete components and the steel are also shown.

Wall 11

![Load-Strain Curve](image)

Figure B.4 Load-Strain Curve of Wall 11, Sheikh and Uzumeri Model
B.2.4 Parameters $\varepsilon_{s1}$, $\varepsilon_{s2}$, $\varepsilon_{s85}$ and $K_s$ for All Columns

The parameters for the square and rectangular columns of this study are given in Table B.2. Values for Sheikh and Uzumeri columns were obtained in part from the author's paper published in 1982. The remaining were computed by the writer using the method outlined in the preceding sections.

**TABLE B.2 Parameters for Sheikh-Uzumeri Model**

<table>
<thead>
<tr>
<th>Column (1)</th>
<th>$K_s$ (2)</th>
<th>$\varepsilon_{s1}$ (3)</th>
<th>$\varepsilon_{s2}$ (4)</th>
<th>$\varepsilon_{s85}$ (5)</th>
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<tbody>
<tr>
<td>2A1-1</td>
<td>1.22</td>
<td>0.00268</td>
<td>0.00410</td>
<td>0.00800</td>
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<tr>
<td>4C8-5</td>
<td>1.60</td>
<td>0.00352</td>
<td>0.01600</td>
<td>0.02850</td>
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<td>2C5-17</td>
<td>1.41</td>
<td>0.00310</td>
<td>0.00540</td>
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<td>4B3-19</td>
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<td>0.00297</td>
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<td>1.28</td>
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<td>0.01150</td>
<td>0.01870</td>
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</table>
B.3 Razvi and Saatcioglu Model

The confinement model proposed by the authors (Razvi and Saatcioglu 1999) was briefly summarized in the literature review in Chapter 2. The model is applicable to normal and high-strength concrete, and to circular and rectangular sections (including square sections). The tie arrangement may be hoops, cross ties, spirals or welded wire fabric. This model was derived through an extensive statistical study of specimens tested by the researchers and other groups. The following calculations were made for Wall 11.

B.3.1 Parameters

- Geometry (using Razvi and Saatcioglu’s notation)

Taking x-x axis in the long direction (i.e., along 700-mm side) and y-y axis in the short direction (i.e., along 150-mm side):

Centre-to-Centre distance of outermost hoop: \( b_{ox} = 644 \text{ mm}, \ b_{oy} = 94 \text{ mm} \)

Tie spacing: \( s = 50 \text{ mm} \)

Area of transverse steel: in x-x: \( A_{ox} = 283 \text{ mm}^2 \), in y-y \( A_{oy} = 56.6 \text{ mm}^2 \)

Spacing between supported long. bars: \( s_{x} = 88.9 \), \( s_{y} = 72 \text{ mm} \)

- Transverse steel ratio \( \rho_c \):

\[
\rho_c = \frac{A_{ox} + A_{oy}}{3(b_{ox} + b_{oy})}
\]

For wall 11 \( \rho_c = 0.0092 \).

- Reduction coefficient of average lateral pressure \( k_l \):
\[ k_2 = 0.15 \sqrt{\left(\frac{\alpha}{90}\right)^2} \leq 1.0 \]  
\hspace{2cm} (B.12)

Thus, for x-x direction \( k_{2x} = 1.0 \), in y-y direction \( k_{2y} = 0.24 \)

- Tie stress \( f_s \) at maximum confinement pressure:

\[ f_s = E_s \left( 0.0025 + 0.04 \sqrt{\frac{k_2 \rho_c}{f'_{\infty}}} \right) \leq f_{yr} \]  
\hspace{2cm} (B.13)

where \( f_{yr} \) is the tie yielding stress, and \( f'_{\infty} = 0.85 f_c \). In this case \( f_s = f_{yr} = 310 \) MPa

- Average lateral pressure \( f_l \):

\[ f_l = \frac{\sum_{i=1}^{q} (A_s f_s \sin \alpha)_i}{s b_c} \]  
\hspace{2cm} (B.14)

where \( q \) number of tie legs that cross through the direction being analyzed (i.e., x-x or y-y), \( A_s \) is the area of a tie leg, \( \alpha \) is the angle of inclination of the leg with respect to the crossing direction (e.g., if the tie leg is perpendicular to the x-x direction \( \alpha = 90^\circ \)).

For wall 11, \( A_s = 28.3 \) mm\(^2\) in both directions, \( \alpha = 90^\circ \), \( q_x = 10 \), \( q_y = 2 \), thus \( f_{lx} = 2.72 \) MPa, \( f_{ly} = 3.73 \) MPa.

- Equivalent lateral pressure in each direction \( f_{le} \):

\[ f_{le} = k_2 \cdot f_l \]  
\hspace{2cm} (B.15)

and a weighted average may be used when \( f_{le} \) is different in both directions:
\[ f_{es} = \frac{f_{ls} \cdot b_x + f_{lsy} \cdot b_y}{b_x + b_y} \]

(B.16)

Thus, \( f_{ls} = 2.72 \) MPa, \( f_{lsy} = 0.88 \) MPa, and the average \( f_{es} = 2.49 \) MPa.

