FINITE ELEMENT ANALYSIS OF TRANSIENT NON-LINEAR COUPLED FIELD PROBLEMS

by

Jinbo Kuang

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of Electrical and Computer Engineering
University of Toronto

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ABSTRACT

With recent advances in computer software and hardware technology, numerical simulation of transient nonlinear coupled field problems is becoming less costly and less time consuming than analytical and experimental study. Thus many problems of economic importance can be addressed with transient nonlinear finite element analysis which cannot be addressed experimentally, for example because the scale on which they occur is too small for experimental study. To address such problems, a 2-D finite element program has been developed for the numerical study of transient nonlinear coupled (electrical, magnetic, thermal and mechanical) field problems. The program uses the Crank-Nicolson time stepping scheme for the time domain discretization with adaptive time-stepping, accommodates position, field, and temperature-dependent material properties, time-dependent boundary conditions (e.g., time-dependent applied voltage), and allows coupling of at least two fields, e.g., electric and thermal. The program has been tested against experimental data in several contexts, with good agreement. The program has been applied successfully to the analysis of defect-induced high electric field phenomena in cross linked polyethylene (XLPE), conversion of water trees to electrical trees in distribution cable, and electro-thermal phenomena in ZnO surge arrester disks, magnetic losses in pipe-type cable, etc.

During studies of high field phenomena in XLPE electric and mechanical fields were computed in the vicinity of a conducting defect for a wide range of conditions, including AC and impulse waveforms. The study shows that substantial mechanical
stresses are generated by the thermally-induced expansion of the XLPE during an impulse, and the space charge-induced local temperature rise also causes the yield stress of XLPE to drop precipitously as the XLPE crystallites melt. Yielding of the XLPE is likely to create a cavity surrounding the defect which is capable of supporting partial discharge.

Field experience indicated that lightning impulses can convert water trees to electrical trees which then grow to failure. The numerical study suggests that during a lightning impulse, polarization current in the water within a water tree channel can cause the water temperature to rise to the point that the water pressure causes the XLPE to yield, generating a cavity large enough to support partial discharge, which would lead to conversion of the water tree to a fault-inducing electrical tree.

Pipe type transmission power cables are based on a dielectric fluid-filled steel pipe which contains the three cable phases. Magnetic losses in the pipe are a significant factor in thermal design of such systems. However, the permeability of the pipe is strongly field-dependent, which, combined with the three phase current within the pipe, makes computation of pipe losses difficult. Pipe loss computations with the transient nonlinear field program have produced good agreement with available data, and we have been able to extend computations to the case of unbalanced phase currents which could not be computed previously.

The nonlinear I-V characteristic of ZnO is used to protect everything from consumer products to high voltage transmission systems from transient over-voltages. The large magnitude and short duration of lightning current impulses causes nonuniform heating of ZnO elements. Numerical studies show that mechanical stress caused by localized thermal expansion can cause damage or failure of the ZnO disk. Nonlinear aspects of this computation include the field-dependent conductivity, temperature-dependent heat capacity, and temperature-dependent thermal conductivity of the ZnO. Computations of the thermal fields and resulting mechanical stresses have provided useful data for ZnO disk design. As a result of this work, the energy absorption capability of commercial ZnO disks has been increased by 50%.
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CHAPTER ONE
INTRODUCTION

This chapter answers a simple question: "Why this study has been made?"

1.1 Area of Applications

Many engineering problems of economic importance are nonlinear. Electric machines and transformers are examples, where the eddy current is governed by the diffusion equation

\[-\nabla \times \left( \frac{1}{\mu} \nabla \times A \right) + \sigma \frac{\partial A}{\partial t} = J_s\]  \hspace{1cm} (1-1)

where \(A\) is the magnetic vector potential, \(J_s\) is the source current density, \(\mu = \mu_r \mu_0\) is the magnetic permeability, and \(\sigma\) is the electric conductivity. In practice, the magnetic permeability \(\mu = \mu (B, T)\) of a magnetic core is a function of the magnetic flux density and temperature.

Induction heating is also a typical application of eddy current phenomena. If the object to be heated is ferromagnetic material, which is true in many cases, the nonlinear permeability of the material must be considered, and the thermal dependence of permeability is significant.

The above two examples are eddy current phenomena coupled with a thermal field. For both examples, the thermal field is governed by

\[-\nabla \cdot (k \nabla T) + \rho c \frac{\partial T}{\partial t} = q\]  \hspace{1cm} (1-2)

where \(T\) is the temperature, \(k\) is the thermal conductivity, \(c\) is the heat capacity, \(\rho\) is the mass density, and \(q\) represents the heat source in \(\text{Watt/m}^3\) which results from the eddy current losses. \(k\) and \(c\) are often temperature dependent, especially over wide temperature ranges.

From equations (1-1) and (1-2), the eddy current is coupled with the thermal field in two ways: (1) the eddy current loss is the source of the power dissipation, (2) the temperature influences the eddy current power dissipation through both magnetic permeability \(\mu\) and the heat transfer \((k \text{ and } c)\). Thus the temperature-dependent properties and the nonlinear magnetic permeability of the heated material should be taken into account.

At high electric fields in solid dielectrics, the electric field is governed by Poisson's equation

\[\nabla \cdot (\varepsilon \nabla V) = -\rho\]  \hspace{1cm} (1-3)
where \( \varepsilon = \varepsilon_r \varepsilon_0 \) is the dielectric constant, \( V \) is the electric potential and \( \rho \) the is volume space charge density. The space charge distribution is governed by the continuity equation

\[
\frac{d\rho}{dt} + \nabla \cdot J = 0
\]  

(1-4)

where \( t \) is time and \( J \) is current density. The current density \( J \) is related to the potential by the current density equation

\[
J = \sigma E = -\sigma \nabla V
\]  

(1-5)

where \( \sigma \) is the electric conductivity which for polymeric dielectrics is often a function of electric field strength \( E \) and temperature \( T \) [Boggs 1995]

\[
\sigma(E,T) = \frac{e^a}{E} e^{bT} \sinh \left( k \frac{|E|}{T} \right)
\]  

(1-6)

where \( a, b \) and \( k \) are constants. \( T \) is the temperature in K.

As an example, under high field condition, polyethylene (PE), which is used as dielectric insulation in power cable, has a highly field and temperature dependent conductivity. Under lightning impulse conditions the temperature rise near a defect in the dielectric can be substantial.

In high field gas breakdown, the processes of positive and negative ion generation, ion recombination, and ion drift lead to ion number volume densities \( n = n(x, y, z, t) \) as a function of space and time \( t \) given by the following coupled equations [Wiegart 1988]

\[
\frac{dn^+}{dt} = N_0 - k_r n^+ n^- - \nabla \cdot (\mu^+ E n^+ - D^+ \nabla n^+)
\]  

(1-7)

\[
\frac{dn^-}{dt} = N_0 - k_r n^+ n^- - \nabla \cdot (\mu^- E n^- - D^- \nabla n^-)
\]

where \( n^+ \) and \( n^- \) are the densities of positive and negative ions, \( N_0 \) is the ionization rate, \( k_r \) the ion-ion recombination constant, \( \mu \) the ion mobility, \( E \) is the electric field strength, and \( D \) is the diffusion coefficient. The above equations are coupled nonlinear equations.

To simulate these coupled nonlinear physical phenomena with reasonable precision, nonlinear properties must be included in the model formulation. In some cases, the problem must be solved in the time domain, such as the gas breakdown and dielectric insulation problems, and in some other cases, the problem can be solved in the time periodic domain, as such a solution is normally computational more efficient.

Analytical study of transient nonlinear coupled problem is difficult. Experimental study is difficult, expensive and time consuming, especially with large or complicated
device, such as large electric machines which are very expensive and time consuming to build a prototype.

With decreasing cost of computational power, numerical simulation is an emerging technique for the study of many engineering problems. For example, in the area of micro-electro-mechanical systems (MEMS), the use of CAD tools are essential for the product design, verification and simulation. FEM tools are also essential for the mechanical and civil engineering design. Numerical field simulation is also becoming essential for many electrical applications as the computer hardware is becoming more powerful and affordable.

To simulate coupled nonlinear physical phenomena with reasonable precision, nonlinear properties must be included in the model formulation. In some cases, the problem can be solved in the time periodic domain, as such a solution is normally computational more efficient. In more general transient cases, the problem must be solved in the time domain, such as gas breakdown and dielectric insulation problems.

At the time when this study was started, no commercial program was available for the solution of transient nonlinear coupled field problems. Some commercial programs capable of solving transient, nonlinear and coupled problems became available a few years after the beginning of this study, but they are either general purpose PDE solvers, which tend to be slow, difficult to use for complicated field problems and requires more user's expertise in numerical computation, or special purpose programs for some specific applications, which is hard to apply for other applications. No general purpose commercial numerical tool is available for application of transient, nonlinear and coupled (electro-thermal-mechanical) problems, which occur in high power apparatus application such as dielectric insulation. The development of such a tool would substantially enhance the capability for the study of this area.
1.2 Applications to be Studied

The applications to be studied in this work have a common nature, that is they are transient, nonlinear, and coupled. They can be classified into three major areas: electrical insulation, ZnO nonlinear ceramic surge arrester and eddy current phenomena in power cables. Analytical study of these problems is difficult, while experimental study is expensive and time consuming. Especially as we move to the level of molecular and quantum engineering, increasing numbers of technically important problems are experimentally untractable and can only be investigated through numerical simulation and analytic approximation.

1.2.1 Electrical Insulation

Electrical insulation usually fails as a result of high electric field phenomena caused by imperfections which result in highly inhomogeneous field regions. Thus an understanding of what goes on in such microscopic high field regions is of great practical interest. Experimental study is inhibited by the extremely small physical extent of such regions (usually a few μm), although measurements have been made of electrode currents caused by high field conduction in needle-plane geometries [Hibma et al. 1984, Baumann et al. 1985]. The region cannot be made larger for experimental purposes, as the power densities involved in the high field phenomena would cause thermal runaway if dissipated throughout a larger volume.

The basic goal of numerical studies is to provide a basis for testing hypotheses concerning functional relationships among parameters under high field conditions. For example, the conductivity as a function of field and temperature can be modeled in various ways, and computations using the various models can be compared with experimental data such as terminal current from needle-plane experiments. Such comparisons will extend our knowledge of the functional relationships among high field parameters in dielectrics. The ability to compute high field phenomena in dielectrics also facilitates "what if" testing of the effect of various modifications of dielectric high field properties on dielectric performance. High field dielectric failure under impulse conditions is thought to result from thermal condensation of the high field-induced current into a filament. The increasing conductivity in the highest field regions results in increased current density and increased heating which, given the highly positive temperature coefficient of the conductivity, is a positive feedback situation which results in the current density condensing into a filament. Such condensation causes catastrophic failure through electrical and thermal phenomena. We hope to simulate such thermal condensation through a coupled finite element solution of the equations for electrical and thermal fields.
Our study will focus on the electrical effects of space charge and the thermal effects of current density, as the creation of substantial space charge essentially defines the boundary between low and high field conditions in dielectrics. The difference between impulse and dc electric strength, the influence of prestressing and impulse rise time on breakdown voltages, and the polarity dependence of the breakdown voltage in the needle-plate geometry all have been explained by space charge build up [Hibma et al. 1985]. Space charge is clearly a major factor in defect-driven failure of solid dielectrics. Many needle-plate geometries result in a geometric field at the needle tip in the range of 1000 to 2500 kV/mm, and such stresses are supported without failure for some time in good solid dielectrics. Such fields are above the inherent dielectric strength of any organic solid. Clearly, localized high field conductivity results in the generation of space charge which limits the maximum field in the dielectric to much lower values, in the range of 200kV/mm for polyethylene at power frequencies [Boggs et al. 1994]. The study of space charge formulation and distribution appears to be the key to a better understanding of dielectric insulation.

The theoretical modeling of space charge generation under inhomogeneous field conditions is difficult for many reasons. The conductivity \( \sigma(E) \), which is a function of field under high field conditions, is not well known. [Boggs 1994] assumes that \( \sigma(E) \) is governed by

\[
\sigma(E) = \sigma_0 e^{k|E|}
\]  

(1.8)

Simulation of inhomogenous field conditions with field dependent conductivity involve the solution of Poisson's equation (1-3) with time-varying applied voltage and field-dependent conductivity.

Previous work [Boggs 1994] has solved the Poisson's equation in one dimension with time-varying applied voltage and field-dependent conductivity through use of an iteration method to compute the time-dependent field and space charge distributions in one dimension. At each time step, a new field distribution is computed by iteration, the space charge distribution is updated according to the current density (1-5), where conductivity \( \sigma \) is a function of electric field \( E \). For power frequency voltage, the time step was chosen in the region of 2 to 5 \( \mu \)s during which the applied voltage is assumed constant. Good agreement with experimental results was obtained.

1.2.2 ZnO Surge Arrester Element

The non-linear I-V characteristic of ZnO is used to protect power transmission and distribution systems from transient over-voltages, especially those caused by lightning-induced current surges. The large magnitude and short duration of the lightning current impulses cause substantial and, under some conditions, nonuniform heating of ZnO elements, which can result in local temperature rise, thermal run-away, and
element failure. These effects are investigated using a 2-D axisymmetric finite element program which solves for electric field and thermal field simultaneously for systems with temperature and/or electric field-dependent conductivity, thermal conductivity, heat capacity, etc. [Kuang & Boggs 1996]

Mechanical stress plays a very important role in the failure of ZnO elements and cracking can result from severe mechanical stresses. The mechanical stress is caused by non-uniform temperature in the material, and the difference in thermal expansion between different materials. Computation of mechanical stress using the FEM will also be an important part of this study.

1.2.3 Power Cable

Metal pipes are used to minimize the time that trenches must remain open during installation of transmission class power cables. The pipe can be installed quickly, so that the trench can be covered, and the cable can be pulled in later. The steel pipe also has the function to enclose the 3 phase conductors in pressurized dielectric fluid, and also to shield the magnetic field inside the pipe. The pipe has the disadvantage that it increases electrical losses as a result of eddy current losses in the pipe. The losses in the pipe are comparable to the skin and the proximity effect losses in the conductors and, in many cases, are larger than the losses caused by all the other AC effects combined. This is especially true for pipe-type cable systems with stranded-segmented conductors. Segmenting greatly reduces the skin effect and the proximity effect, but this change in the conductor structure has very little effect on the pipe losses. The energy losses in the magnetic pipe enclosing a power cable arise from eddy current induced in the pipe by the electromagnetic field created by the alternating current in the conductors.

The highly field dependent permeability of the steel makes the field problem a nonlinear one. Also the coupling of the thermal field with the magnetic field makes it a coupled field problem. Analytical study of the problem has been limited by the difficulty of dealing with the complexity of the three phase structure of the power cable and the nonlinearity of the pipe permeability. Semi-empirical approximate expressions have been used containing factors derived from measurements on a limited number of cables sizes and configurations, but this approach is of limited value for new cable designs with large conductors carrying high current [Mekjian & Sosnowski 1983, Kawasaki et al. 1981]. The numerical analysis of the problem is valuable for the design of power cable, so that the geometric configuration can be optimized. The 2-D finite element approach to this problem is found in the literature applying effective permeability method to accommodate the nonlinear pipe permeability [Labridis & Dokopoulos 1992]. The temperature dependence of pipe permeability has not been considered.
For the pipe magnetic problem, we are interested in the loss distribution and the temperature distribution in the pipe and conductors for steady state cable operation. These are useful data for the design of a power cable. The power loss due to eddy current is converted into heat, resulting temperature rise in the cable. The prediction of temperature rise requires the solution of a coupled set partial differential equations representing magnetic and thermal diffusion phenomena. This finite element computation of the pipe magnetic problem involves a coupled set of magnetic and thermal diffusion equations to model the fast magnetic transient and the slow thermal transient.
1.3 Solution by FEM and Time Stepping

The finite element method (FEM) was first used in the mechanical engineering community to solve stress and strain problems. In the 1960's, FEM spread to the electrical engineering community to solve electromagnetic problems. The FEM has been applied in many areas such as electric machine design and induction heating. Numerical methods associated with the FEM have improved together with advances in computing power. Nonlinear 2D static problems were first solved in the early 1970's, while 2D nonlinear steady state problems have been solved since the late 1970's. Transient nonlinear problems have been solved since 1982 [Boggs 1995]. Coupled field problems have also been solved using FEM in recent years [Wiegart 1988, Lavers 1985, Yamada & Bessho 1988, Trowbridge 1988, Lavers 1983]. The time domain solution of coupled nonlinear field problems has not appeared in the literature prior to the present work but will expand as a result of the reduced cost of computing power.

Fundamentally, the FEM provides a way to determine the values of a field parameter at arbitrary discrete locations in space which minimize a functional, the minimization of which corresponds to the way in which nature organizes the field parameter throughout space. This is achieved by taking the derivative of the functional at each discrete location in space with respect to the field parameter and using an iterative procedure to minimize the overall value of the functional with respect to the field parameter at the discrete locations in space. Obviously, if the discrete locations are not sufficiently dense in a region where the field parameter varies rapidly, an incorrect solution is obtained. One of the major advantages of the FEM is that the density of the discretization (mesh density) can be varied easily.

In finite element analysis, the problem domain is divided into sub-domains or elements, and a matrix relationship is derived among the element vertices from the physical nature (or the governing equation) of the problem. Solution of the problem is based on this matrix relationship and boundary conditions. With FEM, linear behavior is often assumed for the potential variables (i.e. voltage, temperature, etc.) within each element [Jack & Mecrow 1990]. This results in errors when the field parameters are interpolated from the vertices to the space within elements. Such errors can be reduced by using greater numbers of elements, or by assuming quadratic behavior for the potential variables within each element.

The advantage of the FEM for the time domain solution of coupled nonlinear problems is that FEM can deal with the complex geometries, and the problem nonlinearity can be accommodated easily by the FEM formulation. An additional advantage with the FEM for highly inhomogeneous problems is that a mesh can be formulated with
very small elements in areas with rapidly changing field parameters and very large elements for areas where the field changes slowly.

Time domain solutions are essential to address transient nonlinear field problems but are computationally intensive. As an alternative to the time domain solution, people have tried to use many other numerical methods for the solution of nonlinear problems, such as effective permeability and harmonic balance, but these methods can be applied only to time periodic problems. The time stepping approach produces time domain solutions at the expense of substantial computation time. This is due to the large numbers of simultaneous equations which must be solved at each time step and to the related problems of convergence and stability for the time domain solution. Due to the advances in computer power that permit large storage and rapid execution, time domain finite element and finite difference solutions are being used more frequently to solve transient and coupled field problems.
1.4 Thesis Objective

1. Develop a 2-D FEM Software Tool

The objective of this study is to develop an efficient strategy using the finite element method to solve the coupled transient nonlinear problem (coupled electro-magnetic and thermal fields) in the time domain, and to compute the thermal expansion-induced mechanical stress in postprocessing; to develop a computer code to implement the FEM and time stepping and various post-processing capability; and using the computer code to compute results of some examples for which experimental data or analytical solution are available for verification.

2. Transient Nonlinear Dielectric Problem

Simulate the space charge distribution in XLPE under various AC and impulse conditions, analyze the effect of space charge caused by field and temperature dependent conductivity, thermal field caused by power dissipation and mechanical stress as a result of non-uniform thermal expansion.

3. ZnO Surge Arrester Element Analysis

Simulate the ZnO surge arrester element under over-voltage and lightning impulse conditions, the effect of mechanical stress caused by thermal expansion, the effect of positive temperature coefficient (PTC) of the ZnO conductivity.

4. Magnetic Problem With Permeability as a Function of Magnetic Induction

The objective is to study losses in a magnetic pipe containing three phase cable.
1.5 Original Contributions

The contributions of this thesis are to develop algorithm using finite element method and time stepping for the numerical analysis of transient nonlinear coupled field problems in time domain; to implement and verify the algorithm in a software package including a user-friendly interface; to apply the software package to applications including: study of field induced electro-thermal phenomena in cable dielectrics, study of ZnO surge arrester elements, and study of eddy current losses in high pressure fluid filled cable pipe.
1.6 Thesis Outline

Chapter 1. The chapter introduces the background knowledge to the thesis, which includes the purpose of the study, introduction to the applications which require a new software tool to be studied and the mathematic background to the numerical methods to be applied. After that the thesis objective and contributions are presented. The chapter ended by the outline of the thesis.

Chapter 2. The chapter presents the details of the numerical methods that have been applied in the thesis. Existing methods for the study of the nonlinear transient coupled field problems and their limitations are reviewed. It then reviews the governing equations of electro-magnetic field, thermal field, and mechanical stress field, time stepping as a time domain solution of transient problem, the finite element method, the FEM formulation for the 2-D axisymmetric mechanical stress problem, the numerical algorithms for the solution of nonlinear systems. Introduction to the finite element software has been given at the end of the chapter.

