VARIATION IN THE LOCAL STRUCTURE AND PROPERTIES OF PAPER

Baohua Shen

A thesis submitted in conformity with the requirement for the Degree of Master of Applied Science
Department of Chemical Engineering and Applied Chemistry
University of Toronto

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June 30, 2000

ABSTRACT

The rupture of paper is controlled not by the mean strength, but by the minimum strength regions in the sheet. In order to fully understand which aspect of structure controls these weak regions and their effect on the local strength of paper, an effort was made to correlate variations in two microstructural features (fibre orientation and bonded area) of the sheet with the variability of local properties. The objective of this study was to identify if the structure varied significantly from point to point and to attempt to correlate the structural features of paper with properties in a deterministic way. By using a new analytical method for the estimation of fibre orientation, a relatively strong correlation was observed between local variation in mean fibre orientation and the local strength of paper. By considering fibre orientation and bonding simultaneously, a relative strength model was developed in this study in order to improve the correlation between local tensile strength and local structure.
ACKNOWLEDGEMENT

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Finally I appreciate my family, my parents and my sister, for their spiritual support and great help at every turn throughout my life. To them I dedicate this thesis.
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NOMENCLATURE

\( \Delta g_j \) the distribution of density gradient magnitudes at window j

\( g(x, y) \) the average density in a window about the location \((x, y)\)

\( \alpha_i \) the associated dominant gradient orientation at window \( j \), defined by Equation 2.8. \( \alpha_i \in [0, \pi] \)

\( \alpha \) the dominant gradient orientation for each subzone. \( \alpha \in [0, \pi] \)

\( w \) the size of the domain of the operator

\( N \) the subzone size. \( N = 10 \text{pixel} \times 10 \text{pixel} = 2 \times 2 \text{windows} \)

\( C \) the local coherence of subzone

\( S \) the sum of the square of the projections in a subzone (Equation 2.10)

\( S \) the scattering coefficient of a sheet (Equation 2.12)

\( S_o \) the scattering coefficient of the fibres in a completely unbonded state (Equation 2.12)

\( T \) the light transmission in each specimen

\( T_g \) the mean value of light transmission for all specimens with the same grammage

\( T/T_g \) the local bonding index

\( P_b \) force per unit width at failure. \( kN/m \)
$F_b$ breaking force of the specimen, $N$

$T$ breaking length of the specimen, $km$

$w$ the width of the specimen, $mm$ (Equation 3.1)

$g$ the mean grammage over the entire zone of interest, $kg/m^2$

$CV_{\infty}$ the coefficient of variation of grammage

$L$ fibre length, $m$

$ar{L}$ the average fibre length, $m$

$L_r$ the total length of projection on the $X$-axis of $N_{1,0}$ (Equation 3.3), $m$

$h(L)$ the probability density of the fibre length ($L$)

$\theta$ the local dominant fibre orientation (Equation 3.3 and 3.13)

$g(\theta)$ the probability density of the fibre orientation ($\theta$)

$\phi$ a dimensionless function of fibre length and other fibre, matrix and specimen properties (Equation 3.3)

$\varepsilon_o$ strain applied to the specimen

$\nu_i$ Poisson's ratio of the specimen

$E_f$ Young's modulus of the fibre, $N/m^2$

$A_f$ the area of cross-section of the fibre, $m^2$

$V_f$ the fibre volume fraction in a rectangular plate with three dimensions $a, b$ and $c$
SumC90 the mean fibre orientation with respect to the tensile axis in the entire region of interest

$C_{rp}$ the fibre orientation factor (Equation 3.11)

$D_{rp}$ the fibre length factor (Equation 3.12)

$n_f$ the number of fibres crossing the failure line that take the load at failure and then break

$n_p$ the number of fibres crossing the failure line that pull out intact due to prior bond breakage and hence carry no load at failure

$n_i$ the number of the fibres spanning the failure line

$Z_c$ finite-span tensile strength of the strip expressed as breaking length if no bond breakage had occurred. $km$

$Z$ zero-span breaking length of the sheet. $km$

$\phi$ mean fibre strength. $N/m^2$ (Equation 3.20)

$\beta$ mean force applied along the fibre axis and across the failure line required to pull a fibre from the sheet, $N$

$f$ dimensionless function (Equation 3.20)

$P$ perimeter of the cross-section of fibres, $m$

$l_c$ critical value of fibre length when the force for fibre pull out equates to the force for fibre breakage crossing the failure line. $m$

$RBA$ relative bonded area of the sheet
density of the fibre, $g/m^3$

$g$  
acceleration due to gravity. $m/s^2$

$\tau_s$  
shear strength of the inter-fibre bonds. $N/m^2$

$\sigma_f$  
average tensile strength in a fibre of length $L$, cross-section $P$. average area of the cross-section $A$, $N/m^2$
CHAPTER ONE
INTRODUCTION

1.1 BACKGROUND

Uniformity of the physical properties of paper continues to be one of the most serious quality issues in today's paper mills. However, because of the paper manufacturing process itself, defects can never be eliminated completely from paper. Catastrophic failure may occur around weak regions if paper cannot tolerate the stresses induced by modern high-speed printing processes. Current quality control in the paper industry includes online monitoring of paper weight, thickness, water content, opacity, gloss, etc. The resulting large-scale "average" profiles can often be controlled effectively with today's technologies. On the other hand, to detect and control features with sizes from several microns to a few centimetres remains the industry's biggest challenge. However, even within this range, the structure of paper can be described in many different ways, depending on the scale of inspection [1].

On the microstructural level, which is from 1μm to 10mm, paper is a vast network of ribbon-like cellulosic fibres bonded together by hydrogen bonds. In this sense, paper is porous and inhomogeneous. The microstructure of the sheet is defined by a set of variables including fibre morphology and the details of local fibre
connectedness and distribution. A primary goal of paper physics is the prediction of the mechanical properties of the sheet, such as tensile strength and modulus, in terms of microstructural features. However, in previous studies, a lot of simplification and idealization was applied in these predictions to make the problem solvable. Also, the measurement of many microstructural features, such as fibre orientation and interfibre bond strength over a wide range of conditions, is often difficult.

Recent studies [2] have found that "the rupture of paper is controlled not by the mean strength, but by the minimum strength regions in the sheet". and suggested that these "minimum strength regions" can be described by the *mesostructure* of paper. Physically, when examined with the naked eye, paper appears more like a continuum, and a different kind of non-uniformity from that on the microstructural level emerges. Looking through a sheet of paper under light reveals a variation in opacity that corresponds to the variation in local grammage. This kind of non-uniformity on the square millimeter scale can be considered to be the *mesostructure* of paper: it originates from the non-uniform distribution of fibres during the forming process. Therefore the prediction of strength, printability and appearance (opacity) of paper on this scale is critical.

A systematic structural hierarchy of paper was built up by Kortschot [1]. In his work, the mesostructure is characterized by such structural features as *local variations* in *grammage*, *fibre orientation* and *bonded area*, etc., at the millimeter
scale. Note that although the grammage, fibre orientation and bonding are the elements of the microstructure, the variations of these features can be described as mesostructural variables.

Wong et al. attempted to correlate the strength of the sheet directly to the grammage variation [3]. In this case, grammage was treated as the only variable, and the sheet was considered to be a homogeneous film with varying thickness. The results showed that grammage alone is not sufficient for predicting failure, and the incorporation of other mesostructural variables, such as variability in mean local fibre orientation and bonded area, was suggested.

Quite a lot work has been done to relate the global variations in fibre orientation and bonding property to paper strength and stiffness. However, few studies have focused on the variations in local structural parameters and their effect on local properties. Thus, in this research, an effort was made to correlate not only local grammage, but also variations in the local mean fibre orientation and bonded area with the variability in local properties at the millimeter scale, and further to predict the stress, strain or strength field when a tensile load is imposed on the paper sheet.
1.2 HYPOTHESIS AND OBJECTIVES OF THE THESIS

This study was founded on the hypothesis that there are significant local variations in structure that can affect local properties in a deterministic way. The local structural variations studied here include variations in grammage, fibre orientation and relative bonded area (RBA). In this work, microtensile specimens with $2mm \times 2mm$ testing zone were used to obtain the local breaking length. Mean grammage, mean fibre orientation and mean RBA were computed for each specimen, and a comparison between these structural parameters and local mechanical properties was made.

There are three objectives of this thesis: first, to develop algorithms for local fibre orientation evaluation; second, to correlate three major microstructural features (grammage, fibre orientation and bonded area) of the sheet with the variability of local properties; third, to make a prediction of sheet failure using relative strength models based on measurable local structural properties. Unfortunately, due to the difficulty in characterizing local variations in bonded area, the improvement based on the relative strength model which was developed in this study is trivial.
CHAPTER TWO
LITERATURE REVIEW

2.1 INTRODUCTION

Sheet strength is a function of two factors: pulp fibre properties, such as individual fibre strength; and sheet structure, such as grammage distribution, fibre orientation distribution and fibre-fibre bond strength. In order to produce sheets with desired properties and to diagnose paper machine performance, the relationship between these structural variables and sheet strength must be derived. Several models, for example, the Page equation [4] or the Kallmes-Bernier-Perez model [5], have related the failure to sheet structure by treating paper as a bonded random network of natural cellulosic fibres which is defined on the microstructural level. There have not been many studies dealing with tensile strength on the mesostructural level. Therefore, this study concentrates on the effect of local structural variables at the millimeter scale (the mesostructural level). An effort is made to establish whether or not local variations in relative bonded area and fibre orientation are significant enough to affect local properties and further to build up a model relating the local structure to paper failure.
2.2 MESOTRUCTURAL VARIABLES

In Kortschot's paper [1], the term 'mesostructure' (1mm to 10cm) was used to describe those intermediate aspects of structure which exist between the microstructure (1μm to 10mm) and the macrostructure (5mm to 30m). In paper, mesostructural features can be defined at the millimeter scale, including the local variations in grammage, fibre orientation and bonding, etc. Although fibre orientation and RBA are actually microstructural elements, the point-to-point variations in their local mean values can be considered as mesostructural variables [1].

It is well known that the inhomogeneous structure of paper at this level is caused by flocculation of higher consistency furnishes in a papermaking suspension. An overview of forming physics was provided by Norman [6].

The study of local variability in paper on the mesostructural level is an essential element in monitoring the quality of paper production due to the effect of formation variations on mechanical properties and the small-scale dimensional stability problems such as cockling or bagging. These local variations in dimensional stability are often caused by local variations of quantities such as grammage, anisotropy, and dominant fibre orientation [7][8]. There have been some studies relating the mesostructure to cockling or the stress-strain properties of paper.
However, a complete understanding of which aspect of structure at this level controls these properties has not been obtained.

In previous work, the effect of structure, such as grammage, fibre orientation, or bonded area, on paper strength focused on global variations instead of local ones. Here, global variation means a variation in structural properties obtained over the whole sheet, for example by changing wet pressure to change degree of bonding. There have been few studies of the effect of local variations in structure on the local properties in a sheet with variable local structure. In this study, three aspects of mesostructure are considered: local variations in grammage, fibre orientation and bonding.

2.2.1 Grammage

Because it is extremely significant for optical and printing properties, grammage variation in the sheet has received the most attention. Certainly, its effect on mechanical properties has been well studied.

From previous studies of Norman and Dodson et al. [12][13], it is known that breaking length is reduced as grammage variability increases. The decrease in strength with increasing variability in local grammage has been explained by the ‘weakest link concept’ [13][14]. Norman considered the strength of the weakest part in specimen as the tensile strength of standard tensile strength of a sheet [13]. The
coefficient of variation of grammage (CV) is widely used to characterize the uniformity of formation, and is calculated over square zones of a finite size. Generally speaking, the CV of grammage of a sheet decreases with an increase in the size of the inspection zones, and the rate of decrease is governed by the fibre geometry as well as the paper formation [15]. Recent work by Wong et al. [3] attempted to relate the strength directly to the grammage variation, treating the sheet as a homogeneous film, and only considering local grammage variation at the millimeter scale. However, the results showed that the grammage alone is not sufficient for modelling local failure and other structural variables need to be considered.

A background survey of measurements for grammage variations can be found in Norman’s paper [16]. Table 2.1 gives a summary of the advantages and disadvantages of the various detection methods. In this study, video beta-radiography (VBR) was chosen over other methods because of a strong correlation between grey level and grammage, good contrast and insensitivity to the effect of other structural variations in a sheet.
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<th>Disadvantages</th>
<th>Advantages</th>
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<td>Low spatial resolution; Influence from variations in bonding and the scattering coefficient</td>
<td>High contrast</td>
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<td>Soft X-ray Imaging</td>
<td>Measuring the transmitted X-rays intensity on the film</td>
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<td>Measuring the attenuation of β-rays by mass on the film</td>
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<td>Short exposure time; Large grammage range</td>
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*Table 2.1 Techniques Used To Detect Local Grammage [17][18][19][20][21]*

### 2.2.2 Local Fibre Orientation

#### 2.2.2.1 Introduction

Because of the paper manufacturing process, paper is anisotropic. Most of this anisotropy comes from the fibre orientation distribution in the plane of the sheet. The amount of anisotropy depends mostly on the relative velocity of the pulp jet and the wire, the jet-to-wire ratio. Generally, the fibre-orientation distribution shows a
maximum in the direction parallel to the wire velocity, the machine direction (MD). and a minimum in the direction orthogonal to this velocity, the cross-machine direction (CD). This fibre orientation distribution is an important factor affecting the physical properties of paper sheets [22].