- **Strength enhancement factor \( k_1 \):**

\[ k_1 = 6.7(f_{es})^{-0.17} \]

(B.17)

Thus, \( k_1 = 5.74 \) for wall 11.

- **Confinement Strength \( f_{cc} \):**

\[ f'_{cc} = f_{cc} + k_1 \cdot f_{es} \]

(B.18)

For the example, \( f_{cc} = 49.1 \) MPa.

- **Factor that accounts for type of concrete (i.e., normal or high strength concrete) \( k_3 \):**

\[ k_3 = \frac{40}{f'_{cc}} \leq 1.0 \]

(B.19)

Thus, \( k_3 = 1.0 \) \( (f_{cc} = 0.85(41) = 34.9 \) MPa)

- **Factor that accounts for lateral steel strength \( k_4 \):**

\[ k_4 = \frac{f_{sl}}{500} \geq 1.0 \]

(B.20)

Thus, \( k_4 = 1.0 \) \( (f_{sl} = 310 \) MPa)

- **Parameters for unconfined concrete curve:**

  Strain at maximum peak unconfined stress \( f_{cc}, \epsilon_{01}: \)
\[ \varepsilon_{01} \leq 0.0028 - 0.0008k_3 \]  
(B.21)

If \( \varepsilon_{01} \) was provided from tests of plain concrete columns, it should not exceed the value computed with the above equation. For wall 11, \( \varepsilon_{01} = 2.0 \text{ mm}/\text{m} \).

Strain at 0.85 \( f_{\text{cov}} \) in the descending branch, \( \varepsilon_{35} \)

\[ \varepsilon_{35} = \varepsilon_{01} + 0.0018k_3^2 \]  
(B.22)

This equation is valid only if

\[ \rho_e \leq 0.03 - 0.01k_3 \]  
(B.23)

Thus, as \( \rho_e = 0.0092 < 0.02 \) o.k., then \( \varepsilon_{35} = 3.8 \text{ mm}/\text{m} \).

- Parameters for confined concrete curve:

Strain at maximum peak confined stress \( f_{\text{cov}} \), \( \varepsilon_1 \):

\[ \varepsilon_1 = \varepsilon_{01}(1 + 5k_3K) \]  
(B.24)

where

\[ K = \frac{k_1f_c}{f_{\text{cov}}} \]  
(B.25)

For this case, \( K = 0.41 \), and \( \varepsilon_1 = 6.1 \text{ mm}/\text{m} \).

Strain at 0.85 \( f_{\text{cov}} \) in the descending branch, \( \varepsilon_{35} \)

\[ \varepsilon_{35} = 260k_3\rho_e\varepsilon_1[1 + 0.5k_2(k_4 - 1)] + \varepsilon_{35} \]  
(B.26)

Thus, \( \varepsilon_{35} = 18.39 \text{ mm}/\text{m} \)
B.3.2 Stress-Strain Curves

- Confined and Unconfined Concrete

The researchers proposed the use of similar shapes for both types of concrete.

For the ascending branch, the Popovics curve in the form:

\[ f' \alpha = \left( \frac{\varepsilon_\alpha}{\varepsilon_1} \right)^r \]

\[ f = \frac{f' \alpha}{1 + \left( \frac{\varepsilon_\alpha}{\varepsilon_1} \right)^r} \]

(B.27)

where,

\[ r = \frac{E_\alpha}{E_\alpha - E_\infty} \]

(B.28)

and

\[ E_\alpha = 3320 \sqrt{f'} + 6900 \]

(B.29)

For the confined core and unconfined cover, the equations are valid up to the strain \( \varepsilon_1 \) and \( \varepsilon_{01} \), respectively.

The post-peak branches follow a straight line with a slope defined by the strain \( \varepsilon_{25} \) (or \( \varepsilon_{025} \)). As the concrete stress reaches 20% of the peak stress, the model follows a horizontal line afterwards (only for confined concrete).

The stress-strain curve for longitudinal steel is as for the Sheikh and Uzumeri model; elasto-plastic with strain hardening. Figure B.5 shows the stress-strain curves for concrete of wall 11 computed using the Razvi and Saatcioglu model.
Figure B.5 Strain Curves for Concrete, Wall 11, Razvi and Saatcioglu Model

B.3.3 Axial Load versus Axial Strain Curve

The Equation (B.10) is used to compute the contributions of concrete and longitudinal steel to the total axial load. Figure B.6 shows the axial response of wall 11 predicted by the Razvi and Saatcioglu model.

Figure B.6 Load-Strain Curve of Wall 11, Razvi and Saatcioglu Model
### B.3.4 Razvi and Saatioglu Parameters for All Columns

Table B.3 shows the parameters necessary to compute the stress-strain curves for all of the columns analyzed in this work.

