Pole-Potential Mapping and Synthetic Arrays in Electrical Exploration

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of Physics
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Abstract

Many interesting geological features can be characterized by the electrical resistivity. Consequently, the resistivity and induced polarization methods are extensively applied to map subsurface electrical properties and often provide a suggestive outline of geological structure.

Standard survey methods collect data using linear configurations and, in 1d or 2d environments, survey and interpretation methods are well developed. However, in the complex geological settings often encountered the responses of most linear array configurations suffer from an extreme sensitivity to near-surface structure which can obscure the more desirable response from deeper structure. Furthermore, linear profiles and sections can be ambiguous when coarse sampling intervals are used, a problem compounded by the lack of information from between survey lines.

Improving the accuracy of electrical prospecting methods requires that a more complete map of field measurements be taken over a two dimensional set of source and receiver positions. I suggest a 3d mapping method using a 2d surface grid, a method I name pole-potential mapping. To interpret the data from such a survey, I suggest several approaches to defining apparent resistivity in terms of gridded potential maps about a distributed
grid of current poles. This approach significantly improves our ability to resolve subsurface features. Sensitivity analysis, numerical and analytic model studies clearly show the sensitivity of circular arrays to be more localized in one region in the earth than traditional configurations and the location and depth extent of the high sensitivity region is easily controlled by varying the location and dimension of the synthetic arrays.

Finally, to show that systematic pole-potential mapping could practically be carried out on a regular basis with an efficient field methodology, I designed and constructed suitable instrumentation. From practical tests in a scale modelling tank and in a small scale field experiment at Fort York, I show measurement error in synthetic data can be made low enough to exploit the improved signal of deep targets to surficial noise. A roll-along approach, analogous to that used in seismic surveying, is a very efficient field methodology. The results are consistent with predictions from the numerical studies indicating the utility of pole-potential mapping.
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Chapter 1

Earth Resistivity and its Measurement

1.1 Introduction

In applied geophysics the resistivity methods are used to map sub-surface geology by injecting electrical currents into the earth, measuring the resulting potentials, and converting them to a set of profiles, sections or maps of the apparent earth resistivity. These observations are then interpreted in geological terms. The success of the method is in part a consequence of the enormous range of resistivities found in earth structures and materials which vary over 18 orders of magnitude, a range far greater than any other physical property that can be sensed geophysically. While a knowledge of resistivity is seldom diagnostic in itself, the variation in resistivity across a map in many instances does provide a suggestive outline of the geologically interesting features. In addition, when a low frequency alternating current is used, we may observe an additional electro-chemical effect called induced polarization (IP) in which some of the resistivities may be frequency or time dependent. The causes of IP are related to the conduction mechanisms operating in rocks and minerals and additional complementary information to be obtained. Originally employed in mineral exploration, the resistivity and IP methods have proved suitable for solving many geological problems. Today, a partial list of applications includes: mapping rock lithology; detecting pollution plumes and leakage from waste dumps; detection of faults and foundation integrity; and investigating archaeological sites.
While successful in many areas, the resistivity methods have several well known drawbacks. The enormous range of natural resistivities seems useful in characterizing a material or lithologic unit; however, response saturation occurs for resistivity contrasts of as little as one hundred to one, making it hard to distinguish an excellent conductor from a relatively good conductor. Perhaps of greater importance, traditional data collection methods tend to be very sensitive to resistivity structures near the measuring electrodes. This geologic noise often obscures deeper more interesting features. Furthermore, resistivity surveys are one of the more labour intensive and therefore expensive of common, non-seismic, exploration methods. For these reasons, they are not as often used as might be.

Despite many years of development, resistivity methods still possess great potential for improvement. With this in mind, recent advances in electronics present many opportunities to improve instrumentation and survey methodology for resistivity mapping so that much better information can be obtained with only a moderate increase in field effort. In this thesis, I present instrumentation and a field methodology appropriate for mapping the 3d distribution of resistivity and also its low frequency variation, the induced polarisation (IP) effect. The solution to making the observations more sensitive to deeper structures and easier to interpret is found in collecting a greater volume of high quality data from a 2d grid of source and receiver electrodes. The instrument and field methodology are designed to minimise the labour and cost involved in conducting such a field survey. Greater data coverage contrasts with the present minimalist approach to surveying, but it is justified by the great reduction in unwanted near-surface effects, a vastly improved presentation, and it can be implemented by an approach to data collection that avoids excessive costs.

1.2 Low Frequency Electrical Properties of Rocks

When an electric current is introduced to the earth, (for instance, by applying a potential difference to a pair of current electrodes) it distributes electric charge wherever there is
a non-zero component of the electric current perpendicular to the conductivity gradient. The electric field of these charges, together with those on the source electrodes is what we measure in the direct current experiment. The electrical conductivity of a material is defined by Ohm’s law, which is one of Maxwell’s constitutive equations,

\[ J = \sigma E \]  

(1.1)

Since \( J \) and \( E \) are vector quantities, \( \sigma \) is a 2nd rank tensor property of the material. Ohm’s law, an empirical statement about common materials, is that \( \sigma \) is independent of the electric field intensity vector \( E \) and the current density vector \( J \) and thus a property of the material only. In galvanic and IP surveys, the alternative term electrical resistivity is often used and, for an isotropic material, it is the reciprocal of the conductivity. For anisotropic materials where the conductivity depends on the direction of the applied electric field, the resistivity tensor is the inverse of the conductivity tensor. Materials may be anisotropic at the molecular level (for instance, along planes in a crystal lattice) or at the local structural level (for example, interlayered sedimentary beds of different resistivities). Many geophysical methods sense only the spatial average of structures, thus we find it is sufficient to assume the earth is locally isotropic, and then treat conductivity and resistivity as scalar properties.

Resistivity is the local property of a material we wish to discover; however, it is the circuit quantity resistance (or more aptly, when IP is considered, the impedance) which we measure in any experiment. To determine the resistivity from resistance, we must consider the dimensions of the measuring system and of the object. For example, consider two metal disks of cross-sectional area \( A \) separated by a long, uniform conductive cylinder, and a pair of potential electrodes separated by a of length \( L \), (the upper part of Fig. 1.1). The resistance \( R \) observed in this experiment is defined as \( V/I \). Because the current in the uniform cylinder will be uniformly distributed across the cylinder and axially directed, we can immediately determine the resistivity of the cylinder material as \( \rho = \left( \frac{L}{A} \right) R \).

In a similar fashion, when determining the resistivity of the earth, we make measure-
Figure 1.1: The parameters used in defining resistivity in the laboratory and field.

ments of resistance on the earth’s surface that are a function both of the measuring system dimensions and of the distribution of earth resistivities. One well know measuring setup is the equi-spaced "Wenner" array (Wenner 1912), shown in the lower part of Fig. 1.1. For this arrangement, and when the earth resistivity is homogeneous and isotropic, the measured resistance is related to the resistivity simply by \( \rho = (2\pi L)R \). When the measurement is made on heterogeneous or anisotropic earth, this quantity is called the apparent resistivity. Apparent resistivities are a useful data normalization as they make a first order allowance for variations in measurement geometry. The values vary in a systematic fashion across a geologic section and can be diagnostic of both layering and the presence of conductive or resistive inhomogeneties. The normalization facilitates data comparison under a variety of conditions and is important in transforming experience and intuition from previous surveys. However, apparent resistivity is still a function of the geometry and location of the electrodes as well as the region in which the measurements are made and this must be accounted for in quantitative interpretation of the data.

The resistivity structure of the ground is rarely the ultimate target of geophysical surveys. It usually is sought because it is a proxy for more geological properties such as water content, presence of semi-conducting minerals etc. In most rocks, water and the mineral surfaces it interacts with, are the only constituent with significant conductivity. A satisfactory empirical expression relating the resistivity of porous sedimentary rocks
to porosity and water properties was given by Archie (1942),

\[ \rho = a \rho_w S^{-n} \phi^{-m} \]  \hspace{1cm} (1.2)

where \( \rho \) is the overall resistivity of the rock, \( \rho_w \) is the resistivity of the electrolyte in the pore structure, \( S \) is the fraction of the pore space filled with water, \( \phi \) is the volume fraction of void space in the rock, and \( a \) and \( m \) are empirically determined parameters of a given rock. The exponent \( m \), the cementation factor, is dependent on the shape of the particles, and increases as they become less spherical. Typical values range from 1.20 for spheres to 1.85 for plates. Values may reach 3 or 4 in rocks with poorly connected pore space.

The resistivity of low porosity crystalline rocks is usually high. However, unless they contain semiconducting mineral or are at a very high temperature (> 500 C) the value of resistivity still depends almost entirely on their water content and its degree of ionization and the configuration of the pore spaces, be they microfractures or macro fractures. This can make it difficult to directly relate the value resistivity measurements to standard geological parameters such as lithological composition.

Induced polarization can be thought of as replacement of the usual frequency (or time) independent resistivity by a slightly frequency dependent impedance (mainly resistive). The electrochemical phenomena that cause it exists over a wide frequency range, (direct field observation is generally only possible at low frequencies, typically 0.03 to 3 Hz). Broadly speaking, there are two IP mechanisms, electrode and membrane polarization. With electrode polarization, we find that the normally ionic electric conduction paths through fluid in the ground are interrupted by mineral particles in which the carriers of current are electrons, (Fig. 1.2A). Although charge crosses the boundary, negative and positive ions accumulate on either side of the grain, because the ions either release electrons to the grain or to accept electrons conducted through the grain, Fig. 1.2B. If current flow stops, the electric field due to these ions (and the charge layers they create) decays slowly as the ions slowly diffuse back to their original locations. Electrode
polarization is typically observed in the presence of disseminated sulphides or graphite as these minerals conduct current electronically.

An IP response may also be observed in the absence of electronically conducting mineral. Called background or membrane polarization, most rock-forming minerals exhibit a net negative surface charge on their interface with the pore fluid, and this attracts positive ions onto this surface, Fig. 1.2C. When pore surfaces are covered with a thin layer of clay minerals the effect is especially strong. The concentration of positive ions extends about 100 μm into the pore fluid and, if this distance is of the same order as the diameter of the pore throats, the movement of ions in the fluid is inhibited, (Fig. 1.2D). When an impressed current flows in the pore fluid, it slowly disturbs the distribution of charges and creates a temporary set of charge dipoles that slightly affect the electric field. Membrane
polarization is most pronounced in the presence of clay minerals where the pores are sheet-like and particularly thin.

The frequency dependent impedance observed in geology can usually be well described by the Cole-Cole empirical model. It was originally proposed by Cole & Cole (1941) to predict complex dielectric behaviour, but Pelton et al. (1978) adapted it to representing IP and suggested a simple circuit model (Fig. 1.3) to describe the effect. The impedance for this circuit (using the positive sign convention for harmonic time dependence, $e^{i\omega t}$) is just,

$$Z(\omega) = R_0 \left[ 1 - m \left(1 - \frac{1}{1 + (i\omega \tau)^c}\right) \right] \quad (m, \tau, c > 0) \quad (1.3)$$

where $m$ is the parameter that controls the strength of the phenomenon, $\tau$ positions the frequency variation in the frequency spectrum while $c$ controls the spectral extent. Alternatively, $m$ describes the extreme change in the magnitude of $Z$ over the frequency spectrum where,

$$m = \frac{1}{1 + \frac{|Z(\infty)|}{|Z(0)|}} \quad (1.4)$$

and $\omega = \frac{1}{\tau}$ is the angular frequency of maximum phase shift where,

$$\tau = X \left(\frac{R_0}{m}\right)^{\frac{1}{c}} \quad (1.5)$$
The resistance $R_0$ simulates unblocked pore paths and is a dc resistance, $R_1$ represents the electrical resistance of the blocked paths, and $\frac{1}{(\omega t)^c}$ is the impedance produced by interface charge storage. In an experiment where the current is switched on and off, the chargeability $m$ is the ratio of the residual voltage to the voltage drop at turn off, Seigel (1959). The time constant, $\tau$, determines the average rate of decay of the residual voltage (the decay will be exponential only for $c=1$) and increases with the average size of semimetallic grains. Increasing the concentration of polarizable material increases both the chargeability and the time constant, whereas increasing the grain size lengthens the time constant but decreases the chargeability, (Wong, 1979). Finally, the exponent $c$ controls the curvature of the decay curve (in log-log space), and physically relates to the size range of the particle distribution. Most observed $c$ values are in the range 0.1 to 0.3, with bounding limits of 0.0 to 1.0, Pelton et al. (1978). As with resistivity measurements, IP field surveys generally measure an apparent chargeability. While it is closely related to the Cole-Cole parameters in different parts of the ground, it is also a function of the array and target geometry and the spectral characteristics of the measuring system. Only in a homogeneous medium do the intrinsic and apparent values coincide and then only for responses within the spectral window of the measuring system.

Ohm’s law (with or without the addition of the Cole-Cole model to represent the IP effect) is a linear relationship between the current (or current density) and potential difference (or electric field). However, the resistivity of a material can be a non-linear property (a function of the current density or electric field intensity) under certain special circumstances usually related to high current densities or electric field strengths. In general, linearity is a valid assumption in geophysical resistivity measurement for typical voltage and current levels; however, induced polarization effects are more prone to non-linearity. "IP effects are greatest and are linear with current density at low current densities and, as a general rule, fall off markedly as densities increase.” (Sumner (1978)).

In practice, resistivity and IP field methods employ source currents of milliamps to amps, and these currents spread out over very large areas of $10^3 - 10^8$ m². At such low current densities (typically $<10^{-6}$ A/m²), all phenomena generally are linear.
In summary, the bulk electrical properties of the earth are a function of many variables usually only a part of which can be attributed to a specific mineral or rock type. However, "since the factors which generally do determine the average conductivity of a rock formation very often are preserved throughout the bulk of the formation, it is natural that geophysical methods should be used to exploit these effects as an aid in mapping subsurface geological contacts", Grant & West (1965). Qualitatively maps that maintain the spatial and relative distribution of earth resistivities answer many questions we ask in electrical prospecting. This is one reason results from electrical surveys are often displayed in the form of contoured apparent resistivities profiles, sections and maps.

1.3 Traditional Interpretation Techniques

Ideally, the result of a resistivity survey should be a set of profiles, maps or sections that provide a pictorial representation of the average resistivity structure in a selected set of depth ranges. To try to arrive at this goal, resistivity surveying has traditionally been a multi-step process, firstly the collection and display of relatively basic data and secondly, of physical interpretation of the data wherein some kind of estimates of \( \rho(x,y,z) \) are adduced. The final step, drawing geological conclusions from the physical interpretation requires specific knowledge of the exploration problem and will not be discussed here. A number of approaches to physical interpretation, estimating \( \rho(x,y,z) \) from the measurements, have proved successful in the past, in order of increasing mathematical complexity they are:

1. **Qualitative Analysis:** Apparent resistivity profiles, sections, and maps are prepared from the field data. With minimal effort, visual inspection of this data can often provide qualitative information about the depth and location of resistivity structures.

2. **Type curves for simple models:** As an adjunct to 1, catalogues of the apparent resistivity response of typical target structures are assembled as "signatures" that
can be looked for in the field data.

3. **Forward Modelling:** Compute results for analytical solutions to the forward problem for simple model such as a layered space or a space containing brick like anomalous zones. Manually adjust the parameters (iteratively), such that the forward modelled data match the observed apparent resistivities.

4. **Parametric Inversion:** Automate the forward modelling method for models with only a few parameters, such as a layered space. This is currently the standard approach for interpreting sounding data.

5. **Formal Multi-dimensional Inversion:** These methods attempt to automate interpretation where the earth model is given a general form such as \( \rho(x,z) \) or \( \rho(x,y,z) \). The forward computations are numerically intensive and the inverse algorithms must be guided to an acceptable stable solution.

In all but formal inversion, it is the usual practice to convert the field data to an apparent resistivity before an interpretation is made, and even in this case it is helpful to do so. Much of our knowledge of the forward behaviour is obtained through an understanding of the apparent resistivity data transformation. For various earth models, table 1.1 outlines the dimensions required to describe the earth’s resistivity structure as well as the collection and presentation methods commonly employed in these environments. Each measurement in the set of measurements collected for a given mode, is typically made using two current and two potential electrodes. Where the modes differ is in the electrode geometry and relocation of the electrodes between measurements. Spatial information is obtained by moving all electrodes across the surface with a fixed geometry. Depth information is obtained by increasing the separation between all or some of the electrodes, (unlike other electromagnetic methods where the frequency can be adjusted to vary the depth of exploration).

Sounding methods seek information about the vertical electric structure beneath a single location and have optimistically been referred to as electric drilling. In field work, the
Table 1.1: Traditional exploration modes and data presentation methods for various earth models. Volume mapping has only recently become feasible and no traditional display methods exist. Part of the intent of this thesis is to fill this void.

midpoint of the array is fixed while the electrode separations are increased in steps. Typical measuring configurations are the Wenner or Schlumberger arrays, which provide information from successively greater depths as they are expanded. (standard electrode layouts and separations are defined in Appendix C). The results are displayed initially as apparent resistivities versus horizontal separation (in log scale) and interpreted in terms of horizontally or gently dipping layers. For many years the interpretation of sounding curves was performed by matching observed curves with theoretical curves found in catalogues, but manual curve matching techniques have recently fallen into disuse. Today virtually all sounding is performed by computational inversion, typically, a least squares fit between the observed values of $\rho_a$ versus separation and calculated values.

In many exploration scenarios, sounding methods have limited application since the earth is not sufficiently plane-layered and the data are insufficient to understand lateral effects. Where lateral variations in structure are significant we must employ profiling or mapping methods. In particular, for the common case where the geology being surveyed has an anisotropic texture in the horizontal plane, data are typically collected along traverses perpendicular to the regional geologic strike (the horizontal direction of maximum data
correlation). The array is moved (physically, electrically, or computationally) along a line with a selected electrode spacing and sampling interval and thus a single profile of lateral variation is obtained. Additional profiles for different electrode separations give apparent resistivities that characterize different exploration depths. For interpretation, sets of profiles may be converted to contour or colour sections of apparent resistivity versus profile coordinate and electrode separation. Since the electrode separation is some sort of proxy for exploration depth, this presentation format is called a pseudosection of apparent resistivity. Profiling techniques are often useful as a variety of situations can be considered two dimensional (2d) (having a constant cross-section extended enough in strike that it may be considered infinite). For example, steeply dipping contacts of contrasting resistivity; and dikes, fractures, and faults are often aligned with the regional strike.

In mapping methods, the array is moved over a 2d area on the surface in order to investigate buried 3d structures. The simplest mapping methods, such as the gradient array and the fixed bipole-moving dipole method (Bibby & Hohmann, 1993) use a remote, stationary transmitter and a roving receiver. Since the effective transmitter-receiver separation is large and roughly constant, no depth variation is measured and a simple plan map is produced corresponding to one broad range of depths at each receiver location. In another approach, data are gathered sequentially by profiling along several parallel lines to produce a pseudosection for each line. The set of adjacent sections are stacked to represent the 3d earth. Since pseudosections are essentially a 2d method, this can lead to ambiguities in data interpretation. To truly map a volume of resistivity, it is necessary to move both transmitter and receiver and vary their separation. This can, in theory, provide data for plan maps of resistivity at several exploration depths; however, the data are dependent not only on the earth structure but also on the source receiver configuration. Consequently, they may be difficult to interpret. Furthermore, field work is complicated when electrodes are not confined to a single line.
1.4 **Progress in Resistivity/IP surveying**

Over the last decade technological advances in digital data sampling/storage and processing have lead to many developments that make it easier and cheaper to gather extensive data sets of high quality. With higher quality data from instruments measuring the instantaneous decay voltage in the time domain, various mineral discrimination techniques through spectral IP measurements are possible. Seigel (1990) discusses the implementation of a microprocessor-based multichannel time-domain IP receiver, IPR-11, that has the ability to measure several input channels simultaneously. The receiver records the data and processes it into apparent resistivities, printing the data in standard pseudo-section format.

The introduction of computer controlled multichannel resistivity meters using multielectrode arrays has greatly simplified and automated traditional electrical collection and presentation techniques. Griffiths and Turnbull (1985) and Barker (1992) with the Campus Imager System, connect a linear electrode array of tens of electrodes to a single meter. Apparent resistivity measurements are recorded sequentially by appropriate automatic switching of electrodes. As a result, high definition pseudo-sections with dense sampling of apparent resistivity variation for shallow depth ranges (0-100m) are obtained in a relatively short time. Automatic data logging and interpretation have significantly reduced the time and expense of surveying.

Multi-electrode capabilities have enabled areally distributed grids of electrodes to be simultaneously monitored in a pole-pole format. Suzuki (1996) develops a 512 electrode real-time data acquisition system for measuring the electrical resistivity. Analogue integration of the waveform is employed in order to monitor all electrodes simultaneously. Shore (1996) has developed E-Scan as a resistivity system using areally distributed pole-pole measurements to accommodate mountainous terrain. Both these systems use stationary measurement arrays. However, for extensive surveys, it would be unrealistic to expect a data acquisition system to cover the entire area of interest at once. Often one requires the ability to conduct a survey over a larger area or extend a search into an
interesting area. Tabbagh (1996) has developed a variety of towed arrays. In one, a V shaped electrode arrangement is used to search for buried building foundations beneath a relatively benign surface, a farmers field. Panissod (1997) suggested an interesting refinement to Tabbagh’s V shaped array based on the profiling microlaterlog tool which uses a circular electrode pad. His I.F.M.P.P, or Isotropic Focusing Multi-Pole-Pole array, consists of two rigid concentric rings of four electrodes per ring, (a scaled microlaterlog), which is towed to conduct a near surface mapping survey. Though portable, the maximum dimensions of this system is limited because the electrodes are rigidly linked to one another and the two rings provide only two effective measures of depth.

Developments in modelling and inversion, and in methods of dealing with specific problems like topography and geologic noise have paralleled developments in instrumentation. Barker (1981), has sought to reduce the geologic noise produced by surface inhomogeneities in a Wenner array sounding experiment. In an arrangement he calls the "Offset Wenner" array, two successive determinations of Wenner apparent resistivity (at the same array spacing but offset from each other along a profile by one electrode spacing) are averaged. In the average, surface effects are significantly reduced. This new array is most useful in the context of a multiplexed electrode layouts such as the Campus Imager System. However, the data are still from a single profile, and off-line lateral effects still may be troublesome. Ritz (1999) suggests use of a data filter similar in concept to the offset Wenner array to improve resistivity pseudosection modelling by removing near-surface effects from Schlumberger sounding/profiling data.

Numerical modelling techniques for solving the 3d forward problem, for earth models of modest complexity, have been available for many years. But until recently they have not been incorporated into iterative inversion routines because of limited computer capabilities. However, it is now practical to employ numerical modelling techniques in the inversion of small resistivity data sets for 3d structures, and during the last decade, several methods have been suggested. Park and Van (1991) presented a 3d inversion algorithm that uses the finite-difference method for the forward problem. Sasaki (1994) describes an approximate inversion method using a 3d finite element forward solution capable of
Li and Oldenburg (1992) have developed an inversion method based on a conjugate gradient to the Born approximation. Barker (1997) also uses a conjugate gradient approach to interpret data collected from the Campus Imager System. Park (1998) linearises the problem and performs a least squares inversion to solve for perturbations in conductivity.

In general, the quality of modern instrumentation has greatly improved, but most research and development is still directed towards traditional 1d collection modes suitable for 1d or 2d earth models. The geology of the real earth is, however, often far more complex and heterogeneous than a simple set of strike-extended horizontal or vertical contacts as are assumed in much physical interpretation. Thus a true 2d, or areally distributed, data collection mode is required. Research into 2d collection methods suggest either static monitoring arrays or small scale towed systems for archaeological investigation are practical. For interpretation of areally distributed data, computer inversion methods are currently considered as the best (perhaps the only) way to deal satisfactorily analyse these data sets. Although full inversion is always a laudable goal, such a computationally intensive method may be too much of a "black-box" technique to be appropriate in the early stages of data assessment. It is widely agreed that in order to avoid pitfalls, interpretation should progress from rough estimates made in the field (to determine for example, appropriate electrode separations, electrode density and area where intense investigation is warranted), toward more sophisticated methods of interpretation eventually based on the complete survey. While in principle a much more reliable picture of electrical structure of the ground should be obtainable from areal surveys, data collection and interpretation must both be developed further before such an electrical mapping method can become a routine prospecting tool.

1.5 Approach to Areal Arrays and Thesis Outline

Recent developments in electrical methods recognize the possibility of collecting much more complete data by use of automated instrumentation. However, most practitioners
still employ single profile data collection except where array dimensions are small enough large for the array to be towed. Profile data collection is made efficient by employing the roll-along technique perfected by the seismic exploration industry where an extended line of sensors connected to an automated recording system, (Fig. 1.4).

![Diagram](image)

**Figure 1.4: Efficient electrode deployment with a 1d roll-along array.**

The actual recording portion is only a subset of this field array and moves electrically as source points are successively stepped through the array. As the measuring system rolls along by electrical switching from one shot to the next, sensors no longer needed on one side of the array are detached from the field array then moved and reconnected to the array where they will soon be needed.

As the seismic industry has shown, this technique can also be applied to survey areas rather than profiles. Whereas a survey on a 1d profile can in the right circumstances estimate 2d resistivity sections of the ground, a 2d areal survey should be able to estimate a 3d model of the ground without any assumptions about target geometry. To implement an areal electrical survey, one straightforward approach is to measure potential with respect to a fixed remote reference electrode simultaneously over a substantial network of points about a current source pole, moving and remeasuring the potential grid as the source is moved over the survey grid. The data set collected in this pole-potential mapping survey is thus a set of maps of the pole potential where the pole source is moved systematically over the whole survey grid, as illustrated in Fig. 1.5.
Figure 1.5: Patch 1 is a 7x7 map of the potential field from a current pole $I_1$. The patch is moved to a new source position $I_2$ for another measurement.

