The Investigation of Turbulent Statistics of a Fibre Suspension Undergoing Mixing

by

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A thesis submitted in conformity with the requirements for the degree of Masters of Applied Science
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Abstract

This thesis investigates the statistical properties of a fibre suspension in a turbulent flow. An experiment in a mixing jar was performed using Particle Image Velocimetry (PIV) optimized with particle tracking and refractive index matching. The experimental Reynolds numbers range from 13,300 to 53,400 (Re= NR²/ν, where N is the rotation speed of the impeller in that tank, R is the radius of the impeller which is 0.0125m, ν is the kinetic viscosity which is 15.6 ×10⁻⁶ m²/s). The effect of various mass concentrations of fibres on the flow properties were investigated and the flow patterns in the mixing tank were analyzed. Turbulence quantities were strongly influenced by the existence of the fibres in the liquid.

Key words

Flocculation, PIV, LDV, Turbulence.
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Nomenclature

\text{Re} \quad \text{Reynolds Number}

\text{R} \quad \text{Radius of the impeller}

\text{L} \quad \text{length of fibre}

\text{d} \quad \text{diameter of fibre}

\text{r} \quad \text{the ratio of length to diameter of fibre}

\text{N}_c \quad \text{crowd factor}

\bar{G} \quad \text{global velocity gradient as defined by Cheng et al.}

\bar{u} \quad \text{instantaneous velocity}

\text{U} \quad \text{mean velocity}

\text{u} \quad \text{fluctuation velocity}

\bar{p} \quad \text{instantaneous pressure}

\text{P} \quad \text{low-frequency pass pressure}

\text{p}' \quad \text{fluctuation pressure}

u_i \quad \text{velocity for component } i

\text{N} \quad \text{Rotation Speed of the impeller (rpm)}

u_{\text{rms}} \quad \text{Root mean square of fluctuation velocity of x component}

v_{\text{rms}} \quad \text{Root mean square of fluctuation velocity of y component}

\text{d} \quad \text{diameter of the impeller}

\rho \quad \text{density}

\nu \quad \text{dynamic viscosity}

\text{c}_m \quad \text{mass concentration}
\[ \text{mass of the fibre} \]
\[ \text{mass of the oil} \]
\[ \varepsilon \quad \text{Dissipation} \]
\[ k \quad \text{turbulent kinetic energy} \]
\[ \eta \quad \text{Kolmogorov Scale} \]
\[ \overline{uv} \quad \text{Reynolds shear stress} \]
\[ \partial \quad \text{partial derivative} \]
\[ e_{ij} \quad \text{strain} = \frac{\partial u_i}{\partial x_j} \]

\[ \zeta \quad \text{Error in function} \]
\[ \sigma_{\zeta} \quad \text{Uncertainty in the function } \zeta \]
\[ \gamma_i \quad \text{Independent variable} \]
\[ I(i,j) \quad \text{Gray matrix of images} \]
\[ D(i,j) \quad \text{Displacement matrix of the flow field} \]
\[ A_i \quad \text{Area of ith face.} \]
Chapter 1 Introduction

1.1 Background

Fibre suspensions are fundamental to papermaking. A fibre suspension enters the papermachine (Figure 1.1) at the headbox, and, in a simplified description, paper making requires the removal of moisture from the suspension, resulting in paper. The initial orientation and distribution of the fibres at the head box exit will determine the final quality of the paper. The fibre suspension enters the headbox through a header pipe (Figure 1.1) and every effort is made to ensure:

(a) the fibres are evenly distributed across the width of the machine,
(b) fibre flocs are eliminated,
(c) the fibre stock is evenly discharged from the slice exit (Figure 1.2)
(d) the fibre slurry impinges on the forming fabric at the designated location and angle (Smook (29)).

Figure 1.1 Headbox

Figure 1.2 Enlarged view of Jet exit of headbox
Fibres used in the Pulp and Paper industry have a large aspect ratio \( L/d \geq 60 \), (where \( L \) is the length of the fibre and \( d \) is the diameter of the fibre). As the fibres have large aspect ratios, mechanical entanglement causes the fibres to interlock to form flocs (fibre networks) as the fibres are mixed within the headbox. Flocculation is a fundamental characteristic of fibre suspensions, and is responsible for uneven light dispersal qualities in the final sheet, an important aspect of quality as defined within the Pulp and Paper industry. Turbulent dispersion has been widely used to break up flocs. It is not understood how fibre and floc interactions affect turbulence within this flow.

Understanding fibre suspension dynamics is of great interest to the Pulp and Paper industry. A numerical model of turbulent fibre suspension flow would be useful for greater control of flocculation and the improving of paper quality.

Turbulence mixing in a tank has been of interest to researchers from many fields because the stirred vessel is commonly used in industry to promote reactions and enhance mixing, e.g. waste water process. Mixing research is important in the Pulp and Paper industry because the motion of the fibre suspension in the headbox can be viewed as a mixture process of fibre in water.

1.2 Previous work

1.2.1 Theoretical work
Mason (1, 2, 3) studied flocs in a fibre suspension and found mechanical entanglement of fibres (figure 1.3) was the dominant factor in the formation of flocs in shear flows. The relative velocity between fibres in shear flow ensures that fibres will contact each other and form flocs as well as ensuring that flocs disperse. Mason (4) defined a dynamic equilibrium of the continuous formation and dispersion (flocs ruptured by shear forces that are stronger than floc strength) of the flocs in dilute fibre suspension flow, e.g. mass consistency of 0.1%. An increase in velocity will ensure higher dispersion than formation because the flocs undergo higher shear than the floc can withstand.

Meyer and Wahren (6) developed a model for fibre network strength, see Figure 1.3. Figure 1.3a is a stable formation structure, figures 1.3b and 1.3c show contact types that cannot form a stable floc. When a fibre contacts with three or more other fibres, a floc is formed.

![Figure 1.3 Fibre network structure (Kerekes (14))](image)

The network strength is a function of the normal and frictional forces transmitted at the contact points between fibres, \( F = kN \), where \( k \) is the friction coefficient between fibres, \( N \) is the normal force between fibres and \( F \) is the tangential force between fibres (Figure 1.3a).
Andersson (21) stated that to break a floc one has to apply a force greater than the network strength.

Lee and Brodkey (19) studied the dispersion of flocs in high and low speed flow and determined that there were two major mechanisms observed in the dispersion process of a pulp floc. One mechanism involves large-scale, global phenomena that includes deformation, breaking, and fragmentation of the entire floc (shown in Figure 1.4a). The other is the small-scale local phenomenon of surface erosion which involves only individual surface fibres (shown in Figure 1.4b). They also found that dispersion time of flocs decreases as the turbulent intensity increases.

Robertson and Mason (5) have discussed flocculation for different turbulence intensities and length scales of the turbulent suspension. In the turbulent regime, they proposed that the length scale of turbulence was related to the degree of flocculation, and that the presence of flocs inhibited the development of small length scale turbulence because the flocs are larger than the small scale eddies. Parker (24) concluded that for a turbulent fibre suspension, the smaller scale of the turbulence causes better dispersion of flocs.
In decaying turbulence downstream of a grid in a pipe, Kerekes (13) observed that flocs can travel a large number of pipe diameters (>5) before they break up, rather than immediately after formation. Kerekes (13) also observed the continuous formation and rupture of flocs. Transient flocs were observed where the turbulent intensity was high. Further downstream of the grid where the energy decayed, the transient flocs turned into static or stable flocs.

Kerekes and Schell (18) have characterized flocculation regimes by the crowding factor \( N_c \). The crowding factor is the number of fibres inside a sphere with diameter equal to the fibre length,

\[
N_c = \frac{1}{3} c_v r^2
\]  

(1.1)

where \( c_v \) is the volumetric concentration of fibres \( (kg/m^3) \) and \( r \) is the length to diameter ratio. They reported that coherent flocs (stable flocs) formed when \( N_c > 130 \). The crowding factor inside the flocs is higher than the crowding factor in the surrounding regions, and when \( N_c > 60 \) in these surrounding regions, flocs formed by collisions appeared and coherent flocs formed.

1.2.2 Previous measurements in fibre suspensions

Direct quantitative measurement in fibre suspension flows are needed in order to obtain information about the influence of fibres and flocs on the turbulence. The nature of fibre suspensions makes it very difficult to perform velocity measurements within the mixture.
Anderson (28) reported that Hot Film Anemometer (HFA) failed because fibres immediately clogged the sensors, rendering any calibration useless.

Kerekes (15) performed Laser Doppler Velocimetry (LDV) measurements in a 0.5% (mass concentration, fibre mass/fluid mass) wood fibre suspension. Kerekes (15) could not obtain a velocity field in the fibre suspension and was only able to obtain data close to the wall (less than 1 cm from the wall). Since wood fibres are opaque in water, it is impossible to illuminate the fluid suspension. An additional difficulty was that both the fibres and the seed particles backscatter, thus making it difficult to differentiate between the signal from wood fibres and seeding particle. No reliable measurements could be obtained with this method.

Steen (10) performed LDV measurements in a fibre suspension flow where the indices of refraction of the liquid and fibre were matched. Steen measured mean velocities and turbulent velocities along the radial direction of a pipe with various fibre mass consistencies (0.05% - 0.24%), fibre lengths (1mm, 3mm) and Reynolds number (8,500, 65,000). The turbulence in the centre of the pipe was found to increase with increasing mass concentration of the fibres and turbulence increased in the area close to the wall with decreasing mass concentration of the fibres.

Li et al. (25) used Nuclear Magnetic Resonance Imaging measurements to measure mean velocities in a wood fibre suspension and observed a flow pattern transition from steady plug flow to a fully turbulent flow through a mixed flow regime with a steady plug core in the centerline regions. But the low frequency response (10 Hz) and low image resolution (256 by
256 pixels in each image, where each pixel measures 2 cm by 2 cm) of this method limited its capability to reveal the turbulent structure. Only the mean velocities of long time period and large area were obtained in the experiment (Li et. al. (25)).

Moayed and Kuhn (32, 33) investigated the behavior of pulp suspensions in a grid generated turbulence flow field over a velocity range of 0.26 to 0.50 m/s using Dynamic Panoramic View. They found that the flocculation intensities decrease while the turbulent intensities increase, and vice versa.

1.2.3 Index matching

In an index matched mixture, the fibres are transparent and are suitable for optical measurements. Steen (10) used glass fibres and a mixture of benzyl and ethyl alcohol to match the refractive indices for his LDV measurements. Steen was able to adjust the refractive index of a mixture of Benzyl (refractive index = 1.54) and ethyl alcohol (refractive index = 1.36) and the glass fibres (refractive index = 1.495) to within 0.001. Thus, the fibre suspension was transparent, allowing velocity measurements with Laser Doppler Velocimetry. Andersson(20) performed LDV measurements in a mixing tank using index matching in a fibre suspension with mass consistencies up to 5%. Andersson found that the turbulence intensity increases with time after the starting of stirring.
1.2.4 Numerical modeling

Aidun and Kovacs (27) have used $k-\varepsilon$ model and found secondary flows which determine the cross direction orientation of fibre orientation in a headbox. In this model, the fibre is treated as passive in the flow. Thus, it does not provide any feedback to the turbulence as Steen (10) has observed in his measurements.

An added difficulty of modeling this flow is due to the fact that the fibres are very large with respect to most grids. Thus it is necessary to combine Lagrangian and Eulerian representations to model the flow properly. This has not been done in the literature.

1.2.5 Research in mixing

The mixing jar is widely used in water processing and fibre suspension (Andersson (20, 21)) related research.

Cheng and Atkinson (26) performed measurements of the turbulent structure in a mixing jar using Particle Image Velocimetry (PIV). They estimated the mean velocity gradient $\tilde{G}$ as:

$$\tilde{G} = \left( \frac{\varepsilon}{v} \right)^{1/2}$$

(1.2)

where $\tilde{G}$ is used to represent the mixing level in a mixing jar. Isotropy was used to calculate dissipation, which is unnecessary with PIV.
Kresta (30) reviewed current research on turbulence in stirred tanks and proposed that the observed anisotropy and unsteadiness was the nature of the flow in a mixing jar. She also discussed the effects of the various characteristic scales on the turbulence.