Chapter 3. Studies of ZnO surge arrester elements are presented which include characteristic of ZnO, electric field distribution due to impulses, thermal field due to electric power dissipation, and stress distribution due to thermal expansion.

Chapter 4. Studies of field induced electro-thermal phenomena in cable dielectrics are presented which include space charge distribution due to field dependence of dielectric conductivity, electro-thermal phenomena due to defect in XLPE under AC and surge conditions, initiation of an electrical trees from a defect in a dielectric and detection of defect in XLPE cable.

Chapter 5. Studies of eddy current losses in high pressure fluid filled cable pipe are presented which include study of eddy current in cable pipe under balance and unbalanced three phase current conditions.

Chapter 6. Original contributions, list of publications, conclusions and recommendations for future work are presented.
References


CHAPTER TWO
NUMERICAL SOLUTION OF COUPLED TRANSIENT NONLINEAR FIELD PROBLEMS

2.1 Introduction

Many problems of technical interest require the solution of coupled, transient, nonlinear field problems. For example, when a high field is applied to a dielectric, the electrical conductivity becomes (roughly) an exponential function of the field. The conductivity can become substantial resulting in very large power dissipation and temperature rise. The electrical conductivity is also a strong function of temperature, so that the electric field distribution can become a function of the electric field and temperature. Thermal properties are usually temperature dependent, so that the solution to the thermal field is also nonlinear. Dielectric systems are often subjected to non-periodic electrical transients, and the solution for the electric and thermal fields as a function of time requires some form of stepping through time. The spatially nonuniform temperature rise and space charge formation in high electric field regions can result in substantial mechanical stresses which are also of interest. The above considerations apply equally well to nonlinear electrical materials such as ZnO, which is used in transient surge suppressers from household voltages to 500 kV transmission systems.

Two fundamental problems are involved in the solution of transient, nonlinear field problems. The first is computing a solution to the nonlinear field problem, i.e., arriving at a self-consistent solution for the field when the material parameters on which the field depends are a function of the field, and the field distribution is a function of the material parameters. The second problem is to accommodate the time-dependent boundary conditions, time-varying sources and material properties which make the problem transient. This requires some form of time-stepping which maintains a stable solution to the field problem over wide ranges of rates of change in boundary conditions, time-varying sources and material properties. This Chapter will provide a historical overview of approaches to the solution of nonlinear field problems and will then concentrate on methods for addressing the above two issues.
2.2 Historical Review

Nonlinear static problems have been solved using the FEM in two dimensions since the early 1970's, and nonlinear steady state problems have been solved in two dimensions since the late 1970's. Transient nonlinear problems have been solved since 1982 [Trowbridge 1988]. Reports of coupled problems being solved by the finite element method have been published in recent years [Garg et al. 1987, Molino & Repetto 1990, ter Maten & Melissen 1992, Tsukerman et al. 1993]. The finite element solution of nonlinear and coupled field problems in the time domain was limited by the computer power [Lavers et al. 1985, Chari et al. 1993]. The time domain solution of coupled nonlinear problems has become increasingly common with advances in computer hardware that permit large storage and execution speed.

2.2.1 Nonlinear Time Periodic Magnetic Problems

Steady state nonlinear eddy current problems have been solved in one dimension in both the time domain and periodic domain in [Lavers et al. 1985], where the time domain solution was obtained by discretizing the diffusion equation in space (FEM and/or finite difference) and time (time stepping). Time discretization was based on a Crank-Nicolson implicit procedure which will be introduced in detail in the following sections. A time domain solution is very demanding of computer power, so in the past, the time periodic method has often been employed for the solution of nonlinear problems.

Two methods dominated the time periodic solution of nonlinear field problems. One is the effective permeability method which has been applied in [Lavers et al. 1985] to implement a time periodic solution of the eddy current problem. According to this method, permeability $\mu$, which is a nonlinear function of $H$, is represented by an effective permeability $\mu_{eff}$ which changes in space but does not change with time. Several methods of determining an effective permeability from the nonlinear magnetization characteristics have been proposed in the literature. These methods include the computation of $\mu$ based on the average stored energy density, based on time averaging $\mu$ over a cycle of the $B$ and $H$ waveform, and based on the rms values of $B$ and $H$. In the case of sinusoidal $H$ with reasonable saturation levels, the simplest model is to assume that the flux density waveform is a square wave and to define effective permeability $\mu_{eff}$ as

$$\mu_{eff} = \frac{4}{\pi} \frac{B}{H}$$

(2.1)

where $H$ is the peak field strength and $B$ is the corresponding flux density, as determined from the magnetization characteristics.
Harmonic Balance is another method for obtaining a time periodic solution. The steady state nonlinear eddy current problem has been solved using the harmonic balance method by [Yamada 1988] and [Lu 1990]. In the harmonic balance method, all variables, i.e., vector potentials $A$, flux densities $B$, and applied current $J_s$ are expressed by Fourier series

$$A^i = \sum_{n=1,3,5,...}^{\infty} A_{ns}^i \sin(n\omega t) + A_{nc}^i \cos(n\omega t)$$

$$B_x^e = \sum_{n=1,3,5,...}^{\infty} B_{xns}^e \sin(n\omega t) + B_{xnc}^e \cos(n\omega t)$$

$$B_y^e = \sum_{n=1,3,5,...}^{\infty} B_{yns}^e \sin(n\omega t) + B_{ync}^e \cos(n\omega t)$$

$$J_s^i = \sum_{n=1,3,5,...}^{\infty} J_{ns}^i \sin(n\omega t) + J_{nc}^i \cos(n\omega t)$$

(2.2)

The magnetic reluctivity $\nu$, which is obtained from the Fourier expansion, has the form

$$\nu = \frac{1}{\mu} = \frac{H(B(t))}{B(t)} = \nu_0 + \sum_{n=2,4,6,...}^{\infty} (\nu_{ns} \sin(n\omega t) + \nu_{nc} \cos(n\omega t))$$

(2.3)

The harmonic balance method is based on substitution of Eqn. (2.2) and (2.3) into Eqn. (1.1), and discretizing in space by implementing the FEM. Equating the coefficients of $\sin(n\omega t)$ and $\cos(n\omega t)$ on both sides of the resulting matrix equation, we get a selected number of different harmonic components of the $A$ for all the nodes. The harmonic balance method provides all the harmonic components of interest, but when two harmonic components (the fundamental and third harmonic components) are considered, the number of elements in the system matrix is four times the number of nodes. The effective permeability and harmonic balance method can solve nonlinear periodic problems effectively, but can not solve transient nonperiodic problems.

### 2.2.2 Coupled Problems

Coupled fields are the rule rather than the exception in nature, although such coupling is often approximated out of existence during engineering to facilitate analysis. Recent decreases in the cost of computer power as well as improved algorithms have made computation of coupled field problems increasing practical.

Coupled electromagnetic-thermal problems have been treated by the finite element method in recent years [Garg et al. 1987, Molfino & Repetto 1990, ter Maten & Melissen 1992, Tsukerman et al. 1993]. Several strategies have been suggested for the numerical solution of coupled equations, depending on the time scales of coupled.
processes [Molfino & Repetto 1990]. The most obvious approach from a mathematical standpoint is to solve the algebraic system of equations defined by the electro-magnetic and thermal fields as fully coupled equations. This approach is very intensive computationally and is only used when the two coupled process have the same time scales. The second approach, applicable to processes of differing time scales, is to assume that the input from the slower process is constant during computation of the faster process. At each time step of the slower process, the input from the faster process can be chosen from the latest previous step of the fast process.
2.3 FEM Solution

The finite element method has the advantage of allowing for a variable mesh size, i.e., the spatial discretization can vary with the rate of change of the field parameter(s). It also has the advantage of accommodating material property nonlinearities relatively easily. The other important advantage of FEM is the ability to accommodate complicated geometries. For these reasons, we have adopted the FEM approach to the solution of transient, nonlinear field problem.

The present formulation of the basic FEM solution of electromagnetic (EM) field and thermal field problems follows closely that formulated by [Silvester 1990]. A second order FEM solution has been implemented because the electric field and space charge must be derived from the first and second derivatives of the FEM solution for the potential. Even though many references [Zhu 1990, Hinton 1974, Silvester 1991] discuss smoothing of derivatives of the FEM solution, the simpler methods only give a pleasing graphical display, but their numerical values may not be any better than those obtained from direct differentiation of the FE solution [Silvester 1992]. The more complicated methods can give better derivatives but the computation is so intensive that they are impractical to implement during the time domain solution of transient nonlinear coupled field problems.

The finite element method is relatively simple to formulate compared with other numerical methods (such as finite difference) which might be applied to the computation of coupled and nonlinear systems. This results from the way in which each element is represented by its own matrix equation, and the assembly of these matrices forms the system equation which allows field or temperature dependent material properties and material nonlinearities to be handled with relative ease. However these coupled systems usually lead to prohibitively large matrices which for serial computation (single cpu) may not be cost effective [Trowbridge 1988, Lavers 1983]. In the applications treated in this thesis, we assume that the electromagnetic and thermal material properties are not strong functions of mechanical stress. The thermal field normally varies slower than the electromagnetic (EM) field, so that the solution of the EM field and thermal field need to be coupled only every a few time steps.
2.4 The Numerical Solution of Nonlinear Systems

A system of nonlinear equations must be solved to implement either a time domain or time-periodic solution of a nonlinear field problems. As noted above, the fundamental problem is to iterate to a self-consistent solution for the field when the material parameters are a function of the field, and the field is a function of the material parameters.

Suppose we are solving the non-linear Poisson's equation

$$\nabla \cdot (\nu \nabla A) = q$$

where $\nu$ is a function of $A$. The FEM results in a non-linear system of equations described by

$$R(A) = [S][A] - [Q] = 0$$

where vector $[A]$ is the unknown to be solved, matrix $[S]$ is a function of $[A]$ and $[Q]$ is the source vector. Several iteration methods have been proposed to solve such a system of equations [Jack & Mecrow 1990, Chari et al. 1993, Konrad 1993, Kanai et al. 1987, Albanese & Rubinacci 1992], with the "simple iteration" and Newton-Raphson method being most common.

2.4.1 Simple Iteration

The so-called simple iteration method starts with an estimated value for $\nu$ and proceeds successively by solving the resulting equation from Eqn.(2.5) and then updating $\nu$ according to the new solution for $A$. More specifically, simple iteration can be described by the following steps [Konrad 1993], where the superscript indicates the iteration number

1) Assume $[A]^{k-1} = [A]^0$, where $[A]^0$ is an initial guess;
2) Find $\nu(A)$ with $A = [A]^{k-1}$;
3) Assemble matrix equation $[S]^k[A]^k = [Q]^k$;
4) Solve for $[A]^k$;
5) Compare $[A]^k$ with $[A]^{k-1}$;
6) If the difference in 5) is larger than a tolerance go back to step 2);
   Else
   solution converged;
End
The simple iteration method is simple, not always stable, and time consuming. A relaxation method as eqn.(2.6) which modifies step 2) can be used to overcome these problems

\[ v^{k+1} = v(\rho[A]^k + (1-\rho) [A]^{k-1}) \]  \hspace{1cm} (2.6)

where 0<\rho<1. This method is applicable to both gradual and quickly saturating materials for ferromagnetic problems without oscillation or divergence [Kanai et al. 1987].

At each iteration of the simple iteration method, a full solution of the latest updated linear system is necessary, although the solution need not to be computed to very high accuracy in the early iterations. An iteration method for the solution of the linear system of equations is therefore preferable to a direct method such as Gaussian elimination.

2.4.2 Newton-Raphson Iteration

The simple iteration method is not always stable. For this reason, the Newton-Raphson method has been used frequently for the solution of nonlinear field problems as follows [Lavers 1983, Labridis & Dokopoulos 1992]. If Eqn (2.5) is the nonlinear system of equations to be solved and if \( A^k \) is an intermediate solution, an improved solution \( A^{k+1} \) can be expressed as

\[ A^{k+1} = A^k + \delta A \]  \hspace{1cm} (2.7)

Eqn (2.5) can be expanded as a Taylor's series

\[ R(A^k + \delta A) = R(A^k) + \frac{\partial R}{\partial A} \delta A + ... \]  \hspace{1cm} (2.8)

We want to compute \( R(A^k + \delta A) \) and thus want the right hand side of Eqn.(2.8) to be zero. Neglecting the second and higher order terms, we obtain

\[ R(A^k + \delta A) = R(A^k) + \frac{\partial R}{\partial A} \delta A = 0 \]  \hspace{1cm} (2.9)

so that

\[ \delta A = -J_{ac}^{-1} R(A) \]  \hspace{1cm} (2.10)

where \( J_{ac} \) is the Jacobian matrix evaluated at \( A = A^k \)
then Eqn (2.7) becomes

\[
A^{k+1} = A^k - J_{ac}^{-1} \cdot R(A^k)
\]  

(2.12)

The iteration of Eqn (2.12) is repeated until \(|A^{k+1} - A^k|\) is less than the convergence criterion. The Newton-Raphson method can be summarized in the following steps:

1) Assume \([A]^k = [A]^0\);

2) Find residual \([R] = [S][A]^k - [G]\);

3) Find Jacobian matrix \([J_{ac}] = \begin{bmatrix} \partial R_1 \\ \partial A_1 \\ \partial R_2 \\ \partial A_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \partial R_n \\ \partial A_n \end{bmatrix}\);

4) \([\delta A] = [J_{ac}]^{-1}[R]\);

5) New iteration \([A]^{k+1} = [A]^k + [\delta A]\);

6) Compare \([A]^{k+1}\) with \([A]^k\);

7) If the difference in 6) is larger than tolerance:

   go back to step 2);

Else

   the solution has converged;

End.

In forming the Jacobian matrix, the process is problem dependent, i.e., for different nonlinear models, the user must derive analytically the derivative of the parameter \(v\) with respect to \(A\) and change the program for each model. This makes the FEM using Newton-Raphson method rather cumbersome.

If a sufficiently good starting value is provided, the Newton-Raphson method converges quickly, at the cost of large computational effort at each iteration to construct the Jacobian matrix. For materials with strong nonlinearity, the Newton-Raphson method does not always converge [Kanai et al. 1987], and the Newton-Raphson method requires the nonlinear characteristics to be monotonic [Konrad 1993]. Some modifications of Newton-Raphson method have been proposed [Albanese & Rubinacci 1992. Henneberger et al. 1990] to improve performance.
An indirect iteration algorithm, named the Incomplete Cholesky Conjugate Gradient algorithm (ICCG), is widely used to solve the linear systems arising from the Newton-Raphson method. ICCG is an efficient method to solve large, sparse linear systems, as it requires less memory fewer computations, and is less affected by round off error than direct iterative algorithms [Henneberger et al. 1990, Barrett et al. 1992].

In this thesis the simple iteration method has been applied for the solution of non-linear systems.
2.5 Time Stepping and Time Domain Solutions

Linear time periodic problems can be solved very economically using complex arithmetic to eliminate time as a variable. However for a nonlinear system, excitation in one harmonic will produce a response in other higher order harmonics. Time periodic approaches, such as effective permeability and harmonic balance have been proposed to solve nonlinear problems. For nonlinear eddy current problems, the time periodic approach computes the loss fairly accurately but is less accurate when computing other parameters [Jack & Mecrow 1990]. These approaches are only effective for time periodic problems.

Discretization in time (time stepping) is an approach which produces time domain solutions to nonlinear problems at the expense of substantial computation time due to the large numbers of simultaneous equations which must be solved at each time step and to the related problems of convergence and stability [Chari & Silvester 1980]. In view of the advances in computer power that permit large storage and speedy execution, finite element time domain (FETD) and finite difference time domain (FDTD) solutions are used increasingly frequently for solution of linear and nonlinear problems.

To solve a nonlinear problem in the time domain by the time stepping approach, the partial differential equations must be discretized in time. For a diffusion equation, such as Eqn.(2.13)

\[- \nabla (k \nabla T) + \rho_m c \frac{\partial T}{\partial t} = q\]  

(2.13)

the FEM discretization results in a matrix equation

\[RT(t) + S \frac{dT}{dt} + Q(t) = 0\]  

(2.14)

where \(R\) is related to thermal conductivity \(k\), \(S\) is related to heat capacity \(\rho_m c\), and \(Q(t)\) is the excitation vector that comes from heat density \(q\).

We discretize Eqn. (2.14) in time and express \(T(t)\) and \(Q(t)\) by linear interpolation in time [Zienkiewicz and Morgan 1983]. For \(t_{m-1} < t < t_m\), we have the following interpolation relation [Trowbridge 1991]

\[T(t) = (1 - \Theta \tau) T_{m-1} + (\Theta \tau) T_m\]

\[Q(t) = (1 - \Theta \tau) Q_{m-1} + (\Theta \tau) Q_m\]  

(2.15)

where

\[\tau = \frac{t - t_{m-1}}{t_m - t_{m-1}} = \frac{t - t_{m-1}}{\delta t}\]  

(2.16)
\( \theta \) is a dimensionless parameter which is discussed below.

Combining Eqn.(2.15) and (2.16)

\[
\frac{\partial T}{\partial t} = \frac{T_m - T_{m-1}}{\delta t}
\]  

(2.17)

As Eqn (2.14) is always satisfied, its integral weighted by any bounded function will be zero. If we integrate Eqn.(2.14) with \( \tau \) as weighting function, we obtain

\[
\int_0^1 \tau \left( R \left[ T_m \theta \tau + T_{m-1} (1-\theta \tau) \right] + S \frac{T_m - T_{m-1}}{\delta t} + Q_m \theta \tau + Q_{m-1} (1-\theta \tau) \right) d\tau = 0
\]  

(2.18)

From the above integral equation, the recurrence relation for successive iterations in time is obtained

\[
\left[ R(1-\theta) - \frac{S}{\delta t} \right] T_{m-1} + \left[ R\theta + \frac{S}{\delta t} \right] T_m + Q_{m-1} (1-\theta) + Q_m \theta = 0
\]  

(2.19)

Several implicit and explicit algorithms as proposed in [Tsukerman et al. 1993] can be derived from Eqn. (2.19) for various values of \( \theta \). When \( \theta = \frac{1}{2} \), the algorithm is known as Crank-Nicolson; when \( \theta = \frac{1}{3} \), we obtain the central difference form; when \( \theta = 0 \), we obtain the explicit method; and when \( \theta = 1 \), we obtain the implicit method. The explicit method is computationally simple, but the time step, \( \delta t \), must be very small in order to ensure stability. The Crank-Nicolson algorithm is an implicit algorithm which is relatively simple to implement. Although Crank-Nicolson algorithm may be subject to oscillations, it is always stable and converges [Lienhard 1987] and is widely used.

An implicit Runge-Kutta (IRK) time stepping method was introduced by [Tsukerman 1993] and [Nicolet 1996]. IRK has the advantage of greater stability than the Crank-Nicolson method at the expense of greater computational effort. Due to the high stability of the IRK method, it can be used for problems such as nonlinear electromagnetic field coupled with mechanical systems with moving parts.

For the problems treated in this thesis, the Crank-Nicolson method is sufficiently stable and has therefore been employed.
2.6 Mathematical Modeling

In this section, we derive and review the mathematical formulas for the description of the physical (electrical, thermal and mechanical) phenomena we have studied in this thesis.

2.6.1 Electric Field with Field-Dependent Conductivity

For the applications (XLPE and ZnO) where the permeability and conductivity are low, the magnetic field has a negligible effect on electric field. The electrical field can be modeled by Poisson's equation

\[ \nabla \cdot (\varepsilon_e \nabla V) = -\rho_c \]  

(2.20)

where \( \varepsilon_e \) is the dielectric constant, \( \rho_c \) is the volume space charge and \( V \) is the electric potential. The space charge distribution is governed by the continuity equation

\[ \frac{d\rho_c}{dt} + \nabla \cdot J = 0 \]  

(2.21)

where \( t \) is the time and \( J \) is the resistive current density. The resistive current density is related to the potential by

\[ J = \sigma_e(E,T) E = -\sigma_e(E,T) \nabla V \]  

(2.22)

where \( \sigma_e \) is the conductivity which is a function of electric field \( E \), temperature \( T \), it may also varies with time and position. Combining Eqn. (2.20), (2.21) and (2.22) to eliminate \( \rho_c \), we obtain a diffusion equation

\[ \nabla \cdot (\sigma_e \nabla V) + \frac{\partial}{\partial t} \nabla \cdot (\varepsilon_e \nabla V) = 0 \]  

(2.23)

Solving Eqn. (2.23) produces the electrical potential \( V \) as function of time and position. Since Eqn. (2.23) is a non-linear partial differential equation, an iterative method has to be applied for the solution. We start with an initial guess of \( \sigma_e \), and solve Eqn. (2.23) by using the FEM. The electric field is thus computed from the first derivative of potential which allows the field-dependent electric conductivity \( \sigma_e \) to be updated. Through an iterative process, a self-consistent electric field and field-dependent conductivity can be computed. The space charge \( \rho_c \) can be computed as the second derivative of potential as shown by Eqn. (2.20). With the knowledge of space charge distribution, the stress caused by space charge can be calculated.