In order to control anisotropy, the paper industry requires accurate methods for determining the fibre orientation distribution and its effect on physical properties. Image analysis techniques used to extract information about paper local structure have rarely been employed in the pulp and paper industry due to high cost and complicated techniques required. However, the techniques are powerful engineering tools for the extraction of paper mesostructural properties. Our interest in the variability of paper is concerned with the variance of averages measured in finite inspection zones, in other words, on mesostructural level. Therefore, this study focuses on exploring application of image analysis methods to paper formation evaluation, especially fibre orientation investigation.

2.2.2.2 Fibre Orientation Measurement

Various techniques have been proposed for measuring fibre orientation but, except for the measurement of the orientation of dyed fibres, they are all indirect. It is difficult to analyze quantitatively the relationship between the measured anisotropy and the actual fibre orientation. A background survey of the various methods for fibre
orientation measurement can be found in [23][24][25][26][27][28]. Table 2.2 gives a brief summary.

<table>
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<th>Disadvantages</th>
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<tr>
<td>Zero-span Tensile Strength</td>
<td>The angular variation of mechanical properties</td>
<td>The destructive method is time-consuming and influenced by the anisotropy of fibre floc distribution and drying stresses</td>
<td>Insensitive to interfibre bonding</td>
</tr>
<tr>
<td>Ultrasonic Technique</td>
<td>The ratio of ultrasonic velocity in the machine and the transverse directions</td>
<td>The method has a low resolution and depends on paper density and internal stresses, etc.</td>
<td>Useful for measuring elastic modulus</td>
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<td>X-ray Diffraction &amp; Microwave Transmission</td>
<td>Light microscopy for X-ray; Polarized microwaves for Microwave</td>
<td>The cellulose molecular chain orientation is measured instead of real fibre orientation</td>
<td>Strongly correlated to pulp fibre orientation</td>
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<td>Light Diffusion &amp; Diffraction</td>
<td>Transmitted light intensity</td>
<td>The measurement is sensitive to paper surface and influenced by the light scattering ability</td>
<td>Fast and non-destructive</td>
</tr>
<tr>
<td>Far-infrared Polarization (FIR or SMM)</td>
<td>Polarized far-infrared wave transmission</td>
<td>Multiple light scattering must be avoided to obtain real fibre orientation distribution</td>
<td>High spatial resolution and non-destructive</td>
</tr>
</tbody>
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*Table 2.2 Fibre Orientation Measurements [23][24][25][26][27][28]*
Scharscanski and Dodson [29] recently proposed a new method for analyzing anisotropy in paper from greylevel images obtained by β-radiography. It provides insight into the problem of characterizing anisotropy and local density gradient variability of a stochastic planar structure, for instance, paper. However, the methodology was used only for the estimate of local anisotropy of grammage gradients [2][3][29]. In this study, a new method for characterizing local fibre orientation was developed based on Scharscanski and Dodson's theory [29]. By measuring the direction corresponding to the maximum density gradient, and the magnitude of that gradient, a measure of mean local fibre orientation can be obtained.

To further confirm the new method developed in this study, the results for the global images are compared with those obtained from a polarized far-infrared (FIR) wave transmission measurement [27][28]. The FIR characterization technique is based on the observation of the dichroic behaviour of paper in far-infrared. The underlying principle is that minimum transmission occurs when the fibres are mostly oriented parallel to the plane of polarization of the beam and hence the angle where this minimum is observed indicates the dominant fibre orientation in the sheet [27][28]. The main advantages of the method are that it is nondestructive and highly reproducible.

2.2.2.3 Local Gradient Function – Scharcanski and Dodson’s Theory [29]

Considering paper as a planar stochastic structure with random variables,
Scharcanski and Dodson used a *local gradient function* to describe the anisotropy features of the paper. Through computation of the local gradient as a measure of mass variation, both global and local uniformity and anisotropy of texture, such as paper, can be extracted. By using the *local dominant orientation* and the degree of agreement between the orientation of gradients and this local dominant orientation, defined as *coherence*, the local anisotropy of the paper can be quantified [29]. The method for local fibre orientation developed in this study is based on Scharcanski and Dodson's theory. Thus, here we give a brief review of the original theory.

For the measurement of the mass variation in paper, a digitized radiograph of a paper sample can be used as a digital image. This image can be described as a set of pixels arranged into a matrix. Each pixel is assigned a gray level corresponding to its grammage. The columns and rows of the digital radiograph can be viewed as grammage readings along CD and MD axes respectively.

By averaging each window (5\text{pixel} \times 5\text{pixel}) over the subzone (10\text{pixel} \times 10\text{pixel}), the subzone dominant orientation and its magnitude can be calculated. Local dominant gradient orientation and local coherence can be obtained by averaging each subzone over the entire zone size (200\text{pixel} \times 200\text{pixel}). Fig. 2.1 gives the relationship among window, subzone and zone.
Figure 2.1  Window, Subzone and Zone

Window = 5 pixel x 5 pixel; Subzone = 10 pixel x 10 pixel;
Zone = 200 pixel x 200 pixel

Using this method, local anisotropy and local density variability can be extracted from 2-dimensional digital images (2mm x 2mm or 200 pixel x 200 pixel)
zone in this study) based on local gradient variations. The Prewitt Operator is used in the following gradient calculation. The window gradient magnitude, \( \Delta g \), is defined as:

\[
\Delta g = \sqrt{H^2(x, y) + V^2(x, y)}
\]

2.1

where:

\[
H(x, y) = H_+(x, y) - H_-(x, y)
\]

2.2

\[
V(x, y) = V_+(x, y) - V_-(x, y)
\]

2.3

\[
H_+(x, y) = \frac{1}{w(2w+1)} \sum_{x'=-w}^{x+w} \sum_{y'=-w}^{y+w} g(x', y')
\]

2.4

\[
H_-(x, y) = \frac{1}{w(2w+1)} \sum_{x'=-w}^{x+w} \sum_{y'=-w}^{y+w} g(x', y')
\]

2.5

\[
V_+(x, y) = \frac{1}{w(2w+1)} \sum_{x'=1}^{x-w} \sum_{y'=-w}^{y+w} g(x', y')
\]

2.6

\[
V_-(x, y) = \frac{1}{w(2w+1)} \sum_{x'=-1}^{x+w} \sum_{y'=-w}^{y+w} g(x', y')
\]

2.7

and \( g(x, y) \) denotes the average density in a window about the location \( (x, y) \), \( w \) is the size of the domain of the operator (\( w = 2 \) in this study). At each window location
there is an associated angle $\alpha_j$ which maximizes the magnitude of the gradient (see Fig. 2.1). This angle is obtained by:

$$\alpha_j = \tan^{-1} \frac{V(x,y)}{H(x,y)} \tag{2.8}$$

The distribution of these angles over measured zone ($2mm \times 2mm$ in this study) gives the local anisotropy in grammage gradients. In the case where all angles $\alpha_j$ are equally probable, the sample is isotropic.

Window anisotropy can be represented by the window dominant gradient orientation ($\alpha_j$). In the window dominant gradient orientation, the density tends to vary rapidly and consequently, this orientation has the largest spatial variance in mass. In an image location $(x,y)$, the subzone coherence measures the degree of agreement between the orientation of the gradient in each subzone $10 \text{pixel} \times 10 \text{pixel}$ and the subzone dominant gradient orientation ($\alpha$) which gives the maximum mass variation, as shown in Fig. 2.1. Thus, the subzone coherence $C$ is defined as:

$$C = \frac{1}{N} \sum_{j=1}^{N} |\Delta g_j \cos(\alpha, -\alpha)| \tag{2.9}$$

where,
\( \Delta g_j \): the distribution of density gradient magnitudes at window \( j \)

\( \alpha \): the dominant gradient orientation for each subzone, \( \alpha \in [0, \pi] \)

\( \alpha_j \): the associated dominant gradient orientation at window \( j \), defined by

\( Eq. [2.8], \alpha_j \in [0, \pi] \)

\( N \): the subzone size. \( N = 10 \text{ pixel} \times 10 \text{ pixel} = 2 \times 2 \text{ windows} \)

Each subzone has associated one subzone dominant orientation \( \alpha \). High subzone coherence values indicate large gradient magnitude along the subzone dominant orientation. The value \( C \) is a measure of the strength of the local formation anisotropy.

In \( Eq. [2.9] \), when \( \alpha_j - \alpha \in \left[ \frac{\pi}{2}, \pi \right] \) or \( \left[ -\frac{\pi}{2}, -\pi \right] \), \( \Delta g_j \cos(\alpha_j - \alpha) \leq 0 \). So it is possible to have \( C \leq 0 \), which is contradictory to the negative exponential model for the behaviour of \( C \) where \( C \geq 0 \). Therefore, the subzone coherence should be defined as its absolute value instead [30], which is different from [29].

By maximizing the absolute values of the projections with respect to \( \alpha \), we can find the best estimate for the subzone dominant orientation \( \alpha \).

\[
S = \sum_{j=1}^{N} \Delta g_j^2 \cos^2(\alpha_j - \alpha)
\]

2.10
which is the sum of the square of the projections in a subzone size of $N$.

Differentiating the equation above and setting to zero to find maximum $S$, we obtain:

\[
\tan(2\alpha) = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{\sum_{j=1}^{N} \Delta g_{j}^2 \sin 2\alpha}{\sum_{j=1}^{N} \Delta g_{j}^2 \cos 2\alpha},
\]

where $\alpha$ maximizes $S$ and is the best estimate of the subzone dominant orientation.

Based on this computation, local dominant gradient orientation and local coherence can be obtained by averaging each subzone over the entire zone ($200 \text{ pixel} \times 200 \text{ pixel}$ or $2 \text{ mm} \times 2 \text{ mm}$).

The local gradient analysis gives the local dominant orientation which corresponds to the maximum mass gradient, and the magnitude of that gradient. In the previous study [2][3][29], the local gradient analysis method was used only for the estimate of local anisotropy of mass gradients. In this study, a new implementation for characterizing local fibre orientation was developed based on Scharcanski and Dodson's theory [29] (see Section 3.3).
2.2.3 Relative Bonded Area

2.2.3.1 Introduction

The properties of paper depend on both the arrangement and the properties of the building blocks, i.e., fibres. In this sense, paper is really 'an engineered stochastic structure' [15] rather than a material. Thus the structure and properties of the bonded region affect almost all of the mechanical, optical, thermal and electrical properties [31]. Certainly, bonding is one of the critical factors which determine the weak regions in the sheet [32]. Therefore, in order to increase the minimum local strength of the sheet, first of all, we need to improve our fundamental understanding of the effect of bonded regions on the fibre network. Although many studies have attempted to relate bonding to mechanical properties, such as sheet strength, there have been few studies of the effect of local bonded area on the local properties within a single sample and no studies of the point to point variation of bonded area. Therefore, this work focused on local variation in bonded area and its effect on local tensile strength of microtensile specimen. Hence, a sensitive method to evaluate this variable local structure is necessary.

Bonded area is usually described in terms of the fraction of the fibre surface that is bonded to other fibres, i.e., the relative bonded area or RBA. The usual method for calculating RBA begins with the assumption that portions of a fibre not bonded to another fibre scatter light, while the bonded portions (having no interface with air) do
not. Haselton [33] provided a linear relationship between the scattering coefficient and the surface area which can be described as

\[ RBA = (S_o - S)/S_o \]  \hspace{1cm} (2.12)

where \( S \) is the scattering coefficient of the sheet, and \( S_o \) is the scattering coefficient of the same fibres in a completely unbonded state. The major difficulty here is in obtaining \( S_o \) value since it could be quite different for a wide range of fibre characteristics. Ingmanson and Thode [34] suggested extrapolating a plot of \( S \) against tensile strength to zero tensile strength for a series of sheets made from the same pulp but with different degrees of wet pressing and further calculated RBA. The drawback is that such plots involve long extrapolations. Uesaka [35] showed a linear relation between the sheet modulus and the RBA when \( RBA << 1 \). This means that a plot of \( S \) against modulus should be linear and extrapolate to \( S_o \) at zero modulus. Stratton [36] measured RBA by using the in-plane longitudinal (ultrasonic) stiffness. However, for these methods, a large range of bonded area was obtained by changing wet pressure. Therefore, they are global variations instead of local ones. There have been no studies related to local RBA measurement. Moreover, for the 2
\textit{mm} × 2
\textit{mm} testing zone which is used in this study, it is impossible to measure the absolute value of local RBA.

Wu and Kortschot et al. [32] developed a novel method to compute the variability in local bonded area by introducing the \textit{relative} local RBA, i.e., the
bonding index \((T/T_g)\), instead of an \textit{absolute} value of RBA. A detailed review can be found in Section 2.2.3.3.

