An Investigation of Photothermal Radiometric Thermal Diffusivity Measurements of Thin Steel Samples

by

Francis Elek

A thesis submitted in conformity with the requirements for the Degree of Master of Applied Science, Graduate Department of Mechanical and Industrial Engineering, University of Toronto

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Abstract

In this work, a completely noncontact experimental configuration has been utilized to calculate the thermal diffusivity of thin steel samples. The technique, photothermal radiometry, is based on the periodic infrared emission of a periodically heated sample. Three frequency domain experimental techniques- (a) distance scans, (b) frequency scans and (c) pump-probe separated frequency scans- have been used to obtain thermal diffusivity values for the tested samples. In addition, a thermo-acoustic theory has been examined, to see if such a theory may be used to obtain thermal diffusivity when acoustic vibrations convolute the photothermal signal, by taking the sample in and out of focus with the detection optics.
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### Nomenclature

- $k$: thermal conductivity
- $\alpha$: thermal diffusivity
- $c$: specific heat
- $T$: Spatial temperature field
- $I$: Incident optical irradiance
- $f$: frequency
- $G$: Green function
- $S$: Surface
- $w$: laser beam spot size
- $\mu$: thermal diffusion length
- $\sigma$: complex thermal wave number
- $\rho$: density
- $\delta$: delta function
- $\omega$: angular frequency
- $\Phi$: thermal wave flux
- $\varepsilon$: emissivity
Chapter 1

Introduction

1.1 Background

Over the past twenty years there has been a huge increase in the development of thin films for both thermal and electronic applications. Although not a strict definition, a thin film may be identified as a freestanding sample of uniform thickness that is between 5 \( \mu m \) and 2 mm thick. This interest in such films has in turn necessitated the accurate characterization of the physical properties of such materials. Many of the techniques do not require physical contact with the specimen and thus allow measurements on components without the contamination inherent using contact methods.

Thermal wave techniques are among the most prevalent in this field, and by generating and then detecting such waves the thermal diffusivity of the film under investigation may be obtained.
1.2 A History of thermal parameter measurements

1.2.1 Thermal Conductivity

Direct measurement of thermal conductivity depends on the accurate determination of the heat flux through a material along with the associated temperature gradient, and these two quantities are difficult to determine in bulk samples and even more so in thin samples. One such example is Cahill’s DC technique (1) (Fig 1.1) which consists of two parallel metallic strips deposited on the surface of the sample with current (I) and voltage probes at each end. A large current is passed along the left strip which acts as heater (i.e. -source) and it’s temperature is determined from the subsequent change in resistance. The right strip then acts as a resistive temperature sensor. Knowing the temperature of both strips, their separation and the power supplied it possible to solve the thermal conductivity equation and determine the thermal conductivity. Because a physical contact

![Diagram of Cahill's DC method](image)

Figure 1.1 Cahill’s DC method.
is established between sample and probe this technique is classified as a contact method, although since the sample is not destroyed in the process it is designated as a nondestructive one as well. In the field of sample characterization contact is to be avoided if possible as it may introduce contaminants, or in general may make accurate measurements dependent on the physical parameters of the acting probe.

1.2.2 Thermal Diffusivity

Thermal diffusivity is another useful thermal parameter, which is related to the thermal conductivity by the equation (2):

\[ k = \alpha c \rho \]  

(1.1)

where \( k \) is the thermal conductivity, \( \alpha \) is the thermal diffusivity, \( c \) is the specific heat, and \( \rho \) is the density.

In contrast to the thermal conductivity, the thermal diffusivity depends on the time dependence of the temperature field in the sample. Such a time dependent field may be induced by providing a transient or periodic thermal perturbation at a specimen boundary (boundary condition-derived source) or in general through out a portion of the sample volume (volume source). Since time is the measured parameter, thermal diffusivity is easier to measure, and to a greater precision, than thermal conductivity. The conductivity may then be derived through equation (1.1). Of the nondestructive techniques relevant to
the measurement of thermal diffusivity one may subdivide them into two groups—pulse techniques and thermal wave techniques, which will be examined more closely now.

1.2.2. Pulse (or flash) techniques

These techniques make use of a short duration (1 \( \mu \)s to 1 s) heat pulse (or flash) applied to one surface (3) (or if the heat pulse is localized sufficiently, to one small point on the sample). The corresponding change in temperature as a function of time is measured to determine the thermal diffusivity (Fig. 1.2). The temperature change can be monitored either on the same side as the applied heat pulse (front surface or backscatter technique) or the opposite side (rear surface or transmission technique). The most common heat sources are an electron beam, a high-power xenon flash tube, or a pulsed laser. If a very
localized heat source is required the laser becomes the preferred source since it is possible to focus the beam size to a magnitude on the order of a wavelength of light, and thus to a minimum of 0.4 – 0.8 μm as a minimum, although several thermal studies have used beam size in the 5 - 50 μm range. The temperature change may be monitored using a thermocouple, and infrared (IR) detector or by laser beam probe. The first technique is a contact method whereas the later two are non-contact. With a pulsed laser as a heat source and an infrared detector to monitor the temperature change, the thermal diffusivity can be measured without any physical contact with the sample. This technique is usually called a non-destructive, non-contact method, although with a proviso. Since the laser beam may be focused to such a narrow beam size, optical power densities on the order of $10^6 - 10^{12}$ W/m² are possible. Therefore care must be taken to ensure that the sample is not damaged under such high optical density.

The transmission method of thermal diffusivity measurements was first performed by Parker et al (4) and will briefly be reviewed now. A 400 J xenon flash lamp was used to provide the heat pulse and the resulting temperature rise on the rear surface of the metal sample was measured by two thermocouple wires separated by a distance of 1 to 2 mm. By assuming an instantaneous pulse, completely absorbed at the surface of the sample and neglecting heat loss from the sample, they were able to derive a simple relation between the temperature rise of the rear surface and the thermal diffusivity:

$$\alpha = \frac{1.38L^2}{\pi^2 t_{1/2}}$$  \hspace{1cm} (1.2)
where \( L \) is the thickness of the sample and \( t_{1/2} \) is the time for the rear surface temperature to reach half its maximum value.

### 1.2.2.b Thermal Wave (Modulated) techniques

These techniques precede the pulsed method by 100 years, as Angstrom in 1861 determined the thermal conductivity of copper and iron by what is now known as the Thermal Wave method of diffusivity measurement (5). Widespread interest in thermal waves has lead to revival of the field (6-15). The phenomena of thermal waves will be examined now in a quantitative manner using the simplest possible model, which will still yield the key features of thermal waves. Such a model is shown in Fig. 1.3, which depicts a semi-infinite medium in air, with a periodic source of radiation impinging upon the sample and being absorbed at the surface. When a source of energy is focused onto the surface of an opaque solid (actually on a surface that will absorb a part of the energy), a heat flow will occur following a non-radiative de-excitation process. If the source of energy is modulated, then a periodic heat flow is produced in the sample (16). The heat flow in the sample is a diffusive process (as all conductive heat flow must be via the heat diffusion equation), and the resulting periodic (in time) scalar temperature field is called a thermal wave (17).

### 1.3 Thermal Wave Theory

In this model it is assumed that the energy is absorbed within a very thin layer of the sample surface. This allows for a portion of the incident energy to be reflected, which is
what occurs in the case of metal samples. For the regions 1 (surrounding gas) and 2 (the medium) of Fig. 1.3 the heat diffusion equation is satisfied (18):

\[
\frac{\partial^2 T_g(x,t)}{\partial x^2} - \frac{1}{\alpha_g} \frac{\partial T_g(x,t)}{\partial t} = 0
\]  \hspace{1cm} (1.3)

\[
\frac{\partial^2 T_m(x,t)}{\partial x^2} - \frac{1}{\alpha_m} \frac{\partial T_m(x,t)}{\partial t} = 0
\]  \hspace{1cm} (1.4)

where \( g \) and \( m \) represent gas and medium, respectively.
Note that the above equations are homogeneous—they have no source terms. The source of the thermal wave field is incorporated into the boundary conditions at \( x = 0 \).

Here continuity of temperature and heat flux is assumed:

\[
T_s(x = 0) = T_m(x = 0) \tag{1.5}
\]

\[
k_s \frac{\partial T_s(x = 0)}{\partial x} = k_s \frac{\partial T_s(x = 0)}{\partial x} + I e^{i\omega t} \tag{1.6}
\]

where \( I \) is the absorbed incident power density and \( \omega = 2\pi f \) is the angular frequency of the modulated heat source.

If separable solutions periodic in time are assumed, with periodicity equal to that of the incident radiation, the following solutions result:

\[
T_m(x, t) = T_m(x) e^{i\omega t} \quad \Rightarrow \quad T_m(x) = C_2 e^{\sigma x} \quad \tag{1.7}
\]

\[
T_s(x, t) = T_s(x) e^{i\omega t} \quad \Rightarrow \quad T_s(x) = C_1 e^{\sigma x} \quad \tag{1.8}
\]

where \( \sigma \) is the complex thermal wave number given by:

\[
\sigma = (1 + i) \sqrt{\frac{\omega}{2\alpha_{s,m}}} \quad \tag{1.9}
\]
Since the complex thermal wave vector contains both a real and an imaginary part it is seen that the magnitude of the thermal wave must decay as a function of distance. Thus unlike in the case of propagating wave fields thermals wave will have an appreciable magnitude only for very short distances from the excitation point. Furthermore the thermal diffusion length may be defined as:

$$\mu = \sqrt{\frac{2\alpha}{\omega}} = \sqrt{\frac{\alpha}{nf}}$$  \hspace{1cm} (1.10)

A sample of thickness L is classified as thermally thin if \( \mu > L \), and thermally thick otherwise. When \( x = \mu \) the magnitude of the thermal wave decreases to \( e^{-1} \) (37 %) of its value at \( x = 0 \) and thus the thermal diffusion length gives a good measure of the distance from excitation at which the magnitude of the thermal wave is still appreciable. Beyond this distance the wave amplitude decays rapidly. For a steel with thermal diffusivity \( \alpha = 3.5 \times 10^{-6} \text{ m}^2/\text{s} \), with modulation frequency varying from 10 to 1000 Hz, the thermal diffusion length is found to vary from 333 to 33 \( \mu \text{m} \). From the thermal diffusion length one may draw two general conclusions that apply to all thermal wave phenomena. First, the thermal diffusivity gives a measure of the distance to which thermal waves will propagate into a material, with higher diffusivity materials obtaining deeper penetration depths. Also it is seen that by changing the modulation frequency, the depth of
penetration can be controlled. This is represented in Fig. 1.4, which shows that as modulation frequency increases the depth of penetration decreases, a phenomena that is also observed for electromagnetic waves that impinge upon conducting surfaces.

![Diagram showing frequency dependent depth of penetration of a thermal wave](image)

**Figure 1.4** Frequency dependent depth of penetration of a thermal wave

In the theory above one is able to draw several general conclusions about thermal wave phenomena from a very general model. One property that couldn’t be inferred from the above model is that of thermal wave reflections. If the material media had been of finite thickness, rather than semi-infinite, then the theory of thermal waves would support a reflection coefficient for the interface between media 1 and media 2. Since thermal waves are not propagating waves they don’t reflect as would light for instance, but interference phenomena are possible. If the material is surrounded by air the thermal wave reflection coefficient for the interface between the sample and air is given by:
$$R = \frac{1 - b_{\text{mm}}}{1 + b_{\text{mm}}} \quad (1.11)$$

where

$$b_{\text{mm}} = \sqrt{\frac{(\rho c k)_{\text{m}}}{(\rho c k)_{\text{m}}}} \quad (1.12)$$

is the square root of the ratio of the effusivities of the two media.

### 1.4 Objectives

In this investigation the applicability of measuring thermal diffusivity of thin films by photothermal radiometry is examined. Photothermal radiometry (19) is a thermal wave technique in which a thermal wave is launched into the sample by a periodically modulated laser beam source, and is then measured via the modulated radiometric (infrared) emission. In this investigation both the source (laser) and the detection (radiometric) will be localized. Three dimensional thermal wave fields will be derived, using rigorous Green function based methods, with different boundary conditions being considered. The detection of the radiometric signal will be divided into three categories:
(1) Distance scans (2) conventional frequency scans, and (3) Pump-probe separated frequency scans. For the distance scan the laser will be scanned across the sample surface, while the detector remains fixed. By scanning along a distance a decaying radiometric signal will be observed. The signal has an amplitude and a phase associated to it. The conventional frequency scan is one in which the excitation and the detection are coincident. By scanning the frequency from low to high values a decaying amplitude and phase are recorded. The pump-probe separated frequency scans are similar to conventional frequency scans but in this case the excitation and detection are at different positions.