Pole-potential mapping can become a very efficient method of data collection when reciprocity is invoked to reduce the number of electrodes deployed at any one time (described in Chapter 4). It is also a very flexible data set, because pole-pole data can be formed into linear combinations that simulate the measurements obtainable from any other array configurations. I have investigated simple "synthetic arrays" and propose several direct methods of data examination which are akin to the apparent resistivity pseudosection of a 2d survey. For example, the electrode layouts for a circular bipole-dipole array which is a generalization of the linear Schlumberger or Wenner array, and a pole-circular pole array which is a generalization to the linear pole-pole or unipole array are shown in Fig. 1.6.

In the pole-circular pole array, a central current pole is surrounded by a gridded ring of potential measuring electrodes. The scalar apparent resistivity is derived from the average potential (relative to an infinite potential reference electrode) measured on this ring. Additional information can be obtained by analysing the observed potentials into Fourier components with respect to angle about the source. In naming the arrays, I follow the usual convention of transmitter-receiver configuration. For example, the pole-circular pole array consists of a single current (source) pole surrounded by a circle of potential
Figure 1.6: The electrode layout for two synthetic arrays in the context of a pole-potential map survey. The pole-circular pole is shown on the left and the circular bipole-dipole on the right.

(receiver) poles.

In general, an apparent resistivity can (at least for moderate resistivity contrasts) be thought of as a weighted average of the actual resistivity throughout the earth, and the distribution of weights to be a fixed sensitivity array whose space average is one. In Chapter 2, with careful design, I show how the sensitivity of a distributed array can be centered under a single point on the ground and distributed over a desired depth range. 3d maps of apparent resistivity then can provide a low resolution, approximate image of earth structure without the kind of major artifacts that are common in pseudosections of standard profile data. Many comparisons have been made between the performance characteristics of different arrays and their associated apparent resistivities. Such studies can be very misleading as many arbitrary assumptions are made to define a basis for the comparison, especially about what are equivalent noise levels and what are equivalent size parameters. In particular, I show the virtues of the pole-pole array have been obscured.

Detection of resistivity features (anomalous bodies) that are buried an appreciable fraction of their depth extent is usually more of a question of recognizing the targets response in a background of strong responses from near surface features than of detecting its response in a background of measurement noise. But, in either case, an apparent resistivity calculated by averaging numerous potentials will fluctuate less as the array is moved than
will the resistivity from a simple array (assuming the same fractional individual measurement error). As long as the averaged data all are significantly sensitive to the desired part of the ground, a better detection is obtained. Furthermore, profile surveying (2d), especially when lines are not very close together, may suffer interference from surface features between the lines further obscuring the response from deeper targets. In this case, cross-line data can be important in resolving ambiguities in the physical interpretation. I examine this in Chapter 3 through a study of the comparative sensitivity of areal arrays to surface and buried structures.

As I have indicated above, any array resistance measurement (potential difference measured, fixed currents injected) can theoretically be synthesised from linear combinations of pole-pole measurements as long as the measurements are sufficiently accurate and the reference potentials and currents can be replicated (current and potential reference electrodes should be at infinity but necessarily are separated by a finite distance). Since reciprocity can also be invoked, pole-potential map roll-along data acquisition appears to be very versatile. However, because of the fixed dynamic range and limited accuracy of real acquisition hardware and because of the presence of noise, such mathematical constructs could be of limited practical value. Thus, I have constructed instrumentation suitable for scale modelling and a small field experiment and carried out a series of experiments to test for error accumulation and field practicality. The results were favourable and are discussed in Chapter 4.

In summary, I propose pole-potential mapping as an effective method for measuring the near-surface electrical resistivity of the earth which provides as complete an observational data set as it is possible to obtain from the surface. In Chapter 2, I examine the sensitivity of surface potential measurements to the earth's resistivity and suggest an approach whereby a virtual array is created whose sensitivity is centered on a specific region in the earth. Unfortunately, although sensitivities are primarily a function of array geometries, they are only approximately independent of the actual ground structure. Thus, in Chapter 3 I investigate the comparative sensitivity of traditional and synthetic apparent resistivities to a set of extreme earth models. In several cases, apparent re-
sistivities derived from synthetic arrays have a greatly reduced sensitivity to unwanted surface structures, yet retain meaningful information from structures at greater depth. While the mathematical analysis found in Chapters 2 and 3 was encouraging, I felt it necessary to design and construct suitable instrumentation to test the applicability of pole-potential mapping under real-world conditions. This instrumentation along with the favourable results from scale model and a small field experiment are discussed in Chapter 4. In Chapter 5, I summarise my results and make suggestions for future work.
Chapter 2

Theory

2.1 Introduction

Many authors have studied the relationship between apparent resistivity observations obtained with a specific electrode array and idealised earth resistivity structures, especially in relationship to the detectability of layers. A general term for this is sensitivity analysis. When interpreting data, a knowledge of the array sensitivity distribution suggests which portions of the ground most influence a given measurement. Seemingly complicated responses, such as the sharp fluctuations in apparent resistivity profiles crossing a resistivity contact are easily explained by the multi-lobed 3D sensitivity distributions of most linear arrays. Sensitivity functions are also an integral part of any inversion method where they provide the quantitative link between the data and the model parameters i.e., of the so-called Jacobian matrix $[\delta \mathbf{m}/\delta \mathbf{p}]$ where $\mathbf{m}$ is the vector of measurements and $\mathbf{p}$ is the vector of model parameters.

Sensitivity studies have been carried out by many authors, the earliest contribution is by Roy and Apparao (1971). Spitzer (1998) reviews four different criteria for calculating the sensitivity in three dimensions. Numerical solutions are suggested for arbitrary resistivity structures and an analytic solution is given for a homogeneous half-space. Park & Van (1991) look at the sensitivity of pole-pole arrays in the context of inverting data collected on small 3D grids. Boerner & West (1989) present a simplified analysis.
for EM sensitivity to a layered earth and successfully compare the DC expressions to the one for a 1d earth described by Oldenburg (1978). Barker (1979) looked at the sensitivities of specific arrays to variations in a homogeneous earth and suggests an approach to reducing the effect of near-surface resistivity structure in sounding surveys.

To investigate how the apparent resistivity response of an array and earth structure are related, I have used the formalisms of sensitivity analysis to study the dc electrical sensitivity of arbitrary arrays to resistivity perturbations in an otherwise homogeneous half-space. Where most previous studies seek to derive or display (for a specified array) the sensitivity functions to varied earth structures, it is my aim to tailor the sensitivity distribution to focus on a chosen region within the earth. This is accomplished by constructing the geometrical configuration of the transmitter and receiver electrodes to produce the desired results, namely, that the new sensitivity distributions should be (to the extent possible) unimodal, centered on the array’s reference point in the earth (the location with which we associate the apparent resistivity), and concentrated at the desired depth of exploration. I start by examining the response of standard arrays and my new virtual arrays to a layered earth. Several parameters are defined as an aid to survey design. A subsequent 3d analysis provides insight into many of the idiosyncrasies of pseudosection plots and suggests how virtual arrays might provide an improved presentation through virtual arrays.

2.2 Theory of 1d Sensitivity Analysis

It is easy to develop the concept of sensitivity for a host earth model composed of thin horizontal layers. Following the example of Boerner and West (1989), the sensitivity of an observation $\phi(r)$ of potential about a cylindrically symmetric current source (eg. a simple electrode), is the variation in the observation $\delta \phi(r)$ for a variation in resistivity inside the earth $\delta \rho(z)$. The perturbed model is then $\rho(z) + \delta \rho(z)$, and the perturbed observation is $\phi(r) + \delta \phi(r)$ and $\delta \phi$ and $\delta \rho$ can be related by a linear integral equation in
the form,

\[
\delta \phi(r) = \int_z F(r, z, \rho) \delta \rho(z) dz + O[||\delta \rho(z)||^2]
\]  \hspace{1cm} (2.1)

where \( F(r, z, \rho) \) is a Green’s function calculable from EM theory for the given reference model \( \rho(z) \). \( F(r, z, \rho) \) expresses the contribution to \( \delta \phi(r) \) made by a local differential change in resistivity \( \delta \rho(z) \) at each depth. It is easy to see that the perturbation in potential in a thin layer of thickness \( dz \) is proportional to both \( \delta \rho \) and \( dz \), i.e., to the resistivity thickness product of the perturbing layer. The remainder (in Eqn. 2.1) has been shown to be second order in model perturbations by Parker (1977) and Chave (1984) for low frequency EM fields. When it can be neglected, \( \delta \phi(r) \) is said to be Fréchet differentiable (i.e., the function \( F(r, z, \rho) \) is said to be the Fréchet derivative of \( \phi(r) \) with respect to \( \rho(z) \)). It is equivalent to the earlier defined Depth of Investigation Characteristic (DIC) of Roy and Apparao (1971), although they considered their DIC to be a function of array size for a layer perturbation at fixed depth. A widely used alternative formulation by Gomez-Trevino and Edwards (1983) describes the resistivity perturbation in terms of fractional changes, \( \delta \rho/\rho \),

\[
\delta \phi(r) = \int_z S(r, \rho(z)) \frac{\delta \rho(z)}{\rho} dz
\]  \hspace{1cm} (2.2)

where \( S = \rho F \). Sensitivity functions are thus related to the Fréchet derivatives by \( S(r, \rho(z)) = \rho F(r, \rho(z)) \). A third alternative defines \( \delta \phi \) as the fractional change in a standard observable quantity like apparent resistivity, and is called the normalized depth of investigation characteristic or NDIC (Roy and Apparao 1971). It has the helpful effect of making the analysis independent of the background earth resistivity (of the amplitude but not the structure) and the source current. Only when this is done is comparison between arrays of different type or size meaningful.

The 1d sensitivity function for any dc resistivity array can be derived from the simple case of the potential at distance \( a \) from a current source \( I \) over a homogeneous half-space of resistivity \( \rho \) in which the resistivity is perturbed in a thin layer at depth \( h \). The model
Figure 2.1: A homogeneous earth with a thin perturbed layer. The pole potential is measured at a radial separation $a$ from a pole current source.

is shown in Fig. 2.1. The derivation of this is given in Appendix B and follows Banerjee and Pal (1986). The result is,

$$ dV(a, z) = \frac{\rho_1 I}{\pi} \frac{2z \, dz}{(a^2 + 4z^2)^{3/2}} \quad (2.3) $$

Roy and Apparao (1971) derived an equivalent expression by evaluating the integrated contribution of individual points in a thin layer through the concept of electrostatic equivalence (reciprocity), and presented it as the depth of investigation characteristic, DIC or NDIC.

$$ NDIC(a, z)_{pp} = \frac{4az}{(a^2 + 4z^2)^{3/2}} \, dz \quad (2.4) $$

The NDIC for a four electrode array is found by considering linear combinations of pole-pole observations. For example, the layer sensitivity of a four electrode array with current electrodes at A and B and potential electrodes at M and N is

$$ NDIC(z) = \frac{dV(r_{AM}, z) - dV(r_{AN}, z) - dV(r_{BM}, z) + dV(r_{BN}, z)}{\rho_1 I \left( \frac{1}{r_{AM}} - \frac{1}{r_{AN}} - \frac{1}{r_{BM}} + \frac{1}{r_{BN}} \right)} \quad (2.5) $$

Merrick (1997) shows that, to calculate the cumulative response for that part of the earth below depth $z$,

$$ C(z) = \int_{z'}^{\infty} NDIC(z') \, dz' \quad (2.6) $$

For a general array, the cumulative response is,
L.S. Edwards (1977) and Merrick (1997) make an argument for an alternate measure of investigation depth based on the cumulative response function; the median depth $Z_e$, also named the effective depth, taken as the depth at which half of the cumulative sensitivity comes from above and half from below. This proves to be a less ambiguous measure in a multi-layered earth where the DIC exhibits discontinuities at layer boundaries and may have several maxima.

2.2.1 Layer Resolution

Several concepts are developed as an aid to understanding array behaviour from the cumulative and characteristic response functions. From the DIC or NDIC response functions, resolution of a layer can be defined as the inverse of the full-width at half-maximum of the characteristic response. A narrower peak gives rise to a higher number corresponding to better vertical resolution. It can also be defined in terms of the effective thickness $h_e$, the thickness of material below the median depth accounting for an additional contribution of 10% to the measured voltage or apparent resistivity (a measure advocated by Merrick (1997)). A fractional resolution index combining the median depth, $Z_e$, and effective thickness, $h_e$, is then $R = Z_e/(Z_e + h_e)$. Figure 2.2 illustrates for two pole-pole arrays the median depth and two measures of layer resolution.

While precise definitions of exploration depth or resolution in a layered earth are not of paramount importance, they do provide a qualitative understanding of how arrays respond to any sort of anomalous resistivity structure.

Figure 2.3 demonstrates the effect of increasing source-receiver separation for a single current and potential electrode. In the left plot, the sensitivity for pole potential measurements taken at successively greater radial displacements from the source are shown.
Figure 2.2: The pole-pole DIC and cumulative DIC for a 1 and 2 m separation. The median depth is indicated as are two possible definitions for layer resolution, the effective thickness $h_e$ from the cumulative distribution and the full-width-at-half-maximum from the DIC.

In the right plot, each curve rises from zero to a maximum sensitivity and then decreases in sensitivity to zero at infinite depth. The depth to maximum sensitivity increases with increasing source receiver separation and is linearly proportional to the electrode separation at $0.35a$. The proportionality constant for the median depth is $0.87a$ (where $a$ is the pole separation) indicated by solid vertical lines. With increased electrode separation there is a broadening of the peak corresponding to the decrease in the vertical resolution. To detect a layer at greater depths, it must be correspondingly thicker. The total voltage measured on the ground surface is given by the area under the curve, $\frac{a^2 L}{2\pi Z}$, which falls off with the inverse of the radius. Where the NDIC is plotted, the area must be multiplied by the background potential ($\frac{a^2 L}{2\pi Z}$) to give the measured potential. With array separation increasing linearly, the response for larger separations becomes increasing similar, and little new information is added. Oldenburg (1978) in a paper on the generalized inversion of resistivity sounding contemplates the question of optimal electrode spacing for maximum resolution at a given depth for a limited number of electrode positions providing a theoretical basis for the maximum expansion factor of $\sqrt{2}$ commonly used in Schlumberger soundings.

In differencing a measurement made at a large separation from that of a smaller separa-
Figure 2.3: The DIC for three pole-pole arrays (on the left) and the DIC (with the peak response of the 4.8 m pole-pole normalized to 1) for two pole-pole arrays and a pole-dipole array (on the right). The area between two pole-pole gives the response for the pole-dipole array. The solid vertical line indicates the median depth.

tion, the common effects of deeper layers are eliminated while the effects of near-surface layers are accentuated. In theory, this can be done after the survey from individual pole-pole readings, or during the survey with a closely spaced pair of in-line potential electrodes as would be measured with a pole-dipole array. Shown graphically in the right plot of figure 2.3, the pole-dipole sensitivity is given by the area between the pole-pole curves. The fraction of the measured signal coming from the near-surface is increased, effectively increasing the resolution but decreasing the depth of investigation. Although the signal is smaller, the background potential is also smaller.

2.2.2 Comparison of Array Sensitivities (1d)

Many authors have used sensitivity measures to compare the layer sounding capacities of various standard arrays and try to decide which is best. Although this is a straightforward mathematically and computationally, it requires that a specific single measure
of performance be chosen (objective function) and this generally influences the results. In this section, traditional four traditional linear arrays: the Wenner, Schlumberger, dipole-dipole, and pole-pole are compared to one another, and by extension to the circular pole-pole and pole-dipole arrays defined in Chapter 1. The task of computing the response of different arrays to a layer is simplified by noting that many configurations are equivalent when reciprocity is considered. For any system where the current electrodes are symmetrically deployed about the potential electrodes (or vise versa), only one current pole has to be considered when evaluating the layer response. The return current electrode serves only to double the measured potential, but does not otherwise affect the response. Also, the pole-pole and pole-circular pole have identical response to horizontal layers because all the potential electrodes have the same radial separation from the moving source. The same is true for the pole-dipole and pole-circular dipole. Furthermore, the response of a pole-dipole of a given length and very small dipole separation is the same as that of a Schlumberger array of twice the length.

Figure 2.4 depicts the NDIC of these four arrays. In the uppermost plot, the common plotting element is the total distance L between the furthest active electrodes in the movable part of the array (i.e., excluding remote electrodes). At first inspection, the Wenner array has the best resolution (the narrowest peak) while the pole-pole array has the worst. However, since array resolution decreases for all arrays as the depth of investigation increases, a better measure for comparison is either the depth to peak sensitivity or the median depth. From a practical standpoint, either of these two measures is a better choice than the array length as the expected target depth, and hence the depth of exploration, is the important factor in determining electrode spacings and station intervals.

In the middle and lower plots of Fig. 2.4 the peak response scaled to unity and the array lengths are chosen to give an equivalent depth to peak sensitivity and equivalent median depth in the middle and lower plots respectively. For the middle plot, the near-surface rise is almost identical for each array. The pole-pole response decays slowly and the total measure potentials influenced by a broad average of the deep structure. Introducing a
Figure 2.4: Depth of investigation characteristic for 4 standard arrays, the Schlumberger (S), Wenner (W), pole-dipole (D), and the pole-pole (P) arrays. In the upper plot the length of each array is the same. In the lower two plots the peak response is normalized to one. In the middle plot, the array length is chosen such that each array has a common depth to peak sensitivity; and in the lower plot, it is chosen such that the median depths are equivalent.

Local return current or potential reference, as in the Schlumberger configuration, curtails the sensitivity to deeper structure resulting in a narrower response with greater layer resolution. Since improved layer resolution is achieved by increasing the array length while measuring the potential with closely spaced electrodes, the measurable signal is reduced. Not only is the measurable signal smaller, but the 1d model must be valid over a larger area, increasing the susceptibility to geological noise.

Table 2.1 summarizes the important characteristic array responses to a layered earth. The depth to maximum and median depth are useful in survey design and provide a better plotting depth for apparent resistivity pseudosections than an arbitrarily chosen value. Over a homogeneous earth this is a better normalization as it allows surveys conducted
Table 2.1: The 1d exploration depths for standard arrays, from Roy (1971) and Merrick (1997) where \( Z_i \) is the depth to peak sensitivity. The pole-circular pole and circular dipole-dipole arrays are equivalent to the pole-pole and Schlumberger arrays respectively.

with different arrays to be displayed in a common section plot, for example, gradient and Schlumberger sounding surveys. For the virtual arrays the exploration depths and resolutions provide a qualitative guide for electrode spacing one might need in a field survey. The true exploration depth is model dependent, especially to any significant layering above the exploration depth.

2.3 3d Sensitivity Analysis

To understand how lateral anomalies might influence a measurement we need to more than a simple 1d analysis. Sensitivity has been extended (to resistivity perturbations at local points or along lines) to include two and three dimensions by several authors including Park and Van (1991), Li and Oldenburg (1992), and Loke and Barker (1995). Local sensitivity was implicit in the method used by Roy and Apparao to obtain layer DIC’s and NDIC’s but was not further exploited by them. Following Loke and Barker (1995) and Park and Van (1991), I start from the homogeneous earth model and consider the potential \( \phi \) arising from a point current source \( I \), located at \( r \), in a homogeneous half-space of resistivity \( \rho \). Poisson’s equation for a uniform \( \rho \) is,
\[ \nabla^2 \phi = \rho I_s \delta(r_s) \] (2.8)

A perturbation \( \delta \rho \) of the resistivity \( \rho \) in an elementary volume \( \delta \tau \) produces a change in potential \( \delta \phi \) which is,

\[ \delta \phi = \delta \rho \rho^{-2} \int_{\tau} \nabla \phi \cdot \nabla \phi^* d\tau \] (2.9)

where \( \delta \rho \) is constant in a volume element \( V \) and zero elsewhere. \( -\nabla \phi \) is the electric field of the source in the homogeneous background model and \( -\nabla \phi^* \) is the electric field that the current perturbation in \( \delta \tau \) produces at the potential electrode. It is most easily calculated by reciprocity as the electric field produced at \( \delta \tau \) by a fictitious unit current source at the potential electrode. For a current source located at the origin \((0,0,0)\) and the potential electrode at \((a,b,0)\) we have

\[ \phi = \frac{\rho I_s}{2\pi [x^2 + y^2 + z^2]^{1/2}} \] (2.10)

\[ \phi^* = \frac{\rho}{2\pi [(x-a)^2 + (y-b)^2 + z^2]^{1/2}} \] (2.11)

Calculating the divergence of \( \phi \) and \( \phi^* \) and substituting into 2.9 we get

\[ \delta \phi = \int_{\tau} \frac{I_s}{4\pi^2 (x^2 + y^2 + z^2)^{3/2}} \frac{x(x-a) + y(y-b) + z^2}{[x^2 + y^2 + z^2]^{3/2}} \rho dx dy dz \] (2.12)

where the intergrand is the 3d Fréchet derivative for a homogeneous half-space. The sensitivity to resistivity variation in other larger volumes can be derived by integrating 2.12 over the desired space. This approach is usually taken when inverting data. The earth is divided into a large number of finite cubic blocks, (e.g., Loke 1996 and Sasaki 1994) with the potential \( \phi \) evaluated over the volume of the block. This is not necessary here and much can be learnt directly from the Fréchet kernel.

I computed the 3d local sensitivity for the most standard arrays present it using two orthogonal sections, one along the surface and the other vertically through the source.
and receiver. These plots were named signal contribution sections by Barker (1981).

It should be noted that the sensitivity of any linear array is cylindrical about the array line. Sensitivity is defined as the contribution per unit resistivity*volume of perturbation at a point in the model. It contains no normalization for the expected large differences in volume of perturbation necessary to produce a measurable change when the perturbation volume is near or far from an array electrode. Thus it has a very large range of values and can be negative or positive. For better illustration of its variation, I have contoured the inverse hyperbolic sine of the sensitivity normalized by the background potential. This gives a linear scale for small values and a logarithmic scaling for larger ones. Since the sensitivity functions become infinite at the source and receiver contact points, the shallowest evaluation depth is limited to 0.05 m for arrays with spacings of order 1 m. Finally, the polarities of transmitting electrodes are chosen to provide positive sensitivity (an increase in resistivity perturbation produces an increased potential) in the zone of interest around the data plotting point (reference point).

Figure 2.5 shows for a pole-pole array a vertical section through the sources and plan view for depth \( z = 0.05/Z_e \). The sensitivity pattern is symmetric about the axis connecting the electrodes such that the plan view is identical to the section view with the addition of a mirror image. The contribution of a buried element to the total response is equal to the contribution of a surface element, or any other element, the same radial displacement from the electrode axis. Interestingly, this means the lateral sensitivity to a thin vertical sheet parallel to the electrode axis, is equal to the vertical sensitivity to a horizontal layer. This is true for any source receiver configuration where the active electrodes are confined to a line.

Figure 2.6 shows the sensitivity for three linear arrays. Again, distinct regions of negative and positive polarity appear between the electrodes. The main region of positive sensitivity is between the dipole pair. In general, there are \( n + 1 \) such regions where \( n \) is the number of active electrodes (electrodes in the moving portion of the array). A survey taken across a contact is characterized by apparent resistivity profile of \( n + 1 \) segments.
Figure 2.5: Section and plan views of the 3d sensitivity of a surface pole-pole array normalized by the background potential. Contour values are the inverse hyperbolic sine of the sensitivity.

due to the distinct lobes of sensitivity.

For the gradient array, the uppermost plot of Fig. 2.6, the sensitivity pattern is symmetric about the dipole midpoint. Since the current source and sink are far away, they have no influence on the normalized sensitivity measurement and there is only one zone of positive contribution. Maximum positive sensitivity is thus unambiguously located symmetrically beneath the array midpoint. The middle plot for a pole-dipole array shows the effect of bringing one current pole near the potential pair. The section loses symmetry and there are now two zones of positive contribution. However, the sensitivity is greater beneath the potential dipole. The twinning of the region of positive sensitivity is what some authors refer to this as the AB and MN effect, Van Zijl (1985) and Ward (1990). For the pole-dipole array, the plotting point is often taken in the zone of negative sensitivity at the array midpoint; although, it usually would be better to plot the measured apparent resistivity below the center point of the dipole pair, the location suggested in
Figure 2.6: The 3d sensitivity sections of the gradient, pole-dipole and dipole-dipole arrays from top to bottom respectively. Contour values are the inverse hyperbolic sine of the sensitivity.


The lowermost plot in Fig. 2.6 depicts the dipole-dipole array with its two regions of positive sensitivity lying beneath the dipole pairs and joining together at depth under the centre of the array. Although the sensitivity is symmetric about the array midpoint suggesting this to be an appropriate plotting point, the sensitivity is low near the array midpoint.

