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DEVELOPMENT AND VALIDATION OF A MODEL
FOR ANALYZING THE STABILITY OF A POWER WHEELCHAIR

by

Jason A. Young

A thesis submitted in conformity with the requirements
For the degree of Master of Applied Science
Graduate Department of Mechanical and Industrial Engineering
University of Toronto

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"Development and Validation of a Model for Analyzing the Stability of a Power Wheelchair"
Jason A. Young, M.A.Sc., 1998
Department of Mechanical and Industrial Engineering, University of Toronto

Abstract

Wheelchair stability is a growing concern. Between 25 and 38 percent of serious wheelchair accidents result from forward and backward tips [3, 5, 24]. A two dimensional model of a power wheelchair was developed and then validated using a series of static and dynamic tests. Certain wheelchair properties were varied to ensure that the predictions were valid for a wide range of initial conditions. The model was less stable than the actual wheelchair in both static and dynamic scenarios. The model was able to predict static tips within 4.4 degrees and dynamic tips within 3.4 degrees. The model demonstrated that the wheelchair would not tip over upon encountering curbs of up to 7.1 cm high and sudden drops of up to 20.0 cm. The utility of computer modeling as a tool in the design of safer wheelchairs was demonstrated.
Acknowledgements

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- **Dr. William Cleghorn**, for his thoughtful advice and support over the course of this project;

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- **Mr. Alex Mihailidis**, for his assistance in the validation tests and thesis presentation;

- **Mr. Stephen Perry**, for his advice and assistance in digitizing the dynamic trials;

- **Lisa Young**, for her very significant role in editing the thesis and her unending support;

- **Susan Young**, for her tremendous support and strength which should continue to be a source of personal inspiration;

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Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>iii</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>iv</td>
</tr>
<tr>
<td>List of Tables</td>
<td>vi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vii</td>
</tr>
<tr>
<td>List of Appendices</td>
<td>x</td>
</tr>
<tr>
<td><strong>Chapter 1: Introduction</strong></td>
<td>1</td>
</tr>
<tr>
<td>1.1 Objective</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Scope</td>
<td>1</td>
</tr>
<tr>
<td>1.3 Hypothesis</td>
<td>2</td>
</tr>
<tr>
<td>1.4 Approach</td>
<td>2</td>
</tr>
<tr>
<td><strong>Chapter 2: Background</strong></td>
<td>4</td>
</tr>
<tr>
<td>2.1 Wheelchair Use in North America</td>
<td>4</td>
</tr>
<tr>
<td>2.2 Wheelchair Characteristics</td>
<td>5</td>
</tr>
<tr>
<td>2.3 Incidence of Wheelchair Accidents</td>
<td>7</td>
</tr>
<tr>
<td>2.4 Addressing the Issue of Wheelchair Stability</td>
<td>9</td>
</tr>
<tr>
<td>2.5 Current Measures of Wheelchair Stability in the Fore-Aft Plane</td>
<td>13</td>
</tr>
<tr>
<td>2.6 Overview of ANSI/RESNA Wheelchair Stability Standards</td>
<td>16</td>
</tr>
<tr>
<td>2.7 Overview of Modeling Theory and Numerical Methods</td>
<td>19</td>
</tr>
<tr>
<td>2.8 Human Segment Models</td>
<td>23</td>
</tr>
<tr>
<td><strong>Chapter 3: Method</strong></td>
<td>26</td>
</tr>
<tr>
<td>3.1 Development of Integrated Wheelchair Model</td>
<td>26</td>
</tr>
<tr>
<td>3.1.1 Spatial Model</td>
<td>29</td>
</tr>
<tr>
<td>3.1.2 Wheels</td>
<td>32</td>
</tr>
<tr>
<td>3.1.3 Drive Spring</td>
<td>38</td>
</tr>
</tbody>
</table>
List of Tables

<table>
<thead>
<tr>
<th>#</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2a</td>
<td>Definitions of wheelchair dimensions</td>
<td>6</td>
</tr>
<tr>
<td>2b</td>
<td>Comparison of segment mass models</td>
<td>25</td>
</tr>
<tr>
<td>3a</td>
<td>Physical components of the wheelchair model</td>
<td>30</td>
</tr>
<tr>
<td>3b</td>
<td>Geometric properties of the wheelchair model</td>
<td>31</td>
</tr>
<tr>
<td>3c</td>
<td>Finalized segment masses used for human occupant model</td>
<td>44</td>
</tr>
<tr>
<td>4a</td>
<td>Data from wheel stiffness test</td>
<td>55</td>
</tr>
<tr>
<td>4b</td>
<td>Predicted vs. actual ground contact forces for each set of wheels</td>
<td>61</td>
</tr>
<tr>
<td>5a</td>
<td>Actual vs. maximum possible predicted frame angle at the critical points</td>
<td>82</td>
</tr>
<tr>
<td>5b</td>
<td>Predicted extent of tipping for the prototype – encountering bumps in forward motion on level ground</td>
<td>88</td>
</tr>
<tr>
<td>5c</td>
<td>Predicted extent of tipping for the prototype – dropping off a curb in forward motion from level ground</td>
<td>89</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>#</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2a</td>
<td>Illustration of wheelchair dimension definitions</td>
<td>6</td>
</tr>
<tr>
<td>2b</td>
<td>Illustration of an instability point and two equilibrium states using a potential energy curve</td>
<td>11</td>
</tr>
<tr>
<td>2c</td>
<td>Point of static instability</td>
<td>12</td>
</tr>
<tr>
<td>2d</td>
<td>General summary of the modeling process</td>
<td>20</td>
</tr>
<tr>
<td>2e</td>
<td>Effect of integration step size on truncation error, round-off error, and ill-conditioning</td>
<td>22</td>
</tr>
<tr>
<td>3a</td>
<td>Photograph of the prototype power chair used to develop the computer model in this thesis</td>
<td>27</td>
</tr>
<tr>
<td>3b</td>
<td>Spatial model of the prototype wheelchair</td>
<td>30</td>
</tr>
<tr>
<td>3c</td>
<td>Wheel stiffness test apparatus</td>
<td>34</td>
</tr>
<tr>
<td>3d</td>
<td>Coefficient of restitution test apparatus</td>
<td>36</td>
</tr>
<tr>
<td>3e</td>
<td>Trigonometric relationships between release angle, rebound angle, and height of pendulum</td>
<td>38</td>
</tr>
<tr>
<td>3f</td>
<td>Free body diagram of the tractor assembly and drive wheels at rest on level ground</td>
<td>39</td>
</tr>
<tr>
<td>3g</td>
<td>Configuration of the seat and backrest cushion model</td>
<td>41</td>
</tr>
<tr>
<td>3h</td>
<td>Human occupant model provided by Working Model 2D</td>
<td>43</td>
</tr>
<tr>
<td>3i</td>
<td>The final integrated model used throughout the remainder of the thesis</td>
<td>46</td>
</tr>
<tr>
<td>3j</td>
<td>Top and side view sketches of the CG platform</td>
<td>48</td>
</tr>
<tr>
<td>3k</td>
<td>Free body diagram of the CG platform in use</td>
<td>49</td>
</tr>
<tr>
<td>3l</td>
<td>A scale recessed into a floor cavity was used to measure the normal force between each of the wheelchair wheels and a level ground surface</td>
<td>51</td>
</tr>
<tr>
<td>3m</td>
<td>Schematic of the tilt platform used to find the static stability angles</td>
<td>52</td>
</tr>
<tr>
<td>3n</td>
<td>The runway and camera setup for the dynamic tests</td>
<td>54</td>
</tr>
<tr>
<td>4a</td>
<td>Results of the wheel stiffness test</td>
<td>57</td>
</tr>
<tr>
<td>4b</td>
<td>Effect of impact speed on coefficient of restitution</td>
<td>58</td>
</tr>
<tr>
<td>4c</td>
<td>Effect of loading on coefficient of restitution</td>
<td>58</td>
</tr>
<tr>
<td>4d</td>
<td>Effect of impact surface on coefficient of restitution</td>
<td>59</td>
</tr>
<tr>
<td>#</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>-----</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4e</td>
<td>Effect of tire pressure on coefficient of restitution</td>
<td>59</td>
</tr>
<tr>
<td>4f</td>
<td>Predicted vs. actual CG location</td>
<td>61</td>
</tr>
<tr>
<td>4g</td>
<td>Actual vs. predicted static stability angles</td>
<td>62</td>
</tr>
<tr>
<td>4h</td>
<td>Matrix of condition used to validate the dynamic predictions of the model</td>
<td>63</td>
</tr>
<tr>
<td>4i</td>
<td>Predicted and actual vertical displacements of front wheel marker (SFF)</td>
<td>64</td>
</tr>
<tr>
<td>4j</td>
<td>Predicted and actual vertical displacements of drive wheel marker (SFF)</td>
<td>65</td>
</tr>
<tr>
<td>4k</td>
<td>Predicted and actual vertical displacements of rear wheel marker (SFF)</td>
<td>66</td>
</tr>
<tr>
<td>4l</td>
<td>Predicted and actual vertical displacements of head marker (SFF)</td>
<td>67</td>
</tr>
<tr>
<td>4m</td>
<td>Predicted and actual horizontal velocity of drive wheel marker (SFF)</td>
<td>67</td>
</tr>
<tr>
<td>4n</td>
<td>Predicted and actual vertical displacements of front wheel marker (LFF)</td>
<td>68</td>
</tr>
<tr>
<td>4o</td>
<td>Predicted and actual vertical displacements of drive wheel marker (LFF)</td>
<td>69</td>
</tr>
<tr>
<td>4p</td>
<td>Predicted and actual vertical displacements of rear wheel marker (LFF)</td>
<td>69</td>
</tr>
<tr>
<td>4q</td>
<td>Predicted and actual vertical displacements of head marker (LFF)</td>
<td>70</td>
</tr>
<tr>
<td>4r</td>
<td>Predicted and actual vertical displacements of front wheel marker (SLF)</td>
<td>71</td>
</tr>
<tr>
<td>4s</td>
<td>Predicted and actual vertical displacements of drive wheel marker (SLF)</td>
<td>71</td>
</tr>
<tr>
<td>4t</td>
<td>Predicted and actual vertical displacements of rear wheel marker (SLF)</td>
<td>72</td>
</tr>
<tr>
<td>4u</td>
<td>Predicted and actual vertical displacements of head marker (SLF)</td>
<td>72</td>
</tr>
<tr>
<td>4v</td>
<td>Predicted and actual vertical displacements of front wheel marker (SFH)</td>
<td>74</td>
</tr>
<tr>
<td>4w</td>
<td>Predicted and actual vertical displacements of drive wheel marker (SFH)</td>
<td>74</td>
</tr>
<tr>
<td>4x</td>
<td>Predicted and actual vertical displacements of rear wheel marker (SFH)</td>
<td>75</td>
</tr>
<tr>
<td>4y</td>
<td>Predicted and actual vertical displacements of head marker (SFH)</td>
<td>75</td>
</tr>
<tr>
<td>4z</td>
<td>Predicted and actual horizontal velocity of drive wheel marker (SFH)</td>
<td>76</td>
</tr>
<tr>
<td>5a</td>
<td>Geometric relationship between predicted and actual wheelbase angles</td>
<td>81</td>
</tr>
<tr>
<td>5b</td>
<td>Comparison of the spring compression during static testing and during dynamic tipping</td>
<td>83</td>
</tr>
<tr>
<td>5c</td>
<td>Comparison of the maximum extent of tipping for the four dynamic tests</td>
<td>84</td>
</tr>
<tr>
<td>5d</td>
<td>Comparing the susceptibility of four vs. six wheeled wheelchairs to tipping when encountering bumps</td>
<td>90</td>
</tr>
<tr>
<td>#</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>-----</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>A2a</td>
<td>Sample illustration of the Working Model environment</td>
<td>104</td>
</tr>
<tr>
<td>A4a</td>
<td>The rise in CG height as a wheelchair on level ground tips forward and reaches the instability point</td>
<td>115</td>
</tr>
<tr>
<td>A4b</td>
<td>The rise in CG height as a wheelchair on a downhill slope tips forward and reaches the instability point</td>
<td>116</td>
</tr>
</tbody>
</table>
# List of Appendices

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix 1 C.S.i.A. Facility</td>
<td>102</td>
</tr>
<tr>
<td>Appendix 2 Introduction to Working Model® 2D</td>
<td>103</td>
</tr>
<tr>
<td>Appendix 3 PEAKS Motion Tracking System</td>
<td>108</td>
</tr>
<tr>
<td>Appendix 4 Derivation of Equation (2)</td>
<td>114</td>
</tr>
<tr>
<td>Appendix 5 Wheelchair Model Data File</td>
<td>117</td>
</tr>
</tbody>
</table>
Chapter 1  Introduction

1.1 Objective
1.2 Scope
1.3 Hypothesis
1.4 Approach

1.1 Objective

Wheelchair stability is a growing concern. Published studies to date have been limited to manual wheelchairs under static conditions. This thesis extends the scope of wheelchair research by specifically addressing dynamic stability in electrically powered wheelchairs, or 'power chairs' for short.

The objectives of this research were to:

1. develop a two dimensional computer model of a particular power wheelchair;
2. validate the accuracy of the model predictions under various static and dynamic conditions;
3. evaluate the model predictions specifically with respect to wheelchair stability;
4. use the model, if possible, to determine the safety limits for forward tipping of that particular power wheelchair;
5. evaluate the effectiveness of computer modeling as a tool in wheelchair design.

1.2 Scope

The wheelchair model developed in this thesis was limited to a two dimensional side view model; instability and tipping of the wheelchair were only considered in the forwards and backwards directions. Therefore, any effects due to lateral motion or rotation of the wheelchair about the vertical axis were not considered. This research is directly applicable to situations in which a power wheelchair is at risk of tipping forwards or backwards due to any one of the following environmental hazards: bumpy ground conditions; indoor or outdoor ramps; curbs;
transitions from the sidewalk to the road and visa versa (referred to as 'curb cuts'); sudden bumps or dips in the pavement; and steep outdoor hills.

The model was based on one particular power wheelchair due to the diversity in power wheelchair designs. As will be discussed in subsequent chapters, there is no generic stability formula that is appropriate for all power chair configurations; each power chair design has unique structural and mechanical characteristics that fundamentally determine how its stability is defined. Hence, we attempted to develop a consistent approach for analyzing dynamic stability through modeling which was independent of any particular design. In this way, the implications of the research would be relevant to all power wheelchairs, regardless of how unconventional their particular designs might be.

1.3 Hypothesis

It was hypothesized that a particular power wheelchair could be modelled in the sagital plane with sufficient accuracy to be both useful and practical in the testing of static and dynamic stability, as well as in the optimizing of design variables. Modeling is one way of dealing with large numbers of design variables in an integrated fashion. It was hoped that the potential benefits of the modeling process (i.e. safe testing methods, visualization of complex motions, ease of manipulating design variables) would sufficiently offset the drawbacks (i.e. the time required to become familiar with software, the effort required to obtain an accurate enough model, the necessity to validate results).

1.4 Approach

A prototype wheelchair in development at the Centre for Studies in Aging (C.S.i.A.) (see Appendix 1) was selected as the wheelchair to be modelled. The software tool selected for the creation of the two dimensional model was Knowledge Revolution's Working Model® 2D v.4.0 [40] (see Appendix 2).

The model was developed by identifying the significant components of the wheelchair-occupant system and representing their geometric, inertial, and mechanical properties within the simulation environment in a simplified form using basic shapes and constraints. The model was
built up stage by stage, as described in Section 3.1, with the focus on keeping the model as simple as possible for the level of accuracy that was deemed necessary. The initial values used for each of these properties were obtained from either direct measurements, empirical data, literature sources, or trial and error. The model was developed through numerous iterations and was slowly revised by adding complexity in areas which required more detailed representation. As a basis for revising the model, both qualitative and quantitative observations were used.

When the model predictions were sufficiently close to the expected results, a series of static and dynamic validation tests (described in Section 3.2) was conducted and used to marginally adjust the initial values of the mechanical properties of the model one at a time, in a logical iterative sequence, until all the properties of the model were set at their finalized, or 'refined' values. The adjustment of previously measured quantities, even if only by a few percent, avoided the need to introduce "black box" components (which would further complicate the model) to make up for differences between the simplified model constraints and the actual wheelchair properties.

To ensure that the refinement of wheelchair properties was not used excessively just to obtain accurate predictions in the early tests, the final wheelchair model was validated over a wide range of dynamic conditions (Section 4.3). The accuracy of the predictions of the final model were then evaluated in terms of forward and rearward stability (both static and dynamic) (Section 5.2). Before the model was used to predict trends or safety limits, we familiarized ourselves with both the limitations of the predictions and the range over which they could be trusted.

Once a level of confidence was established in the model predictions, the model was then used to determine the safety limits for forward tipping for the prototype chair (Section 5.3); these limits could not be determined from laboratory tests due to the danger involved.

Finally, the effectiveness of using computer modeling as a design tool in the development of power wheelchairs was evaluated (Chapter 6).
Chapter 2 Background

2.1 Wheelchair Use in North America

Estimates of the number of wheelchair users vary considerably but there is no question that the percentage of wheelchair users in the United States is increasing annually. With over 1.4 million wheelchair users in 1990, the percentage of the U.S. population relying upon wheelchairs has increased steadily, almost doubling from 0.33 percent of the population in 1980 to 0.57 percent in 1990 [38]. According to other sources, the percentages have increased more moderately, from 0.33 percent in 1984 to 0.42 percent in 1992 [1,12]. In comparison, 0.71 percent of the Canadian population in 1986 were wheelchair users [13]. It is reasonable to suggest that over the past decade, that percentage may have increased as well.

In general, wheelchairs can be categorized into either manually propelled or electrically powered devices (includes scooters). The sources cited above do not specify exact percentages for power chair/scooter users versus manual chair users, but they do suggest that less than half of all wheelchair users utilize power chairs/scooters [13]. Hence, the number of power wheelchair/scooter users can be estimated approximately, as follows:

Using the percentages cited above (0.57% and 0.71%) and the estimated 1998 populations of the United States and Canada (270,000,000 and 30,675,000, respectively [32, 34]), the estimated number of wheelchair users at present in the United States and Canada is about 1.54 million and 218,000 respectively. Hence, the estimated number of power
wheelchair/scooter users at present is at most 770,000 in the United States, and at most 109,000 in Canada. These numbers are in agreement with rough estimates by the authors, based on annual sales over the past decade, which place the number of power wheelchair users in North America at roughly 300,000, and the corresponding number of scooter users at 500,000 [65].

2.2 Wheelchair Characteristics

The characteristics of a wheelchair-occupant system for power wheelchairs are quite distinct from those of manual chairs and scooters. Power wheelchairs/scooters are much heavier, ranging from 55 to 135 kg (87 kg mean mass) as compared to 9 to 20 kg for manual chairs (15 kg mean mass) [21]. Power chairs/scooters reach higher speeds during normal use (9 km/h vs 6 km/h for manual chairs) [15, 16, 31]. Mechanically, power chairs are more complex than both scooters and manual chairs, often incorporating suspension elements within their design. As well, occupants of power wheelchairs usually have far less upper body control than occupants of manual wheelchairs or scooters, who rely on their upper body strength and/or dexterity to propel the wheelchair or steer the scooter. While users of manual chairs and scooters can alter their upper body posture to stabilize the vehicle when performing difficult manoeuvres or negotiating steep slopes [7], most occupants of power wheelchairs do not have this option and cannot stabilize the wheelchair in a given situation. Worse still, depending on the degree of restraint, their bodies tend to move in a way that increases the tendency to tip (e.g. spasticity).

The fact that no single wheelchair design is suited for all wheelchair users [14] has led to an enormous variety of wheelchair designs and configurations. A 1995 survey identified 47 different manual wheelchair designs, each of which with many configurations and adjustments [11]. The overall range of characteristics and configurations for power wheelchairs is even greater. Depending on whether a chair is a front-wheel, centre-wheel or rear-wheel drive design, the centre of gravity position varies considerably, as do the wheel characteristics. Both the centre of gravity location and the wheel properties contribute significantly to the determination of stability, as will be discussed in the following sections.

Some definitions of wheelchair dimensions [45] relevant to this thesis are given in Figure 2a and Table 2a.
Figure 2a: Illustration of wheelchair dimension definitions (refer to Table 2a)

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Definition</th>
<th>Typical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a Backrest angle</td>
<td>angle between plane of backrest and vertical (positive convention for reclined backrest)</td>
<td>5 - 20°</td>
</tr>
<tr>
<td>b Seat-plane angle</td>
<td>angle between plane of seat and horizontal (positive convention for reclined seat)</td>
<td>0 - 10°</td>
</tr>
<tr>
<td>c Legrest angle</td>
<td>angle between legrest and vertical (positive convention for elevated legrest)</td>
<td>10 - 30°</td>
</tr>
<tr>
<td>d Wheelbase length</td>
<td>distance between centres of forward most and rearward most wheels</td>
<td>60 - 75 cm</td>
</tr>
<tr>
<td>e Wheelchair length (not including footrests)</td>
<td>forward most point to rearward most point, not including footrests</td>
<td>80 - 90 cm</td>
</tr>
<tr>
<td>f Wheelchair length (including footrests)</td>
<td>forward most point to rearward most point</td>
<td>110 - 130 cm</td>
</tr>
<tr>
<td>g Footrest clearance</td>
<td>minimum height from footrest to ground</td>
<td>5 - 10 cm</td>
</tr>
<tr>
<td>h Seat height</td>
<td>height of front of seat above ground, not including cushions</td>
<td>46 - 50 cm</td>
</tr>
</tbody>
</table>

Table 2a: Definitions of wheelchair dimensions (refer to Figure 2a) [45]
2.3 Incidence of Wheelchair Accidents

Despite one opinion that manufacturers of power chairs have been conservative in introducing new designs and ideas to the market, regardless of the many innovations which have developed in recent years [23], the majority opinion is that power wheelchairs have advanced tremendously over the past decade, undergoing huge transformations in performance [20, 21]. Since wheelchair users depend on their chairs for mobility and independence, these design improvements have resulted in a higher quality of life for those individuals [9]. However, those same advances in wheelchair technology and accessibility standards that have helped wheelchair users play more of an active role in society have increased opportunities for accidents and injuries [24].

Environmental hazards leading to tips and falls of either the wheelchair, the user, or both have been a leading source of injuries among both manual and power wheelchair users. At the top of the list of environmental hazards are curb-cuts (transitions from sidewalks to roads), curbs (which people attempt to climb over or drop off of when normal access routes are blocked) [10, 18], steep outdoor slopes, and adverse weather conditions [17].

Specific details of wheelchair accidents are often not described in the references. Because of the difficulty in distinguishing between tips of the chair and falls of the person out of the chair, most authors combine data for tips and falls into one category. As well, the distinction between less severe instability incidents, where the wheels lift off the ground for a certain amount of time without tipping the chair, and more severe instability incidents, where the entire chair tips over completely, is often unclear. Hence, the generic term 'instability incident' in this section refers to both categories of tips.