The three different methods will be investigated to determine whether (1) thermal diffusivity can be calculated and (2) one method is more sensitive than the others. In addition to the thermal wave theory, a thermo-acoustic signal generation mechanism will be examined. Thermal gradients in a sample may cause acoustic vibrations, which may in turn be detected by the radiometric detector. The feasibility of calculating thermal diffusivity in the case where thermo-acoustic vibrations occur will be examined.

1.5 Layout of Thesis

Chapter 2 presents the theoretical basis for thermal wave theory, with fields developed using Green’s function formalism. In addition, a thermo-acoustic theory is presented. Chapter 3 presents the experimental set-up used, as well as a description of radiometric measurement. Samples tested are described. Chapter 4 presents experimental results and theoretical simulations, used to calculate thermal diffusivity. The results are discussed.
and deviations from theory are examined. Finally, Chapter 5 contains the summary and conclusions of the thesis.
Chapter 2

Thermal Wave Theory: Green Function Formalism of Thermal waves

2.1 Introduction

In this chapter the relevant thermal wave equations will be formulated and solved for the geometry’s being used. The mathematical formulation of thermal wave propagation will be presented using Green functions for the relevant geometry’s and boundary conditions.

2.2 Geometry and Boundary conditions

The geometry of the system used is depicted in Fig.2.1. The sample is represented as a slab of uniform thickness, $L$. The coordinate axes are aligned so that the uniform thickness lies along the $z$ axis, while the two largest surfaces are in the $x,y$ plane. The medium surrounding the sample is air. The two types of boundary conditions that will be considered in the theory are the following: (1) continuity of temperature and heat flux at
the air-sample interface, designated by C.B.C. and (2) combined convection-radiation heat transfer boundary conditions at the air-sample interface, designated by R.B.C.

Figure 2.1 Geometry of the sample with laser excitation and detection depicted

The combined convection-radiation boundary condition is used in thermal wave theory because temperature deviations from the ambient temperature are small relative to the ambient temperature and thus the radiation heat loss term may be linearized and combined with the usual convective heat loss term. The mathematical expression for the two boundary conditions are given by Eqns (2.1 – 2.6). The sample and the air are designated by the subscripts 1 and 0, respectively.
\begin{align}
T_0(z = 0) &= T_1(z = 0) \quad (2.1) \\
T_1(z = L) &= T_0(z = L) \quad (2.2) \\
k_0 \frac{\partial T_0(z = 0)}{\partial z} &= k_1 \frac{\partial T_1(z = 0)}{\partial z} + \Phi(r, z = 0) \quad (2.3) \\
k_1 \frac{\partial T_1(z = L)}{\partial z} &= k_0 \frac{\partial T_0(z = L)}{\partial z} \quad (2.4)
\end{align}

\begin{align}
- k_1 \frac{\partial T_1(r, z = 0)}{\partial z} &= -h[T_1(r, z = 0) - T_\infty] + \Phi(r, z = 0) \quad (2.5) \\
- k_1 \frac{\partial T_1(r, z = L)}{\partial z} &= h[T_1(r, z = L) - T_\infty] \quad (2.6)
\end{align}

where \( T_\infty \) is the ambient temperature far from the sample.

The above figure (Fig. 2.1) illustrates a laser beam of Gaussian radius \( w \) impinging on the plane interface of the material. The detector is focused onto the same side of the specimen as the excitation laser beam and hence the system is one that will measure the back-scatter thermal wave field. This means that all thermal wave fields will be measured on the plane \( z = 0 \). Because of the fact that metallic samples will be considered exclusively, a large fraction of the incident laser radiation is reflected from the sample surface. This fact will not alter the thermal parameter calculations, as in the final analysis only the overall line shape of the fields and not their absolute magnitudes will be needed.
The reason that only the absolute line shape is required is that the assumption of linearity will be made. Under linear conditions the absolute magnitude of the thermal field is proportional to the laser power absorbed in a linear manner. That is, if twice the power is absorbed, the value of the field is two times as large. This allows one to normalize the fields in suitable manner for comparison. In the experimental section the assumption of linearity will have to be verified.

Another important property of metallic substances is that they absorb (20) within a very short distance, whatever energy is not reflected away. The higher the frequency of the radiation the shorter the absorption (skin) depth and typically for light (f on the order of $10^{15}$ Hz) this depth is on the order of 1 μm or less. The samples considered here will range in thickness from 50 to 500 μm, thereby permitting the use of a model in which no volume sources are considered. In this case the generation of thermal waves in a sample occurs entirely from the conversion of optical energy to thermal energy at the air-sample interface, with the laser beam acting as a surface-source of thermal energy. The choice of coordinate system is dictated by the laser beam's cylindrical symmetry. It is further assumed that the lateral ($x,y$ plane or $r,\theta$ in cylindrical coordinates) dimensions are such that the thermal wave field will have decayed to zero at the boundary of edge of the sample and in practice this is an assumption which is readily achieved due to the highly dissipative nature of thermal waves. This enables one to mathematically set up the problem as infinite in the lateral domain and finite in the $z$ direction. If material isotropy is assumed then there is no angular dependence, and hence the three dimensional thermal wave field is a function of $r$ and $z$ only in cylindrical coordinates. Because the laser radiation is assumed to be absorbed at the boundary surface $z = 0$ the boundary value
problem that will have to be solved is a homogeneous differential equation, involving an inhomogeneous boundary condition. An inhomogeneous boundary condition is one in which the unknown function sought as a solution, or it's normal derivative is specified as a function of position on the boundary, S of the domain, D in which the solution is defined. In this case the heat flux (from the laser) being specified on the boundary implies that the normal derivative of temperature is specified. The most versatile method to solve such a problem is by the use of Green functions, which will be examined now.

2.3 Green function formulation of thermal wave fields

Thermal waves generated in metallic samples have boundary sources. The equation describing the thermal wave field with no volume source is the following:

\[ \nabla^2 T(r; \omega) - \sigma^2 T(r; \omega) = 0 \quad (2.7) \]

\[ \sigma^2 = (1 + i) \left( \frac{\omega}{2\alpha} \right)^{1/2} \quad (2.8) \]

A solution is sought for \( T(r; \omega) \) in a domain \( D \) subject to some boundary conditions given for \( T(r; \omega) \) or it's normal derivative on the surface \( S \), surrounding the domain. The time dependence is periodic with frequency \( \omega \) and may be neglected, as was done in the introduction. The solution the spatial part of the thermal wave field will be given in terms of the \( G(r, r') \) which is called the Green function of the problem. The Green function is
defined as the solution of the same equation that \( T(r;\omega) \) satisfies, but with a delta-function source (21):

\[
\nabla^2 G(r, r') - \sigma^2 G(r, r') = -\frac{1}{\alpha_1} \delta(r, r')
\]  
(2.9)

where thermal conductivity of the medium, \( \alpha_1 \), has been added to give temperature fields with the correct units. In addition suitable boundary conditions for \( G \) over \( S \) will be chosen later.

\( G(r, r') \) is a function of the unprimed coordinates, \( r \), having the primed coordinates (source) \( r' \), as parameters. However, since \( \delta(r, r') \) is by definition symmetric in the coordinates \( r, r' \), they may be reversed to give:

\[
\nabla'^2 G(r', r) - \sigma^2 G(r', r) = -\frac{1}{\alpha_1} \delta(r', r)
\]  
(2.10)

where \( \omega' \) is the variable and \( r \) is the source coordinate. The original equation in the primed coordinates is given as:

\[
\nabla'^2 T(r';\omega) - \sigma^2 T(r';\omega) = 0
\]  
(2.11)
Multiplying Eqn. (2.11) by \( G(r', r) \) and Eqn. (2.10) by \( T(r'; \omega) \) and then subtracting the first result from the second and integrating over the volume of the sample, \( V \) yields:

\[
\iiint (r') \nabla'^2 G(r', r) - G(r', r) \nabla'^2 T(r'; \omega) \, d\tau' = -\frac{1}{a_i} \iiint (r') \delta(r', r) \, d\tau' \tag{2.12}
\]

Applying Green's Theorem (22) to the volume integral on the left transforms it to a surface integral over the source coordinates. Applying the definition of the \( \delta \) function to the integral on the right the solution to the temperature field may be expressed as the sum of two surface integrals over the source coordinates:

\[
T(r) = -a_i \iiint_{S'} (r') \nabla' G(r', r) - \nabla' T(r') G(r', r) \cdot dS' \tag{2.13}
\]

where the gradient and the surface element point out of the sample surface.

The solution of the thermal wave field is given in terms of a surface integral of the Green function and the desired temperature field and their respective outward gradients. To complete the solution the Green function for the given geometry must be determined. A fundamental property of Green functions is that they satisfy homogeneous boundary conditions, even though the differential equation that the Green function satisfies is
inhomogeneous (23) (Eqn. 2.9). To determine the appropriate boundary condition satisfied by the Green function, the boundary condition of the temperature field is calculated. The Green function then satisfies the homogeneous version of the same boundary condition.

The thermal wave flux, \( \phi(\mathbf{r}', z', t) \), is supplied at the interface, \( z' = 0 \):

\[
\Phi(\mathbf{r}', t) = \Phi_o e^{-\left|\mathbf{r}'\right|^2/\omega^2} e^{i\omega t} \quad (2.14)
\]

This equation describes the thermal wave flux generated by a Gaussian laser beam of spot size \( w \). The surface integral element contained in Eqn. 2.13, must be replaced by an oppositely directed surface element to indicate in-flow of thermal energy. Furthermore

\[
\nabla' = -z' \frac{\partial}{\partial z'} \quad (2.15)
\]

so that from the definition of thermal flux,

\[
\Phi(\mathbf{r}', t) = \Phi(\mathbf{r}', z' = 0, t) = -k \frac{\partial T(\mathbf{r}', z')}{\partial z'} \bigg|_{z' = 0} = \Phi_o e^{-\left|\mathbf{r}'\right|^2/\omega^2} e^{i\omega t} \quad (2.16)
\]

or

\[
\nabla' T(\mathbf{r}', z') \bigg|_{z' = 0} = -z' \frac{\partial T(\mathbf{r}', z')}{\partial z'} \bigg|_{z' = 0} = z' \Phi_o e^{-\left|\mathbf{r}'\right|^2/\omega^2} e^{i\omega t} \quad (2.17)
\]
Since the temperature field satisfies a non-homogeneous (Neumann) boundary condition on the surface, the Green function satisfies the homogeneous version of this boundary condition:

\[
\frac{\partial G(r,z,r',z';\omega)}{\partial n'} = 0
\]  
(2.18)

The axially symmetric Green function then satisfies the above homogenous boundary condition. The cylindrical coordinate representation of this Green function is given by:

\[
G(r,z,r',z';\omega) = \frac{1}{4\pi \alpha_t} \int_0^\infty \frac{1}{\xi_i} J_\nu(\xi r) J_\nu(\xi r_r') \lambda d\lambda \times \\
\left[ e^{-\xi \|r-r'\|} + M^2 e^{-\xi (2L \|r-r'\|)} + M e^{-\xi (z+z')} + M e^{\xi (z-z')} \right] \\
\frac{1 - M^2 e^{-2\xi L}}{1 - M^2 e^{-2\xi L}}
\]  
(2.19)

where

\[
\xi_{0,1}(\lambda) = \sqrt{\lambda^2 + i \omega / \alpha_{0,1}}
\]  
(2.20)

\[
M_{01}(\lambda) = \frac{1 - \gamma_{01}(\lambda)}{1 + \gamma_{01}(\lambda)}
\]  
(2.21)

\[
\gamma_{01}(\lambda) = \frac{k_{0} \xi_{0}(\lambda)}{k_{1} \xi_{1}(\lambda)}
\]  
(2.22)
In the above equation there is only an integration over $\lambda$. This is due to the fact that $\lambda$ is the associated continuous eigenvalue of the radial part of the differential equation, with eigenfunction $J_0(\lambda r)$. In the $z$ direction the domain is finite, and hence over $z$ the expansion of the Green function will be over an infinite sum of eigenfunctions with appropriate eigenvalues. This infinite sum has been (21) evaluated and reduced to the expression involving four terms. By expressing the Green function for the given geometry in cylindrical coordinates the infinite sum is avoided, which is not the case for spherical coordinates. It is true that there is still an infinite integral to evaluate, but the excellent convergence qualities of the Bessel functions makes this integral readily amenable to numerical calculation.