Sensitivity is per unit volume of perturbation (considered always to be in a small volume). However, the practical meaning of small varies. Near an electrode, only a very small region $\sim \left(\frac{a}{30}\right)^3$ can be fitted in. At a depth of $z=a$ it is easy to envisage $\left(\frac{a}{3}\right)^3$ as behaving as small. Thus, sensitivity values must be considered in relation to the depth at which they are situated in order to tell whether they are significant or not.
2.3.1 Improving Array Design

Sensitivity analysis clarifies much of the complicated behaviour observed in apparent resistivity pseudosections and profiles across real geological structures using standard arrays. It should therefore be possible to use the concept to design an improved measurement technique for 3d resistivity. Ideally this should produce a suite of output observations that describe earth structure in a relatively simple, easy to understand, manner and taking sensible account of the method's fundamental limitations. The suite of measurements might be divided into (a few) primary and (possibly more) auxiliary measurements, and the primary measurements should preferably be imaginable as approximate (e.g., smoothed) maps or cross-sections of the earth parameter(s) that affect the measurements. The ancillary measurements then should give additional information useful in resolving ambiguities and details. The features desirable of a primary measurement are the following:

1. Its sensitivity distribution should be unimodal, that is, it should contain but one distinct zone of high sensitivity.

2. The distribution should be symmetric about the point of maximum sensitivity and exhibit only low values (+ or -) everywhere away from the main lobe. Preferably, it should be axially symmetric about a vertical axis through the maximum and the symmetry axis will be the data plotting point at the surface.

3. The suite of primary measurements (e.g., the suite obtained by expanding the size of the array) should exhibit different vertical extents of the zone of high sensitivity, both for 1d and 3d cases (this is automatic, if every part of the array is similarly scaled).

4. A reduced sensitivity to surface structure is desirable so the main sensitivity lobe will peak at depth.
The first item ensures that any anomaly in measurement is most likely due to anomalous structure near the plotting point of the array measurement. Thus, anomaly profiles over a narrow resistivity feature will have the correct polarity and exhibit minimal side oscillations. The width of the anomaly (lateral resolution of the measurement) will be the width of the unimodal distribution. Regarding the second item, it is clearly desirable that symmetry of the anomaly reflect symmetry of the causative structure. Thus, the array's sensitivity should best be symmetric about a vertical axis through the measuring point rather than the horizontal axis characteristic of linear arrays. A single lobe with symmetry about a vertical rather than horizontal axis clearly is more desirable. The third item is difficult to address through sensitivity analysis since it involves assumptions about what anomalous volumes are equivalent at different depths. I address the issue in Chapter 3 by using analytic models of strongly contrasting conductivities. The fourth item is achieved by combining into the total measurement components whose surface sensitivity will cancel but whose sensitivity at depth will add.

Barker (1981) has applied an understanding of signal contributions sections to reduce surface effects. He has considered combining two successive measurements made with a Wenner array, the second measurement offset along the profile one electrode spacing from the first, as illustrated in Fig. 2.7. The ground between any two electrodes provides either a positive or negative contribution to the measured voltage and at depth the contribution is everywhere positive. Thus, a small anomalous body in a negative zone in the first reading falls within a positive zone of the second reading. On averaging the two measurements, the local surface sensitivity is attenuated while the deeper sensitivity remains. The lowermost plot illustrates the combined contribution. The Wenner array is particularly easy to implement in its offset form with a multi-conductor cable, though the concept can be applied to other linear arrays.

Another array with reduced sensitivity to surface structure is the unipole array. As originally proposed, two moving current poles of the same sign and equal in magnitude, were deployed on either side of a potential pole referenced to a remote potential. According to Gupta (1963), the method has increased depth of investigation and sensitivity
Figure 2.7: In the upper two plots are the sensitivity distributions of two Wenner measurements offset on electrode spacing from one another. The lowermost plot shows the average of the two and is called the offset Wenner array by Barker (1981). Contour values are the inverse hyperbolic sine of the sensitivity.

to deeper structure and decreased sensitivity to near-surface inhomogeneities. Gupta’s unipole array is the reciprocal of an array in which the average potential on either side of a single current pole is observed with respect to a remote reference on either side of it. This is a much easier to use form of the array, in practice or for theory and its sensitivity can be constructed by averaging two successive pole-pole measurements on a profile as show in Fig. 2.8. The combined sensitivity is symmetric about the axis through the electrodes. A single large positive lobe exists beneath the current pole but the side lobs, although reduced, are not negligible. An similar arrangement has recently been suggested by Kampke (1999).
Figure 2.8: Sensitivity for individual pole-pole readings and their average. The averaged plot is symmetric about the source point and less sensitive to surface structure. Contour values are the inverse hyperbolic sine of the sensitivity.

2.4 Circular Arrays

The offset Wenner array successfully reduces interference from small superficial anomalies and is easily implemented when using a multi-core cable and electrode switching. Expanding the inter-electrode spacing offers the ability to simultaneously sound and profile, but the continued presence of appreciable side lobes associated with its length limits lateral resolution. Both the offset Wenner and modified unipole arrays have symmetric sensitivity distributions about an axis through the electrodes and thus suffer from effects due to off-line, lateral anomalies and the array to target orientation. More importantly, it is impossible to fully suppress side lobe sensitivities near the individual electrodes if only linear arrays on coarsely spaced sampling points are considered. For gridded, pole-potential map data, forming the measurements into virtual arrays based on a series of
expanding rings, removes these problems. Simply, put, the 3d sensitivity lobes (necessarily) associated with individual electrodes become averaged for each rotated electrode.

![Pole-Circular Pole, R=(0.4, 0.8, 1.2)m](image)

Figure 2.9: Pole-circular pole 3d sensitivity sections through the current pole for 3 different array spacings. Contour values are the inverse hyperbolic sine of the sensitivity.

The simplest virtual array consists of a central current pole surrounded by a ring of potential receivers (Fig. 1.6). Figure 2.9 shows sensitivity cross-sections through the current pole and two of the potential electrodes. The section is axially symmetric about a vertical axis through the current pole. Thus there is no angular dependence on the sensitivity. At depth the sensitivity is everywhere positive with the greatest contribution in the central lobe beneath the source pole. As the radius expands, the central lobe extends to greater depth and expands laterally indicating a greater exploration depth, but also a loss of resolution. The region of negative sensitivity is confined to a near-surface disk between the current pole and the potential ring. Directly beneath the potential ring
Figure 2.10: Differenced pole-circular pole 3d sensitivity distributions. In the three plots, the potential on an outer ring is differenced from that on an inner ring. The rings are separated by 0.25 m with the inner ring taking on values 0.25, 0.5, and 0.75 m.

Electrodes are small dipolar regions of sensitivity. The 1d depth of investigation and the median depth are unchanged from those of a simple pole-pole measurement; however, the 3d sensitivity distribution is now unimodal and axially symmetric; consequently, the resulting apparent resistivity measurements are much simpler to interpret.

On increasing the potential ring radius, the response from successive rings become increasingly similar and suggests that differencing of rings to remove the influence of deeper structure. Figure 2.10 depicts the result when the potential of a large ring at 1.25 m is differenced from that of 0.5, 0.75, and 1.0 m. As with linear arrays, sensitivity to near-surface structure is increased particularly in the zone beneath and between the receiving rings. Layer resolution is better, but lateral resolution is decreased. A significant benefit is found in that the influence of the remote reference voltage is removed from the data.
2.5 Homogeneous, Anisotropic Earth

Since pole-circular pole averages prove to be very effective at suppressing near-surface features, I have looked at other circular arrays. Dipole potential measurements are particularly sensitive to the ground between the dipole electrodes. To synthesise an array with only one main sensitivity, methods of combining pole-pole measurements in a fashion similar to the pole-circular pole array were investigated. A circular ring of current electrodes and two perpendicular dipole potential measurements can make a tensor apparent resistivity measurement.

Tensor apparent resistivity measurement means measuring 2 vector components of $\vec{E}$ for each of two orthogonal directions of current excitation, as in $\vec{E} = \vec{p} J$. In measurement of apparent resistivity, $\vec{E}$ is an observed vector whereas $J$ is an assumed current flux created in a uniform reference medium. Bibby & Hohmann (1993) have advocated making tensor measurements where the source is a fixed pair of dipoles and the receiver a roving dipole pair, as illustrated in Fig. 2.11.

![Figure 2.11: The bipole-dipole array layout after Bibby and Hohmann (1993). Using two perpendicular potential dipoles at site P, the measurement of the two electric field vectors $E_{ab}$ and $E_{cd}$ for two remote bipole source AB and CD is sufficient to determine the apparent resistivity tensor at P.](image)

The potential differences can be taken as a measure of the electric field, provided the dipole lengths are small compared to $r_a$ and $r_b$. To derive an apparent resistivity the observed electric field $E$ at the surface is compared to the current density vector $J$ that would be generated at the measurement location by an equivalent current source in a uniform half-space (note that, $E$ is an observation and $J_0$ a model value, i.e. $E_{obs} = \rho_a J_0$). The current density vector is a theoretical quantity only dependent on the geometry of
the array and the current used and can be expressed as,

\[ \mathbf{J} = \left( \frac{\mathbf{r}_a}{R_a^3} - \frac{\mathbf{r}_b}{R_b^3} \right) \frac{I}{2\pi} \cong \frac{I}{2\pi} (\mathbf{AB}) \nabla \left( \frac{1}{R_0} \right) \quad \text{when } AB \ll R_0 \quad (2.13) \]

where \( \mathbf{r}_a, \mathbf{r}_b \) are the position vectors of length \( R_a, R_b \) of the field station relative to each of the current electrode positions and \( I \) is the applied current (not the actual current density in the earth).

In usual practice \( \mathbf{E} \) is measured in the direction of \( \mathbf{J} \) and then,

\[ \rho_a = \frac{|\mathbf{E}|}{|\mathbf{J}|} \quad (2.14) \]

where only the magnitudes of electric field and current flux vectors are considered and the resultant \( \rho_a \) is a scalar. But, if \( J_0 \) can be applied in two orthogonal directions, we may determine \( \bar{\mathbf{E}} \) for both cases and define,

\[ \mathbf{E} = \rho_a \mathbf{J}_0 \quad (2.15) \]

where \( \rho \) is an apparent resistivity tensor. The rotational invariants of the tensor can then be expressed as,

\[ P_1 = \frac{1}{2} \text{trace}[\rho] = \frac{1}{2}(\rho_{11} + \rho_{22}) \quad \text{2x2 case} \quad (2.16) \]
\[ P_2 = (\text{det}[\rho])^{\frac{1}{2}} = (\rho_{11}\rho_{22} - \rho_{21}\rho_{12})^{\frac{1}{2}} \quad (2.17) \]
\[ P_3 = \frac{1}{2}(\rho_{21} - \rho_{12}) \quad (2.18) \]

where \( P_1, P_2, P_3 \) have dimensions of resistivity. The orientations of the principal axes are also determined. For a uniform earth of resistivity \( \rho \), \( P_1 = P_2 = \rho \), and \( P_3 = 0 \). Thus \( P_1 \) and \( P_2 \) are some kind of average resistivities of the earth. In Appendix E is found a discussion on the physical interpretation of the \( P_2 \) invariant. Other forms of the invariants can be derived from combinations of two of these parameters. Unlike the apparent resistivities derived from each of the current bipoles alone, these properties are independent of the direction of the electric field measurements.
Circular Bipole-Dipole

Figure 2.12: Circular bipole-dipole 3d sensitivity distributions. The radius of the transmitting ring varies from 1.00, 1.25, to 1.50 m.

In the circular bipole-dipole array, an orthogonal pair of dipoles in the center of the array observes the electric field set up by a gridded circle of contact electrodes (Fig. 1.6). To synthesise a uniform current density within the ring in a laterally uniform earth a cosine weighting is applied to the source currents in the ring to account for their spacing and angle with respect to the potential dipole. The apparent current field is easily rotated by changing the weights, so we are able to calculate a tensor apparent resistivity, with invariants of the tensor providing scalar measures.

Figure 2.12 depicts the sensitivity section of a circular dipole-dipole array through the ring center and parallel to the current density vector. As with any dipole measurement, the sensitivity is highest in the zone between the dipole pair regardless of ring radius. Thus a large response may be observed to structure local to one dipole pair in the ring; however, this response will be reduced in the average.
While I discuss scalar measures of apparent resistivity (the P2 invariant for the circular bipole-dipole array and pole-circular pole apparent resistivity found from the average potential measured on a circular ring) in the body of this thesis, additional information can be obtained from these arrays. For the pole-circular pole array, by analysing the potentials into Fourier components with respect to angle about the source, information can be obtained about the amplitude of the local apparent resistivity, the direction of the gradient in apparent resistivity, and the degree of anisotropy. For the circular bipole-dipole array, the apparent resistivity tensor can be displayed graphically as an ellipse. The direction of the major and minor axis can be indicative of lateral inhomogeneities. While these aspects of pole-potential mapping were investigated, and there is undoubtedly considerable information available from a harmonic analysis, I did not find a single and simple presentation method to display these data. Some results and discussion are found in Appendix E.

2.6 Summary

When the exploration model is a layered earth, it is possible to increase the vertical resolution by differencing closely spaced potential readings. This reduces the measurable signal but increases the layer response relative to that of the background. As the distance to the source increases, the investigation depth also increases; however, this does not imply that the systems are insensitive to material very close to the ground surface. The contributions from the elements at the electrodes themselves reach extremely large magnitudes (singularities), though they cancel each other in the summation process.

When the earth is not layered, the sensitivity to material very close to the ground surface does not cancel. In this case, looking to the 3d spatial distribution of sensitivity explains much of the complicated behaviour often observed in apparent resistivity plots. Differencing a closely spaced pair of potential electrodes produces a large lobe of near-surface sensitivity between the dipole pair. Furthermore, this lobe does not disappear with increasing separation to the source. The advantage of increased vertical resolution
is now frustrated by possible contamination from geological noise.

Careful consideration of the geometrical configuration of transmitter and receiver electrodes suggests a circular arrangement of electrodes greatly simplifies array response. For the pole-circular pole array, the sensitivity distribution is unimodal with the maximum value located directly beneath the central current pole. The single lobe eliminates apparent resistivity oscillations and removes any ambiguity in choice of plotting points. Expanding the ring radius increases the depth of investigation and simultaneously reduces the near-surface response. Symmetry in the sensitivity distribution about a vertical axis negates array to target orientation and lateral effects. The circular bipole-dipole array is similarly immune to orientation effects, but is more sensitive to the near-surface effects than the pole-circular pole array.
Chapter 3

Detecting Deep Resistivity Structure in the Presence of Near-Surface Structure

3.1 Introduction

In Chapter 2, I used linear sensitivity analysis to study one of the most troublesome aspects of electrical exploration, the extreme sensitivity to shallow structures in the vicinity of the measuring electrodes. With most traditional measuring arrays, near-surface structures crossed by the array can contribute so strongly to the response that the interpretation of deeper structure becomes difficult, if not impossible. The sensitivity study suggested that several synthetic distributed arrays have a reduced sensitivity to the near-surface while retaining sensitivity to deeper structures. Though encouraging, sensitivity analysis deals only with resistivity perturbations of elemental volumes, usually in a homogeneous background; whereas the objective of many field surveys is to identify the signal from large structures of a high resistivity contrast situated in a heterogeneous background.

In this chapter, I more thoroughly investigate by forward modelling the origin and characteristics of near-surface noise, especially when the resistivity contrast is large. I examine some very simple models whose responses are easy to compute and are very illustrative of general principles. A small scale heterogeneity at the surface is modelled with a hemispherical resistivity anomaly and a subsurface target body is modelled with a
highly conductive sphere. Using the hemisphere, we can determine for arrays of various geometries the nature of the response of a surface heterogeneity and how large it must be to have a serious effect on the apparent resistivity data. The sphere model for subsurface targets provides a way of measuring the limits of target detectability for a compact equidimensional body. Analytic solutions to the forward response of spherical and hemispherical features in a uniform host are discussed by numerous authors, Van Nostrand & Cook (1966), Grant & West (1965), and Wait (1982), to name a few. From Van Nostrand & Cook (1966), the sphere and hemisphere solutions are given in Appendix B. Where both surface bodies and deep targets coexist nearby in the same model, analytic solutions are insufficient, and I resort to a finite-difference method by Bailey and Cheesman (1996), described in Appendix D. Analysis of data from a pole-potential map survey generated by forward modelling of such a complex model show that the broad weak response of circular arrays to deep structure is clearly evident despite the presence of near-surface structure.

3.2 Effects of Surficial Heterogeneity

I begin by showing the hemisphere response of various arrays to introduce the concept of locally produced distortions. Results are presented in two ways: first as continuous profiles at constant array spacing taken across a hemisphere; secondly, as suites of profiles at several array spacings which may be combined into pseudosection plots.

The pole-dipole linear array is in common use for gathering apparent resistivity profile data. In Fig. 3.1, I examine the response of three variations of this array (the configurations are shown at the bottom of the figure) to a 3.33 Ω.m hemisphere in a 10 Ω.m half-space. From the profiles we see the apparent resistivity fluctuates above and below the background value by many 10's of percent, although the hemisphere is half as resistive as the host medium. Where only the dipole or pole is moved (the solid and dashed lines respectively), the data plotting point is taken at the center of the moving electrode(s) as this produces the best correlation between observed anomalous response and the lo-
cation of the hemisphere. When both the pole and dipole are moved together (dotted line), there is no longer an unambiguous choice of apparent resistivity plotting point. Because the response is largest when the dipole passes over the hemisphere, the dipole center is chosen. The responses in Fig. 3.1 will be the same whether the dipole electrodes are current or potential electrodes. To see this, we invoke the principle of reciprocity to invert the configuration by interchanging all current and potential electrodes.

![Continuous pole-dipole profiles over a conductive hemisphere.](image)

Figure 3.1: Pole-dipole profiles across a 3.33 Ω·m hemisphere of 0.75 m radius in a 10 Ω·m half-space. The profiles are shown at the top for the pole-dipole configurations beneath.

In Fig. 3.2, a continuous sampled pseudosection is depicted for a dipole-dipole survey over a highly conductive hemisphere of radius 1 m. The dipole spacing is 0.5 m with the n spacing varied from 1 to 8. The vertical scale corresponds to the n spacing (separation...
Figure 3.2: A dipole-dipole pseudosection showing a classic pant-leg response to a near-surface highly conductive hemisphere (radius 1 m, in a 10 Ω.m background). The dipole spacing is 0.5 m with n varied from 1 to 8.

between the dipole pairs) and is related to the depth. The data are plotted at the array midpoint. Since the response is greatest when one of the dipole pairs is near the hemisphere, this plotting point results in the well known pant-leg effect seen in Fig. 3.2. This strong response to surface structure can obscure the response to deeper structure.

Another way surficial structures interfere with the interpretation of deeper structure is by causing a so called static shift in the apparent resistivity (i.e., a raising or lowering of the background level on which the response of the deeper structure is superimposed). The magnitude of the static shift due to local surficial structure is a complicated function of array geometry and the geometry of the local structure and it need not be a simple additive or multiplicative shift. In Schlumberger soundings it is easy to identify the shift. Thus I begin with an example of a Schlumberger sounding survey near a small hemisphere and proceed to a more general analysis of static shifts cause by a hemisphere adjacent to a variety of arrays.

In Schlumberger sounding, a potential dipole pair is kept stationary while the current poles are expanded symmetrically about the potential pair (Schlumberger 1927). Eventually the voltage becomes to small to measure and the potential dipole separation is increased. In Fig. 3.3 the solid line is the Schlumberger sounding for a 100 Ω.m layer of 10 m thickness over a 500 Ω.m half-space. The remote current poles are expanded.
Figure 3.3: Schlumberger soundings near a 5 $\Omega$.m hemisphere of 0.75 m radius in a 10 $\Omega$.m, 10 m thick layer over a 500 $\Omega$.m half-space. The segments correspond to potential dipole spacings of 2.54, 5.12, and 10.24 m.

in logarithmic intervals with the potential dipole separation increased 3 times. When a small conductive hemisphere is introduced near the potential dipole, separate segments in the sounding curve appear for each dipole separation. The segments are offset from one another by a static shift; so named as the influence of the local anomaly does not depend on the distance to the current electrodes. The standard approach to interpreting such data is first to ensure some overlap in the segments then to shift the individual segments up or down to align with either the first or last segment prior to making a layered earth interpretation.

For a more general analysis, a small hemisphere (diameter < array spacing) is considered in situations in which one of the electrodes is at the hemisphere edge (this leads to maximum anomalous response). Since my concern is with maximum effect, only inhomogeneities that are highly conducting or insulating have been considered. Van Nostrand
Figure 3.4: Electrode positions used in the preparation of Table 3.1. For each array, one electrode is situated on the hemisphere edge. Schlumberger arrays use the same arrangement as the pole-dipole array with the addition of a current electrode symmetrical opposite to the pole shown. The hemisphere is not drawn to scale.

and Cook (1966) undertook a similar study; however, their objective was to identify which electrode, the current or potential electrode, was more disturbed by a near-surface hemisphere (by reciprocity, they will be equally affected). Fig. 3.4 shows the cases considered and the results are shown in Table 3.1. The diameter of the hemisphere has been varied to find the value (relative to array spacing) that produces a noticeable 10% anomalous response. The hemisphere diameter as a percentage of the separation between the electrode touching the hemisphere and the next nearest electrode is tabulated (the separation is "a" except for the pole-dipole (B) case where it is "na").

In Table 3.1 we see the size of a noticeable hemisphere is roughly proportional to the distance between the touching electrode and the closest electrode. A highly resistive hemisphere has less effect than a highly conductive hemisphere and must be larger to produce a 10% anomalous apparent resistivity. Independent of the array type, increasing the distance between the electrodes straddling the hemisphere reduces the apparent resistivity response in an inverse proportion. If the electrodes straddling the hemisphere are of fixed separation while the separation to the remaining electrodes are increased,
Table 3.1: Effect of a highly resistive or conductive hemisphere in a uniform half-space, on the apparent resistivity. The hemisphere diameter (for which the apparent resistivity is 10% anomalous) is expressed as a fraction of the electrode separation between the adjacent electrode and the next nearest electrode.

<table>
<thead>
<tr>
<th>Array</th>
<th>Size of noticeable hemisphere</th>
<th>2R/separation % (insulating case)</th>
<th>2R/separation % (conductive case)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole-Pole</td>
<td></td>
<td>17.48</td>
<td>8.93</td>
</tr>
<tr>
<td>Wenner</td>
<td></td>
<td>16.89</td>
<td>8.55</td>
</tr>
<tr>
<td>A) Pole-dipole n=2</td>
<td></td>
<td>13.59</td>
<td>6.84</td>
</tr>
<tr>
<td>A) Pole-dipole n=6</td>
<td></td>
<td>16.78</td>
<td>6.62</td>
</tr>
<tr>
<td>A) Schlumberger n=6</td>
<td></td>
<td>18.80</td>
<td>6.80</td>
</tr>
<tr>
<td>A) Schlumberger n=30</td>
<td></td>
<td>18.94</td>
<td>9.90</td>
</tr>
<tr>
<td>Dipole-dipole n=2</td>
<td></td>
<td>12.04</td>
<td>6.21</td>
</tr>
<tr>
<td>Dipole-dipole n=6</td>
<td></td>
<td>16.02</td>
<td>8.20</td>
</tr>
<tr>
<td>B) Pole-dipole n=6</td>
<td></td>
<td>19.38</td>
<td>10.04</td>
</tr>
</tbody>
</table>

the hemisphere response decreases at first then tends to the constant value by a uniform horizontal field.

A simple expression describes the observed potentials for a current pole inside or just touching a highly conductive hemisphere (and cases that can be turned into this by reciprocity). In this case, potentials measured outside the hemisphere are equivalent to those generated by a current pole displaced from its original position to the center of the hemisphere, as in the upper plot of Fig. 3.5. The anomalous potential expressed as a geometrical error for a virtual displacement collinear with the poles is,

\[
V(a) = \frac{\rho l \frac{1}{2\pi a}}
\]

Figure 3.5: Geometrical description of response to a highly conductive hemisphere.
where $V(a)$ and $V(a-R)$ for the pole-pole array are defined in Fig. 3.5.

A pole located in the vicinity of a hemisphere of radius $R$, experiences a maximum virtual displacement when it just touches the hemisphere. It is displaced a distance $R$ to the hemisphere centre. These results agree with the data in table 3.1. A positive error in the potential results when the displacement is toward the potential electrode, and a negative potential error results when the displacement is away. If the electrode is not immediately adjacent to the highly conductive hemisphere, the virtual displacement decreases roughly proportional to $\frac{R^3}{D^2}$ where $D$ is the radial displacement of the pole from the hemisphere center. In the limit, errors due to an inaccurate geometrical description and errors due to local anomalies produce identical responses.

A small hemisphere adjacent to one electrode in a dipole (the lower plot of Fig. 3.5), will produce an error that is more closely related to the dipole spacing $a$ than to the pole-dipole separation $n$. For $R \ll a$, only the immediately adjacent potential electrode is displaced and the potential error is,

$$\frac{\Delta V}{V} = \frac{(na + R) - na}{(n+1)a} \frac{\left((n+1)a\right)}{na + R}$$

(3.4)

$$= \frac{(n+1)R}{na + R}$$

(3.5)

$$= \frac{R}{na}$$

(3.6)

where the limit is taken with $n$ increasing. The pole-pole and pole-dipole anomalous readings tend to $R/a$ for $R < a \ll na$.

Returning to the profiles of surficial response in Figures 3.1, 3.2, and the Schlumberger sounding (Fig. 3.3), we can explain these observations in greater detail. In the pole-dipole response of Fig. 3.1, the hemisphere is a greater fraction of the dipole separation than it is of the pole-to-dipole separation. Thus the response is greatest when the dipole is near
the hemisphere. For the dipole-dipole sounding/profiling survey of Fig. 3.2, the peak and trough anomalous apparent resistivities comprising the pant-leg pattern do not attenuate with increasing spacing because dipole separation is fixed. In a Schlumberger sounding, Fig. 3.3, the distance from the pole to the central dipole is great. Local structures near the current poles have to be a significant proportion of this distance before we can observe their presence. Local structure near the dipole can produce a large response which is independent of the distance to the current poles; however, on changing the potential dipole separation, that response changes. Hence the static shifts in the sounding curve when the potential dipole is enlarged.