2.6.2 Thermal Field with Temperature-Dependent Material Properties

The thermal field is governed by the diffusion equation

\[ -\nabla \cdot (k \nabla T) + \rho_m c \frac{dT}{dt} = q \]  

(2.24)
where $k$ is the thermal conductivity. $T$ is the temperature, $\rho_m$ is the mass density, and $c$ is the heat capacity. $q$ is the volume thermal power dissipation in (W/m$^3$), usually caused by Joule heating in which case it can be expressed as

$$
q = \mathbf{J} \cdot \mathbf{E} = \sigma_{ij} E^2
$$

(2.25)

The thermal induced mechanical stress which results from the inhomogeneous temperature rise can be computed after the temperature distribution is known.

### 2.6.3 Mechanical Stress Caused by Thermal Expansion and Space Charge

In the study of dielectric under high field condition, the existence of space charge and thermal expansion can cause substantial local mechanical stress inside the dielectric, which has been computed in post-processing.

For a 2D axisymmetric mechanical stress problem for which the object, the initial strain (due to thermal expansion) as well as the loads (due to the force acting on space charge) acting thereon are axially symmetric, the equilibrium equations in cylindrical coordinates are [Backstrom 1994. Shames 1989]

$$
\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} + \rho_c E_r = 0
$$

$$
\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \tau_{zr}}{\partial z} + \tau_{rz} + \rho_c E_z = 0
$$

(2.26)

where $\sigma$'s represent the normal and $\tau$ the shear components of mechanical stress. $\rho_c E_r$ and $\rho_c E_z$ are the $r$- and $z$-components of the force density caused by the action of the electric field on space charge. The stress is related to the strain by Hook's law [Backstrom 1994. Shames 1989]

$$
\begin{bmatrix}
\varepsilon_r - \alpha \Delta T \\
\varepsilon_\theta - \alpha \Delta T \\
\varepsilon_z - \alpha \Delta T
\end{bmatrix} = \frac{1}{E_y} \begin{bmatrix}
1 & -\nu_p & -\nu_p \\
-\nu_p & 1 & -\nu_p \\
-\nu_p & -\nu_p & 1
\end{bmatrix} \begin{bmatrix}
\sigma_r \\
\sigma_\theta \\
\sigma_z
\end{bmatrix}
$$

(2.27)

where $\varepsilon$'s are the strains, $E_y$ is Young's modulus, $\alpha$ is the coefficient of linear thermal expansion, $\Delta T$ is the temperature rise, and $\nu_p$ is Poisson's ratio.
2.7 2-D Axisymmetric FEM Solution of Mechanical Stress

The FEM solution for mechanical stress is provided for readers who are not familiar with solid mechanics and the implementation of FEM in this context. We define the nodal displacement as

\[ w_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} \]  

(2.28)

where \( u_i \) and \( v_i \) are the displacement in \( r \)- and \( z \)-direction, respectively. The element displacement is given by the following vector which has 6 elements

\[ w^e = \begin{bmatrix} w_i \\ w_j \\ w_m \end{bmatrix} \]  

(2.29)

\((\varepsilon_z, \varepsilon_r, \varepsilon_\theta, \gamma_{r\theta})\) are the four components of strain which are non-zero for axisymmetric problems. They are related to the element displacement \( w^e \) by

\[ \varepsilon = \begin{bmatrix} \varepsilon_z \\ \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{r\theta} \end{bmatrix} = B w^e = [B_i, B_j, B_m] w^e \]  

(2.30)

in which

\[ B_i = \frac{1}{2A} \begin{bmatrix} 0 & c_i \\ b_i & 0 \\ c_i & b_i \end{bmatrix} \]  

(2.31)

where \( A \) is the area of the triangular element, \( r \) and \( z \) are the coordinates of the point where the strain is computed, and \( a_i, b_i \) and \( c_i \) represent

\[ a_i = r \varepsilon_m - r_m \varepsilon_j \]
\[ b_i = z_j - z_m \]
\[ c_i = r_m - r_j \]  

(2.32)

The stress \( \sigma \) and strain \( \varepsilon \) are related by Eqn. (2.33)
\[ \mathbf{\sigma} = \begin{bmatrix} \sigma_\iota \\ \sigma_r \\ \sigma_\theta \\ \tau_{r\iota} \end{bmatrix} = D(\mathbf{\varepsilon} - \alpha \cdot \Delta T) \] (2.33)

where \( D \) is the elasticity matrix and \( \alpha \Delta T \) is the strain caused by thermal expansion, and \( \Delta T \) is computed from the FEM solution of temperature distribution.

For an isotropic material, \( D \) is symmetrical and is given by

\[
D = \frac{E_I(1-v_p)}{(1+v_p)(1-2v_p)} \begin{bmatrix}
1 & \frac{v_p}{1-v_p} & \frac{v_p}{1-v_p} & 0 \\
\frac{v_p}{1-v_p} & 1 & \frac{v_p}{1-v_p} & 0 \\
\frac{v_p}{1-v_p} & \frac{v_p}{1-v_p} & 1 & 0 \\
0 & 0 & 0 & \frac{1-2v_p}{2(1-v_p)}
\end{bmatrix}
\] (2.34)

where \( v_p \) is the Poisson's ratio and \( E_I \) is the Young's modulus. As \( D \) is symmetrical, only the upper right half of the matrix is shown. The overall matrix equation for the FEM solution of the displacement \( \mathbf{w} \) is

\[
[\overline{\mathbf{B}}' \overline{D} \overline{\mathbf{B}} \overline{r} \mathbf{A}_\Delta]\mathbf{w} - [\overline{\mathbf{B}}' \overline{D} \alpha \Delta T \overline{r} \mathbf{A}_\Delta] = \left\{ \begin{array}{c} b_r \\ b_\theta \end{array} \right\} \overline{r} A_\Delta = 0
\] (2.35)

where \( \overline{r} \) is the \( r \)-coordinate for the center of element, \( \overline{B} \) (bar over B) means it is evaluated at element center and \( \overline{B}' \) is the transpose of \( \overline{B} \). The displacement can be solved from Eqn.(2.35) with the appropriate boundary conditions. Stress can then be derived from Eqn.(2.30) and (2.33).
2.8 Introduction to the FEM Program

A 2-D axis-symmetric FEM program has been developed for the solution of transient nonlinear coupled field problems based on the techniques mentioned in the previous sections. The program applies the simple iteration method for the solution of nonlinear field problems, because simple iteration has the advantage of simplicity and generality. The program is aimed at solving a variety of nonlinear field problems, such as those resulting from nonlinear conductivity and nonlinear thermal properties of XLPE and ZnO, which behave very differently in their nonlinearity. The Newton-Raphson method has not been applied as it requires deriving Jacobian matrix for the range of nonlinearities treated in this thesis, which means for each problem, the user must derive the Jacobian matrix for that problem, which is not a trivial task.

The FEM program can be applied to the solution of transient nonlinear electric/magnetic field coupled with thermal field. When the field problem is an electric one, it has to be governed by eqn. (2.23); when it is a magnetic field problem, it has to be governed by eqn. (1.1). The post-processing of the tool can calculate the mechanical stress as a results of space-charge and/or the thermal expansion.

The transient nonlinear coupled field problems treated in this thesis are solved in the time domain. The program adapts the Crank-Nicolson (CN) method for the time stepping, which provides a better compromise between stability and efficiency. With consideration to the problems to be solved in this thesis, where the electromagnetic field is coupled with a slower thermal field and there is no coupling between the electromagnetic and mechanical problems, the CN method satisfies stability requirements. The stability of the FEM and time stepping solution is assured by multiple factors:

1. The number of iterations
2. The size of time step
3. The mesh size

With certain mesh and time step, the number of iterations is decided by the difference of two consecutive solutions. If the difference is smaller than a pre-defined tolerance, the solution is considered convergent. If the number of iterations exceeds a limit, the time step has to be decreased. If decreasing time step can not ensure convergence, the mesh size has to be decreased. With the problems we are going to solve in this thesis, which are very stable, if the numerical solution is convergent, then the solution will be the true solution.

Normally, we have to use different meshes for the electromagnetic field and thermal field, but for this study, we will construct the same mesh but with different boundary conditions for the two fields, as we will have finer mesh for the area where extensive
current exists, the thermal field will also be extensive and need a finer mesh in the same area.

The program has been written in Matlab, a matrix-oriented programming environment which is widely used in educational and industrial community. Later versions of the program have used functions from the Matlab PDE toolbox, which is a collection of functions for the solution of partial differential equations (PDE). PDE toolbox is capable of solving 2-D elliptic (static and steady-state problems), parabolic (transient problems) and hyperbolic (wave problems) PDE's. The PDE Toolbox has a nonlinear solver for elliptic equations, and can solve the parabolic equation in time domain but without the nonlinear capability. It does not support the solution of coupled problems. The Toolbox includes an adaptive mesh generator along with postprocessor and graphics display functions. PDE toolbox became available at a late stage of this study. The program developed for this thesis takes the advantage of the PDE Toolbox by using its mesh generator and postprocessor for graphics display.
2.9 References


CHAPTER THREE

FAILURE MECHANISM AND IMPROVEMENTS IN ZnO ARRESTER ELEMENTS

3.1 Introduction

ZnO is widely used in surge arresters which range in rating from a few volts to 1200 kV. The material is well suited to this application as a result of its highly non-linear conduction with voltage. Figure 3.1 shows the typical current density vs field for a transmission class arrester element for a range of conduction threshold fields. In the most nonlinear region of the curve, the current increases as $\approx V^{100}$.

Of course, the ZnO grains within the arrester element are essentially conducting, with a conductivity of about 1000 S/m. The nonlinear characteristics come from nm thick grain boundary regions between the ZnO grains and the amorphous ceramic material between the grains. These grain boundaries form p-n junctions: however, the nonlinearity of the grain boundaries is substantially greater than is obtained from a p-n junction (nonlinearity coefficient in the range of 40, i.e., in the region of greatest nonlinearity, the current density goes as $\approx V^{10}$). The additional nonlinearity in ZnO comes from a feedback which occurs as electrons start to go over the potential barrier formed by strong trapping at the interface [Greuter & Blatter1990, Blatter & Greuter 1986]. As the electrons go over this potential barrier, they gain enough energy cascading down the potential on the other side that they create electron-hole pairs. The holes propagate back to the barrier where they combine with trapped electrons to cause the barrier to collapse, thus increasing the non-

![Figure 3.1. Typical current density vs electric field for a ZnO arrester elements with different conduction threshold fields.](image-url)
linearity from the range of 40, as would be expected for a p-n junction, to greater than 100 as is typical of ZnO. As a result of this complex operating "mechanism", manufacture of high quality ZnO arrester elements which are stable over time, temperature, etc. is an art, with as many as 15 materials going into the formulation, many at trace levels.

ZnO arrester elements can fail by a number of mechanisms, including through electro-thermal instability due to the positive temperature coefficient of ZnO conductivity and thermal stress-induced mechanical cracking as a result of excessive mechanical stress in this brittle ceramic. Breakdown channels at the edge of the electrode are among the most common failure mechanisms at high currents. Such failures take the form of a melt through puncture of the element from the electrode edge to the counter electrode or to the edge of the disk. This failure mechanism limits the energy absorption capability of ZnO arrester elements.

Using transient, nonlinear finite element analysis (FEA) with coupled thermal and electric fields, we have quantified the temperature enhancement as a function of edge margin and optimized the electrode topology.
3.2 Increasing Energy Absorption of ZnO

The thickness of ZnO element required for a given voltage class application is determined by the threshold field for conduction. The threshold field for conduction can be increased by increasing the number of grain boundaries through which the current passes between electrodes. This can be accomplished in two ways:

(i) by creating smaller grains, and

(ii) by creating a larger number of nonconducting grain boundaries so that the number of grain boundaries along the percolation path between electrodes increases.

In practice, the two mechanisms inevitably act together, as smaller grains are generally created by using more nonconducting material to limit grain growth, and this increases the number of nonconducting grain boundaries and, therefore, the number of grain boundaries along the percolation path between electrodes. Thus the threshold field of ZnO arrester elements can be increased.

Increasing the threshold field of the arrester element is of little value if the energy absorption capability is not increased at the same time, as for a given current waveform, the energy absorption obviously increases with the threshold field. Thus to make practical improvements in arrester technology, such as going from the typical three ZnO stacks within a metal enclosed arrester to a single stack, both the threshold field for conduction and the energy absorption capability of the arrester elements must be increased.

In this chapter, we investigate the electro-thermal phenomena which cause punctures at the electrode edge, and we determine how these can be minimized through control of the electrode topology. Since the electrothermal phenomena at the electrode edge take place on a scale of tens of micrometers which make them almost impossible to measure directly, we investigate these phenomena through transient nonlinear finite element computations in which the material properties (e.g., electrical and thermal conductivity, heat capacity, etc.) can be a function of electric field, temperature, position, etc. and the applied voltage or current can be a function of time. The numerical techniques employed have been described in the previous chapter. Figures 3.2 and 3.3 show the temperature dependent volumetric heat capacity and thermal conductivity employed in the present computations.
3.3 Failure by Thermal Runaway

Figure 3.4 shows the current-voltage characteristics of typical ZnO nonlinear dielectric. Note that the I-V characteristics at 25 C and 450 C are almost identical, while those at 500 C do not differ greatly. However, at 550 C and above, the temperature coefficient of the conductivity becomes substantially positive. Such a positive temperature coefficient means that the current density will increase in the high temperature region relative to the surrounding lower temperature regions which exacerbates the temperature difference and leads to thermal runaway. Thus stable operation of ZnO above about 550 C does not seem likely, and in order to maximize
energy absorption, we must endeavor to minimize hot spots so that the temperature of the entire ZnO element will rise together.

For lightning surge current waveforms which last only a few microseconds, we cannot count on thermal diffusion to smooth the temperature distribution. The thermal diffusivity vs temperature of ZnO is shown in Figure 3.5. In the region of thermal instability, the thermal diffusivity $\kappa_d$ is about $2e-6 \text{ m}^2/\text{s}$, where the thermal diffusivity $\kappa_d$ is defined as

$$\kappa_d = \frac{k}{\rho c}$$

(3.1)

Figure 3.4 Measured data for voltage vs current for a typical ZnO arrester element over the temperature range from $25 \degree C$ to $650 \degree C$. Note the substantially positive temperature coefficient above $550 \degree C$. Data courtesy of Toshiba.

Figure 3.5 Thermal diffusivity of ZnO as a function of temperature. Note that the thermal diffusivity drops substantially with increasing temperature, which means that the thermal time constant decreases with increasing temperature. Data courtesy of Toshiba.
In eqn. (3.1) $k$ is the thermal conductivity, $\rho c$ is the volume heat capacity in [J/m$^3$-K].

The distance heat diffuses in a time, $t$, is approximately given by

$$d(t) = \sqrt{k_d t}$$

(3.2)

Thus in 10 $\mu$s at about 550 C, heat will only diffuse about 4.5 $\mu$m. Thus the temperature rise for impulse current waveforms is essentially adiabatic, and to maximize energy absorption of the arrester element, hot spots must be avoided.

In the following computations, we treat the ZnO as a homogeneous, nonlinear dielectric. Clearly this is an approximation as all the power dissipation actually occurs at the grain boundaries, which constitute only a miniscule fraction of the total volume. We have conducted computations including individual grain boundaries, cavities, etc.; however, such computations are impractical on the scale of a full ZnO arrester element, which is the scale required in the present study as we wanted to compute thermally-induced mechanical stresses as well as electrothermal phenomena.
3.4 Electrode-Related Thermal Phenomena

Temperature Gradient at an Electrode

The sprayed Al electrode acts as a heat sink on the surface of the ZnO. The typical electrode thickness is 0.1 mm, and dissipation in the electrode is negligible. The electrode has an order of magnitude greater thermal conductivity than the ZnO, so that the electrode tends to be at relatively uniform temperature. Under these conditions, the ZnO has a substantial temperature gradient at the electrode, as seen in Figure 3.6. Again, the field applied to the ZnO is 456 V/mm for 0.2 ms resulting in a power dissipation of about 500 J/cm$^3$ in the ZnO and a bulk temperature rise of 175 $^\circ$C. For a relatively thick electrode (in the range of 100 $\mu$m), this temperature gradient can result in appreciable mechanical stress in the arrester element. However, such temperature gradients are not a major cause of failure in ZnO arrester elements.

Electrode Edge Effects

The arrester element electrodes are normally sprayed with metal with an appreciable margin between the edge of the electrode and the edge of the arrester element. The margin is left

(i) to avoid the possibility of the electrode spilling over the edge of the arrester element and reducing the dielectric withstand of the element through surface flashover along the edge of the element, and

![Figure 3.6 Plot of temperature vs position through the Al electrode and into the bulk of the ZnO. The data are plotted for 10 $\mu$m and 100 $\mu$m thick Al electrodes. The applied field to the ZnO is 456 V/mm for 0.2 ms which results in a power dissipation of about 500 J/cm$^3$ and a temperature rise in the bulk of 175 K from an initial temperature of 300 K. Note that for the 100 $\mu$m thick electrode, the temperature gradient is about 125 K over a distance in the ZnO of about 60 $\mu$m.](image-url)
Figure 3.7a Top. equithermal plot for a dissipation of 250 J/cm³ with an initial temperature of 70 °C for electrode edge margin of 2 mm. Below, a temperature profile is shown along the line indicated in the equithermal plot.
Figure 3.7b Top, equithermal plot for a dissipation of 250 J/cm³ with an initial temperature of 70 °C for electrode edge margin of 1 mm. Below, a temperature profile is shown along the line indicated in the equithermal plot.
Figure 3.7c Top, equithermal plot for a dissipation of 250 J/cm\(^3\) with an initial temperature of 70 °C for electrode edge margin 0.5 mm. Below, a temperature profile is shown along the line indicated in the equithermal plot.
Figure 3.7d Top, equithermal plot for a dissipation of 250 J/cm³ with an initial temperature of 70 °C for electrode edge margin of 0.2 mm. Below, a temperature profile is shown along the line indicated in the equithermal plot.
Figure 3.8 Temperature rise in vicinity of electrode edge above bulk ZnO temperature for a current impulse which dissipates 250 J/cm$^3$ in the bulk of the ZnO.

(ii) to increase the dielectric withstand of the element. If the electrode margin were zero, i.e., the electrode came to the edge of the nonlinear ZnO dielectric, then the current density in the ZnO arrester element would be essentially uniform.

However with increasing electrode margin, current can spread from the edge of the electrode into the ZnO beyond the electrode which results in an appreciable increase in the current density and temperature rise of the ZnO at the electrode edge. In addition, any imperfections at the electrode edge such as protrusions of the metalization into cavities in the ZnO surface exposed by grinding, unevenness of the metalization at the electrode edge (thin protrusions of the metalization in the radial direction), or worst, combinations thereof, will enhance the local temperature rise at the electrode edge.

Figure 3.7a through Figure 3.7d show the equithermal plots in the region of the electrode edge for margins of 2, 1, 0.5, and 0.2 mm. Below each equithermal plot is a temperature profile plot along the line indicated on the equithermal plot which was selected to go through the region of maximum temperature near the electrode edge. The computation was made on a full sized ZnO arrester element with a thickness of 22 mm and a radius of 23 mm. The energy dissipated during the impulse discharge was 250 J/cm$^3$ which results in an adiabatic temperature rise of 82 °C over the initial temperature of 70 °C (343 K). Thus the final temperature in the bulk of the ZnO after the current pulse is 425 K.

In the transient nonlinear finite element computations, the mesh was made much finer near the electrode edge than elsewhere, which explains the somewhat coarse nature of the equithermal lines away from the electrode edge region. Each computation required between several days and a week to compute using a fast PC.
The computations clearly indicate the effect of the edge margin on the temperature rise at the edge of the electrode. For an edge margin of 2 mm, the maximum temperature in the region of the electrode edge is 250 °C above the temperature in the bulk while for an edge margin of 0.5 mm this is reduced to 50 °C above the bulk temperature of 425 K. An edge margin of 0.2 mm reduces the temperature rise to a negligible 10 °C. Data were also computed for an edge margin of 0.1 mm but are not shown in Figure 3.7. Figure 3.8 provides these data in graphical form.