The Green function may then be substituted into Eqn. 2.13 to obtain the solution to the thermal wave equation. Since the Green function satisfied a homogeneous boundary Neumann boundary condition the first term in the Eqn. 2.13 drops out, with the gradient of the temperature at the boundary given by Eqn. 2.17. The temperature is evaluated at $z = 0$ because the back-scatter detection mode is being employed throughout. The result is an integral over $r'$ and $\lambda$:

$$T(r, z = 0; \omega) = \frac{\Phi_o}{2k_1} \int_0^\infty e^{-(r')^2/\omega^2} \frac{1}{\xi_1} J_0(\lambda r)J_0(\lambda r') r' dr' \int \lambda d\lambda \times$$

$$\left[ e^{-\xi[z-r']} + M^2 e^{-\xi[2L-z+r']} + Me^{-\xi[z-r']} + Me^{-\xi[2L-z+r']} \right]$$

$$1 - M^2 e^{-2\xi L}$$

$$\text{(2.23)}$$
This expression may be simplified by utilizing the useful Hankel transform relation:

\[ \int_0^\infty e^{-r'/r'^2} J_0(\lambda r') r' \, dr' = 2w^2 e^{-2^w^2/4} \quad (2.24) \]

The final expression for the temperature field assuming C.B.C. then becomes:

\[ T(r, z = 0, \omega) = \frac{\Phi_0 w^2}{k_1} \int_0^\infty \frac{1}{\xi_1(\lambda)} \left( \frac{1 + M_{01}(\lambda)}{1 - M_{01}^2(\lambda) e^{-2^\omega^2 L}} \right) \times e^{-2^w^2/4} J_0(\lambda r) \lambda d\lambda \quad (2.25) \]

If the R.B.C. are assumed the above expression remains the same if \( M_{01}(\lambda) \) is replaced by \( R(\lambda) \):

\[ R(\lambda) = \frac{k_1 \xi_1(\lambda) - h}{k_1 \xi_1(\lambda) + h} \quad (2.26) \]

The numerical code for these simulations for the photothermal signal was written in Fortran (28). The program for the CBC distance scan theory is shown in the Appendix. The input information is the sample thickness, thermal conductivity, thermal diffusivity, frequency, laser beam radius, and if RBC are used the combined heat loss coefficient.
Sample thickness and laser beam spot size are measured, and it was observed that the field was not sensitive to sample conductivity. Thus the only two fitting parameters are thermal diffusivity and the h coefficient, if RBC are used and just the thermal diffusivity if CBC are used. The frequency is not a varying parameter because it is set by the experimenter.
2.4 Thermo-acoustic theory of temperature gradient induced acoustic vibrations

In considering the thermo-acoustic expansion of a solid, resulting from a temperature gradient proportional forcing term, the following assumptions are made. First the material is assumed to be mechanically isotropic in addition to the thermal isotropy that was assumed to calculate the photothermal field in the previous section. In addition the theory will assume that the thermal field is not coupled to the elastic field (25). In this approximation the temperature field that has been derived in the previous section is assumed to be unperturbed. The temperature field though assumed to be unperturbed, acts as a source for acoustic vibrations in the sample. The time dependence of the thermal field is repeated in the acoustic field, since the source of the acoustic field is proportional to the gradient of the temperature field. The derivation of the acoustic field will be performed in the one dimensional case, and then because of material isotropy a Hankel transform at the center of the laser beam spot will be used to recast the final field in a three-dimensional format (26). The one dimensional form will have the coordinate \( z \), with sample thickness designated \( L \). As in the three dimensional case subscripts 1 and 0 refer to the sample and air, respectively. In the presence of both heating and actual stress, the stress-strain relation in the polymer can be written in the following form (27):

\[
\Delta p(z,\omega) = (\lambda^* + 2\mu^*)\epsilon(z,\omega) - B\alpha, T(z,\omega) \tag{2.27}
\]
where the following definitions are made:

\[ \lambda^*: \text{Lame}' \text{ constant} \]
\[ \mu^*: \text{modulus of strain} \]
\[ \varepsilon(z; \omega): \text{modulated strain} \]
\[ \Delta p(z; \omega): \text{modulated stress} \]
\[ T(z; \omega): \text{thermal-wave intensity} \]
\[ B: \text{bulk modulus} \]
\[ \alpha: \text{coefficient of thermal expansion}. \]

Using

\[ \varepsilon(z; \omega) = \frac{\partial}{\partial z} \Delta U(z, \omega t) \quad (2.28) \]

and assuming a time harmonic expansion of the solid

\[ \Delta U(z, \omega t) = \Delta u(z) e^{i\omega t} \quad (2.29) \]

the equation of motion of an acoustic wave in the solid:

\[ \rho \frac{\partial}{\partial t^2} \Delta U(z, \omega t) = \frac{\partial}{\partial z} \Delta p(z, \omega) \quad (2.30) \]

becomes after using Eqn. 2.27

\[ \frac{\partial}{\partial z^2} \Delta u(z, \omega) + k^2 \Delta u(z, \omega) = \eta \alpha \frac{\partial}{\partial z} T(z, \omega) \quad (2.31) \]
where:
\( k = \omega/c_0 \), and \( c_0 \) is the speed of sound in the sample

\( \rho \) is the density of the sample

\( \eta = B/\rho c_0^2 \) (a dimensionless constant)

It is observed that the source of the acoustic field is proportional to the gradient of the temperature field as mentioned above. The expansion of the sample due to heating is unconstrained, leading to the following boundary conditions:

\[
\Delta p(z = 0; \omega) = \Delta p(z = L; \omega) = 0
\]

(2.32)

The one dimensional thermal wave field for a solid, assuming surface absorption with CBC is:

\[
T(z) = \frac{A[(b_{01} - 1)e^{-(\alpha_1 z)} - (b_{01} + 1)e^{\alpha_1 z}]}{(b_{01} - 1)^2 e^{-\alpha_1 z} - (b_{01} + 1)^2 e^{\alpha_1 z}}
\]

(2.33)

where \( A = \frac{\Phi_o}{k_1\sigma_1} \)

with \( b_{01} = k_0\sigma_0 / k_1\sigma_1 \)
The solution to the inhomogeneous acoustic wave equation involves any particular solution plus the solution to the homogeneous equation. The final expression for the acoustic displacement at $z = 0$ is given as:

$$\Delta u(z = 0) = \frac{ik\eta}{\sigma_1(e^{-ikL} - e^{ikL})(k^2 + \sigma_1^2)} \left[ S(e^{-ikL} - e^{ikL}) + R(e^{-\sigma_1L} - e^{ikL}) + \right]$$

$$+ \frac{\eta S}{k^2 + \sigma_1^2} + \frac{\eta S}{k^2 + \sigma_1^2}$$

(2.34)

where

$$S = \frac{\alpha_1 A \sigma_1 (b_{10} - 1)e^{-\sigma_1L}}{(b_{01} - 1)^2 e^{-\sigma_1L} - (b_{10} + 1)^2 e^{\sigma_1L}}$$

$$R = \frac{\alpha_1 A \sigma_1 (b_{10} + 1)e^{\sigma_1L}}{(b_{01} - 1)^2 e^{-\sigma_1L} - (b_{10} + 1)^2 e^{\sigma_1L}}$$

To cast this solution in 3 dimensional form the following substitutions are made:

$$\sigma_{0,1}^2 \rightarrow \xi_{0,1}^2 = \lambda^2 + i \omega / \alpha$$

(2.35)

$$M_{0,1} = \frac{-(b_{01} - 1)}{(b_{01} + 1)}$$

(2.36)

$$1 + M_{0,1} = \frac{2}{(b_{01} + 1)}$$

(2.37)

With the above substitutions an infinite Hankel transform (26), centered on the laser beam spot center is taken to give the final solution:
\[ \Delta u(r, z = 0) = \frac{\alpha_\xi \Phi \sigma T^2}{2k_1} \int_0^\infty \frac{1}{(\xi^2 + k^2)^{[1 + M]}} \left[ 1 - M e^{-2\xi L} \right] \times \left[ 1 - Me^{-2\xi L} \right] \frac{k}{\xi_1 \sin(kL)} (\cos(kL)[1 + Me^{-2\xi L}] - [1 + M]) \times e^{-\xi^2/4} J_0(\lambda r) \lambda d \lambda \]

(2.38)

The coefficient of thermal expansion, \( \alpha_\xi \) is outside of the integral, and hence is not a fitting parameter. Since the experiment is run under linear conditions the field may be normalized, just as the thermal fields are normalized. It is seen from Eqn. 2.38 that the only additional fitting parameter relative to the thermal only theory is the acoustic wave number, \( k \). Numerically, the integral was seen to be insensitive to \( k \), and hence the only fitting parameter is the thermal diffusivity. The total radiometric signal detected by the infrared detector (Chapter 3) is a combination of the thermal-wave temperature field (Eqn. 2.25) and the acoustic field (Eqn. 2.38). The signal captured by the detector is:

\[ S(r, z = 0) = s_1 T(r, z = 0) + s_2 \Delta u(r, z = 0) \]

(2.39)

where \( s_1 \) and \( s_2 \) determine the contribution from each component of the signal. These two parameters may be reduced to a single parameter \( s \), plus an arbitrary
normalization constant. The parameter $s$ determines only the amount of acoustic component that must be added to the thermal component to correctly predict the experimental data. In general $s$ may be a complex number to allow for a phase lag of the acoustic field relative to the thermal field.
2.5 Broadening effects due to detector convolution

To account for the possible broadening of the signal due to the convolution of the detector with the temperature field an integration is performed over an area “effective detector size”, which may be larger or smaller than the actual size of the detector:

\[ T(r, z = 0) = \int_{-a/2}^{a/2} \int_{-a/2}^{a/2} \text{d}x \times T(x, y, z = 0) \]  

(2.40)

\[ T(r, z = 0) = \sum_{i=1}^{n} \sum_{j=1}^{n} T \left( r - a/2 + \left( \frac{a}{n-1} \right) (j-1), -a/2 + \left( \frac{a}{n-1} \right) (i-1) \right) \]  

(2.41)

The second equation was obtained from the first by performing a sum over equally spaced points over a square, the size of the detector. It is known that if the detection optics are not adequately focused onto the sample that broadening occurs. The above model will be used to calculate and effective detector size, if such a broadening is observed.
Chapter 3

Experiment

3.1 Introduction

In this chapter the experimental technique utilized to acquire the data from which the relevant thermal wave fields may be inferred is presented. All of the experimental techniques utilize the same basic set-up, with only simple adjustments to the system to change from one set-up to another. The experimental method employed utilizes radiometric detection, making it a completely non-contact method.

3.2 Experimental Method

3.2.1. Photothermal analysis via radiometric detection

As the title indicates this technique is (29,30) one which infers the photothermal field from data concerning the induced electromagnetic emissions, which are monitored using an infrared detector. It is a property of all bodies above 0 K that they emit electromagnetic radiation due to their temperature. This type of electromagnetic radiation is called thermal radiation. The amount of emitted radiation is governed by the Stefan-Boltzmann law, which relates the total flux of emitted radiation density, $W$, to the temperature, $T$, of the emitter surface. This relationship is given by
\[ W(T) = \varepsilon \sigma T^4, \] (3.1)

Where \( \varepsilon \) is the emissivity of the surface and \( \sigma = 5.67 \times 10^{-8} \text{Wm}^{-2} \text{K}^{-4} \) is the Stefan-Boltzmann constant. The above expression is theoretically exact and the fourth order temperature dependence is obtained by integrating over the total spectral range of the emitted radiation. In practice the detector has a finite bandwidth and thus the total radiant energy captured in the process of detection will not be as described above. At or around room temperature the maximum thermal radiant energy is emitted in the infrared range, and hence to measure thermal properties of material electromagnetically an infrared detector is required. If (hypothetically, of course) these experiments were performed on the sun, then a significant amount of the thermal radiation would be in a higher frequency region and its detection would require an instrument with the ability to measure part optical, part infrared radiation.

If there is no change in temperature then the quantity of emitted radiation remains constant. On the other hand if the temperature is not constant but varies in time, a Taylor expansion may be made of the total emitted radiant energy:

\[ W(T) = \varepsilon \sigma T^4 + 4 \varepsilon \sigma T^3 \delta T \] (3.2)

In the above expression the first term represents a constant D.C. radianc, while the second term is a time dependent radiance. In photothermal experiments the temperature varies periodically at the same frequency as the incident modulated laser beam \( \omega \), so there is an A.C. component to the emitted radiance that is written as:
\[ \delta W(\omega) = 4 e \sigma T_0^3 \delta T(\omega) \] (3.3)

This frequency dependent radiance is the measured quantity. It is measured by using a lock-in amplifier, which has the ability to suppress all signals outside of a narrow bandwidth centered on the modulation frequency, \( \omega \).