Sheng et al. (34) compared results from a CFD simulation based on the Reynolds Average Navier-Stokes Equation (RANS) to PIV measurements in a stirred vessel. The magnitude of kinetic energy found from CFD simulation was one order lower than the results of their PIV measurements.

Andersson (20) performed LDV measurements in a mixing container filled with pulp fibre suspension and compared it to a numerical model. His numerical results were only for the pure liquid flow and therefore did not match the experimental results obtained in the pulp suspension.

1.3 Overview of work presented in the thesis

Previous measurements have not provided enough information to fully understand the effect of the fibre suspension on turbulence. This is largely due to the fact that only point measurements are available and they cannot provide an understanding of the influence of fibre on the flow due to their strong effect on the mean flow. NMR can be used for large scale measurements but the time interval between two successive measurements is at least 100 ms compared with PIV measurements where averaging times are of the order of 100 $\mu$s.
Thus the averaging time for NMR is too long for most turbulent flows and the image resolution is not sufficient.

A mixing jar experiment using refractive index matching has been performed and velocity data were obtained using PIV. Acrylic fibres used in the paper industry with an index of refraction of 1.49 (from Sterling Fibre) were used in the experiment. The PIV experiments provided spatial scale information on the turbulence structure.

These measurements are the first to provide spatial information of the turbulence in fibre suspensions. In addition, these results provide a statistical database for use in numerical models of fibre suspension flow.
Chapter 2 - Experimental methodology

2.1 Experimental setup

This chapter describes the design and setup of the flow cell used to examine fibre suspension flow in a glass mixing jar. Measurements were made within a glass flow cell with Particle Image Velocimetry (PIV). Turbulence was generated by using an impeller, operated at three speeds, 1000, 2000 and 4000 RPM. The plane was illuminated with a dual cavity Nd:YAG laser, and images captured with a progressive scan CCD camera. All components are described below.
2.1 Locations of the centre of each field of view

The closed jar used in these experiments measured 28 cm (y) × 18 cm (x) × 18 cm (z) made with Crystal Clear™ glass. The coordinate system and a diagram of the flow cell are shown in figure 2.1. Measurements were taken at two planes: Plane 1 is located 6 cm from the front wall, plane 2 is located 3 cm from the front wall. There measurements were then obtained using six fields of view located within each plane. The field of view each measured 1.2 cm (x) × 0.9 cm (y) and are listed in table 2.1.

The fluid used in the measurements was Duo Prime 90 oil (Lyondell, Texas, USA). This fluid is Newtonian with a viscosity of 15.6×10⁻⁶ m at 20°C. The refractive index of the oil is 1.479. The fluid is mixed with various mass consistencies (0%, 0.01%, 0.025%, 0.05% and 0.1%) of acrylic fibre (Sterling fibre, New Jersey). The mass consistency is defined as

$$C_m = \frac{m_f}{m_f + m_o}$$  \hspace{1cm} (2.1)

Where $C_m$ is the mass percent, $m_f$ is the mass of fibres and $m_o$ is the mass of oil. The volume consistency is given by

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<th>position</th>
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<th>y (cm)</th>
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<tr>
<td>1</td>
<td>6</td>
<td>11</td>
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<tr>
<td>2</td>
<td>8.5</td>
<td>11</td>
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<td>3</td>
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<td>6</td>
<td>8.5</td>
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Table 2.1 Locations of the centre of each field of view

2.1.1 Mixing Jar
\[ C_v = C_m \times \frac{\rho_o}{\eta} = 0.71 \]  

The crowding factor given by equation 1.1 are 2, 5, 10 and 20 for 0.01\%, 0.025\%, 0.05\% and 0.1\% respectively.

When fibres are immersed into the oil, their size will become larger as they swell. This is a picture of a clump of fibres at rest at 0.025\% mass consistency.

2.1.2 Impeller

A 2.5 cm long and 0.5 cm width impeller was used for mixing. At the maximum rotation speed of the impeller (4,000 rpm) the Reynolds number was 53,400, where \( \text{Re} = N R^2 / \nu \), and \( N \) is the rotation speed of the impeller in that tank, \( R \) is the radius of the impeller, \( \nu \) is the kinetic viscosity of the oil.
2.2 PIV system

The PIV system consists of a Continuum dual cavity Nd:YAG laser, Stanford DG535 Digital Delay / Pulse Generator, Pulnix TM-9700 progressive scan CCD camera, Data Raptor frame grabber and a personal computer.

![Diagram of the PIV setup](image)

Figure 2.4 Diagram of the PIV setup (Buffle (31))

The acquired image data stored consisted of digital image pairs sampled with time separations of 500-1,500 μs (based on the impeller rotation speed). The data were analyzed using a cross-correlation PIV and improved with a Particle Tracking Velocimetry (PTV) algorithm developed by Marxen et al. (24), described below.
2.2.1 Laser

The Nd:YAG laser has two power supplies and a shared optical head. Each power supply is capable of outputting a beam of 15 mJ for 9 ns duration in each pulse. The refresh rate for the system is 15 Hz, allowing a maximum of 15 image pairs per second. The wavelength of the laser is 532 nm.

2.2.2 Optics

The laser sheet is produced by first sending the beam through a cylindrical lens and then through a spherical lens resulting in a plane of laser light one millimeter thick. The seeding particles were spherical, silver coated hollow glass spheres with a specific gravity of 1.65 and a mean diameter of 15μm. There were more than 7 particles in 32×32 pixel area which satisfies the high density criteria for PIV measurement (Adrian (36)), and the error criteria set by Marxen et al. (24).
The delay generator generates four pulses based on the camera timing. The 1st and the 2nd pulses trigger the flash lamp and trigger the Q-Switch of cavity 1 respectively (A for 1st, B for 2nd), the 3rd and 4th pulses trigger the flash lamp and trigger Q-Switch of cavity 2 (C for 1st, D for 2nd). As shown in figure 2.5, the result is that the 1st laser exposes the CCD before the end of the first frame and the second laser exposes the CCD at the beginning of the second frame. No other light enters the system, ensuring a very short exposure time. Table 2.2 shows the settings for the Stanford delay generator used in the experiment.
Table 2.2 Time setting of the laser pulses

<table>
<thead>
<tr>
<th>Rotation Speed (rpm)</th>
<th>A=T+</th>
<th>B=A+</th>
<th>B=A+</th>
<th>D=C+</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,000</td>
<td>0.050211s</td>
<td>0.000152s</td>
<td>0.0005s</td>
<td>0.000152s</td>
</tr>
<tr>
<td>2,000</td>
<td>0.050211s</td>
<td>0.000152s</td>
<td>0.0008s</td>
<td>0.000152s</td>
</tr>
<tr>
<td>1,000</td>
<td>0.050211s</td>
<td>0.000152s</td>
<td>0.0015s</td>
<td>0.000152s</td>
</tr>
</tbody>
</table>

2.2.4 Camera and Frame Grabber

The Pulnix TM-9700 Progressive CCD camera has an image resolution of 640×480 and the field of view was set to 11×9 mm. The time between images was determined from the rotation speed of the impeller; electronic limitations that would limit the experiment to time separation greater than 25 μs were not a consideration.

The Data Raptor frame grabber digitizes the analog video signal from the TM-9700. The camera operates in full-frame or progressive scan mode and therefore the entire video image is digitized at once i.e. there is no time lag between the odd and even fields.
2.2.5 Cross-Correlation Algorithm.

The 640 x 480 pixel images are decomposed into small interrogation areas to ensure that there are five particle pairs in each interrogation area. Typically the size of an interrogation region is 32 x 32 pixels. An digitized image pair is shown in figure 2.5. The images have been digitized to show the seeding particles. Each interrogation area (Figure 2.5a) in the first image of an image pair is cross-correlated to a larger (double size in each dimension, Figure 2.5b) area in the second image, and a peak is located the cross-correlation function indicating the path of the ensemble of particles. Therefore, the displacement of the center of the interrogation area is the distance traveled. The velocity is determined by dividing this distance by the time between the paired images.

Figure 2.6 Schematic of the Cross-Correlation between two successive images (digitized images)
2.2.6 PIV/PTV Algorithm

A combination of PIV and PTV is used to improve the determination of particle displacement. Since PIV is used to track ensembles of particles, the accuracy is limited to one pixel. To obtain improved velocity estimates, it is necessary to be able to track the actual displacement of individual particles. Unlike other PIV experiments, the measurement region is small to optimize measurements. The resolution of the Kolmogorov scale is available with an accuracy less than 1 pixel as 18.5 μm.

2.2.7 PIV/PTV algorithm

(i) Digitize the two successive images to the gray level matrix $I_1(i,j)$ and $I_2(i,j)$.

(ii) Decompose images into small interrogation area of 32 by 32 pixels,

(iii) Cross-correlate the interrogation area of the first image with a larger area (64 by 64 pixels) in the second image,

(iv) Obtain the displacement of PIV for each interrogation area and interpolate the displacement to each pixel to obtain the displacement $D_0(i,j)$ for all pixels in flow field,

(v) Apply PIV to optimize PTV.

step a. Apply $D_0(i,j)$ to $I_1(i,j)$ by displacing each pixel of $I_1(i,j)$, $I_1'(i,j)$ is obtained.

step b. Compare $I_1'(i,j)$ and $I_2(i,j)$, check if all particles are at the same position between the two images. If the difference is less than the criteria, the current displacement is the result. If the difference is larger than the criteria, Cross-correlate $I_1'(i,j)$ and $I_2(i,j)$ to obtain $D_1(i,j)$, and repeat step a with new displacement of $D_0(i,j) + D_1(i,j)$.

(vi) Linearly Interpolate the velocities onto a grid of 16 by 16 pixels each.
2.2.8 2-D coordinate system

The camera is mounted on a 2-D traverse allowing positioning the entire region of interest (Figure 2.6). To determine the region of measurement, a scale was placed in the mixing jar in the field of view (figure 2.7) to determine the size of each pixel (18.5 μm).

![Diagram of the 2-D coordinate system](image)

**Figure 2.7** Diagram of the 2-D coordinate system

![Calibration of pixel size](image)

**Figure 2.8** Calibration of pixel size. (1 pixel = 5/270 = 18.5 μm)

2.3 Index Matching

Refractive index matching was achieved in the experiment with Acrylic fibres (Sterling Pulp Fibres) and Duo Prime 90 Oil (Lyondell, Texas). The refractive index of the acrylic fibre is 1.49 and that of the Duo Prime 90 oil is 1.48 resulting in a semi-transparent medium for measurement. It was found that PIV is not as sensitive to differences in the index of refraction as Laser Doppler Velocimetry, and therefore, it was possible to obtain good measurements, despite the not so perfect matched indices of refraction. As a result, PIV
measurements were only possible in planes that were less than 3 cm and 6 cm from the near wall of the jar for suspensions with mass consistencies of 0.1% and 0.025% respectively.

<table>
<thead>
<tr>
<th></th>
<th>Density</th>
<th>viscosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.0 g/cm³</td>
<td>1×10⁻⁶ m²/s</td>
</tr>
<tr>
<td>Duo Prime 90 Oil</td>
<td>0.87 g/cm³</td>
<td>15.6×10⁻⁶ m²/s</td>
</tr>
</tbody>
</table>

Table 2.3 Comparison of the properties of water and Duo Prime 90 oil

The acrylic fibre has similar properties to wood fibre and is used to make transparent paper. Duo Prime 90 Oil is a colorless, odorless and stable liquid that allows the experiment to be performed at a large scale and under normal conditions with minimal risk. Properties of the fluid compared with water are shown in table 2.3, and properties of the fibre with respect to wood fibre are shown in Table 2.4.

<table>
<thead>
<tr>
<th></th>
<th>Length</th>
<th>Density</th>
<th>Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood Fibre</td>
<td>3 cm</td>
<td>1.5 g/cm³</td>
<td>30 μm</td>
</tr>
<tr>
<td>Acrylic Fibre</td>
<td>3 cm</td>
<td>1.5 g/cm³</td>
<td>10 μm</td>
</tr>
</tbody>
</table>

Table 2.4 Comparison of the properties of wood and acrylic fibres (Sterling Fibres, New Jersey)
2.4 Experimental procedure and data analysis

Five consistencies were measured in the experiment, and the condition of measurement are listed in table 2.5. In plane 1 which is 6 cm to the wall, 0.0%, 0.01% and 0.025% mass consistencies at all fields of view were measured. At plane 2, which is 3 cm from the wall, 0.0%, 0.01%, 0.025% and 0.05% mass consistencies were measured in all fields of view. Only 2 fields of view (1, 4) were measured in 0.1% fibre suspension. For each case, 100 image pairs in sequence were captured. The separation time between two successive image pairs is 1/15 s.