Defects at the edge can increase the temperature even further. We are restricted in the types of defects we can investigate by the axisymmetric 2-D symmetry required by the transient nonlinear finite element program. Figure 3.9 shows a defect consisting of an annular protrusion 20 μm into the ZnO at a distance of 50 μm from the electrode edge. This 2-D axisymmetric allows the current at the defect to spread in two directions, toward and away from the electrode edge. Still, it is undoubtedly less severe than a radial filamentary protrusion from the electrode with a protrusion into the ZnO at its end. Such a defect is very realistic given the typical roughness of the metalization at the edge of the electrode and the typical porosity of the ZnO. The defect of Figure 3.9 causes a maximum temperature of 500 °C for a current pulse which dissipates 250 J/cm³ in the bulk. This is in the region required for thermal instability of the ZnO, as discussed above and make edge-related failures by the postulated mechanism entirely plausible.
Figure 3.9 Equithermal and thermal profile plots for a defect slightly off the edge of the electrode and slightly (20 μm) protruding into the ZnO surface. This geometry allows the current to spread in 2 directions which is still less than for a filamentary defect protruding off the edge of the electrode. The geometry is intended to simulate, to some degree, such a defect within the limitations of axisymmetric 2-D. For a lightning impulse current waveform which dissipates 250 J/cm$^3$ in the bulk of the ZnO resulting in a temperature rise of 82 °C above the initial temperature of 70 °C, this defect causes a maximum temperature rise of 430 °C or 350 °C above the temperature in the bulk. This is in the range necessary to cause thermal runaway.
3.5 Practical Application

In developing a new generation of ZnO arrester elements suitable for a metal-enclosed high voltage arrester with a single stack of elements (rather than the three stack which is typical), a number of improvements were made to the elements to improve uniformity [Imai et al. 1998]. In addition, a new method of metalization was adopted which results in a much smoother electrode edge (Figure 3.10) and which facilitates a substantially reduced edge margin, in the range of 0.3 to 0.6 mm. The reasons for this range were implied above. Beyond a margin of 0.6 mm, the temperature rise at the edge of the electrode causes excessive temperature in the ZnO and reduces the energy absorption capability. Below 0.3 mm margin, the probability of breakdown across the exterior surface increases. Figure 3.11 shows the

Figure 3.10 Electron micrographs (viewing from the top of ZnO disk) showing the large edge of conventional metal sprayed electrode (upper photo) and the smooth, narrow edge of the new, high energy arrester elements (lower photo). Photographs courtesy of Toshiba.
Figure 3.11 Percent of ZnO arrester elements which pass a multiple discharge test (with a constant injected energy of 250 J/cm\(^2\) per shot) as a function of the electrode edge margin. Failures at 0 to 0.3 mm are caused by breakdown along the interface between the ZnO and sintered protection layer on the outer radius of the disk.

Figure 3.12 Element failure rate as a function of energy dissipation. The tests consists of 18 rectangular, 2.5 ms current pulses. The new, high performance elements can dissipate about 50% more energy without failure than can the older units.
element failure rate during application of a series of eighteen. 250 J/cm³ current impulses as a function of the edge margin and indicates that the range from 0.3 to 0.6 mm is optimum. The data show that the new disks with controlled edge margin achieve 50% greater energy absorption than the older disks without edge margin control. Figure 3.12 shows similar data but as a function of the energy per shot and indicates that the elements with the new edge can dissipate about 50% more energy than the elements with a conventional edge. Most recently, the quality of the interface between the ZnO and sintered layer at the outside diameter of the disk has been improved so that the Al electrode can be brought to the edge of the disk, which has resulted in a 100% increase in energy absorption relative to the conventional technology.
3.6 Mechanical Stresses

We have the capability to compute thermally-induced mechanical stresses in post processing of our thermal data, and we did compute the mechanical stresses in the vicinity of the electrode edge. Figure 3.13 shows an example of such a computation for the mechanical stress in the electrode edge region with a 2 mm margin. The radially-directed stress is greater than 0.25 GPa over an extended region although the peak stresses occur over dimensions typical of a ZnO grain.
Figure 3.13 Top shows detail of the 2-directed mechanical stress in the region of the electrode edge for a 2 mm margin. Figure 3.13 (below) shows a stress profile along the line indicated in the upper graph. The maximum radially-directed stress (solid line) is nearly 1 GPa, while the Z-directed stress (dash-dot) peaks at about 0.6 GPa and the \( \theta \)-directed stress (dash) peaks at about 0.4 GPa. Note that the R-directed stress is above 0.25 GPa and the Z and \( \theta \) directed stresses are above 0.1 GPa over an extended region which could pose a risk of mechanical damage to the element.
3.7 Conclusion

Edge margin control has proved to be one of the keys to increasing the energy absorption capability of ZnO arrester elements, along with improved material uniformity, both of which reduce hot regions within the material by making power dissipation more uniform throughout the material. Through such improvements, the energy absorption capability of ZnO arrester elements has been increased by 50%, to the point that high gradient elements can be used to make single-column, metal-enclosed high voltage arresters for use in gas-insulated substations. Such arresters have about 40% of the weight and volume of a standard metal-enclosed arrester and about 50% of the parts, resulting in substantial savings in cost, materials, and structural complexity [Andoh et al. 2000].

We do not have an accurate “budget” for the temperature rise in ZnO arrester elements. The maximum temperature in the element would be the result of (i) defects at the electrode edge, (ii) thermal inhomogeneity caused by dissipation being restricted to grain boundaries, the limited thermal diffusion during a lightning surge waveform, and numerous nonconducting grains and cavities within the ZnO. Increasing the voltage gradient (threshold field) of the ZnO element usually results in increased thermal inhomogeneity as a result of the increased number of nonconducting grains which contribute to the greater conduction voltage gradient through increased conduction percolation path through the ZnO. Some optimum compromise must exist among these factors which maximizes arrester element utility, but the optimum compromise is not yet clear. Of course, the best solution is to increase the temperature at which the temperature coefficient of conductivity becomes substantially positive, likely by increasing the band gap of the material, which has to be further investigated and is beyond the scope of this thesis work.
3.7 References


CHAPTER FOUR
STUDY OF DIELECTRICS UNDER HIGH FIELD CONDITIONS

4.1 Introduction

High field phenomena in solid dielectrics have become increasingly important with the increased operating stresses (electric field) in such applications as transmission class cross linked (XLPE) power cable. Electrical insulation often fails as a result of phenomena caused by a large, defect-induced highly inhomogeneous electric field such as can be caused by a conducting contaminant in the dielectric, asperity at an electrode dielectric interface, etc. An understanding of the physical phenomena which occur in the microscopic high field region surrounding such a defect is therefore of great interest.

Space charge plays an important role in the high field region surrounding a defect. Many needle-plane geometries result in a geometric field at the needle tip greater than 1000 kV/mm, and such stresses are supported without failure for some time in good solid dielectrics such as polyethylene (PE). Yet such fields are clearly above the inherent dielectric strength of any organic solid. At high electric fields, the resistive current increases greatly and can become equal to the displacement current. When this occurs, the electric field is limited by mobility of charge carriers, which generates space charge. Space charge limits the the electric field to much lower values, in the range of 200 kV/mm for polyethylene at power frequency [Boggs 1994]. Space charge is also responsible for the difference between impulse and DC electric strength, the influence of prestressing and impulse risetime on breakdown voltage, and the polarity dependence of breakdown voltage in needle-plane geometry [Kuang & Boggs 1994].

In the study of polyethylene (PE) under high field conditions, the effect of space charge on the breakdown must be analyzed in the microscopic region around a small defect. Experimental study of such phenomena is very difficult, because the defects and space charge limited region around them are very small, in the μm region, and very difficult to measure. Coupling of electrical phenomena to the thermal field is important under impulse conditions, where substantial power is dissipated over a short time in the space charge limited region so that the dielectric heats more or less adiabatically. As well, the parameters which determine both the electric and thermal fields are temperature dependent. The mechanical stress is caused by the electric field acting on space charge and by the inhomogeneous thermal field (in the case of impulse conditions) is also of great interest.
4.2 Modeling of XLPE Conductivity

In this thesis work, the study of dielectrics is focused on but not limited to the study of cross-linked PE (XLPE). At low electric fields, the voltage-current relation of XLPE tends to follow Ohm's law [O'Dwyer 1973]. The exact form of the high field conductivity for XLPE is not known. From theoretical considerations, we expect the high field conductivity to go with the field somewhere between \( \exp(E) \) and \( \exp(\sqrt{E}) \). Very few measurements of high field conductivity have been published. [Tokoro et al. 1992] have published some data for conductivity as a function of temperature and field measured using AC voltage. Basically, Tokoro measures \( \tan(\delta) \) vs field and interprets the incremental increase in \( \tan(\delta) \) from its low field trend to be caused by high field conductivity. Curve fits to his data for high field conductivity vs field and temperature along with the known (measured) point given by \( \sigma(E_{\text{lim}}) = \varepsilon_0 (E_{\text{lim}}) \approx 150 \) to \( 260 \) kV/mm for XLPE suggest that a formula in which conductivity goes as \( \exp(E) \) fits better than when the conductivity goes as \( \exp(\sqrt{E}) \) [Boggs 1995]. Based on an analysis of Tokoro's and other data, we normally use a formula of the following form for high field conductivity of XLPE as a function of temperature and field [Tokoro et al. 1992, Boggs 1995, Jiang et al. 1997, Kuang & Boggs 1997] with the coefficient of the field in the exponential adjusted to give a limiting field between 150 and 250 kV/mm, corresponding to the range of limiting field reported for XLPE.

![Figure 4.1](image-url)  
**Figure 4.1.** Measured threshold field for high carrier mobility for an industry standard XLPE cable compound. The histograms show the measured threshold field measured at a large number of locations with both positive (left) and negative (right) polarity voltage. The threshold field on each figure was determined using a three-parameter Weibull fit to the data. Note that the threshold field is very similar for positive and negative polarity. As well, the statistical distribution of the threshold field is quite broad (±10%), which reflects the inhomogeneous nature of this semicrystalline dielectric on the scale of the 5 μm tip radius needle used to make the measurement. Data courtesy of ABB Corporate Research, Baden, Switzerland.
Figure 4.2 Typical plot of the Laplacian (if there were no space charge effect) and Poisson (with space charge) electric field on the axis of symmetry vs distance from a needle tip when the Laplacian field at the needle tip is appreciably above the limiting field value defined by $\sigma(E_{\text{lim}}) = \varepsilon \omega$. Note the space charge limited field region, which is the focus of the present investigation. The space charge limited field region extends from the tip of the needle (distance = 0) to the point at which the Poisson field starts dropping rapidly, slightly over 3 μm in the above graph.

\[
\sigma(E,T) = \frac{62.24}{|E|} \exp\left(-\frac{6945.71}{T}\right) \exp\left(7.79602 \times 10^{-8} |E|\right)
\]  

(4.1)

The parameters as shown in Equ. (4.1) result in a limiting field of 250 kV/mm at power frequency of 60Hz (the concept of space charge limiting field can be found in the next section). Good agreement to Equ. (4.1) has been obtained by hopping theory over a wide range of electric field [Jiang 1996]. Hopping theory says at zero field, thermally activated hopping exists to transport between potential wells. The motion is biased by the electric field at low field level, which gives rise to linear (ohmic) conductivity. Once the field is high enough to affect the probability of hopping (rather than just the direction), conduction becomes non-ohmic and we get into the high field region (around 10 to 20 kV/mm for XLPE).

Previous computations [Kuang & Boggs 1996] indicate that the exact nature of the equation for $\sigma(E)$ has little effect on the power dissipation, electromechanical forces, etc., as long as the limiting field, $E_{\text{lim}}$, defined by $\sigma(E_{\text{lim}}) = \varepsilon \omega$, remains constant. In particular, we have investigated the case of $\sigma(E)$ being proportional to $\exp(k\sqrt{|E|})$ rather than $\exp(k|E|)$ and have found only minor differences in the resulting phenomena. The underlying phenomena are so fundamental that details such as the functional dependence of conductivity on the electric field have little effect. Basically, either the conductivity must rise to the point that space charge limits the field, or the field will go to totally unreasonable levels. The only effect of the functional dependence of the conductivity on the field is in the slope of $\sigma(E)$ vs $E$ in the region where $\sigma(E) = \varepsilon \omega$, and this has a minor effect on the change in the limiting
field with the Laplacian field. Thus we have reasonable confidence in our computations even though we do not have high confidence in our knowledge of the exact functional relationship between the conductivity and the field other than the condition that $\sigma(E) = \varepsilon \omega$ for a field, $E$, of approximately 250 kV/mm.

The value of this limiting field has been confirmed both theoretically and experimentally. It is known that the dielectric properties are dominated by impurity states, often called "traps", within the band gap. These impurity states have a density in the range of $5e25 \text{ m}^{-3}$ (mean distance between "traps" of about 2.8 nm) in a wide range of organic polymers. The typical trap depth (from the conduction band) is about 0.8 eV in polymers ranging from polypropylene to XLPE. A simple criterion for high field charge carrier mobility is that the charge carrier must be able to gain enough energy in going from one impurity state (trap) to the next to escape from the trap. For the above parameters, this means that the field for high mobility, $E_L = 0.8 \text{ eV}/2.8e-9 \text{ m} = 285 \text{ kV/mm}$, which is very close to the measured value for XLPE which typically has a statistical distribution ranging from about 250 kV/mm to 350 kV/mm (Figure 4.1). The source of the impurity states is not at all clear. In any case, the trap density and depth is similar in a wide range of amorphous and (the amorphous part of) semicrystalline dielectrics.
4.3 Space Charge Limiting Field

The current density flowing in a dielectric consists of two parts, the resistive current density \( J_r = \sigma E \) and displacement current density \( J_d = \varepsilon \frac{dE}{dt} \). For sinusoidal voltage excitation, the displacement current density is \( J_d = j\omega \varepsilon E \), where the \( j \) indicates the \( 90^\circ \) leading phase shift which results from the time derivative of the \( \sin(\omega t) \). In the above formulation, \( \varepsilon \omega \) can be treated as the "capacitive conductivity", which has the same units as the conductivity, \( \sigma \). The grading of the dielectric depends on the relative values of these two conductivities. For dielectrics at typical engineering electric fields, \( \sigma \) varies from \( 10^{-13} \) S/m to \( 10^{-20} \) S/m [Boggs 1994], while \( \varepsilon \omega \) is typically \( 6 \times 10^{-9} \) S/m at 50 or 60 Hz. Thus grading is normally dominated by the capacitive current, which is typically more than four orders of magnitude greater than the resistive current. However at high electric fields, the resistive conductivity increases greatly and can become equal to \( \varepsilon \omega \). When this occurs, the electric field is limited by mobility of charge carriers, which generates space charge. Thus the field is limited by the condition that

\[
\sigma(E_{\text{lim}}) = \varepsilon \omega \tag{4.2}
\]

An analogy to that is the rolloff with frequency of a RC circuit. In a RC circuit, the frequency response starts to drop off when the capacitive impedance becomes less than the resistive impedance. We can think of the dielectric as a series of layers, each of which has a resistance in parallel with a capacitance. Normally the capacitive impedance is much less than the resistive impedance. The space charge limited field region (limiting of the field) occurs when the resistive impedance decreases to become equal to the capacitive impedance, in direct analogy to the low pass filter.

If we take a simple model for \( \sigma(E) \), viz.,

\[
\sigma(E) = \sigma_0 \exp(k|E|) \tag{4.3}
\]

We can substitute eqn.(4.3) for \( \sigma(E_{\text{lim}}) \) in eqn. (4.2) and find that

\[
E_{\text{lim}} = \frac{1}{k} \ln \left( \frac{\varepsilon \omega}{\sigma_0} \right) \tag{4.4}
\]

which is obviously unique to the above assumption for \( \sigma(E) \). However, the high field conductivity generally does go as the exponential of the field. The space charge limiting field for the conductivity as eqn.(4.1), solved by Maple, a symbolic manipulation software, is

\[
E_{\text{lim}} = \frac{-6.24 k}{\varepsilon \omega} \exp\left(\frac{6945.71}{T}\right) \tag{4.5}
\]
where $W(x)$ is a function known to Maple.

Physically, the basis for this limiting field is that the conductivity increases to the point that charge carriers can redistribute in a manner which limits the electric field to this value ($E_{\text{lim}}$). According to the above formula, the space charge limited field goes as the log of the dielectric constant and frequency. Thus as the frequency increases, the space charge limited field increases, although not very rapidly. The reason for the increase in $E_{\text{lim}}$ with frequency is the need for a reduced dielectric time constant ($\tau = \varepsilon / \sigma = \text{the time for charge to redistribute}$) with increasing frequency. Note that the constant, $k$, multiplying the field in the exponential has the dominant effect on $E_{\text{lim}}$.

Another way to look at the determination of $E_{\text{lim}}$ is to plot $\sigma(E)$ vs $E$ and $\varepsilon\omega$ vs $E$ on the same graph (Figure 4.3). The latter is a straight, horizontal line, while the former is an exponential curve. The field at which these intersect is the limiting field, $E_{\text{lim}}$. The constant, $k$, has a large effect on the field of intersection, while changes in $\varepsilon$ and $\omega$ have much smaller effects as they simply shift the horizontal line up or down.

From eqn.(4.5), we can see that $E_{\text{lim}}$ is dependent on temperature, it increases as temperature $T$ decreases. At $T = 300$ K, the model of eqn.(4.1) results in a limiting field of 250 kV/mm at power frequency. The conductivity of XLPE as well as $E_{\text{lim}}$ can be adjusted by doping concentration as well as the density of PE. With increased value of $E_{\text{lim}}$, dielectric properties will improve for some applications but worsen for others [Jiang 1996].

![Figure 4.3. Conductivity (eqn. (4.1) at 300 K) and the product of $\varepsilon\omega$ vs electric field. The point at which the two curves intersect gives the space charge limited field, as at this field the resistive current density equals the displacement current density.](image-url)
4.4 Space Charge Distribution in High Field Dielectrics

This section describes the application of the FEM program to the needle-plane geometry with XLPE dielectric as sketched in Figure 4.4. An experimental study of this geometry has been carried out by [Zeller & Zubakov 1984]. We employ Zeller's data to test the validity and accuracy of our numerical simulation.

4.4.1 Introduction

Zeller et al. [Hibma et al. 1985, Zeller & Schneider 1984] have published a field limiting space charge (FLSC) model for high field dielectrics. This model assumes that no space charge is generated for a field, \( E \), which is below some critical field, \( E_c \). For a field above \( E_c \), space charge is generated to limit the field to \( E_c \). The FLSC concept is simple and facilitates analytical computation of the space charge density from Poisson's equation. Given the spatial distribution of the space charge density, other parameters can be estimated, such as the peak force density and maximum mechanical stress within the dielectric. However, the method does not facilitate computation of power densities, temperature rises, etc. as these parameters depend on the conductivity which goes exponentially with the electric field resulting in large changes in these parameters with small errors in the electric field. Computation of these parameters requires solution of eqn. (2.7) in the time domain with field-dependent conductivity and transient applied voltage, which is the task undertaken in this study.

Figure 4.4 Sketch of needle-plane geometry with XLPE between upper and lower planes, with defect on lower plane. The needle has a tip radius of 5 \( \mu \text{m} \), a cone angle of 30\(^\circ\), and the distance between the needle tip and plane is 35 \( \mu \text{m} \).
Figure 4.5a FEM results of time integral of the space charge-induced terminal current (induced charge) vs applied voltage. The applied voltage waveform is shown in the inset.

Figure 4.5b Test results from [Baumann et al. 1985], the time integral of the space charge-induced terminal current (induced charge) vs applied voltage.
The space charge distribution for PE in a needle-plane geometry has been computed to compare with experimental data provided in [Baumann et al. 1985]. In this case, the needle has a tip radius of 5 μm, a cone angle of 30°, and the distance between the needle tip and plane is 35 μm. The applied voltage is triangular, with a rise to 1800 V over 15 ms and a return to zero over an equal time.

4.4.2 Simulation

The test in [Baumann et al. 1985] is simulated by numerical computation as described in the previous sections. The charge on the plate which does not have the needle is the integration of terminal resistive current that comes from the voltage source. The integral of the resistive terminal current during this voltage waveform is provided in [Baumann et al. 1985] and is shown in Figure 4.5b. The test was done in the way that needle is guarded and compensation techniques are employed to null the residual displacement current from the needle tip so that the resistive (space charge-induced) terminal current can be measured. In our computations, we compute the space charge-induced terminal current from Gauss’ law [Boggs 1994], i.e., by differentiating the “image” charge (hereafter termed “induced charge”) induced in the conducting ground plane by the space charge.