3.2.2 Experimental System

The experimental setup for the thin-film photothermal radiometry experiments that are performed is shown in Figure 4.1 (31, 32). The thermal wave source is a 514.5 nm wavelength cw Innova Ar+ laser which is modulated and then focused onto the surface of a sample, to a spot size of approximately 30 \( \mu \)m, at output powers that vary from 0.2 to 1 W. The modulation is performed by an external acousto-optic (A/O) modulator (ISOMET 1201E-1) which is controlled by the internal oscillator of the lock-in amplifier. The thermal radiation emitted from the sample is focused onto a liquid nitrogen cooled HgCdTe (Mercury-Cadmium-Telluride) detector (33) (EG & G Judson Model J15D12-M204-S050U). The optical excited spot of the sample is placed at the focal point of the first reflecting objective, and the detector at the focal point of the second reflecting objective. The thermal wave excitation and detection both occur on the same side of the sample and hence the technique is back-scattering. The sample is not moved and hence once it is brought into focus it will remain that way. The photothermal field can be
Figure 3.1 Schematic diagram of the experimental set-up
scanned, due to the motor that controls the micrometer stage containing both the mirror and lens that are outlined in the figure. By moving this stage the sample is optically excited at different lateral points, while emitted radiation is focused from a point on the sample which does not change. The mirror-lens combination is mounted on a stage that is controlled by a computer, allowing for automated distance scans.

The HgCdTe detector is a photoconductive element that undergoes a change in resistance proportional to the incident infrared radiation. The active size of the detector is a square of dimensions 50 μm by 50 μm, and its detection bandwidth is from 2 – 12 μm. The efficiency of the detector increases with decreasing temperature, so the detector is operated at a cryogenic temperature of 77 K. An argon coated germanium window with a transmission bandwidth of 2-14 μm is mounted in front of the detector to block any visible radiation from the laser. The signal, which is proportional to the modulated infrared radiation focused onto the detector, was amplified by a preamplifier (EG&G) Judson Model PA-300), before being sent to the digital lock-in amplifier (Stanford Research System Model SR 850) (34). The low-noise preamplifier ensures proper performance for subsequent signal processing with the lock-in amplifier. A “Coarse Adjustment” resistor was installed to roughly compensate for the DC offset level produced by the detector in the absence of the optical signal. Since this DC offset changes with time and background conditions, a “DC OFFSET” dial on the preamplifier case allows periodic manual tuning (35).

The lock-in amplifier, which is interfaced with a personal computer, receives the preamplifier output and rejects all stray signal except for those with the same modulation
frequency of the excitation beam. This process of data acquisition, storage, and scanning is fully automated.

3.2.3 Sample description

The thermal diffusivity of different materials was measured in this work. The samples considered were Zr alloy stainless steel AISI-302, and mild steel. The Zr alloy sample was 3.8 mm thick and approximately 25 by 25 mm wide. For the frequency values used in this work the sample was semi-infinite and used as a reference sample to calibrate the system. It had a literature value for thermal diffusivity of $9.2 \times 10^{-6} \text{m}^2/\text{s}$.

The mild steel sample was approximately 1 mm thick and 15 mm by 15 mm wide. It had a bulk thermal diffusivity value of $15.5 \times 10^{-6} \text{m}^2/\text{s}$.

The stainless steel AISI-300 samples were also approximately 25 by 25 mm wide, but came in a series of thicknesses. The thicknesses were: 500, 210, 110, and 55 um. These samples were the primary ones being investigated. By having a set of samples of the same composition but of different thickness, the applicability of photothermal radiometry could be examined. The bulk thermal diffusivity value for this steel is $3.5 \times 10^{-6} \text{m}^2/\text{s}$.
3.3 Experimental procedure

As was previously mentioned the same experimental set-up was used for all photothermal radiometric measurements. A sample was secured in the sample holder in the open laboratory environment. The experiments were all performed at room temperature. The laser was let to run for at least an hour before experiments where started, to stabilize the laser cavity. When the signal had stabilized, it was necessary to adjust the detector position so that the maximum signal could be obtained.

3.3.1 Frequency Scan Methods

The frequency scan method is a method of obtaining a photothermal signal profile by scanning a sample at a fixed position while the modulation frequency changes. The thermal wave will have its largest magnitude at the lowest frequency that is scanned and hence the signal is usually normalized to this value. This normalization is valid since only the shape of the signal is required, if the experiment is performed in the linear region of laser excitation power. If the lowest frequency is too low (below 1 Hz) the detector exhibits 1/f noise which causes extreme degradation to the signal to noise ratio. For this reason the lowest frequency chosen was 10 Hz, as at this level 1/f noise is small compared to the signal and hence the signal to noise ratio was high (>100). In addition if the frequency is too large the naturally decaying photothermal signal will begin to approach the environment noise level, which also causes a degradation of signal to noise. This was the reason that the highest frequency scanned was 1000 Hz, except for a couple
of cases. In this investigation two different types of frequency scans are performed. The first is the conventional frequency scan, in which the laser and the detector are coincident. The second type of frequency scan is one which is much more infrequently used, the laser-detector (pump-probe separated) frequency scan method. The reason for its infrequent use is that the dissipative nature of thermal waves causes scans with pump-probe separation to yield small signals, especially at higher frequencies, and hence lower signal to noise ratios.

In order to obtain useful information from frequency scans in a photothermal experiment a reference sample must chosen and scanned. This is done in order to calculate the instrumental transfer function that is then utilized to normalize all subsequent frequency scans. The reference sample was chosen to be a semi-infinite Zr alloy sample with known thermal diffusivity. Using the C.B.C. theory with the known thermal properties of the sample, theoretical scans were computed. The theoretically calculated thermal wave field is sensitive to the sample thickness, thermal diffusivity and laser beam spot size. The thickness and beam size are known measured quantities, so that the thermal diffusivity is the only fitting parameter. The amplitude of the experimental signal is divided by the theoretically calculated amplitude for Zr alloy (Eqn. 3.4).

\[
\frac{amplitude_{\text{Theoretical} \ Zr \ alloy}}{amplitude_{\text{Experimental} \ Zr \ alloy}} = (3.4)
\]
This reference normalization factor is multiplied to all subsequent data before theoretical fits are attempted. For the phase the transfer function is the theoretically calculated phase minus the experimentally calculated phase for Zr alloy (Eqn. 3.5).

\[
\text{phase}_{\text{Zr alloy}}^{\text{Theory}} - \text{phase}_{\text{Zr alloy}}^{\text{Experimental}}
\]  

(3.5)

This quantity is added to all subsequent experimental phase data before theoretical fits are attempted.

3.3.2 Distance Scan Method

The distance scan method entails scanning a laser across a sample, at a fixed frequency. Unlike the frequency method the pump-probe distance changes during the scan, while the in frequency scans it remains fixed.

In the distance scan method measurements are again made with respect to a thermally thick reference sample, but in this case it is not a transfer function that is sought, as all scans are made at a single frequency. Instead it is the effect of the broadening of the signal due to convolution with the detector, which was discussed in Chapter 2 (2.5). Distance scans are again made with the semi-infinite Zr alloy sample, and these are compared with the theoretically calculated scans. If signal broadening is found the detector size that is calculated is then applied to all subsequent theoretical simulations before fits with data are attempted.
There is one other impediment to the interpretation of experimental data that does not occur in frequency scans that will now be discussed. In the expression
\[ \delta W(\omega) = 4\pi \sigma T_0^4 \delta T(\omega) \]
the term \( T_0 \) is not the ambient temperature. In the analysis in chapter two of the effects of a periodically supplied source of thermal energy its effect on the steady state temperature distribution was neglected. A complete analysis reveals that \( T_0 \) is the ambient temperature plus a distance dependent steady-state rise in temperature. If this steady state rise varied significantly with distance then it is obvious that the experimentally obtained distance scans would not behave as expected as the radiant energy emitted at different radial positions would vary not only due to the modulated temperature change, \( \delta T(\omega) \), but also due to the distance dependent term \( T_0 \). In most photothermal experiments this is neglected because the steady state rise is not large (on the order of 1 to 10 K), but it should still be remembered.

3.3.3 System Linearity

A crucial requirement for the photothermal measurements performed in this study is that of signal linearity. When signals that are linear as a function of laser power are observed, they may be normalized and compared with theoretical simulations, to obtain thermal diffusivity. If the signal is not linear (and the signal phase not stable) as a function of power, a non-linear (higher order harmonic) theory would be needed to explain the results. To ensure that the experiments were run under linear conditions, signal magnitude vs. laser power was checked for each sample. Laser powers that appeared in the domain of linearity could then be used to perform subsequent
photothermal experiments. An example of a check for linearity is depicted in Fig. 3.2. In addition the phase was observed to remain constant as a function of laser power, which is also necessary for linearity to be valid.

![Signal vs. Laser Power for the Mild Steel Sample at f = 10 Hz, showing linearity up to 0.6 W](image)

**Figure 3.2 Verification of system linearity**

The standard procedure for normalization of distance scan data was to multiply the signal at its maximum value (r=0 mm) by a constant, so that the new maximum value is one. For frequency scans, the data was normalized to its maximum value (which occurs at the lowest frequency), but if the data was too noisy at this frequency, the data could be
normalized starting at any other suitable frequency. The phase channel data could be shifted up or down by adding a constant to all phase data. This shift does not change the shape of the phase curve (for either distance or frequency scans), but only moves it vertically up or down.
Chapter 4

Experimental results, thermal diffusivity calculations and discussion

The purpose of this chapter is to compare the three different photothermal techniques in the measurement of the thermal diffusivity of thin steel samples. First, the experimental data and theoretical fits are presented which required a thermal-only theory. In the second section the combined thermo-acoustic theory is examined.

4.1 Measurements of thermal diffusivity using a thermal theory

4.1.1 Semi-infinite reference samples

The semi-infinite reference sample used was Zr alloy as mentioned in chapter 3. The literature value for the thermal diffusivity of Zr alloy was $9.2 \times 10^6$ m$^2$/s. The main reason that a semi-infinite sample is preferred as a reference sample is that it is expected to obey a purely thermal model.

4.1.1.a Distance scan normalization (Test for broadening)

The first measurements to be made were radial distance scans. As mentioned in chapter 3 the amplitude and phase best fits will subsequently be used to normalize the frequency scan data. Two distance scans were performed for Zr alloy - one at 50 Hz and
Figure 4.1  Distance Scan of Zr Alloy at 50 Hz; $\alpha = 9.2 \times 10^{-6} m^2/s$; CBC, no detector
(a) Normalized amplitude
(b) Phase
Figure 4.2  Distance Scan of Zr Alloy at 100 Hz; $\alpha = 9.3 \times 10^{-6} \text{m}^2/\text{s}$; CBC, no detector
(a) Normalized amplitude
(b) Phase
another at 100 Hz. At these frequencies the thermal diffusion lengths were 243 μm and 172 μm, respectively, if the literature thermal diffusivity value was assumed. The theoretical best fits for amplitude and phase for the scans is shown in Fig. 4.1 and 4.2. To obtain theoretical fits for this data the following parameters were considered. Firstly the diffusivity could be varied. Secondly the boundary conditions could be varied to examine whether or not the R.B.C. were applicable. Thirdly, the possibility of experimental broadening due to detector convolution was considered. It was observed that the best fits were obtained using C.B.C. without detector broadening. From this it was concluded that all subsequent experimental data were to be fit without using a broadening due to detector convolution. The values of the thermal diffusivity where found to be $9.2 \times 10^{-6}$ m$^2$/s for the 50 Hz scan and $9.3 \times 10^{-6}$ m$^2$/s for the 100 Hz scan. In addition the effect of varying the boundary conditions was examined. It was also observed that for values of $h$ below 10,000 W/m$^2$K at 50 Hz the R.B.C. and C.B.C. boundary conditions gave results that were indistinguishable for the semi-infinite Zr alloy sample. At 100 Hz the $h$ coefficient below which the two theories converged was 20000 W/m$^2$K.