<table>
<thead>
<tr>
<th>Mass Consistency (%)</th>
<th>Volume Consistency (%)</th>
<th>Crowding Factor</th>
<th>Plane</th>
<th>Position</th>
<th>Rotation Speed N (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1, 2</td>
<td>1, 2, 3, 4, 5, 6</td>
<td>4,000, 2,000, 1,000</td>
</tr>
<tr>
<td>0.01</td>
<td>0.007</td>
<td>2</td>
<td>1, 2</td>
<td>1, 2, 3, 4, 5, 6</td>
<td>4,000, 2,000, 1,000</td>
</tr>
<tr>
<td>0.02</td>
<td>0.014</td>
<td>4</td>
<td>1, 2</td>
<td>1, 2, 3, 4, 5, 6</td>
<td>4,000, 2,000, 1,000</td>
</tr>
<tr>
<td>0.05</td>
<td>0.035</td>
<td>10</td>
<td>2</td>
<td>1, 2, 3, 4, 5, 6</td>
<td>4,000, 2,000, 1,000</td>
</tr>
<tr>
<td>0.1</td>
<td>0.07</td>
<td>20</td>
<td>2</td>
<td>1, 4</td>
<td>4,000, 2,000, 1,000</td>
</tr>
</tbody>
</table>

Table 2.5 Measurement Locations of each consistency
Chapter 3 Data Processing

The instantaneous velocity fields obtained in the experiment are processed to obtain the various statistical turbulence characteristics. As the flow is strongly non-stationary, filtering of individual velocity fields is used to decompose the flow into high and low frequency components corresponding to turbulent and mean velocity. The method and equations used are described in this chapter.

3.1 Turbulence Decomposition

Instantaneous velocity fields are decomposed with the Reynolds decomposition into mean and fluctuation components,

\[ \tilde{u} = U + u \]  \hspace{1cm} (3.1)

In a steady flow, \( U \) is the time average velocity, \( \tilde{u} \) is the instantaneous velocity and \( u \) is the fluctuation velocity. As the flow pattern in the experiment changes considerably, this makes time averaging inappropriate. From Monin and Yaglom (37), a fundamental definition of turbulence assumes that the frequency component of the mean and the turbulence are separate and exist in distinct frequency ranges. Thus, to decompose the flow field a procedure is used that is similar to cyclic averaging (see Sullivan et al. (36)) to study flow in engines.

1. The instantaneous flow field is Fourier transformed.

2. A power spectral density (PSD) is calculated.

3. The high frequency components determined from PSD (see figure 3.1 a) are removed from the original Fourier transform signal.

4. The mean flow (shown in figure 3.1 b) is found from the inverse Fourier transform of the low frequency components.
5. The fluctuating component (shown in figure 3.1 c) is found by subtracting the mean flow field from the instantaneous flow field. The chosen cut-off wave number is the wave number which divides the energetic low wave number components from the less energetic high wave number components. The cut-off wave number is selected at the wave number where the energy is 5% of the maximum magnitude of the energy. From the PSD of most of the velocity fields, the cut-off wave number is at $4 \times 10^4$ \text{1/m}.

Figure 3.1a is the PSD for the instantaneous velocity field.; Figure 3.1b is the low frequency components. Figure 3.1c is the high frequency components that remain after subtracting the low frequency flow from the instantaneous velocity field.

This procedure is diagrammed in figure 3.2. Figure 3.2 (d) and (e) are the fluctuating velocity fields found by subtracting the low-pass velocity field and the time average velocity field from the instantaneous velocity field plotted in figure 3.2 (a) respectively. A comparison of figures 3.2 (d) and (e) shows that the magnitudes of the fluctuating velocity field obtained by subtracting the time average velocity field are more significantly than that of the instantaneous velocity (figure 3.2 (a)). In addition, its flow pattern is very different from that of the instantaneous velocity field. This is because the flow in the tank is unsteady and the flow pattern shifts dramatically from one velocity field to another as shown in the comparison between figure 3.2 (a) and 3.2 (b). As a result, the time averaged velocity is meaningless and it cannot be used to calculate the fluctuating velocity field or to compute the turbulent statistics.
The low frequency pass filtered flow field is subtracted from the instantaneous velocity field to compute the fluctuating velocity field. From figure 3.2 (d), it is shown that the small vortex and smaller scale motion in the flow is exposed by the fluctuating velocity field.

3.2 Turbulence Statistics

3.2.1 Turbulent Kinetic Energy

The turbulent kinetic energy is defined by

\[ k = \frac{1}{2} (\bar{u_1}^2 + \bar{u_2}^2 + \bar{u_3}^2). \]  \hspace{1cm} (3.2)

Thus, all three components of the velocity are required to determine turbulence kinetic energy. Due to the nature of PIV, \( u_3 \), the third component of velocity is not measured directly. If the flow is isotropic, the fluctuating components should all be equal, \( i.e. \)

\[ \bar{u_1}^2 = \bar{u_2}^2 = \bar{u_3}^2. \]  \hspace{1cm} (3.3)

and for this work it is assumed that

\[ \bar{u_3}^2 = \frac{1}{2} (\bar{u_1}^2 + \bar{u_2}^2). \]  \hspace{1cm} (3.4)

This was found to be better than assuming the value was equal to any one component of velocity as isotropy was not correct (see chapter 4).

Therefore, the turbulence kinetic energy is given by
3.2.2 Turbulent dissipation

It is possible to define turbulent dissipation starting with the equations of motion. From Kundu (38), the equation of motion in an incompressible flow is:

\[
\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = - \frac{1}{\rho_0} \frac{\partial \bar{\rho}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_i \partial x_j} \tag{3.6}
\]

where \( \bar{u}_i \) and \( \bar{\rho} \) are the instantaneous velocity and pressure, respectively.

The equation describing the conservation of mass is given by (Kundu (38)),

\[
\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{3.7}
\]

This can be simplified using Reynolds decomposition,

\[
\bar{u}_i = U_i + u_i \tag{3.8}
\]

\[
\bar{\rho} = P + p \tag{3.9}
\]

Where \( U_i \) is the low frequency velocity component and \( u_i \) the high frequency component of the instantaneous velocity, \( \bar{u}_i \) is the instantaneous velocity and similarly for the pressure.

Overbar are used to note an ensemble average. As \( \bar{u}_i \) is zero, substituting (3.8) into equation (3.7), ensemble averaging, results in (Kundu (38)):

\[
\frac{\partial U_i}{\partial x_i} = 0 \tag{3.10}
\]

\[
\frac{\partial u_i}{\partial x_i} = 0 \tag{3.11}
\]
Substitute (3.7) (3.8), (3.9), (3.10) and (3.11) into (3.6) and ensemble averaging (Kundu (38)),

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} + \frac{\partial (u_i u_j)}{\partial x_j} = \frac{1}{\rho_0} \frac{\partial p}{\partial x_i} + v \frac{\partial^2 U_i}{\partial x_i \partial x_j} \tag{3.12}
\]

Multiplying (3.6), (3.12) by \( U_i \) (and summing over all \( i \)), the equation for the conservation of kinetic energy can be derived (Kundu (38)),

\[
\frac{D}{Dt}(\frac{1}{2}u_i^2) = \frac{\partial}{\partial x_j} \left( \frac{\bar{\rho} u_i}{\rho_0} + \frac{1}{2} v u_i^2 u_j - u_i u_j U_i \right) - 2 v e_{ij} e_{ij} + \bar{u}_i u_j \frac{\partial U_i}{\partial x_j} \tag{3.13}
\]

\begin{align*}
I & \\
II & \\
III & \\
IV & \\
\end{align*}

Where term I is the rate of change of mean kinetic energy, term II is the transport of energy, term III is dissipation of turbulent energy due to viscous forces, and term IV is the transport of energy from the mean motion to turbulence \( \frac{D}{Dt} \) is substantial derivative. The dissipation term in the mean kinetic energy equation is defined as (Kundu (38)),

\[
\varepsilon = 2 v e_{ij} e_{ij} = 2 v \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} \tag{3.14}
\]

As it is only possible to measure the two-dimensional velocity field with planar PIV measurements \( \frac{\partial u_1}{\partial x_1}, \frac{\partial u_1}{\partial x_2}, \frac{\partial u_2}{\partial x_1} \) and \( \frac{\partial u_2}{\partial x_2} \) are found directly from the experiment.

To determine \( \frac{\partial u_3}{\partial x_3} \), the conservation of mass equation (3.7) is used,

\[
\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0, \tag{3.15}
\]

Thus, this gradient is

\[
\frac{\partial u_3}{\partial x_3} = -\left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} \right) \tag{3.16}
\]

The flow is assumed to be isotropic in order to compute the other terms of the equation that were not obtained directly from measurements or conservation of mass,
Thus, turbulence dissipation is

\[ \varepsilon = 2
\frac{\partial u_1}{\partial x_1}^2 + \left( \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right)^2 + 3
\frac{\partial u_1}{\partial x_1}^2 + \left( \frac{\partial u_2}{\partial x_2} \right)^2 + 6
\frac{\partial u_1}{\partial x_1} \frac{\partial u_2}{\partial x_2} \] 

(3.19)

3.2.3 Kolmogorov Scale

The Kolmogorov scale is the smallest scale associated with turbulence, and is calculated from Tennekes and Lumley (36),

\[ \eta = \nu^{3/4} \varepsilon^{-1/4} \] 

(3.20)
3.1a Spectrum of $u$ for instantaneous velocity field, cut-off wave number is selected when the magnitude of the energy is 5% of the maximum magnitude of energy.

3.1b Spectrum of $u$ for low-pass velocity field after cut high frequency component from instantaneous velocity field.

3.1c Spectrum of $u$ for fluctuating velocity field by subtracting the low-pass component from the instantaneous velocity field.

Figure 3.1 Diagram of the Spatial Filtering
Figure 3.2 Velocity fields in comparing time averaging to low-pass filtering, the mass consistency was 0.025%
Chapter 4 Results and Discussions

The TKE, Kolmogorov scale, dissipation Reynolds shear stress are calculated by using the turbulence and calculated value at each pixel for all 100 realizations. To show the result and the distribution of TKE, Kolmogorov scale, dissipation, Reynolds shear stress in the flow field, a histogram was used. The histogram was calculated by dividing the range of the values in the flow field into 100 segments, thus, the distributions shown are values occurring for each bin is referred to as frequency on the y-axis.

4.1 Time Average Velocity Field

Figure 4.1 shows a typical time average velocity profile. The time average velocity fields were calculated by ensemble averaging 100 successive velocity fields at each. Flow in the mixing jar is strongly unsteady, requiring filtered averages. The time average velocity field cannot be used to represent the flow pattern of the flow in a mixing jar.

4.2 Turbulent Kinetic Energy

The turbulent kinetic energy (TKE) was calculated using equation 3.5. The results are presented in figures 4.2 - 4.7.

Figures 4.2a - 4.2f are graphs for all measurement positions at Plane 1 and N=4000 rpm. Measurements are presented at mass concentrations of 0.0%, 0.01% and 0.025%. This location was located furthest from the camera and as the refractive index matching was not ideal, it was impossible to perform measurements at the two highest concentrations. For
position 1, 4 and 5 (figure 4.2a, 4.2c, and 4.2e) which is located nearest the side wall, the introduction of fibres results in a significant displacement of the peak of the TKE, from $3.4 \times 10^{-4}$ m$^2$/s$^2$ at 0.0% to $1.7 \times 10^{-4}$ m$^2$/s$^2$ in figures 4.2a and 4.2c. At the position nearest the lower wall (figure 4.2e), the displacement of the peak TKE is not as large; however, 0.0% concentration continues to have the largest magnitude TKE compared with the remaining two concentrations. Examining the measurement regions closest to the impeller, positions 2 and 3 (figures 4.2b and 4.2d), the peak of the TKE for all concentrations (0%, 0.01% and 0.025%) are seen to not vary but are located at $3 \times 10^{-4}$ m$^2$/s$^2$. At position 6 (figure 4.2f), the distribution of the TKE is again clearly separated between 0% (peak TKE is $\sim 2 \times 10^{-4}$ m$^2$/s$^2$) and 0.01% and 0.025% concentration (peak TKE is $\sim 0.7 \times 10^{-4}$ m$^2$/s$^2$). These results suggest that the influence of the impeller is not strong for position 5, but greater at all remaining positions. The influence of the fibres is to strongly diminish the turbulence.