Figure 4.6 The variation with time of the electric field at various locations along the axis of the needle of Figure 4.4. The effect of space charge in limiting the field is clearly evident. The figure indicates a limiting field of 150 kV/mm for the model parameters employed.
Figure 4.5a plots the square root of the induced charge vs applied voltage. According to the FLSC model [Zeller & Schneider 1984, Hibma & Zeller 1986], this plot should have a linear rise, as the charge should increase as \((V - V_c)^2\), where \(V_c\) is the critical voltage, i.e., the voltage at which the critical field is reached at the tip of the needle. Our computations (Figure 4.5a) show such a linear rise. The experimentally measured charge to the peak of the voltage waveform is \(3.1 \times 10^{-7}\sqrt{C}\), while the computed value is \(2.0 \times 10^{-7}\sqrt{C}\). Figure 4.6 shows the variation in the electric field at various points on the axis of symmetry of the needle. The effect of space charge in limiting the field is clearly evident, and the limiting field is about 150 kV/mm, in excellent agreement with the 149.5 kV/mm given by the criterion that \(\sigma(E_{lim}) = \varepsilon_0\). In these computations, the value of the coefficient multiplying the field in the exponential of Equ. (4.1) is \(1.264\times10^{-7}\), resulting in a limiting field of about 150 kV/mm, which was selected on the basis of the data published by [Baumann et al. 1985].

Zeller et al. [Zeller & Schneider 1984, Hibma & Zeller 1986] could not explain the increase in the integrated terminal current which takes place after the peak of the waveform. This appears to be a phenomenon peculiar to dimensions higher than one and appears to result from the spreading of space charge away from the needle axis as a result of the spatial variation in conductivity and field. Zeller et al. considered their model in various one-dimensional approximations in which such a phenomenon could not occur, and it is a phenomenon not observed in previous numerical solutions of Poisson’s equation in one dimension [Boggs 1994]. The spreading of space charge is very sensitive to the functional dependence of the conductivity on the field in the region of the limiting field. By matching the experimentally observed increase in charge after the peak voltage with that computed for various functional relationships between conductivity and field in the region of the limiting field, some idea can be gained of this functional relationship. Such analysis suggests that the conductivity may vary as something like the cube root of the electric field at fields close to the limiting field [Jiang, 1999].

**4.4.3 Discussion**

Agreement between the measured data and the computations is acceptable for a “first try” but leaves room for improvement. The extrapolation of the linearly rising portion of Figure 4.5, as discussed above, to the voltage axis results in an intercept of about 1100 V as compared to roughly 930 V as measured from Zeller’s data. According to Figure 4.6, the limiting field appreciably is below the geometric field on axis of the tip for 930V. i.e., about 160 kV/mm. Thus the formula for \(\sigma(E)\) needs to be changed to (1) lower the critical voltage from 1100V to 930V and raise the limiting field from about 150 kV/mm to 160 kV/mm. The induced charge is very sensi-
tive to the limiting field and will increase substantially with such a change. This is likely to bring the computed data in line with the published experimental data. It also provides a basis for improving the formula for $\sigma(E) \text{vs} E$.

### 4.4.4 Conclusions

Our preliminary results indicate the ability to compute space charge-induced phenomena using the FEM and time-stepping in two dimensions. This opens many possibilities, including that of simultaneous solution of thermal and electrical fields. As the thermal field changes much more slowly in time than the electrical field, the thermal field need to be solved much less frequently. Thus addition of the thermal field to the problem would probably not slow down computation a great deal. With the addition of the thermal field, we could include the temperature dependence of conductivity along with field dependence. Since the conductivity is strongly dependent on temperature, this is likely to alter the solution under some high field conditions. Computation of the thermal field along with the electrical field would also allow mechanical yield stresses to be estimated for the temperatures which actually occur within the high field region, and the thermally-dependent yield stress could be compared with the space charge-induced mechanical stresses to assess more accurately the mechanical contribution to electrical tree initiation.

Also, these techniques allow us to conduct “what if” analyses to investigate the effect of modifying the high field conductivity on “defect tolerance”. The literature provides clear evidence that defect tolerance can be improved through such modifications. However, the work to date has been conducted through empirical “trial and error”. Expansion of the present work should provide an analytical basis for such investigations.
4.5 Defect in PE under AC and Surge Conditions

4.5.1 Introduction

In this section, we show that under AC conditions, the temperature rise around a defect is small. For normal overvoltage (less than 5 p.u.) condition, the heat produced at the defect will diffuse sufficiently during a power frequency cycle to result in negligible temperature rise (much less than 1 K). Under surge conditions, the local temperature rise is substantial, as is the mechanical stress caused by thermal expansion which is larger than space charge-induced stress [Kuang 1997].

4.5.2 Material Parameters

XLPE has a field and temperature dependent electric conductivity which varies with the nature of the polymer. Analysis of one set of data resulted in the relationship [Kuang & Boggs 1997]

\[ \sigma(E,T) = \frac{62.24}{|E|} \exp\left(-\frac{6946}{T}\right) \exp(1.124 \times 10^{-7} |E|) \]  (4.6)

![Graph showing Heat capacity and thermal conductivity of XLPE vs temperature. Data courtesy of Union Carbide.](image)

Figure 4.7 Heat capacity and thermal conductivity of XLPE vs temperature. Data courtesy of Union Carbide.
Figure 4.8 Space charge distribution along axis of symmetry from the tip of the defect into the XLPE at four times during an half AC cycle. The space charge at the boundary of the defect and XLPE is not shown but is included in the computation of mechanical stress.

where $T$ is in Kelvins and $E$ is in V/m. The space charge limited field [Hibma et al. 1985, Baumann et al. 1985 and Hibma & Zeller 1986] is given by the condition that $\sigma(E,T) = \varepsilon \omega$ and is about 170 kV/mm. The thermal conductivity and heat capacity vs temperature are shown in Figure 4.7. The linear thermal expansion coefficient is $4 \times 10^{-4}$/K in the temperature range of 333 K to 366 K, and Young's modulus is $1.03 \times 10^9$ Pa. The defect is assumed to be semiconducting polymer with the same thermo-mechanical properties as the XLPE.

As a basis for investigating electro-thermo-mechanical phenomena at a stress enhancement in PE, we take a conical defect of 300 $\mu$m height, 30° cone angle, and 25 $\mu$m tip radius situated on a plane electrode in a background of 40 kV/mm for lightning impulse and 14.5 kV peak/mm for AC voltage. The 40 kV/mm lightning impulse represents 4 pu relative to a rated AC field of 10 kV rms/mm, which is a typical operating stress for transmission class solid dielectric cable. In all the computations, an initial temperature of 333 K is assumed.

4.5.3 Results under AC Condition

With a 50 Hz AC voltage which produces a background peak field of 14.5 kV peak/mm, the slow variation of the AC voltage results in no appreciable tempera-
Figure 4.9 Stress distribution along axis of symmetry from 2 μm inside the defect to 8 μm away from the tip of the defect at time instance of 4.7 ms when the magnitude of the voltage is maximum. Under AC condition, the temperature rise is negligible, and the existence of space charge is the only reason for the stress. It can be seen that space charge induced stress is too small to cause mechanical failure.

Figure 4.8 shows the variation of space charge with time along the axis of symmetry at four time instances during an AC half cycle after the cyclic space charge equilibrium has been established. It can be seen that the space charge in front of defect varies in space and time as a "wave". Before time = 0, there is a big chunk of negative space charge accumulated in front of defect, but about 11 μm from defect tip, there is some positive charge which was the leftover from the previous positive half cycle of the applied voltage. At time = 0 ms, the applied voltage on the defect change polarity from negative to positive, as a result, some positive charge has been injected just at the tip of defect. As time goes on at time = 2.9 ms, the front of the positive charge moves to 7.5 μm from the defect tip, pushing the negative charge to a smaller area but with a larger charge density. At time = 7.5 ms, while the positive applied voltage is declining, the positive charge has penetrated to 10 μm from the defect tip, when the time goes to 10 ms, the voltage goes back to 0 volt, the space charge presents the same distribution pattern as time = 0 ms, but with a reversed polarity. The space charge at the interface between the semiconducting defect and XLPE has not been shown as it is much larger than the space charge in XLPE. In the computation of mechanical stress, the space charge
at the boundary has been considered. Figure 4.9 shows the space charge-induced mechanical stress at 4.7 ms, close to the negative peak of applied AC voltage. At this time, the mechanical stress should be near maximum. Four stress components are shown in the figure. They are the normal stress components in the r, θ and z-direction, and the shear stress component. A positive normal component implies that the material is in tension, while a negative stress is compressive. The figure indicates that the maximum stress is 0.75 MPa which is well below the yield stress of XLPE at temperature of 333 K, as seen in Figure 4.10.

4.5.4 Lightning Impulse Voltage

Figure 4.11 shows the space charge distribution on axis for a 1.2/50 μs lightning impulse which produces a maximum background field of 40 kV/mm (4 p.u.). The numerically computed space charge is compared with the space charge distribution computed analytically assuming that the electric field is constant at the space charge limited field, and the defect tip can be approximated as a sphere for calculating the field near the defect surface.

![Graph](image)

Figure 4.10 Yield stress of XLPE as a function of temperature. The yield stress drops precipitously above 360 K. Data courtesy of Union Carbide.
Figure 4.11 The jagged line shows the space charge distribution on axis from the tip of the defect into the XLPE at 1.7 μs into the 1.2/50 μs impulse compared to analytical approximation.

Figure 4.12 3D plot of temperature vs time and position on the axis of symmetry. The maximum temperature is 366K at about 1.7 μs.
As a result of the short duration of the pulse rise time, the temperature increases rapidly in a small region just outside the defect tip. Figure 4.12 shows a 3D plot of temperature vs time and position on the axis of symmetry. The data show that the temperature is maximum (366 K) at about 1.7 μs.

The mechanical stress at 1.7 μs into the lightning impulse is shown in Figure 4.13. Four stress components are shown in the figure. they are the normal stress components in the r, θ and z-direction, and the shear stress component. A positive normal component implies that the material is in tension, while a negative stress is compressive. The maximum temperature rise of 33 K is localized, resulting in a very large compressive stress. The figure indicates that the maximum stress is 17 MPa which is higher than the yield stress of XLPE at temperature of 366 K, as seen in Figure 4.10.

Comparing the mechanical stress for the lightning impulse and AC conditions, we see that space charge is a relatively minor effect for the lightning impulse case but the entire effect for the AC case. In the AC case, an appreciable and cyclic stress is generated. Eventually the dielectric might be degraded to the point that it yields mechanically to the stresses computed here. Under lightning impulse conditions, larger space charge limited fields (which increase roughly as the log of fre-

![Figure 4.13 Stress distribution on axis from 2 μm inside the defect to 8 μm outside the defect at 1.7 μs into a 1.2/50 μs lightning impulse. σ_r, σ_θ and σ_z are normal stress components, and τ_{rz} is shear component.](image-url)
4.5.5 Temperature Rise vs. Waveform Risetime

*J-E*-induced heating takes place primarily during the rise of the waveform, when the creation of space charge density within the dielectric results in substantial current density [Kuang & Boggs 1996b]. If the applied voltage results in a Laplacian field (i.e. assuming no space charge) more than twice the limiting field, then a second redistribution of space charge takes place during the tail of the waveform; however, as the fall time of the tail is usually much longer than the risetime, this redistribution takes place over a much longer time with the result that the temperature rise induced thereby is "diluted" by thermal diffusion. Thus for waveforms...
Figure 4.15 Above, electric field profiles on axis for the 10 ns risetime waveform. Below, profile near time of maximum temperature rise plotted to the same distance scale (from 300K) for risetime (10 to 90%) of 10 ns, 100 ns, and 1.2 µs (lightning impulse). The transition from zero field indicates the edge of the tip in both the thermal and electric field profiles.
Figure 4.16. Temperature rise (from 300 K) on axis from 2 µm within the semiconducting stress enhancement to 8 µm into the dielectric for 100, 200, and 300 µm protrusions for lightning impulse, 100 ns step wave, and 10 ns step wave, each of which produces a maximum background field of 40 kV/mm. The total energy dissipated during the rise of the voltage waveform appears to be fairly constant. The peak temperature increases with decreasing risetime as a result of reduced diffusion of heat away from the region of maximum power dissipation.

such as a standard 1.2/50 µs voltage lightning impulse, the maximum temperature rise occurs near the peak of the pulse, as seen in Figure 4.14.

The effect of risetime on temperature rise is interesting in a number of respects. As the risetime is reduced (from 1.2 µs to 100 ns and 10 ns) for a constant voltage amplitude (which produces a background field of 40 kV/mm), the limiting field increases as a result of the effective higher frequency which reduces the space charge density required to achieve the limiting field. On the other hand, the field increases so that in considering the time integral of $JE$ the integral of $J$ decreases but the integral of $E$ increases. If the waveform levels off at the crest, then the effective frequency drops and the space charge relaxes to a higher density while the limiting field drops. As can be seen from Figure 4.15, the net effect of these opposing "forces" is to increase the temperature rise with decreasing waveform risetime. The
total energy dissipated in all cases is probably very similar; however, with longer risetime, the heat has more time to diffuse away from the region of maximum power dissipation, which results in reduced peak temperature. Examination of Figure 4.15 indicates that the reason for the difference in peak temperatures is the degree of diffusion of heat. The maximum power dissipation is in the dielectric immediately adjacent to the tip; however, no power is dissipated in the tip as it is conducting. The very sharp power gradient at this boundary results in an extremely sharp temperature gradient for short risetime pulses, in the range of 500 K/µm for the 10 ns risetime waveform. As a result of this situation, the thermal distribution is far from adiabatic even for a 100 ns risetime waveform. The distribution of temperature vs distance (as the temperature falls off in the dielectric) have little difference between 3 cases because the power dissipation in all cases is similar so that the temperature falls off smoothly which limits dT/dr and the resulting diffusive heat flow.

4.5.6 Temperature Rise vs. Defect Severity

Figure 4.16 shows temperature rise vs. defect severity. For the smallest defect (100 µm) height, the field drops from the space charge limited value about 8 µm away from the tip. The most remarkable feature of the data in Figure 4.16 is the very similar temperature rise with a change of two orders of magnitude in waveform risetime, from 1.2 µs for the lightning impulse to 10 ns for the shortest risetime pulse. Each curve represents the temperature profile on the axis of symmetry at a time corresponding to approximately the 95% voltage on the rise of the waveform. The total power dissipated (roughly proportional to the area under each curve) appears to be very similar in all cases, which means that the power density increases approximately inversely with waveform risetime. Note that for the 10 ns risetime, the temperature must have changed at a rate of about 3 GK/s! Such a rapid change in temperature by tens of K should produce an acoustic shock wave which is readily detectable.

4.5.7 Conclusions

Computations of the thermal and space charge induced mechanical stresses at a defect in XLPE indicate that under AC conditions, the temperature rise and thermally-induced mechanical stresses are negligible. However, cyclic space charge formation results in a cyclic stress in the range of 750 kPa which is small compared to the room temperature yield stress of XLPE but about half the yield stress at 100 °C.

Under lightning impulse condition, the current density necessary for space charge formation results in substantial power dissipation in the immediate vicinity of a defect. The resulting temperature rise and thermal expansion cause a large,
compressive mechanical stress in the immediate vicinity of the defect which far exceeds the space charge induced tensile stress, as seen in Figure 4.13.
4.6 Conversion of Water Trees to Electrical Trees

4.6.1 Introduction

Water (or electrochemical) trees are dendritic patterns of electro-oxidized polymer which grow in hydrophobic polymers in the presence of AC electric field and water. Recent evidence [Moreau et al. 1993] indicates that in the growth region, water trees consist of "tracks" (about 10 nm in diameter) of oxidized polymer which connect microvoids. Water treeing is a self-propagating pattern of electro-oxidation, self-propagation results from the fact that electro-oxidation of a wide range of hydrophobic polymers causes the oxidized polymer to become much more hydrophillic which results in condensation of moisture from the hydrophobic polymer into the hydrophillic electro-oxidized region. This results in extension of the water-filled region, facilitates distortion of the electric field, continued electro-oxidation, and self-propagation through the polymer in the form of a tree-like structure. The relevance of the microvoids is very likely statistical in nature, i.e., the likelihood of survival of a track is increased if it encounters a microvoid from which it can continue to grow.

Electrical trees are dendritic patterns of electro-mechanical damage caused by high field phenomena and the chemical effects of discharge within the tree channels. Electrical trees start from a stress enhancement. Initially charge cycling in and out of the dielectric results in electron-hole recombination, generation of UV photons (resulting in breaking of chemical bonds), and degradation of the dielectric in the immediate region of the stress enhancement. Eventually a cavity is generated which is large enough to support partial discharge, initially in the fC region. This initiates an electrical tree. An electrical tree grows through electro-mechanical and electrochemical phenomena based on gas discharge, charge injection and space charge limited fields (under impulse conditions of sudden discharge, etc.)

In general, water trees do not cause failure of in-service XLPE cable, rather electrical trees initiate from water trees as a result of lightning surges. The circumstantial evidence for this assertion is strong, in that cable failures often occur days after heavy summer lightning activity in the southeast of the United States. Also, electrical trees have been found growing from water trees in XLPE cable removed from service.

We have carried out computations which shed light on the means by which electrical trees are initiated from water trees. We believe that this occurs as a result of electro-thermo-mechanical phenomena induced by the lightning voltage surge. For moderate water conductivity, the high electric field caused by the combination of the lightning-induced voltage and the distortion of the electric field in the polymer by a large water tree results in power dissipation in water tree channels near the tip.
100 µm long, 0.5 µm radius channel protruding from water tree

Water Tree (2.5 mm long x 0.5 mm radius)

Figure 4.17 Geometric model employed for finite element analysis. The water tree channel protruding from the tip of the water tree is too fine to be visible.

of the water tree sufficient to boil the water therein, presumably exploding the polymer and leaving behind a cavity sufficient to support partial discharge and initiate an electrical tree. For higher water conductivity, the water can act as a conductor and cause an electric field at the tip of a water tree sufficient to raise the temperature of the XLPE through the region in which the XLPE expands by about 20%, which would cause (i) several orders of magnitude reduction in the yield stress of the XLPE, (ii) substantial electromechanical stresses as a result of the action of the space charge limited field in the XLPE (about 250 kV/mm at power frequency and proportional to the logarithm of frequency) [Jiang et al. 1996, Fukawa et al. 1996] with the space charge [Boggs 1995], and (iii) a mechanical shock wave as a result of the substantial thermal expansion of a micrometer region of the XLPE over a period of a few microseconds. Either mechanism would be sufficient to initiate an electrical tree from a water tree; however, we feel that heating of the water is the more likely mechanism under most conditions.

4.6.2 Dielectric Model for a Water Tree

At the growth front, water trees probably consist of tracks of oxidized (and therefore hydrophillic) polymer within the hydrophobic XLPE [Zeller 1991]. These oxidized, hydrophillic tracks result in condensation of water from the hydrophobic matrix into the hydrophillic track. Producing a chemical potential sufficient for water tree growth requires substantial conductivity in such very narrow tracks [Zeller 1991], which means that a substantial supply of ions is probably required. The
growth front of the water tree is only visible as a series of water-filled microvoids; the tracks are not normally visible [Moreau et al. 1993].

The visible tubules in the majority of the water tree are presumably tracks which have been oxidized to form water-filled channels. If the conductivity of the water in these channels is similar to that of the water in the tracks, then the channels should be substantially conducting (i.e., they should have a dielectric time constant which is short compared to power frequency). Given the dense pattern of visible channels in a typical water tree, we can reasonably model the region as conducting, at least at power frequency. Thus a thin, water-filled tubule protruding from a conducting, water tree-filled region is one logical model for investigating initiation of an electrical tree from a water tree.

Here, we model a 15 kV XLPE cable with an insulation thickness of 4.51 mm (Figure 4.17). We take the bulk of the water tree as a conducting cylinder with a hemispheric tip with its cylindrical axis perpendicular to the semiconducting layer. We assume that a water tree channel which is 0.5 μm in radius and 100 μm long is protruding from the tip of the conducting water tree region. Obviously, this is only one of many models which could be employed. We can model the treed region with a range of properties, from fully conducting, to dielectric constant of 80 with no conductivity, to intermediate value of conductivity and dielectric constant; however, similar results are obtained if the water tree region (but not the channel protruding therefrom) is taken as having a dielectric constant of 80 with no conductivity compared to fully conducting water tree region.