4.1.1.b Frequency scan normalization (Instrumental Transfer Function)

The next step was the calculation of the instrumental transfer function. Using the average value obtained for thermal diffusivity via the distance scan, $9.25 \times 10^{-6}$ m$^2$/s, theoretical frequency scans where calculated and plotted using this value along with the experimentally determined frequency scan of Zr alloy. From this data the transfer function for amplitude and phase was determined. This transfer function was subsequently applied to all other frequency scans using Eqn.'s 3.4 and 3.5 from chapter 3.
Figure 4.3 Frequency Scan of Zr Alloy, 10 – 1000 Hz, to determine instrumental Transfer Function; $\alpha = 9.25 \times 10^{-8} \text{m}^2/\text{s}$; CBC
(a) Normalized amplitude
(b) Phase
Figure 4.3 shows the amplitude and phase of both experimental and theoretical plots. Since it was determined that there was no broadening, and as CBC were used in the distance scans, these stipulations were repeated in the frequency scans. It is seen that the experimental phase is slightly below the theory, and the difference obtained by subtracting the experimental data from the theoretical data is added to all subsequent experimental frequency scans, before fits are attempted. This difference is positive, indicating that experimental frequencies are slightly raised before fits are attempted. The experimental amplitude is slightly above the theoretical one and in this case the ratio of the theoretical divided by the experimental amplitude is multiplied by all subsequent experimental amplitudes before fits are attempted. This ratio is slightly less than 1, indicating that experimental amplitudes are slightly decreased before fits are attempted.

4.1.2 Distance Scan Thermal diffusivity measurements

4.1.2.a Mild Steel

The first sample that was examined was a Mild Steel sample of thickness 1 mm. The literature value of bulk thermal diffusivity for this sample is $1.55 \times 10^{-5}$ m$^2$/s. Distance scans for this sample where performed at frequencies of 10, 100, and 500 Hz, which corresponded to thermal diffusion lengths of 702, 222, and 99 µm, respectively. For the frequency values used the sample was thermally thick. The experimental data was fit by varying the thermal diffusivity and the boundary conditions. At each frequency the thermal diffusivity value that produced the best fit for both amplitude and phase
simultaneously was found so that the results would be consistent. The results are shown in Fig. 4.4–4.6. At 10 Hz it is seen that the phase was very noisy at the edge of the scan, allowing a fit only in the middle portion. The optimal thermal diffusivity value for the phase was found to be $1.45 \times 10^{-5}$ m$^2$/s. To obtain this value the C.B.C. theory was used. It was observed that for values of $h$ below 10000 W/m$^2$K the C.B.C. and R.B.C. theories gave identical results, but for $h$ values higher than this the amplitude and phase profiles began to diverge. This conclusion relies on the fact that both amplitude and phase should be fit. It is important to obtain theoretical fits for both amplitude and phase, for the following reason. Increasing the value of the heat transfer coefficient has opposing effects on the amplitude and phase profiles. For the amplitude profile the effect of increasing $h$ is to make the distance scan linewidth narrower, while the phase profile becomes broader. The effect is monotonic, and thus decreasing the $h$ coefficient (from a value above zero) results in opposite trend occurring in both amplitude and phase. The effect of changing the thermal diffusivity is not the opposite of this. Increasing the thermal diffusivity broadens the phase profile, but it also broadens the amplitude profile, since higher thermal diffusivity implies deeper depth of penetration. This implies that if a fit is made assuming C.B.C. with a given thermal diffusivity, an alternate thermal diffusivity fit can be achieved for one (and at most one) channel, by adjusting the $h$ term. For example suppose that a fit was made with a given thermal diffusivity for both amplitude and phase, using C.B.C. If the value of the thermal diffusivity is decreased the theoretically calculated amplitude and phase profiles are both narrowed. Increasing $h$ from 0 up will broaden the phase, and for a certain value of $h$ a fit of the phase channel is
Figure 4.4  Distance Scan of Mild Steel at 10 Hz; $\alpha = 1.45 \times 10^{-5} \text{m}^2/\text{s}$; CBC
  (a) Normalized Amplitude
  (b) Phase
Figure 4.5  Distance Scan of Mild Steel at 100 Hz; $\alpha = 15.0 \times 10^{-6} \text{m}^2/\text{s}$; CBC

(a) Normalized Amplitude

(b) Phase
Figure 4.6  Distance Scan of Mild Steel at 500 Hz; $\alpha = 1.55 \times 10^{-6} \text{m}^2/\text{s}$; CBC
(a) Normalized Amplitude
(b) Phase
possible. On the other hand, the already narrowed amplitude scan will be further narrowed and it will not be possible to fit amplitude channel.

An example of this is depicted on Fig. 4.7 for the thermal diffusivity fit of mild steel at 100 Hz. Theoretically the distance scan was fit for both amplitude and phase with a thermal diffusivity of $1.5 \times 10^{-5}$ m$^2$/s with CBC and this is plotted on the figure. Also plotted is a simulation of the CBC theory with diffusivity of $1.2 \times 10^{-5}$ m$^2$/s, which is narrower in both amplitude and phase. A fit of the phase channel for $\alpha = 1.2 \times 10^{-5}$ m$^2$/s is possible if RBC are incorporated. The figure shows two simulations of this case, with $h = 50000$ W/m$^2$K and $h = 100000$ W/m$^2$K. For the higher value of the heat transfer coefficient the phase profiles overlap, but the amplitude profile diverged even further away.

If on the other hand the thermal diffusivity is increased, it will be possible to fit the amplitude by increasing $h$, so as to narrow it, but the phase profile will be further broadened, thus making it impossible to fit the phase (Fig. 4.8). These arguments allow one to obtain both the correct thermal diffusivity and the appropriate boundary conditions (in this case CBC are valid) if the fitting of experimental data is done on both channels, and eliminate the ambiguity inherent using single channel fits (35).

The distance scan at 100 Hz is shown on Fig. 4.5. At this frequency the fits were made using CBC with a best fit value of $1.5 \times 10^{-5}$ m$^2$/s. The amplitude and phase proved to be less noisy at this frequency as is seen from the graphs. There was a slight asymmetry in the data, with the experimental phase being slightly broadened on the right side of the scan. At this frequency the CBC and RBC theories were indistinguishable for values of $h$ below 30000 W/m$^2$K. Since there is no reason to assume a frequency dependent heat loss
Figure 4.7 Distance Scan Simulation of Mild Steel Sample showing effects of changing thermal diffusivity and heat transfer coefficient, h. Decreasing the thermal diffusivity, and increasing h, resulting in the overlap of the RBC and CBC theories on the phase channel.
(a) Normalized Amplitude
(b) Phase
Figure 4.8  Distance Scan Simulation of Mild Steel Sample showing effects of changing thermal diffusivity and heat transfer coefficient, h. Increasing the thermal diffusivity, and increasing h, resulting in the overlap of the RBC and CBC theories on the amplitude channel
(a) Normalized Amplitude
(b) Phase
Figure 4.9  Distance Scan of 500 μm Steel AISI 302 at 10 Hz; $\alpha = 3.7 \times 10^{-6} \text{m}^2/\text{s}$; CBC
(a) Normalized Amplitude
(b) Phase
Figure 4.10  Distance Scan of 500 µm Steel AISI 302 at 100 Hz; $\alpha = 3.5 \times 10^{-6} \text{m}^2/\text{s}$; CBC
(a) Normalized Amplitude
(b) Phase
Figure 4.11  Distance Scan of 500 μm Steel AISI 302 at 500 Hz; $\alpha = 3.7 \times 10^{-6} \text{m}^2/\text{s}$; CBC
(a) Normalized Amplitude
(b) Phase
factor, it is concluded that to determine the boundary conditions accurately, low frequency distance scans must be performed, even if noise is a limiting factor. Distance scans at 500 Hz resulted in an optimal thermal diffusivity value of $1.55 \times 10^{-6}$ m$^2$/s, using the CBC theory. At this frequency the CBC and RBC theories gave indistinguishable simulations for $h$ values below 75000 W/m$^2$K. The average value of thermal diffusivity found from the three different distance scans was $1.5 \times 10^{-5}$ m$^2$/s, which is within 4% of literature value for this steel’s bulk diffusivity.

4.1.2.b Steel Samples #AISI 302

The next set of samples were stainless steel shim (#AISI 302), all made from the same manufacturer with thickness’ of 110 µm, 210 µm and 500 µm. The bulk thermal diffusivity value of this steel was reported as $3.5 \times 10^{-6}$ m$^2$/s. For these samples distance scans were performed at 10, 100, and 500 Hz as before. At these frequencies the thermal diffusion lengths are 333, 105, and 47 µm, respectively.

The 500 µm thick sample is thermally thick for all of the frequencies scanned. The scans along with the best fits are shown in Fig. 4.9-4.11. The curves were fit with CBC theory. The optimal value for thermal diffusivity at 10 Hz was found to $3.7 \times 10^{-6}$ m$^2$/s with amplitude and phase both conforming to theory. Both the amplitude and phase were observed to have a slight asymmetry, being broadened on the right side of the scan. This was a system asymmetry due to a slight misalignment of the small reflecting mirror.
For 100 and 500 Hz the value of thermal diffusivity was found to be $3.5 \times 10^{-6}$ m$^2$/s and $3.7 \times 10^{-6}$ m$^2$/s, respectively, with the system asymmetry being observed again.

As mentioned above CBC were employed to obtain the fits, but there was a region of overlap if the RBC theory was used. At 10 Hz the two boundary conditions converged for values of $h$ below 3000 W/m$^2$K, whereas for the 1 mm thick sample it occurred for the $h$ value below 10000. At 100 and 500 Hz, these convergence's occurred for values of $h$ below 10000 W/m$^2$K and 40000 W/m$^2$K, respectively, again below the corresponding values for the thicker 1 mm sample. The average value of thermal diffusivity calculated from the three distance scans is $3.63 \times 10^{-6}$ m$^2$/s, which is within 4 % of the bulk diffusivity.

The next sample investigated was the 210 μm sample, which was thermally thin at 10 Hz, but thermally thick at 100 and 500 Hz. The data along with the theoretical fits are shown in Fig. 4.12-4.14. At 10 Hz, the thermally thin frequency, the best fit was calculated to be $3.5 \times 10^{-6}$ m$^2$/s. For this fit it is seen that the amplitude is slightly narrowed, while the phase fits excellently on the left side of the scan. The right side of the scan is broadened in both amplitude and phase due to asymmetry again. The probable cause of the amplitude being slightly narrowed is the steady state rise in temperature. Since there is a different steady state rise on concentric circles around the laser beam excitation point, such a factor would lead to slightly varying amounts of infrared power being emitted from the sample. In particular the photothermal signal amplitude would be narrowed, since the steady state rise decreases as a function of distance from the laser. The phase on the other hand is a measure of the lag of the field at spatially separated positions, and as such variances in the flux of emitted radiation would not affect it. The
Steel AISI 302
210 μm thick
Distance Scan, 10 Hz
$\alpha = 3.5 \times 10^{-6} \text{m}^2/\text{s}$

(a) Normalized Amplitude

(b) Phase (degrees)

Figure 4.12 Distance Scan of 210 μm Steel AISI 302 at 10 Hz; $\alpha = 3.5 \times 10^{-6} \text{m}^2/\text{s}$; CBC
(a) Normalized Amplitude
(b) Phase
Steel AISI 302
210 μm thick
Distance Scan, 100 Hz
$\alpha = 3.5 \times 10^{-6} \text{m}^2/\text{s}$

(a) Normalized Amplitude

(b) Phase (degrees)

Figure 4.13  Distance Scan of 210 μm Steel AISI 302 at 100 Hz; $\alpha = 3.5 \times 10^{-6} \text{m}^2/\text{s}$, CBC
(a) Normalized Amplitude
(b) Phase
Figure 4.14  Distance Scan of 210 μm Steel AISI 302 at 500 Hz; $\alpha = 4.0 \times 10^{-6} \text{m}^2/\text{s}$; CBC
(a) Normalized Amplitude
(b) Phase
Figure 4.15  Distance Scan of 110 μm Steel AISI 302 at 10 Hz; $\alpha = 3.2 \times 10^{-6}$ m$^2$/s; CBC

(a) Normalized Amplitude

(b) Phase
Distance Scan [mm]

Figure 4.16 Distance Scan of 110 µm Steel AISI 302 at 100 Hz; $\alpha = 4.0 \times 10^{-6} \text{m}^2/\text{s}$; CBC
(a) Normalized Amplitude
(b) Phase

Steel AISI 302
110 µm thick
Distance Scan, 100 Hz
$\alpha = 4.0 \times 10^{-6} \text{m}^2/\text{s}$
Figure 4.17  Distance Scan of 110 μm Steel AISI 302 at 500 Hz; $\alpha = 4.0 \times 10^{-6} \text{m}^2/\text{s}$; CBC
(a) Normalized Amplitude
(b) Phase
Distance scans at 100 and 500 Hz had optimal thermal diffusivity fits of 3.5 and $4.0 \times 10^{-6} \text{ m}^2/\text{s}$, which gave an average value for the diffusivity of $3.66 \times 10^{-6} \text{ m}^2/\text{s}$, which is within 5% of the bulk value.