Figures 4.3a - 4.3f are graphs for all measurements positions at Plane 1, N=2000 rpm. Here the influence of the impeller is not as significant, resulting in the fibres not changing the peak TKE as much. At position 1 (figure 4.3a), the distribution of peak TKE goes as $\sim 1.4 \times 10^{-4}$ m$^2$/s$^2$ (0.01%), $\sim 1 \times 10^{-4}$ m$^2$/s$^2$ (0%) and $\sim 0.8 \times 10^{-4}$ m$^2$/s$^2$ (0.025%). For position 4 (figure 2c), the distribution is the same. At positions 2 (figure 4.3b), the distribution of the fibres is similar with the largest magnitude TKE peak associated with 0.01% consistency, the next with 0% concentration, and finally 0.025% concentration. For position 3, the results show a peak at $\sim 1.2 \times 10^{-4}$ m$^2$/s$^2$ for all concentrations. At the two remaining positions (figures 4.3e and 4.3f), the peaks for all concentrations has been shifted down to $\sim 0.7 \times 10^{-4}$ m$^2$/s$^2$. There is still a slightly higher value of the peak TKE for position
16, however, it is not as great as it was for previously described positions. It is interesting to note that aside from position 3, the peak TKE associated with 0.025% concentration does not shift, and the influence of the impeller at this concentration is not great on the flow.

The TKE was also determined for Plane 1, N=1000 rpm (figures 4.4a - 4.4f). For all positions, the peaks of the TKE separated into 0.0% ~0.1×10^-4 m^2/s^2, 0.01% ~ 0.12×10^-4 m^2/s^2, and 0.025% ~0.25×10^-4 m^2/s^2. For positions 2 and 3 (figures 4.4b and 4.4d), the peaks are more distinct with the greatest magnitude peak associated with 0.1% ~ 0.35×10^-4 m^2/s^2, then 0.025% ~ 0.2×10^-4 m^2/s^2 and 0.0% ~ 0.15×10^-4 m^2/s^2.

The TKE for Plane 2, N=4000 RPM are presented in figure 4.5. For all positions measurements were done for mass concentrations of 0.0%, 0.01%, 0.025%, and 0.05%. For positions 1 and 4 (figures 4.5a and 4.5c), results for 0.1% consistency are also presented. At position 1 (figure 4.5a), the peak magnitudes of the TKE are ~4.3×10^-4 m^2/s^2 (0.1%), ~3.7×10^-4 m^2/s^2 (0.01%),~2.7×10^-4 m^2/s^2 (0.0%), ~2.2×10^-4 m^2/s^2 (0.05%) and ~1.1×10^-4 m^2/s^2 (0.025%). At position 4 (figure 4.5c), all concentrations, except 0.01%, have a peak TKE at ~2.2×10^-4 m^2/s^2. At 0.01%, the peak occurs at ~1.4×10^-4 m^2/s^2. For position 5 (figure 4.5e), there are two distinct peaks ~1.8×10^-4 m^2/s^2(0.0% mass concentration) and ~1.6×10^-4 m^2/s^2 (0.01%, 0.025% and 0.05% mass concentration). Closer to the impeller, position 2 (figure 4.5b), the distribution of the peak magnitude shows peak regions~1.5×10^-4 m^2/s^2 (0.025%) ~2.5×10^-4 m^2/s^2 (0.01%) and ~4×10^-4 m^2/s^2 (0.0% and 0.05%). At position 3 (figure 4.5d), the peak magnitude of the distribution at 0%, 0.025% and 0.05% concentration occurs at ~2.5×10^-4 m^2/s^2 and ~3.5×10^-4 m^2/s^2(0.01%
concentration). At position 6 (figure 4.5f), the peak value of the distribution occurs at \(-1.5 \times 10^{-4} \text{ m}^2/\text{s}^2\) for 0.0% and 0.01% mass concentration, \(-2.1 \times 10^{-4} \text{ m}^2/\text{s}^2\) for 0.025% concentration and \(-2.5 \times 10^{-4} \text{ m}^2/\text{s}^2\) for 0.05% concentration. The effect of the concentration is not as apparent here as it had been for Plane 1.

Figure 4.6 is the distribution of the TKE for Plane 2, N=2000 rpm. Figure 4.6a, position 1, there is a distinct separation between the 0.1% concentration which has a peak of the distribution at \(-1.7 \times 10^{-4} \text{ m}^2/\text{s}^2\) and the remaining consistencies where the peak occurs at \(-0.6 \times 10^{-4} \text{ m}^2/\text{s}^2\). The peak of the distribution for position 4 (figure 4.6c) occurs at \(-0.5 \times 10^{-4} \text{ m}^2/\text{s}^2\) (0.01% and 0.1% mass concentration), \(-1 \times 10^{-4} \text{ m}^2/\text{s}^2\) (0% and 0.05% mass concentration) and \(-1.5 \times 10^{-4} \text{ m}^2/\text{s}^2\) (0.025% mass concentration). At position 5 (figure 4.6e), 0%, 0.01% and 0.025% have a peak of the distribution that occurs at \(-0.4 \times 10^{-4} \text{ m}^2/\text{s}^2\) and \(-1 \times 10^{-4} \text{ m}^2/\text{s}^2\) for 0.05% mass concentration. At position 2 (figure 4.6b), the peak of the distribution is at \(-0.9 \times 10^{-4} \text{ m}^2/\text{s}^2\) (0.01%), \(-1.5 \times 10^{-4} \text{ m}^2/\text{s}^2\) (0.025%) and \(-1.7 \times 10^{-4} \text{ m}^2/\text{s}^2\) (0% and 0.05%). At position 3 (figure 4.6d), the peaks of the distribution occur at \(-1.7 \times 10^{-4} \text{ m}^2/\text{s}^2\) (0.025%), \(-1.3 \times 10^{-4} \text{ m}^2/\text{s}^2\) (0.01%), \(-1.5 \times 10^{-4} \text{ m}^2/\text{s}^2\) (0%) and \(-2.2 \times 10^{-4} \text{ m}^2/\text{s}^2\) (0.05%). For position 6 (figure 4.6f), the peaks of the distributions are at \(-0.8 \times 10^{-4} \text{ m}^2/\text{s}^2\) for all concentrations, similar to what was seen in figure 2f.

Figure 4.7 is the distribution of TKE for Plane 2, N=1000 rpm. For all figures, the peak of the TKE occurs at approximately \(-0.3 \times 10^{-4} \text{ m}^2/\text{s}^2\) with very little difference between positions. The distributions are slightly wider for 0.05% concentration than for the other
concentrations examined here. At position 1 (figure 6a), this behaviour is also seen with 0.1% concentration.

4.3 Kolmogorov scale

Figures 4.8 - 4.13 present histograms of the distribution of Kolmogorov scales calculated from equation 3.20.

Figure 4.8 is the distribution of Kolmogorov scales for Plane 1, N= 4000 rpm. Figure 4.8a, 4.8c, 4.8e (positions 1, 4, 5), the peaks of the distribution are ordered (minimum Kolmogorov scale to maximum Kolmogorov scale) 0%, 0.025% and 0.01%. For position 1 (figure 4.8a), the peak distributions of Kolmogorov scale is 0.35 mm (0%), 0.4 mm (0.025%) and 0.47 mm (0.01%). For position 4 (figure 4.8c), the distribution of the peak values is 0.36 mm (0%), 0.44 mm (0.025%) and 0.49 mm (0.01%). At position 5 (figure 4.8e), the distribution is not as widely separated; however, it does show the same distribution with a value around 0.46 mm for all concentrations. Figure 4.8b (position 2), 0.0% shows a slightly higher magnitude for its peak with respect to the other concentrations (0.36 mm vs. 0.34 mm). At position 3 (figure 4.8c), all distributions show a peak at 0.36 mm and there is no apparent difference for the various concentrations. At position 6 (figure 4.8f), the distributions of the peak clearly separate into two distributions, for 0% concentration there is a peak at 0.42 mm and for the remaining concentrations 0.52 mm.
Figure 4.9 is the distribution of the Kolmogorov scales for Plane 1, N=2000 rpm. Figure 4.9a (position 1), shows the peak of the distributions for 0.01% to be at approximately 0.45 mm and for the remaining concentrations to be at 0.52 mm. The distribution of scales is also found to be attenuated for 0.01% concentration, notably narrower with a higher peak in comparison to the other two concentrations. For figure 4.9c (position 4), the location of the peak of the distributions occur (from minimum to maximum) at 0.44 mm (0.0%), 0.45 mm (0.01%) and 0.54 mm (0.025%). At the location, position 5 (figure 4.9e), furthest from the impeller, the peak of the distribution separates into two distinct peaks, 0.5 mm (0%) and 0.62 mm (0.01% and 0.025%). At position 2 (figure 4.9b), the distribution shows the same separation as found in figure 4.9a, i.e., the peak of the distribution for 0.01% occurs at 0.4 mm and 0.48 mm for 0% and 0.025%. Figure 4.9d (position 3), the peak of the distribution occurs at 0.43 mm, it is also noted that the distribution of 0.01% is slightly narrower than the other two concentrations. This behavior is also noted in figure 4.9f, where the peak occurs at 0.52 mm, with a narrower expanse for 0.01% concentration.

Figure 4.10 is the distribution of Kolmogorov scales for Plane 1, N= 1000 rpm. At this impeller speed, the behaviour of the suspension has changed significantly. In figure 4.10a (position 1), the peak of the distributions occur at 0.8 mm (0.025%), 0.9 mm (0.01%) and 1.1 mm (0%). Note also that the distribution of the Kolmogorov scales has been significantly attenuated for the fibre suspension with respect to the 0% case. At position 4 (figure 4.10c), there are two distinct peaks for the distributions, 0.7 mm (0.01%) and 0.8 mm (0% and 0.025%). At position 5 (figure 4.10e), the distributions are the same as those found in figure 4.10c except peaks changes to 0.75 mm (0.0%) and 0.85 mm (0.01 and 0.025%) . There are
slight differences in the peak of the Kolomogorov scales found at position 2 (figure 4.10b), 0.5 mm (0.01%), 0.75 mm (0.025%) and 0.8 mm (0%). Similar behavior is found in figure 4.10d (position 3); the peaks of the distributions are 0.56 mm (0.01%), 0.65 mm (0.025%) and 0.75 mm (0%). For position 6 (figure 4.10f), the peaks of the distributions occur at 0.7 mm (0.01% and 0.025%) and 0.9 mm (0%). The effect of the fibres at this velocity to generally diminish the size of the Kolmogorov scales. It is also noted that there is a general widening of the Kolmogorov scales for 0.01% with respect to the other two consistencies.