Of particular interest, though, is the prevalence of stability-related accidents serious enough to cause injuries, regardless of whether the injury was caused by a partial tip, a full tip, a fall out of the chair, or a combination of the above. The difficulty in defining wheelchair stability is discussed further in the next section. To further complicate the issue, injury data are often reported with little or no distinction between manual and power wheelchairs. Hence, every effort was made here to differentiate between combined data and power wheelchair data whenever possible. When unspecified, the data in this section refer to both manual and power chairs.
There were an average of 36,000 wheelchair accidents annually in the United States between 1986 and 1992 that resulted in emergency hospital visits [4]. In 1992 alone there were 55,000 such accidents [14]. Included in these figures are an annual average of 51 wheelchair-related fatalities [3]. A number of studies have confirmed that the proportion of reported accidents is equally split between manual and power chairs and that tips and falls are equally prevalent among both classes of wheelchair [22, 24].

Whereas the majority of injuries and deaths in motor vehicle accidents are due to collisions with other vehicles or stationary objects, the majority (50 to 75 percent) of wheelchair accidents resulting in serious (hospital-requiring) injuries are due to tips/falls [3, 5, 24]. In power chairs, half of reported tips/falls are in the fore-aft direction (flipping forwards or backwards over the front or rear wheels) and half are in the lateral direction (rolling sideways over the outer wheels) [24].

According to Gaal's detailed survey of wheelchair accidents [24], 60 percent of forward tips/falls are due to either a sudden transition from downhill to uphill (e.g. at the bottom of a curb-cut) or a sudden bump; only 10 percent of forward tips/falls occur on downhill slopes where no bump or transition is encountered. Since more weight is shifted to the front casters when inclined on a downhill slope, the front wheels are less able to overcome bumps or transitions. As expected, the majority (65 percent) of backwards tips/falls occur on uphill slopes. Clearly then, front and rear anti-tippers have not solved the problem of fore-aft tipping. Kirby found that because of reduced manoeuvrability and the potential for 'getting stuck' on the anti-tipper wheels, wheelchair users often position them ineffectively or remove them entirely from the wheelchair frame [29].

From Kirby's studies of tips and falls [4, 6, 14], he estimated that 5.2 percent of manual wheelchair users were seriously injured in 1994 because of an instability related accident (up from a reported 3.3 percent of wheelchair users in 1990 [3]). This estimate was verified for power wheelchairs by Gaal's 1997 study [24] in which, on average, 13 percent of power wheelchair users reported an instability incident each year, and in one third of those incidents (i.e. about 4.3 percent), medical attention was required.

When one compares the frequency of these injuries with those caused by motor vehicle accidents, the significance of these figures becomes clear. Each year, about 1.2 percent of all people in the United States are injured seriously in motor vehicle accidents (a total of 3,228,000
occupant injuries in 1995 not including pedestrians [33] among 260,000,000 people [32]). The prevalence of serious injuries among wheelchair users due to instability accidents alone is four times greater than the prevalence of serious injuries among vehicle occupants due to motor vehicle accidents. Clearly then, there is adequate motivation to address the issue of wheelchair stability and develop means to design more stable wheelchairs.

2.4 Addressing the Issue of Wheelchair Stability

Over the past decade, the issue of wheelchair stability has become a serious concern among both manual and power wheelchair developers because of the growing population of wheelchair users, the push for higher performance chairs, and the proportion of reported wheelchair accidents resulting from tips and falls. The majority of research in this field to date has focussed on the static stability of manual wheelchairs. In recent years, attention has begun to shift towards dynamic stability and power wheelchairs to some extent.

The inherent stability of a wheelchair-occupant system depends on three key factors, namely:

1. the location of the centre of gravity of the system with respect to the wheels;
2. the number, size and mechanical properties of the wheels; and
3. the presence and configuration of suspension elements and/or anti-tippers.

Each of these factors plays an important role in determining how safely the wheelchair will perform under a variety of situations. To improve the stability of a wheelchair, one may adjust any number of these parameters at the expense of manoeuvrability and/or cost. Often, the extent to which these parameters can be adjusted is also limited by spatial constraints. To cite a few common examples:

- lowering the seat to lower the centre of gravity is limited by the ground clearance required by the footrests;
- extending the wheelbase to improve stability makes the wheelchair more bulky, requiring more room for turns and manoeuvres;
- increasing the diameter of the front caster wheels to help overcome obstacles is limited by interference of the footrests;
using a sophisticated suspension system to provide a safe and comfortable ride may be too expensive;

Incorporating stability into a wheelchair is therefore a matter of trading off design requirements in search of an optimal balance. To achieve this optimal balance, one would typically require mathematical relationships between the above mentioned parameters and stability. For manual wheelchairs, where the number of wheels and lack of complicated suspension systems are consistent from design to design, generic mathematical expressions have been developed to describe stability in terms of wheelchair parameters [5, 7, 9, 29, 30, 48]. These formulae are applicable to all manual wheelchairs regardless of where the seat is, how large the wheels are, where the anti-tippers are positioned, etc. For power wheelchairs, such generic equations have not be developed. The number of wheels, location of wheels, number of suspension elements, and configuration of suspension elements are all variable from one design to the next. The mathematical expressions obtained when studying power wheelchair stability are fundamentally different for each particular design. Hence, manual wheelchair research to date has focussed primarily on understanding stability parameters, while power wheelchair research has focussed primarily on developing minimum stability criteria (i.e. 'pass-fail' performance standards).

The definition of "stability" as it applies to wheelchairs has itself been the subject of much confusion. Ambiguous definitions of instability within the literature range from outright tips where the entire chair flips over, to incidents where one wheel loses contact with the ground even for a moment. At times, the terminology used may be misleading. A particular wheelchair may be more stable (i.e. less likely to tip) than another wheelchair. However, it would be incorrect to generally describe a wheelchair as being 'stable' or 'unstable' without qualifying the statement. A system may be more stable than another, or stable under certain conditions, but not absolutely stable. It is therefore beneficial to establish a fundamental understanding of the terms stability, instability, and instability point before proceeding.

*Stability* is classically defined as describing an object or system which will always return to its starting state if perturbed from equilibrium. *Instability*, on the other hand, implies that an object or system will move to a new equilibrium state if perturbed even slightly from its starting position. Therefore, the *instability point* is that crucial moment at which a system will no longer tend towards its previous equilibrium state but will instead move to a new equilibrium state [35].
Graphically, this is represented by the potential energy curve illustrated in Figure 2b, in which the system (currently at point 'C') will only reach the global equilibrium state (point 'A') if it is given enough activation energy to reach the instability point (point 'B').

![Potential Energy Curve](image)

**Figure 2b:** Illustration of an instability point (point B) and two equilibrium states (points A and C) using a potential energy curve. The system is currently at point C.

When considering a wheelchair in the transverse plane, there are three equilibrium states: the normal upright position, and the two tipped over positions - forwards and backwards. If the wheelchair is tilted to the point that the centre of gravity (CG) of the wheelchair-occupant system is no longer vertically above the "footprint" of the wheelbase (see Figure 2c), the wheelchair will tend to tip over completely. Unless a sufficient restoring moment is applied, tipping will occur as soon as the CG crosses the edge of the footprint. The edge of the wheelbase footprint is in fact the locus of instability points. The location of the wheelchair-occupant CG on level ground with respect to the outer wheels dictates the degree to which that wheelchair-occupant system can be tilted before reaching the instability point (angle $\alpha$ in Figure 2c).

The "footprint" is formed by the base points of the outermost wheels. For a wheel which is locked, such as a geared drive wheel or a locked caster wheel, the base point is the ground contact point. For a wheel which is free to rotate, the base point is the centre of the wheel, since
the chair is free to pivot about that point. The difference between the two scenarios can be illustrated using Figure 2c. A static stability test will yield different results depending on the method used to secure the downhill wheels. If the wheels were secured using a block, the chair would tend to rotate about an axis formed by the wheel centres, and tipping would not be imminent. However, since the wheels are locked and are not free to rotate, the chair will tend to pivot about the ground contact point and tipping is imminent.

**Figure 2c:** Point of static instability. Centre of gravity of wheelchair-occupant system is directly vertical of footprint edge, formed by the ground contact points (both wheels are locked), and tipping is imminent.
2.5 Current Measures of Wheelchair Stability in the Fore-Aft Plane

Based on the understanding of stability developed in Section 2.4, the following fundamental point is clear: *A wheelchair stability measure should give some indication of how close a particular wheelchair comes to its instability point under given conditions.* This approach, albeit specific to wheelchairs, is the suggested method of measuring the stability of a non-linear system with multiple equilibrium states [35].

In the static case, fore-aft stability is measured by the forward and backward tipping angles, which indicate the degree of inclination that the wheelchair can encounter while at rest before tipping over completely. These parameters allow wheelchairs of various designs and configurations to be compared under standard test conditions - the higher the tipping angle, the more *statically stable* the wheelchair. Tipping angles are easily measured and have been used by researchers, designers and manufacturers as a benchmark measure for many years.

In the dynamic case, fore-aft stability can be measured in two ways. In the first approach, the minimum rotational impulse required to tilt a wheelchair beyond its instability point is formulated from dynamic principles. In the second approach, the maximum extent of tipping of a wheelchair is observed during standardized testing and recorded.

Cooper [30] applied the first approach to the specific case of a wheelchair that collides with an obstacle while travelling forward on level ground. Neglecting energy losses and damping (due to suspension elements, motion of the occupant, frame vibration, etc.), the resulting formula gives a conservative estimate of the velocity required for a wheelchair to potentially flip over under these conditions. As long as the wheelchair is kept under the critical velocity, it will not be capable of tipping over completely. The critical velocity for a forward collision on level ground is given by [30]:

\[
V_c = \sqrt{\frac{2gI_o}{My}} \left(\sqrt{1 + \frac{x^2}{y^2}} - 1\right) \tag{1}
\]

where

- \(V_c\) = critical velocity at impact
- \(M\) = mass of wheelchair-occupant system
- \(I_o\) = moment of inertia of system about axis formed by front caster centres
- \(x\) = horizontal distance between CG of system and front caster centres
- \(y\) = vertical distance between CG of system and front caster centres
- \(g\) = acceleration due to gravity
Cooper's formula can be generalized\(^1\) to include the effect of a sloped terrain, yielding the following critical velocity equation for forward impacts:

\[
v_c = \sqrt{\frac{2gI_0}{My}} \left( \sqrt{1 + \frac{x^2}{y^2}} - \cos\theta - \frac{x}{y} \sin\theta \right)
\]

where \( \theta = \) angle of ground incline (\(+\theta\) convention for downhill slope)

\( \) all other variables as described in equation (1)

While these two formulae provide a rough measure of dynamic stability, there are a number of serious limitations that should be noted, including the fact that:

- the moment of inertia of a wheelchair-occupant system is difficult to measure accurately;
- the formula is too conservative because of neglected damping and energy losses;
- the CG and moment of inertia of the occupant with respect to the wheelchair may change significantly when the wheelchair is subjected to large impacts;
- the CG and moment of inertia of the wheelchair itself may change significantly as tipping proceeds if the wheelchair has telescoping components;
- the issue of bumpy ground conditions (i.e. where a resonance effect may be enough to tip the chair at a lower speed than expected) is not addressed.

The second approach involves determining the maximum rotation of a wheelchair during standard testing relative to its instability point. By measuring how close a wheelchair comes to tipping over under a variety of conditions, the dynamic performance of any chair can be assessed in a consistent manner. The resulting scores, or 'instability indices', ranging from 0 to 100 percent, would indicate how well each wheelchair handled each test. A score of 100 percent would indicate that the instability point was reached or exceeded during a test.

This approach avoids both the limitations listed above and those mentioned in Section 2.4 for categorizing the diverse parameters of power wheelchairs. All wheelchairs can be compared on a consistent basis, regardless of their complexity. In fact, the need to relate wheelchair

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\(^1\) See Appendix 4 for the derivation of equation (2), developed by the author
parameters to stability through mathematical expressions is completely avoided. In place of such formulae, dynamic stability tests can be repeated using different wheelchair configurations to empirically determine the relationships between design parameters. The advantage of this simple but robust measure of dynamic stability is its applicability to all wheelchair types and situations.

The principal drawback of this approach is that motion analysis equipment is required to track the wheelchair throughout the tests. To determine the maximum degree of tipping observed, the angular position of the wheelchair in the sagittal plane must be measured at all critical points during the trials (i.e. those points where proximity to tipping is greatest). Motion tracking systems and/or high-speed cameras can be expensive endeavors, both in terms of capital investment and in terms of analysis time. For this reason, no wheelchair research group to date has pursued the use of 'instability indices' as described above, despite the promising advantages.

However, based on this approach Kirby did develop a limited but practical "ordinal scale" [37] to measure the degree of instability observed during testing. The scale, from 0 to 4, describes various degrees of tipping observed visually. A score of 4 ("no tip") indicates that even the uphill wheels did not leave the ground, while a score of 0 ("full tip") indicates that a complete tip occurred. Scores of 3 to 1 indicate the degree to which the anti-tippers came into play during the test (3 = not required at all; 2 = tip hindered by anti-tippers; 1 = wheelchair stuck on anti-tippers). Using this scale, dynamic stability measures can be obtained with much greater speed and ease. The current ANSI/RESNA wheelchair standards (see Section 2.6) use this scale to measure dynamic stability.

While Kirby's scale is both practical and useful, it does have limitations. If the uphill wheels on a six wheeled chair momentarily lift off the ground during a test and the other four wheels remain in contact with the ground, the resulting score of 3 ("transient tip") would incorrectly imply that some degree of tipping had occurred. A consumer comparing wheelchairs by their dynamic test scores would be led to believe that the six wheeled chair is less safe than a four wheeled chair which scored 4 ("no tip") in the same test. Yet in both cases, each wheelchair had four wheels on the ground at all times. Similarly, a consumer would assume that a six wheeled chair which scored 3 in a particular test is no safer than a four wheeled chair which also scored 3; however, the four wheeled chair was left with only two wheels in contact with the ground during the partial tip, whereas the six wheeled chair was left with four.
Therefore, even Kirby's ordinal scale, which is a better measure of dynamic stability than Cooper's formula, yields misleading results for unconventional wheelchair designs.

2.6 Overview of ANSI/RESNA Wheelchair Stability Standards

The first wheelchair standards, developed during the 1960's, were based solely on strength, endurance/durability, controllability and energy consumption, with no consideration given to definitions of stability [30]. Since 1982, the American National Standards Institute (ANSI) and the Rehabilitation Engineering and Assistive Technology Society of North America (RESNA) have been working together to develop more comprehensive wheelchair standards and testing procedures [20]. The ANSI/RESNA Wheelchair Standards Committee was formed with the goal of providing consumers with as much comparative information about wheelchairs as possible. Wheelchair standards have since expanded to include both static and dynamic stability criteria. Although the ANSI/RESNA standards are not obligatory on wheelchair manufacturers, the industry has adopted them as the “golden rule” for competitiveness. These standards have been developing concurrently into international standards through the International Standards Organization (ISO) [28].

In recent years, the ANSI/RESNA wheelchair stability standards have undergone considerable revision, largely as a result of the increased research focus on wheelchair stability. As the stability standards have evolved, more accurate predictors of how stable a wheelchair will be under typical conditions have been obtained. Well-defined restrictions on apparatus and testing methods have reduced the variability of results obtained by different researchers.

Manufacturers must comply with the following apparatus and test method guidelines when testing wheelchairs against the stability standards [8] (in general, all pertinent wheelchair settings during testing should be reported):

- ensure that coefficient of friction between floor and tires is high (~0.75)
- for pneumatic tires, maintain tire pressure at maximum recommended setting
- use standard 100 kg anthropomorphic test dummy (ATD) if possible; otherwise, use additional masses to increase mass of human test subject to 100 kg ± 2 kg
- conduct static tests with caster wheels in forward trailing position (i.e. following position); if caster wheels can be locked, test wheelchair with both locked and unlocked wheels

- conduct static tests using the most stable and least stable user-adjustable settings (i.e. backrest angle, legrest angle, seat adjustment, anti-tipper adjustment)

- conduct dynamic tests using the least stable user-adjustable settings only (i.e. backrest angle, legrest angle, seat adjustment, anti-tipper adjustment, speed, acceleration)

- note any unusual characteristics or settings not in compliance with these guidelines

Indoor ramps range from 5 to 7 degrees but it is not uncommon for wheelchairs to encounter outdoor slopes greater than 12 degrees [20, 30]. The 1998 ANSI/RESNA static stability standard [8] contains no minimum performance requirements for a wheelchair at rest on an incline. Instead, the standard requires that the tipping angles for each wheelchair be determined so that different chairs can be compared to one another. These angles correspond to the point at which the uphill wheels lift off the ground (i.e. no ground contact force) and the point at which the wheelchair is supported solely by the anti-tippers (i.e. the downhill wheels lift off the ground and tipping is imminent). The tipping angles are determined for three wheelchair orientations - facing uphill, facing downhill, and facing parallel to the contour of the incline. Each orientation must be tested in its most and least stable configuration.

With respect to the dynamic stability standard, even the most recent revisions have some limitations. The 1990 dynamic stability standard was written with broad definitions in order to be applicable to all power wheelchairs. The standard [26, 27] consisted of minimum test criteria that a wheelchair had to be able to encounter while in motion without becoming "unstable" (definitions of instability have evolved, c.f. Section 2.4). The problem with this approach was that it only determined whether a power wheelchair passed or failed each test instead of measuring instability. Since the standard was met by most wheelchair designers, clinicians could not use the results to compare or prescribe wheelchairs. The dynamic stability standard lacked a quantitative measure that was comparable to the tilt angles used to measure static stability.

In response to this issue, the dynamic stability standard was recently modified by replacing the minimum performance requirements with the "ordinal scale" described in Section 2.5. This scale classifies the degree of tipping observed during testing according to five visually distinguishable categories. By quantifying the degree of tipping, a basic dynamic stability
measure is obtained for comparing different wheelchairs. The limitations of the scale were discussed in Section 2.5. The 1998 dynamic stability standard [8] uses the scale to measure the forward and rearward stability of power wheelchairs while undergoing the following tests:

Forward dynamic stability tests
- braking abruptly while travelling forward at maximum speed down inclines of 0, 3, 6, and 10 degrees
- making the transition from a downhill incline to a horizontal plane when travelling forward at maximum speed down inclines of 3, 6, and 10 degrees
- encountering sudden steps of 1.2, 2.5, and 5.0 cm while travelling forward at maximum speed on level ground
- encountering sudden drops of 1.2, 2.5, and 5.0 cm while travelling forward at minimum speed on level ground

Rearward dynamic stability tests
- braking abruptly while travelling backward at maximum speed down inclines of 0, 3, 6, and 10 degrees
- accelerating forward from rest up inclines of 0, 3, 6, and 10 degrees
- rolling back slightly after travelling forward at maximum speed up inclines of 0, 3, 6, and 10 degrees and then stopping
- climbing sudden steps of 1.2, 2.5, and 5.0 cm from rest at maximum acceleration

During dynamic testing, all user-adjustable wheelchair parameters should be set to the least stable configuration for the test being conducted. For the forward dynamic stability tests, the seat should be in the most forward position with the backrest upright; for the rearward dynamic stability tests, the seat should be in the most rearward position with the backrest reclined. Regardless of the dynamic test being conducted, the speed and acceleration controllers should be set to their maximum settings, anti-tippers should be in their least effective positions, and the seat height should be set to its maximum recommended setting.
2.7 Overview of Modeling Theory and Numerical Methods

The statement that "all models are wrong, but some are useful" (G.E.P. Box) [63] is a concise overview of the approach to modeling. Models can be useful tools in studying complex systems if care is taken not to make overstated conclusions. As long as the limitations of the model are taken into account, modeling can be a helpful part of the design process. Models help visualize complex motion, safely test new designs, and determine trends caused by changing specific design variables. Modeling reduces the number of prototypes that need to be built and reduces the number of adjustments that need to be incorporated into them. Ideally, the benefit of using modeling is a shorter, cheaper product development cycle.

The modeling process may be summarized in a number of general steps which must be followed if a model is to be useful. These steps include (see Figure 2d):

Step 1: Understanding the system being considered and identifying significant variables
Step 2: Creating the model and testing it qualitatively at each stage of development
Step 3: Verifying the complete model quantitatively against empirical data
Step 4: Verifying the sensitivity of the model to changes in input parameters
Step 5: Using the model to predict unknown scenarios

The first step, understanding the system and identifying parameters of significance, is probably the most underestimated stage. During this stage, the following questions should be addressed: What are the implications of modeling the system in one way and not another? Which aspects of the system are not relevant to the hypothesis and should be neglected? Which variables are most likely to affect the parameter being investigated? Are the assumptions of the study reasonable?

Einstein's modeling philosophy that "everything should be made as simple as possible but not simpler" [63] should be kept in mind when addressing these questions. Models should be developed using simple elements and constraints; complex mechanisms should be represented by simplified linkages that provide the desired function. Extra care should be taken when modeling elements or constraints in the vicinity of applied forces, since oversimplification in these areas will lead to errors that will be propagated throughout the model. One way to test
assumptions regarding the relative significance of variables is to create rough models and determine how changes to individual parameters affect the entire system. By balancing judgement with trial and error, one can develop a model that suits the particular purpose without being unnecessarily detailed [41].

**Figure 2d**: General summary of the modeling process.

In general, modeling is an iterative process, as illustrated in Figure 2d. During each loop of model development (Step 2), the model is refined, tested, and validated qualitatively against expected results. Mechanical properties and other inputs are obtained through empirical testing, engineering texts, previous studies, or trial and error. As the model becomes more sophisticated, its behaviour should begin to resemble that of the system in question. Documentation of minor changes as the model evolves during this stage allows one to revert to a previous model if necessary. Limitations within the modeling environment or within the model itself should be documented as well [2].

The 'final' integrated model must be validated quantitatively (Step 3), even if individual components were validated separately. Validation consists of comparing the predicted simulation data to empirical data gathered under the same test conditions. Evaluation criteria should be established to determine whether the predictions are close enough to be useful or whether another iteration is in order. The sensitivity of the model to a variety of initial conditions should also be validated (Step 4) to ensure that the model is robust enough to be useful for more than just the original set of conditions [41].
Once a level of confidence in the model has been established and the validity of its predictions has been determined, it can then be used for its original purpose (Step 5), namely:

- simulation of untested initial conditions
- determination of relationships between design variables (within specified range)
- determination of safety limits (with caution paid to limitations of model)

It is reasonable to expect a model to yield more accurate predictions as it becomes more refined. However, models which utilize numerical methods will never yield perfect predictions simply due to the fact that numerical methods provide approximations instead of exact solutions. Numerical methods determine finite variable values at discrete times and locations in space in order to simulate continuous systems. Numerical errors can be minimized, but not eliminated entirely, by matching the settings of the numeric algorithm to the particular system being represented [41]. In general, numerical errors can be subdivided into round-off errors, truncation errors, and errors arising from ill-conditioned systems.

Round-off errors arise due to the limited ability of computers to keep track of significant digits during calculations [2]. For extended precision calculations, where the number of significant digits maintained by a computer is increased, the error due to rounding off is reduced. Under normal circumstances these errors are negligible, but with the thousands of calculations performed during computer simulations, round-off errors can accumulate and become significant. As the 'step size' of a simulation (i.e. the time interval considered as one discrete point) is decreased, the number of calculations increases and the round-off error increases.