A large surface effect does not necessarily change the response contribution from deeper structure; however, it will obscure it in most traditional apparent resistivity pseudosections where there is insufficient sampling to allow short wavelength (< a) anomalies to be spatially filtered out. Because of aliased near-surface noise \( \rho_a \) contours normally are drawn at resistivity changes of 20% or more.
3.3 Subsurface Structure and Apparent Resistivity

Although far too regular in shape to geologically resemble a typical ore body, a sphere is a useful model for bounding the detection limits of any compact, equidimensional conductor or resistor. In the literature, for various arrays Van Nostrand (1953) and Apparao (1992, 1997) determine the depth of detection to a highly conductive sphere (where a 10% anomalous apparent resistivity was deemed detectable). I extend this analysis to circular arrays and determine the depth of detection for a deep sphere and investigate the interfering effects of a shallow sphere on linear array response. Several response characteristics are desirable:

1. Unimodal response (apparent resistivity vs. position) which peaks when the array reference point is on the epicentre of the target and is of the same polarity as the anomalous resistivity of the target.

2. The response varies with array spacing as follows:
   a. The response geometry is mainly unimodal and of the same sign from very small to very large separations.
   b. The response amplitude maximises at an array spacing relating to the depth of the body. (A simple proportionality would be nice). Thus, the depth of exploration is controlled by array size. Larger arrays should give a broad weak response to deeper targets and minimal response to small shallow targets.

Profiles of $\rho_a/\rho_0$ as a function of the array plotting point taken over a shallow sphere for several arrays are shown in Fig. 3.6 with additional arrays presented in Appendix C. These profiles are computed from image theory as outlined in Appendix B. Each family of profiles is for a highly conductive sphere of unit radius with its center at a depth to top of 0.25 units in a half-space of resistivity $\rho_0$.

For the pole-dipole array of Fig. 3.6, the plotting point is the dipole midpoint. With the dipole separation fixed and for small n (the pole is a small integer number of dipole
Figure 3.6: Pole-dipole, pole-pole, and Wenner profiles for several separations over a highly conductive sphere. The depth to top is 0.25 the sphere radius.

separations from the dipole), the dipole and pole contributions merge, producing a single large minimum slightly displaced from the sphere epicentre towards the dipole midpoint. A better plotting point for a deep target would be midway between the dipole and pole, but this shifts the response of near-surface structure. Since, for the majority of $n$ spacings shown (with an $a$ spacing of 0.5 m), the sphere is a shallow target, I have chosen the dipole midpoint as the plotting point. For intermediate $n$ spacings, the response is asymmetric and the pole and dipole associated responses begin to separate. At large $n$ spacings the target is excited by a uniform horizontal field and the largest response is observed when the dipole is above the target. The dipole associated part of the response is symmetric about the dipole midpoint at the sphere epicentre. Further increase in $n$ leaves the dipole response unchanged (producing a static shift in an apparent resistivity section), while the
pole response diminishes. Although at large n, the diameter of the sphere is much less than the length of the array, the response is still strong and could mask long wavelength weak response from deeper features.

For the pole-pole response in Fig. 3.6 (the middle plot), the plotting point is midway between the poles. The suite of profiles for increasing pole separation is symmetric about the epicentre. When the poles are close together, the sphere is large and deep (when compared to the array length) and the response is a single trough in the profile. At larger pole separations, the profile becomes inverted, overshooting the background value (an inverted response to shallow structure between the poles is often cited as a drawback to the pole-pole array). While the maximum amplitude is less than that for the pole-dipole array, the anomalous apparent resistivity decreases in amplitude and broadens as the pole separation increases. Since we are sampling a deeper, larger volume of the earth, we expect a broad weak sensitivity to big structures, so this is a desirable result.

For the Wenner array in Fig. 3.6 (lowest plot), the overall shape of the profile is a strong function of the relative electrode separation a/R. The response is symmetric about the sphere epicentre with two positive overshoots and three minima for large separations. The global minimum occurs directly above the sphere. Positive peaks occur on either side of the sphere which significantly exceed one when a<R. For smaller separations, a single minimum is observed directly over the sphere. Since the separation between all electrodes is increased simultaneously, the anomaly produced by the sphere decreases as a increases. A large Wenner array is, therefore, less sensitive to local near-surface structure; however, the anomaly profile produced always has a lateral extent equal to about 6 a and thus has limited spatial resolution.

For the short arrays (small a or n spacing), each of the profiles in Fig. 3.6 show a pronounced trough in apparent resistivity. The trough is centered over the sphere for the symmetric pole-pole and Wenner arrays, and slightly displaced towards the dipole for the asymmetric pole-dipole array. When the array is short, the trough is not flanked by apparent resistivity overshoots as is the case for longer arrays. From the sensitivity
analysis of Chapter 2 we know that the sensitivity of any array is positive beneath the centre of the array at depths which are large compared to the array length. The oscillatory responses seen in Fig. 3.6 arise when the arrays are lengthened and the sphere is small and shallow and passes through zones of positive and negative sensitivity between the various electrodes. Whether a sphere is considered to be a deep or shallow is dependent on the type of array and its length.

Several general conclusions can be drawn from the profiles in Fig. 3.6 and from the additional examples presented in Appendix C. The greatest anomaly over a shallow sphere in the apparent resistivity is always of the correct sense and occurs with the array plotting point at the epicentre. Where a 10% anomalous apparent resistivity is deemed detectable, and in the absence of geologic and measurement noise, the maximum depth of detection (depth to top) is from 1.1 to 1.5 times the sphere radius. The lateral resolution (the full anomaly width at half-maximum) is very similar for each array when the array length is chosen to given the greatest response. For a sphere at depth 1.1 the radius, the optimal array length varies from slightly less than the sphere diameter to roughly twice the diameter for the pole-pole and dipole-dipole arrays respectively. For all arrays, as the depth increases the size of the array needed for maximum response increases.

Pole-dipole profiles near a conductive hemisphere (R=1)

Figure 3.7: Pole-dipole profiles for a traverse passing a very conductive hemisphere (offset 0.25 R from the edge of the hemisphere at the closest approach). The dipole separation is 0.5 R.
So far I have only considered profiles passing directly through the epicentre of the hemisphere or sphere. In Fig. 3.7, I show a model of a pole-dipole traverse which passes close to, but not over a very conductive hemisphere. At its closest approach the traverse is 1.25 the hemisphere radius from the epicentre. To facilitate comparisons, the geometry of the array is the same as that for the pole-dipole of Fig. 3.6. Visual inspection of the figures reveals the profiles to have a very similar shape, but the anomalous magnitude (the difference in apparent resistivity from the background of 1 Ω.m) for the sphere below the traverse is about twice that of the hemisphere. This is understandable from image theory. The buried sphere interacts with the insulating earth’s surface, with horizontal component horizontal fields thus being raised and vertical components cancelled. The effect is like that of an image sphere existing above the earth’s surface and the background medium being a whole space instead of a half-space. From a single linear profile, it is impossible to uniquely determine the direction to anomalous structure.

3.4 Summary and Analysis of Traditional Arrays

The observations so far can be summarized in 6 statements. In these, the term ”response” pertains to the observed apparent resistivity. To say a response is large is to say that observed apparent resistivity differs considerably (higher or lower) from the background apparent resistivity. The definition for the length of each array is defined in Appendix C. The size of an array refers to its length. For symmetric arrays, the array mid-point is the chosen as the plotting point.

1. A sphere can only be detected (at a 10% level) if the depth to top is at most 1.5 times the radius.

2. For a given sphere and array type, the maximum observable response will be greater the shallower the sphere. For an array of fixed length, the shape of the anomaly may vary with size and depth of burial of the sphere. However, the length of the array which gave the maximal response will directly vary with the sphere’s diameter.
and depth.

3. If the array length is quite small relative to the sphere radius, the observed response decreases with decreasing array length. Also, the profile takes on a simple form, a single trough centered over the sphere.

4. For most traditional arrays, if the array is longer than the one that gives maximal response, the largest response will arise where the epicentre of the sphere lies between the most closely spaced electrodes. The magnitude of this response is independent of the distance to the remaining electrodes. A profile over the sphere will oscillate, the magnitude of the oscillation a function of the spacing of the two electrodes closest to the sphere.

5. A hemisphere whose epicentre is offset from the axis of a linear array by an amount D will behave as described in items 1 to 4 for a sphere at depth D.

Of the traditional linear arrays, only for the pole-pole and Wenner arrays (in which every part of the array is similarly scaled) will the response to structures smaller than the array decrease as the array separations are increased. This is important as it is near-surface structures smaller than the array which are usually responsible for what we term geologic noise.

### 3.5 Circular Arrays

While the linear pole-pole and Wenner arrays are less sensitive to near-surface geologic structure than other linear arrays, the profiles obtained still leave much to be desired. In a profile over a small shallow sphere, the Wenner array has reduced but multiple lobes and the pole-pole array gives an inverted resistive response when straddling the sphere. These observations can be confusing when dealing with field data, as one is usually faced with inadequately sampled data where all the high spatial frequency components are aliased beyond recovery. Through circular arrays it is possible to improve the signal-to-
noise ratio by averaging out the response due to small, high spatial frequency structures while making the observation.

As was suggest from the sensitivity analysis of Chapter 2 and described in detail there, the pole-circular pole array performs the antialiasing by averaging each pole-pole response with others. The contribution to the response originating from the earth near the central current pole is reinforced in the average while the contribution from the potential electrodes on the ring is diminished. This also makes the measurement configuration axially symmetric (with a vertical axis). This significantly reduces the possibility of aliasing the response of surface structures outside the scope of the array with the response from structure below the array.

<table>
<thead>
<tr>
<th>Array</th>
<th>Size of noticeable hemisphere</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2R/separation %</td>
</tr>
<tr>
<td></td>
<td>(insulating case)</td>
</tr>
<tr>
<td>Offset Wenner</td>
<td>44.4</td>
</tr>
<tr>
<td>Pole-circular pole</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td>2R/separation %</td>
</tr>
<tr>
<td></td>
<td>(conductive case)</td>
</tr>
<tr>
<td></td>
<td>22.8</td>
</tr>
<tr>
<td></td>
<td>70.0</td>
</tr>
</tbody>
</table>

Table 3.2: The diameter of the hemisphere at which the anomalous apparent resistivity is 10% of the background expressed as a fraction of the separation between the two closest electrodes.

The effectiveness of the circular arrays in rejecting the response of small surface features can be assessed by finding the diameter of hemisphere required to produce a 10% anomalous response (see the second line in Table 3.5). It is immediately apparent that the sensitivity to a hemisphere is far less than for any linear array. In fact, a highly conductive hemisphere must have a diameter fully 70% of the ring radius before a 10% anomalous apparent resistivity is observed. This is considerably better than the offset Wenner arrangement, which also is designed to reduce near-surface noise. For a resistive hemisphere, the diameter is equal to the ring radius. However, large structures beneath the source still produce a notable response as seen in Fig 3.8 where pole-circular pole apparent resistivity profiles over a highly conductive sphere of constant radius R and at a depth to top of 0.25 the radius are shown for several ring radii. In general, for analytic modelling with the circular arrays I use $2\pi R_i$ potential electrodes where $R_i$ is the index.
of the ring, that is \( I = \frac{R}{a} \) where \( R \) is the radius of the ring and \( a \) is the grid spacing. This gives a close approximation to the actual number of potential electrodes that would be used in a gridded pole-potential map survey.

**Pole–circular pole profiles over a sphere D=0.25 R**

![Graph showing pole-circular pole profiles over a sphere](image)

Figure 3.8: Pole-circular pole profiles over a sphere at a depth to top of 0.25 \( R \) for rings of several different radii over a highly conductive sphere. The profiles are nearly unimodal from very large to very small separations. The exploration depth is controlled by the array size.

The apparent resistivity profile for each radius is symmetric and the minimum occurs at the epicentre. Unlike the linear pole-pole profile of Fig. 3.6, no overshoots appear when the ring radius exceeds the diameter of the sphere. The anomaly amplitude changes little with ring radius in the range \( a/R < 0.75 \) and decreases rapidly for larger rings. The sphere response has a limited lateral extent and at the optimal electrode separation, \( a=0.25 R \), the width of the pole-circular pole response is comparable to that of the optimal pole-dipole or dipole-dipole array in Fig. 3.6. While the lateral resolution is comparable, the sensitivity of the pole-circular pole array is seemingly poorer than the pole-dipole array. Of the arrays considered in Fig. 3.6, the pole-dipole is most sensitive to a highly conductive sphere and shows an 80% drop in apparent resistivity compared to the 50% for the \( a=0.25R \) pole-circular pole anomaly (Fig. 3.8). However, its higher sensitivity comes at high cost; the sensitivity to near-surface structure is also vastly larger.

I illustrate this through a simple case study in which a pole-circular pole and a linear
Figure 3.9: Electrode separations for pole-circular pole and pole-dipole surveys over a highly conductive sphere whose depth to top equals the sphere radius and a small but highly conductive hemisphere.

pole-dipole survey are compared. Two structures are considered, a very conductive sphere at a depth to top equal to the sphere radius (the limiting depth of detection for the pole-circular pole array) and a small highly conductive hemisphere. The model and electrode geometries are shown in Fig. 3.9. Here, $a=0.5\ R$ for a $n=4$ pole-dipole array, and $a=1.4\ R$ for a pole-circular pole array. For clarity, a continuous series of measurement points are evaluated where in fact measurements would be recorded only at some multiple of the electrode separation. This is to the benefit of the pole-dipole array, because the high frequency anomaly component are then not aliasing.

The profiles are shown in Fig. 3.10. The solid line indicates the response of the target (the signal) while the dashed line indicates the influence of the near-surface hemisphere (the geological noise). The pole-circular pole trough is somewhat broader than the pole-dipole anomaly of 23%. The pole-dipole minimum is slightly displaced from the epicentre and, since the array parameters were optimised to this target, there is little overshoot in the apparent resistivities. For the pole-circular pole array, the geological noise is just two small oscillations with a high spatial frequency. In the pole-dipole case, the interference is greater in magnitude to that of the target response with considerable overshoots. While the hemisphere’s anomaly is of much shorter wavelength than the deep sphere, spatial
Figure 3.10: Pole-dipole and pole-circular pole profiles over a highly conductive hemisphere and sphere (for model in Fig. 3.9). The signal-to-noise ratio (response of the deep sphere to near-surface hemisphere) is far better for the pole-circular pole array than the pole-dipole array.

aliasing of practical field data would prevent use of a simple low pass filter.

3.6 Interference Test Model

The sphere and hemisphere models clearly suggest that circular arrays will be better able to resolve deep structure in the presence of near-surface structure. However, these models do not test the interaction between the noise and target structures. Numerical modelling is required if complicated cases are to be investigated. A code devised by Bailey and Cheesman (1996) and operable on a PC computer was available and is described in Appendix D. The accuracy of the finite-difference code was tested by comparing analytic and numerical solutions for the hemisphere and sphere (see Appendix D). Numerical
results accurate to with a few percent even for highly contrasting bodies could be obtained so long as the bodies were relatively small, roughly equidimensional and removed from all but the surface boundary.

A test model containing both surficial and deeper structure and suitable for finite-difference modelling with a 64x64x64 grid was constructed. It is very similar to one used by Li and Oldenburg (1992) to illustrate reformation of their inversion algorithms. The multigrid solution space was gridded into 64³ nodes of nominal 0.5 m spacing and with a pole source located at any of the surface nodes. A pole-potential map data set was then recorded over a 21 x 21 grid of nominal 1 m spacing. The model consists of the five prisms listed in Table 3.3 in a uniform half-space of 1 Ω.m. Prisms B1 and B2 are buried target bodies while S1, S2, and S3 simulate near-surface interference structure (geological noise). A plan and section view of the model are shown in Fig. 3.11.

<table>
<thead>
<tr>
<th>Prism</th>
<th>x-dimension (m)</th>
<th>y-dimension (m)</th>
<th>z-dimension (m)</th>
<th>Resistivity (Ω.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>11-13</td>
<td>11-13</td>
<td>0-1</td>
<td>0.1</td>
</tr>
<tr>
<td>S2</td>
<td>14-16</td>
<td>12-20</td>
<td>0-1</td>
<td>0.2</td>
</tr>
<tr>
<td>S3</td>
<td>19-20</td>
<td>13-20</td>
<td>0-1</td>
<td>2.0</td>
</tr>
<tr>
<td>B1</td>
<td>12-14</td>
<td>12.5-19</td>
<td>1-4</td>
<td>2.0</td>
</tr>
<tr>
<td>B2</td>
<td>16-20</td>
<td>14-16</td>
<td>1.5-4.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 3.3: Parameters of the five-prism interference model. The background resistivity is 1 Ω.m.

Plan maps of response from three x-directed linear arrays are shown in Figure 3.12. The strong influence of the surface bodies and the effect of array-body orientation is clearly evident. For the the dipole-dipole array, the response to the surface conductors S1 and S2 is pronounced in both the n=1 and n=4 maps. The response to the deep conductive body B2 is concealed by the response to surface body S2. The surface anomalies decrease for the larger pole-pole and Wenner arrays but oscillations due to the surface structures are present. For each of the linear arrays, surface interference obscures and complicates the response to the deeper target structures.
The results from a pole-potential map survey traditional results are in sharp contrast to Fig. 3.13. The small 1 m ring gives a reasonable geometrical description of the surface prisms. At a 2 m radius the surface influence has all but vanished and an anomalous low is centered over the buried conductor B2. The image conductivity becomes less sharp and the magnitude of the anomaly decreases as the ring radius increases. Because of loss of resolution, the bottom of the buried prism B2 is poorly defined and there is a smooth transition from the anomaly to the average background conductivity. Overall, the recovered apparent resistivity map appears as a depth-filtered version of the true model where the amount of filtering is proportional to array radius. Despite the influence of the surface conductivity variations, the buried conductive prism is clearly defined. The resistive prism is somewhat harder to discern, but this limitation is common to all resistivity methods. There is no dependence between the array "direction" and the corresponding apparent resistivity.

For the interference test the pole-circular dipole (left) and circular bipole-dipole (right) response maps are shown in Fig.3.14. The pole-circular dipole array is more sensitive to the surface blocks than the pole-circular pole array due to the dipole potential ring; however, the array is also more sensitive to deep conductor B2. The P2 invariant apparent resistivity for the circular bipole-dipole array cleanly describes the surface features and
Figure 3.12: Apparent resistivities over the interference test model for the linear arrays; pole-pole, Wenner, and pole-dipole. The strong influence of the surface bodies is present in all plots and obscures the deeper target bodies.
Figure 3.13: Pole-circular pole apparent resistivities over the interference test model. The surface anomalies have all but vanished from the 2 m ring. The deeper block B2 produces a well defined low centered over the block.
Figure 3.14: Pole-circular dipole (left) and circular bipole-dipole results (right) over the interference test model of Fig. 3.11. The pole-circular dipole is more sensitive to surface structure than the pole-circular pole array but also has a greater sensitivity to the deep conductive target. Surface features are cleanly described by the circular-bipole dipole array.
is unaffected by orientation of the array.

### 3.7 Summary

To avoid near-surface geologic noise in a surveys to detect deep features, we would like to simultaneously reduce sensitivity to the near-surface local features while increasing sensitivity at depth. If the measuring array involves a small, fixed separation between two electrodes, such as in pole-dipole, dipole-dipole, Schlumberger or gradient array, then the anomalous apparent resistivity produced by a local hemisphere or shallow sphere near the fixed electrode pair is independent of the array's source-receiver separation. This phenomenon is observed as a static shift in Schlumberger soundings when the dipole spacing is changed. In dipole-dipole pseudosections, it is observed in the pant-leg effect where increasing the n spacing does not decrease the pant-leg anomaly.

Linear arrays in which all electrode separations are not scaled simultaneously have a high sensitivity to geologic noise. For shallow structures, the greatest sensitivity is located between the closely spaced pair or pairs, but the region of maximum sensitivity for deeper objects is located at the array midpoint. For interpretation, data are usually plotted vs. profile coordinate and vs spacing sometimes restated as the depth of investigation of the array. However, the variation in location of maximum sensitivity makes the choice of a single plotting point inadvisable. Only when all inter electrode separations (excluding remote electrodes) are increased simultaneously will the apparent resistivity anomaly produced by local features decrease.

Anomalies due to surface effects can be cancelled by averaging the appropriate measurements as is done with the pole-circular pole array. Response to near-surface structure is then reduced as the array expands, while the response to deeper structure is preserved. The pole-circular pole array has a unique optimum plotting point for all depths because the response is unimodal at all ring sizes. Thus, a set of maps of apparent resistivity for different ring radii appear roughly like a set of (smoothed) resistivity maps at different
levels, or a 3d (x,y,a) volume of $\rho_a$ is qualitatively similar to a 3d volume of $\rho(x,y,z)$ with smoothing applied. Examples computed by 3d numerical forward modelling show that the signatures of shallow and deep structures can be simpler in 3d pseudo-volumes than in commonly used 2d array pseudosections and that deeper features of the earth can be better discerned in spite of the interference effects from variable resistivity of the surficial layers.
Chapter 4

Field and Laboratory Measurement

4.1 Introduction

In Chapters 2 and 3, I have provided the theoretical foundation for reliable 3d resistivity exploration through the use of several synthetic arrays. What remains to show is that systematic pole-potential mapping can practically be carried out on a regular basis with an efficient field methodology and suitable instrumentation, and that measurement error in the synthetic array data can be made low enough to exploit the improved S/N ratio of deep targets and surficial noise.

Practical testing can be conducted at two levels, as full scale field tests and as laboratory scale physical modelling. Since it was impractical to do many field tests I have chosen to concentrated on laboratory modelling and on one small scale field problem in archaeological investigation at Fort York, Toronto. This has made it possible to use the same apparatus for field and physical model studies, and still allows me to investigate a variety of situations encountered in surveying; such as the effect of random, systematic and geometrical errors; the influence of remote electrodes, and the practicality and speed of field surveying.

Resistivity and IP scale models require a conductive host medium. An electrolyte such as water is convenient and when mixed with sand forms a conductive matrix into which conductive and resistive targets may be easily placed. In my experiments, the lateral
extent of a survey in the modelling tank is of order 100 cm, with electrode separations on the order of a centimeter. The anticipated background resistivity is tens of $\Omega \cdot m$ in the water and sand matrix of the modelling tank.

The conditions in the models are similar (with scale reduction) to those one might encounter in a small field experiment such as the archaeological study I carried out at Fort York. The lateral extent of the field survey was tens of meters with electrode separations on the order of tens of centimeters. In fresh water dampened soil one can expect background resistivities to range from tens to hundreds of Ohm meters.

4.2 Instrumentation: Design Criteria

The objective is to acquire high quality resistivity measurements (along with IP data) simultaneously on several hundred potential electrodes. To this end, a multi-channel resistivity/IP meter was designed that consists of: an array of electrodes, a multichannel receiver for monitoring the potential on the electrode array, and a current transmitter. Obviously some form of automated, continuously sampling A/D converter system is required and a mechanically efficient procedure for moving the electrode patch. I discuss some of the more important aspects of the instrumentation here, details of the electronics are found in Appendix A.

4.2.1 The Electrode Array

Decisions about data acquisition geometry inevitably are compromises between survey cost (effort) and the need for maximum data. With the increased capabilities of modern instrumentation, it has become possible to increase data coverage without substantially increasing data cost, at least for linear multi-electrode arrays employing a roll-along technique as shown in Chapter 1, Fig. 1.4. This approach is well developed for linear arrays and I extended the concept to incorporate areal electrode grids. In these arrangements typically at least 8 electrodes are within the active section of electrode line at any given
Figure 4.1: In the modelling tank the active patched is moved as a unit. Shown is a 7x7 patch mapping the potential for the current pole $I_{E5}$. The patch is moved to a new source position $I_{F6}$ for another measurement.

time. As with the linear arrays used for resistivity profiling/mapping, I found measuring at seven radial separations from a point source to be completely sufficient to achieve adequate lateral and vertical coverage. This suggested that a 15x15 patch of 225 electrodes is more than adequate pole-potential map.

In the modelling tank with its uppermost layer of water, a roll-along array is not necessary because the survey grid can moved as a unit. Thus a rigid but shiftable 15x15 electrode grid was constructed with the current pole was located at the center. In a survey, the grid was moved in steps of a grid unit from source point to source point, with the potentials measured at each location. Each electrode grid or, in analogy to seismic exploration terminology, ‘active patch’ is named after the column and row position of the source.
pole, that is $I_{\text{column,row}}$. In Fig. 4.1, measurements were taken on the 7x7 patch E5, a future patch F6 is shown in a lighter shade. The remote current and potential electrodes are in the corners of the tank diametrically opposite one another.

![Image](image)

Figure 4.2: The electrode grid used in scale modelling. The gold plated wire-wrap pins are equally spaced in both directions 1.66 cm apart. The source electrode is located at the center of the grid.