Measurements made at plane 2 are presented in figure 4.10 for N=4000. Position 1 (figure 4.11a), the distribution of the peaks of the Kolmogorov scales are found to be: 0.31 mm (0.1%), 0.35 mm (0.01%), 0.4 mm (0% and 0.05%) and 0.5 mm (0.025%). Note also the wider distribution associated with the 0.025% consistency histogram. At position 4 (figure 4.11c), peak distribution occurs at 0.34 mm (0.025%), 0.38 mm (0.0% and 0.05%) and 0.4 mm (0.01%). For position 5 (figure 4.11e), the peak of the distribution is at 0.42 mm and the distribution is slightly wider for 0.05% consistency. For positions 2 and 3 (figures 4.11b), the peak of the distributions occurs at around 0.37 mm. For positions 3 (figures 4.11d), the peak of the distributions occurs at 0.32 mm (0.01%), 0.35 mm (0.025% and 0.05%) and 0.48 (0%). For positions 6 (figures 4.11f), the peak of the distributions occurs at 0.35 mm (0.025% and 0.05%) and 0.4 mm (0.0% and 0.01%). The Kolmogorov Scales in position 5 and 6 are larger than that at other locations as the flow speed at the filed of view 5 and 6 which are furthest from the impeller.
The Kolmogorov scales for Plane 2, \( N=2000 \) rpm are presented in figure 11. At position 1 (figure 4.12a), the distribution again shows a separation between the behavior at 0.1% consistency with a peak of the distribution at 0.42 mm and the remaining consistencies: 0.52 mm (0% and 0.025%), 0.58 mm (0.01% and 0.05%). The peak of the distribution occurs at 0.42 mm (0.025%), 0.47 mm (0.0% and 0.05%), 0.55 mm (0.01% and 0.1%) at position 4 (figure 4.12c). The peaks of the distributions changes at position 5 (figure 4.12e) and are: 0.45 mm (0.05%), 0.5 mm (0% and 0.025%) and 0.57 mm (0.01%). At position 2 (figure 4.12b), the distributions have two distinct peaks: 0.42 mm (0%, 0.025%, 0.05%) and 0.48 mm (0.01%). At position 3 (figure 4.12d), the peaks separate into three distinct values: 0.38 mm (0.05%), 0.44 mm (0.0% and 0.01%) and 0.52 mm (0.025%). At position 3 with respect to position 2, the variation of the Kolmogorov scales are significantly greater. At position 6 (figure 4.12f), there seems to be a merger of the two distributions. However, the peak at 0.01% remains distinct with respect to the other three concentrations.

Figure 4.13 presents the Kolmogorov scales for Plane 2, \( N=1000 \) rpm. At position 1 (figure 4.13a), the distributions have three distinct peaks: 0.67 mm (0.05% and 0.1%), 0.9 mm (0.025%) and 0.95 mm (0% and 0.01%). At position 4 (figure 4.13c), the distribution associated with 0.025% consistency has a distinct peak with respect to the remaining consistencies. There is a peak at 0.64 mm and the remaining consistencies have a peak in the distribution at ~0.8 mm. This behavior is also noted in position 5, where the peak of the distribution associated with 0.025% (0.72 mm) is distinct from all other consistencies (0.85 mm). The distributions of peaks at position 2 (figure 4.13b) is found to be: 0.6 mm (0.025% and 0.01%), 0.7 mm (0.05%) and 0.8 mm (0%). The distributions at position 4 (figure 4.13d)
the peaks of the distributions occur at: 0.57 mm (0.05%), 0.7 mm (0% and 0.01%) and 0.85 mm (0.025%). At position 6 (figure 4.13f), the distributions are not as distinct and the peaks occur at: 0.55 mm (0.05%), 0.7 mm (0.025%), 0.8 mm (0.01%) and 0.9 mm (0%).

The scales within the flow have been found to have been affected by the introduction of fibres. The large scale motion is destroyed by the fibre existing in the suspension when fibre consistency is greater than 0.025%, thus, the length scales in 0.05% and 0.1% suspension are found to be the smallest in the most locations.

4.4 Reynolds Stress

The Reynolds Stress, $\overline{\vec{u}\vec{v}}$, have been plotted in figures 4.14 - 4.19.

Figure 4.14 presents Reynolds stresses for Plane 1, N= 4000 rpm. The peaks of the Reynolds stresses occur at $-1 \times 10^{-4}$ m$^2$/s$^2$ (0%, 0.025%) and $-0.3 \times 10^{-4}$ m$^2$/s$^2$ (0.01%). In all of the figures, the Reynolds stresses have near zero distributions, with the expected negative values for the peak values. The general behavior of the fibres is to attenuate the range of possible Reynolds stress values (figures 4.14b, 4.14c and 4.14d) near the impeller. In figure 4.14d (position 4), the histogram has the greatest expanse for 0.025%.

Figure 4.15 presents the Reynolds stress for Plane 1, N= 2000 rpm. At this impeller speed, the distributions of the peaks of the Reynolds stress is significantly lower magnitude with magnitudes of $-0.5 \times 10^{-4}$ m$^2$/s$^2$. Nearest the impeller (position 2, figure 4.14b), the
histogram is widest for 0% consistency. Further away, the histogram is widest for 0.01% consistency (positions 1 and 4, figures 4.15a and 4.15c). The histogram is widest for 0.025% consistency at position 3 (figure 4.15d). While there are some slight differences at the remaining positions (5 and 6, figure 4.15e and 4.15f), the histograms show the same behavior.

Figure 4.16 presents the Reynolds stress for Plane 1, N = 1000 rpm. The peaks are near zero for all of the distributions except 0.025%, with a slight widening of the histogram for 0.025% consistency at all positions. The peaks for 0.025% is \(-0.5 \times 10^{-5}\) m\(^2\)/s\(^2\) which is one order less than those at higher rotation speed, as expected due to the diminished turbulence energy.

Figure 4.17 presents the Reynolds stress for Plane 2, N = 4000 rpm. The peak values of the Reynolds stress for all positions nearest the side wall (positions 1, 4, and 5, figures 4.16a, 4.16c, and 4.16e), have values of zero. The histograms are also slightly wider for 0.05% and 0.1% consistency. At positions 2, 3 and 6 (figures 4.16b, 4.16d and 4.16f), the peak values are positive but of small magnitude. The histograms do not show the symmetry found at positions 1, 4 and 5. At positions 3 and 6, the histograms associated with the higher fibre concentrations are wider.

Figures 4.18 and 4.19 presents the Reynolds stress for Plane 2, N = 2000 rpm and Plane 2, N=1000 rpm respectively. The behavior of the histograms is similar to that seen in figure 4.17.
4.5 Dissipation

The dissipation is plotted in figures 4.20 - 4.25. It is calculated using equation 3.19.

Figure 4.20 presents the dissipation for Plane 1, N= 4000 rpm. The dissipation decreases while the consistency of fibres increases. The results show that the action of the fibres is to diminish the dissipation, see figures 4.20a, 4.20c, 4.20e and 4.20f. Positions closer to the impeller (figures 4.20b and 4.20d) have the same distributions with peak values at approximately the same value.

Figure 4.21 presents the dissipation for Plane 1, N= 2000 rpm. Here the behavior is different from figure 4.20, the highest dissipation is associated with the 0.01% consistency. The dissipation is increases from 0.025%, 0% and 0.01% consistency.

Figure 4.22 presents the dissipation for Plane 1, N= 1000 rpm. The behavior is similar to that found in figure 4.21, with increasing dissipation from 0.025%, 0% to 0.01% consistency.

Figure 4.23 presents the dissipation for Plane 2, N= 4000 rpm. The dissipation at position 1 (figure 4.23a), increases from 0.025%, 0% and 0.05%, 0.01% and 0.1%. At position 3 (figure 4.23c), the distributions show peak values that increase from 0.01% and 0.05%, 0.025% and 0.1%, and 0%. The dissipation at the remaining positions show less variation.
Figure 23 presents the dissipation for Plane 1, \( N = 2000 \) rpm. The dissipation is higher with the introduction of fibres for all positions. Close to the impeller (figures 4.24b and 4.24d), there is again some differences in the dissipation distribution than that found further away.

Figure 4.25 presents the dissipation for Plane 1, \( N = 1000 \) rpm. Here the dissipation of significantly lower magnitude, however, the effect of the fibres continues to increase dissipation.

4.6 Ratio of \( u_{rms} \) to \( v_{rms} \)

To determine the isotropy of the turbulence within this flow, the ratio of \( u_{rms} \) to \( v_{rms} \) has been presented in figures 4.26 - 4.31. For all figures presented here, there is no apparent isotropy. The values of the peaks generally vary between 0.8 and 1.5. This suggests that \( u_{rms} \) is generally greater than \( v_{rms} \), and the corresponding turbulence energy is generally directed towards the x-direction. The action of the fibres is generally to attenuate the histograms.
Figure 4.1 Plane 1 - Mean Velocity, Rotation Speed 1 (4,000 rpm) for 0.0%
Figure 4.2 Plane 1 - TKE, Rotation Speed 1 (4,000 rpm)
Figure 4.3 Plane 1 - TKE, Rotation Speed 2 (2,000 rpm)
Figure 4.4 Plane 1 - TKE, Rotation Speed 3 (1,000 rpm)
Figure 4.5 Plane 2 - TKE, Rotation Speed 1 (4,000 rpm)
Figure 4.6 Plane 2 - TKE Rotation Speed 2 (2,000 rpm)
Figure 4.7 Plane 2 - TKE Rotation Speed 3 (1,000 rpm)
Figure 4.8 Plane 1 - Kolmogorov Scale Rotation Speed 1 (4,000 rpm)
Figure 4.9 Plane 1 - Kolmogorov Scale Rotation Speed 2 (2,000 rpm)
Figure 4.10 Plane 1 - Kolmogorov Scale Rotation Speed 3 (1,000 rpm)
Figure 4.11 Plane 2 - Kolmogorov Scale  Rotation Speed 1 (4,000 rpm)
Figure 4.12 Plane 2 - Kolmogorov Scale Rotation Speed 2 (2,000 rpm)
Figure 4.13 Plane 2 - Kolmogorov Scale Rotation Speed 3 (1,000 rpm)
Figure 4.14 Plane 1 Reynolds Shear Stress, Rotation Speed 1 (4,000 rpm)
Figure 4.15 Plane 1 Reynolds Shear Stress, Rotation Speed 2 (2,000 rpm)
Figure 4.16 Plane 1 Reynolds Shear Stress, Rotation Speed 3 (1,000 rpm)
Figure 4.17 Plane 2 Reynolds Shear Stress, Rotation Speed 1 (4,000 rpm)
Figure 4.18 Plane 2 Reynolds Shear Stress, Rotation Speed 2 (2,000 rpm)
Figure 4.19 Plane 1 Reynolds Shear Stress, Rotation Speed 3 (1,000 rpm)
Figure 4.20 Plane 1 - Dissipation Rotation Speed 1 (4,000 rpm)
Figure 4.21 Plane 1 - Dissipation Rotation Speed 2 (2,000 rpm)
Figure 4.21 Plane 1 - Dissipation Rotation Speed 2 (2,000 rpm)
Figure 4.22 Plane 1 - Dissipation Rotation Speed 3 (1,000 rpm)
Figure 4.23 Plane 2 - Dissipation Rotation Speed 1 (4,000 rpm)
Figure 4.24 Plane 2 - Dissipation Rotation Speed 2 (2,000 rpm)
Figure 4.25 Plane 2 - Dissipation Rotation Speed 3 (1000 rpm)

(i) Position 2-6

(e) Position 2-5

(d) Position 2-3

(c) Position 2-4

(b) Position 2-2

(a) Position 2-1
Figure 4.26 Plane 1 Ratio of $u_{rms}$ to $v_{rms}$ Rotation Speed 1 (4,000 rpm)
Figure 4.27 Plane 1 Ratio of $u_{rms}$ to $v_{rms}$ Rotation Speed 2 (2,000 rpm)
Figure 4.28 Plane 1 Ratio of $u_{rms}$ to $v_{rms}$, Rotation Speed 3 (1,000 rpm)
Figure 4.29 Plane 2 Ratio of $u_{rms}$ to $v_{rms}$ Rotation Speed 1 (4,000 rpm)
Figure 4.30 Plane 2 Ratio of $u_{rms}$ to $v_{rms}$ Rotation Speed 2 (2,000 rpm)
Figure 4.31 Plane 2 Ratio of $u_{rms}$ to $v_{rms}$ Rotation Speed 3 (1,000 rpm)
Chapter 5 Conclusions and Recommendations

This thesis has investigated the stirred flow in a square tank for zero concentration flow and various mass consistencies of a fibre suspension flow. The spatial filtering method has been applied to subtract the turbulence information from the unsteady flow. The turbulence statistics and the effect of fibre on the turbulence statistics characteristics have been investigated.

5.1 Conclusions

PIV measurements were performed in fibre suspension in a tank using refractive index matching. Spatial filtering was used to extract the turbulence information from the instantaneous velocity field in this highly unsteady flow.

The turbulence statistics characteristics, turbulence kinetic energy, Reynolds shear stress, dissipation, and Kolomogorov scale were obtained from the measurement results.