4.6.3 Lightning Surge-Induced Phenomena

4.6.3.1 Power Dissipation vs Water Conductivity

When a water-filled, partly conducting tubule is exposed to a transient high field, charge redistributes in the tubule to cancel the field within the partially conducting fluid (water). This occurs with a time constant which depends on the conductance of the tubule, i.e., the product of the water conductivity times the cross section of the tubule, and the geometry of the system. If the time constant is long compared to the duration of the impulse, then the current density is low and little power is dissipated.

If the time constant is comparable to the risetime of the impulse, then the current density is high and the resistance is appreciable, which results in substantial power dissipation in the water within the tubule. As well, charge accumulation at the ends of the tubule (one end where the tubule meets the XLPE and the other end
where the tubule meets the high conductivity water tree region) along with the high dielectric constant of the water in the tubule relative to that of the XLPE result in substantial field enhancements at the ends of the tubule. This causes space charge injection in the XLPE, electromechanical forces, and heating as a result of $JE$ power dissipation therein.

If the time constant for redistribution of charge in the water of the tubule is very short, then the current density is high during the rise of the impulse, but resistance is very low, and little power is dissipated. However in this case, the water acts as a conducting protrusion, and the field at the tip of the tubule will be very high which results in substantial space charge formation, heating in the XLPE as a result of field-dependent conductivity and $JE$ (field times current density) power dissipation, and electromechanical forces as a result of the electric field acting on the injected space charge [Boggs 1995, Boggs et al. 1994]. Given the reduction in the yield stress of the XLPE caused by the $JE$ heating, the XLPE is likely to yield.

From the above considerations, we expect the conductivity of the water in a channel must change in inverse proportion to the channel cross section to maintain the time constant for redistribution of charge within the channel.

Our choice of a 1 μm diameter water tree channel is dictated mainly by practical considerations associated with application of finite element analysis. For a 1 μm diameter channel, the smallest mesh elements must in the range of 0.1 μm in extent, while the largest dimension in the problem is about 5 mm. The range of geometry requires in the range of 2000 mesh elements. However, if we were to conduct the analysis for a 0.1 μm diameter channel, we would expect that the optimum water conductivity would be 100 times greater, but we would also expect a substantial temperature rise in the water over a greater range of water conductivity.

4.6.3.2 Space Charge Formation

In most solid dielectrics, space charge results from redistribution of charge within a dielectric. However, in the presence of a water tree, two forms of space charge will oscillate with the AC cycle. The dominant form is probably ionic space charge which forms and oscillates at the ends of the water tree tubules as the result of ion motion in the water during the AC cycle (the ions are the result of impurities in the semicon dielectric interface. The semicon is filled with carbon black - at one time rather dirty furnace black, which has a lot of ionic impurities. Today, much cleaner forms of carbon black are used). The second form is space charge injected from the tip of the water tree channels. The space charge density vs time for these two forms of space charge are predictable and will differ substantially.
Figure 4.18 Electric Field and Temperature profiles from just inside the conducting water tree region (left), along the axis of the water tree channel, and into the XLPE beyond the tip of the water tree channel. The temperature rise (from an initial 330 K) is greatest at the base of the water tree channel for reasons discussed in the text.

4.6.4 Model for XLPE

Based on measurements [Boggs et al. 1994] and data in the literature for XLPE cable dielectric [Boggs 1995], we employ the following formula for the conductivity, $\sigma$ (S/m) of the XLPE vs temperature, $T$ (K), and electric field, $E$ (V/m).

$$\sigma(E,T) = \frac{62.24}{|E|} \exp\left(-\frac{6945.71}{T}\right) \exp\left(7.796 \times 10^{-8} |E|\right)$$ (4.7)

As this equation blows up at small $E$, we assume that $\sigma(E,T) = 0$ for $|E| \leq 10$ kV/mm. This formula is used in all computations but those of Figure 4.23. The extreme nonlinearity caused by XLPE conductivity can slow the computations greatly.

In the present analysis, system material properties are modeled as follows:

- XLPE Heat Capacity and Thermal Conductivity as a function of temperature are modeled as per Figure 4.7.
- XLPE Electrical Conductivity is modeled as per eqn. (4.7).
- Water Volumetric Heat Capacity is given by eqn. (4.8), the second term of which is a Gaussian with a standard deviation of 1 K centered at 373 K which integrates to the heat of vaporization of water
  $$RC(T) = 4.18 \times 10^6 + 9 \times 10^5 \exp\left(-\frac{(T-373)^2}{2}\right) \frac{J}{m^3 K}$$ (4.8)
- Water Thermal Conductivity is modeled by
\[ K(T) = 6.1 + 0.01(T-300) \frac{W}{m \text{ K}} \]  \hspace{1cm} (4.9)

- We model the water conductivity in the range of 0.001 S/m to 100 S/m, that range of water conductivities are very reasonable. For example, at low to moderate NaCl concentrations, the derivative of the conductivity with respect to ion concentration is about 540, so that a concentration of 20 ppm NaCl results in a conductivity of about 0.01 S/m at 300 K. Thus the above conductivities are easily achieved and do not require a very high ion concentrations.

- Water Electrical Conductivity is modeled as increasing by 2.5%/K from a nominal value at 300 K. This is a typical temperature coefficient for ionic conductors. Note that this implies that the conductivity at 330 K is about 2.1 times that at 300 K while the conductivity at 373 K is 6.1 times that at 300 K. The nominal conductivity given on the Figures is at 300 K.

The applied voltage is a standard lightning impulse (1.2 /50 µs) of 80 kV magnitude which is appreciably less than the (Basic Insulation Level) BIL of 15 kV cable (95 kV).

### 4.6.5 Results

Figure 4.17 shows the model employed in these computations. As noted above, the 100 µm long, 0.5 µm radius channel protruding from the tip of the conducting water tree region is not visible in the figure.

Figures 4.18 shows the electric field profile and temperature profile along the axis of the 100 µm long water tree channel, from slightly inside the conducting water tree region (to the left) to beyond the tip of the water tree channel for the case of 0.01 S/m water conductivity at 3 µs into a standard lightning impulse, near the 80 kV peak magnitude. Current in the water tree channel increases from the tip of Figure 4.19 Equithermal plot at tip of water tree channel for the parameters of Figure 4.18. Note that the peak temperature at the tip of the water tree channel caused by high field conductivity in the XLPE occurs slightly off the tip, i.e., not at the point of highest electric field and power dissipation because the water in the tree channel acts as a heat sink. The temperature gradient along the water tree channel near the tip is clearly visible, as is the radial temperature gradient from the water into the XLPE. The channel radius is 0.5 µm.
Figure 4.20 Profile of the temperature along a radial line through the water tree channel near the region of maximum temperature rise. The temperature (and electric field) are relatively constant through the 0.5 μm of water. Thermal diffusion during the 3 μs of the lightning impulse is substantial.

Figure 4.20 shows a radial profile of the temperature near the location of maximum temperature rise along the water tree channel and indicates an appreciable effect from thermal diffusion.

the water tree channel to the base, which results in the increasing electric field and increasing temperature from the tip to the base, as seen in Figure 4.18.

The field in the lower part of the water tree channel is about 150 kV/mm, which would result in a power dissipation (σE^2) of about 10^{16} W/m, taking into account the temperature dependence of the water conductivity. Over a period of 1 μs this would dissipate 7.9 nJ per μm of water tree channel length (for a 1 μm diameter water tree channel). Given a heat capacity for water of 4.18 x 10^6 J/K·m^3 and heat of vaporization of 2.26 x 10^9 J/m^3, 0.14 nJ per μm of channel length is required to take the water from 330 K to boiling, and 1.77 nJ per μm of channel length is required to vaporize the water in the channel. Thus we might expect the pressure of the water in the channel to rise to well above atmospheric. However, the above computation overestimates the power dissipation as (i) the water conductivity increases with increasing water temperature, and (ii) the field increases during the impulse waveform and we have taken the value at the peak of the waveform. Figure 4.18 indicates that the water has reached the boiling point (at atmospheric pressure) along most of the length of the water tree channel. The temperature and field drop near the tip of the channel as a result of reduced current near the tip of the channel as mentioned above.

Figure 4.19 shows an equithermal plot of the tip region of the water tree channel. The longitudinal temperature gradient near the tip of the water tree channel is apparent, as is the high field conductivity-induced power dissipation in the XLPE at the tip of the water tree. Figure 4.20 shows a radial profile of the temperature near the location of maximum temperature rise along the water tree channel and indicates an appreciable effect from thermal diffusion.
Figure 4.21 Electric field and temperature profiles for water conductivity of 0.001 S/m at 10 μs into the lightning impulse waveform. The temperature rise is modest at this relatively low water conductivity.

Figure 4.22 Electric potential, electric field, and temperature profiles for water conductivity of 0.1 S/m at 1 μs into the lightning impulse waveform for a computational model which includes high field conductivity in the XLPE. Even before the peak voltage of the impulse, the water is boiling over most of the tree channel length, and the high field conductivity-induced temperature rise in the XLPE at the tip of the water tree channel is over 140 °C.
Figure 4.23 Electric field and temperature profiles for water conductivity of 0.1 S/m at 1 μs into the lightning impulse waveform for a computational model in which the XLPE conductivity is assumed to be zero. The data are unphysical in that XLPE cannot support a field of 3000 kV/mm. The large field at the tip distorts the field along the water tree channel and reduces power dissipation therein so that the water boils over a reduced longitudinal extent of the water tree channel relative to the data of Figure 4.22 which include the effect of high field conductivity in the XLPE.

The sudden change and peak in the electric field at about 0.115 mm in Figure 4.18 occurs at the transition from the water tree channel to the XLPE. The space charge limited field of about 350 kV/mm is typical for lightning impulse conditions. The high field conduction in the XLPE results in an appreciable temperature rise at the boundary between the two materials, as indicated by Figures 4.18 and 4.19.

Figures 4.21 shows similar data for water conductivities of 0.001 S/m. The temperature rise is modest, at about 30 K and peaks at the base of the water tree channel as a result of the capacitive current coupling described above. The temperature rise at 10 μs is greater than that at 5 μs, which indicates that the dielectric time constant for redistribution of charge in the water tree channel is long compared to the risetime of the lightning impulse, so that substantial current continues to flow in the water tree channel even during the fall of the lightning impulse waveform. The temperature peak from high field conductivity of the XLPE is small, as a large voltage drop occurs along the water tree channel resulting in a modest field at the tip.

Figures 4.22 and 4.23 show electric field and temperature profiles for a water conductivity of 0.1 S/m (at 300 K) and 1 μs into an 80 kV lightning impulse. The data of Figure 4.22 were computed with field dependent conductivity for the XLPE.
Figure 4.24. Electric field and temperature profiles on axis of a 1 μm diameter water tree channel for a water conductivity of 100 S/m at 3 μs into a standard lightning impulse of 80 kV magnitude. The water boils over most of the length of the water tree channel.

while those of Figure 4.23 were computed with zero conductivity assumed for the XLPE. Comparing the two figures, we see that assuming zero conductivity for the XLPE results in unphysical data and substantially different results. Without high field conductivity in the XLPE, the field must drop suddenly at the tip of the water tree channel where charge accumulates as a result of the much larger conductivity

Figure 4.25 Electric field and temperature profiles on axis of a 1 μm diameter water tree channel for a water conductivity of 1000 S/m at 3 μs into a standard lightning impulse of 80 kV magnitude. The conductivity is sufficiently high that the water does not boil during the lightning impulse.
Figure 4.26. Electric field and temperature profiles for water conductivity of 0.1 S/m at 2.5 μs into the lightning impulse waveform for a computational model which assumes that the water is confined at constant volume so that it cannot boil and which also includes high field conductivity in the XLPE. As seen from Figure 4.27, the maximum temperature in the channel of about 480 K would result in a pressure throughout the channel of about 1.6 MPa or 16 bars.

Figure 4.27. Water vapor pressure as a function of temperature for the roughly 420 K temperature over most of the water tree channel in Figure 4.26, the pressure would be about 0.4 MPa or about 4 bar. However, the pressure would be determined by the highest temperature in the channel of about 480 K which occurs near the tip. This would result in a pressure of over 1.6 MPa or 16 bars in the channel which is well above the yield stress of the polyethylene.
in the water tree channel relative to the XLPE. This results in a field of over 3000 kV/mm in the XLPE at the tip of the water tree, which is totally unphysical. The large field results in a change in the field distribution which reduces the capacitively coupled current near the tip of the tree with a resulting reduction in the temperature in the tip region relative to Figure 4.22. Thus in Figure 4.22, the water is boiling over a larger longitudinal extent of the tree channel, and the peak temperature occurs in the XLPE at the tip of the water tree where the peak temperature of about 415 K (142 °C) would probably reduce the yield stress of the XLPE sufficiently that it will yield as a result of the boiling water-induced pressure and the electromechanical forces. Later in the chapter, we will present the results from the modified program to compute electro-thermo-mechanical stresses.

Data have also been computed for a water conductivity of 1 S/m, for which the water is already boiling at 0.44 μs into the lightning impulse, at which time the peak temperature in the XLPE is about 420 K. The water also boils for a water conductivity of 100 S/m (Figure 4.24); however, it does not boil for a conductivity of 1000 S/m (Figure 4.25). In none of the cases examined, does the computation “run away” in the sense that the water vaporizes totally and the temperature increases rapidly thereafter. However, for all conductivities between 10^{-2} and 100 S/m, the water boils over most of the tree channel length. This very broad range of conditions suggests that lightning impulse-induced boiling of the water in water tree channels could create cavities capable of supporting partial discharge and initiating an electrical tree.

The data of Figure 4.26 were computed assuming that the water is confined in the water tree channel at constant volume so that the water cannot boil. In this case, the maximum temperature in the water tree channel is in the range of 480 K which results in a pressure within the water tree channel in the range of 1.6 MPa or 16 bars (Figure 4.27). Obviously, this combination of temperature and pressure could not occur, which implies that at some combination of temperature and pressure, the water tree channel must expand, resulting in some boiling of the water and increase in the water tree channel volume. The heat required to boil the water would reduce the temperature rise of the water and polyethylene but would result in expansion of the water tree channel in a manner which is likely to leave a partially filled cavity capable of supporting partial discharge and initiating an electrical tree after the lightning surge.

**4.6.6 Conclusion**

The above results suggest a mechanism for conversion of water trees to electrical trees under impulse conditions. The impulse voltage-induced transient current can cause the water in a tree channel to boil (although not necessarily at 373 K or 1
Figure 4.28 Graph of Laplacian field for electric tree initiation vs. defect (needle) tip radius, reproduced from [Fukawa et al. 1996, Kubota et al. 1994].

atmosphere). the temperature of the surrounding XLPE to increase sufficiently to reduce yield stress substantially, a temperature increase at the tip of the water tree channel as a result of high field conductivity in the XLPE, and substantial electromechanical forces as a result of the action of the field on the space charge. The overall result is yielding of the XLPE, generation of a cavity which can sustain partial discharge, and initiation of an electrical tree.
Figure 4.29 Typical plot of the Laplacian (without the effect of space charge) and Poisson (with space charge) electric field on the axis of symmetry vs distance from a needle tip when the Laplacian field at the needle tip is appreciably above the limiting field value defined by $\sigma(E_{\text{lim}}) = \varepsilon \omega$. Note the space charge limited field region of the Poisson field, which is the focus of the present investigation. The space charge limited field region extends from the tip of the needle (0 distance) to the point at which the Poisson field starts dropping rapidly, slightly over 3 µm in the above graph. The data in Figure 4.28 are for the Laplacian field, which is not the true field within the dielectric.

4.7 Criteria for Electrical Tree Formation in XLPE

Published data for electrical tree inception field at a defect in XLPE vs. the Laplacian field at the tip of the defect are explained on the basis of a minimum distance which the space charge limited field must extend from the defect tip into the field direction to facilitate PD inception and tree initiation.

4.7.1 Introduction

In two recent publications [Fukawa et al. 1996, Kubota et al. 1994], a graph (Figure 4.28) based on limited measured data was presented for the Laplacian field in XLPE required for tree initiation vs defect tip radius. No indication was provided for the geometry in which the experiments were carried out; however, the tip radii employed suggest the use of Ogura needles in a needle-to-plane geometry. No hypothesis is provided for the data other than the vague statement that they probably result from space charge at the tip of the needle. The purpose of this work is to provide a physical and analytic basis for Figure 4.28, and in doing so provide an indication if the figure is likely to be accurate for small tip radii.

For any dielectric, the conductivity increases with the electric field at very high fields. With increasing field under AC conditions, a point is reached where $\sigma(E_{\text{lim}}) = \varepsilon \omega$, where $\sigma(E)$ is the field-dependent conductivity, $\varepsilon$ is the absolute dielectric constant, and $\omega$ is the angular frequency. When this condition is reached, the resistive
current density is equal to the capacitive current density, and space charge is generated to limit the field to approximately $E_{\text{lim}}$. Thus as the applied voltage on a stress enhancement is increased, a point is reached at which the peak field at the defect reaches $E_{\text{lim}}$. As the applied voltage is increased above this value, the field at the tip no longer increases but rather a region of approximately constant field ($E_{\text{lim}}$) spreads out from the tip. This can be seen in Figure 4.29, which shows the field vs position on axis as computed both without the effect of space charge (the Laplacian field) and with the effect of space charge (the Poisson field) for a field-dependent conductivity given by Eqn. (4.10) below.

Our hypothesis to explain Figure 4.28 is that initiation of an electrical tree requires sufficient damage to the XLPE at the defect tip that electro-chemo-mechanical (or in the case of impulse waveforms, electro-thermo-chemo-mechanical) effects create a cavity at the defect tip which can support partial discharge. In previous publications [Boggs et al. 1994, Kuang & Boggs 1996a, Jiang & Boggs 1995], we have pointed out that it is easier for electrical trees to grow than to initiate as a result of the transient nature of the electric field at the tree tip during tree growth. Presumably, the cavity created at the defect tip must have a minimum extent along the direction of the electric field in order to support partial discharge of sufficient energy to facilitate tree growth. We assume that the region of appreciable damage in the XLPE is limited to the space charge limited field region, in which the mobility of charge carriers is relatively high. The combination of UV photons and “hot electrons” generated in this region break chemical bonds which weakens the dielectric [Baumann et al. 1985, Hibma et al. 1985]. The combination of substantial space charge (several thousand coulombs/m$^3$) and electric field ($E_{\text{lim}} = 250$ kV/mm) results in an electromechanical force density and mechanical stress which can be a few percent of the yield stress of XLPE at room temperature, rising to 20% of the room temperature yield stress for an impulse waveform. In addition, impulse waveforms can cause substantial heating above the normal dielectric operating temperature [Kuang 1996a] which can lower the yield stress of XLPE by a factor of 100.

4.7.2 Computational Approach

As noted in the introduction, our hypothesis is that a minimum space charge limited field region is required in the direction of the electric field in order to initiate an electrical tree. Based on this hypothesis, we can compute data corresponding to Figure 4.28 for various needle tip radii and various extents of space charge limiting field in the direction of the applied electric field. To do this, we must assume a conductivity vs electric field for the XLPE. Based on both the data of Figure 4.28 and measurements of the threshold field for high charge mobility for two commercial XLPE compounds [ABB], we deduce that the power frequency limiting field for cable
XLPE is approximately 250 kV/mm and that the conductivity vs field is given by eqn.(4.10), which is derived from eqn.(4.7) when $T = 300$ K.

$$\sigma(E) = \frac{5.497 \times 10^{-9}}{|E|} \exp(7.796 \times 10^{-8} |E|)$$  \hspace{1cm} (4.10)

Previous computations [Kuang 1996a] indicate that the exact nature of the equation for $\sigma(E)$ has little effect on the power dissipation, electromechanical forces, etc., as long as the limiting field, $E_{\text{lim}}$, defined by $\sigma(E_{\text{lim}}) = \epsilon \omega$, remains constant. In particular, we have investigated the case of $\sigma(E)$ proportional to $\exp(k \sqrt{|E|})$ rather than $\exp(k |E|)$ and have found only minor differences in the resulting phenomena. The underlying phenomena are so fundamental that details such as the functional dependence of conductivity on the electric field have little effect. Basically, either the conductivity must rise to the point that space charge limits the field, or the field will go to totally unreasonable levels. The only effect of the functional dependence of the conductivity on the field is in the slope of $\sigma(E)$ vs $E$ in the region where $\sigma(E) = \epsilon \omega$, and this has a minor effect on the change in the limiting field with the Laplacian field. Thus we have reasonable confidence in our computations even though we do not have high confidence in our knowledge of the exact functional re-

![Figure 4.30](image_url)  

Figure 4.30. Extent of (space charge) limited field region vs applied voltage for various tip radii.
The relationship between the conductivity and the field other than the condition that \( \sigma(E) = \varepsilon \omega \) for a field, \( E \), of approximately 250 kV/mm.