The potential electrodes used in the model tank were gold plated wire-wrap pins suspended with 1 mm of the pin in the water. This small contact area was necessary to better approximate point electrodes since the separations between electrodes was just 1.66 cm. The grid construction is pictured in Fig. 4.2. The grid framework was constructed from a lucite sheet as it was insulating (not the case for the original wooden base which was conductive when damp) and transparent so that the structures beneath the grid can be seen. One unforeseen draw-back was encountered; when suspended just above the surface the lower side of the sheet did slowly absorbed enough water to cause it to warp upwards, lifting the outermost electrodes from the tank (the sheet's insulating properties were not appreciably affected). Metal supports around the outer edge helped prevent the warping as did removing the electrode array from the tank when not in use. Lines of electrodes are connected to 8-wire, RJ-45 telephone jacks at one end of the grid and then connected to the receiver through 8 conductor level-5 telephone cable.

To make data collection efficient at Fort York, I designed a roll-along technique in which lines of electrodes were successively deployed that were much longer than required in
Figure 4.3: A roll-along technique for pole-potential mapping in the field. Reciprocity of source potential measurements is used to recover the full active patch.

Along each line in use, each electrode was monitored for potential and the source was switched from one electrode to another, thus moving the active patch electrically in the y direction through the deployed electrodes. This minimised the time required for a survey because the instrument operator made measurements on one set of active lines while the field crew laid out the next line of electrodes. As measurement progressed from line to line the field crew removed those lines of electrodes no longer in use and planted them ahead of the active region. An additional simplification was realised through eliminating observations rendered redundant by reciprocity. The maximum number of independent measurements that can be made with \( N \) electrodes is given by \( N_{max} = N(N - 1)/2 \), [Xu and Noel 1993]. Thus, it was necessary to
measure only the potentials at electrodes with a column letter lower or equal to the present electrode column. For the 7x7 patch shown in Fig. 4.3, four lines of 16 electrodes are connected to the receiver at once. The source is switched along column \( D \) from electrode positions \( I_{D1} \) through \( I_{D16} \) while the potential is measured in the active patch in columns \( A \) through \( D \). It is sufficient to measure only the 4x7 electrode grid depicted in Fig. 4.3 recovering the other potentials through reciprocity from the active patch of the appropriate reciprocal source. For instance, potential \( P_{E6} \) for active patch \( D5 \), is obtained from the potential measured at \( P_{D5} \) for source \( I_{E6} \). In a similar fashion, every potential in the right half of patch \( D5 \) is found. When all electrodes in column \( D \) have taken a turn as the source electrode, the operator switches the source to \( I_{E1} \), the field crew can now detach column \( A \) and so forth.

The advantage gained from reducing the number of needed measurements is that more electrodes in each column can be attached to potential measuring channels. Switching connectors or replacing electrode wires is only necessary after the source point has moved by a substantial amount. I conducted my field survey with 8 active electrode columns within which 16 electrodes could be monitored without wiring changes.

4.2.2 The Receiver and Transmitter

To exploit the benefits of roll-along surveying and to minimise the effects of telluric noise, it is necessary to monitor many electrode voltages simultaneously. Several approaches to accomplishing this task are mentioned in the literature. Hasegawa et al (1996) of OYO Corporation digitises the dipole potential difference between adjacent electrodes in a collinear array and transmits the digitised signal back to a central monitoring computer. This method has the advantage of reducing the amount of wire deployed in the field as no universal potential reference is required; however, I require pole potential measurements and would need to construct these values from the observed differences (this would require perpendicular networks of wire to measure differences in two directions). K. Suzuki (1996) collected pole resistivity data by multiplexing electrodes onto a single A/D converter. I
use a similar approach, but with the addition of individual sample and hold (S&H) amplifiers on each electrode so as to capture the instantaneous potential across the patch. The stored potentials are then multiplexed into a single A/D channel. The S&H unity gain amplifiers had an additional benefit, a very high 10 MΩ input resistance which minimised the possibility of grounding loops occurring through the potential electrodes. To allow some flexibility in electrode arrangements, provision was made to sample up to 256 electrodes at time.

Figure 4.4: The receiver is housed in a tool box for portability in the field. A total of 256 electrodes can be connected (in groups of 8) to the 256 S/H amplifiers in the box by the grid of telephone jacks on the left. Two of the cables (16 electrodes) were fed through the current pole switch box on the right. It permits any of the 16 to be a current or potential electrode.

In the Fort York survey the groups of 8 electrodes (2.5 inch galvanized nails) were wired to 8 conductor telephone jacks and then through 8 wire telephone cables to to a switch box on the receiver. The switch box (Fig. 4.4), assisted the roll-along of electrodes and was the interface between the S&H amplifiers and the deployed electrodes. The pluggable connectors on the box allowed one column of electrodes to be easily detached while the next is connected. The electrodes in the source column were fed through a single pole double throw switches. One of the switches was selected to connect it to the source current electrode. Simultaneously the S&H amplifier is disconnected from this electrode and connected to the remote current monitor.
A reasonably high speed A/D converter was needed to sample the potentials at a sufficiently high rate to be able to detect waveform distortion due to the IP decay. A Spectrum A/D card, TMS320C30, with two 16 bit ADCs with their own sample-and-hold circuits, each with 153 ksamples/s throughput [Burr-Brown PCM78 (5μs) ADC with Burr-Brown SHC5320 (1.5 μs)] over a +/- 3 volt range, proved suitable to the task. The software drivers for the Spectrum A/D card and a windows program for displaying the raw potentials was created by Ken Nurse (1992). Due to limitations in data transfer to the computer, for 256 electrodes the maximum electrode sampling frequency was 320 Hz with a net conversion rate of 81.920 ksamples/s.

The transmitter was designed to provide an 16 Hz controlled current square waveform, which was time synchronised to the receiver sampling and has its amplitude adjustable through a potentiometer located on the transmitter. This relatively low frequency was chosen so that a waveform distortion measurement of IP was possible with a < 320 Hz sampling rate. The precise current injected into the tank or the ground is also monitored by the receiver system by passing it through a sense resistor and presenting the voltage across it to one of the sample-and-hold inputs. The transmitter unit is primarily a ground isolated, high voltage power supply consisting of two isolating dc-dc converters and a high voltage/current integrated circuit operational amplifier configured as a voltage controlled current source. In the modelling tank a source current of 10 mA will produce, with a background resistivity of 20 Ω.m, a voltage of 1.5 volts (half the maximum of the A/D converter) at the closest distance of 1.66 cm. Since the current electrodes can have equivalent resistances up to about 1 kΩ a voltage supply of about +/-50 V is necessary to drive this source current (the electrodes have little surface area through which to inject the current giving them a moderately high input impedance). Currents of < 5 mA were adequate in the model tank, but larger values of about 25 mA were needed for the field survey. The model tank electrodes have much higher resistances than the field electrodes because of the small contact area. It was also noted that the current electrodes developed a significant polarization voltage (i.e., their impedance was not resistive and frequency independent so dynamic control of the current waveform was definitely required.
The IP transmitter is powered from two dc supplies. A ground isolated 6 volt battery powers the digital circuitry and the two 12 volt batteries in series power the receiver circuitry and also supply the dc-dc converters in the transmitter. They convert the 24 volt input into an isolated +/-48 volt supply. An important consideration in choosing the dc to dc converter was the high degree of resistive and capacitive isolation provided on the output, ensuring that no appreciable ground loops existed between the source and potential electrodes.

4.3 The Modelling Tank

Aside from the multi-channel resistivity-IP meter, the scale modelling apparatus consists of a sand and water-filled tank of fibreglass polyester coated wooden construction in which conductive and polarizable objects are buried. The 1.25x1.25x0.25 m³ tank simulates ground conditions that might be met in practice. The upper water layer is of variable depth (usually about 1 cm depth) and is more conductive than the sand-water matrix and simulates the conductive overburden that is often present in nature. The sand-water medium below, about 17 cm in thickness, serves as a host medium for various more conductive or more resistive local features inserted into it as blocks or plates. The tank and electrode grid are shown in Fig. 4.5.

The tank surveys contained 11x11 or 13x13 source nodes, with a 15x15 potential map collected for each source. The electrode spacing was 1.66 cm and the remote electrodes were in opposite corners of the tank. A 4 mA source current was used.

In both model and field experiments the source waveform was a square wave at 8Hz, and for every source cycle the potential was sampled 20 times with about a 0.25 ms offset from the source cycle in the modelling tank data. This ensured that no measurements were taken directly on the source transition. This feature was not used at Fort York (it had not been tested at the time) and consequently the data point sampled on the source transition is ignored. To reduce 60 Hz noise, 128 stacks were averaged over 8 seconds.
The modelling tank is 1.25x1.25x0.25 m$^3$. The electrode grid is wired back to the transmitter on the left through 32 eight-conductor telephone cables. The remote potential reference is in the lower left corner while the remote current pole is in the upper right corner. The tank common of the +/-12 V receiver power supply is grounded to the tank at a point in the lower right corner.

Thus, after stacking, the raw potential record, for each receiver electrode and for a single source position was a time series of 20 potentials, 10 per half-cycle. Several estimators were applied to the stacked signals to reduce the quantity of data prior to interpretation. For each estimator a set of weights was defined, for example,

$$W^s = (0, 1, 1, 1, 1, 1, 1, 1, 0, -1, -1, -1, -1, -1, -1, -1, -1)$$ (4.1)

where zero weights mean simply that this value is not used. All estimators are variations of the basic weighted average or rms average,

$$Avg[W, S] = \frac{\sum_{i=1}^{2n} W_i S_i}{\sum_{i=1}^{2n} |W_i|}$$ (4.2)

$$rms[W, S] = \left( \frac{\sum_{i=1}^{2n} |W_i S_i|^2}{\sum_{i=1}^{2n} |W_i|^2} \right)^{\frac{1}{2}}$$ (4.3)

Let $S_i$ be $i^{th}$ element of the total input signal $S^I$. With this, in Table 4.1 I define the estimators. The result for the IP estimator is a weighted measure of the average slope $\Delta v/\Delta t$ on the waveform in units of volts per $\Delta t$ (usually $\Delta t = 3.125$ ms where the sampling frequency is 320 hz).
As an example of data quality, two sets of results for repeated measurement are shown in Fig. 4.6. In the left graph the electrode array was placed in the center of the tank and a potential grid sampled 30 times without repositioning the array. The graphed quantity is the standard deviation of the individual pole-circular pole apparent resistivities expressed as a percentage of their mean value and as a function of receiver radius. In the right graph, measurements were made over the small conductive graphite block of Fig. 4.8 with the grid repositioned between successive samples. This time the errors are an order of magnitude larger, but still less than 2%. Since the graphite block is small and near the surface, the greatest errors are observed at small radii and diminish at a rate proportional to the ring radius, as predicted by the modelling results of Chapter 3.

The results from my modelling tank surveys are displayed as plan sections of apparent resistivity and percent IP effect. I synthesize the apparent resistivity and percent IP effect results for several linear and circular arrays from pole-potential map surveys. The values contoured are the log of the apparent resistivity ratioed to the mean apparent resistivity (the background value) for that plan section. This facilitates the comparison of results from different surveys where the background resistivities may differ. The IP parameter is first calculated using the estimator of Table 4.1 over a half-cycle and then normalised in the same way as the potential measurements are converted to apparent resistivities to account for the current amplitude and the geometrical factor of the array (i.e. they are in units of $\Delta \rho_a/\Delta t$ where $\Delta t$ equals the half-cycle time. Finally, the fraction $(\Delta \rho_a/\Delta t)/\rho_a$ is expressed in percent.

Table 4.1: Estimators applicable to stacked signals.

<table>
<thead>
<tr>
<th>Estimators</th>
<th>Formulae</th>
<th>Arguments, $1 \leq i \leq 2n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean DC level, $S$</td>
<td>Avg</td>
<td>$W_i = 1$, $S_i = S_i'$</td>
</tr>
<tr>
<td>Square wave signal, $S_s$</td>
<td>Avg</td>
<td>$W_i = W_i^s$, $S_i = S_i^t$</td>
</tr>
<tr>
<td>AC input (rms), $S_{rms}$</td>
<td>rms</td>
<td>$W_i = 1$, $S_i = S_i - S$</td>
</tr>
<tr>
<td>AC noise, $N_{rms}$</td>
<td>rms</td>
<td>$W_i = W_i^s$, $S_i = S_i^t - S_i^s W_i^s - S$</td>
</tr>
<tr>
<td>IP signal, $S_{ip}$</td>
<td>Avg</td>
<td>$W_i^{ip} = (0, -4, -3, -2, -1, 0, 1, 2, 3, 4, 0, 4, 3, 2, 1, 0, -1, -2, -3, -4)/3$</td>
</tr>
</tbody>
</table>
Figure 4.6: Observed measurement errors in pole-circular pole apparent resistivity in the scale modelling tank. Standard deviations are calculated for 30 measurements for a stationary array (left) and for an array repositioned between readings (right).

In the absence of anomalous bodies, the sand and water creates a layered structure with a fairly strong regional gradient due to variations in the layering, the outer walls of the tank, and the influence of the fixed remote poles. Figure 4.7 shows plots of the apparent resistivity and IP background response for two pole-circular pole arrays. The regional trend is clearly evident in two upper apparent resistivities and corresponds to about 25% maximum lateral change in the apparent resistivity. The two center plots show that only a weak average background IP effect is present and no regional trend. In the lower plots the background pole-circular dipole apparent resistivity is compared to the theoretical layered earth response to the model on the right calculated using the Hankel transform method of Appendix C. The model was adjusted manually to obtain best visual fit, as is shown, and it consists of 4 separate layers. The deepest 'layer' is the highly resistive boundary of the tank bottom. Since the thickness and resistivity of the water layer, and the total thickness of the sand layers are known, one can determine the remaining layer parameters with fair confidence. The tap water comes from Lake Ontario, originally about 40 Ω.m at 25 Celsius, [J.D. McNeill, 1980] and a value of 12.2 Ω.m was observed.
Figure 4.7: The background response of the tank, the pole-circular pole apparent resistivity (top plots) show a regional gradient introduced by the remote potential reference at about (-32,-32) and the remote current return at (44,44) source nodes. The percent IP effect (middle plots) is weak and randomly scattered. At the bottom the pole-circular dipole apparent resistivities are compared to those obtained for the layered earth model on the right. At least four layers were found to be necessary to describe the data. The real sand water structure is likely more gradational.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Depth (cm)</th>
<th>Resistivity (Ωm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>0.9</td>
<td>12.2</td>
</tr>
<tr>
<td>Sand &amp; Water</td>
<td>3.0</td>
<td>19.5</td>
</tr>
<tr>
<td>Sand &amp; Water</td>
<td>14.0</td>
<td>105</td>
</tr>
<tr>
<td>Insulating tank bottom</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
for its resistivity in the lab after city chlorination and after minor addition of salt. Two layers were found necessary to describe the water-sand matrix, a relatively conductive upper layer over deeper more resistive layer; though the real situation is probably more gradational. The rise in resistivity in the lower part of the tank is believed due to difficult-to-release trapped air bubbles, and possibly an artifact of the walls of the tank.

The first model studied was a small conductive graphite block 2.1 cm square by 1.0 cm laid flat with its upper face at the sand water interface at a depth of 0.9 cm. The resistivity of the graphite is of order $10^{-4}$ to $10^{-5}$ Ω.m, so the block is effectively a perfect conductor except for possible surface (IP) effects. A sphere of equivalent volume has a radius of 1 cm and would be expected to produce a maximum anomaly of 10% at this depth, thus the block is near the depth of detection limits. The apparent resistivities for several synthetic arrays are displayed in Fig. 4.8. The block is outlined with a solid black line. The plots on the left are for pole-circular pole arrays of increasing ring radius from top to bottom. A localized anomalous response centered over the block is clearly evident at the smallest radius. For larger radii, a broader response is observed. The apparent resistivity of the anomaly for the smallest ring is 9.5% lower than the background value and very close to the predicted anomaly of 10% for a very conductive sphere. The diameter of the smallest ring is 3.32 cm, larger than the length of the block. This suggests that any further increase in the ring diameter should decrease the anomalous pole-circular pole response, and this is observed in the lower two plots on the left.

In the pole-circular pole apparent resistivities, especially for larger rings, the regional trend is plainly evident. It is due in part to close proximity of the 'remote' potential to which each individual potential measurement is referenced and in part to heterogeneity of the tank medium and the walls. It is largely removed when the potentials observed on adjacent rings are differenced as shown in the plots on the right of Fig. 4.8. The anomalous response due to the block then is better centered over the block and just broadens as the ring radius increases.

For the block model, the percent IP effect is displayed in Fig. 4.9 for the pole-circular
Figure 4.8: Pole-circular pole (left) and pole-circular dipole (right) apparent resistivities from a 15x15 potential grid collected over a 11x11 source grid. A small 2.1x2.1x1.0 cm$^3$ conductive graphite block, outlined in black, was placed with it's top at the sand water interface, a depth of 0.9 cm. The background response of the tank (Fig. 4.7) has not been removed, and accounts for the drifting of peak response to the bottom left as separation is increased.
Figure 4.9: The percent IP parameter for the pole-circular pole (left) and the pole-circular dipole (right) arrays over a small conductive graphite block (2.1x2.1x2.1 cm³) at a depth-to-top of 0.9 cm, in 0.9 cm of water.
Figure 4.10: The apparent resistivity for a y-directed pole-dipole (left) and the circular bipole-dipole arrays (right). A small shallow conductive graphite block, 2.1x2.1x2.1 cm$^3$, at a depth to top of 0.9 cm, in 0.9 cm of water.
pole (left) and pole-circular dipole arrays (right). Because there is very little background IP response, the anomalous percent IP response due to the block is easy to identify even for large rings. The pole-circular dipole percent IP responses are even better resolved. The anomalous response increases with the ring dimension and changes from a response localized over the block to a halo around the block. The halo is produced when a portion of the potential ring passes over the block and is analogous to the classic pant- leg signature seen with dipole-dipole array. However, unlike the dipole-dipole pant- leg signature, the pole-circular pole or dipole halo anomaly decreases once the size of the dipole ring becomes sufficiently large because an increasing number of dipoles are averaged in the observed response and only those over the body contribute much.

Apparent resistivity for the linear pole-dipole and the circular-bipole dipole (P2 invariant) arrays are shown in Fig. 4.10. The pole-dipole responses in the left plots are for an array running in the positive y direction with the pole leading and the n-spacing of 1 in the top plot to 3 in the bottom plot. The apparent resistivity is plotted at the pole location. For n=1, the anomalous response is mostly localized over the block; however, for larger spacings the response is displaced towards the pole. For n=3, oscillations are observed with a positive overshoot occurring when the array straddles the block and a large minimum when the dipole is over the block. These arrays do not suffer from background effects due to remote potential reference as this is mostly eliminated in the dipole difference.

The right plots display the P2 invariant calculated from the circular bipole-dipole array tensor apparent resistivities. Since the potential grid, 15x15, was larger than the source grid, 11x11, I invoked reciprocity and took the central dipole pairs to be the sources to obtain a full survey. Accordingly, the potentials measured on the outer ring of electrodes were weighted with cosine and sine weights as the central electrode pairs were sources. The ring radius varies from 2 in the top plot to 4 in the bottom plot. The geometric location of the conductive block is clearly defined in all three apparent resistivity plots, and as predicted from analytic modelling, the amplitude of the anomaly does not decrease with increasing ring size. As the outer ring radius is made larger, the observation becomes
equivalent to the scalar potential gradient anomaly of the block in a uniform horizontal current field. With increasing ring radius, the anomaly intensity always increases, since increasing the source separation increases the depth to which the current field penetrates without altering the intensity of the near-surface part.

In the next model, a large graphite block 7.5x7.5x2.5 cm³ was placed with its upper surface at the sand water interface at a depth-to-top of 0.9 cm. Figure 4.11 are the apparent resistivities (left) and the percent IP effect (right) for the pole-circular pole array. From top to bottom are shown the responses from rings of increasing radius. The anomalous response to this large structure is confined to the region above the block for smaller rings, and broadens as the ring radius increases. The regional background trend is observed with increasing radius but is not as evident as with the small block in Fig. 4.8 where the contour interval is smaller. The location of the block is also clearly indicated by its percent IP effect (there is little background IP response). Interestingly, while the IP and apparent resistivity anomaly of the large block have much greater extent than that of the small block, the peak amplitude of the IP response is not correspondingly larger. A variety of explanations can be advanced including experimental factors such as a slightly greater depth of water over the large block. However, the most likely explanation is related to the surface area of the blocks. The IP effect here is likely to be largely a surface interaction on the block.

The usual approach in mapping structures using a linear array is to survey a sequence of profiles along lines perpendicular to the predominant strike of the target. This assumes both that the strike is known and that all geological features in the region of interest have the same strike. A significant advantage of pole-potential mapping is that a-priori knowledge of the strike direction is unnecessary. In the model of Fig. 4.12, four small graphite block 2.1 cm square by 1.0 cm were laid on the 1.0 cm edge end-to-end at a 30 degree angle to the x-axis. The composite slab was buried with the top edge of the blocks at the sand water interface, a depth of 1.2 cm. Pole-circular dipole apparent resistivities and percent IP effect are shown. There is an excellent agreement between the orientation of the anomalies and the true orientation of the slab. While the apparent
Figure 4.11: Pole-circular pole apparent resistivities (left) and percent IP effect (right) from a 15x15 potential grid collected over a 11x11 source grid. The structure is a large 2.1x2.1x1.0 cm$^3$ conductive graphite block, outlined with a black line, with its top at the sand water interface at a depth of 0.9 cm.
Figure 4.12: Four graphite blocks 2.1 cm square by 1.0 cm were laid on their edge end to end at a 30 degree angle to the x-axis with a depth to top at the sand water interface of 1.2 cm. For the pole-circular dipole array, the apparent resistivity on the left and the percent IP parameter on the right.
resistivity anomaly is somewhat larger than the anomaly produced by the single block of Fig. 4.8, the percent IP anomaly is considerably larger. For larger rings, the greatest anomaly is not at the center of the slab, but towards the ends of the slab. An explanation is found from the sensitivity analysis from which we know the greatest near-surface sensitivity is found when the anomalous feature is between the potential pairs. However, more current travels through the anomalous body when a source is near the end of the slab and thus a maximum response is observed when several potential pairs are over the center of the slab.

In the modelling tank, the importance of IP is very apparent because the background IP response is small so in many cases the signal-to-noise ratio of the IP effect is considerably better than with resistivity. To illustrate the advantage of a combined resistivity/IP survey, I examined the detectability of a small conductive block near a very large conductive block, both with upper face at the sand water interface at a depth of 0.9 cm. The large graphite block 7.5x7.5x2.5 cm³ was laid flat with its upper right corner beneath source location (4, 4). The small block 2.1x2.1x1.0 cm³ was situated beneath electrode location (8, 8) as outlined with the solid black lines in the plots of Fig. 4.13. Four responses are shown, the apparent resistivities from the circular bipole-dipole, pole-circular pole, and pole-circular dipole arrays, and finally the percent IP effect from the pole-circular pole array. The large block is easily identified in the three apparent resistivity plots, but with uniform contouring and a reasonable number of contours, the anomaly produced by the small block would be easy to miss. If the small block were shallower than the large block, its presence could easily be identified (as I show in the next model). On the other hand, the IP response from both blocks is more similar and the location of each block can be easily identified from these data.

To test the the ability of the synthetic arrays to distinguish deeper structures in the presence of smaller near surface structures, I conducted two studies of interference. The results are shown figures 4.14 and 4.15. The deep structure was a large graphite block 7.5x7.5x2.5 cm³ laid flat at a depth-to-top of 2.0 cm, the upper right corner at source location (5,5). The near surface structures in the first test were 5 scattered small graphite
Figure 4.13: A small block 2.1x2.1x1.0 cm³ in the presence of a large one 7.5x7.5x2.5 cm³ with their upper surface at the sand water interface at a depth of 0.9 cm. Shown are, the apparent resistivities from the circular bipole-dipole, pole-circular pole, and pole-circular dipole arrays, and finally the percent IP effect for the pole-circular pole array.
blocks 2.1x2.1x1.0 cm³ with a depth to top of 0.6 cm at locations indicated by the solid black squares on Fig. 4.14. In the second test, two of the small blocks were replaced with one long insulating PVC block 15.0x2.0x1.0 cm³ stretching diagonally from source location (0,5) to (8,1), as outlined by the solid rectangle in Fig. 4.15. The figures show apparent resistivity maps for the linear pole-dipole (y directed with pole in the direction of increasing y value), circular bipole-dipole, and the pole-circular pole array. The plotting point for the linear pole-dipole array is the array midpoint (chosen to help correctly position deep structure).

For the first test (Fig. 4.14), the pole-dipole (n=1) and circular bipole-dipole (r=2) apparent resistivities both clearly indicate the near surface conductive blocks. The linear pole-dipole near-surface response is greater than that from the circular bipole-dipole since the potential dipole length is double in the latter. This is large enough to partly attenuate anomalies from very shallow sources. The pole-circular pole array exhibits very little response to the surface blocks but clearly locates the deeper block. Each of the larger arrays also responds to the deep block. The pole-dipole n=4 shallow response is displaced towards the pole. The circular bipole-dipole deep response, mid-right plot, is positioned over the deep block. The response to shallow structures is still present, but it is at least in the correct location above the shallow source. The larger (n=6) pole-circular pole array (lower right) still shows a modest broad response to the deep block, but an n = 6 array is now too large for this target.