The anisotropy of the stirred flow in the tank was shown in this experiment with instantaneous velocity field results in which the Reynolds shear stress is not zero and the ratio between the fluctuation velocity of \( u_{rms} \) and \( v_{rms} \) components differs significantly from 1 in most cases.

An increase of the turbulence energy was noted in the fibre suspension at fibre consistencies equal to or greater than 0.05%. A decrease of the turbulence energy is found for fibre suspensions in which the fibre consistency is less than 0.025%.
5.2 Recommendations

The refractive index difference between Duo Prime 90 oil and the acrylic fibre is 0.01 which only allows the measurement in fibre suspension consistence of 0.1% as maximum. To obtain higher consistence measurement, a mixture of the new nontoxic fluid is needed to use for better refractive index matching.

The flow in the tank is strongly three dimensional and anisotropic flow; therefore, stereoscopic PIV technique are required for the information on the flow field and the effect of the fibre on the turbulence of the flow in the tank.

In future work, the second phase in the flow should be considered in the design stage. The introduction of fibres precludes the use of conservation of momentum (3.12) to describe velocity gradients. Thus using the conservation of mixture momentum from Gidaspow (40).

\[
\sum_{i=1}^{n} \rho_i \varepsilon_i \frac{dv_i}{dt} = \sum_{i=1}^{n} \nabla \cdot T_i + \sum_{i=1}^{n} \rho_i \varepsilon_i f_i + \sum_{i=1}^{n} p_i \tag{5.1}
\]

where \( n = 2 \) in two-phase flow, \( \rho_i \) is the density of phase \( i \), \( \varepsilon_i \) is volume concentration of phase \( i \) in total volume, which is defined in

\[
V_i = \iiint_{V(i)} \varepsilon_i dV \tag{5.2}
\]

where

\[
\sum_{i=1}^{n} \varepsilon_i = 1. \tag{5.3}
\]
$T_i$ is the surface forces of phase i, $f_i$ is the force acting on phase i, $p_i$ is the interaction forces of phase I. In the $(x, y, z)$ coordinate system the stress tensor $T_i$ for phase I is given by the matrix (Gidaspow (40)),

$$T_i = \begin{bmatrix} T_{ixx} & T_{ixy} & T_{ixz} \\ T_{iyx} & T_{iyy} & T_{iyz} \\ T_{izx} & T_{izy} & T_{izz} \end{bmatrix}$$

(5.4)

Where its typical element $T_{ixx}$ is the $i$th force ($F_{ix}$) in the $x$ direction per unit area ($A_y$) of the $y$th face (Gidaspow (40)),

$$T_{ixx} = \frac{\partial F_{ix}}{\partial A_y}$$

(5.5)
Appendix A Programs

A.1 Turbulence calculation

% Calculate turbulence with spatial filtering
% Result: Turbulence Kinetic energy, Dissipation, Kolmogorov Scale.

% doing 2d fft on instantaneous velocity field. Then cut the high frequency part, do the inverse ffts to do the low-band pass filtering, the filtered output is saved in .tur
% subtract the low-pass field from instantaneous velocity field

% input the file names

clear all;
Miu=15.6e-6;
Cut_FrequencyX=15;
Cut_FrequencyY = 12;
Cut_FrequencyX2 = 22;
Cut_FrequencyY2 = 19;
fname3 = 'f';
gd=16; % gd is the distance between the grid points in pixels
    dd = 18.5e-6;
    dx=(gd*dd); %dx is the distance between the grid points in m
    dy=(gd*dd); %dy is the distance between the grid points in m

path(path,'d:\data\f1\plane2\pos1\v1');

path(path,’d:\repetitive\');
fp = ’d:\repetitive\’;
for kkk=2:2
    ss0 = int2str(kkk);
for iii=1:1 %'1:6
    ss1=int2str(iii);
for jjj=1:1
    ss2=int2str(jjj);
    if jjj==1
        dt = 5e-4;
    end
    if jjj == 2
        dt = 8e-4;
    end
    if jjj ==3
        dt =15e-4;
    end
fff=strcat(ss0,ss1);
fff=strcat(fff,ss2);
fname00=strcat(fname3,fff);
ffreq=strcat(fff,'freq.txt');
ff1=strcat(fff,'tur.txt');
ff2=strcat(fname3,ff1);
fmean=strcat(fpath,ff2)
ffreq = strcat(fname3,ffreq);
ffreq = strcat(fpath,ffreq);
%fname00=input('Input the mean velocity lile name prefix, fname= ')
%dt=input('Input the pulse separation in milliseconds, dt= ')
%initialize the mean values (velocity, dissipation, .....)
%input file series:

for i=1:1131
    Diss(i)=0;
    Epsilon(i)=0;
    uu(i)=0;
    uv(i)=0;
    v(i)=0;
    Ruu(i)=0;
    Ruv(i)=0;
    Rw(i)=0;
end

is=û %input(' Enter the start lile number, is= ');
ie=û0 % 100 %input(' Enter the ending lile number, ie= ');
N =ie-is+1;
%input the beginning file No. and end file No.
    for I = is:ie, %add the No. of the files
ss=int2str(I);
if I>=0 & I<=9
    ss=strcat('00',ss);
end
if I>=10 & I<=99
    ss=strcat('0',ss);
end
if I>=100 & I<=999
    ss=strcat(ss);
end

fname=strcat(fname00,ss);
% add the No. of the liles
fx=fname;
% the file to save vorticity graph
fname = strcat(fname, '.vec');
eval(['load 'fname]);
eval(['v='fx',']);
R=size(v,1);
x=v(:,9) * dd;
y=v(:,10) * dd;

% for contour
counter=R+1;
for i=1:29
    for j=39:-1:1
        counter=counter-1;
        U1(i,j)=v(counter,11)*dd/dt;
        V1(i,j)=v(counter,12)*dd/dt;
        UU1(i,j)=U1(i,j);
        VV1(i,j)=V1(i,j);
    end
end

% ccc='before filtering'
[lfu,lfv,hfu,hfv]=filtering(UU1, VV1, Cut_FrequencyX, Cut_FrequencyY, Cut_FrequencyX2, Cut_FrequencyY2);
counter=0;
for i=1:29
    for j=1:39
        counter=counter+1;
        temp1(counter)=real(lfu(i,j));
        temp2(counter)=real(lfv(i,j));
        % fluctuating velocities
        fu(i,j)=U1(i,j)-temp1(counter);
        fv(i,j)=V1(i,j)-temp1(counter);
        uu(counter)=uu(counter)+fu(i,j)*fu(i,j)/N;
        uv(counter)=uv(counter)+fu(i,j)*fv(i,j)/N;
        vv(counter)=vv(counter)+fv(i,j)*fv(i,j)/N;
        Ruu(counter)=Ruu(counter)+sqrt(fu(i,j)*fu(i,j))/(abs(temp1(counter))*N);
        Rvv(counter)=Rvv(counter)+sqrt(fv(i,j)*fv(i,j))/(abs(temp2(counter))*N);
    end
end
% end of the i, j loop through each flow field
[fux,fuy,fvx,fvy]=Gradient2(fu,fv,dx);
counter=0;
for i=1:29
    for j=1:39
        counter=counter+1;
\[
\text{Diss}(\text{counter}) = \text{Diss}(\text{counter}) + \text{Dissipation}(\text{fux}(i,j),\text{fuy}(i,j),\text{fvx}(i,j),\text{fvy}(i,j),\text{Miu})/\text{N};
\]
end
end
end% end of the i, j loop through each flow field
end  %end of time average for 100 series of the velocity fields
for \(i = 1:1131\)
    \(\text{Epsilon}(i) = (\text{Miu}^{(3/4)}) * (\text{Diss}(i))^{(-1/4)};\)
    \(\text{Ruv}(i) = \text{uv}(\text{counter})/\text{uu}(\text{counter});\)
end
\(\text{Outdata1}(:,1) = x;\)
\(\text{Outdata1}(:,2) = y;\)
\(\text{Outdata1}(:,3) = \text{uu}';\)
\(\text{Outdata1}(:,4) = \text{uv}';\)
\(\text{Outdata1}(:,5) = \text{vv}';\)
\(\text{Outdata1}(:,6) = \text{Ruu}';\)
\(\text{Outdata1}(:,7) = \text{Ruv}';\)
\(\text{Outdata1}(:,8) = \text{Rvv}';\)

save(fmean, 'Outdata1', '-ascii');

[\(\text{DF},\text{FDiss}\)] = \text{Histoo}(\text{Diss});
[\(\text{Ef},\text{FEpsi}\)] = \text{Histoo}(\text{Epsilon});
[\(\text{Ruuf},\text{FRuu}\)] = \text{Histoo}(\text{Ruu});
[\(\text{Ruvf},\text{FRuv}\)] = \text{Histoo}(\text{Ruv});
[\(\text{Rvrf},\text{FRvv}\)] = \text{Histoo}(\text{Rvv});

\(\text{Outdata2}(:,1) = \text{DF}';\)
\(\text{Outdata2}(:,2) = \text{FDiss}';\)
\(\text{Outdata2}(:,3) = \text{Ef}';\)
\(\text{Outdata2}(:,4) = \text{FEpsi}';\)

\(\text{Outdata2}(:,5) = \text{FRuu}';\)
\(\text{Outdata2}(:,6) = \text{Ruvf}';\)
\(\text{Outdata2}(:,7) = \text{FRuv}';\)
\(\text{Outdata2}(:,8) = \text{Rvrf}';\)
\(\text{Outdata2}(:,9) = \text{FRvv}';\)

save(ffreq, 'Outdata2', '-ascii');
end
end
end

A.2 Functions
1 Filtering
% filter the high-frequency components of velocity field
function [out1,out2,out3,out4]=filtering(in7,in8,in3,in4,in5,in6)
for i=1:64
    for j=1:128
        in1(i,j)=0;
        in2(i,j)=0;
        end
    end
for i=1:29
    for j=1:39
        in1(i,j) = in7(i,j);
        in2(i,j) = in8(i,j);
        end
    end

fUU=fft2(in1);
fVV=fft2(in2);

N=length(fUU);

ABSXX= abs(fUU(1:N/2)).^2;
ABSYY = abs(fVV(1:N/2)).^2;

IUU=fUU;
IVV=fVV;
hUU=fUU;
hVV=fVV;
for i=in3:64
    for j = in4:128
        IUU(i,j)=0;
        IVV(i,j)=0;
        end
    end
out1=ifft2(IUU);
out2=ifft2(IVV);

for i=1:in5
    for j = 1:in6
        hUU(i,j)=0;
        hVV(i,j)=0;
        end
    end
out3=ifft2(hUU);
out4 = ifft2(hVV);

2. Histoo.m
% the program to calculate the histogram.
function [out1,out2]=Histoo(In1)
N = 1131;
for i = 1:N
    step = (max(In1)-min(In1))/100;
end
for j = 1:100
    out2(j)=0;
    out1(j)=min(In1)+(j-1)*step;
end
for i = 1:N
    for j = 1:100
        if ((In1(i)-out1(j))<step & (In1(i)-out1(j))>=0)
            out2(j)=out2(j)+1;
        end
    end
end

3. Vorticity
% the function to calculate the vorcity field
function m = vorticity (p1,p2,dx)
for i=2:28
    for j=2:38
        dvdx = 1/8/dx*((2*p2(i,j+1)+p2(i-1,j+1)+p2(i+1,j+1))-(2*p2(i,j-1)+p2(i-1,j-1)+p2(i+1,j-1)));
        dudy = 1/8/dx*((2*p1(i+1,j)+p1(i+1,j-1)+p1(i+1,j+1))-(2*p1(i-1,j)+p1(i-1,j-1)+p1(i-1,j+1)));
        m(i,j) = dvdx-dudy;
    end
end
% for corner (1,1) and (39,29)
dvdx = 1/dx*(p2(1,2)-p2(1,1));
dudy = 1/dx*(p1(2,1)-p1(1,1));
m(1,1) = dvdx-dudy;
dvdx = 1/dx*(p2(29,39)-p2(29,38));
dudy = 1/dx*(p1(29,39)-p1(28,39));
m(29,39) = dvdx-dudy;
% for edge
for i=2:28
m(i,1) = 1/2/dx*((p2(i,2)-p2(i,1))-1/2*(p1(i+1,1)-p1(i-1,1)));
m(i,39) = 1/2/dx*((p2(i,39)-p2(i,38))-1/2*(p1(i+1,39)-p1(i-1,39)));
end

for j=2:38
m(1,j) = 1/2/dx*((p2(2,j)-p2(1,j))-1/2*(p1(1,j+1)-p1(1,j-1)));
m(29,j) = 1/2/dx*((p2(29,j)-p2(28,j))-1/2*(p1(29,j+1)-p1(29,j-1)));
end