Under AC conditions, the temperature rise is negligible, so that we can ignore the temperature dependence of the high field conductivity. Given eqn.(4.10), we can employ the program for transient, nonlinear finite element analysis to solve for the field distribution around tips of various radii. The geometry employed is needle-to-plane with a tip to plane separation of 3.25 mm. The geometry is not critical since the data are plotted as a function of the Laplacian field, as in Figure 4.28. The primary parameter which determines the distance to which the space charge limited field extends is the ratio of Laplacian field to space charge limited field. For a “large” needle tip radius, the Laplacian field near the tip is relatively constant, as very near the tip surface, the tip looks like a plane. The distance from the tip surface to which the field is relatively constant decreases with the tip radius. On the other hand, smaller tip radii result in a large Laplacian field at the tip surface for a given geometry and applied voltage. As a result, the distance to which the space charge limited field extends is a complicated function of the tip radius, Laplacian field at the tip, and, to a much lesser degree, the geometry.

The present analysis consists of the following steps:

1. The geometry is defined, and the voltage-normalized Laplacian field is determined at the tip surface.

![Figure 4.31. Laplacian field at the needle tip vs extent of (space charge) limited field region for various needle tip radii.](image)
2. The space charge limited field region in the direction of the field is determined through use of the transient, nonlinear FE program with an applied voltage which increases linearly over a period of 5 ms. This is a cumbersome computation, as the finite element solution for the electric field is computed thousands of times as the voltage is increased, so that the high field conduction-induced space charge developed between two consecutive solutions affects the field distribution during the next solution. A ramp voltage is employed rather than a sine wave, as for a sine wave, the space charge relaxes and the extent of the field limited region increases as dV/dt drops near the peak of the sine wave. This makes a definitive determination of the extent of the space charge limited region difficult. We therefore employed a linearly increasing voltage with a dV/dt similar to that for a power frequency sine wave (i.e., a voltage rise time of 5 ms). Since the limiting field goes as ln(ω), the effect of the slight change in dV/dt is immaterial. A typical plot of the field vs distance on axis from the needle tip is shown in Figure 4.29.

3. A linear least squares fit is made to data for the voltage vs radial extent of the space charge limited field for each tip radius. As seen in Figure 4.30, the slopes for the various tip radii are similar. We have also demonstrated that data for various peak voltages (differing dV/dt) overlap for a given tip radius.

![Laplacian electric field vs tip radius](image_url)

**Figure 4.32.** Plot of Laplacian field for tree initiation vs needle tip radius for various assumed space charge limited field extents. The line from Figure 4.28 is superimposed on the data.
4. The applied voltage of Figure 4.30 is converted to the Laplacian field at the needle tip by multiplying the applied voltage by the voltage-normalized Laplacian field as determined in step 1. Figure 4.31 shows the resulting graph.

5. From Figure 4.31, the Laplacian field and extent of the space charge limited field region can be determined for each needle tip radius. This allows the data to be plotted as the Laplacian field vs tip radius for various space charge limiting field extents, as seen in Figure 4.32, which also shows the line of Figure 4.28.

4.7.3 Discussion

In examining Figure 4.32, we note that a limiting field extent in the range of 1 to 1.5 μm provides the best fit in the range of 5 to 10 μm tip radius. For tip radii below 4 μm, the computed data fall seriously below the data of Figure 4.28 and the line drawn therefrom. We must therefore consider two aspects of the problem, viz., (i) the meaning of the data points and the line fitted thereto in Figure 4.28 and (ii) the meaning of the present computations.

The data of Figure 4.28 represent the Laplacian field at the needle tip at which tree initiation occurred for various needle tip radii. The experimental procedure is
not provided. but one must assume that the experiments were carried out by applying successively higher voltages to the needle until tree initiation occurred. Presumably, the needle was left a relatively long time at each voltage level. This experiment is subject to several forms of variability, including

1. The tree initiation field varies with the local morphology of the polyethylene. Figure 4.33 shows a histogram of the tree initiation field vs position for a commercial XLPE cable compound as measured with a needle tip radius of 3.8 μm. Note that the tree initiation field (space charge limited field) varies greatly with position as a result of inhomogeneity of the polyethylene, i.e., variability of the local environment at the needle tip depending on the location at which the needle is inserted in the XLPE [ABB].

2. As the needle tip radius is reduced, the volume of XLPE “sampled” by the tip decreases, and we expect the variation in the electrical tree initiation field with position to increase.

3. Obviously, needle-based tree inception experiments can never determine the “minimum possible” tree initiation field. Experiments are doomed to measure a greater tree initiation field, and how much greater is unknown. However, the variability undoubtedly increases with decreasing tip radius, with the result that we place little confidence in the very few measurements at small tip radii shown in Figure 4.28.

Finally, the Laplacian field at the needle tip is a poor variable with which to correlate electrical tree inception, as the Laplacian field never occurs. In reality, only the space charge limited field occurs, and as seen in Figure 4.33, this field varies with local morphology throughout the dielectric. We have no idea how the variation in space charge limited field goes with the volume of material over which we average the measurement; however, we expect the variability to increase as the volume decreases.

Thus we have reason to question the data of Figure 4.28, both in terms of the relevance of the coordinates on which the data are plotted and the degree to which the data approximate the minimum tree initiation field. We can be quite certain that the reliability of the data decreases with decreasing needle tip radius; however, we cannot estimate the degree to which the reliability decreases.

In the present computations, we have based our estimate of the minimum Laplacian field for electrical tree initiation on the assumption of a minimum extent of space charge limited field region in which material damage occurs at a relatively high rate. We have assumed a value of power frequency limiting field based on both experimental measurements of the minimum limiting field for a commercial XLPE cable compound and the data in Figure 4.28. Based on our computations as pre-
presented in Figure 4.30 and through comparison with the data of Figure 4.28, especially at 5 and 10 μm tip radius where the data are well clustered, we can suggest that the extent of space charge limiting field region (from the surface of the needle-tip) required for electrical tree initiation is in the range of 1 to 2 μm. If this is true and if our hypothesis is correct, then the minimum Laplacian field required to initiate electrical treeing for small needle tip radii is substantially less than predicted by Figure 4.28, with the Laplacian field increasing by about a factor of 2, from 5 μm tip radius to 1.5 mm tip radius, rather than the factor of 3 suggested by Figure 4.28. The difference is even more dramatic for smaller tip radii.

**4.7.4 Conclusion**

Our analysis, based on the hypothesis that a minimum extent of material along the direction of the electric field must be damaged through exposure to a space charge limited field within the dielectric in order to initiate an electrical tree, suggests that the tolerable Laplacian field at a defect increases by roughly a factor of two as the defect radius decreases from about 5 μm to about 1.5 μm. At 1 μm tip radius, the minimum Laplacian field for tree initiation would be about 2.25 times the field for tree initiation at a 5 μm radius tip, which is well below the >4 times suggested by the curve in Figure 4.28. We thus suggest more conservative design rules than those suggested by the authors of [Fukawa et al. 1996, Kubota et al. 1994].
4.8 Application of Nonlinearity Conductivity to Defect Location in XLPE Cable

When a surge is applied to a solid dielectric cable with a stress-enhancement-containing defect, the current which flows to produce a field-limiting space charge surrounding the defect generates a signal on the conductor of the cable. Based on transient, nonlinear FE computations of the defect-induced current, we predict the nature of this signal as a function of the applied surge waveform and defect severity, and we investigate implications for defect location during a factory cable reel test.

4.8.1 Introduction

Recent publications concerning the design of 500 kV cable which operates at stresses in the range of 15 kV/mm indicate that the defects which limit the operating stress take the form of asperities at the semicon-dielectric interface and impurities in the dielectric [Kubota et al. 1994, Fukawa et al. 1996]. Some defects, such as hollow cotton fibers which go into partial discharge (or can be made to go into partial discharge with x-ray irradiation) might be detectable using standard PD detection techniques, although the high frequency attenuation caused by the semiconducting layers of a power cable limit sensitivity with which PD can be detected [Stone & Boggs 1982, Boggs & Stone 1982]. However, defects such as asperities at the conductor-semicon interface and small conducting impurities in the dielectric are usually assumed to be undetectable, as they will not generate partial discharge during a factory test of reasonable duration since the time for initiation of an electrical tree exceeds the factory test time. Any attempt to raise the factory test voltage or extend the factory test time runs the risk creating an incipient failure during the test process which goes to failure in the field.

As is well known, the electrical tree initiation process involves charge cycling from a defect to the dielectric with the applied AC field. The currents involved at normal AC test levels are in the fA range and are too small to detect except under well-controlled laboratory conditions [Baumann et al. 1985, Himba & Zeller 1986]. However, under fast-rising impulse conditions, the currents become substantial and the voltage on the cable resulting therefrom, which are analogous to a partial discharge pulses but of substantially different shape, may be detectable. In this contribution, we present a preliminary investigation of defect location in transmission class solid dielectric cable based on detection of charge injection currents which do not involve partial discharge. In XLPE power cable, the high field conductivity is highly field dependent [Tokoro et al. 1992, Boggs 1995, Jiang et al. 1996]. When a defect causes severe field enhancement, space charge is generated which limits the
field according to the condition that $\sigma(E_{lim}) = \varepsilon \omega$, where $\sigma(E)$ is the field-dependent conductivity. The current required to generate such space charge represents a resistive current which generates a voltage on the cable conductor given by $V(t) = I(t) Z$, where $I(t)$ is the resistive current caused by space charge generation and $Z$ is the cable impedance as seen from the point at which the space charge is generated. A typical case is that the space charge is generated in mid-cable, so that $Z$ is half the characteristic cable impedance, i.e., about 12 ohms.

As noted above, this phenomenon has the potential to facilitate the detection of defects in power cable. To this end, we compute the phenomena which occur if we apply a step voltage with a relatively fast risetime (20 ns) to a cable with a defect. Computations are based on transient, nonlinear FE analysis with coupled thermal and electric fields as mentioned in the early part of this thesis, the material electrical and thermal properties can be a function of the electric field, temperature, space, time, etc. and the boundary conditions (i.e., applied voltage) can be time-dependent. This program is used to compute the resistive current waveform generated by application of a step wave to a defect in polyethylene.

4.8.2 Modeling of XLPE and Defect

In this section, we consider a 230 kV power cable with 900 kV BIL designed for 12 kV rms/mm maximum field at the maximum rated voltage of 242 kV. We assume a 1250 mm$^2$ (2500 kcmil) conductor cross section. The thickness of the dielectric is 15.8 mm. We consider the signal which will be generated from needle-like defects at the surface of the semicon-dielectric interface with heights of 50 and 100 µm and tip radii of 5 and 10 µm. In order to reduce the geometry to axi-symmetric 2-D, we conduct the computations in a parallel plane geometry with the needle in the same background field as would be encountered at the conductor semicon-dielectric interface of the cable. In order to reduce the ratio of the largest to smallest feature in the problem, we conduct the computations for electrode separations much less than those of the cable and then scale the result to the electrode separation of the cable. As is well known for the theory of PD detection [Boggs et al. 1987], the current in the electrode goes inversely with the electrode separation, i.e., the PD magnitude generated by a given defect goes inversely with the size of the apparatus tested. The same is the case for the present phenomenon, as was verified by conducting computations for two electrode separations. The electrode separation was generally 2.4 mm except for a computation for 4.8 mm to verify the scaling rule described above. In this section, we present the data as current waveforms computed for an electrode separation of 2.4 mm. When we discuss defect location, we scale these results to the case of the 230 kV cable described above (conductor cross section of 1250 mm$^2$, dielectric thickness of 15.8 mm, and impedance of 24 ohms).
Figure 4.34. Resistive current signal as function of time for defects of 100 μm and 50 μm height, 5 μm tip radius, for an electrode separation of 2.4 mm. The background field is 77.3 kV/mm. The resistive current waveforms for the two defects are very similar in shape but differ greatly in magnitude. The signal for the 100μm defect occurs at a slightly shorter time as a result of the lower applied voltage required to reach the Laplacian field threshold for charge injection.

Figure 4.35. Resistive current as function of time for defects with tip radii of 5 μm and 10 μm, defect height of 100 μm, and electrode separation of 2.4 mm for a background field of 77.3 kV/mm. The time at which charge injection begins is later for the 10μm tip radius (lower Laplacian field), but the volume into which charge is injected is larger than for the 5 μm tip radius. The two effects tend to cancel.
The high field conductivity of XLPE is modeled as field and temperature dependent as given by eqn.(4.1). This model results in a limiting field of 250 kV/mm at power frequency.

4.8.3 Computation of Terminal Current

In principle, a cable defect can be located by applying a step voltage with a risetime in the range of 20 ns. The risetime is chosen as short as possible but to ensure that it will propagate with reasonably low wavefront distortion due to the typical semicon-induced high frequency attenuation of power cable [Boggs et al. 1996]. We assume that a short risetime, short duration pulse can be applied at the BIL. For 230 kV power cable, the BIL is 6.44 times the maximum RMS operating voltage, which, for a cable designed with a maximum operating field of 12 kV rms/mm, results in a field at the semicon-dielectric interface in the range of 77 kV/mm. We therefore conduct our computations at this background field, where background field means the field when there is no defect.

At each time step we compute the image charge on the terminal without defect that is caused by the space charge surrounding the defect (i.e., we subtract the displacement current-induced terminal charge from the total terminal charge). The time derivative of the resistive image charge provides the resistive terminal current. As noted above, this current flows through the impedance of the cable (12 ohms as seen at a defect location in mid-cable) to produce a voltage pulse on the cable conductor, just as for a partial discharge pulse. As the conductivity of XLPE is temperature dependent, we have coupled the thermal field in the computation; however, the effect of temperature rise is negligible, although under other similar conditions temperature rise can be substantial [Kuang & Boggs 1996b]. To study the effect of defect size and radius of curvature, we have computed the problem for several defect heights and tip radii.

4.8.4 Results

Figures 4.34 to 4.37 show computed data from the FE program. Figure 4.34 shows the resistive terminal current for two defect heights (50 and 100 μm, defect radius 5 μm). As can be seen from the figure, the resistive terminal current decreases rapidly with defect height. The slightly different front time for the two defects results from the difference in applied field required to reach the charge injection threshold field, in combination with the 20 ns risetime of the applied voltage. However, the waveforms are very similar, which could make it possible to apply a matched filter using cross correlation techniques for signal detection.

Figure 4.35 shows the resistive current signal for tip radii of 5 and 10 μm (defect height 100 μm). For this defect, the resistive terminal current is not a strong
Figure 4.36 Resistive current as function of time for electrode separations of 2.4 mm and 4.8 mm with the same background field (77.3 kV/mm), defect height (100 μm), and tip radius (5 μm). As noted in the text, the resistive current goes inversely with the electrode separation, as for partial discharge detection.

Figure 4.37 Resistive current as function of time for a defect of 50 μm height, 5 μm tip radius, and electrode separation of 2.4 mm in a background field of 77.3 kV/mm. A second impulse of reverse polarity has been applied at 1.0 μs, which results in a resistive current magnitude about twice that caused by the initial surge.
function of the tip radius, although the difference in Laplacian field at the tip is demonstrated by the difference in time at which the charge injection field is reached for the two tip radii. Basically, the field is greater for the smaller tip radius, but the volume over which space charge is injected is larger for the larger tip radius, and the two effects tend to cancel, at least for the present case. Figure 4.36 shows the resistive terminal current vs time for the same background field at two electrode separations (2.4 mm and 4.8 mm). This figure demonstrates the validity of the scaling rule described above. Figure 4.37 shows the resistive current vs time for a defect of 50 µm height and 5 µm tip radius. A second impulse of reverse polarity has been applied after 1 µs. As a result of the space charge which remains from the first impulse, the second current signal magnitude is twice that of the first.

4.8.5 Discussion

4.8.5.1 Defect Detection

The ability to detect a signal in noise depends on what we know about the signal. Useful information includes bandwidth, time interval in which the signal will be received, pulse shape, etc. In the present case, we know both the pulse shape and the time interval in which the pulse will be received (if any pulse is generated). In principle, such information can be used to improve the sensitivity and probability with which we can detect a signal. Given the investigatory nature of the present contribution, we will restrict our analysis to simple considerations of signal and noise power. The noise power per Hz at 300 K is 4 kBT or about 1.66e-20 W/Hz. For the 24 Ω cable under consideration, the resistive current-induced voltage pulse caused by application of a 20 ns risetime, 77 kV/mm background field step wave to a 100 µm height, 5 µm tip radius defect at the conductor semiconductor interface has an energy of about 8.2e-20 J after scaling to a dielectric thickness of 15.8 mm, which produces a signal-to-noise ratio of 4.9 or 6.9 dB, which is adequate for detection. For a 50 µm defect (Figure 4.35), the energy in the initial voltage pulse is 8.1e-22 J, for a S/N ratio of -13.1 dB, which is not likely to be detectable. The second pulse in Figure 4.35 has an energy of 4.2e-21 J, which is about 5 times larger than the initial pulse but still produces a S/N ratio of only -6 dB, which is not likely to be detectable without signal averaging. Signal detection can be enhanced through averaging of repeated measurements with the S/N ratio increasing with the square root of the number of measurements for uncorrelated noise and reproducible data. In the present context, such an approach has two limitations. First, the number of high voltage impulses which can safely be applied is limited. More than 10 impulses is probably unacceptable. Second, the first pulse injects space charge at a defect, which is the basis of detection. Some of this space charge will relax (spread out away from the defect and return into the defect)
as the voltage falls at the end of the impulse. However, some of the space charge will remain. This has two implications, viz., (i) a second impulse of the same polarity will result in smaller resistive terminal current than the first impulse and (ii) a second impulse of reverse polarity will result in a larger resistive terminal current than the first impulse on the "virgin" cable, as seen in Figure 4.37. Thus signal averaging is not a good prospect for enhancing detection, unless it is applied in conjunction with voltage impulses of alternating polarity. Reverse polarity surges at the BIL are considered a severe test of a cable; however, the purpose of the test technique is to detect any defects which could cause in-service failure. If a test is capable of detecting any such defects and will not damage the cable for defects which will not cause failure, then by definition it is safe. In summary, an optimal detection scheme would probably employ our knowledge of the pulse waveshape and the time range during which a signal would be received (which limits the data to be processed) to implement a matched filter through cross correlation.

4.8.5.2 Defect Location

Defect location requires not only detection of a defect-induced signal but also location of that signal in time so that defect location can be determined through measurement of the pulse time delay relative to the impulse voltage application. Fundamental and practical aspects of partial discharge location in solid dielectric cable have been treated recently [Boggs et al. 1996, Kreuger et al. 1993]. The ability to locate a pulse in time depends on the pulse width and the signal-to-noise ratio. An approximate formula for Gaussian pulses is given by Vainstein (or "Wainstein" in the Dover edition) and Zubakov [Vainstein, 1962], viz., \( \Delta = \sqrt{2 \mu} \beta \), where \( \Delta \) is the standard deviation in the time of arrival measurement, \( \beta \) is the pulse width (FWHM: full width half maximum), and \( \mu \) is the signal-to-noise ratio. Thus a greater S/N ratio is required for reasonably accurate pulse location than is required for a reasonable probability of pulse detection. For a pulse width of 15 ns and a pulse S/N ratio of 6, the standard deviation in time-of-arrival measurement will be about 9 ns. The finite risetime of the applied voltage impulse will cause an additional uncertainty so that the total uncertainty might be in the range of 15 ns, which would result in a total standard deviation for location in the range of \( \pm 1 \) m. Thus the 90% confidence interval on location would be in the range of \( \pm 2 \) m, which is acceptable.

4.8.6 Conclusions

Conventional wisdom holds that solid dielectric cable can contain fatal stress enhancements which will not be detected during a conventional factory partial discharge test, as the electrical tree initiation time exceeds the partial discharge test time. In this thesis, we provide a preliminary theoretical investigation of a technique which could detect and locate such defects. The present investigation is con-
fined to defects at the semicon-dielectric interface. The data indicate that detection sensitivity is probably inadequate when a single, fast rising surge at the BIL is employed; however, sensitivity is increased substantially through the use of alternating polarity surges. The case of floating defects in the dielectric has not yet been treated.
ABB, Data provided by the ABB Corporate Research Laboratory, Baden, Switzerland.