In second interference model, Fig. 4.15, a long, insulating slab of PVC was placed over the conductive target block in order to investigate possible shielding effects. Unlike a near-surface conductor or moderately resistive body, the near-surface insulating slab does have a significant impact on the response of the pole-circular pole array, particularly when the source is directly over it. It is evident in the pole-circular pole response of both the smaller (lower left plot) and larger receiver ring (lower right plot), although the response of the underlying deep block response can also be seen in the latter. With the pole-circular pole arrays, the normal current density vector is near vertical beneath the pole source and thus it is not surprising that the array is very sensitive to resistive
Figure 4.14: Interference test 1. A 2.0 cm deep conductive block, 7.5x7.5x2.5 cm$^3$ (outlined with a dashed line), in the presence of five shallow (0.6 cm deep) conductive blocks, 2.1x2.1x1.0 cm$^3$ (outline with solid lines) in 1.7 cm of water. From top to bottom are shown the the apparent resistivities for a y-directed pole-dipole array, circular bipole-dipole array, and the pole-circular pole array.
Figure 4.15: Interference test 2. A deep, 2.0 cm, conductive block, 7.5x7.5x2.5 cm³ (outlined with a dashed line), three shallow, 0.6 cm, conductive blocks, 2.1x2.1x1.0 cm³ (outlined with solid lines), and a long shallow, 0.8 cm, resistive block 15.0x2.0x1.0 cm³ (outlined with a solid line), in 1.7 cm of water. From top to bottom, the apparent resistivities for a y-directed pole-dipole array, circular bipole-dipole array, and the pole-circular pole array.
structures with appreciable horizontal extent and a high resistivity thickness product. On the other hand, at the center of the circular bipole-dipole array, the current density vector is primarily horizontal and tends to remain horizontal as the ring size is increased. A horizontal current is less sensitive to a thin horizontal sheet than a vertical current, as evidenced in the middle right plot. The current density vector will also tend to be horizontal beneath the dipole of the pole-dipole array, when \( n \) is large and in the upper right plot of Fig. 4.15 \( (n=4) \) there is little response to the slab. However, the responses to the near-surface conductive blocks oscillates from a high to low apparent resistivity and are displaced from the location of the blocks.

4.4 Summary of Modelling Tank Experiments

From the pole-potential map surveys in the modelling tank several general statements can be made:

1 Precise measurement is easily possible in modelling tank conditions. Observations from the traditional and the newly defined synthetic arrays can be reliably created from a set of pole-potential maps where the measuring array is translated for each measurement.

2 The responses predicted by theoretical modelling Chapter 3 are easily observable in the modelling tank. The predicted responses of linear and circular arrays to small near-surface anomalous resistivity structure and to larger deeper conductive objects are verified in the tank. This includes: 1) the conditions which lead to an oscillatory response profile, 2) the spatial relationship between observed response and target location/orientation, and 3) the magnitudes of response to near-surface and deep structures.

3 Circular arrays tend, for any given target, to display a peak apparent resistivity anomaly that is a smaller fraction of the host resistivity than does a simple linear array, but the sensitivity to local surficial variations is much more strongly reduced.
Thus, there is a marked increase in signal-to-geological noise sensitivity for deeper targets even though there is a decrease in the dynamic range of the 'observed' data. There is no evidence from the model tank experiments that experimental errors seriously degrade the performance of synthetic array data.

4 When polarizable objects or materials are involved, induced polarization is an important effect to observe in addition to the resistance. The parameters which determine the IP and resistivity response, though related, can be sufficiently differentiated to permit discrimination of structure. For instance, a small graphite block could be easily discerned in close proximity to a larger graphite block through the IP response.

The scale models demonstrate that pole-potential maps are a very versatile data set from which any kind of array response in any desired orientation can be reconstructed. Selective reconstruction enables one to discriminate various earth structures based upon their conductivity, their geometry, and the low frequency variation in their conductivity (IP).
4.5 Fort York, a Field Study

To see if field measurement would give similar good results, a small field study was carried out at an archaeological site at Fork York, downtown Toronto. In 1793 John Graves, the first Lieutenant Governor of Upper Canada, established Fort York as the Garrison for the town of York. At the time the site lay on the western shore of Lake Ontario, now it is more than 1 km inland at the foot of Bathurst Street. The original construction was earthwork perimeter walls and log cabin buildings with stone fireplaces and hearths.

Figure 4.16: Fort York in 1842 from Carl Benn, 1993, pp. 107. The site of the survey is the circled area over the former Cook House against the south wall.

The study was undertaken along the south wall and just west of the central bastion in the area circled in figure 4.16. The Lieutenant Governor's house, a large U-shaped structure, stood against the southern embankment until it was destroyed by fire in 1813. Though all remnants of the wood frame have since disappeared, some indication of the foundation might have survived [C. Webb 1989]. A year after the fire in 1814, a row of splinter-proof barracks, including a "Cook House" was constructed along the south wall. These temporary structures were demolished in the 1840s, and in 1861 a row of gun batteries was excavated into the south wall. The fort fell into disuse around the turn
of the century as the city encroached and the shoreline receded. Not until the 1930s did restoration begin in earnest with government work programs. The gun batteries were filled in at this time and the perimeter walls restored with stone work, though originally they were lined with wood.

The Cook House of 1814 had been partially trenched and surveyed in a prior study by Breede 1973, and was thought to represent a suitable target. Though only a “temporary construction” with ”no proper foundations”, [ibid Carl Benn 1993, pp 157], the Cook House floor was laid with stone [Carl Benn 1993, pp 143] and housed a large stone hearth on the east side and a boiler on the west, (Fig, 4.17). The figure displays the outer dimensions of the structure, about 56' by 22', and superimposed survey grids of Breede and Fisher.

![Figure 4.17: A floor plan of the Cook House adapted from Breede, 1973. The perimeter of the survey grid is outlined and the grid coordinates are shown extending 14 m west and 8 m north with a small section extending 14 m north. Breede's survey area is indicated in grey dashes.](image)

The probable location of the house and hearth were surmised from a trenching study, Breede, 1977. A 1x2 m² trench with the long axes aligned N-S at the east end of the hearth was opened to an average depth of 50 cm. The uppermost layer was a 15 cm sod composed of dark brown soil and organic matter. Beneath this lay another 15 cm of topsoil, lighter in shade than the organic layer and loose. A single lens of clay was
found in this layer, possibly overspill from some excavation. Next, a layer of gravel from 5 to 15 cm in thickness and running the full length and breadth of the trench overlying a cement block surrounded with brick rubble and some smaller flagstones. Through the center of the trench at a depth of 50 to 60 cm ran a waterlogged sleeper (buried log), probably part of the Cook House foundation.

The baseline was located 1.25 m from the wall face with the survey area extending to the west of the central bastion. The grid of source points spanned 50 columns to the west by 32 rows to the North with a 25 cm node separation. Readings were taken from east to west. Starting with the first column of 32 sources at 0 W the source column leads 7 receiver columns of 32 potential electrodes each. This was more than sufficient to make a 15x15 patch where half the patch was generated through reciprocity. To complete the patches for column 44-50 W it was necessary to continue source rows up to 57 W. All potentials are determined with respect to the instrument common which is grounded at a point within 2 m west of the active source column. The remote current electrode was placed at (-11.8 W, 6.75 N)m and the remote potential electrode at (23.85 W, 7.75 N)m. Although a controlled current source was used, a monitor electrode was placed at (23.85 W, 4.75 N)m to provide a record of the injected current amplitude.

The survey proceeded smoothly after an initial learning period. Data were collected on 6 separate days over a 10 day period. With the average temperature hovering just above freezing, the moisture content of the soil remained relatively constant aside from the downpour on the first day. On that day, several pools of water gathered within the survey grid requiring the RJ-45 jacks be protected in plastic bags and the low contact resistance made a current amplitude of 28 mA possible. For most of the remainder of the survey a current amplitude of 22 mA was used except for a few at 15 mA where the source voltage was insufficient to inject 22 mA.

The roll-along worked as planned, while the operator sat frozen at the instrument located at the western end of the columns, the one man field crew, (many thanks to Graeme Cairns and Mladen Nedimovic), planted electrodes (2.5" galvanized nails). The operator
Figure 4.18: A view of the electrode grid at Fort York looking South from (0W, 8N). There are 8 columns of 32 electrodes each. Eight electrodes are connected to a single white telephone jack. The array was moved to the west, to the right in the photo.

moved the instrument and instrument ground after sampling 5 columns. Care was taken to avoid systematic errors in line spacings by marking the outer boundary of the survey area first, then filling in the grid. For future archaeological surveys, movable rigid columns of electrodes would reduce the amount of labour in the field. An even greater time saving would be achieved if the source was automatically rather than manually switched.

In the modelling laboratory, the data quality are assessed during the collection process through direct interpretation of a graphical display while the source waveform is observed on an oscilloscope. During field work, only alphanumeric computer data were accessible.
Thus, several diagnostic routines were coded to check data quality in the field and provide a spread sheet of parameters for each current electrode. The "ac noise" and "ac input" defined in Table 4.1 proved the most useful parameters for monitoring problems. Automatic generation of file names simplified data recording. A LED on the current source monitored the source voltage and indicated when insufficient voltage was provided to inject the requested current.

Identical potentials should be measured when the transmitting and receiving poles are interchanged, if we have true pole-pole measurements and the data are of good quality. In fact, potentials from a true pole-pole electrode configuration are impossible to measure since, at best, the remote current and voltage electrodes only approximate the desired infinitely remote case.

The influence of remote poles is easily calculated. In Fig. 4.19 electrodes 1 and 2 are the poles to be interchanged, 0 is the instrument ground, 3 and 4 the remote current and potential electrodes respectively. All potentials are sampled and held with respect to the instrument ground, then differenced from the remote potential and passed through the A/D converter. Using the convention \( P_{\text{current,potential}} \) (eg. \( P_{30} \) is the potential at pole 0 due to source at pole 3), I solve for the potential at 1 for source at 2, \( V_A \), and the potential at 2 for source at 1, \( V_B \).

\[
V_A = [(P_{21} - P_{31}) - (P_{20} - P_{30})] - [(P_{24} - P_{34}) - (P_{20} - P_{30})] \quad (4.4)
\]

\[
V_B = [(P_{12} - P_{32}) - (P_{10} - P_{30})] - [(P_{14} - P_{34}) - (P_{10} - P_{30})] \quad (4.5)
\]
This is a complete description of the actual measured potential in terms of true, unmeasurable, pole-pole potentials. On simplifying, the terms with respect to the instrument ground cancel and we have,

\[ V_A = (P_{21} + P_{34}) - (P_{31} + P_{24}) \]  \hspace{1cm} (4.7)

\[ V_B = (P_{12} + P_{34}) - (P_{32} + P_{14}) \]  \hspace{1cm} (4.8)

where the first bracketed term is common to both, since \( P_{21} = P_{12} \). The second bracketed term gives the difference and involves potentials from the electrodes in the grid to the remote electrodes. They present problems only if the 'poles' within the grid are separated by a distance which is a sizable portion of the distance to one or both remote electrodes. Since the remote poles are generally at distances of order 10x to 20x the distances between local poles, the second term will be smaller than the first by a similar factor (i.e., of order 10%). More importantly, the fractional difference in the distances between local and remote poles will vary by a similar order of 10% over a given map. Thus the neglected second terms in \( V_A \) and \( V_B \) can be considered as uniform base level shifts (of about 10%) in a given potential map with a 10% change in this level between adjacent maps.

In my lab and field surveys, I covered only a small area and it was never necessary to move the remote electrodes. But, if a sufficiently large area were to be covered the remote poles would have to be relocated and this might necessitate a corrective shift in base level. This shift can be removed from the data if we remeasure several source points or, preferably, a complete source column for the new reference pole position as well as the old. Measurements made for the original and new remote pole locations can then be related by a correction factor taken from the difference of the duplicate measurements.

In summary, several factors must be considered in conjunction with the location of the remote poles. The simplest method of coping with the possible interference cause by the remote poles is to record the location of the poles and account for their influence. With GPS measurements we can record the position of the remote poles to a fair degree of
Figure 4.20: The fractional error in reciprocity as a function of electrode separation for the source line 9.75 W. The theoretical value for the layered earth model of Fig. 4.21 is compared to the mean and rms values from the field data.

accuracy; however, to account for the pole influence an assumption must be made about the regional resistivity structure. If the regional structure is homogeneous or horizontally layered then the background response is easily calculated (the presence of 3d structures in the vicinity of the remote pole will have no significant impact, to see this, refer to the hemispherical model studies in Chapter 3). 2d structures can have a greater influence, for instance, a vertical dike of thickness $a$ and conductivity $\sigma_1$ which passes very close to the remote pole and to the survey area has an influence proportional to $\frac{\sigma_1 a^2}{\sigma_2 R_2}$ where $\sigma_2$ is the background conductivity and $R_2$ is the distance from the survey grid to the remote pole. While this will certainly produce a shift in the apparent resistivity, I am most interested in the fractional change in apparent resistivity over the area surveyed. The response from a large structure will not, in general, change significantly over the relatively small survey area.

At Fort York, it was possible to estimate the level of both measurement and remote pole error, even though only half the potential map was measured. Reciprocal measurements could be compared along the source column, and the remote pole errors could be esti-
mated using a layered earth model. In Fig. 4.20 the mean and rms fractional reciprocity errors are shown, as a function of electrode separation, from a column of field data at 9.75 W along with the fractional difference due to the remote poles as calculated for the layered earth of Fig. 4.21. One sees immediately that remote pole error does become significant compared to measurement error for local pole separations more than 15 grid units. However, the influence of random fluctuations and remote electrodes is less than about 1% within the 15x15 patch used in the synthetic circular arrays.

Pole-circular pole apparent resistivity plan maps for 3 radii (from top to bottom, 0.25, 0.50, and 0.75 m) are shown in Fig. 4.22. I have contoured the negative log of the ratio of apparent resistivity to average apparent resistivity of the 0.5 m radius map. Thus the positive contour intervals (mostly the red, orange, and yellow shading) corresponds to a conductivity high. The average resistivity decreases with increasing radii as would be expected from the layer model. At the first radius, the array is sensitive to small near-surface features producing fluctuations with a relatively short spatial wavelength. At the west end a surface resistivity low is observed and is most likely associated with the pools of rain water that collected in that area on the surface on the day it was surveyed. Although the pools disappeared on the other days of surveying, all the ground remained consistently damp. In each plot, especially the 0.5 m radius (a=2) map, a resistivity low is observed running the length of the survey.

A simple layer over a half-space describes the regional background observed in the Fort York data. In Fig. 4.21, I compare the pole-circular pole apparent resistivity of a model, a 1 m, 60 Ω.m layer over a 19.1 Ω.m half-space, to the average response from the field data. The depth to the water table taken from an open grating about 10 m northwest of the grid at (21.6W, 14.2N)m, was about 1.12 m and provides a reasonable explanation for the layering.

There is no clear evidence of the foundations unless the conductive anomaly is interpreted as the wooden waterlogged sleeper as described by Breede (1973). This might be an attractive hypothesis, but its location does not seem to correspond with the floor plan.
Figure 4.21: The solid line is the average pole-circular pole apparent resistivity at each radii from 0.25 m to 1.75 m over the main 18x50 grid of 0.25m source points. The dashed line is the best visual fit consisting of a 1 m thick 60 \( \Omega \cdot m \) layer over a half-space of 19.1 \( \Omega \cdot m \).

in Fig. 4.17. Also, although no record of locations exist, drains have been installed to prevent flooding at the fort. Thus, I feel it is more likely that a metal pipe was buried running parallel to the wall.

Further evidence supporting the postulated presence of a metal pipe is afforded by the plan maps of pole-circular pole percent IP effect in Fig. 4.23. The surface has a generally higher response, possibly due to the electrode array itself. The conductive anomaly at the western end of the survey is has no corresponding anomaly in the IP response. This suggests the source is not a clay lens or metal artifact and supports the suggestion that the rainfall experienced on the first day of the survey produced the low in resistivity. On the other hand, the long conductive "pipe" is clearly evident in the IP response. An electrolytic interaction at the pipe-earth interface would be expected.

In an attempt to accentuate the near surface effects, the P2 invariant from the circular bipole-dipole array is graphed in Fig. 4.24. The conductive anomaly at the west end and the linear anomaly are visible in the contour plots, but again there is no clear evidence of the Cook-House foundations. The spatial frequency of variations is still high at large
Figure 4.22: Pole-circular pole apparent resistivities at Fort York.
Figure 4.23: Pole-circular pole percent IP effect at Fort York.
Figure 4.24: Plan maps of the P2 invariant apparent resistivity circular bipole-dipole at Fort York.
source radii indicating the higher sensitivity to the near-surface or greater sensitivity to geologic noise.

4.6 Summary

The test conducted at Fort York, although unsuccessful in finding the buried structure sought, did provide some insight into the earth resistivities at Fort York. A single 1 m thick layer of 60 $\Omega\cdot$m over a half-space of 19.1 $\Omega\cdot$m describes the regional structure. A linear conductive anomaly runs the length of the surveyed area through its center. There is an IP effect associated with this conductive anomaly. The most probably source is a metal pipe. Several conclusions regarding field techniques suitable to pole-potential mapping can be drawn from the field work:

1. The devised roll-along approach was very successful and allowed close to 1,200 source pole potentials maps to be collected quickly and efficiently after an initial learning period. The task of moving the source location along lines and from line to line was simplified by taking advantage of reciprocal measurements (which allowed a 15x15 electrode patch to be emulated from 8 columns of 32 electrodes) and through the aid of a switch box. The field crew and operator were able to worked independently reducing the number of man-hours required for field work.

2. A high quality data can be collected in an archaeological setting. Reciprocity tests along the source line demonstrated the majority of reciprocal measurements in the active patch were within 1% of each other.

3. The character of the results are consistent with predictions from the numerical studies. For example, the spatial frequency decreases with increasing ring radius for the pole-circular pole array in accordance with analytic and model tank observations.

For future archaeological surveys several modifications can be suggested. First, the ground was exceptionally damp providing good electrical contact; however, to supply
an appropriate current in a wider range of earth conditions a source capable of driving
current at several hundred volts is desirable. Secondly, the electrodes were placed indi-
vidually, but for a smooth or gently undulating surface, such as that at Fort York, it
would be faster to place electrodes in prefabricated strips. Finally, the source switching
and data collection could be further automated through software control and electronic
relays. Then, it might be possible to complete a similar survey of 1,200 source points in
a single day with the limiting factor being the time required to achieve the desired stack
depth at the chosen source frequency.
Chapter 5

Conclusions

5.1 Summary

Identifying structure within the upper 200 m of the earth’s surface is of interest in many areas from mining exploration to environmental and geotechnical site evaluations. To gain a detailed understanding of this region in the earth, geophysical methods are an increasingly attractive exploration alternative to drilling as they are both non-invasive and can be relatively inexpensive. In mining exploration, where electrical properties are often characteristic of structure, the galvanic methods are used with exploration objectives ranging from simple anomaly detection to detailed mapping for lithological discrimination and ore zone characterization. In the geotechnical and environmental areas, knowledge of the electrical properties is relevant to issues such as the delineation and evaluation of landfills, detection of contamination, permafrost evaluation, and archaeological site examination. Within the past decade there has been a strong development in 2d data acquisition and inversion techniques for galvanic resistivity/IP surveys. Often a number of 2d sections can be merge together to form reasonably accurate images of 3d structures. However, in the widely variable environments typically encountered, 3d techniques may have to be considered where a detailed mapping of the sub-surface is required.

Despite the recent improvements in instrumentation and processing capabilities, the underlying survey techniques remain unchanged with linear profiling predominant. It is
natural to consider whether a 3d mapping method using a 2d surface grid, might improve resolution of the structure near the earth’s surface. There are two central issues we need to address before we might adopt a 3d surveying technique:

1. Will acquiring a 3d data set significantly improve our ability to resolve surface structures?

2. If so, are 3d mapping methods technically, logistically and economically feasible for near-surface studies?

In answer to the first question, I provide in my research an approach to interpreting data collected from a 2d grid through apparent resistivities defined for synthetic arrays. This approach greatly enhances our ability to resolve features in electrically complex environments. In answer to the second question, I have designed and constructed a small scale modelling apparatus to collect 2d grids of data in a roll-along mode I name pole-potential mapping. With this design, pole-potential mapping was easy to implement in the field and provided a high quality data set. The introduction of this new mode of exploration greatly increases the utility of electrical mapping methods particularly in electrically complex environments.

Data Interpretation

From a pole-potential map survey, any imaginable array configuration can be generated. I have demonstrated two circular configurations, the pole-circular pole and the circular bipole-dipole, provide a data presentation that closely resembles the actual subsurface distribution of resistivities and chargeabilities. The important imaging properties are:

- Tight control over the region of the earth probed. The zone of maximum sensitivity exists at the center. Since the response from inhomogeneities displaced from the ring center is minimal, the greatest anomalous observation occurs directly over the anomalous resistivity.
- Deep probing measurements are insensitive to small, near-surface structure. Compared to the circular bipole-dipole and most linear arrays, the pole-circular pole has a reduced sensitivity to near-surface inhomogeneities while retaining sensitivity to deeper inhomogeneities of interest. The sensitivity within a particular range can be controlled by altering the spacing between the current pole and potential ring.

- Strong prior assumptions about the ground structure are not required to interpret the data. Responses are independent of the array to target orientation and a simple relationship exists between array spacing and measured response.

- The data are of a high quality. The anomalous potential measured with the pole-circular pole array is the largest of any array confined to the earth’s surface. Random errors in the measured potentials and electrode locations are reduced in the average without reducing the response from deep structure.

- The pole-circular pole response has the largest depth of investigation of any array for a given interelectrode separation (excluding the separation to the remote poles).

- The apparent resistivities can be evaluated extremely quickly.

Interpretation through synthetic arrays is a potent approach for the field geophysicist to obtain a quasi-quantitative interpretation of his data. The resulting image of ground structure resembles a plan view of the sub-surface that can be readily used and understood by geophysicists and nongeophysicists alike. This is not only easier to interpret, but also a much better starting point for inversions than the raw data. Moreover, the ideas developed here are easily extended to other array definitions, each with a unique set of advantages and disadvantages.
Data Collection

The advantages of a 2d grid deployed in a roll-along fashion with a multicore cable system are summarized as follows:

- No increase in manpower over traditional methods, at least in an archaeological setting. The cables are easily laid out with a two person team. A single line takes no longer to deploy than it would with traditional linear sampling methods.
- A reduced data collection time over traditional methods as the source is electronically switched from source point to source point.
- Improved data quality. Since the field crew can work independently of the instrument operator, the operator has the freedom to resample and stack until the desired data quality is reached.

Unless environmental conditions prohibit the use of remote pole references, the pole-potential map technique is ideally suited to mining, hydrological, engineering and similar applications.

5.2 Recommendations

The equipment for this work was intended only as a prototype for scale modelling. As technology is advancing rapidly, it is possible to improve upon the design and construct a field instrument for conducting geotechnical, environmental and archaeological surveys. A more powerful transmitter is required and greater integration of the source switching and computer controller. An increased sampling rate would provide a better description of the IP curve and EM coupling. Since a fair quantity of wire is deployed at any given instant, sufficient data to accurately identify its signature is desirable. A/D converters are cheaper now, higher precision is obtainable, possible digitising at every electrode, increasing sample rate. More surveys in a controlled environment to test the results of interpretation methods.
Numerical modelling is necessary step in any inversion and interpretive process. Presently, the numerical models are poor when an object is coarsely gridded which is always the case when both object and receiver are close to the source, the most desirable location. There is a need for adaptive gridding to multigrid methods. Adaptive gridding would allow higher conductivity contrasts as inaccuracies in gridded secondary sources limit present methods to contrasts of preferably less than 10:1. The boundaries would also be effectively further away.

Finally, the ideas developed here are easily extended to other array definitions, each with a unique set of advantages and disadvantages. Additional information can be obtained by analysing the potentials into fourier components with respect to angle about the source. While this possibility was investigated, no one measure of the parametric data stood out.
References & Bibliography


Boerner, D.E., A Generalized Approach to the Interpretation of Controlled Source Elec-


Li, Y., Oldenburg, D.W., Approximate inverse mappings in DC resistivity problems,


Mauriello P., Monna D., Patella D., 3D geoelectric tomography and archaeological applications., Geophysical Prospecting, 46, 543-570, 1998.


Nabighian M. (Editor), Electromagnetic Methods in Applied Geophysics, Investigations

123


Spitzer, K. Kumpel, H., 3D FD resistivity modelling and sensitivity analyses applied to


Appendix A

Instrumentation

The construction of the transmitter, interface to the laptop, ground connections, power supplies and powerline filtering are discussed in this appendix. For a time domain system this necessitates measuring the transient IP curve as frequently as possible for as long a period as possible. An attempt has been made to achieve these basic objectives in a time domain induced polarization transmitter and receiver described in detail below.

A.1 Functional Description

The general layout of the instrument is outlined in figure A.1. All aspects of the data acquisition are software driven with the exception of the current source amplitude, which is selected by a switch on the source itself. Through the software the source frequency and waveform along with the number of receiving electrodes and sampling frequency are chosen. These parameters are downloaded through the serial port of the Spectrum A/D card to the "external timing logic". This programmable logic array controls the source and multiplexing of potential electrodes to the A/D converter. The source is power amplifier configured as a constant current supply. It is powered by a +/- 50 volt DC-DC converter, for safety the current is limited to 50 mA. The receiver consists of 256 sample and hold amplifiers which are multiplexed to two instrumentation amplifiers, a high gain and a low gain amplifier. The amplifiers connect to the Spectrum A/D card in the computer, a 16 bit card with input range of +/- 3 Volts. The instrument ground,
or common, is connected to a stake located near the survey grid. Potentials are sampled and held with reference to this common point. A differential input is connected to the spectrum A/D converter, both sides referenced to the common. Data are stored on the computers hard disk by the DMA controller on the A/D card. Several diagnostics are available to check the data quality and will be fully described in a following section.