5 Gradient
% the program to calculate the gradient of a velocity field

function [ux,uy,vx,vy] = Gradient2(p1,p2,dx)
% calculate the vorticity
for i=2:28
for j=2:38
vx(i,j) = 1/8/dx*((2*p2(i,j+1)+p2(i-1,j+1)+p2(i+1,j+1))-(2*p2(i,j-1)+p2(i-1,j-1)+p2(i+1,j-1)));
uy(i,j) = 1/8/dx*((2*p1(i+1,j)+p1(i+1,j-1)+p1(i+1,j+1))-(2*p1(i-1,j)+p1(i-1,j-1)+p1(i-1,j+1)));
ux(i,j) = 1/8/dx*((2*p1(i,j+1)+p1(i-1,j+1)+p1(i+1,j+1))-(2*p1(i,j-1)+p1(i-1,j-1)+p1(i+1,j-1)));
vx(i,j) = 1/8/dx*((2*p2(i+1,j)+p2(i+1,j-1)+p2(i+1,j+1))-(2*p2(i-1,j)+p2(i-1,j-1)+p2(i-1,j+1)));
end
end

% for corner (1,1) and (29,39)
vx(1,1) = 1/dx*(p2(1,2)-p2(1,1));
uy(1,1) = 1/dx*(p1(2,1)-p1(1,1));
ux(1,1) = 1/dx*(p1(1,2)-p1(1,1));
vx(1,1) = 1/dx*(p2(2,1)-p2(1,1));
uy(1,1) = 1/dx*(p1(1,2)-p1(1,1));
ux(1,1) = 1/dx*(p1(2,1)-p1(1,1));
vy(29,1) = 1/dx*(p2(29,1)-p2(28,1));

vx(1,39) = 1/dx*(p2(1,39)-p2(1,38));
uy(1,39) = 1/dx*(p1(2,39)-p1(1,39));
vx(1,39) = 1/dx*(p1(1,39)-p1(1,38));
uy(1,39) = 1/dx*(p2(2,39)-p2(1,39));
% for edge j=1 , j= 39
for i=2:28

vx(i,1)=1/dx*(p2(i,2)-p2(i,1));
uy(i,1)=1/2/dx*(p1(i+1,1)-p1(i-1,1));
ux(i,1)=1/dx*(p1(i,2)-p1(i,1));
vy(i,1)=1/2/dx*(p2(i+1,1)-p2(i-1,1));

vx(i,39)=1/dx*(p2(i,39)-p2(i,38));
uy(i,39)=1/2/dx*(p1(i+1,39)-p1(i-1,39));
ux(i,39)=1/dx*(p1(i,39)-p1(i,38));
vy(i,39)=1/2/dx*(p2(i+1,39)-p2(i-1,39));
end
% for edge i=1 , j= 29

for j=2:38

vy(1,j)= 1/dx*(p2(2,j)-p2(1,j));
ux(1,j)= 1/2/dx*(p1(1,j+1)-p1(1,j-1));
uy(1,j)= 1/dx*(p1(2,j)-p1(1,j));
vx(1,j)= 1/2/dx*(p2(1,j+1)-p2(1,j-1));

vy(29,j)= 1/dx*(p2(2,j)-p2(1,j));
ux(29,j)= 1/2/dx*(p1(1,j+1)-p1(1,j-1));
uy(29,j)= 1/dx*(p1(2,j)-p1(1,j));
vx(29,j)= 1/2/dx*(p2(1,j+1)-p2(1,j-1));
end

6. Dissipation
% calculate Turbulent Dissipation
function m=dissipation(In1,In2,In3,In4,Miu)
m = 2*(In1^2+In2^2+(In1+In2)^2);
m = m+3*(In3^2 + In4^2);
m = m+6*In3*In4;
m = Miu*m;

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Appendix B  Error Analysis

B.1 Methodology

The uncertainty in the measurements (\( \sigma_{\zeta}^2 \)) of \( \zeta = f(\gamma_1, \gamma_2, \gamma_3, ..., \gamma_n) \) can be approximated as (Bevington, 1969)

\[
\sigma_{\zeta}^2 \approx \sum_{i=1}^{n} \left( \sigma_{\gamma_i}^2 \left( \frac{\partial \zeta}{\partial \gamma_i} \right)^2 \right)
\]

(B.1)

where \( \zeta \) is the function with variances of \( \gamma_1, \gamma_2, \gamma_3, ..., \gamma_n \).

B.2 Instantaneous Velocity Error

The instantaneous velocity of a particle, \( \bar{u}_i \) is,

\[
\bar{u}_i = \frac{d_i}{M \Delta t}
\]

(B.2)

Where \( d_i \) is the particle displacement in pixels, \( M \) is the magnification factor of the CCD camera in pixels/mm, and \( \Delta t \) is the time interval in seconds.

From equation B.1, the uncertainty in the instantaneous velocity data is,

\[
\sigma_{\bar{u}_i}^2 = \sigma_{d_i}^2 \left( \frac{\partial \bar{u}_i}{\partial d_i} \right)^2 + \sigma_{\Delta t}^2 \left( \frac{\partial \bar{u}_i}{\partial \Delta t} \right)^2 + \sigma_M^2 \left( \frac{\partial \bar{u}_i}{\partial M} \right)^2
\]

(B.3)

where \( \sigma_{\bar{u}_i}^2 \) is the variance in the particle displacement measurement, \( \sigma_M^2 \) is the variance in the magnification factor, and \( \sigma_{\Delta t}^2 \) is the variance in the time interval.

Noting that

\[
\frac{\partial \bar{u}_i}{\partial d_i} = \frac{1}{M \Delta t}
\]

(B.4)
Using \( M = 54 \) pixel/mm (the pixel size is 18.5 \( \mu\text{m} \)), \( \sigma_M = 0.2 \) pixel/mm, \( \Delta t = 0.5 \) ms, \( \sigma_st = 6.4 \) \( \mu\text{s} \), \( \sigma_{sl} = 0.3 \) pixel, and noting that \( \sigma_{sl} = \bar{u}_i M\Delta t \), we have

\[
\sigma_{ai} = \bar{u}_i \sqrt{\left( \frac{\sigma_{ai}}{\bar{u}_i} \right)^2 + \left( \frac{\sigma_M}{M} \right)^2 + \left( \frac{\sigma_{sl}}{\Delta t} \right)^2}
\]

The fluctuating velocity \( u_i \) defined as

\[
u_i = \bar{u}_i - U
\]

And the fluctuating variance \( \sigma_{ui} \) is approximately \( \sigma_{ai} \).

\[
\sigma_{ui} \approx \sigma_{ai} = 6 \text{ mm/s}
\]

where the uncertainty is 15% of the real value of fluctuating velocity.

B. 4 Turbulence Kinetic Energy (TKE) Error

TKE is defined as (equation 3.5)

\[
k = \frac{3}{4}(u_1^2 + u_2^2)
\]

The variance in the quantity \( \sigma_k \) can be estimated as \( \sigma_{u1}^2 \), since \( u_1^2 \) and \( v_1^2 \) are approximately same. Thus apply equation B.1 to equation (3.5), we have
\[ \sigma_{u_i^2} = \sigma_{u_i}^2 \left( \frac{\partial u_i}{\partial x} \right)^2 = \sigma_{u_i}^2 \times 4u_i^2 \]  
(B.12)

\[ \sigma_k = \sigma_{u_i^2} = \sigma_{u_i}^2 \times 4u_i^2 = 24 \frac{mm^2}{s^2} \]  
(B.13)

where the typical \( u_i \) is 1 mm/s. The relative error of TKE is 20%.

**B.5 Velocity Gradient Error**

In order to determine the gradient of an arbitrary function, \( \zeta \), the central scheme used in the data analysis of this experiment. The error produced in these schemes is caused by the accuracy of the accuracy of the velocity measurements and the truncation error resulting from the different scheme used.

The central difference scheme for the first order gradient is written as,

\[ \left( \frac{\partial \zeta_i}{\partial x_j} \right)_k = \frac{(\zeta_i)_k+(\zeta_i)_{k-1}}{2\Delta x} + O(\Delta x^2) \]  
(B.14)

The \( O(\Delta x^2) \) represents the truncation error. For this scheme the truncation error is found as

\[ O(\Delta x^2) = \frac{\Delta x^2}{3!} \frac{\partial^3 \zeta_i}{\partial x_j^3} + \frac{\Delta x^4}{5!} \frac{\partial^5 \zeta_i}{\partial x_j^5} + \ldots \]  
(B.15)

Using \( \Delta x = 1.08 \) mm and assuming that all gradients are approximately equal, then the truncation errors can be written as,

\[ O(\Delta x^2) \approx 0.2 \frac{\partial \zeta_i}{\partial x_j} \]  
(B.16)
Gradient calculations neglect the truncation errors, thus equation (B.14) reduces to

$$\left( \frac{\partial \xi}{\partial x} \right)_k = \frac{(\xi)_{i+1} - (\xi)_{i-1}}{2\Delta x}$$

(B.17)

The variance $\sigma^2_{\xi}$ (neglecting the truncation error) can be calculated (assuming a spatially constant $\sigma^2_{\xi}$)

$$\sigma^2_{\xi} = \frac{1}{2(\Delta x)^2} \sigma^2_{\xi},$$

(B.18)

Next, if the truncation error is assumed to have a Gaussian distribution, then the variance for this error can be written as,

$$\sigma_i = \left[ \frac{0.2 \left( \frac{\Delta x}{\Delta y} \right)}{2.33} \right]^2 = 0.007 [\frac{\xi_i}{\xi_j}]^2$$

(B.19)

$$\sigma_i^2 = 0.1 \left[ \frac{\xi_i}{\xi_j} \right]^2$$

(B.20)

Finally the complete estimate for the variance in the gradient calculation is found by summing the first gradient estimate with the truncation estimate and noting $\Delta x = 0.6$ mm.

Thus

$$\sigma^2_{\varphi} = \sigma^2_{\varphi_{\xi}} + \sigma^2_i + 0.007[\frac{\xi_i}{\xi_j}]^2$$

(B.21)

$$\sigma^2_{\varphi} = \sigma^2_{\varphi_{\xi}} + \sigma^2_i + 0.4 \sigma^2_{\xi} + 0.0007[\frac{\xi_i}{\xi_j}]^2$$

(B.22)

The variance for the fluctuating velocity gradients can now be determined by inserting equation B.9 into the error in above equation,

$$\sigma^2_{\varphi_{ui}} = 0.24 + 0.007[\frac{\xi_i}{\xi_j}]^2$$

(B.22)

in which values of $\sigma_{\varphi_{ui}}$ ranges from minimum value 4.2 s$^{-2}$ to maximum value 42 s$^{-2}$ with respect to the minimum velocity gradient 50 and maximum velocity gradient 500 where the relative error is about 8.4%.
B.6 Turbulent Dissipation Error

Turbulent dissipation is calculated as

$$
\varepsilon = 2\nu\left(\frac{\partial u_1}{\partial x_1}\right)^2 + \left(\frac{\partial u_2}{\partial x_2}\right)^2 + \left(\frac{\partial u_3}{\partial x_3}\right)^2 + 3\nu\left(\frac{\partial u_1}{\partial x_1}\right)^2 + \left(\frac{\partial u_2}{\partial x_2}\right)^2 + 6\nu\frac{\partial u_1}{\partial x_1}\frac{\partial u_2}{\partial x_1}
$$

We assume that all the terms in equation (3.19) have approximately same error, thus

$$
\left(\frac{\partial u_1}{\partial x_1}\right)^2 = \left(\frac{\partial u_2}{\partial x_2}\right)^2 = \left(\frac{\partial u_3}{\partial x_3}\right)^2 = \left(\frac{\partial u_1}{\partial x_1}\right)^2 = \frac{\partial u_1}{\partial x_1}\frac{\partial u_2}{\partial x_1}
$$

$$
\sigma_\varepsilon^2 = 12\nu \sigma_{\varepsilon^2}^2
$$

(B.23)

Substitute equation B.22 into above equation B.23, we have

$$
\sigma_\varepsilon^2 = 12\nu \left(\frac{\partial u_i}{\partial x_j}\right)^2(0.24 + 0.007\left[\frac{\partial u_i}{\partial x_j}\right]^2)(1/\frac{\partial u_i}{\partial x_j})
$$

(B.24)

Substituting the known values of the gradients, we have the value of \(\sigma_\varepsilon\) range from 7 \times 10^{-6} m^2/s^3 to 7 \times 10^{-5} m^2/s^3, where the relative error is about 20%.