Truncation errors result from using numerical algorithms to approximate exact mathematical procedures [2]. These errors are usually related to estimating the slope at a point (i.e. evaluating derivatives) or estimating the area under a curve (i.e. evaluating integrals). For example, when evaluating a Taylor Series, only a certain number of terms are evaluated; all other high order terms are neglected. The closer the Taylor Series approaches the function, the more accurate the estimation will be. Therefore, truncation errors can be minimized either by using a more sophisticated algorithm or by reducing the step size of the calculation to "straighten out" the function in the time interval being considered.

Ill-conditioning refers to a system which is naturally prone to irregular or unpredictable behaviour (e.g. physical instability, non-linearity, mathematical instability, etc.). An ill-
conditioned system may produce significantly different results under near-identical initial conditions. Even physically stable systems may unintentionally be transformed into ill-conditioned models (e.g. when division or multiplication by near-zero values causes 'negligible' round-off and truncation errors to be magnified [2]). Examples of ill-conditioned systems include a ball which just barely rolls to the top of a hill, colliding objects with erratic velocities, objects with sharp corners, double pendulums, and a swinging object whose support rope is at the point of failure. When predicting the behaviour of an ill-conditioned system, algorithms will often yield inconsistent results that are highly sensitive to initial conditions and the step size selected. A smaller step size will reduce the errors arising from ill-conditioning [41]. Unstable systems should be modelled with caution and tested to determine the range of reliable predictions.

With the above understanding of how round-off error, truncation error and ill-conditioning are all affected by step size, one can see that some optimal step size exists at which the combined numerical error is minimized (Figure 2e). Hence, simulations should be run with different step sizes to determine the optimal setting.

Using modeling to analyze complex systems is not an exact science, but with responsible error management, it can be a powerful tool in dealing with otherwise unsolvable systems [2].

Figure 2e: Effect of integration step size on truncation error, round-off error, and ill-conditioning (refer to text). At some optimal step size, total numerical error is minimized.
2.8 Human Segmentation Models

Anthropometric data and human segment model data (focusing on various segments of the population and different postures) have been reported by a number of studies [46, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64]. Upon reviewing these references, one will note a great deal of discrepancy in how the body should be modelled, specifically with regard to dividing up the body into segments and assigning mass to each segment. The differences are significant enough that before one can use a model of the human body in a study, one must decide which source is most appropriate for the purpose at hand. If the applicability of two sources are both close but not perfect for the needs of a study, judgement must be used to decide which is most fitting or whether a combination of the sources might be more appropriate.

Models for studying overall body motion begin with a division of the body into the following 15 basic segments:

- head
- neck
- upper arms (2)
- lower arms (2)
- hands (2)
- torso
- upper legs (2)
- lower legs (2)
- feet (2)

As is true with all models, a human model should be only as sophisticated as necessary for the motion being considered. When vertebral forces are not of interest, the head and neck are often combined into one segment [49, 55, 59, 62]. Similarly, the hands and lower arms are often combined into one segment [52, 53, 54, 58, 64]. The major source of contention within the literature is how to model the torso. Whereas some models have considered the entire torso as one segment [46, 49, 50, 51, 55, 59], some subdivide it into three segments (upper thorax region, central abdominal region and lower pelvic region) [52, 53, 54, 62, 64]. Surprisingly, the overall torso mass in proportion to the total body mass is not consistent regardless of which model is used. The source of the discrepancy seems to be in defining the boundaries of the torso, since in models where less mass is attributed to the torso, more mass is attributed to the neck and/or upper legs.
When comparing the segment mass models developed by different researchers, it is useful to normalize the segments by expressing them in terms of the total body mass. Table 2b compares the normalized segment mass data for four commonly cited models: Nigam (1987), Amirouche (1988), Dempster (1955), and Chandler (1975).

*Nigam's geometric model* [64], which evolved from the work of Bartz [52] and Drillis [51], was based on determining segment volumes from the anthropometric dimensions of a 50th percentile male. The mass of each segment was then calculated with the assumption that each segment is elliptical in shape and that the density of the body is uniform. Three torso segments were used in the model.

*Amirouche's anthropometric model* [53, 54] was based on direct segment mass measurements taken from a 50th percentile male anthropometric dummy, part 572. Amirouche's model also made use of three torso segments.

*Dempster's specimen model* [55] was based on measurements taken from 8 male U.S. Air Force cadavers. Dempster's identification of the single torso segment using anatomical landmarks was somewhat unclear. Nevertheless, his model has been cited more than any of the others listed here. *Chandler's specimen model* [59] was also based on male U.S. Air Force cadavers (n = 6) of roughly the same weight and stature.

Comparing the four models (refer to Table 2b), the segments masses within a few percent of each other are the upper arms, lower arms, hands, lower legs and feet. The segments in contention are the head/neck, torso and upper legs. The difference between Chandler's results and Dempster's reveals the difficulty in identifying the neck-torso boundary, even when using the same research method. Amirouche's model emphasizes the difference that can result in the upper leg mass due to different definitions of the torso-upper leg boundary. Finally, Nigam's model is based on the same anthropometric data as Amirouche, yet it is more in agreement with the results of Chandler and Dempster.
Table 2b: Comparison of Segment Mass Models
(all data have been normalized and expressed as a percentage of total body mass)

<table>
<thead>
<tr>
<th>Segment</th>
<th>Nigam(^1)</th>
<th>Amirouche(^2)</th>
<th>Dempster(^3)</th>
<th>Chandler(^4)</th>
<th>(\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head/Neck</td>
<td>4.3</td>
<td>7.2</td>
<td>8.1</td>
<td>6.1</td>
<td>41.66</td>
</tr>
<tr>
<td>Torso</td>
<td>51.1</td>
<td>43.0</td>
<td>49.7</td>
<td>52.2</td>
<td>49.13</td>
</tr>
<tr>
<td>Upper Arms</td>
<td>6.2</td>
<td>6.0</td>
<td>5.6</td>
<td>5.8</td>
<td>0.26</td>
</tr>
<tr>
<td>Lower Arms/Hands</td>
<td>5.1</td>
<td>5.8</td>
<td>4.4</td>
<td>4.6</td>
<td>0.62</td>
</tr>
<tr>
<td>Upper Legs</td>
<td>20.9</td>
<td>26.1</td>
<td>20.0</td>
<td>20.4</td>
<td>22.86</td>
</tr>
<tr>
<td>Lower Legs/Feet</td>
<td>12.4</td>
<td>11.9</td>
<td>12.2</td>
<td>10.8</td>
<td>0.71</td>
</tr>
<tr>
<td>TOTAL</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>99.9</td>
<td></td>
</tr>
</tbody>
</table>

\(^1\) 1987 Geometric model [64]
\(^2\) 1988 Anthropometric model [53, 54]
\(^3\) 1955 Specimen model [55], n=8
\(^4\) 1975 Specimen model [59], n=8
Chapter 3  Method

3.1 Development of Integrated Wheelchair Model

3.1.1 Spatial Model
3.1.2 Wheels
3.1.3 Drive Spring
3.1.4 Cushions
3.1.5 Motors
3.1.6 Human Subject
3.1.7 Integration of Model Components

3.2 Development of Validation Protocol

3.2.1 Static Validation Tests
3.2.2 Dynamic Validation Tests

3.1 Development of Integrated Wheelchair Model

This section details the development of the two dimensional computer model. Following a brief overview of both the simulation environment and the prototype wheelchair that served as the template for the model, each stage of the model development will then be discussed.

As mentioned in Chapter 1, the model was constructed based on a side view of the wheelchair. Only those properties of the wheelchair relevant to fore/aft tipping were modelled. The model was developed through numerous iterations in which the wheelchair was represented in increasing detail and complexity.

The software tool selected for creating the integrated wheelchair model and running the dynamic simulations was Knowledge Revolution's Working Model® 2D v.4.0 [40]. This software was chosen over other dynamic simulation software packages based on considerations of cost, ease of use, recommendations, and availability of trial versions. A pilot study verified that the capabilities of the software were adequate for the purpose of simulating power wheelchair dynamics. That is, the software was sufficient for the level of complexity being modeled. Working Model 2D is a Windows®-based application which simulates the kinematics and dynamics of any number of colliding or non-colliding objects in a two dimensional plane. The software uses a Kutta-Merson numerical integration algorithm (described in Appendix 2) to compute the discrete frame-by-frame movements of all objects in the simulation. A full description of the software features is outlined in Appendix 2.
The power wheelchair selected as the basis for this thesis was a leading edge prototype design developed by the mobility research team at the Centre for Studies in Aging (C.S.i.A.) at Sunnybrook Health Science Centre, in coordination with the Ontario Rehabilitation Technology Consortium (ORTC). The C.S.i.A. focuses on understanding and responding to the needs of elderly people with respect to loss of balance, mobility, and quality of life issues. An overview of the C.S.i.A. facility and research team is included in Appendix 1.

The prototype chair used in this study is illustrated in Figure 3a. This power wheelchair is a central drive six-wheeled chair. Its unconventional design is part of a small but growing family of power wheelchairs which are designed with more than four wheels to improve manoeuvrability without sacrificing stability.

Figure 3a: Photograph of the prototype power chair used to develop the computer model in this thesis. The wheelchair is a centre drive six wheeled chair with freely rotating front and rear caster wheels.
The central drive wheels are located directly below the centre of gravity (CG) of the occupant. In this prototype, the drive wheels are slightly forward of midway between the front and rear wheels. A compressed spring, located within a telescoping shaft below the seat, keeps the drive wheels in constant contact with the ground. The shaft compresses and extends as the frame of the wheelchair moves relative to the drive wheels. The front and rear wheels, which are freely rotating pneumatic caster wheels, are connected directly to the wheelchair frame without any suspension. The method by which each wheelchair component was represented within the model will be discussed in the subsections that follow.

The initial values for the properties of each component were obtained either from direct measurement, empirical data, literature sources, or trial and error. Some values were later refined during the many iterations of model development, thereby improving the accuracy of the predictions. The iterations of model development were conducted in a logical, ordered sequence. Those values which had been assigned using trial and error were finalized based on static and dynamic tests before any measured quantities were adjusted. Measured quantities were adjusted as little as possible and only when necessary to account for discrepancies between the simplified model and the actual wheelchair. For example, the stiffness and coefficient of restitution of the wheels were lowered by 17 percent and 3 percent, respectively, in the final model to account for energy losses in the wheels and frame which were not represented in the model. Rather than adding in additional components to account for these differences and further slow down the model, specific values were adjusted.

The static properties were finalized first, one at a time, based on the measurement of the CG of the wheelchair and the measurement of the normal force at each of the wheels. Using these tests, we were able to assign mass to components which were not directly weighed, position the occupant properly in the seat model, and check the calculation of the initial spring compression. The time-dependent properties were then finalized, one at a time, based on measurements obtained during the dynamic trials. Using these tests, we were able to determine appropriate damping coefficients, which were neither measured nor found in the literature. As well, these tests were the basis for adjusting the motor torque, wheel stiffnesses, maximum spring compression, and coefficients of restitution from their measured values. The initial and refined values, when adjusted, were indicated in each case in the subsections that follow.
3.1.1 Spatial Model

The first stage in developing the computer model was creating a geometric representation of the prototype wheelchair which included all inertial properties. Based on recommendations [41] to keep the number of objects in the model to a minimum, components which did not contribute significantly to the mass of the chair (i.e. less than 6 kg or 5 percent of the unoccupied chair mass) were not modelled precisely. The mass, geometry, and centroid coordinates of each significant component were carefully measured. All 'non-significant' components, aside from the exceptions mentioned below, were combined with adjacent components. The CG of each component was assumed to coincide with its centroid (i.e. its geometric centre). Since many of the components were each uniformly dense, this assumption seemed reasonable. By combining some of the smaller masses together and assuming constant density for individual components, it was expected that the CG, when measured from the front caster, and the moment of inertia of the simplified model would both be within 10 percent of the actual value.

The wheelchair was subdivided into the following components for modeling purposes (refer to Figure 3a): front wheels; front caster fork assemblies; rear wheels; rear caster fork assemblies; drive wheels; tractor assembly; wheelchair frame; batteries; seat assembly (includes cushions, armrests and joystick controller); electronic circuitry; legrests; and additional weight (located under seat cushion to weigh down front casters; not visible in the figure).

Although the wheels did not contribute much to the wheelchair mass, they were carefully modelled, since wheel properties are crucial in determining stability (c.f. Section 2.4). The caster forks were modelled separately, despite their small mass, in order to connect the wheels to the frame. The electronics and legrests were modelled separately (unlike the cushions, armrests and joystick controller) because of their relative large distance from the wheelchair CG.

The two heaviest components, the batteries and tractor assembly, each contributed 34 kg (27 percent) to the total wheelchair mass of 124.9 kg. Unlike the batteries, the tractor assembly was not symmetrically shaped. The tractor was modelled in three rectangular sections - an upper part (representing the clutch and telescopic shaft), a lower part (representing the motors), and a 'motor mount' (representing the connection from the motors to the drive wheels).

---

1 The moment of inertia of an object is related to the square of the distance between each point mass and the total CG (i.e. objects of equal mass may have unequal inertial properties) [39].
Hence, the computer model (shown in Figure 3b) consisted of the fourteen masses, or 'bodies' as referred to in Working Model, listed in Table 3a.

Table 3a: Physical Components of the Wheelchair Model (refer to Figure 3b)

<table>
<thead>
<tr>
<th>Model ref. #</th>
<th>Name</th>
<th>Mass (kg)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body [48]</td>
<td>Front wheel</td>
<td>0.5</td>
<td>represents two wheels</td>
</tr>
<tr>
<td>Body [16]</td>
<td>Front caster fork</td>
<td>3.0</td>
<td>represents two caster forks</td>
</tr>
<tr>
<td>Body [62]</td>
<td>Rear wheel</td>
<td>0.5</td>
<td>represents two wheels</td>
</tr>
<tr>
<td>Body [78]</td>
<td>Rear caster fork</td>
<td>3.0</td>
<td>represents two caster forks</td>
</tr>
<tr>
<td>Body [49]</td>
<td>Drive wheel</td>
<td>0.5</td>
<td>represents two wheels</td>
</tr>
<tr>
<td>Body [61]</td>
<td>Lower tractor</td>
<td>13.0</td>
<td>represents motors, gears</td>
</tr>
<tr>
<td>Body [63]</td>
<td>Upper tractor</td>
<td>18.5</td>
<td>represents telescopic shaft and clutch</td>
</tr>
<tr>
<td>Body [36]</td>
<td>Motor mount</td>
<td>2.65</td>
<td>represents motor connections; used to achieve correct total mass</td>
</tr>
<tr>
<td>Body [50]</td>
<td>Frame</td>
<td>19.0</td>
<td>—</td>
</tr>
<tr>
<td>Body [30]</td>
<td>Batteries</td>
<td>34.0</td>
<td>represents two batteries</td>
</tr>
<tr>
<td>Body [6]</td>
<td>Seat</td>
<td>12.0</td>
<td>represents seat, armrests, joystick &amp; cushions</td>
</tr>
<tr>
<td>Body [57]</td>
<td>Added weight</td>
<td>10.25</td>
<td>used to add load to the front casters; weighed precisely</td>
</tr>
<tr>
<td>Body [3]</td>
<td>Legrest</td>
<td>3.0</td>
<td>represents two legrests</td>
</tr>
<tr>
<td>Body [1]</td>
<td>Electronics</td>
<td>5.0</td>
<td>represents circuitry behind backrest</td>
</tr>
</tbody>
</table>

Figure 3b: Spatial model of the prototype wheelchair (refer to Table 3a).
Table 3b lists the geometric properties of the wheelchair model components. The centre of the front caster wheel (in the trailing position) was selected as the origin of the local coordinate system (positive 'x' pointing to rear of wheelchair; positive 'y' pointing upwards).

Two additional objects were created for the simulations, namely the ground and the test bump, which were fixed in place and assigned arbitrary masses of 100 kg and 10 kg respectively. Testing confirmed that the mass of an anchored object does not affect the simulation in any way. The coefficients of static and kinetic friction and the coefficient of restitution for these two objects were set at the maximum values so as not to interfere with the results of the simulation (Working Model looks for the lowest value when dealing with interacting objects).

It should be noted that Working Model considers all objects as rigid bodies. Deflections of a significant nature must be modeled via springs, dampers, or other constraints. In the present case, deflection of the wheelchair frame was negligible relative to deflections in the drive spring, wheels and cushions. Therefore, neglecting frame deformation was reasonable. The same could not be said for energy absorption due to potential frame rattling (e.g. vibrations at the seat-frame pin connection) which is also neglected by Working Model. In fact, for that very reason, the coefficients of restitution of the wheels were slightly adjusted, as discussed above.

Table 3b: Geometric Properties of the Wheelchair Model (refer to Figure 3b)

<table>
<thead>
<tr>
<th>Name</th>
<th>Shape</th>
<th>Dimensions (all in cm)</th>
<th>(x, y) Centroid Coordinates (in cm from centre of front wheel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front wheel</td>
<td>circular</td>
<td>radius: 9.5</td>
<td>(0.0, 0.0)</td>
</tr>
<tr>
<td>Front caster fork</td>
<td>rectangular</td>
<td>length: 29.0, width: 8.0</td>
<td>(-0.1, 12.5)</td>
</tr>
<tr>
<td>Rear wheel</td>
<td>circular</td>
<td>radius: 9.5</td>
<td>(69.4, 0.0)</td>
</tr>
<tr>
<td>Rear caster fork</td>
<td>V-shaped*</td>
<td>width: 8.0 (see figure)</td>
<td>(65.3, 15.9)</td>
</tr>
<tr>
<td>Drive wheel</td>
<td>circular</td>
<td>radius: 15.9</td>
<td>(33.7, 6.4)</td>
</tr>
<tr>
<td>Lower tractor</td>
<td>rectangular</td>
<td>length: 25.0, width: 15.0</td>
<td>(26.2, 11.4)</td>
</tr>
<tr>
<td>Upper tractor</td>
<td>rectangular</td>
<td>length: 22.0, width: 10.0</td>
<td>(33.8, 24.9)</td>
</tr>
<tr>
<td>Motor Mount</td>
<td>rectangular</td>
<td>length: 20.6, width: 5.0</td>
<td>(30.6, 6.1)</td>
</tr>
<tr>
<td>Frame</td>
<td>triangular</td>
<td>base: 75.0, height: 12.5</td>
<td>(30.6, 30.2)</td>
</tr>
<tr>
<td>Batteries</td>
<td>rectangular</td>
<td>length: 21.0, width: 13.0</td>
<td>(62.5, 34.0)</td>
</tr>
<tr>
<td>Seat</td>
<td>L-shaped</td>
<td>width: 13.0 (see figure)</td>
<td>(30.6, 53.0)</td>
</tr>
<tr>
<td>Added weight</td>
<td>rectangular</td>
<td>length: 17.5, width: 4.0</td>
<td>(14.0, 35.7)</td>
</tr>
<tr>
<td>Legrest</td>
<td>rectangular</td>
<td>length: 50.0, width: 3.0</td>
<td>(-20.1, 18.2)</td>
</tr>
<tr>
<td>Electronics</td>
<td>rectangular</td>
<td>length: 20.0, width: 10.0</td>
<td>(58.7, 72.6)</td>
</tr>
</tbody>
</table>

* Since the rear wheel was shifted during the validation stage, a V-shaped caster fork was used for convenience
3.1.2 Wheels

Preliminary simulations confirmed that wheelchair behaviour under dynamic conditions was highly sensitive to changes in wheel parameters. Hence, a great deal of effort was put into designing tests which would empirically determine the necessary wheel input parameters. The wheel parameters of interest were the stiffness and damping coefficients, the coefficient of friction, and the coefficient of restitution. Since each set of wheels was represented by one wheel in the two dimensional model, the equivalent values for two wheels in parallel were used. Hence, the wheel stiffness and damping were both doubled (c.f. two equal springs or dampers in parallel). The coefficient of restitution remained unchanged, since two wheels in parallel rebound equally high when connected as when separate. Since all six wheels of the prototype wheelchair were pneumatic, it was important to determine the sensitivity of these parameters to tire pressure as well.

The effect of rolling resistance was not taken into account. Rolling resistance is a measure of the energy lost as a wheel deforms while rolling, which is compensated for in a power wheelchair by the batteries. The efficiency of the wheelchair was not of interest in this thesis.

Stiffness and Damping Coefficients

All objects deflect to some extent under static loading depending on the resistance, or stiffness, of the object to deflection in the direction of loading. The stiffness coefficient is the proportionality constant that relates the deflection to the applied force. This relationship can have a linear form (e.g. \( F = kx \) for an ideal spring\(^1\)) or a more complex form (e.g. \( F = kx^2 \)).

An object which, when loaded, deflects at a velocity proportional to the loading exhibits damping. The damping coefficient is the proportionality constant that relates the velocity of deflection to the applied force. Again, this relationship can have a linear form (e.g. \( F = cv \) for an ideal damper\(^2\)) or a more complex form (e.g. \( F = cv^2 \)).

---

\(^1\) where \( F \) is the applied force, \( x \) is the deflection in that direction, and \( k \) is the stiffness coefficient in units of force per distance

\(^2\) where \( F \) is the applied force, \( v \) is the rate of deflection in that direction, and \( c \) is the damping coefficient in units of force per velocity
In the prototype wheelchair, the wheel stiffness represented the degree to which the pneumatic wheels resisted vertical deflection as they bore the wheelchair-occupant load. The approximate stiffness of the caster wheels was determined empirically using the simple load-deflection apparatus shown in Figure 3c. One of the caster wheels at full tire pressure (250 kPa or 36 psi) was loaded through its centre with various masses by means of an inverted caster fork. The masses were placed in a steel pan in increments of approximately 20 kg and hung from the caster fork, up to a maximum load of 51.2 kg (twice the load borne by any of the caster wheels under static conditions). A 5 cm by 4 cm solid steel support bar was used as a reference surface. The fixed supports were close enough that the support bar did not sag. At each stage of loading, the wheel was checked for perpendicularity with respect to the support bar and the height of the wheel centre was measured. The test was repeated with a different set of masses at low tire pressure (165 kPa or 24 psi) up to a maximum of 55.8 kg load. The results were used in the initial wheelchair model. The wheel stiffnesses were later refined during the dynamic validation stage. The results are presented in Section 4.1.

In our model, the wheel damping represented the degree to which the wheels resisted the rate of deflection as they bore dynamic loading. The damping coefficient for a wheel in this situation was not found in the literature, nor were we able to determine it empirically. As a result, it was left as one of the variables to be determined during the dynamic validation stage. The initial estimate for wheel damping (8 Ns/cm) was obtained via simulations in which the wheelchair was dropped from a height of a few millimetres. The resulting oscillations of the wheels for different damping values were inspected for overdamping or underdamping. Since the actual wheelchair would be expected to exhibit one or two such oscillations, the value which resulted in near-critical damping was selected as the initial estimate. After many iterations of model refinement, the final values used in the model for each set of wheels were 12 Ns/cm for high tire pressure (250 kPa) and 10 Ns/cm for low tire pressure (165 kPa).
Figure 3c: Wheel stiffness test apparatus. The wheel is loaded through its centre by hanging masses from the end of an inverted caster fork. The wheel is supported by a solid steel bar.