#### TABLE B.3 Razvi and Saatioglu Parameters

<table>
<thead>
<tr>
<th>Column Label</th>
<th>$f_0$ [MPa]</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$k_4$</th>
<th>$K$</th>
<th>$e_{01}$ [mm/m]</th>
<th>$e_{02}$ [mm/m]</th>
<th>$e_1$ [mm/m]</th>
<th>$e_2$ [mm/m]</th>
<th>$f_{cc}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.92</td>
<td>6.80</td>
<td>0.48</td>
<td>1.00</td>
<td>1.06</td>
<td>0.20</td>
<td>2.000</td>
<td>3.800</td>
<td>3.954</td>
<td>8.685</td>
<td>38.1</td>
</tr>
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### B.4 Axial Shortening Curves

The experimental curves for the columns of Chapter 4 and 5 are compared with the SPARCS analyses and the predicted load versus deformation curves using the models described in this Appendix.
B.4.1 Sheikh and Uzumeri Columns

Figure B.7 Column 2A1-1

Figure B.8 Column 4C6-5
Figure B.9 Column 2C5-17

Figure B.10 Column 4B3-19
Figure B.11 Column 4B4-20

Figure B.12 Column 4D3-22
Figure B.13 Column 4D6-24

B.4.2 Liu et al Columns

Figure B.14 Column 2C60-10S50-15
Figure B.15 Column 2C60-10S100-15

Figure B.16 Column 2C60-150-15
Figure B.17 Column 2C80-10S50-15

Figure B.18 Column 2C80-6S50-15
B.4.3 Scott Column (Mander)

![Scott Column Diagram](image)

**Figure B.19 Scott Column**
Appendix C

Variable Poisson's Ratio in Liu et al. Columns

The Poisson's ratio, ν, was calculated for ten of the columns tested by Liu et al. (1998). Plots for eight of these specimens are shown in this appendix, the remaining two were shown in Chapter 4. The graphs include load-strain curves, and normalized curves of the variable Poisson's ratio versus axial and volumetric strain as well as the variation of the lateral expansion with respect to the applied axial load.

It was concluded that the model for variable Poisson's ratio proposed by Vecchio (1992) and implemented in SPARCS predicted reasonably well this effect on confined concrete.
Figure C.1 Load-Strain Curve of 2C60-10S50-15

Figure C.2 Load-Poisson's Ratio Curve of 2C60-10S50-15
Figure C.3 Poisson's Ratio-Axial Strain Curve of 2C60-10S50-15

Figure C.4 Volumetric-Axial Strain Curve of 2C60-10S50-15
Figure C.5 Load-Strain Curve of 2C60-10S100-15

Figure C.6 Load-Poisson's Ratio Curve of 2C60-10S100-15
Figure C.7 Poisson's Ratio-Axial Strain Curve of 2C60-10S100-15

Figure C.8 Volumetric-Axial Strain Curve of 2C60-10S100-15
Figure C.9 Load-Strain Curve of 2C60-10S150-15

Figure C.10 Load-Poisson’s Ratio Curve of 2C60-10S150-15
Figure C.11 Poisson's Ratio-Axial Strain Curve of 2C60-10S150-15

Figure C.12 Volumetric-Axial Strain Curve of 2C60-10S150-15
Figure C.13 Load-Strain Curve of 2C80-10S50-15

Figure C.14 Load-Poisson's Ratio Curve of 2C80-10S50-15
Figure C.15 Poisson's Ratio-Axial Strain Curve of 2C80-10S50-15

Figure C.16 Volumetric-Axial Strain Curve of 2C80-10S50-15
Figure C.17 Load-Strain Curve of 2C80-6S100-15

Figure C.18 Load-Poisson's Ratio Curve of 2C80-6S100-15
Figure C.19 Poisson's Ratio-Axial Strain Curve of 2C80-6S100-15

Figure C.20 Volumetric-Axial Strain Curve of 2C80-6S100-15
Figure C.21 Load-Strain Curve of 2C90-6S100-25

Figure C.22 Load-Poisson's Ratio Curve of 2C90-6S100-25
Figure C.23 Poisson's Ratio-Axial Strain Curve of 2C90-6S100-25

Figure C.24 Volumetric-Axial Strain Curve of 2C90-6S100-25
Figure C.25 Load-Strain Curve of 2C90-6S50-25

Figure C.26 Load-Poisson's Ratio Curve of 2C90-6S50-25
Figure C.27 Poisson's Ratio-Axial Strain Curve of 2C90-6S50-25

Figure C.28 Volumetric-Axial Strain Curve of 2C90-6S100-25
Figure C.29 Load-Strain Curve of 2C90-10S100-0

Figure C.30 Load-Poisson's Ratio Curve of 2C90-10S100-0
Figure C.31 Poisson’s Ratio-Axial Strain Curve of 2C90-10S100-0

Figure C.32 Volumetric-Axial Strain Curve of 2C90-10S100-0