Kuang, Jinbo and Steven A. Boggs, "Computation of Thermal, Electrical, and Mechanical Fields at a Defect in Polyethylene", CEIDP, October, 1997.


CHAPTER FIVE

EDDY CURRENT LOSSES IN 3-PHASE PIPE TYPE POWER CABLE

5.1 Introduction

The energy which can be transmitted through a buried power cable is limited by the maximum operating temperature of the cable, typically in the range of 100°C. Power dissipated by the cable must be conducted away by the surrounding soil, but that is another topic. Here, we are interested in computing the power dissipated by the cable, especially as a result of eddy current losses in the cable pipe caused by the complex magnetic field generated by the currents carried by 3-phase conductors within the pipe (Figure 5.1). These losses can be comparable to resistive losses in the conductor. Normally cables are placed in closed triangular or cradle configuration. The cradle configuration is the most practical one, since it does not require spacers, but the losses are higher for the cradle configuration than the closed triangular configuration because the 3-phase cables are further apart in a cradle configuration than the closed triangular configuration.

Metal pipe (normally ferro-magnetic steel) is used to facilitate the rapid installation by minimizing the time that trenches must remain open during the installation. The pipe can be installed quickly, so that the trench can be covered, and the cable can be pulled in later. The pipe also serves as a magnetic shield which prevents interference to communication cables and electronic devices and reduce the environmental concern from the public. The steel pipe also facilitates operation of the cables at about 1.5MPa oil pressure, which improves dielectric integrity.

The AC current in the 3-phase cables induces eddy currents in the pipe, thus causing resistive loss in the pipe. Actually for each cable phase, the two other cables phases and the pipe are acting as a short-circuited secondary of a transformer. The losses in the pipe are comparable to the losses due to the skin and the proximity effects in the cables, and in many cases are larger than losses caused by all the other AC effects combined. This is especially true for pipe-type cable systems with stranded-segmented conductors. These losses will produce heat in the pipe, reducing the ampacity of the cable.

Transmission of AC current in power cables is associated with losses which originate from electromagnetic interactions in the system. These phenomena include the interaction of the current in a conductor with its own electromagnetic field known as the skin effect, as well as the interaction of the current in the conductor with the electromagnetic fields of the other conductors, named the proximity
Figure 5.1 Cross-section of pipe-type three phase cable configuration.
effect. The picture becomes more complicated when a system includes other elements such as cable metallic ground shields, and the magnetic pipe containing the three phase cable system. Alternating current in the conductors induces eddy currents in the cable shields and pipe which not only are the source of power losses, but also influence the current distribution in the conductors. All of these effects are absent when a DC current is passed through the system. In DC case, the only power losses result from the resistivity of the conductors.

**Classification of Losses**

Skin effect is caused by the interaction of AC current in a conductor with its own magnetic field. For a conductor with a circular cross-section, this interaction changes the radial distribution of the current density from a uniform distribution for the direct current, to a distribution characterized by concentration of current near the conductor surface for an AC current. With increased frequency, the current concentrates in a thinner layer or "skin" at the conductor surface. The skin depth can be approximated as

\[ \delta = \left( \frac{2}{\omega \sigma \mu} \right)^{\frac{1}{2}} \]  

(5.1)

For example for copper at frequency of 60 Hz, the skin depth is in the order of 1 cm, while for steel with a relative permeability of 850, the skin depth is about 1 mm.

The proximity effect is due to the interaction of current in a conductor with the magnetic field created by the currents in other conductors in its proximity. The effect influences both the radial and the angular distribution of current density in the conductors. The penetration depth is also computed by eqn. (5.1).

The pipe proximity effect is due to a similar phenomena. The current distribution in a conductor is affected by the magnetic field of the pipe. This field is associated with eddy currents induced in the pipe by the currents in conductors enclosed by the pipe. The losses in the pipe are due to the ohmic losses of eddy currents in the resistive material of the pipe and depend on both the resistivity and the permeability of the pipe material. These (skin and proximity) loss effects co-exist in a multiple conductor system such as a three phase power cable and make an analytical solution to such a problem impossible.

Loss caused by the proximity effect can be reduced by:

1. Use of Al stranded conductor for which the oxidized layer on the surface of the strands inhibits greatly current transfer between strands.

2. Use of enameled Cu stranded conductor, which is effective but very expensive.
3. Segmenting the conductor into three or more triangular segments allows conductor strands to be circulated from the outside diameter of the cable to the center which, if the strands are insulated, reduces both skin and proximity effects.

Shields losses are similar but much smaller as a result of the thin, stainless steel tape employed.

**Nonlinearity**

Figure 5.2 shows the magnetic flux density B as function of magnetic field intensity H of a ferromagnetic material that is used to make the pipe. The permeability of a material is the ratio of the magnitudes of the magnetic flux density B and the magnetic field intensity H

\[ \mu = \mu_0 \mu_r = \frac{B}{H} \]  

where \( \mu_0 \) is the permeability constant, \( \mu_0 = 4\pi \times 10^{-7} \) H/m, \( \mu_r \) is the relative permeability. The permeability \( \mu \) is highly (magnetic) field dependent. When solving an AC or tran-

![Figure 5.2 Magnetic flux density as function of magnetic field intensity for ferromagnetic pipe.](image-url)
sient magnetic field in the time domain, the differential permeability should be used. Differential permeability is defined as

\[ \mu = \frac{dB}{dH} \]  

which is the slope of B-H curve. Compared with the permeability defined by eqn. (5.3), the permeability defined by eqn. (5.2) is called ordinary permeability [Kraus 1992]. Ordinary and differential permeability of pipe material have been plotted in Figure 5.3 as a function of flux density.

The permeability of ferromagnetic pipe is a strong function of the magnetic field intensity. The major difference between ordinary permeability and differential permeability is their peak magnitude. Permeability is also highly temperature dependent. When the temperature is high enough, the ferromagnetic material will lose its ferromagnetic property, and its relative permeability becomes unity. For the application of the cable pipe, the temperature will be below 200 °C, and the experimental test shows that the temperature dependency of \( \mu \) can be neglected.

![Figure 5.3](image)

Figure 5.3 Relative differential permeability (circles) vs. magnetic flux density of the ferromagnetic pipe compared with ordinary relative permeability (squares).
Technical Problem

As analytical study of the problem has been limited by the difficulty of dealing with the complexity of the three phase structure of power cable and the nonlinearity of the pipe permeability. Semiempirical approximate expressions have been used containing factors derived from measurements on a limited number of cable sizes and configurations, but this approach is of limited value for new cable designs with large conductors carrying high current [Mekjian & Sosnowski 1983, Kawasaki et al. 1981]. The finite element method approach is found in the literature with the nonlinearity of the pipe permeability treated in two dimensions using the effective permeability method [Labridis & Dokopoulos 1992]. We employ a time domain solution to solve the problem.
5.2 Numerical Modeling

The magnetic vector potential \( A \) is related to magnetic flux density \( B \) by

\[
B = \nabla \times A
\]  
(5.4)

If \( A \) is to be defined, the divergence of \( A \) has also to be defined. When a Coulomb gauge [Silvester 1990] is employed, \( A \) satisfies

\[
\nabla \cdot A = 0
\]  
(5.5)

When the magnetic vector potential \( A \) is used, the magnetic field of an eddy current problem is governed by the diffusion equation

\[
-\nabla \cdot \frac{1}{\mu} \nabla \times A + \sigma \frac{\partial A}{\partial t} = J_s
\]  
(5.6)

where \( J_s \) is the source current density. In general, the magnetic permeability \( \mu = \mu(B,T) \) of a ferromagnetic material is a function of the magnetic flux density and temperature.

Eqn.(5.6) can not be solved directly, because the \( J_s \) is not known. What is known is the net conductor current \( I_c \). Integro-differential finite element formulation is widely applied for solving 2D multi-conductor eddy current problems [Konrad 1981, Chen 1995]. The integro-differential formulation can be described by

\[
\nabla \cdot \left\{ \frac{1}{\mu} \nabla A \right\} - \sigma \frac{\partial A}{\partial t} + \sigma \frac{\int \sigma \frac{\partial A}{\partial t} dS}{\int \sigma dS} = - \frac{\sigma I_c}{\int \sigma dS}
\]  
(5.7)

the integration is over the conductor area. Eqn.(5.7) can be solved using the finite element method [Silvester, 1992] and time stepping as mentioned in Chapter 2.

In the steel enclosed three-phase cable problem, the conductors in the cables are stranded in order to reduce the skin effect losses. As a result of stranded conductor, the current in the conductors is nearly uniform. The three phase conductors thus can be modeled as having zero conductivity but with uniform current density, so that no eddy current will exist in conductors.

The temperature rise of the steel pipe problem is insufficient to affect eddy current phenomena within the pipe. Thus magnetic losses in the pipe can be computed without coupling a thermal field.

A number of inevitable uncertainties attend computation of magnetic pipe losses including:
Permeability of the Pipe

The magnetic losses depend on the permeability vs. field. Very few data exist, and we employ those measured by Kuffel, Model 1 and presented in EPRI Report No. EL-1125, 1979 (page 10-A-1) [Hohendorf 1979], as shown in Figure 5.2. For the time domain solution of an AC or transient magnetic field, the differential permeability, dB/dH, as shown in Figure 5.3 is applied. However at a given value of B, numerous values of dB/dH exist on the various hysteresis loops. Thus completely accurate computation of the losses would require knowing the full set of possible hysteresis loops. In fact, we have only one set of data for B vs. H from which we can compute the permeability and differential permeability as shown in Figure 5.2, and we have no choice but to use these data. If we multiply the differential permeability by 0.8, 1, and 1.2 and compute the pipe loss at 1440 A in an equilateral configuration, we obtain 28.874, 29.185, and 29.75 W/m, respectively, which suggests that the effect of moderate variations in the permeability is not large. However, the effect may differ depending on the conductor configuration. Finally, we note that the cable pipes are not specified for magnetic properties. Given the very small number of pipes which have been characterized for magnetic properties and the wide range of time over which pipes have been installed, we have no idea how pipe magnetic properties may have changed over time (e.g., with changes in steel or pipe production technology) or from one manufacturer to another.

Conductivity of the Pipe

The pipe resistivity is not specified. Measurements in the literature vary over a range of about 2:1 with most of the reported measurements in the US literature around 12E-8 Ω-m (8.2E6 S/m). The measurements reported in the Japanese literature tend to differ from those in the US literature. Since this value is not well known, we have treated it as a free variable within the reported range. For a closed pipe, we find best agreement between our computations of pipe loss and reported measurements at a conductivity of about 11.5E6 S/m (or resistivity of 8.6E-8 Ω-m). The conductivity has a substantial effect on the pipe loss, as seen in Figure 5.4.
5.3 Losses for a Balanced Load

Computations have been made to calculate the eddy current loss in the pipe due to balanced 3-phase currents. The pipe under consideration has an outer diameter of 10.75" and a wall thickness of 0.25". The diameter of the conductors is 1.824" and the diameter of the single phase cable is 3.876". The 3 cables are installed in a cradle configuration as shown in Figure 5.1, with conductor-to-conductor (center-to-center) spacing in the cradle configuration of 3.876", 3.876", and 6.0874". We assume the pipe has a conductivity of $11.5 \times 10^6$ S/m. The B-H curve of the pipe is as shown in Figure 5.2, which is Model 1 as measured by Kuffel and presented in EPRI Report No. EL-1125, 1979 (page 10-A-1) [Hohendorf 1979]. The differential permeability of the pipe is derived from the B-H curve and plotted in Figure 5.3.

We have modeled the above pipe-type three phase cable. Balanced three phase current is injected into the three phase cable, and we compute the magnetic vector potential as function of position and time. The magnitude of AC current is increased exponentially from zero to its full value, otherwise the program will not converge. The time constant of the exponential function is 5 ms. Substantial transients components persist for the first two AC cycles. This can be seen from the fact that the loss computed from the solution of the 3rd AC cycle is different from that of 2nd cycle by 5% or so. In the 3rd AC cycle, the transients are small enough
Figure 5.5 FE modeling vs. experimental data for an 10" (254 mm) pipe, with a wall thickness of 0.25"(6.35 mm), and cable diameter of 3.876" (98.45 mm).

to be neglected. Power loss per unit length of the pipe can be computed from the solution of vector potential $A$.

An AC/DC resistance measurement data of a 10" pipe has been given in EPRI Report No. EL-1125, 1979 [page 5-17] [Hohendorf 1979]. The pipe under test has the same configuration and size as the pipe we have modeled except the pipe under test has a conductor cross-section area of 1015 mm$^2$ (2000 kcm), while that of the pipe being modeled has a conductor cross-section area of 1267 mm$^2$ (2500 kcm). The comparison of the two results has been shown in Figure 5.5. The conductor loss has been adjusted for the pipe we have modeled to an equivalent of a 1015 mm$^2$ (2000 kcm) conductor. The figure shows that the computational results agree very well with that of the test results for the range of current ($\leq 800$A) that is available from the test.

The FE modeling indicates that the AC/DC resistance ratio increases when the current is larger than 800 A. This is unexpected, as the AC/DC resistance ratio was expected to remain relatively constant beyond 800 A. The reason for the increase in AC/DC resistance ratio can be deduced from Figure 5.3. The permeability of the pipe is low at low flux density, reaches a maximum at about 0.3 Tesla, and then drops as flux density increases further. The steel pipe provides a magnetic path for the flux that is generated by 3-phase currents, which are almost if not completely balanced. At certain current level when the permeability of the pipe is high, the flux generated by 3-phase currents has a better passage in the pipe to cancel each other, which reduces the eddy current loss, while when the current is very low
or very high the permeability is low, the flux generated by each 3-phase conductor is more concentrated in the pipe around its source conductor, which results in greater eddy current losses.

What should also be mentioned here is that we have not modeled the hysteresis loss in the calculation, but rather we have used an equivalent conductivity of the pipe so that the loss matches the test. As a result, this equivalent conductivity is higher than a typical conductivity of a steel pipe. This equivalent conductivity seems to be effective during the available range of the test current.
5.4 Unbalanced Three Phase Current

Under ideal operating conditions, the current in the 3-phase power cable are balanced, so that the magnetic field in the pipe caused by 3-phase current tends to cancel each other out. As a result the eddy current loss in the pipe is relatively low. But when the currents in 3-phase conductors are unbalanced, there is a net current from the three phase cables that will generate more eddy current loss in the pipe.

The ratio of current unbalance \( K_{unbal} \) in a three phase system is defined as the ratio of net current of the 3 phase currents over the average current in the 3 cables. For a case study, we assume two of the three phases (B and C) have the same current magnitude. We define \( I_{av} \) as the average three phase current, \( I_a, I_b \) and \( I_c \) as the three phase currents. The magnitude of three phase currents can be derived as

\[
\begin{align*}
I_a &= I_{av} (1 + \frac{2}{3}K_{unbal}) \\
I_b &= I_{av} (1 - \frac{1}{3}K_{unbal}) \\
I_c &= I_{av} (1 - \frac{1}{3}K_{unbal})
\end{align*}
\] (5.9)

According to eqn. (5.9), unbalanced currents are injected into three cables for the computation of losses in the pipe. It is assumed that there is no net current in the pipe, that is the return current will go through earth. The computation is made for the cradle configuration as shown in Figure 5.1. The largest current \( I_a \) among the three phase is applied to the center phase cable. Figure 5.6 shows the computation results for balanced and unbalanced three phase currents in the cables. For the unbalanced currents, computation are made for 5% and 10% unbalanced ratio. The upper plot in Figure 5.6 is the absolute loss while the lower plot is the ratio of pipe loss to conductor loss. From Figure 5.6 it is seen that balanced current produces the lowest loss, while 10% unbalance produces more loss than 5% unbalance. The other interesting phenomena discovered from this study is that the ratio of pipe loss to conductor loss decreases as current increases when \( I \leq 600 \) A, and reaches its minimum when \( I = 700 \) A, and then rises again as current increases when \( I \geq 800 \) A. This is similar to what we have found for the balanced 3-phase currents in the previous section.
Figure 5.6 Eddy current loss in a 10" pipe as function of cable current for balanced and (5% and 10%) unbalanced three phase currents. The wall thickness of the pipe is 0.25", the conductor diameter 1.824", the cable diameter 3.876".
5.5 Conclusions

The FE modeling of the cable pipe for the calculation of pipe loss is valid. It has been verified by the experimental test data.

By FE modeling, it has been discovered that the pipe/conductor loss ratio will increase beyond certain current level, for the 10" pipe with cable OD of 3.876". This current level is about 800A.

An unbalanced 3-phase current will produce more pipe eddy current loss than a balanced 3-phase current due to the non-zero 3-phase net current.
References


6.1 List of Contributions

An efficient strategy was developed using the finite element method in axisymmetric 2D to solve the coupled transient nonlinear field problems (coupled electro-magnetic and thermal fields) in time domain and to compute the thermal expansion-induced mechanical stress in postprocessing.

A computer code was developed to implement the algorithms (FEM, coupling of fields, adaptive time stepping, post-processing, etc.) [9,21]. The code was verified against problems with analytic solutions and with experimental data in the field of electrical insulation and magnetic shielding. The reference numbers refer to the publication list provided under 6.2.

The computer code was applied to the study of field induced electro-magneto-thermal phenomena including:

- computation of the temperature rise at a conducting defect as a function of impulse voltage risetime [8,12,16,22]
- computation of the space charge distribution around a conducting defect and the mechanical stress caused by the action of the space charge limited field on the space charge under AC and impulse conditions [6,8,21]
- computation of the electrical signal generated by formation of space charge during a step wave voltage application, which has the potential to facilitate location of defects within transmission class solid dielectric cable [14,20]
- computation of the temperature rise in a water tree channel which results from application of a lightning impulse, which explains how water trees convert to electrical trees and cause failure in distribution cables [10,13]
- computation of the space charge limited field region around a defect in polyethylene and correlation of such computations with Japanese data for the Laplacian field at which electrical tree initiation occurs as a function of needle tip radius to explain the mechanism behind the Japanese data which show a large increase in the Laplacian initiation field with decreasing needle tip radius [11,15]
- computation of the electrostatic energy available for electric tree channel formation under impulse conditions and development of a self-consistent theory of channel formation [2]
- computation of the electric and thermal field distributions in ZnO arrester elements, which facilitated optimization of the element topology and a 50% increase in the energy absorption capability of the elements [1,3,4,5,7]
- computation of the thermal field in a distribution lightning arrester during multiple impulse current applications [18]
• computation of the electric field distribution in a joint which employs nonlinear grading materials [19]

• computation of the transient errors in a precision resistive divider caused by thermal and electromagnetic effects [17]

• computation of the magnetic losses in steel HPFF cable pipes caused by three phase eddy currents, to investigate the effects of unbalanced currents and the loss as a function of cable current (as yet unpublished).

• computation of the chemical potential which drives water tree formation in a range of geometrical configurations (unpublished).

• computation of the coupled thermal, dielectric loss, electrical, and moisture fields in oil-paper insulation operated at very high temperatures (where decomposition of the paper creates moisture) and in the presence of a conducting defect. In this work, the dielectric loss was a function of moisture and electric field and the moisture evolution rate was a function of temperature. Failure could occur as a result of thermal runaway or through saturation of the dielectric with CO, the rate of evolution of which was also computed but was not coupled into the field equations (as yet unpublished).

6.2 Publications


6.3 Concluding Remarks

Advances in computer power have made the time domain solution of transient, nonlinear coupled field problems economically advantageous using FEM and time stepping. This has facilitated development of software tools for field simulation which provide new opportunities for the development and design of electric apparatus. Many of the technically important phenomena and systems are difficult, expensive, or impossible to study experimentally but can be simulated through the use of such software. For example in ZnO arrester elements, the thermal phenomena which limit energy absorption take place on a scale of about 10 μm, too small to be measured. Yet the understanding which resulted from simulation of these phenomena resulted in a 50% increase in energy absorption. This work was described at senior levels of Toshiba management as the most successful project which Toshiba ever conducted with an outside organization, as it resulted in a major and immediate improvement in the product and the economic position of Toshiba in the ZnO arrester market. This is only the most practical example of the wide range of computational results which have resulted from the work in this thesis.
6.4 Future Work

Future work will focus on improving the computational efficiency with which highly nonlinear problems can be addressed. Computation of the electro-thermal field distribution in a ZnO arrester element takes nearly a week on a fast PC, even after a number of approximations and other devices to reduce the computational load. Clearly, improved computational approaches are needed.

In the present algorithm, the material properties are constant within an element. In regions of rapidly changing field, this results in a “jagged” field distribution, as the material properties should be changing within the element. The algorithm needs to be improved so that both the field and the material properties can change within an element, and this will be a focus of future work.