Coefficient of Friction

At the point of contact between a rolling wheel and the ground there is no relative motion between the wheel and the ground, assuming that slipping does not occur. The coefficient of friction, \( \mu \), defined by \( \mu = F/N \), is a measure of the resistance of an object to sliding along a contact surface. It is an empirical quantity whose value depends on the material properties, geometry and slickness of the contacting surfaces. The static value, \( \mu_s \), determines the force necessary to overcome the initial resistance and initiate sliding, while the kinetic value, \( \mu_k \), determines the force necessary to keep an object sliding at a constant speed. For a pneumatic wheel on a smooth dry surface, the static and kinetic coefficients of friction were listed as 0.85/0.80, respectively, depending on the tire pressure [39]. The upper end of the range corresponds to a lower tire pressure (since more tire surface area is in contact with the ground when underinflated). In the final model, the coefficients used were 0.80/0.75 for the front and rear tire and 0.85/0.80 for the drive wheel, respectively. For the trial in which the front tire pressure was decreased, the coefficients used for the front tire were 0.85/0.80.

---

1 where \( F \) is the friction force and \( N \) is the normal force between contacting surfaces
Coefficient of Restitution

The coefficient of restitution, e, is a measure of the kinetic energy lost during a collision due to plastic deformations. It is a scalar quantity, defined by the ratio of post-impact relative speeds to pre-impact relative speeds between the two objects. That is, \( e = \frac{(v_{\text{bl}2})}{(v_{\text{bl}1})} \), where \((v_{\text{bl}2})\) and \((v_{\text{bl}1})\) are the relative velocities, after and before the collision respectively, of one object with respect to the other. The coefficient of restitution is an empirical quantity whose value depends on the geometries, material properties and speeds of the colliding bodies. A value of 1 implies a perfectly elastic collision in which the objects rebound with the same speed as that with which they collide; a value of 0 implies a perfectly plastic collision in which the two objects move in unison after the collision. For a bouncing wheelchair wheel, where one of the colliding objects is fixed, the coefficient of restitution is simply equal to the ratio of post-impact to pre-impact wheel speed perpendicular to the ground.

The approximate coefficient of restitution for a pneumatic wheel under typical wheelchair conditions was determined experimentally using the carefully calibrated apparatus shown in Figure 3d. One of the caster wheels was fixed to the centre of a pendulum which swung about a supporting axis. The pendulum was counter-balanced such that its centre of gravity coincided exactly with the centre of the wheel. When released from a given height, the wheel collided perpendicularly with the solid concrete pillar at the bottom of its swing arc at a known speed. A board marked off in 1 degree increments was placed behind the pendulum to determine the angles of release and maximum rebound. An axle which extended through the centre of the wheel assisted in reading off these angles. Since the pendulum was motionless for an instant at the point of maximum rebound, the rebound angle could be read visually to the nearest half degree (i.e. \(\pm 0.25\) degrees). The pendulum support axis was lubricated to minimize pin friction. The rest of the apparatus was tightly connected to minimize rattling.

The test was conducted under the following conditions (each of the 60 trial condition combinations was repeated 3 times for averaging, for a total of 180 trials):

- 5 tire pressures ranging from fully inflated (250 kPa) to underinflated (165 kPa)
- 3 impact speeds (0.2, 0.4, 0.6 m/s) corresponding to 5, 10, and 15 degree release angles
- 2 impact surfaces (vinyl tile, plywood) both of which were fastened to the concrete pillar
- 2 loading conditions (32.8, 15.5 kg) corresponding to the front and rear caster static loads
For this experiment, the coefficient of restitution can be expressed directly in terms of the release and rebound angles, as follows. The results of this experiment, and the refined values used in the final model are both reported in Section 4.1.
Since the concrete pillar is fixed,

\[ e = \frac{v_2}{v_1} \]  \hspace{1cm} (3)

where \( v_2 \) and \( v_1 \) are the respective wheel velocities after and before impact. But the velocity of the wheel (at its centre) is at all times perpendicular to the pendulum and is equal to:

\[ v = \omega r \]  \hspace{1cm} (4)

where \( \omega \) is the pendulum rotational velocity and \( r \) is the distance from the wheel centre to axis of rotation. Thus,

\[ e = \frac{\omega_2}{\omega_1} \]  \hspace{1cm} (5)

where the subscripts 2 and 1 denote the post and pre-impact states respectively. Now, neglecting pin friction, rattling and air resistance, the potential energy of the pendulum at its release angle is completely converted into rotational kinetic energy just before impact. That is,

\[ mgh = I_0\omega_1^2/2 \]  \hspace{1cm} (6)

where \( I_0 \) is the moment of inertia of the pendulum about the axis of rotation, and \( h \) is the release height of the pendulum CG with respect to its lowest swing point. Rearranging this equation yields the rotational velocity of the pendulum just before impact:

\[ \omega_1 = \sqrt{2mgh/I_0} \]  \hspace{1cm} (7)

Similarly, the reduced rotational kinetic energy after impact is completely converted back into potential energy at the point of maximum rebound. Hence,

\[ \omega_2 = \sqrt{2mgh'/I_0} \]  \hspace{1cm} (8)

where \( h' \) is the maximum rebound height of the pendulum CG with respect to its lowest point. Therefore, substituting equations (7) and (8) into equation (5) and simplifying yields:

\[ e = \sqrt{h'/h} \]  \hspace{1cm} (9)

Thus, for this experiment the coefficient of restitution can be expressed in terms of the ratio of heights reached by the pendulum CG before and after impact. Since the wheel centre coincided exactly with the pendulum CG, these heights can be described in terms of the rebound and release angles, as illustrated in Figure 3e. Hence,

\[ e = \sqrt{\frac{1 - \cos \theta_2}{1 - \cos \theta_1}} \]  \hspace{1cm} (10)

Finally, equation (10) can be simplified using the identity \( \sqrt{1 - \cos \theta} = \sin(\theta/2) \) [43] to obtain:

\[ e = \frac{\sin(\theta_2/2)}{\sin(\theta_1/2)} \]  \hspace{1cm} (11)
3.1.3 Drive Spring

The suspension elements in the prototype wheelchair consisted of the drive spring, the pneumatic tires, and the seat cushions. Models of the tires and cushions are described in Sections 3.1.2 and 3.1.4 respectively. This section focuses specifically on the pre-compressed central drive spring that was aligned parallel to the telescoping shaft in the tractor assembly. As discussed above, this spring was used to maintain traction between the drive wheels and the ground, regardless of ground conditions.

The spring used in the prototype wheelchair was a SAE6150 (chrome vanadium steel) helical compression spring with a spring constant of 70 N/cm, a free length of 30.48 cm (12.0”), and a minimum length of 15.24 cm (6.0”). To represent this spring in the model, a 70 N/cm spring element was used to connect the upper tractor to the frame along a non-pivoting vertical slot. A rope element and separator element were used to constrain the compression and extension of the spring to its total range of motion. Rope elements do not exert tension until a
maximum prescribed length is reached, at which point they exert as much tensile force as necessary to prevent that length from being exceeded. Separator elements do not exert repulsion until a minimum prescribed distance is reached, at which point they exert as much repulsive force as necessary to maintain that minimum distance.

In order to assign rope and separator lengths, the initial compression of the spring had to be determined. This was estimated using a free body diagram of the tractor assembly and drive wheels at rest on level ground (Figure 3f). Neglecting shaft friction, summing the forces in the vertical direction yields:

\[ R_d = F + W \]  \hspace{1cm} (12)

where \( R_d \) is the ground contact force at the drive wheels

\( F \) is the force being exerted by the drive spring

\( W \) is the combined weight of the tractor assembly and wheels

![Figure 3f: Free body diagram of the tractor assembly and drive wheels at rest on level ground (vertical forces only). By measuring the normal force (\( R_d \)) at the drive wheels arising from the combined effect of the drive spring (\( F \)) and gravity (\( W \)), the compression of the spring can be estimated. The tractor components are rigidly connected.](image)

Since the force exerted by the spring is proportional to its deflection (i.e. \( F = kx \)), equation (12) can be rearranged to yield the initial spring compression, given a measurement of the normal force between the drive wheels and the ground (the ground contact forces were measured for a 100 kg occupant on level ground using a recessed scale in Section 3.2.1):
\[ x = (R_d - mg) / k \]  
(13)

where  \( x \) is the compression of the spring, in cm  
\( R_d \) is the normal force between the drive wheels and the ground, in N  
\( m = 34.6 \, \text{kg} \) is the mass of the tractor and drive wheels  
\( g = 9.8 \, \text{m/s}^2 \) is the acceleration due to gravity  
\( k = 70 \, \text{N/cm} \) is the spring constant

The original and refined values for spring compression, rope slack, and separator clearance are reported in Section 4.1.

When the original spring model was tested, the tractor oscillated endlessly. Friction between the telescopic shaft components had not been taken into account. Unfortunately, despite numerous attempts we were unable to introduce additional frictional elements to the model without overcoming the limitations of the software (i.e. the simulation either failed or slowed down to an unfeasible pace). Therefore, a damper element was used instead. The damper element, placed in parallel with the drive spring, was given an initial value that slightly overdamped the tractor oscillations. This value was left as another of the variables to be fine-tuned during the dynamic validation stage. In the final model, the damping was not changed from its original value of 7.5 Ns/cm.

### 3.1.4 Cushions

Preliminary versions of the wheelchair model did not account for damping in the seat cushion or backrest. Instead, the occupant was rigidly connected to the seat. Consequently, these versions predicted wheelchair oscillations which were much more pronounced than that of the actual wheelchair. The predicted vertical velocities and accelerations were 2 to 10 times greater than the corresponding actual values. Hence, the effect of cushion damping was significant and could not be neglected.

The two cushions (seat and backrest) were simulated using spring-damper elements between the upper legs and seat and between the torso and backrest. Two equal and parallel elements were used in each location, as shown in Figure 3g. The initial placement of elements and stiffness/damping coefficients were selected using trial and error. The damping coefficients were then increased until overdamping was observed (i.e. until oscillations were no longer...
observed when the occupant sat down). In the final configuration of spring damper elements, shown in the figure, the occupant seemed to 'sink back' into the seat in a realistic manner. After numerous iterations of model refinement, the spring/damper coefficients of each element were set to their final values of 400 N/cm and 20 Ns/cm, respectively, with no initial spring compression. Hence the equivalent coefficient for each cushion as a whole were 800 N/cm and 40 Ns/cm, respectively.

Figure 3g: Configuration of the seat and backrest cushion model. Two spring-damper elements were used in parallel to model each cushion. Parameter values were initially determined using trial and error, and were later finalized at 400 N/cm and 20 Ns/cm for each element.

3.1.5 Motors

The prototype wheelchair utilized two DC permanent magnet geared motors with the following individual specifications [44]:


The maximum combined torque that the two motors could apply to the drive wheels was 27.8 Nm. The actual torque at any given moment varied depending on the power setting and the acceleration setting (via two dial controllers), as well as the applied voltage (via the joystick).

The motors were initially represented in the computer model using a 27.8 Nm torque motor element which was connected between the drive wheel and the motor mount. Since the dynamic validation tests would be conducted at a roughly constant speed, the motor element was configured to produce a constant torque whenever the wheelchair velocity was below a specified setting, and turn off once the desired velocity was reached or exceeded. Motor regulation was originally achieved by calculating wheelchair velocity from the rotational velocity of the drive wheels. In the jargon of Working Model, this condition read:

\[
\text{Torque:} \quad 27.8 \text{ Nm} \\
\text{active when:} \quad \text{Body [49] v.r.< x}
\]

which translated to "set the motor torque to 27.8 Nm, unless the drive wheels are rotating at or above x°/s". During the dynamic validation phase it was found that regulation of speed could be improved by using the horizontal velocity of the drive wheel as the basis for motor control, rather than rotational velocity. As well, the motor torque in the model was increased by 26 percent to 35.0 Nm when the predicted velocity was observed to be slower than that of the actual chair by up to 100 cm/s at times. Further increases in motor torque did not further improve the regulation of wheelchair speed in the simulation.

3.1.6 Human Subject

In compliance with the testing methods recommended by the ANSI/RESNA wheelchair standards (see Section 2.6), a 100 kg occupant was used in both the model and the validation stage. A 15 segment anthropometric model was provided by the software. In this model, shown in Figure 3h, the head and neck were considered as one segment, the torso was considered as two
segments (upper torso and abdomen/pelvis), and the hands and feet were both considered individually. The right and left sides of the upper and lower extremities were combined within the software model (i.e. only 9 polygonal segments were provided).

Figure 3h: Human occupant model provided by Working Model 2D. The 9 segments included the head/neck, upper torso, abdomen/pelvis, upper arms, lower arms, hands, upper legs, lower legs and feet. The left and right limbs were combined within the model.

The anthropometric dimensions of the segment model shown above were in agreement with the four models discussed in Section 2.8; the segment masses were not. The segment masses were changed to those of Nigam's geometric model [64]. Nigam's model was chosen over those of Amirouche, Dempster and Chandler for the following reasons:

- The geometric/anthropometric models (Nigam, Amirouche) were more generalizable than those developed using cadavers from specific populations (Dempster, Chandler)
- Nigam's recent model was an evolution of earlier, well-established models
• Amirouche's upper leg mass differed significantly from the other models which were all in close agreement regarding that segment.

• The division of segments provided by the software was visually very similar to the segment division illustrated in Nigam's model.

The masses were input in terms of total body mass so that the mass of the occupant could be adjusted proportionately with ease (power wheelchair users include the entire population spectrum from children to elderly [38]).

Two thirds of Nigam's combined torso mass was assigned to the upper torso; the remaining third was assigned to the abdomen/pelvis. Following Dempster and Chandler, three quarters of Nigam's lower arm/hand segment mass was assigned to the lower arm; the remaining quarter was assigned to the hand. Table 3c summarizes the final segment masses used.

**Table 3c:** Finalized segment masses used for human occupant model (refer to Figure 3h).

<table>
<thead>
<tr>
<th>Model ref. #</th>
<th>Segment Name</th>
<th>Segment Mass (percentage of total body mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body [160]</td>
<td>Head/Neck</td>
<td>4.3</td>
</tr>
<tr>
<td>Body [151]</td>
<td>Upper Torso</td>
<td>33.7</td>
</tr>
<tr>
<td>Body [143]</td>
<td>Abdomen/Pelvis</td>
<td>17.4</td>
</tr>
<tr>
<td>Body [153]</td>
<td>Upper Arms</td>
<td>6.2</td>
</tr>
<tr>
<td>Body [155]</td>
<td>Lower Arms</td>
<td>3.7</td>
</tr>
<tr>
<td>Body [158]</td>
<td>Hands</td>
<td>1.4</td>
</tr>
<tr>
<td>Body [162]</td>
<td>Upper Legs</td>
<td>20.9</td>
</tr>
<tr>
<td>Body [147]</td>
<td>Lower Legs</td>
<td>9.2</td>
</tr>
<tr>
<td>Body [149]</td>
<td>Feet</td>
<td>3.2</td>
</tr>
</tbody>
</table>

As mentioned in Section 2.6, wheelchair tests should be conducted with the occupant staying as still as possible. Therefore, after positioning the human subject model in a seated posture (Figure 3h), rigid joints were used to connect the segment masses at the 8 joints (neck, shoulder, elbow, wrist, hip, greater trochanter, knee, and ankle). This approach was reasonable for the wrist, greater trochanter and ankle joints, since the hands, thighs and feet of a seated subject can be easily controlled. However, the head, upper arms, lower arms, torso and lower legs are harder to control when one is subjected to vibrations. We originally attempted to use pins constrained by rotational spring-damper elements for those joints. However, the additional constraints drastically slowed down the simulation; for the sake of computing time, the joints were connected rigidly in the final model.
3.1.7 Integration of Model Components

As the model developed, it was tested periodically to identify components that were not created or connected properly, thereby minimizing the number of errors that would have to be diagnosed at a later time. Constraint properties were also checked periodically to ensure that the software did not inadvertently change specified parameters (e.g. rope slack, separator clearance, initial spring deflection), as it was prone to do. Each updated wheelchair model was qualitatively tested during simulations to see:

- how well the simulation handled the increasing number of constraints
- which elements were sources of errors or problems
- which variations most/least affected the dynamic performance of the chair

One particular challenge in integrating the components of the model was positioning the occupant in the seat. No suitable data were found for the horizontal and vertical location of the greater trochanter of a seated subject with respect to the seat corner. Approximate values of 30 cm and 52 cm were obtained for the respective horizontal and vertical distances between the greater trochanter and the centre of the front wheels. These values were used in the initial positioning of the occupant, and were later adjusted to 25.0 cm and 51.0 cm based on the measured CG of the occupied chair (Figure 3i). The data file for the final model, including all parameter values, is presented in Appendix 5.
Figure 3i: The final integrated model used throughout the remainder of the thesis.
3.2 Development of Validation Protocol

Validation of the computer model involved two stages:

1. Refining a few specific parameters until the static and dynamic predictions of the model paralleled the actual wheelchair behaviour under similar conditions within a certain tolerance ('Step 3' of the modeling process - see Section 2.7)

2. Testing the final model under different initial conditions to verify the sensitivity of the model to changes in those parameters ('Step 4' of the modeling process)

A 100 kg healthy male test subject was used for the entire series of static and dynamic tests. The exact mass of the test subject was measured before each set of trials. The subject was instructed to remain as still as possible during testing. The backrest angle, seat-plane angle and legrest angle were set to 15°, 0° and 30° respectively throughout the tests (refer to Table 2a for a definition of these terms). The caster wheels were in the trailing position throughout the tests.

The series of static and dynamic tests used to validate the model are described in the following sections. The results of these tests are reported in Chapter 4. The evaluation criteria and implications of the validation tests are discussed in Chapter 5.

3.2.1 Static validation tests

Three static tests (i.e. the wheelchair was motionless throughout the test) were used to verify and refine the basic geometric and inertial properties of the integrated model. Each test was simulated under identical conditions using the computer model. The static tests comprised:

1. Determination of the horizontal CG location with respect to the front wheels
2. Measurement of the ground contact force at each set of wheels on level ground
3. Determination of the static stability angles in the fore and aft directions

Using the first test, the position of the occupant relative to the seat was refined and finalized. Using the second test, the initial compression of the drive spring was refined and finalized. The third and final test, determination of stability angles, was used to check the predictions of the final model near the point of instability. All static tests were conducted at full tire pressure: 250 kPa (36 psi) for the caster wheels and 275 kPa (40 psi) for the drive wheels.
Static Test #1: Centre of gravity location

A platform was constructed to measure the horizontal location of the CG of the wheelchair. The platform, illustrated in Figures 3j and 3k, was essentially a reinforced wooden floor, elevated at its two ends by supports of uneven heights. The platform was leveled ($\pm 0.5^\circ$) by placing a scale under the lower end. The supports were rounded to pinpoint the exact ground contact points. A reference line, marked off along the platform surface, was used to locate the CG of any object placed on the platform.

![Figure 3j: Top and side view sketches of the CG platform (as constructed by the author).](image)
From a free body diagram analysis of the platform in use, the horizontal distance between the CG of any object and the reference line can be determined from the scale reading as follows (refer to Figure 3k):

**Figure 3k**: Free body diagram of the CG platform in use. The object can be placed anywhere on the platform surface. A and X are the vertical reaction forces at the support points; m and m_p are the masses of the object and the platform, respectively. The CG of the object and of the platform are indicated. CG_x is the local platform coordinate, measured from the reference line.

Summing moments about left support point (point A) yields

\[ mgd_1 + m_p gd_2 = Xgd_4 \]  \hspace{1cm} (14)

or

\[ md_1 + m_p d_2 = Xd_4 \]  \hspace{1cm} (15)

where m is the mass of the object whose CG is to be determined  
m_p is the mass of the platform 
g is the acceleration due to gravity  
X is the scale reading in kg  
d_1, d_2, d_3, and d_4 are the dimensions as shown in Figure 3k

Rearranging equation (15) in terms of d_1 yields

\[ d_1 = (Xd_4 - m_p d_2) / m \]  \hspace{1cm} (16)
Now, expressing \( d_1 \) in terms of the local platform coordinate,

\[
d_1 = d_3 - CG_x
\]  
(17)

Finally, substitution of equation (16) into (17) yields

\[
CG_x = d_3 - (Xd_4 - m_Pd_2) / m
\]  
(18)

where \( CG_x \) is the location of the object's CG with respect to the reference line.

The platform mass \( (m_P) \) and the distance between the reference line and the left support point \( (d_3) \) were 22.05 ± 0.05 kg and 84.0 ± 0.1 cm respectively. The values for \( d_2 \) and \( d_4 \) were obtained by calibrating the platform. Masses with precise CG locations were placed at various distances from the reference line. The resulting scale readings were used to obtain \( d_2 \) and \( d_4 \) values of 65.9 ± 1.1 cm and 142.8 ± 0.1 cm, respectively. The calibrated formula for the CG platform was therefore:

\[
CG_x = 84.0 - (142.8X - 1453) / m
\]  
(19)

where

- \( m \) was the mass of the object being tested, in kg
- \( X \) was the scale reading, in kg
- \( CG_x \) was the horizontal distance from the object's CG to the reference line, in cm

For objects between 150 and 250 kg, the margin of uncertainty in Equation (19) was less than 0.5 cm. A ramp was used to drive the wheelchair onto the elevated platform. The wheelchair was positioned such that the front wheel centres were directly over the reference line. Hence, the calculated CG position corresponded to the CG location with respect to the front wheels. The tests were conducted for both the occupied and unoccupied cases.

**Static Test #2: Ground contact forces**

In order to measure the normal force between the ground and each wheel, a scale was placed in a recessed cavity such that the surface of the loaded scale was level with the floor, as shown in Figure 31. The displacement of the scale under a 75 kg load was less than 0.1 cm. The floor surface was level within half a degree. The test subject maintained a relaxed but upright posture.

Each wheel was positioned over the centre of the scale in succession. The six scale
readings corresponded to the normal force between the ground and each wheel. The test was conducted for both the occupied and unoccupied case. Each measurement was repeated with the wheelchair facing the opposite direction on the test surface. The average reading was recorded.

**Figure 31:** A scale recessed into a floor cavity was used to measure the normal force between each of the wheelchair wheels and a level ground surface.

**Static Test #3: Static stability angles**

A tilt platform was used to determine the static stability angles of the wheelchair in the fore and aft directions. An illustration of the platform is shown in Figure 3m. A hand crank allowed angular increments of less than 0.25° up to a maximum 25.0° inclination. The critical angles were measured with a certainty of ± 0.5°. The coefficient of static friction between the plywood surface and the wheels was estimated to be about 0.7 [39]. A 5.1 cm (2.0") high rectangular support block was used to prevent the wheelchair from sliding down the platform; trials for which the block was required were noted. In the simulated trials, the platform was tilted at a speed that was slow enough (less than 1.5°/s) to be considered quasi-static.