Continuity boundary conditions were used to fit the data, with RBC converging for small values of $h$. Simulations revealed that for the 210 $\mu$m steel sample boundary conditions converged for the 10, 100, and 500 Hz at the $h$ values 2000, 6000, and 30000 W/m$^2$K. These values were below the ones calculated for the 500 $\mu$m sample.

The 110 $\mu$m sample was thermally thin at 10 Hz, and just at the boundary of thermal thinness at 100 Hz. The scans are shown in Fig. 4.15-4.17. For this sample the distance scan at 10 Hz revealed an optimal value of $3.2 \times 10^{-6} \text{ m}^2/\text{s}$. At both 100 and 500 Hz the optimal value of thermal diffusivity was found to be $4.0 \times 10^{-6} \text{ m}^2/\text{s}$. The effects of the steady state temperature rise and system asymmetry where also observed for this sample.

Continuity boundary conditions were again used to fit the data with the trend of overlap for CBC and RBC conditions continuing. For the 110 $\mu$m sample the $h$ values were found to be 1000, 4000, and 20000 W/m$^2$K. From this it is concluded that the radiation heat loss conditions would be most easily observable in thinner samples where the thermal gradients are larger due to energy confinement in a smaller spatial region. This is consistent with previous findings (36). The average value of the thermal diffusivity from the distance scans is $3.73 \times 10^{-6} \text{ m}^2/\text{s}$, which is within 7% of the bulk value.
4.1.3 Frequency scan (standard)

The standard frequency refers to the one which is done with the pump (laser) and probe (detector) at the same position. These frequency scans were performed starting at a frequency of 10 Hz and ending at 1000 Hz, which corresponded to thermal diffusion lengths of 702 and 70.2 um for the mild steel sample, and 333 and 33 um for the stainless steel AISI 302 samples. The lowest frequency chosen for the distance scan was 10 Hz since detector 1/f noise for values below 10 Hz would yield signals that were not faithful to the true photothermal field.

4.1.3.a Mild Steel

The mild steel sample was observed to be noisy in the frequency range of 10 to 100 Hz. The higher thermal diffusivity of this sample meant that the absolute signal levels were lower, leading to a smaller signal to noise ratio. The amplitude and phase were fit using CBC as was done with the distance scans (Fig. 4.18). There was a difference with respect to the convergence of the continuity and radiation theories in this case. Since the frequency scan by definition scans over a range of frequencies it can be guessed that the CBC and RBC theories would only converge for values of h below which the two theories would converge in a distance scan for the lowest value of frequency. For this sample the two theories converged for values of h less than 10000 W/m²K for the distance scan at 10 Hz. Due the noise level of the signal in the range from 10 to 100 Hz, the experimental signal was only fit for the range 100 to 1000 Hz. For this frequency range the two theories converged for values of h below 20000 W/m²K. From this it is
Figure 4.18  Frequency Scan of Mild Steel, 10 – 1000 Hz, $\alpha = 1.5 \times 10^{-5} \text{m}^2/\text{s}$; CBC
(a) Normalized amplitude
(b) Phase
concluded that the frequency scan is a less sensitive technique to detect the presence of RBC, if noise levels prevent fits in the lower-end of the frequency range. By fitting the frequency scan in the range of 100 to 1000 Hz, the thermal diffusivity was calculated to be $1.5 \times 10^{-5}$ m$^2$/s, with both amplitude and phase having excellent agreement with the theory.

4.1.3.b Steel AISI-302

The stainless steel samples gave signals that were smoother than the 1 mm sample, mainly due to the lower diffusivity which lead to higher absolute signal values and hence a higher signal to noise ratio. The experimental data and theoretical fits, using CBC's are shown in Fig. 4.19-4.21. For the 500, 210 and 110 μm sample the best fit thermal diffusivity values were 4.0, 3.5, and $3.8 \times 10^{-6}$ m$^2$/s, respectively. The percentage difference between the calculated values and the bulk diffusivity ($3.5 \times 10^{-6}$ m$^2$/s) for the three samples was 14, 0, and 8.5 %. Even though there was a greater difference on average between the bulk values and the frequency scan fit values for diffusivity, the frequency scan had fits with smaller deviations between amplitude and phase. This may be explained by recognizing that the frequency scan was being measured at a single position. Thus the infrared emission didn't have any distance varying effects which were the cause of the variations between amplitude and phase for the distance scans. Since the signal was smooth for the entire frequency range over which the scan was taken, the whole scan was fit, unlike for the 1 mm sample. This meant that RBC and CBC theories converged for the same factor that lead to convergence of the distance scans at 10 Hz.
Figure 4.19  Frequency Scan of 500 μm Steel AISI 302, 10 – 1000 Hz, \( \alpha = 4.0 \times 10^{-9} \text{m}^2/\text{s} \), CBC
(a) Normalized amplitude
(b) Phase
Figure 4.20  Frequency Scan of 210 μm Steel AISI 302, 10 – 1000 Hz, $\alpha = 3.5 \times 10^{-6} \text{m}^2/\text{s}$; CBC

(a) Normalized amplitude
(b) Phase
Figure 4.21  Frequency Scan of 110 μm Steel AISI 302, 10 – 1000 Hz, $\alpha = 3.8 \times 10^{-6} \text{m}^2/\text{s}$, CBC
(a) Normalized amplitude
(b) Phase
4.1.4 Frequency scan with pump probe separation

4.1.4.a Mild Steel

The frequency scans with pump probe separation were performed to determine whether or not such scans reveal any additional sensitivity in the determination of thermal diffusivity. The scans were made with a pump probe separation of 0.2 mm. The mild steel sample was fit with the continuity theory with a best fit thermal diffusivity of $1.45 \times 10^{-5} \text{m}^2/\text{s}$ (Fig. 4.22). The signal is observed to contain a higher noise level throughout the entire scan, due to the pump probe separation, which leads to a smaller signal.

4.1.4.b Steel AISI-302

The scans for the stainless steel samples are shown in Fig. 4.23-4.25. The thermal diffusivity calculated for the 500, 210, and 110 μm samples were found to be 3.5, 4.0, and $3.5 \times 10^{-6} \text{m}^2/\text{s}$, although there was higher uncertainty due to the narrower frequency region over which the fits were made. The higher photothermal signal of these samples meant that the scans were less noisy, although when the frequency reached approximately 200 to 300 Hz the signal to noise ratio was extremely degraded, which made fitting throughout the whole frequency range impossible.

There was one additional scan made for only one sample which, after additional analysis, revealed some interesting features of high precision photothermal systems. This was a pump-probe separated frequency scan made with the 110 μm steel sample at a pump-probe separation of $r = 0.05$ mm. As was previously determined this sample was
Figure 4.22  Pump Probe separated frequency Scan of Mild Steel \( r = 0.2 \text{ mm}, 10 - 1000 \text{ Hz}, \alpha = 14.5 \times 10^6 \text{m}^2/\text{s}; \text{CBC}

(a) Normalized amplitude
(b) Phase
Figure 4.23  Pump Probe separated frequency Scan of 500 μm Steel AISI 302

r = 0.2 mm, 10 – 1000 Hz, α = 3.5 x 10^{-6} m²/s; CBC

(a) Normalized amplitude
(b) Phase
Figure 4.24  Pump Probe separated frequency Scan of 210 \( \mu \)m Steel AISI 302  
\( r = 0.2 \text{ mm}, 10 - 1000 \text{ Hz}, \alpha = 4.0 \times 10^{-6}\text{m}^2/\text{s}; \text{ CBC} \)  
(a) Normalized amplitude  
(b) Phase
Figure 4.25  Pump Probe separated frequency Scan of 110 µm Steel AISI 302 r = 0.2 mm, 10 - 1000 Hz, $\alpha = 3.5 \times 10^{-4}\text{m}^2/\text{s}$; CBC

(a) Normalized amplitude

(b) Phase
the one for which CBC were the most well justified since thin samples are more sensitive to boundary conditions. Using the CBC fits were attempted for this sample, with the amplitude being fit with a parameter of $3.7 \times 10^{-6}$ m$^2$/s. Using this parameter it was not possible to fit the phase, which was observed to contain a minimum. In fact for a large range of thermal diffusivity values fits of the data were not possible (Fig. 4.26). The effect of changing the thermal diffusivity was a shift of the frequency at which the phase minimum occurred to a higher value, and also to slightly shift down the value of the phase at this minimum frequency. Since the boundary conditions were known, it appeared that there were no additional parameters to vary, as thermal diffusivity and boundary conditions were the only fitting parameters. It was noted though that the phase was above the theoretical phase, and thus one way to raise the theoretical phase would be to decreasing the pump probe separation distance in the theoretical calculation, since smaller separations give smaller phase lags. It should be noted that in all previous simulations the pump-probe parameter was not a fitting parameter and was assumed to be a known constant set by the experimenter. At this point a digression will be made concerning the stages and the motor for the experiment. The stage that was used in this experiment was known to have a $1 \ \mu$m minimum increment of motion. This was not applicable for this system since the stage was attached to a motor that had a minimum increment of $2.5 \ \mu$m. Since the precision of the system is dictated by the precision of its least accurate component the effective resolution of the system was seen to be at least $2.5 \ \mu$m. The above analysis allows one to consider the pump-probe separation as a parameter, albeit within limits relating to the resolution of this system. Theoretical fits were then attempted using pump-probe separations of $0.04 \ \text{mm}$ (10 $\mu$m away from $0.05 \ \text{mm}$),
Figure 4.26  Pump Probe separated frequency Scan of 110 μm Steel AISI 302
r = 0.05 mm, 10 – 10000 Hz, showing deviation in Phase from that which is to be expected using α = 3.7 x 10⁻⁶ m²/s; CBC
(a) Normalized amplitude
(b) Phase
0.045 mm (5 μm from 0.05 mm) and 0.047 mm (3 μm from 0.05 mm). These theoretical simulations (along with the experimental data) are shown on Fig. 4.27. It is seen that changing the pump-probe separation parameter in the vicinity of a phase minimum has a very large effect on the phase profile and a smaller effect on the amplitude profile, which reveals no minimum. Decreasing the pump-probe separation has the effect of shifting the frequency at which the minimum occurs to larger values of frequency, in addition to raising the phase (that is decreasing the phase lag). The large effect on the phase regarding the pump-probe separation was not revealed for the distance scans at r = 0.2 mm. Since this distance is four times further than the r = 0.05 mm scan which revealed the phase discrepancy it was predicted that the discrepancy was due mostly to the occurrence of the minimum. For the r = 0.2 mm scan there was no phase minimum in the frequency range where the signal was stable and simulations revealed this fact (Fig. 4.28). Simulations which were made with differing pump-probe separation (Fig. 4.27) revealed that a pump probe separation of r = 0.047 mm had a fit that was close to the experimental data but the position of the minimum was not correct. By varying the thermal diffusivity a best fit of 4.0 x 10^{-6} m^2/s was obtained (Fig. 4.29). This value of diffusivity had a phase minimum at 4475 Hz. For the two other curves plotted in Fig. 4.29, with thermal diffusivities of 3.5 and 3.8 x 10^{-6} m^2/s the minima occurred at 3910 and 4260 Hz respectively. For the simulation with 4.0 x 10^{-6} m^2/s both the position of the minimum and the shape of the curve matched the experimental data most closely. A problem with the experimental data that was used for this experiment was that it was somewhat noisy in the frequency region of the phase minimum. This is attributed to the high frequency at which the minimum occurs and it is also due to the fact that the
Figure 4.27 Pump Probe separated frequency Scan (Phase) Simulations of 110 \( \mu \text{m} \) Steel AISI 302; \( r = 0.05, 0.047, 0.045, 0.040 \text{ mm} \), 10 – 10000 Hz, with \( \alpha = 3.7 \times 10^6 \text{m}^2/\text{s} \); CBC
Figure 4.28 Pump Probe separated frequency Scan (Phase) Simulations of 110 µm Steel AISI 302; \( r = 0.2, 0.195, 0.205 \text{ mm} \), 10 – 1000 Hz, with \( \alpha = 3.5 \times 10^{-4} \text{ m}^2/\text{s} \); CBC
Figure 4.29  Pump Probe separated frequency Scan (Phase) of 110 μm Steel AISI 302; Experiment and theoretical simulations for $r = 0.047$ mm, 1000 - 10000 Hz, with $\alpha = 3.5, 3.8, \text{ and } 4.0 \times 10^{-6} \text{m}^2/\text{s}$; CBC
experiments were run at laser powers that were at or near the minimum of the level where a stable signal is achievable. In other words the power could have been increased while maintaining signal linearity. This increased power would have given an enhanced signal, which would allow for more accurate delineation of the data's features.