2.2.3.2 Relative Bonded Area Measurement

<table>
<thead>
<tr>
<th>Methods</th>
<th>Principles</th>
<th>Disadvantages</th>
<th>Advantages</th>
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<tbody>
<tr>
<td>Direct Microscopic Observation</td>
<td>Light and electron microscopy over the cross-section of paper</td>
<td>Destructive, affected by surface fibres and difficult to get clear images of paper</td>
<td>Direct observation of the nature of fibre-fibre bonds</td>
</tr>
<tr>
<td>Gas Adsorption &amp; Optical Scattering methods</td>
<td>Measurement of the surface area of the constituent fibres</td>
<td>Time-consuming and limited by several assumptions</td>
<td>Non-destructive measurement</td>
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<tr>
<td>Digitizing method</td>
<td>Analysis of photomicrographs of sheet cross-section on a digitizer</td>
<td>Difficult to identify bonded areas, time-consuming and destructive</td>
<td>Analysis of internal fibre network geometry</td>
</tr>
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\textit{Table 2.3 Relative Bonded Area Measurement [37][38][39]}

Methods for measuring the bonding properties of paper can be divided into four categories: microscopic observation; gas adsorption and optical scattering; digitizing; and other methods [37][38][39], as shown in Table 2.3. Obviously, the applications of these existing methods are limited by their complicated, destructive,
or time-consuming measurements. To investigate local variations in bonded area instead of global variations, Wu and Kortschot's method [32] was adopted in this study by using a high quality image acquisition system. The local bonded area measurement is based on a simple comparison of local grammage (obtained by β-radiography) and local light transmission within a single sample.

2.2.3.3 Bonding Index Theory and Implementation [32]

Wu and Kortschot et al. [32] simplified the problem of characterizing bonding by using a relative value of local RBA, termed the bonding index, instead of an absolute value of RBA. On the basis of comparisons within a single sample of paper, the bonding index provides a suitable estimation of local RBA. In this study, the local bonding index [32] has been determined and compared to the local strength for sheets with constant grammage. Also, the results of global variations in bonding index obtained by changing wet pressure for handsheets are presented in this study to further confirm the method.

The bonding index measurement is based on a comparison of local grammage (obtained by β-radiography) and local light transmission [32]. The underlying principle is that more poorly bonded regions permit less light transmission than highly bonded regions with equivalent grammage. Therefore, for the regions with the same grammage, the ratio of β-ray to optical transmission can be directly interpreted as the relative local bonded area, referred to as the bonding index \( T/T_g \) in this
study. Although it does not provide an absolute value of RBA, theoretically, this measurement of local bonding should be able to identify relatively weak regions by comparing the bonded area of regions with different grammage in a single sheet.

\[ y = 1348.4x^{0.5803} \]

\[ R^2 = 0.952 \]

**Figure 2.2**  *Transmitted light vs. Grammage for Krafi-pulp Handsheets [32]*

In reference [32], an experimental normalization procedure was developed to compare the bonded index of regions with differing grammage. First, the light transmission is plotted as a function of grammage for all specimens, as shown in Fig. 2.2. If the relative bonded area was constant everywhere in a sheet, as expected, all the data would fall on the regression curve through these data. The local bonding index was deduced by dividing the light transmission \( T \) in each specimen by the mean value of light transmission for all specimens of the same grammage \( T_g \).

Therefore, any deviation from the regression line should, in theory, represent local
fluctuations in bonded area.

In Fig. 2.3, the ratio of the transmitted light intensity to the mean value for each grammage is plotted as a function of grammage. This ratio is proportional to the local relative bonded area and hence it can represent the local bonding index of each specimen. Areas with greater relative bonded area, i.e., higher value of $T/T_g$, are more transparent than the others of equivalent grammage. The bonding index as a measure of the bonded area is limited to comparisons within a single sheet. This measurement of local bonding should identify fluctuations in local bonded area. Furthermore, microtensile specimens can be used to determine if the breaking length is correlated with the local bonding determined in this way.
2.3 STRENGTH MODELS

Paper is a bonded random network of natural cellulosic fibres. Because of its inhomogeneous structure, it is difficult for paper scientists to predict paper failure. The mechanism of failure depends on the balance between the strength of the fibres and the strength of the fibre-fibre bonds. Failure is usually initiated by bond failure in weakly bonded paper and by fibre failure in strongly bonded paper [40][41]. In other words, the number of broken fibres across the failure line of the sheet is greater than the number of fibres pulled out for a strongly bonded sheet; whereas, fibre pull out dominates in weakly bonded sheet [42].

Many attempts have been made to predict the tensile strength of paper from its structure by means of a closed-form network model [42]. The pioneering research of Cox [43] provided the first micromechanical structural model of paper in 1952. The model considered paper as a network of continuous fibres randomly distributed in a two-dimensional planar mat with a known angular distribution function. Based on this concept of an ideal paper sheet, the Cox model accounted for the effect of fibre orientation on the stiffness and strength of paper. Since then, the Cox model has been cited extensively by both paper scientists and composite material researchers. Several variations of the Cox-type models have been developed. A comprehensive review of these closed-form network models for the tensile strength of paper can be found in Jayaraman and Kortschot’s paper [42]. Here we only give a brief summary.
Kane [44][45] provided the first mathematical analysis for the tensile strength of paper by calculating the amount of force required to create a rupture line resulting in the breaking or pulling out of fibres across the failure line. New concepts, such as the "critical fibre length" and the "effective length" used in Kane model, were widely accepted in paper physics and fibre-reinforced composite material fields later on. Kallmes et al. [5] expanded the original Cox model by considering the behaviour of inter-fibre bonds and using a definition of critical event in the sheet: either fibre failure or bond failure. This model used sheet strain as the basis for its failure criterion and was based on the concept of an ideal paper sheet with defined fibre orientation as postulated by Cox [43].

Page developed a semi-empirical expression for the breaking length of a sheet with random fibre orientation, which has been widely cited [4]. It incorporated the fibre strength and the bond strength in a simple closed-form equation, known as the Page equation.

By considering the tensile strength as a function of the force required to both pull out and break fibres spanning a rupture line. Shallhorn and Karnis [46] incorporated that shear strength of the bonds between fibres in their prediction of tensile strength. Their model gives some trends in relationship between tensile strength and microstructural variables such as fibre length and RBA, but is limited by the assumption of all the fibres in the sheet are oriented perpendicular to the rupture
line. Most recently, Kärenlampi [47][48] expanded models for changed distribution of fibre properties by means of statistical simulation.

These models all use a simplified microstructure and assume that the fibres are ideal by ignoring the effect of curl or crimps. Although the simplification of the microstructure and idealization of fibre and bond properties was applied to make the problem solvable, it is the major limitation of the existing network models. Also, a complete test of these models is quite difficult since it requires the measurement of many micromechanical parameters, such as fibre, bond and sheet properties over a wide range of conditions. In particular, the interfibre bond strength and relative bonded area are notoriously difficult to measure. As a result, it is exceedingly difficult to produce quantitative predictions for tensile strength as a function of the structural variables of paper. Therefore, the applications of these models in material engineering fields are limited.

However, the theory together with the present models can be used to predict trends associated with changes in the distribution of fibre properties, such as fibre orientation and bond properties. The theory can be verified experimentally through such treatments, and can further improve our understanding of the origin of the properties of paper. In this study, based on the experimental results from both global and local variations in paper structure, we intend to analyze the effect of the local structural properties on the tensile strength and develop the existing models to predict
the tensile strength of paper so that different kinds of distributions of structural properties may then be inserted.

In this study, the fundamental work of Cox is applied to explore the effect of fibre orientation on the tensile strength. The study of RBA is based on the Kane theory and the Page equation which have been widely accepted to predict tensile strength of a sheet by including the fibre strength and the bond strength in a simple closed form equation. The expanded implementations of these theories are introduced in Section 3.3 and Section 3.4 respectively.
CHAPTER THREE
THEORETICAL

3.1 INTRODUCTION

In this study, an effort is made to develop a correlation between breaking length and local structural variations in grammage, fibre orientation and RBA at the millimetre scale. The quantitative correlation between these local structural variations and local mechanical properties, such as tensile strength, has not been studied previously. Since these structural factors cannot be varied independently in experiments, a comprehensive model which includes all the three variables is necessary to predict the local strength. Therefore, in our study, the paper strength modelling concentrates on predicting relative changes in breaking length based on changes in local grammage, local fibre orientation and local RBA. Eventually, we intend to make a prediction of sheet failure using relative strength models based on these local structural properties. In short, we expect to build up such relationships as:

\[ \text{Predicted Tensile Strength} = f (\text{grammage}, \text{fibre orientation}, \text{RBA}) \]
3.2 GRAMMAGE

The effect of grammage on mechanical properties has been recognized for a long time. Obviously, force per unit width at failure, $P_b (kN/m)$, should increase as grammage increases, which can be described as

$$P_b = \frac{F_b}{w} \propto \text{Grammage}$$  \hspace{1cm} (3.1)

where, $F_b$ is breaking force, $N$; $w$ is the width of the specimen, $mm$. As a standardized measure of tensile strength, breaking length ($km$) can be calculated as

$$\text{Breaking Length} = \frac{F_b \cdot w}{9.8 \times \bar{g}}$$  \hspace{1cm} (3.2)

where, $\bar{g}$ is the mean grammage over the entire zone of interest for each specimen, $kg/m^2$. Through dividing by the grammage value, the trivial effect of compaction is factored out of the calculation of strength, and the force is normalized by the cross-sectional area of load bearing fibres.

From previous studies, it is known that breaking length is reduced as grammage variability, i.e., the coefficient of variation of grammage ($CV_{\text{var}}$), increases.
In order to minimize the zone size of specimens so that all aspects of structure would be constant over the zone, microtensile specimens with a 2\text{mm} \times 2\text{mm} testing zone were used in this study instead of standard tensile testing samples. The square millimetre scale was believed to be most appropriate primarily because it is of the same order of magnitude as the fibre length [20][49]. Therefore, the distribution of grammage can be regarded as uniform and the mean values can be used over the zone.

3.3 FIBRE ORIENTATION FACTOR

3.3.1 Introduction

Many attempts have been made to incorporate the fibre orientation factor in paper strength models (see Section 2.3). In this study, by using Cox's theory for fibre orientation factor and a new measurement (see Section 4.4), local variations in fibre orientation on the mesostructural level are quantified. Furthermore, the relationship between local tensile strength and local fibre orientation is investigated. Variations in \textit{global} fibre orientation are examined by comparing specimens cut from machine made paper in the MD to CD directions. In the case of specimens for all along CD or MD direction within a single machine made paper, or for a random handsheet, point-to-point variations in the \textit{local} orientation are examined.

In order to compare fibre orientation to strength, the focus of this study was to
characterize fibre orientation in a way relating orientation to the sheet strength directly. Many attempts have been made to model this relationship. The fundamental point of Cox theory as well as the improved theories is to calculate the force sustained by the fibres across a scan line. The Cox model was derived for an ideal paper using a force formulation and was based on several assumptions, which are discussed in Jayaraman and Kortschot's paper [42]. A modified Cox theory, introduced by Kallmes and Perez [50] and Kallmes et al. [5] is developed below. The specific approach used in this study is based on work done by Jayaraman and Kortschot [42].

3.3.2 General Theory [51]

The specimen is in the shape of a thin, rectangular plate with dimensions $a$, $b$ and $c$, where dimension $c$ is parallel to the $X$-axis direction as shown in Fig. 3.1. The fibre orientation is defined as the angle between the axial direction of the fibre and the $X$-axis direction, $\theta$. $N$ straight fibres are bonded together and dispersed in the specimen with a fibre volume fraction $V_f$. The independent probability density functions, $h(L)$ and $g(\theta)$, represent the statistical variation in the length and orientation of the fibres respectively. Applied load on the specimen is parallel to the $X$-axis direction with strain, $\varepsilon_x$. Here the strain is underlying sheet strain including the total strains of specimen together with strain in fibre. Based on this definition, the average axial stress in a fibre of length $L$ and orientation $\theta$ in the specimen is given by
Fukuda and Kawata [52] as

$$\bar{\sigma}_f = \phi E_f \varepsilon_a (\cos^2 \theta - \nu, \sin^2 \theta)$$  \hspace{1cm} (3.3)

where,

\(\phi\) : a dimensionless function of fibre length and other fibre, matrix and specimen properties; for very long fibres, \(\phi = 1\)
\( E_r \): Young's modulus of the fibre. \( N/m^2 \)

\( \varepsilon_o \): strain applied to the specimen

\( \nu \): Poisson's ratio of the specimen

The average axial force in a fibre of length \( L \) and orientation \( \theta \) is \( \overline{F}_r \). And the average axial force projected in the direction of the applied strain (\( X \)-axis direction) is

\[
F_i = \overline{F}_r \cos \theta = A_i \overline{\sigma}, \cos \theta = A_i \phi \varepsilon_o (\cos^2 \theta - \nu \sin^2 \theta) \cos \theta
\]

where,

\( A_i \): the area of cross-section of the fibre. \( m^2 \)

For the two-dimensional probability density function of fibre orientation distribution, assuming that the distribution of fibre orientation is symmetric with respect to the applied strain along the \( X \)-axis direction, there is

\[
\int_0^{\pi/2} g(\theta) d\theta = 1
\]

\[
\int_0^\infty h(L) dL = 1
\]
Thus, the number of fibres of length between $L$ and $(L + dL)$, orientation between $\theta$ and $(\theta + d\theta)$ is

$$N_{L,\theta} = Nh(L)dLg(\theta)d\theta = \frac{V',abc}{A, L} \times h(L)dL \times g(\theta)d\theta$$ \hspace{1cm} 3.7$$

where,

$\bar{L}$: the average fibre length, i.e., $\bar{L} = \int_0^L h(L)dL$, $m$

$V_f$ : the fibre volume fraction, i.e., $V_f = \frac{NA_f \bar{L}}{abc}$

Thus, the number of fibres of length between $L$ and $(L + dL)$ and orientation between $\theta$ and $(\theta + d\theta)$ that cross a scan line is

$$N_{\text{scan}} = \frac{L_r}{c} = \frac{V',ab}{A, \bar{L}} \times h(L)dL \times g(\theta)d\theta \cos \theta$$ \hspace{1cm} 3.8$$

where,

$L_r$ : the total length of projection on the X-axis of the $N_{L,\theta}$. $L_r = L \cos \theta$, $m$