B.7 Kolmogorov Scale Error

Kolmogorov scale is defined by (3.20)

$$
\eta = \nu^{3/4} \varepsilon^{-1/4}
$$

(3.20)

The uncertainty of Kolmogorov scale \(\sigma_\eta^2\) is calculated with equation B.1

$$
\sigma_\eta^2 = \nu^{3/4} \sigma_\varepsilon^2 \left(\frac{d\eta}{dx}\right)^2
$$

(B.25)

$$
\sigma_\eta^2 = -\frac{5}{16} \times \nu^{3/4} \sigma_\varepsilon^2 \varepsilon^{-9/4}
$$

(B.25)

Substitute (B.24) into (B.25), we have

$$
\sigma_\eta^2 = -3.75\nu^{7/4} \varepsilon^{-9/4} \left(\frac{d\eta}{dx}\right)^2(0.24 + 0.007\left[\frac{d\eta}{dx}\right]^2)(1/\frac{d\eta}{dx})
$$

(B.26)
Substituting the values of $\varepsilon = 4 \times 10^{-4}$ and the known velocity gradients into equation (B.26), we have, the values of $\sigma$ range from 0.006 to 0.05 mm. The relative errors in Kolmogorov Scale is about 25%.

\textbf{B.8 Reynolds Shear Stress Error.}

Reynolds Shear Stress is defined as $\overline{u_i u_j}$.

Apply equation (B.1)

$$
\sigma_{\overline{u_i u_j}}^2 = \sigma_{\overline{u_i}}^2 \left( \frac{\partial \overline{u_i u_j}}{\partial \overline{u_i}} \right)^2 + \sigma_{\overline{u_j}}^2 \left( \frac{\partial \overline{u_i u_j}}{\partial \overline{u_j}} \right)^2
$$

(B.27)

where the $\sigma_{\overline{u_i}}^2$ and $\sigma_{\overline{u_j}}^2$ are approximately equal to $\sigma_{\overline{u_i}}^2 = 6$ mm/s, from equation (B.11).

$$
\sigma_{\overline{u_i u_j}}^2 = \left( \frac{\partial \overline{u_i u_j}}{\partial \overline{u_i}} \right)^2 + \left( \frac{\partial \overline{u_i u_j}}{\partial \overline{u_j}} \right)^2 \times 0.6 \text{mm/s}
$$

(B.27)

Substituting the experiment values in the experiment and with the calculation of the Matlab, the values of $\sigma_{\overline{u_i u_j}}^2$ range from $1 \times 10^{-5}$ m$^2$/s$^2$ to $4 \times 10^{-5}$ m$^2$/s$^2$. The relative errors in this case are around 30%.

\textbf{B.9 Ratio of $u_{rms}$ to $u_{rms}$ Error}

Apply equation (B.1)

$$
\sigma_{u_{rms}/v_{rms}}^2 = (1/v_{rms})^2 \times \sigma_{u_{rms}}^2 + (u_{rms} \times \ln(v_{rms}))^2 \times \sigma_{v_{rms}}^2
$$

(B.28)
The value of \( \sigma_{u_{rms}/v_{rms}} \) which range from 0.1 to 0.3 are obtained by substituting the value ranges of the terms in (B.28). The relative error in this case are around 30%.

<table>
<thead>
<tr>
<th>Errors</th>
<th>Value</th>
<th>Relative Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instantaneous Velocity Error</td>
<td>6 mm/s</td>
<td>3%</td>
</tr>
<tr>
<td>Fluctuating Velocity Error</td>
<td>6 mm/s</td>
<td>15%</td>
</tr>
<tr>
<td>TKE Error</td>
<td>24 mm(^{2}/s)</td>
<td>20%</td>
</tr>
<tr>
<td>Velocity Gradient Error</td>
<td>50 - 1750 s(^{-1})</td>
<td>8.4%</td>
</tr>
<tr>
<td>Turbulent Dissipation Error</td>
<td>(7 \times 10^{-6} m^{2}/s^{3}) to (7 \times 10^{-5} m^{2}/s^{3})</td>
<td>20%</td>
</tr>
<tr>
<td>Kolmogorov Scale Error</td>
<td>0.006 to 0.05 mm</td>
<td>25%</td>
</tr>
<tr>
<td>Reynolds Shear Stress Error</td>
<td>(1 \times 10^{-5} m^{2}/s^{2}) to (4 \times 10^{-5} m^{2}/s^{2})</td>
<td>30%</td>
</tr>
<tr>
<td>Ratio of (u_{rms}/v_{rms}) Error</td>
<td>0.1 to 0.3</td>
<td>30%</td>
</tr>
</tbody>
</table>

Table B.1 Table of Errors.
Appendix C The Investigation of the Repetitiveness of the Experiment

The TKE, Kolmogorov Scale, Dissipation, Reynolds shear stress and the ratio of $u_{rms}$ to $v_{rms}$ were calculated from the data used in the experiment and were plotted in Figure C.1 to C.10. Each set has 40 velocity fields to obtain the turbulence statistics. Case 1 are 1 - 40 velocity fields and case 2 are 60 - 100 images in the 100 successive velocity fields. Table C.1 - C.5 lists all the peak value comparison of the distribution of TKE, Kolmogorov Scale, Dissipation, Reynolds shear stress and the ratio of $u_{rms}$ to $v_{rms}$ respectively.

For most consistencies, the difference of TKE between two cases is less than $1.5 \times 10^{-4} \text{m}^2/\text{s}^2$ which is the 20% of maximum value (table C. 1); the difference of Kolmogorov Scale between two cases is less than 0.1 mm which is the 25% of the maximum value (Table C.2); The difference of Dissipation between two cases is less than $0.04 \times 10^{-4} \text{m}^2/\text{s}$ which is 10% of the largest peak value (table C.3); The difference of Reynolds shear stress between two cases is less than 30% (Table C.4); The difference of Kolmogorov Scale between two cases is less than 30% (Table C.5).

The results of the comparison show reproducibility was achieved.
### Table C.1 The comparison of TKE peak distribution in two cases

<table>
<thead>
<tr>
<th>Consistency</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>$4 \times 10^{-4} \text{m}^2/\text{s}^2$</td>
<td>$6.2 \times 10^{-4} \text{m}^2/\text{s}^2$</td>
</tr>
<tr>
<td>0.01%</td>
<td>$5.5 \times 10^{-4} \text{m}^2/\text{s}^2$</td>
<td>$5.5 \times 10^{-4} \text{m}^2/\text{s}^2$</td>
</tr>
<tr>
<td>0.025%</td>
<td>$4.5 \times 10^{-4} \text{m}^2/\text{s}^2$</td>
<td>$2.6 \times 10^{-4} \text{m}^2/\text{s}^2$</td>
</tr>
<tr>
<td>0.05%</td>
<td>$7 \times 10^{-4} \text{m}^2/\text{s}^2$</td>
<td>$6.5 \times 10^{-4} \text{m}^2/\text{s}^2$</td>
</tr>
<tr>
<td>0.1%</td>
<td>$3.5 \times 10^{-4} \text{m}^2/\text{s}^2$</td>
<td>$5.5 \times 10^{-4} \text{m}^2/\text{s}^2$</td>
</tr>
</tbody>
</table>

### Table C.2 The comparison of Kolmogorov scale peak distribution in two cases

<table>
<thead>
<tr>
<th>Consistency</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>$4 \times 10^{-4} \text{m}$</td>
<td>$4.2 \times 10^{-4} \text{m}$</td>
</tr>
<tr>
<td>0.01%</td>
<td>$3.5 \times 10^{-4} \text{m}$</td>
<td>$5 \times 10^{-4} \text{m}$</td>
</tr>
<tr>
<td>0.025%</td>
<td>$4.5 \times 10^{-4} \text{m}$</td>
<td>$4.6 \times 10^{-4} \text{m}$</td>
</tr>
<tr>
<td>0.05%</td>
<td>$5.2 \times 10^{-4} \text{m}$</td>
<td>$4.5 \times 10^{-4} \text{m}$</td>
</tr>
<tr>
<td>0.1%</td>
<td>$3.5 \times 10^{-4} \text{m}$</td>
<td>$3.6 \times 10^{-4} \text{m}$</td>
</tr>
</tbody>
</table>

### Table C.3 The comparison of dissipation peak distribution in two cases

<table>
<thead>
<tr>
<th>Consistency</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>0.12 m$^2$/s$^3$</td>
<td>0.08 m$^2$/s$^3$</td>
</tr>
<tr>
<td>0.01%</td>
<td>0.25 m$^2$/s$^3$</td>
<td>0.06 m$^2$/s$^3$</td>
</tr>
<tr>
<td>0.025%</td>
<td>0.02 m$^2$/s$^3$</td>
<td>0.02 m$^2$/s$^3$</td>
</tr>
<tr>
<td>0.05%</td>
<td>0.04 m$^2$/s$^3$</td>
<td>0.12 m$^2$/s$^3$</td>
</tr>
<tr>
<td>0.1%</td>
<td>0.4 m$^2$/s$^3$</td>
<td>0.25 m$^2$/s$^3$</td>
</tr>
</tbody>
</table>

Table C.1 The comparison of TKE peak distribution in two cases

Table C.2 The comparison of Kolmogorov scale peak distribution in two cases

Table C.3 The comparison of dissipation peak distribution in two cases
### Table C.4 The comparison of Reynolds shear stress peak distribution in two cases

<table>
<thead>
<tr>
<th>Consistency</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>0.1 \times 10^{-4} m^2/s^2</td>
<td>0.5 \times 10^{-4} m^2/s^2</td>
</tr>
<tr>
<td>0.01%</td>
<td>0.1 \times 10^{-4} m^2/s^2</td>
<td>0.6 \times 10^{-4} m^2/s^2</td>
</tr>
<tr>
<td>0.025%</td>
<td>0.2 \times 10^{-4} m^2/s^2</td>
<td>1 \times 10^{-4} m^2/s^2</td>
</tr>
<tr>
<td>0.05%</td>
<td>2 \times 10^{-4} m^2/s^2</td>
<td>1.5 \times 10^{-4} m^2/s^2</td>
</tr>
<tr>
<td>0.1%</td>
<td>0 \times 10^{-4} m^2/s^2</td>
<td>0 \times 10^{-4} m^2/s^2</td>
</tr>
</tbody>
</table>

### Table C.5 The comparison of Ratio of $u_{rms}$ to $v_{rms}$ peak distribution in two cases

<table>
<thead>
<tr>
<th>Consistency</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>0.25</td>
<td>0.12</td>
</tr>
<tr>
<td>0.01%</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>0.025%</td>
<td>0.12</td>
<td>0.2</td>
</tr>
<tr>
<td>0.05%</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>0.1%</td>
<td>0.3</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Figure C.1 Kinetic Energy in case 1

Figure C.2 Kinetic Energy in case 2

Figure C.3 Kolmogorov Scale in case 1

Figure C.4 Kolmogorov Scale in case 2
Figure C.5 Dissipation in case 1

Figure C.6 Dissipation in case 2

Figure C.7 Reynolds Shear Stress in case 1

Figure C.8 Reynolds Shear Stress in case 2
Figure C.9 Ratio of $u_{rms}$ to $v_{rms}$ in case 1

Figure C.10 Ratio of $u_{rms}$ to $v_{rms}$ in case 2
Reference:


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