Each trial was conducted twice. The subject was instructed not to compensate for the tilt by leaning forward or backward. The trials were conducted in compliance with the ANSI/RESNA standard for determining static stability, described in Section 2.6, with one exception - the wheelchair was tested with the backrest, seat and footrests in midway positions rather than in the least and most stable positions. The purpose of this static test was not to test the prototype using the standards but rather to test the validity of the model at the point of instability.
Since the six wheeled prototype did not utilize anti-tipper wheels, the two recorded angles were the uphill wheel lift-off angle and the tipping angle. The lift-off angle (i.e. the angle at which the uphill wheels lift off the ground) was determined by attempting to pass a piece of paper under the uphill wheels. The tipping angle (i.e. the angle at which a full tip is initiated) was determined with the help of safety straps. Since the drive spring was capable of extending, the uphill wheels rose up to 30 cm above the ramp surface before reaching the tipping point. The safety straps were gradually loosened as the wheels rose off the ground to provide enough slack to not hinder the extension of the spring, without endangering the test subject. Assistants were positioned behind the wheelchair as an added precaution in case of an unexpected tip. No dangerous situations were encountered during the tests.

3.2.2 Dynamic validation tests

The dynamic tests consisted of tracking the motion of four wheelchair markers as the wheelchair drove over a 4.6 cm (1.8") high rounded bump under various test conditions. The path lines formed by markers on the wheelchair were then compared to the predicted path lines.
generated in the computer simulation. Using the dynamic validation data, the time-dependent properties of the wheelchair model (i.e. damping coefficients for the wheels, cushion and drive spring; coefficients of restitution of the wheels; motor torque), were adjusted one by one and finalized over the course of numerous iterations. The original damping values were assigned by trial and error; the adjustments in the coefficients of restitution and motor torque were discussed earlier. It should be clearly noted that even after all the adjustments were made and the mechanical properties of the model were finalized, neither the static nor the dynamic predictions were perfect.

Reflective markers were placed at the centre of each of the left-hand wheels, such that the markers would be visible to the stationary camera as the wheelchair drove by. The fourth marker was placed on the left side of the occupant's head, between the temple and the ear canal. The wheels and head were selected for marker placement because they were closest and furthest from the externally applied force, the ground. As well, by tracking the motion of the head, the overall effect of errors in the model as they propagated up from the wheels could be assessed.

The markers were tracked by a video camera, positioned perpendicular to the plane of motion. The video was digitized using the PEAK5 motion analysis software [19]. The position, velocity and acceleration data for each of the four markers were then calculated using the PEAK5 system. A step by step description of the digitization process is included in Appendix 3.

The bump height was selected based on preliminary trials in which a 2.5 cm (1.0") high rounded bump was found to be too small to produce significant marker oscillations. In those trials, the vertical oscillations were not much larger than the total experimental error of approximately 1.0 cm. A larger bump was required to significantly disturb the wheelchair motion. The bump height was therefore increased to 4.6 cm (1.8"). A wooden beam of circular cross-section (5.1 cm diameter) was cut once along its length to obtain both the desired bump height and a flat bottom surface for fixation to the floor. The bump was attached to the floor using industrial strength tape. Before filming the tests, practice runs were conducted at reduced speeds to ensure that the wheelchair could safely handle the bump.

The runway and camera setup are illustrated in Figure 3n. A 10 m by 1 m indoor area was designated as the runway, with the bump was located roughly at the midway point. Since only one camera was used to film the wheelchair motion, it was imperative to minimize the effect of parallax by positioning the camera as far away from the runway as possible using the
highest possible zoom setting. The camera (JVC model TK-S300, 60 Hz) was placed 5.5 m from the centre of the runway and 1.2 m above the ground. The camera pointed perpendicular to the plane of motion and was leveled in the other two planes. The lens was zoomed until 2 m of the runway were visible within the field of view, with the bump centrally located. At that setting, the wheelchair-occupant system occupied approximately two thirds of the vertical field of view, with sufficient room available to capture vertical oscillations of the head marker. An 'F-stop' of 11 was used during filming.

![Diagram of runway and camera setup](image)

**Figure 3a:** The runway and camera setup for the dynamic tests (aerial view).

Between the two days of filming, the zoom setting was slightly adjusted. Hence, the camera was recalibrated to obtain the correct scaling factor for the second set of data (refer to Appendix 3 for a discussion of scaling factors).

The wheelchair speed in the computer simulation could be set arbitrarily. Therefore, it was not critical that the dynamic trials be conducted at any specific speed. However, it was critical that the wheelchair speed remain roughly constant throughout the trial and roughly consistent between trials. The test zone was found to be sufficiently long for the wheelchair to
reach a preset speed before entering the field of view and safely slow down after leaving the field.

A number of control runs in which no bump was present were recorded. The control runs allowed for an analysis of the total experimental error due to the combined effect of unevenness in the ground, rocking of the wheelchair frame under normal conditions, lack of camera levelness, parallax error, and digitization error.

The dynamic tests were conducted with the following combinations of wheelchair parameters, for a total of eight trial conditions:

- 2 wheelchair speeds: 'full' (2.0 m/s; 7.2 km/hr) and 'half' (1.3 m/s; 4.7 km/hr)
- 2 front wheel tire pressures: 'full' (250 kPa) and 'underinflated' (165 kPa)
- 2 wheelbase lengths: 'long' (80.7 cm) and 'short' (69.2 cm)

Qualitative notes were kept during the dynamic tests to identify trials that should be discarded due to uneven or unsteady driving. Each combination of trial conditions was repeated until at least two good trial runs were observed. A 'good' trial run was one in which the wheelchair speed seemed to be constant prior to impact, the front wheels hit the bump simultaneously, and the wheelchair did not veer to either side after encountering the bump.

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1 In the long configuration, the rear wheels were shifted 11.5 cm further back
Chapter 4  Results

4.1 Wheelchair Model Inputs

4.2 Static Validation Results

4.3 Dynamic Validation Results

4.1  Wheelchair Model Inputs

Wheel stiffness

The method used to estimate the stiffness of each wheel's spring element was described in Section 3.1.2. The results are reported in Table 4a and Figure 4a. In the range tested, the wheel stiffness was found to be nearly constant at high pressure with a value of 603 N/cm. At low pressure, the stiffness was also found to be nearly constant with a value of 511 N/cm. Since each set of wheels in the model represented two wheels connected in parallel, the values used for high and low tire pressure were doubled to 1206 N/cm and 1022 N/cm, respectively. During the validation iterations, these values were lowered to 1000 N/cm and 900 N/cm, respectively, accounting for the fact that the simplified model wheels were not able to deflect horizontally.

Table 4a: Data from wheel stiffness test

<table>
<thead>
<tr>
<th></th>
<th>m (± 0.1 kg)</th>
<th>F (N) = mg</th>
<th>Δx (± 0.05 cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High T.P. (36 psi)</td>
<td>14.5</td>
<td>142</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>25.8</td>
<td>253</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>51.2</td>
<td>502</td>
<td>0.60</td>
</tr>
<tr>
<td>Low T.P. (24 psi)</td>
<td>20.7</td>
<td>203</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>43.3</td>
<td>425</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>51.6</td>
<td>506</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>55.8</td>
<td>547</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Coefficient of restitution

The method used to determine the coefficient of restitution of the wheels, $e$, was described in Section 3.1.2. Using equation (11), the average $e$ values for 60 different collision
The increasing load was plotted against wheel displacement, giving the spring constant value $K$ (where $K = \Delta F/\Delta x$) for the wheel in the range of loading. $R^2$ was the correlation coefficient of the linear slope to the data.

Combinations were calculated from the release angles and maximum rebound angles. For example, when testing the 15.5 kg load condition against the vinyl tile surface at full tire pressure from a 15.0° release angle, the maximum rebound angle observed (average of 3 trials) was 12.8°, corresponding to an $e$ value of 0.85. The margin of error in the determination of $e$ was ± 0.03. The variation in $e$ was smaller than expected. All 60 test combinations yielded $e$ values between 0.80 and 0.90. In particular:

- $e$ was nearly constant (0.85 ± 0.02) at a high impact speed, regardless of other test conditions (see Figure 4b, data pairs were presented in ascending order of $e$ values for convenience of identifying trends in the difference between each pair).
- There was no effect of wheel loading on $e$ (see Figure 4c; pairs arranged in ascending order).
- $e$ values for the vinyl tile surface were only 0.01 to 0.02 higher than those for the plywood surface which rattled slightly upon impact (see Figure 4d; pairs arranged in ascending order).
- There was a demonstrable but weak linear relationship ($R^2 = 0.40$) between tire pressure and $e$, corresponding to a 0.0022 decrease in $e$ per psi decrease in tire pressure (Figure 4e).
The combined results of Figures 4b and 4e suggested that the coefficient of restitution of the wheels in the model be set to 0.87 ± 0.03 to simulate full tire pressure and 0.84 ± 0.03 to simulate low tire pressure. During the numerous iterations of the dynamic validation phase, these values were lowered slightly and eventually set at the following final values:

Front wheel: 0.85 (fully inflated); 0.82 (underinflated)
Drive/rear wheel: 0.82 (fully inflated); 0.79 (underinflated)

**Effect of Impact Speed on Coefficient of Restitution**

![Graph showing the effect of impact speed on coefficient of restitution.](image)

**Figure 4b:** Effect of impact speed on coefficient of restitution. Twenty ordered pairs of trial data (slow impact speed: 1 km/h; fast impact speed: 2 - 3 km/h) were organized in ascending order for ease of comparison. Error bars (± 0.03) were omitted for clarity. As the collision speed increased, \( e \) approached a nearly constant value of 0.85.

**Effect of Wheel Loading on Coefficient of Restitution**

![Graph showing the effect of wheel loading on coefficient of restitution.](image)

**Figure 4c:** Effect of loading on coefficient of restitution. The ordered pairs of trial data (lightly loaded: 15.5 kg; heavily loaded: 32.8 kg) were organized in ascending order for ease of comparison. Each pair of \( e \) values differed by at most 0.03. There was no distinguishable difference between the two sets of data.
Effect of Impact Surface on Coefficient of Restitution

Figure 4d: Effect of impact surface on coefficient of restitution. The ordered pairs of trial data were organized in ascending order for ease of comparison. The vinyl surface yielded $e$ values which were consistently higher than those of the plywood surface. The noticeable outlier (fourth pair from the right) was attributed to human error.

Effect of Tire Pressure on Coefficient of Restitution

Figure 4e: Effect of tire pressure on coefficient of restitution. All 60 $e$ values were plotted against tire pressure, ranging from 20 to 36 psi (many points represent multiple data). A weak linear relationship ($R^2 = 0.40$) was demonstrated between $e$ and tire pressure corresponding to a positive slope of 0.0022 per unit increase in psi.
Initial compression of drive spring

The method used to estimate the initial compression of the drive spring on level ground was described in Section 3.1.3. Using the ground contact force at the drive wheels, measured during the second static test (see Section 3.2.1) to be 1141 N, equation (13) was solved to yield the approximate initial spring compression, \( x \), in cm:

\[
x = \left( 1141 - 34.6 \times 9.8 \right) / 70 \text{ cm} = 11.5 \text{ cm}
\]

The central drive spring was originally modelled as having an initial compression of 11.5 cm with a maximum compression of 15.24 cm (via the separator element) and a minimum compression of 1.5 cm (via the rope element). The ground contact force test was used to fine tune the initial compression to 11.95 cm. During the dynamic validation iterations, the separator clearance was increased slightly (from 3.74 cm to 4.0 cm), resulting in a maximum potential compression of 15.95 cm in the final model.

4.2 Static Validation Results

The test methods for the series of static validation tests were described in Section 3.2.1. Equivalence of test conditions was maintained between each test and the corresponding simulation. The mass of the test subject during the static and dynamic tests was 102.0 ± 1.0 kg.

The simulation predictions reported in this section and the following section were all obtained using the final wheelchair model. That is, the final set of revised inputs (obtained via numerous iterations of model refinement) was used consistently in the final validation loop. The numerical integration settings used to run the simulations (e.g. step size, integration error, number of significant digits etc.) are described in Appendix 2.

Static test #1: Centre of gravity location

The CG of the prototype wheelchair was found to be 39.3 ± 0.5 cm horizontally distant from (i.e. behind) the centre of the front wheels when unoccupied and 30.5 ± 0.5 cm behind the centre of the front wheels when occupied. The corresponding CG of the computer model for the unoccupied and occupied cases was respectively 38.5 ± 0.1 cm (2 percent too far forward) and 30.3 ± 0.1 cm (1 percent too far forward). These results are illustrated in Figure 4f.
**Figure 4f:** Predicted vs. actual CG location. The horizontal CG locations were accurate within 2 percent both when occupied and unoccupied. The model CGs were within 0.2 and 0.8 cm respectively of the actual locations. The error bars did not quite overlap for the unoccupied case.

**Static test #2: Ground contact forces**

The actual and predicted ground contact forces for the three sets of wheels, for both the occupied and unoccupied cases, are presented in Table 4b. The degree of uncertainty in each measurement is indicated. The predictions were all within 0.5 kgf, or 1.0 percent, of the corresponding actual forces. Each prediction was well within the margin of error for that set of wheels.

**Table 4b: Predicted vs actual ground contact forces for each set of wheels**

(in kg of force)

<table>
<thead>
<tr>
<th></th>
<th>Actual (+/-)</th>
<th>Predicted</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unoccupied</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Front wheels</td>
<td>0.0 0.0%</td>
<td>0.0</td>
<td>0.0%</td>
</tr>
<tr>
<td>Drive wheels</td>
<td>107.8 0.2%</td>
<td>107.9</td>
<td>-0.1%</td>
</tr>
<tr>
<td>Rear wheels</td>
<td>17.1 1.2%</td>
<td>17.0</td>
<td>-0.6%</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>124.9</td>
<td>124.9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Actual (+/-)</th>
<th>Predicted</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Occupied</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Front wheels</td>
<td>69.8 0.3%</td>
<td>69.9</td>
<td>0.1%</td>
</tr>
<tr>
<td>Drive wheels</td>
<td>116.4 4.5%</td>
<td>115.9</td>
<td>-0.4%</td>
</tr>
<tr>
<td>Rear wheels</td>
<td>40.7 2.0%</td>
<td>41.1</td>
<td>1.0%</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>226.9</td>
<td>226.9</td>
<td></td>
</tr>
</tbody>
</table>

61
Static test #3: Stability angles

The uncertainty in the measurement of critical angles was ± 0.5 degrees for the actual tilt platform and ± 0.2 degrees for the simulation. For trials in which the critical angles were greater than 16.0 degrees, use of the support block was required to prevent the wheelchair from sliding down the platform. The wheelchair did not tip backwards even at the maximum platform inclination of 25.0 degrees.

The actual and predicted static stability angles in the forward and rearward directions are presented in Figure 4g. The computer model predicted uphill wheel lift off angles within ± 3 degrees of the observed occurrences: 2.8 degrees (17 percent) too late when facing uphill and 1.3 degrees (10 percent) too early when facing downhill. The instability points, or points of imminent tipping, were predicted on the conservative side in both cases: at least 3.1 degrees (12 percent) too early when facing uphill and exactly 4.4 degrees (23 percent) too early when facing downhill. The static model therefore erred on the side of safety since it was less stable than the actual wheelchair, with a margin of safety of at least 12 percent.

![Actual vs. Predicted Static Stability Angles](image)

**Figure 4g**: Actual vs. predicted static stability angles. The point at which the uphill wheels lifted off the platform ("wheel lift") was predicted by the model 2.8 degrees too late when facing uphill and 1.3 degrees too early when facing downhill. The instability point ("full tip") was predicted to the conservative side in both cases by at least 3.1 degrees. The wheelchair did not tip backwards at the maximum platform inclination.
4.3 Dynamic Validation Results

The dynamic validation trials were described in Section 3.2.2. Three wheelchair parameters were varied between two settings each during the dynamic tests, for a total of eight trials. Qualitative and quantitative results from four of those trials are reported in this section (the summary, evaluation, and implications of the results are discussed in Chapter 5):

- Short wheelbase; Full front tire pressure; Full speed (Trial SFF)
- Long wheelbase; Full front tire pressure; Full speed (Trial LFF)
- Short wheelbase; Low front tire pressure; Full speed (Trial SLF)
- Short wheelbase; Full front tire pressure; Half speed (Trial SFH)

The trial corresponding to the least stable configuration, trial SFF (short wheelbase, full tire pressure, full speed) was used as the basis for refining the dynamic properties of the model (i.e. 'Step 3' of the modeling process). Once the model was finalized, the three initial condition parameters (wheelbase, tire pressure, and speed) were varied individually and the predictions were compared to the corresponding data ('Step 4'). Hence, dynamic validation of the model comprised the use of four trials as illustrated in Figure 4h.

Figure 4h: Matrix of conditions used to validate the dynamic predictions of the model. Three wheelchair parameters were varied between two settings each for a total of eight possible combinations. The four trials reported in this thesis are labelled in the figure. Trial SFF was used to refine the model. Trials LFF, SLF and SFH were used to validate the sensitivity of the model to changes in initial conditions.
The actual and predicted vertical displacements of the four markers for trial SFF are illustrated in Figures 4i, 4j, 4k and 4l. In each figure, the rise and falls of that particular marker were compared. The time scales were consistent across the four graphs and from marker to marker. Once the simulations were shifted to begin at the same time as the trials, no further shifting of data was done. Error bars of ± 0.9 cm, illustrated in each graph, represented the overall degree of uncertainty in actual data due to the factors discussed in Appendix 3.

The strengths and weaknesses of the model will become apparent as the various graphs are analyzed. In general, the predicted curves for the drive wheel and rear wheel paralleled the observed data more closely than the curves for the front wheel and head. The figures tell volumes, both qualitatively and quantitatively, about how the wheels and occupant responded as the wheelchair encountered the 4.6 cm high bump. As the front wheels encountered the bump, the large rise and fall of Figure 4i was initiated, followed immediately by secondary displacements of Figures 4k and 4l as the rear wheels and head "kicked" forward. The drive wheels displaced sharply as they drove over the bump due to the compressed spring (Figure 4j), again causing small secondary reactions at the rear wheels and head. Finally, as the rear wheels encountered the bump (the large displacement of Figure 4k), a slightly delayed large reaction was observed at the head. The front wheels and drive wheels seemed to be unaffected by impacts at other wheels.

![Front wheel marker](image)

**Figure 4i:** Predicted and actual vertical displacements of front wheel marker (Trial SFF). Range of uncertainty for actual data was ± 0.9 cm in this and the following three figures. (Refer to text)
The predicted displacements for all four markers were within +3.0/-1.9 cm of the observed data at all times and were often within the error range (± 0.9 cm). The wheel data in particular were never more than 1.0 cm below the actual data during this trial.

The overly high and overly smooth predicted rise and fall of the front wheels (2.0 cm or 50 percent difference in peak amplitude) was consistent throughout the trials. This discrepancy was attributed to the vertical spring element in the wheel model not being allowed to deflect horizontally. In reality, the wheels deformed with components in both the vertical and horizontal axes. Hence, the real wheels absorbed more of the initial impact (causing smaller, more delayed peaks) and rebounded upwards only after reaching the crest of the bump (causing plateau-shaped peaks). This difference was also visible in the drive and rear wheels, but to a much lesser extent (1.2 cm or 35 percent too high and 0.8 cm or 15 percent too high respectively).

The low predicted oscillation frequency of the drive wheels was attributable to using damping to represent shaft friction. Nevertheless, aside from the initial rise and fall, the predicted displacements were within the margin of measurement error. The error in the initial displacement was magnified by the difference in peak amplitudes (discussed above) and the difference in wheelchair speeds at the time of impact of the drive wheels (up to 50 cm/s or 30 percent too low in the simulation, as illustrated in Figure 4m).

Figure 4j: Predicted and actual vertical displacements of drive wheel marker (Trial SFF). (Refer to text)

1 The drive wheels were considerably larger than the front wheels; the rear wheels bore considerably less weight.
The predicted data for the *rear wheels* were within the measurement error margin at almost every point. The predicted peak amplitude was 15 percent above the actual peak amplitude. The secondary rise and fall, initiated by the front wheel impact, occurred earlier in the simulation in direct response to the fact that the front wheel also reacted earlier to the impact. The main impact, on the other hand, occurred slightly later due to the fact that the wheelchair was travelling at a slower average speed in the simulation.

![Rear wheel marker](image)

**Figure 4k:** Predicted and actual vertical displacements of rear wheel marker (Trial SFF). (Refer to text)

The low predicted oscillations of the *head* (1.2 cm or 25 percent difference in peak amplitude) were attributed to the spring-damper elements used to represent the cushions. In the actual trials, the subject was not restricted from bouncing out of the seat. However, the spring-damper elements in the model constrained the relative motion of the occupant in all directions. Hence, the resulting curve was more damped and the peaks were lower.

For this trial, the *peak vertical velocity* and *peak vertical acceleration* of the head marker were (predicted/actual) 0.45/0.68 m/s and 11.4/14.3 m/s² respectively. The *root mean square (RMS) vertical velocity* and *RMS vertical acceleration* of the head marker were (predicted/actual) 0.24/0.38 m/s and 4.9/6.9 m/s² respectively. (RMS values yield an 'average' value for curves which oscillate about an axis.)
Figure 41: Predicted and actual vertical displacements of head marker (Trial SFF). (Refer to text)

Wheelchair velocity during trial SFF

Figure 4m: Predicted and actual horizontal velocity of drive wheel marker (Trial SFF). The average velocities during the trial were 177 cm/s and 186 cm/s respectively. The predicted data were 10 to 30 percent too low as the drive wheels went over the bump (0.30 > t > 0.40) and 10 to 25 percent too low as the rear wheels went over the bump (0.55 > t > 0.65).

The qualitative and quantitative results of the remaining three trials will now be reported. Because of the similarity between data sets, qualitative comments will be kept to a minimum, highlighting only results unique to each data set.
The vertical displacement curves for trial LFF (i.e. long wheelbase) are illustrated in Figures 4n, 4o, 4p and 4q. This set of trial conditions yielded the closest agreement between the data, mainly due to the fact that the average speed of the wheelchair during the trial was only 2 cm/s (1.1 percent) lower in the simulation than during the actual trial. Consequently, the front wheel, drive wheel and head predictions were much improved. Specifically,

- the displacements were within +2.8/-1.4 cm of the observed data at all times
- the front wheel and drive wheel predictions were accurate within ±1.4 cm
- the peak amplitudes were all within 1.1 cm (within 28 percent)

The fact that the predicted period of oscillation of the front wheels was shorter than the actual oscillation (unique to this trial) corresponded to the fact that in this trial only, the wheelchair speed at the beginning of the simulation was too fast (7 percent too high).