Then, the total load that fibres crossing the scan line carry is
\[ F_z = \sum_{L} \sum_{\theta} N_{\text{scan}} F_z \]

\[ = \int_0^{\pi/2} \int_0^\pi V_{iJ} \frac{\phi_{L}}{A_{ij}} h(L) dLg(\theta) d\theta d\theta \cos \theta \times [A_{ij}, \phi E_r, \varepsilon_0 (\cos^2 \theta - \nu, \sin^2 \theta) \cos \theta] \]

\[ = \frac{E_r V_{ij} \varepsilon_0 ab}{L} \int_0^{\pi/2} \phi L h(L) g(\theta) (\cos^4 \theta - \nu, \sin^2 \theta \cos^2 \theta) d\theta dL \quad 3.9 \]

Since \( h(L) \) and \( g(\theta) \) are independent of each other, Jayaraman and Kortschot [42] defined two new terms to characterize the local structure. Then the following equation can be obtained:

\[ F_r = E_r V_{ij} \varepsilon_0 [\int_0^{\pi/2} g(\theta) (\cos^4 \theta - \nu, \sin^2 \theta \cos^2 \theta) d\theta \times \left[ \frac{1}{L} \int_0^{\pi} \phi L h(L) dL \right] \]

\[ = E_r V_{ij} \varepsilon_0 ab \times [C_{rr}] \times [D_{rr}] \quad 3.10 \]

where \( C_{rr} \) is the fibre orientation factor; \( D_{rr} \) is the fibre length factor, defined as:

\[ C_{rr} = \int_0^{\pi/2} g(\theta) (\cos^4 \theta - \nu, \sin^2 \theta \cos^2 \theta) d\theta \quad 3.11 \]

\[ D_{rr} = \frac{1}{L} \int_0^{\pi} \phi L h(L) dL \quad 3.12 \]
In the limiting cases, if all the fibres in the specimen are aligned with the X-axis, i.e., for all fibres: $\theta$ equals 0. Eq. [3.11] leads to

$$ C_{pp} = 1 $$

For a sheet with randomly orientated fibres, the value of $C_{pp}$ is $1/3$ [43][51]. The values of Poisson ratio ($\nu_r$) are discussed in Section 3.3.3.

Based on the mechanical interpretation (see Eq. [3.11]). $C_{pp}$ is the single parameter characterizing $g(\theta)$ which should be best correlated to strength. Therefore, $C_{pp}$ introduced here was to build up a proportional correlation between fibre orientation and paper strength for the relative strength model developed in this study (see Section 3.5).

The calculation of fibre length factor $D_{pp}$ is very complicated since the microstructural dimensions and shear modulus of matrix need to be computed. In this study, since $h(L)$ will not vary from specimen to specimen. $D_{pp}$ is assumed to be constant from point to point. Therefore, only the proportional relationship between breaking length and fibre orientation factor $C_{pp}$ is used in the relative strength model.

### 3.3.3 Local Dominant Fibre Orientation Analysis, SumC90 and $C_{pp}$

A new method developed in this study to characterize the local fibre
orientation is based on an analysis of the mass variation. As introduced in Section 2.2.2.3, from the grammage map for each specimen, local gradient and local dominant orientation are analyzed based on the Scharscanski and Dodson's theory [29]. The local dominant gradient orientation plot is realized by the Quiver Plot (see Fig. 3.2), which is built-in function of MATLAB (see Section 4.3 and 4.4). By converting the local dominant gradient orientation and normalizing its magnitude, we obtained a Polar Plot of local dominant gradient orientation by using localGrad.m program (see Appendix B), as shown in Fig. 3.3.

In Fig. 3.2, the magnitudes of the arrows represent the subzone coherence (C) values: the angles of them are the subzone dominant orientations which correspond to the maximum mass gradients. For example, \( i \) arrow denotes the subzone dominant...
orientation and its length gives the magnitude along that direction.

![Local Dominant Gradient Orientation Polar Plot of TMP Newsprint](image)

**Figure 3.3** *Local Dominant Gradient Orientation Polar Plot of TMP Newsprint 1452-26-2 (MD) in 2mm x 2mm (Specimen#21)*

In this application, the length in each radial direction denotes the probability of subzone dominant orientation vectors along this radial direction.

Our interest here is the local mean fibre orientation in the whole testing region. As introduced in Section 2.2.2, the underlying principle is that in an anisotropic sheet, there will be higher gradients in grammage in a direction perpendicular to the direction of fibre orientation, i.e., cross-fibre direction. Therefore, local fibre orientation (θ) can be estimated by simply rotating the local mass gradient orientation (α) by 90°, shown as
\[ \theta = \alpha - \frac{\pi}{2} \] 3.13

**SumC90** is defined as a simple summation with respect to the direction of applied tensile stress. In the *Polar Plot of Local Dominant Orientation* (see Fig. 3.4), *SumC90* is described as the sum of the projection of each point on the plot on 90° direction. In other words, it can be computed by the sum of the projections of each arrow, for example, in the *Quiver Plot of Local Dominant Orientation* (see Fig. 3.2) on 90° and is then normalized over the entire zone. The computation of *SumC90* can be realized by using MATLAB program (see Appendix B). Based on this definition, *SumC90* represents the mean fibre orientation with respect to the tensile axis in the entire region of interest and the results measured in this way have been compared to the local tensile strength in a microtensile specimen.

In *Eq. [3.11]*, the probability density of the fibre orientation, \( g(\theta) \), can be approximated as the probability of local dominant gradient (mass) orientation, which equals the radius along \( \alpha \) on the *Polar Plot of Local Dominant Orientation* (see Fig. 3.3). Then, by summation from \(-\frac{\pi}{2}\) to \(\frac{\pi}{2}\) over the testing zone instead of the integral in *Eq. [3.11]*, the fibre orientation factor \( C_{pp} \) is obtained for each specimen. The computation of \( C_{pp} \) is realized by using MATLAB program (see Appendix B).
SumC90 is a simple sum of the projections of local dominant gradient orientation on 90°, i.e., the mean fibre orientation with respect to the tensile axis in the entire zone. Whereas, based on mechanical interpretation in Cox Theory, \( C_{pp} \) is defined as the theoretical contribution of fibre orientation distribution to tensile strength.

In the limiting case of a specimen with unidirectional orientation along the testing direction (see Fig 3.4), i.e., \( \theta_i = 0°, \alpha_i = 90° \) for all fibres, then \( \text{SumC90} = 1, C_{pp} = 1 \). On the other hand, if all the fibres are transverse to the testing direction, theoretically \( \text{SumC90} = 0, C_{pp} = 0 \).

As introduced in Section 3.3.2, \( D_{pp} \) can be treated as a constant since there should be no significant difference in \( h(L) \) from specimen to specimen. Therefore, \( D_{pp} \) is not considered as a structural variable. From Eq. [3.10], we can get the relationship as:

\[
F_r \propto C_{pp} \quad \Rightarrow \quad T \propto C_{pp}
\]

where, \( T \) is the breaking length of a specimen.
In this approach, the Poisson’s ratio of the specimen is considered to be $1/3$ for a sheet with linear elastic fibres and random fibre orientation distribution [43]. However, for oriented paper, especially commercial newsprint paper, the value of Poisson’s ratio is not $1/3$ any more. Furthermore, later work [53][54][55] has conclusively shown that the Poisson’s ratio of a sheet is dependent on the applied strain and the fibril angles of the individual fibres. Thus, the assumption that Poisson’s ratio of a sheet will equal $1/3$ will introduce an error in the expressions for strength provided here. To simplify the calculation in this study, we only consider three cases for the variation in Poisson’s ratio of a sheet.
1. In the case of regular handsheets, since random fibre orientation distribution can be assumed, as a general principle, the variation in Poisson's ratio is ignored and taken as a constant value of $1/3$.

2. In the case of machine made paper on the CD direction which corresponds to $SumC90 = 0$ theoretically, the Poisson's ratio is assumed to be 0.

3. In the case of machine made paper on the MD direction which corresponds to $SumC90 = 1$ theoretically, the Poisson's ratio is also taken as $1/3$.

3.4 RELATIVE BONDED AREA

3.4.1 Introduction

The mechanism of failure depends on the balance between the strength of the fibres and the strength of fibre-fibre bonds. Thus RBA is an important variable to predict the strength and the stress-strain behaviour of paper. Based on the bonding index theory (see Section 2.2.3), local variations in bonding have been represented. By combining the Page equation and Kane theory, a relationship between local tensile strength and local RBA is built up. In this study, the mean bonding index ($T/T_x$) over the entire testing zone ($2mm \times 2mm$) is used to represent local RBA value for each specimen. Global variations in RBA were obtained by changing wet pressure for handsheets.
3.4.2 General Theory

The Page equation is also based on the scan line concept introduced by Cox [43]. There are two important premises resulting in this semi-empirical expression for the breaking length of a sheet with random network structure. The first premise is that only the fraction of fibres that break across the failure line bear stress at rupture and the strength is therefore based only on the zero-span strength and the fraction of fibres that break. Using these assumptions, the mathematical expression can be derived

\[ T = Z_c \frac{n_f}{(n_f + n_p)} \]  

where,

- \( n_f \): the number of fibres crossing the failure line that take the load at failure and then break
- \( n_p \): the number of fibres crossing the failure line that pull out intact due to prior bond breakage and hence carry no load at failure
- \( T \): breaking length of the sheet, \( km \)
- \( Z_c \): finite-span tensile strength of the strip expressed as breaking length if no bond breakage had occurred, \( km \)

Here, \( Z_c \) can be obtained from the zero-span tensile test since it was assumed that the formation of the sheet is uniform and hence the number of fibres crossing the failure
line is same as the number that cross any scan line. The relationship is

\[ Z_c = Z(1 - \nu_i^2) \]  

where,

\( \nu_i \): the Poisson's ratio

\( Z \): zero-span breaking length of the sheet, km

For a sheet with random fibre orientation distribution, \( \nu_i \) is theoretically 1/3. Eq. [3.15] may be expressed as follows:

\[ T = \frac{8Z}{9} \times \frac{n_f}{(n_f + n_p)} \]  

The second premise is that the ratio of pulled-out fibres to broken fibres across the failure line is dependent only on the ratio of fibre strength to bond strength. This may be expressed as

\[ \frac{n_p}{n_f} = f(\phi/\beta) \]  

where,
\[ \phi : \text{mean fibre strength, } N/m^2 \]
\[ \beta : \text{mean force applied along the fibre axis and across the failure line required to pull a fibre from the sheet, } N \]
\[ f : \text{dimensionless function} \]

Here, \( \beta \) is a function of the relative bonded area of a sheet (RBA), the bond strength per unit area, and fibre length. Assuming that all fibre-to-fibre bonds act cooperatively along the length of a fibre, the bond strength \( \beta \) is given by:

\[ \beta = \tau_s P \frac{L}{4} (RBA) \quad 3.19 \]

where,
\[ \tau_s : \text{shear strength of the inter-fibre bonds, } N/m^2 \]
\[ P : \text{perimeter of the cross-section of fibres, } m \]
\[ L : \text{length of the fibre, and hence } L/4 \text{ is the mean pulled length, } m \]
\[ RBA : \text{relative bonded area of the sheet} \]

Through some assumptions and simplifications [4], the Page equation can be finally defined as
where,

\[ \frac{1}{T} = \frac{9}{8Z} + \frac{12A_f \rho g}{\tau_b PL(RBA)} \] 3.20

\[ \rho : \text{ density of the fibre, } g/m^3 \]

\[ g : \text{ acceleration due to gravity, } m/s^2 \]

\[ A_f : \text{ average area of the cross-section of fibres, } m^2 \]

The Page equation is semi-empirical and based on the theory of zero-span strength and two important premises. Although these are the major limitations of the equation, it has been proven remarkably effective in predicting experimental trends of tensile strength of paper by including fibre strength and bond strength in a simple closed form equation.