It should be noted that frequency scans were made for the pump-probe coincident configuration in the 10-10000 Hz range, but that no minimum in phase was observed in the signal. The minimum in the phase for the $r = 0.05$ mm separation scan revealed a sensitivity that was not previously believed to exist for such a system. For values of $r$ less that 0.03 mm, theoretical simulations revealed no phase minimum.
4.2 Measurements of thermal diffusivity using a thermo-acoustic theory.

There was one steel sample that yielded data that was not consistent with a thermal theory, at least for the case where radiometric measurements are used as was the case in this investigation. This was for the stainless steel sample of thickness 55 μm, which was the same composition as the 110, 210, and 500 μm samples that were analyzed using a thermal only theory. Distance scans for this sample at 10, 100, and 500 Hz revealed amplitude and phases which were much broader than would be theoretically predicted with a thermal theory using a value for thermal diffusivity similar to that calculated for the thicker samples. It has been noted (37) that as sample thickness decreases it is possible that thermal parameters decrease, but this conclusion wouldn’t be supported from this experiment’s data which would imply the opposite trend if only purely thermal theory is used. In addition, varying the boundary conditions by incorporating a heat loss factor would be inconsistent with the amplitude becoming broader. The persistence of a radiometric (not necessarily meaning thermal) signal at distances far from the pump is usually a sign that there is an acoustic component to the signal in addition to the thermal component. As a complement to this argument two additional features were noted from the data which support this conclusion. First the magnitude of the signal (read from the lock-in amplifier) was observed and this revealed an interesting trend. Table 4.1 shows the magnitude of the signal for the 500, 210, 110 and 55 μm steel samples at 50 Hz.
Table 4.1 Signal level for various thickness steel samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>Signal (µV) $r = 0$ mm</th>
<th>Signal (µV) $r = 0.2$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 µm</td>
<td>250</td>
<td>7</td>
</tr>
<tr>
<td>210 µm</td>
<td>270</td>
<td>7</td>
</tr>
<tr>
<td>110 µm</td>
<td>320</td>
<td>10</td>
</tr>
<tr>
<td>55 µm</td>
<td>77</td>
<td>8</td>
</tr>
</tbody>
</table>

For the samples of thickness 500, 210, and 110 µm the signal level increases as the sample thickness decreases, which is consistent with theory. This information was not used in the thermal fits since all of the data was normalized, which is a legitimate procedure if the experiment was run under linear conditions which is known to be the case. The data for the 55 µm sample shows that at $r = 0$ the signal of 77 µV is reduced to approximately 20% of the value it would have if a thermal only theory was applicable, which would have been approximately 400 µV. At $r = 0.2$ mm though, the signal (8 µV) is approximately 66% of the value it would have had otherwise (12 µV). This is consistent with the fact that at pump-probe incident ($r=0$ mm) positions the thermal component is going in and out of focus more than at the edge where the signal is closer to the thermal only value. This is due to the fact that the amplitude of the acoustic vibrations are larger at $r = 0$ than at $r = 0.2$ mm. Even though the signal at $r = 0.2$ mm is close to the
value it would have for the thermal theory, it can not be concluded that at this point the signal is due solely or mostly to the thermal component. It may only be the case that the thermal component is degraded to a lesser extent at $r = 0$ mm than it is at $r = 0$ mm. If one was to look only at the normalized signal with its broadened amplitude it would be argued that the thermal component dominated at the position $r = 0$ mm, and that the acoustic component was dominant at $r = 0.2$ mm. The argument that is made by looking at the absolute signal implies that the convolution of signals may be more complex than that. Laser power linearity versus signal level was examined for this sample and it was found that the experiment was run under linear conditions. Since linearity was verified a linear combination of the thermal and acoustic signal was used to attempt theoretical fits as given by equation 2.39, chapter 2.

The other experimental observation that was used to deduce the existence of an acoustic component to the theory was from observations made while focusing the sample. For the thicker samples it was observed that when focusing the sample so that the maximum amount of infrared radiation was impinging on the detector the depth of focus seemed to be smaller than when the same was attempted with the 55 µm sample. To test this observation data was taken on the magnitude of the signal as the sample was brought in and out of maximum focus with the detection optics. As was mentioned before, the absolute value of the signal was greater for the thicker samples, so the observed signals were normalized. In this case the normalization is legitimate as the only concern is the relative focusing ability for samples of different thickness. The data is represented in Fig. 4.30. As is seen from this data the signal from the thicker samples goes out of focus more sharply than does the signal from the thinner sample. This would lead one to the
Comparison of Signal as Sample is brought in and out of focus

Figure 4.30  Comparison of apparent depth of focus of 55 and 110 μm Steel AISI 302 Samples
conclusion not that the thinner sample has a larger depth of focus, but that in fact the signal is never truly in focus since the sample is vibrating. This vibration in effect convolutes the signal leading to the seemingly larger depth of focus. It is important to note that the above two conclusions are made with respect to the radiometric detection scheme that is used in our experimental setup. The degradation of the photothermal signal is not meant to suggest that the thermal field is degraded (at least not substantially) but only that the detection scheme used in the measurement is sensitive to other effects, which convolute with the thermal field. If the detection system was one which measured only the thermal signal, such as a localized thermo-couple it would be expected that the data would fit the thermal theory that worked for the thicker samples.

Attempts to fit the distance scan data to the combined thermal theory are presented in figures Fig. 4.31-4.33. The figures reveal that the data could not be fit to the theory with accuracy, but certain trends were revealed. The figures depict the acoustic theory, the thermal theory, and the thermo-acoustic theory, with varying phase factors revealed for the phase coupling parameter used in the thermo-acoustic theory. The amplitude of the acoustic field is broader than that of the thermal field, which is to be expected since the acoustic field is a propagating field, while thermal waves are diffuse in nature. The acoustic phase was also seen to be broader than the thermal phase. At 10 Hz the theoretical acoustic amplitude was broader than the experimental data, while the theoretical thermal amplitude was narrower than the acoustic amplitude (Fig. 4.31). It is thus possible in principle to obtain a fit if a suitable fraction of the acoustic theory is added to the thermal theory. The amplitude was fit by using the thermo-acoustic model with an acoustic component of 0.4 and a phase factor of 0 radians. With this choice of
Figure 4.31 Distance Scan of 55 μm Steel AISI 302 at 10 Hz; Attempted fits using the Thermo-acoustic theory
(a) Normalized Amplitude
(b) Phase
Figure 4.32  Distance Scan of 55 µm Steel AISI 302 at 100 Hz; Attempted fits using the Thermo-acoustic theory
(a) Normalized Amplitude
(b) Phase
Figure 4.33  Distance Scan of 55 μm Steel AISI 302 at 500 Hz; Attempted fits using the Thermo-acoustic theory
(a) Normalized Amplitude
(b) Phase
parameters the thermo-acoustic phase was observed to lag even further than the thermal only theory alone would predict. An attempt was then made to fit the phase by changing the phase factor, leaving the relative amount of acoustic component the same. An optimal phase factor of 1.4 radians was found and the data was fit with this factor. The new amplitude was virtually the same as before, and the phase was now broader, but still not a good fit. This trend was repeated in subsequent trials which revealed that the amplitude fit was sensitive mainly to the relative amount of acoustic to thermal component, but that the phase was also very sensitive to the phase factor between the two signals. Even for this optimal parameter the phase was not a good fit, certainly in comparison to the thermal only fits with the thicker samples. The attempted phase fit was broader in the pump-probe coincident region, but then decayed more sharply away from this region than did the experimental phase.

Distance scans at 100 Hz and 500 Hz revealed similar trends but fits even for amplitude were not attainable. At 100 Hz the acoustic component had approximately the same line shape as the data, while at 500 Hz the acoustic line shape was even narrower than the data. Since the thermal theory predictions were much narrow in line shape than the data a fit of the amplitude was not possible in these two cases. By changing the phase factor the theoretical phase was broadened, but still did not fit the data. The phase factor that gave the broadest profile for the theoretical phase was 1.4 radians at 10 Hz, 2.0 radians at 100 Hz, and 2.5 radians at 500 Hz.

The frequency scans are shown in figures 4.34 and 4.35. At \( r = 0 \) mm the attempted fit with an acoustic component of 0.4 and a phase lag of 1.4 radians was much better than the thermal only attempt, especially in phase. At \( r = 0.2 \) mm the attempted fits did not
conform to the predicted theory. The fits were performed assuming that the thermal field obeyed CBC. If the boundary conditions were varied so that RBC were used, fits were still not possible, and hence the thermo-acoustic theory is not adequate for this sample.
Figure 4.34  Frequency Scan of 55 μm Steel AISI 302, 10 – 1000 Hz; Attempted fits using the Thermo-acoustic theory
(a) Normalized amplitude
(b) Phase
Figure 4.35  Pump Probe separated frequency Scan of 55 µm Steel
AISI 302  r = 0.2 mm, 10–1000 Hz,
Attempted fits using the Thermo-acoustic theory
(a) Normalized amplitude
(b) Phase
Chapter 5

Conclusions and Recommendations

5.1 Overview of photothermal radiometry of thin film thermal diffusivity measurements

In this work the applicability of measuring thermal diffusivity of thin steel samples via photothermal radiometry was examined. Simulated theoretical scans were fit to the experimental data to obtain thermal diffusivity values for a series of samples of decreasing thickness.

Theoretically the simulations were made using a Green function based approach that allowed for two types of boundary conditions: (1) continuity of temperature and heat flux across a boundary and (2) heat loss by a combined radiation-convection linearized heat transfer term. The presence of a thermal wave in a medium was observed by focusing the periodically emitted infrared radiation from the sample onto an infrared detector. Both the excitation (laser) and detection (infrared detector) are non-contact and non-destructive, making photothermal radiometry an attractive technique if sample contamination needs to be avoided.

In addition to the thermal theory described above, a thermo-acoustic theory was examined and used to try to fit the data that could not be fit with a thermal only theory. In
this theory an acoustic disturbance propagates in the medium, due to induced thermal gradients in the sample. Since the detection used in this work is radiometric, with infrared radiation being focused onto an infrared detector, these acoustic vibrations cause the sample to go in and out of focus with the detection optics, leading to an acoustic component to the measured photothermal signal. Since thermal gradients are larger in thinner samples of a given material, acoustic vibrations are expected to be observed in the thinnest sample first. To fit the data to the theory in the case where acoustic vibrations were observed, a linear combination of the thermal-only and the thermo-acoustic theory was attempted.

Experimentally, three different types of photothermal experiments were performed on the same system: (1) distance scans (2) conventional frequency scan and (3) pump-probe separated frequency scans. Only minor adjustments needed to be made to switch between the three set-ups. The scanned photothermal fields (both distance and frequency) were normalized and fit to the theoretical formulas. The normalization was a valid procedure, since system linearity was checked for all samples before experiments were run. The theoretical simulations that best fit the experimental data simultaneously for both the amplitude and the phase channel were used to obtain the thermal diffusivity.

5.2 Overall Conclusions

The analysis revealed that the semi-infinite Zr alloy sample fit to a thermal theory with no broadening due to convolution with the detector. Thus all of the data was fit to theories that had no broadening. The mild steel sample of thickness 1 mm gave values for diffusivity that were within 7% of the bulk value. The series of steel AISI 302 samples
had thicknesses of 500, 210, 110, and 55 μm. For all but the thinnest of these samples a thermal-only model was seen to fit the experimental data. The distance scans had best fits with larger discrepancies between amplitude and phase compared to the frequency scans. This could be explained by the presence of distance dependent steady state temperature rises that would lead to greater infrared emission at the center of the scan, leading to slightly narrower amplitude scans. The frequency scans gave results with better conformity between the channels, as frequency scans are taken at a fixed position, and hence have no distance dependent effects. The pump-probe separated frequency scans revealed no additional sensitivity to thermal diffusivity, except for one scan- a pump-probe separated frequency scan of the 110 μm AISI 302 sample with pump-probe separation of r = 0.05 mm. This scan revealed that the phase data, which had a minimum, was extremely sensitive to the pump-probe separation. In addition the frequency at which the phase was a minimum was sensitive to changes in thermal diffusivity. For all other scans, both distance and frequency, the amplitude and phase decayed monotonically from maximum values, with no characteristic minimum values in either amplitude or phase. This makes a true determination of the accuracy of the calculation difficult. By revealing a minimum in the phase, the r = 0.05 mm pump-probe separated scan revealed a characteristic value that could be compared to the theoretically calculated one. The observations made with respect to this scan are believed to reveal a sensitivity previously not observed in photothermal radiometry.