Similarly, the time discrepancy between the large oscillation of the rear wheels was attributed to the difference in wheelchair speeds at that point. Although the average predicted speed during the trial was only 1.1 percent too low, the time in question corresponded to the time at which those speeds were most different (30 percent too low).
For this trial, the peak vertical velocity and peak vertical acceleration of the head marker were (predicted/actual) 0.45/0.47 m/s and 8.8/15.7 m/s$^2$ respectively. The RMS vertical velocity and RMS vertical acceleration of the head marker were (predicted/actual) 0.18/0.24 m/s and 3.9/5.8 m/s$^2$ respectively.
The vertical displacement curves for trial SLF (i.e. low front tire pressure) are illustrated in Figures 4r, 4s, 4t and 4u. This set of trial conditions yielded results which were almost identical to trial SFF. In particular, the lowering of the front tire pressure from 36 to 24 psi caused almost no difference in the predicted curves and only slight differences in the actual curves (compare with Figures 4i, 4j, 4k, 4l). The corresponding predicted and actual averages wheelchair speeds during the two trials were nearly identical as well (178 and 188 cm/s in this trial as opposed to 177 and 186 cm/s, respectively). In this trial,

- the displacements were within +2.2/-2.2 cm of the observed data at all times
- the drive wheel and rear wheel predictions were accurate within +1.2/-2.0 cm
- the peak amplitudes were all within 2.4 cm (within 50 percent)

The underinflated front wheels displayed slightly more of a plateau than in trial SFF, reinforcing the observation that the actual wheels were more yielding to deformation than in the simulation, allowing absorption of the initial horizontal impact and causing a subsequent rebound. Consequently, the front wheels were elevated off the ground for a slightly longer time period.
Comparing the rear wheel reactions to those of trial SFF, the peak amplitudes as the rear wheels encountered the bump were higher by 1 cm in both the predicted and actual curves. Regardless of the cause of the more pronounced reaction, both the simulation and the actual trial displayed the same effect (the higher peaks were probably due to the fact that with underinflated front wheels, the rear wheels bore slightly less of the total mass as the front wheels sagged).
same was not true of the resulting head oscillation. The actual peak amplitude increased by 2 cm whereas the predicted peak amplitude was only 0.5 cm higher.

For this trial, the peak vertical velocity and peak vertical acceleration of the head marker were (predicted/actual) 0.46/0.76 m/s and 9.7/11.4 m/s² respectively. The RMS vertical velocity and RMS vertical acceleration of the head marker were (predicted/actual) 0.23/0.36 m/s and 4.6/6.7 m/s² respectively.

**Rear wheel marker**

![Graph showing predicted and actual vertical displacements of rear wheel marker.](image)

*Figure 4t:* Predicted and actual vertical displacements of rear wheel marker (Trial SLF).

**Head marker**

![Graph showing predicted and actual vertical displacements of head marker.](image)

*Figure 4u:* Predicted and actual vertical displacements of head marker (Trial SLF).
The vertical displacement curves for trial SFH (i.e. reduced impact speed) are illustrated in Figures 4v, 4w, 4x and 4y. Of the four trials, the predicted data for this trial were least in agreement with the observed data. Not coincidentally, the average predicted wheelchair speed during this trial was also least in agreement with the average actual speed. The average predicted speed during this trial was 10 percent too low (c.f. 5 percent or less in the other trials), reaching a maximum of 75 cm/s (60 percent) too low at times (c.f. 30 percent or less in the other trials). Even so,

- the displacements were within +3.8/-2.6 cm of the observed data at all times
- the front wheel and drive wheel predictions were accurate within +3.5/-0.8 cm
- the peak amplitudes were within 2.7 cm (within 70 percent)
- the model consistently predicted overly high wheel peaks and low head oscillations

Reducing the speed of the wheelchair at impact affected both the magnitude and shape of the actual data. The peak amplitudes were smaller in this trial than in trial SFF by 0 to 2 cm. The differences in the shapes of the displacement curves (c.f. Figures 4i, 4j, 4k, 4l) were particularly noteworthy. These differences were most apparent for the drive wheels, rear wheels and head between t = 0.3 and t = 0.5 seconds (i.e. as the drive wheels drove over the bump). The predicted data did not resemble the actual data at all during that same time period. The source of the discrepancy became clear upon watching the animated simulation. The drive wheels rolled over the bump far too slowly, allowing the chair to rock forward and the rear wheels to lift off the ground. That observation (see Figure 4z and note the drastically reduced speed at that point) explained many of the discrepancies in the predicted data, namely:

- the unexpected rocking of the front wheels (Figure 4v);
- the overly long displacement of the drive wheels (Figure 4w);
- the unexpected second displacement of the rear wheels (Figure 4x);
- the large time shift in the final displacement of the rear wheels (Figure 4x);
- the unusual shape of the head motion at that point (Figure 4y).
Figure 4v: Predicted and actual vertical displacements of front wheel marker (Trial SFH).

Figure 4w: Predicted and actual vertical displacements of drive wheel marker (Trial SFH).
Figure 4x: Predicted and actual vertical displacements of rear wheel marker (Trial SFH).

Figure 4y: Predicted and actual vertical displacements of head marker (Trial SFH).
Wheelchair velocity during trial SFH

![Wheelchair velocity during trial SFH](image)

**Figure 42**: Predicted and actual horizontal velocity of drive wheel marker (Trial SFH). The average velocities during the trial were 115 cm/s and 128 cm/s respectively (10 percent too low). The predicted data were up to 60 percent too low in the regions 0.3 > t > 0.4 and 0.65 > t > 0.75.

For this trial, the *peak vertical velocity* and *peak vertical acceleration* of the head marker were (predicted/actual) 0.42/0.58 m/s and 9.0/9.9 m/s² respectively. The *RMS vertical velocity* and *RMS vertical acceleration* of the head marker were (predicted/actual) 0.20/0.27 m/s and 4.6/4.8 m/s² respectively.
Chapter 5 Discussion

5.1 Overview of Validation Results

5.2 Evaluation of the Model

5.3 Predicted Safety Limits for Forward Tipping

5.4 Recommendations for Future Work

5.1 Overview of Validation Results

Static Tests:

Both the predicted CG of the wheelchair-occupant system in the horizontal direction and the ground contact forces at the wheels (for the occupied and unoccupied cases) were accurate within 2 percent. The predictions were within the range of measurement error in every case except one.

The predicted uphill wheel lift off angles were predicted within 2.8 degrees of the actual lift off angles. The static stability (i.e. tipping) angles were predicted to be lower than the actual angles by at least 3.1 degrees, or 12 percent too early.

Dynamic Tests:

In general, the predicted vertical displacement curves for the drive and rear wheels paralleled the observed data more closely than the curves for the front wheels and head. In particular,

- the predicted peak wheel displacements were consistently high by up to 2.9 cm;
- the front wheels exhibited flattened peaks which were not predicted by the model;
- the predicted drive wheel curves were consistently overly damped;
- the predicted peak head displacements were consistently low by up to 2.5 cm.

The predicted RMS vertical velocities of the head were consistently low by 25 to 37 percent. The predicted RMS vertical accelerations of the head were consistently low by 5 to 33 percent.

1 The predicted CG when unoccupied was 2 mm outside of the measurement error range.
The predictions were most accurate when the horizontal speed of the wheelchair at any particular moment paralleled the actual speed at that same moment. Many discrepancies between the time scales and shapes of the displacement curves corresponded to large differences in the predicted and actual wheelchair speeds at those moments. The average predicted wheelchair speeds for the four trials ranged from 1 percent to 10 percent too low. The maximum difference observed during a given trial ranged from 30 percent to 60 percent too low.

The vertical displacement curves were more sensitive to changes in the speed of the wheelchair at impact than to changes in either the wheelbase length or the front tire pressure. The accuracy of the simulation improved when the wheelbase was lengthened, worsened when the impact speed was lowered, and neither improved nor worsened when the front tire pressure was lowered.

In the most accurate simulation (LFF):

- the vertical marker displacements were at all times accurate within +2.8/-1.4 cm;
- the front wheel displacements were at all times accurate within +1.4/-1.4 cm;
- the peak wheel displacements were at most 1.1 cm (up to 28 percent) too high;
- the vertical velocity of the head marker (RMS/peak) was 25/4 percent too low;
- the vertical acceleration of the head marker (RMS/peak) was 33/44 percent too low.

In the least accurate simulation (SFH):

- the vertical marker displacements were at all times accurate within +3.8/-2.6 cm;
- the front wheel displacements were at all times accurate within +3.5/-0.8 cm;
- the peak wheel displacements were at most 2.9 cm (up to 70 percent) too high;
- the vertical velocity of the head marker (RMS/peak) was 26/28 percent too low;
- the vertical acceleration of the head marker (RMS/peak) was 5/9 percent too low.

5.2 Evaluation of the Model

Evaluation of static predictions

The results of the first two static validation tests clearly indicated that the geometric and inertial properties of the wheelchair-occupant system had been modelled with sufficient accuracy. Furthermore, by obtaining highly accurate results in both the CG test and the ground contact force test, a strong level of confidence was established in more than just the spatial
model. For a four wheeled wheelchair, the correct positioning of the CG automatically implies a proper static distribution of load between the front and rear wheels. However, for a six wheeled chair this is not the case. Because there are three sets of wheels, the system is statically indeterminate. One cannot predict the load that will be borne by the wheels simply from knowledge of the CG position. Rather, one must know how each set of wheels will deflect as the load on it is increased. In the case of the prototype wheelchair, this depends on the stiffness of the wheels, the spring constant of the drive spring, the initial compression of the drive spring, and the ratio of tractor assembly mass to total wheelchair mass. By obtaining excellent results in both tests, we were able to confirm that these parameters, in addition to the geometric and inertial properties, had been modelled with appropriate values.

The static stability test revealed the first limitation of the model, namely that the damper element used to simulate shaft friction caused the model to be overly prone to tipping under static conditions. The wheelchair tipped at least 3.1 degrees too early when facing uphill and exactly 4.4 degrees too early when facing downhill. In reality, friction in the shaft held back the compressed spring for a few extra degrees from tipping over the chair until the tilt angle was high enough that the friction force was overcome. In the model, when the critical angle was approached, the damper was able to slow down the rate of tipping but not stop it completely. As discussed in Chapter 3, we were unable to model the shaft friction feasibly using sliding surfaces and opted for the damper element as the next best option (no mention was made of modeling shaft friction in the Working Model literature). Hence, the final model was less stable than the actual wheelchair under static conditions. There was an inherent safety factor in the static stability of the model of at least 3.1 degrees when facing uphill and exactly 4.4 degrees when facing downhill. While more accurate static predictions would be preferable, it was reassuring to know that the error was on the side of safety.

Evaluation of dynamic predictions

The discrepancies between the head marker data were attributed to the fact that the occupant did not wear a shoulder restraint during testing (the prototype was not equipped with one), whereas the cushion model restrained the occupant in both the upwards and downwards directions. As a result, the errors in the predicted RMS vertical velocity and acceleration of the head were consistently low by $31 \pm 6$ percent and $19 \pm 14$ percent, respectively.
The discrepancies between the predicted wheel data and the actual data were mostly attributed to differences between the simulated and actual instantaneous speeds of the wheelchair. In general, as the predicted speed approached that of the actual speed at any given moment, the error decreased. The other discrepancies regarding the wheel data were attributed to either the front wheel model (vertical stiffness components only, as opposed to both vertical and horizontal components) or the use of damping to represent shaft friction as discussed above.

The results for the wheel data, summarized in the previous section, were used to analyze the validity of the model predictions for forward motion. This analysis and its implications will now be discussed, specifically in regard to:

- the accuracy of the maximum extent of tipping predicted by the model
- the sensitivity of the model accuracy to changes in initial conditions
- the range over which the accuracy of the predictions was demonstrated

Predicted extent of tipping

The overall accuracy of the dynamic predictions of the model was gauged in terms of the maximum angle of the wheelchair frame, relative to the horizontal, at the critical points. The critical points were those points at which the wheelchair frame was tilted furthest back or furthest forward. Visual inspection of the simulations confirmed that the two critical points in each trial were the moment at which the vertical displacement of the front wheels was greatest and the moment at which the vertical displacement of the rear wheels was greatest.

Using the vertical displacement curves of the front and rear wheels presented in Section 4.3, including the measurement error and repeatability error in the actual data (described in Appendix 3), it was possible to calculate the maximum error in the predicted frame angles at the critical points.

The front wheel peak displacement predictions were between 0.8 and 2.9 cm too high throughout all four trials. At those points, the predicted rear wheel displacements were between 1.0 cm too high and 0.9 cm too low. Hence, the maximum predicted height of the front wheels above the rear wheels at the first critical point ranged from -0.2 to 3.8 cm too high.

The rear wheel peak displacement predictions were between 0.3 and 1.5 cm too high throughout all four trials. At those points, the predicted front wheel displacements were between
0.5 cm too high and 1.0 cm too low. **Hence, the maximum predicted height of the rear wheels above the front wheels at the second critical point ranged from -0.2 to 2.5 cm too high.**

Since the base of the wheelchair frame was parallel to the imaginary wheelbase line, these results could be used to calculate the maximum difference between the predicted and the actual peak frame angles, as follows. Referring to Figure 5a, where \( P \) is the predicted location of the higher wheel relative to \( L \), the lower wheel, and \( A \) is the location of the actual wheel,

\[
\theta_P = \sin \left( \frac{y_P}{69.2} \right) \tag{20}
\]

or

\[
\theta_P = \sin \left( \frac{(y_P - y_A)}{69.2} + \sin \theta_A \right) \tag{21}
\]

where \( \theta_P \) is the predicted wheelbase (and frame) angle at the critical point 
\( \theta_A \) is the actual wheelbase (and frame) angle at the critical point 
\( y_P \) is the predicted height of the higher wheel relative to the lower wheel 
\( y_A \) is the actual height of the higher wheel relative to the lower wheel

![Figure 5a: Geometric relationship between predicted and actual wheelbase angles. The subscripts \( P \) and \( A \) refer to the predicted and actual dimensions of the higher, respectively, \( L \) refers to the lower wheel, and \( \theta \) is the wheelbase angle. The wheelbase length is constant (69.2 cm).](image)

Replacing \((y_P - y_A)\) in equation (21) with 3.8 cm and 2.5 cm (for partial tips in the rearward and forward directions respectively), and solving for angles ranging from 0 to 25 degrees yields Table 5a, which illustrates the maximum possible difference between the actual
and predicted peak frame angles. The predicted peak frame angles were up to 3.5 degrees too high for rearward partial tips and up to 2.3 degrees too high for forward partial tips in the range calculated (using a value of -0.2 cm in equation (21) yields a minimum difference between the predicted and actual extent of tipping of -0.2 degrees; in other words, the predicted extent of tipping could only be up to 0.2 degrees too low at either critical point).

Table 5a: Actual vs. maximum possible predicted frame angle at the critical points [calculated from equation (21)].

<table>
<thead>
<tr>
<th>Actual peak frame angle at either critical point (degrees)</th>
<th>Maximum possible frame angle prediction: rearward tilt (degrees)</th>
<th>Maximum possible frame angle prediction: forward tilt (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.1</td>
<td>2.1</td>
</tr>
<tr>
<td>5</td>
<td>8.2</td>
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<td>23.4</td>
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</tr>
<tr>
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<td>28.5</td>
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</tr>
<tr>
<td>Difference within:</td>
<td>3.5 degrees</td>
<td>2.3 degrees</td>
</tr>
</tbody>
</table>

In trial SFF, the predicted frame angle reached a peak value of 4.5 degrees (tilted backward) when the front wheels were most elevated and 5.3 degrees (tilted forward) when the rear wheels were most elevated. Based on the margins of uncertainty established above, the predicted frame angles at those points were up to 3.2 and up to 2.1 degrees too high respectively, and up to 0.2 degrees too low. That is, the maximum extent of tipping was predicted with a 100 percent confidence interval of ± 1.7 degrees for the rearward partial tip and ± 1.15 degrees for the forward partial tip.

For most wheelchairs, the maximum frame angle reached during a partial tip can be compared directly to the forward or rearward static stability angle for that wheelchair. If the partial tip proceeds beyond the static stability angle, a full tip will occur (unless a strong enough stabilizing moment is applied, e.g., accelerating quickly during a forward tip or braking quickly during a rearward tip). Therefore, knowledge of the predicted extent of tipping during a given test and of the static stability angle in that direction will yield a predicted tipping index.

For this particular wheelchair, the same cannot be said. In the static stability tests, the drive wheels were supported by the platform and the spring was almost fully compressed. At the
critical points of the simulations, however, the drive wheels were level with the ground and the spring was not nearly as compressed (see Figure 5b). As well, the CG was shifted 1.5 cm downward along the line of action of the spring. Therefore, the static stability angles did not represent the dynamic instability points for this wheelchair. This was confirmed by the simulations of Section 5.3 in which the maximum frame angle far exceeded the static stability angle without a full tip occurring. Hence, the extent of tipping predictions could be used to compare how well the prototype would handle different tests, but not to calculate a tipping index.

Figure 5b: Comparison of the spring compression during static testing and during dynamic tipping. During static stability testing (upper illustration), the drive wheels are supported by the platform at the point of imminent tipping. During an equally severe rearward partial tip while in motion (lower illustration), the drive wheels remain in contact with the ground and a full tip is not imminent (due to the reduced spring compression and lowered CG).
Sensitivity analysis: Dynamic stability

A comparison of the predicted peak frame angles for all four trials is illustrated in Figure 5c. The figure describes the relative extent of tipping in more detail than would the 'ordinal scale' discussed in Chapter 2, which would simply yield a value of 3 ("transient tip") for all four trials. Although the peak frame angles could have been measured and compared directly from the video footage, the entire purpose of the validation stage was to establish confidence in the model predictions so that the simulations could be used to test and compare new scenarios.

![Comparison of Peak Frame Angles During Dynamic Trials](image)

**Figure 5c:** Comparison of the maximum extent of tipping for the four dynamic tests. The margins of uncertainty for trial SFF using a 100 percent confidence interval overlapped with those of the other three trials.

Since the margins of uncertainty in the peak frame angle for trial SFF overlapped those of the other three trials for both forward and rearward tipping, it could not be conclusively stated from Figure 5b alone that the dynamic stability of the chair improved or worsened as a result of the changes. The results did, however, support certain observations made in Chapter 4, namely:

- that the wheelchair seemed to be more dynamically stable in the long wheelbase configuration (LFF), as expected;
- that the front tire pressure did not significantly affect the stability of the wheelchair when encountering obstacles in the range of speeds tested, unlike what we had expected;
that significantly reducing the impact speed (SFH) did not significantly reduce the extent of tipping of the wheelchair - the wheelchair was equally (if not more) stable at high speed.

**Sensitivity analysis: Model accuracy**

The confidence intervals established for the prediction of both vertical wheel displacement and extent of tipping were valid for the entire range of conditions tested. The magnitude of these intervals could have been reduced by limiting future use of the model to certain conditions only. For example, at speeds above 1.7 m/s or 6 km/h (i.e. the three higher speed trials), the regulation of wheelchair speed in the simulation was more smooth as the bump was encountered. Consequently, the maximum differences between the predicted and actual wheel data were smaller. The resulting 100 percent confidence intervals for those trials, obtained via the analysis method described above, were ± 1.4 degrees (18 percent smaller) and ± 1.05 degrees (9 percent smaller), respectively. The same argument could be made to further limit use of the model to the long wheelbase configuration at high speeds. Based on the displacement data of trial LFF, the resulting 100 percent confidence intervals were ± 1.0 degrees (an additional 23 percent smaller) and ± 1.05 degrees (no additional reduction), respectively.

Nevertheless, it was preferable to allow the model to be used over the entire range of conditions. A robust model, which can be used to test a wide variety of scenarios, is much more useful as a design tool than one which is limited. Therefore, it was necessary to rely upon the original, larger confidence intervals of ± 1.7 degrees and ± 1.15 degrees, respectively.

**Validated range of model predictions**

In summary then, the dynamic predictions for the prototype wheelchair were validated (within the 100 percent confidence intervals described above) for the following range of conditions:

- forward motion between 1.3 m/s (4.7 km/h) and 2.0 m/s (7.2 km/h)
- the full range of realistic tire pressures, from underinflated to fully inflated
- any wheelbase length between 69.2 cm and 80.7 cm

It should be noted that running simulations in which these ranges are exceeded does not necessarily imply that the predictions will have a greater margin of error. What has been
established is simply that the confidence intervals for the model were not validated beyond these ranges. In fact, it appeared that the model predictions would be *more accurate* as the speed of the wheelchair increased and *more accurate* as the wheelbase length was increased. However, since one can only guess as to how long these trends would continue, one must be cautious in interpreting predictions obtained beyond the validated range. For example, at some point the increased speed might cause the tires to blow out upon impact, which would not be predicted by the model. Similarly, at some point the impact might be sufficient to throw an unrestrained subject out of the chair, which would also not be predicted by the model. Therefore, the limitations of the model were:

1. the bounds within which its predictions were validated;

2. the margins of uncertainty established above, within which the precise head velocity, head acceleration, wheel displacements, and extent of tipping could not be determined;

3. the judgement that would have to be exercised if the model were used to run simulations outside of those bounds.

### 5.3 Predicted Safety Limits for Forward Tipping

The validated model was used to study the dynamic stability and predicted safety limits of the prototype wheelchair for:

- forward collisions with a fixed obstacle while travelling on level ground
- dropping off a curb while travelling forward on level ground

Both of these scenarios are included in the current dynamic stability standard described in Section 2.6. As well, each of these scenarios is associated with a high proportion of reported forward tips and falls, as described in Section 2.3.

In both sets of tests, the footrest was set to not collide with the ground so that it would not mistakenly act as a stabilizer when the wheelchair tipped forward. In reality, the footrests may act as either stabilizers or tip instigators, depending on whether they scrape against the ground, dig into a soft surface, or catch against a rough surface. Since the footrests did not come into contact with the ground during the validation tests, their lower contour was not modelled in great
Hence, the following simulations were conducted as if the footrests were elevated.

The purpose of the first test was to determine the safety limit of the prototype upon encountering a raised fixed obstacle, such as a curb, speed bump or transition in sidewalk height. Trial SFF was used as the baseline reference. Three test parameters were varied within the maximum limits which might reasonably be encountered in such a scenario:

1. the mass of the occupant (increased 50 percent to 150 kg; decreased 50 percent to 50 kg)
2. the speed at impact (increased 25 percent to 2.5 m/s; decreased 35 percent to 1.3 m/s)
3. the size of the bump (increased 50 percent to 7.1 cm high; no need to decrease bump size)

The minimum speed tested was limited to 1.3 m/s, corresponding to trial SFH, beyond which the model predictions were not validated. The bump height was not increased beyond 7.1 cm; at that height, the wheelchair was no longer able to overcome the bump at low speed.

The results are reported in Table 5b. The error margins were +0.2/-3.2 degrees for the rearward partial tip and +0.2/-2.1 degrees for the forward partial tip. Simulations in which the extent of tipping (either fore or aft) was significantly different from trial SFF were indicated. In no cases did the wheelchair approach the point of complete tipping. The wheelchair was slightly more stable for the heaviest occupant and significantly less stable for the lightest occupant.

Increasing the impact speed yielded mixed results - the wheelchair tipped slightly less as the front wheels impacted and significantly more as the rear wheels impacted. Decreasing the impact speed also yielded mixed results, but with the opposite effect - the wheelchair tipped slightly more as the front wheels impacted and slightly less as the rear wheels impacted.

Increasing the size of the bump to 7.1 cm only slightly increased the extent of tipping at high speed (i.e. 2.0 m/s) but significantly affected the performance of the wheelchair at low speed (i.e. 1.3 m/s or less). At low speed, the obstacle-climbing limit of the wheelchair had been reached; the wheelchair was unable to overcome the bump, with or without a running start. Even with a running start, however, the ensuing large forward partial tip of 13.9 +0.2/-2.1 degrees did not approach the point of a complete tip.