3.4.3 Implementation

From the definition of \( n_f \) and \( n_p \) in the Page equation, we can define

\[ \frac{n_f}{n_f + n_p} = \frac{n_f}{n_i} = f(\tau_b, \sigma_f, RBA, L, P, A_f) \] 3.21

where,

\[ \sigma_f : \text{ average tensile strength in a fibre of length } L, \text{ cross-section } P, \text{ average} \]
area of the cross-section $A$, $N/m^2$

$n_f$: the number of the fibres spanning the failure line, $n_f = n_p + n_r$

Here, the total number of fibres crossing the failure line, $n_f$, includes broken fibres and pulled-out fibres. The paper model considered here is a thin, randomly oriented, fibre-reinforced network structure. The tensile failure load for the sheets is obtained by summing the force required to either break or pull out the fibres crossing the failure line. In Page equation, a proportional relationship was built up between $\left( \frac{1}{T} \right)$ and $\left( \frac{1}{RBA} \right)$. In order to combine RBA with fibre orientation factor ($C_{pp}$) in one equation, a direct relationship between breaking length and local RBA ($T/T_k$) was expected. Thus, different from Page equation, Kane theory [45] was applied in the later sections. We consider that fibres across the failure line break or pull out depending on their embedded length $l$ [44][45]. In the case of the failure due to fibre pull out, the fibre embedded length is less than half the critical length, $l_c$. $l_c$ is the critical value of fibre length when the force for fibre pull out equates to the force for fibre breakage crossing the failure line. Fibres will break when the embedded length is greater than half the critical length. For weakly bonded sheets, all the fibres crossing the failure line will pull out; while for strongly bonded sheets, both fibre breaks and pullouts can occur. Fig. 3.5 gives the physical interpretation of the theory.
Figure 3.5  Broken and Pulled-out fibres crossing the failure line during the rupture event

When the fibre embedded length is equal to half the critical length, i.e., fibre $A$ in Fig. 3.5, the forces required for fibre pull out and fibre breakage across the failure line have the same value. At this point, we have

$$l_c \times P \times RBA \times \tau_b = \sigma_f \times A_f$$  \hspace{1cm} 3.22

where,
\( l_c \): critical value of fibre length when the force for fibre pull out equates to the force for fibre breakage crossing the failure line, \( m \)

Rearranging Eq. [3.22]

\[
l_c = \frac{\sigma_f A_f}{\tau_g P} \times \frac{1}{RBA}
\]

By applying the Kane theory [45], the following expression can be derived

\[
\frac{n_f}{n_t} = \frac{n_f}{n_r + n_p} = \frac{L - 2 - l_c}{L} \\
= \frac{L}{2} - \frac{\sigma_f A_f}{\tau_g P} \times \frac{1}{RBA} \times \frac{L}{2} \\
= 1 - \frac{2\sigma_f A_f}{\tau_g LP} \times \frac{1}{RBA}
\]

Here, \( \left( \frac{n_f}{n_r + n_p} \right) \) represents the fraction of fibres crossing the failure line with effective length longer than half the critical length \( l_c \), which all break. \((L/2)\) is half the mean embedded length of \( N_f \) fibres which is the number of fibres across the
failure line. Then mathematically, \( \left( \frac{L_2 - l_e}{L_2} \right) \) can be regarded as the fraction of fibres with the embedded length longer than half the critical length which is equal to \( \left( \frac{n_f}{n_f + n_p} \right) \). Combining Eq. [3.17] and Eq. [3.24], we arrived at final equation for the tensile strength of paper:

\[
T = \frac{8Z}{9} \left( \frac{n_f}{n_f + n_p} \right) = \frac{8Z}{9} \left( 1 - \frac{2\sigma_f A_f}{\tau_n LP} \times \frac{1}{RBA} \right)
\]

In this new approach, there are several assumptions built into the theoretical result:

1) The approach is based on a thin, randomly oriented, fibre-reinforced network structure with a Poisson's ratio of 1/3. "A scan line" perpendicular to the direction of applied load is introduced into this structural model. And the uniform structure is obtained by assuming that the number of fibres crossing any scan line, e.g., the failure line, is approximately the same and all the bonds act cooperatively along the length of a fibre.

2) Only the fraction of fibres that break across the failure line bear stress at rupture and the strength of the sheet is therefore dependent only on the zero-span strength and the fraction of fibres that break. This came from the first
premise of the *Page* equation.

3) The number of fibres perpendicular to the failure line is only considered here to contribute to the tensile strength of paper. Thus, the effect of fibre orientation on the tensile strength of paper is not fully taken into account.

Thus in this study, the implementation of Page’s and Kane’s theories is limited to only two cases: the fibres that carry load at the instant of failure and break across the failure line; the fibres that pull out due to bond breakage and therefore carry no load at failure. However, in real sheets, especially for weakly bonded sheets, there are a certain percentage of fibres that carry load at the instant of rupture but subsequently pull out at failure. Furthermore, this kind of theories is based on a “scan line” model that assume the number of fibres crossing any line in the sheet transverse to the direction of applied load is approximately the same and all fibre-to-fibre bonds act cooperatively along the length of a fibre. In this case, the effect of fibre imperfections, such as microcompressions, curls and crimps, was ignored. However, in real sheets, this kind of microstructural features changes the stress-strain behaviour of the fibre. These are the major limitations of this study.

From this approach, we intend to correlate the relationship between the tensile strength of paper and *RBA*. Then, from *Eq. [3.25]*, we have
Here, \( k (k > 0) \) is a function of the microstructural parameters, i.e., \( \left( \frac{2A_r}{LP} \right) \), and the fibre and bond properties, i.e., \( \left( \frac{\sigma_r}{\tau_b} \right) \). Since this study mainly focuses on the impact of local structural variations in fibre orientation and RBA on the tensile strength of paper, an effort is made to correlate these two local structural properties with the local tensile strength in a single sheet and \( k \) can be treated as a constant. As a result, instead of an absolute correlation between the tensile strength and local RBA, a proportional relationship referred to Eq. [3.26] is obtained.

### 3.5 RELATIVE STRENGTH MODEL

In this study, the problem of modelling local bonding and local fibre orientation simultaneously can be simplified by combining Eq. [3.14] and Eq. [3.26].

From

\[
F_r \propto C_{pp} \quad \Rightarrow \quad T \propto C_{pp} \quad \text{3.14}
\]

\[
T \propto (1 - \frac{k}{RBA}) \quad \text{3.26}
\]
then, we have the final expression for the tensile strength or breaking length:

\[ T \propto C_{pp}(1 - \frac{k}{RBA}) \quad \text{or} \]

\[ \text{Breaking Length} \propto C_{pp}(1 - \frac{k}{RBA}) \quad 3.27 \]

Here, to simplify measurement and calculation, \( k \) is considered as an unknown constant value. It can be calculated by plotting \( \frac{T}{C_{pp}} \) against \( \frac{1}{RBA} \) with the maximum correlation coefficient value. Thus, we have

\[ \text{Predicted Breaking Length} = mC_{pp}\left(1 - \frac{k}{RBA}\right) \quad 3.28 \]

where \( m \) is an unknown constant. The comparison between the experimental data and theoretical results is obtained by plotting \( \text{Breaking Length (Exp.)} \) vs. \( \text{Predicted Breaking Length} \). Again, since we intend to correlate the variability in local tensile strength to local structural features of the sheet, the absolute value of \( RBA \) or \( \text{breaking length} \) is not required. Thus, the local bonding index \( (T/T_x) \) is used instead of local RBA and the calculation of \( m \) is ignored in this study. The Eq. [3.28] may be simplified:
\[ \text{Predicted Breaking Length} \propto C_m \left(1 - \frac{k}{\left(\frac{T}{T_i}\right)}\right) \]
4.1 PREPARATION OF PAPER SPECIMENS

Standard handsheet specimens used in this study were made from softwood Kraft pulp (Aspen) according to CPPA Standard C4. There was one modification in the procedure: in order to control the formation of the sheets, flocculation was increased from standard isotropic handsheets (no settling time) by introducing a two-minute settling time before drainage of water from the handsheet maker.

By modifying CPPA Standard C4, which involves wet pressing the sheets at 400psi, handsheet specimens with changing wet pressure were prepared from softwood Kraft pulp (Aspen) at a series of pressing pressures (300psi, 400psi, and 500psi) to change the global mean bonded areas.

Handsheets with local orientations were obtained from softwood Kraft pulp (Aspen) by stirring the water and introducing vortices, just before draining the handsheet maker. The images of oriented handsheets were analyzed by both the new method developed in this study for the estimation of fibre orientation (see Section 3.3 and Section 4.4) and the polarized far-infrared wave transmission measurement (FIR)
method [27][28].

Two kinds of TMP Newsprint (1452-26-2 and 1452-17-4) machine made paper were used in this study. The specimens were cut along the cross-machine direction (CD) and machine direction (MD) respectively by using Lab made micro-tensile sample cutter with the testing zone of $2mm \times 2mm$.

4.2 METHODOLOGY

In Fig. 4.1. the methodology of this study is presented.
Figure 4.1 Methodology
4.3 GRAMMAGE MEASUREMENT

The local grammage distribution in each sheet was measured using video beta-radiography (VBR). In Fig. 4.2, VBR is a two-stage process consisting of β-radiography and digital image processing. It is a standard technique developed by Ng and Dodson [56]. Specifications of the equipment can be found in Appendix A. β-radiography was taken and calibrated with a Mylar step wedge for the paper sample. By using Optimas image analysis software running on a Pentium computer, the radiographs were digitized with a Kodak Megaplus camera. A thresholding operation was then applied to these digital images with a moving window with the size of 10 grey levels to identify regions of relatively uniform grammage, as shown in Fig. 4.3. Based on the threshold results, microtensile specimens (see Fig. 4.4) with relatively
constant grammage in their central $2mm \times 2mm$ zone were identified and cut by a specially designed micro-tensile sample cutter. These specimens were then radiographed and digitized to produce a grammage map individually. Resolution of the resulting grammage maps is $10 \ pixels/mm$ for big paper sample ($100mm \times 74mm$) and $114 \ pixels/mm$ for microtensile specimen ($2mm \times 2mm$).

![Selected Threshold (10)](image)

*Figure 4.3  Histogram of the Threshold*

![Microtensile Specimen](image)

*Figure 4.4  Microtensile Specimen*
From the video image of the microtensile specimen, the texture-related information in the greyscale image is digitized and recorded as a bitmap of grey levels, which is in fact a two-dimensional histogram of image based on the density variation. To extract local variations in grammage, we have developed a recursive algorithm for the computation of local gradients based on Scharscanski and Dodson’s theory (see Section 2.2.2).

MATLAB programming is used in this study to implement this image and signal processing on the grammage maps. The grey scale image is converted to grammage map using the calibration curve. Thus grammage of the calibration wedges and their corresponding gray levels on the β-radiograph form a group of calibration values in the format: (gray level, grammage). Polynomial fitting is used to connect these points into a curve, as shown in Fig. 4.5. With this calibration curve, gray scale image can be converted to a corresponding grammage map. The computed results of mean grammage (\( \bar{g} \) and \( CV_{g} \)) for each specimen are used for strength modelling later on. For details of the program localGrad.m, see Appendix B for the source code and comments.

For large handsheet or machine made samples, a similar MATLAB program (globGrad.m) was applied except lower resolution and larger size of operator window and inspection zone. The detailed information can be found in Appendix C.
4.4 FIBRE ORIENTATION MEASUREMENT

Local fibre orientation can be obtained by applying image analysis to radiograph images for each specimen. To minimize vibration effects and electronic noise, image acquisition was controlled by Optimas 6.0 via a macro program (Appendix D). 16 images were captured at 2 s intervals and an average image was computed. From this average grammage map for each microtensile specimen, local gradient and local dominant orientation were analyzed based on the Scharscanski and Dodson's theory [29]. The local dominant gradient orientation plot was presented as either a Quiver Plot (see Fig. 3.2) or a Polar Plot (see Fig. 3.3), using built-in MATLAB functions.
As introduced in Section 3.3.3, the mean fibre orientation factors ($C_{pp}$ and $SumC^90$) for each specimen were computed and used for strength modelling later on. For details of the program localGrad.m and globGrad.m. see Appendix B and C respectively for the source code and comments.

To calibrate the mean fibre orientation factors ($C_{pp}$ and $SumC^90$) used in this study, the optical images for oriented handsheet and machine made samples were obtained from the Chemistry Department in Laval University, Quebec. A polarized far-infrared wave transmission measurement was based on the observation of the dichroic behaviour of paper in the far infrared [27][28]. The technique has a spatial resolution varying from 1mm$^2$ to several square centimeters. The results for both oriented handsheet samples (10cm$\times$10cm) and machine made samples (7.6cm$\times$10cm) are presented in Section 5.4.3. Here, the resolution of the FIR measurement was 1.96 mm/poin.t; the inspection zone was 3mm$\times$3mm square; each sampled point was characterized for three parameters: basis weight, the magnitude of anisotropy of fibre orientation and the angle of the dominant fibre orientation. Thus one arrow on the graphical images represents a window size around 5mm$\times$5mm square.
The microtensile specimens were placed on a fluorescent light box and an image of the transmitted light was captured for the purpose of RBA analysis. To minimize vibration effects and electronic noise, image acquisition was controlled by Optimas 6.0 via a macro program (Appendix D). 16 images were captured at 2 s intervals and an average image for each specimen was computed. Based on the theory introduced in Section 2.2.3, measured light transmission \((T)\) for each specimen was plotted as a function of mean grammage from the \(\beta\)-radiograph and hence the bonding index \((T/T_e)\) can be deduced by simple computations (see Section 2.2.3). This local optical property measurement has been well-established in our laboratories. The set up is as shown in Fig. 4.6 for the local optical measurement.

\[\text{Figure 4.6} \quad \text{Local Optical Measurement}\]
4.6 MICROTENSILE TESTING

To investigate local variation in properties, a new microtensile testing stage was used in this study instead of a standard tensile testing machine. The machine was connected to a desktop computer and controlled by MTS Sintech TestWorks 2.1. The test speed used in this study was 0.7573 mm/min, and the gage length was 15 mm. Fig. 4.7 shows the microtensile testing stage constructed in our laboratories. From the camera, the initialization and propagation of the failure can be obtained for the fracture analysis. The results for breaking force ($N$) were obtained from the desktop computer. Hence, breaking length ($km$) for each microtensile specimen can be computed followed Eq. [3.2].