The transmitter is powered from the analogue supply, two 12V, 66 Amp Hour car batteries in series as depicted in figure A.2. To minimise ground loops between the transmitter and the rest of the circuit and to provide sufficient voltage for current injection, two Vicor DC-DC converters, VI-J14-CZ, are connected in series. This provides 25 Watts at +/-48 volts (with an accessible ground) to the power amp with input to output isolation of 3000 Vrms and 75 pF. The computer and receiver also run off the analogue power supply. EMI filters are installed to reduce high frequency noise on the supply itself. The digital power for the receiver is provided by a 6 volt 6 Amp hour battery.

There are two separate grounds, the analogue and digital grounds. The digital ground is loosely connected to the analogue ground through a 47 Ohm resistor. The analogue

Figure A.1: Conceptual diagram of 256 channel IP data acquisition system
ground is common to the A/D converter, the instrumentation amplifiers on the receiver and the sample and hold circuits. It is further connected through a 500 Ohm resistor to a point near the survey grid providing an instrument ground.

The transmitter provides a microprocessor controlled current waveform the amplitude of which is adjustable through a switched feedback resistor located on the transmitter. The amplitude of the load current is given by,

\[ i_{load} = \frac{V_{in}}{R_{in}} + \frac{V_{in}R_f}{R_{in}R_s} \]  

(A.1)

where \( R_{in} \) is the input resistance, \( R_f \) is the feedback resistance, and \( R_s \) is the resistance varied to select the current. Since \( R_{in} = 390 \ k\Omega \) the first term is very small and the load current is approximately proportional to the input voltage,

\[ i_{load} = \frac{V_{in}R_f}{R_{in}R_s} \]  

(A.2)

The load current is varied by changing \( V_{in} \) and by adjusting \( R_s \), the sense resistor. In the first case, voltages are gated to \( V_{in} \), the gating occurring via two optically coupled diode transistor pairs. The optically coupled logic levels are connected to the switch select lines of an analogue multiplexed (DG508). The gated input voltage is further filtered by an
RC network. Three voltage levels are used for Vin, zero volts when the transmitter is off, a positive voltage for a positive current and negative voltage for a negative current. The second option for varying the current is through the sense resistor. To prevent possible damage during a short circuit, the amplifier is current limited to 40 mA.

Two voltage regulators provided a +/- 12 volt supply for the CMOS multiplexer and precision voltage source as well as the unity gain inverting amplifier for negative voltage source. A further regulator provides a digital TTL logic supply of +5 volts, required by the opto couplers.

A.2 The Receiver

The receiver is capable of monitoring up to 256 simultaneous time series on separate sample and hold, (S&H) amplifiers. A S&H amplifier is simply a voltage memory device in which an input voltage is stored on a high quality capacitor through the action of an electronic switch. There are two modes of operation for a S&H circuit; sample mode (or tracking mode) when the switch is closed or hold mode, when the switch is open. The important characteristics of these functions are illustrated in figure A.4.

Perhaps the most important characteristic of the S&H circuit is the acquisition time. The definition is similar to that of settling time for an amplifier. It is required, after the sample command is given, for the hold capacitor to charge to a full-scale voltage change and remain within a specified error band around some final value. The time required to
Figure A.4: The important characteristics in a sample and hold amplifier.

acquire a signal may be calculated by first finding the voltage on the hold capacitor, that is

\[ V(t) = V_0 - V_0 \exp\left(-\frac{t}{RC}\right) \]  \hspace{1cm} (A.3)

where \( V_0 \) is the input voltage to the capacitor, \( R \) is the input resistance and \( C \) is the capacitor value. The time required to charge the capacitor to \( V(t) \) is then,

\[ t = -RC \ln\left[\frac{V_0 - V(t)}{V_0}\right] \]  \hspace{1cm} (A.4)

A 16 bit system provides resolution of one part in 65536 or approximately 91.6\( \mu \)V for a full scale of +/- 3 volts. To track the input signal we desire \( V(t) \) to be within one LSB of \( V_0 \) within some specified time period, in this case 100\( \mu \)S. For a full scale voltage change of 6 volts, \( R = 300\Omega \) (from the LF398 specification sheets), and \( C = 27nF \), an acquisition time of 89\( \mu \)S is obtained. The hold mode droop rate is the output voltage change per unit time when the sample switch is open. The droop is caused by leakage currents of the capacitor, switch, and output bias current. At room temperature with the 27\( nF \) hold capacitor, the LF398 has an output droop rate of 0.001 mV/ms. Thus
Figure A.5: S&H circuit with input protection

for the voltage to droop 1 LSB, the LF398 would have to be in hold mode for 91 ms, the maximum hold time expected is 3 ms.

Hold mode feed through is the percentage of input voltage signal transferred to the output when the sample switch is open, due to capacitive coupling. For the LF398, and 27 nF hold capacitor the feed through rejection ratio is roughly -100 dB for the worst case scenario where a 16 bit system has about 90 dB of dynamic range.

The most critical phase of the sample-hold operation is the transition from sample mode to hold mode. Here the offset error is the change in output voltage with a constant input voltage caused by the switch transferring charge into the capacitor. This is rated at 0.2 mV limiting the capacitor to 15 bit resolution.

The S&H amplifiers have a high input impedance, $100M\Omega$ so as not to load the modelling tank. A polystyrene hold capacitor was chosen with low leakage and low dielectric absorption and temperature characteristics. Overvoltage protection circuitry was included in the form of a 24k series input resistor and diode to the rails. The tank ground electrode is connected to the analogue ground restricting the input voltage levels. The final circuit is shown in figure A.5.
Appendix B

Analytic Solutions

B.1 Layered Earth Response

I derive an expression for the potential of a point current electrode on the surface of a horizontally layered earth in an approach similar to Parasnis 1986, (Pg. 371), and Nigel Edwards (personal communication). Choosing a cylindrical coordinate system \((r, \theta, z)\), with the current electrode at the origin, \(z\) positive downwards and an earth of \(n\) layers over an \(n+1\) layer of infinite thickness, (Fig B.1).

![Diagram](image)

Figure B.1: Point electrode on a stratified earth and the cylindrical coordinate system.

Charge accumulates only where gradients in conductivity are parallel to electric field gradients, therefore, no charge accumulates within a layer and Laplace’s equation describes
the electric potential $U$,

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial z^2} = 0 \tag{B.1}$$

where, by symmetry, $U$ is independent of $\theta$, or $U(r, \theta, z) = U(r, z)$. This equation is most easily solve in wavenumber space and the solution transformed back to the spatial domain with the zero order Hankel transform pair,

$$\tilde{A} (\lambda) = \int_0^\infty r A(r) J_0(\lambda r) dr \tag{B.2}$$
$$A(r) = \int_0^\infty \lambda \tilde{A} (\lambda) J_0(\lambda r) d\lambda \tag{B.3}$$

where $J_0$ is the Bessel function of order zero. In the Hankel domain Laplace's equation becomes,

$$\frac{\partial^2 \tilde{U}}{\partial z^2} - \lambda^2 \tilde{U} = 0 \tag{B.4}$$

two simple solutions are,

$$\tilde{U} (\lambda z) = A \cosh(\lambda z) + B \sinh(\lambda z) \tag{B.5}$$
$$\tilde{U} (\lambda z) = A e^{-\lambda z} + B e^{+\lambda z} \tag{B.6}$$

I shall continue using the first solution. Solving for the potential, it is convenient to choose $z = 0$ as the $i + 1^{th}$ surface, figure B.1,

Now the potentials at the top and bottom of the $i^{th}$ layer are,

$$i^{th} surface, z = -d_i : \tilde{U}_i (\lambda, -d_i) = A \cosh(-\lambda d_i) + B \sinh(-\lambda d_i) \tag{B.7}$$
$$i + 1^{th} surface, z = 0 : \tilde{U}_{i+1} (\lambda, 0) = A \tag{B.8}$$
To further simplify by eliminate the coefficients A and B, a ratio \( \tilde{Q} \) is defined,

\[
\tilde{Q} = \frac{\tilde{U}}{\tilde{J}_z}
\]  

(B.9)

From the potential \( \tilde{U} (\lambda z) \) we can find \( \tilde{J}_z (\lambda z) \),

\[
\tilde{J}_z (\lambda z) = \frac{-\lambda}{\rho} [A \cosh(\lambda z) + B \sinh(\lambda z)]
\]  

(B.10)

since \( \tilde{J}_z = -\sigma \nabla \tilde{U} \). Evaluating \( \tilde{Q} \) at the boundaries we have,

\[
\tilde{Q} = \frac{-\rho}{\lambda} \left[ \frac{A \cosh(\lambda z) + B \sinh(\lambda z)}{A \sinh(\lambda z) + B \cosh(\lambda z)} \right]
\]  

(B.11)

\[
\tilde{Q}_i \big|_{z=-d_i} = \frac{-\rho_i}{\lambda} \left[ \frac{A \cosh(-\lambda d_i) + B \sinh(-\lambda d_i)}{A \sinh(-\lambda d_i) + B \cosh(-\lambda d_i)} \right]
\]  

(B.12)

\[
\tilde{Q}_{i+1} \big|_{z=0} = \frac{-\rho_i}{\lambda} \left[ \frac{A}{B} \right]
\]  

(B.13)

noting that since both \( \tilde{U} \) and \( \tilde{J}_z \) are continuous across the boundary, \( \tilde{Q} \) must also be continuous. Substituting \( \tilde{Q}_{i+1} \) into \( \tilde{Q}_i \) we find a recurrence relationship,

\[
\tilde{Q}_i = \frac{\rho_i}{\lambda} \left[ \frac{\frac{\lambda}{\rho_i} \tilde{Q}_{i+1} + \tanh(\lambda d_i)}{\frac{\lambda}{\rho_i} \tilde{Q}_{i+1} \tanh(\lambda d_i) + 1} \right]
\]  

(B.14)

where I use the relation \( \tanh(-\lambda) = -\tanh(\lambda) \). Thus, given \( \tilde{Q}_{i+1} \), the effect of the layers beneath me, I can find \( \tilde{Q}_i \), the effect of my layer and the layers beneath. For the starting condition of the recursion I choose an infinite layer or halfspace for which the solution to Laplace's equation and \( \tilde{Q}_{n+1} \) are,

\[
\tilde{U}_{n+1} = Ae^{-\lambda z}
\]  

(B.15)

\[
\tilde{Q}_{n+1} = \frac{\rho_{n+1}}{\lambda}
\]  

(B.16)
since \( \tanh(\lambda d_{n+1}) \to 1 \) as \( d_{n+1} \to \infty \). To match a specific source onto the stack of layers an alternate expression for \( \tilde{Q}_1 \) must be found.

\[
\begin{align*}
J_R &= \frac{I}{2\pi R^2} \\ J_z &= \frac{I}{2\pi R^2} \left( \frac{z}{R} \right) = \frac{I}{2\pi} \left[ \frac{z}{(r^2 + z^2)^{3/2}} \right] 
\end{align*}
\] (B.17) (B.18)

to find the Hankel transform for \( \tilde{J}_z \) we observe from Parseval’s Integral,

\[
\frac{1}{\sqrt{r^2 + z^2}} = \int_0^\infty e^{-\lambda z} J_0(\lambda r) d\lambda 
\] (B.19)

now taking the partial derivative of both sides with \( z \),

\[
\frac{\partial}{\partial z} \Rightarrow \frac{-z}{(r^2 + z^2)^{3/2}} = \int_0^\infty -\lambda e^{-\lambda z} J_0(\lambda r) d\lambda 
\] (B.20)

then,

\[
\frac{z}{(r^2 + z^2)^{3/2}} = \int_0^\infty \lambda \left[ e^{-\lambda z} \right] J_0(\lambda r) d\lambda 
\] (B.21)

thus, by inspection, we have the Hankel transformed current \( \tilde{J}_z = \frac{I}{2\pi} e^{-\lambda z} \). On taking the limit as we approach the surface,
\[ \tilde{J}_z \bigg|_{z=0} = \left[ \frac{I}{2\pi} e^{-\lambda z} \right] \xrightarrow{z \to 0} \frac{I}{2\pi} \]  \hspace{2cm} (B.22)

and finally,

\[ \tilde{U}_1 (\lambda) = \frac{I}{2\pi} \tilde{Q}_1 (\lambda) \]  \hspace{2cm} (B.23)

\( \tilde{Q}_1 \) provides a measure of potential for a pole source. The potential in the spatial domain is just,

\[ U(r, 0) = \frac{I}{2\pi} \int_0^\infty \lambda \tilde{Q}_1 (\lambda) J_0(\lambda r) d\lambda \]  \hspace{2cm} (B.24)

To evaluate this an iterative routine is coded to find \( Q_1 \) from B.14 and the integral solved numerically with the Hankel transform routine written by Alan Chave, 1986.

### B.1.1 Stefanescu’s Form

In chapter two I derive array sensitivity from Stefanescu’s solution (Stefanescu 1930) to the layered earth. For the simple case of a layer of thickness \( d_1 \) and resistivity \( \rho_1 \) over a half-space of resistivity \( \rho_2 \), I show Stefanescu’s solution can be derived from B.24. First, we find the kernel function from B.14. The potential on the surface is then,

\[ U(r, 0) = \frac{I}{2\pi} \int_0^\infty \lambda \frac{\rho_1}{\lambda} \left[ \frac{\rho_2 + \tanh(\lambda d_1)}{\rho_2 \tanh(\lambda d_1) + 1} \right] J_0(\lambda r) d\lambda \]  \hspace{2cm} (B.25)

From the relation,

\[ \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = 1 - \frac{2}{e^{2x} + 1} \]  \hspace{2cm} (B.26)

you can see the bracketed term in B.25 is just,
Defining a reflection coefficient \( k \) such that,

\[
k = \frac{(\rho_2 - \rho_1)}{(\rho_2 + \rho_1)}
\]  

(B.28)

B.27 can be rearranged to,

\[
\frac{1 + ke^{-2\lambda d_i}}{1 - ke^{-2\lambda d_i}} = 1 + \frac{2ke^{-2\lambda r}}{1 - ke^{-2\lambda d_i}}
\]  

(B.29)

Thus,

\[
U(r, 0) = \frac{I}{2\pi} \int_0^\infty \frac{\rho_1}{\lambda} \left[ 1 + \frac{2ke^{-2\lambda d_i}}{1 - ke^{-2\lambda d_i}} \right] J_0(\lambda r) d\lambda
\]  

(B.30)

Observing the integral relationship,

\[
\frac{1}{r} = \int_0^\infty J_0(\lambda r) d\lambda
\]  

(B.31)

we have,

\[
U(r, 0) = \frac{I\rho_1}{2\pi} \left[ \frac{1}{r} + 2 \int_0^\infty \frac{ke^{-2\lambda d_i}}{1 - ke^{-2\lambda d_i}} J_0(\lambda r) d\lambda \right]
\]  

(B.32)

where \( K(\lambda) \) contains all the information, resistivities and thicknesses, of the layers beneath. The integral represents the disturbance potential due to the layered earth. The operator, \( \int_0^\infty \text{Func}[ ]J_0(\lambda r) \, d\lambda \), samples Func[ ] preferentially near \( \lambda r = 1 \). This is observed from the basic form of Bessel function \( J_0 \), (Fig. B.1.1),

where most of the value is within \( \lambda r = 1 \). Suppose then that \( r \ll d_1 \), then \( \lambda d_1 \) is large for \( \lambda r = 1 \), thus \( e^{-2\lambda d_1} \) will be small for a range of \( \lambda \) which contributes to the integral,
\[ \rho_a = \rho_1 r \int_0^\infty [1 + small] J_\nu(\lambda r) d\lambda = \rho_1 r \frac{1}{r} = \rho_1 \]  
(B.33)

Hence for small \( r \), the potential is sensitive to \( \rho_1 \) only.

The more general form for a multilayered earth is easily derived starting from the second Laplacian solution B.6 and applying the appropriate boundary conditions to determine the unknown functions A and B (see Parasnis, 1986 and Wait, 1982).

### B.1.2 Sensitivity to a Layer.

Banerjee and Pal (1986) derive the 1d sensitivity function for the dc method which I include here for completeness. Consider the potential at distance \( a \) from a current source \( I \) over a homogeneous half-space of resistivity \( \rho \). The total potential may be viewed as the sum of individual contributions from many infinite horizontal sheets of infinitesimal thickness \( dz \) at every depth \( z \). The model is shown in figure B.3. Starting from Stefanesco's equation, the electrical potential for a thin layer is,

![Figure B.3](image)

Figure B.3: An earth model with a thin perturbed layer in an otherwise homogeneous half-space. The pole potential is measured at a radial separation \( a \) from a pole current source.
where the kernel function, \( \phi(\lambda) \), for 2 layers over a half-space is,

\[
\phi(\lambda) = \frac{k_1 e^{-2\lambda h_1} + k_2 e^{-2\lambda h_2}}{1 - k_1 e^{-2\lambda h_1} - k_2 e^{-2\lambda h_2} + k_1 k_2 e^{-2\lambda(h_2-h_1)}}
\]  

For the model shown, the reflection coefficient \( k_2 = -k_1 \) so we can simplify the kernel function to

\[
\phi(\lambda) = \frac{2k_1 h}{1 - k_1^2} e^{-2\lambda h_1}
\]  

Starting with the Weber integral identity,

\[
\int_0^\infty e^{-\lambda x} J_0(\lambda y) d\lambda = 1/(x^2 + y^2)^{1/2}
\]  

and on differentiating it with respect to \( x \),

\[
\int_0^\infty \lambda e^{-\lambda x} J_0(\lambda y) d\lambda = x/(x^2 + y^2)^{3/2}
\]  

we can reduce Stefanesco’s equation (B.34) in our case to,

\[
V(a) = \frac{\rho_1 I}{2\pi} \left[ \frac{1}{a} + \frac{8k_1 h}{1 - k_1^2 (a^2 + 4h_1^2)^{3/2}} \right]
\]  

Simplifying the nomenclature, depth \( h_1 \) is replaced with \( z \), and thickness \( h \) with \( dz \), then equation B.39 becomes,

\[
V(a, z) = \frac{\rho_1 I}{2\pi} \left[ \frac{1}{a} + \frac{8k_1 z}{1 - k_1^2 (a^2 + 4z^2)^{3/2}} \right]
\]  

Evaluating the reflection coefficient \( k_1 \) for \( \rho_2 = \rho_1 + d\rho_1 \), the right hand side of equation B.40 is just

\[
\frac{I\rho_1}{2\pi a} + \frac{I}{2\pi d\rho_1} \frac{4z}{(a^2 + 4z^2)^{3/2}}
\]  

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The first term of this expression is the potential due to a half-space. The second term is the effect of an infinitesimal sheet of thickness \( dz \) and resistivity contrast \( d\rho_1 \). The Fréchet kernel of a pole-pole array is obtained by integrating the second term with respect to \( \rho_1 \),

\[
dV(a, z) = \frac{\rho_1 I}{\pi} \frac{2z \ dz}{(a^2 + 4z^2)^{3/2}}
\]

(B.42)

Through the concept of electrostatic equivalence, Roy and Apparao (1971) derived an identical expression by evaluating the integrated thin layer contribution of individual layers at varying depths which they named the depth of investigation characteristic, or DIC. When given as a fraction of the half-space, it is called the normalized depth of investigation characteristic, or NDIC. Thus, for the two electrode or pole-pole array, the NDIC is

\[
NDIC(z)_{pp} = \frac{4az}{(a^2 + 4z^2)^{3/2}} \ dz
\]

(B.43)

where \( a \) is the distance between the electrodes. The sensitivity function for a four electrode array is found by considering linear combinations of pole-pole observations. For example, the layer sensitivity of a four electrode array with current electrodes at A and B and potential electrodes at M and N is

\[
NDIC(z) = \frac{dV(r_{AM}, z) - dV(r_{AN}, z) - dV(r_{BM}, z) + dV(r_{BN}, z)}{\rho_1 I \left( \frac{1}{r_{AM}} - \frac{1}{r_{AN}} - \frac{1}{r_{BM}} + \frac{1}{r_{BN}} \right)}
\]

(B.44)

It is sometimes useful (Merrick 1997), to calculate the cumulative response for that part of the earth below depth \( z \),

\[
c(r, z) = \int_{z' = z}^{\infty} NDIC(z') \ dz'
\]

(B.45)

where for the general array, the cumulative response is,

\[
C(z) = \frac{\left[ (r_{AM}^2 + 4z^2)^{-1/2} - (r_{AN}^2 + 4z^2)^{-1/2} - (r_{BM}^2 + 4z^2)^{-1/2} + (r_{BN}^2 + 4z^2)^{-1/2} \right]}{\left[ \frac{1}{r_{AM}} - \frac{1}{r_{AN}} - \frac{1}{r_{BM}} + \frac{1}{r_{BN}} \right]}
\]

(B.46)
B.2 Forward Solutions for a Hemisphere and Sphere.

Solutions to the hemisphere and sphere problems are discussed by numerous authors, Van Nostrand & Cook (1966), Grant & West (1965), and Wait (1982), to name a few. From Van Nostrand & Cook (1966), I use the image solution for the model in Fig. B.4, their derivation follows. A single source, \( I_0 \), is located on the surface of a halfspace of resistivity \( \rho \) outside a perfectly conducting hemisphere of radius \( R \). Two images, \( I_1 \) and \( I_2 \), are sufficient to solve this problem. The image \( I_1 \) is of opposite sign to the source and satisfies the boundary condition of zero potential on the hemisphere surface (and everywhere inside for a perfectly conducting body). Solving for this condition gives an image of magnitude \(-\frac{R}{x_0}I_0\) located at \( x_1 = \frac{R^2}{x_0} \). Since there are no sources within the hemisphere, no net current flows into or out of the hemisphere. To satisfy this conditions an additional source \( I_2 \), of opposite sign to the first image, is added at the center of the hemisphere (the only location from which this source can influence all boundaries equally). Thus, the potential outside the hemisphere is due to: the original source, an image \( I_1 \), and a second image at the center of the hemisphere \( I_2 \). On the surface of the earth, outside the hemisphere we have,

\[
U(x, y) = \frac{I_0\rho}{2\pi} \left[ \frac{1}{\sqrt{(x_0 - x)^2 + y^2}} - \frac{R}{x_0\sqrt{(\frac{R^2}{x_0} - x)^2 + y^2}} + \frac{R}{x_0\sqrt{x^2 + y^2}} \right] \tag{B.47}
\]

Figure B.4: Plan view of a point current source \( I_0 \) and its images, near a perfectly conducting hemisphere.
Figure B.5: Cross-sectional view of a point source of current $I_0$ and its images, near a buried perfectly conducting sphere.

For a current source located inside the hemisphere, the potential is uniform and found from reciprocity. The current source will appear to originate from the center of the hemisphere and the potential just outside is then $\frac{I_0 \rho}{2\pi R}$. This is the potential to which the hemisphere is raised.

Van Nostrand & Cook (1966) derive the image solution to the buried sphere, their derivation follows. As before, we start with a source $I_0$ and create an image $I_1$ in the sphere such that the sphere boundary is at zero potential, figure. B.5. Taking the earth’s surface as $z = 0$, and setting up the condition that no current may cross this boundary, we place an image $I_1'$ above the surface and equidistance from $z = 0$ as $I_1$, such that no current crosses the surface. The new image in turn is reflected in the sphere at $I_2$. This process establishes an infinite series of images. All images lie in the plane $y = 0$ and are described using the recursion relationships,

$$I_1 = -\frac{RI_0}{\sqrt{x_0^2 + D^2}}$$

(B.48)
In order that no net current crosses the sphere boundary, a source is placed at the sphere center equal in magnitude but of opposite sign to the sum of all images so far located in the sphere, \( J_0 = -\sum_{n=1}^{\infty} I_n \) at \( z = D \). \( J_0 \) is reflected in the earth’s surface producing another series of images,

\[
I_n = -\frac{RI_0}{\sqrt{x_{n-1}^2 + (D + Z_{n-1})^2}} \quad \text{(B.49)}
\]

\[
x_1 = \frac{x_0 R^2}{x_0^2 + D^2} \quad \text{(B.50)}
\]

\[
x_n = \frac{x_{n-1} R^2}{x_{n-1}^2 + (D + z_{n-1})^2} \quad \text{(B.51)}
\]

\[
z_1 = D \left( \frac{x_0^2 + D^2 - R^2}{x_0^2 + D^2} \right) \quad \text{(B.52)}
\]

\[
z_n = D - \frac{(D + z_{n-1}) R^2}{x_{n-1}^2 + (D + z_{n-1})^2} \quad \text{(B.53)}
\]

This second image series produces a net current flow from the sphere such that another image is required at the sphere center. This in turn produces a series of images, and so on. The sum of all images is a convergent series. Finally, the potential on the earth’s surface is found by considering all images beneath the surface and doubling the potential to account for the surface,

\[
U(x, y) = \frac{\rho}{2\pi} \left[ \frac{I_0}{R} + 2 \sum_{n=1}^{\infty} \frac{I_n}{\sqrt{(x_n - x)^2 + y^2 + z_n^2}} + 2 \sum_{n=0}^{\infty} \frac{J_n}{\sqrt{x^2 + y^2 + z_{Jn}^2}} \right] \quad \text{(B.58)}
\]

The computational effort in evaluating the sphere potential is directly proportional to the number of terms included. For a conductive sphere buried at a depth greater than
1.3R, only a few iterations are required as the anomalous charge distribution is relatively undisturbed by the surface. As the sphere approaches the surface, an increasing number of images are necessary making burial depths of less than 1.1R impractical.