The purpose of the second test was to determine the safety limit of the prototype upon suddenly dropping off a steep curb. For this scenario, the baseline reference trial was a 10.0 cm drop at slow speed (1.3 m/s). Again, three test parameters were varied within the maximum limits which might reasonably be encountered in such a scenario:
Table 5b: Predicted extent of tipping for the prototype - encountering bumps in forward motion on level ground.

<table>
<thead>
<tr>
<th>Simulation conditions</th>
<th>Extent of rearward/forward tipping (+0.2°, -3.2°/-2.1°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference trial: 100 kg, 2.0 m/s, 4.6 cm bump</td>
<td>4.7/5.3</td>
</tr>
<tr>
<td>- heavier subject: 150 kg</td>
<td>2.8/4.5 (slightly more stable)</td>
</tr>
<tr>
<td>- lighter subject: 50 kg</td>
<td>8.5*/8.4* (less stable)</td>
</tr>
<tr>
<td>- higher impact speed: 2.5 m/s</td>
<td>3.5/8.9* (mixed results; see text)</td>
</tr>
<tr>
<td>- lower impact speed: 1.3 m/s</td>
<td>6.0/4.5 (mixed results; see text)</td>
</tr>
<tr>
<td>- bigger bump: 7.1 cm</td>
<td>4.9/6.6 (slightly less stable)</td>
</tr>
<tr>
<td>- bigger bump/lower speed: 7.1 cm, 1.3 m/s</td>
<td>1.1/13.9* (unable to overcome bump; no tip)</td>
</tr>
</tbody>
</table>

1. the mass of the occupant (increased 50 percent to 150 kg; decreased 50 percent to 50 kg)
2. the speed prior to the drop (increased 50 percent to 2.0 m/s; speeds below 1.3 m/s not tested)
3. the size of the drop (increased 100 percent to 20.0 cm; no need to decrease drop)

The baseline speed in this test was lowered from that of the previous test; a wheelchair would not normally drop off a curb even at half speed. The speed corresponding to the low end of the validated range (i.e. 1.3 m/s or 4.7 km/h) was selected for the reference trial. The magnitude of the drop was not increased beyond 20.0 cm (7.9") since at the higher speed, the wheelchair almost reached the point of tipping over during the simulation.

The results are reported in Table 5c. The maximum extent of rearward tipping was less than 2.0 degrees in all six trials; those results were therefore not reported. The error margins in the final two trials (20 cm drop) were +0.2°/-2.2 and +0.2°/-2.3 degrees respectively, due to the larger predicted angles (c.f. Table 5a in Section 5.2). Simulations in which the extent of forward tipping was significantly different from the reference trial were indicated. In the final trial, the wheelchair reached the verge of tipping forwards completely but did not tip over.

The extent of tipping during the curb drop was significantly higher for the heaviest occupant and significantly lower for the lightest occupant. As the wheelchair rotated forward and the front wheels dropped off the curb, the rider mass increased the angular momentum of the chair, increasing the extent of forward tipping that occurred before the front wheels could stabilize the chair. In the previous test, the increased rider mass loaded down the caster wheels, increasing the resistance to vertical perturbation as the bump was encountered, thus reducing the
extent of tipping that occurred. Even so, the difference in the extent of tipping due to rider mass in each of these extreme scenarios was at most 5.7 degrees.

**Table 5c:** Predicted extent of tipping for the prototype - dropping off a curb in forward motion from level ground.

<table>
<thead>
<tr>
<th>Simulation conditions</th>
<th>Extent of forward tipping (+0.2°/-2.1°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference trial: 100 kg, 1.3 m/s, 10.0 cm drop</td>
<td>10.7</td>
</tr>
<tr>
<td>- heavier subject: 150 kg</td>
<td>13.3* (less stable)</td>
</tr>
<tr>
<td>- lighter subject: 50 kg</td>
<td>8.6* (more stable)</td>
</tr>
<tr>
<td>- higher speed prior to drop: 2.0 m/s</td>
<td>11.1 (slightly less stable)</td>
</tr>
<tr>
<td>- bigger drop: 20.0 cm</td>
<td>19.7*(+0.2/-2.2) (much less stable; no tip)</td>
</tr>
<tr>
<td>- bigger drop/higher speed: 20.0 cm, 2.0 m/s</td>
<td>25.8*(+0.2/-2.3) (much less stable; tip imminent)</td>
</tr>
</tbody>
</table>

The wheelchair tipped forward by at most 13.3 degrees for the 10 cm drop and handled the transition easily, with no signs of tipping. The simulated response of the chair was precisely what was observed in actual tests using more recent versions of the prototype. As the drive wheels reached the edge of the curb, they dropped sharply due to the compressed spring, and then supported the majority of the wheelchair mass as the rear wheels passed over the edge of the curb. The chair then continued forward smoothly, with the rear wheels suspended in the air, until the spring slowly compressed, bringing the rear wheels back to the ground.

Increasing the wheelchair speed prior to the drop did not significantly increase the extent of tipping for the 10 cm drop but did significantly increase the extent of tipping for the 20 cm drop. At half speed, the wheelchair tipped forward 19.7 degrees (c.f. the forward static stability angle of 19 degrees) during the 20 cm drop without reaching the instability point. At the higher speed, the stability limit of the wheelchair had been reached. The wheelchair tipped forward 25.8°+0.2°/-2.3 degrees (well beyond the static stability angle) and approached the verge of tipping over completely but did not tip (i.e. the instability point for this scenario).

In summary then, it was determined using the validated model that the wheelchair:

- would most likely not need to be adjustable based on the occupant mass;
- would not tip over forward on level ground when colliding into curbs up to 7 cm high;
would only be able to overcome 7 cm high curbs at or near full speed; 
would be able to handle step drops of up to 20 cm, provided that the step is perpendicular to 
the motion of the chair

In general, it was observed that a six wheeled chair is more dynamically stable than a 
four wheeled chair, all other things being equal, just as a tank with many rollers on its gear track 
is more stable than one with few rollers. With additional support points, the mass is more evenly 
distributed and the shock of an impact is minimized. A six wheeled chair benefits from the same 
principle (refer to Figure 5d). When a bump is encountered, the front wheels absorb the impact 
while the majority of the wheelchair mass is supported by the remaining four wheels. The chair 
tends to tip \textit{backward} slightly, shifting mass away from the front wheels and allowing the 
wheelchair to overcome the bump with the help of its forward momentum. The front wheels of a 
four wheeled chair support more mass than those of a six wheeled chair. When a bump is 
encountered, the impact is more severe and the chair tends to tip \textit{forward}, shifting even more 
mass to the front wheels. The wheelchair is thereby prevented from overcoming the bump and is 
susceptible to a full forward tip because of its forward momentum.

\textbf{Figure 5d:} Comparing the susceptibility of four vs. six wheeled wheelchairs to tipping when encountering bumps. 
Four wheeled chairs tend to tip forward when encountering bumps, making them susceptible to full forward tips. Six 
wheeled chairs tends to tip backwards in the same situation, allowing them to overcome obstacles. (see text)
5.4 Recommendations for Future Work

The model developed in this thesis and the method with which it was validated have a wide number of potential applications. Besides its value as a quantitative analysis tool, the model was particularly useful in visualizing and understanding the dynamics of the statically indeterminate six wheeled chair. Some logical recommendations for improving future versions of the model and extending the scope of this thesis to other applications will now be discussed.

1. Attempt to incorporate shaft friction into future wheelchair models

The model predictions could have been improved by replacing the damping element in the telescopic shaft with a truer representation of friction. Since damping forces are velocity-dependent and friction forces are roughly independent of velocity, the accuracy of the shaft model was affected by the velocity of the tractor motion. For simulations in which the tractor velocities were relatively high (e.g. the high speed dynamic trials and the curb drop), the tractor model yielded very close results. However, in the static stability tests and the slow speed trial, the damper was a more significant source of error. Hence, improving the shaft friction would improve the accuracy of the model under static or quasi-static conditions.

Since we were unable to incorporate shaft friction using conventional means without overburdening the model, one possible solution might be to use the built-in programming language to represent the shaft friction. The Working Model literature discussed modeling pin friction. It is reasonable to assume that in coordination with Working Model's technical support staff, an analogous method could be used to model shaft friction. This solution would probably yield quicker results on the Pentium used in this thesis (32-bit, 64 MB, 133 MHz) than would adding frictional components to the model and running the simulation on a faster computer.

2. Improve regulation of the wheelchair speed in the simulation

The dynamic predictions were most accurate when the instantaneous wheelchair speed in the simulation matched that of actual wheelchair. In the high speed simulations, the model was able to maintain a somewhat smooth speed regulation. Even so, the predictions would have been more accurate had the speed not dropped drastically as the wheels impacted. In the slow speed
simulation, the wheelchair speed was even more erratic as the bumps were encountered.

Improving the speed regulation could be achieved by using multi-level motor control (c.f. 'on-off' control used in this model). With multi-level motor control, the torque would be highest when the wheelchair speed is well below the target speed and lowest when just below the target speed. The decisions regarding whether to use two- or three-level control (and where to set the cut-off speeds) would be made using trial and error. One would ideally select the motor control model which is sufficiently complex without excessively slowing down the simulation.

3. *Improve the front wheel model by incorporating horizontal stiffness*

The predicted front wheel peak displacements were consistently overly high and rounded, whereas the actual data were plateau-shaped. This difference was attributed to the fact that in reality, the wheels deformed in both the horizontal and vertical axes, whereas in the model, they were constrained to deflect along a vertical slot only. Improving even the front wheel model alone to reflect the true behaviour would greatly improve the model predictions.

One way to address this issue would be to rotate the vertical slot while increasing the wheel spring constant (to maintain an equivalent vertical component), thereby creating a horizontal component of stiffness in the wheel model. The slot rotation could be varied between 0 and 45 degrees and the simulation results could be compared to the validation data. Other approaches are surely possible; it is left to the ingenuity of the designer to try various approaches, true to the modeling process, and see which works best for the given situation.

4. *Use the model to test the prototype against the complete ANSI/RESNA dynamic standards*

Since the model was validated for a wide variety of initial conditions, it could be used to test the prototype according to the remainder of the ANSI/RESNA stability standards. The majority of these tests are within, or close to, the validated bounds of the model. Using simulations to test the stability of the prototype, as was done in the previous section for two particular tests, is quicker and cheaper at this point than conducting the tests manually (in terms of building test apparatus, running tests, checking data, etc.). Any remaining tests not within the validated bounds of the model can of course be conducted manually. By having a basic model which can be tested, adapted, and re-tested, one can determine the effect of various design
changes before building the modifications into the wheelchair. Before the final wheelchair is constructed and tested, the results can already be anticipated within some degree of certainty.

5. *Use the model to study head vibrations and associated ergonomic issues*

It was demonstrated that the RMS values for the vertical velocities and accelerations of the head were consistently low, throughout the range of tested conditions, by $31 \pm 6$ percent and $19 \pm 14$ percent, respectively. This implied that the model could be used to study the impact of wheelchair vibrations on the rider. The variance in the ratio between the actual and predicted data were small enough that actual velocities and accelerations could be estimated from the predicted data in future simulations reasonably well. For example, a long set of identical sidewalk blocks could be substituted for the ground and the effect of continuous wheelchair travel could be studied with respect to nausea. Alternatively, the discomfort due to oscillations caused by rough ground could be predicted with the help of ergonomic data. It should be recalled that the predicted head marker data would have been more accurate had the occupant been wearing a shoulder restraint or lapbelt.

6. *Develop lateral stability models*

In order to model the lateral stability of power wheelchairs (lateral tips and falls were also associated with a high incidence of power wheelchair accidents, as described in Section 2.3), one would require either a three dimensional wheelchair model or a two dimensional front view model. A lateral model would be useful in determining both the lateral static stability angle and the effect of turning sharply on level or inclined ground. Similarly, for omni-directional wheelchairs (i.e. wheelchairs which can move in any direction), it may be particularly useful to model lateral motion of the chair.

Both the wheelchair parameter data and the validation methods presented in this thesis could be applied to the development of a lateral wheelchair model (either 2-D or 3-D). With respect to our prototype, for example, the only geometric measurements required to develop a lateral model from the present one would be the 'widths' of each wheelchair component and of each body segment. Since the local and global coordinates of each component were already determined in the vertical plane, and since the wheelchair is roughly symmetric when viewed
from the front, development of the spatial model would be simplified. Similarly, the stiffnesses of the wheels, drive spring and cushions in the vertical direction would all be unchanged, requiring only a few remaining tests for mechanical properties in the lateral direction (e.g. lateral coefficient of friction, lateral cushion model, etc.). Upon developing the initial model, the steps for refining and testing the model, outlined generally in Section 2.7 and developed specifically in Chapter 3, could then be used as the basis for revising and validating the lateral model.
Chapter 6  Conclusions

The two dimensional model was consistently less stable than the actual wheelchair in both static and dynamic scenarios. The inaccuracies in modeling various components of the wheelchair combined to create an overly unstable model. Nevertheless, the model proved very useful in analyzing stability for forward and rearward tips. While a more accurate model (in which factors of safety could be added at will) would have been preferable to our less stable model (with a "built-in" factor of safety), the speed with which results could be obtained using the model was a very strong balancing point. At this level of accuracy, simulations could be run within ten minutes, as opposed to the anticipated overnight requirements for more complex models (e.g. if frictional elements were used rather than damping in the telescopic shaft).

It was reassuring to know that the errors in the predictions were on the side of safety. The very nature of approximating reality by simplifying complex systems necessitates some degree of error, and one would rather err on the side of caution. The model was used to simulate a variety of test conditions knowing that the actual wheelchair would not tip over as long as the simulation did not indicate a complete tip. By pushing the model to its limits for sudden bumps and step drops, we were able to determine a range of conditions within which the safety of the actual wheelchair was ensured (i.e. bumps up to 7.1 cm and step drops up to 20.0 cm).

The exact safety margin between the predicted point of tipping and actual point of tipping varied depending on the type of scenario. In static or quasi-static situations, the model tipped at least 3.1 degrees too early when facing uphill and exactly 4.4 degrees too early when facing downhill. For forward collisions, the predicted extent of tipping was up to 3.2 degrees too high when tipping backwards up to 2.1 degrees too high when tipping forwards. The dynamic predictions were more accurate with a longer wheelbase and with an increased impact speed. Changing the front tire pressure did not affect the accuracy of the model.

The model demonstrated that the degree of inflation of the front wheels (barring a totally flat wheel) was not significant at all in terms of medium to high speed collisions with obstacles, contrary to what one might reasonably suppose. This conclusion was completely supported by the actual data. In the range of collision speeds tested, the system was not at all sensitive to changes in the combined stiffness-damping of the wheels. At slower impact speeds, this observation would not be expected to hold true.
The usefulness of the validated model was clear. Within days we were able to:

- determine two dynamic safety limits, neither of which could have been tested easily or quickly in the lab;
- study whether or not the central spring would need to be adjusted based on the occupant mass, which also would not be easy to determine in the lab;
- come up with immediate practical applications for the model, including studying the effect of oscillations on the rider, studying lateral stability, and conducting additional dynamic tests.

The utility of having a validated model was never in doubt. However, the ultimate question remained: did the benefits of having a validated model justify the investment costs (in terms of time and effort) in developing and validating that model?

In short, the answer to that question in the opinion of the author was yes, especially in the case of the prototype chair. Even before being properly validated, the model yielded important information regarding the unique stability characteristics of a central-drive wheelchair. Six wheeled chairs, being statically indeterminate, are very difficult to analyze mathematically. Thus, the outcome of a design change is difficult to predict. Using the model, we were able to observe and analyze trends caused by design changes even in early versions of the model. This improved our understanding of the dynamics of six wheeled chairs and proved useful when modifying the prototype chair, even prior to the completion of the thesis (e.g. we focused on moving the CG directly over the central spring and lowering it as much as possible).

In summary then, all five objectives of the research were accomplished, namely:

- the development of a two dimensional model for a particular power chair;
- the validation of the model predictions over a wide range of static and dynamic conditions;
- the evaluation of the model predictions with respect to fore-aft stability;
- the use of the validated model to determine safety limits for forward tipping;
- the evaluation of the utility of computer modeling as tool for wheelchair design.

With the completion of the thesis, the full benefits of having a validated dynamic model are only beginning to be realized and the true rewards of the invested effort still lie ahead.
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Appendix 1: C.S.i.A. Facility

The Centre for Studies in Aging (C.S.i.A.), located in Toronto's Sunnybrook Health Science Centre, U-Wing, is one of the rehabilitation engineering research centres of the Institute of Biomedical Engineering (IBME) at the University of Toronto. In addition, the centre is also the location of the mobility team of the Ontario Rehabilitation Technology Consortium (ORTC).

The focus of the C.S.i.A. is understanding and addressing the needs of elderly people with respect to balance, mobility, independence and quality of life issues. Both basic and applied research are conducted to understand and address these needs. The C.S.i.A. team consists of a wide spectrum of professional staff from a variety of backgrounds including engineers, scientists, physical therapists, technologists, industrial artists and research students. Ongoing graduate projects contribute to a variety of research fields including balance, caregiving and mobility.

The research facilities at the C.S.i.A. include:

- multi-axis moving balance platforms
- video and electromagnetic motion tracking systems
- computer aided design (CAD) workstations
- a wide variety of design software
- machine shop facilities
- computer aided manufacturing (CAM) technology
- a display showroom for finished products and prototypes

Most of the design, manufacturing, electronics and computer programming needed at the C.S.i.A. for research and prototyping purposes is done in-house; most of the integrated control systems are contracted out.

For an updated description of ongoing research projects, ready-to-order assistive technology products, ready-to-license prototypes, on-line resources, publications, staff information, graduate opportunities, or contact information, please refer to the C.S.i.A. website: http://www.sunnybrook.utoronto.ca:8080/~csia.
Appendix 2: Introduction to Working Model® 2D

This appendix provides an introduction to the Working Model software, and is based largely on the on-line help guide [40] and User's Manual [41]. The purpose of this appendix is to give the reader an appreciation for the features and limitations of the software by:

- describing some of its features, including the numerical integrator algorithm
- describing the simulation settings used for the validation tests
- discussing some of the difficulties encountered and how they were overcome

Knowledge Revolution's Working Model® 2D v.4.0 was the dynamic simulation software used in this thesis to create and test the power wheelchair model. Working Model is a Windows-based application which allows the user to create, modify and analyze complex dynamic motions. It provides advanced motion simulation and editing capabilities via a simple graphical user interface. In the 2D version, objects are only considered in the plane of the computer screen, and can therefore only be moved in three independent orientations: up/down, left/right, and rotation about an axis perpendicular to the computer screen. Unless specified otherwise, gravity acts towards the bottom of the screen.

Simulation data may be plotted directly within the interface, allowing one to view the quantitative data before exporting it as a data file. Alternatively, the simulation can be exported in audio-visual format and replayed within other multi-media software.

Working Model allows constraints in the form of linear spring-dampers, rotational spring-dampers, ropes, separators, gears, pivoting slots, fixed slots, pin joints and rigid joints. Applied forces and torques can be created in the form of actuators, motors, external forces, and force fields (e.g. electrostatic).

Geometric properties of objects can be assigned using the 'click-and-drag' paradigm, by specifying exact coordinates, or by specifying parametric dimensions. Default values are assigned for the inertial and mechanical properties of each object or constraint until the user assigns a revised value. For physical objects, these properties include the mass, local CG location, type of mass distribution (solid, shell or spherical), static and dynamic coefficients of friction, coefficient of restitution, and electrical charge (if any). Certain properties are more crucial than others for particular objects. For example, the mass and CG location for fixed
objects are completely irrelevant. Similarly, it is not necessary to know the coefficients of friction for non-colliding bodies. For constraints, these properties vary. Spring-damper elements require a spring constant, free length, starting length, and damping coefficient, whereas motor elements only require specification of a motor type (torque, velocity, or rotation) and output value. Each property can be assigned a fixed value, conditional value (e.g. using Boolean operators), or parametric value (e.g. time-dependent).

One of the most useful features of the software was the ease with which simulation parameters and initial conditions could be adjusted. This made it possible to test a wide variety of configurations in the early stages of model development and decide, based on visual inspection, which approaches should be pursued. As well, by observing the effect of changing one variable at a time, we were able to estimate the relative sensitivity of the model to each variable. Therefore, this feature allowed us to narrow in on both the number of unknowns and the range of values being considered.

The simulation environment is illustrated in Figure A2a. The upper toolbar allowed for file handling, cutting and pasting, printing, zooming in and out, and controlling the simulation. The left toolbar allowed for creation of the objects and constraints listed above. The lower toolbar allowed for a frame by frame viewing of the simulation. The 'Properties' and 'Geometry' windows contained the inertial, mechanical and geometric properties of each object.

Figure A2a: Sample illustration of the Working Model environment. (refer to text)
Some additional noteworthy features of the software included the ability to:

- view a simulation from a moving frame of reference (e.g. one which follows the object)
- program simulations which needed extended periods of computing time to stop automatically
- play simulations frame by frame or in reverse to help understand the behaviour of the model
- identify and track the overall CG (the masses of fixed objects were automatically excluded)
- hide particular objects or constraints to simplify the simulation window

Collisions of objects are handled by Working Model using the coefficient of restitution parameter, referred to misleadingly in the Properties window as 'elasticity'. This empirical property describes the ratio of post-collision relative velocities to pre-collision relative velocities for two colliding objects. A value of 0 corresponds to a perfectly plastic collision, whereas a value of 1 corresponds to a perfectly elastic collision. When two objects interact, the lesser of the two values assigned to the bodies is used.

When Working Model detects a collision, the impulse necessary to change the momentum of the colliding bodies (to prevent them from penetrating each other beyond a specified overlap error) is calculated. Since impulse is a function of both time and force, the contact forces generated by the simulation will vary somewhat depending on the integration time step specified by the user. Velocities and positions are then computed by integrating the accelerations obtained from the dynamic equations of the system.

Working Model has two setting for performing numerical integration - the Euler method ('fast' setting) and the Kutta-Merson method ('accurate' setting). The Euler method is based on a straightforward integration algorithm using rectangular boxes to approximate the area under the acceleration curve. The Euler method is best suited for smooth curves; it is not appropriate for regions of erratic or sharp curvature. The Kutta-Merson method, also known as the fifth-order Runge-Kutta method [41], is more robust and accurate than the Euler method. It is more well suited for regions of erratic curvature or ill-conditioned systems. The Kutta-Merson method uses a fifth-order polynomial when calculating the area under each region of the curve, thereby rounding the top of each integration "rectangle" to more closely match the actual function.

The step size setting determined the block of time that was lumped together and considered as one moment or frame. The integration error setting determined the maximum
acceptable absolute error for all computations involving length. If that value was exceeded, a variable step size setting automatically cut the step size in half and recalculated the frame. The assembly error setting determined how well connections were maintained during the simulation. Unless modified, the values for each of these parameters were automatically tailored by the software to match the simulation.

The following simulation settings were used in the validation tests:

- air resistance: off
- gravity: on
- integrator: Kutta-Merson
- step size: 1/30 second
- variable step size: on
- integration error: 0.8 cm
- overlap error: 0.5 cm
- assembly error: automatic
- significant digits: automatic

The simulations were run at 30 frames per second rather than at 60 (the speed of the video camera) after determining, via trial and error as recommended, that 1/30 second was the optimal step size for these simulations. The relationship between integration step size and total numerical error was discussed previously in Section 2.7. The results obtained at 30 frames per second were more accurate than those obtained at either 60 or 20 frames per second. At 45 frames per second, the results were equally accurate but the simulation took 50 percent longer to compute.