![Microtensile Testing Machine](image)

**Figure 4.7 Microtensile Testing Machine**
CHAPTER FIVE
RESULTS AND DISCUSSION

5.1 INTRODUCTION

For each handsheet and machine made specimen, the mean grammage ($\bar{g}$), coefficient of variation of grammage ($CV_g$), fibre orientation factors ($SumC_{90}$ and $C_{rs}$), bonding index ($T/T_s$), and predicted and experimental breaking length were computed. We then tried to correlate these structural parameters to local breaking length of $2mm \times 2mm$ specimens.

To confirm our methods, global variations in bonding and fibre orientation were measured. In this study, global variations in bonding were created by changing the wet pressure for handsheets, and global variations in fibre orientation were created by selecting the specimens from MD to CD directions in commercial machine made paper. Comparisons between the results of our new method for local fibre orientation and the polarized far-infrared wave transmission measurement (FIR) of local fibre orientation (from Laval University in Quebec) are described in Section 5.4.3.
5.2 GRAMMAGE

There have been many studies relating grammage variation to the mechanical properties of paper. Previous results have shown that fracture is initiated from low grammage regions of sheet [3] and tensile strength is reduced as grammage variability increases [3][12]. As mentioned in Section 2.2.1, local grammage variation is a significant factor affecting the strength of paper, but it is not the only one. In this study, we focus on other local structural variations instead of local grammage by using microtensile specimen with a $2\text{mm} \times 2\text{mm}$ testing zone. We assume that the grammage is uniform and use the mean grammage value in the zone for subsequent calculations.

Obviously, the regions with larger grammage have a greater number of fibres and more chances of fibre-fibre bonding. As a result, a larger breaking load is obtained, which can be seen from Fig. 5.1 for machine made paper in CD and also from Fig. 5.2 for handsheet specimens.
Figure 5.1  Breaking Load vs. Grammage for TMP Newsprint 1452-26-2 (CD)

Figure 5.2  Breaking Load vs. Grammage for Kraft-pulp Handsheets
It is generally accepted that failure in paper is initiated at weak or low grammage spots along the edge of the sheet where stress is concentrated [2][3]. During the rupture of paper, the weak low grammage areas serve as nuclei for cracks. As a result, when the weakest local spot reaches the breaking load, it may not relate to the simultaneous failure of all fibres or bonded areas across the specimen's width. Therefore, breaking load drops to zero before mean grammage reaches zero since there are always some relatively weaker points within the testing zone, which is evident in the negative intercept in Fig. 5.1 for machine made paper. For handsheets, since it is isotropic (ideal), it is relatively more uniform and has less "weak spots" compared to machine made paper. Thus, it is not as evident as for machine made paper that the breaking load drops to zero before mean grammage reaches zero (see Fig. 5.2).

In Fig. 5.3 and Fig. 5.4, no correlation was observed between breaking length and grammage for both handsheet and machine made specimens. As shown in Eq. [3.2], by taking care of the grammage effect, breaking length is a simplified stress norm. Therefore, the model for the relationship between breaking length and grammage is trivial.
Figure 5.3  Breaking Length vs. Grammage for TMP Newsprint 1452-26-2 (CD)

Figure 5.4  Breaking Length vs. Grammage for Kraft-pulp Handsheets
5.3 RELATIVE BONDED AREA

As introduced in Section 2.2.3, the bonding index \( T/T_e \) is used to characterise the degree of local bonding instead of the absolute value of RBA in this study. Although this measurement of local bonding is indirect, it should, in theory, be able to identify locally strongly bonded or weakly bonded regions within a single sample of paper.

5.3.1 Local Variations in Bonding

\[ \frac{T}{T_g} \text{ vs. Grammage} \]

![Graph showing \( \frac{T}{T_g} \) vs. Grammage for Kraft-pulp Handsheets [32]](image)

*Figure 5.5  \( \frac{T}{T_g} \) vs. Grammage for Kraft-pulp Handsheets [32]*

From the previous results, no correlation was observed between the mean grammage and local RBA for handsheet specimens [32], as shown in Fig. 5.5.
Because a stack of handsheets is essentially pressed in a soft platen press so that all areas are subjected to the same pressure, they have relatively small fluctuations in local bonded area.

![T/Tg vs. Grammage](image)

**Figure 5.6**  \( T/T_g \) vs. Grammage for TMP Newsprint 1452-26-2

For machine made paper, a larger range of bonding index was expected. Fig. 5.6 gives the relationship between the bonding index \( (T/T_g) \) and grammage for the machine made paper specimens. Again, there is no correlation obtained between the mean grammage and local RBA which is defined by \( T/T_g \).
Theoretically, for a set of microtensile specimens with constant grammage,
those with a greater degree of bonding (and hence a higher value of $T/T_K$) should be
stronger. In other words, local breaking length is expected to be proportional to local
RBA. However, from the current results for machine made paper of 1452-26-2 in CD
(see Fig. 5.7) and previous results for handsheets (see Fig. 5.8), there is no correlation
observed between local RBA and local breaking length. The original hypothesis of
this study was that there was significant local variation in bonded area that could
affect local breaking length in a deterministic way. Unfortunately, the results did not
support this hypothesis. We consider two possibilities here: first, there are relatively
small fluctuations in local bonded area for both handsheets and machine made paper
used in this study; second, the measurement itself might not be sensitive enough to
accurately identify these local variations in structure. To further determine whether or
not this method can characterize bonded area property in this way, global variations
in bonding were induced by varying the wet pressure during sheet consolidation.

5.3.2 Global Variations in Bonding

Based on the fact that sheet strength generally improves as wet pressure (and
hence bonding) is increased [4], breaking length has been plotted as a function of
$T/T_K$ for handsheet samples made at a variety of wet pressures (see Fig. 5.9). In this
case, $T_K$ is the mean value of light transmission for the standard pressure handsheet
specimens with the same grammage. The variation in bonding was global rather than
local. In Fig. 5.9, the positive correlation between Breaking Length and $T/T_K$. 

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indicates that the measurement used in this study is sound; however, it is not sensitive enough to characterize local bonding variation within a single sample of paper.

![Break Length vs. T/Tg](image)

**Figure 5.9**  *Breaking Length vs. *(T/T_g)* for Handsheets with changing wet pressure*

### 5.4 FIBRE ORIENTATION

In previous work [30], an attempt was made to estimate fibre orientation distribution based on the two-dimensional power spectrum theory [23]. Unfortunately, the original results were not very convincing (see Fig. 5.10). There was no strong correlation observed between fibre orientation, characterized by the parameter *Coherence90*, and local breaking stress. In this study, two new analytical methods (see *Section 3.3* and *4.4*) were derived for the estimation of both local and
global fibre orientation distribution by gradient variation in grammage map. Furthermore, comparisons between the results of this new method and the polarized far-infrared wave transmission measurement (FIR) of local fibre orientation are described in Section 5.4.3.

In this study, we used SumC90 and $C_{pp}$ (see Section 3.3) to characterize local fibre orientation.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{plot}
\caption{Breaking Stress vs. Coherence90 for Kraft-pulp Handsheets, based on two-dimensional power spectrum [30]}
\end{figure}
5.4.1 Local Variations in Fibre Orientation

Local variation in fibre orientation is independent of mean grammage. In Fig. 5.11 and Fig. 5.12, there is no correlation between $\text{SumC90}$ and grammage for either handsheets or machine made paper. Also no relationship is observed between $C_{np}$ and grammage, as shown in Fig. 5.13 and Fig. 5.14. On the other hand, we can see that the variation in fibre orientation is much larger for machine made paper than for handsheets. This is because turbulent eddies die out prior to deposition in forming process of handsheet making, which is therefore less likely to induce orientation.

![SumC90 vs. Grammage](image)

*Figure 5.11 SumC90 vs. Grammage for Kraft-pulp Handsheets*
Figure 5.12  SumC90 vs. Grammage for TMP Newsprint 1452-26-2

Figure 5.13  $C_{pp}$ vs. Grammage for Kraft-pulp Handsheets
The local variation in fibre orientation is expected to affect the anisotropy of paper properties, such as local tensile strength. Fibres are usually stronger than fibre bonds, and machine made sheets show much higher breaking length in the MD than in the CD. Thus, a correlation between the local breaking length and fibre orientation factor (SumC90 or $C_{pp}$) is expected. Fig. 5.15, Fig. 5.16, Fig. 5.17 and Fig. 5.18 show the variation of microtensile specimen strength as a function of SumC90 or $C_{pp}$ respectively for specimens taken from various positions in handsheets. A relatively strong correlation was observed between local breaking length and $C_{pp}$ for these handsheet specimens. In Fig. 5.19 and Fig. 5.20, the results for the CD specimens from the machine made sheets are plotted. In this case, variations in SumC90 or $C_{pp}$ represent the local variations in fibre orientation, since all specimens were cut parallel.
to the CD direction. A weak correlation was obtained in this set of experiments.

Figure 5.15  Breaking Length vs. SumC90 for the same Handsheet specimens as those in Fig. 5.10, but based on the new method for local fibre orientation analysis.

Figure 5.16  Breaking Length vs. Cpp for the same Handsheet specimens as those in Fig. 5.10 [30], but based on the new method for local fibre orientation analysis.
**Figure 5.17**  
*Breaking Length vs. SumC90 for Kraft-pulp Handsheets*

**Figure 5.18**  
*Breaking Length vs. $C_{pp}$ for Kraft-pulp Handsheets*
**Figure 5.19**  Break Length vs. SumC90 for TMP Newsprint 1452-26-2 (CD)

**Figure 5.20**  Break Length vs. $C_{pp}$ for TMP Newsprint 1452-26-2 (CD)
The current results indicate that local fibre orientation factor is an additional factor affecting paper strength. The scattering of data implies that local fibre orientation is not the only factor governing paper strength.

As a single parameter characterizing $g(\theta)$, $C_{pp}$ was derived on the basis of mechanical interpretation (see Section 3.3.2). Therefore, $C_{pp}$ should be better correlated to strength compared to $SumC90$ and give a proportional relationship with strength (see Eq. [3.10]). This can be seen in Fig. 5.17, Fig. 5.18, Fig. 5.19 and Fig. 5.20.

5.4.2 Global Variation

As introduced in Section 4.1, if specimens for machine made sheet are cut at a variety of angles to the machine direction, a global variation in fibre orientation is obtained. Two kinds of machine made paper were used in this study to confirm the new measurement for fibre orientation. Fig. 5.21, Fig. 5.22, Fig. 5.23 and Fig. 5.24 show a fairly positive correlation between breaking length and fibre orientation ($SumC90$ and $C_{pp}$).
Figure 5.21  
**Breaking Length vs. SumC90 for TMP Newsprint 1452-26-2**

Figure 5.22  
**Breaking Length vs. \( C_{pp} \) for TMP Newsprint 1452-26-2**
Figure 5.23  Breaking Length vs. SumC90 for TMP Newsprint 1452-17-4

Figure 5.24  Breaking Length vs. $C_{pp}$ for TMP Newsprint 1452-17-4

SumC90 represents the mean local value of fibre orientation in the entire
region of interest for each microtensile specimen (see Section 3.3.3). $C_{pp}$ was defined as a fibre orientation factor in the sheet strength equation based on mechanics (see Section 5.4.1) and a linear relationship was derived between breaking length and $C_{pp}$ (see Eq. [3.10]). From the current results, a relatively stronger correlation was observed between breaking length and $C_{pp}$. This implies that $C_{pp}$ is more suitable to characterize local fibre orientation in the prediction of sheet strength though $\text{SumC90}$ directly corresponds to local fibre orientation in the zone.

5.4.3 Global Images

The visualized global images of fibre orientation distribution in a paper sample were studied by both the FIR measurement and our new image analysis method based on gradient variation. The arrows on the following figures give the direction and magnitude of mean fibre orientation over a given measurement region. Fig. 5.25 presents the fibre orientation distribution for oriented handsheets measured by the FIR.

In this study, we have developed algorithms for extracting local fibre orientation from $\beta$-radiographs based on image analysis techniques. In Fig. 5.26, a Quiver Plot of Local Dominant Fibre Orientation for the same sample is presented.
Figure 5.25  Global Image for Oriented Handsheets with resolution = 1.96 mm/point, Area = 100mm × 100mm, one arrow = 2.55 × 2.55 point s = 5mm × 5mm
Figure 5.26  Global Image for Oriented Handsheets with
resolution = 0.1 mm/pixel. Area = 100 mm × 74 mm. one
arrow = 50 × 50 pixels = 5 mm × 5 mm

Note: Since it was difficult to locate the exact same position of paper sample as Fig. 5.25, there is some difference between two images (experimental error).

Fig. 5.27 and Fig. 5.28 give the results for machine made paper.
Figure 5.27  Global Image for TMP Newsprint 1452-17-4 with
resolution = 1.97 mm/point. Area = 76mm \times 100mm,
one arrow = 2 \times 2 points = 4mm \times 4mm
Figure 5.28 Global Image for TMP Newsprint 1452-17-4 with
resolution = 0.1mm/pixel. Area = 100mm \times 74mm.

One arrow = 50 \times 50 pixels = 5mm \times 5mm

A comparison of Fig. 5.25 and Fig. 26 shows that both methods produce qualitatively similar images. These results indicate that our new method for the estimation of fibre orientation distribution is sound. It is a non-destructive
measurement based on (mass) gradient variation [29]. Fibre orientation distribution is quantified by image analysis techniques. Therefore, it has potential to be applied online monitoring paper quality.