Where arbitrary conductivity contrasts are desired the solution is found through Laplace’s equation. The basic problem is one of determining the potential in the vicinity of a single point source of current when in the presence of an anomalous conductivity. If the potential at a measurement point is influenced by two or more sources, the total potential at that point can be computed by algebraic addition of the separate potentials due to each of the sources acting alone. This is simply an application of the principle of superposition. After computing the potential at each potential electrode due to all sources, the potential differences are easily obtained, if so desired.

To solve Laplace’s equation in the context of a resistivity problem, Van Nostrand & Cook (1966) outline four steps to be followed. The first step is to select the proper coordinate system, one in which we can describe the existing boundaries of our idealised geologic situation in terms of the variables chosen. Preferably, these boundaries should be surfaces over which one of the three variable coordinates remains constant. The surface of the earth must also be similarly described. Since no current flows across the earth surface, it is usually chosen as a plain of symmetry in the coordinate system. The second step is to establish the proper form of Laplace’s equation in the coordinate system chosen. The third step consists of finding the necessary expansion of the reciprocal distance from the point source to the point at which the potential is to be calculated. This expansion is carried out either in terms of an infinite series or an integral. In the fourth and final step, we identify the appropriate general solution by consideration of the relevant boundary conditions which are:

- As the point source is approached from any direction on the earth’s surface, the potential must become infinite as \( \frac{r \rho}{2\pi R} \).
- Far from the source the potential vanishes as \( \frac{1}{R} \).
Plan View

Cross Sections

Figure B.6: Plan view of hemisphere showing the relationship to an arbitrary resistivity traverse. Cross sections with the current source outside or inside the hemisphere show the regions in which a particular solution is valid. Adapted from Van Nostrand and Cook (1966).

- The potential is everywhere continuous except at the current sources and sinks.
- The normal component of the current density is continuous at the boundaries.
- The solution is everywhere finite except at the sources and sinks.

Specific examples are solved in Van Nostrand and Cook (1966). For example, a hemisphere embedded in the surface of a semi-infinite space, shown in figure B.6 for which the potential functions with the source lying on the surface of the ground along the polar axis are,

\[
U_{1A} = \frac{I \rho_1}{2\pi} \left[ \frac{1}{R} + k \sum_{n=0}^{\infty} \frac{2nr_1^{2n+1}}{(2n + 1 + k)(\tau_0 r)^{n+1}} P_n(\cos \theta) \right] \tag{B.59}
\]

\[
U_{2A} = \frac{I \rho_1}{2\pi} \left[ \frac{1 + k}{\tau_0} \sum_{n=0}^{\infty} \frac{(2n + 1)}{(2n + 1 + k)} \left( \frac{r}{\tau_0} \right)^n P_n(\cos \theta) \right] \tag{B.60}
\]
where the reflection coefficient $k = \frac{\rho_s - \rho_1}{\rho_s + \rho_1}$. The solution for a sphere embedded in a whole space is found by changing the factor $2\pi$ to $4\pi$. To the extent that the sphere does not interact with its image, we can use the above equations to solve for a buried conductive or resistive sphere. For a conductive sphere, a burial depth of 1.3 $R$ leads to a 10% error. The error for a resistive sphere at an equivalent burial depth is always less. In applying these equations care should be taken as $r$ approaches $r_0$ in magnitude since there is poor convergence leading to an excessive number of iterations.
Appendix C

Standard Array Geometries and Profiles over Simple Models

C.1 Profiles over a Near-Surface Highly Conductive Hemisphere

The following figures profiles for a variety of arrays across an near-surface highly conductive hemisphere of radius 1m in a 10 Ω.m half-space. The traverses bisect the hemisphere. The plotting point is chosen at the array center in all but the pole-dipole array where the the mid-point of the dipole is used. Profile lines are shown for several electrode separations and combined into pseudosection format. In the pseudosections, the effective depth, the depth to median layer sensitivity, is used for the vertical scale. A limitation exists for dipoles when inside the near-surface hemisphere. The apparent resistivity remains zero until the potential electrodes are out of the hemisphere because the potential difference is zero.
Figure C.1: Pole-pole profiles over an near-surface hemisphere. The pole spacing varies from 0.5 to 4.0 m. For small separations the hemisphere is well defined; however, for separations much larger than the hemisphere there is a considerable negative overshoot in the response. The sense of the anomaly produced by most traditional arrays is strongly dependent on the ratio of array length to anomaly size. This is especially confusing if the profile is sparsly sampled or incomplete.
Figure C.2: Wenner profiles over an near-surface hemisphere. The $a$ spacing varies from 0.5 m to 4.0 m. Large overshoots, cusps, appear for all electrode spacings. The location of the hemisphere is well defined for the smaller spacings as the apparent resistivity is zero when all electrodes are within the hemisphere. The 4 m spacing produces a profile which bears little resemblance to a conductive hemisphere and has a large amplitude over a extended horizontal range.
Figure C.3: Pole-dipole profiles over an near-surface hemisphere. The dipole spacing is 0.5 m with $n$ varied from 1 to 8. Plotting the apparent resistivity under the dipole ensures the anomaly is almost centered over the hemisphere. Since the dipole spacing is less than the radius of the hemisphere, the anomaly does not decrease with increased $n$ spacing. The effect due to the pole passing over the hemisphere is seen as a diagonal conductive line whose magnitude decreases in proportion to the $n$ spacing. A large overshoot exists when the pole and dipole are on opposite sides of the hemisphere.
Figure C.4: Dipole-dipole profiles over an near-surface hemisphere. The dipole spacing is 0.5 m with $n$ varied from 1 to 8. A classic pant-leg effect is observed when either dipole is located within the hemisphere. A large overshoot exists when dipole pairs are on opposite sides of the hemisphere reaching a maximum when one dipole pair is just outside the hemisphere. The amplitude of the overshoot decreases with increasing $n$ spacing.
Figure C.5: Pole-circular pole profiles over an near-surface hemisphere. The ring radius varies from 0.5 m to 4 m. A profile across the hemisphere consists of a single trough, particularly for small rings, when all electrodes electrodes are within the hemisphere. Even then, there is still a measureable potential to the infinite return; though, this limits the maximum amplitude of the anomaly. The overshoot is everywhere less than 10% reaching a maximum for a source just outside the hemisphere. The anomaly falls off rapidly for source positions outside the hemisphere. The edge of the hemisphere clearly defined.
Figure C.6: Pole-circular dipole profiles over an near-surface hemisphere. The inner ring radius varies from 0.5 m to 4 m with a constant dipole separation of 0.5 m. The greater sensitivity under the dipole pair leads to a larger overshoot when the ring of potential electrodes passes over the hemisphere. The overshoot decreases with ring size at first, then remains constant as the inter ring spacing is constant as is the number of dipole pairs in the rings.
Figure C.7: Circular dipole-dipole profiles over an near-surface hemisphere. The outer ring radius varies from 0.5 m to 4 m with a constant potential electrode separation of 0.5 m. Since the dipole spacing is smaller than the hemisphere diameter, we observe a zero apparent resistivity for all ring sizes. A large overshoot occurs when the dipole pairs are adjacent to the hemisphere. Little anomalous response is observed when part of the current ring passes over the hemisphere.
C.2 Profiles over a Subsurface Conductive Sphere

The following figures profiles for a variety of arrays across highly conductive sphere of radius 1m buried at a depth of 1.25m in a 10 Ω.m half-space. The traverses along the surface pass over the center of the sphere. The plotting point is chosen at the array center in all but the pole-dipole array where the the mid-point of the dipole is used. Profile lines are shown for several electrode separations and combined into pseudosection format. In the pseudosections, the effective depth, the depth to median layer sensitivity, is used for the vertical scale.
Figure C.8: Pole-pole profiles over an sphere. The pole spacing varies from 0.5 to 4 m. For small separations the sphere is well defined; however, for separations much larger than the sphere, there is a considerable negative overshoot in the response; though, the cusps that appear for an near-surface structure are smoothed.
Figure C.9: Wenner profiles over an sphere. The \( a \) spacing varies from 0.5 m to 4 m. Large overshoots, cusps, appear for all electrode spacings. The location of the sphere is well defined for the smaller spacings as the apparent resistivity is zero when all electrodes are within the sphere. The 4 m spacing produces a profile which bears little resemblance to a conductive sphere.
Pole-Dipole Profiles over a Sphere (D=1.25, R=1), a=0.75, n={1,...,8}

Figure C.10: Pole-dipole profiles over an sphere. The dipole spacing is 0.5 m with n varied from 1 to 8. Plotting the apparent resistivity under the dipole ensures the anomaly is almost centered over the sphere. With the dipole centered over the sphere, the observed anomaly does not decrease with increased n spacing. The effect due to the pole passing over the sphere is seen as a diagonal conductive line whose magnitude decreases in proportion to the n spacing. A overshoot exists when the pole and dipole are on opposite sides of the sphere.
Figure C.11: Dipole-dipole profiles over an sphere. The dipole spacing is 0.5 m with \( n \) varied from 1 to 8. A classic pant-leg effect is observed when either dipole is located within the sphere. A small overshoot exists when dipole pairs are on opposite sides of the sphere reaching a maximum when one dipole pair is just outside the sphere. The anomalous amplitude beneath the dipole pair does not decrease with increased \( n \) spacing.
Figure C.12: Pole-circular pole profiles over a sphere. The ring radius varies from 0.5 m to 4 m. A profile across the sphere consists of a single trough, particularly for small rings, when all electrodes are within the sphere. There is no overshoot.
Figure C.13: Pole-circular dipole profiles over an sphere. The inner ring radius varies from 0.5 m to 4 m with a constant dipole separation of 0.5 m. The greater sensitivity under the dipole pair leads to a small oscillation in the apparent resistivity, but very little overshoot. The oscillation amplitude remains constant as the inter ring spacing is constant as is the number of dipole pairs in the rings.
Figure C.14: Circular bipole-dipole profiles over a sphere. The outer ring radius varies from 0.5 m to 4 m with a constant potential electrode separation of 0.5 m. Since the dipole spacing is smaller than the sphere diameter, we observe a zero apparent resistivity for all ring sizes. A large overshoot occurs when the dipole pairs are adjacent to the sphere. Little anomalous response is observed when part of the current ring passes over the sphere.
C.3 Depth of Detection

The following are profiles for the dipole-dipole, pole-dipole, Wenner, pole-pole, and pole-circular pole arrays across a highly conductive sphere of radius R scaled to unity and buried in a 10 $\Omega$.m half-space. The traverses along the surface pass over the center of the sphere. Each curve corresponds to a different depth of burial. The electrode spacings are varied and the apparent resistivity evaluated at the plotting point.
Figure C.15: Dipole-dipole (n=2, and n=6), depth of detection for a sphere sounding curves over a highly conductive sphere. The depth D/R is taken as the depth to the sphere center.
Figure C.16: Pole-dipole (n=2) and Wenner depth of detection for a sphere sounding curves over a highly conductive sphere. The depth D/R is taken as the depth to the sphere center.
Figure C.17: Pole-pole and Pole-circular pole depth of detection for a sphere sounding curves over a highly conductive sphere. The depth D/R is taken as the depth to the sphere center.
C.4 Array Definitions

Figure C.18: Electrode geometries for traditional arrays on a regular grid. The arrays are shown aligned on their usual data plotting point and scaled to their usual array size parameter $L$. For the pole-pole: $L=a$, pole-dipole: $L=(n+1)a$, Wenner $L=3a$, Schlumberger: $L=(2n+1)a$. For surveys on a uniformly gridded profile $a$ is a simple multiple of the grid spacing (usually 1) and $n$ is a positive integer multiplier. At full expansion, electrical sounding (VES) surveys (usually Schlumberger or Wenner array), the spacings are varied logarithmically over at least a 30:1 ratio and are unrelated to gridding.
Appendix D

Numerical Solution to the Forward Problem

Computer modelling appears to be a relatively straightforward way of obtaining the dc response of arbitrary conductivity distributions in three dimensions. The resistivity of an anomalous body and the host medium are easily varied in a computer package and a noise-free data set should be obtained. Two methods are used in numerical modelling; the differential equation and integral equation methods. Differential equation solutions, either finite element or finite difference, are easiest to implement in terms of mathematical complexity and were the preferred choice for 2D studies in the mid 1970s, (Hohmann, 1975). One frequently referenced example is Dey & Morrison (1979). Originally, differential equation methods were found to be demanding both in CPU time and memory requirements for use on 3D problems. However, this is no longer the case, and they are now the preferred method when resistivity structure is complicated. Several multigrid solvers were available, a generic package of FORTRAN subprograms, MUDPACK (Adams, 1989), and a multigrid method specifically for 3D resistivity, (Bailey and Cheesman, 1996).

D.1 Numerical Formulation

The forward modelling or numerical solutions in MUDPACK (Adams), and Multigrid (Bailey et al), are formulated in terms of the anomalous conductivity and the anomalous potential in the self-adjoint form. The anomalous potential can generally be obtained
with greater accuracy than the total potential method since it falls off more rapidly with distance from the source and thus is less likely to be affected by errors in applying boundary conditions.

The self-adjoint solution is easily derived (Li, and Oldenburg 1991). A steady state electric current introduced into the ground sets up a distribution of charges both on and beneath the earth’s surface. The electric potential may be evaluated by using Maxwell’s equations, conservation laws, and constitutive relations with the appropriate boundary conditions. From the conservation of charge,

\[ \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} = I \delta(\vec{r} - \vec{r}_s) \tag{D.1} \]

where \( \vec{J} \) is the current due to free charges and for steady state, the time rate of change of current is zero everywhere except at the source and hemisphere locations, \( I(\vec{r}_s) \). From the constitutive relation, \( \vec{J} = \sigma \vec{E} \), and expressing the electric field in terms of the electric potential, \( \vec{E} = -\nabla \phi \), we have,

\[ \nabla \cdot (\sigma \nabla \phi) = -I \delta(\vec{r} - \vec{r}_s) \tag{D.2} \]

where the homogeneous background conductivity \( \sigma \) and potential \( \phi \). Adding anomalous values, \( \sigma^a \) and \( \phi^a \) to the background case, substituting the total values into D.2 and subtracting the background case, we have,

\[ \nabla \cdot (\sigma \nabla \phi^a) = -\nabla \sigma^a \nabla \phi^* - \sigma^a \nabla^2 \phi^* \tag{D.3} \]

The right hand side is known as it involves terms in the background potential and the anomalous conductivity. The left hand side, has a term involving the anomalous potential which we seek and can be viewed as the anomalous or induced source term. They are determined by the influence of the primary potential on the anomalous conductivity and represent the accumulation of charge on the gradient in conductivity.

Boundary conditions for the mudpack routines may be any combination of periodic, Dirchlet and mixed-derivative. Multigrid uses impedance boundary conditions in which
an extra layer of cells surrounding the solution space is added. The influence of the primary and secondary sources on the inner boundary are summed assuming a uniform medium. The outer boundary is set to zero potential and impedance of the cell layer separating the two is adjusted to the ratio of the potential difference and its normal derivative.

D.2 Model Tests

Bailey et al have examined the accuracy of the multigrid method for a number of test cases, a uniform medium, a double half space and a resistive prism. Their numerical test is on a coarse grid of $16^3$ nodes, a more stringent test than the possible solution region of $64^3$ nodes. Apparent resistivities derived from the electric field along a borehole through the solution space are compared to analytic solutions and were only a few percent in error. For a homogeneous space a maximum error of 6% was found close to the source, associated with the limitations of cartesian gridding, and a similar error close to the boundary. Everywhere else, errors were found to be less than 1%. To test the impedance boundary conditions an anomalous resistive prism model was used. Moving the prism from the center to the edge of the solution space changed the resulting electric field only a few percent, though comparison to an analytic solution was not made.

I have tested the conductive targets for which I have an analytic solution, the outcropping hemisphere and the surface and the buried sphere. The solution space is 32m cubed composed of $64^3$ of separation 0.5m. One boundary is a free surface with impedance boundary conditions on the remaining faces. The radius of the 1 σm hemisphere and sphere is 2.5m, the hemisphere outcorps while the top of sphere is 1m below the surface. Aside from the adjacent free 'surface' the anomalous conductivity is otherwise centrally located in the solution space. A cube was deemed to be in the anomalous zone if the radial displacement of its center is less than or equal to the sphere radius. For a radius of 2.5m interpreted on a 0.5m grid, the resulting gridded anomalies are 5% larger than the theoretical volume.
Figure D.1: A section taken through the center of the hemispherical model. The node separation is 0.5m with electrode separation of 1m. An ideal hemisphere is outlined.

Though the anomalous potential can be generally be obtained with greater accuracy an exception occurs when anomalies are much more conductive than the reference value. In this case small percentage errors in the secondary potential may result in large errors in the final result, Bailey (1996). This problem is somewhat resolved by choosing the highest conductivity present as the reference value. The analytic image solutions are for perfectly conducting bodies in a 100 \( \sigma m \) halfspace the numerical model is a 1 \( \sigma m \) in 100 \( \sigma m \) halfspace. Figure D.1 shows the bisected test hemisphere. Node separations are 0.5m with an electrode spacing of 1m.

Several controls exist to fine-tune the multigrid method. The number of multigrid cycles and the number of relaxation cycles may be altered. After numerous runs it was found that the default values, 2 cycles with 2 relaxations at the beginning and end of each cycle, generally produced an error level far less than 1%. The error level is determined by the ratio of original sources to residual sources, that is,

\[
\text{errorlevel} = \frac{\sum |S_r|}{\sum |S_o|} \tag{D.4}
\]

where \( S_r \) are the residual sources and \( S_o \) are original sources. If this error level were exceeded, increasing the number of cycles greatly increased the run time (proportional to the number of cycles times the sum of pre and post relaxations), but produced no or marginal improvement in the result.

Multigrid performs calculations using either the original or the induced sources calculated
Figure D.2: Pole-circular pole profile across a conductive hemisphere. The numeric solution show spurious results.

on the surfaces of conductivity anomalies. “By default, induced sources are calculated, since the procedure is more accurate. (If induced sources are not used, the first cycle of the multigrid process basically creates them anyway, so you lose a cycle’s worth of accuracy in the results” Cheesman (1996). However, for objects an order of magnitude mode conductive than the reference conductivity, this is not the case as demonstrated by the hemispherical and to lesser extent, spherical models. Figure. D.2 depicts a 3m radius pole-circular pole profile with a source separation of 1m taken across the center of the hemisphere and sphere (Fig D.1). With the source just inside the 2.5m radius hemisphere spurious numerical results occur despite setting the background conductivity at 1 σm. With the source just inside the anomalous zone at the 2m mark, the secondary sources are underestimated, while the opposite is true with the source located at the 3m mark just outside the anomalous zone. A similar result is obtained for a source located directly above a buried sphere, (Fig, D.2 thought to a lesser degree. The deviation is more pronounced at larger radii. Increasing the number of cycles had no appreciable influence on the results. Further calculations were based on the primary field with induced sources turned off for such extreme conductivities.

Figure D.3 illustrates the results of pole-circular pole profile across the conductive hemi-
Figure D.3: A pole-circular pole survey across an outcropping 1 \( \sigma m \) hemisphere in 100 \( \sigma m \) background. Apparent resistivities for numeric and analytic solutions are compared at several radii.

sphere. The smallest ring of radius 1m, or two nodes from the source, shows apparent resistivities about 4% above the background reading of 100 \( \sigma m \). This is due to the finite grid size, and is not observed on potential rings taken further from the source. The apparent resistivities for the radius 3 ring are a distinct improvement over figure D.2 where induced sources were employed. Discrepancies may be attributed to the course grid interpretation of the hemisphere and, where the analytic model is perfectly conducting, the numeric solution is 1 \( \sigma m \). Nevertheless, the apparent resistivities are everywhere with 5% of the analytic values, a fairly reasonable result given the source is in direct contact with the anomalous zone.

Figure D.3 illustrates the results of pole-circular pole profile across the conductive sphere. For the 1m ring the apparent resistivities are again too high due to the limitations of finite gridding. The numerical results are in close agreement with the analytic result with errors less than 2% aside from those closest to the source where the error is just over 4%.
Figure D.4: A pole-circular pole survey across a 1 σm sphere in 100 σm with depth to top of 1m. Apparent resistivities for numeric and analytic solutions are compared at several radii.

The sphere is further from the source and receiver points which improves the numerical approximation.
Appendix E

The Apparent Resistivity Ellipse and Harmonic Analysis

In the body of this thesis I define several scalar measures of apparent resistivity: the P2 invariant for the circular bipole-dipole array and for the pole-circular pole array the apparent resistivity found from the average potential measured on a circular ring. These measures describe some sort of average apparent resistivity at the measurement location. Additional information can be obtained from these arrays regarding the directional dependence of the apparent resistivity.

For the circular bipole-dipole array an apparent resistivity tensor is calculated by synthesizing two orthogonal fields of current excitation and measuring two vector components of the electric field. Dipole potential differences are taken as proportional to the electric field for small dipole separations. The the $P_1$ and $P_2$ rotational invariants (equations 2.16) provide a good indication of the average apparent resistivity at the array center and are independent of the direction of the electric field. The directional information in the tensor measurement is easily displayed in a graphical format as an apparent resistivity ellipse. The local length of the ellipse is proportional to the apparent resistivity that would be found for a potential dipole measurement at the ring center in that direction. For more information on the P2 invariant see Bibby and Hohmann (1993) who discuss it in regards to the multiple-source bipole bipole technique.

Figure E.1 illustrates the results for a numerically modelled pole-potential map survey
Apparent resistivity ellipses provide a graphical display of the circular bi-pole-dipole apparent resistivity tensor. The contoured values are the $P_2$ invariant proportional to the root of the area of the ellipse. The ellipses in the figure are scaled by the $P_2$ invariant. Note, the central ellipse 16,16 at the corner of the anomalous section has been evaluated by the code but is inaccurate. One would expect the ellipse to align with the major axis directed towards the high resistivity section.

over an eighth-space of 15 Ohm.m in a 1 Ohm.m host. From the original 15x15 pole-pole grid, a 13x13 map of the $P_2$ invariant for the circular bi-pole-dipole array is contoured for a source ring of 4 m diameter. Every 3rd ellipse is shown with the dimensions scaled by the $P_2$ invariant (the root of the geometric mean of the area of the ellipse). Away from the resistivity contact the ellipses are equidimensional; however, just outside the 15 Ohm.m region the ellipticity is large with the major axes perpendicular to the resistivity contact. While the ellipticity is a strong function of the position of the measurement, the $P_2$ invariant is a smoothly varying representation of the average apparent resistivity at the measurement location.

From the pole-circular pole array, information can be obtained about the amplitude of the local apparent resistivity, the direction of the gradient in apparent resistivity, and the degree of anisotropy by analysing the array potentials into Fourier components with respect to angle about the source. Over a homogeneous half-space, the potential at each
Figure E.2: Pole-circular pole apparent resistivities plotted as a function of angle (scaled by their average size). The contoured value is the average size of the figure. The crosses indicate the ellipticity of the figure with the bold axis representing the major axis. There are 24 potential electrodes for the 4 m radius ring where the grid spacing is 1 m.

Thus for a given ring a measurement, the apparent resistivity as a function of angle can be obtained by simply evaluating the individual pole-pole apparent resistivities. The size of the figure is proportional to the pole-pole apparent resistivity that would be measured in any given direction. The apparent resistivity figure generally results in a distorted ellipse and in most cases a parametric model has been found to be an excellent representation of the measured apparent resistivity, that is

\[ V_i = \frac{\rho l}{2\pi} \left( \frac{1}{r} \right) \]  

(E.1)

\[ \rho(\phi) = \rho_{A0} + \rho_{A1}\cos(\phi) + \rho_{A2}\sin(\phi) + \rho_{A3}\cos(2\phi) + \rho_{A4}\sin(2\phi) \]  

(E.2)

where the coefficients are found by performing a least squares fit of equation E.2 to the observed figures. Interpretations could be made directly from a graphical representation of these figures. The parameter \( \rho_{A0} \) represents the average size of the apparent resistivity figure. The parameters \( \rho_{A1} \) and \( \rho_{A2} \) are combined to form a vector indicating the displacement of the center of the figure from the plotting point (source location). Param-
Figure E.3: The vectors are proportional to the displacement of the figures of Fig. E.2 from the plotting point (source location). They are greatest where there is a strong gradient in the apparent resistivity and align perpendicular to the contours.

Parameters $\rho_{A3}$ and $\rho_{A4}$ describe the ellipticity of the figure, in particular, the major and minor axes correspond to the directions of maximum and minimum apparent resistivities.

Figures E.2 and E.3 illustrate the pole-circular pole figures for a 4 m diameter potential ring over the eighth-space model. The contoured values in the figures are average size of the figure, $\rho_{A0}$, and correspond to the average apparent resistivity at the plotting point. The polar figures in Fig. E.2 are scaled by the average length $\rho_{A0}$. The length of the figure in a given direction correspond to the pole-pole apparent resistivity in that direction. As for the apparent resistivity ellipse, the figures are equidimensional when not adjacent to the resistivity contact. The cross indicates the ellipticity of the figure from the parameters $\rho_{A3}$ and $\rho_{A4}$ where the darker line is the major axis.

In Fig. E.3, the contoured value is $\rho_{A0}$. The vectors are proportional to the displacement of the figures of Fig. E.2 from the plotting point and are found from $\rho_{A1}$ and $\rho_{A2}$. In general, the figure is only displaced when there is marked gradient in the apparent resistivity and thus the vectors tend to align perpendicular to the $\rho_{A0}$ apparent resistivity contours.