While the temptation was to set the integration error very low, the User's Manual recommended that using a "reasonable" setting would greatly speed up the simulations while sacrificing very little accuracy, as long as a large number of significant digits were used. Using the same trial and error approach, the integration error was set to 0.8 cm. The default setting for the number of significant digits was 7.

The major software limitations included:

1. The lack of any debugging or diagnosis tools

During development of the model, generic error messages were often encountered. The software did not provide a means of identifying the specific problem or problem component.
Finding the source of the error, which was often an extraneous joint or an improperly connected component, was usually very difficult and time consuming. If error messages were ignored, ridiculous results occurred (e.g. neglected constraints, high accelerations). This limitation was handled by testing the model at each stage of development to isolate problem areas. If errors persisted (e.g. when modeling the tractor), complete rebuilding of components was necessary. Experience also assisted in the diagnosis of problem areas.

2. The somewhat limited modeling capability

Once the model exceeded 20 objects and 10 constraints, the simulation significantly slowed down. The simulation speed was particularly sensitive to the use of polygons, ropes, pin joints, rotational constraints, and sliding bodies. The simulation speed was improved somewhat by replacing pin joints with fixed joints whenever possible, setting all non-contacting objects to 'do not collide', using only one colour in the animations, and changing the accuracy settings to those listed above. As mentioned previously, by the time shaft friction in the tractor was created (using two sliding contacts and a spring in tension), the simulation had reached its limits.

3. Occasional resetting of spring, rope and separator properties

When springs, ropes or separator elements (or the objects connected to them) were manipulated using the mouse, the software tended to reset their initial properties. That is, the initial spring displacement, rope slack, or separator clearance was often reset to zero. Using the CTRL-SHIFT keys while manipulating these objects often solved this issue; nevertheless, the initial lengths were checked before each simulation.

4. Specific software glitches

While using the software, some annoying 'bugs' were discovered, including unexplained errors which shut down the program on rare occasions and an inability to send images to the printer. The printer issue was resolved by either copying the images to other software programs or hiding the constraints before sending the image to the printer.
Appendix 3: PEAK5 Motion Tracking System

This section provides a general overview of the process of tracking two dimensional motion using the PEAK5 system. This information is based to a large extent on the PEAK5 User's Manual [25], with additional comments and recommendations based on experience gained during this research. A brief discussion of resolution error, digitization error, and parallax error as they pertain to the wheelchair trials is included as well.

PEAK5 v. 5.1.0, released in July 1993 by Peak Performance Technologies Inc. [19], is a DOS-based software application used to digitize and display the motion of objects captured on video. The PEAK system is typically used in the study of the kinematics of human motion. Both two and three dimensional motion can be digitized using the software, yielding linear and rotational data for position, velocity and acceleration. The system components included a video cassette recorder, a video monitor, a digitizing board, and a computer system (i.e. monitor, hard drive, keyboard, mouse).

Tracking the two dimensional motion of an object using the system involves:

Step 1: Setting up a video camera to film the trial and calibrating the apparatus
Step 2: Conducting the trials and videotaping the moving object
Step 3: Digitizing the marker positions from the videotape
Step 4: Filtering and scaling the data
Step 5: Calculating velocities and accelerations from the coordinate data
Step 6: Plotting and analyzing the results

Step 1: Setup

Since only one camera is used in two dimensional motion analysis, the following recommendations should be observed for maximum accuracy:

- face the camera perpendicular to the plane of motion of the object
- level the camera in the other two planes
- set the camera as far back from the test area as possible and use the maximum possible zoom setting to minimize parallax errors
- use high quality videotapes since the tape quality will degrade to some extent during the digitization process
film a large stationary square object in the plane perpendicular to the line of sight of the camera in order to calculate the aspect ratio of the camera (length to width ratio in the field of view) during Step 3

film a large rod of known length in the centre of the plane of motion, perpendicular to the line of sight of the camera in order to calculate the scaling factor (the proportional length of objects in the plane of motion) during Step 4

ensure that the lighting is consistent across the field of view (important for digitization)

ensure that the reflective markers clearly stand out against the background; mask any reflective surfaces (important for digitization)

Step 2: Conducting the trials

At least three minutes of recording time should be left before and after the trials so that the computer will be able to search beyond the frames of interest during digitization. An indication of which trial is being conducted should also be made so that one will be able to easily identify specific trials. Detailed notes should be kept during testing, identifying any trials which should be disregarded for specific reasons. The plane of motion of the markers should coincide with the plane that was calibrated in Step 1.

Step 3: Digitizing the trials

This step is by far the most time consuming stage. When digitizing the trials, the marker positions can be tracked either automatically (by the computer) or manually (using the mouse). Automatic digitization is much faster but only works when the markers contrast well with the background. When the computer is unable to track a marker in a particular frame, the user must identify the marker until the computer is able to resume tracking it. The software prompts the user for information based on the number of markers being tracked.

Before digitization of the trials may begin, the aspect ratio of the camera (i.e. vertical to horizontal field of view ratio) must be calculated by digitizing the four corners of the square object filmed in Step 1. The software prompts the user with the necessary steps. The aspect ratio of the camera used in the trials was 0.841.

The settings for automatic data capture must be tailored to the trial conditions. A careful selection of these settings will result in successful tracking of the markers without a great deal of user intervention. The following parameters must be set for each marker:
- minimum and maximum marker outline size
- search radius size
- threshold value
- prediction algorithm value
- colour

Minimum and maximum marker outline size refer to the pixel size of the cluster identified as the marker. The computer searches for clusters within that range and then identifies the midpoint of the cluster as the marker position. For the front and rear wheel markers (2 cm diameter), minimum and maximum values of 1 and 8 were used, respectively; for the drive wheel and head markers (3 - 4 cm diameter), respective values of 1 and 10 were used.

The search radius refers to the maximum area surrounding the previous marker location in which to search for the marker in the present frame. This parameter should be set based on the speed of the marker during the trial (larger search radius for higher marker speed). An overly large search radius will result in other points being mistakenly identified as the marker. A search radius of 5 was used for the trials.

The threshold setting refers to the level of brightness below which all pixels will be ignored. All pixels below the threshold level are filtered out, leaving only the brightest clusters as potential marker locations. The system will differentiate between 256 shades of brightness for each pixel. An appropriate level must therefore be set to successfully filter out the background while leaving the markers visible. Different threshold levels can be tested out before selecting a value, but once digitization begins only one value per marker is allowed. Hence, markers which are more bright at one end of the field of view than another will typically require manual digitization during parts of the trial. Threshold values ranging from 52 to 155 were used for the trials (the background illumination varied during filming).

The prediction algorithm refers to the method used by the software to predict marker locations in successive frames. The linear setting is most appropriate for motions that are linear in nature, the parabolic setting is most appropriate for curvilinear motion, and the spline setting is most appropriate for cyclic motion. The parabolic setting was used for the trials.

The colour setting refers to whether white or black clusters are to be identified as markers. The white setting was used for the trials.

Once appropriate settings have been selected, digitization of the trial may proceed. As the marker positions are acquired automatically by the computer, the data should be checked
visually to ensure that the points being tracked are indeed the markers and not nearby reflective surfaces.

**Step 4: Data filtering and scaling**

After the raw data are acquired and checked, they may then be filtered to remove high frequency, low amplitude noise (originating from video signal flicker or incorrect digitization of pixels above/below the actual marker position). The PEAK5 software includes three high frequency filters. The *Butterworth* filter is best suited for removing constant frequency random amplitude noise; the *Fast Fourier Transform* filter is best suited for removing noise from cyclic data; the *Cubic Spline* filter is best suited for removing noise from parabolic motion [25]. Each of these low-pass filters may be adjusted to determine the extent of filtering. The software allows one to preview the results of various filtering schemes before saving the filtered data.

For the wheelchair trials, discretion was used to select the correct Fast Fourier Transform setting which would remove the unwanted high frequency noise without affecting the low frequency oscillations of interest. The vertical data were filtered using a setting of 3 for the drive wheels and 4 for the other markers (the higher setting filtered out some of the useful higher frequency drive wheel oscillations); the horizontal data were filtered using a setting of 1 (hardly any filtering at all).

The filtered coordinate data are then scaled to the correct units. The scaling factor is calculated by digitizing the endpoints of the rod of known length filmed in Step 1. The software prompts the user with the necessary steps. The scaling factor for the short wheelbase trials was 2.603 and for the long wheelbase trials was 2.868 (the zoom setting was unintentionally adjusted between days of filming).

**Step 5: Calculating velocities and accelerations**

Once the data have been scaled, the marker velocities and accelerations may then be calculated. One may also calculate the velocities and accelerations of imaginary segments, created by joining two markers. PEAK5 uses a first-order central difference approximation to calculate velocities and accelerations from the position data. That is, the average slope and average acceleration at each point are calculated using the position data from the previous and following frames.
Step 6: Plotting and analyzing the results

The contents of PEAK5 data files can be identified by their file name extensions. The two dimensional file name extensions are as follows:

- .rda: raw pixel coordinate data
- .rca: filtered coordinate data
- .cda: scaled coordinate data
- .vda: linear velocity data
- .lda: linear acceleration data
- .ada: angular displacement data
- .wda: angular velocity data
- .mda: angular acceleration data

The kinematic data can be plotted and examined within the software, and then exported as a data file. PEAK5 automatically extrapolates the velocity and acceleration data in the first and last frames of each trial and in frames for which there is no position data, often with poor results. Hence, the built-in features can be used to check the results and identify data points which should be ignored.

Resolution, Digitization and Parallax Error

Two sets of trials were digitized to determine the repeatability of the data. The vertical displacement data were found to be almost identical from trial to trial (recall that only trials in which the wheelchair maintained a nearly straight path and nearly constant speed were used). Thus, repeatability was not an issue in determining the magnitude of measurement error.

The resolution of the digitization screen for automatic data capture was listed as being approximately 1 in 1000 [25]. Resolution refers to the minimum distinguishable difference between two adjacent points and is equal to half of the smallest possible measurement. Since the digitization board had a total of 508 pixels in both the horizontal and vertical directions, the resolution of x and y coordinates was therefore 1 in 1016. In the plane of wheelchair motion, the field of view consisted of a 2.0 m by 1.7 m rectangular space. Therefore, the minimum uncertainty in the coordinate data due to resolution error alone was ± 2 mm in both the horizontal and vertical directions. Any addition human error (e.g. incorrectly identifying a marker 1 full pixel too high or low, which was certainly possible for markers not in the vicinity of the bump)
would double the total digitization error for that point to \( \pm 4 \) mm.

The consistent high frequency (~ 60 Hz), low amplitude (\( \pm 1-2 \) mm) noise in the raw data was attributed to flicker in the video signal between frames. The vast majority of this noise was filtered out during Step 4, and was not considered to be a source of uncertainty in the results.

The issue of camera parallax added an additional degree of uncertainty to the data. Parallax error occurs when a single camera is used to measure distances. Since the edges of the field of view are slightly farther away from the camera than the centre of the field of view, objects (and displacements) appear to be slightly smaller as the edge of the field is approached. The scaling factor, when measured at the edge of the field, was found to be 1 percent lower than when measured at the centre of the field. Therefore, vertical displacements of 5 cm in that area would be 0.5 mm too low. Similarly, for every 5 cm that the wheelchair accidentally veered towards/away from the camera (i.e. out of the specified plane) due to control problems, the vertical displacements were an additional 1 percent too high/low, bringing the total uncertainty due to parallax error to at least \( \pm 1 \) mm. Since the bump was located in the centre of the field of view, and we were careful to drive the wheelchair over that part of the bump which was marked as being in the test plane, neither of these sources of error were applicable to the wheel markers at the point of encountering the bump.

Therefore, the total measurement error in the marker displacements due to resolution, digitization, and parallax was approximately \( \pm 2 \) mm for the wheels as they encountered the bump, and at least \( \pm 5 \) mm for all the other data points. This estimate was confirmed by control trials in which the wheelchair was driven down the runway at constant speed without the bump in place. The results consistently showed vertical displacements between \( \pm 2 \) mm (exactly at the centre of the field of view) up to \( \pm 9 \) mm (as either edge of the field of view was approached). The additional degree of error was attributed to the levelness of the camera relative to the floor and to small bumps in the floor. Hence, the degree of uncertainty in the measurement of vertical displacement data was found to be \( \pm 2 \) mm for the wheels as the bump was driven over and up to \( \pm 9 \) mm at all other points.

In the displacement graphs reported in Chapter 4, the error bars were simply shown as being \( \pm 9 \) mm at all points for convenience. In the determination of the extent of tipping at the critical points (Chapter 5), the proper error margins were used.
Appendix 4: Derivation of Equation (2)

In this appendix, equation (2) in Section 2.5 is derived from first principles. This formula is a generalization of equation (1) for a forward moving wheelchair on a downhill slope. The assumptions and limitations of the formula have been discussed previously.

During an impact in which the linear motion of the wheelchair-occupant system is completely converted to rotational motion, we can state from the conservation of angular momentum that:

\[ I_0 \omega = M v y \]  

(A4.1)

where \( I_0 \) is the moment of inertia of the system about the point of impact
\( \omega \) is the rotational velocity of the system immediately after impact
\( M \) is the mass of the system
\( v \) is the velocity of the system immediately before impact
\( y \) is the height of the CG above the point of impulse

That is, the rotation of the wheelchair immediately after the impulse will be larger for a higher CG and smaller for a lower CG. Now, since the rotational kinetic energy of the wheelchair-occupant system is converted to potential energy as the CG rises above the ground, we can determine the highest point that will be reached (where there is no remaining rotational kinetic energy) using:

\[ \frac{1}{2} I_0 \omega^2 = M g \Delta h \]  

(A4.2)

where \( \Delta h \) is the total increase in CG height at the maximum point of rotation

By solving for \( \omega \) in equation (A4.1) and substituting that value into equation (A4.2), we can express the relationship between the velocity at impact and the maximum increase in CG height after a full frontal collision as:

\[ v = \sqrt{\frac{2 g I_0}{M y} \left( \frac{\Delta h}{y} \right)} \]  

(A4.3)
On level ground, the increase in CG height required to reach the instability point is shown in Figure A4a to be

$$\Delta h = \left( \sqrt{x^2 + y^2} - y \right) \quad (A4.4)$$

where x and y are the respective horizontal and vertical CG positions with respect to the centre of the front wheels. Substituting this value into equation (A4.3) yields equation (1) upon simplification.

Figure A4a: The rise in CG height as a wheelchair on level ground tips forward and reaches the instability point. The horizontal and vertical positions of the CG (relative to the front wheels) before tipping begins are indicated by x and y, respectively. At the instability point, the CG is directly above the centre of the front wheels.
For a wheelchair traveling on a downhill incline of $\theta$ degrees, however, the increase in CG height required to reach the instability point is

$$\Delta h = \left( \sqrt{x^2 + y^2} - (y \cos \theta + x \sin \theta) \right)$$  \hspace{1cm} (A4.5)

as shown in Figure A4b. Substituting this revised expression into equation (A4.3) yields equation (2), the generalization of Cooper's formula for a downhill slope:

$$v_c = \frac{2gI_o}{My} \left( \sqrt{1 + \frac{x^2}{y^2} - \frac{x}{y} \cos \theta - \frac{x}{y} \sin \theta} \right)$$  \hspace{1cm} (A4.6)

**Figure A4b:** The rise in CG height as a wheelchair on a downhill slope tips forward and reaches the instability point. The vertical position of the CG, relative to the front wheels, before tipping begins is no longer simply $y$ but rather $x \sin \theta + y \cos \theta$, where $x$ and $y$ are unchanged from the previous figure and $\theta$ is the angle of the downhill incline. The greater the downhill incline, the less the CG will have to rise to reach the instability point.
Appendix 5:  Wheelchair Model Data File

Simulation File:  777
(P1x, P1y, P2x, P2y are local coordinates, relative to centroid of object)

Unit System

Distance:  centimetres
Mass:  kilograms
Force:  newtons
Energy:  joules
Power:  watts
Time:  seconds

Integration Settings

Integrator  Kutta-merson
Variable/Fixed  Variable
Animation step  0.033
Overlap error  0.5
Assembly error  0.08
Gravity  Linear
Gravity constant  980.67

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| [147] Lower Legs | poly | 22.994 | 0.2 | X1: -0.02 | Y1: -25.469 |
| | 0.092M | 31.549 | 0.2 | X2: 9.696 | Y2: 18.241 |
| | | 34.543 | 0.6 | X3: 6.457 | Y3: 24.485 |
| | | -0.02 | | X4: -2.558 | Y4: 24.485 |
| | | -6.497 | | X6: -6.497 | Y6: 2.63 |
| | | -6.497 | | X7: -6.497 | Y7: -16.102 |
| | | -6.497 | | X8: -6.497 | Y8: -28.591 |

| [149] Feet | poly | 43.145 | 0.2 | X1: -4.718 | Y1: 3.919 |
| | 0.032M | 7.662 | 0.2 | X2: 2.75 | Y2: 3.919 |
| | | 25.332 | 0.6 | X3: 9.684 | Y3: 0.698 |
| | | 12.351 | | X4: 12.351 | Y4: -1.88 |
| | | 8.617 | | X7: 8.617 | Y7: -5.102 |
| | | 3.817 | | X8: 3.817 | Y8: -3.169 |
| | | -12.185 | | X11: -12.185 | Y11: 1.342 |
| | | -10.052 | | X12: -10.052 | Y12: 5.208 |
| | | -7.385 | | X13: -7.385 | Y13: 5.852 |

| [151] Upper Torso | poly | -36.193 | 0.2 | X1: -5.927 | Y1: -25.299 |
| | 0.337M | 86.3 | 0.2 | X2: -8.18 | Y2: -20.19 |
| | | 8.8 | 0.6 | X3: -8.631 | Y3: -10.538 |
| | | &yen;0.012 | | X4: -9.982 | Y4: -1.454 |
| | | 11.785 | | X5: -11.785 | Y5: 11.321 |
| | | 4.125 | | X7: -4.125 | Y7: 23.243 |
| | | -0.521 | | X8: -0.521 | Y8: 23.243 |
| | | 11.193 | | X11: 11.193 | Y11: 13.592 |
| | | 12.094 | | X12: 12.094 | Y12: 7.062 |
| | | 11.193 | | X13: 11.193 | Y13: 0.533 |
| [153] Upper Arms | poly | -34.449 | 0.2 | X1:  -6.001 | Y1:  14.7 |
| | 0.062M | 89.87 | 0.2 | X2:  -5.728 | Y2:  4.556 |
| | | 13.575 | 0.6 | X3:  -4.364 | Y3:  -7.675 |
| | | | | X4:  -2.182 | Y4:  -19.31 |
| | | | | X5:  -0.272 | Y5:  -20.802 |
| | | | | X6:  2.183 | Y6:  -20.802 |
| | | | | X7:  4.092 | Y7:  -16.625 |
| | | | | X8:  5.183 | Y8:  -6.78 |
| | | | | X9:  6.001 | Y9:  4.855 |
| | | | | X10:  4.365 | Y10:  13.208 |
| | | | | X11:  0 | Y11:  18.578 |
| | | | | X12:  -4.091 | Y12:  18.28 |
| | | | | X13:  -4.91 | Y13:  15.893 |

| [155] Lower Arms | poly | -15.789 | 0.2 | X1:  17.366 | Y1:  -1.233 |
| | 0.037M | 74.871 | 0.2 | X2:  0.932 | Y2:  -3.595 |
| | | 2.622 | 0.6 | X3:  -8.459 | Y3:  -4.382 |
| | | | | X4:  -14.563 | Y4:  -3.988 |
| | | | | X5:  -15.502 | Y5:  -2.807 |
| | | | | X6:  -15.502 | Y6:  0.735 |
| | | | | X7:  -14.093 | Y7:  3.097 |
| | | | | X8:  -6.581 | Y8:  3.884 |
| | | | | X9:  5.158 | Y9:  3.097 |
| | | | | X10:  17.366 | Y10:  3.097 |
| | | | | X11:  18.774 | Y11:  2.704 |
| | | | | X12:  19.244 | Y12:  0.735 |

| [158] Hands | poly | 7.077 | 0.2 | X1:  10.639 | Y1:  -3.558 |
| | 0.014M | 77.314 | 0.2 | X2:  9.192 | Y2:  -0.497 |
| | | 3.749 | 0.5 | X3:  6.825 | Y3:  1.953 |
| | | | | X4:  3.273 | Y4:  3.381 |
| | | | | X5:  -1.988 | Y5:  3.586 |
| | | | | X6:  -9.091 | Y6:  1.34 |
| | | | | X7:  -9.223 | Y7:  -1.313 |
| | | | | X8:  -6.592 | Y8:  -2.742 |
| | | | | X9:  -1.988 | Y9:  -3.558 |
| | | | | X10:  0.643 | Y10:  -2.538 |
| | | | | X11:  1.563 | Y11:  -1.109 |
| | | | | X12:  4.983 | Y12:  -1.313 |
| | | | | X14:  9.719 | Y14:  -4.987 |

| [160] Head/Neck | poly | -42.435 | 0.2 | X1:  -6.828 | Y1:  -12.958 |
| | 0.043M | 123.295 | 0.2 | X2:  -6.341 | Y2:  -6.106 |
15.616 0.7  X3: -8.289 Y3: -0.836
X4: -9.75 Y4: 3.381
X5: -9.263 Y5: 9.705
X6: -8.289 Y6: 11.813
X7: -5.367 Y7: 14.976
X8: -0.009 Y8: 17.084
X9: 3.888 Y9: 16.03
X10: 7.784 Y10: 12.34
X11: 10.22 Y11: 8.651
X12: 10.22 Y12: 3.381
X13: 11.681 Y13: 0.218
X14: 11.194 Y14: -1.363
X15: 10.22 Y15: -3.471
X16: 9.733 Y16: -7.16
X17: 8.759 Y17: -9.269
X18: 5.836 Y18: -9.796
X19: 1.94 Y19: -9.796
X20: -0.496 Y20: -11.904
X21: -0.983 Y21: -18.755
X24: -9.263 Y24: -17.701

[162] Upper Legs  poly  -8.968 0.2  X1:  2.506 Y1: -29.685
0.209M  51.855 0.2  X2:  4.649 Y2: -23.834
90.048 0.6  X3:  6.792 Y3: -15.058
X4:  8.935 Y4:  5.421
X5:  6.792 Y5:  22.974
X6:  2.506 Y6:  28.825
X7: -3.923 Y7:  25.9
X8: -8.21 Y8:  17.123
X9: -8.21 Y9:  -6.281
X10: -6.067 Y10: -20.909
X11: -1.78 Y11: -32.611

Pin Constraints

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Spring/Damper Constraints

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**Rod/Rope/Separator Constraints**

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**Motor/Actuator Constraints**

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**Linear Slot Constraints**
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**Round and Square Points** *(only significant points have been included)*

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<th>Px</th>
<th>Py</th>
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<td>Head marker</td>
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<td>[117]</td>
<td>Test bump anchor</td>
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