However, there is some difference in the comparison between Fig. 5.27 and Fig. 5.28. Some variation in local fibre orientation, as shown in Fig. 5.28, was not evident on the FIR image. Relative to the fibre width (i.e., 0.02mm), the resolution of the FIR measurement (1.97mm/point) is too large to investigate local variation in fibre orientation of paper. On the FIR image, one point extracted from a window size of 1.97mm x 1.97mm may represent thousands of fibres. It gives a mean fibre orientation value for the entire region. Although the fibre orientation distribution based on a local gradient variation also represents the mean fibre orientation over a certain region with 0.1mm/pixel resolution, it can give more precise information compared to the FIR measurement for local variation analysis. According to Fig. 5.27, we cut the specimens all perfect parallel to the MD direction which have some variation in our results (Fig. 5.28). In Fig. 5.29, the specimens cut along the MD based on the FIR image are plotted by using our new algorithms analysis for fibre orientation. The scattering of data implies that the resolution used in this study is better than that of the FIR measurement for local fibre orientation study.
Because the local grammage, fibre orientation and bonding vary simultaneously from point to point in the sheet, it is difficult to isolate the relationship between just one aspect of structure and the local strength of paper. As a normalized measure of tensile strength, breaking length can be used to factor out the effect of grammage and further to see if correlations can be improved by considering other local structural factors simultaneously, such as fibre orientation and bonded area.

In Chapter Three, a model was developed to deal with the relationship between breaking length and relative bonded area and fibre orientation. Based on Eq. [3.28], and to simplify the measurement and calculation, $k$ is considered to be a

---

**Figure 5.29**  Breaking Length vs. $C_{pp}$ for TMP Newsprint 1452-17-4 (MD)
constant in this study and therefore it can be computed by plotting \( \frac{T}{C_{pp}} \) against

\( \frac{1}{RBA} \) with the maximum correlation coefficient value. As introduced in Section 2.2.3, the bonding index \( \frac{T}{T_s} \) was used in this study to characterize bonding instead of an absolute value of \( RBA \). Thus, \( k \) can be obtained by plotting \( \frac{\text{BreakLength}}{C_{pp}} \) against \( \frac{1}{T \ T_s} \) for two kinds of machine made paper.

In Fig. 5.30 and Fig. 5.31, the experimental data for breaking length of specimens have been plotted against the results predicted by the relative strength model. As the results in these figures, a relatively strong correlation was observed.

However, compared to \( R^2 \) values in those plots (see Fig. 5.22 and Fig. 5.24) for individual structural parameter \( C_{pp} \), there is no significant difference observed in Fig. 5.30 and Fig. 5.31 for the relative strength model. The improvement is trivial since there is no correlation obtained between bonding and local strength of paper in this study.
Figure 5.30  Breaking Length (Exp.) vs. Predicted Breaking Length for TMP
Newsprint 1452-26-2

Figure 5.31  Breaking Length (Exp.) vs. Predicted Breaking Length for TMP
Newsprint 1452-17-4
5.6 SUMMARY AND DISCUSSION

This study was founded on the hypothesis that there were significant local variations in structure that could have a significant effect on local property variations of paper. The results presented above indicate that

\[
\text{Predicted Breaking Length} \propto C_p \left(1 - \frac{k}{T/T_k}\right)
\]

The simplified Eq. [3.30] gives a multivariable correlation between local tensile strength and local structural properties. Previous studies, such as those of Page [4] or Kallmes, Bernier and Perez [5], dealt with the failure of random network structures at the global level. There have not been any previous models relating local RBA and fibre orientation to the local tensile strength of sheet.

The current results, while quite limited, do support the original hypothesis of this study. On the mesostructure level (2\text{mm} \times 2\text{mm} in this study), local breaking length can be plotted as a function of local fibre orientation \((C_{pp})\) and local bonding \((T/T_k)\), with local grammage, \(C_{pp}\) and \(T/T_k\) are assumed to be constant in microtensile specimen. Particularly, for the local fibre orientation factor, the strong trend obtained here is in agreement with the generally accepted view that fibre orientation is an important factor affecting the strength of paper. It also indicates that
there is a significant local variation in mean fibre orientation, which can influence local strength of paper. However, the scattering of data points indicates that there are other unknown factors:

- The scale of the inspection and testing zone might be too large to be taken as uniform distribution of grammage and bonding
- Other structural variations, e.g., drying shrinkage might cause the defects
- The experimental method might not be sensitive enough to the local structural variations in RBA and fibre orientation

To investigate the first possibility, the coefficient of variation (CV) of grammage was calculated over the testing zone of $2\text{mm} \times 2\text{mm}$ for each specimen. The breaking length for machine made 1452-26-2 (CD) specimens is plotted against coefficient of variation of grammage in Fig. 5.32.

If the size of the specimens was too large to assume uniformity, a decrease of the strength with increasing CV of grammage is expected. However, no trend was observed in Fig. 5.32. The results indicate that the effect of $CV_{\lambda g}$ on breaking length is minor at the measurement scale ($2\text{mm} \times 2\text{mm}$) used in this study and the mean grammage value can be used over the zone.
Figure 5.32  Breaking Length vs. $CV_{\lambda}$ for TMP Newsprint 1452-26-2 (CD)

Since this study concentrated on the effect of three local structural properties on tensile strength of the sheet, other structural variations were ignored. For example, drying of commercial paper has the effect of increasing the coefficient of variation because thinner regions suffer strain caused predominantly by the lateral shrinkage of fibres in denser regions [58][59]. Also, there might be some structural variations in grammage, RBA and fibre orientation due to macroscopic forming process. These simplifications and idealizations of the structure, which are done to make the problem solvable, are the limitations of this study.

In Section 5.3, the relationship between relative bonded area and local strength in $2mm \times 2mm$ zones is explored. Unfortunately, no correlation was observed in the study of local variability, although the experiments conducted with
global variations in bonding demonstrates that our indirect measurement of RBA is sound. One possibility we consider here is that there are relatively small fluctuations in local density and bonded area within one single sample of paper. In other words, the experimental method used in this study might not be sensitive enough to investigate local structural variation in RBA.
6.1 CONCLUSIONS

(1) A new method based on Scharscanski and Dodson's theory is proposed and applied in this study. Fibre orientation is quantified by \( \text{SumC90} \) and \( C_{pp} \) through local mass gradient variation analysis. Comparisons between the results of our new method and the polarized far-infrared wave transmission measurement (FIR) for local fibre orientation confirmed that the method developed in this study for the estimation of fibre orientation distribution is sound.

(2) The current results, while quite limited, do support the original hypothesis: there were significant local variations in structure that could have a significant effect on local tensile strength of sheet. On the mesostructural level, local strength can be plotted as a function of local fibre orientation \( (C_{pp}) \) and local bonding \( (T/T_g) \) while local grammage can be assumed as a constant value for microtensile specimens. A strong trend obtained in this study (see Section 5.4 for
local and global variations) indicates that local variation in mean fibre orientation can affect the local strength of paper in a deterministic way. However, a weak correlation was observed between local strength and local bonded area. The scattering of data points implies that there are other unknown factors affecting the strength of paper.

(3) A relative strength model (see Eq. [3.30]) gives a multivariable correlation between local strength and local structural properties, such as fibre orientation and bonded area. This is the first time that both local RBA and fibre orientation variables are related to local strength of sheet on the mesostructural level. However, due to no correlation obtained between local strength and bonding in this study, there is no significant improvement on the correlation in the relative strength model by considering fibre orientation and bonding simultaneously.

6.2 RECOMMENDATIONS

(1) There was a weak correlation between local strength and local bonded area \((T/T_g)\) observed in this study for both handsheets and machine made paper. Relatively small fluctuations in bonded area are considered to be one of the possible reasons. Although the experimental method for characterizing RBA as local bonding index
was confirmed to be sound through experiments involving global variations in bonding, the current results for local variations are not very convincing. Further study, such as comparisons with other methods for measuring local bonding, is necessary.

(2) Although the relative strength model developed in this study gives a strong trend which is in agreement with the hypothesis of this work, the scattering of data points suggests that other unknown factors should be considered, such as drying shrinkage which is unavoidable in forming process.

(3) The effect of inspection zone size on local structural properties is critical for this study since uniform distribution of grammage and bonded area is assumed. From the current results, the effect of coefficient variation of grammage on local strength can be ignored and uniformity can be assumed for grammage distribution of microtensile specimens. However, there is no better method of examining local bonding variation at present.
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APPENDIX A

Imaging and Testing System

1. β-radiography
   • Type: $^{14}$C PMMA Sheet (Du Pont)
   • Decay: $\beta^-$
   • Max. emission: 0.156 Mev
   • Total activity: 11.9 mCi

2. X-ray Film
   • Structrix D7, Agfa
   • Expansion Time: 30 min

3. Imaging System
   • Camera: Kodak 1.61 CCD camera with AF micro Nikkor 60mm lens
   • Cope maker stand: Mini Repro 1192
   • Light table: Knox 6002
   • Computer: Pentium Pro 200 PC
   • Software: Optimas 6.0 and ITEX-IC-PCI 2.7.2.0

4. Testing System
   • Tensile testing machine: Lab made microtensile stage controlled by 486 PC
   • Software: MTS Sintech TestWorks 2.1
   • Test Method: TAPPI T494 om-88 (changed for microtensile testing)
- Test Speed: 0.7573 mm/min, experimentally discovered speed
- Gage Length: 15 μm
- Test Condition: 50.0% ± 2.0% RH and 23.0 ± 1.0 °C
- Cutter: Lab made micro-tensile sample cutter specially for 2mm x 2mm microtensile specimen
APPENDIX B

Local Fibre Orientation MATLAB Program
Yinglin Chen [30]
Modified by Baohua Shen

% localGrad.m--1452-26-2(Jan) film1

%Read in .tiff image file and convert the gray level image to g rammage map
grammage=[162.0 145.8 129.6 113.4 97.2 81.0 64.8 48.6 32.4 16.2];
gray=[233.762 231.34 224.153 214.07 199.2 174.377 143.411 103.096 61.6639
30.7941];
gram=[];
p1=polyfit(gray(5:10). grammage(5:10), 1);
p2=polyfit(gray(1:5). grammage(1:5), 2):

A=double(X);
for i=1:200
    for j=1:200
        if A(i,j) < 201
            G(i,j)=p1(1)*A(i,j)+p1(2);
        else
            G(i,j)=p2(1)*A(i,j)^2+p2(2)*A(i,j)+p2(3);
        end
    end
end

gmean=mean(mean(G)); %Mean grammage value
% Given G (the grammage map) already
tic;
resolution=0.008772; % (mm/pixel)
imgLen=1.7544; % length of the original image (mm)
suvLen=1.7544; % length of suveyed area (mm)
suvWid=1.7544; % width of surveyed area (mm)
%insLen=0.02; % inspection zone side length (mm)
x1=1; % starting row of the surveyed area
y1=1; % starting column

x2=x1+suvWid/resolution; % ending row
y2=y1+suvLen/resolution; % ending column
imgDim=imgLen/resolution; % dimension of the original image
suvWDim=y2-y1; % dimension of surveyed area length
suvLDim=x2-x1; % dimension of surveyed area width
opOrder=5; % order of operator, odd number
w=(opOrder-1)/2;
tSetup=toc;

%tic;
deltg=zeros(suvLDim-2*w,suvWDim-2*w);
theta=zeros(suvLDim-2*w,suvWDim-2*w); % initialize matrices for deltg and theta

PrewCol(1:opOrder,1:w)=ones(opOrder,w);
PrewRow(1:w,1:opOrder)=ones(w,opOrder);
PrewRow(w+1,1:opOrder)=zeros(1,opOrder);
PrewCol(1:opOrder,w+1)=zeros(opOrder,1):
PrewCol(1:opOrder,w+2:opOrder)=(-1)*ones(opOrder,opOrder-w-1):
PrewRow(w+2:opOrder,1:opOrder)=(-1)*ones(opOrder-w-1,opOrder):
% define prewitt row and column operators

GradRow=conv2(G,PrewRow,'valid'); % gradient in row direction
GradCol=conv2(G,PrewCol,'valid'); % gradient in column direction

deltg=sqrt(GradRow.^2+GradCol.^2);
Mudeltg=mean(deltg);

theta=atan2(GradRow,GradCol):

validLDim=suvLDim-2*w;
validWDim=suvWDim-2*w;
Unit=ones(validLDim,validWDim);
MbyN=suvLDim*validWDim;

Mux=mean(mean(GradCol));
Muy=mean(mean(GradRow));
Vxx=sum(sum((GradCol-Mux*Unit).^2))/MbyN;
Vyy=sum(sum((GradRow-Muy*Unit).^2))/MbyN;
Vxy=sum(sum((GradCol-Mux*Unit).*(GradRow-Muy*Unit)))/MbyN;
Lumdmax=(Vxx+Vyy+sqrt((Vxx-Vyy)^2+4*Vxy^2))/2;
Lumdmin=(Vxx+Vyy-sqrt((Vxx-Vyy)^2+4*Vxy^2))/2;
e2=Lumdmax^2/Lumdmin^2; % eccentricity number e2

insDim=2*opOrder;