The 55 μm steel sample revealed normalized data that was broader in both amplitude and phase. The absolute signal level, though, was approximately one tenth of the value it would have had, if thermal-only phenomena occurred. This sample also appeared to have
a larger depth of focus. These observations lead to the belief that an acoustic component was being added to the thermal field. The infrared detector detects the acoustic component, as the sample goes in and out of focus with the detection optics, due to acoustic vibrations. Attempted fits using a thermo-acoustic theory approximated the data more closely than a thermal-only theory, but were not adequate to obtain thermal diffusivity values.

5.3 Future Work

The current study has successfully obtained the thermal diffusivity of a series of thin steel samples, but not for the thinnest sample tested.

This work can be extended by:

(i) further examining pump-probe separated frequency scans, and in particular looking for such scans that reveal phase minima, to observe whether the sensitivity revealed by the sample examined in this study may be used to determine thermal diffusivity more accurately.

(ii) deconvolution of the thermo-acoustic field obtained radiometrically, by performing experiments in a mode that is not sensitive to acoustic vibrations. A localized thermo-couple could be used, for instance. Even though this introduces a contact into the experimental signal, delineation of the thermal component could verify the belief that the thermal field is not degraded.

(iii) Examination of the mechanical component. Previous thermal wave studies which included mechanical components in the total signal looked mainly at acoustic vibrations. By considering more general elastic vibrations in a
medium, the mechanical component of the signal may give a more accurate fit with the experimental data.
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Appendix
C THIS PROGRAM SOLVES A 3 DIMENSIONAL CONTINUITY
C THEORY WITH POINT DETECTION
C
C written by Lena Nicolaides and modified by Francis Elek
C
DECLARING VARIABLES

IMPLICIT REAL*4 (A-H,O-Z)
REAL*4 KO,K1
COMMON /CONST/ KO,K1,ALFAO,ALFA1,W,PI,F,H,R,TH

COMPLEX*8 INTG,INTT
DIMENSION DIS(200),AM(200),PH(200),SAA(200),SBB(200)
EXTERNAL FUNKA, FUNKB
PI= 3.141592654
KO=0.0262
ALFA= 2.2*1E-5
OPEN (UNIT=13,FILE='3DCPN.DAT',STATUS='UNKNOWN')

WRITE(*,*)'ENTER W,ALFA1,F RESPECTIVELY'
READ(*,*)W,ALFA1,F
WRITE(*,*)'ENTER THERMAL CONDUCTIVITY [W/cm K]'
READ(*,*)K1
WRITE(*,*)'ENTER SAMPLE THICKNESS IN mm'
READ(*,*)TH
WRITE(*,*)'ENTER UPPER LIMIT [>0] IN mm'
READ(*,*)RU
WRITE(*,*)'NUMBER OF SYMMETRIC SUBDIVISIONS'
READ(*,*)N

RU=RU*1E-3
TH=TH*1E-3
W=W*1E-6
K1=K1*100
ALFA1=ALFA1/100**2

R=RU

DX=(ABS(XL)-1./1E6)/(N-1)
DR=-RU/N

DO 10 I=1,N+1

CALL QROMO(FUNKA,0E0,5E5,S1,1)
CALL QROMO(FUNKA,5E5,1E20,S2,2)
CALL QROMO(FUNKB,0E0,5E5,S3,1)
CALL QROMO(FUNKB,5E5,1E20,S4,2)
SA=S1+S2

109
SB=S3+S4
INTG= CMPLX(SA,SB)
INTT= INTG

C WRITE(*,*)R,SA,SB
APLT=SQR(SA**2+SB**2)

IF ((SB.LT.0).AND.(SA.EQ.0)) THEN
    PHAS=-90
ELSE IF ((SB.EQ.0).AND.(SA.EQ.0)) THEN
    PHAS=0
ELSE IF ((SB.GT.0).AND.(SA.EQ.0)) THEN
    PHAS=90
ELSE IF ((SB.GT.0).AND.(SA.GT.0)) THEN
    PHAS=ATAN(SB/SA)*180/PI-360
ELSE IF ((SB.LT.0).AND.(SA.LT.0)) THEN
    PHAS=ATAN(SB/SA)*180/PI-180
ELSE IF ((SB.GT.0).AND.(SA.LT.0)) THEN
    PHAS=ATAN(SB/SA)*180/PI-180
ELSE
    PHAS=ATAN(SB/SA)*180/PI
END IF

do j=1,3
    IF (PHAS.GT.0) THEN
        PHAS=PHAS-360
    END IF
end do

C WRITE(13,*)X,SA,SB,PHAS
DIS(I)=R
AM(I)=APLT
PH(I)=PHAS

SAA(I)=SA
SBB(I)=SB
R=R+DR

10 CONTINUE

DO 20 I=1,N+1

    WRITE(13,*)DIS(I),AM(I)/AM(N+1),PH(I)-PH(N+1)
C WRITE(*,*)DIS(I),SAA(I),SBB(I),AM(I),PH(I)

20 CONTINUE

DO 30 I=1,N

    WRITE(13,*)-DIS(N+1-I),AM(N+1-I)/AM(N+1),PH(N+1-I)-PH(N+1)
C WRITE(*,*)-DIS(N+1-I),SAA(N+1-I),SBB(N+1-I),AM(N+1-I),PH(N+1-I)

30 CONTINUE
CLOSE(UNIT=13)

STOP
END

SUBROUTINE QROMO(FUNC,A,B,SS,L)
IMPLICIT REAL*4 (A-H,O-Z)

INTEGER JMAX,JMAXP,K,KM,J,L
EXTERNAL FUNC
PARAMETER (EPS=1E-6, JMAX=20, JMAXP=JMAX+1, K=5, KM=K-1)

REAL*4 DSS,H(JMAXP),S(JMAXP)
H(1)=1

DO 10 J=1,JMAX
  IF (L.EQ.1) CALL MIDPNT(FUNC,A,B,S(J),J)
  IF (L.EQ.2) CALL MIDINF(FUNC,A,B,S(J),J)
  IF (J.GE.K) THEN
    CALL POLINT(H(J-KM), S(J-KM), K, 0E0, SS,DSS)
    IF (ABS(DSS).LE.EPS*ABS(SS)) RETURN
  ENDF
  S(J+1)=S(J)
  H(J+1)=H(J)/9
10 CONTINUE

PAUSE 'TOO MANY STEPS IN QROMO'
END

SUBROUTINE MIDINF(FUNC,AA,BB,S,N)
IMPLICIT REAL*4 (A-H,O-Z)

INTEGER N,IT,J
EXTERNAL FUNC

CHANGE OF VARIABLES FOR THE FUNCTION

  FUNC(P) = FUNK(1./P)/P**2

B=1./AA
A=1./BB

IF (N.EQ.1) THEN
  S=(B-A)*FUNK(0.5*(A+B))
ELSE
  IT=3**N-2
  TNM=IT
  DEL=(B-A)/(3.*TNM)


111
DO 10 J=1,IT
    SUM=SUM+FUNC(P)
    P=P+DDEL
    SUM=SUM+FUNC(P)
    P=P+DEL
10      CONTINUE

S=(S+(B-A)*SUM/TNM)/3.

ENDIF
RETURN
END

SUBROUTINE MIDPNT(FUNC,A,B,S,N)

IMPLICIT REAL*4 (A-H,O-Z)
INTEGER N,IT,J
EXTERNAL FUNC
IF (N.EQ.1) THEN
    S=(B-A)*FUNC(0.5*(A+B))
ELSE
    IT=3**(N-2)
    TNM=IT
    DEL=(B-A)/(3.*TNM)
    DDEL=DEL+DEL
    X=A+0.5*DEL
    SUM=0.0
    DO 10 J=1,IT
        SUM=SUM+FUNC(X)
        X=X+DDEL
        SUM=SUM+FUNC(X)
        X=X+DEL
    10      CONTINUE
    S=(S+(B-A)*SUM/TNM)/3
ENDIF
RETURN
END

C ROUTINE FOR POLYNOMIAL INTERPOLATION OR EXTRAPOLATION

SUBROUTINE POLINT(XA,YA,N,X,Y,DY)

IMPLICIT REAL*4 (A-H,O-W)
INTEGER N,NMAX
REAL*4 DY,X,Y, XA(N),YA(N)
PARAMETER (NMAX=10)
GIVEN ARRAYS XA AND YA, EACH OF LENGTH N, AND GIVEN A VALUE X,
THIS ROUTINE RETURNS A VALUE Y, AND AN ERROR ESTIMATE DY. IF
P(X) IS THE POLYNOMIAL OF DEGREE N-1 SUCH THAT P(XA(I))=YA(I),
i=1,2, ..., N, THEN THE RETURNED VALUE Y=P(X).

INTEGER I, M, NS
REAL*4 DEN, DIF, DIFT, HP, HO, W, C(NMAX), D(NMAX)
NS=1
DIF= ABS(X-XA(1))

DO 10 I=1,N
    DIFT= ABS(X-XA(I))
    IF (DIFT.LT.DIF) THEN
        NS=I
        DIF=DIFT
    ENDIF
    C(I)=YA(I)
    D(I)=YA(I)
10 CONTINUE

Y=YA(NS)
NS=NS-1

DO 11 M=1,N-1
    DO 12 I=1,N-M
        HO=XA(I)-X
        HP=XA(I+M)-X
        W=C(I+1)-D(I)
        DEN=HO-HP
        IF (DEN.EQ.0.) PAUSE 'FAILURE IN POLINT'
        DEN=W/DEN
        D(I)=HP*DEN
        C(I)=HO*DEN
12 CONTINUE

IF (2*NS.LT.N-M) THEN
    DY=C(NS+1)
ELSE
    DY=D(NS)
    NS=NS-1
ENDIF
Y=Y+DY
11 CONTINUE

RETURN
END

FUNCTION bessj0(x)
REAL*4 bessj0, x
REAL*4 ax, xx, z

RETURNS THE BESSEL FUNCTION J0(X) FOR ANY REAL X
DOUBLE PRECISION p1, p2, p3, p4, p5, q1, q2, q3, q4, q5, r1, r2, r3, r4, r5, r6,
        s1, s2, s3, s4, s5, s6, y
SAVE p1, p2, p3, p4, p5, q1, q2, q3, q4, q5, r1, r2, r3, r4, r5, r6, s1, s2, s3, s4,
        s5, s6
DATA p1, p2, p3, p4, p5/1.d0, -.1098628627d-2, .2734510407d-4,
    -.2073370639d-5, .2093887211d-6/, q1, q2, q3, q4, q5/-1.562499995d-1,
FUNCTION FUNKA(P)

IMPLICIT REAL*4 (A-H,O-Z)
REAL*4 KO,K1
COMMON /CONST/KO,K1,ALFAO,ALFAL,W,PI,F,H,R,TH
COMPLEX*8 FUNK1,Z0,Z1,Y01,M01,R1

BP=P*R
Z0 = SQRT(P**2 + CMPLX(0,1)*2*PI*F/ALFAO)
Z1 = SQRT(P**2 + CMPLX(0,1)*2*PI*F/ALFAL)
Y01 = (KO*Z0)/(K1*Z1)
M01 = (1-Y01)/(1+Y01)

C R1 = (K1*Z1-H)/(K1*Z1+H)

FUNK1 = (1+M01)*(1+M01*EXP(-2*Z1*TH)) * EXP((-P**2)*(W**2) */4) * BESJ00 (BP) * P/(Z1*(1-M01**2*EXP(-2*Z1*TH)))
FUNKA=REAL(FUNK1)
RETURN
END

FUNCTION FUNKB(P)

IMPLICIT REAL*4 (A-H,O-Z)
REAL*4 KO,K1
COMMON /CONST/KO,K1,ALFAO,ALFAL,W,PI,F,H,R,TH
COMPLEX*8 FUNK1,Z0,Z1,Y01,M01,R1

BP=P*R
\[ Z_0 = \sqrt{P^{*2} + \text{CMPLX}(0,1)*2*\text{PI}*F/\text{ALFAO}} \]
\[ Z_1 = \sqrt{P^{*2} + \text{CMPLX}(0,1)*2*\text{PI}*F/\text{ALFA1}} \]
\[ Y_01 = (K_0*Z_0)/(K_1*Z_1) \]
\[ M_01 = (1-Y_01)/(1+Y_01) \]
\[ C \quad R_l = (K_1*Z_1-H)/(K_1*Z_1+H) \]

\[ \text{FUNK1} = (1+M_01)*(1+M_01*\text{EXP}(-2*Z_1*\text{TH}))*\text{EXP}((-P^{*2})*(W^{*2})*4)*\text{BESSJO}(BP)*P/(Z_1*(1-M_01^{*2})*\text{EXP}(-2*Z_1*\text{TH})) \]

\[ \text{FUNKB=IMAG(FUNK1)} \]
\[ \text{RETURN} \]
\[ \text{END} \]