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LoDoLDim=floor(validLDim/insDim);
LoDoWDim=floor(validWDim/insDim);
Thetad2=zeros(LoDoLDim,LoDoWDim);
Thetad=zeros(LoDoLDim,LoDoWDim);
Coherence=zeros(LoDoLDim,LoDoWDim);

for i=1:LoDoLDim
    for j=1:LoDoWDim
        yO=(i-1)*insDim+1;
        xO=(j-1)*insDim+1;
        sinSum=sum(sum(deltg(yO:yO+insDim-1,xO:xO+insDim-1).^2.*sin(2*theta(yO:yO+insDim-1,xO:xO+insDim-1))));
        cosSum=sum(sum(deltg(yO:yO+insDim-1,xO:xO+insDim-1).^2.*cos(2*theta(yO:yO+insDim-1,xO:xO+insDim-1))));
        Thetad2(i,j)=atan2(sinSum,cosSum);
        if Thetad2(i,j)>=0
            Thetad(i,j)=0.5*Thetad2(i,j);
        end
        if Thetad2(i,j)<0
            Thetad(i,j)=0.5*(2*pi+Thetad2(i,j));
        end
        deltgc=deltg(yO:yO+insDim-1,xO:xO+insDim-1);
        thetac=theta(yO:yO+insDim-1,xO:xO+insDim-1);
        Thetadc=ones(insDim,insDim)*Thetad(i,j);
        Coherence(i,j)=sum(sum(abs(deltgc.*cos(thetac-Thetadc))));
        Coherence(i,j)=Coherence(i,j)./(insDim^2);
    end
end

% find the local dominant orientation & coherence (absolute values of projections)
Muc = mean(mean(Coherence));
%Muc
%tAlgrhm = tic;
%tic;
% quiver plot of the whole surveyed area with Thetad
ThetadX = Coherence.*cos(Thetad);
ThetadY = Coherence.*sin(Thetad);
quiver(ThetadX, ThetadY, 1);
title('Subzone Dominant Orientation of 1452-26-2 Ts1')
%print -deps LoDoOrTs1.eps
%pause
%tPlot1 = toc;
pause;

%tic;
% polar plot of dominant orientation
pThetad = zeros(181, 1);
alph = 0:179;
alph = alph.*pi./180;
for i = 1:LoDoLDim
    for j = 1:LoDoWDim
        floThetad = floor(Thetad(i, j)/pi*180+1);
        pThetad(floThetad) = pThetad(floThetad) + Coherence(i, j);
    end
end
pThetad(1) = pThetad(1) + pThetad(181);
pThetad = pThetad(1:180);
pnorm = pThetad./sum(pThetad);
polar(alph, pnorm);
gtext('Probability Distribution of Subzone Dominant Orientation of 1452-26-2 Ts1')
%print -deps PolarLoDoOrTs1.eps

%tic;
%Ccpp Calculation
Ccpp=zeros(1,1);
theta90=alph-ones(180.1)*0.5*pi;
costheta90=cos(theta90);
sintheta90=sin(theta90);
cos_2=costheta90(1:180).*costheta90(1:180);
cos_4=cos_2(1:180).*cos_2(1:180);
sin_2=sintheta90(1:180).*sintheta90(1:180);
Ccpp=sum(abs(pnom).*(cos_4-(1/3)*cos_2.*sin_2)); %MD or Random
%Ccpp=sum(abs(pnorm).*cos_4); % CD Direction
Ccpp % Cox Theory -- Fibre Orientation Factor

%tic;
%sum the Local Dominant Fibre Orientation
SumC90=zeros(1,1);
SumC90=sum(abs(pnorm).*cos(alph-ones(180.1)*0.5*pi));
SumC90 %Coherence90

%Total=tSetup+tMass+tGray+tAlgrhtm+tPlot1+tPlot2;

% write result to file localGrad.mat
%save localGrad Thetad tSetup tMass tGray tAlgrhtm tPlot1 tPlot2 tTotal
APPENDIX C

Global Fibre Orientation MATLAB PROGRAM
Yinglin Chen [30]
Modified by Baohua Shen

% globalGrad.m--Regular Handsheets R

% Read in .tiff image file and convert the gray level image to grammage map
grammage=[162.0 145.8 129.6 113.4 97.2 81.0 64.8 48.6 32.4 16.2];
gray=[255.987 254.980 251.249 242.938 221.25 195.411 155.332 112.148 64.6232
31.9927];
gram=[];
p1=polyfit(gray(6:10), grammage(6:10),1);
p2=polyfit(gray(1:6), grammage(1:6),2);

[X,map]=imread('/u/grad/shenb/matlab/bigimage/r1.tif');
A=double(X);
for i=1:740
    for j=1:1000
        if A(i,j) < 223
            G(i,j)=p1(1)*A(i,j)+p1(2);
        else
            G(i,j)=p2(1)*A(i,j)^2+p2(2)*A(i,j)+p2(3);
        end
    end
end
gmean=mean(mean(G));
gmean  % Mean grammage value
% save -v4tensile1 G

% Given G (the grammage map) already
tic;
resolution=0.1; % (mm/pixel)
imgLen=100; % length of the original image (mm)
suvLen=100; % length of surveyed area (mm)
suvWid=74; % width of surveyed area (mm)
% insLen=0.02; % inspection zone side length (mm)
x1=1; % starting row of the surveyed area
y1=1; % starting column

x2=x1+suvWid/resolution; % ending row
y2=y1+suvLen/resolution; % ending column
imgDim=imgLen/resolution; % dimension of the original image
suvWDim=y2-y1; % dimension of surveyed area length
suvLDim=x2-x1; % dimension of surveyed area width
opOrder=5; % order of operator. odd number
w=(opOrder-1)/2;
tSetup=toc;

% tic;
deltg=zeros(suvLDim-2*w,suvWDim-2*w);
theta=zeros(suvLDim-2*w,suvWDim-2*w); % initialize matrices for deltg and theta

PrewCol(1:opOrder,1:w)=ones(opOrder,w);
PrewRow(1:w,1:opOrder)=ones(w,opOrder);
PrewRow(w+1,1:opOrder)=zeros(1,opOrder);
PrewCol(1:opOrder,w+1)=zeros(opOrder,1);
PrewCol(1:opOrder,w+2:opOrder)=(-1)*ones(opOrder-opOrder-w-1);
PrewRow(w+2:opOrder,1:opOrder)=(-1)*ones(opOrder-w-1,opOrder);
% define prewitt row and column operators

GradRow=conv2(G.PrewRow,'valid'); % gradient in row direction
GradCol=conv2(G.PrewCol,'valid'); % gradient in column direction

validLDim=suvLDim-2*w;
validWDim=suvWDim-2*w; %size of deltg and theta
Unit=ones(validLDim, validWDim);
MbyN=suvLDim*validWDim;

deltg=sqrt(GradRow.^2+GradCol.^2);
Mudeltg=mean(mean(deltg));
%Mudeltg % mean magnitude of gradient
Sigmdeltg2=mean(mean((deltg-Mudeltg*Unit).^2));
Sigmdeltg=sqrt(Sigmdeltg2)/10;
%Sigmdeltg % variance of magnitude of gradient
CVdeltg=sqrt(Sigmdeltg2)/Mudeltg;
%CVdeltg % coefficient of variation--deviation of gradient magnitude
Kdeltg=Mudeltg^2/Sigmdeltg2;
%Kdeltg % parameter of gamma distribution
theta=atan2(GradRow,GradCol);

validLDim=suvLDim-2*w;
validWDim=suvWDim-2*w; %size of deltg and theta

Mux=mean(mean(GradCol));
Muy=mean(mean(GradRow));
\[ V_{xx} = \frac{\text{sum}(\text{sum}((\text{GradCol} - M_{xx} \cdot \text{Unit})^2))}{M \times N} \]
\[ V_{yy} = \frac{\text{sum}(\text{sum}((\text{GradRow} - M_{yy} \cdot \text{Unit})^2))}{M \times N} \]
\[ V_{xy} = \frac{\text{sum}(\text{sum}((\text{GradCol} - M_{xx} \cdot \text{Unit}) \cdot (\text{GradRow} - M_{yy} \cdot \text{Unit})))}{M \times N} \]
\[ L_{\text{umdmax}} = \frac{(V_{xx} + V_{yy} + \sqrt{(V_{xx} - V_{yy})^2 + 4 \times V_{xy}^2})}{2} \]
\[ L_{\text{umdmin}} = \frac{(V_{xx} + V_{yy} - \sqrt{(V_{xx} - V_{yy})^2 + 4 \times V_{xy}^2})}{2} \]
\[ e^2 = \frac{L_{\text{umdmax}}^2}{L_{\text{umdmin}}^2} \]

\%e2 \quad \% \text{eccentricity number } e^2

\text{insDim} = 10 \times \text{opOrder};
\text{LoDoLDim} = \text{floor}(\text{validLDim}/\text{insDim});
\text{LoDoWDim} = \text{floor}(\text{validWDim}/\text{insDim});
\text{Thetad2} = \text{zeros}(\text{LoDoLDim}, \text{LoDoWDim});
\text{Thetad} = \text{zeros}(\text{LoDoLDim}, \text{LoDoWDim});
\text{Coherence} = \text{zeros}(\text{LoDoLDim}, \text{LoDoWDim});

\text{for } i=1: \text{LoDoLDim}
\text{for } j=1: \text{LoDoWDim}
\quad y_0 = (i-1) \times \text{insDim} + 1;
\quad x_0 = (j-1) \times \text{insDim} + 1;
\quad \text{sinSum} = \text{sum}(\text{sum}(\text{deltg}(y_0:y_0+\text{insDim}-1,x_0:x_0+\text{insDim}-1)^2 \times \text{sin}(2 \times \text{theta}(y_0:y_0+\text{insDim}-1,x_0:x_0+\text{insDim}-1))));
\quad \text{cosSum} = \text{sum}(\text{sum}(\text{deltg}(y_0:y_0+\text{insDim}-1,x_0:x_0+\text{insDim}-1)^2 \times \text{cos}(2 \times \text{theta}(y_0:y_0+\text{insDim}-1,x_0:x_0+\text{insDim}-1))));
\quad \text{Thetad2}(i,j) = \text{atan2}(\text{sinSum}, \text{cosSum});
\quad \text{if } \text{Thetad2}(i,j) \geq 0
\quad \quad \text{Thetad}(i,j) = 0.5 \times \text{Thetad2}(i,j);
\quad \text{end}
\text{if } \text{Thetad2}(i,j) < 0
\quad \quad \text{Thetad}(i,j) = 0.5 \times (2 \times \pi + \text{Thetad2}(i,j));
\text{end}
end
deltgc=deltg(y0:y0+insDim-1,x0:x0+insDim-1);
theta=theta(y0:y0+insDim-1,x0:x0+insDim-1);
Thetad=ones(insDim,insDim)*Thetad(i,j);
Coherence(i,j)=sum(sum(abs(deltgc.*cos(theta-Thetad))));
Coherence(i,j)=Coherence(i,j)/(insDim^2);
end
end

% find the local dominant orientation & coherence (absolute values of projections)
Muc=mean(mean(Coherence));
%Muc % mean magnitude of coherence
%tAlgrthm=toc;

%tic;
% quiver plot of the whole surveyed area with Thetad
ThetadX=Coherence.*sin(Thetad);
ThetadY=Coherence.*cos(Thetad);
quiver(ThetadX,ThetadY,1);
title('Local Dominant Fibre Orientation of Regular Handsheet R')
%print -deps LoDoOrR1.eps
%pause
%tPlot1=toc;
pause;

% polar plot of dominant orientation
pThetad=zeros(181,1);
alph=0:179;
alph=alph.*pi./180;
for i=1:LoDoLDim
for j=1:LoDoWDim
    floThetad=floor(Thetad(i,j)/pi*180+1);
    pThetad(floThetad)=pThetad(floThetad)+Coherence(i,j);
end
end
pThetad(1)=pThetad(1)+pThetad(181);
pThetad=pThetad(1:180);
phnorm=pThetad./sum(pThetad);
polar(alph, pnorm);
gtext('Probability Distribution of Local Dominant Orientation for Regular Handsheet R')
%print -deps PolarLoDoOrR1.eps
% tTotal=tSetup+tMass+tGray+tAlgrthm+tPlot1+tPlot2;
% write result to file globalGrad.mat
% save globalGrad Thetad tSetup tMass tGray tAlgrthm tPlot1 tPlot2 tTotal
APPENDIX D

dNoise Macro-Program
Standard Program in Optimas 6.0

// AVERAGING 16 IMAGES TOGETHER

// Define variables
CHAR cA, cImage;
CHAR cB = "IMAGE";
CHAR cD = "AVG";

// turn off undo
Undo(FALSE);

// Acquire and average the first image
Acquire();
Freeze();

ROIToList("PRINoise");
Acquire();
Freeze();

ArithmeticOp("Average", "PRINoise");

ROIToList("AVG1");

for (y=2;y<9;y++)
{
}
Acquire();
Freeze();
ROIToList(NULL,"frame");
Acquire();
    Freeze();
ArithmeticOp("Average", "frame");
cA = ToText(y);
    cImage = cB:cA;
    ROIToList(cImage);;

    z = ToText(y-1);
    CHAR avg = cD:z;
    ArithmeticOp("Average", cD:z);
    ROIToList(cD:cA);
    DeleteImage(cImage);
    DeleteImage(cD:z):
}

DeleteImage("frame");